A STATISTICAL ANALYSIS
OF WORK-TIME DISTRIBUTIONS

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ABSTRACT

This thesis attempts to answer some of the questions that must be answered before a formal mathematical model can be determined for performance times. Several characteristics of this model have been studied in answering the following questions:

(1) Do operators follow any one pattern of performance or a work curve throughout the day?

(2) Do unadjusted performance times tend toward any formal distribution or could they be made to form any model?

(3) Are operator's cycle times statistically stable?

(4) What is the relationship of variation without a period to the relationship between periods?

Nearly 3200 observations of a short cycle, manually controlled assembly operation, gathered by the use of a decimal minute stop watch, were analyzed statistically using control chart procedures. There was a total of nineteen different operators studied. These operators were distributed over three shifts.

From the results of the analysis, the following conclusions were drawn within the limitations set forth:

(1) The operators on this operation did not follow any particular work curve.

(2) The unadjusted performance times tend to form a positively skewed distribution.

(3) The performance times of sixteen out of nineteen operators were not statistically stable.

(4) The variation within a period was significantly greater than the variation between periods.

(5) Stop watch performance time data do not give sufficient information to separate chance causes from assignable causes of variation.

It is recommended that a similar study be made using high speed motion pictures to collect the data. By using micromotion study it should be possible to isolate many of the assignable causes of variation.

CHAPTER I

INTRODUCTION

Ever since Frederick W. Taylor expounded his philosophy of scientific management, engineers have endeavored to replace opinion with fact, and to develop many of the functions of management as a science rather than as an art.

One field of industrial engineering has lagged in this movement. Time study today is still an art based upon a science. Wilkinson says that time study engineers have continued to shy away from the mathematics in their solutions. He also states that, "Science constantly strives to clothe its conclusions in more and more vigorous mathematical formulation and to eliminate all possible sources of subjective bias."[1] One place in which subjective bias is introduced in time study is in the form of a rating or leveling factor. This factor of subjective judgment makes present day standards liable to the criticism that they lack scientific validity.[2] Thus, time study has failed to keep pace with other sciences in the quest for validity of results.

Currently, standards set by a stop watch are obtained by first dividing the cycle into elements. End points for these elements are established. The time study man then observes the operator, records a

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[1]G. D. Wilkinson, "Application of Statistical Techniques in Time Study," Mechanical Engineer, 73: 906-9, November, 1951.

[2]Harold O. Davidson, Functions and Bases of Time Standards, A Research Report (Columbus, Ohio: American Institute of Industrial Engineers, 1952), p. 906.

time for each element and also determines a performance rating. This performance rating is an estimate of the operator's performance based upon a previously determined scale. The normal elemental time is established for each element by multiplying the rating factor by the performance time. These times for the same elements are then averaged. Experience is the usual determinate of the number of element reading taken. Allowances are then added to the total of the averaged elemental times in order to set a standard time for the operation.

Since the publication of Walter A. Shewhart's[3] original book on the use of statistical methods in the physical sciences, a considerable amount of work has been done in applying these techniques to performance times scientifically. W. A. Gomberg[4] has constantly tried to advance this movement toward scientific time study.

Statistical methods allow one to predict future performance based upon an analysis of present performance. However, there are certain criteria that must be met before prediction can be made.

Statistical stability is a prerequisite of this type of prediction. Stability requires a system of chance causes to be in operation. Assignable causes must be determined and eliminated before stability can be assumed. Presently these assignable causes of variation have not been determined for performance times.

The second prerequisite of this type of prediction is the determination of a formal distribution. Before a particular procedure of

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[3] Walter A. Shewhart, Economic Control of Quality of the Manufactured Product (New York: D. Van Norstrand and Company, Inc., 1931).

[4] W. A. Gomberg, A Trade Union's Analysis of Time Study (Chicago: Science Research Associates, 1948).

analysis is used, something must be known of the formal distribution. Until recently, a normal distribution has been frequently assumed. Davidson states that "the assumption of normal distribution of relative production rates of industrial workers is operationally invalid. The development of any general rule for the statistical definition of a normal worker should be approached with great caution."[5]

If this statement is true, then the theory of wage incentives, based upon a normal distribution of performance times, may be invalid. This statement also indicates that the steps of the incentive plans do not include the actual percentage of workers that they theoretically include.[6] If Davidson's statement is correct, it would also mean that the Gaussian distribution[7] function does not hold true for performance times. This rule roughly states that any measurable capacity or trait tends to approximate a normal distribution.

Rothe,[8] in his studies, found distributions that were in opposition to the results reported by Davidson.[9] In two studies, Rothe found definite indications of stable distributions. Of the first eight operators studied, none of the individual distributions differed significantly from the normal. This test was made using a 1 per cent

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[5]Harold O. Davidson, op. cit., p. 332.

[6]Ibid., p. 317.

[7]Ibid., p. 319.

[8]H. F. Rothe, "Output Rates Among Butterwrappers--II Frequency Distributions and a Hypothesis Regarding Restriction of Output," Journal of Applied Psychology, Vol. 30, 1946, p. 323.

[9]Harold O. Davidson, op. cit., p. 332.

significance level with the Chi Square Test.[10] The beta method showed that three of these eight were significantly leptokeutic.

In the second test of eight operators, both the Chi Square and the beta test showed that the distributions did not differ from the normal significantly. In summary, Rothe states that distributions tend to be bell-shaped and approximate a normal distribution.[11]

The question is not entirely one of a formal distribution, but rather one of statistical stability. If performance times are found to be statistically stable, then the procedures formulated by Shewhart may be applicable.

Adam Abruzzi divides this problem of statistical stability into two parts. Studies reported in Work Measurement[12] were grouped according to "local" and "grand" stability. Local stability is the stability of a continuous series of items produced during a period of several hours, or during a complete shift, while grand stability is the study of the production rates represented by items produced over a protracted period; these rates are examined in terms of small samples taken from successive production units made under essentially the same conditions.

Abruzzi's work has been solely in the garment making industry. The data were collected on various sewing operations including different sized garments. The grouping of the various sizes into a sample is permissible in this case, if the size has no effect on the operation.

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[10] H. F. Rothe, op. cit., p. 323.

[11] Ibid., p. 324.

[12] Adam Abruzzi, Work Measurement (New York: Columbia University Press, 1952), 37 pp.

This work and that of Rothe's are two studies in which previous production records were not used. Abruzzi objected to the use of past production records, because many variables are lost when such data are used. Also, it is impossible to be assured that similar conditions existed during the entire study. The required amount of data is usually not available in the form desired in such records.

Rothe did not obtain cycle times, but rather he collected rates of production for a 15 minute period.[13] This method tends to eliminate variations and thus to destroy the actual distribution. Rothe also states that all operators used a slightly different method.[14] The magnitude of this variation is very significant since changes in methods can be considered assignable sources of variation, and thus will tend to make the performance time unstable.

The use of past production data must be discouraged because of the effect of delays on performance times. Much research has been reported on effect of delays on cycle times. Davis and Josselyn, in a study of an assembly operation, report that there was no significant difference in effective operation time[15] throughout the hours of the day or between morning and afternoon. They also concluded that the operator continued to use essentially the same method and to work at the same rate of speed, but the operators introduced more and longer

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[13] H. F. Rothe, "Ouput Rates Among Butterwrappers--I Work Curves and Their Stability," Journal of Applied Psychology, Vol. 30, 1946, 202 pp.

[14] Ibid.

[15] Effective operation time is equal to cycle times minus delays.

work stoppages.[16] This work illustrates that the use of production records will give no insight into cycle time variations. The magnitude of introduced delays is such that they will tend to overshadow any variation or trend of cycle times obtained from production records.

Closely allied to the study of stability and work-time distributions has been the work of many people on daily production curves. These curves are not only important in the determination of a sampling procedure, but it also might be of value in the setting of control limits if a stable work curve is developed. Abruzzi reports that his work suggests that time of day does not have nearly as important an effect on work performance as is generally believed to day.[17] In support of Abruzzi, Wiberg states that he may have a reason for the misunderstanding of the effect of time of day or day of the week. He found that there was no statistical difference in production per day of the week. This, he says, is contrary to previous findings; however, the previous data were not subjected to significance tests in most cases.[18] Again the use of production records and the use of measures, such as production per hour or day, are questioned. Production rates such as these are subject to many variables not recorded in the data.

Davidson, in a study of an assembly operation, found a tendency

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[16] Louis E. Davis and P. Dudley Josselyn, "An Analysis of Work Decrement Factors in a Repetitive Industrial Operation."

[17] Adam Abruzzi, op. cit., p. 63.

[18] Martin Wiberg, "The Work Time Distribution--A Technique for Analyzing Performance Differences," Motion and Time Study Notes, C. B. Gordy and Others (Ann Arbor, Michigan: The Edwards Letter Shop, 1949) C-29 pp.

for performance times to increase from Monday through Friday. However, he did not believe that this increase was a basis for saying that it represented a tendency of performance times to increase toward the end of the week. It may have been due to some other cause or combination of unknown causes.[19]

Rothe, in the study of butterwrappers, found no predictable pattern. He found that the curves had many different forms.[20] Thus, the use of control limits that follow a work curve are not feasible, since a stable work curve has not been developed.

There are many questions that must be answered before any attempt is made to satisfy Gomberg's appeal for a scientific time study. Davidson sets forth two such questions when he asks, "What variability do performance time phenomena exhibit?" and "Do they tend toward any sort of stability?"[21] These questions, the author will attempt to answer in this thesis. In basis statistical language the question might be stated to read, "Is the occurrence of a performance time like or nearly like the drawing of a chip (numbered) from a bowl containing a large number of chips?"

Before the study proceeds, a working definition of statistical control is sought. Shewhart defines it in the following terms: "A phenomenon will be said to be controlled when, through past experience,

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[19] Harold Davidson, op. cit., p. 369.

[20] H. F. Rothe, "Output Rates Among Butterwrappers--I Work Curves and Their Stability," Journal of Applied Psychology, Vol. 30, 1946, p. 209.

[21] Harold Davidson, op. cit., p. 304.

we can predict, at least within limits, how the phenomenon may be expected to vary in the future. Here it is understood that prediction within limits means that we can state, at least approximately, the probability that observed phenomenon will fall within given limits.[22]

To establish a scientific method of setting standards is far beyond the scope of this thesis. The author will, however, try to answer a few of the basic questions involved in such a program.

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[22]Walter A. Shewhart, _op. cit._, p. 6.

CHAPTER II

OBJECTIVE

Before any scientific time study can be attempted, certain questions must be answered regarding operator's performance times. It is the objective of this thesis to answer many of these questions. Specifically these are:

(1) Do operators follow any pattern of performance or work curve throughout the day?

(2) Do unadjusted performance times tend toward any formal distribution or could they be made to form any model?

(3) Are operator's cycle times statistically stable?

(4) What is the relationship of variation within a period to the variation between periods?

CHAPTER III

PROCEDURE

Selecting an Operation

The operation used in this study was a short cycle manually controlled assembly operation in a local plant. It was selected partially because of the large concentration of operators which, it was felt, would facilitate obtaining a representative sample. There was about six or seven girls on each of three shifts assembling the unit. The experience of the girls on this operation varied from about two weeks to one year. It is estimated that two months are required for a new girl to reach the plateau of productivity.

The operation was quite well established, having been in existence about one year prior to the study, and the method used had been standardized within limits. The work place was essentially the same for all operators. Although each girl had been instructed as to the method used, many of the girls had a tendency to make slight changes in the standard method. These were noted in the data whenever possible. However, the individual method used by each girl was practically constant throughout the study, i.e. no noticeable changes were detected.

The company does not employ an incentive system of wage payment in this department; however, it does have a standard or goal that the girls are expected to approach or reach. The operators that do not attain this standard risk possible transfer or lay-off. Production records are available to each operator throughout the day. By watching the posting, it is possible for an operator to pace herself quite accurately so as to attain a particular goal of production. It was felt that this is standard practice for all of the operators. All girls on the operation receive the same hourly wage. No shift differentials are paid. The estimated average pace of the operators was between 125 and 130 per cent of normal. Personal relations are extremely good.

The assembled pen consists of four groups or parts. They are: (1) barrel and clip assembly, (2) writing unit, (3) ferrule or sleeve, and (4) drive cap. In the present set-up, each hand picks up a barrel and clip assembly and places it in the jig. The hands then place a writing unit in each barrel and clip assembly. A driving cap is placed on each writing unit and simultaneously turned down. Next, the sleeve is added to each unit before it is removed from the jig. Each hand grasps a complete unit and guides it into the staking die. A foot pedal is actuated, which causes an air hammer to stake the parts in place. Lastly, the assemblies are removed and transferred to one hand which asides the parts. (See Figure 1.)

The variation in method previously mentioned is encountered in the last element. Some of the girls held the completed units in the right hand until a full hand of units had been assembled, and then aside the units. This variation may cause a slight decrease in the performance times of certain operators, but it would be a constant factor and, therefore, would only effect the variation between operators and not the individual distributions.

1. Grasping Barrel

2. Assembling Writing Unit

Figure 1. Assembly Operation and Layout

3. Grasping Drive Nut

4. Assembling Drive Nut

Figure 1. Assembly Operation and Layout
(Continued)

5. Grasping Ferrule

6. Removing Complete Unit

Figure 1. Assembly Operation and Layout
(Continued)

7. Staking Unit

8. Asiding Unit

Figure 1. Assembly Operation and Layout (Continued)

Recording the Data

A representative sample of cycle times of each operator was desired. It was decided that a sample of 25 cycles would be observed as often as possible in the before and after lunch sections of each of the three shifts.

The day shift operation is as follows:

7:00 - 9:30	A.M.	Work
9:30 - 9:35	A.M.	Morning break
9:35 - 11:40	A.M.	Work
11:40 - 12:00	Noon	Lunch
12:00 - 1:45	P.M.	Work
1:45 - 1:50	P.M.	Afternoon break
1:50 - 3:20	P.M.	Work

The second shift operation is as follows:

3:20 - 6:10	P.M.	Work
6:10 - 6:30	P.M.	Lunch
6:30 - 9:00	P.M.	Work
9:00 - 9:05	P.M.	Break
9:05 - 11:40	P.M.	Work

The third shift operation is as follows:

11:40 - 3:00	A.M.	Work
3:00 - 3:20	A.M.	Lunch
3:20 - 5:00	A.M.	Work
5:00 - 5:05	A.M.	Break
5:05 - 7:00	A.M.	Work

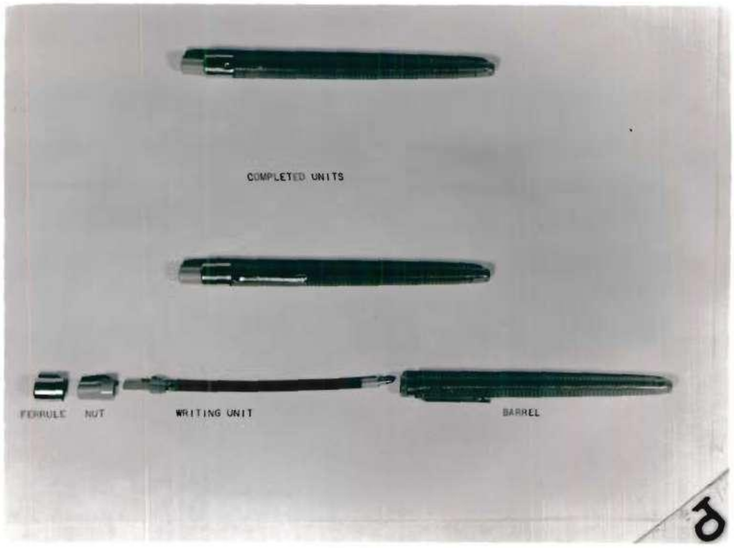
The total working time on the first shift is 470 minutes, on the second it is 475 minutes, but on the third it is only 415 minutes.

The operators on each shift were told by the recorder that the data to be collected was for research purposes only, and their supervisors would not see the data. They were also asked to work at their normal pace just as if the recorder were not present. Each girl was approached individually and was asked to tell the recorder at any time she became nervous or bothered in any way.

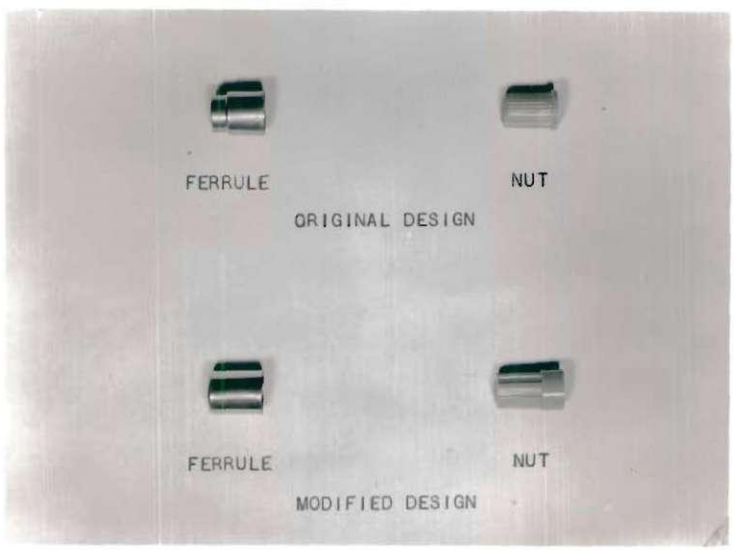
On Wednesday, October 22, 1952, the date of the study of the day shift, the department was working six days per week and was running about 2,000,000 units behind in production. Daily production was about 75,000 units.

The study of the second shift was made on Wednesday, December 17, 1952, and the department was still behind in production. Experimental units of a new ink were being run, and this had curtailed production on certain days. Three of the regular girls, being ill the night of this study, were not present.

On the evening of Wednesday, February 4, and the morning of February 5, 1953, the third shift was studied. The department had been producing only intermittently since December 23, because of inventory and a parts change. However, the girls on this shift had worked continually for about a week on this modified unit. A few of the girls complained that they were not completely familiar with it and that the new driving cap made their fingers sore at times, since the serrations were more pronounced and sharper. (See Figure 2 for Comparison of Parts.)



Parts



Original and Modified Parts

Figure 2. Comparison of Old and New Parts

During the first shift the recorder was able to observe the operators about once each hour. At this time five groups of five cycles each (continuous) were recorded. Each operator was observed according to a set pattern i.e., operators 1, 2, 3, 4, 5, 6, and 7 respectively.

The first sample was taken at 8:45 A.M. from operator 1 and the last sample was recorded at 2:55 P.M. No samples were taken between 11:30 A.M. and 12:45 P.M. of this shift. This eliminated any possible effect of the lunch hour on the samples.

The first sample of the second shift was taken fifteen minutes after the girls reported to work. The average girl took about ten minutes to get settled and to stock up with parts prior to beginning. Four observations on each operator were made before supper at intervals of between 30 to 40 minutes. Four more observations were made before the five minute break at 9:00 P.M. Three observations were made between 9:05 and quitting time. The last sample was taken at about 11:05 P.M. The girls usually begin to clear up at about 11:30, and therefore, another group of observations could not have been made.

It took each operator on the third shift about five to ten minutes to do the preliminary work, prior to actually producing a unit. About five minutes after operator 13 began producing, the first sample was taken. Three observations were made of each operator before the break, except operator 19. The break came soon after the observation had begun and thus a complete sample was not taken. The sample was completed, however, immediately after the lunch period. Three more observations were made after the meal, making a total of six from each operator.

The last of these was taken at about 6:30 A.M. The remaining time did not allow a complete sample of all operators to be made.

When recording the data, the following notations were made:

DP - Dropped part

BP - Bad part

GP - Get parts

MP - Moved parts

AD - Avoidable delay

S - Turned cap to inspect finished part

I - Inspected part or unit

CM - Changed method

RP - Released parts

An "X" was placed on the data sheet adjacent to all cycles containing deviations from the standard procedure. These cycles were not used in the summary of data presented. Only cycles which contained one of the extraneous elements mentioned were eliminated. This procedure required a decision during the cycle as to whether the delay encountered should be considered extraneous, and thus the cycle time eliminated from the analysis. Such a procedure is current practice in stop watch time study.

A split hand decimal minute stop watch (capable of accuracy approaching .001 minutes) was used to determine the cycle times. The data was recorded to the nearest hundreth of a minute, since the short cycles did not allow sufficient time for an interpolation of the watch. All times recorded are hundreth of a minute. This was done to simplify calculations. In addition, it was desired that the recorder observe the operator as constantly as possible in order to detect variations in methods. A few cycles in the first study were recorded to the nearest .005 minute, but this proved to be time-consuming and was later abandoned.

Definition of Terms

Observation - one cycle

Subgroup - five consecutive observations

Period - five subgroups (25 observations)

X - observation (one cycle time)

$\bar{X}$ - mean of subgroup of five observations

$\bar{\bar{X}}$ - mean of period of 25 observations

$\bar{\bar{\bar{X}}}$ - operator's mean cycle time

$\bar{\bar{\bar{\bar{X}}}}$ - mean cycle time of all operators on shift

$\bar{\bar{\bar{\bar{\bar{X}}}}}$ - mean cycle time of all operators

R - range of subgroup of five cycles

$\bar{R}$ - average range of period (5 subgroups)

$\bar{\bar{R}}$ - average range of subgroups for operator

$\bar{\bar{\bar{R}}}$ - average subgroup range for all operators of the shift

$\bar{\bar{\bar{\bar{R}}}}$ - grand average subgroup range

σ_w^2 - variation within a period

σ_b^2 - variation between periods

σ_T^2 - total variation of an operator

n_s - number of subgroups per period

N - total number of observations of operator

n_B - number of observations in a subgroup

n_o - number of periods of operator observed

n_k - number of operator on shift

$R_{\bar{X}}$ - range of subgroup means for a sample

$R_{\bar{\bar{X}}}$ - range of sample means for an operator

$R_{\bar{\bar{\bar{X}}}}$ - range of operator's mean

$UCL_{\bar{X}}$ - upper control limit of X values

$LCL_{\bar{X}}$ - lower control limit of X values

$UCL_{\bar{\bar{X}}}$ - upper control limit of X values

$LCL_{\bar{\bar{\bar{X}}}}$ - lower control limit of X values

UCL_R - upper control limit of R values

LCL_R - lower control limit of R values

The definition of work-time distribution as presented by Wiberg is:

> A work-time distribution is defined as a frequency distribution of a specified number of time values obtained through time study as actual unadjusted watch readings, on an element of repetitive work for the purpose of relating the distribution characteristics to the various influences which the worker, the work method, and the work environment exert upon the efficiency of a particular work situation.[23]

All other terms used are standard statistical terms and a definition can be secured from any statistics text, such as Grant.[24]

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[23]Martin Wiberg, *op. cit.*, C-29 pp.

[24]Eugene L. Grant, *Statistical Quality Control* (New York: McGraw Hill Book Company, Inc., 1952).

Method of Analysis

Each period sample of twenty-five observations was broken into five subgroups of five observations each. Thus, a period sample contained five sequential subgroups each containing five observations.

Mean Cycle Time.

The mean cycle time at several levels was determined. These are:

1. Subgroup mean
2. Period mean
3. Operator mean
4. Shift mean
5. Operation mean

The subgroup mean was calculated using

$$\bar{X} = \frac{\Sigma X}{n}$$

where n equals the number of observations per subgroup. (n = 5)

The period mean was determined by combining the subgroup means:

$$\bar{\bar{X}} = \frac{\Sigma \bar{X}}{n_s}$$

where n_s is the number of subgroups in a period. (n = 25)

The operator's mean was calculated in a similar manner, i.e.

$$\bar{\bar{\bar{X}}} = \frac{\Sigma \bar{\bar{X}}}{n_o}$$

where n_o is the number of periods observed.

The shift mean ($\bar{\bar{\bar{X}}}$) was calculated by combining the operator's means. Since the number of periods observed from each operator on a shift was constant, it was not necessary to weigh the individual means in obtaining this value, i.e.

$$\bar{\bar{\bar{X}}} = \frac{\Sigma \bar{\bar{\bar{X}}}}{n_k}$$

where n_k is the number of operators on the shift.

To obtain a cycle mean time for the operation, a weighted average of each shift was used.

$$\bar{\bar{\bar{\bar{X}}}} = \frac{\Sigma (n_{k_1} \bar{\bar{\bar{X}}}_1 + n_{k_2} \bar{\bar{\bar{X}}}_2 + n_{k_3} \bar{\bar{\bar{X}}}_3)}{\Sigma (n_{k_1} + n_{k_2} + n_{k_3})}$$

Variance of Cycle Times.

Variation within a Period.--Variation within each period (σ_w) was determined by

$$\sigma_w = \frac{R}{d_2}$$

Since the range had been used in the control chart, and had already been calculated, it was used to calculate σ_w rather than

$$\sigma_w = \sqrt{\frac{(X - X)^2}{n-1}}$$

Variation within operator, between periods observed.--A method similar to that used to determine σ_w was used to calculate σ_B, i.e.,

$$\sigma_B = \frac{R_{\bar{\bar{X}}}}{d_2}$$

where $R_{\bar{\bar{X}}}$ is the range of the mean of the period averages for an operator.

Stability of Operator's Cycle Time and Variance of Cycle Times

Use of Control Charts.--Control chart procedures were used to analyze stability of an operator's cycle times within a period and within a shift. Limits of

$$\bar{\bar{X}} \pm \frac{3\sigma_B}{\sqrt{n_B}}$$

(where $\bar{\bar{X}}$ is the grand average of an operator's cycle times and n_B is the size of the subgroup) were used as criteria in analyzing within period stability, whereas, control limits of

$$\bar{\bar{X}} \pm \frac{3\sigma_B}{\sqrt{n_B \cdot n_s}}$$

(where n_s is the number of subgroups in each period) were used in testing stability between periods for a particular operator.

The range of the observations within a subgroup were analyzed in a similar manner. Using a five observation subgroup, the limits for the control chart are:

$$LCL_R = D_3\bar{\bar{R}} = 0(\bar{\bar{R}}) = 0$$

$$UCL_R = D_4\bar{\bar{R}} = 2.09(\bar{\bar{R}})$$

Runs.--Significant runs on one side of the center line (mean value) were analyzed on the X and R charts. A run of eight values was taken to be significant, the probability of this happening due to chance being less than one per cent.[25]

Distribution

Each operator's cycle times were plotted as a histogram. A visual interpretation of the patterns was made. A histogram of the distribution of cycle times for each shift was plotted. A visual inspection indicated that the calculation of all four moments was not necessary.

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[25] $(\frac{1}{2})^7$.

CHAPTER IV

RESULTS

Summary.--All operators tended to follow individual work curves. There was no particular pattern common to any group of operators.

All except three operators tended toward a positively skewed distribution which was quite evident visually.

Eighteen out of the nineteen operators studied exhibited one or more indications of a lack of stability in either the cycle mean or the cycle range values.

The variation within a period was always larger than the variation between periods. (See Tables 1 through 6 and Figures 3 through 6.)

Operator 1.--This operator tended to be stable except for two values of $\bar{X}$. The data sheet shows that the first unstable value was caused by a faulty staking machine. The operator was forced to restake many of the assemblies in this subgroup. During the third subgroup of the 12:55 P.M. period, the operator obtained parts twice. This tends to destroy rhythm and acts as a delay or rest.

No obvious trend of the performance times is illustrated.

The cycle time histogram exhibits a positive skewness. This is due in part to the values previously discussed.

There were no other indications of an unstable distribution. (See Table 7 and Figure 6.)

Table 1. Final Results

Operator	$\bar{\bar{X}}$	$\bar{\bar{R}}$	σ_w	σ_B	Points Out of Control $\bar{X}$	$\bar{\bar{X}}$	R	Runs $\bar{X}$	R	Distribution
1	19.484	5.04	2.166	1.238	2	1	0	0	0	pos. skewed
2	16.264	4.46	1.917	.447	0	0	2	0	0	pos. skewed
3	18.776	7.00	3.009	2.080	2	3	2	8 below	8 below	pos. skewed
4	26.568	9.14	3.929	1.220	1	0	1	0	0	
5	18.176	5.32	2.287	1.057	0	0	1	0	0	pos. skewed
6	21.320	12.58	5.408	2.717	1	1	0	0	0	pos. skewed
7	16.748	5.70	2.450	1.238	1	1	3	8 below	8 below	pos. skewed
Shift	19.619	7.034	3.023	1.443						
8	21.163	7.92	3.405	2.579	2	2	1	13 above 8 below 13 below	0	pos. skewed
9	16.473	6.04	2.592	1.049	2	0	4	9 below 8 above 8 below	0	pos. skewed
10	16.880	7.47	3.213	1.049	1	0	3	9 below	0	pos. skewed

(continued on next page)

Table 1. Final Results
(continued)

Operator	$\bar{\bar{X}}$	$\bar{\bar{R}}$	σ_w	σ_b	Points Out of Control $\bar{X}$	$\bar{\bar{X}}$	R	Runs $\bar{X}$	R	Distribution
11	16.205	5.44	2.337	1.668	3	2	1	0	0	pos. skewed
12	19.247	7.31	3.142	1.685	2	1	3	11 below	0	pos. skewed
Shift	17.994	6.836	2.937	1.685						
13	18.607	5.90	2.537	.361	0	0	0	0	0	
14	19.707	5.90	2.537	.791	0	0	0	9 below	0	
15	16.787	4.20	1.806	.773	1	0	1	0	0	pos. skewed
16	17.060	5.33	2.291	.894	1	0	0	0	0	pos. skewed
17	20.727	6.87	2.952	1.100	0	0	0	9 below	0	pos. skewed
18	19.385	5.15	2.214	1.014	1	0	2	0	0	pos. skewed
19	22.693	8.63	3.711	1.539	0	1	0	9 above	0	not normal
Shift	19.280	5.997	2.578	.924						
Average	18.834	6.621	2.858	1.4229						

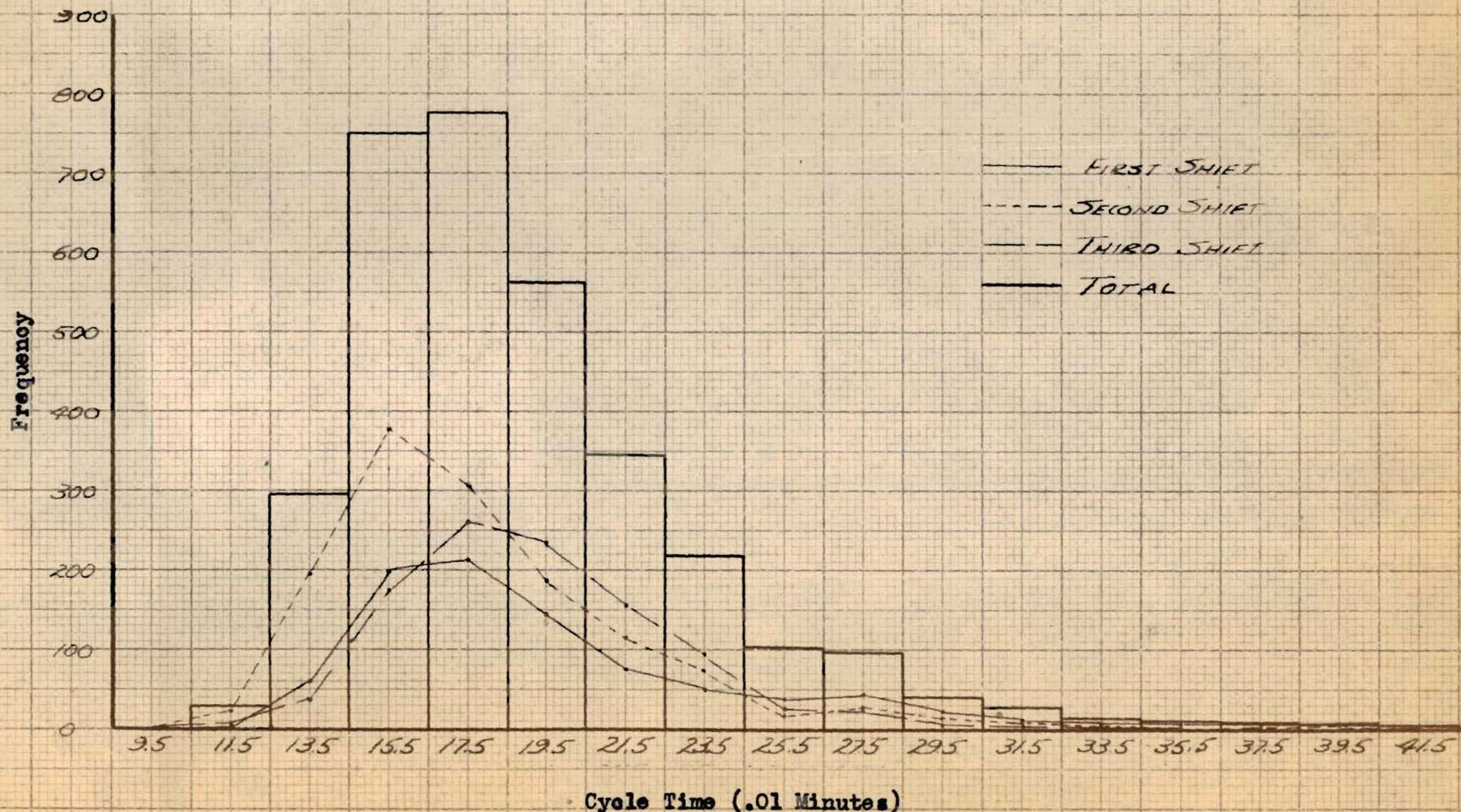


Figure 3. Performance Time Histogram

Figure 4. Operator Control Chart

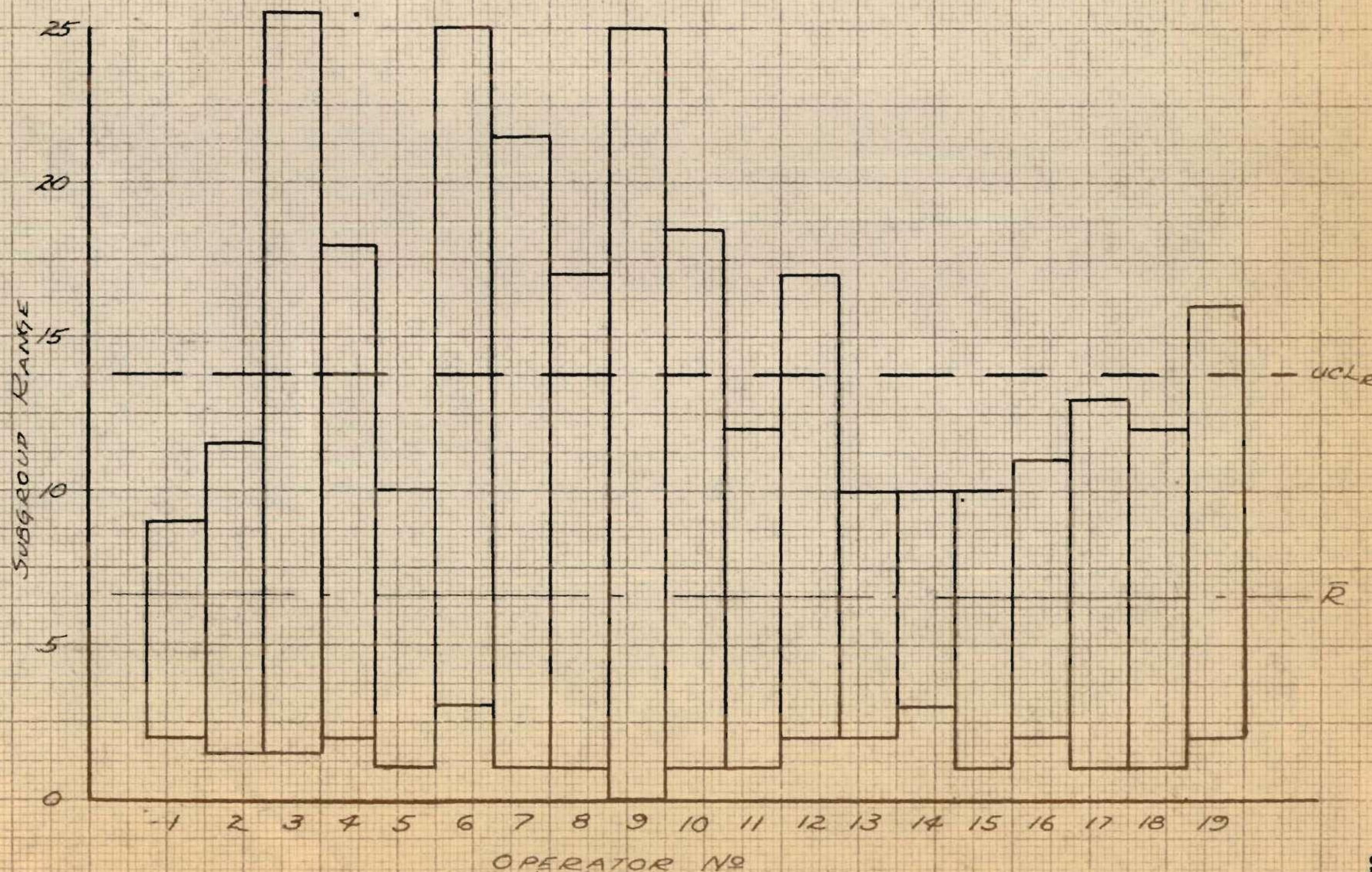


Figure 5. Operator's Cycle Range

Table 2. Control Limits

Shift	$UCL_{\bar{\bar{X}}}$	$LCL_{\bar{\bar{X}}}$
1	19.610	18.088
2	19.392	18.318
3	19.556	18.154

$$\bar{\bar{X}} = 18.834$$

$$\bar{\bar{R}} = 6.621$$

Table 3. Total Frequency

Cycle Time	1st Shift	2nd Shift	3rd Shift	Total
11	1		1	2
12	1	21	5	27
13	12	67	4	83
14	48	129	35	212
15	82	182	69	333
16	116	196	106	418
17	124	173	116	413
18	87	131	144	362
19	91	105	128	324
20	53	82	105	240
21	42	68	91	201
22	32	48	65	145
23	27	41	62	130
24	23	30	34	87
25	26	19	11	56
26	11	20	15	46
27	24	17	10	51
28	20	11	15	46
29	14	9	2	25
30	8	4	3	15
31	6	5	4	15
32	5	3		8
33	5	2	1	8
34	2	1	1	4
35	2		1	3
36	3		2	5
37	2			2
38	3	1		4
39	1	1		2
40	3			3
41	1			1

Table 4. Frequency Distribution by Operator - First Shift

Cycle Time	Operator "1"	"2"	"3"	"4"	"5"	"6"	"7"
11		1					
12							1
13		4			2	1	5
14		19	5		6	5	13
15	1	25	15	1	7	9	24
16	10	29	28		11	12	26
17	22	19	15	1	30	12	25
18	16	12	14	1	23	8	13
19	28	7	16		14	18	8
20	12	5	5	5	16	7	3
21	11	1	11	4	6	8	1
22	10	1	4	5	2	9	1
23	4	2	1	12	1	7	
24	3		1	12	2	4	1
25	5			16	3	2	
26				8		3	
27	3		3	14	1	2	1
28			1	13		4	2
29			3	8	1	2	
30				7		1	
31				5		1	
32			1	1		3	
33				3		2	
34			1	1			
35				1			1
36				3			
37				1		1	
38			1	1		1	
39						1	
40				2		1	
41						1	

Table 5. Frequency Distribution by Operator - Second Shift

Cycle Time	Operator "8"	"9"	"10"	"11"	"12"
11					
12		4	4	13	
13	2	12	25	27	1
14	4	42	40	35	8
15	10	55	50	45	22
16	16	64	36	54	26
17	25	39	31	35	43
18	21	19	24	23	44
19	36	13	13	10	33
20	25	4	18	10	25
21	20	7	9	7	25
22	17	5	8	4	14
23	23	5	4	5	4
24	22			1	7
25	10		4	3	2
26	9	2	1	1	7
27	10		2	1	4
28	4	1	3		3
29	6			1	2
30	2		1		1
31	1	1			3
32	2		1		
33	1				1
34	1		1		
35	1				
36					
37					
38		1			
39		1			
40					
41					

Table 6. Frequency Distribution By Operator - Third Shift

Cycle Time	Operator "13"	"14"	"15"	"16"	"17"	"18"	"19"
11				1			
12	1			4			
13		1		3			
14	2	2	13	15	2	1	
15	9	6	33	17	2	1	1
16	17	8	32	32	3	12	2
17	21	13	27	25	12	13	5
18	27	24	16	18	14	34	11
19	21	22	13	11	23	26	12
20	15	24	7	3	23	18	15
21	17	13	6	11	17	11	16
22	11	16	2	2	11	8	15
23	6	10		4	9	9	24
24	2	7		2	9	1	13
25		2		1	4	1	3
26	1		1	1		2	10
27		1			3	1	5
28		1			6		8
29					1	1	
30					1		2
31						1	3
32							
33							1
34							1
35							1
36							
37							
38							
39							
40							
41							

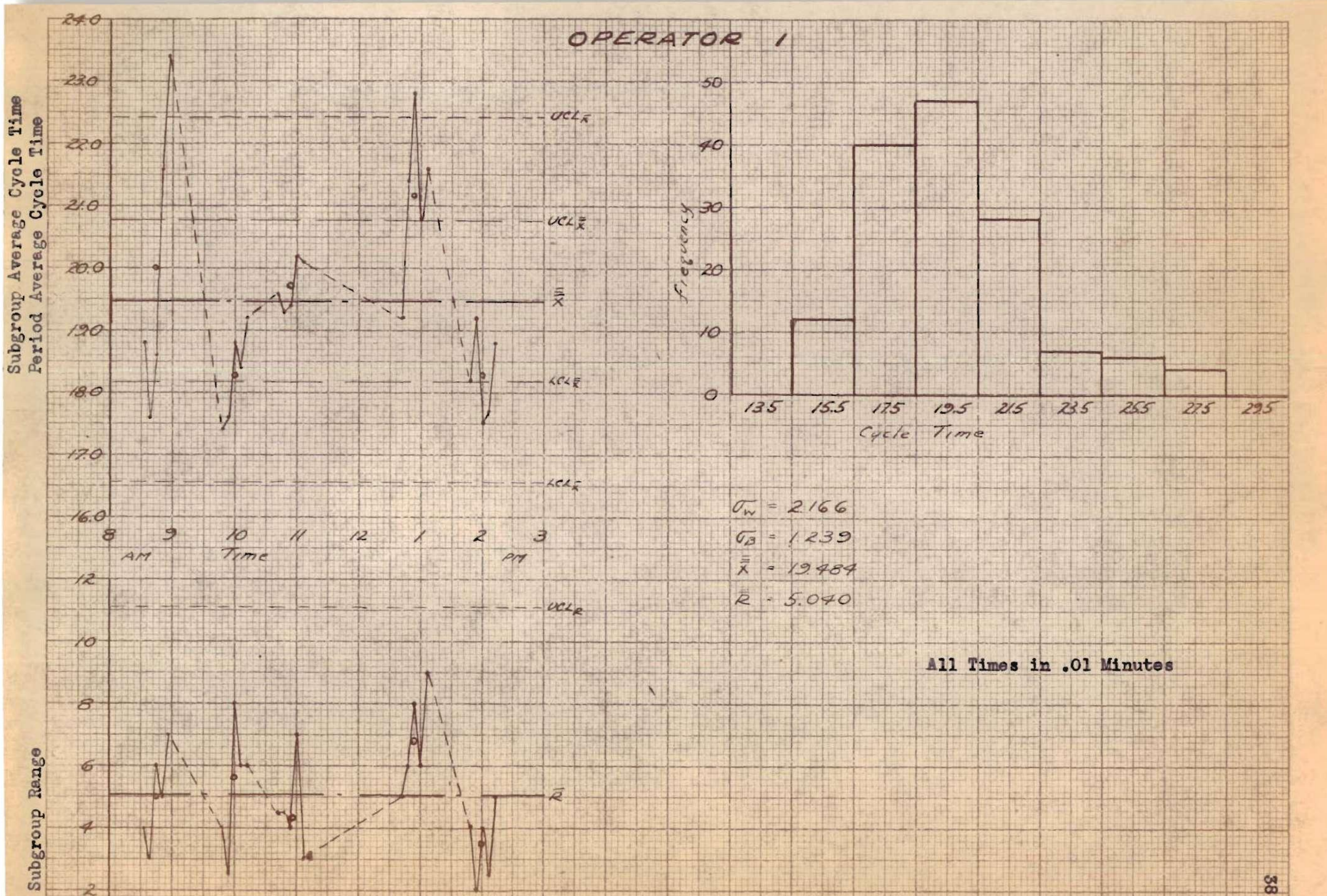


Figure 6. Control Charts and Performance Time Histogram--Operator 1

Operator 2.--The $\bar{X}$ and $\bar{\bar{X}}$ control charts showed no points out of control. On the range chart, two points fell outside of the 3σ limits. The first of these points was caused by a bad part. No definite cause was indicated for the second point. With the exception of these two points, the operator's performance tended to be statistically stable.

The cycle time histogram exhibited a slight positive skewness. This is attributed in part to the two points previously discussed.

There were no significant runs or other indications of an unstable distribution. (See Table 8 and Figure 7.)

Operator 3.--This operator had a relatively high variation between periods. There was a tendency for the cycle times to increase throughout the work day.

The $\bar{X}$ control chart points out two values which lack stability. The first of these was due to an extremely bad part which caused the cycle time to nearly double. The second value was the result of a dropped part and a bad part inspection, both within one subgroup.

Because of the variation of cycle times between periods, three period values fell outside of the 3σ limits on the $\bar{\bar{X}}$ chart. The initial one of these was the first period of the morning while the other two were the last periods of the day.

Two values pointed to a lack of stability on the range chart. The first of these corresponded to the first $\bar{X}$ point out of control, which was due to the bad parts, while the second value was also caused by a bad part.

There were two significant runs of values below the centerline. This indicates a lack of stability and non-randomness. (See Table 9 and Figure 8.)

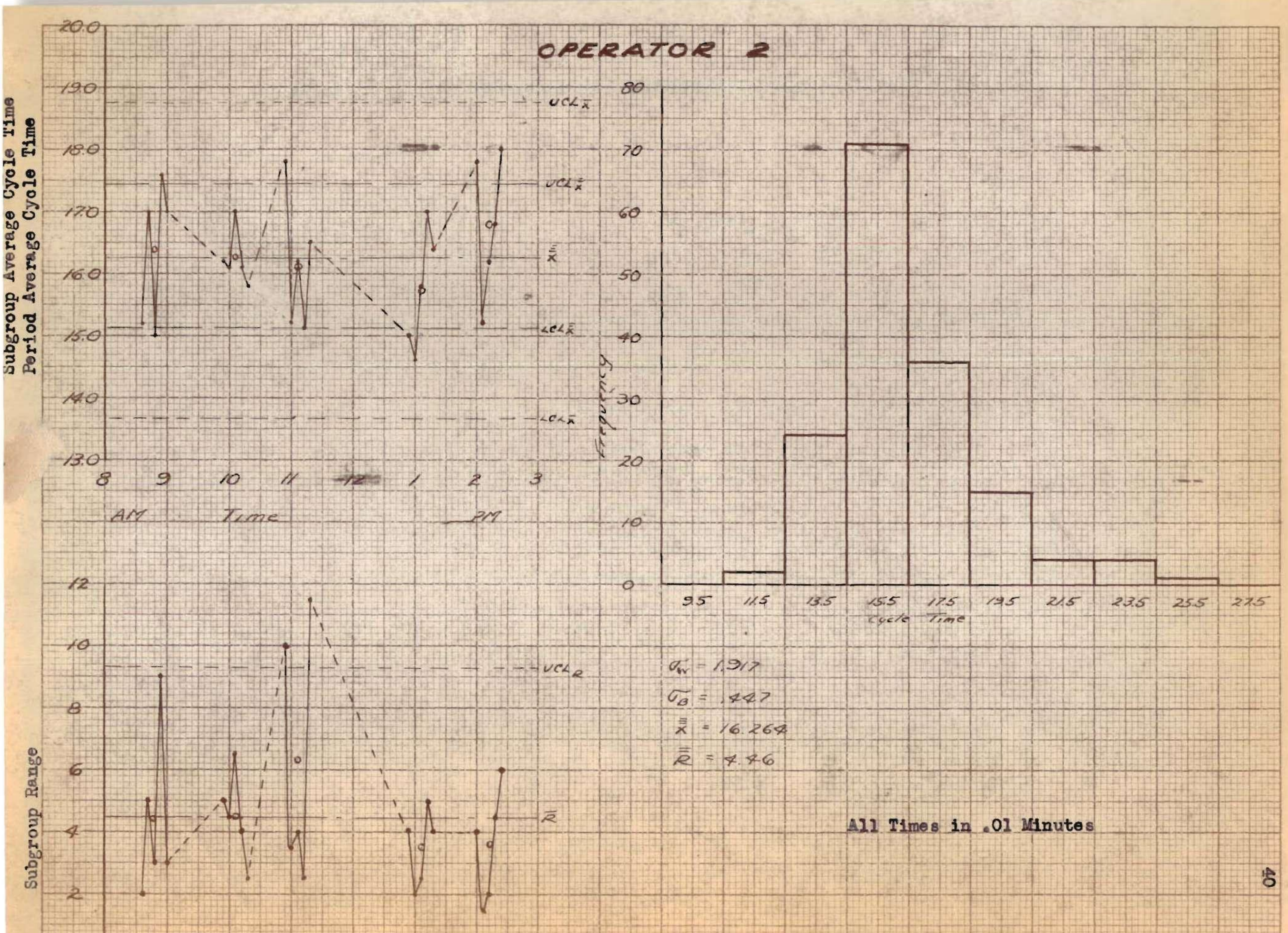


Figure 7. Control Charts and Performance Time Histogram--Operator 2

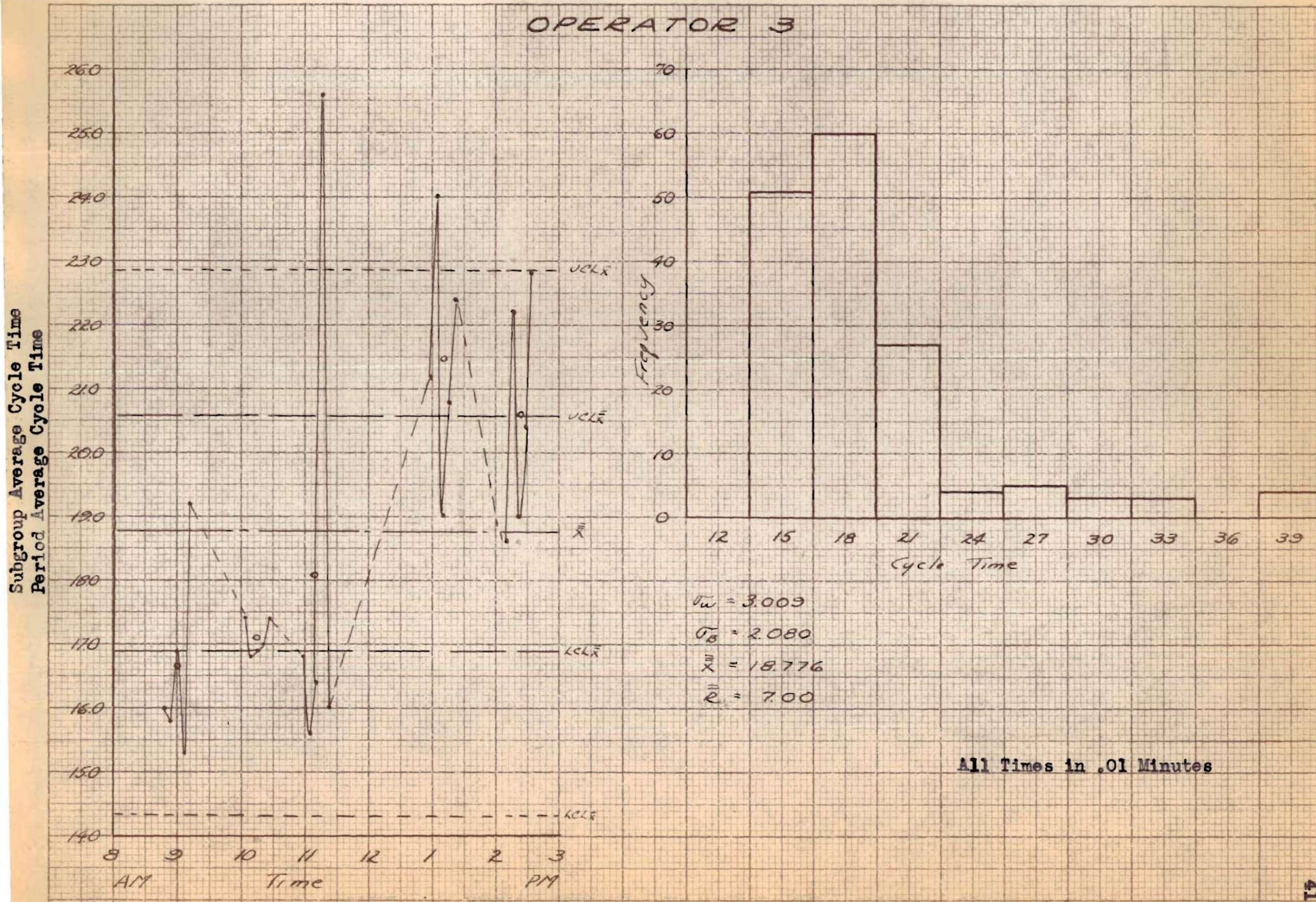
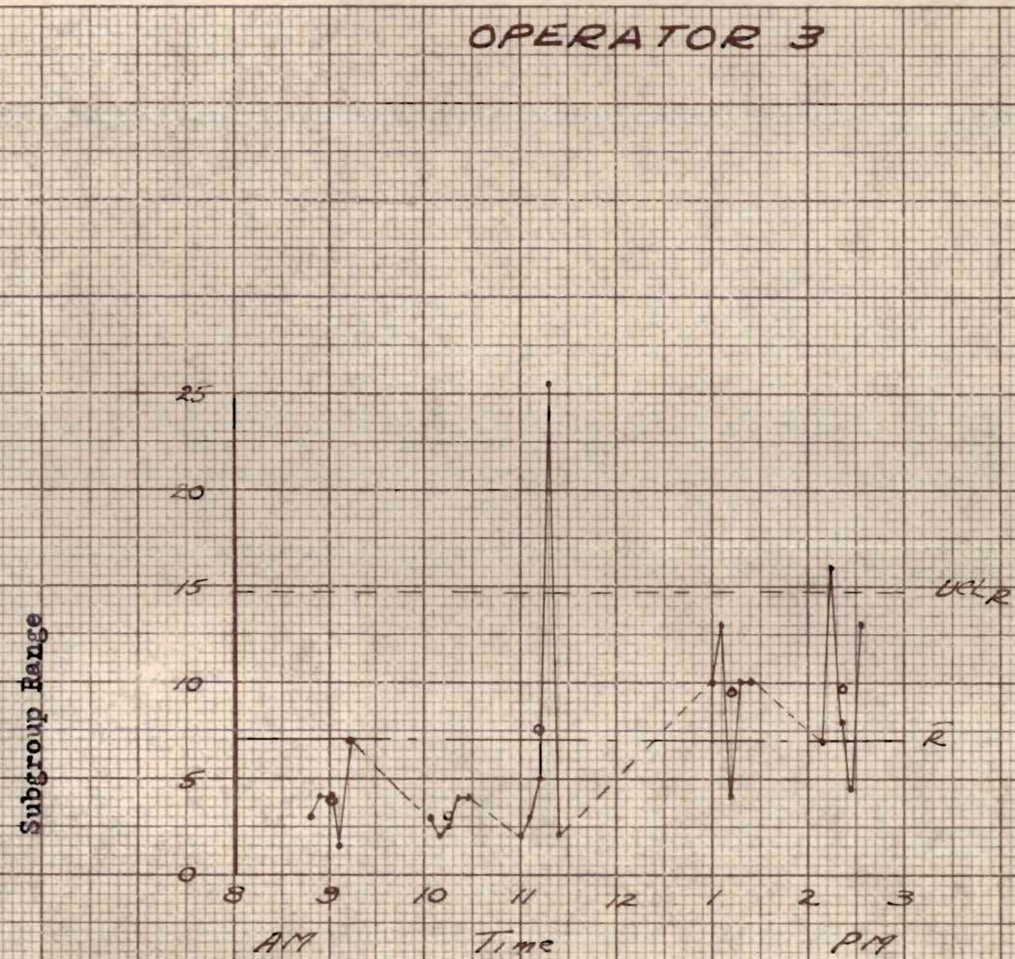


Figure 8. Control Charts and Performance Time Histogram--Operator 3



All Times in .01 Minutes

Figure 8. Control Charts and Performance Time Histogram--Operator 3
(Continued)

Operator 4.--This operator tended toward a stable distribution except for one subgroup in the last period. This lack of stability was caused by three successive bad parts. This also caused a value to go out of control on the range chart. No other signs of a lack of stability were present.

The distribution tended toward a bell-shaped curve.

No overall daily trend of cycle times was evident, although cycle times did increase during the morning and then again in the afternoon. This tendency followed the hypothetical work curve.

This operator had only two weeks experience on the operation and therefore, was producing at a much lower level than were the other operators. There was a tendency for this operator to have more and longer delays and to have more "bad parts" than the average girl. The latter was probably due to her inexperience. (See Table 10 and Figure 9.)

Operator 5.--This operator had no points outside of the 3 limits on either the $\bar{X}$ or $\bar{\bar{X}}$ control charts. However, one point did fall outside of these limits on the range chart. No assignable cause for this lack of stability was found.

The distribution of cycle times tends toward a positive skewness. This was caused by a comparatively few values as the result of bad part, restaking, and others.

There are no other indications of a lack of stability. (See Table 11 and Figure 10.)

Operator 6.--There is a definite positive skewness present in the distribution of cycle times as shown by the histogram.

One point lacked control on the $\bar{X}$ control chart and this was

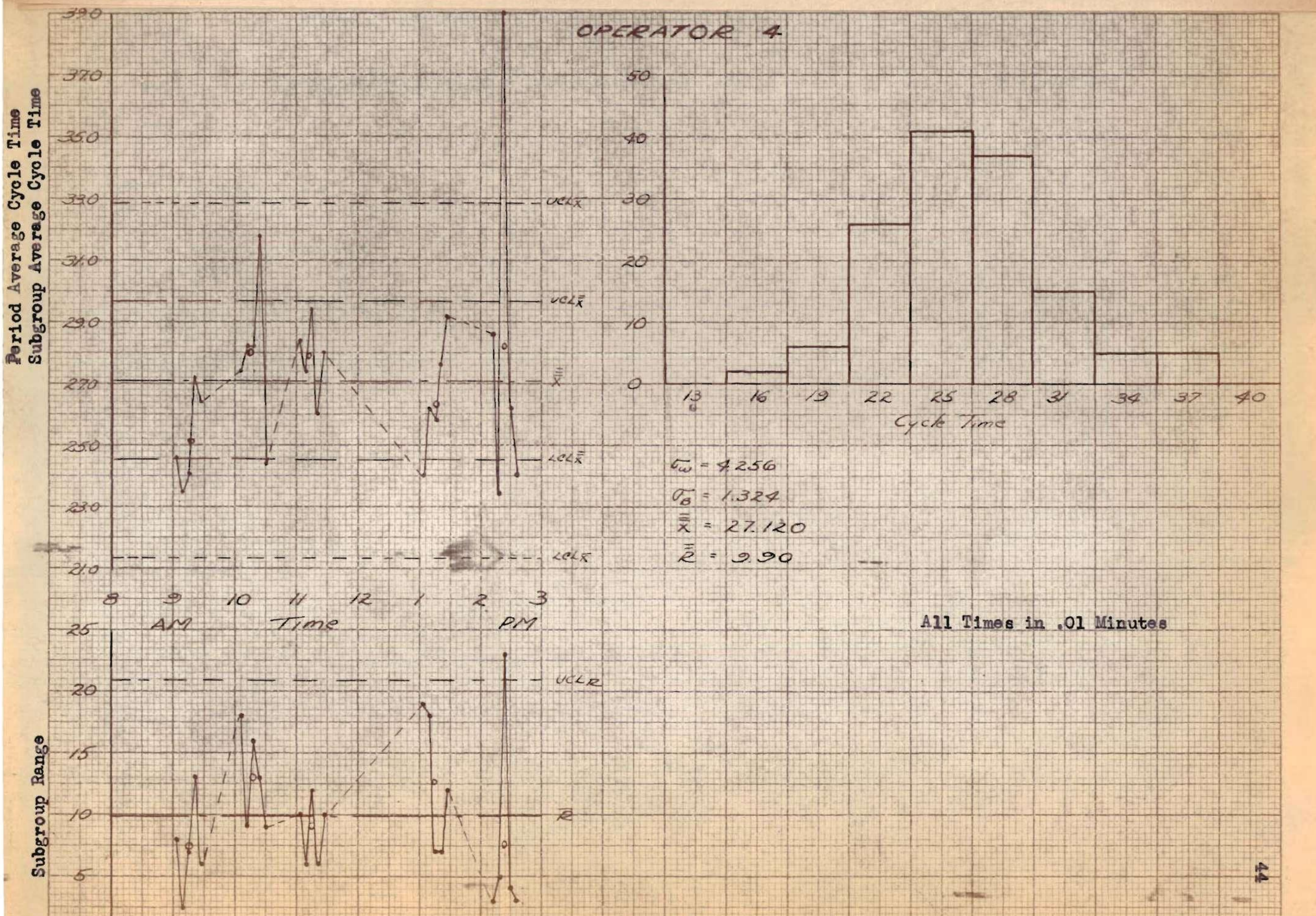


Figure 9. Control Charts and Performance Time Histogram--Operator 4

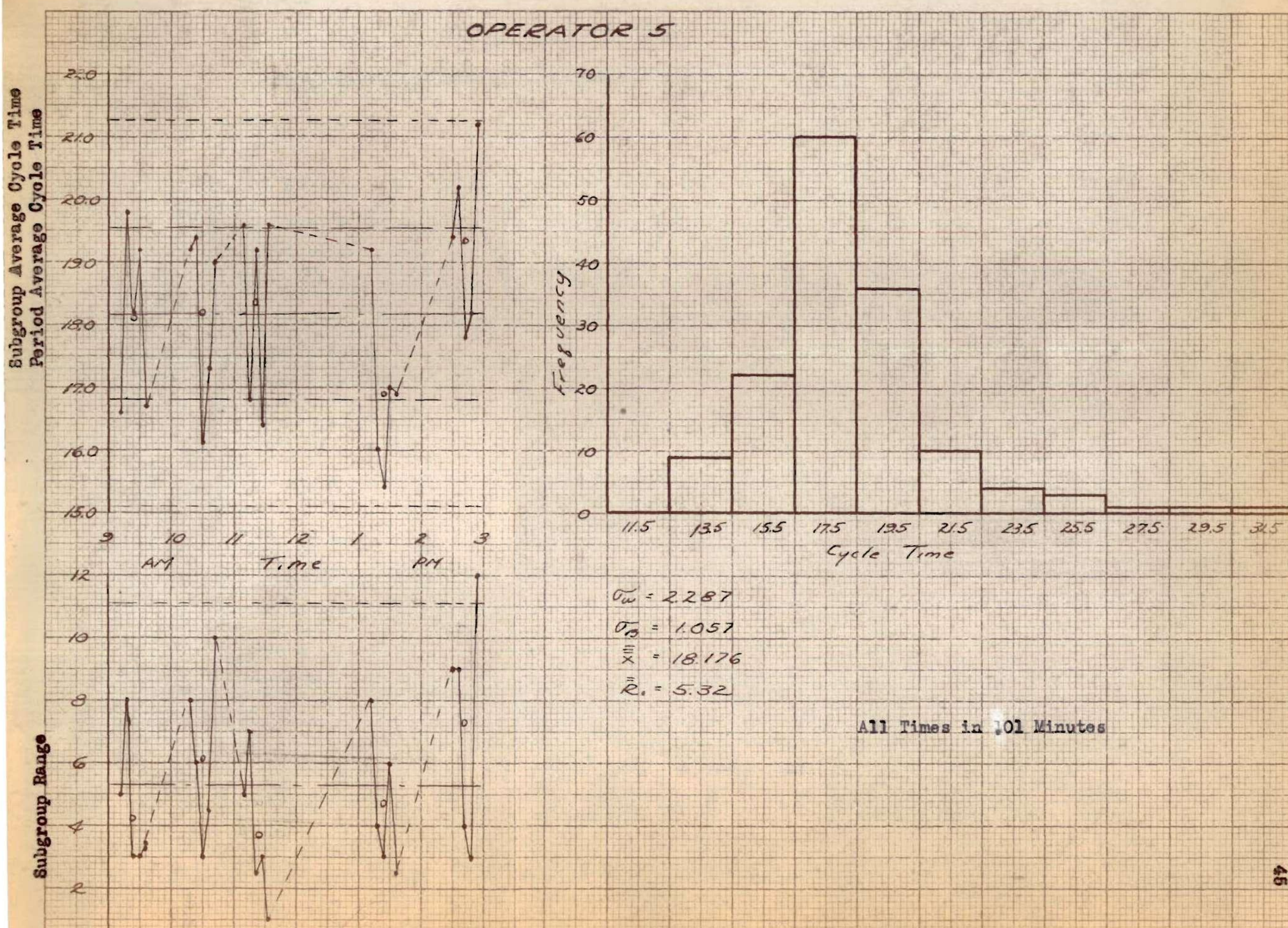


Figure 10. Control Charts and Performance Time Histogram--Operator 5

found to be due to a dropped part and also two bad parts in this subgroup.

The corresponding period average was likewise out of control on the $\bar{\bar{X}}$ control chart. This lack of control was due in part to the large number of bad parts encountered in this particular period.

This operator tends to be statistically stable in regards to means and ranges, but a trend is indicated in cycle times by the fact that the cycle time increased both in the morning and in the afternoon.

There are no other indications of a lack of control. (See Table 12 and Figure 11.)

Operator 7.--The cycle time distribution of this operator exhibited a very positive skewness. It could be due partially to the number of bad parts encountered in the third period. Both the $\bar{X}$'s and the $\bar{\bar{X}}$ for this period were outside of their respective control limits. The range of these values also showed a lack of control. All other period data are well within the control limits.

Two significant runs were present, one on each control chart. This is an indication of the lack of stability of the values. (See Table 13 and Figure 12.)

Operator 8.--There was a definite trend for performance times of this operator to decrease during the day. Because the control limits are determined by the subgroup range, a trend this pronounced will cause points on both ends to be outside of the 3σ control limits. Four $\bar{\bar{X}}$ points show a lack of stability while two $\bar{X}$ points are above the control limit. One value on the range chart was also out of control.

Another indication of the lack of stability is the presence of

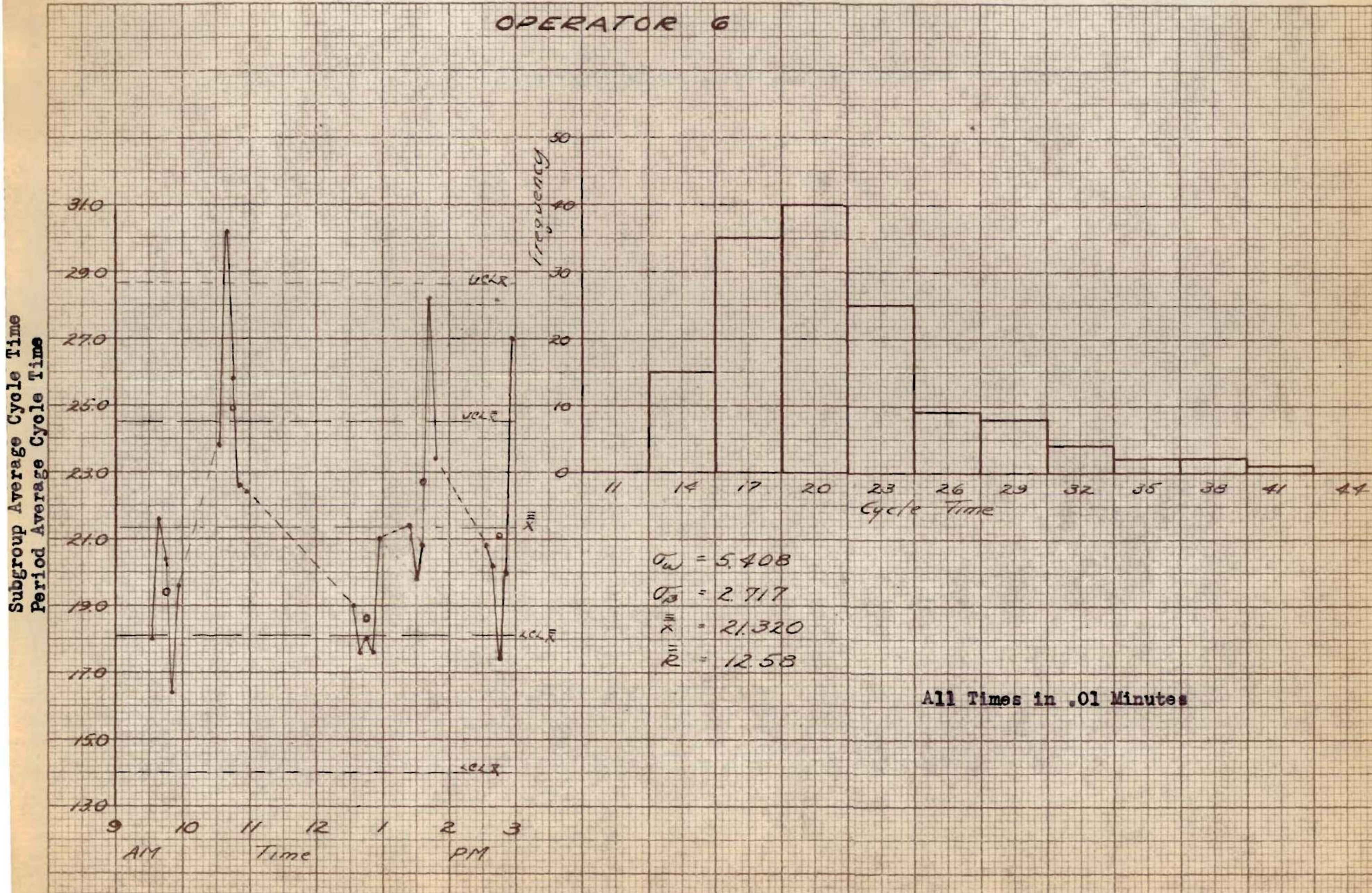
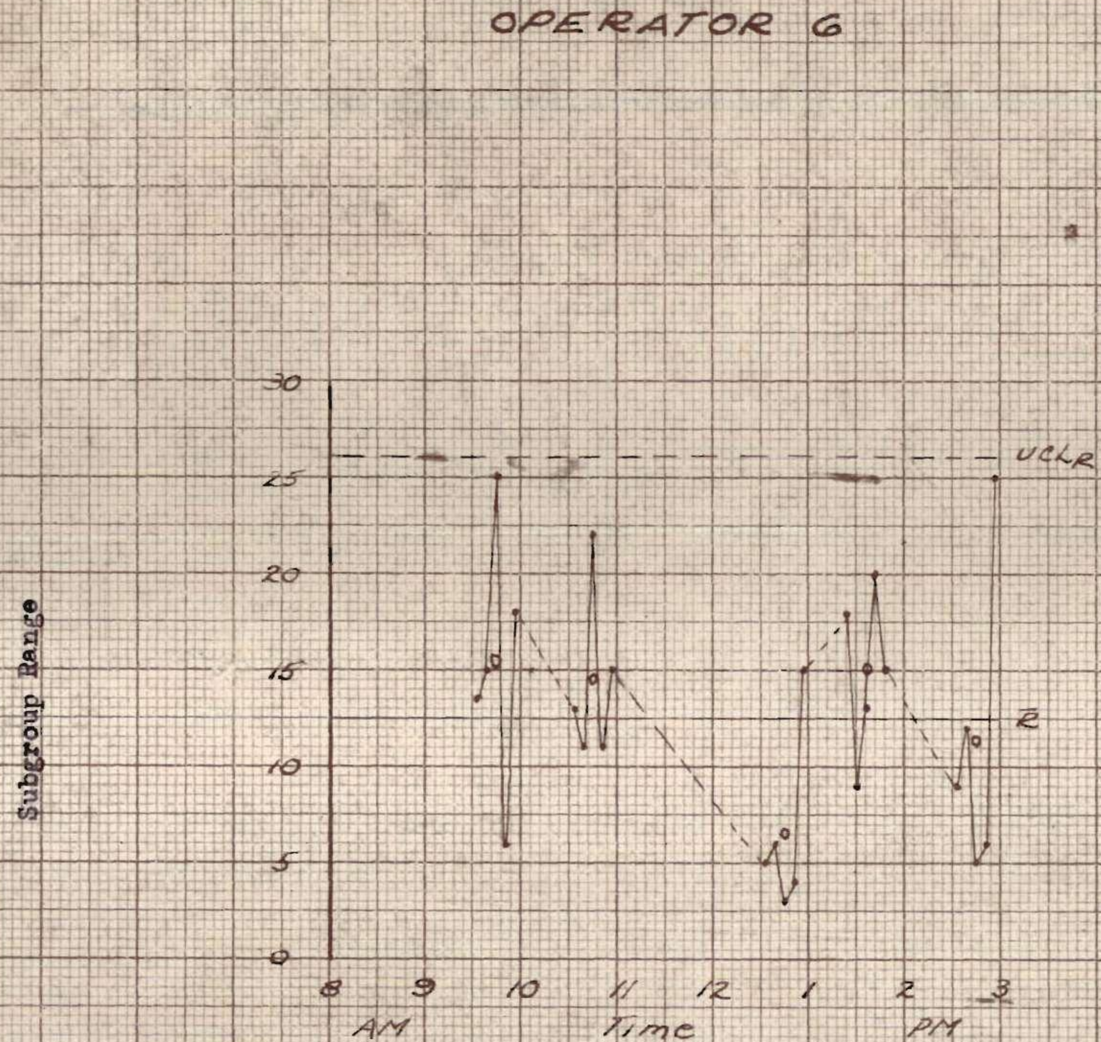


Figure 11. Control Charts and Performance Time Histogram--Operator 6



All Times in .01 Minutes

Figure 11. Control Charts and Performance Time Histogram--Operator 6
(Continued)

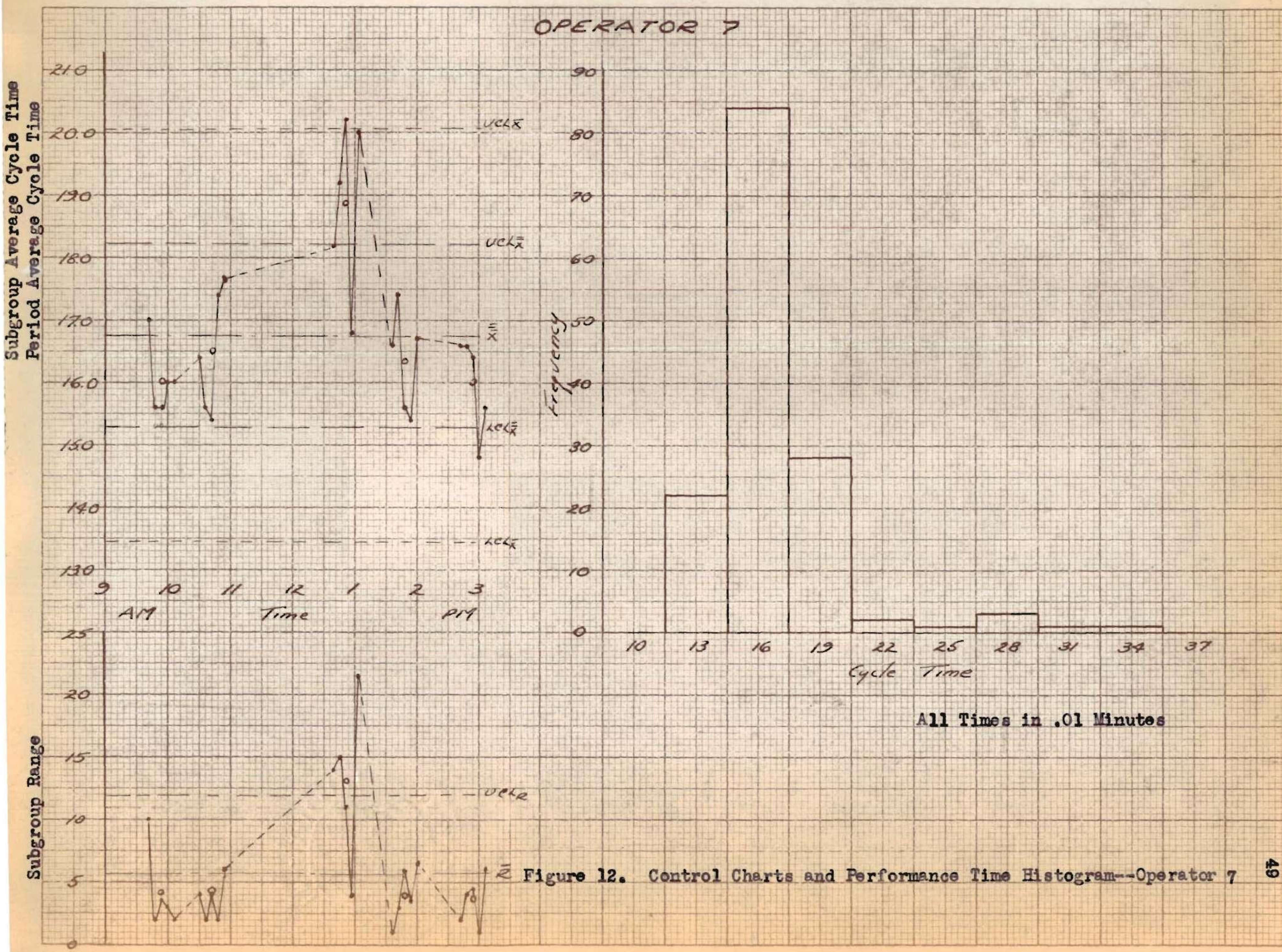


Figure 12. Control Charts and Performance Time Histogram--Operator 7

several significant runs on one side of the centerline. There was a run of 13 values above the centerline and also runs of 13 values and 8 values below the centerline of the $\bar{X}$ chart.

The cycle time histogram showed a positive skewness. Both the σ_w and σ_b were relatively high. (See Table 14 and Figure 13.)

Operator 9.--There was a slight tendency for the performance time of this operator to increase as the work day advanced.

Two points on the $\bar{X}$ chart were outside of the established limits. There were both in the last period observed. The cause of both of these values can be traced to bad parts encountered by the operator.

The range chart showed four points out of control. Two of these correspond to those mentioned above. An analysis of the observation indicates that the other two were also the direct result of bad parts.

There are three runs of points of significant length on the $\bar{X}$ chart. No runs were found on the range chart.

The frequency distribution has a definite positive skewness. This was at least partially the result of the bad parts encountered. (See Table 15 and Figure 14.)

Operator 10.--This operator illustrated good stability of mean cycle times. Only one value was out of control and the observation sheet pointed out that this was caused by a bad part. No $\bar{\bar{X}}$ values were outside of the 3σ limits established.

Three values on the range chart showed a lack of control. All of these, however, were traced to a bad part in the assembly.

The frequency distribution for this operator was positively skewed. Most of the extremely high values were caused by bad parts.

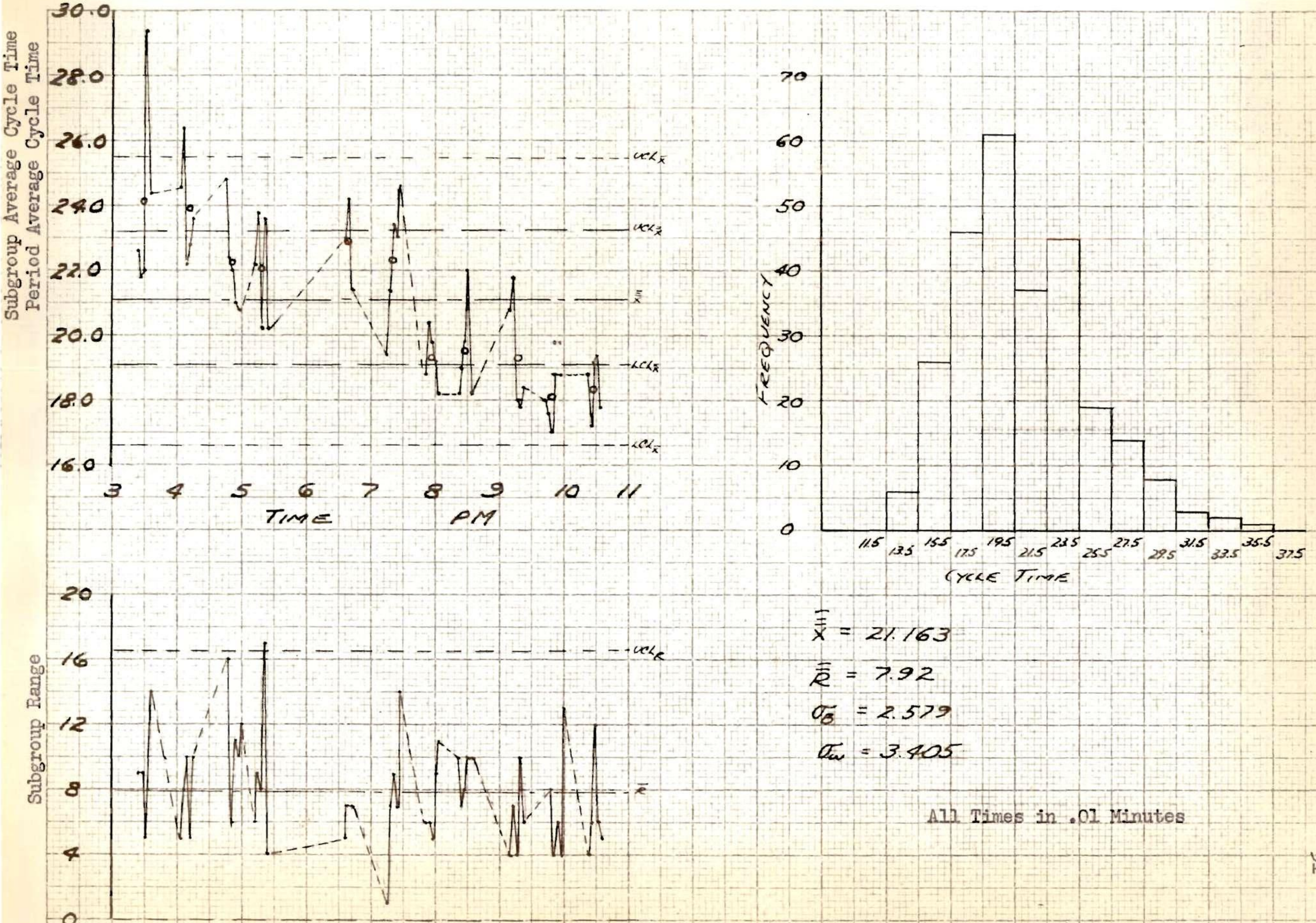


Figure 13. Control Charts and Performance Time Histogram--Operator 8

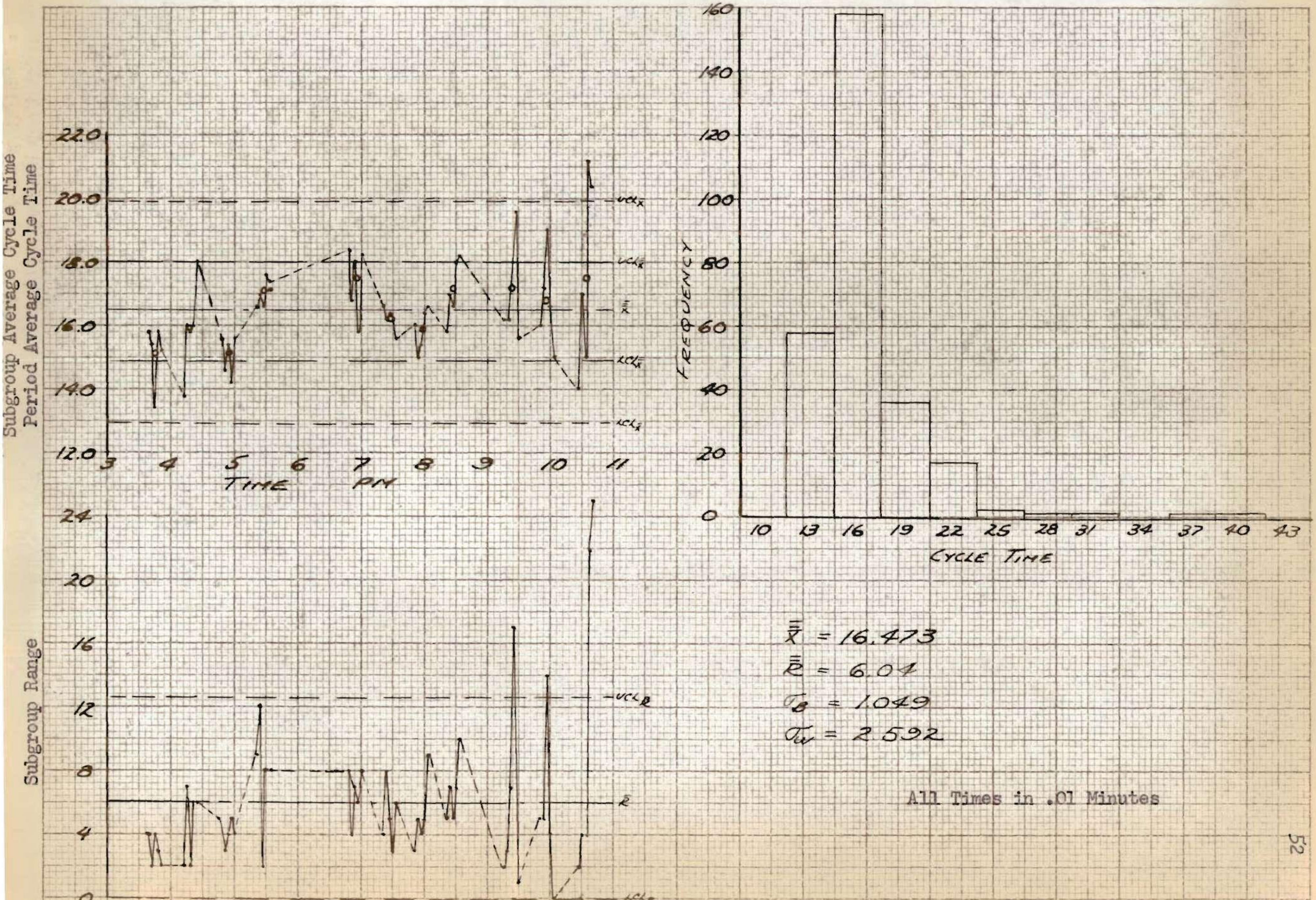


Figure 14. Control Charts and Performance Time Histogram--Operator 9

The σ_B of this operator is rather low in comparison with the σ_w, thus illustrating that the operator's performance times varies considerably within a period in relation to the variation between periods. (See Table 16 and Figure 15.)

Operator 11.--The control chart of subgroup means for this operator had three points outside of the 3σ limits. The first two can be attributed to bad parts while the third point was a result of talking to other operators (avoidable delay).

Two period mean values were also out of control. The first of these two points was caused by bad parts. The second is below the lower control limit, and no reason was recorded for this value.

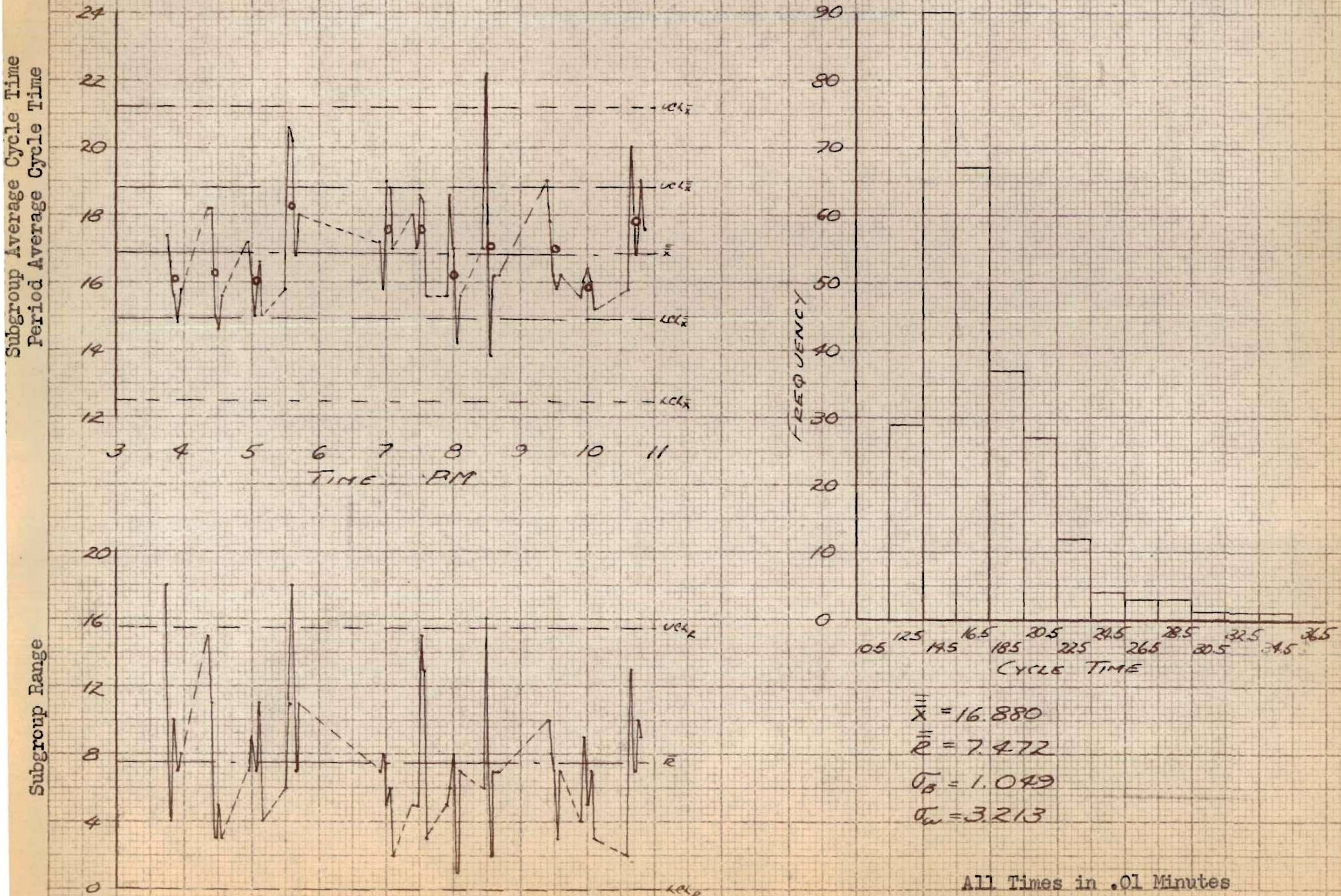
Only one point fell outside of the limits on the range chart. This point corresponded to the talking period mentioned above.

There are no other indications of non-stability except for the positive skewness shown by the frequency distribution.

The variance within periods is also very high compared to the variance between periods. (See Table 17 and Figure 16.)

Operator 12.--There was a general tendency for the operator's cycle time to decrease as the work day progressed. Two points on the $\bar{X}$ chart were outside of the 3σ control limits. These were both in the first period of the work day. The period average for this time was also beyond its control limit. Three values on the range chart lacked stability. A check of the observation sheet showed that two of these points were caused by bad parts.

The cycle time distribution showed a positive skewness. Both the variance within and between periods were above the average of all the



All Times in .01 Minutes

Figure 15. Control Charts and Performance Time Histogram--Operator 10

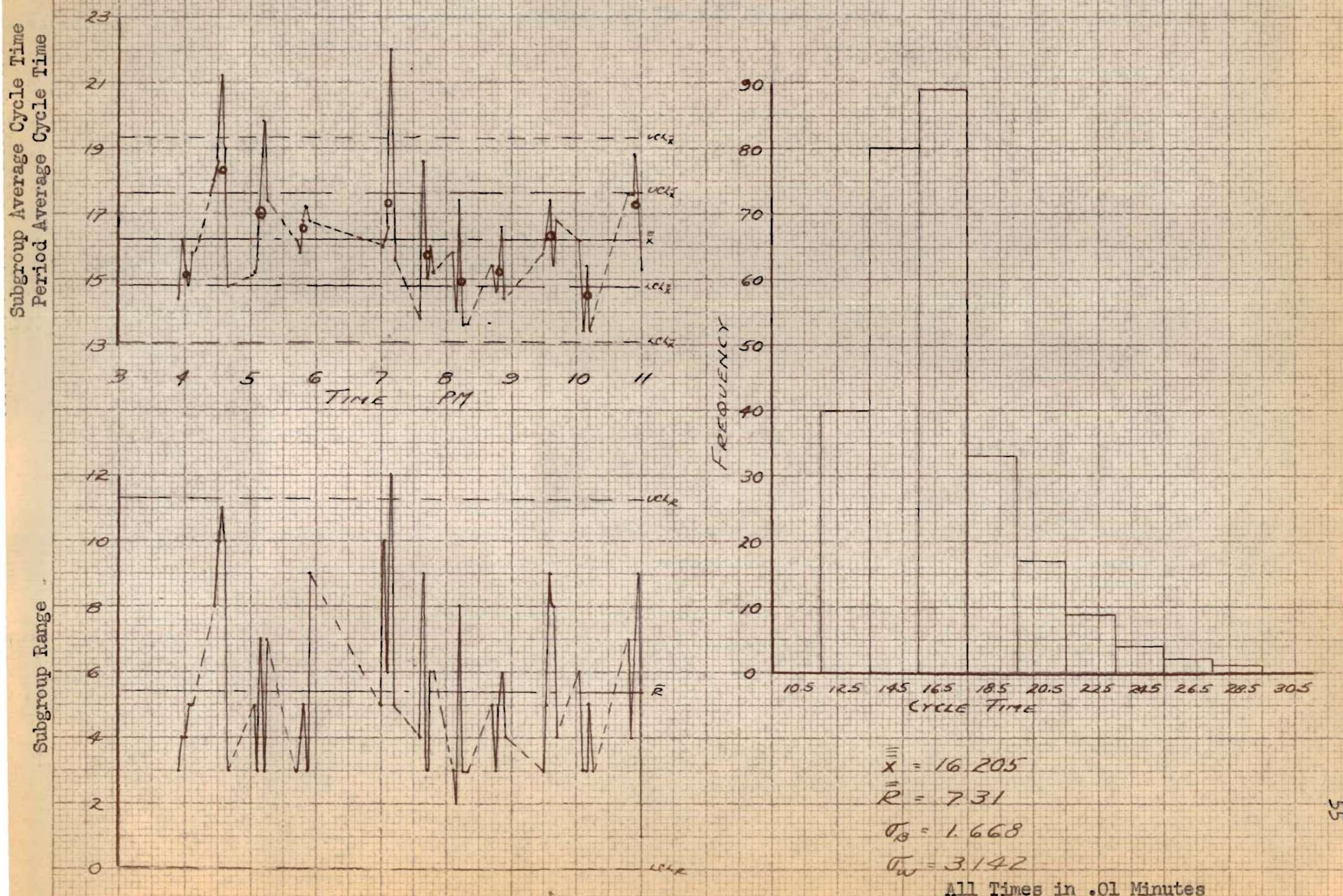


Figure 16. Control Charts and Performance Time Histogram--Operator 11

operators. The $\bar{X}$ chart had a significant run of 11 points below the centerline. (See Table 18 and Figure 17.)

Operator 13.--This operator showed very good stability. There were no points out of control on any of the control charts. The distribution exhibited no large skewness in either direction. There were no other indications of a lack of stability. (See Table 19 and Figure 18.)

Operator 14.--This operator also indicated good control. The only indication of a lack of stability was a run of 9 values below the centerline on the $\bar{X}$ chart. There were no points outside of the 3σ limits on any of the control charts.

The frequency distribution did not exhibit any large skewness in either direction. (See Table 20 and Figure 19.)

Operator 15.--The cycle times of this operator did not follow any formal trend. One value fell outside of the control limits on the $\bar{X}$ chart. An irregularity in the previous cycle may have caused the operator to lose her rhythm. The second point which is close to the control limit in this period was traced to a similar situation. Two avoidable delays were encountered in previous cycles.

The cycle time distribution exhibited a definite positive skewness. There is no apparent reason for this in the data presented. Also, there were no other indications of a lack of stability. (See Table 21 and Figure 20.)

Operator 16.--One point on the $\bar{X}$ chart fell outside of the 3σ limits. This point was traced to a bad part in the assembly. Two points on the range chart were very close to the upper control limit. This is an indication of a lack of control. All other points on all charts were in

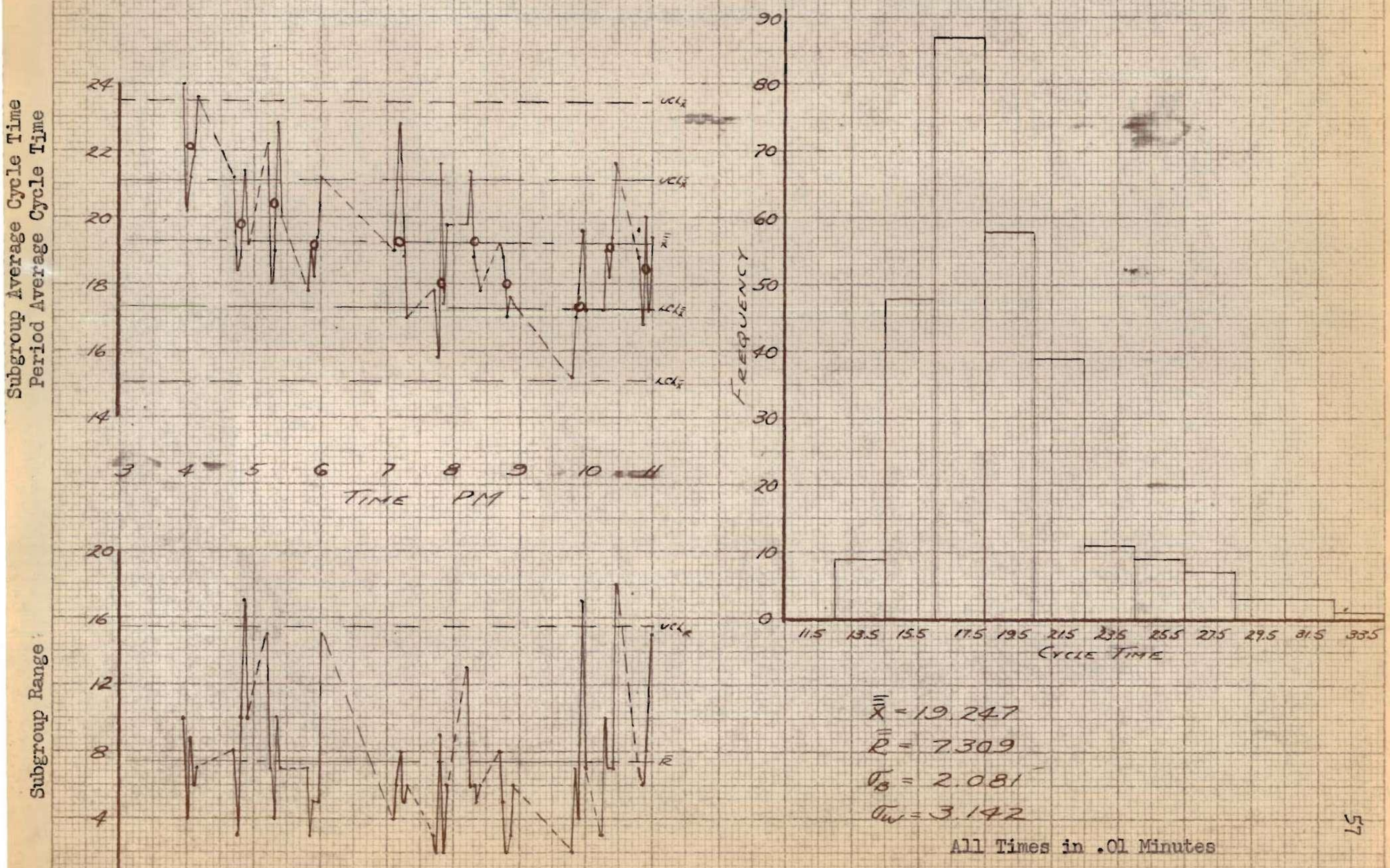


Figure 17. Control Charts and Performance Time Histogram--Operator 12

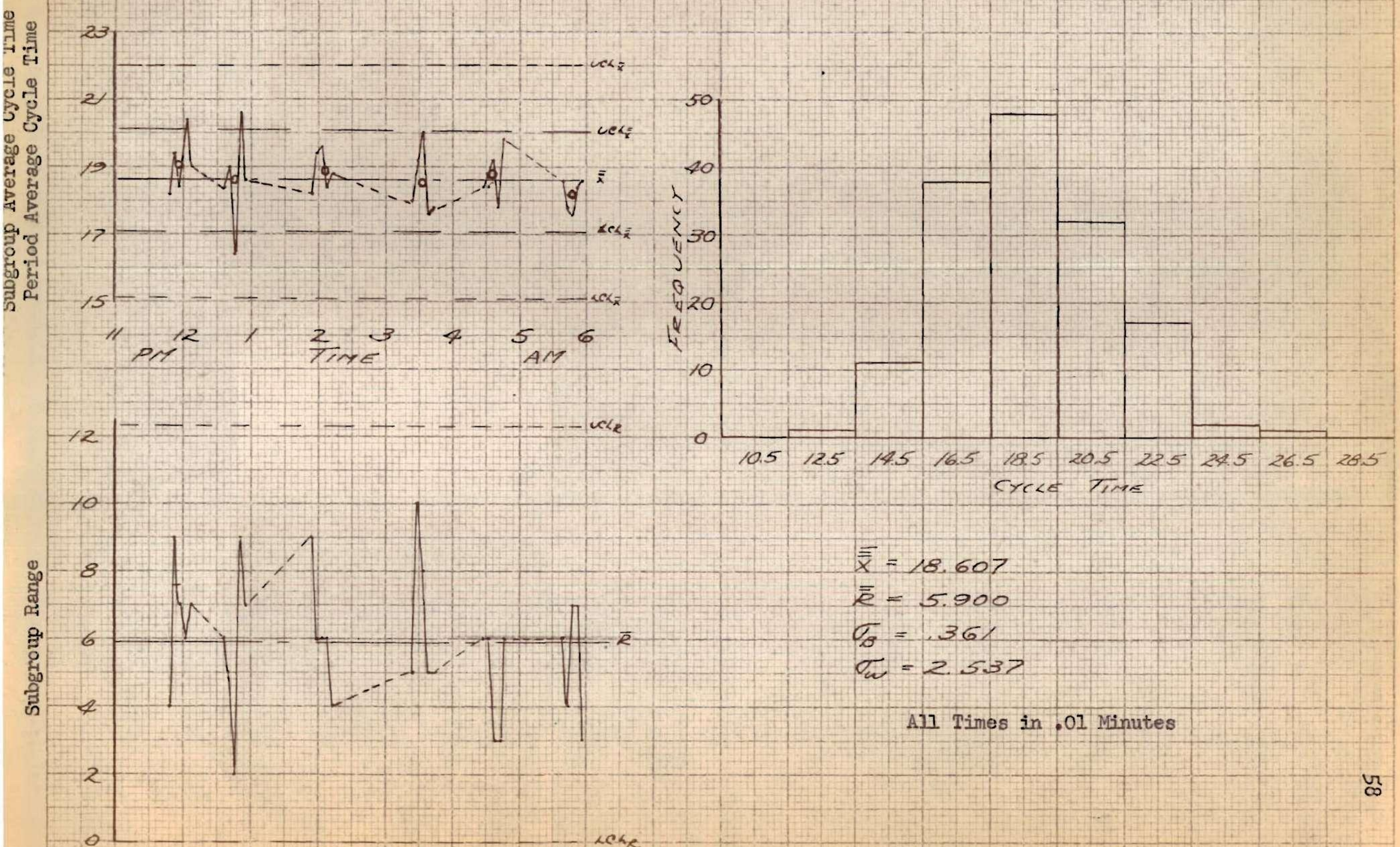


Figure 18. Control Charts and Performance Time Histogram--Operator 13

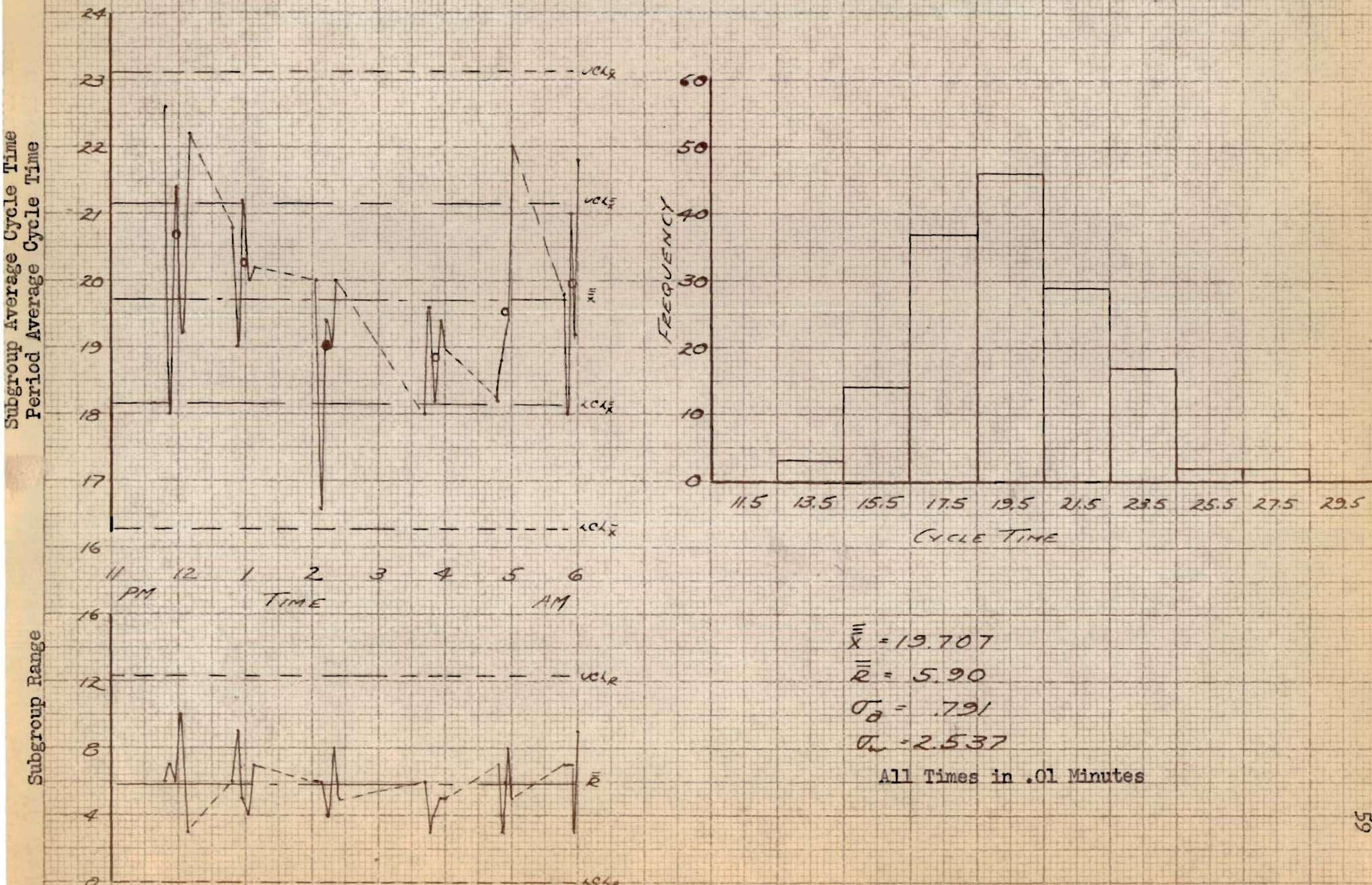


Figure 19. Control Charts and Performance Time Histogram--Operator 14

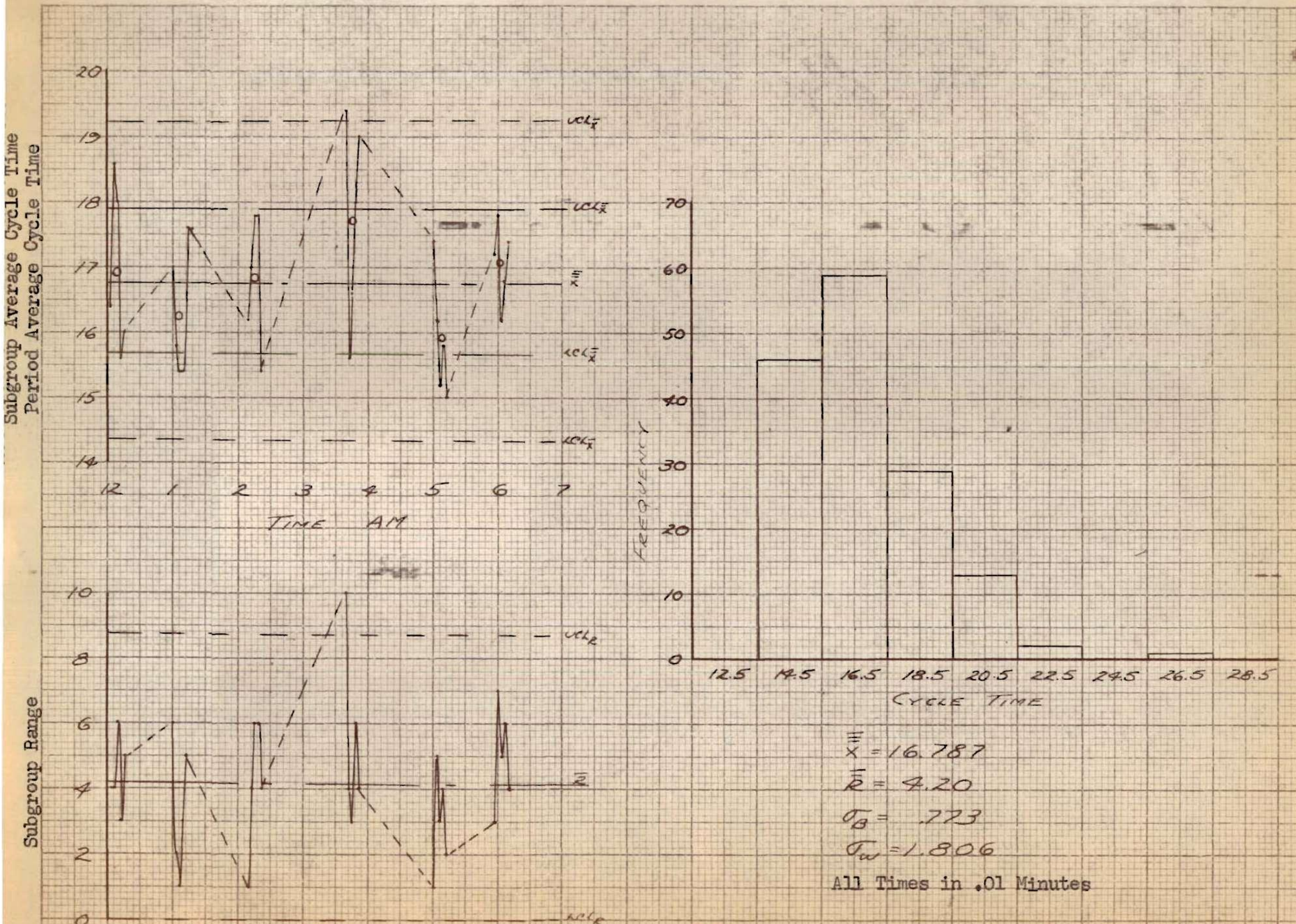


Figure 20. Control Charts and Performance Time Histogram--Operator 15

control.

The cycle time histogram shows a positive skewness. This was due to a few high cycle times. There were no other indications of a lack of stability. (See Table 22 and Figure 21.)

Operator 17.--All points were within the 3σ control limits on all charts. There was one run of none points below the centerline on the $\bar{X}$ chart. The cycle time histogram exhibited a definite positive skewness. Both the σ_w and σ_B values were less than the average value for all operators.

The performance time followed the trend that has been expressed by some, in that it increased during the first part of the shift, dropped slightly after the meal break and then increased toward the end of the work day. However, this trend cannot be taken as being the same for all operators. (See Table 23 and Figure 22.)

Operator 18.--One point on the $\bar{X}$ chart was outside of the control limits. This was traced to a bad part in the assembly. This same point also lacked control on the range chart. The second value of the range chart which lacked stability was found also to be caused by an irregularity in the cycle.

There is a positive skewness shown by the cycle time histogram. This is due, in part at least, to the times which resulted from these irregularities. There were no other indications of a lack of stability. Both the variance within and between periods were below the average of all the operators. (See Table 24 and Figure 23.)

Operator 19.--There was a general tendency for this operator's cycle time to increase as the work day progressed. No $\bar{X}$ or range values

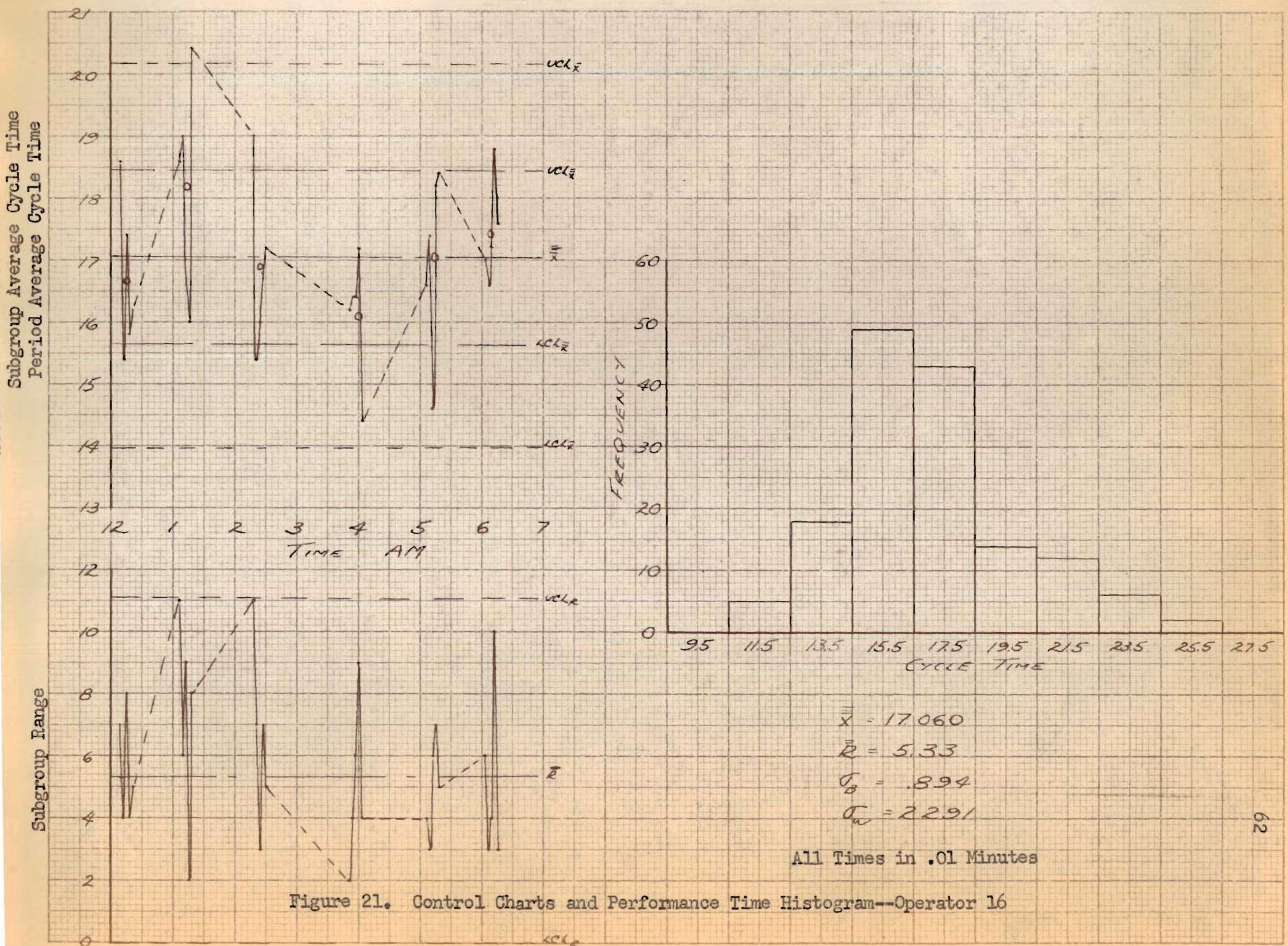


Figure 21. Control Charts and Performance Time Histogram--Operator 16

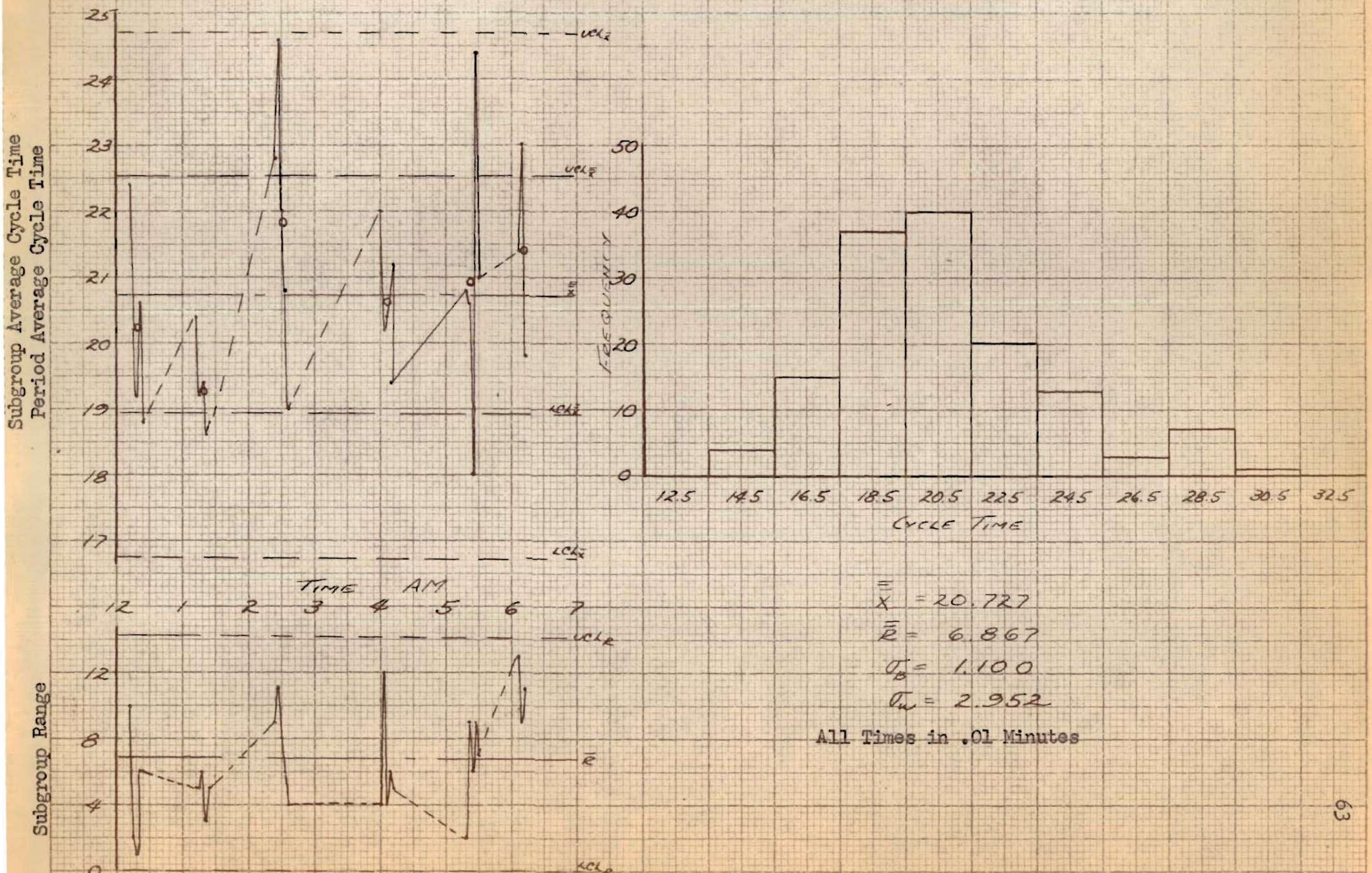


Figure 22. Control Charts and Performance Time Histogram--Operator 17

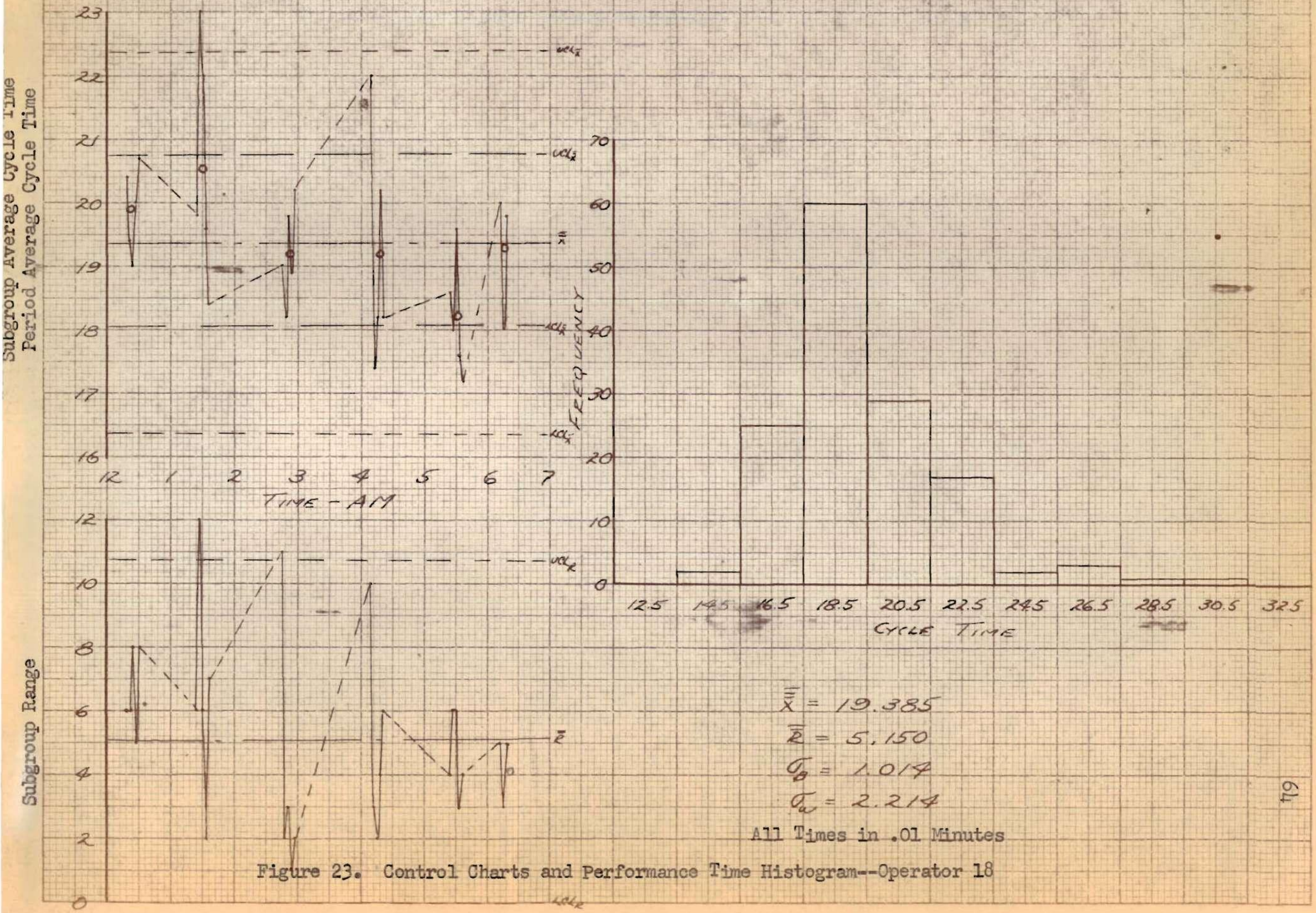


Figure 23. Control Charts and Performance Time Histogram--Operator 18

were out of control, but one period mean showed a lack of stability. This was due to the cycle time trend. This trend also caused a significant run of nine values above the centerline on the $\overline{X}$ chart.

The frequency distribution exhibited a positive skewness. The trend in cycle times probably caused this. There were no other indications of a lack of control. (See Table 25 and Figure 24.)

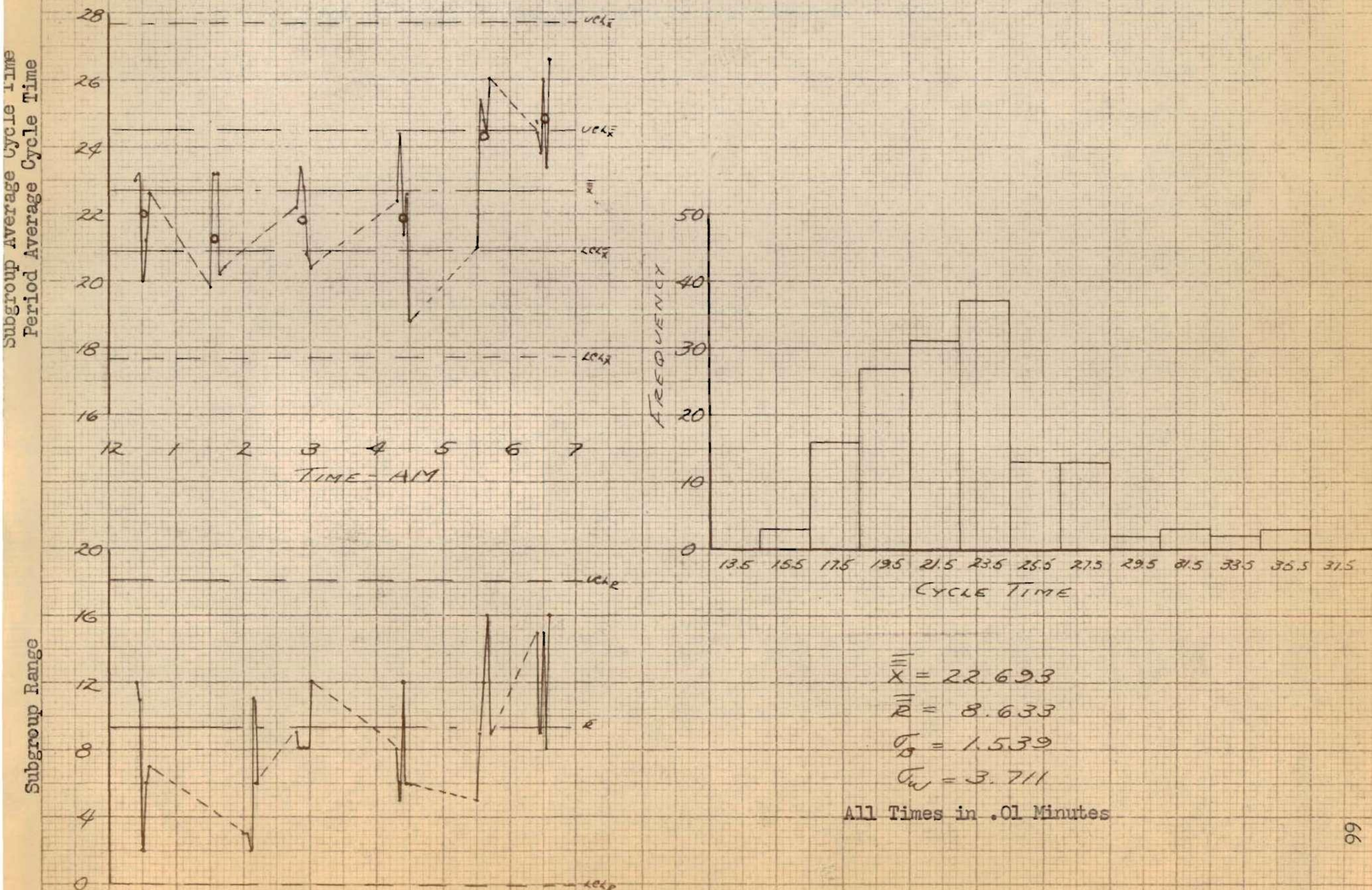


Figure 24. Control Charts and Performance Time Histogram--Operator 19

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

There are several limitations which should be attached to the results of this study. These are:

(1) The data covers only one operation.

(2) The operation is operator controlled except for the staking element.

(3) It was taken from within one plant.

(4) It covers only a limited number of operators (although this number was quite large).

(5) There was a separation of time between phases of the study.

(6) There was a modification of parts during the study.

In the light of the foregoing, the following conclusions can be drawn:

(1) The operators on this operation did not follow any theoretical work curve.

(2) The unadjusted performance times formed a positively skewed distribution. An unadjusted performance time is one to which no rating or performance factor has been attached. It is the observed cycle time. A positively skewed distribution is one that is shewed to the right or positive side of the mean value, with a shifting of the peak to the left.

(3) The performance times of these operators lacked stability in all except one case.

(4) The variation within a period was significantly greater than the variation between periods.

Each operator tended to follow a somewhat different pattern of performance throughout the day. From the data gathered, it is impossible to say whether these are stable work curves which the operator follows from day to day or whether an operator's performance does not follow any set pattern. In order to answer these questions, a similar study should be extended to cover all the days of the week and also a number of weeks. This would make it possible to answer two other questions. These are:

(1) Are work curves stable for a particular operator from day to day?

(2) What is the performance pattern for a particular operator during a week and between weeks?

The control charts for the individual operators exhibited one or more indications of a lack of stability in all except one case. Of the fifty-six values which lacked control, thirty-five could be traced to some irregularity in the cycle or subgroup, while the lack of control for twenty-four values could not be traced directly. Using accepted control chart procedures, after eliminating all values with assignable causes of variations, new limits are established and the data is plotted in respect to these modified limits. Thus, if this procedure is continued, after first determining just what are assignable causes, a stable distribution should result. This procedure was not followed because the data did not supply the necessary criteria for differentiating between assignable and chance causes and also because another project had been initiated using motion picture film to collect similar data.

The difference of the variation within and between periods was significant at the one per cent level using the F-ratio. The F-ratio was 2.008 with 156 degrees of freedom. This means difference in variations could not be due to chance alone. Only six operator's mean performance times were in control on the operator's mean performance time control chart. This shows that thirteen of the operator's mean performance times varied to an extent where it could happen due to chance only three times in a thousand. This means that the operators do not form a homogenous group in respect to their cycle times. The reason for this is obviously due to an assignable cause of variation in the form of experience, motivation or ability. (See Figure 4 for Control Chart.)

All except three performance time distributions had a positive skewness that was definitely visible. It might be expected that a skewed distribution will result from unadjusted performance times since there seems to be a physiological speed limit on which chance factors cannot operate in the lower direction, while at the other end, any chance factor can operate to add to performance time, thus, if we can eliminate all of these factors (such as bad parts, inspections, restakes, etc.) we may expect a theoretical distribution such as the normal distribution. But, elimination of what we now call assignable causes may still produce a skewed distribution because we have not determined what should be labeled assignable causes of variation.

In order to learn more about these causes of variation in performance times, a similar study should be conducted using high speed motion pictures, preferably taken at 2000 frames per minute or at a higher rate of speed. This would facilitate the determining and classifying of the causes of variations. Such a study, in order to obtain sufficient data, would require in the neighborhood of twenty thousand feet of motion picture film. The cost of the film alone would be nearly one thousand dollars.

In addition this film would give insight into the time distribution of many of the catagories of the predetermined time systems. This could serve to substantiate or refute the basis of the data of these systems which is now being used quite extensively in industry.

APPENDIX

Table 7. Operator 1's Period and Grand Cycle Time Mean and Range

8:45 to 8:50 AM

Subgroup	$\bar{X}$	R
1	18.8	4
2	17.6	3
3	18.6	6
4	21.6	5
5	23.4	7

$\bar{\bar{X}} = 20.0$

$\bar{R} = 5.0$

$\sigma_w = 2.149$ $\sigma_{\bar{x}} = \frac{2.149}{\sqrt{25}}$

10:00 to 10:05 AM

Subgroup	$\bar{X}$	R
1	17.4	4
2	17.6	2.5
3	18.8	8
4	18.4	6
5.	19.2	6

$\bar{\bar{X}} = 18.28$

$\bar{R} = 5.3$

$\sigma_w = 2.278$

10:55 to 11:05 AM

Subgroup	$\bar{X}$	R
1	19.6	4.5
2	19.3	4.5
3	19.4	4
4	20.2	7
5	20.0	3

$\bar{\bar{X}} = 19.70$

$\bar{R} = 4.6$

$\sigma_w = 1.977$

Table 7. Operator 1's Period and Grand Cycle Time Mean and Range (continued)

12:55 to 1:05 PM

Subgroup	$\bar{X}$	R
1	19.2	5
2	21.4	6
3	22.8	8
4	20.8	6
5	21.6	9

$\bar{\bar{X}} = 21.16$

$\bar{R} = 6.8$

$\sigma_w = 2.948$

2:00 to 2:10 PM

Subgroup	$\bar{X}$	R
1	18.2	4
2	19.2	2
3	17.5	4
4	17.7	2.5
5	18.8	5

$\bar{\bar{X}} = 18.28$

$\bar{R} = 3.5$

$\sigma_w = 1.504$

$\bar{\bar{\bar{X}}} = 19.484$

$\bar{\bar{R}} = 5.040$

$R_{\bar{\bar{X}}} = 2.88$

$$\sigma_B = \frac{R_{\bar{\bar{X}}}}{d_2} = 1.238$$

$$\sigma_w = \frac{\bar{\bar{R}}}{d_2} = 2.166$$

Table 7. Operator 1's Period and Grand Cycle Time Mean and Range (continued)

$N = 125 \qquad R = 13$

$$\text{Class Interval} = \frac{13}{1 + 3.322\ (\log_{10} 125)} = 2$$

Control limits for subgroup averages

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2 \bar{\bar{R}} = 19.484 + .58\ (5.04) = 22.407$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2 \bar{\bar{R}} = 19.484 - .58\ (5.04) = 16.561$$

Control limits for period averages

$$UCL_{\bar{\bar{X}}} = \bar{\bar{X}} + \frac{A_2 \bar{\bar{R}}}{\sqrt{n_s}} = 19.484 + \frac{.58\ (.5040)}{\sqrt{5}} = 20.784$$

$$LCL_{\bar{\bar{X}}} = \bar{\bar{X}} - \frac{A_2 \bar{\bar{R}}}{\sqrt{n_s}} = 19.484 - \frac{.58(.5040)}{\sqrt{5}} = 18.185$$

Control limits for subgroup ranges

$$UCL_R = D_4 \bar{\bar{R}} = 2.09\ (5.040) = 10.533$$

$$LCL_R = D_3 \bar{\bar{R}} = 0\ (5.040) = 0$$

Table 8. Operator 2's Period and Grand Cycle Time Mean and Range

8:50 to 9:00 AM

Subgroup	$\bar{X}$	R
1	15.2	2
2	17.0	5
3	15.0	3
4	17.6	9
5	17.0	3

$\bar{\bar{X}} = 16.36$

$\bar{R} = 4.4$

$\sigma_w = 1.891$

10:05 to 10:15 AM

Subgroup	$\bar{X}$	R
1	16.2	5
2	16.1	4.5
3	17.0	6.5
4	16.1	4
5	15.8	2.5

$\bar{\bar{X}} = 16.24$

$\bar{R} = 4.5$

$\sigma_w = 1.934$

11:05 to 11:10 AM

Subgroup	$\bar{X}$	R
1	17.8	10
2	15.2	3.5
3	16.2	4
4	15.1	2.5
5	16.5	11.5

$\bar{\bar{X}} = 16.16$

$\bar{R} = 6.3$

$\sigma_w = 2.708$

Table 8. Operator 2's Period and Grand Cycle Time Mean and Range (continued)

1:05 to 1:10 PM

Subgroup	$\bar{X}$	R
1	15.0	4
2	14.6	2
3	15.8	2.5
4	17.0	5
5	16.4	4

$\bar{\bar{X}} = 15.76$

$\bar{R} = 3.5$

$\sigma_{\omega} = 1.504$

2:10 to 2:20

Subgroup	$\bar{X}$	R
1	17.8	4
2	15.2	1.5
3	16.2	2
4	16.8	4.5
5	18.0	6

$\bar{\bar{X}} = 16.80$

$\bar{R} = 3.6$

$\sigma_{\omega} = 1.547$

$\bar{\bar{\bar{X}}} = 16.264$

$\bar{\bar{R}} = 4.46$

$R_{\bar{\bar{X}}} = 1.04$

$\sigma_{\beta} = .447$

$\sigma_{\omega} = 1.917$

Table 8. Operator 2's Period and Grand Cycle Time Mean and Range (continued)

N = 125 R = 14

Class Interval = 2

Control limits for subgroup averages

$UCL_{\bar{X}} = 18.750$

$LCL_{\bar{X}} = 13.678$

Control limits for period averages

$UCL_{\bar{\bar{X}}} = 17.414$

$LCL_{\bar{\bar{X}}} = 15.114$

Control limits for subgroup ranges

$UCL_R = 9.321$

$LCL_R = 0$

See Table 7. for Sample Calculations.

Table 9. Operator 3's Period and Grand Cycle Time Mean and Range

9:00 to 9:10 AM

Subgroup	$\bar{X}$	R
1	16.0	3
2	15.8	4
3	16.9	4
4	15.3	1.5
5	19.2	7

$\bar{\bar{X}} = 16.64$

$\bar{R} = 3.9$

$\sigma_w = 1.676$

10:15 to 10:20 AM

Subgroup	$\bar{X}$	R
1	17.4	3
2	16.8	2
3	16.9	2.5
4	17.0	4
5	17.4	4

$\bar{\bar{X}} = 17.10$

$\bar{R} = 3.1$

$\sigma_w = 1.332$

11:10 to 11:15 AM

Subgroup	$\bar{X}$	R
1	16.8	2
2	15.6	3
3	16.4	5
4	25.6	25.5
5	16.0	2

$\bar{\bar{X}} = 18.08$

$\bar{R} = 7.5$

$\sigma_w = 3.224$

Table 9. Operator 3's Period and Grand Cycle Time Mean and Range (continued)

1:10 to 1:15 PM

Subgroup	$\bar{X}$	R
1	21.2	10
2	24.0	13
3	19.0	4
4	20.8	10
5	22.4	10

$\bar{\bar{X}} = 21.48$

$\bar{R} = 9.4$

$\sigma_w = 4.041$

2:20 to 2:25 PM

Subgroup	$\bar{X}$	R
1	18.6	7
2	22.2	16
3	19.0	8
4	20.4	4
5	22.8	13

$\bar{\bar{X}} = 20.60$

$\bar{R} = 9.6$

$\sigma_w = 4.127$

$\bar{\bar{\bar{X}}} = 18.780$

$\bar{\bar{R}} = 7.00$

$R_{\bar{\bar{X}}} = 4.84$

$\sigma_B = 2.080$

$\sigma_w = 3.009$

Table 9. Operator 3's Period and Grand Cycle Time Mean and Range

N = 125 R = 25

Class Interval = 3

Control limits for subgroup averages

$UCL_{\bar{X}} = 22.840$

$LCL_{\bar{X}} = 14.720$

Control limits for period averages

$UCL_{\bar{\bar{X}}} = 20.585$

$LCL_{\bar{\bar{X}}} = 16.975$

Control limits for subgroup ranges

$UCL_R = 14.630$

$LCL_R = 0$

See Table 7. for Sample Calculations.

Table 10. Operator 4's Period and Grand Cycle Time Mean and Range

9:15 to 9:25 AM

Subgroup	$\bar{X}$	R
1	24.6	8
2	23.5	2.5
3	24.1	7
4	27.2	13
5	26.4	6

$\bar{\bar{X}} = 25.16$

$\bar{R} = 7.3$

$\sigma_w = 3.138$

10:20 to 10:30 AM

Subgroup	$\bar{X}$	R
1	27.4	18
2	28.2	9
3	28.2	16
4	31.8	13
5	24.4	9

$\bar{\bar{X}} = 28.00$

$\bar{R} = 13.00$

$\sigma_w = 5.588$

11:15 to 11:23 AM

Subgroup	$\bar{X}$	R
1	28.4	10
2	27.4	6
3	29.4	13
4	26.0	6
5	28.0	10

$\bar{\bar{X}} = 27.84$

$\bar{R} = 9.0$

$\sigma_w = 3.869$

Table 10. Operator 4's Period and Grand Cycle Time Mean and Range (continued)

1:15 to 1:25 PM

Subgroup	$\bar{X}$	R
1	23.0	19
2	26.2	18
3	25.8	7
4	27.6	7
5	29.2	12

$\bar{\bar{X}} = 26.36$

$\bar{R} = 12.6$

$\sigma_\omega = 5.417$

2:25 to 2:35 PM

Subgroup	$\bar{X}$	R
1	28.6	3
2	23.4	5
3	27.8	3
4	23.8	3
5	23.8	5

$\bar{\bar{X}} = 25.48$

$\bar{R} = 3.8$

$\sigma_\omega = 1.633$

$\bar{\bar{\bar{X}}} = 26.568$

$\bar{\bar{R}} = 9.14$

$R_{\bar{\bar{X}}} = 2.84$

$\sigma_\beta = 1.220$

$\sigma_\omega = 3.929$

Table 10. Operator 4's Period and Grand Cycle Time Mean and Range (continued)

N = 125 R = 25

Class Interval = 3

Control limits for subgroup averages

$UCL_{\bar{X}} = 31.869$

$LCL_{\bar{X}} = 21.267$

Control limits for period averages

$UCL_{\bar{\bar{X}}} = 28.938$

$LCL_{\bar{\bar{X}}} = 24.198$

Control limits for subgroup ranges

$UCL_R = 19.102$

$LCL_R = 0$

See Table 7 for Sample Calculations.

Table 11. Operator 5's Period and Grand Cycle Time Mean and Range

9:25 to 9:30 and 9:35 to 9:40 AM

Subgroup	$\bar{X}$	R
1	16.6	5
2	19.8	8
3	18.2	3
4	19.2	3
5	16.7	3.5

$\bar{\bar{X}} = 18.10$

$\bar{R} = 4.5$

$\sigma_w = 1.934$

10:30 to 10:45 AM

Subgroup	$\bar{X}$	R
1	19.2	8
2	19.4	6
3	16.1	3
4	17.3	4.5
5	19.0	10

$\bar{\bar{X}} = 18.2$

$\bar{R} = 6.3$

$\sigma_w = 2.708$

11:23 to 11:28 AM

Subgroup	$\bar{X}$	R
1	19.6	5
2	16.8	7
3	19.2	2.5
4	16.4	3
5	19.6	1

$\bar{\bar{X}} = 18.32$

$\bar{R} = 3.7$

$\sigma_w = 1.594$

Table 11. Operator 5's Period and Grand Cycle Time Mean and Range (continued)

1:25 to 1:35 PM

Subgroup	$\bar{X}$	R
1	19.2	8
2	16.0	4
3	15.4	3
4	17.0	6
5	16.9	2.5

$\bar{\bar{X}} = 16.90$

$\bar{R} = 4.7$

$\sigma_w = 2.020$

2:40 to 2:45 PM

Subgroup	$\bar{X}$	R
1	19.4	9
2	20.2	9
3	17.8	4
4	18.2	3
5	21.2	2

$\bar{\bar{X}} = 19.36$

$\bar{R} = 7.4$

$\sigma_w = 3.181$

$\bar{\bar{\bar{X}}} = 18.176$

$\bar{\bar{R}} = 5.32$

$R_{\bar{\bar{X}}} = 2.46$

$\sigma_\beta = 1.057$

$\sigma_w = 2.287$

Table 11. Operator 5's Period and Grand Cycle Time Mean and Range (continued)

N = 125 R = 16

Class Interval = 2

Control limits for subgroup averages

$UCL_{\bar{X}} = 21.262$

$LCL_{\bar{X}} = 15.090$

Control limits for period averages

$UCL_{\bar{\bar{X}}} = 19.548$

$LCL_{\bar{\bar{X}}} = 16.804$

Control limits for subgroup ranges

$UCL_R = 11.119$

$LCL_R = 0$

See Table 7. for Sample Calculations.

Table 12. Operator 6's Period and Grand Cycle Time Mean and Range

9:45 to 9:55 AM

Subgroup	$\overline{X}$	R
1	18.0	13.5
2	21.6	15
3	20.4	25
4	16.4	6
5	19.6	18

$\overline{\overline{X}} = 19.2$

$\overline{R} = 15.5$

$\sigma_w = 6.663$

10:45 to 10:50 AM

Subgroup	$\overline{X}$	R
1	23.8	13
2	30.2	11
3	25.8	22
4	22.6	11
5	22.4	15

$\overline{\overline{X}} = 24.96$

$\overline{R} = 14.4$

$\sigma_w = 6.190$

12:45 to 12:50 PM

Subgroup	$\overline{X}$	R
1	19.0	5
2	17.6	6
3	18.0	3
4	17.6	4
5	21.0	15

$\overline{\overline{X}} = 18.64$

$\overline{R} = 6.6$

$\sigma_w = 2.862$

Table 12. Operator 6's Period and Grand Cycle Time Mean and Range (continued)

1:35 to 1:45 PM

Subgroup	$\bar{X}$	R
1	21.4	18
2	19.8	9
3	20.8	13
4	28.2	20
5	23.4	15

$\bar{\bar{X}} = 22.72$

$\bar{R} = 15.0$

$\sigma_w = 6.465$

2:45 to 2:55 PM

Subgroup	$\bar{X}$	R
1	20.8	9
2	20.2	12
3	17.4	5
4	20.0	6
5	27.0	25

$\bar{\bar{X}} = 21.08$

$\bar{R} = 11.4$

$\sigma_w = 4.901$

$\bar{\bar{\bar{X}}} = 21.320$

$\bar{\bar{R}} = 12.58$

$R_{\bar{\bar{X}}} = 6.32$

$\sigma_B = 2.717$

$\sigma_w = 5.408$

Table 12. Operator 6's Period and Grand Cycle Time Mean and Range (continued)

$N = 125$ $\bar{R} = 28$

Class Interval = 3

Control limits for subgroup averages

$UCL_{\bar{X}} = 28.616$

$LCL_{\bar{X}} = 14.024$

Control limits for period averages

$UCL_{\bar{\bar{X}}} = 24.565$

$LCL_{\bar{\bar{X}}} = 18.075$

Control limits for subgroup ranges

$UCL_{R} = 26.292$

$LCL_{R} = 0$

See Table 7. for Sample Calculations.

Table 13. Operator 7's Period and Grand Cycle Time Mean and Range

9:55 to 10:00 AM

Subgroup	$\bar{X}$	R
1	17.0	10
2	15.6	2
3	15.6	3.5

$\bar{\bar{X}} = 16.04$

$\bar{R} = 4.1$

$\sigma_w = 1.762$

10:40 to 10:55 AM

Subgroup	$\bar{X}$	R
1	16.4	4
2	15.6	2
3	15.4	4
4	17.4	2
5	17.6	6

$\bar{\bar{X}} = 16.48$

$\bar{R} = 3.6$

$\sigma_w = 1.547$

12:50 to 12:55 PM

Subgroup	$\bar{X}$	R
1	18.2	14
2	19.2	15
3	20.2	11
4	16.8	4
5	20.0	21.5

$\bar{\bar{X}} = 18.88$

$\bar{R} = 13.1$

$\sigma_w = 5.631$

Table 13. Operator 7's Period and Grand Cycle Time Mean and Range (continued)

1:50 to 2:00 PM

Subgroup	$\bar{X}$	R
1	16.6	1
2	17.4	3
3	15.6	6
4	15.4	3.5
5	16.7	6.5

$\bar{\bar{X}} = 16.34$

$\bar{R} = 4.0$

$\sigma_w = 1.719$

2:55 to 3:00 PM

Subgroup	$\bar{X}$	R
1	16.6	2
2	16.6	4
3	16.4	4.5
4	14.8	2
5	15.6	6

$\bar{\bar{X}} = 16.00$

$\bar{R} = 3.7$

$\sigma_w = 1.590$

$\bar{\bar{\bar{X}}} = 16.748$

$\bar{\bar{R}} = 5.7$

$R_{\bar{\bar{X}}} = 2.88$

$\sigma_B = 1.238$

$\sigma_w = 2.450$

Table 13. Operator 7's Period and Grand Cycle Time Mean and Range (continued)

N = 125 R = 23

Class Interval = 3

Control limits for subgroup averages

$UCL_{\bar{X}} = 20.054$

$LCL_{\bar{X}} = 13.442$

Control limits for period averages

$UCL_{\bar{\bar{X}}} = 16.748$

$LCL_{\bar{\bar{X}}} = 15.278$

Control limits for subgroup ranges

$UCL_R = 11.913$

$LCL_R = 0$

See Table 7. for Sample Calculations.

Table 14. Operator 8's Period and Grand Cycle Time Mean and Range

3:35 to 3:44 PM

Subgroup	$\bar{X}$	R
1	22.6	9
2	21.8	9
3	22.0	5
4	29.4	8
5	24.4	14

$\bar{\bar{X}} = 24.04$

$\bar{R} = 9.0$

$\sigma_w = 3.869$

4:10 to 4:18 PM

Subgroup	$\bar{X}$	R
1	24.6	5
2	26.4	8
3	22.2	10
4	22.8	5
5	23.6	10

$\bar{\bar{X}} = 23.92$

$\bar{R} = 7.6$

$\sigma_w = 3.267$

4:50 to 4:57 PM

Subgroup	$\bar{X}$	R
1	24.8	16
2	22.4	6
3	22.0	11
4	21.0	10
5	20.8	12

$\bar{\bar{X}} = 22.20$

$\bar{R} = 11.0$

$\sigma_w = 4.729$

Table 14. Operator 8's Period and Grand Cycle Time Mean and Range (continued)

5:20 to 5:28 PM

Subgroup	$\bar{X}$	R
1	22.2	6
2	23.8	9
3	20.2	8
4	23.6	17
5	20.2	4

$\bar{\bar{X}} = 22.00$

$\bar{R} = 8.8$

$\sigma_w = 3.783$

6:42 to 6:51 PM

Subgroup	$\bar{X}$	R
1	23.0	5
2	24.2	7
3	21.4	7
4	Delays	
5		

$\bar{\bar{X}} = 22.87$

$\bar{R} = 6.3$

$\sigma_w = 2.708$

7:18 to 7:25 PM

Subgroup	$\bar{X}$	R
1	19.4	1
2	21.4	7
3	23.4	9
4	23.0	7
5	24.6	14

$\bar{\bar{X}} = 22.36$

$\bar{R} = 7.6$

$\sigma_w = 3.267$

Table 14. Operator 8's Period and Grand Cycle Time Mean and Range (continued)

7:52 to 7:58 PM

Subgroup	$\bar{X}$	R
1	18.8	6
2	20.4	6
3	19.8	5
4	19.2	9
5	18.2	11

$\bar{\bar{X}} = 19.28$

$\bar{R} = 8.0$

$\sigma_w = 3.439$

8:23 to 8:28 PM

Subgroup	$\bar{X}$	R
1	18.2	10
2	19.0	7
3	19.8	8
4	22.2	10
5	18.2	10

$\bar{\bar{X}} = 19.48$

$\bar{R} = 9.0$

$\sigma_w = 3.869$

9:15 to 9:22 PM

Subgroup	$\bar{X}$	R
1	20.6	4
2	21.8	7
3	18.0	4
4	17.8	10
5	18.4	6

$\bar{\bar{X}} = 19.32$

$\bar{R} = 6.2$

$\sigma_w = 2.666$

Table 14. Operator 8's Period and Grand Cycle Time Mean and Range (continued)

9:47 to 9:52 PM

Subgroup	$\bar{X}$	R
1	18.0	8
2	17.6	4
3	17.0	6
4	18.8	4
5	18.8	13

$\bar{\bar{X}} = 18.04$

$\bar{R} = 7.0$

$\sigma_w = 3.017$

10:25 to 10:32 PM

Subgroup	$\bar{X}$	R
1	18.0	4
2	17.0	6
3	19.2	12
4	19.4	6
5	17.8	5

$\bar{\bar{X}} = 18.28$

$\bar{R} = 6.6$

$\sigma_w = 2.837$

$\bar{\bar{\bar{X}}} = 21.163$

$\bar{\bar{R}} = 7.92$

$R_{\bar{\bar{X}}} = 6.00$

$\sigma_B = 2.579$

$\sigma_w = 3.405$

Table 14. Operator 8's Period and Grand Cycle Time Mean and Range (continued)

$N = 275 \qquad R = 22$

Class Interval = 2

Control limits for subgroup averages

$UCL_{\bar{X}} = 25.757$

$LCL_{\bar{X}} = 16.569$

Control limits for period averages

$UCL_{\bar{\bar{X}}} = 23.218$

$LCL_{\bar{\bar{X}}} = 19.108$

Control limits for subgroup ranges

$UCL_R = 16.55$

$LCL_R = 0$

See Table 7. for Sample Calculations

Table 15. Operator 9's Period and Grand Cycle Time Mean and Range

3:44 to 3:49 PM

Subgroup	$\bar{X}$	R
1	15.8	4
2	15.4	2
3	13.4	4
4	15.8	3
5	15.2	2

$\bar{\bar{X}} = 15.12$

$\bar{R} = 3.0$

$\sigma_w = 1.290$

4:18 to 4:25 PM

Subgroup	$\bar{X}$	R
1	13.8	2
2	16.0	7
3	15.8	2
4	16.0	6
5	18.0	6

$\bar{\bar{X}} = 15.92$

$\bar{R} = 4.6$

$\sigma_w = 1.938$

4:57 to 5:02 PM

Subgroup	$\bar{X}$	R
1	15.6	5
2	14.6	3
3	15.4	4
4	14.2	5
5	15.6	4

$\bar{\bar{X}} = 15.08$

$\bar{R} = 4.2$

$\sigma_w = 1.806$

Table 15. Operator 9's Period and Grand Cycle Time Mean and Range (continued)

5:29 to 5:35 PM

Subgroup	$\bar{X}$	R
1	16.6	9
2	17.0	12
3	16.6	2
4	17.6	8
5	17.4	8

$\bar{\bar{X}} = 17.04$

$\bar{R} = 7.8$

$\sigma_w = 3.353$

6:52 to 6:56 PM

Subgroup	$\bar{X}$	R
1	18.4	8
2	16.8	4
3	18.0	7
4	15.8	
5	18.2	8

$\bar{\bar{X}} = 17.44$

$\bar{R} = 6.6$

$\sigma_w = 2.837$

7:25 to 7:30 PM

Subgroup	$\bar{X}$	R
1	16.6	4
2	16.2	8
3	16.4	5
4	16.2	3
5	15.6	6

$\bar{\bar{X}} = 16.20$

$\bar{R} = 5.2$

$\sigma_w = 2.236$

Table 15. Operator 9's Period and Grand Cycle Time Mean and Range (continued)

7:59 to 8:03 PM

Subgroup	$\bar{X}$	R
1	16.0	3
2	15.0	5
3	15.4	4
4	16.4	5
5	16.6	9

$\bar{\bar{X}} = 15.88$

$\bar{R} = 5.2$

$\sigma_w = 2.236$

8:29 to 8:35 PM

Subgroup	$\bar{X}$	R
1	15.8	5
2	17.0	7
3	16.6	5
4	18.0	7
5	18.2	10

$\bar{\bar{X}} = 17.12$

$\bar{R} = 6.8$

$\sigma_w = 2.923$

9:22 to 9:28 PM

Subgroup	$\bar{X}$	R
1	16.2	2
2	16.2	3
3	18.0	7
4	19.6	17
5	15.6	1

$\bar{\bar{X}} = 17.12$

$\bar{R} = 6.0$

$\sigma_w = 2.580$

Table 15. Operator 9's Period and Grand Cycle Time Mean and Range (continued)

9:52 to 9:58 PM

Subgroup	$\overline{X}$	R
1	16.0	5
2	17.2	5
3	19.0	14
4	16.6	4
5	15.0	0

$\overline{\overline{X}} = 16.76$

$\overline{R} = 5.6$

$\sigma_w = 2.408$

10:33 to 10:39 PM

Subgroup	$\overline{X}$	R
1	14.0	2
2	17.0	4
3	15.0	4
4	21.2	22
5	20.4	25

$\overline{\overline{X}} = 17.52$

$\overline{R} = 11.4$

$\sigma_w = 4.901$

$\overline{\overline{\overline{X}}} = 16.473$

$\overline{\overline{R}} = 6.04$

$R_{\overline{\overline{X}}} = 2.44$

$\sigma_B = 1.049$

$\sigma_w = 2.592$

Table 15. Operator 9's Period and Grand Cycle Time Mean and Range (continued)

$N = 275$ $\bar{R} = 27$

Class Interval = 3

Control limits for subgroup averages

$UCL_{\bar{X}} = 19.976$

$LCL_{\bar{X}} = 12.970$

Control limits for period averages

$UCL_{\bar{\bar{X}}} = 18.040$

$LCL_{\bar{\bar{X}}} = 14.906$

Control limits for subgroup ranges

$UCL_R = 12.624$

$LCL_R = 0$

See Table 7. for Sample Calculations.

Table 16. Operator 10's Period and Grand Cycle Time Mean and Range

3:50 to 3:55 PM

Subgroup	$\bar{X}$	R
1	17.4	18
2	16.6	4
3	15.6	10
4	14.8	7
5	15.8	8

$\bar{\bar{X}} = 16.04$

$\bar{R} = 9.4$

$\sigma_w = 4.041$

4:25 to 4:30 PM

Subgroup	$\bar{X}$	R
1	18.2	15
2	18.2	11
3	15.0	3
4	14.6	5
5	15.6	3

$\bar{\bar{X}} = 16.32$

$\bar{R} = 7.4$

$\sigma_w = 3.181$

5:03 to 5:07 PM

Subgroup	$\bar{X}$	R
1	17.2	7
2	16.2	9
3	15.0	5
4	16.6	11
5	15.0	4

$\bar{\bar{X}} = 16.00$

$\bar{R} = 7.2$

$\sigma_w = 3.095$

Table 16. Operator 10's Period and Grand Cycle Time Mean and Range (continued)

5:35 to 5:42 PM

Subgroup	$\bar{X}$	R
1	15.8	6
2	20.6	11
3	20.2	18
4	16.8	7
5	18.0	11

$\bar{\bar{X}} = 18.28$

$\bar{R} = 10.6$

$\sigma_\omega = 4.557$

6:58 to 7:05 PM

Subgroup	$\bar{X}$	R
1	17.2	7
2	15.8	8
3	19.0	5
4	18.8	6
5	17.0	2

$\bar{\bar{X}} = 17.56$

$\bar{R} = 5.6$

$\sigma_\omega = 2.408$

7:30 to 7:38 PM

Subgroup	$\bar{X}$	R
1	18.0	5
2	17.0	5
3	18.6	15
4	18.4	13
5	15.6	3

$\bar{\bar{X}} = 17.52$

$\bar{R} = 8.2$

$\sigma_\omega = 3.525$

Table 16. Operator 10's Period and Grand Cycle Time Mean and Range (continued)

8:04 to 8:09 PM

Subgroup	$\bar{X}$	R
1	15.6	5
2	18.6	6
3	17.0	8
4	14.2	1
5	15.6	7

$\bar{\bar{X}} = 16.20$

$\bar{R} = 5.4$

$\sigma_w = 2.322$

8:35 to 8:43 PM

Subgroup	$\bar{X}$	R
1	17.0	6
2	22.2	16
3	13.8	2
4	16.2	7
5	16.2	7

$\bar{\bar{X}} = 17.08$

$\bar{R} = 7.6$

$\sigma_w = 3.267$

9:29 to 9:35 PM

Subgroup	$\bar{X}$	R
1	19.0	10
2	17.8	8
3	16.2	7
4	15.8	3
5	16.2	7

$\bar{\bar{X}} = 17.00$

$\bar{R} = 7.0$

$\sigma_w = 3.009$

Table 16. Operator 10's Period and Grand Cycle Time Mean and Range (continued)

10:00 to 10:05 PM

Subgroup	$\bar{X}$	R
1	15.6	4
2	16.0	9
3	16.4	5
4	16.0	7
5	15.2	3

$\bar{\bar{X}} = 15.84$

$\bar{R} = 5.6$

$\sigma_\omega = 2.408$

10:40 to 10:46 PM

Subgroup	$\bar{X}$	R
1	15.8	2
2	20.0	13
3	16.8	7
4	19.0	10
5	17.6	9

$\bar{\bar{X}} = 17.84$

$\bar{R} = 8.2$

$\sigma_\omega = 3.525$

$\bar{\bar{\bar{X}}} = 16.880$

$\bar{\bar{R}} = 7.472$

$R_{\bar{\bar{X}}} = 2.44$

$\sigma_B = 1.049$

$\sigma_\omega = 3.213$

Table 16. Operator 10's Period and Grand Cycle Time Mean and Range (continued)

N = 275 R = 22

Class Interval = 2

Control limits for subgroup averages

$UCL_{\bar{X}} = 21.214$

$LCL_{\bar{X}} = 12.546$

Control limits for period averages

$UCL_{\bar{\bar{X}}} = 18.818$

$LCL_{\bar{\bar{X}}} = 14.942$

Control limits for subgroup ranges

$UCL_{R} = 15.616$

$LCL_{R} = 0$

See Table 7 for Sample Calculations.

Table 17. Operator 11's Period and Grand Cycle Time Mean and Range

3:57 to 4:02 PM

Subgroup	$\bar{X}$	R
1	14.4	3
2	16.2	4
3	15.4	4
4	14.8	5
5	15.8	5

$\bar{\bar{X}} = 15.12$

$\bar{R} = 4.2$

$\sigma_\omega = 1.806$

4:37 to 4:43 PM

Subgroup	$\bar{X}$	R
1	18.0	8
2	18.6	10
3	21.2	11
4	19.0	10
5	14.8	3

$\bar{\bar{X}} = 18.32$

$\bar{R} = 8.4$

$\sigma_\omega = 3.611$

5:07 to 5:13 PM

Subgroup	$\bar{X}$	R
1	15.2	5
2	15.6	3
3	17.0	7
4	19.8	3
5	17.4	7

$\bar{\bar{X}} = 17.00$

$\bar{R} = 5.0$

$\sigma_\omega = 2.150$

Table 17. Operator 11's Period and Grand Cycle Time Mean and Range (continued)

5:43 to 5:48 PM

Subgroup	$\bar{X}$	R
1	16.2	3
2	15.8	4
3	16.8	5
4	17.2	3
5	16.8	9

$\bar{\bar{X}} = 16.56$

$\bar{R} = 5.2$

$\sigma_w = 2.236$

7:05 to 7:11 PM

Subgroup	$\bar{X}$	R
1	16.0	5
2	16.2	10
3	16.6	6
4	22.0	12
5	15.6	5

$\bar{\bar{X}} = 17.28$

$\bar{R} = 7.6$

$\sigma_w = 3.267$

7:39 to 7:44 PM

Subgroup	$\bar{X}$	R
1	13.8	4
2	18.6	9
3	15.0	3
4	16.0	6
5	15.2	6

$\bar{\bar{X}} = 15.72$

$\bar{R} = 5.6$

$\sigma_w = 2.408$

Table 17. Operator 11's Period and Grand Cycle Time Mean and Range (continued)

8:10 to 8:15 PM

Subgroup	$\bar{X}$	R
1	15.8	3
2	14.0	2
3	17.4	8
4	13.6	3
5	13.6	3

$\bar{\bar{X}} = 14.88$

$\bar{R} = 3.8$

$\sigma_\omega = 1.634$

8:43 to 8:47 PM

Subgroup	$\bar{X}$	R
1	15.4	5
2	14.6	3
3	15.4	5
4	16.6	6
5	14.4	4

$\bar{\bar{X}} = 15.28$

$\bar{R} = 4.6$

$\sigma_\omega = 1.978$

9:35 to 9:40 PM

Subgroup	$\bar{X}$	R
1	15.8	3
2	16.2	5
3	17.4	9
4	15.4	8
5	16.8	4

$\bar{\bar{X}} = 16.32$

$\bar{R} = 5.8$

$\sigma_\omega = 2.494$

Table 17. Operator 11's Period and Grand Cycle Time Mean and Range (continued)

10:05 to 10:10 PM

Subgroup	$\bar{X}$	R
1	16.2	6
2	13.4	3
3	15.4	3
4	13.4	5
5	13.8	3

$\bar{\bar{X}} = 14.44$

$\bar{R} = 4.0$

$\sigma_w = 1.720$

10:47 to 10:53 PM

Subgroup	$\bar{X}$	R
1	17.6	7
2	17.6	4
3	18.8	7
4	17.4	9
5	15.3	1

$\bar{\bar{X}} = 17.34$

$\bar{R} = 5.6$

$\sigma_w = 2.408$

$\bar{\bar{\bar{X}}} = 16.205$

$\bar{\bar{R}} = 5.436$

$R_{\bar{\bar{X}}} = 3.88$

$\sigma_B = 1.668$

$\sigma_w = 2.337$

Table 17. Operator 11's Period and Grand Cycle Time Mean and Range (continued)

N = 275 R = 17

Class Interval = 2

Control limits for subgroup averages

$UCL_{\bar{X}} = 19.358$

$LCL_{\bar{X}} = 13.052$

Control limits for period averages

$UCL_{\bar{\bar{X}}} = 17.606$

$LCL_{\bar{\bar{X}}} = 14.804$

Control limits for subgroup ranges

$UCL_R = 11.361$

$LCL_R = 0$

See Table 7. for Sample Calculations.

Table 18. Operator 12's Period and Grand Cycle Time Mean and Range

4:00 to 4:10 PM

Subgroup	$\bar{X}$	R
1	24.0	10
2	20.2	4
3	21.2	9
4	21.8	6
5	23.6	7

$\bar{\bar{X}} = 22.16$

$\bar{R} = 7.2$

$\sigma_w = 3.095$

4:44 to 4:49 PM

Subgroup	$\bar{X}$	R
1	21.2	8
2	18.4	3
3	18.8	10
4	21.4	17
5	19.2	10

$\bar{\bar{X}} = 19.8$

$\bar{R} = 9.6$

$\sigma_w = 4.127$

5:14 to 5:20 PM

Subgroup	$\bar{X}$	R
1	22.2	15
2	18.0	7
3	19.0	4
4	22.8	10
5	20.0	7

$\bar{\bar{X}} = 20.4$

$\bar{R} = 8.6$

$\sigma_w = 3.697$

Table 18. Operator 12's Period and Grand Cycle Time Mean and Range (continued)

5:48 to 5:55 PM

Subgroup	$\bar{X}$	R
1	17.8	7
2	19.0	3
3	18.2	5
4	19.4	5
5	21.2	15

$\bar{\bar{X}} = 19.12$

$\bar{R} = 7.0$

$\sigma_w = 3.009$

7:12 to 7:17 PM

Subgroup	$\bar{X}$	R
1	19.0	4
2	20.8	6
3	22.8	8
4	18.8	5
5	17.0	6

$\bar{\bar{X}} = 19.68$

$\bar{R} = 5.8$

$\sigma_w = 2.494$

7:45 to 7:51 PM

Subgroup	$\bar{X}$	R
1	17.8	3
2	15.8	2
3	21.6	9
4	17.4	2
5	19.8	6

$\bar{\bar{X}} = 18.48$

$\bar{R} = 4.4$

$\sigma_w = 1.892$

Table 18. Operator 12's Period and Grand Cycle Time Mean and Range (continued)

8:15 to 8:22 PM

Subgroup	$\bar{X}$	R
1	19.8	13
2	21.4	6
3	18.8	6
4	18.4	5
5	17.8	6

$\bar{\bar{X}} = 19.24$

$\bar{R} = 7.2$

$\sigma_w = 3.095$

8:48 to 8:55 PM

Subgroup	$\bar{X}$	R
1	19.2	8
2	18.8	5
3	17.0	2
4	17.6	3
5	17.4	6

$\bar{\bar{X}} = 18.00$

$\bar{R} = 4.8$

$\sigma_w = 2.064$

9:40 to 9:46 PM

Subgroup	$\bar{X}$	R
1	15.2	2
2	17.0	7
3	17.6	4
4	19.6	17
5	17.2	7

$\bar{\bar{X}} = 17.32$

$\bar{R} = 7.4$

$\sigma_w = 3.181$

Table 18. Operator 12's Period and Grand Cycle Time Mean and Range (continued)

10:10 to 10:15 PM

Subgroup	$\bar{X}$	R
1	17.2	3
2	19.0	10
3	18.2	7
4	19.4	7
5	21.6	18

$\bar{\bar{X}} = 19.08$

$\bar{R} = 9.0$

$\sigma_\omega = 3.869$

10:53 to 10:58 PM

Subgroup	$\bar{X}$	R
1	18.8	7
2	16.8	6
3	20.0	8
4	17.2	11
5	19.4	15

$\bar{\bar{X}} = 18.44$

$\bar{R} = 9.4$

$\sigma_\omega = 4.041$

$\bar{\bar{\bar{X}}} = 19.247$

$\bar{\bar{R}} = 7.309$

$R_{\bar{\bar{X}}} = 4.84$

$\sigma_\beta = 2.081$

$\sigma_\omega = 3.142$

Table 18. Operator 12's Period and Grand Cycle Time Mean and Range (continued)

N = 275 R = 20

Class Interval = 2

Control limits for subgroup averages

$UCL_{\bar{X}} = 23.487$

$LCL_{\bar{X}} = 15.007$

Control limits for period averages

$UCL_{\bar{\bar{X}}} = 21.143$

$LCL_{\bar{\bar{X}}} = 17.351$

Control limits for subgroup ranges

$UCL_R = 15.276$

$LCL_R = 0$

See Table 7. for Sample Calculations

Table 19. Operator 13's Period and Grand Cycle Time Mean and Range

<u>11:50 to 11:57 PM</u>

Subgroup	$\bar{X}$	R
1	18.2	4
2	19.4	9
3	18.2	7
4	20.4	6
5	19.0	7

$\bar{\bar{X}} = 19.04$

$\bar{R} = 6.6$

$\sigma_w = 2.837$

<u>12:45 to 12:53 AM</u>

Subgroup	$\bar{X}$	R
1	18.4	6
2	19.0	5
3	16.4	2
4	20.6	9
5	18.6	7

$\bar{\bar{X}} = 18.60$

$\bar{R} = 5.8$

$\sigma_w = 2.494$

<u>1:54 to 2:05 AM</u>

Subgroup	$\bar{X}$	R
1	18.2	9
2	19.4	6
3	19.6	6
4	18.4	6
5	18.8	4

$\bar{\bar{X}} = 18.88$

$\bar{R} = 6.2$

$\sigma_w = 2.666$

Table 19. Operator 13's Period and Grand Cycle Time Mean and Range (continued)

3:34 to 3:41 AM

Subgroup	$\bar{X}$	R
1	18.0	5
2	19.2	10
3	20.0	8
4	17.6	5
5	17.8	5

$\bar{\bar{X}} = 18.52$

$\bar{R} = 6.6$

$\sigma_w = 2.838$

4:35 to 4:42 AM

Subgroup	$\bar{X}$	R
1	18.4	6
2	18.8	6
3	19.2	3
4	17.8	3
5	19.8	6

$\bar{\bar{X}} = 18.80$

$\bar{R} = 4.8$

$\sigma_w = 2.064$

5:45 to 5:52 AM

Subgroup	$\bar{X}$	R
1	18.6	6
2	17.8	4
3	17.6	7
4	18.4	7
5	18.6	3

$\bar{\bar{X}} = 18.20$

$\bar{R} = 5.4$

$\sigma_w = 2.322$

Table 19. Operator 13's Period and Grand Cycle Time Mean and Range (continued)

$\bar{\bar{X}} = 18.607$

$\bar{\bar{R}} = 5.900$

$R_{\bar{X}} = 0.84$

$\sigma_B = .361$

$\sigma_w = 2.537$

N = 150 R = 14

Class Interval = 2

Control limits for subgroup averages

$ULC_{\bar{X}} = 22.029$

$LCL_{\bar{X}} = 15.185$

Control limits for period averages

$UCL_{\bar{\bar{X}}} = 20.137$

$LCL_{\bar{\bar{X}}} = 17.077$

Control limits for subgroup ranges

$UCL_R = 12.331$

$LCL_R = 0$

See Table 7. for Sample Calculations.

Table 20. Operator 14's Period and Grand Cycle Time Mean and Range

11:57 to 12:04 AM

Subgroup	$\bar{X}$	R
1	22.6	6
2	18.0	7
3	12.4	6
4	19.2	10
5	22.2	3

$\bar{\bar{X}} = 20.68$

$\bar{R} = 6.4$

$\sigma_w = 2.752$

12:54 to 1:02 AM

Subgroup	$\bar{X}$	R
1	20.8	6
2	19.0	9
3	21.2	5
4	20.0	4
5	20.2	7

$\bar{\bar{X}} = 20.24$

$\bar{R} = 6.2$

$\sigma_w = 2.666$

2:05 to 2:16 AM

Subgroup	$\bar{X}$	R
1	20.0	6
2	16.6	6
3	19.4	4
4	19.0	8
5	20.0	5

$\bar{\bar{X}} = 19.00$

$\bar{R} = 5.8$

$\sigma_w = 2.494$

Table 20. Operator 14's Period and Grand Cycle Time Mean and Range (continued)

3:42 to 3:48 AM

Subgroup	$\bar{X}$	R
1	18.0	6
2	19.6	3
3	18.2	4
4	19.4	5
5	19.0	5

$\bar{\bar{X}} = 18.84$

$\bar{R} = 4.6$

$\sigma_w = 1.978$

4:52 to 5:00 AM

Subgroup	$\bar{X}$	R
1	18.2	7
2	18.8	3
3	19.2	6
4	19.4	8
5	22.0	5

$\bar{\bar{X}} = 19.52$

$\bar{R} = 5.8$

$\sigma_w = 2.494$

5:52 to 5:59 AM

Subgroup	$\bar{X}$	R
1	19.8	7
2	18.0	7
3	21.0	7
4	19.2	3
5	21.8	9

$\bar{\bar{X}} = 19.96$

$\bar{R} = 6.6$

$\sigma_w = 2.837$

Table 20. Operator 14's Period and Grand Cycle Time Mean and Range (continued)

$\bar{\bar{\bar{X}}} = 19.707$

$\bar{\bar{R}} = 5.90$

$R_{\bar{\bar{X}}} = 1.84$

$\sigma_{\beta} = .791$

$\sigma_{\omega} = 2.537$

N = 150 R = 15

Class Interval = 2

Control limits for subgroup averages

$UCL_{\bar{X}} = 23.129$

$LCL_{\bar{X}} = 16.285$

Control limits for period averages

$UCL_{\bar{\bar{X}}} = 21.237$

$LCL_{\bar{\bar{X}}} = 18.177$

Control limits for subgroup ranges

$UCL_{R} = 12.331$

$LCL_{R} = 0$

See Table 7 for Sample Calculations.

Table 21. Operator 15's Period and Grand Cycle Time Mean and Range

12:05 to 12:12 AM

Subgroup	$\bar{X}$	R
1	16.4	4
2	18.6	4
3	18.0	6
4	15.6	3
5	16.0	5

$\bar{\bar{X}} = 16.92$

$\bar{R} = 4.4$

$\sigma_w = 1.892$

1:04 to 1:10 AM

Subgroup	$\bar{X}$	R
1	17.0	6
2	15.8	2
3	15.4	1
4	15.4	3
5	17.6	5

$\bar{\bar{X}} = 16.24$

$\bar{R} = 3.4$

$\sigma_w = 1.462$

2:16 to 2:23 AM

Subgroup	$\bar{X}$	R
1	16.2	1
2	17.0	4
3	17.8	6
4	17.8	6
5	15.4	4

$\bar{\bar{X}} = 16.84$

$\bar{R} = 4.2$

$\sigma_w = 1.806$

Table 21. Operator 15's Period and Grand Cycle Time Mean and Range (continued)

3:48 to 3:56 AM

Subgroup	$\bar{X}$	R
1	19.4	10
2	15.6	4
3	16.6	3
4	18.0	6
5	19.0	4

$\bar{\bar{X}} = 17.72$

$\bar{R} = 5.4$

$\sigma_w = 2.322$

5:06 to 5:12 AM

Subgroup	$\bar{X}$	R
1	17.4	1
2	16.2	5
3	15.2	3
4	15.8	4
5	15.0	2

$\bar{\bar{X}} = 15.92$

$\bar{R} = 3.0$

$\sigma_w = 1.290$

6:00 to 6:05 AM

Subgroup	$\bar{X}$	R
1	17.2	3
2	17.8	7
3	16.2	5
4	16.8	6
5	17.4	3

$\bar{\bar{X}} = 17.08$

$\bar{R} = 4.8$

$\sigma_w = 2.064$

Table 21. Operator 15's Period and Grand Cycle Time Mean and Range (continued)

$\bar{\bar{\bar{X}}} = 16.787$

$\bar{\bar{R}} = 4.20$

$R_{\bar{\bar{X}}} = 1.80$

$\sigma_{B} = .773$

$\sigma'_{w} = 1.806$

N = 150 R = 13

Class Interval = 2

Control limits for subgroup averages

$UCL_{\bar{X}} = 19.223$

$LCL_{\bar{X}} = 14.351$

Control limits for period averages

$UCL_{\bar{\bar{X}}} = 17.876$

$LCL_{\bar{\bar{X}}} = 15.698$

Control limits for subgroup ranges

$UCL_{R} = 8.778$

$LCL_{R} = 0$

See Table 7. for Sample Calculations.

Table 22. Operator 16's Period and Grand Cycle Time Mean and Range

12:12 to 12:17 AM

Subgroup	$\bar{X}$	R
1	18.6	7
2	15.4	4
3	17.4	8
4	15.8	4
5	16.2	5

$\bar{\bar{X}} = 16.68$

$\bar{R} = 5.6$

$\sigma_w = 2.408$

1:10 to 1:19 AM

Subgroup	$\bar{X}$	R
1	18.6	11
2	19.0	6
3	17.0	9
4	16.0	2
5	20.4	8

$\bar{\bar{X}} = 18.20$

$\bar{R} = 7.2$

$\sigma_w = 3.095$

2:23 to 2:32 AM

Subgroup	$\bar{X}$	R
1	19.0	11
2	15.4	7
3	16.0	3
4	16.8	7
5	17.2	5

$\bar{\bar{X}} = 16.88$

$\bar{R} = 6.6$

$\sigma_w = 2.837$

Table 22. Operator 16's Period and Grand Cycle Time Mean and Range (continued)

3:56 to 4:03 AM

Subgroup	$\bar{X}$	R
1	16.2	2
2	16.4	4
3	16.4	6
4	17.2	9
5	14.4	4

$\bar{\bar{X}} = 16.12$

$\bar{R} = 5.0$

$\sigma_\omega = 2.149$

5:12 to 5:19 AM

Subgroup	$\bar{X}$	R
1	16.6	4
2	17.4	3
3	14.6	6
4	18.2	7
5	18.4	5

$\bar{\bar{X}} = 17.04$

$\bar{R} = 5.0$

$\sigma_\omega = 2.149$

6:06 to 6:11 AM

Subgroup	$\bar{X}$	R
1	17.0	6
2	16.6	3
3	17.2	4
4	18.8	10
5	17.6	3

$\bar{\bar{X}} = 17.44$

$\bar{R} = 5.0$

$\sigma_\omega = 2.149$

Table 22. Operator 16's Period and Grand Cycle Time Mean and Range (continued)

$\bar{\bar{X}} = 17.060$

$\bar{\bar{R}} = 5.333$

$R_{\bar{\bar{X}}} = 2.08$

$\sigma_{B} = .894$

$\sigma_{\omega} = 2.291$

$N = 150$ $R = 15$

Class Interval = 2

Control limits for subgroup averages

$UCL_{\bar{X}} = 20.153$

$LCL_{\bar{X}} = 13.967$

Control limits for period averages

$UCL_{\bar{\bar{X}}} = 18.443$

$LCL_{\bar{\bar{X}}} = 15.677$

Control limits for subgroup ranges

$UCL_{R} = 11.146$

$LCL_{R} = 0$

See Table 7. for Sample Calculations.

Table 23. Operator 17's Period and Grand Cycle Time Mean and Range

12:17 to 12:24 AM

Subgroup	$\bar{X}$	R
1	22.4	10
2	20.2	2
3	19.2	1
4	20.6	6
5	18.8	6

$\bar{\bar{X}} = 20.24$

$\bar{R} = 5.0$

$\sigma_w = 2.149$

1:19 to 1:27 AM

Subgroup	$\bar{X}$	R
1	20.4	5
2	19.2	5
3	19.4	6
4	18.6	3
5	18.8	5

$\bar{\bar{X}} = 19.28$

$\bar{R} = 4.8$

$\sigma_w = 2.063$

2:33 to 2:45 AM

Subgroup	$\bar{X}$	R
1	22.8	9
2	24.6	11
3	22.0	8
4	20.8	6
5	19.0	4

$\bar{\bar{X}} = 21.84$

$\bar{R} = 7.6$

$\sigma_w = 3.267$

Table 23. Operator 17's Period and Grand Cycle Time Mean and Range (continued)

4:05 to 4:13 AM

Subgroup	$\bar{X}$	R
1	22.0	4
2	20.2	12
3	20.4	4
4	19.4	6
5	21.2	5

$\bar{\bar{X}} = 20.64$

$\bar{R} = 6.2$

$\sigma_w = 2.665$

5:20 to 5:27 AM

Subgroup	$\bar{X}$	R
1	20.8	2
2	20.6	9
3	18.0	6
4	24.4	9
5	21.0	7

$\bar{\bar{X}} = 20.96$

$\bar{R} = 6.6$

$\sigma_w = 2.837$

6:12 to 6:15 AM

Subgroup	$\bar{X}$	R
1	21.4	13
2	23.0	9
3	19.8	11
4	Not taken	
5		

$\bar{\bar{X}} = 21.40$

$\bar{R} = 11.0$

$\sigma_w = 4.729$

Table 23. Operator 17's Period and Grand Cycle Time Mean and Range (continued)

$\overline{\overline{\overline{X}}} = 20.727$

$\overline{\overline{R}} = 6.867$

$R_{\overline{\overline{X}}} = 2.56$

$\sigma_{B} = 1.100$

$\sigma_{w} = 2.952$

N = 140 R = 16

Class Interval = 2

Control limits for subgroup averages

$UCL_{\overline{X}} = 24.710$

$LCL_{\overline{X}} = 16.744$

Control limits for period averages

$UCL_{\overline{\overline{X}}} = 22.508$

$LCL_{\overline{\overline{X}}} = 18.946$

Control limits for subgroup ranges

$UCL_{R} = 14.352$

$LCL_{R} = 0$

See Table 7. for Sample Calculations.

Table 24. Operator 18's Period and Grand Cycle Time Mean and Range

12:24 to 12:31 AM

Subgroup	$\bar{X}$	R
1	20.4	6
2	19.4	6
3	19.0	8
4	20.0	5
5	20.6	8

$\bar{\bar{X}} = 19.88$

$\bar{R} = 6.6$

$\sigma_{\omega} = 2.837$

1:28 to 1:36 AM

Subgroup	$\bar{X}$	R
1	19.8	6
2	23.0	12
3	22.0	6
4	19.6	2
5	18.4	7

$\bar{\bar{X}} = 20.56$

$\bar{R} = 6.6$

$\sigma_{\omega} = 2.837$

2:51 to 2:57 AM

Subgroup	$\bar{X}$	R
1	19.0	11
2	18.2	2
3	19.8	3
4	18.8	1
5	20.2	2

$\bar{\bar{X}} = 19.20$

$\bar{R} = 3.8$

$\sigma_{\omega} = 1.633$

Table 24. Operator 18's Period and Grand Cycle Time Mean and Range (continued)

4:14 to 4:20 AM

Subgroup	$\bar{X}$	R
1	22.0	10
2	17.4	3
3	18.2	2
4	20.2	4
5	18.2	6

$\bar{\bar{X}} = 19.20$

$\bar{R} = 5.0$

$\sigma_\omega = 2.149$

5:28 to 5:34 AM

Subgroup	$\bar{X}$	R
1	18.6	4
2	18.0	6
3	19.6	6
4	17.6	3
5	17.2	4

$\bar{\bar{X}} = 18.20$

$\bar{R} = 4.6$

$\sigma_\omega = 1.977$

6:15 to 6:20 AM

Subgroup	$\bar{X}$	R
1	20.0	5
2	18.0	3
3	19.8	5
4	Not taken	
5		

$\bar{\bar{X}} = 19.27$

$\bar{R} = 4.3$

$\sigma_\omega = 1.848$

Table 24. Operator 18's Period and Grand Cycle Time Mean and Range (continued)

$\bar{\bar{\bar{X}}} = 19.385$

$\bar{\bar{R}} = 5.150$

$R_{\bar{\bar{X}}} = 2.36$

$\sigma_B = 1.014$

$\sigma_\omega = 2.214$

N = 140 R = 17

Class Interval = 2

Control limits for subgroup averages

$UCL_{\bar{X}} = 22.372$

$LCL_{\bar{X}} = 16.398$

Control limits for period averages

$UCL_{\bar{\bar{X}}} = 20.720$

$LCL_{\bar{\bar{X}}} = 18.050$

Control limits for subgroup ranges

$UCL_R = 10.764$

$LCL_R = 0$

See Table 7. for Sample Calculations

Table 25. Operator 19's Period and Grand Cycle Time Mean and Range

12:32 to 12:45 AM

Subgroup	$\bar{X}$	R
1	23.0	12
2	23.2	11
3	20.0	2
4	21.2	6
5	22.6	7

$\bar{\bar{X}} = 22.00$

$\bar{R} = 7.6$

$\sigma_\omega = 3.267$

1:37 to 1:45 AM

Subgroup	$\bar{X}$	R
1	19.8	3
2	23.2	3
3	23.2	2
4	20.2	11
5	20.4	6

$\bar{\bar{X}} = 21.26$

$\bar{R} = 5.0$

$\sigma_\omega = 2.149$

2:57 to 3:00 AM

Subgroup	$\bar{X}$	R
1	22.2	9
2	23.4	8
3	22.8	8
4	20.8	8
5	20.4	12

$\bar{\bar{X}} = 21.82$

$\bar{R} = 9.0$

$\sigma_\omega = 3.869$

Table 25. Operator 19's Period and Grand Cycle Time Mean and Range (continued)

4:20 to 4:30 AM

Subgroup	$\bar{X}$	R
1	22.4	8
2	24.4	5
3	21.4	12
4	22.6	6
5	18.8	6

$\bar{\bar{X}} = 21.92$

$\bar{R} = 7.4$

$\sigma_{\omega} = 3.181$

5:35 to 5:42 AM

Subgroup	$\bar{X}$	R
1	21.0	5
2	25.4	9
3	24.8	12
4	24.4	16
5	26.0	9

$\bar{\bar{X}} = 24.32$

$\bar{R} = 10.2$

$\sigma_{\omega} = 4.385$

6:20 to 6:36 AM

Subgroup	$\bar{X}$	R
1	24.4	15
2	23.8	9
3	26.0	15
4	23.4	8
5	26.6	16

$\bar{\bar{X}} = 24.84$

$\bar{R} = 12.6$

$\sigma_{\omega} = 5.417$

Table 25. Operator 19's Period and Grand Cycle Time Mean and Range (continued)

$\bar{\bar{\bar{X}}} = 22.693$

$\bar{\bar{R}} = 8.633$

$R_{\bar{\bar{X}}} = 3.58$

$\sigma_B = 1.539$

$\sigma_\omega = 3.711$

N = 150 R = 21

Class Interval = 3

Control limits for subgroup averages

$UCL_{\bar{X}} = 27.700$

$LCL_{\bar{X}} = 17.686$

Control limits for period averages

$UCL_{\bar{\bar{X}}} = 24.485$

$LCL_{\bar{\bar{X}}} = 20.901$

Control limits for subgroup ranges

$UCL_R = 18.043$

$LCL_R = 0$

See Table 7. for Sample Calculations.

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