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Viable Designs Through a Joint Probabilistic Estimation Technique

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ABSTRACT

A key issue in complex systems design is measuring the 'goodness' of a design, i.e. finding a criterion through which a particular design is determined to be the 'best'. Traditional choices in aerospace systems design, such as performance, cost, revenue, reliability, and safety, individually fail to fully capture the life cycle characteristics of the system. Furthermore, current multi-criteria optimization approaches, addressing this problem, rely on deterministic, thus, complete and known information about the system and the environment it is exposed to. In many cases, this information is not be available at the conceptual or preliminary design phases. Hence, critical decisions made in these phases have to draw from only incomplete or uncertain knowledge. One modeling option is to treat this incomplete information probabilistically, accounting for the fact that certain values may be prominent, while the actual value during operation is unknown. Hence, to account for a multi-criteria as well as a probabilistic approach to systems design, a joint-probabilistic formulation is needed to accurately estimate the probability of satisfying the criteria concurrently. When criteria represent objective/ aspiration functions with corresponding goals, this 'joint probability' can also be called viability. The proposed approach to probabilistic, multi-criteria aircraft design, called the Joint Probabilistic Decision Making (JPDM) technique, will facilitate precisely this estimate.

INTRODUCTION

"Decision making is characterized by its involvement with information, value assessments, and optimization. Thus whereas inventiveness seeks many possible answers and analysis seeks one actual answer, decision making seeks to choose the *one best answer*."[1] But the 'one best answer' can be difficult to obtain when the decision is based on several objectives. Techniques that aid the decision-maker in determining the best or a set of 'best' solutions have been developed over the past three decades. Excellent references that give a detailed overview of most available techniques can be found in References 2, 3, 4, 5, 6, 7, and 8. Hwang, in particular, introduced two classifications, the Multi-Attribute Decision Making (MADM) and Multi-Objective Decision Making (MODM) techniques.[2,3]

According to Hwang, multiple attribute decision problems involve the selection of the 'best' alternative from a pool of preselected alternatives described in terms of their attributes.[2] Attributes are generally defined as characteristics that describe in part the state of a product or system. Objectives can then be defined as attributes that are associated with a goal and a direction 'to do better' as perceived by the decision maker. Goals are thus desired (target) levels in terms of a specific state in space and time. In many cases, however, the terms 'objective' and 'goal' are used interchangeably. With this definition for objectives in mind, multi-objective decision problems involve the *design* of alternatives which optimize or 'best satisfy' the objectives of the decision maker.[2] In other words, multi-attribute decision problems are *product selection* problems, multi-objective decision problems are *optimization* problems. Together all techniques for solving these problems can be classified as Multi-Criteria Decision Making (MCDM) techniques. While criteria typically describe the standards of judgment or rules to test acceptability, here, they simply indicate attributes and/or objectives. In general an MCDM problem is described by $\max\{f_1(\underline{x}), f_2(\underline{x}), \ldots, f_k(\underline{x})\}$, where f_i, $i = 1, .., k$, are the criteria and $\underline{x}$ is the n-dimensional vector of design variables the criteria depend on. If a given vector $\underline{x}$ satisfies all specified goals for the criteria, then it is considered to be a *viable* solution.

Furthermore, most existing MCDM techniques require deterministic information about the criteria in order to find the 'best' or set of 'best' solutions. This requirement presents a significant restriction for systems design, which is closely related to, if not identical with decision making (see Ref. 9). A deterministic formulation, though, implies complete and known information about the system and the environment it is exposed to. This information, however, may not be available in the conceptual or preliminary design phases. Critical decisions made in these phases often draw from only incomplete or uncertain knowledge. One modeling option is to treat this incomplete information probabilistically, accounting for the fact that certain values may be prominent, while the actual value during operation is unknown. By assigning probability estimates to the values within the range of interest, the method guarantees that all values are kept as possible solutions. In other words, a probabilistic design method yields the aircraft's attributes, and thus the decision criteria, as random variables. Even though probabilistic design techniques have gained much popularity in recent years,[10,11,12,13,14] they typically treat one criterion at a time only. This apparent shortcoming for realistic systems design situations calls for a new technique that treats all specified decision/design criteria concurrently and as random variables. The technique presented in this paper employs a joint probabilistic formulation of the decision problem in order to address this deficiency, thereby determining the viability of a system as the probability of satisfying all criteria goals concurrently.

JOINT PROBABILITY THEORY

Definition: Let $X_1, X_2, \ldots, X_n$ be a set of random variables defined on a (discrete) probability space Ω. The probability that the events $X_1 = x_1, X_2 = x_2, \ldots$, and $X_n = x_n$ happen *concurrently*, is denoted by $f(x_1, x_2, \ldots, x_n) = P(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n)$[*] for the set of desired solutions $A \subseteq \Omega$. If the function $f(x_1, x_2, \ldots, x_n)$ is discrete, it is called the *joint probability mass function* of $X_1, X_2, \ldots, X_n$ and has the following properties:[15]

$$0 \le f(x_1, x_2, \ldots, x_n) \le 1$$

$$\sum_{(x_1,x_2,\ldots,x_n)} \sum_{\in\Omega} f(x_1, x_2, \ldots, x_n) = 1 \qquad (1)$$

$$P[(X_1, X_2, \ldots, X_n) \in A] = \sum_{(x_1,x_2,\ldots,x_n)} \sum_{\in A} f(x_1, x_2, \ldots, x_n), \; A \subseteq \Omega$$

If $f(x_1, x_2, \ldots, x_n)$ is continuous, it is called *joint probability density function* of $X_1, X_2, \ldots, X_n$ and has the following properties:[15]

$$0 \le f(x_1, x_2, \ldots, x_n)$$

$$\int_\Omega \cdots \int f(x_1, x_2, \ldots, x_n)\,dx_1 dx_2 \ldots dx_n = 1 \qquad (2)$$

$$P[(X_1, X_2, \ldots, X_n) \in A] = \int_A \cdots \int f(x_1, x_2, \ldots, x_n)\,dx_1 dx_2 \ldots dx_n, \; A \subseteq \Omega$$

If the lower bound of A, the set of desired solutions, is equal to the infimum[†] of Ω for all X_i, i.e. if $A = (\inf_i(\Omega), a_i]$, for all $i = 1, 2, \ldots, n$, a function $F(a_1, a_2, \ldots, a_n)$ can be defined, such that:

$$F(a_1, a_2, \ldots, a_n) = P[(X_1, X_2, \ldots, X_n) \in A] = \sum_{(x_1,x_2,\ldots,x_n)} \cdots \sum_{\in A} f(x_1, x_2, \ldots, x_n), \; A \subseteq \Omega \qquad (f \text{ is discrete}) \quad (3)$$

$$F(a_1, a_2, \ldots, a_n) = P[(X_1, X_2, \ldots, X_n) \in A] = \int_A \cdots \int f(x_1, x_2, \ldots, x_n)\,dx_1 dx_2 \ldots dx_n, \; A \subseteq \Omega \qquad (f \text{ is continuous}) \quad (4)$$

F is called the joint cumulative probability distribution function.[16] For $\Omega = \Re^n$ and a continuous function f:[‡]

$$\begin{aligned} F(a_1, a_2, \ldots, a_n) &= P[(X_1, X_2, \ldots, X_n) \in ((-\infty, -\infty, \ldots, -\infty), (a_1, a_2, \ldots, a_n))] \\ &= \int_{-\infty}^{a_n} \cdots \int_{-\infty}^{a_1} f(x_1, x_2, \ldots, x_n)\,dx_1 \ldots dx_n \end{aligned} \qquad (5)$$

The common notation $F(a_1, a_2, \ldots, a_n) = P(X_1 \le a_1, X_2 \le a_2, \ldots, X_n \le a_n)$ will be used subsequently also.

The univariate probability function f_{Xi} for each criterion X_i, obtained from the traditional probabilistic design process, can also be generated with the joint probability function f. f_{Xi} is called *marginal probability mass or density function* of X_i and is defined by:

$$f_{X_1} = \sum_{(x_2,\ldots,x_n)} \cdots \sum_{\in R} f(x_2, \ldots, x_n) \qquad (f \text{ is discrete}) \quad (6)$$

$$f_{X_1} = \int_R \cdots \int f(x_2, \ldots, x_n)\,dx_2 \ldots dx_n \qquad (f \text{ is continuous}) \quad (7)$$

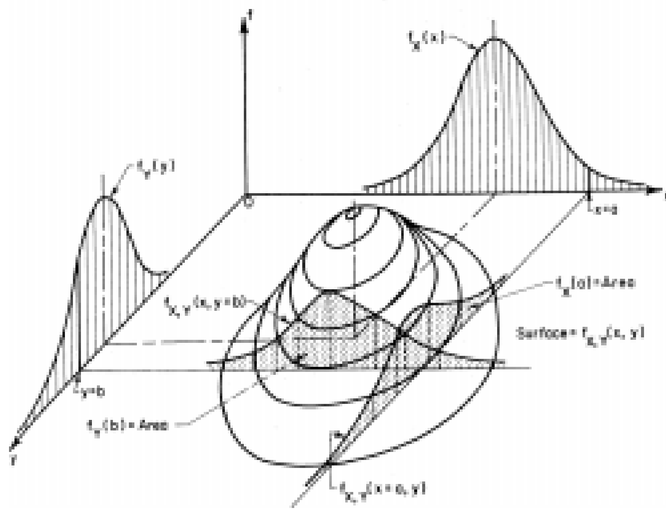

Figure 1: Joint and marginal probability density function of continuous criteria X and Y[17]

To further illustrate the concept of joint probability, an example for two continuous criteria, X and Y, is displayed in Figure 1. The joint probability function, $f_{X,Y}(x,y)$, creates the surface of a probability 'hump' in the x-y-f-space, characterized by rings of constant probabilities. The distribution curves over the x- and y-axis are the aforementioned marginal probability functions $f_X(x)$ and $f_Y(y)$, respectively. Also displayed in Figure 1 are two 'cuts' through the probability 'hump', marking the conditional probability distributions $f_{X,Y}(x = \mathrm{a}, y)$ and $f_{X,Y}(x, y = \mathrm{b})$ and their respective areas underneath $f_X(\mathrm{a})$ and $f_Y(\mathrm{b})$.

The last necessary concept to mention here for the development of a joint probabilistic formulation is the concept of *dependence of criteria*. Two random variables X and Y are said to be independent, if $f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$ otherwise X and Y are said to be dependent. This dependence is a mathematical notion and should not be confused with 'causal dependence'. A simple example for mathematical dependence without causal dependence is the number of times a person takes an umbrella to work and the number of times he wears long pants in a given month. The two numbers increase similarly with the number of rainy days in that month, i.e. they are (mathematically) dependent. They are, however, not causally dependent, since wearing pants does not depend on taking an umbrella or vice versa, but rather on the rain the person has to face on the way to work.

From here on, mathematical dependence will be referred to as *correlation*. Correlation is measured by the covariance of two criteria, X and Y, defined by

$$Cov(X,Y) = E[XY] - E[X]E[Y].^{17} \qquad (8)$$

It is more convenient, however, to use a covariance normalized by the standard deviations, σ_X and σ_Y, for both criteria, called *correlation coefficient*:[17]

$$\rho = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}. \qquad (9)$$

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[*] $P(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) = P[(X_1 = x_1) \cap (X_2 = x_2) \cap \ldots \cap (X_n = x_n)]$.

[†] Greatest lower bound.

[‡] $\Re^n$ denotes the set of all real valued n-touples.

The correlation coefficient is defined over the interval [-1,1], indicating strongly positively correlated criteria at values close to 1 and strongly negatively correlated criteria at values close to –1. The criteria are independent, if $\rho = 0$. In aerospace systems design ρ can be quite difficult to calculate by Equation 9. It is much more effective to view the correlation coefficient differently for calculation purposes. Jointly collected data from a probabilistic or any other analysis can be thought of as vectors of numbers. The correlation coefficient measures the orthogonality, i.e. independence, of both vectors. ρ is simply the cosine of the angle between the two criterion vectors, indicating their alignment. For $\rho = 1$, vectors are parallel and point in same direction, for $\rho = -1$, vectors are parallel and point in opposite direction. For $\rho = 0$, vectors are orthogonal and the criteria are independent. The correlation coefficient plays a significant role in the formulation of joint probability distribution models as described in the next section.

PROBABILITY FUNCTIONS

Attention is now directed to the implementation of this probabilistic formulation in the design process. The necessary transition from the mathematical formulation above to a probabilistic model that yields the information relevant for multi-variate decision making is described in this section. There are two alternatives for this task.

Joint Probability Model

The first joint probability density function introduced here is an analytical probability model for criteria whose univariate distributions and their corresponding means and standard deviations are known. All necessary information for the model can be generated by the traditional probabilistic design process, using its output of univariate criterion distributions. A particular model for two criteria with normal distributions, represented by Equation 10, has been introduced by Garvey and Taub.[18] Garvey further generated models for two criteria with combinations of normal and lognormal distributions, which are summarized in Ref. 19.

$$f_{XY}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}\exp\{\frac{1}{2\rho^2-2}[(\frac{x-\mu_X}{\sigma_X})^2 - 2\rho(\frac{x-\mu_X}{\sigma_X})(\frac{y-\mu_Y}{\sigma_Y}) + (\frac{y-\mu_Y}{\sigma_Y})^2]\} \quad (10)$$

Note that the only information needed for the Joint Probability Model consists of the means μ_X and μ_Y, the standard deviations σ_X and σ_Y, and the correlation coefficient ρ for the criteria X and Y.[§] The model variables, x and y, are defined over the interval of all possible criterion values. The advantage of this model is the limited information needed, which makes it very flexible for use and application. For example, if only expert knowledge and no simulation/modeling is available in the early stages of design, educated guesses for the means, standard deviations, and the correlation coefficient can be used to execute the joint probability model. It also lends itself to use in combination with increasingly important fast probability integration techniques.

Empirical Distribution Function:

The second probability function identified is the Empirical Distribution Function (EDF), named after the empirically collected data samples on which it is based. The *joint probability mass function* can be formulated as:

$$f(x_1, x_2, .., x_m) = \tfrac{1}{n}\sum_{i=1}^{n} I\left((a_{i1}, a_{i2}, .., a_{im}) = (x_1, x_2, .., x_m)\right) \quad (11)$$

and $I\left((a_{i1}, a_{i2}, .., a_{im}) = (x_1, x_2, .., x_m)\right) = \begin{cases} 1 & \text{for } (a_{i1}, a_{i2}, .., a_{im}) = (x_1, x_2, .., x_m) \\ 0 & \text{otherwise} \end{cases}$

a_i are the criterion sample values derived from a sampling method such as the Monte-Carlo simulation (MCS), while x_i are the criterion values of interest.

Consequently, the *joint cumulative probability distribution function* can be formulated as:

$$F(x_1, x_2, .., x_m) = \tfrac{1}{n}\sum_{i=1}^{n} I(a_{i1} \le x_1, a_{i2} \le x_2, .., a_{in} \le x_n) \quad (12)$$

and $I(a_{i1} \le x_1, a_{i2} \le x_2, .., a_{in} \le x_n) = \begin{cases} 1 & \text{for } (a_{i1}, a_{i2}, .., a_{im}) = (x_1, x_2, .., x_m) \\ 0 & \text{otherwise} \end{cases}$

The joint EDF depends on joint samples for the criteria only, and is not limited by any assumptions about variable distributions made beforehand. It does not rely on any particular sampling method either and can be used as long as sample data is available. The need for this data, on the other hand, is its very limitation, since it can only be used in a design process with available simulation data. Given enough sample data, however, the joint EDF yields the most accurate joint distribution prediction, since it does not rely on any approximation to generate the criterion statistics needed. Its greatest advantage lies in its lack of a requirement for a correlation coefficient, which can be difficult to estimate reliably in a design for new unconventional or revolutionary products. For very large numbers of sample data, the joint EDF can yield the exact solution for the joint distribution. However, in product design, a large number of process evaluations may not be a feasible option. In this case, the prediction accuracy of the Joint Probabilistic Model and joint Empirical Distribution Function are similar.[20]

JOINT PROBABILISTIC DECISION MAKING TECHNIQUE

A key issue in complex systems design is measuring the ‘goodness’ of a design, i.e. finding a criterion through which a particular design is determined to be the ‘best’. Traditional choices in aerospace systems design, such as performance, cost, revenue, reliability, and safety, individually fail to fully capture the life cycle characteristics of the system. Thus, a common approach has been to combine all criteria together into one equation termed the overall evaluation criterion, OEC. This equation is often very simple in its mathematical structure due to lack of any better model for the decision process. Recognizing this lack of proper decision process modeling, a different approach is proposed here, using the system attributes concurrently as *decision criteria* for the evaluation of designs. This evaluation is not based on a summation of criteria, but rather the probability of satisfying all criteria at the same

§ The normality assumption for the attribute distributions is already part of the model, however, it may also be regarded as information about the output distribution that helps to select the model.

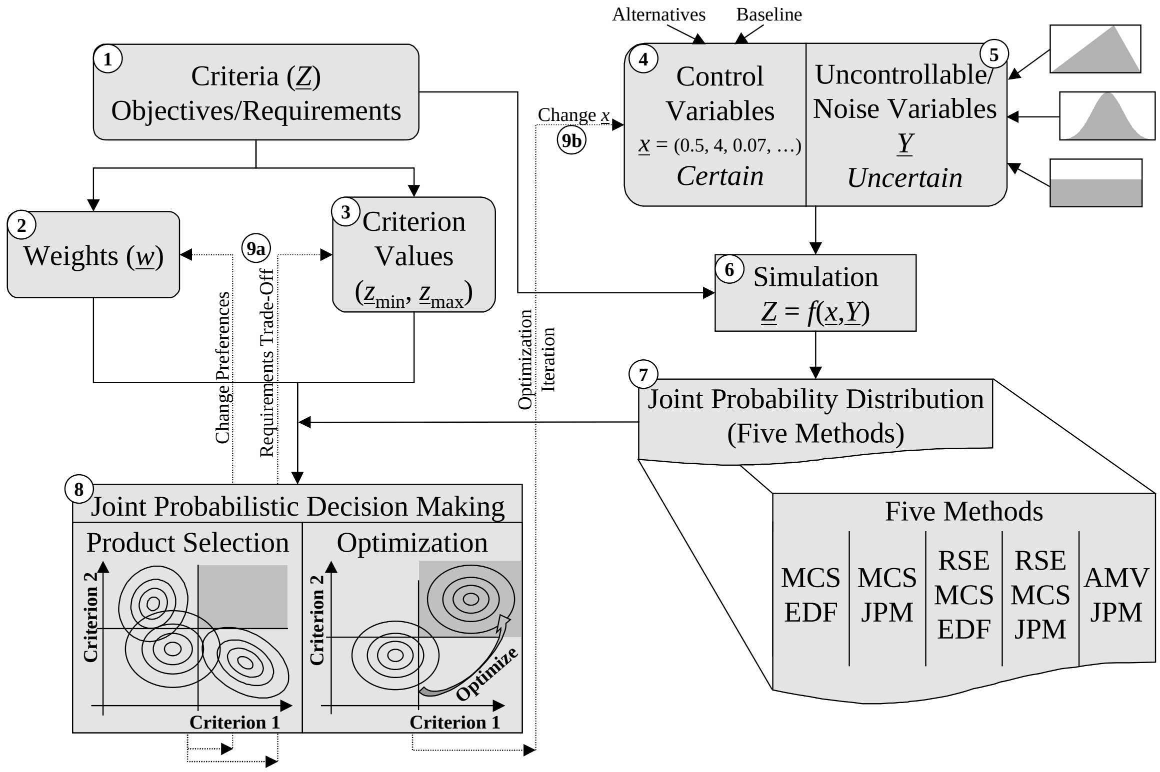

Figure 2: Joint Probability Decision Making Technique

time, a notion similar to a Pareto-optimality.** The main difference with respect to Pareto-optimality lies in the optimizable objective function, called probability of success (in satisfying all criteria).

This multi-criteria approach to decision making lends itself more suitably to aircraft design than a probabilistic single-criterion approach, since customers typically like to see all decision criteria satisfied. For example, a probabilistic multi-criteria approach can yield the design solution, which maximizes the probability of low cost, high capacity, speed, and dependability, while a single objective design will only yield an optimum in one of these criteria, neglecting all others.

An outline for determining the probability of success, assumptions its calculation is based on, as well as its use for product selection and optimization is presented in Figure 2. This nine step process is described in detail in the following sections.

STEP 1 – DEFINE CRITERIA

First, the criteria for the decision making process need to be determined. They are typically comprised of customer requirements†† or desirements‡‡ as well as objectives that need to be satisfied from the designer's perspective. These criteria are usually established in the early conceptual design phase of the product. The Joint Probabilistic Decision Making technique treats this set of criteria as a random vector,§§ represented by $\underline{Z}$.

STEP 2 – DETERMINE PREFERENCES AMONG CRITERIA

Next, customer or designer supplied preferences are established for each criterion. They are usually represented by a set, or vector $\underline{w}$, of (preference) 'weights' which are normalized to sum to 1, signifying the relative importance of each criterion.[3] If no criterion is associated with a prevalent preference over other criteria, all weights w_i, i = 1, 2, …, N, are assigned a value of 1/N, with N being the number of criteria.

STEP 3 – ESTABLISH CRITERION VALUES

Subsequently, values need to be identified which are considered sufficient for satisfying the particular criterion. These values include a minimum and a maximum, or infimum and supremum in the case of minus and plus infinity, listed in vectors $\underline{z}_{min}$ and $\underline{z}_{max}$. The space limited by these numbers is called the *area of interest* for multiple criteria. The probability for a design solution to produce criterion values within this area is called *probability of success* and denotes the objective function in this decision making technique.

STEP 4 – FIX CONTROL VARIABLES

Values for variables that are under the control of the designer are considered known (with certainty). For each probabilistic analysis, these control variables need to be held constant, while the vector $\underline{x}$ with their values is a representation of each alternative under consideration.

** State of economic affairs where no one can be made better off without simultaneously making another worse off.[21]

†† Customer supplied needs that must be fulfilled.

‡‡ Customer supplied wants that impact product design.

§§ Vector of random variables.

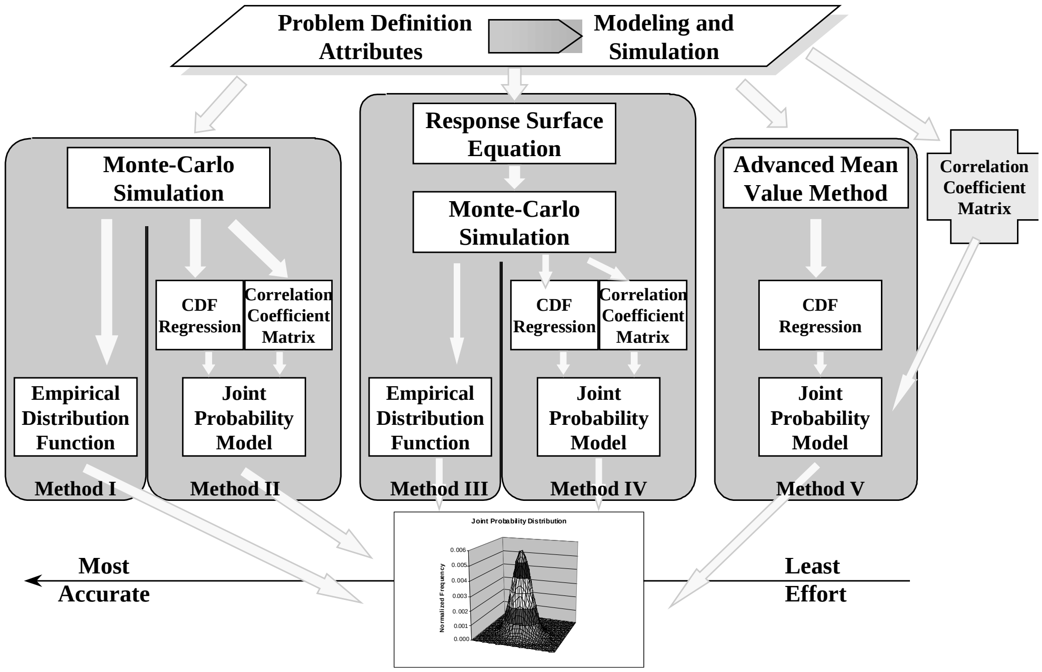

Figure 3: Step 7 - Five Methods for the Evaluation of the Joint Probability Distribution

STEP 5 – ASSIGN PROBABILITY DISTRIBUTIONS

All other variables that are not under the control of the designer, i.e. their values are not known with certainty, need to be assigned probability distributions that represent the likelihood of taking on certain values. These variables are often called noise variables. This step allows the subsequent simulation to use a range of values for these variables rather than a single value.

STEP 6 – SIMULATION

Next, a computer supported system analysis has to be identified that evaluates the criterion values based on the control and noise variables.

STEP 7 – EVALUATION OF THE JOINT PROBABILITY DISTRIBUTION

Step 7 consists of the determination of the joint probability distribution for all criteria. This step is the cornerstone of this technique and provides the objective function for the decision making process, Step 8. Five methods for the evaluation of the joint distribution have been developed so far, outlined in Figure 3 and the subsequent sections.

Method I – Monte-Carlo Simulation with Empirical Distribution Function

The first method uses data from the system analysis generated by a Monte-Carlo simulation as samples for the Empirical Distribution Function. This method is the most accurate, but also requires the most system analysis, which can be computationally expensive.

Method II – Monte-Carlo Simulation with Joint Probability Model

For the second method, a Monte-Carlo simulation is utilized to generate the data, which is regressed for each criterion individually to obtain a probability distribution function and it's associated mean and standard deviation. In addition, a correlation coefficient needs to be determined for each pair of criteria. The joint probability distribution can then be determined by using the correlation coefficient matrix and the statistics for each criterion distribution in a Joint Probability Model.

Method III – Response Surface Equation/Monte-Carlo Simulation with Empirical Distribution Function

Similarly to Method I, Method III employs a Monte-Carlo simulation, whereas the data is generated with a Response Surface Equation (RSE)*** rather than the computationally more expensive system analysis. While this method requires significantly less analysis to generate the RSE, its accuracy in predicting the joint probability distribution depends heavily on the prediction accuracy of the RSE itself. The obtained data is then used for an Empirical Distribution Function.

Method IV – Response Surface Equation/Monte-Carlo Simulation with Joint Probability Model

Similarly to Method II, Method IV employs a Monte-Carlo simulation and regresses the data in order to obtain mean and standard deviation for the individual criterion distributions. However, for the data generation an RSE is used rather than the computationally more expensive system analysis. For the Joint Probability Model necessary correlation coefficients can be obtained from

*** System analysis approximation based on Taylor series expansion.[11,12]

either the systems analysis data used for the generation of the RSE, or the Monte-Carlo simulation data based on the RSE. When the DOE table calls for only a few system simulations, the Monte-Carlo simulation data provides the greater statistical significance for the correlation coefficients and is to be preferred, despite the fact that the data is based on a system analysis approximation. When the DOE requires a significant amount of systems analysis already, the data might as well be used for the correlation coefficient estimation, since it is more accurate than the Monte-Carlo simulation data.

Method V – Advanced Mean Value Method with Joint Probability Model

The fifth method employs the Advanced Mean Value (AMV)[22] method, or one of its derivatives, to obtain an approximate cumulative distribution function for each criterion and its associated mean and standard deviation, which subsequently can be used in the Joint Probability Model. A disadvantage of this particular method, however, is its inability to provide the correlation coefficient, required by the JPM, which needs to be determined by some other means.

STEP 8 – JOINT PROBABILISTIC DECISION MAKING

The eighth step finally combines the criterion values, the weights, and the joint probability distribution function. All alternative solutions under consideration are ranked by their *probability of success* (*POS*), i.e. probability of satisfying all criteria. The higher the probability the better the solution. Mathematically, the joint probability of success is determined by:

$$POS = \frac{1}{M}\sum_{j=1}^{M} I(z_{j\,\min} \le z_j \le z_{j\,\max}), \tag{13}$$

for the EDF with M = number of samples, and

$$POS = \int_{\underline{z}_{\min}}^{\underline{z}_{\max}} f(\underline{z})\,d\underline{z}, \tag{14}$$

for the JPM.

Visually, the 'goodness' of a solution can be determined by how much its probability 'hump' overlaps with the area of interest, depicted in Figure 4. This, however, can obviously be exercised with two criteria at a time only.

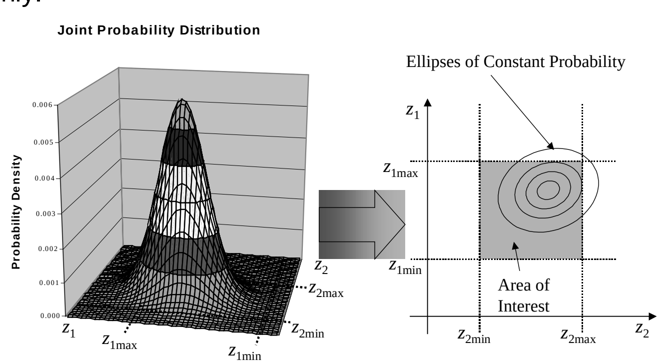

Figure 4: Display of a Sample Joint Probability Distribution for Two Criteria

Weight Adjusted Target Values

While the aforementioned probability of success has a particular meaning to the decision-maker, namely the chance the product has of satisfying the needs of the customer and the company, it does not account for certain preferences the decision-maker may have among the criteria. However, these preferences can be accounted for, if the probability of success is being evaluated based on *weight adjusted target values* $\underline{t}$ rather than pure criteria values. One proposed adjustment could be:

$$\underline{t}_{\min} = (\underline{w} \cdot N) \cdot \underline{z}_{\min}, \text{ and} \tag{15}$$

$$\underline{t}_{\max} = \frac{\underline{z}_{\max}}{(\underline{w} \cdot N)}, \tag{16}$$

with N = number of criteria.

This formulation essentially narrows the target range of interest for the criteria with high preference weights and widens its range of interest for the ones with low weights. The formulation for the probability of success consequently changes to:

$$POS = \frac{1}{M}\sum_{j=1}^{M} I(t_{j\,\min} \le z_j \le t_{j\,\max}), \tag{17}$$

for the EDF with M = number of samples, and

$$POS = \int_{\underline{t}_{\min}}^{\underline{t}_{\max}} f(\underline{z})\,d\underline{z}, \tag{18}$$

for the JPM.

It is essential to note that the *POS* in this case does not represent the numerical probability of achieving values within the area of interest spanned by the criteria, but rather yields a measured 'goodness' for the design alternatives, accounting for preferences among decision criteria. Hence, products fielded based on this formulation promise to yield a higher customer satisfaction than products which were fielded based on the aforementioned probability without criteria *preferencing*.[†††] This concept is also visualized in Figure 5, where one design solution (Alternative 1) appears to be the better one when no preferencing is applied, while the other solution (Alternative 2) appears to be the better one when preferencing is applied.

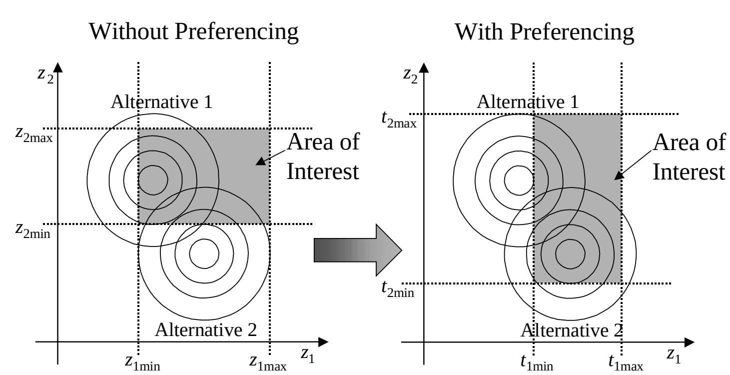

Figure 5: Comparison of Alternatives Based on Joint Probability With or Without Preferencing

[†††] Preferencing = Assigning weighting values to criteria in a decision making environment.

Product Selection

If several alternative design solutions are being evaluated at the same time, the Joint Probabilistic Decision Making technique can provide the objective function based on which the best alternative is determined. A joint probability of satisfying all criteria, with or without preferencing, can be assessed for each alternative, identifying the solution with the highest *POS*. This principle is displayed for two criteria and three alternatives in Figure 6, with Alternative 2 being the best solution. Subsequently, Step 9a will facilitate a desired change in preferences among the criteria or requirement traded-off study.

Optimization

If the decision making problem at hand is an optimization, i.e. finding the best solution within the design space spanned by a set of *design variables*, the joint probability serves as the objective function for the optimization routine. The optimal solution is found when the design with the highest *POS* is found within the design range. This notion is visualized in Figure 7 for two criteria, shifting a baseline (starting point) solution with low probability to the final (optimal) solution with a high probability of success. This optimization process requires the changing of design variables within an iteration loop, represented by Step 9b.

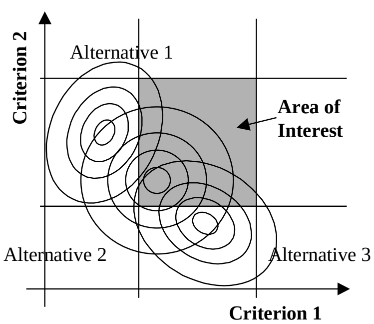

Figure 6: *POS* for Product Selection

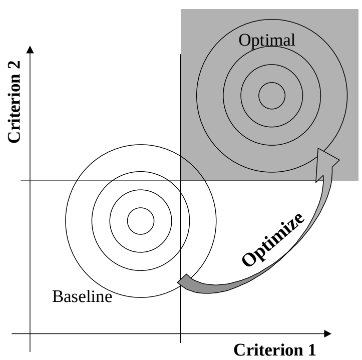

Figure 7: *POS* for Optimization

STEP 9A – CHANGE OF TARGET VALUES

Product selection problems often exhibit a change in criterion target values resulting from a need for '*What-If*' studies executed by the decision-maker. These studies greatly enhance the comprehension of the outcome of the probabilistic analysis and are typically comprised of a change in criteria preferences or a requirements trade-off procedure.

Change of Preferences

If the decision-maker decides to look at a different preference structure among the criteria, new weight values need to be assigned to all criteria. These new weights yield a change in target values, i.e. changing the area of interest. The modified area in return yields a different *POS* for each alternative. This process can be repeated until the decision-maker has a sufficient understanding of the dependence of the best design alternatives on the distribution of weights.

Requirement Trade-Off

A very common approach to reconcile situations in which no alternative yields a sufficiently high probability of success is a trade-off of requirements. In general one or more requirements are being relaxed in order to gain probability or tighten other requirements, while keeping *POS* constant. In other words, values for one requirement (or more) are being 'traded in' for values of another. Both principles are depicted in Figures 8 and 9.

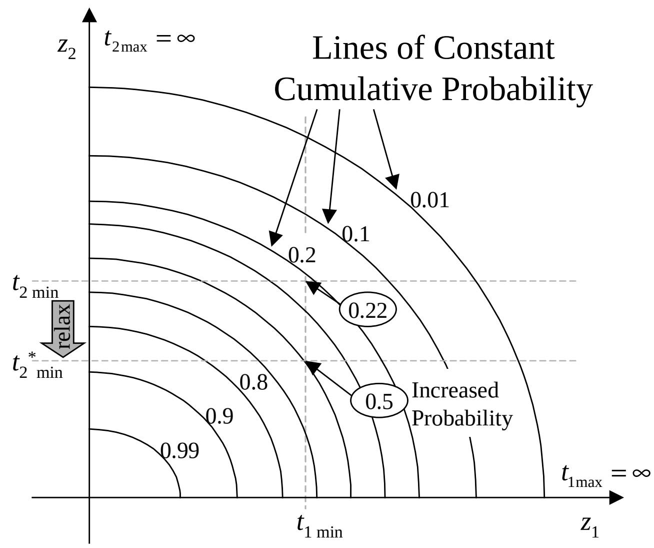

Figure 8: Requirement Trade-Off to Gain Probability

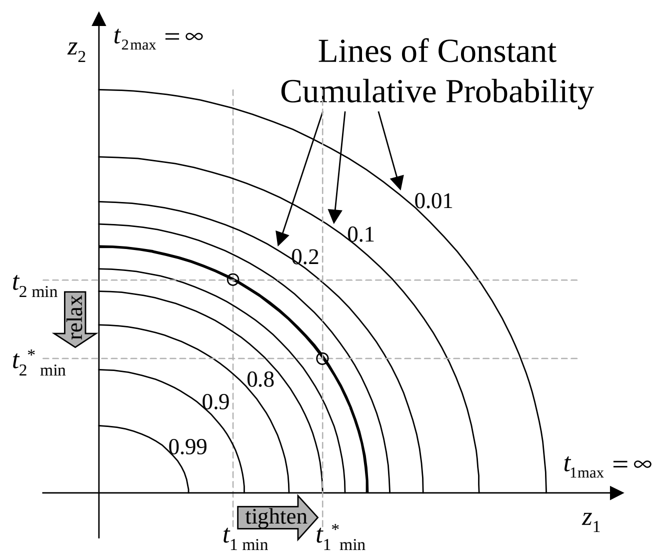

Figure 9: Trade-Off to Tighten Requirement

STEP 9B – OPTIMIZATION ITERATION

Step 9b represents the optimization routine used to change the input variables in the probabilistic design process in pursuit of the optimal solution. Thus, it closes the optimization loop with the nine-step process for joint probabilistic decision making. This iteration loop changes just the design variables. All design assumptions in form of probability distributions remain constant. The optimization process is terminated when no significant improvements to *POS* are possible.

Note that in the product selection process, with its change in criteria preferences and requirement trade-offs, the joint probability for the alternatives stay constant while the criterion targets are changing. Conversely, during the optimization process the joint probability distributions are changing and the targets typically stay constant.

IMPLEMENTATION AND PROOF OF CONCEPT

To demonstrate the application of the Joint Probabilistic Decision Making technique, an example problem has been formulated, determining the viability of three notional supersonic transport concepts. The basic configuration used for these alternative concepts is depicted in Figure 10. The vehicle has an area-ruled fuselage (maximum diameter of 12 ft.), a double delta planform, and four nacelles below the wing, housing mixed flow turbofan (MFTF) power plants. The values for some of the important design parameters are presented in Table I. The mission profile for these aircraft encompasses a split subsonic/supersonic mission, which results from the restriction of subsonic flight over land. The length of the subsonic cruise segment is assumed to be 15% of the design range. All concepts are supposed to carry roughly 300 passengers over a maximum distance of at least 5000 nautical miles. Each concept guarantees an average required yield per revenue passenger mile of ¢13 for the airline. The sale price is based on how much the airline is willing to pay rather then production cost plus profit for the manufacturer.

Table I: Description of the Aircraft's Layout

Parameter	Baseline
Span	77.5 ft.
Inboard Sweep	74 deg.
Outboard Sweep	45 deg.
Wing Reference Area	8,500 ft^2
Design Cruise Mach Number	2.4
Supersonic Cruise Altitude	~63,000 ft.
Sustained Load Factor	2.5 g

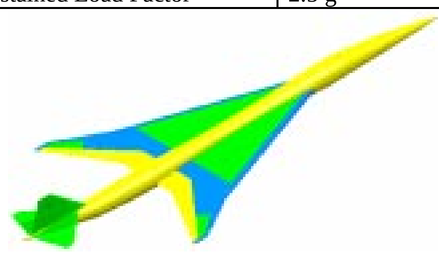

Figure 10: Notional Supersonic Transport

The three concepts considered are 1) a 'Baseline Configuration,' which has been subject to numerous trade-offs and optimization exercises in various publications,[13,20,23,25,26,27] 2) a maximum range design, and 3) a maximum number of passenger design. Together, they predominately constitute a payload-range trade-off. Each aircraft alternative was sized (through fuel balance iteration for the stated mission profile) under a common set of assumptions. For instance, a certain level of low speed aerodynamic capabilities were assumed, thus allowing many of the possible configurations to satisfy FAA constraints on take-off and landing balanced field lengths (less than 10,500 ft) and maximum approach speed (less than 154 kts). However, other key constraints for a supersonic transport, such as community and fly-over noise, were not addressed here. Thus, all designs are feasible, based on the applied constraints approach speed and takeoff and landing field length. To demonstrate the Joint Probabilistic Decision Making technique, each step is applied to the problem and outlined in the following section.

STEP 1 – DEFINE CRITERIA

Viability of a particular design concept is measured by the probability of satisfying certain desired levels of the return on investment for the airline (ROIA) and return of investment for the manufacturer (ROIM). Hence the criteria of the decision making process are *ROIA* and *ROIM*.

STEP 2 – DETERMINE PREFERENCES AMONG CRITERIA

For the first trial no preferences are identified in the decision-making process, i.e. $w_1 = w_2 = 1/N = 0.5$.

STEP 3 – ESTABLISH CRITERION VALUES

Both *ROIA* and *ROIM* are desired to be as large as possible. Therefore, $+\infty$ as a maximum value is assigned to both criteria. The minimum value that needs to be satisfied is identified as 10% for *ROIA* and 12% for *ROIM*.

STEP 4 – FIX CONTROL VARIABLES

This step is particularly important for distinguishing between alternative concepts, since all three have a lot of commonality. Only the number of passengers, the design range and the thrust-to-weight ratio varies between alternatives. Their values are listed in Table II together with the resulting take-off gross weight values. Note that this example constitutes a proof of concept for the JPDM technique only. For a true design problem many more variables and requirements would need to be considered concurrently.

Table II: Control Variable Settings for Each Alternative

Alternatives	Number of Passengers	Design Range	Thrust-to-Weight Ratio	Take-Off Gr. Weight
Baseline Config.	300	5000 nm	0.32	805 Klbs
Maximum Range	280	5185 nm	0.33	807 Klbs
Max # of Passengers	311	5000 nm	0.33	808 Klbs

STEP 5 – ASSIGN PROBABILITY DISTRIBUTIONS

All noise variables employed in this example are manufacturing and operational economic parameters that strongly influence *ROIM* and *ROIA* respectively. Their distributions, listed in Table III, have been selected to represent the likelihood of their values during operation. 'Learning Curve' changes all manufacturing learning curves. 'Load Factor' is the ratio of the equivalent full fare booked seats to the number of available seats, and 'Economic Range' is the average distance between city pairs the supersonic transport is scheduled to connect.

Table III: Noise Variable Distributions

Noise Variable	Range	Unit	Distribution Type	Parameter 1	Parameter 2	Parameter 3
Load Factor	65 - 85	%	Triangular	Mode: 70		
Learning Curve	0.75 - 0.85		Triangular	Mode: 0.8		
Fuel Price	0.6 - 1.20	$/gal	Weibull	Location: 0.6	Scale: 0.20	Shape: 1.6
Economic Range	3000 - Design Range	nm	Triangular	Mode: 3200		
Production Quantity	300 - 800		Weibull	Location: 300	Scale: 225	Shape: 2.25
Aircraft Price	200 - 300	K$	Normal	Mean: 250	Std Dev.: 6	

STEP 6 – SIMULATION

The simulation code used to model the three aircraft alternatives is a combination of the aircraft synthesis/ sizing code FLOPS[28] (FLight OPtimization System) and the Aircraft Life Cycle Cost Analysis (ALCCA) program.[29] It calculates values for *ROIA* and *ROIM* based on input values for the control and noise variables identified in Steps 4 and 5.

STEP 7 – EVALUATION OF THE JOINT PROBABILITY DISTRIBUTION

Method III is executed to determine the joint probability distribution, particularly the Probability of Success (*POS*), i.e. viability. In detail, a response surface equation is created for each criterion *ROIA* and *ROIM* as a function of the control and noise variables. The control variables are set to their corresponding values, and a Monte-Carlo simulation with the indicated noise variable distributions is executed for each design alternative. A regression of the individual sample data for *ROIA* and *ROIM*, to be used in Method IV, yields the sample means and standard deviations listed in Table IV. The correlation coefficient for *ROIA* and *ROIM* is determined to be $\rho = -0.07$. As a first observation, it can be noted that the manufacturer's return of investment varies considerably less between alternatives than the one for the airline. In order to display the distributions in the 2-D space spanned by *ROIA* and *ROIM*, Method IV is used to determine the location of the joint distributions. This transition is necessary to distinguish the different alternatives visually. However, due to the skewness of the data, see Figure 11, Method IV employing the JPM for normal distributions is not accurate enough to determine the *POS* itself. An extension of the JPDM technique is being developed that incorporates non-Gausian distributions for the JPM, capable of handling such skewed data as in this example.

Table IV: Sample Means and Standard Deviations

Alternatives	Mean *ROIA*	Std. Dev. *ROIA*	Mean *ROIM*	Std. Dev. *ROIM*
Baseline Configuration	11.0681	5.9370	11.4530	5.4674
Maximum Range	6.7633	6.4749	10.6292	5.4798
Max # of Passengers	13.4959	5.4863	10.4607	5.4875

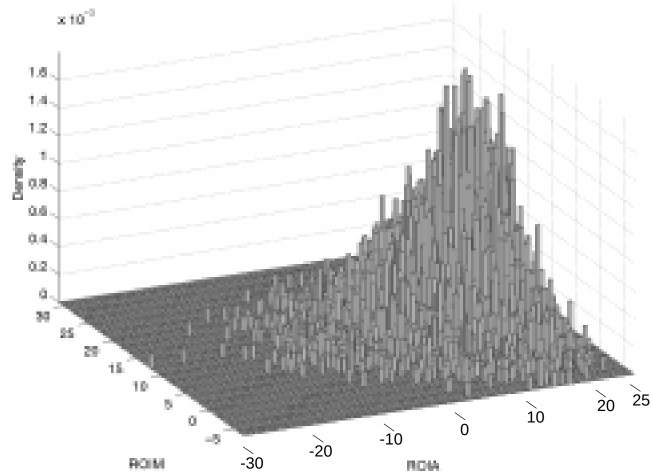

Figure 11: Joint Probability Distribution for the 'Baseline Configuration' Alternative

STEP 8 – JOINT PROBABILISTIC DECISION MAKING

Combining the joint probability distribution established in Step 7, the criterion values of interest from Step 3, and preferencing weights from Step 2, the Probability of Success can be determined for each alternative concept. Table V lists the *POS'* as well as the marginal probabilities for the two criteria to identify the most restrictive one.

Table V: Summary of *POS* for each Alternative

Alternatives	Joint *POS*	Prob. for *ROIA*	Prob. for *ROIM*
Baseline Configuration	0.2864	0.6373	0.4612
Maximum Range	0.1321	0.3473	0.4063
Max # of Passengers	0.2991	0.7750	0.3942

In principle, Table V constitutes the solution to the product selection problem at hand, identifying the 'Maximum # of Passengers' alternative as the best solution with the highest *POS*. This result is also reflected by the marginal distributions for *ROIA* and *ROIM*, since this concept generates the most revenue for the airline, given a fixed average yield and price of the aircraft, per economic scenario. But, more passengers increase the size, i.e. weight, i.e. production cost, of the aircraft, yielding a low return of investment for the manufacturer. The 'Maximum Range' concept suffers from the same disadvantage without the added benefit of more passengers, making it the least desirable solution of the three. However, a possible shift in the Load Factor distribution due to the increased range and hence market share has not been accounted for here. To further increase insight into the decision problem, an additional plot, displaying the location of the joint probability distributions and the area of interest is provided in Figure 12. The probability ellipses clearly indicate that the 'Baseline Configuration' has a larger chance of satisfying high values for *ROIM* than the 'Maximum # of Passengers' concept. Considering *ROIA*, the probabilities are reversed. Hence, with this plot or the marginal distributions alone, it would be difficult to determine which of the two is the 'better' concept. Only the use of the joint *POS* lets the decision-maker discriminate between the two solutions.

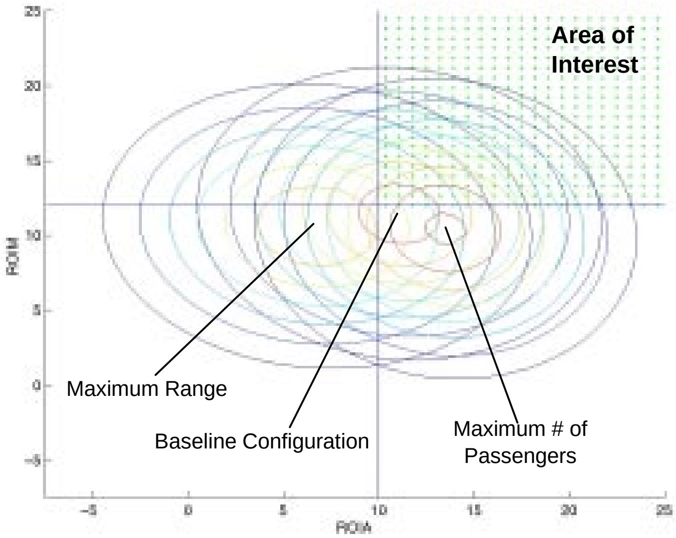

Figure 12: Location of Joint Probability Distributions

STEP 9A – CHANGE OF TARGET VALUES

Change of Preferences

If, based on the previous results, the decision-maker decides to change the preference values, new target values can be established that may yield another 'best' solution. Suppose satisfying the *ROIM* is 1.5 times more important than the *ROIA* criterion. Weighting values of 0.6 are assigned for w_{ROIM} and 0.4 for w_{ROIA}. With Equation 15 and these weights the target values change to

$$t_{ROIA} = (0.4 \cdot 2) \cdot 10\% = 8\%$$

$$t_{ROIM} = (0.6 \cdot 2) \cdot 12\% = 14.4\%.$$

These new target values, indeed, identify the 'Baseline Configuration as the best, since it yields higher *ROIM* values than the 'Maximum # of Passengers' concept and emphasis on *ROIA* has been reduced (compare *POS* values in Table VI). The actual probability of achieving these new target values, however, remains at the *POS* value listed in Table V. The change in target values, i.e. area of interest, is also displayed in Figure 13.

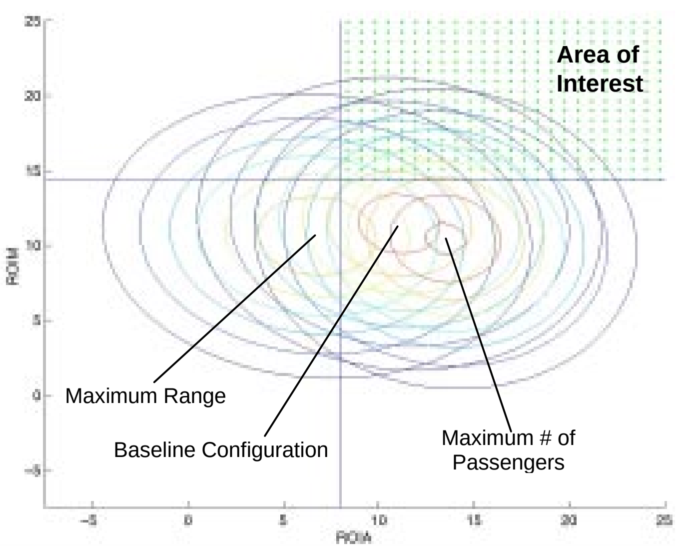

Figure 13: Location of Joint Probability Distributions and "Revised" Area of Interest

Table VI: *POS* Summary with w_{ROIM} = 0.4 and w_{ROIA} = 0.6

Alternatives	Joint *POS*	Prob. for *ROIA*	Prob. for *ROIM*
Baseline Configuration	0.2176	0.7436	0.3018
Maximum Range	0.1165	0.4936	0.2538
Max # of Passengers	0.2044	0.8540	0.2444

Requirement Trade-Off

If, based on the previous results, the decision-maker decides to increase the *ROIA* value that needs to be satisfied to 14% while keeping *POS* constant, *ROIM* has to decline. This process is demonstrated in Figure 14, displaying the joint cumulative distribution for the 'Maximum Number of Passengers' alternative and the two points of interest. The new *ROIM* value that allows the airline to have a return on investment of 14% is 9.4%. If the decision maker simply wants to increase *POS*, holding *ROIA* constant, *ROIM* has to decline. Using the same example in Figure 14, decreasing *ROIM* to 9.4% yields a *POS* of 0.44.

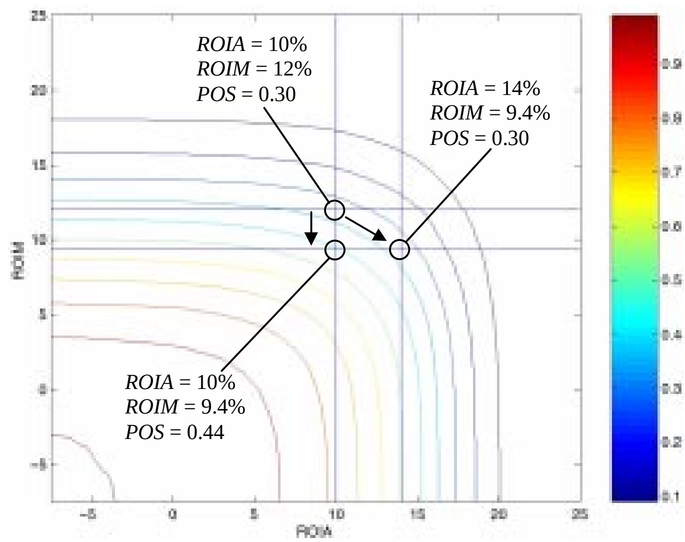

Figure 14: Requirement Trade-Off for *ROIM* and *RIOA*

Step 9b is omitted due to the fact that this example constitutes a product selection problem.

CONCLUSION

A Joint Probabilistic Decision Making (JPDM) technique is developed to help the designer and decision-maker identify the best possible solution for a multi-objective design problem. It utilizes the information generated by modern probabilistic design procedures and comprises this information in one evaluation criterion, the (joint) Probability of Success (*POS*). *POS* is the objective function used by traditional optimization methods for multi-objective optimization, or the selection criterion based on which the best design is identified among a closed set of alternatives. For a given set of values the *POS* is obtained through the use of one of two possible joint probability distributions for the decision-making criteria: the Empirical Distribution Function and Joint Probability Model. Five methods are presented that determine the joint probability distribution. The JPDM technique also allows the decision-maker to gain additional insight about the decision-making problem by facilitating requirements trade-off studies. Given this insight, the decision-maker is able to identify which criterion is the hardest to satisfy and by how much other criteria need to be relaxed in order to increase the chance of meeting the criterion goal. A proposed extension, which evaluates the sensitivity of *POS* with respect to the criteria or weighting factors, will further enhance the technique's capability.

The JPDM technique has been successfully applied to a product selection problem, identifying the best out of three supersonic transport alternatives based on the following criteria: probability of exceeding 10% for the return of investment for the airline (*ROIA*) and 12% the manufacturer (*ROIM*). The results indicate that the concept with the maximum number of passengers yields high values for *ROIA*, while the 'Baseline Configuration' yields high values for *ROIM*. Determining the best solution from just the marginal distributions for *ROIA* and *ROIM* is almost impossible. Only the use of *POS* reveals the 'Maximum Number of Passengers' concept as the better one, provided equal preference weighting for both criteria. If *ROIM* is more preferable than *ROIA* (by a 60/40 ratio), however, the 'Baseline Configuration' appears to be the better design solution. While this conclusion may seem intuitive for the presented example with two criteria, an n-dimensional product selection or optimization problem could prove to be difficult to solve without the use of the joint *POS* as an evaluation criterion.

ACKNOWLEDGEMENTS

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