Performance Limitations of Joint Variable-Feedback Controllers Due to Manipulator Structural Flexibility

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Abstract—The performance limitations of manipulators under joint variable feedback control are studied as a function of the mechanical flexibility inherent in the manipulator structure. A finite dimensional time domain dynamic model of a two-link, two-joint planar manipulator is used in the study. Emphasis is placed on determining the limitations of control algorithms that use only joint variable feedback information in calculations of control decision since most motion control systems in practice are of this kind. Both fine and gross motion cases are studied. Fine motion results agree well with previously reported results in the literature and are also helpful in explaining the performance limitations in fast gross motions.

NOMENCLATURE

$w_i(x_i, t)$ elastic deformation of link i at location x_i and time t

θ vector of joint angles ($[\theta_1, \theta_2]$)

δ vector of flexible-mode generalized coordinates

δ_{st} static values of flexible-mode generalized coordinates

u vector of effective torque at joints

x_θ $[\theta, \dot\theta]$

x_p plant full-state vector

x_m reference model state vector ($[\theta, \dot\theta]_{desired}$)

x $[\theta, \delta, \dot\theta, \dot\delta]$

u_m commanded input vector to the reference model

e error state vector ($x_m - x_\theta$)

v filtered error state

z output vector of the nonlinear time varying feedback block of the standard hyperstability problem

k_{ij} ij component of joint angle feedback gain matrix

c_{ij} ij component joint velocity feedback gain matrix

K_{pn} nominal joint variable (position and velocity) feedback gain matrix

K_{un} nominal feedforward gain matrix

ΔK_p adaptive state feedback gain matrix

ΔK_u adaptive feedforward gain matrix

p_{pi}, p_{ui} positive scalar constants of integral gain adaptation algorithm

w_{st}^* lowest natural frequency of the arm with all joints clamped

w_{bw} closed-loop bandwidth of the feedback-controlled flexible arm

w_{r1} closed-loop bandwidth of the feedback-controlled equivalent rigid arm

w_{mi} desired motion bandwidth (the natural frequency of the reference model that has step command input)

ξ_i damping ratio of mode i

R^n n-dimensional real vector space

$\in$ belongs to symbol

$\exists$ there exists symbol

$\{P\}$ dynamic systems defined by Popov class

$\rightarrow$ approaches symbol

LTI   linear time invariant

NLTV   nonlinear time varying

FFB   feedforward block

FBB   feedback block

AMFC   adaptive model following control

CLS   closed-loop system

I. INTRODUCTION

ROBOTIC manipulators have compliance that is inherent in their links and joints. The compliance becomes significant especially at high manipulation speeds and/or large payload conditions. Today, there is an increasing demand for manipulators with high speed, precision, and payload-handling capabilities as a result of higher productivity needs. Hence, manipulator flexibility and control has become an important problem. In some cases, structural flexibility in manipulators may be desirable. For instance, a manipulator cleaning delicate surfaces or handling household jobs needs to have significant structural flexibility so that errors in position control do not generate large forces that may damage the surface or become dangerous for the people in the house in case of accidents.

Regardless of the reason that the flexibility becomes significant (i.e., due to high speeds, large payloads, inherently very soft links for household services), precision control of the manipulator tip is necessary to accomplish the desired task.

Manipulator motions may be divided into two groups in terms of the range of motion: 1) fine motion and 2) gross motion. In fine motion, the manipulator tip moves in a small region of workspace. Despite high closed-loop bandwidth, absolute velocities do not become very large since the motion occurs in a small region. Therefore, the nonlinear dynamic forces (Coriolis and centrifugal) are generally negligible. In gross motion, the manipulator tip makes large rotational maneuvers in workspace. The large rotations of joints relative to each other are the main source of complicated nonlinear dynamic coupling between the generalized coordinates [39], [45], [46]. Absolute velocities may become large during the fast, large maneuvers to the point that the nonlinear dynamic forces become very dominant [28].

A. Review of the State of Art

The majority of work in control of robotic manipulators ignores the flexibility of the manipulator in the analysis. Therefore, no reference is made to the effect and/or limitations of flexibility in control system performance [2], [15], [16], [23]. In order to avoid the flexibility problem, very conservative controller design rules are suggested [28], [35]. At a time when researchers are striving to design high-performance controllers, it is logical to explicitly study the limitations imposed by the manipulator flexibility instead of taking conservative design measures. Closed-loop bandwidth limitations of nonadaptive joint variable feedback controllers were studied explicitly as function of arm flexibility in fine motion [5]. However, the results cannot be generalized to fast gross motions where dynamic nonlinear effects become significant. The dynamics of flexible manipulators are described by infinite dimensional mathematical models due to their distributed flexibility [4], [27], yet the controllers are designed based on truncated finite-dimensional models. The discrepancy between the designed performance and the actual performance achieved as a result of model truncation for the purpose of controller design is studied, and an iterative design procedure is suggested in [6]. Mathematical modeling of dynamics of flexible manipulators (flexible multibody systems) is studied by many different researchers using a variety of methods [7], [10], [21], [33], [43].

The class of control algorithms studied here, that is, algorithms that use only joint variable measurements, are particularly important since most industrial robots and mechanisms are controlled that way. Tip position measurements [8], [41], strain measurements along the flexible link [20], and tip acceleration measurements [24] are examples of attempts to design so-called *noncolocated* controllers that would achieve performance beyond the traditional limitations of *colocated* controllers. Gebler [18] included flexible dynamic considerations in the trajectory tracking control of a two-link, two-joint manipulator. A similar, but more general three-stage control algorithm, is presented in [37]. Their experimental results have shown that the tip position trajectory tracking of a flexible manipulator can be improved by taking the flexible behavior of the manipulator into account in the control algorithm.

A major problem associated with noncolocated control is the destabilizing effect of observation and control spillover [1],

[19]. Independent joint variable control of multilink manipulators, that is, each joint control action is based on the local measurement information of that joint only (colocated control), does not have this problem since spillover never drives the system unstable in colocated control [19]. This conclusion, however, cannot be extended to the class of joint variable controllers where intrajoint feedback is used to achieve decoupled joint response [5].

In short, joint variable feedback controllers require fewer sensors, have better stability robustness against spillover and unmodelled dynamics, and are widely used in practice. Therefore, it is worthwhile to study their potential use in fine and gross motion control of flexible manipulators, even though their upper limit of closed-loop bandwidth is in general considerably smaller than that of noncolocated controllers. In particular, the adaptive joint variable feedback controllers should be analyzed since they receive increasing interest due to the adaptability of feedback gains as a function of the changing task conditions.

B. Characterization of the Problem and Definitions

The significance of structural flexibility in motion control of a manipulator is a function of the task conditions. Any given manipulator can be moved slowly enough so that the structural flexibility will not cause any significant deviation from the intended motion. Similarly, it can also be moved fast enough so that the structural flexibility will become very apparent in the response of the manipulator (presuming the availability of actuators that can deliver sufficiently high torque/force levels).

Physically, every robotic manipulator has structural flexibility. The question of whether the controller needs to be concerned with it or not varies from task to task. At this point, one must quantify the term slow enough motions such that flexibility does not present any problem; we also must quantify the term fast enough motions, where the flexibility does present a problem.

The speed of motion is quantified as slow or fast (low, medium, or high speed) with respect to the structural flexibility of a manipulator using the lowest structural frequency of the manipulator when all joints are locked (w_{sl}^*) as the reference.

Book *et al.* [5] quantified the speed of a given fine motion relative to the structural flexibility using the ratio of necessary closed-loop bandwidth (w_{bw}) to the lowest structural frequency of the system (w_{bw}/w_{sl}^*). Given a manipulator and a desired fine motion, one can predict whether the structural flexibility will be significant or not during that motion using the ratio of (w_{bw}/w_{sl}^*).

In fast gross motion, where dynamic nonlinearities are dominant due to high joint speeds and large angular rotations, the notion of bandwidth is no longer a well-defined characteristic of the control system. However, in the context of model reference control, the speed of gross motion may be quantified using the bandwidth of the reference model (w_{m_l}) with a step input. Here, the (w_{m_l}/w_{sl}^*) ratio is proposed to quantify the speed of gross motions relative to the structural flexibility.

The essential difference between this work and other works in control of the single link flexible arm is that in the case of

multiple joints (two-joint, two-link example used in this study), there are many nonlinear couplings between the generalized coordinates of different links as a result of large angular rotations of joints. Most of these couplings do not exist in the single link case. Book et al. [5] and Book and Majette [6] studied the control aspects of the two-link, two-joint flexible manipulator example in fine motion using infinite-dimensional linear frequency domain models based on transfer matrices. Here, both fine and gross motion control aspects are studied using a finite-dimensional nonlinear time domain model.

The remainder of this paper is organized as follows: The mathematical model of a two-link, two-joint flexible manipulator is briefly described in Section II. Fine and gross motion control under joint variable feedback controllers are analyzed in Section III, and results are discussed in Section IV. The conclusions of this work are summarized in Section V. Design details of the proposed adaptive model following controller are presented in the Appendix.

II. DYNAMIC MODEL OF A TWO-LINK FLEXIBLE MANIPULATOR

Symbolic derivation details of dynamic models for flexible manipulators are described in [10]. The differences between the different Lagrangian-assumed mode-based modeling approaches come from the kinematic descriptions. Here, the kinematic description will be summarized, and a derivation using the Lagrangian-assumed mode approach will be skipped since it is a well-known standard procedure.

Let $(O_o X_o Y_o)$ be the inertial coordinate frame (Fig. 1). Assign two coordinates for each flexible link; one is fixed to the base (e.g., $O_1 X_1 Y_1$), and the other is fixed to the tip of the link (e.g., $O_2 X_1' Y_1'$). In order to describe the absolute position of any differential element on the links, let θ_1 and θ_2 describe the joint angles, and let $w_1(x_1, t)$, $w_2(x_2, t)$ describe the elastic deformations of links from the undeformed positions.

The spatial variable dependence of the deformation coordinates leads to a mathematical dynamic model that is of partial integro-differential equation form [27]. In order to simplify the model, the deformation coordinates are approximated by a finite series that consists of shape functions multiplied by time-dependent generalized coordinates:

$$w_i(x_i, t) = \sum_{j=1}^{n_i} \phi_{ij}(x_i)\delta_{ij}(t); \quad i = 1, 2$$

$$j = 1, \cdots, n_i$$

where n_i is the number of mode shapes considered in the approximation describing the elastic deformation of link i.

This results in a finite-order dynamic model. Since the spatial variable dependence is already specified through the shape functions, the mathematical model is of ordinary differential equation form. Let us order the generalized coordinates as $q = [\theta, \delta]$, where $\theta = [\theta_1, \theta_2]$ (the joint coordinates), and $\delta = [(\delta_{11}, \cdots, \delta_{1n_1})], (\delta_{21}, \cdots, \delta_{2n_2})]$ (the deformation coordinates). Having uniquely established the kinematic description of the manipulator, the derivation steps of the equations of motion via Lagrangian formulation is straightforward [4], [10]. The dynamic model of a flexible manipulator may be
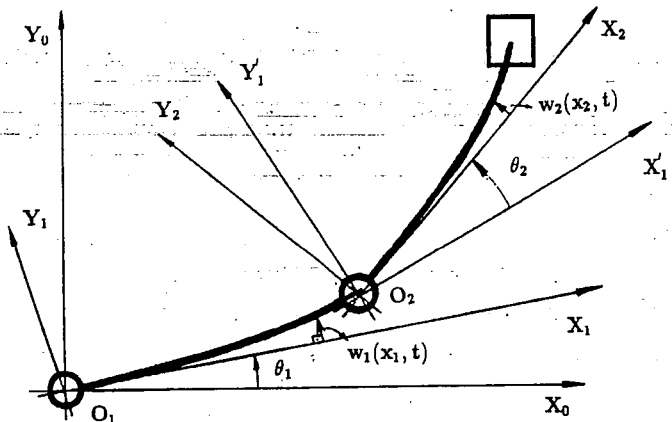

Fig. 1. Two-link flexible manipulator kinematic description.

TABLE I
MANIPULATORS DYNAMIC MODEL PARAMETERS USED IN THE ANALYSIS

Manipulator model parameters	Value
Geometric properties of uniform, slender links (link 1 and 2 are identical)	
Length of link i (l_i)	2.0 m.
Cross-section area of link i (A_i)	7.224×10^{-4} m^2
Cross-section area moment of inertia about z-axis (I_{zi})	7.6190×10^{-9} m^4
Link material properties (Aluminum)	
Mass density (ρ_i)	2768.8 kg/m^3
Young's modulus of elasticity (E_i)	$7.0 \times 10^{+10}$ Nt/m^2
Resultant link inertial and structural properties	
Mass per unit length ($\rho_i A_i$)	2.0 kg/m.
Mass of link i	4.0 kg
Flexural rigidity of link i ($E_i I_{zi}$)	533.33 and 5333.33 $Nt.m^2$
Lowest natural frequency of the arm (w_{cc1}) (both joint are locked, and $\theta_2 = 0$)	3.59 and 11.35 rad/sec
Joint inertial parameters	
Joint 1 and 2 masses (m_{j1}, m_{j2})	0.0
Joint 1 and 2 mass moment of inertia about the joint center of mass (J_{j1}, J_{j2})	0.0
Payload inertial properties	
Mass (m_p)	0.0 to 2.0 kg.
Mass moment of inertia about the center of mass (J_p)	0.0

expressed in the form

$$\begin{bmatrix} m_r(\theta, \delta) & m_{rf}(\theta, \delta) \\ m_{rf}^T(\theta, \delta) & m_f(\theta, \delta) \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\delta} \end{bmatrix} + \begin{bmatrix} f_r \\ f_f \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ [K]\delta \end{bmatrix} + \begin{bmatrix} g_r \\ g_f \end{bmatrix} = \begin{bmatrix} I \\ B_m \end{bmatrix} u \quad (1)$$

where $m_r(\theta, \delta)$, $m_{rf}(\theta, \delta)$, and $m_f(\theta, \delta)$ are partitioned elements of the generalized inertia matrix, which is always positive definite and symmetric, $f_r(\theta, \dot{\theta}, \delta, \dot{\delta})$, $f_f(\theta, \dot{\theta}, \delta, \dot{\delta})$ are Coriolis and centrifugal terms, which are quadratic in the generalized coordinate velocities $(\dot{\theta}, \dot{\delta})$, $g_r(\theta, \delta)$, $g_f(\theta, \delta)$ are gravitational terms, $[K]$ is the structural stiffness matrix associated with arm flexibility and mode shape functions, and u represents the effective torque (or force) vector at the joints. For the two-link arm example considered here, $\theta = [\theta_1, \theta_2]$, and since two mode shapes are used per link, $\delta = [(\delta_{11}, \delta_{12}), (\delta_{21}, \delta_{22})]$.

Equation (1) is a highly nonlinear and coupled ordinary differential equation set. This makes the controller synthesis and design a difficult problem. Furthermore, experiments [20] and analytical studies [12] indicate that the mode shapes of the links quickly converge to the mode shapes of clamped-base beam under joint variable feedback control for even low values of feedback gains of interest. All mode shapes of a clamped-base beam have zero slope at the base; therefore, $B_m = 0$ for the dynamics of flexible manipulators under feedback control. That means the joint variable controller effects the flexible variables through coupling from joint variables but not directly through the input matrix. The dynamic model of a rigid manipulator, in general, has the form

$$[M(\theta)]\ddot{\theta} + f(\theta, \dot{\theta}) + g(\theta) = u. \qquad (2)$$

The structural difference between the dynamics of rigid and flexible manipulator is displayed by (1) and (2).

III. Fine and Gross Motion Control with Joint Variable Feedback

The question of when the arm flexibility becomes significant and what limitations it imposes on the performance of joint variable controllers are first studied in fine motion. The results are valid only when the dynamic nonlinearities are negligible. In order to determine the effect of dynamic nonlinearities, the linear and nonlinear control algorithms are simulated on the nonlinear model (1).

A. Fine Motion Control

The nonlinear model (1) is linearized about a nominal configuration $x_n = [\theta, \delta, \dot{\theta}, \dot{\delta}] = [\theta_{nominal}, 0, 0, 0]$ and nominal input $u_{nominal}$, which compensates for the nominal gravitational loading. Notice that the manipulator is assumed to move in a small region of workspace where manipulation speed does not reach large values. However, in extreme situations where the input torque switches sign at high frequencies, this linearization assumption may no longer hold. Since nonlinear Coriolis and centrifugal terms are quadratic in $\dot{\theta}$, $\dot{\delta}$, they have no contribution to the model that is obtained by linearizing about a nominal configuration where nominal values of velocities are zero ($\dot{\theta} = \dot{\delta} = 0$). Let $\theta = \theta_{nominal} + \Delta\theta$, $\delta = \delta_{nominal} + \Delta\delta$, and $u = u_{nominal} + \Delta u$; then, the linear dynamic model about the nominal configuration $x_{nominal} = [\theta_{nominal}, 0, 0, 0]$ is given by (3):

$$\underbrace{\begin{bmatrix} m_r & m_{rf} \\ m_{rf} & m_f \end{bmatrix}}_{M_{eff}} \left\{ \begin{array}{c} \Delta\ddot{\theta} \\ \Delta\ddot{\delta} \end{array} \right\} + \underbrace{\begin{bmatrix} \partial g_r/\partial\theta & \partial g_r/\partial\delta \\ \partial g_f/\partial\theta & \partial g_f/\partial\delta + [K] \end{bmatrix}}_{K_{eff}}$$

$$\cdot \left\{ \begin{array}{c} \Delta\theta \\ \Delta\delta \end{array} \right\} = \left\{ \begin{array}{c} \Delta u \\ 0 \end{array} \right\}. \qquad (3)$$

In compact form, let $\Delta x = [\Delta\theta, \Delta\delta, \Delta\dot{\theta}, \Delta\dot{\delta}]$. The linear dynamic model about the given nominal configuration can be expressed as

$$\Delta\dot{x} = A\Delta x + B\Delta u \qquad (4)$$

where

$$A = \begin{bmatrix} 0 & I \\ -M_{eff}^{-1}K_{eff} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M_{eff}^{-1}\left(\begin{array}{c} I \\ 0 \end{array}\right) \end{bmatrix}. \qquad (5)$$

The closed-loop eigenstructure of the linear model under linear joint variable feedback controllers is studied as a function the feedback gains. The linear joint variable feedback control has the general form

$$\Delta u = -[K_{ij}]\Delta\theta - [C_{ij}]\Delta\dot{\theta}. \qquad (6)$$

For independent joint control

$$[K_{ij}] = \text{diag}\{k_{ii}\}$$

$$[C_{ij}] = \text{diag}\{c_{ii}\}.$$

For decoupled joint control

$$[K_{ij}] = m_r(\theta_{nominal}, 0)\text{diag}\{k_{ii}\}$$

$$[C_{ij}] = m_r(\theta_{nominal}, 0)\text{diag}\{c_{ii}\}.$$

Independent joint control results are presented here so that they can be compared with the previously reported ones. Position and velocity feedback gains of joint 1 (k_{11}, c_{11}) are set to very high values in order to force joint 1 behave like a clamped base. The loci of closed-loop eigenvalues are studied as a function of joint 2 feedback gains k_{22}, c_{22}. The finite-dimensional linear model should be able to predict at least the dominant behavior of the closed-loop dynamics of the infinite-dimensional actual system, despite the errors introduced due to truncated dynamics. Otherwise, the truncated finite dimensional model would be of no value.

By comparing the root locus behavior of a given flexible manipulator with that of an equivalent rigid manipulator, the conditions at which flexibility becomes significant and the range of conditions where the flexibility can be ignored can be determined and compared with reported results. The study of dominant behavior of closed-loop eigenvalues will determine the best possible performance in fine motion.

B. Gross Motion—Adaptive Model Following Control

The fundamental challenge in the control of industrial and space robots is to provide high-speed, high-precision motion, despite large variations in payload and other task conditions. Extensive research over the past decade has shown that adaptive control methods are more likely to meet that change than are nonadaptive control methods. It is desirable to have an adaptive controller that will achieve the following performance criteria:

1) Good transient and steady-state tracking of desired motion trajectory
2) high-speed and precision manipulation in gross and fine motion (high closed-loop bandwidth) relative to structural flexibility
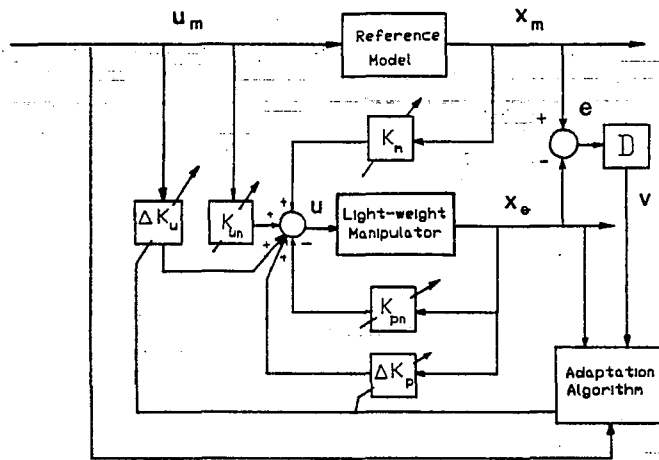
Fig. 2. Generalized inertia matrix-based AMFC.

3) good performance and stability robustness against unknown task condition variations.

An adaptive model following control (AMFC) algorithm is developed based on the hyperstability approach [9]. The design details are presented in the Appendix in order to keep the essential points of this paper in focus. Let us call $x_\theta = [\theta, \dot\theta]$. The adaptive control algorithm is given by (Fig. 2)

$$u = -K_{pn}x_\theta + K_{un}u_m + \Delta K_p(e, t)x_\theta + \Delta K_u(e, t)u_m$$

(7)

where

$$K_{pn} = m_r(\theta, \delta_{st})[[k_{ii}], [c_{ii}]] \qquad (8a)$$

$$K_{un} = m_r(\theta, \delta_{st}) \qquad (8b)$$

$$\Delta K_p = \int_0^t p_{pi}m_r(\theta_0, \delta_{st})\nu x_\theta^T \, d\tau \qquad (8c)$$

$$\Delta K_u = \int_0^t p_{ui}m_r(\theta_0, \delta_{st})\nu u_m^T \, d\tau. \qquad (8d)$$

$[k_{ii}]$ and $[c_{ii}]$ are the reference model dynamic components chosen by the designer, and δ_{st} is the static deflection values of flexible modes. These are obtained from the dynamic model (1) by setting the velocity and acceleration of state variables to zero and calculating the values of the mode shape variables (which correspond to static deflection values) as a function of *static loading* ($g_f(\theta, 0)$), *structural flexibility* ($[K]$), and *joint configuration* (θ vector). Note that as a result of these approximations, the control algorithm requires neither the real-time measurement nor the estimation of the flexible-mode variables.

Here, the reference model is chosen as a decoupled linear system of the form

$$\begin{bmatrix} \dot\theta_m \\ \ddot\theta_m \end{bmatrix} = \begin{bmatrix} 0 & I \\ [-k_{ii}] & [-c_{ii}] \end{bmatrix} \begin{bmatrix} \theta_m \\ \dot\theta_m \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u_m. \qquad (9)$$

The response of the reference model $\theta_m(t)$ to the commanded input $u_m(t)$ is the desired joint response. The reference model

dynamics affects the control through (8a), (8c), and (8d). Using δ_{st} in the control algorithm does not require real-time feedback information about the flexible states. Therefore, the controller is still a joint variable feedback control algorithm. The use of δ_{st} as opposed to 0 (zero) for the flexible modes is more accurate and improves the decoupled control of the flexible manipulator without imposing any significant implementation difficulty. The ν is the filtered tracking error e (Fig. 2). p_{pi} and p_{ui} are arbitrary positive scalar adaptive controller design parameters effecting the convergence rate of the adaptive control system and the transient response of the closed-loop system.

The specific dynamic characteristics of manipulators are utilized in the general context of hyperstability-based design so that the resultant controller is particularly suitable for the control of manipulators by exploiting their specific dynamic characteristics as opposed to treating them as a *black box* dynamic system. Following that philosophy, the generalized inertia matrix plays a significant role in the adaptation algorithm (8c), (8d) and in the nominal control (8a)–(8d). First, the feedback gains are naturally adapted in a manner that preserves the decoupled joint control. Second, arbitrary parameter selection that is generally required in Lyapunov and hyperstability-based designs is reduced to the selection of only *two scalar parameters*, no matter how many joints the manipulator has, as opposed to the usual requirement for selection of two arbitrary positive definite matrices [23]. Notice that the gain adaptation is of integral type (8c), (8d), which is a commonly used adaptation type in model reference adaptive control. Finally, the reference model used here is linear-time invariant. The use of other types of reference models involving dynamic nonlinearities may provide further performance improvements.

IV. RESULTS AND DISCUSSION

A. Fine Motion Control Results and Discussion

Let w_{st}^* be the lowest structural frequency of the manipulator when both joints are clamped and extended (k_{11} and $k_{22} \to \infty$, c_{11} and $c_{22} = 0$) (Fig. 3). Consider an equivalent rigid manipulator with the same inertial and geometric properties of the flexible manipulator. The rigid system with the first joint clamped ($k_{11} \to \infty$) will be a second-order mass-spring system with feedback gains (k_{22}, $c_{22} \neq 0$). Let w_{r1} be the undamped natural frequency of the rigid system for a set of feedback gains k_{22} and c_{22}.

In fine motion, the w_{r1}/w_{st}^* ratio determines the significance of flexibility and the dominant behavior of the closed-loop system. In the rigid manipulator case, it is possible to achieve an arbitrary large closed-loop bandwidth by increasing k_{22} and c_{22} for $w_{r1} = \sqrt{k_{22}/(J_{02})_{eff}}$ and damping ratio $\xi_{r1} = c_{22}/(2.0 \times \sqrt{(J_{02})_{eff} \times k_{22}})$, where $(J_{02})_{eff}$ is the effective moment of inertia of both the link 2 and the payload about the joint 2 axis of rotation.

However, when the same controller is applied to the flexible manipulator, the closed-loop bandwidth w_{bw} will definitely be smaller than w_{st}^* because as $k_{22} \to \infty$, $|w_{bw}| \to w_{st}^*$ with very little damping ratio (Fig. 3). If the servo stiffness is low relative to the structural flexibility, that is, $w_{r1}/w_{st}^* \ll 1/2$,
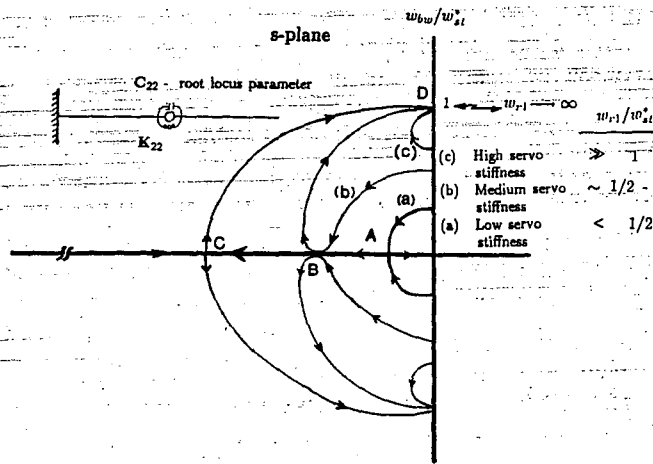
Fig. 3.   Illustration of performance limitations of joint variable feedback controller due to arm structural flexibility.

the locus of closed-loop eigenvalues is indistinguishable from those of the rigid case as c_{22} increases. However, if the velocity feedback gain c_{22} is further increased to large values, the effective result is a stiffening of the joint. One dominant eigenvalue meets with another on the negative real axis and breaks away from the real axis converging to the w_{st}^* magnitude on the imaginary axis as c_{22} increases (see curve (a) of Fig. 3 and Fig. 4(a)). In the rigid case, this phenomenon does not exist for any value of feedback gains. The root locus analysis of fine motion is done as a function of c_{22} for many other values of w_{r1}/w_{st}^*. The basic outcome of this analysis is illustrated in Figs. 3 and 4, where only the dominant regions of the root locus are shown. It is seen from Fig. 4(b) and (c) that above certain values of the w_{r1}/w_{st}^* ratio, the dominant eigenvalues are no longer able to reach the real axis. Physically, that means that if joint position control is too stiff relative to the arm flexibility, it is not possible to provide well-damped dominant modes no matter how large the velocity feedback gain.

For a given manipulator and payload, w_{st}^* is determined by the geometric, inertial, and structural flexibility properties of the manipulator. If a joint-variable controller attempts a closed-loop bandwidth that is larger than $(1/2)w_{st}^*$, the flexibility of the manipulator will be a significant factor during the fine motions. Otherwise, the structural flexibility may be ignored, and the controller may be designed based on rigid manipulator assumptions (see curve (a) of Fig. 3 and Fig. 4(a)). The best performance of a joint variable feedback controller is defined here as the highest possible closed-loop bandwidth (that is, the largest dominant eigenvalue magnitudes with sufficient damping ratios, i.e., 0.707 or more). As shown in Fig. 4(b), an approximately $(2/3)w_{st}^*$ closed-loop bandwidth can be achieved by the appropriate choice of feedback gains. It is equally important, however, to note that the dominant eigenvalues are very sensitive to the variations in feedback gains in the best performance region. In locations 8, 9, and 10 of Fig. 4(b), between each point, the velocity feedback gain is incremented by a constant amount. In practice, it may not be easy to realize that performance due to modeling errors.

The results concerning the effects of structural flexibility in closed-loop performance agree very well with the previously
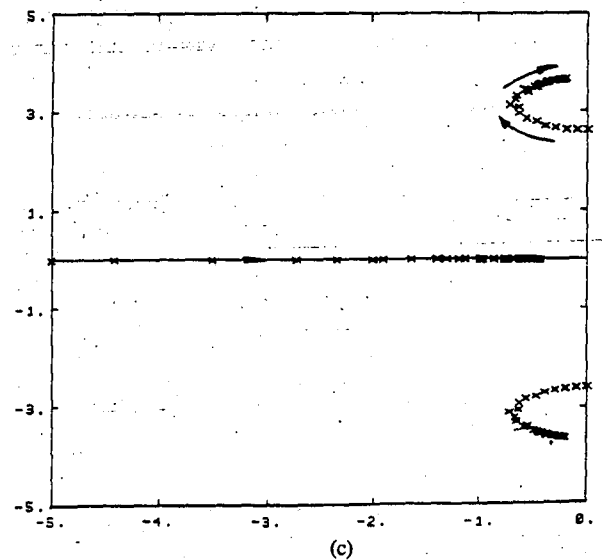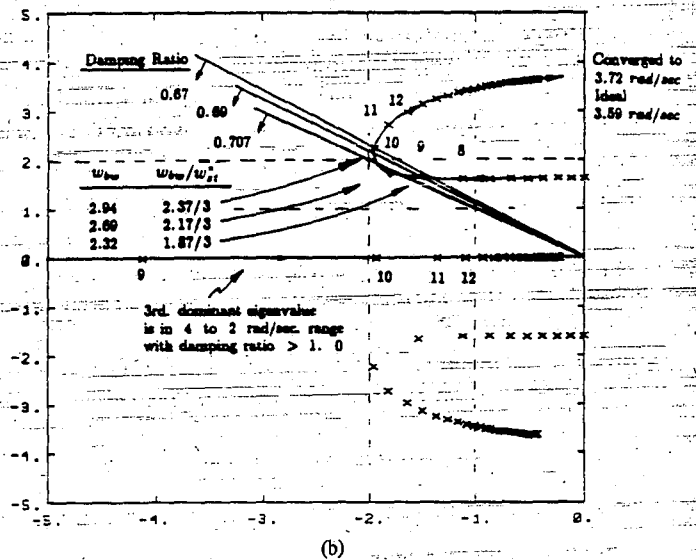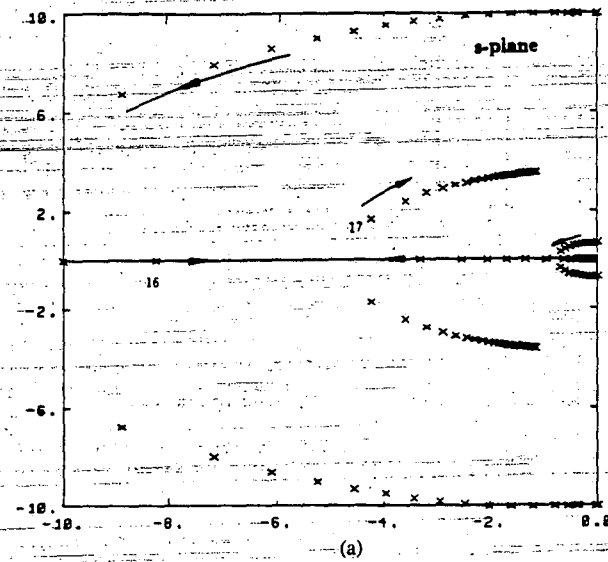

(a)

(b)

(c)

Fig. 4.   Locus of closed-loop eigenvalues as a function of joint 2 velocity position and velocity feedback gains: (a) low servo stiffness case; (b) medium servo stiffness case; (c) high servo stiffness case.
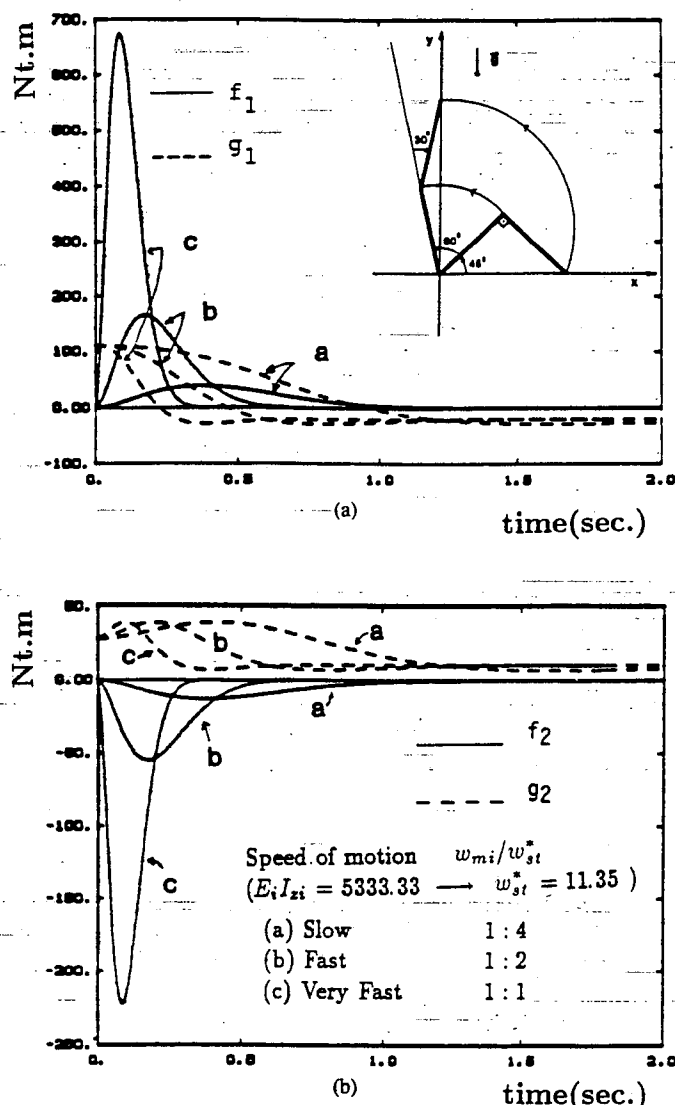
Fig. 5. Relative importance of (a) nonlinear (Coriolis and centrifugal) forces and (b) gravitational forces along different speeds of motion.

reported results based on infinite-dimensional frequency domain results [5], [6].

B. Gross Motion Control Results and Discussion

In order to see the effect of dynamic nonlinearities, the closed-loop system is simulated for two classes of motion: slow motion, where nonlinear forces are small (see curve a of Fig. 5(a) and (b)) and fast motion, where nonlinear forces are significantly larger or of the same magnitude as the other dynamic forces (see curve b of Fig. 5(a) and (b)).

Fig. 6 shows the response of the manipulator with the adaptive controller to the desired slow motion. Two different adaptive control results are shown for slow and fast adaption, which refers to the small and large values of the adaption parameters p_{pi} and p_{ui}, respectively. The appropriate values for these parameters are found by trial and error. This motion has two properties: 1) Dynamic nonlinearities are not significantly large (see curve a of Fig. 5), and 2) the bandwidth of the desired motion is about one-fourth of the lowest natural frequency of the arm. The bandwidth of the desired motion w_{mi} is defined as the bandwidth of the reference model, which

generates the desired motion in response to a step input command (Fig. 2).

Since the adaptive controller essentially tries to make the closed-loop dynamic behavior match that of the reference model, the function of w_{mi} in the nonlinear analysis content is similar to the function of the w_{r1} in the linear analysis. Clearly, Fig. 6(a)–(e) show that flexibility of the arm is not significant in terms of the joint tracking and the setting time of flexible vibrations at the end of motion, which is in agreement with the linear analysis results. When the same system is simulated for motion (b), where $w_{mi}/w_{cc1} = 1/2$ and nonlinearities are significant (curve b in Fig. 5(a) and (b)), the response deteriorates. Persistent, lightly damped oscillations occur in joint and flexible-mode variables (Fig. 7(a)–(e)). The difference between the two simulations (Figs. 6 and 7) is the magnitude of nonlinear forces (curves a and b in Fig. 5). When the nonlinear forces are significant compared with other dynamic forces, the performance is unacceptably poor. Therefore, the nonlinear effects during fast gross motions impose further restrictions on the performance of adaptive joint variable feedback controllers with integral gain adaptation.

The mechanism through which the nonlinear forces affect the joint controller performance can be described as follows with the help of the insights gained from the fine motion analysis: If the nonlinearities are significant, the adaptive controller automatically adjusts its feedback gains through integral adaptation (8c), (8d) to compensate for the tracking errors caused by the nonlinear forces. Increasing the controller gains through the adaptation rule eventually leads to very stiff joints. Linear analysis has shown that very high joint stiffness relative to the flexibility of a given arm results in very lightly damped dominant modes (see curve (c) of Fig. 3 and Fig. 4(c)). Thus, lightly damped dominant modes are generated by the adaptive controller while it is trying to compensate for the joint tracking errors caused by the large nonlinear forces. It is important to note that this mechanism is valid for the class of adaptive controllers that use *integral-type gain adaptation*.

V. CONCLUSION

In fine and slow gross motions, where Coriolis and centrifugal nonlinear forces are negligible, a given manipulator can be considered to be rigid if the controller does not attempt to reach a closed-loop bandwidth of more than half of the lowest structural frequency of the manipulator when all joints are locked (w_{st}^*). In fine motion, the best possible performance of joint-variable feedback controllers may be up to two thirds of w_{st}^* with damping ratios greater than 0.707. However, it is equally important to note that the sensitivity of the dominant eigenvalues to the variations of joint feedback gains are highest in the best performance region (locations 8, 9, and 10 of Fig. 4(b)). Therefore, it may be difficult to achieve a $(2/3)w_{st}^*$ closed-loop bandwidth in a practical situation due to the modeling errors. The fine motion analysis results obtained here based on a finite-dimensional time domain model agree very well with the previously reported results based on infinite-dimensional frequency domain models [5], [6].

The performance of an adaptive controller with integral gain adaptation is also shown to be limited by the structural flex-
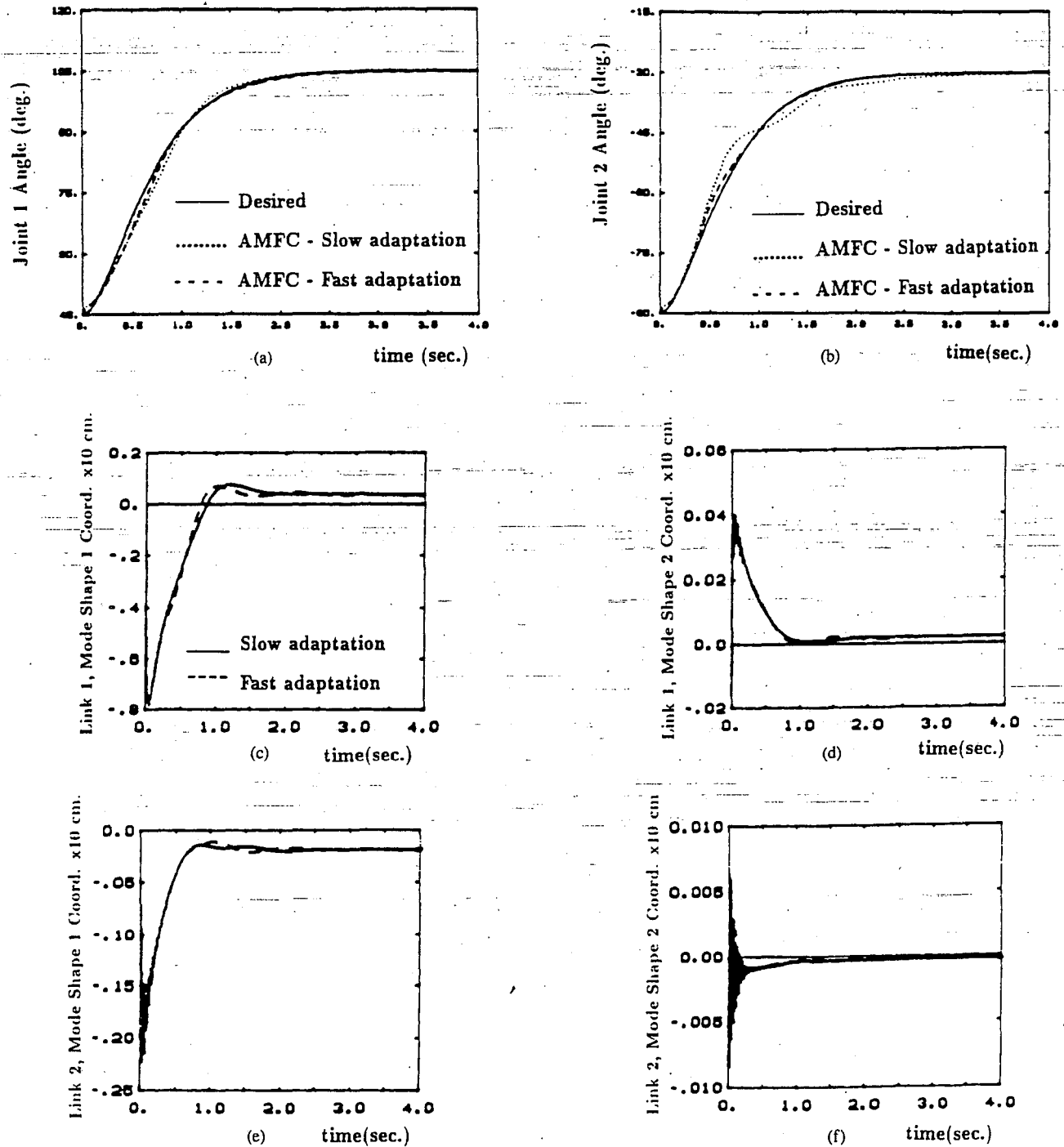
Fig. 6. Joint and flexible mode responses along the motion (a) of Fig. 5 under the AMFC controller.

ibility. Although the adaptation algorithm increases the feedback gains to provide good tracking in joint variables against the large nonlinear forces (curve b of Fig. 5), the same increase in feedback gains will result in very stiff joints, hence, persistent structural vibrations. Through that mechanism, the manipulator flexibility presents potential problems and limitations to the utilization of adaptive controllers with *integral-type gain adaptation*. The conclusions reached here are based on numerical results obtained by using a two-link, two-joint manipulator dynamic model. The general nature of dynamic coupling of multilink manipulators is observed in the model

used here. The only exception is that torsional vibrations may become significant in spatial manipulators. Therefore, it is expected that the conclusions of this work hold for other multilink manipulators with the provision on the significance of torsional vibration effects.

APPENDIX A

AMFC HYPERSTABILITY-BASED DESIGN

The basic idea of AMFC comes from the linear perfect model following control (LPMFC) problem of Erzberger [17].
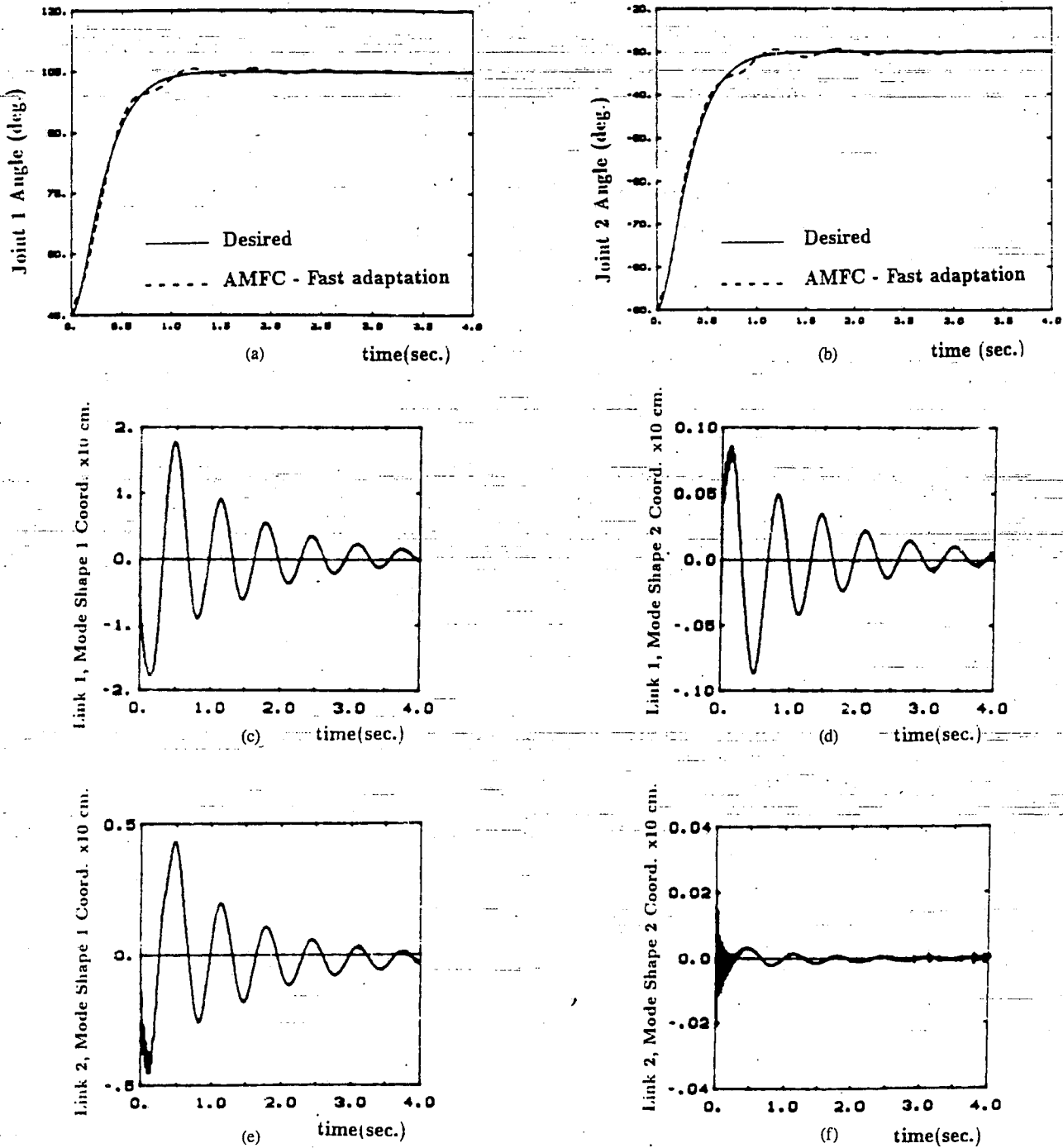
Fig. 7. Joint and flexible mode responses along the motion (b) of Fig. 5 under the AMFC controller.

AMFC attempts to asymptotically realize the same objective of LPMFC for time-varying systems.

Let the reference model be

$$\dot{x}_m = A_m x_m + B_m u_m \tag{10}$$

and the plant dynamics be in time-varying (quasilinear) form

$$\dot{x}_p = A_p(x_p, t)x_p + B_p(x_p, t)u_p \tag{11}$$

with the control algorithm of the form

$$u_p = -K_p x_p + K_u u_m + K_m x_m. \tag{12}$$

Clearly, as the plant dynamics $(A_p(x_p, t), B_p(x_p, t))$ vary, the feedback gains must also vary in order to match the dynamics of the plant to that of the reference model.

There are two basic assumptions associated with the current AMFC designs [25]:

1) There exist K_p, K_u, K_m for every $(A_p(x_p, t), B_p(x_p, t))$ and the given (A_m, B_m) so that at any instant, LPMFC conditions of Erzberger are satisfied.
2) Variations of $A_p(x_p, t)$, $B_p(x_p, t)$ are slower than the speed of adaptation.

Assumption 1) is an expected *existence* condition. AMFC attempts to converge to the ideally correct values of feedback gains through adaptation as the plant dynamics vary. *Existence* of such limit values is the first requirement for the convergence, let alone whether the adaptation algorithm will converge or not.

Assumption 2) is commonly made in most AMFC design methods. During adaptation intervals, it is assumed that time-invariant approximations of the plant model are accurate enough. Therefore, robot motions must be slow compared with the adaptation speed of the adaptive controller. Let us look at the origin of this assumption by going through the derivation steps of hyperstability-based AMFC design.

Letting $K_m = 0$, without loss of generality [25], the error dynamics are described by

$$\dot{e} = A_m e + [A_m - A_p(x_p, t) + B_p(x_p, t)K_p]x_p$$

$$+ [B_m - B_p(x_p, t)K_u]u_m. \quad (13)$$

For $e(t) \to 0$ as $t \to \infty$ for all x_p, u_m that belong to a piecewise continuous bounded class of functions, the coefficients of x_p, u_m must be zero. By assumption 1), there exist K_p^*, K_u^* such that

$$A_p(x_p, t) - A_m = B_p(x_p, t)K_p^* \quad (14a)$$

$$B_m = B_p(x_p, t)K_u^*. \quad (14b)$$

The goal is to develop adaptive control algorithms for K_p, K_u such that K_p, K_u converge to K_p^*, K_u^*. Convergence must be fast enough for assumption 2) to hold. Let the feedback gains be

$$K_p = K_{pn} - \Delta K_p(e, t) \quad (15a)$$

$$K_u = K_{un} + \Delta K_u(e, t) \quad (15b)$$

where K_{pn}, K_{un} are nominal, and $\Delta K_p, \Delta K_u$ are adaptive feedback gain matrices. Following the standard steps of hyperstability-based design [25], it can be shown that the equivalent hyperstable closed-loop system representation of the error dynamics can be expressed as (see Fig. 8)

$$\dot{e} = A_m e + B_p(x_p, t)z_1 \quad (16a)$$

$$v = De \quad (16b)$$

$$z = -z_1 = [K_p^* - K_{pn} + \Delta K_p]x_p$$

$$+ [K_{un} + \Delta K_u - K_u^*]u_m \quad (16c)$$

where D is determined by using the Kalman–Yakubovich–Popov lemma. In order to guarantee the hyperstability of the closed-loop system (CLS), the $\Delta K_p, \Delta K_u$ selection as follows is sufficient (not necessary):
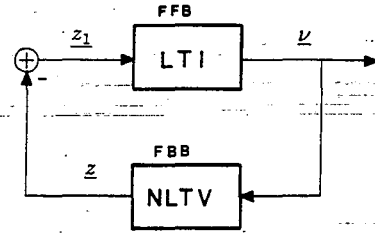

Fig. 8. Problem of hyperstability.

$$\Delta K_p(e, t) = \int_0^t \phi_1(v, t, \tau)\,d\tau + \phi_2(v, t) + \Delta K_p(0) \quad (17a)$$

$$\Delta K_u(e, t) = \int_0^t \psi_1(v, t, \tau)\,d\tau + \psi_2(v, t) + \Delta K_u(0) \quad (17b)$$

where the most general conditions on $\phi_1, \phi_2, \psi_1, \psi_2$ are discussed in [25], and more specific forms are discussed in Appendix B. $\Delta K_p(0), \Delta K_u(0)$ can be chosen as zeros without loss of generality since any nonzero values of them can be included in K_{pn}, K_{un} nominal gains. Substituting (17) into (16c)

$$z = -z_1 = \left[\int_0^t \phi_1(v, t, \tau)\,d\tau + \phi_2(v, t) + \Delta K_p^0 \right] x_p$$

$$+ \left[\int_0^t \psi_1(v, t, \tau)\,d\tau + \psi_2(v, t) + \Delta K_u^0 \right] u_m \quad (18)$$

where

$$\Delta K_p^0 = K_p^* - K_{pn} \quad (19a)$$

$$\Delta K_u^0 = -K_u^* + K_{un}. \quad (19b)$$

The hyperstability of the feedback block (hence, the CLS using Kalman–Yakubovich–Popov lemma) is proven for the $\Delta K_p^0, \Delta K_u^0$ constant case. That is where assumption 2) comes from.

The $\Delta K_p^0, \Delta K_u^0$ constant requirement implies that $(K_p^* - K_{pn})$ and $(K_u^* - K_{un})$ are constants. If K_{pn}, K_{un} are chosen to be *constant* nominal gains, K_p^*, K_u^* must be constant at least during the adaptation intervals. From (14), this implies that $(A_p(x_p, t), B_p(x_p, t))$ must be constant during the adaptation process. Equivalently, $(A_p(x_p, t), B_p(x_p, t))$ must vary slower than the speed of adaptation (which is assumption 2)).

Notice that the condition imposed by the hyperstability is not that K_p^*, K_u^* should be constant but that $(K_p^* - K_{pn})$ and $(K_u^* - K_{un})$ should be constant. If nominal feedback gains are not constant but are somewhat better in helping the plant track the reference model, then assumption 2) would not have been so restrictive. Choosing variable K_{pn}, K_{un} nominal gains based on the decoupled joint control algorithm [49] where generalized inertia matrix plays a significant role, assumption 2) may be *relaxed* as follows: The previous assumption 2) was that the difference between the reference model and the

closed-loop plant dynamics under *constant linear nominal control* should vary more slowly than the speed of adaptation. The new assumption 2) is that the difference between the reference model and the closed-loop plant dynamics under *variable nonlinear nominal control* should vary more slowly that the speed of adaptation.

APPENDIX B

GENERALIZED INERTIA MATRIX-BASED AMFC: APPLICATION TO FLEXIBLE MANIPULATORS

Consider the flexible manipulator model

$$\begin{bmatrix} m_r(\theta, \delta) & m_{rf}(\theta, \delta) \\ m_{rf}^T(\theta, \delta) & m_f(\theta, \delta) \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\delta} \end{bmatrix} + \begin{bmatrix} f_r \\ f_f \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ [K]\delta \end{bmatrix} + \begin{bmatrix} g_r \\ g_f \end{bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix} \quad (20)$$

$$m_r(\theta, \delta)\ddot{\theta} = u - [m_{rf}\ddot{\delta} + f_r + g_r] \quad (21)$$

$$u = \hat{g}_r + u_p \quad (22)$$

$$m_r(\theta, \delta)\ddot{\theta} = u_p + [m_{rf}\ddot{\delta} + f_r + (g_r - \hat{g}_r)] \quad (23)$$

where $\hat{g}_r$ is gravity compensation (feedforward). During the gross motion, nonlinear terms and coupling from the flexible modes to the joint variable dynamics are treated as a disturbance and are to be taken care of by the closed-loop system robustness.

Under the influence of a gravitational field, a flexible arm will deflect. Designing a control system that uses the static deflections as the nominal value for flexible states as opposed to zero would be more accurate.

Let the desired reference model be

$$\begin{bmatrix} \dot{\theta}_m \\ \ddot{\theta}_m \end{bmatrix} = \begin{bmatrix} 0 & I \\ -\Lambda_0 & -\Lambda_1 \end{bmatrix} \begin{bmatrix} \theta_m \\ \dot{\theta}_m \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u_m \quad (24)$$

and the control law

$$u_p = -K_p x_p + K_u u_m + \overbrace{K_m x_m}^{0}$$

$$= \underbrace{-K_{pn} x_p + K_{un} u_m}_{\text{Nominal control}} + \underbrace{\Delta K_p(e, t) x_p + \Delta K_u(e, t) u_m}_{\text{Adaptation algorithm control action}}.$$

$$(25)$$

The nominal control can be chosen in the form (as is used by the computed torque method) [9], [29]

$$u_{pn} = \hat{m}_r(\theta, \delta_{st}) u_m + \hat{m}_r(\theta, \delta_{st})$$

$$\cdot [[c_{ii} - \Lambda_1]\dot{\theta}_m + [k_{ii} - \Lambda_0]\theta_m + [k_{ii}]\theta_0]$$

$$- \hat{m}_r(\theta, \delta_{st})[[c_{ii}]\dot{\theta} + [k_{ii}]\theta]. \quad (26)$$

The nominal gains for the adaptive model following control algorithm based on the generalized inertia matrix is given by

$$K_{un} = \hat{m}_r(\theta, \delta_{st}) \quad (27a)$$

$$K_{pn} = \hat{m}_r(\theta, \delta_{st})[[k_{ii}], [c_{ii}]] \quad (27b)$$

$$K_{mn} = \hat{m}_r(\theta, \delta_{st})[[k_{ii}] - \Lambda_0], [c_{ii}] - \Lambda_1]]. \quad (27c)$$

If error dynamics eigenvalues are equal to those of the reference model, $k_{ii} = \Lambda_0$, $c_{ii} = \Lambda_1 \Rightarrow K_m = 0$. The $\hat{m}_r(\theta, \delta_{st})$ term in the control algorithm is the key for decoupled control of joints. The adaptation algorithm should be designed such that when it is added to the nominal control vector u_{pn}, the decoupled nature of the control is preserved. The adaptive part of the control is

$$\Delta K_p = \int_0^t F_{p1} v [G_{p1} x_p]^T \, d\tau + \underbrace{F_{p2} v [G_{p2} x_p]^T}_{\text{Proportional adaptation; } \Delta K_{pp}}$$

$$\underbrace{}_{\text{integral adaptation; } \Delta K_{pi}}$$

$$(28a)$$

$$\Delta K_u = \int_0^t F_{u1} v [G_{u1} u_m]^T \, d\tau + \underbrace{F_{u2} v [G_{u2} u_m]^T}_{\text{Proportional adaptation; } \Delta K_{up}}$$

$$\underbrace{}_{\text{Integral adaptation; } \Delta K_{ui}}$$

$$(28b)$$

Any positive definite matrix of appropriate dimension for F_{p1}, F_{p2}, G_{p1}, G_{p2}, F_{u1}, F_{u2}, G_{u1}, G_{u2} would be sufficient (but is not neccessary) to guarantee the global asymptotic stability of the control system with an appropriate output filter. For an *n*-degree-of-freedom system with *m* number of inputs F_{p1}, F_{p2}, F_{u1}, F_{u2}, G_{u1}, $G_{u2} \in R^{m \times m}$, and G_{p1}, $G'_{p2} \in R^{n \times n}$. There are too many design parameters that can be chosen arbitrarily from a large admissible class. Neither the hyperstability-based design nor Lyapunov methods give any guidelines for the selection of the elements of these matrices. As the system dimension increases, finding appropriate adaptation algorithm parameters becomes a more serious design problem.

The proposed AMFC design method solves that problem to a great extent. Since decoupled control calls for the use of the generalized inertia matrix, one should utilize this fact in the adaptation algorithm to direct the adaptation algorithm in the right direction. The following adaptation algorithm, which uses the generalized inertia matrix, will guarantee the global asymptotical stability of the closed-loop system:

$$\Delta K_p = \Delta K_{pi} + \Delta K_{pp}$$

$$= \int_0^t p_{pi} \hat{m}_r(\theta_0, \delta_{st}) v x_p^T \, d\tau + p_{pp} \hat{m}_r(\theta_0, \delta_{st}) v x_p^T$$

$$(29a)$$

$$\Delta K_u = \Delta K_{ul} + \Delta K_{up}$$

$$= \int_0^t p_{ui} \hat{m}_r(\theta_0, \delta_{st}) v u_m^T \, d\tau + p_{up} \hat{m}_r(\theta_0, \delta_{st}) v u_m^T.$$

(29b)

The generalized inertia matrix-based AMFC algorithm described by (25), (27), and (29) has the following advantages over previous algorithms:

1) The use of the generalized inertia matrix immediately solves the magnitude selection problem of the adaptation algorithm because it is naturally compatible with the problem in the sense that it preserves the decoupled joint control.

2) The number of design parameters for integral adaptation is only 2, for integral plus proportional adaptation is 4, no matter how many degrees of freedom the system has. Thus, the design problem of finding the good adaptation parameters becomes much simpler.

3) Utilizing the generalized inertia matrix as an integral part of adaptation improves the decoupled response of joint variables.

4) The use of variable nominal gains results in less restrictive conditions on the applications of AMFC to nonlinear systems.

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