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PROJECT ADMINISTRATION DATA SHEET

[X] ORIGINAL [ ] REVISION NO. ______

Project No. E-25-641 R6113-0A0 GTRC/GIT DATE 4 / 16 / 86

Project Director: Dr. Aldo A. Ferri School/Lab ME

Sponsor: Honeywell Avionics Division 13350 U.S. Highway 19 South Clearwater, FL 33546-7290

Type Agreement: P.O. No. 5277145-A and Revisions No. 01 and No. 02 Under NAS9-17367

Award Period: From 10/1/85 To ~~8/1/86~~ ~~9/30/86~~ 11-15-86 (Performance) 8/1/86 (Reports)

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Title: Influence of Nonlinear Joints on the Dynamics of Large Flexible Space Structures.

ADMINISTRATIVE DATA

OCA Contact R. Dennis Farmer X4820

1) Sponsor Technical Contact:

Dr. John F.L. Lee
Honeywell, Inc.
Space & Strategic Avionics Division
M/S 809-5
13350 U.S. Highway 19 South
Clearwater, FL 33546-7290 (813) 539-5667

2) Sponsor Admin/Contractual Matters:

Mr. Greg J. Geurtz
Honeywell, Inc.
Military Avionics Division
M/S 719-4
13350 U.S. Highway 19 South
Clearwater, FL 33546-7290 (813) 539-5842

Defense Priority Rating: DOC9

Military Security Classification: UNCLASSIFIED

(or) Company/Industrial Proprietary: N/A

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COMMENTS:

To initiate subcontracts and consulting agreements contract OCA subcontracts.
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Date 5-22-87

Project No. E-25-641

School/~~Lab~~ ME

Includes Subproject No.(s) N/A

Project Director(s) Dr. A.A. Ferri

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Sponsor Honeywell Avionics Division 13350 U.S. Highway 19 South, Clearwater, FL 33546-7290

Title Influence of Nonlinear Joints on the Dynamics of Large Flexible Space Structures.

Effective Completion Date: 11/15/86 (Performance) 12/15/86 (Reports)

Grant/Contract Closeout Actions Remaining:

- [ ] None
- [x] Final Invoice or Final Fiscal Report
- [x] Closing Documents
- [x] Final Report of Inventions - Questionnaire sent to P.I.
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E-25-641

The Influence of Nonlinear Joints on the Dynamics
of Large Flexible Space Structures

Progress Report Number 1
February 1986

Principal Investigator
Dr. A. A. Ferri

Introduction

This report consists of a review of research accomplished to date and plans for further research on the influence of nonlinear joints on the dynamics of large flexible space structures. A summary of accomplishments is presented first followed by a detailed account of work completed to date. The report concludes with an outline of activities planned for the next phase of research.

Summary of Accomplishments

1. One graduate student has been acquired to aid in the modelling and computational work.
2. A literature review has been conducted to investigate state-of-the-art modelling of space structure joints.
3. A generic model for a sleeve-type joint has been developed.
4. A component mode analysis technique was chosen as the method by which the equations for the three-beam model will be obtained.

Literature Review

A wide range of research has been directed recently to the dynamics of large flexible space structures. Much of this work has addressed the problems of control of large structures and problems of model reduction. See, for example, the proceedings of the VPI & SU/AIAA Annual Symposium on Dynamics and Control of Large Structures. Most of this work, however, has assumed that the structure behaves linearly, and very little material has addressed the problems of modelling and analysis of nonlinear joints.

Several authors have tried to estimate the contribution to a structure's overall damping level from joints. Hertz and Crawley have produced a number of papers dealing with the experimental and analytical determination of damping in jointed structures [1-4]. This work has dealt with actual joint models and with the influence of the joints on the dynamics of structures. Reference [1] also provides an analysis on the effects of size (scale) on the structure's damping level. Ashley [5] has also studied the effects of size on the overall damping of jointed structures, but does not address the problem of modelling. Beards, in References [6] and [7], has studied the contribution to a structure's overall damping by dry friction in joints.

The general problem of analyzing structures with dry friction has received considerable attention. See, for example, the Ph.D. thesis by Ferri [8] and the references cited therein. Many of the analysis techniques developed to analyze structures with nonlinear dry friction joints can be extended to analyze structures with other types of nonlinear joints.

The material most closely related to the present research is the previously cited research by Hertz and Crawley [1-3]. This work presents a number of different types of joint configurations and presents analytical models suitable for studying them. Thus, it provides a sound basis for the present modelling effort. It is felt that this work can be extended in a number of ways.

1. multi-mode descriptions of the linear portions of the structure can be used
2. multi-harmonic and/or time domain, numerical integrations can be used
3. more advanced models for complex connected systems can be used

Progress mode in these three areas will be discussed next in the **Summary of Research** section.

Summary of Research

The research to date has focused on the modelling of sleeve-type joints connecting beam-type elements. The major joint characteristics that need to be modelled in order to develop a general purpose sleeve joint model are:

- Dissipative Effects
  - damping due to impact
  - damping due to dry friction and beam end rotation
  - damping due to dry friction and beam end longitudinal motion
  - material damping due to deformation of the joint

- Geometry and Elastic Effects
  - overall beam/sleeve geometry including possible gaps between beam and sleeve
  - hardening spring characteristics due to large deformations of the beam and/or sleeve

An attempt has been made to account for the possibility of any of the above effects in the joint models. Although, as pointed out in [3], many of these effects are difficult to quantify, it is felt that the qualitative effects of each joint characteristic can be studied through parameter variation.

Joint Model

As mentioned above the type of joint that this study will concentrate on is the sleeve type joint which is commonly used to assemble truss structures. Often the sleeve joint has a "quick-connect" feature which enables it to be handled easily by astronauts in a space environment. Figure 1 shows a sketch of two beams joined together by a sleeve joint. For the purpose of modelling, it is assumed that the sleeve joint is composed of two parts - an outer sleeve which moves with one beam and an inner cylinder that moves with the other beam. Fig. 2 shows a close-up of the sleeve joint model. Six degrees-of-freedom are identified to describe planar motion of the sleeve-joint system. Note that y_1, x_1, and θ_1 are the lateral deflection, longitudinal deflection and rotation of the end of the left hand beam (beam 1) and y_2, x_2 and θ_2 represent the corresponding quantities for the right hand beam (beam 2). These six degrees of freedom fully describe the interaction of the outer sleeve and the inner cylinder. Later it will be explained how these local degrees of freedom can be combined with the beam's generalized coordinates using the technique of Component Mode Analysis. Three coordinate systems can be identified: the $\hat{i}$, $\hat{j}$ coordinate system which remains fixed in inertial space, the $\hat{i}_1$, $\hat{j}_1$ system that is fixed on sleeve 1 and the $\hat{i}_2$, $\hat{j}_2$ system that is fixed on inner cylinder 2. Note that the three systems are related by:

$$\begin{Bmatrix} \hat{i}_1 \\ \hat{j}_1 \end{Bmatrix} = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{Bmatrix} \hat{i} \\ \hat{j} \end{Bmatrix} \quad ; \quad \begin{Bmatrix} \hat{i}_2 \\ \hat{j}_2 \end{Bmatrix} = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{Bmatrix} \hat{i} \\ \hat{j} \end{Bmatrix} \tag{1}$$

Fig. 3 shows the undeflected position of the sleeve (dotted) together with a deflected state (solid lines). It is seen that the distance from point E to side BD can be expressed as:

$$\delta_E = \hat{j}_1 \cdot \vec{r}_{EB} \tag{2}$$

where

$$\vec{r}_{EB} = \vec{r}_{12} + \vec{r}_2 + \vec{r}_{EO_2} - \vec{r}_1 - \vec{r}_{BO_1} \tag{3}$$

and

$$\vec{r}_{12} = \ell \, \hat{i}$$

$$\vec{r}_2 = x_2 \, \hat{i} + y_2 \, \hat{j}$$

$$\vec{r}_{EO_2} = - \ell_2 \, \hat{i}_2 - \frac{d_2}{2} \hat{j}_2 \tag{4}$$

$$\vec{r}_1 = x_1 \, \hat{i} + y_1 \, \hat{j}$$

$$\vec{r}_{BO_1} = - \frac{d_1}{2} \hat{j}_1$$

substituting (1), (3) and (4) into (2) gives:

$$\delta_E = (x_1 - x_2 - \ell) \sin\theta_1 + (y_2 - y_1) \cos\theta_1 + \ell_2 \sin(\theta_1 - \theta_2)$$
$$- \frac{d_2}{2} \cos(\theta_1 - \theta_2) + \frac{d_1}{2} \tag{5}$$

or, using small angle approximations:

$$\delta_E = (x_1 - x_2 - \ell)\theta_1 + (y_2 - y_1) + \ell_2(\theta_1 - \theta_2) + \frac{d_1 - d_2}{2} \tag{6}$$

Similar expressions exist for the distances from F to side AD, δ_F, from D to side FG, δ_D, and from C to side EH, δ_C.

In order to facilitate the determination of contact forces, it is assumed that springs exist between the inner cylinder and the sleeve as shown in Fig. 4. These springs can be characterized as shown in Fig. 5, where $x = \frac{d_1 - d_2}{2} - \delta$. Note that each springs can transmit force in one direction only, has a dead zone or threshold, and has a cubic stiffness characteristics once contact occurs. These springs will allow for the transmission of forces and movements from one beam to the other. They also facilitate the determination of normal forces which are needed to calculate the friction forces.

The friction force can be modelled as shown in Fig. 6. There are two coefficients of friction - a static coefficient of friction, μ_s, which applies when sticking occurs and a dynamic coefficient of friction, μ_d, which applies during slipping. The relative velocity of a potential contact point can be found using vector relations. For example, the absolute velocity of point E of the inner cylinder is given by:

$$\vec{V}_E = \frac{d}{dt}(\vec{r}_2 + \vec{r}_{EO_2}) \tag{7}$$

$$\vec{V}_E = \dot{\vec{r}}_2 + \dot{\vec{\theta}}_2 \times \vec{r}_{EO_2}$$

$$\vec{V}_E = (\dot{x}_2 + \dot{\theta}_2 \ell_2 \sin\theta_2 + \dot{\theta}_2 \frac{d_2}{2} \cos\theta_2)\,\hat{i}$$

$$+ (\dot{y}_2 - \dot{\theta}_2 \ell_2 \cos\theta_2 + \dot{\theta}_2 \frac{d_2}{2} \sin\theta_2)\,\hat{j} \tag{8}$$

Similarly, the absolute velocity of point B on the outer sleeve is given by:

$$\vec{V}_B = (\dot{x}_1 + \frac{d_1}{2}\dot{\theta}_1 \cos\theta_1)\,\hat{i} + (\dot{y}_1 + \frac{d_1}{2}\dot{\theta}_1 \sin\theta_1)\,\hat{j} \tag{9}$$

The relative velocity of point E with respect to point B in the direction of side BC is:

$$V_{EB} = (\vec{V}_E - \vec{V}_B) \cdot \hat{i}_1 = (\vec{V}_E - \vec{V}_B) \cdot (\cos\theta_1\,\hat{i} + \sin\theta_1\,\hat{j}) \tag{10}$$

$$\begin{aligned} V_{EB} &= (\dot{x}_2 - \dot{x}_1)\cos\theta_1 + (\dot{y}_2 - \dot{y}_1)\sin\theta_1 + \dot{\theta}_2 \ell_2 \sin(\theta_2 - \theta_1) \\ &\quad + \dot{\theta}_2 \frac{d_2}{2}\cos(\theta_2 - \theta_1) - \frac{d_1}{2}\dot{\theta}_1 \end{aligned} \tag{11}$$

or, using small angle approximations:

$$V_{EB} = \dot{x}_2 - \dot{x}_1 - \frac{d_1}{2}\dot{\theta}_2 + \frac{d_2}{2}\dot{\theta}_2 + (\dot{y}_2 - \dot{y}_1)\,\theta_1 + \dot{\theta}_2\ell_2(\theta_2 - \theta_1) \tag{12}$$

The friction force of side BC on point E, $\vec{f}_E$ could be described by:

$$\begin{aligned} \vec{f}_E &< N_E\,\mu_s\,(\pm\,\hat{i}_1) && \text{if } V_{BE} = 0 \\ \vec{f}_E &= -\,N_E\,\mu_d\,\text{sign}\,(V_{EB})\,\hat{i}_1 && \text{if } V_{EB} \neq 0 \end{aligned} \tag{13}$$

where the normal force, N_E, is given by

$$N_E = K_E\,(\frac{d_1 - d_2}{2} - \delta_E) \tag{14}$$

or

$$N_E = \begin{cases} 0 & \delta_E \geqslant 0 \\ K_1|\delta_E| + K_3|\delta_E|^3 & \delta_E < 0 \end{cases} \tag{15}$$

Note that $K_E(x)$ is the function of x as described by Fig. 5 and not K_E times x. Similar other expressions can be found for the relative slip velocities and the friction forces at the other potential contact points.

Once accurate models are found for all the contact "spring" forces and friction forces, the overall equations of motion for the beam-sleeve system can be found using Component Mode Analysis (CMA). (See, for example, Refs. [9] and [10] and Appendix A of Reference [8]). This method basically consists of writing expressions for the total kinetic energy, potential energy and virtual work for the entire system and then introducing constraints between the coordinates. For the two beam system being considered here, the constraints would be:

$$f_1 = W_L\ (\xi_1 = L_1, t) - y_1 = 0$$

$$f_2 = X_L\ (\xi_1 = L_1, t) - x_1 = 0$$

$$f_3 = W_{L,\xi}(\xi_1 = L_1, t) - \theta_1 = 0 \tag{16}$$

and

$$f_4 = W_R(\xi_2 = 0, t) - y_2 = 0$$

$$f_5 = X_R(\xi_2 = 0, t) - x_2 = 0$$

$$f_6 = W_{R,\xi}(\xi_2 = 0, t) - \theta_2 = 0 \tag{17}$$

where W_L and W_R are the transverse displacement of the left and right beams respectively; ξ_1 and ξ_2 are the length coordinates along the left and right hand beams respectively as shown in Fig. 1; and X_L and X_R are the longitudinal displacements of the left and right hand beams respectively. Here, we will consider X_L and X_R as being composed of rigid-body longitudinal displacements and shortening effects from transverse beam deflection.

Total kinetic energy, total potential energy, total virtual work plus the constraints and Lagrange multipliers are used to formulate a modified Lagrangian, which is then used to find the equations of motion for the entire system. This type of approach can be used regardless of the type of description used for the beam dynamics - modal description, finite element description or both.

Outline for Future Research

1. An analytical model for impact energy dissipation will be developed and incorporated into the total joint model.
2. Equations of motion will be found for the three beam model interconnected with two sleeve joints. These equations will be computer coded for numerical integration simulations.
3. An attempt will be made to simplify the joint model in order to reduce computation costs and simplify the analysis.
4. Frequency domain analysis of the three beam system will be performed. One harmonic solutions will be compared with time integration results in order to determine the accuracy of the single temporal harmonic solution.
5. An estimate of the modal damping of the system for the first few modes will be determined.

References

1. Hertz, T. F. and Crawley, E. F., "The Effects of Scale on the Dynamics of Flexible Space Structures," Report #SSL 18-83, Space Systems Laboratory, Dept. of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, MA, Sept. 1983.

2. Hertz, T. J. and Crawley, E. F., "Damping in Space Structure Joints," AIAA Dynamics Specialists Conference, May 17-18, 1984, Palm Springs, California.

3. Crawley, E. F., "Nonlinear Characteristics of Joints as Elements of Multi-Body Dynamic Systems," Workshop on Computational Methods for Structural Mechanics & Dynamics, NASA Langley Research Center, Hampton, VA, June 19-21, 1985.

4. Crawley, E. F., Sarver, G. L., and Mohr, D. G., "Experimental Measurement of Passive Material and Structural Damping for Flexible Space Structures," Acta Astronautica, Vol. 10, No. 5-6, 1983, pp. 381-393.

5. Ashley, H., "On Passive Damping Mechanisms in Large Space Structures," AIAA J. of Spacecraft and Rockets, Vol. 21, No. 5, 1984, pp. 448-455.

6. Beards, C. F. and Woowat, A., "The Control of Frame Vibration by Friction Damping in Joints," ASME J. of Vibrations, Acoustics, Stress, and Reliability in Design, Vol. 106, Jan. 1985, pp. 26-32.

7. Beards, C. F., "The Damping of Structural Vibration by Controlled Interfacial Slip in Joints," ASME J. of Vibration, Acoustics, Stress and Reliability in Design, Vol. 105, July 1983, pp. 369-373.

8. Ferri, A. A., "Dynamics of Dry Friction Damped Systems," Ph.D. Thesis, Princeton University, Princeton, NJ, Sept. 1985.

9. Dowell, E. H., "Component Mode Analysis of Nonlinear and Nonconservative Systems," ASME Journal of Applied Mechanics, Vol. 47, March 1980, pp. 172-176.

10. Tongue, B. H. and Dowell, E. H., "Component Mode Analysis of Nonlinear, Nonconservative Systems," ASME Journal of Applied Mechanics, Vol. 50, March 1983, pp. 204-209.

Summary of Expenses Incurred to Date
15 November 1985 - 15 February 1986

Principal Investigator:

Dr. A. A. Ferri, Assistant Professor 30% for three months	$ 3,500
Graduate Research Assistant (one) three months	2,329
Total Personal Services	**5,829**
Fringe Benefits 21.0% of applicable personal services	735
Total Direct Costs	**6,564**
Overhead At predetermined fixed rate of 63.5% of total direct costs less capital equipment	4,168
Total Cost	**$10,732**

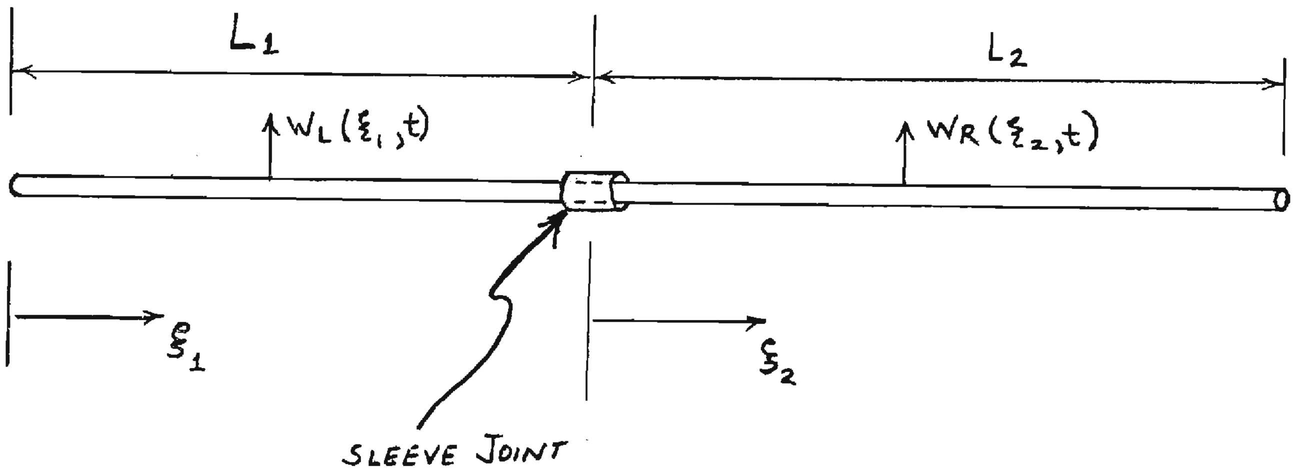


Figure 1. Beam/sleeve joint configuration

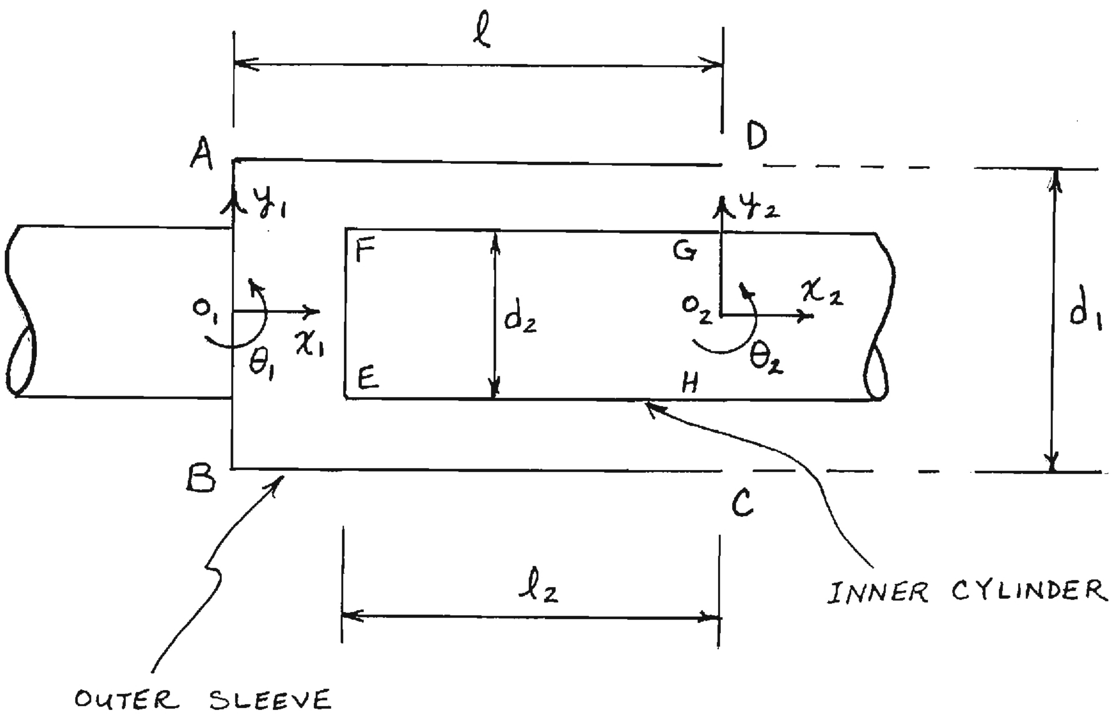


Figure 2. Sleeve joint model

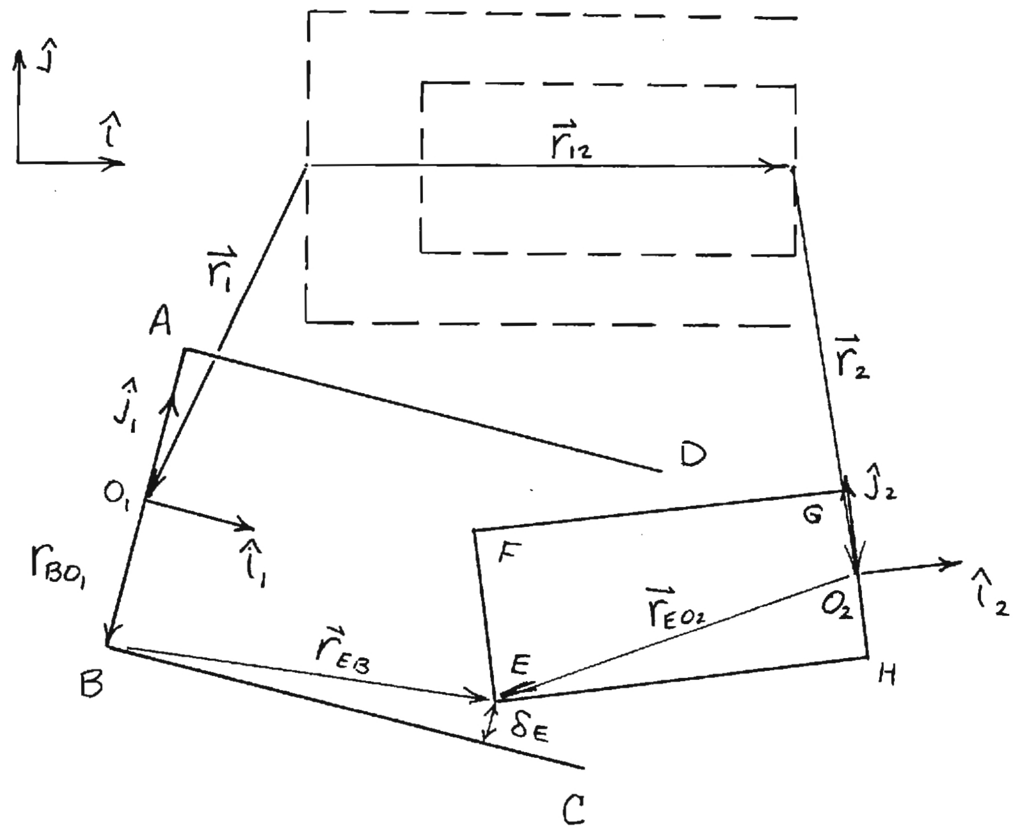


Figure 3. Deflected position of sleeve joint

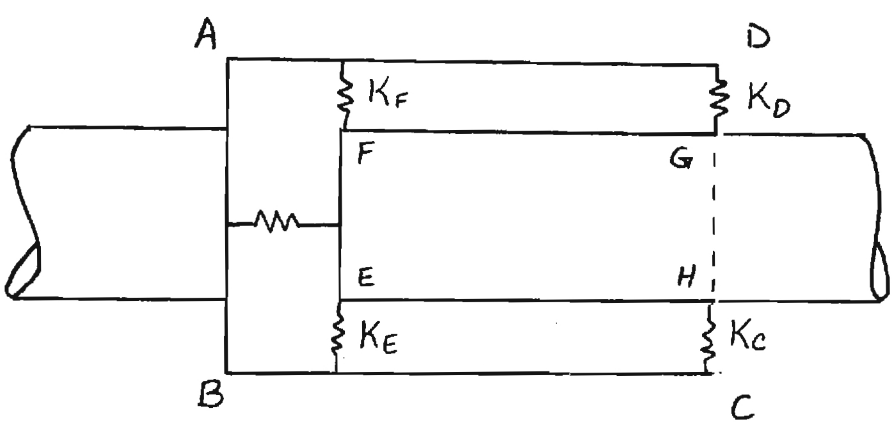


Figure 4. Sleeve joint model with nonlinear springs

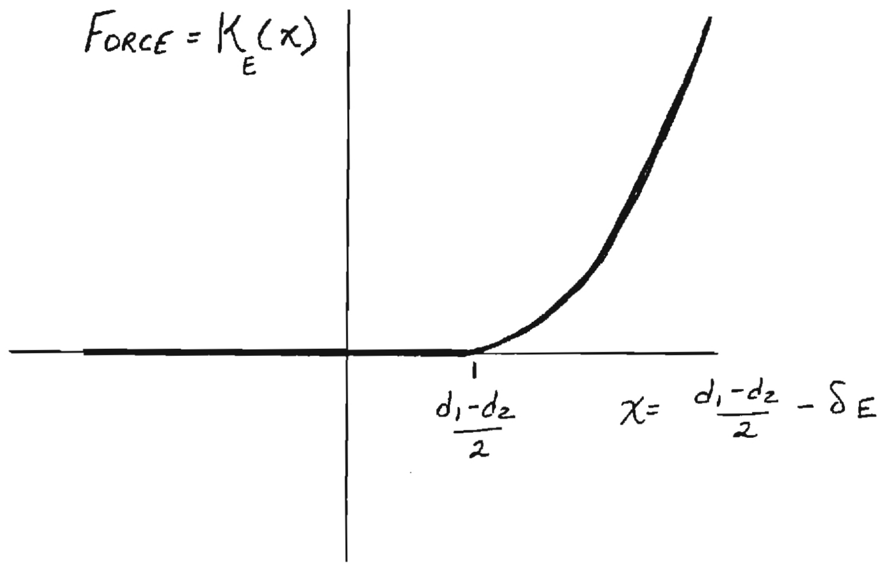


Figure 5. Nonlinear spring characteristic for contact point E

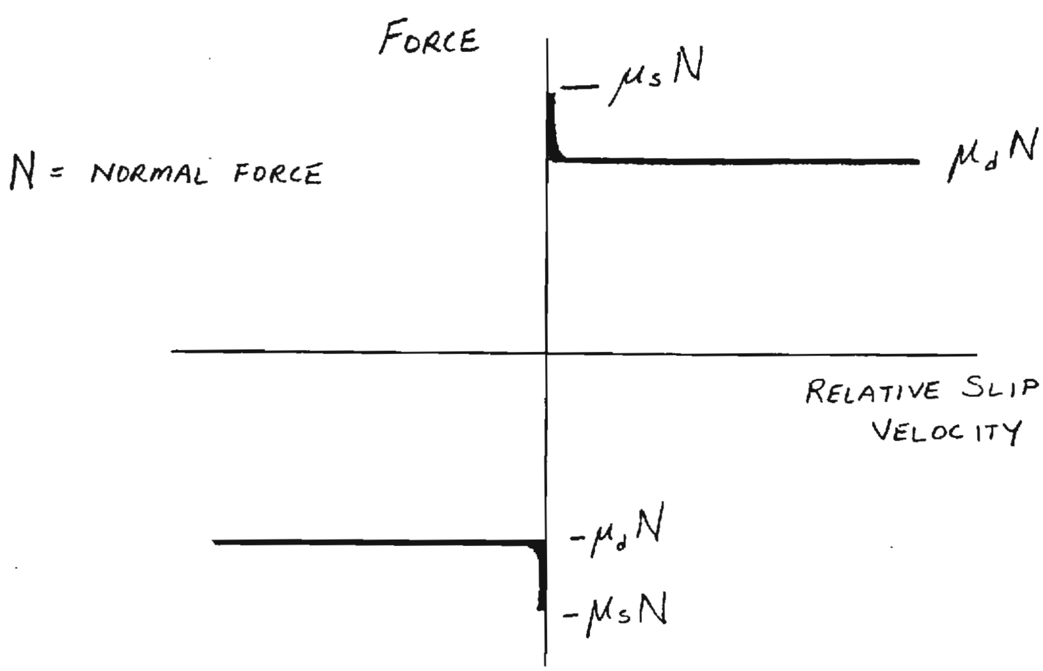


Figure 6. Dry friction damping characteristic

E-25-641

The Influence of Nonlinear Joints on the Dynamics of Large Flexible Space Structures

Progress Report No. 2

April 1986

Principal Investigator

Dr. A. A. Ferri

INTRODUCTION

The effort since progress report No. 1 has been directed towards refining the sleeve-joint model, and testing the design on a simpler beam-sleeve model. In addition, further literature has been examined pertaining to analytical and experimental studies of joints with clearances. These activities will be detailed below, followed by an outline of future activities.

SUMMARY OF ACCOMPLISHMENTS

1. The sleeve joint model was modified to account for material damping losses due to impact and deformation.

2. A rigid beam-sleeve joint model was chosen as the first model to be simulated. The model was chosen on the grounds that it would yield the most information about the beam sleeve interaction.

3. A literature review into analysis of general mechanisms with clearances and impact was performed.

SLEEVE-JOINT DAMPING

Material damping was introduced into the beam-sleeve model as viscous damping elements which are engaged when contact occurs. As seen in Fig. 1, the dampers are placed in parallel with the nonlinear springs (described in Progress Report 1). The viscous dampers transmit a relative force between the beam and sleeve which is proportional to the relative velocity and which is zero when the beam and sleeve are not in contact at that point. This approach was chosen over the coefficient of restitution approach since employing the latter method for eccentric impact is fairly difficult especially when two or more points may contact at once. See, for example, [1] and [2].

RIGID BEAM-SLEEVE JOINT MODEL

In order to verify the sleeve-joint model, a simple beam-sleeve model was developed (See Fig. 2). The model consists of a rigid beam mated with a sleeve which is fixed in inertial space. The beam can be treated as two rigid beam segments, (Fig 3), with component mode synthesis used to constrain the two segments to move together. Segmenting the beam was done to facilitate the extension of the model to the flexible beam case.

This model serves several purposes. First, it isolates the sleeve-joint so that an accurate assessment of its effects can be made. For example, the total damping of this system is due entirely to the dry friction and material damping in the joint, hence the damping contribution of the joint to the system is easily determined. Second, it can be used to determine the amount of computation needed for the three-beam/two-joint model. Since the rigid beam system is essentially three-degree-of-freedom, computer runs will be relatively inexpensive to run. Effects of different joint variables and of different numerical-integration schemes will be studied. Third, the results of this study will serve to modify the sleeve-joint model, perhaps simplifying it, before the large scale computation is performed. For example, if it appears that the fine detail of the joint model is not evident in the global motion, a simpler joint model will be developed. The response of systems with different joint models can be compared to determine their relative accuracy. If it turns out that the damping contribution of the sleeve-joint is too small, the effects of in-plane slip may be incldued in the flexible beam model, or, pin joint structures may be considered.

LITERATURE SURVEY OF JOINTS WITH CLEARANCES

In order to accurately model the damping due to impact in the joints, a literature survey of joints with clearances and impact damping was conducted. Much of the work in this area was performed in connection with mechanical linkages, however, the general modelling techniques and analysis techniques are applicable to the present study.

As mentioned in Progress Report 1, Hertz and Crawley have stated that clearances in sleeve and pin joints may be a factor contributing to overall structure damping [3,4]. However, their analysis of impact damping is much too simple for the present study. (For example, in [3], a one harmonic solution to a single-degree-of-freedom system impacting against rigid constraints is considered.) For this reason, the reserach in the mechanical linkages was investigated.

Dubowsky, et al., has studied the effects of clearances in mechanisms in several papers [5-9]. The joints in linkages (mainly pin joints or "revolute pairs") have many of the features of the joints proposed for flexible spacecraft. Dubowsky and Freudenstein's one DOF model of the "imapct pair", developed in [5,6] and shown in Fig. 4, accounts for the effects of clearances, nonlinear stiffening after contact is made and damping due to material deformation during impact. Numerical simulation results, approximate solutions such as those obtained using describing functions, and experimental results are given for the impact pair. Some of the significant conclusions from these two papers are: 1) The nonlinear stiffening of the joint after contact was made did not greatly influence the behavior of the system; linear force-deflection relations produced results very close to those obtained with the nonlinear stiffness law. 2) Numerical solutons are best performed using predictor-convector methods (For example, Gear Method, Wilson's Theta Method, etc.) rather than Euler and Runge-Kutta Methods. This was due to differences in the natural frequencies of the system depending on whether or not contact had occurred. 3) The nonlinear behavior becomes more pronounced as the amplitude and/or frequency of the motion decreases.

In the later papers of Dubowsky et al., the imapct pair model is extended to multiple link systems [7,8] and even flexible link systems [9]. However, much of this later work in concerned with impact forces and wear in joints with clearances.

OUTLINE FOR FUTURE RESEARCH

1. Perform numerical simulation sof rigid-beam/sleeve joint model using predictor-corrector numerical integration technique (Gear method).

2. Analyze the local behavior of the sleeve joint and global behavior of the rigid beam to determine damping contribution from sleeve and to examine joint mechanics.

3a. Based on results from task 2, determine if simpler models for the joint are possible, and if so, compare the simpler joint behavior to the "full" joint behavior to determine the relative accuracy of simpler models.

3b. If damping contribution is too small, consider in-plane slip in the flexible beam case, or consider alternate joint types.

4. Extend analysis to the flexible beam case.

References

1. Beckett, R. E., Pan, K. C., and Chu, S. C., "A Numerical Method for the Dynamic Analysis of Mechanical Systems in Impact," **ASME J. of Engineering for Industry** (August 1977) 665-673.

2. Hunt, K. H., and Crossley, F. R. E., "Coefficient of Restitution Interpreted as Damping in Vibroimpact," **ASME J. of Applied Mechanics** (June 1975) 440-445.

3. Hertz, T. F. and Crawley, E. F., "The Effects of Scale on the Dynamics of Flexible Space Structures," Report #SSL18-83, Space Systems Laboratory, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, MA (September 1983).

4. Hertz, T. F. and Crawley, E. F., "Damping in Space Structure Joints," AIAA Dynamics Specialists Conference (17-18 May 1984) Palm Springs, California.

5. Dubowsky, S. and Freudenstein, F., "Dynamic Analysis of Mechanical Systems with Clearances, Part 1: Formation of Dynamic Model," **ASME J. of Engineering for Industry** (February 1971) 305-309.

6. Dubowsky, S. and Freudenstein, F., "Dynamic Analysis of Mechanical Systems with Clearances, Part 2: Dynamic Response," **ASME J. of Engineering for Industry** (February 1971) 310-316.

7. Dubowsky, S., "On Predicting the Dynamic Effects of Clearances in Planar Mechanisms," **ASME J. of Engineering for Industry** (February 1974) 317-323.

8. Dubowsky, S., "On Predicting the Dynamic Effects of Clearances in One-Dimensional Closed-Loop Systems," **ASME J. of Engineering for Industry** (February 1974) 324-329.

9. Dubowsky, S. and Moening, M. F., "An Experimental and Analytical Study of Impact Forces in Elastic Mechanical Systems with Clearances," **Mechanisms and Machine Theory 13** (1978) 451-465.

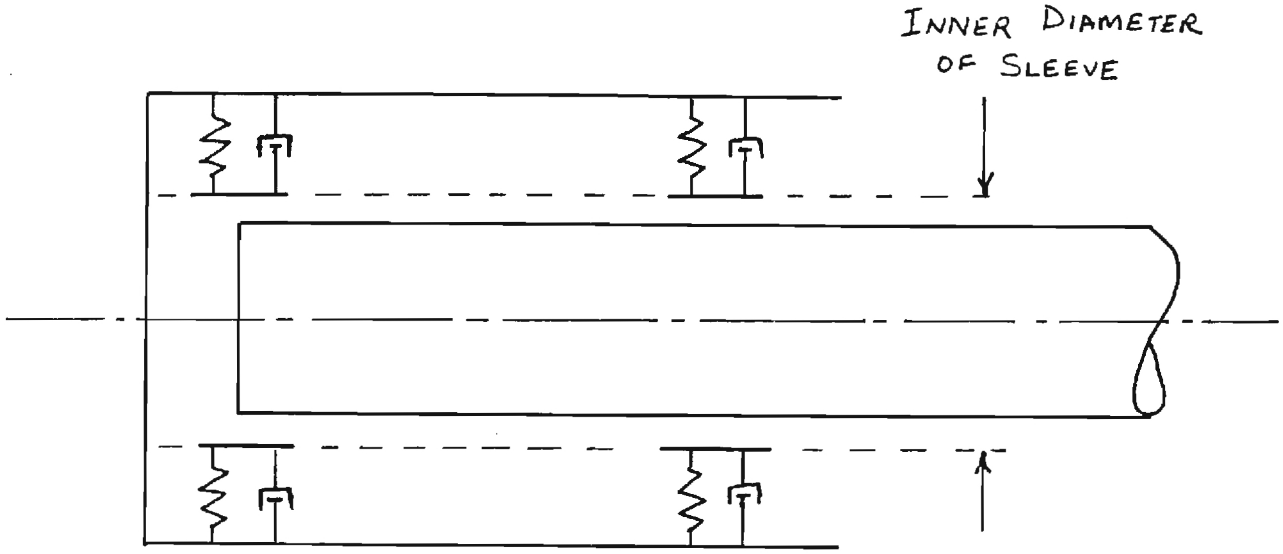


FIGURE 1: Sleeve Joint showing Impact Dampers

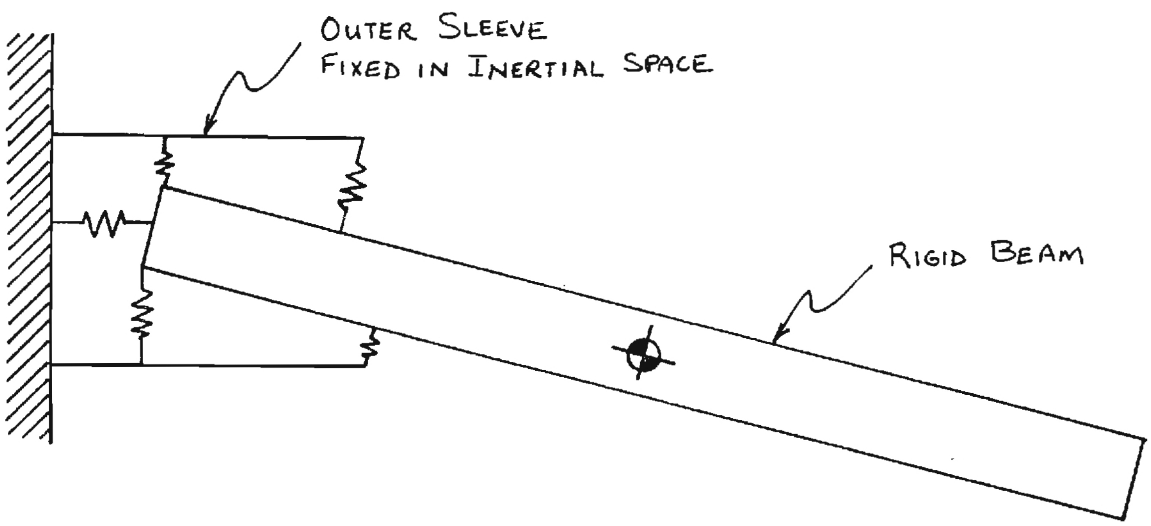


FIGURE 2: Rigid-Beam-Sleeve Model

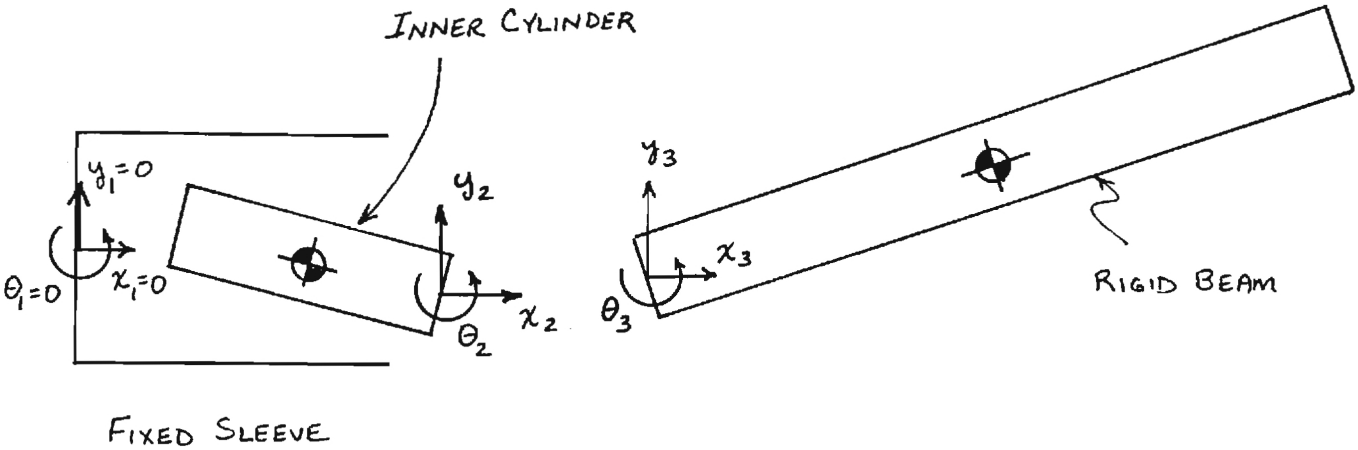


FIGURE 3: Rigid-Beam-Sleeve Model showing Segmentation of Rigid Beam

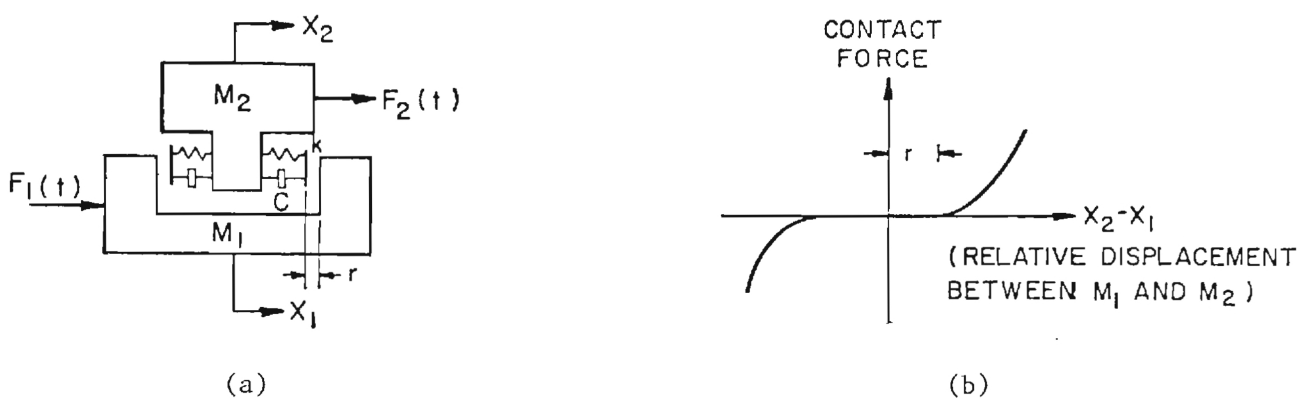


FIGURE 4: a) Impact Pair; b) Force-displacement Characteristic of Impact Pair (from [5])

E-25-641

The Influence of Nonlinear Joints on the Dynamics of Large Flexible Space Structures

Progress Report Number 3

June 1986

Principal Investigator

Dr. A. A. Ferri

INTRODUCTION

This report consists of a review of research accomplished to date and plans for future work. A schedule of work for the proposed tasks is included.

SUMMARY OF ACCOMPLISHMENTS

- Computer code entitled "FORCE2" for simulating 1 inertially fixed sleeve joint together with a rigid beam is complete. Some degree of verification of the code has been completed, and the code and equations appear to be producing reasonable results.

- A second computer code entitled "RIGID2" which models the system shown in Figure 1 is almost complete. This program will be used to develop simple sleeve joint models, based on the actual sleeve joint model.

OUTLINE FOR FUTURE RESEARCH

1. Some final checks will be run to complete the verification for the computer codes described above.
2. Simple models for the sleeve joint will be developed.
3. Parametric studies will be conducted to determine the influence of joint geometry and joint material properties on the damping of the joint.

4. The simple joint models from Task 2 will be incorporated into a three-flexible-beam/two-sleeve-joint model. Computer code will be developed for this system.

5. Parametric studies will be conducted to determine the overall damping of the three-flexible-beam/two-sleeve-joint model. In particular, "modal" damping ratios will be determined as a function of joint characteristics and vibration amplitudes of response.

6. Use Honeywell supplied data for a truss structure and for force-displacement characteristics of the interconnecting joints to determine the behavior of the combined truss structure.

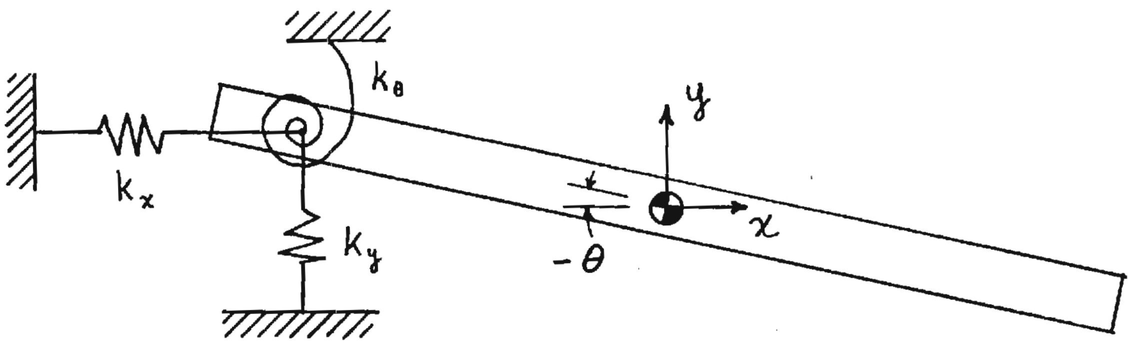
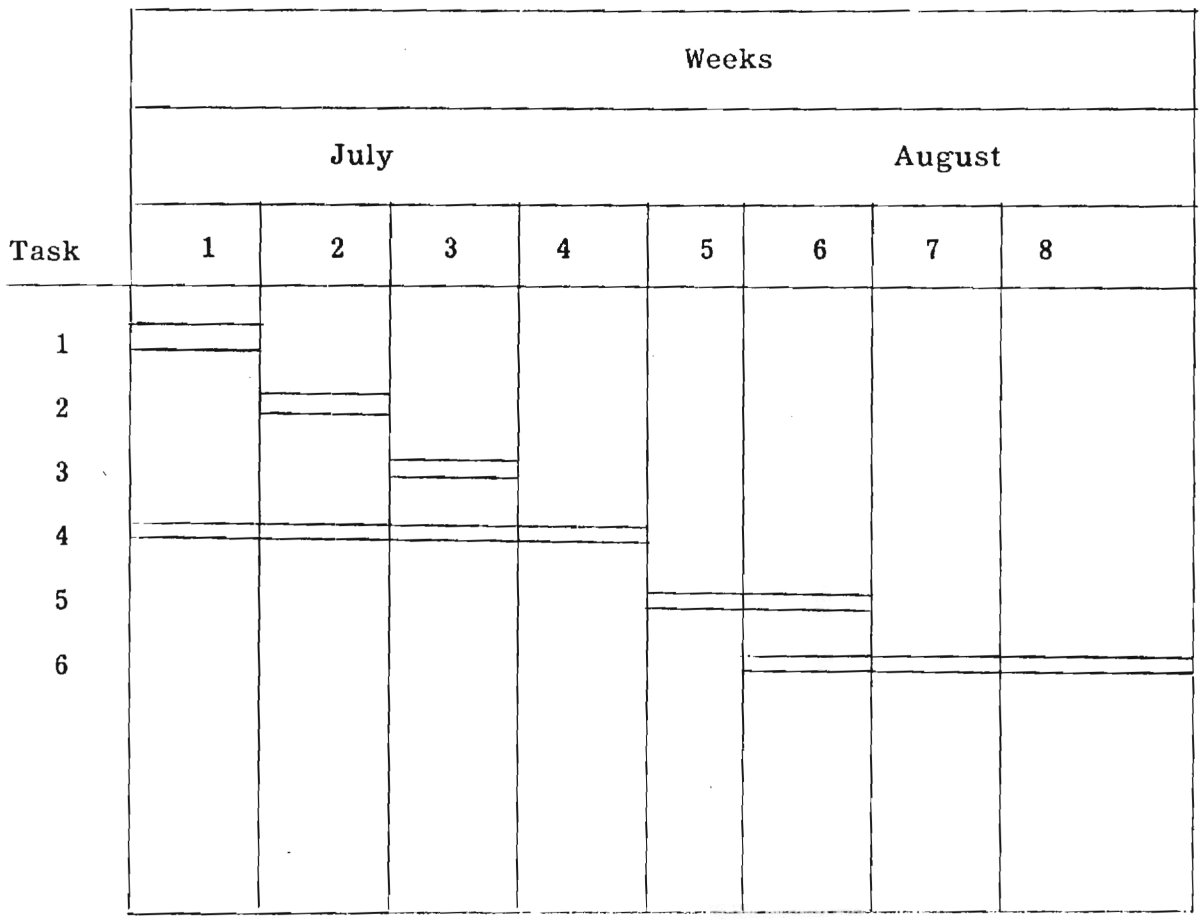


Figure 1. Rigid Beam with Nonlinear Springs.
[Nonlinear dampers in parallel with springs not shown.]

SCHEDULE OF WORK



Progress Report Number 4

The Influence of Nonlinear Joints on the Dynamics of Large Flexible Space Structures

by

Dr. A. A. Ferri

Principal Investigator

Prepared for

Honeywell, Inc.

Space & Strategic Avionics Division

Clearwater, FL 33546-7290

August 1986

The George W. Woodruff School of Mechanical Engineering

Georgia Institute of Technology

Atlanta, Georgia 30332

INTRODUCTION

This report consists of a summary of research accomplished since the completion of Progress Report 3 and an update of work in progress.

SUMMARY OF RESEARCH

Using the "FORCE" computer code (Included as Appendix A), which simulates an inertially fixed sleeve together with a rigid beam, a number of interesting results have been obtained. Baseline physical system parameters are given in Table 1, with nomenclature defined in Figure 1. For a 1 in (2.54 cm) diameter beam, a baseline clearance with the sleeve was chosen to be D = 0.1 mm. The beam is 1 m long, with approximately 1.5 in. (L = 4 cm) of the beam inserted into the sleeve. Mass properties have been calculated assuming the beam to be aluminum.

Some sample time histories are shown in Figures 2 through 5. y_2 refers to the transverse displacement of the beam at a point which lies just outside the sleeve. θ_2 is the angular rotation of the beam in radians. Figures 2 and 3 show the free response of y_2 and θ_2, respective, for the baseline set of parameter values and with $K_1 = K_2 = K_4 = 100$ N/m, $K_3 = K_5 = 0$, and $C_1 = C_2 = 0$. It is seen that each motion is dominated by two frequencies, a high frequency motion due to translation of the beam's end and a low frequency motion due to net rotation of the beam. (A third frequency corresponding to the longitudinal motion of the beam is also present, but is much smaller in amplitude.) The large difference in the two dominant frequencies is responsible for the "stiffness" of the governing equations, making numerical simulations difficult to obtain. To maintain accuracy, without excessive computer costs, the Gear method of solution (from the IMSL subroutine library) was chosen.

Another noticeable feature of the curves shown in Figures 2 and 3 is the exponential envelopes of decay. It should be noted that, for the case shown, viscous impact damping has been set equal to zero. Thus, all damping is due to the dry friction between the sleeve and beam. Although classical dry friction damped systems usually exhibit linear decay envelopes, here we have exponential envelopes because the normal forces (the forces of contact) are proportional to the deflection of the sleeve. It can also be seen that the frequency of the slow motion is noticeably dependent on the amplitude of the response; i.e., as the motion decays, the slow motion gets lower in frequency. This is due to the hardening spring effect associated with the clearance. (Note that in this case, the cubic spring terms, K_3 and K_5, have been set equal to zero; thus, the springs are piecewise linear hardening springs.) Using the log decrement approach, the damping ratio for the low frequency motion was estimated and was found to be amplitude dependent. For example, the high amplitude ($y_2 \sim .003$ m) damping ratio was calculated to be $\zeta = .146$; after the amplitude decreased to $y_2 \sim .0012$ m, the damping ratio was reduced to $\zeta = .079$. Thus, the low amplitude motion is <u>more lightly damped</u> than is the high amplitude motion.

Figures 4 and 5 show the θ_2 motion for two different values of impact damping. The physical parameters for Figures 4 and 5 are equal to those in Figures 2 and 3 except for the viscous damping coefficient; $C_1 = C_2 = 0.1$ N·s/m in Figure 2, and $C_1 = C_2 = 100$ N·s/m in Figure 3. It is seen that the viscous damping has a large effect on the overall damping of the beam/sleeve system. However, this viscous coefficient is extremely hard to quantify physically. For this reason, a baseline value will be chosen based on realistic <u>damping ratio</u> values, and parameter studies will be performed to determine the effects of this parameter on the system damping.

The effect of beam/sleeve geometry on the overall damping is shown in Figure 6. The quantity D/L is essentially a relative clearance, defined as the clearance divided by sleeve length. It is seen that the damping ratio is inversely proportional to D/L. Thus, small clearances are more beneficial for dry friction damping then large clearances. Note that the abcsissa of the graph is the amplitude of y_2, which is required because the damping ratio is a function of the amplitude as well as D/L. Again, it is seen that, for a given D/L, ζ is inversely proportional to amplitude.

Figure 7 shows the effect of sleeve stiffness on damping ratio, ζ. It is seen that ζ is relatively insensitive to sleeve stiffness. However, increasing the sleeve stiffness increases the system natural frequencies, which tends to make the response decay to zero <u>faster</u>. Of course, the number of oscillations before settling takes place remains the same.

A simplified joint model is being developed using the computer program "RIGID" (included as Appendix B), which simulates the system shown in Figure 8. The nonlinear spring and nonlinear damping characteristics are determined by comparing the free response of the simplified model with that the actual model for various levels of initial disturbances. On physical grounds, one can estimate the general form of the stiffness characteristic to be piecewise linear, with the threshold displacement depending on θ. Preliminary tests of this form of stiffness have shown good agreement with the FORCE output for the zero damped case. Damping characteristics are in the process of being developed. In light of the exponential envelopes of decay for the free response, it is likely that a <u>nearly linear</u> viscous damping will be used. It will not be exactly linear, since the damping ratio was found to depend on amplitude.

A computer program which analyzes the 3-flexible-beam/2-sleeve-joint model is nearly completed. Methodology for the development of the equations of motion is shown in Appendix C and a copy of the computer code is included as Appendix D. Tasks 5 and 6 previously defined in Progress Report 3 will be started after the code has been verified.

TABLE 1

Total Beam Length	1 m
Beam Outer Diameter	0.0254 m
Total Beam Mass	1.3 kg
Mass Moment of Inertia of Beam about C.G	.1084 kg-m^2
Coefficient of Sliding Friction, μ (Al on Steel)	.47

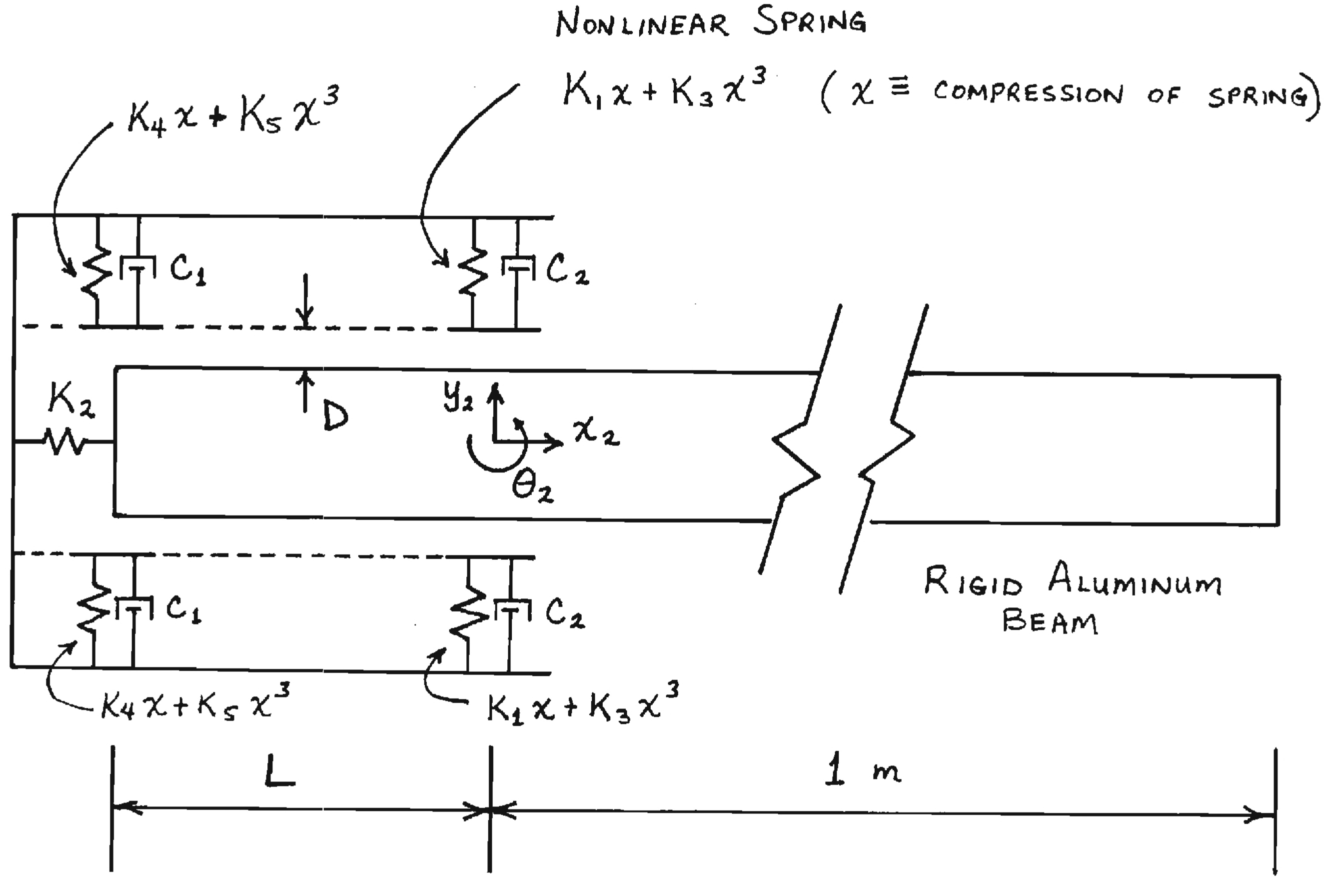


Figure 1. Beam/Sleeve Model

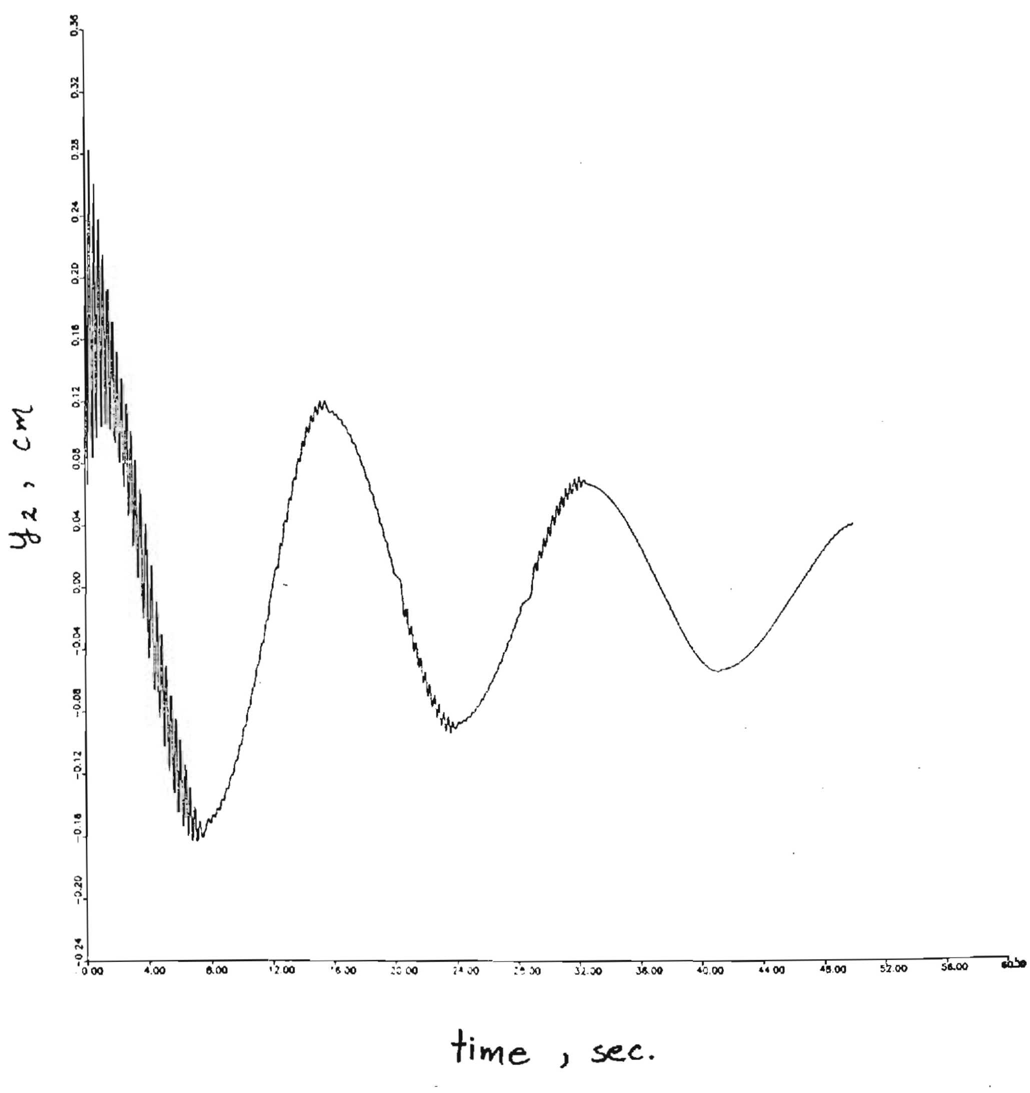


Figure 2. y_2 in cm vs time in seconds; $C_1 = C_2 = 0$, $K_1 = K_2 = K_4 =$ 100 N/m, $K_3 = K_5 = 0$, $\mu = .47$, D = .1 mm, L = 4 cm.

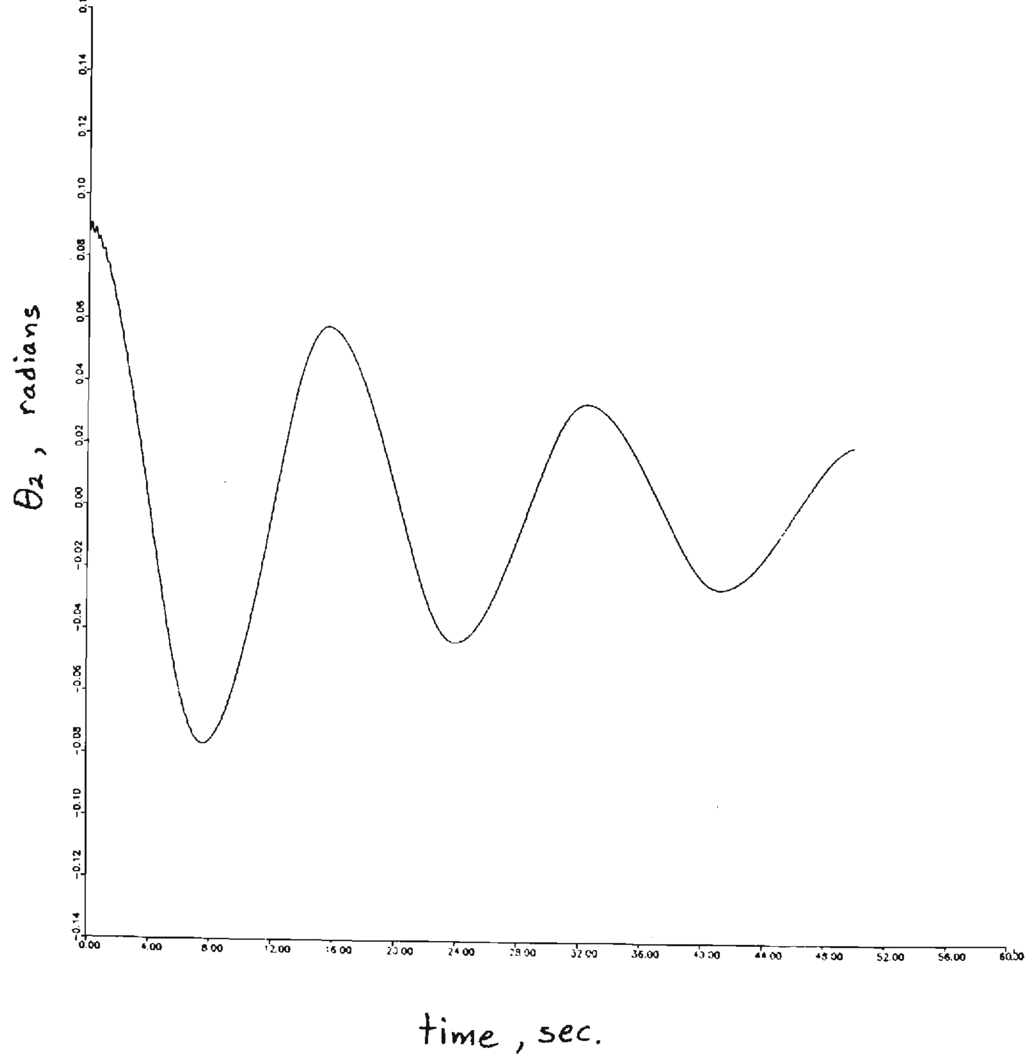


Figure 3. Θ_2 in radians vs time in seconds; $C_1 = C_2 = 0$, $K_1 = K_2 = K_4 = 100$ N/m, $K_3 = K_5 = 0$, $\mu = .47$, D = .1 mm, L = 4 cm.

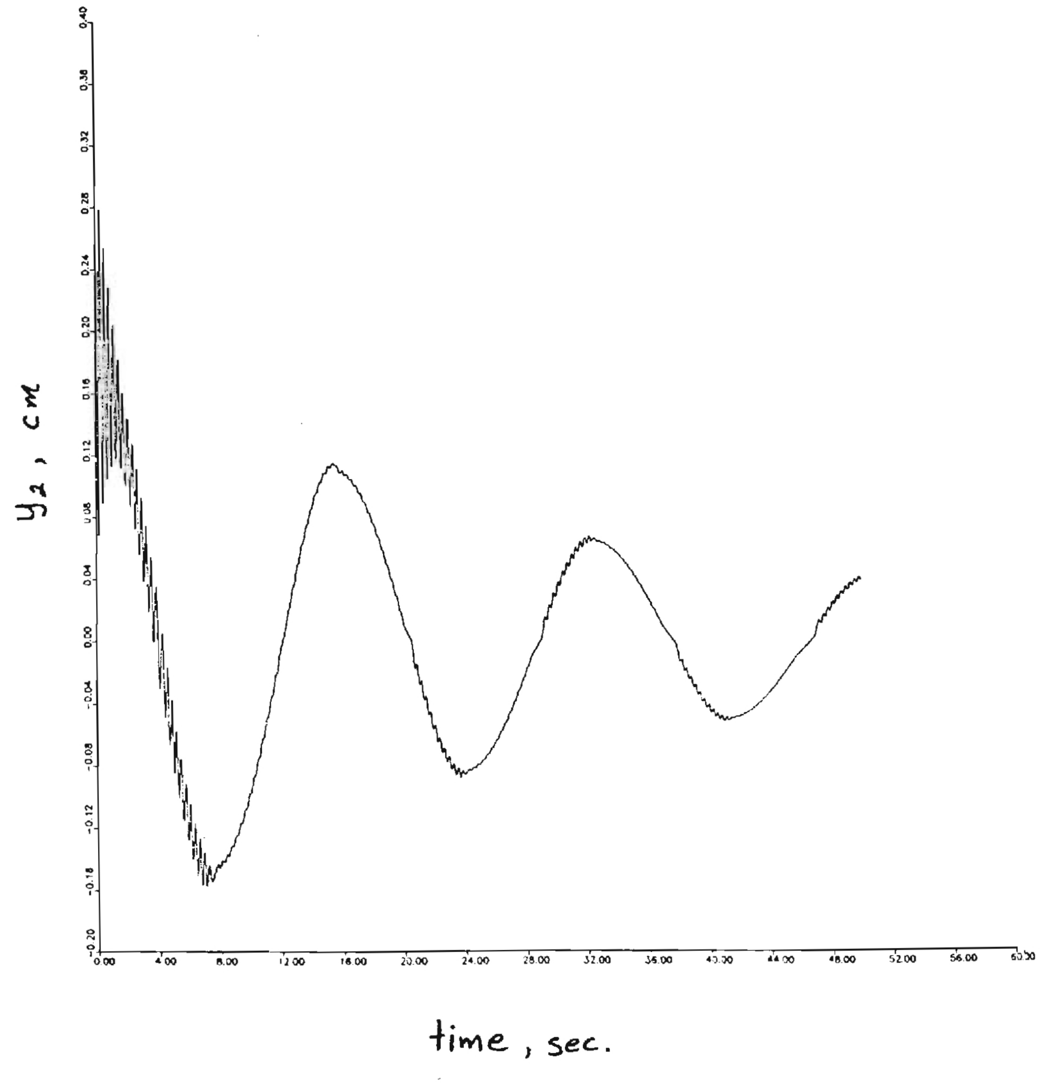


Figure 4. y_2 in cm vs time in seconds; $C_1 = C_2 = 0.1$ N·s/m, $K_1 = K_2 = K_4 = 100$ N/m, $K_3 = K_5 = 0$, $\mu = .47$, $D = .1$ mm, $L = 4$ cm.

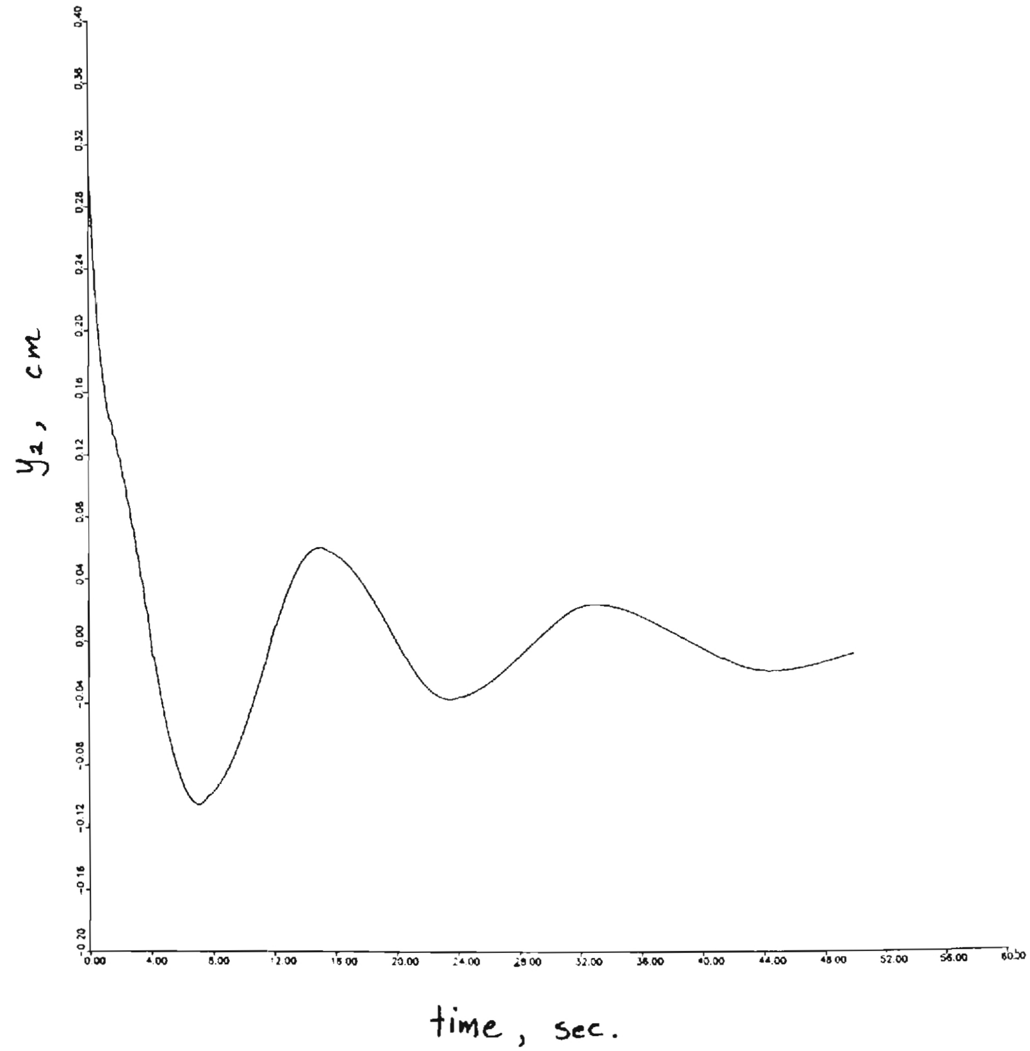


Figure 5. y_2 in cm vs time in seconds; $C_1 = C_2 = 100$ N·s/m, $K_1 = K_2 = K_4 = 100$ N/m, $K_3 = K_5 = 0$, $\mu = .47$, $D = .1$ mm, $L = 4$ cm.

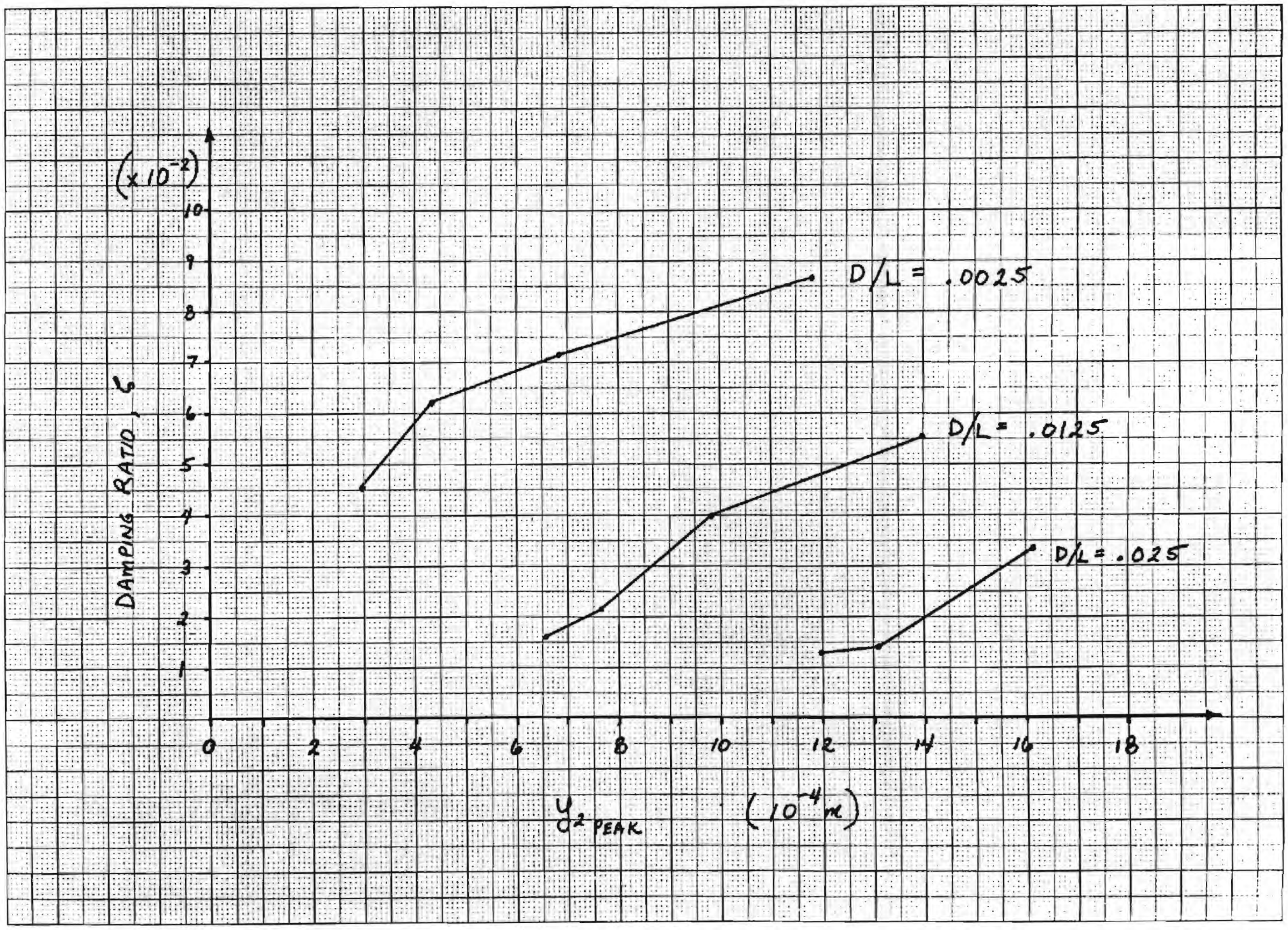


Figure 6. Damping ratio, ζ, vs peak y_2; various levels of relative clearance, D/L; $C_1 = C_2 = 0$, $K_1 = K_2 = K_4 = 10$ N/m, $K_3 = K_5 = 0$, $\mu = .47$.

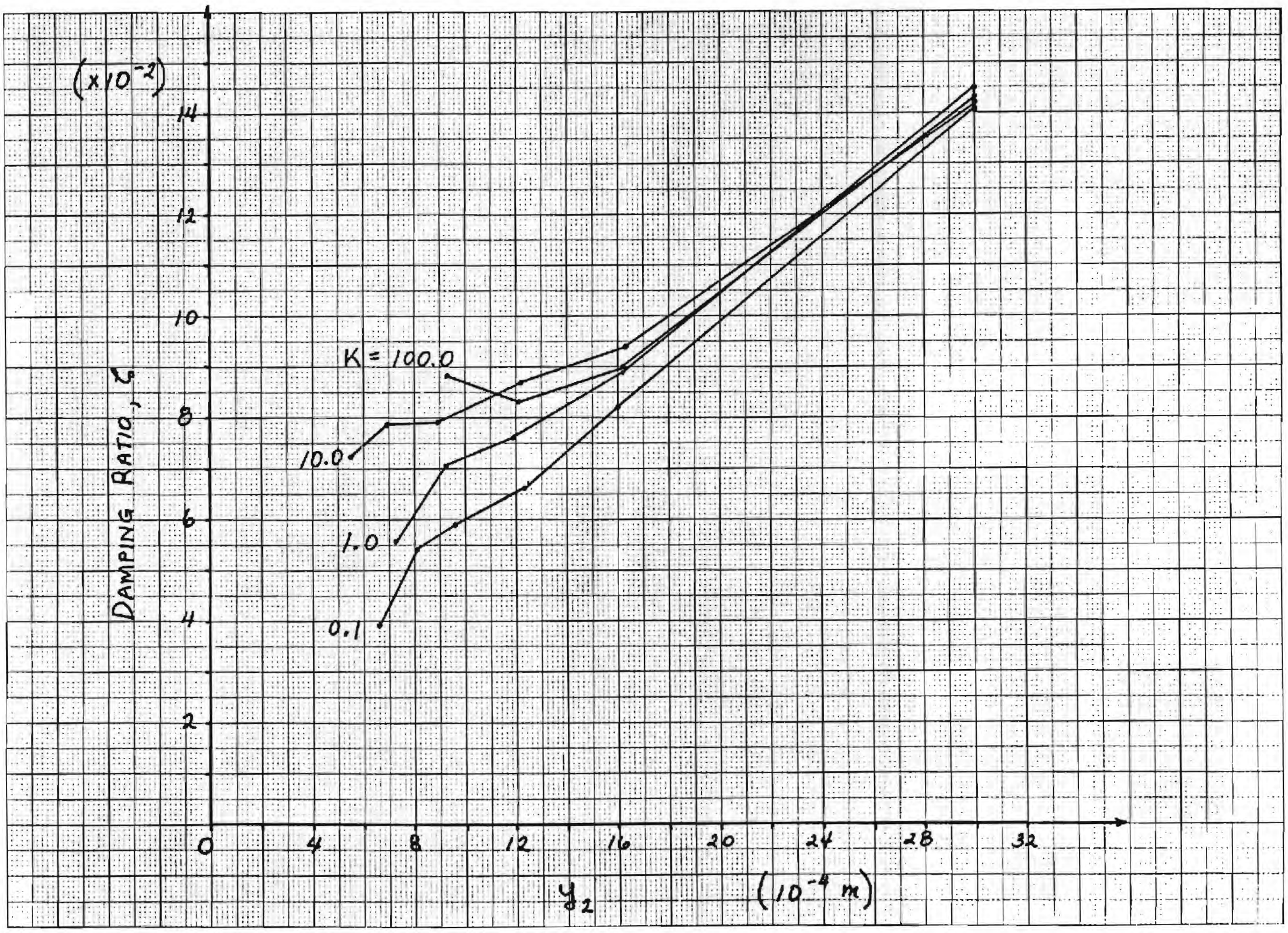


Figure 7. Damping ratio, ζ vs peak y_2; various levels of joint stiffness; $C_1 = C_2 = 0$, $K_1 = K_2 = K_4 = K$, $K_3 = K_5 = 0$, $\mu = .47$, $D = .1$ mm, $L = 4$ cm.

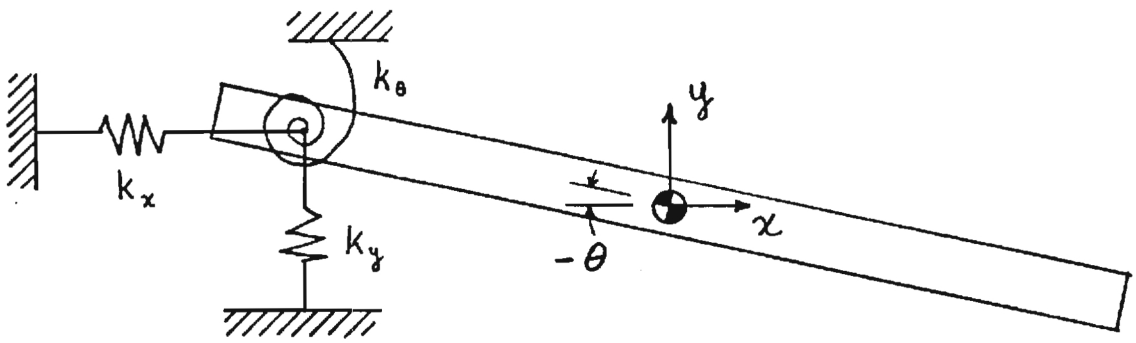


Figure 8. Rigid Beam with Nonlinear Springs.
[Nonlinear dampers in parallel with springs not shown.]

APPENDIX A

"FORCE" COMPUTER CODE

```
      PROGRAM FORCE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      COMMON TT(5,5),MA(5,5),MB(5,5),W1(5,5),B(5,5),S1(5,5),
     &       C0(5,5),C1(5,5),C2(5,5),C3(5,5),C5(5,5)
     &       ,L1,L2,L3,D1,D2,K1,K2,K3,K4,K5,UD,CC1,CC2
      REAL MA,MB,L1,L2,L3,D1,D2,K1,K2,K3,K4,K5,UD
      CALL INITIAL
      CALL MATRIX
      STOP
      END
      SUBROUTINE INITIAL
      COMMON TT(5,5),MA(5,5),MB(5,5),W1(5,5),B(5,5),S1(5,5),
     &       C0(5,5),C1(5,5),C2(5,5),C3(5,5),C5(5,5)
     &       ,L1,L2,L3,D1,D2,K1,K2,K3,K4,K5,UD,CC1,CC2
      REAL MA,MB,L1,L2,L3,D1,D2,K1,K2,K3,K4,K5,UD,MAA,MBB,IAA,IBB
      READ*,K1,K2,K3,K4,K5,MAA,MBB
      READ*,IAA,IBB,L1,L2,L3,D1,D2
      READ*,UD,CC1,CC2
      READ*,((B(I,J),J=1,3),I=1,3)
      READ*,((S1(I,J),J=1,3),I=1,3)
      READ*,((TT(I,J),J=1,3),I=1,3)
      READ*,((MA(I,J),J=1,3),I=1,3)
      READ*,((MB(I,J),J=1,3),I=1,3)
      READ*,((W1(I,J),J=1,3),I=1,3)
      DO 5 I=1,3
      PRINT*, (W1(I,J),J=1,3)
  5   CONTINUE
C
C
      L=3
      M=3
      NN=3
      IA=5
      IB=5
      IC=5
      CALL VMULFF(MB,W1,L,M,NN,IA,IB,C0,IC,IER)
      DO 55 I=1,3
      PRINT*,(C0(I,J),J=1,3)
 55   CONTINUE
      CALL VMULFF(C0,TT,L,M,NN,IA,IB,C1,IC,IER)
      RETURN
      END
C
C
      SUBROUTINE MATRIX
      COMMON TT(5,5),MA(5,5),MB(5,5),W1(5,5),B(5,5),S1(5,5),
     &       C0(5,5),C1(5,5),C2(5,5),C3(5,5),C5(5,5)
     &       ,L1,L2,L3,D1,D2,K1,K2,K3,K4,K5,UD,CC1,CC2
      REAL MA,MB,L1,L2,L3,D1,D2,K1,K2,K3,K4,K5,UD,Y(6),WK(103),T,TOL,TEND,H
     &     ,XX(2000),T1(2000),YY(2000),TH(2000)
      INTEGER N,METH,MITER,INDEX,IWK(2),IER,K,BUF(512),BAF(512),BEF(512)
      EXTERNAL FCN,FCNJ
      N=6
      T=0.0
      Y(1)=0.0
      Y(2)=.003
      Y(3)=.0873
      Y(4)=0.
      Y(5)=0.
      Y(6)=0.
      XX(1)=Y(1)
```

```
      YY(1)=Y(2)
      TH(1)=Y(3)
      T1(1)=0.
      TOL=0.00001
      H=0.00001
      METH=1
      MITER=0
      INDEX=1
      NT=1000
      DO 10 K=1,NT
      TEND=FLOAT(K)*0.025
      CALL DGEAR(N,FCN,FCNJ,T,H,Y,TEND,TOL,METH,MITER,INDEX,IWK,WK,IER)
      IF( IER .GT. 128) GOTO 20
      PRINT 100, T,Y(1),Y(2),Y(3),Y(4),Y(5),Y(6)
  100 FORMAT(1X,F10.5,6(2X,F9.6))
      KP1=K+1
      T1(KP1)=TEND
      XX(KP1)=Y(1)
      YY(KP1)=Y(2)
      TH(KP1)=Y(3)
   10 CONTINUE
      CALL PLOTS(BUF,512,7)
      CALL FACTOR(0.5)
      CALL SCALE(T1,15.,NT+1,1)
      CALL SCALE(XX,15.,NT+1,1)
      CALL AXIS(0.,0.,"TIME",-4,15.,0.,T1(NT+2),T1(NT+3))
      CALL AXIS(0.,0.,"Y(1)",4,15.,90.,XX(NT+2),XX(NT+3))
      CALL PLOT(0.,0.,3)
      CALL LINE(T1,XX,NT+1,1,0,0)
      CALL PLOT(0.,0.,999)
C
      CALL PLOTS(BAF,512,8)
      CALL FACTOR(0.5)
      CALL SCALE(T1,15.,NT+1,1)
      CALL SCALE(YY,15.,NT+1,1)
      CALL AXIS(0.,0.,"TIME(SEC)",-9,15.,0.,T1(NT+2),T1(NT+3))
      CALL AXIS(0.,0.,"Y(2)",4,15.,90.,YY(NT+2),YY(NT+3))
      CALL PLOT(0.,0.,3)
      CALL LINE(T1,YY,NT+1,1,0,0)
      CALL PLOT(0.,0.,999)
C
      CALL PLOTS(BEF,512,9)
      CALL FACTOR(0.5)
      CALL SCALE(T1,15.,NT+1,1)
      CALL SCALE(TH,15.,NT+1,1)
      CALL AXIS(0.,0.,"TIME(SEC)",-9,15.,0.,T1(NT+2),T1(NT+3))
      CALL AXIS(0.,0.,"Y(3)",4,15.,90.,TH(NT+2),TH(NT+3))
      CALL PLOT(0.,0.,3)
      CALL LINE(T1,TH,NT+1,1,0,0)
      CALL PLOT(0.,0.,999)
   20 PRINT*, 'TROUBLE'
      RETURN
      END
C
C
      SUBROUTINE FCN(N,T,Y,YPRIME)
      COMMON TT(5,5),MA(5,5),MB(5,5),W1(5,5),B(5,5),S1(5,5),
     &       C0(5,5),C1(5,5),C2(5,5),C3(5,5),C5(5,5)
     &       ,L1,L2,L3,D1,D2,K1,K2,K3,K4,K5,UD,CC1,CC2
      REAL MA,MB,L1,L2,L3,D1,D2,K1,K2,K3,K4,K5,UD,Y(6),YPRIME(6),
```

```
     &     T,A(3,3),AINV(3,3),WKAREA(3),FX(3)
      INTEGER N,IA,IDGT,IER
C
C       CALCULATE DISPLACEMENTS
C
      DELE=Y(2)-L2*Y(3)+(D1-D2)/2.
      DELF=-Y(2)+L2*Y(3)+(D1-D2)/2.
      DELD=Y(1)*Y(3)-(D2*Y(3)**2)/2.-Y(2)+(D1-D2)/2.
      DELC=-Y(1)*Y(3)-(D2*Y(3)**2)/2.+Y(2)+(D1-D2)/2.
      DELI=Y(1)+L1-L2
C     PRINT 30, DELE,DELF,DELD,DELC
C30   FORMAT( 1X,4(2X,F9.7))
C
C       CALCULATE THE DERIVATIVE OF DISPLACEMENTS.
C
      DELET=Y(5)-L2*Y(6)+D2*Y(6)*Y(3)/2.
      DELFT=-Y(5)+L2*Y(6)+D2*Y(6)*Y(3)/2.
      DELDT=Y(4)*Y(3)+Y(1)*Y(6)-Y(5)+Y(2)*Y(6)*Y(3)-D1*Y(6)*Y(3)/2.
      DELCT=-Y(4)*Y(3)-Y(1)*Y(6)+Y(5)-Y(2)*Y(6)*Y(3)-D1*Y(6)*Y(3)/2.
C
C       CALCULATE THE RELATIVE VELOCITIES.
C
      VELHC=Y(4)+Y(5)*Y(3)+D2*Y(6)/2.
      VELGD=Y(4)+Y(5)*Y(3)-D2*Y(6)/2.
      VELEB=Y(4)+L2*Y(6)*Y(3)+D2*Y(6)/2.
      VELFA=Y(4)+L2*Y(6)*Y(3)-D2*Y(6)/2.
C
C        CALCULATE THE SPRING FORCES.
C
      IF(DELC .LT. 0.0) THEN
       FORC1=K1*ABS(DELC)+K3*(ABS(DELC))**3
      ELSE
       FORC1=0.0
      ENDIF
      IF(DELD .LT. 0.0) THEN
       FORD1=K1*ABS(DELD)+K3*(ABS(DELD))**3
      ELSE
       FORD1=0.0
      ENDIF
      IF(DELE .LT. 0.0) THEN
       FORE1=K4*ABS(DELE)+K5*(ABS(DELE))**3
      ELSE
       FORE1=0.0
      ENDIF
      IF(DELF .LT. 0.0) THEN
       FORF1=K4*ABS(DELF)+K5*(ABS(DELF))**3
      ELSE
       FORF1=0.0
      ENDIF
       FORI=-K2*Y(1)
C
C       CALCULATE THE VISCOUS FORCES.
C
      IF(DELC .LT. 0.0 .AND. DELCT .LT. 0.0)THEN
        FORC2=-CC2*DELCT
      ELSE
        FORC2=0.0
      ENDIF
      IF(DELD .LT. 0.0 .AND. DELDT .LT. 0.0)THEN
        FORD2=-CC2*DELDT
```

```
      ELSE
        FORD2=0.0
      ENDIF
      IF(DELE .LT. 0.0 .AND. DELET .LT. 0.0)THEN
        FORE2=-CC1*DELET
      ELSE
        FORE2=0.0
      ENDIF
      IF(DELF .LT. 0.0 .AND. DELFT .LT. 0.0)THEN
        FORF2=-CC1*DELFT
      ELSE
        FORF2=0.0
      ENDIF
C
C      COMBINE THE SPRING AND VISCOUS FORCES TO GET THE
C      TOTAL NORMAL FORCE AT EACH POINT.
C
      FORC=FORC1+FORC2
      FORD=FORD1+FORD2
      FORE=FORE1+FORE2
      FORF=FORF1+FORF2
C
C        CALCULATE THE FRICTION FORCES.
C
      IF(VELEB .EQ. 0.0) THEN
       FRICE=0.0
      ELSE IF(VELEB .GT. 0.0) THEN
       FRICE=-FORE*UD
      ELSE
      FRICE=FORE*UD
      ENDIF
      IF(VELFA .EQ. 0.0) THEN
       FRICF=0.0
      ELSE IF(VELFA .GT. 0.0) THEN
       FRICF=-FORF*UD
      ELSE
       FRICF=FORF*UD
      ENDIF
      IF(VELHC .EQ. 0.0) THEN
       FRICC=0.0
      ELSE IF(VELHC .GT. 0.0) THEN
       FRICC=-FORC*UD
      ELSE
       FRICC=FORC*UD
      ENDIF
      IF(VELGD .EQ.0.0) THEN
       FRICD=0.0
      ELSE IF(VELGD .GT. 0.0) THEN
       FRICD=-FORD*UD
      ELSE
       FRICD=FORD*UD
      ENDIF
C
C      CALCULATE THE VARIABLE MATRIX ELEMENTS.
C
      B(3,1)=-(0.5)*L2*Y(3)
      S1(3,1)=L3*Y(3)
      L=3
      M=3
      NN=3
```

```
      IA=5
      IB=5
      IC=5
      CALL VMULFF(B,S1,L,M,NN,IA,IB,C2,IC,IER)
      CALL VMULFF(C2,C1,L,M,NN,IA,IB,C3,IC,IER)
      DO 70 I=1,3
       DO 60 J=1,3
        A(I,J)=MA(I,J)-C3(I,J)
 60    CONTINUE
 70   CONTINUE
C
C
      NM=3
      IA1=3
      IB=3
      IDGT=0
      CALL LINV1F(A,NM,IA1,AINV,IDGT,WKAREA,IER)
      CALL VMULFF(TT,AINV,L,M,NN,IA,IB,C5,IC,IER)
C
C      CALCULATE THE COMPONENTS OF THE F MATRIX.
C
      FX(1)=FORI+FRICF+FRICE+FORD*Y(3)+FRICD-FORC*Y(3)+FRICC
      FX(2)=FORE-FORF+FRICD*Y(3)+FORC+FRICC*Y(3)-FORD
      FX(3)=((0.5)*L2+(0.5)*D2*Y(3))*FORF+((0.5)*L2*Y(3)-(0.5)*D2)*FRICF
     &+(-(0.5)*L2+(0.5)*D2*Y(3))*FORE+((0.5)*L2*Y(3)+(0.5)*D2)*FRICE+
     &(0.5)*L2*Y(3)*FORI-(0.5)*L2*FORD-(0.5)*D2*FRICD+(0.5)*L2*FORC+
     &(0.5)*D2*FRICC
C
C
      YPRIME(1)=Y(4)
      YPRIME(2)=Y(5)
      YPRIME(3)=Y(6)
      YPRIME(4)=C5(1,1)*FX(1)+C5(1,2)*FX(2)+C5(1,3)*FX(3)
      YPRIME(5)=C5(2,1)*FX(1)+C5(2,2)*FX(2)+C5(2,3)*FX(3)
      YPRIME(6)=C5(3,1)*FX(1)+C5(3,2)*FX(2)+C5(3,3)*FX(3)
      RETURN
      END
C
C
      SUBROUTINE FCNJ(N,T,Y,PD)
      COMMON TT(5,5),MA(5,5),MB(5,5),W1(5,5),B(5,5),S1(5,5),
     &       C0(5,5),C1(5,5),C2(5,5),C3(5,5),C5(5,5)
     &       ,L1,L2,L3,D1,D2,K1,K2,K3,K4,K5,UD,CC1,CC2
      REAL MA,MB,L1,L2,L3,D1,D2,K1,K2,K3,K4,K5,UD,PD(N,N),T
      RETURN
      END
```

APPENDIX B

"RIGID" COMPUTER CODE

```
      PROGRAM RIGID(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      COMMON  KK(3,3),TT(3,3),MA(3,3),MB(3,3),W1(3,3),T1(3,3),B(3,3),
     &        S1(3,3),C(3,3),C0(3,3),C1(3,3),C2(3,3),C3(3,3),C4(3,3),
     &        C5(3,3),CF(3,3),L2,L3,CX,CY,CO,KX,KY,KO,DELC,THC
      REAL  KK,MA,MB,L2,L3,KX,KY,KO
      CALL INITIAL
      CALL MATRIX
      STOP
      END
C
C
      SUBROUTINE INITIAL
      COMMON  KK(3,3),TT(3,3),MA(3,3),MB(3,3),W1(3,3),T1(3,3),B(3,3),
     &        S1(3,3),C(3,3),C0(3,3),C1(3,3),C2(3,3),C3(3,3),C4(3,3),
     &        C5(3,3),CF(3,3),L2,L3,CX,CY,CO,KX,KY,KO,DELC,THC
      REAL  KK,MA,MB,L2,L3,MAA,MBB,KX,KY,KO,IAA,IBB
      READ*, KX,KY,KO,L2,L3,MAA,MBB,IAA,IBB,CX,CY,CO
      READ* ,DELC,THC
      READ*, ((B(I,J),J=1,3),I=1,3)
      READ*, ((S1(I,J),J=1,3),I=1,3)
      READ*, ((KK(I,J),J=1,3),I=1,3)
      READ*, ((TT(I,J),J=1,3),I=1,3)
      READ*, ((MA(I,J),J=1,3),I=1,3)
      READ*, ((MB(I,J),J=1,3),I=1,3)
      READ*, ((W1(I,J),J=1,3),I=1,3)
      READ*, ((T1(I,J),J=1,3),I=1,3)
      DO 5 I=1,3
       PRINT*, (W1(I,J),J=1,3)
  5   CONTINUE
      L=3
      M=3
      NN=3
      IA=3
      IB=3
      IC=3
      CALL VMULFF(MB,W1,L,M,NN,IA,IB,C0,IC,IER)
      DO 55 I=1,3
       PRINT*, (C0(I,J),J=1,3)
  55  CONTINUE
      CALL VMULFF(C0,TT,L,M,NN,IA,IB,C1,IC,IER)
      CALL VMULFF(KK,T1,L,M,NN,IA,IB,CF,IC,IER)
      RETURN
      END
C
C
      SUBROUTINE MATRIX
      COMMON  KK(3,3),TT(3,3),MA(3,3),MB(3,3),W1(3,3),T1(3,3),B(3,3),
     &        S1(3,3),C(3,3),C0(3,3),C1(3,3),C2(3,3),C3(3,3),C4(3,3),
     &        C5(3,3),CF(3,3),L2,L3,CX,CY,CO,KX,KY,KO,DELC,THC
      REAL  KK,MA,MB,L2,L3,Y(6),WK(103),T,TOL,TEND,H,XX(2000),TT1(2000),
     &      YY(2000),TH(2000),KX,KY,KO
      INTEGER N,METH,MITER,INDEX,IWK(2),IER,K,BUF(512),BAF(512),BEF(512)
      EXTERNAL FCN,FCNJ
      N=6
      T=0.
      Y(1)=0.00
      Y(2)=0.003
      Y(3)=0.0873
      Y(4)=0.0
      Y(5)=0.0
```

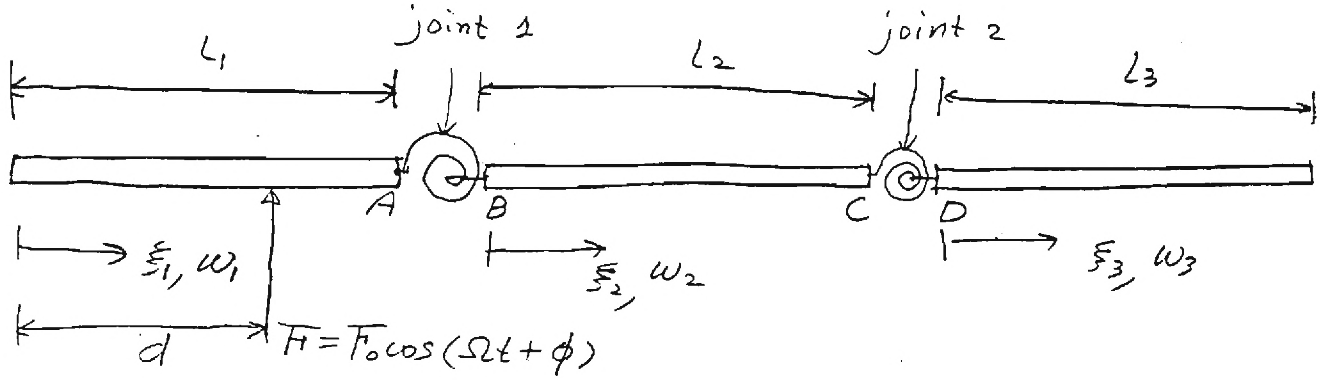
```
      Y(6)=0.0
      XX(1)=Y(1)
      YY(1)=Y(2)
      TH(1)=Y(3)
      TT1(1)=0.0
      TOL=0.00001
      H=0.00001
      METH=1
      MITER=0
      INDEX=1
      NT=1000
      DO 10 K=1,NT
      TEND=FLOAT(K)*0.025
      CALL DGEAR(N,FCN,FCNJ,T,H,Y,TEND,TOL,METH,MITER,INDEX,IWK,WK,IER)
      IF (IER .GT. 128) GOTO 20
      PRINT 100, T,Y(1),Y(2),Y(3),Y(4),Y(5),Y(6)
100   FORMAT(1X,F6.1,6(2X,F9.6))
      KP1=K+1
      TT1(KP1)=TEND
      XX(KP1)=Y(1)
      YY(KP1)=Y(2)
      TH(KP1)=Y(3)
 10   CONTINUE
      CALL PLOTS(BUF,512,7)
      CALL FACTOR(0.5)
      CALL SCALE(TT1,15.,NT+1,1)
      CALL SCALE(XX,15.,NT+1,1)
      CALL AXIS(0.,0.,"TIME",-4,15.,0.,TT1(NT+2),TT1(NT+3))
      CALL AXIS(0.,0.,"Y(1)",4,15.,90.,XX(NT+2),XX(NT+3))
      CALL PLOT(0.,0.,3)
      CALL LINE(TT1,XX,NT+1,1,0,0)
      CALL PLOT(0.,0.,999)
C
      CALL PLOTS(BAF,512,8)
      CALL FACTOR(0.5)
      CALL SCALE(TT1,15.,NT+1,1)
      CALL SCALE(YY,15.,NT+1,1)
      CALL AXIS(0.,0.,"TIME(SEC)",-9,15.,0.,TT1(NT+2),TT1(NT+3))
      CALL AXIS(0.,0.,"Y(2)",4,15.,90.,YY(NT+2),YY(NT+3))
      CALL PLOT(0.,0.,3)
      CALL LINE(TT1,YY,NT+1,1,0,0)
      CALL PLOT(0.,0.,999)
C
      CALL PLOTS(BEF,512,9)
      CALL FACTOR(0.5)
      CALL SCALE(TT1,15.,NT+1,1)
      CALL SCALE(TH,15.,NT+1,1)
      CALL AXIS(0.,0.,"TIME(SEC)",-9,15.,0.,TT1(NT+2),TT1(NT+3))
      CALL AXIS(0.,0.,"Y(3)",4,15.,90.,TH(NT+2),TH(NT+3))
      CALL PLOT(0.,0.,3)
      CALL LINE(TT1,TH,NT+1,1,0,0)
      CALL PLOT(0.,0.,999)
 20   PRINT*,
      RETURN
      END
C
C
      SUBROUTINE FCN(N,T,Y,YPRIME)
      COMMON  KK(3,3),TT(3,3),MA(3,3),MB(3,3),W1(3,3),T1(3,3),B(3,3),
     &        S1(3,3),C(3,3),C0(3,3),C1(3,3),C2(3,3),C3(3,3),C4(3,3),
```

```
     &        C5(3,3),CF(3,3),L2,L3,CX,CY,CO,KX,KY,KO,DELC,THC
      REAL  KK,MA,MB,L2,L3,Y(6),YPRIME(6),T,A(3,3),AINV(3,3),WKAREA(3)
     &       ,KX,KY,KO,FX(3)
      INTEGER N,IA,IDGT,IER
      B(3,1)=L2*Y(3)
      S1(3,1)=-L3*Y(3)
      L=3
      M=3
      NN=3
      IA=3
      IB=3
      IC=3
      CALL VMULFF(B,S1,L,M,NN,IA,IB,C2,IC,IER)
      CALL VMULFF(C2,C1,L,M,NN,IA,IB,C3,IC,IER)
      DO 70 I=1,3
       DO 60 J=1,3
        A(I,J)=MA(I,J)-C3(I,J)
 60    CONTINUE
 70   CONTINUE
      IDGT=0
      CALL LINV1F(A,NN,IA,AINV,IDGT,WKAREA,IER)
      CALL VMULFF(TT,AINV,L,M,NN,IA,IB,C5,IC,IER)
C
      DELMC=-DELC
      THMC=-THC
      AY2=ABS(Y(2))
      AY3=ABS(Y(3))
      IF (Y(2).GT.DELC) FORCE=-KY*DELC+KY*Y(2)
      IF (AY2.LT.DELC)  FORCE=0.
      IF (Y(2).LT.DELMC) FORCE=KY*DELC+KY*Y(2)
      IF (Y(3).GT.THC)  XMOM=-KO*THC+KO*Y(3)
      IF (AY3.LT.THC)   XMOM=0.
      IF (Y(3).LT.THMC) XMOM=KO*THC+KO*Y(3)
      FX(1)=-KX*Y(1)-CX*Y(4)
      FX(2)=-FORCE-CY*Y(5)+L2*CY*Y(6)
      FX(3)=-XMOM-CO*Y(6)
C
      YPRIME(1)=Y(4)
      YPRIME(2)=Y(5)
      YPRIME(3)=Y(6)
      YPRIME(4)=C5(1,1)*FX(1)+C5(1,2)*FX(2)+C5(1,3)*FX(3)
      YPRIME(5)=C5(2,1)*FX(1)+C5(2,2)*FX(2)+C5(2,3)*FX(3)
      YPRIME(6)=C5(3,1)*FX(1)+C5(3,2)*FX(2)+C5(3,3)*FX(3)
      RETURN
      END
C
C
      SUBROUTINE FCNJ(N,T,Y,PD)
      COMMON  KK(3,3),TT(3,3),MA(3,3),MB(3,3),W1(3,3),T1(3,3),B(3,3),
     &        S1(3,3),C(3,3),CO(3,3),C1(3,3),C2(3,3),C3(3,3),C4(3,3),
     &        C5(3,3),CF(3,3),L2,L3,CX,CY,CO,KX,KY,KO,DELC,THC
      REAL  KK,MA,MB,L2,L3,Y(6),PD(N,N),T,KX,KY,KO
      RETURN
      END
```

APPENDIX C

METHODOLOGY FOR THE DEVELOPMENT OF THE EQUATIONS OF MOTION FOR THE THREE-BEAM-TWO-JOINT MODEL

Three beams and Two joints



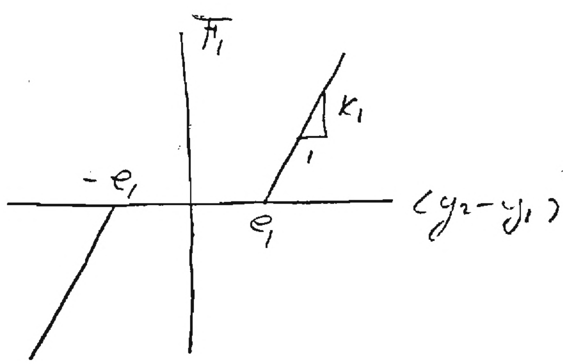
A : (y_1, θ_1)

B : (y_2, θ_2)

C : (y_3, θ_3)

D : (y_4, θ_4)

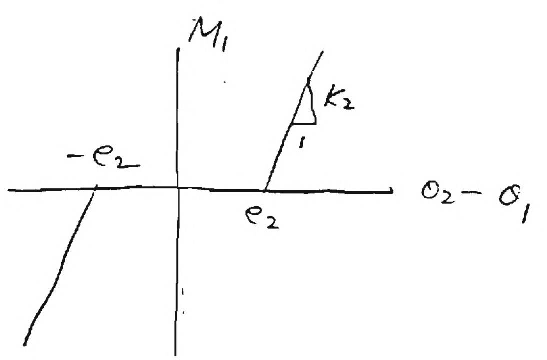
Joint 1, Spring



$y_2 - y_1 > e_1, \quad F_1 = k_1 (y_2 - y_1 - e_1)$

$y_2 - y_1 < -e_1, \quad F_1 = k_1 (y_2 - y_1 + e_1)$

$|y_2 - y_1| \le e_1, \quad F_1 = 0$

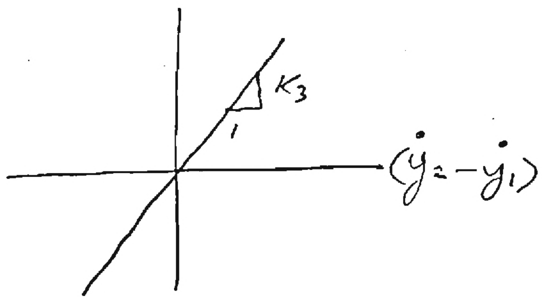


$\theta_2 - \theta_1 > e_2, \quad M_1 = k_2 (\theta_2 - \theta_1 - e_2)$

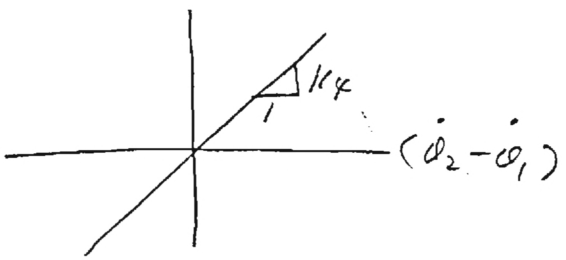
$\theta_2 - \theta_1 < -e_2, \quad M_1 = k_2 (\theta_2 - \theta_1 + e_2)$

$|y_2 - y_1| \le e_2, \quad M_1 = 0$

Viscous damper

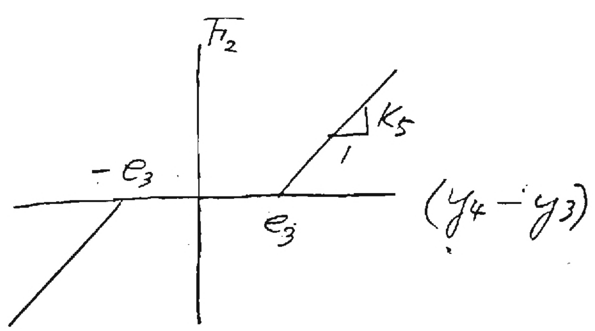


$k_3 = 0$: $|y_2 - y_1| \leq e_1$



$k_4 = 0$: $|\theta_2 - \theta_1| \leq e_2$

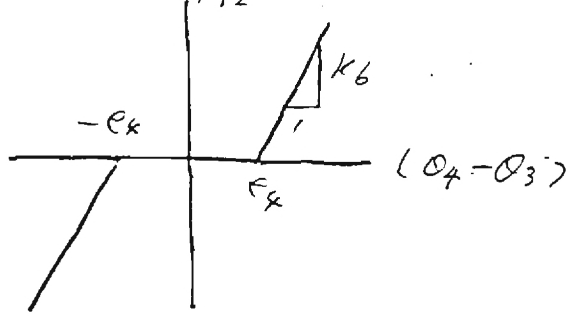
Joint 2.



$F_2 = k_5(y_4 - y_3 - e_1)$; $y_4 - y_3 > e$

$F_2 = k_5(y_4 - y_3 + e_1)$: $y_4 - y_3 < -$

$|y_4 - y_3| \leq e_3$, $F_2 = 0$

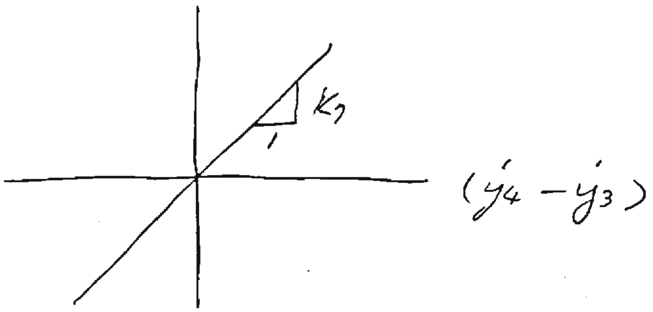


$M_2 = k_6(\theta_4 - \theta_3 - e_2)$; $\theta_4 - \theta_3 >$

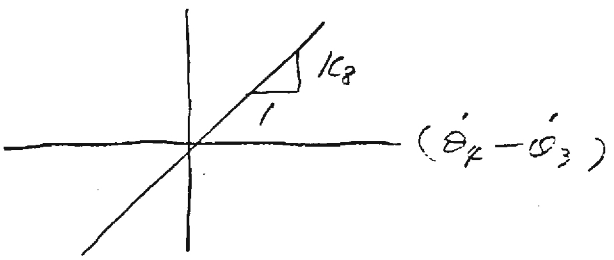
$= K_6(\theta_4 - \theta_3 + e_2)$; $\theta_4 - \theta_3 < -e$

$|\theta_4 - \theta_3| \leq e_4$, $M_2 = 0$

Viscous damper



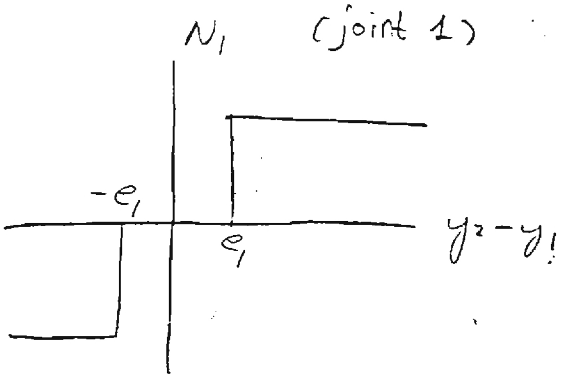
$$k_7 = 0 \quad ; \quad |y_4 - y_3| \le e_3$$
$$k_7 = k_3 \quad ; \quad |y_4 - y_3| > e_3$$



$$k_8 = 0 \quad : \quad |\theta_4 - \theta_3| \le e_4$$
$$k_8 = k_4 \quad ; \quad |\theta_4 - \theta_3| > e_4$$

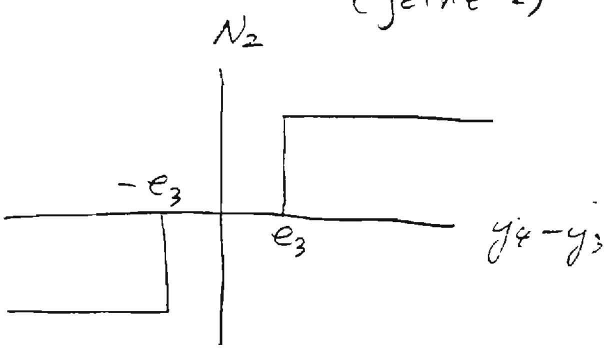
Normal force

(joint 1)



$$N_1 = |F_1|$$

(joint 2)



$$N_2 = |F_2|$$

Kinetic energy.

$$T = \sum_{i=1}^{3} \frac{1}{2} \int_0^{L_i} \rho_i A_i \left(\frac{\partial w_i}{\partial t}\right)^2 d\xi_i$$

Potential energy

$$V = V_{beam} + V_{spring}$$

$$V_{beam} = \sum_{i=1}^{3} \frac{1}{2} \int_0^{L_i} E_i I_i \left(\frac{\partial^2 w_i}{\partial \xi_i^2}\right)^2 d\xi_i$$

$$\delta V_{spring} = F_1 \delta(y_2 - y_1) + M_1 \delta(\theta_2 - \theta_1) + F_2 \delta(y_4 - y_3) + M_2 \delta(\theta_4 - \theta_3)$$

Rayleigh dissipation energy

$$\delta D = k_3(\dot{y}_2 - \dot{y}_1)\delta(y_2 - y_1) + k_4(\dot{\theta}_2 - \dot{\theta}_1)\delta(\theta_2 - \theta_1) + k_7(\dot{y}_4 - \dot{y}_3)\delta(y_4 - y_3) + k_8(\dot{\theta}_4 - \dot{\theta}_3)\delta(\theta_4 - \theta_3)$$

$$w_i(\xi_i, t) = \sum_{j=1}^{N} \eta_{ij}(t)\,\phi_{ij}(\xi_i) \qquad i = 1, 2, 3$$

$$\phi_{ij}(\xi_i) = \cosh\left(\frac{\lambda_j \xi_i}{L_i}\right) + \cos\left(\frac{\lambda_j \xi_i}{L_i}\right) - \sigma_j\left[\sinh\left(\frac{\lambda_j \xi_i}{L_i}\right) + \sin\left(\frac{\lambda_j \xi_i}{L_i}\right)\right]$$

(mode of free-free beam)

$$\sigma_j = \frac{\cosh\lambda_j - \cos\lambda_j}{\sinh\lambda_j - \sin\lambda_j}$$

$$\cos\lambda_j \cosh\lambda_j = 1$$

$\lambda_1 = 4.73004074$ $\qquad$ $\sigma_1 = 0.982502215$

$\lambda_2 = 7.85320462$ $\qquad$ $\sigma_2 = 1.000777312$

$\lambda_3 = 10.9956078$ $\qquad$ $\sigma_3 = 0.999966450$

$\lambda_4 = 14.1371655$ $\qquad$ $\sigma_4 = 1.000001450$

$\lambda_5 = 17.2787597$ $\qquad$ $\sigma_5 = 0.999999937$

$$T = \sum_{i=1}^{3} \sum_{j=1}^{N} \frac{1}{2} m_i \dot{\eta}_{ij}^{2} \qquad (m_i = \rho_i A_i L_i)$$

$$V_{beam} = \sum_{i=1}^{3} \sum_{j=1}^{N} \frac{1}{2} m_i \Omega_{ij}^{2} \eta_{ij}^{2}$$

where $\Omega_{ij}^2 = \left(\dfrac{\lambda_j}{L_i}\right)^4 \dfrac{E_i I_i}{\rho_i A_i}$

Ω_{ij} = jth natural frequency of beam i.

Virtual Work

$$\delta W = \delta W_{ef} + \delta W_s + \delta W_d$$

δW_{ef} = due to exciting force

$$= \sum_j F(t)\, \phi_{ij}(d)\, \delta\eta_{ij}(t)$$

δW_s = due to structural damping

$$= -\sum_{i=1}^{3} \sum_{j=1}^{N} \left[2\xi_{ij} \Omega_{ij}\, m_i\, \dot{\eta}_{ij}\, \delta\eta_{ij} \right]$$

δW_d = due to joint damping

$$\left.\begin{aligned} = &-\mu N_1 \,\mathrm{sign}(\dot{y}_2 - \dot{y}_1)\, \delta(y_2 - y_1) \\ &-\mu r_{e1} N_1 \,\mathrm{sign}(\dot{\theta}_2 - \dot{\theta}_1)\, \delta(\theta_2 - \theta_1) \end{aligned}\right\} \text{joint 1}$$

$$-\mu N_2 \,\mathrm{sign}(\dot{y}_4 - \dot{y}_3)\, \delta(y_4 - y_3)$$

$$-\mu r_{e2} N_2 \,\mathrm{sign}(\dot{\theta}_4 - \dot{\theta}_3)\, \delta(\theta_4 - \theta_3)$$

Constraints

Lagrange's multiplier

$$g_1 = \sum_{j=1}^{N} \eta_{1j}\phi_{1j}(L_1) - y_1 = 0 \quad : \beta_1$$

$$g_2 = \sum_{j=1}^{N} \eta_{1j}\phi_{1j}'(L_1) - \theta_1 = 0 \quad : \beta_2$$

$$g_3 = \sum_{j=1}^{N} \eta_{2j}\phi_{2j}(0) - y_2 = 0 \quad : \beta_3$$

$$g_4 = \sum_{j=1}^{N} \eta_{2j}\phi_{2j}'(0) - \theta_2 = 0 \quad : \beta_4$$

$$g_5 = \sum_{j=1}^{N} \eta_{2j}\phi_{2j}(L_2) - y_3 = 0 \quad : \beta_5$$

$$g_6 = \sum_{j=1}^{N} \eta_{2j}\phi_{2j}'(L_2) - \theta_3 = 0 \quad : \beta_6$$

$$g_7 = \sum_{j=1}^{N} \eta_{3j}\phi_{3j}(0) - y_4 = 0 \quad : \beta_7$$

$$g_8 = \sum_{j=1}^{N} \eta_{3j}\phi_{3j}'(0) - \theta_4 = 0 \quad : \beta_8$$

Eq. of motion

η_{1j} : $m_1[\eta_{1j}'' + 2\xi_{1j}\Omega_{1j}\dot{\eta}_{1j} + \Omega_{1j}^2\eta_{1j}] = F(t)\phi_{1j}(d) + \beta_1\phi_{1j}(L_1)$
$+ \beta_2\phi_{1j}'(L_1)$

η_{2j} : $m_2[\eta_{2j}'' + 2\xi_{2j}\Omega_{2j}\dot{\eta}_{2j} + \Omega_{2j}^2\eta_{2j}] = \beta_3\phi_{2j}(0) + \beta_4\phi_{2j}'(0)$
$+ \beta_5\phi_{2j}(L_2) + \beta_6\phi_{2j}'(L_2)$

η_{3j} : $m_3[\eta_{3j}'' + 2\xi_{3j}\Omega_{3j}\dot{\eta}_{3j} + \Omega_{3j}^2\eta_{3j}] = \beta_7\phi_{3j}(0) + \beta_8\phi_{3j}'(0)$

y_1 : $-F_1 - k_3(\dot{y}_2 - \dot{y}_1) = \mu N_1 \,\text{sign}(\dot{y}_2 - \dot{y}_1) - \beta_1 = 0$

θ_1 : $-M_1 - k_4(\dot{\theta}_2 - \dot{\theta}_1) = \mu r e_1 N_1 \,\text{sign}(\dot{\theta}_2 - \dot{\theta}_1) - \beta_2 = 0$

y_2 : $F_1 + k_3(\dot{y}_2 - \dot{y}_1) = -\mu N_1 \,\text{sign}(\dot{y}_2 - \dot{y}_1) - \beta_3 = 0$

θ_2 : $M_1 + k_4(\dot{\theta}_2 - \dot{\theta}_1) = \mu r e_1 N_1 \,\text{sign}(\dot{\theta}_2 - \dot{\theta}_1) - \beta_4 = 0$

joint 1

y_3 : $-F_2 - k_7(\dot{y}_4 - \dot{y}_3) = \mu N_2 \,\text{sign}(\dot{y}_4 - \dot{y}_3) - \beta_5 = 0$

θ_3 : $-M_2 - k_8(\dot{\theta}_4 - \dot{\theta}_3) = \mu r e_2 N_2 \,\text{sign}(\dot{\theta}_4 - \theta_3) - \beta_6 = 0$

y_4 : $F_2 + k_7(\dot{y}_4 - \dot{y}_3) = -\mu N_2 \,\text{sign}(\dot{y}_4 - \dot{y}_3) - \beta_7 = 0$

θ_4 : $M_2 + k_8(\dot{\theta}_4 - \dot{\theta}_3) = -\mu r e_2 N_2 \,\text{sign}(\dot{\theta}_4 - \dot{\theta}_3) - \beta_8 = 0$

joint

$\beta_3 = -\beta_1, \quad \beta_4 = -\beta_2, \quad \beta_7 = -\beta_5, \quad \beta_8 = -\beta_6$

$$[M]\,\underline{\ddot{\eta}} + [C]\,\underline{\dot{\eta}} + [K]\,\underline{\eta} = \underline{\phi}_d\, F(t) + [B]\,\underline{\beta}$$

$$[M] = \left[\begin{array}{cc|cc|cc}
m_1 & 0 & & & & \\
0 & \ddots\ m_1 & & 0 & & 0 \\
\hline
& & m_2 & 0 & & \\
0 & & 0 & \ddots\ m_2 & & 0 \\
\hline
& & & & m_3 & 0 \\
0 & & 0 & & 0 & \ddots\ m_3
\end{array}\right] \qquad 3N \times 3N$$

$$[C] = \left[\begin{array}{c|c|c}
\begin{matrix} 2\xi_{11}\Omega_{11}m_1 & \\ & \ddots\ 2\xi_{1N}\Omega_{1N}m_1 \end{matrix} & & \\
\hline
& \begin{matrix} 2\xi_{21}\Omega_{21}m_2 & \\ & \ddots\ 2\xi_{2N}\Omega_{2N}m_2 \end{matrix} & \\
\hline
& & \begin{matrix} 2\xi_{31}\Omega_{31}m_3 & \\ & \ddots\ 2\xi_{3N}\Omega_{3N}m_3 \end{matrix}
\end{array}\right]$$

$$[K] = \left[\begin{array}{c|c|c}
\begin{matrix} m_1\Omega_{11}^2 & \\ & \ddots\ m_1\Omega_{1N}^2 \end{matrix} & & \\
\hline
& \begin{matrix} m_2\Omega_{21}^2 & \\ & \ddots\ m_2\Omega_{2N}^2 \end{matrix} & \\
\hline
& & \begin{matrix} m_3\Omega_{31}^2 & \\ & \ddots\ m_3\Omega_{3N}^2 \end{matrix}
\end{array}\right] \qquad 3N \times 3N$$

$3N \times 3N$

$$\underline{\Phi}_d = \begin{bmatrix} \phi_{11}(d) \\ \vdots \\ \phi_{1N}(d) \\ \hdashline 0 \\ \vdots \\ 0 \\ \hdashline 0 \\ \vdots \\ 0 \end{bmatrix} \quad 1 \times 3N$$

$$\underline{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_5 \\ \beta_6 \end{bmatrix}$$

$$[B] = \begin{bmatrix} \phi_{11}(L_1) & \phi_{11}'(L_1) & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \phi_{1N}(L_1) & \phi_{1N}'(L_1) & 0 & 0 \\ \hdashline -\phi_{21}(0) & -\phi_{21}'(0) & \phi_{21}(L_2) & \phi_{21}'(L_2) \\ \vdots & \vdots & \vdots & \vdots \\ -\phi_{2N}(0) & -\phi_{2N}'(0) & \phi_{2N}(L_2) & \phi_{2N}(L_2) \\ \hdashline 0 & 0 & -\phi_{31}(0) & -\phi_{31}'(0) \\ \vdots & \vdots & \vdots & \\ 0 & 0 & -\phi_{3N}(0) & -\phi_{3N}'(0) \end{bmatrix} \quad 4 \times 3N$$

$$\underline{\eta} = \begin{bmatrix} \eta_{11} \\ \vdots \\ \eta_{1N} \\ \hdashline \eta_{21} \\ \vdots \\ \eta_{2N} \\ \hdashline \eta_{31} \\ \vdots \\ \eta_{3N} \end{bmatrix} \quad 1 \times 3N$$

$$\beta_1 = F_1 + k_3(\dot{y}_2 - \dot{y}_1) + \mu N_1 \operatorname{sign}(\dot{y}_2 - \dot{y}_1)$$

$$\beta_2 = M_1 + k_4(\dot{\theta}_2 - \dot{\theta}_1) + \mu r_{e1} N_1 \operatorname{sign}(\dot{\theta}_2 - \dot{\theta}_1)$$

$$\beta_5 = F_2 + k_7(\dot{y}_4 - \dot{y}_3) + \mu N_2 \operatorname{sign}(\dot{y}_4 - \dot{y}_3)$$

$$\beta_6 = M_2 + k_8(\dot{\theta}_4 - \dot{\theta}_3) + \mu r_{e2} N_2 \operatorname{sign}(\dot{\theta}_4 - \dot{\theta}_3)$$

$$\begin{aligned} F_1 &= k_1(y_2 - y_1 - e_1) &&; (y_2 - y_1) > e_1 \\ &= k_1(y_2 - y_1 + e_1) &&; y_2 - y_1 < -e_1 \\ &= 0 &&; |y_2 - y_1| \le e_1 \end{aligned}$$

$$\begin{aligned} M_1 &= k_2(\theta_2 - \theta_1 - e_2) &&; \theta_2 - \theta_1 > e_2 \\ &= k_2(\theta_2 - \theta_1 + e_2) &&; \theta_2 - \theta_1 < -e_2 \\ &= 0 &&; |\theta_2 - \theta_1| \le e_2 \end{aligned}$$

$$\begin{aligned} F_2 &= k_5(y_4 - y_3 - e_3) &&; y_4 - y_3 > e_3 \\ &= k_5(y_4 - y_3 + e_3) &&; y_4 - y_3 < -e_3 \\ &= 0 &&; |y_4 - y_3| \le e_3 \end{aligned}$$

$$\begin{aligned} M_2 &= k_6(\theta_4 - \theta_3 - e_4) &&; \theta_4 - \theta_3 > e_4 \\ &= k_6(\theta_4 - \theta_3 + e_4) &&; \theta_4 - \theta_3 < -e_4 \\ &= 0 &&; |\theta_4 - \theta_3| \le e_4 \end{aligned}$$

$$\begin{aligned} N_1 &= |F_1| &&; |y_2 - y_1| > e_1 \\ &= 0 &&; |y_2 - y_1| \le e_1 \end{aligned}$$

$$\begin{aligned} N_2 &= |F_2| &&; |y_4 - y_3| > e_3 \\ &= 0 &&; |y_4 - y_3| \le e_3 \end{aligned}$$

$$\underline{\beta} = \underline{f}(\underline{g}, \dot{\underline{g}}) \quad , \qquad \text{where} \qquad \underline{g} = \begin{bmatrix} y_1 \\ \theta_1 \\ y_2 \\ \theta_2 \\ y_3 \\ \theta_3 \\ y_4 \\ \theta_4 \end{bmatrix} \; 1\times 8$$

Constraint eq.

$$[G] = \left[\begin{array}{ccc|ccc|ccc} \phi_{11}(L_1) & \cdots & \phi_{1N}(L_1) & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \phi_{11}'(L_1) & \cdots & \phi_{1N}'(L_1) & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \phi_{21}(0) & \cdots & \phi_{2N}(0) & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \phi_{21}'(0) & \cdots & \phi_{2N}'(0) & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \phi_{21}(L_2) & \cdots & \phi_{2N}(L_2) & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \phi_{21}'(L_2) & \cdots & \phi_{2N}'(L_2) & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \phi_{31}(0) & \cdots & \phi_{3N}(0) \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \phi_{31}'(0) & \cdots & \phi_{3N}'(0) \end{array}\right] \; 3N \times 8$$

$$\underline{g} = [G]\,\underline{\eta}$$

$$\underline{\beta} = \underline{f}\{[G]\underline{\eta},\ [G]\dot{\underline{\eta}}\} = \underline{f}_1\{\underline{\eta}, \dot{\underline{\eta}}\}$$

$$[M]\ddot{\underline{\eta}} + [C]\dot{\underline{\eta}} + [K]\underline{\eta} = [B]\,\underline{f}_1\{\underline{\eta}, \dot{\underline{\eta}}\} + \underline{\phi}_1 F(t)$$

$$\Rightarrow \quad \underline{\eta}_1 = \underline{\eta}$$

$$\underline{\eta}_2 = \dot{\underline{\eta}}_1$$

$$\begin{cases} \dot{\underline{\eta}}_1 = \underline{\eta}_2 \\ \dot{\underline{\eta}}_2 = [M]^{-1}\left[-[C]\,\underline{\eta}_2 - [K]\,\underline{\eta}_1 + [B]\,\underline{f}_1\{\underline{\eta}_1, \underline{\eta}_2\} + \underline{\phi}_1 F(t)\right] \end{cases}$$

APPENDIX D

COMPUTER CODE FOR THE SIMULATION OF THE THREE-BEAM-TWO-JOINT MODEL

```
      PROGRAM HH(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)


      DIMENSION RAMDA(5),RHO(3),AREA(3),YOD(3)
      DIMENSION OMEGA(5),PHID(15)
      REAL LENG(3),M(3),MASS(3),MOI(3)
      COMMON/AA/FO,W,THETA
      COMMON/BB/B(15,4),G(8,15)
      COMMON/CONST/XK1,XK2,XK3,XK4,E1,E2,HNU,RE
      COMMON/DIM/XINV(15),C(15),STIFF(15)
      DATA RAMDA/4.73004074,7.85320462,10.9956078,14.1371655,17.27875971

          RAMDA=ROOT OF CHARACTERISTIC EQN. OF BEAM
          RHO=DENSITY OF BEAM

      WRITE(6,5)

        READ INPUT DATA
      N=NUMBER OG MODES

      READ(5,10) N,KMAX,DT
      READ(5,20) FO,W,THETA
      WRITE(6,25) N,FO,W,THETA
      DO 30 L=1,3
      READ(5,30) RHO(L),AREA(L),YONG(L),MOI(L),LENG(L)
      WRITE(6,40) RHO(L),AREA(L),YONG(L),MOI(L),LENG(L)
   30 CONTINUE
      READ(5,50) XK1,XK2,XK3,XK4,E1,E2
      WRITE(6,55)XK1,XK2,XK3,XK4,E1,E2
      READ(5,56)HNU,RE
      WRITE(6,57) HNU,RE

      READ(5,60) ZETA
      WRITE(6,65) ZETA
    5 FORMAT(1H1,///," ** THREE BEAMS AND TWO JOINTS **",///)
   10 FORMAT(2I5,F10.0)
   20 FORMAT(3F10.0)
   25 FORMAT(//,10X,"NUNBER OF MODES=",I5,
     *//,10X,"MAGNITUDE OF EXCITING FORCE=",F10.5
     *//,10X,"CIRCULAR FREQUENCY=",F10.5,"(RAD/S)",
     */,10X,"PHASE SHIFT=",F10.5,"(RAD/S)")
   30 FORMAT(5F10.0)
   40 FORMAT(//,5X," PROPERTY OF BEAM ",/,
     *5X,"DENSITY=",F12.5,/,
     *5X,"AREA=",E12.5,"(IN*IN)",/,
     *5X,"YOUNG'S MODULUS=",E12.5,"(PSI)",/,
     *5X,"MOMENT OF INERTIA=",E12.5,"(IN**4)",/,
     *5X,"LENGTH=",E12.5,"(FT)")
   50 FORMAT(5F10.0)
   60 FORMAT(6F10.0)
   55 FORMAT(//,5X,"PHYSICAL CONSTANTS  OF JOINTS",
     *//,10X,"LATERAL SPRING CONSTANT=",E12.5,"(LB/IN)",
     */,10X,"TORSIONAL SPRING CONS.=",E12.5,"(LB-IN/RAD)",
     */,10X,"LATERAL DAMPING CONST.=",E12.5,
     */,10X,"TORSIONAL DAMPING CONST.=",E12.5,
     */,10X,"LATERAL CLEARANCE=",E12.5,"(IN)",
     */,10X,"TORSIONAL CLEARANCE=",E12.5,"(RAD)")
```

```
 56 FORMAT(2F10.0)
 57 FORMAT(//,10X,"FRICTION COEFF.=",F10.3,/,
   *10X,"EQUIVALENT LENGTH FOR TORSIONAL FRICTION=",E12.5,"(IN)")
 65 FORMAT(//,10X,"MATERIAL DAMPING OF BEAM(PERCENT OF CRITICAL DAMPING
   *)=",F10.4)

       CALCULATE MASS

    DO 80 I=1,3
 80 M(I)=RHO(I)*AREA(I)*LENG(I)

       CALCULATE NATURAL FREQUENCY [DIES

    DO 90 I=1,3
    DO 90 J=1,N

 90 OMEGA(I,J)=(RAMDA(J)/LENG(I))**4*(YOD(I)*MOI(I)/(RHO(I)*AREA(I)))

  CALCULATE MASS ,

    N3=N*3
    DO 100 I=1,N
    MASS(I)=M(1)
    MASS(N+I)=M(2)
    MASS(2*N+I)=M(3)


       CALCULATE C MATRIX

    C(I)=2.*ZETA*OMEGA(1,I)*M(1)
    C(I+N)=2.*ZETA*OMEGA(2,I)*M(2)
    C(I+2*N)=2.*ZETA*OMEGA(3,I)*M(3)
    K(I)=M(1)*OMEGA(1,I)**2
    K(I+N)=M(2)*OMEGA(2,I)**2
100 K(I+2*N)=M(3)*OMEGA(3,I)**2
    DO 120 I=1,N3
120 XINV(I)=1./MASS(I)

      CALCULATE B MATRIX

    DO 110 I=1,N3
    DO 110 J=1,4
110 B(I,J)=0.
    DO 125 J=1,N
    CALL MODE(1,J,PHI,PRIM,1.0)
    B(J,1)=PHI
    B(J,2)=PHI
    CALL MODE(2,J,PHI,PRIM,0.0)
    B(J+N,1)=-PHOI
    B(J+N,2)=-PRIM
    CALL MODE(2,J,PH;PRIM,1.0)
    B(J+N,3)=PHI
    B(J+N,4)=PRIM
    CALL MODE93,J,PHI,PRIM,0.0)
    B(J+2*N,3)=-PHI
    B(J+2*N,4)=-PRIM
120 CONTINUE
    DO 130 J=1,N
    R=D/LENG(1)
    CALL MODE91,J,PHI,PRIM,R)
```

```
      PHID(J)=PHI
      PHID(J+N)=0.
      PHID(J+2*N)=0.
  130 CONTIJNUE

    G MATRAM   IX

      DO 140 I=N3
      DO 140 J=1,8
  140 G(I,J)=0.
      DO 150 J=1,N
      G(1,J)=B(J,1)
      G(2,J)=B(J,2)
      G(3,J+N)=-B(J+N,1)
      G(4,J+N)=-B(J+N,2)
      G(5,J+N)=B(J+N,3)
      G(6,J+N)=B(J+N,4)
      G(7,J+2*N)=-B(J+2*N,3)
      G(8,J+2*N)=-B(J+2*N,4)
  150 CONTINUE

       SOLVE 6*N DIFFERENTIAL EQUATIONS

      NN=6*N
      CALL SOLVE(NN,DT,KMAX)
      STOP
      END

           CALCULATE MODE SHAPE AND DERIVERTIVE

      SUBROUTINE MODE(I,J,PHI,PRIM,R,RAMDA,LENG)
      REAL RAMDA(5),LENG(3)

        I=BEAM I
        J=J TH MODE
        PHI=MODE SHAPE OF A FREE-FREE BEAM
        PRIM=FIRST DERIVERTIVE OF A MODE SHAPE

      XI/LENG(I)=R
      RAM=RAMDA(J)
      SIGMA=(COSH(RAM)-COS(RAM))/SINH(RAM)-SIN(RAM))
      Z=RAM*R
      PHI=COSH(Z)+COS(Z)-SIGMA*(SINH(Z)+SIN(Z))
      PRIM=(RAM/LENG(I))*(SINH(Z)-SIN(Z)-SIGMA*(COSH(Z)+COS(Z)))
      RETURN
      END

            SOLVE DIFF. EQNS. USING RUNGE KUTTER METHOD
            NN=NUMBER OF EQUATIONS

      SUBROUTINE SOLVE(NN,DT,KMAX)
      INTEGER NN
      DIMENSION ETA(NN),CC(24),W(NN,9)
      COMMON/AA/FO,W,THETA
      COMMON/CONST/XK1,XK2,XK3,XK4,E1,E2,HNU,RE
      COMMON/BB/B(15,4),G(8,15)
      COMMON/DIM/XINV(15),C(15),STIFF(15)
      EXTERNAL FUNC
      NW=NN
      X=0.
```

```
        INITIAL CONDITION

    N2=NN/2
    DO 100 I=1,N2
    ETA(I)=0.
100 ETA(I+N2)=0.1
    TOL=0.0001
    IND=1
    DO 200 K=1,KMAX
    XEND=DT*K
    CALL DVERK(NN,FUNC,X,ETA,XEND,TOL,IND,CC,NW,W,IER)
    IF(IND.LT.0.OR.IER.GT.0) GO TO 20
    WRITE(6,300) ETA(1)
300 FORMAT(/,2X,"ETA(1)=",E12.5)
200 CONTINUE
 20 RETURN
    END

        COMPUTES DERIVERTIVES   EPRIME(NN)

    SUBROUTINE FUNC(NN,X,ETA,EPRIME)
    COMMON/DIM/XINV(15),C(15),STIFF(15)
    COMMON/AA/FO/W,THETA
    INTEGER N
    REAL CP(15),KP(15),BP(15),BETA(4)
    DIMENSION ETA(15),EPRIME(NN)
    COMMON/CONST/XK1,XK2,XK3,XK4,E1,E2,HNU,RE
    N2=NN/2
    DO 100 I=1,N2
    EPRIME(I)=ETA(I)
    II=I+N2
    CP(II)=C(II)*ETA(II)
    KP(II)=STIFF(II)*ETA(I)
    CALL NON(BETA,ETA,NN)
    DO 10 L=1,4
 10 BP(II)=B(II,L)*BETA(L)
    FORCE= FO*COS(W*X+THETA)
100 EPRIME(II)=XINV(II)*(-CP(II)-KP(II)+BP(II)+PHID(II)*FORCE)
    RETURN
    END

        NONLIEAR FUNCTION   BETWEEN  LAGRANGE MULTIPLIERS BETA AND
        STATE VARIABLE ETA(NN)

    SUBROUTINE NON(BETA,ETA,NN)
    DIMENSION BETA(4),ETA(30),Q(8),QP(8)
    COMMON/BB/B(15,4),G(8,15)
    COMMON/CONST/XK1,XK2,XK3,XK4,E1,E2,HNU,RE
    REAL N1,N2,M1,M2
    NN2=NN/2
    DO 100 I=1,8
    DO 100 J=1,NN2
    Q(I)=G(I,J)*ETA(J)
100 QP(I)=G(I,J)*ETA(J+NN2)
    CALL CRIT(Q(3),Q(1),XK1,E1,F1)
    CALL CRIT(Q(4),Q(2),XK2,E2,M1)
    CALL CRIT(Q(7),Q(5),XK5,E3,F2)
    CALL CRIT(Q(8),Q(6),XK6,E4,M2)
    N1=ABS(F1)
```

```
N2=ABS(F2)
BETA(1)=F1+XK3*(QP(3)-QP(1))+HNU*N1*SIGN(1.0,QP(3)-QP(1))
BETA(2)=M1+XK4*(QP(4)-QP(2))+HNU*RE*N1*SIGN(1.0,QP(4)-QP(2))
BETA(3)=F2+XK7*(QP(7)-QP(5))+HNU*N2*SIGN(1.0,QP(7)-QP(5))
BETA(4)=M1+XK8*(QP(8)-QP(6))+HNU*RE*N2*SIGN(1.0,QP(8)-QP(6))
RETURN
END

        COMPUTES NORMAL FORCES AND MOMENTS

SUBROUTINE CRIT(Y2,Y1,XK,E,F)
IF((Y2-Y1).GT.E) F=XK*(Y2-Y1-E)
IF((Y2-Y1).LT.-E) F=XK*(Y2-Y1+E)
IF(ABS(Y2-Y1).LE.E) F=0.
RETURN
END
```

E-25-641

Final Report

The Influence of Nonlinear Joints on the Dynamics of Large Flexible Space Structures

by Aldo A. Ferri

Prepared for

Honeywell, Inc.
Space and Strategic Avionics Division
Clearwater, FL 33546-7290

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EXECUTIVE SUMMARY

Problem Statement:

One of the major problems remaining in the development of large flexible space structures such as the NASA sponsored space station is the anticipated low level of passive damping. This low level of damping impacts the feasibility of placing large truss structures in orbit because of the difficulties in designing shape and attitude controllers. In particular, since the open-loop system has low relative stability to begin with, it is quite possible that perturbations to the control scheme, such as observation spillover or plant uncertainty, can drive the closed-loop system unstable.

Joints play a central role in the development of large, light-weight truss structures. It has been suggested that they provide a significant source of damping for the structure. Unfortunately, many of the physical properties of joints are governed by nonlinear mathematical relations. Hence, any accurate structural model of space truss structures must be nonlinear if it is to account for the presence of nonlinear joints.

Main objectives of the current research:

This research addresses the problems of modelling and analysis of large space structures with nonlinear joints. In addition, qualitative information is sought for the dynamic behavior of structures with nonlinear joints. Specific objectives are summarized below.

A primary objective of the present research is the development of a generic mathematical joint model. A model is developed for a rigid beam partially inserted into an inertially fixed sleeve joint. This model is used to study parametrically the system dynamics as a function of various joint properties. The system is also used to develop simplified sleeve joint models to be used in more complex, flexible structures.

A second objective is to incorporate the simplified sleeve joint models into an otherwise linear flexible structure. The equation of motion are developed for a system consisting of three linear flexible beams interconnected with two nonlinear sleeve joints. This model is studied to determine the influence of various joint properties on the overall system dynamics and to observe the qualitative behavior of a flexible structure with nonlinear joints.

Finally, a mathematical model is developed for a pin-point joint based on actual measured test data. This model is incorporated into a three-beam system to determine the behavior of the combined system.

Conclusions:

The major conclusions of this study are:

1. A sleeve joint system damped only with dry friction will exhibit envelopes of decay which are largely exponential. In other words, the damping contributions from dry friction tend to appear like those from linear viscous damping.

2. Much of the behavior associated with systems having nonlinear joints is amplitude dependent. For example, higher amplitudes produce larger damping ratios and higher resonant frequencies than lower amplitudes.

3. The "natural frequencies" of jointed structures are dependent on the joint stiffness. As the joint stiffness increases, the "natural frequencies" increase but, also, the difference between the highest and lowest "natural frequency" grows making numerical integration of the equations of motion difficult to perform.

4. Systems with typical nonlinear joints can exhibit marked hardening spring behavior. This includes the presence of jump phenomenon and multiple steady-state solutions to harmonic excitation.

5. The systems studied in this report exhibited both sub-harmonic and super-harmonic response when forced harmonically.

Items 2 through 5 may have a significant impact on the design and performance of linear control schemes for the shape and attitude control of large flexible truss structures.