"In presenting the dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institution shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.

"

AN ENGINEER'S APPROACH TO THE DYNAMIC BEHAVIOR OF MICROECONOMIC SYSTEMS

A THESIS

Presented to

The Faculty of the Graduate Division

by

Paul Truninger

In Partial Fulfillment

of the Requirements for the Degree of

Master of Science in Industrial Engineering

Georgia Institute of Technology

June 1958

5a
12T

AN ENGINEER'S APPROACH TO THE DYNAMIC BEHAVIOR OF MICROECONOMIC SYSTEMS

APPROVED:

Date Approved by Chairman: May 30, 1958

ACKNOWLEDGMENTS

The author wishes to express sincere appreciation to Professor J. Krol for providing constant encouragement during the course of this study. Thanks are extended to Professors D. C. Ekey and F. O. Nottingham, members of the reading committee, for their many helpful comments. A special word of gratitude is due to R. S. Johnson, of Georgia Tech Experimental Station, for constructive criticism of the analogue model. The author is also indebted to the Georgia Tech World Student Fund, whose contribution helped to make this work possible.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

AN ENGINEER'S APPROACH TO THE

DYNAMIC BEHAVIOR OF MICROECONOMIC SYSTEMS

(59 pages)

by

Paul Truninger

Dr. J. Krol, Thesis Advisor

ABSTRACT

A microeconomic system is an economic system strictly related to a single enterprise. In a free, competitive economy, such a system (as well as any other economic organization) is subject to continuous changes. These variations are due to exterior influences acting upon the system, one of them being the demand for the manufactured goods. A theory of the statics of a microeconomic system cannot take into account rapid variations in time of system parameters. Therefore, the dimension "time" has to be introduced explicitly by studying the dynamics of the system.

The problem is to find an optimum price policy for a product underlying a random demand and manufactured in a selected microeconomic system. This system consists of the following eight components: Management, Production, Inventory, Market, Demand, Cost, Accounting and Forecasting. These components represent all essential functions necessary in a typical production - marketing cycle. The problem is solved quantitatively by mathematical methods. A block diagram based on

the law of supply and demand is derived showing the mechanism of the system. Then, a differential equation expressed in terms of the Laplace Transformation, a so called "operator", is assigned to each block, defining its input - output relationship. Exceptions are the management and the forecasting operators, which are introduced into the equations as unknowns and then determined quantitatively by requiring the system to operate at every instance with the maximum possible profit. Such a solution is found, but it is only stable if the slope of the demand curve is bigger than the slope of the approximated cost curve. With this optimum management operator, the best price policy for any demand can be determined.

An electronic analogue approximating the symbolic model was derived and constructed, using R - C networks and vacuum tubes. The analogue was tested and found to represent the symbolic model with acceptable accuracy. The analogue serves two main purposes: First, it provides for displaying the best price policy for any demand on an oscilloscope, thus eliminating direct calculations. Second, with its aid, effects of variations of system parameters upon different variables can be studied easily.

CHAPTER I

INTRODUCTION

The success engineers have had in the past in developing various physical systems is certainly based on a thorough knowledge of the behavior of such systems under changing conditions. For systems where experimentation is not possible or too costly, only with the aid of mathematics could optimum design specifications be derived. It is not only virtually impossible but also not ethical to experiment in economics without reasonable chances of success. These chances can only be guessed by qualitative verbal reasoning or determined with limited accuracy by mathematical methods. The engineer tends to prefer the latter, not because of a <u>déformation professionelle</u>, but simply because too much is at stake to allow guessing, and because a quantitative analysis can only be done with the aid of mathematics. Therefore, the purpose of this study was to point out the potentialities of a systems engineering approach to economic systems.

In traditional Industrial Engineering, many problems encountered are of the following type: Given a certain production volume or output (supposed to be constant over a long period of time), determine the most economical solution of a certain problem. Now suppose for a specific enterprise all problems of this kind have been solved (which will naturally never be achieved completely in reality), so that the plant is working at its optimum capacity. We have good reason to assume that the output will sooner or later deviate from its optimal value (this means simply that the output generally will not remain constant, instead, it will be a function of time). This change in output is primarily due to fluctuations in demand. Along with the output and demand, other factors vary, such as costs, profit, inventory, management decisions regarding the output, etc. In an actual plant, all these factors are integrated in a system, in this case called a "microeconomic system". The choice of the word explains itself by considering an economy strictly related to a specific enterprise as an "atom" of the overall national economy the latter being called a macroeconomic system. The study of variations of the system with respect to time is the study of its dynamic behavior. The mathematical tools and methods for the study of the dynamics of mechanical and electrical systems are well known and widely used by engineers. The present study was an attempt to apply the same mathematical tools and methods for the analysis of a non-physical microeconomic system.

To point out the need of such an approach and to illustrate the nature of the problem, the following practical example is given:

Consider a product whose demand undergoes seasonal changes, say gloves for men. There will be a high sales volume during the late fall and winter months, and very few sales can be made during the rest of the time. Suppose the manufacturer produces only this item and that the plant formerly was working during the on-season only. Now, for some reason, it is decided to take up year-round production. The capital means to maintain an inventory of a capacity of more than half a year's production are not available. Therefore, a good part of the actual production must be sold during the summer months. To motivate retailers to buy during that time, it is decided to give them an incentive in form of a price reduction for off-season buys. Therefore, the following questions are to be answered: How and how much must the price change during the year? Knowing the expected demand for a fixed price, what would be the demand for the variable price? What profit can be expected if a variable price policy is adopted? Which variable price policy, if any, does yield a maximum profit?

To answer these and similar questions, a thorough understanding of the mechanism of such a single plant system is needed. In this study, the optimum price policy for a selected microeconomic system was derived, and both its mechanism and a mathematical method to attack the problems of its dynamics are presented.

CHAPTER II

ANALYSIS OF A MICROECONOMIC SYSTEM

General Nature of Microeconomic Systems

The reader is undoubtedly familiar with organization charts. They show exactly the relationship between all employees of a factory and the way in which a management decision goes "down the line". Every man in the organization has a superior, except the manager. Now it would be completely false to infer that the manager can do whatever he likes. Instead, a sound management bases its decisions on certain facts, mostly economic in nature. While employees are controlled by their superiors, the manager is controlled by his desire to maintain successful operations, a measure of which is the profit a plant is making. Each management decision will affect in a certain sequence the employees, the output, and the profit. If the actual profit does not attain a desirable level, then the management will undertake corrective action. This again affects the output and the profit, and based on this new profit and the expected demand, a new output level will be set by the management. Clearly there is a closed sequence of interdependence; in other words, we are faced with a closed loop system (also called feedback or servo-system). This suggests the use of the Servomechanisms Theory for the analysis of the system. In order to be allowed to use this theory, we have to formulate the following hypothesis:

The mathematical analysis developed for physical systems (in particular the Servomechanisms Theory) can be applied for the analysis of a microeconomic system.

If this hypothesis is true, we can infer that it must be possible to simulate a microeconomic system by a physical analogue. The realization of a physical analogue of a microeconomic system will be a test of the hypothesis. We shall see that we can do it theoretically, but that practically we can only approximate it. We have good reasons to believe that this generalization holds for all possible microeconomic systems, although we shall show it only for one specific case.

Choice of a Particular Microeconomic System

All that we know so far about our system is that it will contain at least one closed loop. All microeconomic systems certainly have some others common characteristics and parts, but they are connected differently from case to case, thus forming different systems. From the infinity of possible systems we are going to select carefully a system which should be as realistic as possible, and also as simple as possible. Both conditions oppose themselves; therefore we have to choose a compromise. This will be a system which contains all its important factors (important with respect to its dynamic behavior) and in which all details are omitted. This choice demonstrates clearly the purpose of the study:

Emphasis was given to the usefulness of this type of thinking rather than to the solution of an actual problem with all its details.

To meet the described conditions, the following assumptions are made:

1. The enterprise, whose microeconomic system will be investigated, operates in a free, competitive economy.
2. The plant is producing only one item. (For a firm producing a number of items the system could be considered as a part of an overall system related to this specific item.)
3. Management bases its decisions regarding the output entirely on the profit resulting from the manufacturing process, and this profit is measured in dollars per unit of the output. However, this assumption will be dropped later on.

1. <u>The demand- and cost- functions</u>.--Assumption 3 needs to be further investigated. Let us assume a price-demand curve and a cost-output curve for the product to be manufactured (Figs. 1 and 2). The profit then will be the difference between price and costs at the same output or demand (Figs. 3 and 4). For a certain output the profit has a maximum. We assume that the management seeks to maximize the profit in every instance. If there is a deviation in output ΔO_1, the new profit is $P_o - \Delta P$. Notice that the same profit $P_o - \Delta P$

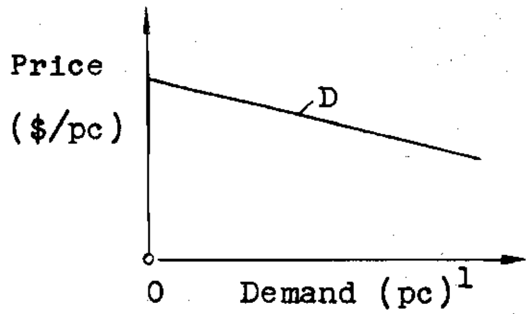


Fig. 1: Demand Function

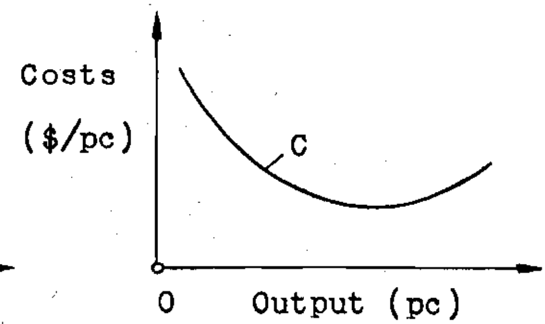


Fig. 2: Cost Function

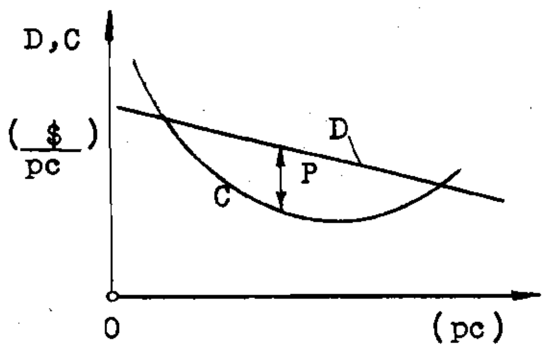


Fig. 3: Combined Demand- and Cost Functions

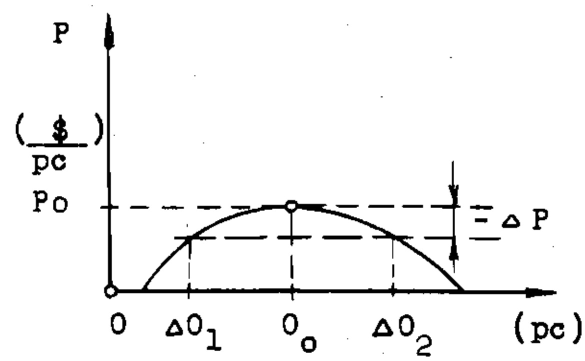


Fig. 4: Profit P as a Function of the Output

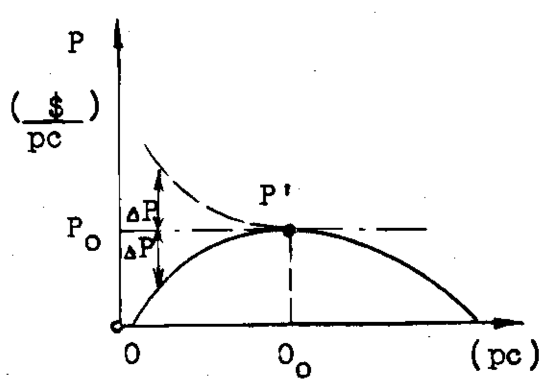


Fig. 5: Construction of P'

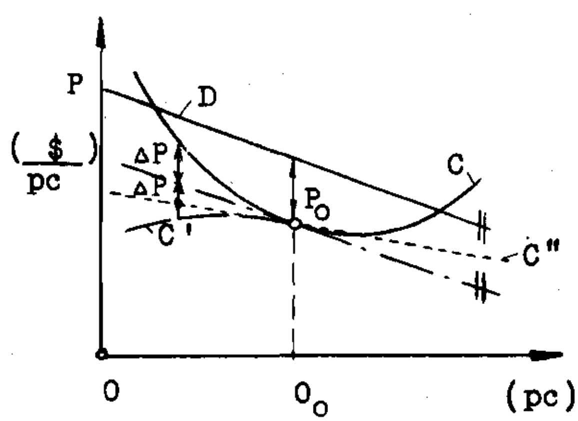


Fig. 6: Construction of C' and C"

---

[1] pc is the abbreviation for "pieces"

would result from an output of $O_o + \Delta O_2$. Therefore, the management has to know two facts:

1. The actual value of the profit $P_o - \Delta P$, and
2. on which side of the O_o - point the process is running.

In other words, the manager must know if the plant is producing too many or too few items. We can (and we shall see later that we have to) combine these two items of information into one, by the following reasoning (see Fig. 5): The upper part of the profit-curve $O < O_o$[1] is rotated around an axis of symmetry as indicated. This means simply that the profit deviation $-\Delta P$ for $O < O_o$ changes the sign from minus to plus. The new curve $P' = f(O)$ is now single-valued, because for every O only one P' corresponds. From P' only, the management knows exactly the output O and the profit, which now is given by $P = P_o + |\Delta P|$.

According to P', we have to define a new cost function C', which will be determined by the equation

$$P' = D - C' \qquad \text{or} \qquad C' = D - P'$$

Fig. 6 shows the graphical derivation of C'.

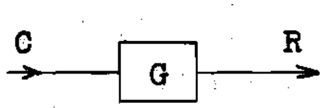
But C' is not yet the final form of the cost function to be used. C' is not a linear function and as such is very difficult to handle mathematically. A common method of overcoming this difficulty is to linearize the curve. Keeping

---

[1] O = Output

in mind that only relatively small variations around a certain point (in this case θ_o) have to be considered, we can replace the actual curve by its tangent at the point considered (θ_o). In our case we choose a straight line C″ having a slope slightly bigger than D, because C' has at θ_o a slope equal the slope of D, and the difference between both lines would be constant. Our final cost curve is then C″. In any further consideration we shall omit the two primes of C″ and speak about C as our cost function (straight line approximation).

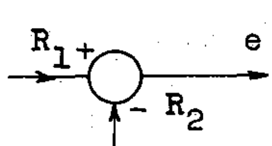
2. Representation of a microeconomic system as a closed-loop system.--We are now prepared to sketch a microeconomic system as a closed-loop relationship. In order to explain the derivation of the system, we first have to establish its graphical representation. Before doing so, we shall give a brief description of the symbols used.

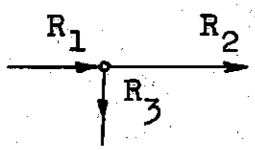
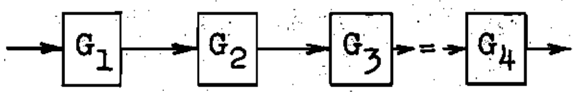


C = Input (Command)

R = Output (Response)

G = Transfer Function

Symbol	Equation	
	$R = CG \qquad G = R/C$	(1)
	$e = R_1 - R_2$ (error)	(2)

Symbol	Equation
	$R_1 = R_2 = R_3$ (3) Information direction = direction indicated by arrows
	$G_4 = G_1 G_2 G_3$ (4) (follows from (1))

Generally, R and C are functions of time. They will be translated with the Laplace-Transformation into complex frequency-functions R (p) and C (p), and for convenience all calculations are made in the p-domain. Then the results will be inverse-transformed into the t-domain, thus making possible their interpretation.

With the symbolic notations described above, a micro-economic system is sketched in Fig. 7. In this system, management bases its decisions regarding the price of the product on the deviation of the actual profit from the desired profit. If there is an error "e", management will take corrective action and adjust the price in order to regulate the demand. The relationship between price and demand is given by the price-demand curve. The difference between supply and demand, e_3, is the number of pieces per day by which the inventory increases. The integral with respect to time of e_3 is the inventory level. This level has to be reported to the production department, and this will be achieved with a certain delay, the inventory-control delay.

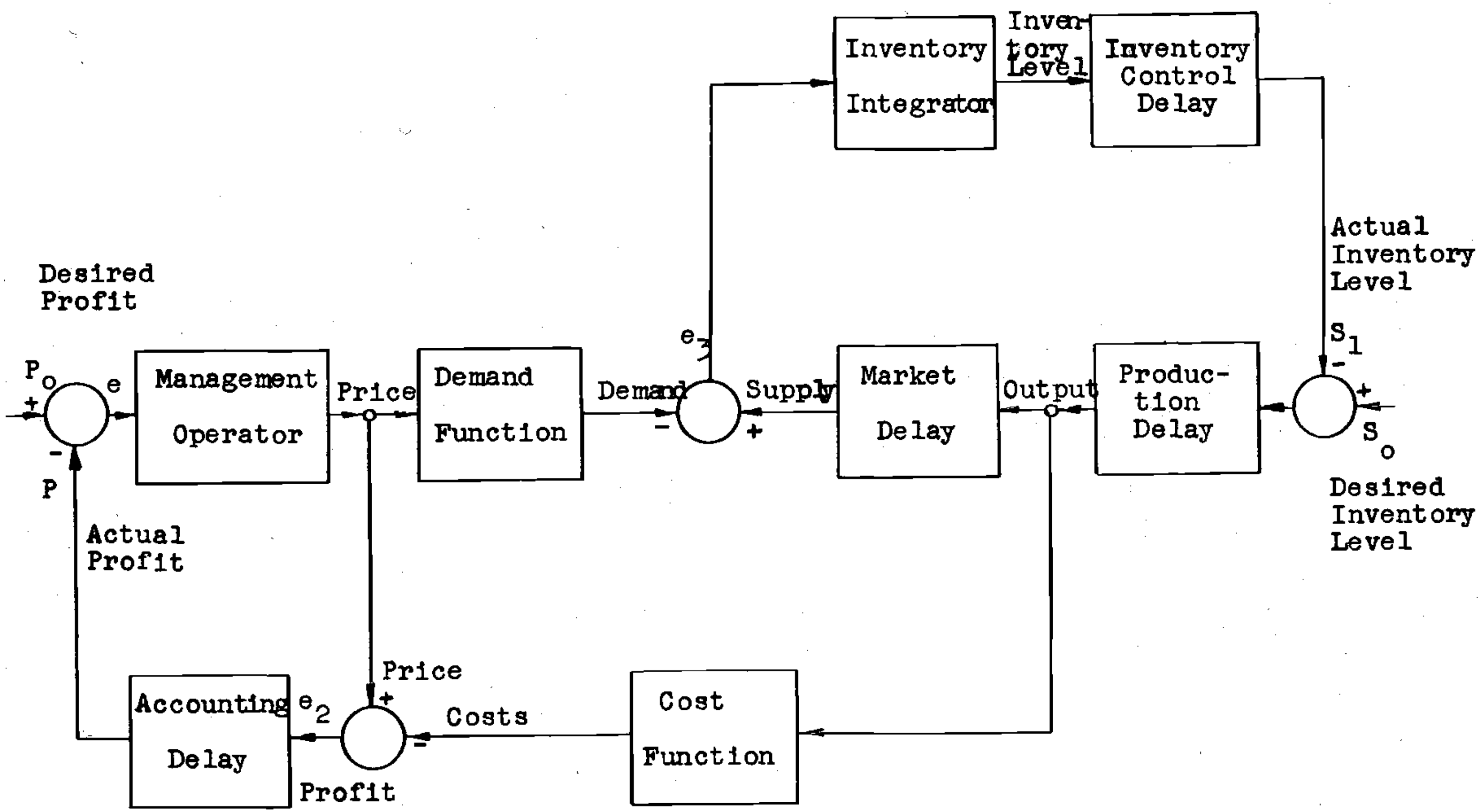


Fig. 7: Block Diagram of a Microeconomic System

We suppose that the production rate is determined so as to keep the inventory at an optimum, constant level S_o. If there is a small error ΔS between the actual and the optimum inventory level, the production rate will be changed in order to bring S_1 back to S_o. After a market delay, the output will be the supply on the market. From the output, we derive with the aid of the cost-curve the costs; and the difference between the price and the costs, e_2, is the profit. The actual profit will be reported to management by the accounting department after a delay, called "accounting delay". Based on this new profit, the management will determine a new price, etc. This is one complete cycle of the system which will be repeated again and again. Thus, the mechanism of the system has been explained.

3. Choice of the operators.--The remaining task is to investigate the nature of the different blocks. Each block is a symbol for a transfer function or an operator. The derivation of an operator follows directly from an experiment. Suppose we would like to determine the production delay operator. We would then have to perform the following experiment: Assume the production rate as determined by management has been constant over a long period of time. Due to a certain change in demand, the management decides that the production department has to proceed at a new output level $0 + \Delta 0$. The production department generally will not be able to follow the new order immediately; instead, it

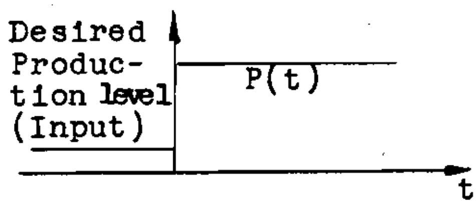


Fig. 8

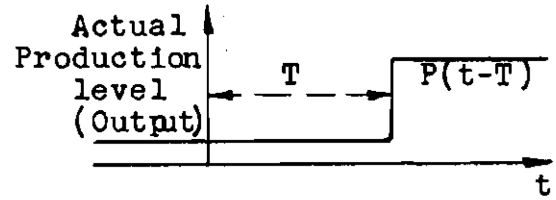


Fig. 9

Input-Output Relationship of the Production Department

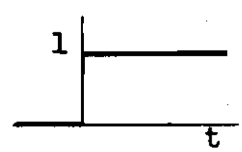
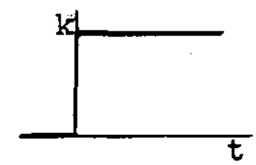
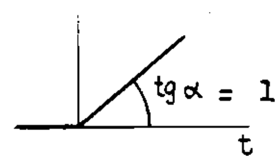
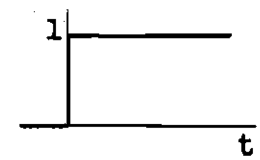
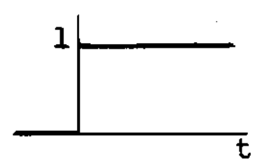
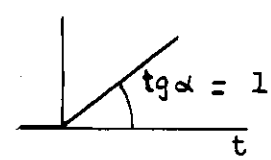
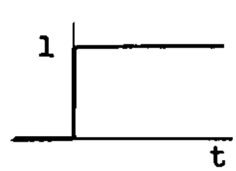
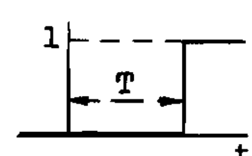
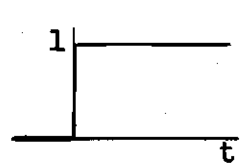
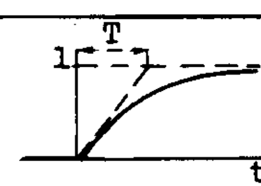
#	Description	Operator F(p)	Example	
			Input	Output
1	Proportional Action	k		
2	Rate Action	p		
3	Integral Action	$\frac{1}{p}$		
4	Time Lag or Transportation Lag	e^{-pT}		
5	Exponential Lag	$\frac{1}{1 + pT}$		

Fig. 10: Basic Unit Processes

has to make preparations over a certain period of time. Only after the time lag "T" can the desired production level be obtained (Figs. 8 and 9).

The operator P will then be such that it delays an input function P(t) to an output function P(t-T). This operator is in terms of the Laplace Transformation e^{-pT} and it is independent of the shape of the input function. The symbol "p" is the Laplace variable (substituted for the time t) which denotes a complex frequency of the type $p = \alpha + j\omega$. The reason for choosing functions of p rather than functions of t lies in the simplicity of the Laplace Transformation approach. The equation C = RG holds only for C(p), R(p) and G(p). C = RG (p. 8) does not hold for C(t), R(t) and G(t). The Laplace Transformation also allows one to transform linear differential equations to common algebraic expressions. Instead of establishing the differential equations governing our system we can directly determine the corresponding algebraic functions of p.

Definition:

$$F(p) = \int_{0}^{\infty} e^{-pt} f(t)dt$$

Symbolically: $L\left[f(t)\right] = F(p)$

There exist tables of the Laplace Transformations in which the F(p) of all possible f(t) can be found. Naturally, the table also can be used to find the inverse transform f(t) if F(p) is known. We shall not go further into this purely mathematical aspect of the problem.

There are only a few possible basic forms the operator can take. These are listed and illustrated in Fig. 10 (p. 13). Of course, there are an unlimited number of other operators, but the basic features of the microeconomic system can be described by these five operators. Prof. D. P. Campbell calls them "unit processes" (Ref. 2). He gives an excellent description of these unit processes:

> "Proportional action" (1) is self-evident. Response is in proportion to command, $R = kC$. The constant k may be greater than, equal to, or less than unity. "Integral action" (3) defines how a quantity builds up in proportion to the integral with respect to time of another quantity, $R = C/p$. Thus, the build up of stock or the accumulation of material between successive stages in production processes illustrates integral action. "Rate action" (2) is best defined in a mathematical sense as a derivative. It signifies a situation where the rate of change of a quantity in a production process is in proportion to another quantity; thus $R = pC$. "Transportation lag" (4), or a dead time, is encountered most commonly when materials are moved from one place to another in a production system, or when the groups, humans or machines possibly computers, responsible for directing the action of human operators, or production machinery show periods of indecision. The mathematical definition becomes $R = e^{-ap}C$, where "a" is the lag or dead time in seconds, minutes or hours depending upon the particular time scale. "Exponential lags" (5) are common throughout the whole of dynamics. They are found whenever the rate of change of a quantity with respect to time is in proportion to the magnitude of the quantity.

In our case, the demand function has this property. Having defined and described the unit-processes, we can assign each block in our system an operator. This is done in the table below (Fig. 11):

Description of blocks	Symbol	Unit process no.	Operator	Dimension of constants	Numerical value assumed
Demand Function	D	1+5	$\frac{-d}{1+pT_1}$	$d = (pc)^2/\text{day}\$$ $T_1 = \text{day}$	$d = 10$ $T_1 = 5$
Integrator	S	3	$1/p$	-	-
Inventory Control Delay	I	4	e^{-pT_o}	$T_o = \text{day}$	$T_o = 1$
Production Delay	P	1+4	be^{-pT_2}	$b = 1/\text{day}$ $T_2 = \text{day}$	$b = 0.125$ $T_2 = 10$
Market Delay	B	4	e^{-pT_3}	$T_3 = \text{day}$	$T_3 = 6$
Cost Function	C	1	$-c$	$c = \$\text{day}/(pc)^2$	$c = 0.08$
Accounting Delay	A	4	e^{-pT_4}	$T_4 = \text{day}$	$T_4 = 1$

Fig. 11: Choice of Operators

In comparing the operators with the definitions given in Fig. 10, the significance of the constants can be seen immediately. The coefficients c and d have to be explained further. Both the cost- and demand functions are straight lines. Since we are concerned only with deviations of the demand and the costs from a constant or static value, the deviation in demand ΔD is equal to $\Delta P \cdot d$, and the deviation of the costs ΔC is equal $\Delta O \cdot c$ (see Fig. 7). The coefficients d and c represent the slope of the D- and C-curves. Because these are assumed to be straight lines, c and d are constant. Furthermore, d is negative, because an increase in price will decrease the demand. The same is true for c, because an increase in output results in a decrease in costs per unit, when using the approximation of the cost curve especially fitted for our purpose.

The constant b can be best explained by interpreting its inverse $1/b$. The value $1/b$ represents the time in which the production department plans to restore any deficiency in the inventory level.

Derivation of the Management Operator

In the previous sections we have completely defined a specific microeconomic system with the exception of the management operator. All other operators have been derived from the result of a simple experiment: We apply a unit step function to the input of a block. This perturbation will have a certain influence on the output of the same block, and knowing this response, together with the input, we were able to derive the corresponding operator. To perform a similar experiment with the management would probably be a very difficult undertaking, and the results would vary greatly from manager to manager. Imagine asking several executives which action they would take when faced with the problem of keeping the production on a standard level and knowing that the demand would fall to a new level one week from now. Suppose all managers would have exactly the same problem to solve, and the product in question has a demand curve similar to that described in Fig. 1. All managers would agree that they have to adjust the price of the product, but they would have different opinions on the question of how to do it. Should the price be reduced by the necessary amount at one time and, if so, at what time? Or would it not be better to make the reduction in steps? How big should the steps be and in which time interval should the part-reduction be granted? Or are other courses of action more effective? All these questions would be answered differently by different persons, and therefore they could not be used for the determination of the best policy. But among the variety of opinions might be one which is in fact the best one. This is the problem to which a good part of the study is devoted, the finding of an optimal management operator.

To start with, the management operator is introduced in the system as an unknown, and it will be determined so as to allow the system to behave in a prescribed manner. A natural condition to impose upon the system is not very difficult to find. Since an enterprise should be a profit-making organization, the system must work <u>at every instance</u> with a maximum profit P_0. Each deviation of the actual profit from the maximum value is a direct loss, and we try to minimize this loss (or error e_2).

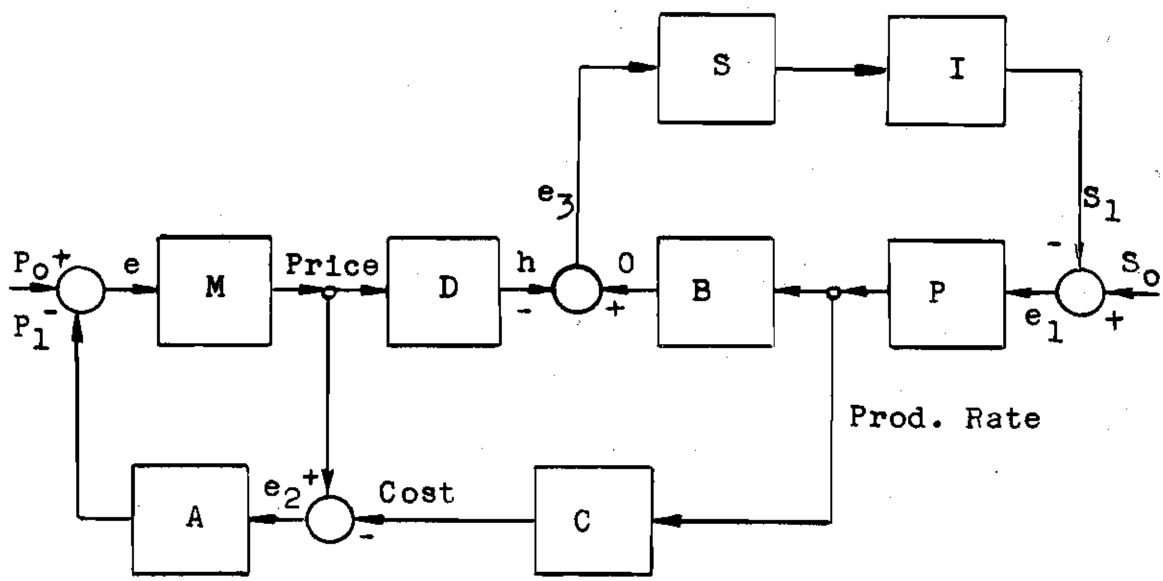


Fig. 12: Block Diagram of a Microeconomic System

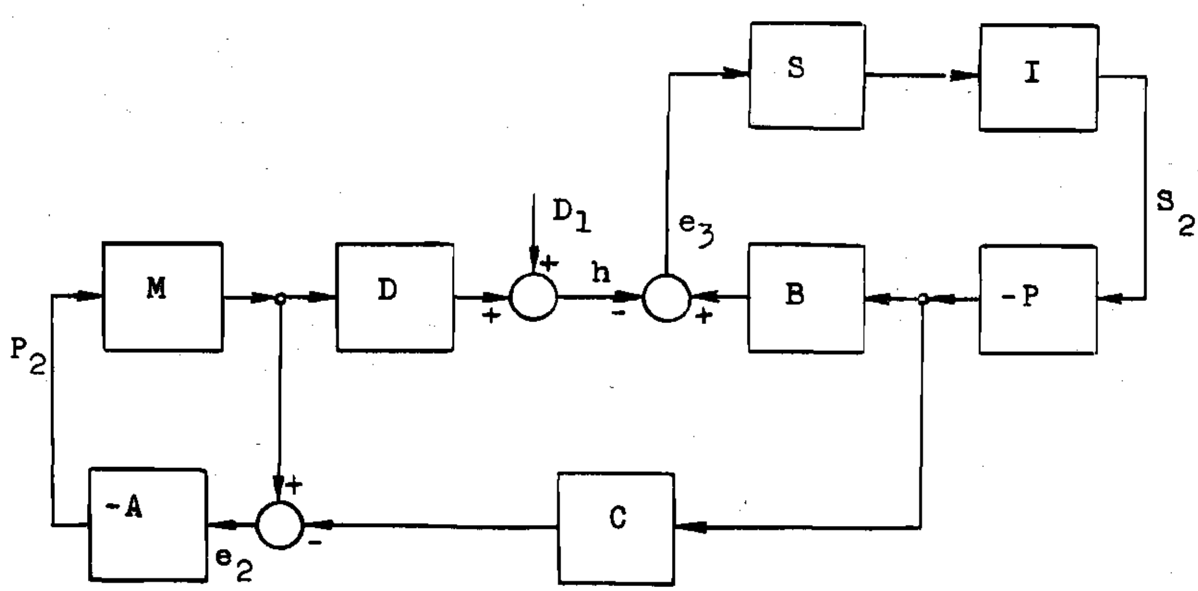


Fig. 13: Block Diagram of a Microeconomic System with Exterior Disturbance

Let us draw again the microeconomic system already sketched in Fig. 7, but instead of using the description of the blocks we use only their symbols as defined in Fig. 11. For the management operator we use the symbol M (Fig. 12). Assume the system to be in a state of equilibrium. Then $P_0 = P_1$ and $S_0 = S_1$. We shall now omit the static terms P_0 and S_0 which determine only the level upon which dynamic fluctuations caused by a variable disturbance are superimposed. The quantities P_0 and S_0 have no influence upon the dynamic behavior. We can represent P_1 and S_1 as being

$$P_1 = P_0 + P_2 \qquad \text{and} \qquad S_1 = S_0 + S_2$$
$$P_1 - P_0 = P_2 = -e \qquad \text{and} \qquad S_1 - S_0 = S_2 = -e_1$$

and introduce new variables P_2 and S_2 for the difference $P_1 - P_0$ and $S_1 - S_0$. To incorporate the equations $e = -P_2$ and $e_1 = -S_2$ into the system, A and P must be multiplied by -1.

We also have to consider external disturbances. In our case this will be a stochastic demand. The mean of the distribution in question is controlled by the management, while the superimposed fluctuations cannot be controlled by the management and act upon the system as an exterior disturbance D_1. Making the adjustments described above, we obtain a new system as sketched in Fig. 13.

For any possible disturbance D_1, the error e_2 must be a minimum. The least value that e_2 can have is 0, therefore we are going to seek an operator for M which will make e_2 = 0. Or in other words, we seek a management policy that results in a maximum profit over a long period of time independent of fluctuations in demand D_1. We realize immediately that a solution to this problem cannot be found with the system of Fig. 13. The error e_2 represents not only the deviation of the actual profit from its maximum level P_0, but also the information upon which management bases its decisions. If e_2 is equal to 0, the management does not get any information, and therefore will not be able to decide anything. This upsets the mechanism of the whole system, and we cannot expect any sound solution. There are only two alternatives to remedy the situation: Either we relax the condition imposed upon the system or we try to find an alternative system to which the original condition can be applied. The preference is given to the second approach, because we would like to keep the condition $e_2 = 0$.

A logical way to alter the system is to provide management with more information about the variations in demand D_1; therefore, a link between the input of the management operator and D_1 has to be installed. The operator in this link will have the symbol F, because it has the function of forecasting the demand fluctuations D_1, as we shall establish later. In Fig. 14, the system of Fig. 13 is sketched in a different manner and the block F is added.

This is the final form of the system which we are going to analyze thoroughly. All transfer functions are known except F and M. We further know that e_2 must be zero independently of what happens to the input of the system. Let us determine the transfer function $e_2/D_1 = G$. According to the definitions given on pp. 9 and 10 we can write:

$$h = d_1 + D_1 \tag{1}$$

$$-e_3 SIPB - h = e_3 \tag{2}$$

$$+e_3 SIPC + M(-e_2 A + D_1 F) = e_2 \tag{3}$$

$$e_3 = \frac{-h}{SIPB + 1} \tag{2}$$

$$D_1 + d_1 = MD\,(-e_2 A + D_1 F) + D_1 \tag{4}$$

$$-e_3 = \frac{(-e_2 A + D_1 F)MD + D_1}{SIPB + 1}$$

$$\frac{-\left[(-e_2 A + D_1 F)MD + D_1\right] SIPC}{SIPB + 1} + D_1 FM = e_2(1 + AM)$$

We obtain finally

$$\frac{e_2}{D_1} = \frac{FM(SIPB + 1) - SIPC(FMD + 1)}{(1 + AM)\,(SIPB + 1) + AMDSIPC} = G$$

For $e_2 = 0$ (independent of D_1) we must have

$$FM(SIPB + 1) - SIPC(FMD + 1) = 0$$

or $$FM = \frac{SIPC}{SIPB + 1 - SIPCD} = \frac{C}{B - CD + 1/SIP}$$

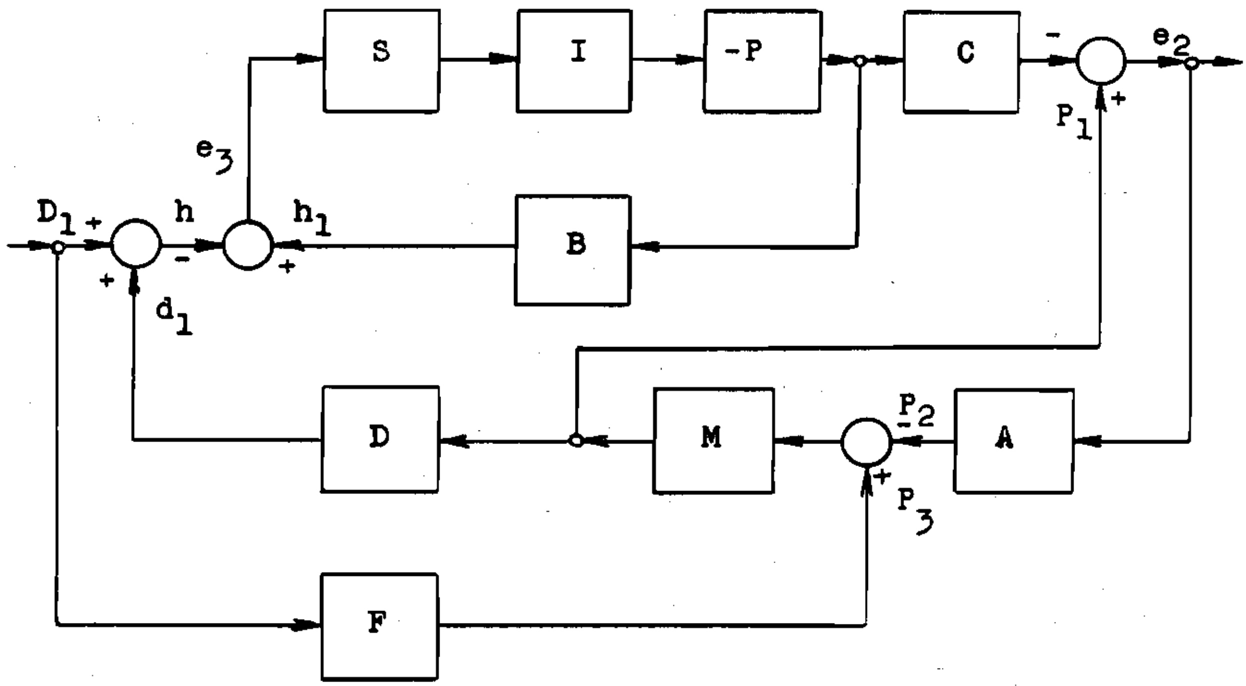


Fig. 14: Microeconomic System with Forecasting Operator

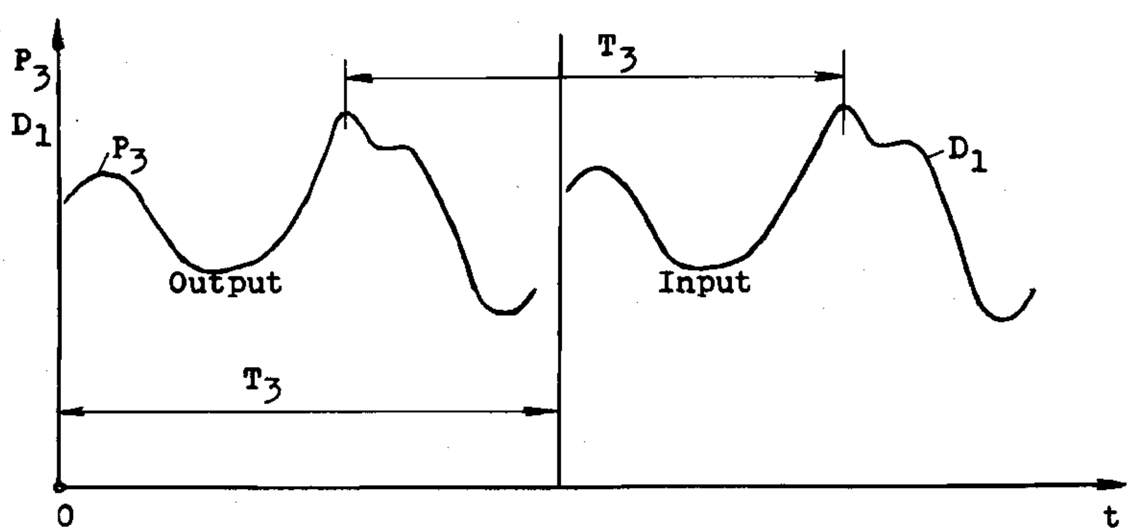


Fig. 15: Explanation of the Forecasting Operator

This result is very interesting, because FM is not a function of A. A glance at Fig. 14 shows that this has to be true, because M does not get any information through A if $e_2 = 0$. Theoretically we could remove A from our block diagram.

Now we replace, in the expression for FM, the symbols by their operators:

$$FM = \frac{ce^{pT_3}}{\frac{cde^{pT_3}}{1+pT_1} - \frac{pe^{p(T_o+T_2+T_3)}}{b} - 1}$$

We choose to assign F and M the following transfer functions:

$$F = ce^{pT_3} \qquad \text{and}$$

$$M = \frac{1}{\frac{cde^{pT_3}}{1+pT_1} - \frac{pe^{p(T_o+T_2+T_3)}}{b} - 1}$$

The Significance of F and M

With these expressions, the system is completely defined. However, both F and M have to be interpreted in terms of their time functions, which will explain their choice and practical significance.

1. The significance of F.--Suppose the input of F, D_1, is a unit step occurring T_3 days after $t = 0$. Then, the Laplace Transform of the output would be:

$$P_3(p) = ce^{pT_3} \frac{e^{-pT_3}}{p} = \frac{c}{p}$$

In tables of the Laplace Transformation we can find the corresponding time function

$$P_3(t) = cU(t)$$

This is the unit step occurring at $t = 0$. More generally, if the input is any function $D_1(t - T_3)\ U(t - T_3)$, its Laplace Transform is

$$\frac{D_1(p)\ e^{-pT_3}}{p}$$

and the output is given by

$$P_3(p) = ce^{pT_3} \frac{e^{-pT_3}}{p} D_1(p) = \frac{c}{p} D_1(p)$$

The inverse transformation of $P_3(p)$ is

$$P_3(t) = cD_1(t)U(t) \qquad \text{(See Fig. 15)}$$

The output P_3 has to know T_3 days in advance what the input is going to do; therefore the function of F is clearly forecasting. Notice that we are able to define the optimal forecasting policy more accurately than the management has to know somewhat in advance the expected variations in demand. In our system management must know today the expected demand

T_3 days from now. It does not need to worry at all about the demand in 5 days or in 7 or 60 days. Hence it is not only possible to establish the need of forecasting, but also to define clearly its function in quantitative terms.

At this point, it is interesting to note a statement of Prof. A. Tustin in his excellent book "The Mechanism of Economic Systems" (Ref. 8):

> It may be that an immediate possibility from the study of dynamic models is to bring about changes in business forecasting that would in themselves contribute greatly to economic stability.

2. <u>The significance of M</u>.--The choice of F is only justified if M is

$$M = \frac{1}{\dfrac{cde^{pT_3}}{1+pT_1} - \dfrac{pe^{p(T_0+T_2+T_3)}}{b} - 1}$$

It is much more difficult to handle this function mathematically than F, but the method of interpretation will be the same. For the input function we choose a unit impulse, which is defined as follows:

$$\delta(t) = \lim_{\Delta t \to 0} \frac{U(t) - U(t - \Delta t)}{\Delta t}$$

where U (t) is the unit step, and

$$\int_{-\Delta t/2}^{+\Delta t/2} \delta(t)dt = 1$$

The Laplace Transform of the unit impulse is $L\left[\delta(t)\right] = 1$. It follows that the Laplace Transform of the response of M to an unit impulse is simply M(p).

It is not possible to find in any table of the Laplace Transformation the inverse transform of the expression M(p). A method of inverse transformation which can be used successfully in this case is the derivation of M(t) by contour integration. The inverse Laplace Transform of any function F(p) is defined as

$$f(t) = \frac{1}{2\pi j} \oint F(p)\, e^{pt}\, dp$$

Therefore, f(t) is the sum of the residues of the function $F(p)e^{pt}$ with respect to all its singular points in the finite complex p-plane. The singular points of $M(p)e^{pt}$ are the solutions of the equation

$$\frac{cde^{pT_3}}{1+pT_1} - \frac{pe}{b}^{p(T_o+T_2+T_3)} - 1 = 0$$

Because this equation is transcendent and complex, it is not possible to solve it analytically. However, we can find a real solution by determining graphically the intersection point of the following two curves:

$$y_1 = \frac{cde^{pT_3}}{1+pT_1}$$

$$y_2 = 1 + \frac{pe^{-p(T_o + T_2 + T_3)}}{b}$$

$$y_1 - y_2 = 0$$

In Fig. 17 the two curves

$$y_1 = \frac{0.8e^{6p}}{1 + 5p} \qquad \text{and} \qquad y_2 = 1 + 8pe^{17p}$$

are drawn and the solution is

$$p_1 = -0.10 \text{ day}^{-1} \qquad |1/p_1| = T_5 = 10 \text{ days}$$

It can be shown that there exist no other solutions. By putting $p = \alpha + j\omega$ in y_1 and y_2, we can find the real and imaginary parts of y_1 and y_2, Re y_1, Re y_2 and Im y_1, Im y_2. Then, in order that $p_i = \alpha_i + j\omega_i$ is a solution, we have to have

$$\text{Re } y_1\Big|_{p_i} = \text{Re } y_2\Big|_{p_i}$$

and

$$\text{Im } y_1\Big|_{p_i} = \text{Im } y_2\Big|_{p_i}$$

Only if for the same p_i both conditions are fulfilled p_i is a singular point of M. This is possible only for $\omega_i = 0$, that is for real values of p_i. Here, a numerical proof is omitted, because it involves lengthy calculations and the result only is important.

In fact, the result is extremely important. Any $\alpha_i > 0$ and $\omega_i \neq 0$ would have questioned seriously the

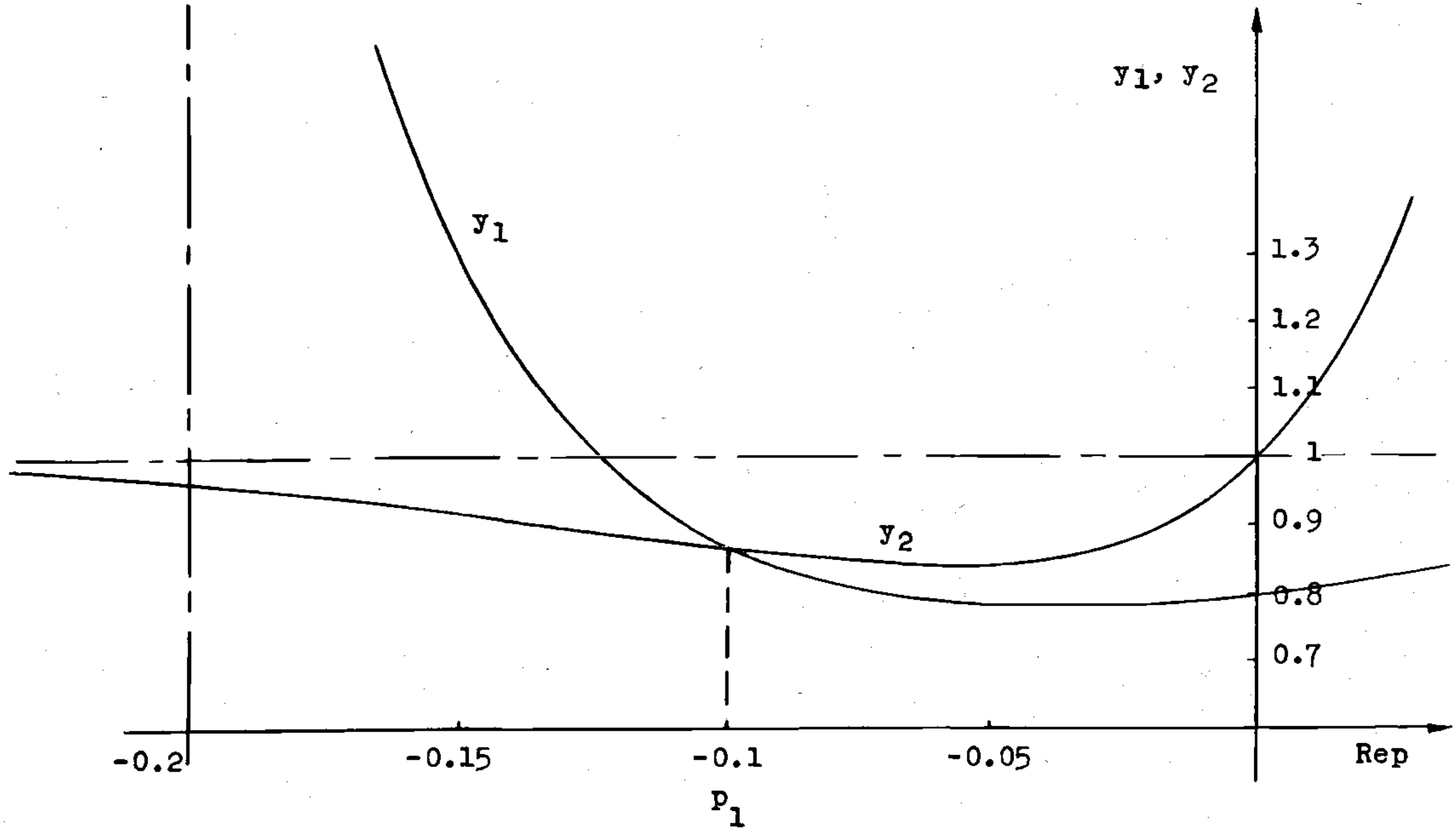


Fig. 16: Construction of the Singular Point of M(p)

soundness of the findings. An $\alpha_1 > 0$ would mean an unstable factor $e^{\alpha_1 t}$ and an $\omega_1 \neq 0$ an oscillatory term in the expression for M(t). But we expect management to be neither unstable nor wavering in its decisions. That is why only negative real solutions for p_i are meaningful. This result gives us good evidence that we are moving on solid ground.

Let us now investigate the conditions under which the system is unstable. For this purpose consider Fig. 17. If $cd > 1$, p_2 is positive (unstable solution). This means that if the slope of the demand function is equal or less than the slope of the cost function, the system would be unstable. If $d = 1/c$, D and C are parallel (compare Fig. 6), and there would no longer be any finite point of equilibrium.

The remaining task is to find the residue of the function

$$\frac{e^{pt}}{\dfrac{cde^{pT_3}}{1+pT_1} - \dfrac{pe^{p(T_o + T_2 + T_3)}}{b} - 1}$$

at p_1. The value of the residue is equal to

$$\left.\frac{e^{pt}}{\dfrac{d}{dp}\left[\dfrac{cde^{pT_3}}{1+pT_1} - \dfrac{pe^{p(T_o + T_2 + T_3)}}{b} - 1\right]}\right|_{p = p_1}$$

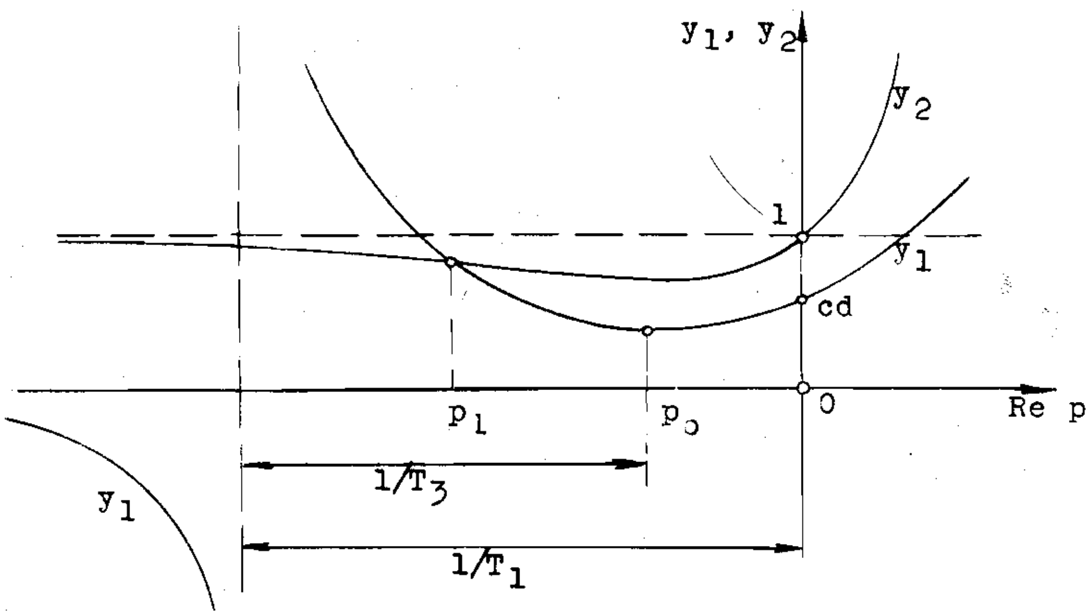


Fig. 17: Solution for M(p)

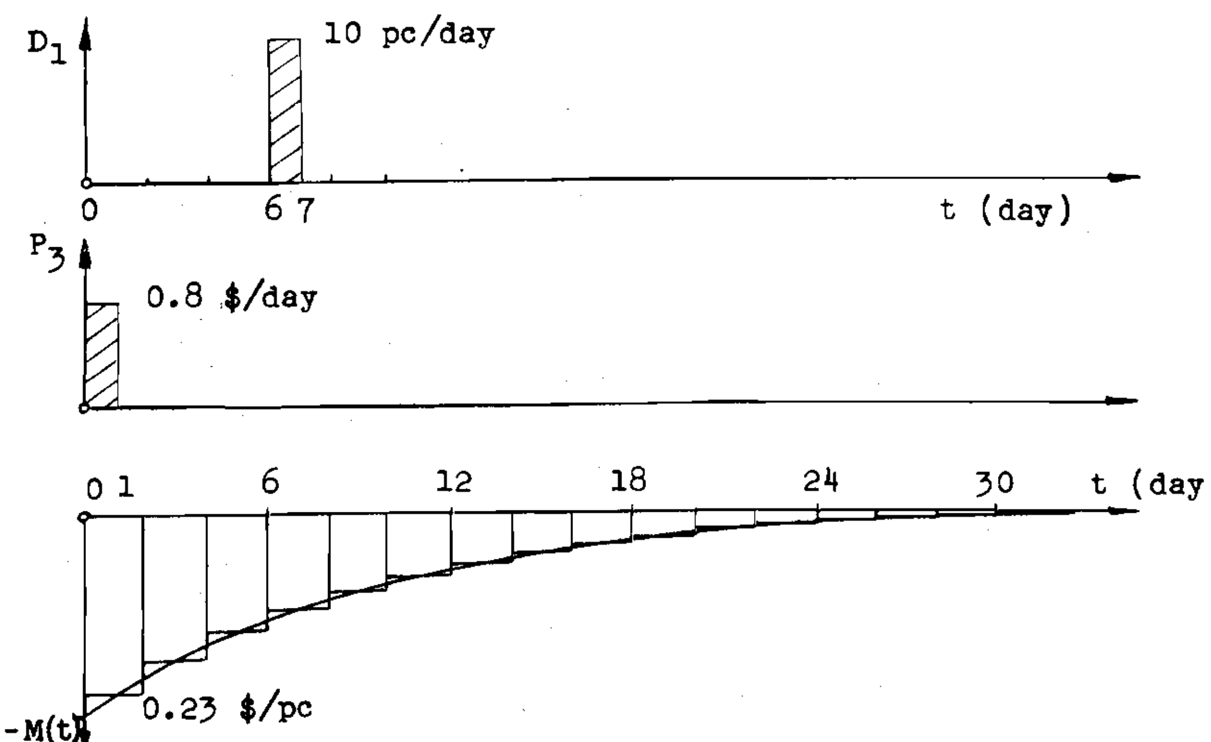


Fig. 18: Optimum Management Decision to Counteract Unit Impulse Disturbance

The numerical evaluation of this expression yields the value:

For $p_1 = -0.10$, Residue $= -0.288e^{-0.10t}$

$$M(t) = -0.288e^{-0.10t}$$

This is the optimal management decision issued to counteract a unit impulse disturbance. The mechanism of this system is illustrated with some oscillograms in Fig. 18. If an increase in demand of 10 pieces per day is expected to take place in 6 days and to vanish again completely the following day, the management would have to decrease the price of the product according to M(t) in Fig. 18. Because a continuously varying price is not practical, M(t) has to be approximated with a time series. If we choose time intervals of 2 days, the price reductions vary from 23¢ per piece at the beginning to 0 after 30 days following the steps indicated in Fig. 18. A management's price policy according to the smooth curve M(t) would cause the error to be 0 at every instance between $t = 0$ and ∞. With the approximated time series, we have good reasons to believe that e_2 (e_2 = deviation from maximum profit) is very small preceding, during and after the occurrance of the disturbance.

Generally, the input of the management operator will not be a unit impulse function. If the input is some arbitrary function $P_3(t)$, then the management decision $M_1(t)$ can be given in terms of a superposition integral

$$M_1(t) = \int_0^t P_3(\tau)M(t-\tau)d\tau.$$

For every specific $P_3(\tau)$, this integral has to be evaluated in order to know the management response for this particular case. However, the calculations may be lengthy or may not be possible of solution with analytical tools. We can avoid this difficulty by constructing a model of the system. It will be a special purpose analogue computer for the investigation of business system behavior.

CHAPTER III

THE ANALOGUE

Derivation of an Analogue of the Symbolic Model

In the preceding part we were concerned with the development of a mathematical or symbolic model of a microeconomic system. The differential equations governing the behavior of the system have been derived in terms of Laplace Transforms, and these equations represent a symbolic model of the system. Based on these, we shall derive an electronic analogue of the symbolic model. In other words, the analogue directly simulates the differential equations of the system, and simulates only indirectly the system itself. This implies that the analogue cannot be a better approximation than the equations, as long as we restrain ourselves to these equations. But it might be approximated better by considering other factors too, which had to be neglected for the derivation of the symbolic model, e.g., we could introduce instead of the linearized cost- and demand curves the actual ones, or we could introduce limitations in the production level or other nonlinearities. In the present analogue, this has not been done, because the first step in the procedure is the construction of an analogue which represents as closely as possible the symbolic model. Only after its verification could we justify proceeding further. However this second step, the introduction of nonlinearities, exceeds the scope of the present study and therefore will not be discussed here.

1. <u>Realization of the management operator</u>.--The problem to be solved is to find an electrical circuit whose response to a unit impulse is the function M(t) sketched in Fig. 18. Since it is in this case easier to find the circuit by knowing its response to a unit step function, we are going to derive this response. Incidentally, we note that we are allowed to do this, because, although the response will vary according to the input, the circuit configuration and the values of the components are not affected by the wave shape. It follows that we have to obtain the same circuit in either way. One may interpret this well known behavior of a physical system in terms of a manager's behavior as follows:

In order to determine the actual price of the product, the manager has to know the past, actual and expected demand. This demand has to be weighted with a certain "weighting function". Generally, the present demand is weighted the most, and the weight-coefficients decrease as time increases (see Fig. 18). The sum (in the discrete case and the integral in the continuous case) of the weighted demands over the complete time range is the price which has to be set for the present time. What is inherent to the manager is the weighting function, it is his decision-making policy which is not changed by the amplitude or waveform of the demand function. We realize that in practice the manager most probably decides intuitively on the basis of some kind of weighting function not exactly defined. An accurate definition can only be formulated by conducting the calculations described on the previous pages.

The Laplace Transform of management's response to a unit step is:

$$M_2(p) = \frac{1}{p} M(p) = L\left[M_2(t)\right]$$

The corresponding time function (inverse transform) can be found by following the same procedure as outlined in the derivation of M(t).

$$M_2(p) = \frac{1}{p\left[\dfrac{cde^{pT_3}}{1+pT_1} - \dfrac{pe^{p(T_o+T_2+T_3)}}{b} - 1\right]}$$

For the two poles of this expression we obtain the following residues:

For $p = 0$, Residue $= 1/(cd - 1) = -5$

For $p_1 = -0.10$, Residue $= 3.85e^{-0.1t}$

$$\underline{M_2(t) = -5(1 - 0.77e^{-0.1t})}$$

$M_2(t)$ can also be written as $M_2(t) = -7(0.714 - 0.55e^{-0.1t})$

Consider the following network:

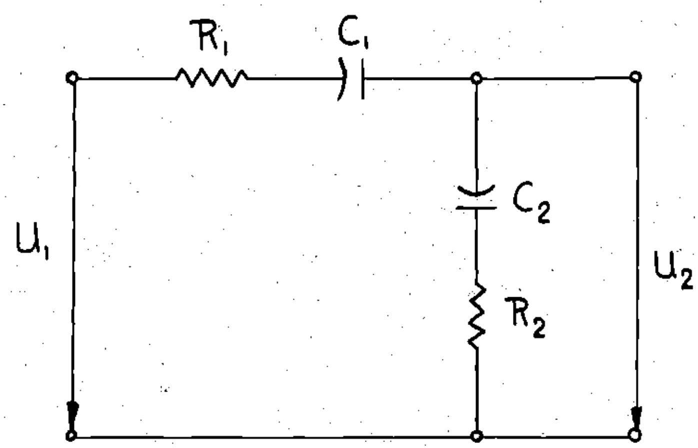


Fig. 19: Network for M

The response of this network to a unit step input is proportional to $M_2(t)$ if the components satisfy the following conditions:

$$\frac{R_2}{R_1 + R_2} = 0.164 \quad \text{and} \quad \frac{C_1}{C_1 + C_2} = 0.714$$

$$\frac{(R_1 + R_2)C_1C_2}{C_1 + C_2} = 10x10^{-5} \text{ sec.}$$

It is necessary to make a time transformation, otherwise the analogue could practically not be realized. We put 1 day $\rightarrow$ 10^{-5} sec. = 10 microseconds. This means that the period of one day is simulated by the analogue in 10 microseconds.

This particular transformation was chosen in order that the analogue can be used in the audiofrequency range.

The following values for the network components satisfy the conditions:

$R_1 = 58.5\ k\Omega$

$R_2 = 11.5\ k\Omega$

$C_1 = 5000$ mmfd

$C_2 = 2000$ mmfd

The network must be preceded by an amplifier which takes care of the factor -7. The complete circuit for the management operator is sketched in Fig. 20.

2. <u>The realization of the forecasting operator</u>.--

$$F(p) = ce^{pT_3} \quad \text{where} \quad c = 0.08\ \$\text{day}/(\text{pc})^2$$
$$\text{and} \quad T_3 = 6\text{x}10^{-5}\text{sec.}$$

Approximation: $F(p) = c(1 + pT_3 + \frac{(pT_3)^2}{2!} + \ldots\ldots)$

If we neglect the second and higher order terms, then F(p) is approximately

$$F(p) \approx c(1 + pT_3) = c + cpT_3$$

This equation can be represented with the following block diagram:

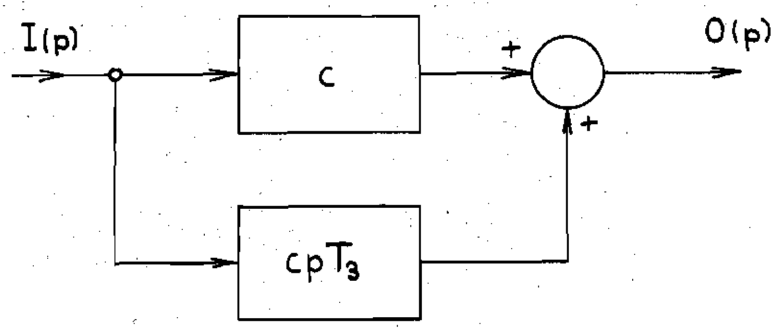


Fig. 21: Block Diagram of the Forecasting Operator

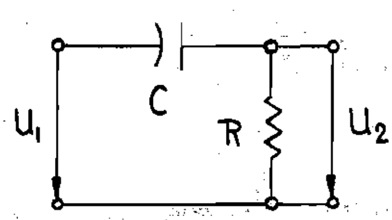
Input = I(p) (Laplace Transform)

Output = $O(p) = cI(p) + cpT_3I(p)$

The inverse transform is

$$O(t) = cI(t) + cT_3\frac{dI}{dt} \quad \text{(Trend forecasting)}$$

Hence we have to find an electrical circuit whose output is equal to the sum of the input and its derivative multiplied by T_3. Consider the network in Fig. 22.



$$\frac{U_2}{U_1} = G = \frac{R}{R + \frac{1}{pC}} = \frac{pT}{1 + pT}$$

where $p = j\omega$ and $T = RC$

Fig. 22: Network for Derivation

If we make pT very small compared with 1, $pT \ll 1$, then we can write approximately $G = pT$. The variable p will have

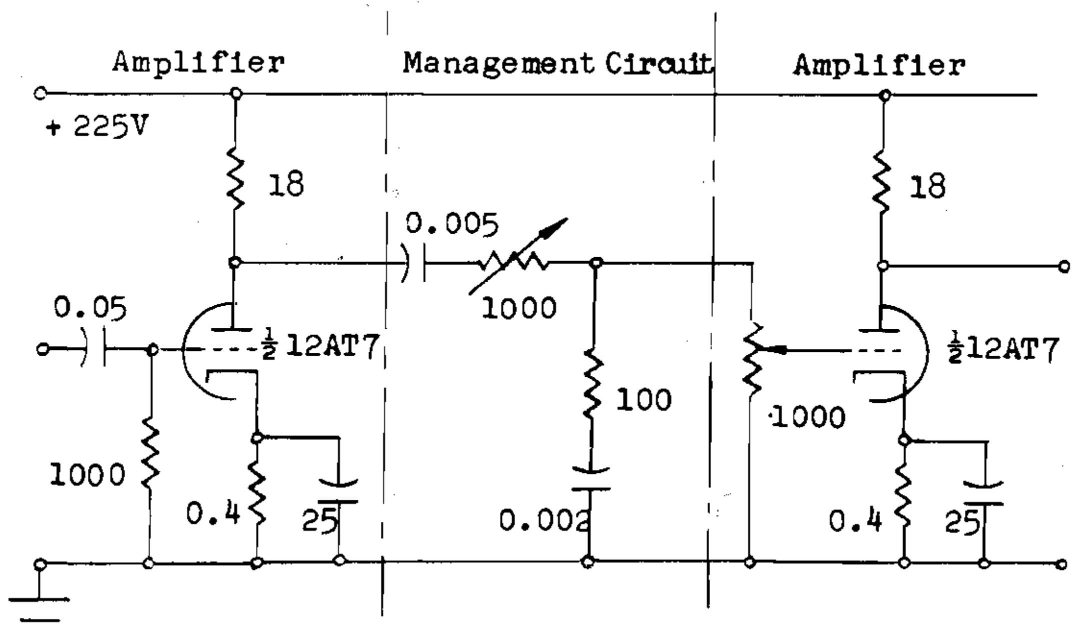


Fig. 20: Schematic of the Management Circuit

In all schematics, R is given in kΩ , and C in mfd.

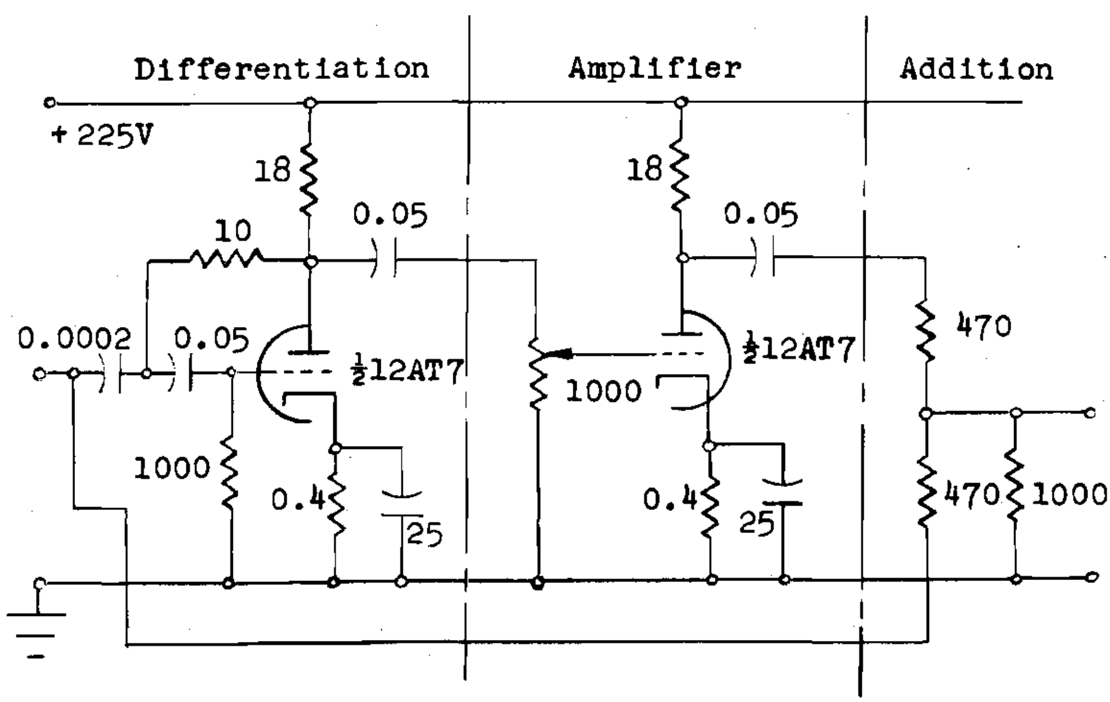


Fig. 23: Schematic of the Forecasting Operator

values between $2x10^3$ to $30x10^3$. To meet the condition $pT \ll 1$, T is chosen as $2x10^{-6}$ sec., then pT will not exceed 0.06. In other words, the maximum error is 6%. However, it is possible to lower this error by exact differentiation through use of a special feedback amplifier. Since after differentiation the signal is low in level and has a negative sign, it has to be amplified. The amplification v must be such that

$$Tv = T_3 \quad \text{or} \quad v = T_3/T = 30$$

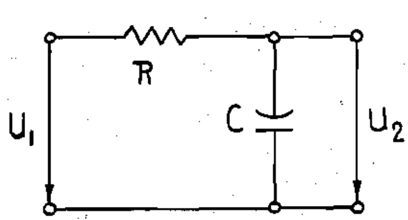
$$T = RC = 2x10^{-6} \text{ sec.}, \quad R = 10 \text{ k}\Omega, \quad C = 200 \text{ mmfd}$$

We are now ready to design the forecasting circuit sketched in Fig. 23. We use the same type of tube (Twin-Triode 12AT7) as was used for the management operator.

3. <u>The realization of the demand operator</u>.--

$$D(p) = \frac{-d}{1 + pT_1} \qquad \text{where} \quad d = 10 \text{ (pc)}^2/\text{day}\$ \quad \text{and} \quad T_1 = 5x10^{-5} \text{ sec.}$$

Consider the following network:



$$\frac{U_2}{U_1} = \frac{1}{1 + pT_1}$$

$$R = 100 \text{ k}\Omega, \quad C = 500 \text{ mmfd}$$

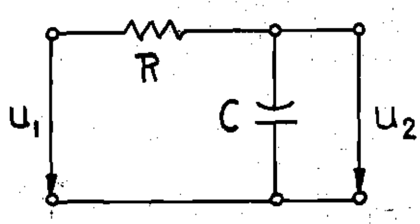
Fig. 24: Network for D

Since the signal is already negative after the output amplifier stage of the management operator, the circuit of Fig. 24 will yield an output proportional to the demand with the correct sign.

4. <u>The realization of the inventory integrator</u>.--

$$S(p) = 1/p$$

Consider the following circuit:



$$G(p) = \frac{U_2}{U_1} = \frac{1}{1 + pT}$$

where $T = RC$

Fig. 25: Network for S

If we make $pT \gg 1$, we can write approximately:

$$G(p) \approx 1/pT = S(p)/T$$

For $T = 10^{-2}$ sec., the minimum of pT is $2x10^3x10^{-2} = 20$, or the error is 5%. However, lower this error by integrating with the aid of a Miller Integrator, which is again a special kind of feedback amplifier.

For $T = 10^{-2}$ sec., $R = 100$ kΩ, $C = 0.1$ mfd

For purely electrical reasons (impedance transformation), the Miller Integrator is followed by a cathode follower (see Fig. 26).

5. <u>The realization of the inventory- and production delay operators</u>.--Because the inventory- and the production delay operators are two consecutive blocks in the diagram of Fig. 14 and have the same type of operators (no. 4 of Fig. 10), we can represent both with only one circuit, having as time lag the sum of the lags of the individual departments.

$$T' = T_o + T_2 = (1 + 10)10^{-5} = 11x10^{-5} \text{ sec.}$$

$$PI = -be^{-p(T_o + T_2)} = -be^{-pT'}$$

The realization of this operator is not as obvious as that of the previous ones. The problem is to find an electrical circuit whose output is delayed T' seconds against the input. This could be approximated very closely by a so called delay line. Since this would need many components, it would disturb the simplicity of the analogue; therefore no attempt was made to seek an appropriate delay line. Another solution would be the recording of the input by means of a tape recorder and the play-back of the information so stored after the elapse time T', but we shall try to solve the problem by a simpler method. This is suggested in an article written by Smith and Erdley (Ref. 10).

Consider the following circuit:

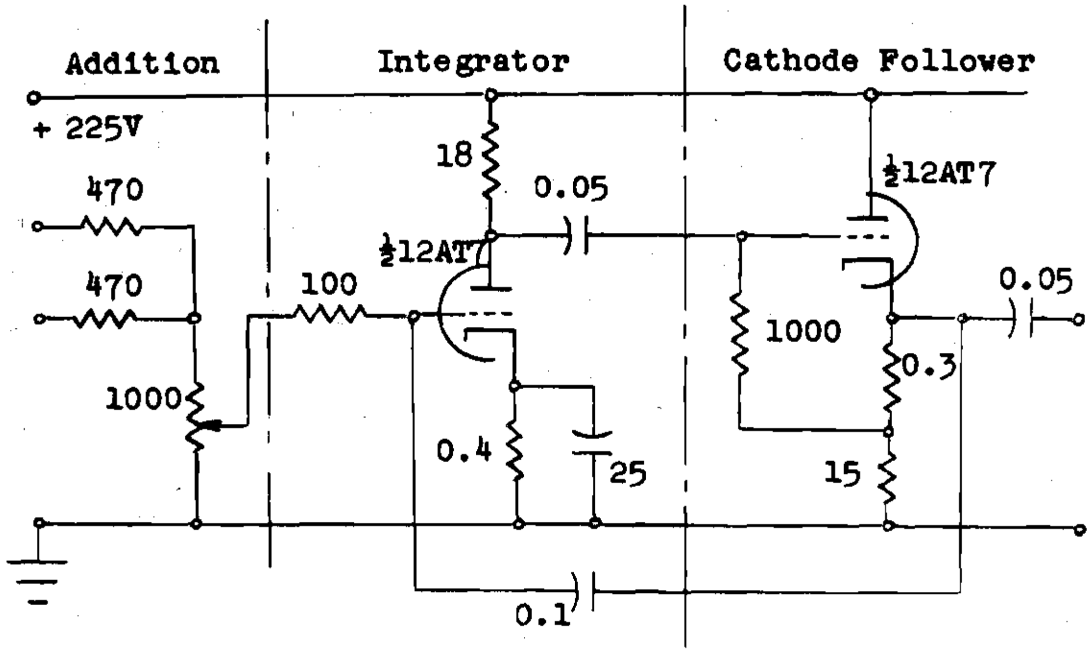


Fig. 26: Schematic of the Inventory Integrator

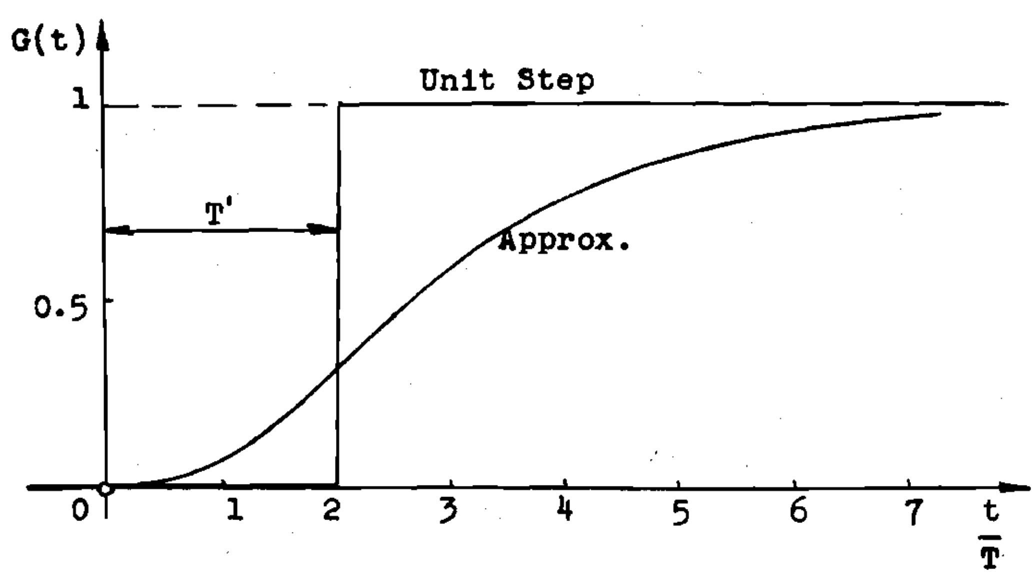


Fig. 28: Approximation of a Delayed Step Function

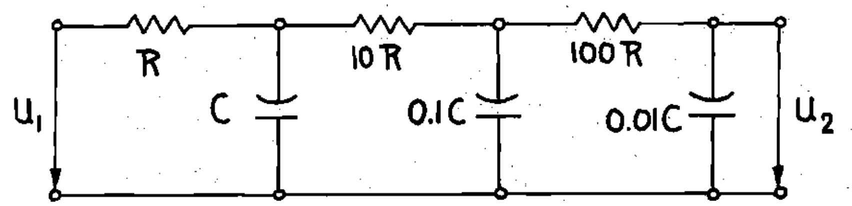


Fig. 27: Delay Circuit

Because the impedance of the first two components R and C is much less than the impedance of its load, we can write approximately

$$G = \frac{1}{(1 + pT)^3} \qquad \text{where} \quad T = RC$$

If U_1 is a unit step time function, its Laplace Transform is $U_1(p) = 1/p$, and the Laplace Transform of the output $U_2(p)$ is then $G(p)U_1(p) = U_2(p)$

$$U_2(p) = \frac{1}{p(1 + pT)^3}$$

To find how close this circuit approximates the ideal case, the time function $U_2(t)$ was derived and plotted in Fig. 28. The approximation is rather rough, but it also has a good feature: For very rapid changes in demand, the production department no longer adjusts the production level to the full extent. This is meaningful, because quick changes in production are costly or perhaps cannot be carried out.

According to Fig. 28 the time lag T' is chosen equal to 2T. The distance $t/T = 2$ is the abscissa of the inflection point of the approximated response.

For the inventory- and production delay, we have

$$T' = 11\text{x}10^{-5}\text{sec.} \quad \text{and} \quad T = 5.5\text{x}10^{-5}\text{sec.} = RC$$
$$R = 10^3\,\Omega \qquad\qquad C = 0.055 \text{ mfd}$$

The corresponding circuit is sketched in Fig. 29. An amplifier is provided for the purposes of amplification and sign change of the signal, and a cathode follower supplies the power for the next stage.

6. <u>Realization of the market- and accounting delay operators</u>.--A similar calculation as described above yields the circuits for the market- and accounting delay operators:

Market delay:	$T_3 = 6\text{x}10^{-5}\text{sec.}$	
	$T_3' = 3\text{x}10^{-5}\text{sec.}$	Fig. 30
Accounting delay:	$T_4 = 1\text{x}10^{-5}\text{sec.}$	
	$T_4' = \frac{1}{2}\text{x}10^{-5}\text{sec.}$	Fig. 31

7. <u>The complete circuit diagram of the analogue</u>.--With the aid of the block diagram of Fig. 14, the different circuits derived above are put in proper sequence to furnish the final circuit plotted in Fig. 32. Generally, the signal changes the sign several times (each amplifier), and special care only has been taken for the signal to have the proper sign when two signals had to be added (or subtracted). Be-

Because e_2 is theoretically equal to 0, the output of the accounting department is not fed to the input of the management operator. Fig. 32 differs from Fig. 14 in this respect.

The analogue makes use of 6 electronic tubes (12 tube functions), but it might be well to emphasize that they have only secondary tasks to fulfill. These are:

1. Amplification
2. Impedance transformation
3. Signal flow direction guard

This last property, however, is important. It means that there is no reaction from the output to the input in amplifiers and cathode followers. This allows us to keep the flow directions as prescribed in Fig. 14.

The main objective, the physical realization of the different operators, is achieved only with RC circuits, with the exception of the management operator, where an amplifier constitutes a part of the operator. The electronic circuitry techniques used are elementary and could be refined.

The Construction and Test of the Analogue

The circuit in Fig. 32 has been built (Fig. 33) and tested (Fig. 34). For this purpose the following apparatus is needed:

a) Power supply (+225V dc, 6.3V ac)

b) Function generator to simulate demand variations. This may be an audio frequency sine and square

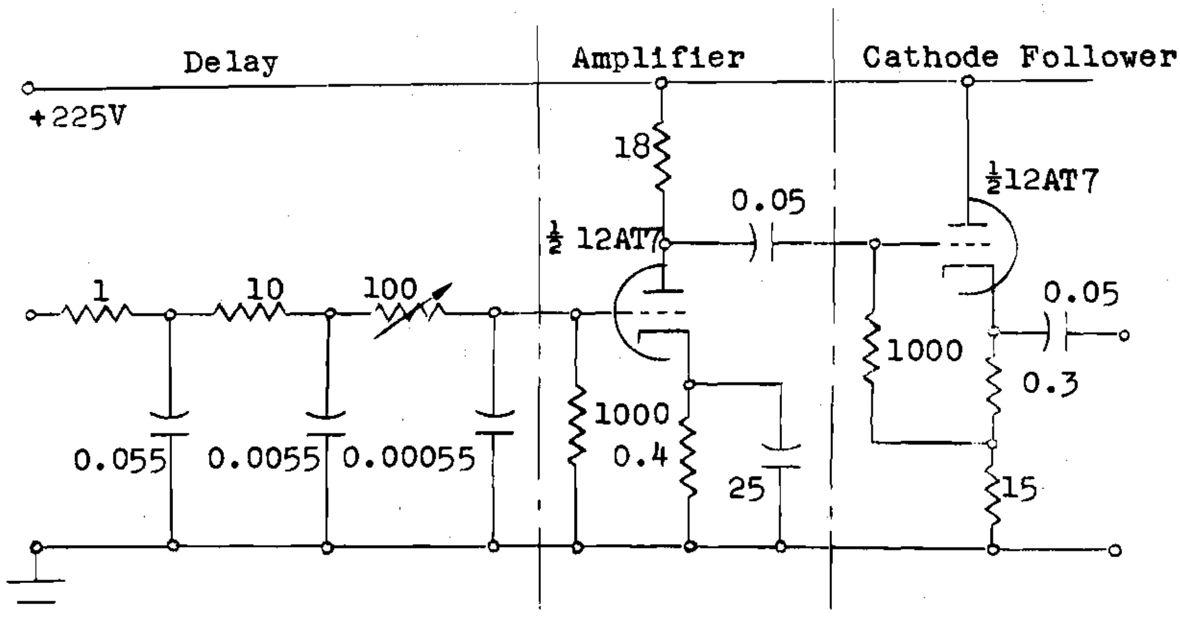


Fig. 29: Schematic of Inventory- and Production Delay Operator

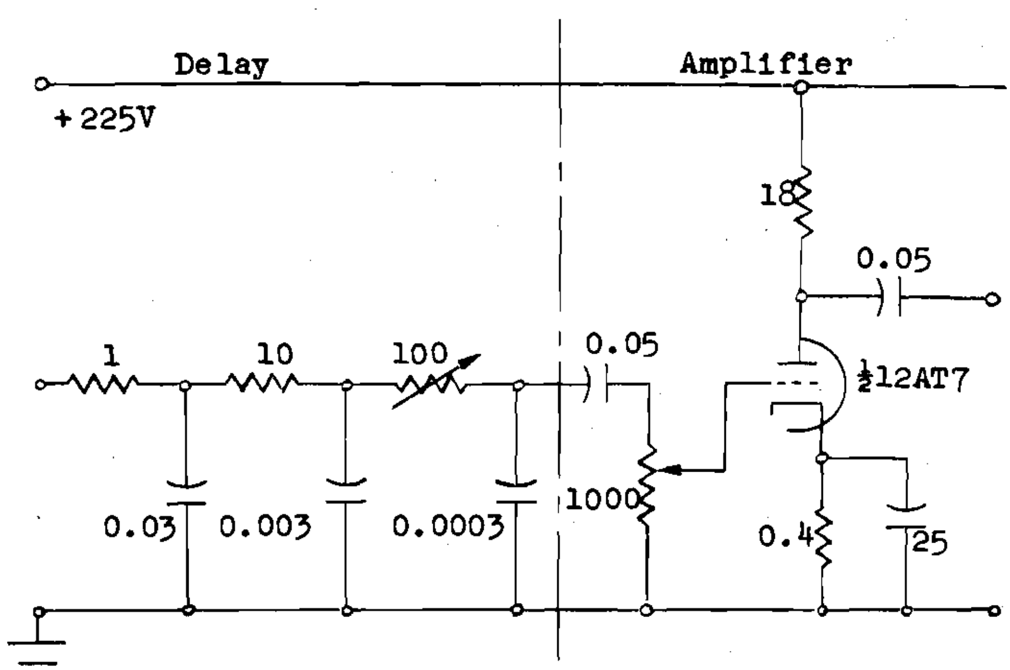


Fig. 30: Schematic of the Market Delay Operator

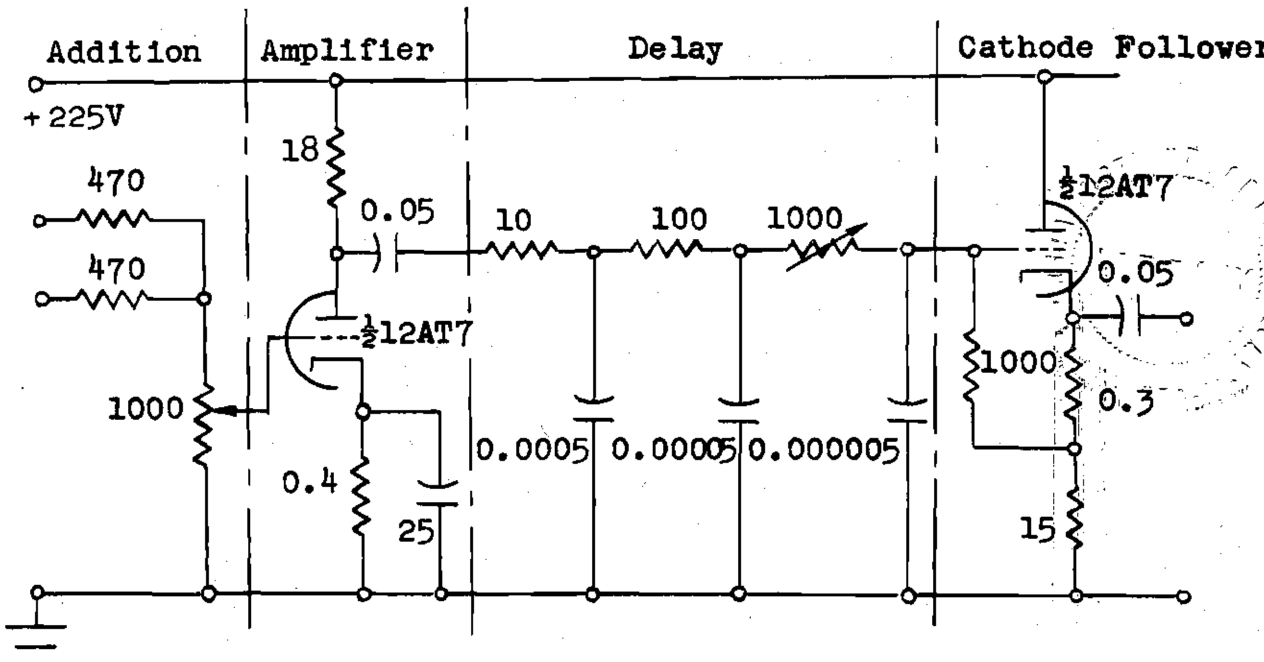


Fig. 31: Schematic of the Accounting Delay Operator

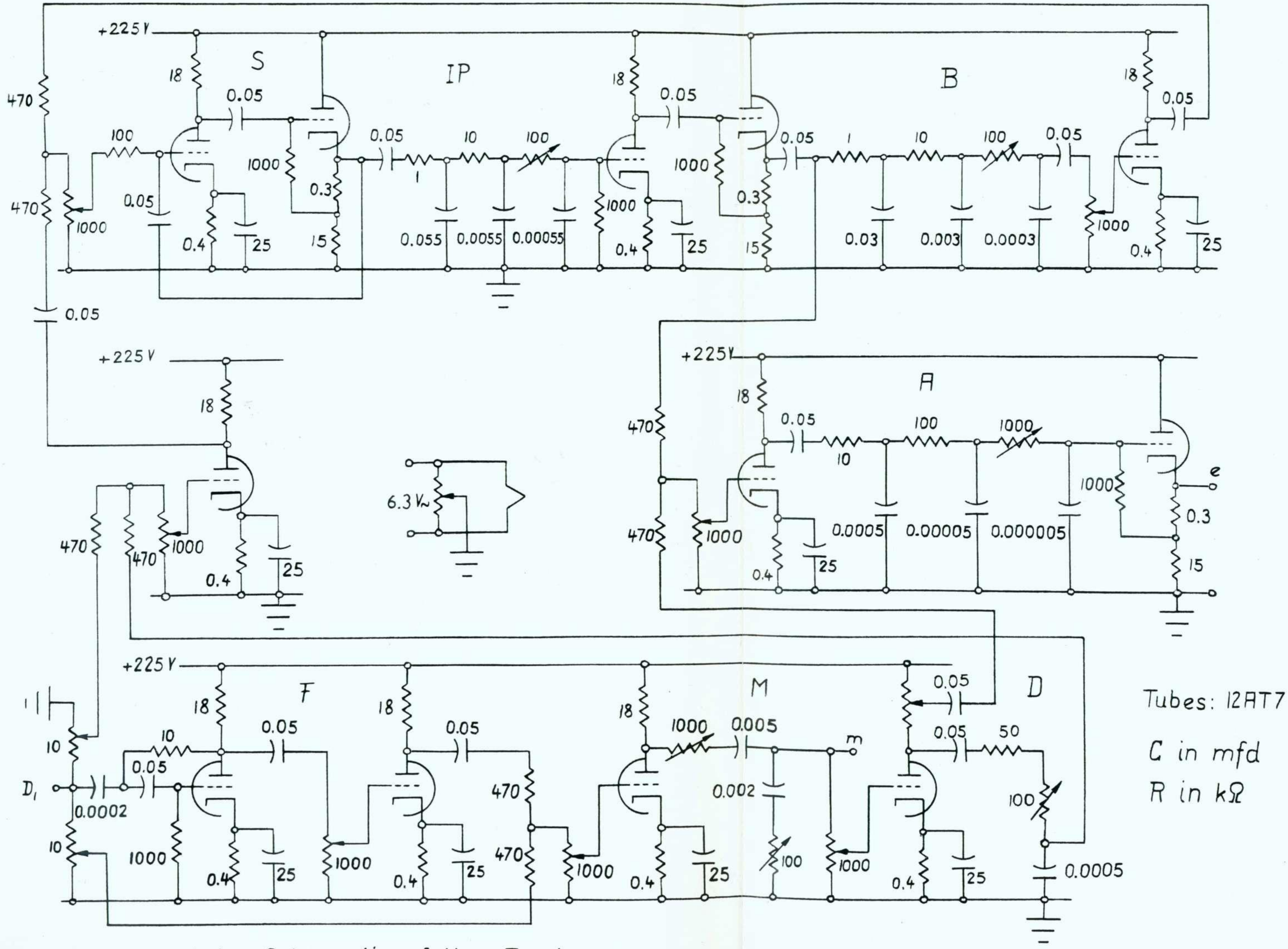


Fig. 32: Complete Schematic of the Analogue

wave generator, a random noise generator or a function generator. Since only sine and square wave generators were available and because the approximated forecasting operator is unable to forecast step functions, the experiments were performed with the sine generator only. Results based upon square wave inputs will have to be interpreted cautiously.

c) Voltmeter to measure the deviation from maximum profit (called error e_2).

d) Oscilloscope to make the price of the product visible as a function of time (terminals m), or to observe the error (terminals e_2). The oscilloscope is preferably a double beam type; otherwise an electronic switch may be provided to trace two curves at a time.

To check the calculations, experiments were performed to test the deviation from maximum profit under given conditions. We recall that the management operator has been derived by putting this error equal to 0. Therefore, by setting all dials to the values used in the calculations, the error has to be theoretically 0 or practically very small. As demand function a sine wave is chosen with:-

a) a frequency of 380 c/s. In reality this corresponds to a business cycle of one year, if a business year is defined to have 260 days.

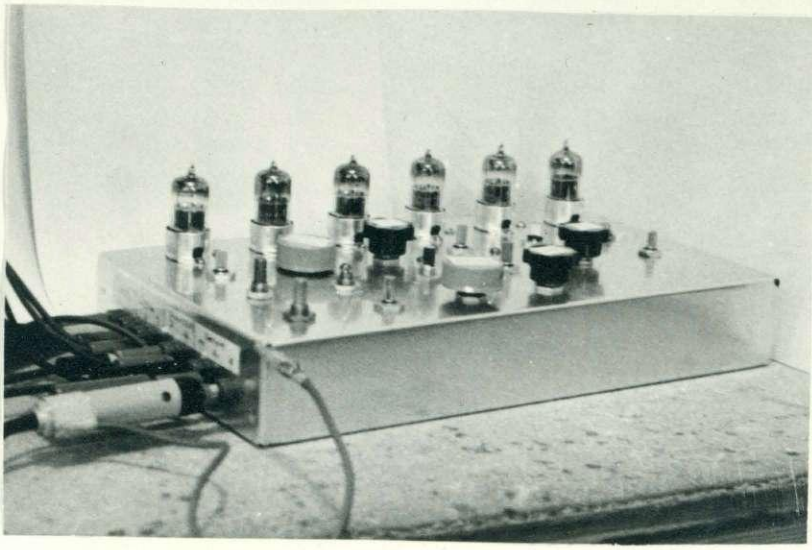


Fig. 33: The Analogue

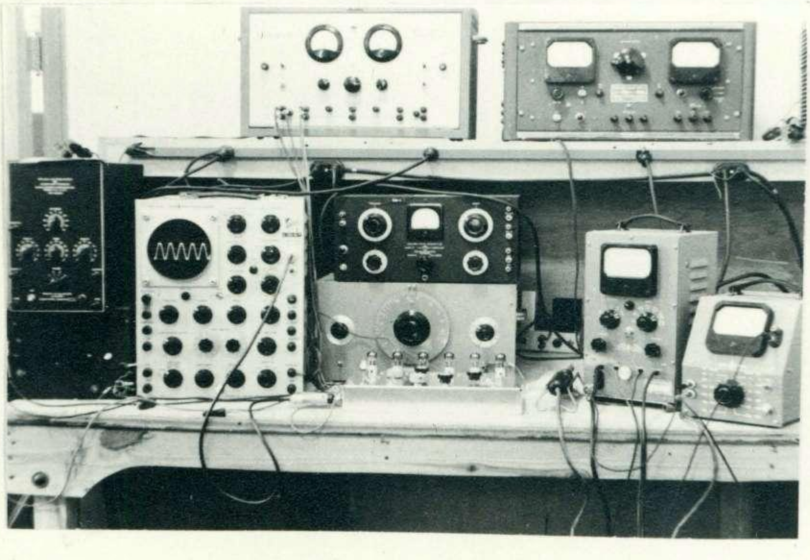


Fig. 34: Test Set-up

b) a frequency of 2250 c/s. In reality this corresponds to
a business cycle of two months.

In both cases the amplitude is kept constant. The corresponding oscillograms are pictured and scaled in Fig. 35a) and 35b). The demand and the resulting price as it should be set by the management can be compared on one picture. The error was measured by a voltmeter and is expressed as a percentage of the price. Remember that the demand, the price and the error represent deviations from static values which, incidentally, need not be known. The percentage of error is very low, especially when we keep in mind that it is expressed in percentage of the price deviation, not in percentage of the maximum profit. The latter percentage would be consistently lower.

The error should be 0 independent of the frequency of the input. However, because of the approximation used for the realization of the forecasting-, inventory-, production-, and market delays, this error increases with the frequency. It could be removed with a slight change in the potentiometer settings. However, these are the same for both frequencies applied, as would be the case in reality.

These tests assure that the analogue is a good approximation to the symbolic model. They also reveal that the analogue and the model yield logical results and that this method has good possibilities.

The experiments performed are not at all exhaustive;

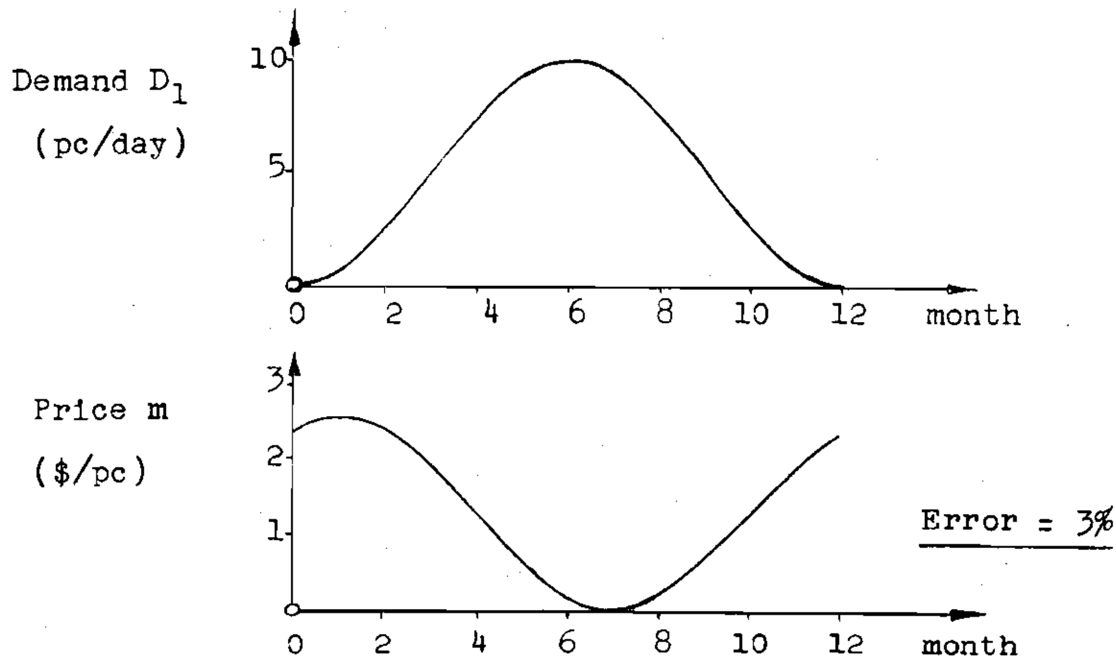


Fig. 35a: Best Price Policy (m) for a Sinusoidal Demand (D_1). Cycle Duration of 1 Year.

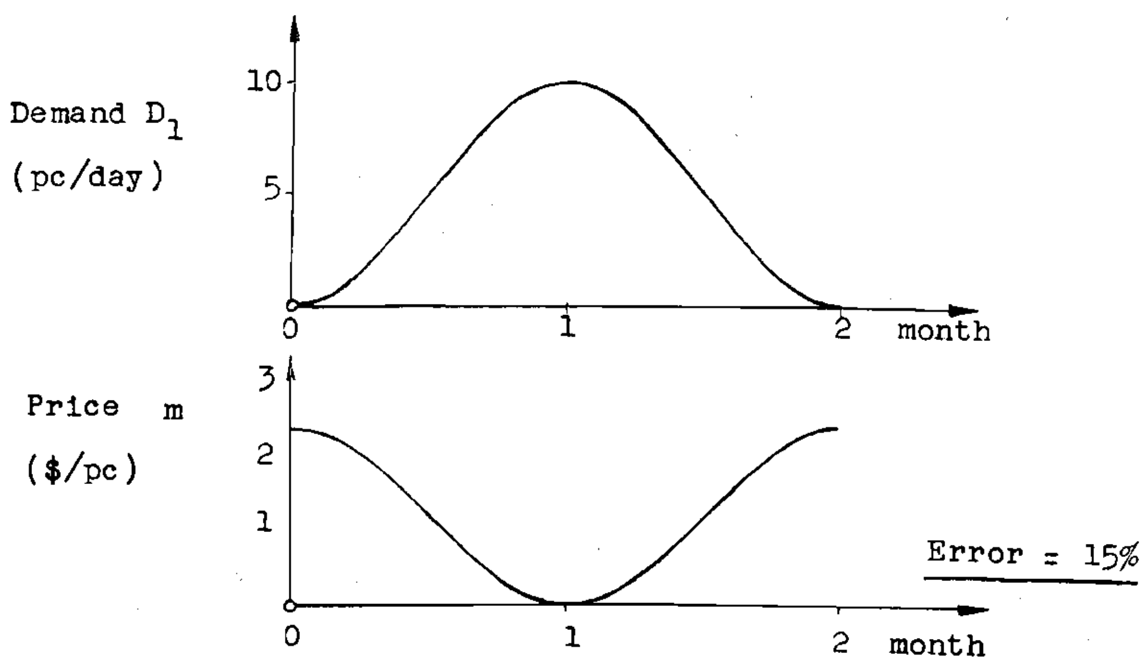


Fig. 35b: Best Price Policy (m) for a Sinusoidal Demand (D_1). Cycle Duration of 2 Months.

the analogue is much more versatile. With its aid, we also could study such matters as how the different time lags affect the optimal management policy or how big the error would be if the policy is not optimal. Although the optimal management policy should be independent of the waveshape, in the analogue this is only approximately true. Therefore the influences of different waveshapes could be investigated. However, further experimentation would only be meaningful if there were a specific problem in mind.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

Conclusions:

For the particular microeconomic system investigated, the following can be concluded:

1. Using simple, fundamental relationships between the causes and effects (as the law of supply and demand), the dynamics of the microeconomic system can be studied qualitatively and quantitatively.
2. It is possible to derive a symbolic model of the microeconomic system.
3. The symbolic model can be approximated by a physical analogue.
4. It is possible to derive quantitatively an optimum price policy for any variation in the demand.
5. This and only this price policy yields a maximum profit.
6. The microeconomic system is only stable if the slope of the demand curve is bigger than the slope of the approximated cost curve.
7. An accurate quantitative forecasting function can be defined.

Recommendations:

It is repeated that the system considered so far is not a general one; it is only claimed that it contains some basic relationships common to all possible microeconomic systems. Naturally, for reasons of simplicity, much had to be neglected, and much more work along these lines remains to be done.

One necessary addition to the system is the procurement loop for raw materials. It should not introduce any insurmountable difficulties in the analysis. Another refinement would be the introduction of a cost-component proportional to the rate of change of the production volume. Every increase in production causes additional costs which are not proportional to the output, and which, incidentally, cannot be redeemed completely by decreasing the output again (Goodwin's nonlinear accelerator principle). This nonlinear relationship, however, is difficult to handle mathematically.

Other criteria could be applied, e.g. instead of using the profit per unit of the output as criterion, the total profit (profit per piece times number of pieces) could be used.

The electronic analogue should be improved to match more closely the symbolic model by better approximating the forecasting and delay operators.

BIBLIOGRAPHY

Microeconomic Models:

1. Simon, H. A., "An Exploration into the Use of Servomechanisms Theory in the Study of Production Control," Econometrica, 20, 1952, 247-268.

2. Campbell, D. P., "Dynamic Behavior of Linear Production Systems," Mechanical Engineering, 75, 1953, 279-283.

3. Smith, O. J. M., "Economic Analogues," Proceedings of the Institute of Radio Engineers, 41, 1953, 1514-1519.

4. Moorehouse, Strotz and Horwitz, "An Electro - Analogue Method for Investigating Problems in Economic Dynamics, Inventory Oscillations," Econometrica, 18, 1950, 313-328.

5. Strotz, Calvert and Moorehouse, "Analogue Computing Techniques Applied to Economics," Transactions of the American Institute of Electrical Engineers, 17, 1951, 557-563.

6. Cooper, W. W., "A Proposal for Extending the Theory of the Firm," Quarterly Journal of Economics, 65, 1951, 87-109.

7. Simon, H. A., "Modern Organization Theories," Advanced Management, 15, 1950, 2-4.

Macroeconomic Models:

8. Tustin, A., The Mechanism of Economic Systems, Cambridge: Harvard University Press, 1953.

9. Goodwin, R. M., "The Nonlinear Accelerator and the Persistence of Business Cycles," Econometrica, 19, 1951, 1-17.

10. Smith, O. J. M., and Erdley, "An Electronic Analogue for an Economic System," Electrical Engineering, 71, 1952, 362-366.

11. Strotz, Anulty and Naines, "Goodwin's Nonlinear Theory of Business Cycles: An Electro-Analogue Solution," Econometrica, 21, 1953, 390-411.

12. Tobin, J., "Dynamic Aggregative Model of the Business Cycle and of Economic Growth," Journal of Political Economy, 63, 1955, 103-115.

13. Enke, S., "Equilibrium Among Spacially Separated Markets, Solution by Electric Analogue," Econometrica, 19, 1951, 40-47.

14. Klein, R. L., "The Use of Econometric Models as a Guide to Economic Policy," Econometrica, 15, 1947, 115-151.

General Theories of Business Cycles:

15. Kalecki, M., "A Macrodynamic Theory of Business Cycles," Econometrica, 3, 1935, 225-239.

16. Kalecki, M., Studies in Economic Dynamics, London: G. Allen and Erwin Ltd., 1943.

17. Leontief, W., The Structure of American Economy 1919 - 1929, New York: Oxford University Press, 1951.

18. Hicks, J. R., A Contribution to the Theory of the Trade Cycle, Oxford: Claredon Press, 1950.

Mathematical Tools:

19. Wiener, M., Cybernetics, New York: John Wiley and Sons, Inc., 1948.

20. Chestnut, H. and Mayer, R. W., Servomechanism and Regulating-System Design, Vol. 1, New York: John Wiley and Sons, Inc., 1951.

21. Brown, G. S., and Campbell, D. P., Principles of Servomechanisms, New York: John Wiley and Sons, Inc., 1948.

22. Bothwell, F. E., "The Method of Equivalent Linearization," Econometrica, 20, 1952, 269-283.

23. Solow, R., "On the Structure of Linear Models," Econometrica, 20, 1952, 29-46.