WATER PRESSURE UNDER DAMS

A Thesis

Submitted for the Degree of

MASTER OF SCIENCE

IN

CIVIL ENGINEERING

By

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Approved by _____

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Several years ago the writer, while working out the design
of the Long Lake Dam for the Washington Water Power Company of
Spokane Washington, came to the conclusion that upward water
pressure under dams had but slight effect on the overturning
moment but did decrease the factor of safety against sliding.
It will be shown later that it is more economical, as far as
safety against sliding is concerned, to batter the upstream
face of the dam and thus enlist the aid of the water pressure
in giving increased safety. When the Long Lake Dam, which at
the time of its construction was the highest spillway dam in
the world, was completed, many inquiries were received as to
the reasons for the batter of the upstream face. It is the
purpose of this paper to show that this is the economical
way of providing for upward pressure, if it is decided to
make such provision by increased weight of masonry in addition
to or instead of drainage and cut-off walls.

While at the present time (1928) this question is not
being discussed much in engineering literature, yet if a fail-
ure of some dam should occur in the future, there would be for
several months a revival of discussion and many advocates ar-
guing that under pressure was the cause of such failure, and
they would probably be right. The failure of the Austin
Pennsylvania Dam and prior to that the Austin Texas Dam brought
out a great deal of discussion on this point. The writer has
read most of this discussion and has studied most of the books
on the design of dams and has come to the conclusion that the
Equations derived for the design of dams with upward pressure
considered are in error. It will be the attempt of this

paper to show wherein these equations are erroneous.

The nearest approach to the writer's views are to be found in Volume I of Hydro-electric Power by Lamar Lyndon published in 1916. Mr Lyndon probably has the same Ideas as the writer but does not carry them out to a complete conclusion. On page 196 of Volume I of the first edition Mr Lyndon says;-

"The conventional and accepted ideas concerning the entrance or percolation of water under the base of a solid dam are;

1. The full pressure, due to the head of water backed up by the dam is exerted under the base, at the upstream edge of the joint.

2. The pressure is zero at the downstream edge of the joint.

3. The diminution of pressure is uniform through the base of dam from the upstream to the downstream side.

4. The total upward force exerted by the pressure taken over the whole base, is added to the reaction of the foundation against the base of the dam.

5. The center of upward pressure is located at a point L/3 from the upstream edge of the dam at the base line, and the net force has therefore a lever arm about the toe of 2L/3 L being the length of the base.

On page 97 we find the following comments by Mr Lyndon;

"But assumption 4 is the one for which the least excuse exists. This assumes that a structure having a certain weight rests on a supporting surface. and that a film of water under pressure interposes itself between the whole area of the base of the structure and that of the support and then adds itself

to the existing reaction of the support.

This would mean;-

(a) That the base of the dam could only rest on a film of water and not touch the foundation.

(b) That the water pressure, plus the reaction of the foundation against the base of the dam is the total pressure against the base of the dam and the total reaction of the foundation , notwithstanding the fact that, according to the theory of an interposed film of water, the dam would not rest on the foundation but on a water cushion, and the only foundation reaction possible would be that of the water.

(c) That water having a given pressure would spread apart surfaces between which a much greater pressure exists, and add its pressure to that greater one which first existed."

All of the above is true and so evident that no proof is necessary. However, if a dam has a resultant passing through the downstream middle third point, thus making the foundation pressure zero at the heel, then at this point and for a short distance into the base the upward pressure would exceed the foundation reaction and there would be a readjustment of the foundation reaction but in no case can the reaction exceed the weight of the dam. We would have then a small triangle of upward pressure and this triangle would be the total effect of upward pressure on overturning moment. To follow Mr. Lyndon's argument to conclusion, if the water pressure under the dam cannot add to the foundation reaction, neither can it add to the overturning moment, excepting the effect of the above mentioned small triangle of pressure.

To make this clear, a dam will be assumed with a vertical upstream face. Let a water tight bulkhead be built parallel to this face at a distance of one foot and the water between this bulkhead and the face of the dam be pumped out. The water pressure will then be against the bulkhead and to support the bulkhead short struts will be placed at intervals between the bulkhead and the face of the dam. The water pressure will now be transmitted through these struts to the dam and will exert practically the same overturning moment on the dam as it would if the bulkhead were removed. Now if water is allowed to flow into the space between the dam and bulkhead, it will be clear that the overturning moment is not increased by this water until the head of the water in this space becomes greater than the head back of the bulkhead and then the increase will be only that due to the excess head. At the same time the stresses in the short struts are being decreased and when the two heads are equal the pressure of the struts against the face of the dam will be zero if no allowance is made for the cross sectional area of the struts. Applying this to the base of a dam, the foundation will take the place of the bulkhead and the points of contact between the foundation and the base take the place of the struts. If water is allowed to percolate into the spaces under the dam, it is then evident that this water cannot increase the overturning moment on the dam until its pressure is equal to the foundation.reaction. Near the heel of the dam where the upward pressure is greater than the reaction, the case is similar to the upper space between the bulkhead and dam and the excess head in both cases does increase the moments.

However, as the stresses in the struts decrease so do the pressures between the dam and its foundation decrease and the safety against sliding is greatly decreased.

When the heads of water on both sides of the bulkhead are equal the total pressure on the upstream side will be greater due to the fact that it acts on a greater area. the area on the other side being reduced by the sum of the cross sectional areas of the struts. Similarly, the water under a dam cannot exert its full pressure on the entire area of the base. The usual practice of assuming it as acting on 2/3 of the area appears to be too high. However, in view of the fact that most dams have a low safety factor against sliding, it is evident that upward pressure is very likely to reduce this factor to less than one. It is the writer's opinion that most of the failures have been due to a low factor of safety against sliding combined with a bad leaky foundation. On such foundations, every attempt should be made to increase the factor of safety against sliding. In nearly every failure the dam has slid out and did not overturn and in nearly every case the foundation was leaky.

In this connection the writer quotes Mr. Edward Godfrey in the discussion of an article "Provisions for Uplift and Ice Pressure in Designing Masonry Dams" by C.L. Harrison in the Transactions of the American Society of Civil Engineers Volume LXXV Dec. 1912. On page 152 Mr Godfrey says;

"One of the arguments which is supposed to show that of the thirty or forty masonry dams that have failed in the last twenty years, under pressure has not been the cause, is the fact that blocks generally slide out and do not overturn. When under-pressure assisted by the horizontal pressure has pried a

dam loose, the former has spent the greater part of its force. It would require time to gain a new momentum as the water can enter but slowly in a narrow slit. The escape of a very small quantity of water in a test under hydraulic pressure drops the gage pressure very quickly. In the case of the dam, there is the ever present horizontal pressure with practically unlimited volume behind it, and this quickly acts to force the dam out in a horizontal direction."

Mr Godfrey is correct in the statement that under pressure assisted by the horizontal pressure pries the dam loose but it is not the overturning moment that is the cause. The writer believes that if the failure were started by moments it would continue by moments notwithstanding the argument that it takes time for water to enter a narrow slit. It appears that the overturning moment would increase once the dam started to rise from its foundation as the pressure would then act upon the whole area of the base instead of only a small percentage of it.

In this same discussion there is the following by Mr. A. P. Davis

" If the dam must be built as a purely gravity structure on a straight plan, the most economical method of meeting this problem is by increasing the batter on the water side of the original gravity structure, such increase of batter to depend on the amount of uplift to be provided against. For dams of moderate heighth, the greatest safety with a given quantity of masonry is attained by a section roughly conforming to a right angled triangle with the hypotenuse on the water slope.

This form enlists the aid of the water to assist in holding the dam in place and the increase of batter may be varied to such a point that this resistance overbalances the tendency of the water to push the dam down stream.

" The reason this principle is inapplicable in so many cases is that the average low masonry dam must serve as a spillway, and the impact of a large volume of water at the downstream toe would be dangerous. Therefore, it becomes necessary to carry the masonry on such a slope as will prevent this 'mpact and carry the water quietly away from the dam, allowing it to expend its accumulated energy in friction on the river bed some distance below. This usually requires enough masonry to fulfill gravity requirements, without much batter on the back."

It might be mentioned here that Mr Davis, who is Chief Engineer of the United States Reclamation Service, approved the writers design of the Long Lake Dam which conforms to the principles advocated in this paper.

The writer found in designing the above mentioned dam that the statement of Mr. Davis is true and that the most economical method of providing for uplift by increased section is to batter the upstream side and this seems to be true even though uplift is considered as adding to the overturning moments. It is even more economical if we consider uplift as affecting mainly the factor of safety against sliding. The following proof will show that for overturning the most economical section is one with the back face vertical but for sliding the most economical section is one with the front

face vertical and rear face battered. Theoretically this batter will be more efficient if it starts near the top of the dam and is gradually increased as the depth increases. The back face then would be a curve approximating a parabola to which a vertical line would be tangent. Care would have to be taken that the vertical shear would not be excessive near the base of the dam. This form will require less masonry and enlist the aid of a greater weight of water than a straight line batter for the whole depth. For practical reasons this batter would consist of a series of straight lines approximating the curved form. The batter of the rear face of the Long Lake Dam is of this type as shown by the cross section shown in FigureI.

In the following discussion a dam of trbangular section is used.

Let s equal the specific gravity of masonry.

h the head of water retained by the dam being equal to the heighth of dam in the discussion.

x and y be the two segments into which the base is divided by a perpendicular dropped from the vertex as shown in Figure II.

Consider moments only, no uplift

The resisting Moment $=$ $sh(y^2/3 + xy/2 + x^2/6)$

The overturning moment $=$ $h^3/6 - hxy/2 - hx^2/6$

If F is the factor of safety

$$F = \frac{s(2y^2 + 3xy + x^2)}{h^2 - 3xy - x^2}$$

If x is 0 $F = \dfrac{2sy^2}{h^2}$ If y is 0 $F = \dfrac{sx^2}{h^2}$

For the same value of F in both cases

$x^2 = 2y^2$ or $x = 1.414y$

This shows that for overturning only it is more economical to

batter the front face keeping the rear face vertical.

For sliding let S be the safety factor

$$\frac{s(x + y)h}{2} + \frac{xh}{2} = \frac{Sh^2}{2}$$
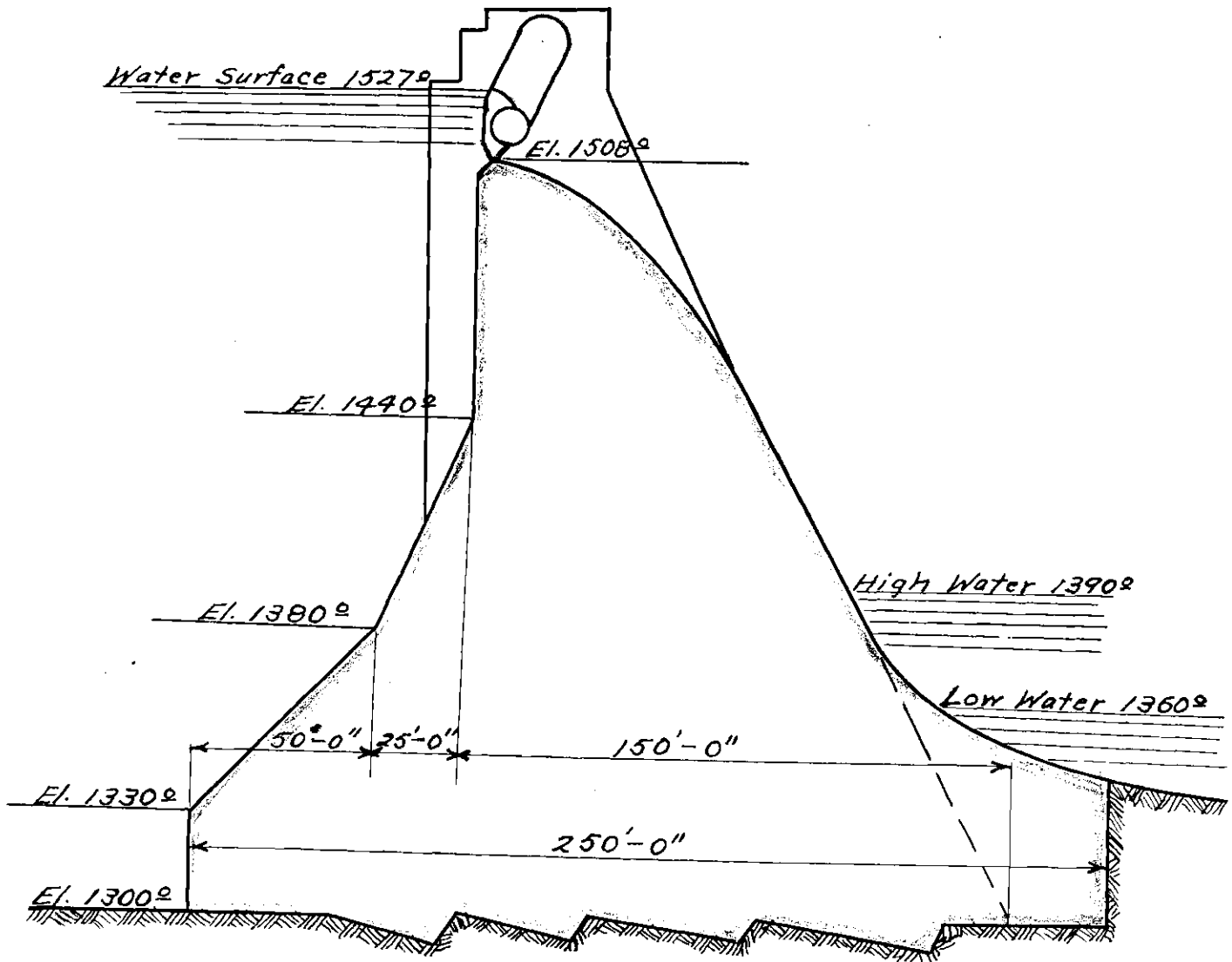
If y is 0 $S = \frac{(sx + x)}{h}$

If x is 0 $S = \frac{sy}{h}$

For the same value of S in both cases $sx + x = sy$

If s is 2.33   y is 1.43 x

In this case it is more economical to batter the rear face and keep the front face vertical.

It is evident then that to offset uplift it is more economical to add masonry to the rear face of the standard section. In doing this it is probable that the batter of the downstream face can be lessened and by battering the rear face in the curved form above mentioned, the increase in section over the standard masonry dam section will not be excessive. In the Long Lake Dam the increase in section was larger but this was due to the company desiring a high safety factor against sliding. The reasons for this were the bad foundation and the fact that a failure would destroy a three million dollar power plant down stream and a probable loss of many lives of inhabitants of the valley below the dam. In the case of this dam, a line of grouted holes forming a cut-off was placed under the heel of the dam and in addition a thorough system of drainage was installed and the section increased. If a solid dam were designed with uplift neglected and with a factor of safety against sliding of 1.4 and comparison made with the Long Lake Dam the increase of section would be found to be

Water Surface 1527º

El. 1508º

El. 1440º

El. 1380º

High Water 1390º

Low Water 1360º

50'-0" 25'-0" 150'-0"

El. 1330º

250'-0"

El. 1300º

LONG LAKE DAM
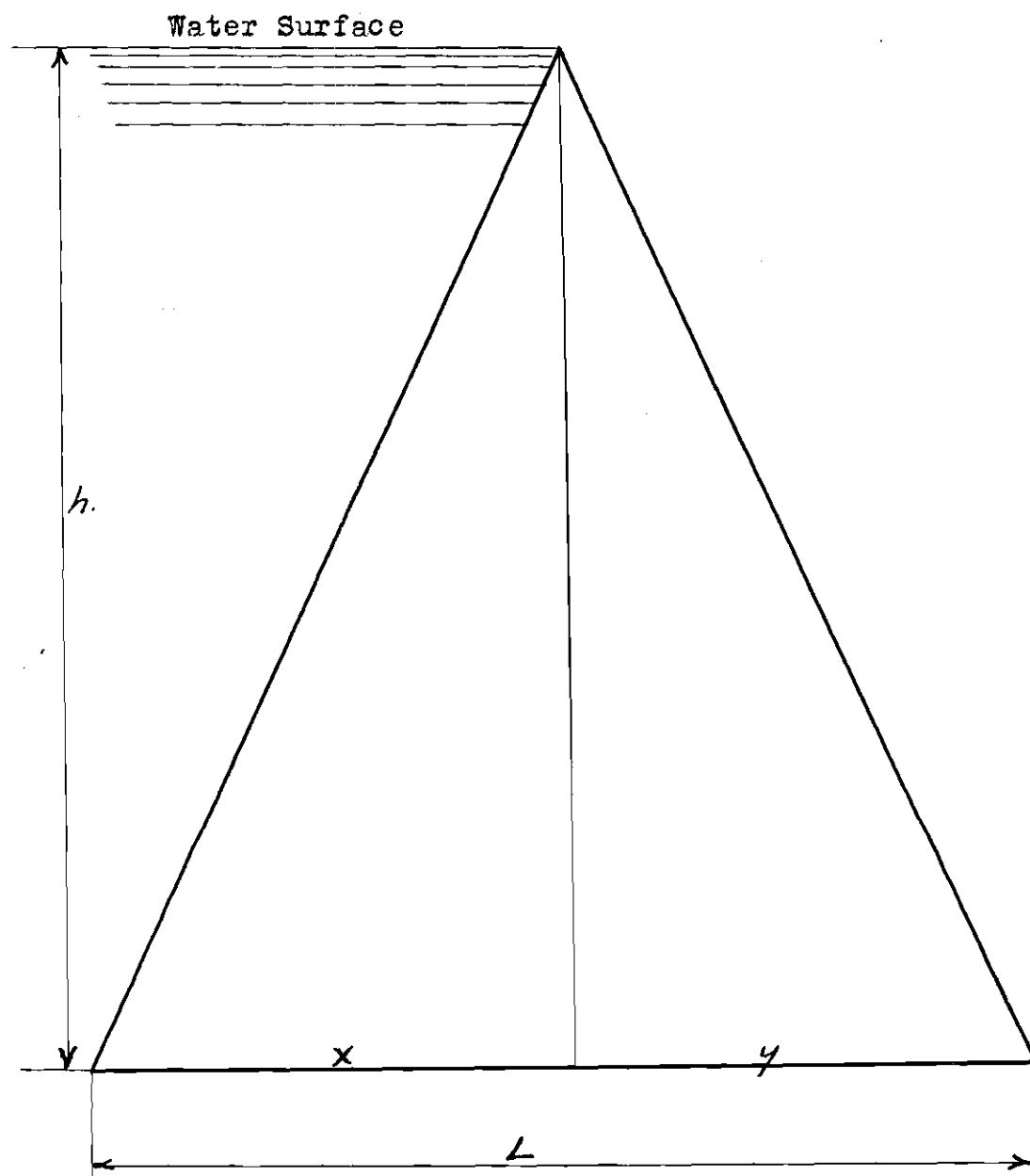
Figure I.

Water Surface

Figure II.

not very great, in the latter case. The first design of this dam had a straight continuous batter on the rear face but for the same safety the curved form as shown effected a considerable saving in concrete.

The writer will now show that the effect of upward pressure on the overturning safety factor is not very great but that the effect on the sliding safety factor may be very great, particularly in the case of low overflow dams subject to excessive floods. These dams, if built on a seamy foundation, should have a much more generous section than is usually allowed and it is very probable that the hollow reinforced concret dam is the safer and more economical type for such conditions. In the case of gravity dams it will be more economical to add the excess material by battering the rear face.

In the following discussion, the upward pressure is taken as acting on two thirds the area of the base, although the writer is of the opinion that this is too high a value. The specific gravity of the masonry will be taken as 2.33 and the weight of a cubic foot of water will be taken as a unit.
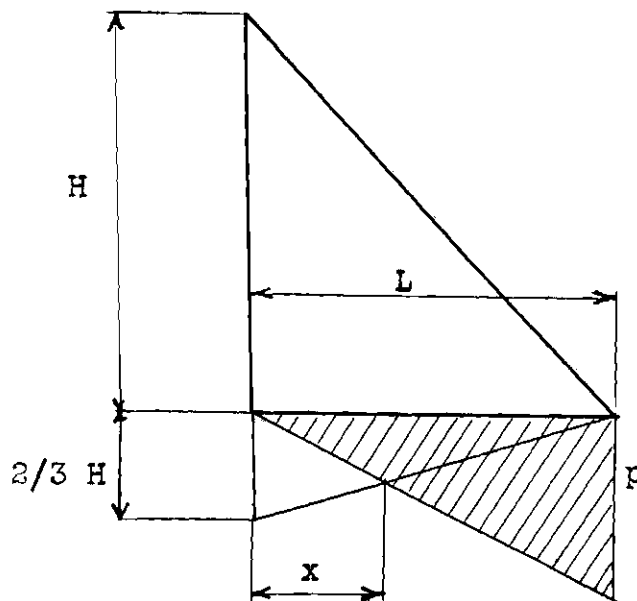
In the diagram, the unshaded triangle of base 2/3 H and heighth x represents all of the upward pressure force that affects overturning. To take the whole triangle of base 2/3 H and heighth L is erroneous and corresponds to the old method of calculating initial tension in bolts which the writer has always maintained was incorrect but which is now correctly stated in Professor Leutwiler's book on Machine Design, in the design of bolts in the cylinder head of a steam engine. The principle is the same in both cases. However, in regard to sliding, the whole triangle should be taken. As the sliding safety factor in most dams is low, this may result in a dangerous reduction in safety and cause failure. This accounts for the fact that all failures of this kind slide out and do not overturn.

Since the foundation reaction equals the Total weight of dam.

Area of shaded triangle $= W - \dfrac{2/3\ Hx}{2} = \dfrac{2.33\ HL - 2/3\ Hx}{2} = \dfrac{pL}{2}$

Then $p = 2.33\ H - \dfrac{2/3\ Hx}{L}$ and by proportional triangles

$$\dfrac{2/3\ H}{2.33\ H - \dfrac{2/3\ Hx}{L}} = \dfrac{x}{L - x}$$

$2/3\ L - 2/3\ x = 2.33\ x - \dfrac{2/3\ x^2}{L}$

$x^2 - 4.5\ Lx - L^2 = 0$ from which $x = 0.23\ L$

A dam of triangular section is taken as this is what dam sections approximate and this section also gives the largest value of x. If a rectangular section is taken x will equal L/8 and the sections of all dams lie between a rectangle and a triangle.

A comparison of the base widths of a triangular dam both without and with upward pressure will now be made.

With no upward pressure $L = zH$, 1/2 the resisting moment being taken for a factor of safety of 2.

$$\frac{2.33\ zH(zH)H}{6} = \frac{2.33\ z^2H^3}{6} = \frac{H^3}{6}$$

$2.33\ z^2 = 1$ from which $z = 0.66$

With upward pressure considered

$$\frac{2.33\ z^2H^3}{6} = \frac{H^3}{6} + \frac{H(.23Hz)}{3}\left(Hz - \frac{.23\ Hz}{3}\right)$$

$$2.33\ z^2 = 1 - .42\ z^2$$

from which $z = .723$

The necessary increase in section is 9.5 per cent while by the conventional method the increase in section is about 20 per cent.

If we consider the foundation reaction with no upward pressure and then consider the triangle of upward pressure acting as above, it will be found that the pressure on the toe of the dam increases, also the area of the triangle and its altitude x but these increments reduce and finally a balance is found.

In working out graphically a dam 90 feet high and of base width 63 feet, the following successive values of p,x,p' and U-p' were found.

p   184.0 188.37 190.7 191.9 192.7 193.4 193.76 193.78
x   9.8   11.86 12.85 13.4 13.7 14.o5 14.19 14.19
p' 26.0   16.47 11.11 8.1 6.25 4.47 3.62   3.60
U-p' 34.0 43.53   48.9 51.9 53.75 55.53 56.41 56.40

It will be noted that p' did not become 0 nor did x

become 0.23 of the base or 14.49 which shows that z may be a little less than 0.7 When x becomes 14.19 a balance is obtained.

It will also be seen that upward pressure increases the pressure on the toe and this would have to be taken into consideration in the design of very high dams. By the usual method of calculating upward pressure, the pressures on the toe of the dam are not increased which is not logical. If p', the pressure at the heel not considering upward pressure exceeds U the intensity of upward pressure at this point, then upward pressure will have no effect on overturning. This same statement will be found in Parker's Control of Water and corroborates the writer's theory.

The following equations have been derived and the solution will have to be made by trial but this will give the value of x more quickly than by making successive trials by graphics as done above.
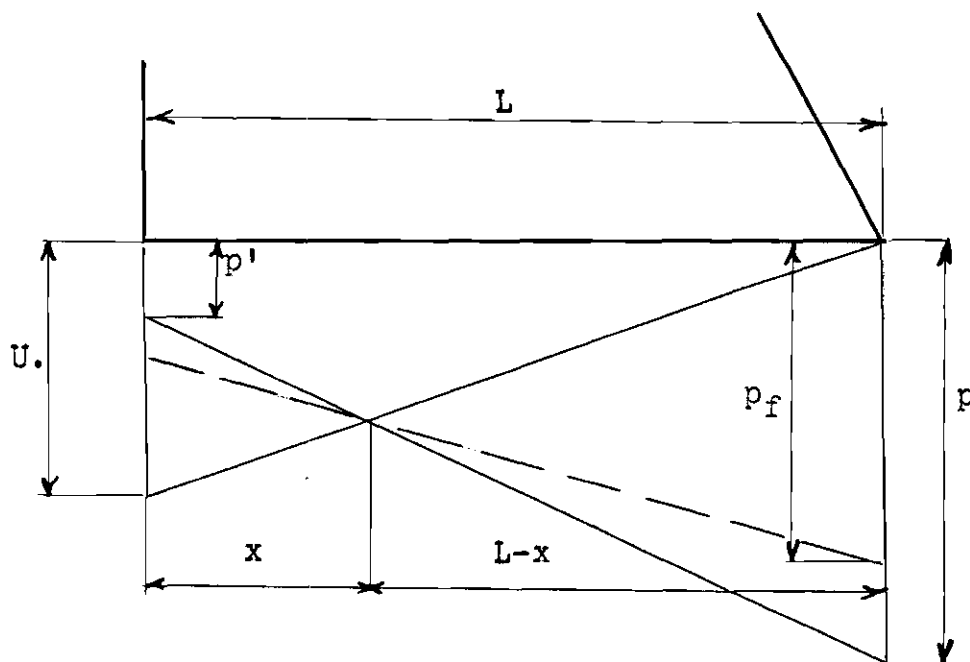
p = pressure at toe, uplift considered

p_f = pressure at toe, uplift not considered

$$p(L-x) = 2W - UL \qquad p = \frac{2W-UL}{L-x}$$

Then $p = p_f$ plus increase due to moment minus decrease due to uplift.

$$p = p_f + \frac{6(U-p')x(L/2-x/3)}{2L^2} - \frac{(U-p')x}{2L}$$

By proportion $\dfrac{U-p'}{x} = \dfrac{p}{L-x}$ or $\dfrac{(U-p')x}{1} = \dfrac{px^2}{L-x}$

Substituting and transposing

$$p = \frac{p_f}{1 - \dfrac{3x^2(L/2-x/3)}{L^2(L-x)} + \dfrac{x^2}{2L(L-x)}}$$

Substituing value of p above and transposing

$$2W-UL = \frac{p_f (L-x)^2 L^2}{L^2(L-x) - 3x^2(L/2-x/3) + 1/2Lx^2}$$

$$= \frac{p_f (L-x)^2 L^2}{L^3 - L^2x - 3/2Lx^2 + x^3 + Lx^2}$$

Now let x equal kL and substitute

$$\frac{2W-UL}{p_f L} = \frac{(1-k)^2}{1-k-k^2+k^3}$$

For the above dam $\dfrac{2W-UL}{p_f L} = \dfrac{12230-3780}{184(63)} = .816$

Then $1-2k-k^2 = .816 -.816k -.816k^2 +.816k^3$

$.816k^3 -1.816k^2 +1.184k -.184 = 0$

From which k equals .2252 or 1   The value .2252 checks the value .23 previously obtained and also .2252 x 63 = 14.188 checking the value 14.19 in the above table obtained by graphical computation.

From this we can see that upward pressure has no serious effect on overturning and will now take up the effect of sliding and find out where the real danger of upward pressure lies.

A series of dams of 20, 40, 60, 80 and 100 feet in heighth will be worked out with flat tops to simplify calculations. All of these dams will have a safety factor against sliding of about 1.23 with five feet of water flowing over the crest and with no upward pressure considered. The large drop in safety will then be noticed when upward pressure is considered and then more of a drop in the safety factor as the heighth of overflow is increased. This latter drop is larger for low dams. The assumption is made, which is logical, that a five foot rise in the head water will cause a five foot rise in the tail water. All dams have the back face vertical and the front face sloping 1 horizontal to two vertical.
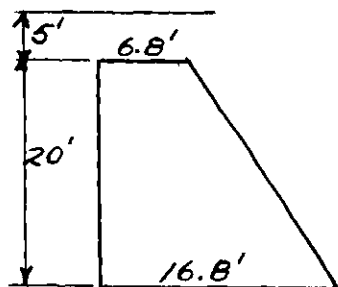
Weight of dam = 6.8(20) 2.33 + 10x10 = 559

Horizontal thrust = 5x20 + $\frac{20 \times 20}{2}$ = 300

Factor Safety (Sliding) $\frac{.66(559)}{300}$ = 1.23

Upward Pressure $(\frac{25+5}{2})16.1 \times 2/3$ = 168

Factor Safety (Upward Pressure) $\frac{.66(559-168)}{300}$ = 0.86

Overflow head of 10 feet, 20 foot dam

Horizontal Thrust = 400

Factor Safety $\dfrac{.66(559)}{400}$ = .924

Upward Pressure 2/3(16.8)20 = 224

Factor Safety (Upward Pressure) $\dfrac{.66(559-224)}{400}$ = 0.553

Overflow head of 15 feet, 20 foot dam

Horizontal Thrust   500

Factor Safety $\dfrac{.66(559)}{500}$ = 0.738

Upward pressure 2/3 (16.8)25 = 280

Factor Safety (Upward Pressure) $\dfrac{.66(559-280)}{500}$ = 0.368

If this dam is totally submerged its weight would be $\dfrac{559(1.33)}{2.33}$ Or 308   In this case of 15 feet overflow, the weight minus upward pressure was less than 308 which cannot be true. Evidently the weight of the water falling over the dam would add to the weight of dam but this is a difficult thing to estimate.

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40 foot dam with 5 foot overflow Top width 9.66 feet, bottom width 29.66 feet.

Weight 9.66(40)2.33 $+$ 20x20x2.33 = 1835

Horizontal Thrust 5x40 $+$ $\dfrac{40(40)}{2}$ = 1000

Factor Safety $\dfrac{.66(1835)}{1000}$ = 1.22

Upward Pressure 29.66(25)2/3 = 495

Factor Safety (Upward Pressure) $\dfrac{.66 (1835-495)}{1000}$ = 0.896

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40 foot dam 10 foot overflow

Horizontal Thrust $10(40) + \dfrac{40 \times 40}{2} = 1200$

Factor Safety $\dfrac{.66(1835)}{1200} = 1.01$

Upward Pressure $29.66(40/2 + 10)2/3 = 595$

Factor Safety (Upward Pressure) $\dfrac{.66 (1835-595)}{1200} = 0.683$

Continuing these calculations in the same manner for the 60 80 and 100 foot dams the following table of safety factors are found and the values are plotted curve sheet No. I

	OVERFLOW HEAD			
20 foot dam	5'	10'	15'	20'
F.S. No up. press.	1.23	0.924	0.738	
F.S. Upward "	0.86	0.553	0.368	
40 foot dam				
F.S. NO. Up. Press.	1.22	1.01	0.865	0.758
F.S. Upward "	0.896	0.683	0.540	0.430
60 foot dam				
F.S. No. Up. Press.	1.23	1.08	0.958	0.861
F.S. Upward "	0.92	0.760	0.645	0.550
80 foot dam.				
F.S. No. Up. press.	1.245	1.123	1.02	0.940
F.S. Upward "	0.942	0.815	0.696	0.626
100 foot dam				
F.S. No Up. press.	1.248	1.142	1.053	0.975
F.S. Upward "	0.940	0.838	0.750	0.674

As the assumption that upward pressure on two thirds the area of the base is excessive, the following safety factors were worked out for a pressure on 40% of the base. Also in case the base is thoroughly drained values are worked out for tail water only acting on two thirds the area of the base. This will cover about all the usual assumptions made as to upward pressure.
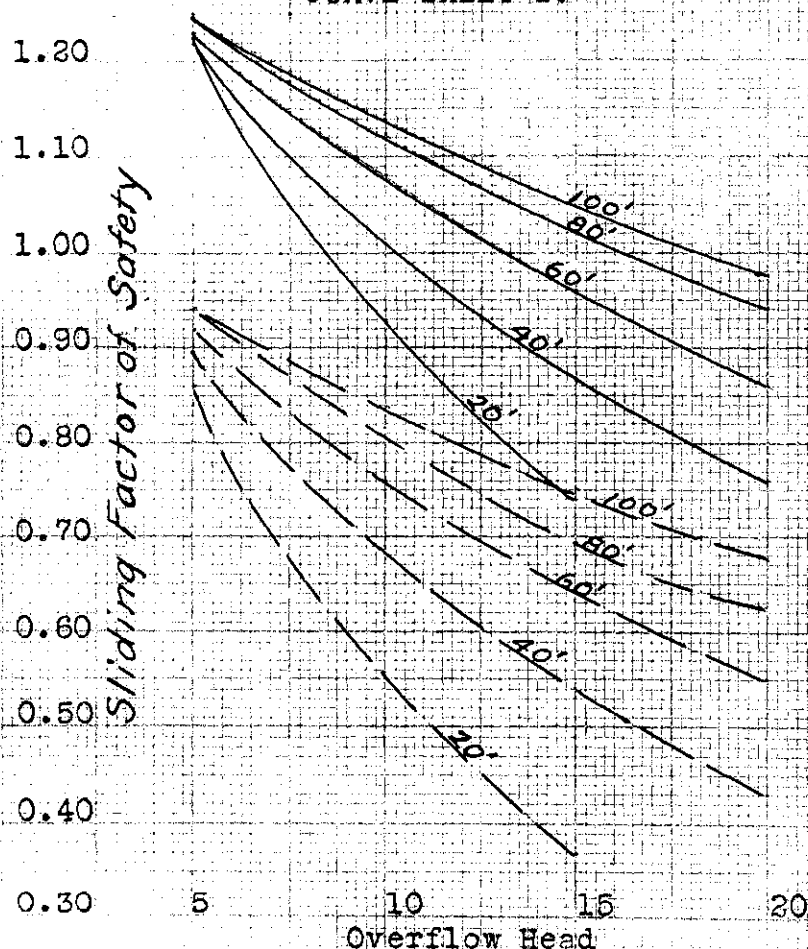
SLIDING SAFETY FACTORS

OVERFLOW HEAD

20 foot dam	5'	10'	15'	20'
No Up. Pressure	1.23	0)924	0.738	
Up. Press. 40%	1.01	0.713	0.516	
Up. Press. Tail water 2/3 base	1.11	0.745	0.525	
40 foot dam				
No. Up. Pressure	1.23	1.010	0.865	0.758
Up. Press. 40%	1.02	0.815	0.670	0.562
Up. Press. Tail water 2/3 base	1.142	0.900	0.776	0.595
60 foot dam				
No. Up. Pressure	1.23	1.080	0.958	0.861
Up. Press. 40%	1.04	0.890	0.770	0.675
Up. Press. Tail water 2/3 base	1.19	1.000	0.855	0.737
80 foot dam				
No Up. Pressure	1.245	1.123	1.020	0.940
Up. Press. 40%	1.060	0.940	0.836	0.750
Up. Press. Tail water 2/3 base	1.215	1.060	0.936	0.834
100 foot dam				
No Up. Pressure	1.248	1.142	1.053	0.973
Up. Press. 40%	1.062	0.960	0.872	0.796
Up. Press. Tail water 2/3 base	1.220	1.090	0.995	0.893

From an examination of these tables and the corresponding curves plotted therefrom, it will be seen that there is a large drop in the safety factor when upward pressure is considered. In addition it will be noted that there is still more of a drop as the overflow head increases. This latter drop is more pronounced the lower the dam. This is to be expected as the rise on a low dam is a larger proportionate rise than on a high dam but it is true that a five or ten foot rise is as likely to occur on a low dam as on a high one. In many cases the probability of a rise on the low dam is greater. The writer believes that this is the reason why

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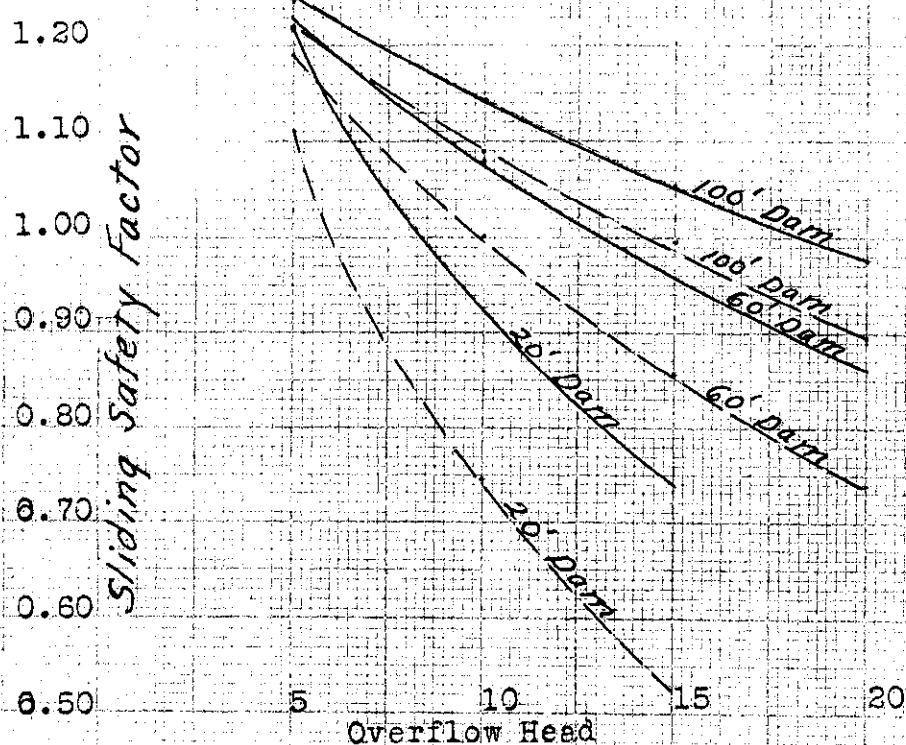
CURVE SHEET I.

Overflow Head

Solid lines, No Upward Pressure
Dotted lines, Upward Pressure due to full
upstream and tailwater head acting on 2/3
the area of the base.

Curves showing drop in Sliding Safety Factor
due to upward pressure combined with abnormal
rise in water overflowing crest.

CURVE SHEET II.

Solid lines, No Upward Pressure

Dotted lines, Upward Pressure Considered
Tailwater only on 2/3 area of base

Curves showing drop in Sliding Safety Factor
due to upward pressure combined with abnormal rise
of water overflowing crest.

the most of these failures have been dams of less than 60

feet in heighth. Of course, it must be considered that there

are more dams of low heighth and that more care is taken in

the design and construction of high dams than of low dams.

One of the purposes of this paper is to show that as much or

even more care should be taken in the design of a low dam as

in the design of a high one.

Another danger due to the rise of overflow head is that

due to the falling sheet of water leaving the face of the dam

and creating a partial vacuum behind it. The amount of this

force is hard to estimate but it is evident that it is a

source of danger when added to the decreased resistance to

sliding shown above.

The safety factor against sliding is very low in most

dams. This is due to the increase in cost necessary to get a

larger safety factor, but in the case of low dams with overflow

the highest possible safety should be obtained. Due to the

necessity of having the downstream face of the dam conform in

slope to the sheet of falling water, low dams usually have a

larger base in proportion to heighth than high dams. This

adds to their safety but if upward pressure is considered as

acting on two thirds of the base area, the section will still

be unsafe. Either this assumption is excessive or most dams

are on good foundations, otherwise the majority of them would

have failed. From the evidence of those that have failed it

seems that the fault has been with the foundation. A good

geologist should be consulted before any dam is built.

From the tables and curves given, it will be seen that

in the case of tailwater only acting on two thirds the area of the base, the lowering of the safety factor is not excessive. As this condition can be realized by cut-off walls and thorough under-drainage, this seems to be the correct thing to do in the case of low dams. As above stated the writer is not aware of this ever having been done in the construction of low dams. In this case, if any additional material is needed to increase the safety, it is more economical to add it by battering the upstream face preferably as shown later by starting the batter at about one half the depth of the dam and thus get the benefit of as much water weight as possible for the concrete used. However, when the expense of this drainage and extra concrete is considered, it will probably be found that the hollow re-inforced concrete dam is to be preferred particularly on foundations that are not first class. The upstream face of hollow dams has a slope of about forty five degrees and for this reason any rise of head water will cause an increase of horizontal pressure but at the same time the vertical weight of the water on the rear face is increased by an equal amount. As the co-efficient of friction is about two thirds this will not entirely compensate for the increased horizontal pressure but will prevent and excessive decrease in the safety factor. The base of a hollow dam is greater than that of a gravity dam but not so large proportionally in the case of low dams as in the case of high dams. In the case of low hollow dams, the base of the buttresses will have only about one sixth the area of the whole foundation and the upward pressure would not be very great. In high dams, this buttress base area will be a larger proportion

and the upward pressure would have to be considered. For this reason, it appears that for low overflow dams the hollow dam is more safe and economical but that for high dams, considering the extra foundation excavation, cost of forms and better grade concrete, the gravity dam is more economical and can be made as safe. Just where the dividing line is would have to be worked out for each particular case as too many factors enter into the case for a general solution. It is the writer's opinion, after investigations made in the design of the Long Lake dam and in the design of hollow dams of clay sand and rock foundations, that above one hundred feet in height, the gravityy dam will usually be the more economical.

It will also be seen from the tables, that excessive overflow is a source of serious danger, especially in the case of low dams. In case the flow records of the river are known for a period of twenty or more years, the dam can be designed for the highest recorded flow but if records are not available, then great caution must be taken. Even flow records of many years are not an absolutely safe guide, as the writer has seen a high water mark of the Colorado River below Austin Texas which was made in 1876 and yet when the Austin Dam failed in 1900, with 13 feet of water flowing over the crest, the resulting flood did not reach this mark which is located several miles below the dam. The flood with the addition of the released impounded water did not reach this mark by twenty feet.

The writer will now compare this dam at Austin Texas, which failed, with the Speir Falls Dam on the Hudson River, New York, both dams being 60 feet high and designed for 15 feet of over-

flow. The latter dam has not failed. This dam has a cross-sectional area of 153 square feet more than the Austin Dam and with upward pressure considered has no larger safety factor than the Austin Dam, showing that little is gained by adding area on the downstream side except possibly preventing the lower nappe of falling water from leaving the face of the dam and producing a vacuum. This probably happened in the case of the Austin Dam, as the writer remembers when a boy of seeing this dam when a flood was passing over the crest and feeling the vibration of the dam caused by the forming and breaking of a vacuum under the lower nappe of falling water. However, the face of this dam cofresponds very closely to the theoretical curve of the lower nappe of falling water. The writer will add 153 square feet of section to the rear of the Austin Dam and compare the results with the Speir Falls Dam.
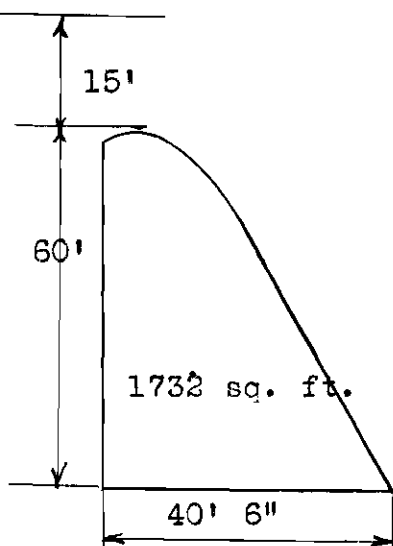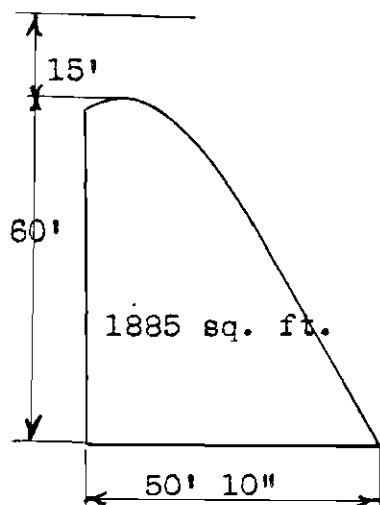
AUSTIN TEXAS DAM

Weight 1732 x 2.33 = 4040

Horizontal Thrust $15(60) + \dfrac{60^2}{2} = 2700$

Factor Safety $\dfrac{.66(4040)}{2700} = 1.00$

Upward Pressure $(60/2 + 15)40.5 \times 2/3 = 1215$

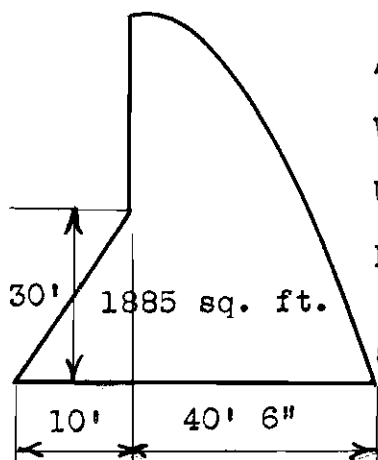Factor Safety with Upward Pressure $\dfrac{.66(4040-1215)}{2700} = 0.69$

Diagram labels: 15', 60', 1732 sq. ft., 40' 6"

SPEIR FALLS DAM

Weight 1885 x 2.33 = 4400

Horizontal Thrust 2700

Factor Safety $\dfrac{.66(4400)}{2700} = 1.072$

Upward Pressure $45(50.83) \times 2/3 = 1520$

Factor Safety with Upward Pressure $\dfrac{.66(4400-1520)}{2700} = 0.705$

Diagram labels: 15', 60', 1885 sq. ft., 50' 10"

AUSTIN DAM (153 square feet added on rear)

Weight 4400 $+$ 60x10 =5000

Upward Pressure 45x50,5 (2/3) $=$1510

Factor Safety $\dfrac{.66(5000-1510)}{2700} = 0.852$

as compared with 0.705 above

- -

Additional material added in the form of a parabola on the rear face (AUSTIN DAM)

Weight 4400 $+$ 2x150 $+$ 7.5(15) $=$4812.5

Upward Pressure 45x48(2/3) $=$ 1440

Factor Safety $.\dfrac{66(4812.5 - 1440)}{2700} = 0.823$

If upward pressure is taken on 40% of the base the upward pressure safety factor would be 0.965 or a little less than one.

If only tail water is taken, acting on two thirds the area of the base the Safety factor will be 1.09 or greater than the Speir Falls Dam safety factor without any upward pressure.

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In order to show by a concrete example, the amount that can be saved by adding material to the rear face of a dam instead of to the front face, a comparison will be made of the designs of the Long Lake Dam, shown in Figure I. and the Kensico Dam of the New York Water Supply, shown in Figure III. Both of these dams were designed for upward water pressure acting under two-thirds the base area, and comparison will be made for the same total head of water retained by each dam. The Long Lake Dam has extra material added on the rear and the Kensico Dam has extra material added on the front face.

LONG LAKE DAM

Cross Sectional Area 25230 square feet

Weight 2.33x25230x62.5 = 58870x62.5

Weight of Water over Rear Face 11525x62.5

Total Vertical Weight 70395x62.5

Upward Pressure 2/3(227)x1/2(225)x62.5 = 17175x62.5

Horizontal Thrust 227^2x1/2 x62.5 = 25760x62.5

Vertical Weight - Upward Pressure 53220x62.5

Sliding Safety Factor $\dfrac{.66 \times 53220 \times 62.5}{25760 \times 62.5}$ = 1.381

Kensico Dam

Cross Sectional Area 25800 square feet

Weight 2.33x 25800x62.5 = 60200 x 62.5

Weight of Water over Rear Face 2100x62.5

Total Vertical Weight 62300x62.5

Upward Pressure 1/3(219.67) 227 x 62.5 = 16620 x 62.5

Weight - Upward Pressure 45680 x 62.5

Factor Safety, Sliding $\dfrac{.66(45680)\ 62.5}{25760 \times 62.5}$ = 1.181

From the above, we find that the Kensico Dam has about the same amount of material as the Long Lake Dam but has a safety factor 17 per cent lower. To get the same safety factor 20 per cent more material would have to be added to the front face or 10 per cent more material to the rear face. The original design of the Long Lake Dam was practically the same as that of the Kensico Dam. The writer, by taking this design and modifying along the lines suggested in this paper, made a saving of 13000 cubic yards of concrete in a total of 275000 cubic yards which at the very lowest estimate meant a saving of at least $100,000 and this was done with no loss of safety. In fact there was an increase in the sliding safety factor as has just been shown.

On the following page, Figure III. the cross sections of both of the above dams are plotted, that of the Long Lake Dam being in Dotted outline. This dam is 208 feet high but retains a total head of 227 feet, there being a head of 19 feet above the dam retained by roller gates. The Kensico Dam is a little higher but the above comparison was made on the basis of both dams retaining the same head of water.

Water Surface
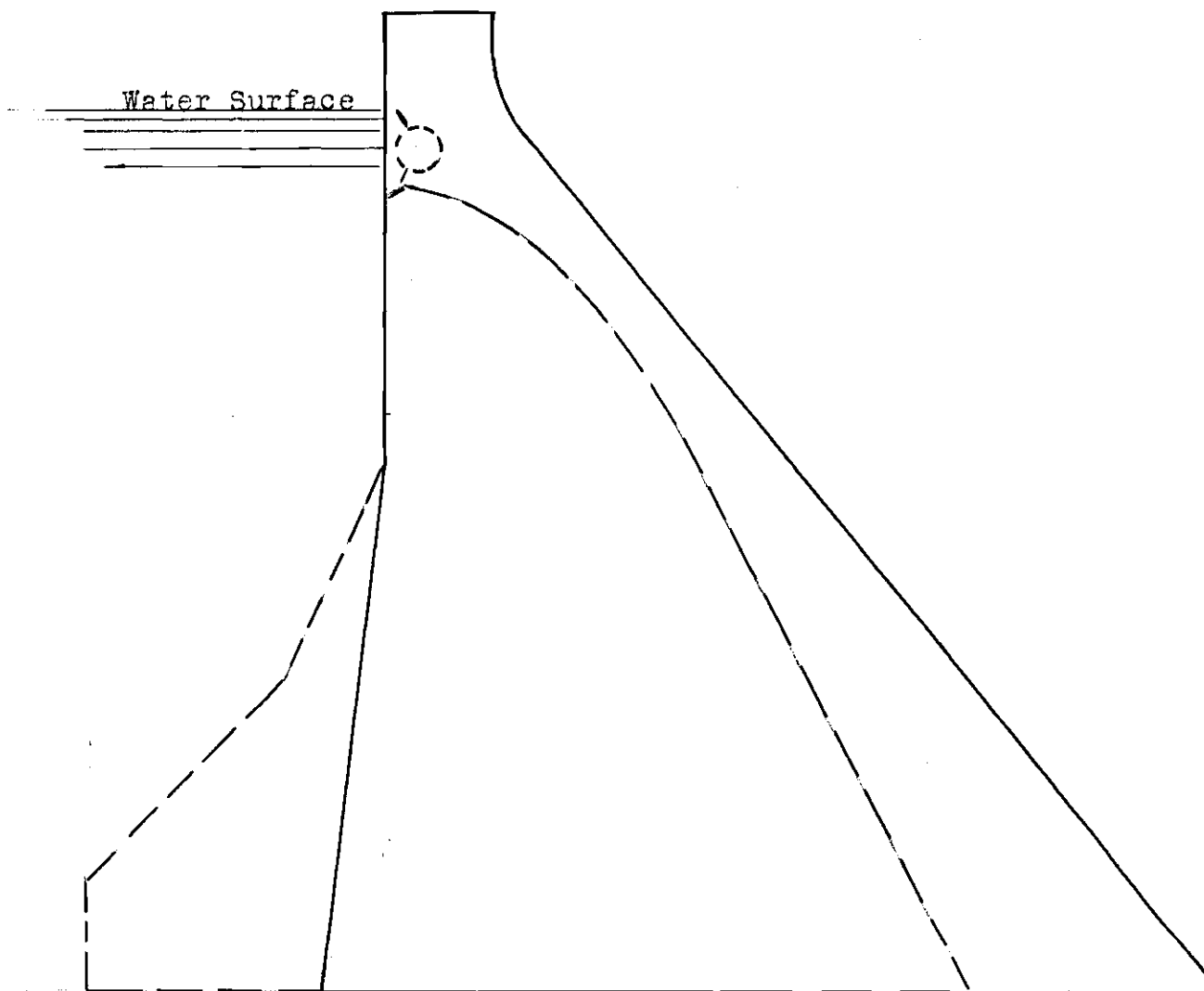
KENSICO DAM ——————
LONG LAKE DAM — — — —

Figure III.

To summarize, it has been the attempt of the writer to show that upward pressure under a dam has no effect on the overturning moment except near the heel of the dam where this pressure exceeds the foundation reaction. This is identically the same proposition as the method of figuring initial tension in the bolts in the cylinder head of a steam engine which for many years was incorrectly given in text books but which is now correctly given in books on machine design. This is the correct theory and is not so radical as it may appear. The effect of upward pressure is in sliding and as most dams have a low safety factor against sliding, the result may be serious. In most texts on the design of dams, the method is to design for overturning and in the case of overflow dams make the downstream face of the dam stay inside of the lower nappe of the falling water. After the dam is designed then it is tested for sliding and the upward pressure is assumed as acting on a small per cent of the base area to make the safety factor stay within limits. The writer contends that for small dams with overflow the method should be to make the dam safe against sliding and then it will be safe against overturning even if the incorrect method of taking all the upward pressure as affecting overturning is used.

If the foundation is faulty the only safe way is to use grout, a cut-off wall and thoroughly under drain. Then only upward pressure due to the tail water need be considered and it will not be necessary to guess as to whether two thirds or one third of the base area should be used. If additional material is needed for sliding safety, it is more economical

to add material to the upstream face by either of the two methods given by the writer in the preceding pages.

In the case of low dams, it is of great importance to know the amount of flood water that may pass over the dam. Even with only the upward pressure due to tail water the safety factor in the case of low dams may be seriously lowered by an unexpected flood. In the case of high dams the danger is not so great. Of course it would not be. advisable to put in under drains where the foundation is first class because in this case one would be adding upward pressure where none would exist.

It is very probable that for low dams on faulty foundations the hollow reinforced concrete dam is safer. Upward pressure is practically reduced to nothing and in addition due to the upstream face having a slope of about 45 degrees, any high rise of water will add as much to the vertical forces as to the horizontal forces and if the coefficient of friction is 0.66 then two thirds of the additional forces will be neutralized.

In the case of high dams, this is not so important and in addition the extra foundation excavation necessary may prove the gravity dam to be the cheaper. This would have to be worked out for the particular case, but it is almost certain that the gravity dam is the more economical for heighths of over 100 feet. In this case it is more economical to add the extra material against sliding to the rear of the dam.

The main point of this article, however, is to show that as much care should be taken in the design of low dams as in the design of high ones. The uncertain factors such as flood

flows, upward pressure allowance and vacuum behind the falling sheet of water have a greater effect on the safety of low dams. The writer believes that this is why the most of the failures have been of low dams even after taking into consideration the the fact that the greater number of dams in existance are low. Of all the structures that an engineer is called upon to design , the loads that are placed on a dam are the most certain and yet failures have been probably more numerous in proportion to numbers than for other structures. It appears then that the greatest uncertainty is in the foundation, and in that the greatest study must be made, and the type of structure to be used must fit the foundation.