Investigation of transport in the DIII-D edge pedestal

W. M. Stacey
Georgia Institute of Technology

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ABSTRACT

A comparison of various heat conduction theories with data from several DIII-D [Luxon, Nucl. Fusion, 42, 614, 2002] shots indicates: 1) that neoclassical theory is in somewhat better agreement with experiment than is ion temperature gradient mode theory for the ion thermal conductivity in the edge pedestal, although both are in reasonable agreement with experiment for most discharges; and 2) that electron temperature gradient theory ($k_{\perp}c_s \leq \omega_{pe}$) is in much better agreement with experiment than is electron drift wave theory ($k_{\perp}c_s \leq \Omega_i$) for the electron thermal conductivity. New theoretical expressions derived from momentum balance are presented for: 1) a 'diffusive-pinch' particle flux, 2) an experimental determination of the momentum transfer frequency, and 3) the density gradient scale length. Neither atomic physics nor convection can account for the measured momentum transfer frequencies, but neoclassical gyroviscosity predictions are of the correct magnitude.

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I. INTRODUCTION

The H-mode (high confinement mode) pedestal is important due to its impact on core performance in tokamaks (e.g. Refs. 1-3). Although the edge pedestal has been the subject of intensive investigation for a number of years (e.g. Refs. 4-6), the causes for the pedestal structure are still not well understood. MHD (magnetohydrodynamic) instability thresholds appear to place an upper limit on pedestal pressure and/or pressure gradient (e.g. Refs. 7 and 8). However, between ELMs (edge localized modes) or in their absence, temperature and density gradients in the edge pedestal, as elsewhere, must satisfy transport relations[9,10], and it is plausible that the structure of the pedestal is controlled by transport and sources. Thus, the transport in the edge pedestal is expected to be an important element in determining the edge pedestal structure in H-mode plasmas.

There has been relatively little comparison of observed transport in the edge pedestal with theoretical predictions, due to various difficulties. Experimentally, it is difficult to separate convective and conductive transport in the pedestal, where the particle source and convection may be large and varying. At present, this can only be done by neutral transport calculations that are coupled to edge plasma transport calculations (e.g. the coupled fluid neutral-fluid plasma calculation of Ref. 11 that was used to infer edge transport coefficients in ASDEX [Axisymmetric Divertor Experiment] Upgrade). Fundamental transport physics models for theoretical studies in the edge pedestal are embodied in some codes used to simulate the Braginskii fluid equations (e.g. Refs 12 and 13). However, such codes are very computationally intensive.

An alternate approach is to use analytical representations of transport coefficients arising from various physical transport phenomena to compare with transport coefficients or rates that are inferred from experiment. While the approximations that are inherent in the development of such analytical expressions may introduce some ambiguity into the interpretation of their comparison with experiment, this approach can provide guidance with regards to which transport phenomena are most promising for more detailed transport calculations. Reference 14 is a recent example of an application of such an alternative approach to study transport in the pedestal region of ASDEX Upgrade and JET (Joint European Torus).

In this paper we make a comparison of analytical transport coefficients with experiment in the edge pedestal in a representative set of DIII-D[15] H-mode plasmas. This work takes advantage of the good spatially-resolved measurements of T_e, n_e, T_i and n_{carbon} in the pedestal in this machine. We first consider thermal transport. Heat transport rates through the edge pedestal are inferred from the conventional heat conduction relationship, using measured pedestal densities and temperatures and their gradients and using particle and power fluxes through the edge calculated from particle and power balance. These experimental rates are then compared with values predicted by analytical expressions derived from various theoretical heat conduction models (neoclassical, ion and electron temperature gradient, electron drift wave). A feature of the transport analysis used in this paper is that atomic physics particle sources and heat losses in the pedestal are taken into account.

Our investigation of particle and momentum transport begins with more fundamental derivations from momentum balance of 1) a generalized diffusion-pinch relation among particle fluxes, density and temperature gradients, and a collection of terms that constitute a pinch velocity, 2) an expression for the calculation of a frequency for the outward radial transfer of toroidal momentum, and 3) an expression for the ion density gradient scale length in the edge pedestal. We compare experimental momentum transfer frequencies with values calculated from atomic physics, convection and neoclassical gyroviscous momentum transfer. Finally, we give an example of how the new theoretical expression for ion density gradient scale length can be used to check the consistency of measurements and theoretical models.

II. THERMAL TRANSPORT

A. Flux-gradient relations

The conventional conductive heat flux closure relation

$$\left(Q_{av} - \frac{5}{2}T\Gamma_{av}\right) \equiv q_{av} = -n\chi_a \frac{dT}{dr} \equiv nT\chi_a L_T^{-1} \tag{1}$$

involving the total heat flux, Q, the particle flux, Γ, and the conductive heat flux, q, can be used to develop flux-gradient relationships. We consider average values over the pedestal, indicated above by the 'av' subscript. We relate these average values of heat and particle fluxes to the values of these fluxes crossing the separatrix (the quantities available from particle and heat balances on the plasma) by taking into account ionization of incoming neutral atoms and cooling by charge-exchange, elastic scattering, ionization and impurity radiation to obtain[9,10] flux-gradient relations in the pedestal for the ions

$$L_{Tj} = \frac{\chi_j^{av}}{\left[\left(\frac{Q_{\perp j}^{sep}}{n_j^{av}T_j^{av}} - \frac{5}{2}\frac{\Gamma_{\perp j}^{sep}}{n_j^{av}}\right) + \frac{1}{2}\Delta\left(\frac{3}{2}\nu_{atj}^{av} + \frac{5}{2}\nu_{ionj}^{av} - \frac{Q_{je}^{av}}{n_j^{av}T_j^{av}}\right)\right]} \tag{2}$$

and for the electrons

$$L_{Te} = \frac{\chi_e^{av}}{\left[\left(\frac{Q_{\perp e}^{sep}}{n_e^{av}T_e^{av}} - \frac{5}{2}\frac{\Gamma_{\perp e}^{sep}}{n_e^{av}}\right) + \frac{1}{2}\Delta\sum_{ions}\left(\frac{n_j^{av}L_j}{T_e^{av}} + \nu_{ionj}^{av}\left(\frac{E_{ionj}}{T_e^{av}} + \frac{5}{2}\right) + \frac{Q_{je}^{av}}{n_e^{av}T_e^{av}}\right)\right]} \tag{3}$$

Here $L_T = -T/(dT/dr)$, ν_{at} is the charge-exchange plus elastic scattering frequency of plasma ions with 'cold' incoming neutrals, ν_{ion} is the electron impact ionization frequency with all neutrals present in the pedestal, E_{ion} is the ionization energy, Q_{je} is the ion-electron equilibration rate of energy exchange, L is the radiation emissivity, and Δ is the pedestal width. The '*av*' and '*sep*' superscripts refer to the average value in the pedestal region and to the value at the separatrix, respectively.

In principle, Eqs. (2) and (3) can be used to infer 'experimental' values of the average ion and electron thermal conductivities from measured and calculated quantities. The densities and temperatures, the temperature gradient scale lengths, and the pedestal width are measured. The main ion particle flux crossing the separatrix can be calculated from the known neutral beam particle source, the calculated inward neutral particle flux and the measured rate of change in the density. The total heat flux, $Q = Q_e + Q_i$, crossing the separatrix can be calculated from the known neutral beam heating power, the measured ohmic heating power, the measured rate of change of the thermal energy and the measured radiation from within the separatrix.

The separation of the total heat flux into ion and electron components and calculation of the ion-electron equilibration are more difficult. In order to avoid these difficulties, albeit at the cost of being unable to distinguish between ion and electron transport, the total conductive heat flux determined from experiment may be compared with the theoretical expression for the combined conductive heat flux due to ions and electrons

$$q \equiv Q - \frac{5}{2}\left(T_e + T_i\right)\Gamma = n\left(T_e \chi_e L_{Te}^{-1} + T_i \chi_i L_{Ti}^{-1}\right) \tag{4}$$

The terms on the left can be determined as described above. Using measured *n, T* and L_T on the right, various theoretical expressions for thermal conductivity can be tested for their ability to predict the measured combined conductive heat flux.

The neutral concentrations needed to evaluate v_{at} and v_{ion} and the recycling neutral influx needed to calculate Γ are obtained using a 2D neutral transport calculation of fueling and recycling neutrals coupled to a "2-point" scrape-off layer and divertor plasma model and to a core plasma particle and power balance model[16]. The plasma ion flux to the divertor plate is recycled as neutral atoms (at a fraction of the incident ion energy) or molecules which are assumed to immediately dissociate into Franck-Condon atoms (at ~ 2 eV). These atoms are transported out of the divertor across the separatrix and into the plasma edge to produce a poloidally distributed neutral density which is averaged to evaluate v_{at} and v_{ion}. Measured plasma densities in the scrape-off layer and pedestal region are used in calculating the penetration of recycling neutrals. Atoms that are ionized inside the separatrix contribute to the neutral source used to calculate Γ, and atoms that are charge-exchanged or scattered are assume to take on the energy of the ions at that location. Although the neutral transport calculation is well-founded, the recycling neutral source is uncertain in such calculations. We normalize the calculations to experiment by adjusting the recycling source so that the calculated core fueling by neutral influx plus neutral beam results in a prediction of the line-average density that agrees with the experimental value. This model has been found to predict neutral densities that are in reasonable agreement with measured values in DIII-D and with Monte Carlo predictions[17].

B. Theoretical heat conductivities

Our objective is to determine which, if any, of the candidate phenomena for causing heat conduction is generally consistent with the values inferred from experiment, and hence a candidate for more detailed transport analyses. For this purpose, we use analytical expressions to characterize the heat conduction produced by the following phenomena.

Neoclassical

The basic neoclassical expression for ion heat conductivity for a two-species (ion-impurity) plasma is

$$\chi_i = \varepsilon^{1/2}\rho_{i\theta}^2\nu_{iI} \tag{5}$$

where $\varepsilon = r/R$ is the ratio of minor and major radii, $\rho_{i\theta}$ is the ion poloidal gyro-radius, and ν_{iI} is the ion-impurity collision frequency.

A more complete expression is given by the Chang-Hinton formula[18]

$$\chi_i = \varepsilon^{1/2}\rho_{i\theta}^2\nu_{ii}\left[a_1 g_1 + a_2\left(g_1 - g_2\right)\right] \tag{6}$$

where the a's account for impurity, collisionality and finite inverse aspect ratio effects and the g's account for the effect of the Shafranov shift. These parameters are given in appendix A.

In the presence of a strong shear in the radial electric field, E_r, the particle banana orbits are 'squeezed', resulting in a reduction in the ion thermal conductivity by a factor of $S^{-3/2}$, where[19]

$$S = \left|1 - \rho_{i\theta}\left(\frac{d\ ln\, E_r}{dr}\right)\left(\frac{E_r}{\upsilon_{th\ i}B_\theta}\right)\right| \tag{7}$$

υ_{thi} is the ion thermal speed, and B_θ is the poloidal magnetic field.

Ion temperature gradient mode

For a sufficiently large temperature gradient ($L_{Ti} < L_{Ti}^{crit} \approx 0.1R$—*Ref. 16*) the toroidal ion temperature gradient (ITG) mode becomes unstable. An estimate of the ion thermal conductivity due to ITG modes is given by[20]

$$\chi_i = \frac{5}{2}\left(\frac{1}{RL_{Ti}}\right)^{1/2}\left(\frac{T_e}{m_i}\right)\left(\frac{m_i}{e_i B}\right)\frac{1}{2}\rho_i \tag{8}$$

where $k_\perp\rho_i = 2$ has been used, with ρ_i being the ion gyro-radius in the toroidal field.

Electron drift waves

The principal electron drift wave instabilities with $k_\perp c_s \leq \Omega_i$ arise from trapped particle effects when $\nu_e^* = \nu_e/(v_{the}/qR)\varepsilon^{3/2} < 1$. In more collisional plasmas the mode becomes a collisional drift wave destabilized by passing particles. An expression for the electron thermal conductivity that encompasses both the dissipative trapped electron mode (TEM) and the transition to the collisionless mode as $\nu_e^* \rightarrow 0$ is given by[21]

$$\chi_e = \frac{5}{2}\frac{\varepsilon^{3/2}}{\nu_e}\frac{c_S^2\rho_S^2}{L_n L_{Te}}\left(\frac{1}{1+0.1/\nu_e^*}\right) \tag{9}$$

where c_s is the sound speed and $\rho_s = c_s/\Omega_i$, with Ω_i being the ion cyclotron frequency.

Electron temperature gradient modes

The electron temperature gradient (ETG) mode (an electron drift wave with $k_\perp c_s \leq \omega_{pe}$) is unstable for $\eta_e = L_n/L_{Te} \geq 1$. An expression for the electron thermal conductivity associated with the ETG mode is given by[21]

$$\chi_e = 0.13\left(\frac{c_S}{\omega_{pe}}\right)^2 \frac{\upsilon_{th\,e} S_m}{qR}\eta_e\left(1+\eta_e\right) \tag{10}$$

where ω_{pe} is the electron plasma frequency and $S_m = (r/q)(dq/dr)$ is the magnetic shear.

C. DIII-D Experimental results

A set of DIII-D shots covering a range of operating parameters and upper (87085) and lower divertor configurations was used for this study, as described in Table 1.

The measured edge pedestal parameters are given in Table 2. The density and temperature given is that at the top of the pedestal. The average density and temperature in the pedestal region (top of pedestal to separatrix) is somewhat greater than half of the values shown. The measured widths (from the top of the pedestal to the separatrix) and average gradient scale lengths ($L_x = \Delta_x ln(x_{ped}/x_{sep})$, where $x_{ped/sep}$ is the value at the top of the pedestal/separatrix) have been mapped to a flux-surface averaged cylindrical model, as described in Ref. 10.

D. Analysis of experimental data

As mentioned previously, it is not possible to separate experimentally the ion and electron components of the heat flux through the pedestal. Yet it is of interest to compare ion and electron heat conductivities separately. Rather than introduce ambiguity into the procedure by making an approximate calculation, we assume for the moment that the ion and electron components of the total heat flux are equal ($Q_i = Q_e$). We further assume that we can neglect the Q_{ie} equilibration term in the heat flux correction terms in Eqs. (2) and (3). Note that this is not equivalent to neglecting equilibration in the pedestal because we use measured ion and electron temperatures in the pedestal which have been affected by equilibration.

The 'experimental' values of χ calculated from Eqs. (2) and (3) by using measured *n, T* and L_T and heat and particle flux balances, as discussed previously, and the 'theoretical' values calculated by using measured quantities to evaluate Eqs. (6)-(10) are given in Table 3.

For the ion thermal conductivity, the neoclassical χ_i is in somewhat better agreement with experiment than the ITG χ_i, although both are in reasonable agreement with experiment. The neoclassical χ_i is calculated from the Chang-Hinton formula and reduced for orbit squeezing (the value without orbit squeezing shown in parentheses is usually closer to the experimental value). The toroidal ITG mode should be unstable ($L_{Ti}/L_{Ti}^{crit} < 1$) for all shots.

For the electron thermal conductivity, the TEM χ_e is clearly orders of magnitude too large at lower collisionality, but is in reasonable agreement with experiment for $\nu_e^* > 1$. For the ETG mode, which should be at least marginally unstable ($\eta_e \geq 1$) for all the shots, the predicted χ_e is reasonably close to the experimental value. The neoclassical χ_e (not shown) is orders of magnitude too small, indicating that even in the H-mode edge transport barrier the electron transport is due to non-classical phenomena (a similar result has been noted for internal transport barriers[22]).

As mentioned and shown explicitly in Eqs. (2) and (3), the total heat and particle fluxes at the separatrix, which can be determined from heat and particle balance on the plasma, are 'corrected' to 'average' values over the pedestal region by calculating the radiative cooling and particle ionization sources between the midpoint of the pedestal region and the separatrix. This correction was 30-40% for the particle flux but only a few percent for the total heat flux.

The ambiguity introduced in the results of Table 3 by the assumption $Q_i = Q_e$ can be removed by comparing the total conductive heat flux predicted by Eq.(4), when evaluated with measure *n*, *T* and L_T and the theoretical expressions for χ, with the 'experimental' conductive heat flux constructed from the power and particle balances on the plasma, as discussed previously. The results are shown in Table 4. The use of either neoclassical or ITG χ_i and ETG χ_e results in a predicted conductive heat flux that is well 'within the ballpark' of the experimental value. The conductive fraction of the total heat flux was 50% for the first two shots and 65-80% for the other shots.

III. PARTICLE AND MOMENTUM TRANSPORT

A. Flux-gradient relations

Our purpose in this section is to derive flux-gradient relations and an expression for the ion density gradient scale length directly from particle and momentum balance, taking into account the various phenomena that are important in the plasma edge. We include neoclassical physics in a fluid formulation by making use of neoclassical expressions for the parallel viscosity, the gyroviscosity and the collisional friction, but refrain from making the approximations needed to obtain analytical solutions for the particle flows that lead to the usual Pfirsch-Schluter and neoclassical components of the particle flux, preferring to solve numerically for the flows in order to retain all important effects.

We first develop an edge transport relation between particle fluxes and gradient scale lengths from momentum balance. Subtracting the ion particle balance equation (including an ionization source) from the ion momentum balance equation (including a charge-exchange and elastic scattering momentum loss term), then taking the vector cross product of ***B****x* the resulting equation and making use of ***B***• the momentum equation leads to two independent equations

$$e_i B_\theta \Gamma_j + M_{\phi j} + n_j e_j E_\phi^A - n_j m_j \nu_{jk}\left(\upsilon_{\phi j} - \upsilon_{\phi k}\right) - n_j m_j \nu_{je}\left(\upsilon_{\phi j} - \upsilon_{\phi e}\right) = n_j m_j \nu_{dj}^* \upsilon_{\phi j} \tag{11}$$

and

$$\upsilon_{\phi j} = f_p^{-1}\upsilon_{\theta j} + \frac{E_r}{B_\theta} - P_j' = f_p^{-1}\upsilon_{\theta j} + \frac{E_r}{B_\theta} + \frac{T_j}{e_j B_\theta}\left(L_{nj}^{-1} + L_{Tj}^{-1}\right) \tag{12}$$

which we shall use in the following derivation and to a third independent equation which we shall use in solving for the poloidal velocities. Here, $f_p = B_\theta/B_\varphi$, M_φ represents the toroidal component of any external momentum input, E_φ^A is the induced toroidal electric field, E_r is the radial electric field, and

$$\nu_{dj}^* = \nu_{dj} + \nu_{at\,j} + \nu_{ion\,j}\xi_j \tag{13}$$

with

$$\nu_{dj} \equiv \left\langle R^2 \nabla\phi \cdot \nabla \cdot \boldsymbol{\pi}_j \right\rangle \Big/ R n_j\, m_j \upsilon_{\phi j} \tag{14}$$

representing the viscous angular momentum transport rate across the flux surface (<X> indicates the flux surface average) and

$$\xi_j \equiv \left\langle R^2 \nabla\phi \cdot m_j \tilde{S}_j \upsilon_{\phi j} \right\rangle \Big/ R\, m_j S_j \upsilon_{\phi j} \approx \tilde{S}_j \Big/ S_j \tag{15}$$

representing the poloidal asymmetry over the flux surface of the ionization source.

There is a pair of Eqs (11) and (12) for each ion species. When there are more than 2 species present the ‘k’ subscript is understood to represent a sum over all other species k ≠ j. Here, the tilde indicates the difference between that local (in poloidal angle) and average (over the flux surface) values of the ionization source.

Using Eq. (12) to eliminate the toroidal velocities, v_φ, from Eq. (11) allows the latter to be reduced to a ‘diffusive-pinch’ flux relationship

$$\Gamma_j = n_j D_{jj}\left(L_{nj}^{-1} + L_{Tj}^{-1}\right) - n_j D_{jk}\left(L_{nk}^{-1} + L_{Tk}^{-1}\right) + \upsilon_{pj} \tag{16}$$

where the diffusion coefficients are

$$D_{jj} \equiv \frac{m_j T_j \left(\nu_{dj}^{*} + \nu_{jk}\right)}{\left(e_j B_\theta\right)^2}, \quad D_{jk} \equiv \frac{m_j T_k \nu_{jk}}{e_j e_k B_\theta^2} \tag{17}$$

and the pinch velocity is

$$\upsilon_{pj} \equiv -\frac{M_{\phi j}}{e_j B_\theta} - \frac{n_j E_\phi^A}{B_\theta} + \frac{n_j m_j \nu_{dj}^{*}}{e_j B_\theta}\left(\frac{E_r}{B_\theta}\right) + \frac{n_j m_j f_p^{-1}}{e_j B_\theta}\left(\left(\nu_{jk} + \nu_{dj}^{*}\right)\upsilon_{\theta j} - \nu_{jk}\upsilon_{\theta k}\right) \tag{18}$$

In deriving Eqs. (16)-(18), we have assumed that the condition $(n_{carbon} Z^2_{carbon}/n_e) >> (m_e/m_D)^{1/2} \approx 0.016$ is satisfied, so that the ion-electron collisions can be neglected relative to the ion-impurity collisions; i.e. the ion-electron friction has been neglected relative to the ion-impurity friction. It is interesting that the atomic physics (ν_{ion} and ν_{at}) and the viscous (ν_d) momentum transfer frequencies, as well as the more familiar interspecies collision frequency (ν_{jk}), enter the expressions for the diffusion coefficients and the pinch velocity; i.e. all modes of momentum transfer to and from ion species ‘j’ are included. The dependence of the pinch velocity on the electric fields, momentum input and poloidal rotation is also noteworthy.

B. Experimental momentum transfer frequency

Equation (11) can be solved directly for the momentum transfer frequency in the pedestal

$$\nu_{dj}^{*} = \frac{\left[e_j B_\theta \Gamma_j + M_{\phi j} + n_j e_j E_\phi^A - n_j m_j \nu_{jk}\left(\upsilon_{\phi j} - \upsilon_{\phi k}\right) - n_j m_j \nu_{je}\left(\upsilon_{\phi j} - \upsilon_{\phi e}\right)\right]}{n_j m_j \upsilon_{\phi j}} \tag{19}$$

Since Γ can be determined from particle balance, M can be calculated, n, E_φ and υ_φ can be measured, and the friction terms can probably be neglected, this expression provides a means to evaluate an experimental momentum transfer rate across the pedestal. This quantity can be directly compared with various theoretical models for momentum transfer frequencies.

C. Neoclassical momentum transport frequencies

We now consider the neoclassical model for the toroidal viscous force, $\langle R^2 \nabla\phi \cdot \nabla \cdot \boldsymbol{\pi} \rangle$, which determines the viscous momentum transport frequency given by Eq. (14). There are three neoclassical viscosity components—parallel, perpendicular and gyroviscous. The 'parallel' component of the neoclassical viscosity vanishes identically in the viscous force term, and the 'perpendicular' component is several orders of magnitude smaller than the 'gyroviscous' component[23]

$$\left\langle R^2 \nabla\phi \cdot \nabla \cdot \boldsymbol{\pi}_j \right\rangle = \frac{1}{2}\tilde{\theta}_j G_j \frac{n_j m_j T_j}{e_j B_\phi}\frac{\upsilon_{\phi j}}{\bar{R}} \equiv R n_j m_j \nu_{dj} \upsilon_{\phi j} \tag{20}$$

where

$$\tilde{\theta}_j \equiv \left(4 + \tilde{n}_j^c\right)\tilde{\upsilon}_{\phi j}^s + \tilde{n}_j^s\left(1 - \tilde{\upsilon}_{\phi j}^c\right) \tag{21}$$

represents poloidal asymmetries [the tilde quantities are the sine (s) and cosine (c) components of the variation over the flux surface of the respective quantities) and

$$G_J \equiv -\frac{r}{\eta_{4j}\upsilon_{\phi j}}\frac{\partial\left(\eta_{4j}\upsilon_{\phi j}\right)}{\partial r} \tag{22}$$

with the gyroviscosity coefficient $\eta_{4j} \approx n_j m_j T_j / e_j B$. The poloidal asymmetries in density needed for the evaluation of Eq. (21) can be calculated from low-order Fourier moments of the third independent (poloidal) component of the momentum balance equation[24].

We note that is has been suggested[25] that the above expression for the gyrovicous toroidal force underestimates the momentum transport rate in regions of steep pressure gradients and low toroidal rotation (e.g. the edge pedestal) because of failure to take into account a drift kinetic correction not present in the original Braginskii derivation. More recent work[26] indicates that the Braginskii derivation is correct when the toroidal flow is of the same order as the thermal velocity, but that when the toroidal flow is much less than the thermal velocity (i.e. in the 'drift' ordering) then an additional heat flux term should appear in the viscosity tensor. It is not clear *a priori* which ordering is more appropriate for the plasma edge. In any case, the above equations have done well in predicting radial momentum transport in the DIII-D core plasma[27], which motivates us to investigate their predictions in the edge pedestal.

D. Evaluation of experimental momentum transfer frequencies

The experimental momentum transfer frequencies were evaluated from Eq. (19) using measured and calculated quantities, as discussed previously. These experimental ν_d^* are shown in Table 5. The atomic momentum transfer frequencies due to charge-exchange, elastic scattering and ionization were calculated and also are shown in the table. The large poloidal asymmetry in the neutral fueling through the x-point region was represented in the calculation by using $\xi = 1$ for the fueling asymmetry factor of Eq. (15). The frequency of momentum convection was also evaluated from the calculated radial particle flux and the measured toroidal velocity in the pedestal. It is clear that the momentum transfer frequencies due to atomic physics and convection are too small by an order of magnitude to account for the observed experimental momentum transfer frequency. On the other hand, the neoclassical gyroviscous momentum transport frequencies evaluated from Eq. (20) are in reasonable agreement with the experimental frequencies. In general, the neoclassical momentum transfer frequencies are somewhat less than the experimental frequencies, perhaps indicating the presence also of an 'anomalous' momentum transport mechanism.

IV. DENSITY GRADIENT SCALE LENGTH

A. Theoretical expression

In order to gain theoretical insight, as well as to obtain an expression for calculating the density gradient scale length, we use Eq. (12) to eliminate v_φ only from the term on the right side in Eq. (11). This leads immediately to an expression for the density gradient scale length of ion species 'j'

$$L_{nj}^{-1} = \frac{e_j B_\theta}{n_j m_j \nu_{dj}^* T_j}\left[e_j B_\theta \Gamma_j + M_{\phi j} + n_j e_j E_\phi^A - n_j m_j \nu_{jk}\left(\upsilon_{\phi j} - \upsilon_{\phi k}\right) - n_j m_j \nu_{je}\left(\upsilon_{\phi j} - \upsilon_{\phi e}\right)\right] - \frac{e_j B_\theta}{T_j}\left(f_p^{-1}\upsilon_{\theta j} + \frac{E_r}{B_\theta}\right) - L_{Tj}^{-1} \tag{23}$$

We will find that the momentum input (M_φ) and toroidal electric field (E_φ) are negligible and would expect that the friction term can also be neglected. The import of Eq. (23) is then that the pressure gradient scale length must be consistent with the particle flux (Γ) and momentum transfer rate (ν_d*), with the poloidal rotation, and with the radial electric field, which latter is related to both poloidal and toroidal rotation velocities and the pressure gradient. Since the temperature gradient scale length is determined by heat transport (i.e. Eqs. [2] and [3]), this momentum balance constraint on the pressure gradient can be considered a constraint on the density gradient scale length.

B. Application to experiment

Since the electron (j-e) friction term can be neglected relative to the ion (j-k) friction terms in plasmas with realistic impurity concentrations, all quantities on the right in Eq. (23) could be determined if E_r could be measured directly and if the rotation velocities and radial particle fluxes could be measured separately for each ion species, which would allow experimental density gradient scale lengths to be determined for the various ion species from Eq. (23). In fact, only carbon rotation velocities are usually measured, the ion particle flux is difficult to determine for the main ion species and can only be estimated for impurity ions, and the 'experimental' E_r is usually calculated from Eq. (12) using measured carbon rotation velocities and electron pressure gradients. Until this situation improves, the best use we can make of Eq. (23) is for a consistency check on the various measurements or theories or combinations thereof for the quantities on the right side.

As an example, we evaluate an average value of the gradient scale length from Eq. (23) for the main ion species as follows. The momentum transfer frequency, ν_{di}*, is calculated from the neoclassical gyroviscous expression plus the atomic physics and convective momentum transfer frequencies. The radial particle flux is determined from particle balance, as discussed previously, and the neutral beam momentum input in the pedestal is calculated directly. The friction terms involving the difference in ion and impurity and in ion and electron toroidal velocities are assumed to be negligible. The E_φ^A term and the temperature gradient scale length term are evaluated from experimental data. The poloidal velocity is calculated by solving coupled Fourier moments of the poloidal momentum balance equation for the poloidal velocities of the ions and impurities and for the sine and cosine components of the ion and impurity density asymmetries which are needed to evaluate the poloidal asymmetry factor of Eq. (21); this calculation is described in detail in Ref. 24.

The radial electric field is calculated by summing the toroidal components of the momentum balance equation for the ions and impurities, and using the toroidal component of the electron momentum balance, to obtain

$$\frac{E_r}{B_\theta}=\frac{\left\{M_{\phi i}+M_{\phi I}\right\}+n_i m_i \nu_{di}^*\left(P_i'-f_p^{-1}\upsilon_{\theta i}\right)+n_I m_I \nu_{dI}^*\left(P_I'+f_p^{-1}\upsilon_{\theta I}\right)}{n_i m_i \nu_{di}^*+n_I m_I \nu_{dI}^*} \tag{24}$$

This expression is evaluated using the calculated values of the $M_{\varphi j}$ and the theoretical values of ν_{dj}*and $\upsilon_{\theta j}$ just discussed, together with the experimental value of P_i' and the assumption that the pressure gradient scale length is the same for the impurity as for the main ions.

The calculated values of the deuterium (i) and carbon impurity (I) poloidal velocities and of the radial electric field are compared with the measured values of the carbon toroidal and poloidal rotation velocities and with the 'experimental' value of E_r in Table 6. The 'calculated' E_r is evaluated from Eq. (24) using the calculated values of $v_{\theta i}$ and $v_{\theta I}$, the theoretical ν_{di} of Eq. (20), the calculated values of ν_{at} and ν_{ion}, the calculated values of $M_{\varphi i}$ and $M_{\varphi I}$, and the experimental values of P'_I and P'_i. The 'experimental' E_r is evaluated from the force balance Eq. (12) using the measured values of the carbon rotation velocities and pressure gradient. The uncertainty in the measured υ_θ's is probably a few km/s, and this introduces a significant uncertainty in the experimental E_r.

Using the calculated values given in Table 6 and determining the other parameters in Eq. (23) as discussed above, the main ion density gradient scale lengths, L_{ni} were calculated, This quantity is compared with the measured value of L_{ne} determined by Thomson scattering in Table 7. We note that the use of the experimental L_{ni} and L_{Ti} to evaluate the P'_i term in the above expression for E_r, which is then used to evaluate L_{ni} from Eq. (24), is somewhat circular. However, this calculation can be used to check the consistency of the measured and calculated quantities because the calculation of E_r depends also on the calculation of the $\upsilon_{\theta j}$ from poloidal momentum balance[24] and on the calculation of the neoclassical momentum transfer frequency from Eq. (20). Furthermore, the calculation of L_{ni} from Eq. (23) also depends directly on the calculation of the $\upsilon_{\theta j}$ from poloidal momentum balance and on the calculation of the neoclassical momentum transfer frequency from Eq. (20), on the calculation of Γ_i from particle balance (including a neutral recycling calculation), on the calculation of the neutral beam momentum deposition in the pedestal ($M_{\varphi j}$) and on the value of E_φ^A, which we take from experiment. The last two terms had a negligible effect on the result. The reasonably good agreement is indicative of 1) the consistency of the experimental measurements with the momentum balances of Eq. (23) and (24) and of 2) the neoclassical calculation models for ν_{dj}[23] and $\nu_{\theta j}$[24] that were used in evaluating these terms in Eqs. (23) and (24).

IV. SUMMARY

Theoretical heat conductivities based on analytical representations of neoclassical and ITG modes for the ions and ETG and TEM modes for the electrons have been compared with measured thermal transport rates. Thermal transport coefficients from the neoclassical, ITG and ETG theories are found to be within at most a factor of 2-3 of values inferred from experiment for most of the discharges considered, with the agreement being significantly better for neoclassical than for ITG ion thermal conductivities. The edge gradients of these discharges are such that ITG and ETG modes are predicted to be unstable. This finding that ETG modes should be unstable in the edge is consistent with previous observation of $\eta_e \approx 2$ in a large number of discharges in ASDEX Upgrade[28]. Furthermore, the results shown in Fig. 16 of Ref. 29 imply that $\eta_e \approx 1.5$ in a large number of DIII-D discharges.

New expressions for a 'diffusive-pinch' form of particle flux, for calculating an experimental frequency for momentum transfer, and for predicting the ion density gradient scale length have been derived from momentum balance. The experimental momentum transfer rates are too large by an order of magnitude to be accounted for by atomic physics and convective momentum transfer, but neoclassical gyroviscous theory predicts frequencies comparable to those found experimentally.

Perhaps the most significant finding of this investigation is that neoclassical theory appears to provide a reasonable representation of ion transport in the edge pedestal. The neoclassical predictions of both ion thermal conductivity and ion momentum transfer frequency were within a factor of 2-3 or better of the experimental values, and the use of neoclassical momentum transfer frequencies in the calculation of density gradient scale lengths results in a prediction that is within a factor of 2 of the directly measured value.

Appendix: Coefficients for Chang-Hinton Formula

The coefficients for the Chang-Hinton expression for ion thermal conductivity given by Eq. (6) are[18]

$$
\begin{aligned}
a_1 &= \frac{0.66\left(1+154\alpha\right)+\left(1.88\sqrt{\varepsilon}-1.54\varepsilon\right)\left(1+3.75\alpha\right)}{1+1.03\sqrt{\mu_j^*}+0.31\mu_j^*} \\
a_2 &= \frac{0.59\mu_j^*\varepsilon}{1+0.74\mu_j^*\varepsilon^{3/2}}\left[1+\frac{1.33\alpha\left(1+0.60\alpha\right)}{1+1.79\alpha}\right] \\
g_1 &= \frac{1+\frac{3}{2}\left(\varepsilon^2+\varepsilon\Delta'\right)+\frac{3}{8}\varepsilon^3\Delta'}{1+\frac{1}{2}\varepsilon\Delta'} \\
g_2 &= \frac{\sqrt{1-\varepsilon^2}\left(1+\frac{\varepsilon\Delta'}{2}\right)}{1+\frac{\Delta'}{\varepsilon}\left(\sqrt{1-\varepsilon^2}-1\right)}
\end{aligned}
\tag{A1}
$$

$\alpha = n_I Z_I^2/n_i Z_i^2$, $\mu_i^* = \nu_{iI} qR/\varepsilon^{3/2}\upsilon_{thi}$ and $\Delta' = d\Delta/dr$, where Δ is the Shafranov shift. The impurity thermal conductivity is obtained by interchanging the i and I subscripts, both in Eq. (6) and in the above expressions.

The Shafranov shift parameter may be evaluated from[21]

$$
\Delta' \equiv \frac{d\Delta}{dr} = -\frac{1}{RB_\theta^2}\left(\frac{r^3}{a^2}\beta_\theta B_{\theta a}^2+\frac{1}{r}\int_o^r B_\theta^2 r' dr'\right) \tag{A2}
$$

where $\beta_\theta = p/(B_\theta^2/2\mu_0)$ and $B_{\theta a}$ denotes the poloidal magnetic field evaluated at $r = a$. Since we need this quantity at $r = a$, we can take advantage of the definition of the internal inductance

$$l_i = \frac{2\int_o^a B_\theta^2 r' dr'}{a^2 B_{\theta a}^2} \tag{A3}$$

where $\beta_{\theta a}$ denotes the quantity evaluated using the average pressure over the plasma and $B_{\theta a}$. Using a parabola-to-a-power current profile $j(r) = j_0(1 - (r^2/a^2))^v$, for which the ratio of the values of the safety factor at the edge to the center is $q_a/q_0 = v + 1$, and a fit[21]

$l_i = ln(1.65 + 0.89v)$ leads to the simple expression

$$\begin{aligned} \Delta' &= -\frac{a}{R}\left(\bar{\beta}_{\theta a} + \frac{1}{2} l_i\right) \\ &= -\frac{a}{R}\left(\bar{\beta}_{\theta a} + \frac{1}{2} ln\left(1.65 + 0.89\left(\frac{q_a}{q_o} - 1\right)\right)\right) \end{aligned} \tag{A4}$$

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Table 1: Operating parameters (R=1.74-1.78m, a = 0.60-0.62m)

Shot	*Time(ms)*	*I(MA)*	*B(T)*	q_{95}	δ	κ	P_{nb}*(MW)*
93045	3700	1.6	2.1	4.1	.41	1.84	5.1
87085	1620	1.2	1.6	5.5	.86	2.04	7.5
97979	3250	1.4	1.6	3.9	.75	1.75	6.5
106005	3000	1.2	1.5	3.1	.14	1.78	5.0
106012	3000	1.2	2.1	4.2	.13	1.78	5.0
92976	3210	1.0	2.1	5.7	.33	1.79	5.0
98893	4000	1.2	1.6	4.2	.14	1.77	2.1

Table 2: Edge pedestal parameters

Shot	n_e^{ped} $(10^{19}/m^3)$	T_e^{ped} *(eV)*	Δ_n *(cm)*	Δ_{Te} *(cm)*	L_n *(cm)*	L_{Te} *(cm)*	L_{Ti} *(cm)*	f_{carbon} *(%)*
93045	4.0	1150	5.1	5.5	2.8	2.2	4.7	4.1
87085	2.8	685	8.1	10.2	4.3	4.5	8.5	5.5
97979	6.3	525	3.5	5.0	3.3	2.6	6.2	1.1
106005	4.6	460	4.6	4.6	2.7	2.1	5.3	1.8
106012	4.6	395	4.4	5.9	2.4	2.0	10.3	2.0
92976	4.9	215	3.6	7.2	6.0	4.2	10.3	1.8
98893	8.3	120	2.2	2.2	1.5	1.5	10.1	0.8

Table 3: Experimental and theoretical thermal conductivities (m^2/s)

Shot	ν_e*	L_{Ti}/L_{Ti}^{crit}	η_e	$\chi_i^{exp,a}$	χ_i^{NEO}	χ_i^{ITG}	$\chi_e^{exp,a}$	χ_e^{TEM}	χ_e^{ETG}
93045	0.10	0.27	1.03	0.20	0.31(0.67[b])	3.7	0.17	>100	2.4
87085	0.28	0.49	0.96	1.1	0.58(0.93)	2.5	1.4	52	3.6
97979	0.40	0.36	1.27	0.80	0.49(0.54)	1.7	0.48	44	1.6
106005	0.30	0.31	1.29	1.1	0.62(0.76)	1.7	0.57	62	2.8
106012	0.62	0.60	1.20	1.6	0.51(0.67)	0.59	0.73	21	1.5
92976	1.53	0.60	1.43	1.5	0.53(0.84)	0.37	1.3	1.6	1.4
98893	4.86	0.59	1.00	1.0	0.62(0.68)	0.20	0.34	1.7	0.55

[a] Experimental values evaluated assuming $Q_i = Q_e$.

[b] Without orbit squeezing correction.

Table 4: Average conductive heat fluxes (10^5 W/m^2) in pedestal

Shot	***Exp.***	χ_i***=NEO,*** χ_e***=ETG***	χ_i***=ITG,*** χ_e***=ETG***	χ_i***=NEO,*** χ_e***=TEM***	χ_i***=ITG,*** χ_e***=TEM***
93045	0.38	2.7(3.0[a])	5.5	> 100	> 100
87085	0.51	0.87(.94)	1.2	> 10	> 10
97979	0.69	1.4(1.4)	1.9	> 10	> 10
106005	0.60	1.6(1.7)	1.9	>10	>10
106012	0.61	0.75(.78)	0.76	9.5(9.5[a])	9.5
92976	0.48	0.36(.40)	0.34	0.41(.46)	0.39
98893	0.25	0.29(.29)	0.24	0.74(.75)	0.70

[a] w/o orbit squeezing.

Table 5: Momentum transfer frequencies

Shot	ν_d* (10^3 s^{-1})			
	Exp.	**Atomic**	**Convect**	**Neo**
93045	5.0	.49	.36	4.9
87085	1.0	.20	.07	2.6
97979	1.6	.13	.08	1.1
106005	2.7	.11	.06	1.3
106012	2.2	.12	.06	1.5
92976	4.5	.16	.11	1.6
98893	2.4	.13	.10	.81

Table 6: Rotation velocities and radial electric fields

Shot	$v_{\varphi I}^{exp}$ *(km/s)*	$v_{\theta i}^{\ a}$ *(km/s)*	$v_{\theta I}^{\ a}$ *(km/s)*	$v_{\theta I}^{exp}$ *(km/s)*	$E_r^{\ b}$ *(kV/m)*	$E_r^{exp\ c}$ *(kV/m)*
93045	5.9	-4.8	-0.2	-1.3	-58	-42
87085	55	3.8	-12	9	-19	-15
97979	17	-1.3	-1.7	3.5	-19	-13
106005	13	-2.3	-1.1	-1.8	-21	-2
106012	17	-2.8	-1.0	-0.3	-27	-7
92976	8.5	-1.5	-1.9	-0.8	-10	-13
98893	13	-1.4	3.5	2.6	-12	-2

[a] calculated from poloidal momentum balance, Ref. 24; [b] calculated from Eq. (24) using calculated velocities and ν_{di}* and experimental electron pressure gradients ; [c] calculated from force balance using measured carbon velocities and pressure gradients.

Table 7: Density gradient scale lengths

Shot	*93045*	*87085*	*97979*	*106005*	*106012*	*92976*	*98893*
Exp. L_{ne}	2.8	4.3	3.3	2.7	2.4	6.0	1.5
Calc. L_{ni}	2.7	3.3	2.4	1.9	1.8	3.3	0.8