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Design Studies of a Robotic Manipulator for Ultrasonic Inspection

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SECTION 1

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1 Introduction and Summary

The mechanical design parameters of a system such as the automated ultrasonic inspection system are related in complex ways to each other and to the performance of the system. Described herein are studies of the structural and actuator parameters as they relate to the rigidity of the manipulator of such a system. The configuration studied is shown schematically in Figure 1. The relationship between structural and motor parameters are sumarized as follows: As the structural mass is increased to increase rigidity the motor size must increase to drive the increased inertia. Increased motor mass in turn increases the need for a rigid structure to maintain high vibrational frequencies. Thus a viscious cycle can be set up resulting in a massive and expensive mechanical system.

The rigidity of the manipulator is most critical in a dynamic sense. The number which summarizes this rigidity is the lowest structural natural frequency. For relatively simple feedback control algorithms of the type almost always used, this frequency must be <u>at least</u> twice the bandwith of the closed loop manipulator. A prediction of this frequency can be obtained by various means such as transfer techniques or finite element techniques. The former was used in this study.

In order to study this design problem, the relationship between motor torque and motor mass must be known. Manufacturer's data for d.c. pancake torque motors of the type studied were obtained. The optimal control of such motors was used to predict the minimum time to move a given inertia a given distance. These results were incorporated into an empirical power law predicting the performance of each motor from quite simple calculations.

The design problem then posed as being most relevant and tractable was the following: Given a required motion time at a standard distance, what are the structural and actuator parameters that maximize the lowest structural natural frequency? To answer this question in a general way, a rather extensive computer program was devised and written to choose design parameters which maximized natural frequency subject to the given constraint on motion time. The results obtained were suprising as described below.

For the configuration proposed (see Figure 1) the motor mass has negligible effect on the lowest natural frequency. This effectively decouples the two design decisions of structural parameters and actuator parameters <u>for this configuration</u>. This greatly simplifies the design process. One must first select the structural parameters. This determines the load inertia to be used in selecting the actuator.

The insensitivity of the lowest natural frequency to motor mass is seen in Figure 2. In fact, the first two natural frequencies are essentially invariant while the third and fourth natural frequencies exhibit the qualitative k/m behavior initially expected of the lowest frequencies. For high enough motor mass these modes will become dominant and motor mass will affect lowest natural frequency. This occurs for parameter values out of the range of interest.

This phenomena is easily understood by looking at mode shapes as sketched in Figure 3. The first two modes involve almost no motion of the motor mass. The first two modes involve almost no motion of the motor mass. The third and fourth modes involve substantial motor movement, hence the sensitivity to motor mass.

Unfortunately the effects of this behavior were not appreciated until much effort had been spent on the coupled optimization problem. It was in an attempt to explain the strange results from these efforts that this effect was finally discovered. The importance of this behavior should be realized by anyone attempting the detailed design of the manipulator.

A simplified design proceedure was performed as follows: For a specified total structural mass, what is the distribution of structural mass which maximizes the first natural frequency. Circular tubes of constant cross section were assumed for both the vertical and horizontal members with inner radius equal to 0.9 times the outer radius. Aluminium and T-300 graphite composite were considered and a motor mass of 70 lbs. was used (though not important).The optimal radii are shown in Figure 4. The resulting frequencies are shown in Figure 5.

The details of these studies are described in later sections.

2 Sensitivity Studies and Simplified Optimization

The work described herein was actually the last undertaken. It was initiated to explain the strange results of the more extensive optimization studies described later. It is placed first since the conclusions are a result of this last work which is largely unrelated to the other work.

2.1 The Effects of Manipulator Compliance

Rigidity of the manipulator affects performance in several ways. Static deflection is the obvious way, but not the most important. Other ways are deflection due to dynamic external disturbances and the interaction of the structure with the control system dynamics. The last of these is the most critical since it can cause system instability. We will consider the first two briefly to demonstrate that they are of minor significance.

Disturbances on the manipulator come primarily from the water through which it must move. These disturbances are both "static" and dynamic. The "static" disturbance is a constant drag force at constant manipulator velocity. The drag forces on a cylinder moving in the water can be closely approximated by the equation[1]

$$F_d = C_d(\rho U^2 A)$$

where:

C_d = drag coefficient dependent on the Reynolds number
ρ = density of the fluid
U = velocity of the cylinder
A = cross sectional area to direction of flow

A quick analysis shows that the resulting drag forces are quite small. For a velocity of 20 in/sec and cylinder diameter of the order of 1 in., C_d is approximately one. For these numerical values the drag force is predicted to be .74 lb. For larger diameters of this order the force is proportional to the diameter. Thus for the diameters resulting from the design study (typically 7 to 8 inches) one could expect the modest force of about 6 lb.

The motion through the water can also produce time varying disturbances through the creation of vortices. The shedding frequency of these vortices is the primary frequency of excitation of the disturbance. If this frequency were in the neighborhood of the lowest structural frequency then significant amplitudes of vibration could build up even though the amplitude of the disturbing force was quite small. The predominant frequency of vortex shedding is

$$f_s = S\,U/D$$

where S is the Strouhal number which depends on cross section shape and the Reynolds number. For a large range of Reynolds numbers in the area of interest the Strouhal number is 0.2. Thus with D = 1in., f_s = 4 rad/sec. The slightly larger diameters of interest result in even lower frequencies, far below the structural or control natural frequencies.

Consequently, the primary constraint on rigidity of interest is the effect on the control system

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[1] R.D. Blevins, Flow Induced Vibrations, Van Nostrand, Rheinhold, 1977.

dynamics. The remainder of the report focuses on that effect. It has been shown by Book[2] that for the common position and velocity feedback control the limitation on rigidity is predicted by the natural frequency of the manipulator with the joints clamped. This natural frequency should be at least twice the bandwidth desired from the closed loop system.

2.2 Optimal Structural Parameters for Constant Motor Mass

If one assumes that motor size has an insignificant effect on the structural parameters, one can optimize these parameters with only a rough approximation of what the motor mass will eventually be. This insensitivity can then subsequently be verified.

One approach is to specify the desired rigidity in terms of the natural frequency desired and solve one optimization problem, say to minimize the structural mass. Unfortunately, the required natural frequency is not a firm specification, but some latitude in the choice is permissible. Also, the solution proceedure makes it easier to find the natural frequency given the structural parameters than to constrain the frequency and find the structural parameters. Consequently, a family of solutions was found. The total structural mass was specified and then the distribution between the two major components was found that maximized the lowest natural frequency.

2.2.1 Design Rules Used in the Optimizaton

In order to carry out the optimization described above, certain rules must be formulated which direct the incorporation of the structural mass. It was decided, for example, that the horizontal and vertical members should be of constant cross section. This is the most practical choice for eventual fabrication. Furthermore it was decided to use tubular members of circular cross section, since they are structurally efficient and generally availlable. The relationship between inner and outer radius must then be established. From the viewpoint of rigidity alone, very thin wall tubes are most effective. However, such tubes are subject to local buckling and also have very large diameters which would obstruct operation. A rather arbitrary choice was made to have the inner radius be a constant fraction of the outer radius. The ratio of 0.9 was used in the results presented here. A detailed design would need to reevaluate this decision in the light of additional information and standard sizes.

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[2] W.J. Book, "Modelling, Design, and Control of Flexible Manipulators," Ph.D. Thesis, Massachusetts Institute of Technology, l974

2.2.2 Justification of Model Studied

A decision on the complexity of the configuration to be studied was made in favor of simplicity. A more complicated model than that shown in Figure 1 at this stage would only create a false sense of security in the results. Conspicuously absent from this model are compliances at the joint between the horizontal and vertical members and at the ends of the horizontal member. Representative numbers could be obtained from a preliminary design but time was not available due to other complications. A rough analysis of the constraints at the ends of the horizontal members indicates that these effects are secondary but significant. This analysis follows.

Since the lowest natural frequency involves primarily twisting of the horizontal member, it is most important to consider the torsional constraints on the end of the member. A hypothetical form of that constraint might be as shown in Figure 6. Assuming the cross section of the constraining member to be identical to the horizontal member, one can find a relationship between the rotational compliances relevant to the motion in the first mode.

The spring constant k_e of the end constraint is found from simple beam theory to be

$$k_e = 8\,EI/L_e$$

while the spring constant of the horizontal member is found to be

$$k_h = 4\,GJ/L_h.$$

For a circular cross section $J = 2I$, and for a material such as Aluminium $E = 2.7\,G$. Thus to close approximation

$$k_h = 3\,EI/L_h$$

and

$$k_e/k_h = (8L_h)/(3L_e)$$

For typical values of $L_e = 12$ in and $L_h = 72$ in one expects a total spring constant of 17/16 of the value without the end constraint consideration and consequently a decrease of the lowest natural frequency to SQRT(17/16) of its previous value--a change of less than four percent.

2.2.3 Materials Considered

Two materials were considered: Aluminum, and a graphite composite designated T-300. Standard density of .1 lbm/in^3, Young's modulus of $1x10^7$ lbf/in^2, and shear modulus of $3.7x10^6$ lbf/in^2 were used for aluminum. Values for the T-300 composite were provided by Bonner Staff of Lockheed. They are: density = .056 lbm/in^3, Young's modulus of $1.8x10^7$ lbf/in^2, and shear modulus of $3.2x10^6$ lbf/in^2.

2.2.4 Results

The results have already been discussed briefly in the Introduction and Summary. Typical results are displayed in Figures 4 and 5. Here the motor mass of 70 lb was used. A range of values of motor mass was used with no noticeable change in the results. The transducer mass at the end of the vertical member was assumed to be 5 lbs. This assumption will have considerable effect on the results and any significant increase over that value this analysis should be repeated. Notice that the use of the graphite composite approximately doubles the natural frequency and greatly increased the size of the members. Since adequate values of natural frequency can be obtained with Aluminum, however, it is questionable if the added expense and complication of fabrication would be justified.

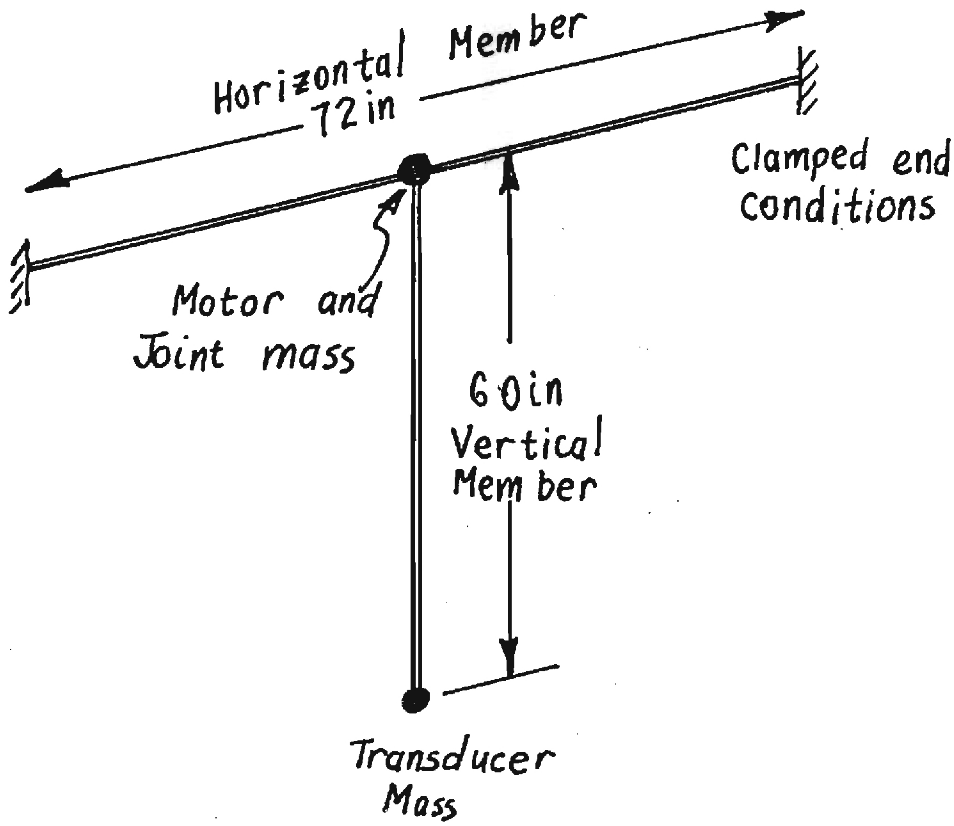
The choice of natural frequency and consequently structural mass remains an open question. Studies of this choice were being performed by a student who left prior to their completion. Typical servo bandwidths of industrial manipulators are 10 Hz and lower. A conservative value of natural frequency would be four time this, or 40 Hz. The resulting structural mass would be a total of 100 lb for an aluminum manipulator. Motors can be readily sized from this value and the radii presented in Figure 4.

It is prudent at this point to check the assumption of negligible static deflection. Assuming the optimal 100 lb structural mass design the vertical member has a diameter of 8 in. The drag force at 20 in/sec is approximately 6 lb. Assuming a worst case with all drag force at the transducer, a deflection of $.0001F_d$ in or .0006 in occurs from the complete structure. Thus the assumption is justified.

2.3 Sensitivity of First Natural Frequency to Motor Mass

For the sensitivity studies a nominal configuration was assumed and motor mass was varied. The nominal configuration was the optimal design for the 70 lb. motor mass using Aluminum as the material. The first four natural frequencies were found using transfer matrix techniques. The results are shown in Figure 2 for a range of motor masses from 70 to 1000 lbs. As explained in the Introduction and Summary the first two natural frequencies are invariant while the frequencies three and four vary considerably. If the vertical member were to be moved from its center position one could expect a greater sensitivity of mode 2 since the motor would not now be located at a node of the modal shape. A higher absolute value of the frequency results due to the greater rigidity of the horizontal member.

Figure 1: Configuration of manipulator studied.



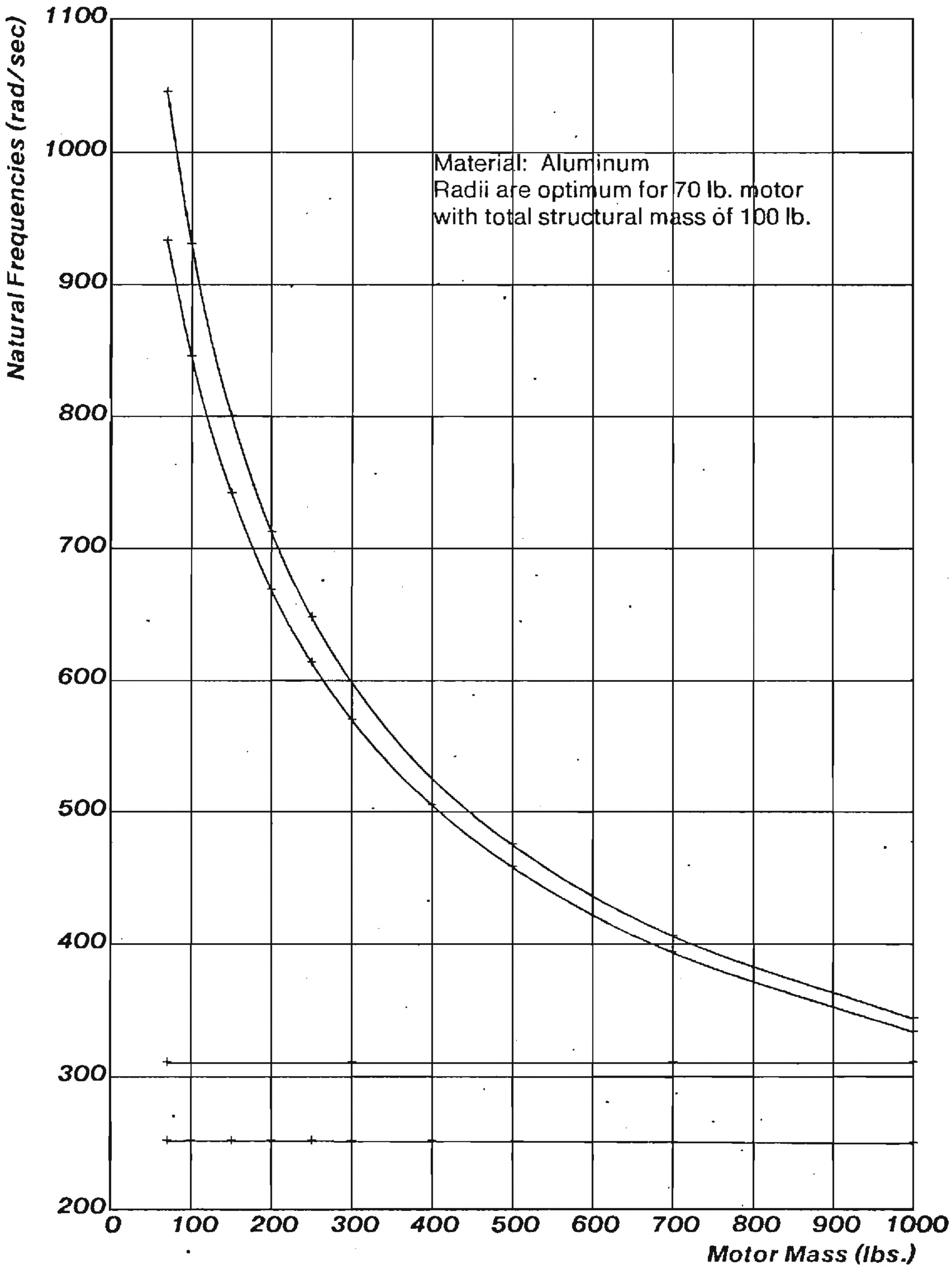


Figure 2. Variation of the first four natural frequencies.

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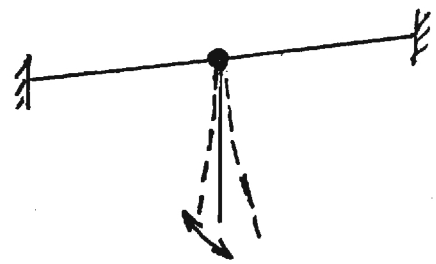
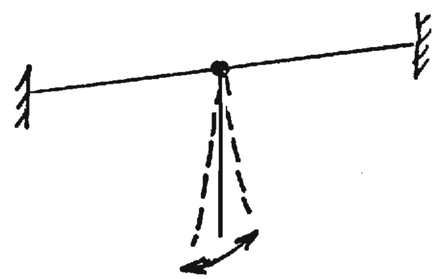
First Mode: Vertical beam moves out of plane of beams.

Horizontal beam primarily twists.

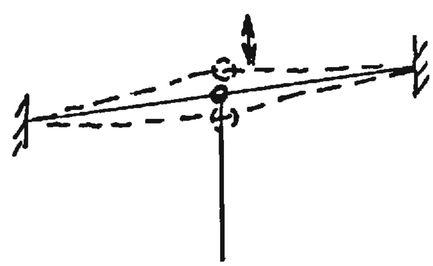


Second Mode: Vertical beam moves in the plane of the beams.

Horizontal beam bends slightly.



Third Mode: Motor moves in the plane of the beams.



Fourth Mode: Motor moves out of the plane of the beams.

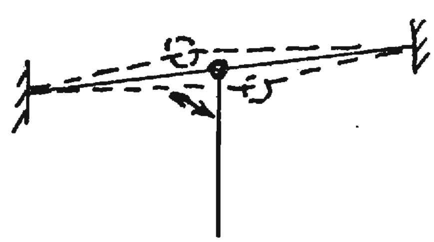


Figure 3: Shape of the first four modes.

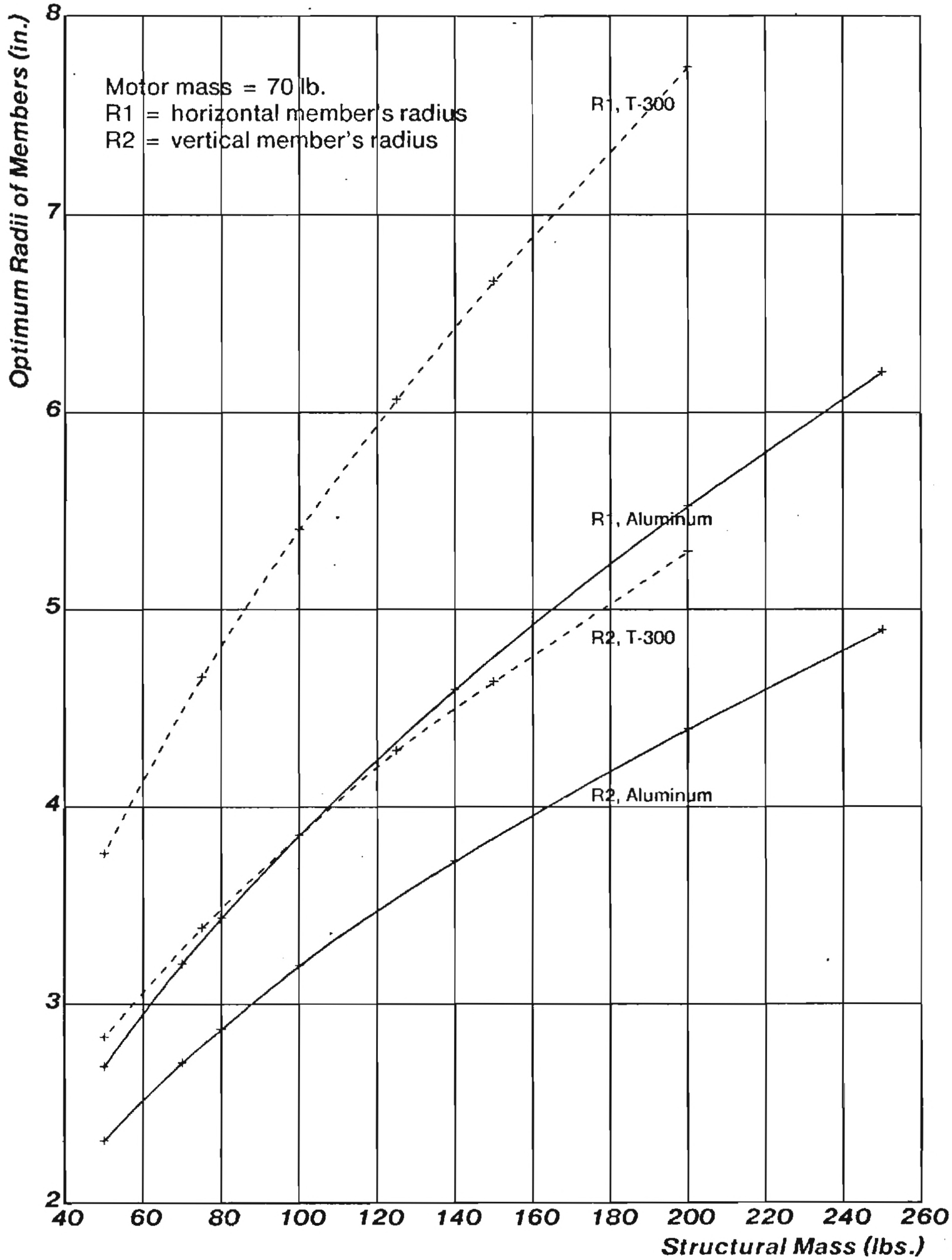


Figure 4. Optimum radii for members.

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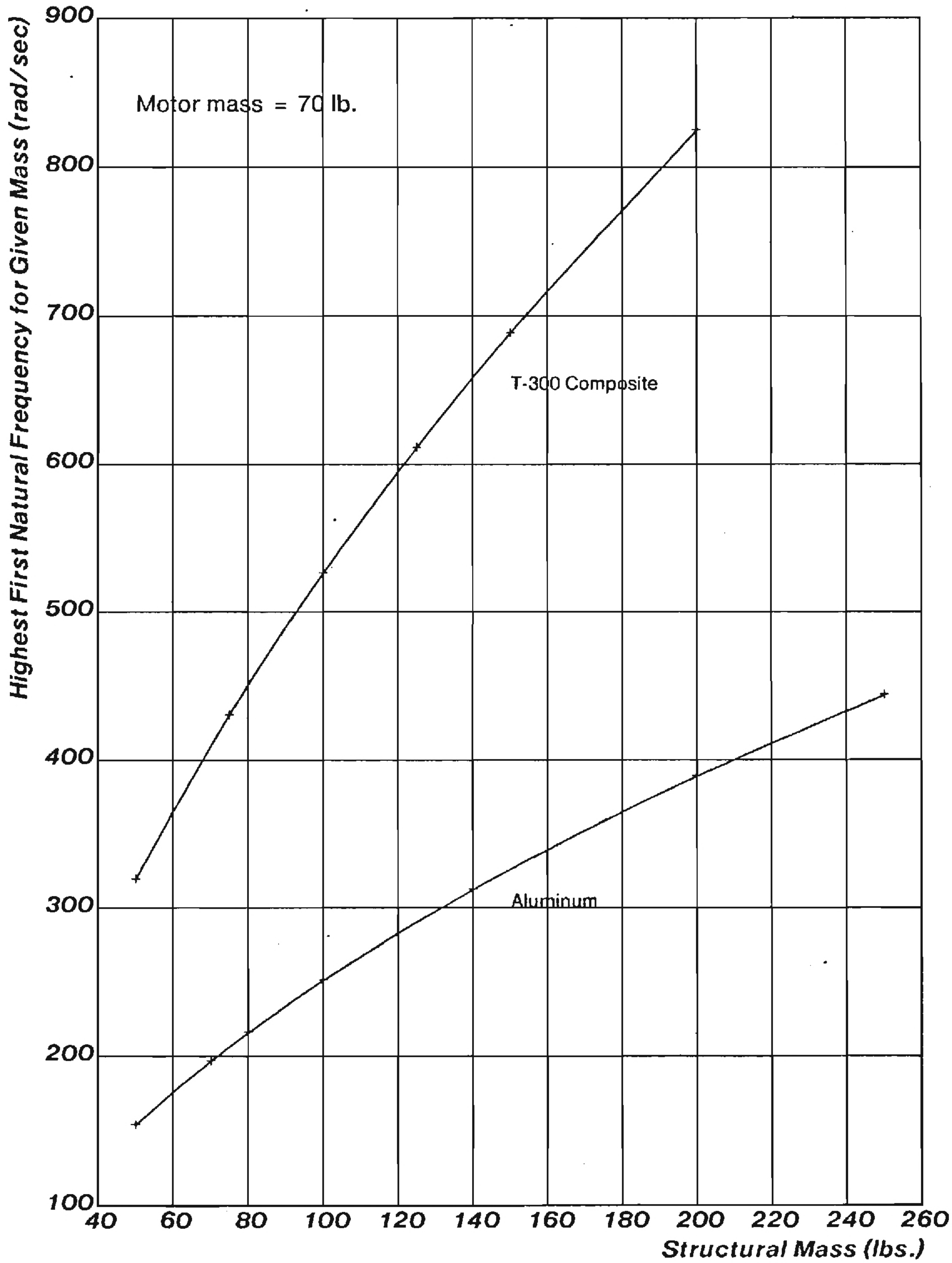

Figure 5. Maximum first natural frequency.

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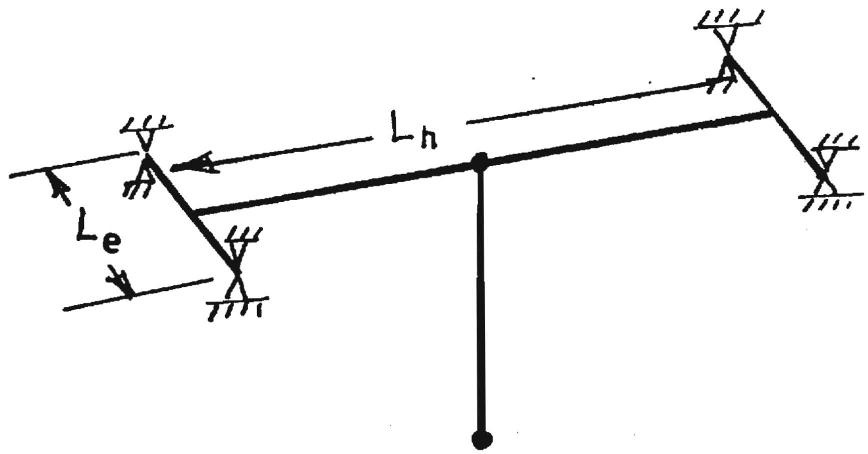
Figure 6: Analysis of clamped end assumption



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SECTION 2

1. Design Parameter of Interest

Design of a complex mechanism involves many choices, not all of which can be reduced to solving an equation. In this section we state the parameters to be determined by this study and other design decisions which are assumed given or are beyond the scope of this study.

The arrangement and type of axes assumed is shown in Figure(12). Three linear motions are of primary interest. Two additional motions, rotations at the distal end of the manipulator, have minimal effect on the decisions of interest and are ignored here. Vertical motion of link 1 is powered by motor 1. The size of the motor, the link cross section, and the speed reduction or effective drive radius are to be determined for each of the three motions. Motor 1 and link 1 ride on link 2. Two configurations are under consideration. Shown in Figure(1) is the configuration in which motor 2 rides on link 2. Also being considered is motor 2 mounted on link 1. To be determined are the size of the motor, the link cross section, the speed reduction, and for the second configuration, the cross section of the drive belt. Motor 3 rides on link 3 and both are to be sized together with the speed reduction in the drive. The linear motions lend themselves to operation in the long ultrasonic immersion tank. The tank length (30.5m) dicates that motor 3 ride with link 3 rather than be mounted in a stationary arrangement.

Selection of a motor specifies motor weight, rotor inertia, and its acceleration and speed characteristics. Within a family of motors only one design decision specifies all three parameters. For the design study permanent magnet direct current motors are considered. relationship between motor parameters has been determined empirically from manufacturer's data as shown in figures 2 and 3. Knowing this approximate relationship we specify a motor in terms of its mass, although any of several other parameters could be used. Motor inertia follows from this specification

Time optimal control of permanent magnet d.c. motors results in movement times dependent on the effective load inertia, the distance traveled and the motor parameters. Time optimal control with voltage limits as solved by Szabados (3,4) was applied to the same motors used in Figure (2) and (3).

2. Feasible Constraints

The designer's delima is that by increasing rigidity he increases mass and movement time. Rigidity is characterized here by lowest natural frequency w_c and end point stiffness k_e. End point stiffness constriants are directly related to accuracy of end point lacation with constant gravitational and/or drag loads. Natural frequency is related in a more complex manner to the dynamic performance. Movement time or its inverse Ω (which has the same units as w_c) is related to performance in a complex dynamic fashion also. We discuss here the determination of feasible values for w and Ω, and the least costly way to provide a feasible value.

To determine the limits of feasibility we propose the following procedure.

Maximize the performance index L

$$L = w_c$$

or

$$\text{Minimize } L = -w_c$$

subject to the equality constraints on = 1/(movement time) which is required to be the same for each of the three axes. Ω for axis i depends on its total effective load inertia, JT_i, which in turn depends on the link masses ml_i and motor masses mm_i, the rotor inertias JT and the effective drive radii r . See Figure(1) for a complete description of the terms in each J . Having empirically related motor mass and inertia we can eliminate J and write

$$\Omega_1(r_1, m\ell_1, mm_1) - \Omega = f_1(r_1, m\ell_1, mm_1) = 0$$

where

$m\ell_1$ = the mass of link 1

mm_1 = the mass of motor 1

r = the effective drive radius of axis i.

Similarly constraints on Ω_2 and Ω_3 can be formally written as

$$f_2(r_2, m\ell_1, m\ell_2, mm_1, mm_2) = 0$$

and

$$f_3(r_3, m\ell_1, m\ell_2, m\ell_3, mm_1, mm_2, mm_3) = 0$$

We adjoin the constraint equations to the performance index by way of Lagrange multipliers λ_i to define

$$H = w_c+\lambda_1 f_1+\lambda_2 f_2+\lambda_3 f_3 \quad = \quad L'(x,u)+\lambda^T f(x,u)$$

We arbitrarily designate six variables of the problem as decision variables variables and form the decision vector u

$$u = (m\ell_1, m\ell_2, m\ell_3, r_1, r_2, r_3)$$

The three remaining variables are designated state variables and form the state vector x

$$x = (mm_1, mm_2, mm_3)$$

To solve this extremization problem we propose the Fletcher -Reeves conjugate gradient algorithm (ref 5) in a modified form

For conjugate gradient we should set the equation

$$x = f(m\ell_1, m\ell_2, m\ell_3, mm_1, mm_2, mm_3, r_1, r_2, r_3)$$

The conjugate gradient algorithm is as follows

a) Select initial values for x

b) Determine X_4, X_5, X_6 from $f_1(x,y), f_2(x,y)$ and $f_3(x,y) = 0$

c) Approximate $\partial L/\partial \underline{X} \approx \Delta L/\Delta \underline{X}$ $\quad (\underline{\nabla} L(\underline{X}))$

(1) d) Let $\underline{X}^o$ denote the first approximation to X

compute $\nabla L(\underline{X}^o)$

and $\underline{V}^o = -\nabla L(\underline{X}^o)$

(2) For i=1,2,...,n-1

e) Set $\underline{X}^i = \underline{X}^{i-1} + \underline{\lambda_{i-1}} \underline{V}^{i-1}$, where λ_{i-1} minimize $L(\underline{X}^{i-1}, \underline{\lambda V}^{i-1})$

with respect to $\underline{\lambda}$ ($\underline{\lambda}$ are the step sizes)

f) Compute $\underline{\nabla} L(\underline{X}^i)$

g) when $i<n$,define

$$\underline{V}^i = -\underline{\nabla}L(\underline{X}^i) + \frac{||\nabla L(X^i)||^2}{||\nabla L(X^{i-1})||^2} \underline{V}^{i-1}$$

(3) Replace $\underline{X}^o$ by $\underline{X}^n$ and go to (1) unless the stopping rule is satisfied.

(Note: In the gradient method, we move $\underline{X}^i$ to $\underline{X}^{i+1}$ along $V^i = -\underline{\nabla}L(X^i)$

for function minimization. In the conjugate-gradient method,

modify the gradient direction by adding

$$\frac{||\nabla L(X^i)||^2}{||\nabla L(X^{i-1})||^2} \quad V^{i-1}$$

The Cost of Achieving w_c and Ω

Given that values of w and w are feasible, what is the best way to achieve them? the obvious answer is: "in the way that minimizes cost". Less obvious is the way in which to compute cost. Empirical relationships for cost are not known expect for the motors and other relatively minor components. The proposed method of ascribing cost is by mass of the components.

A case of practical interest is when the values of w_c and Ω on the feasibility boundary are used. This assumes no cost differential associated with achieving the highest performance.

3. Minimum-Time Position Control Using a Permanent Magnet dc Motor

The control of the angular position of a shaft has been solved in many ways. Minimum-time position control is an interesting technique. One can apply Pontryagin's minimum principle by varying the developed torque in a Bang-Bang manner, thus requiring the step changes in armature or field current. This is acceptable for the permanent-magnet motor which can accept large step changes in applied voltage without the inclusion of current limiting resistors.

Fig 1. shows how the components are related in the plant. In this plant , the mechanical actuator is a permanent magnet dc motor. This motor is a prime mover to move another component such as transducer (For rectilinear movement, we have to convert angular rotation to the rectilinear motion first) in a robotic device for ultrasonic inspection.

A step voltage (U_O) applied to the armature of a permanent magnet dc motor produces a maximum obtainable acceleration on the shaft. The theoritical time-optimal control for providing the maximum deceleration to the shaft, we applied the reverse voltage to the armature. By this method the breaking time is impoved by a factor of ten over just removing the voltage (ref.4).

We store the velocity profiles including the switching point in the programmable controller such as microcomputer . The voltage produced by a tachometer and a voltage measuring the distance from the target can be used to make the switching decision.

3.1 Mathematical Formulation of Minimum Time Control Problem

The second order model represents the PM motor (linear piecewise model)

$$U = K\Omega + Ri + L\,di/dt$$
$$Ki = a\Omega + b + J\,d\Omega/dt \tag{1}$$

where

U = voltage applied to armature

K = voltage constant at a given excitation of field (constant for a PM machine)

R = armature resistance

L = armature inductance

a,b = friction coefficients

J = moment of inertia of motor and load

Ω = angular velocity

i = armature current

θ = angular displacement

Note: The armature current i must exceed b/k in order to overcome the static friction torque (b)

Define the state variables as

$$X_1 = \theta(t)$$

$$X_2 = \Omega(t) = d\theta/dt$$

$$X_3 = i(t)$$

From these state variabls we can obtain the vector differential equation

$$\ddot{\underline{X}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -a/J & K/J \\ 0 & -K/J & -R/j \end{bmatrix} \underline{X} + \begin{bmatrix} 0 \\ -b/J \\ U/J \end{bmatrix} \qquad (3)$$

Initial condition

$$X(0) = \begin{bmatrix} 0 \\ 0 \\ b/K \end{bmatrix} \qquad (4)$$

$$X(t\ ) = \begin{bmatrix} \theta_f \\ 0 \\ b/K \end{bmatrix} \qquad (5)$$

From the vector differential equation and boundary condition, we want to find the control law $U^{*}(t)$.

The constraint is

$$|U(t)| < U_o \quad \text{(constant voltage power supply)} \qquad (6)$$

Performance index (integral cost function)

$$J = \int_{o}^{t_f} dt \qquad (7)$$

The Hamiltonian for the system is

$$H(t) = 1+\lambda_1 X_2+\lambda_2 1/J(KX_3-aX_2-b)+(\lambda_3/L)(U-RX_3-KX_2)$$

where λ_1, λ_2, λ_3 are adjoint or costate variables.

The adjoint differential equation

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ -1 & a/J & K/J \\ 0 & -K/J & R/J \end{bmatrix} \qquad (8)$$

Using the Pontryagin's mimimum principle, the minimum control law is obtained by adjusting $U(t)$ to minimize $H(t)$

$$U^{*} = -U_o \operatorname{sgn} \lambda_3 \quad \text{(a Bang-Bang control)} \qquad (9)$$

The final time t_f is free so we have

$$H(t_f) = 0 \qquad (10)$$

From piecewise linear model, we have

$$U = K\Omega + Ri + L\,di/dt$$

$$Ki = a\Omega + b + J\,d\Omega/dt \qquad (11)$$

Take laplace transform in both equation

$$\Omega(s)K + I(s)(R+Ls) = LI_o + U(s) \qquad (12)$$

$$\Omega(s)(a+Js) - KI(s) = J\Omega_o - b/s \qquad (13)$$

or

$$\begin{bmatrix} K & (R+LS) \\ a+JS & -K \end{bmatrix} \begin{bmatrix} \Omega(S) \\ I(S) \end{bmatrix} = \begin{bmatrix} LI_o + U(S) \\ J\Omega_o - b/S \end{bmatrix}$$

determinant of the system is

$$\Delta = JLs^2 + s(RJ + aL) + aR_o + K^2$$

The poles are

$$s_{1,2} = \frac{-(RJ+aL) \pm \sqrt{(RJ+aL)^2 - 4JL(aR+K^2)}}{2JL} \qquad (14)$$

So the general equation of the laplace transform of speed and current are

$$\Omega(S) = \frac{S^2\Omega_o + S\left[\frac{L}{J} I_o + \frac{R}{L}\Omega_o - \frac{b}{J}\right] - \frac{Rb}{JL} + SU(S)\frac{K}{JL}}{S(S-S_1(S-S_2)}$$

$$I(S) = \frac{S^2 I_o + S\left[\frac{a}{J} I_o - \frac{K}{L}\Omega_o\right] + \frac{bK}{JL} + \frac{SU(S)a}{JL} + \frac{S^2U(S)}{L}}{S(S-S_1)(S-S_2)}$$

When U_o is replaced by $-U_o$, the machine starts working as a generator rather than a motor.

It is seen that an analysis in the phase plane would be more useful. Angular displacement and angular velocity are chosen as the variables for the phase plane. The third state variable, armature current, is studied only to determine whether the mechanical and eletrical constraints are met.

3.2 Acceleration Curve

The trajectory obtained when a positive step-voltage U is applied to the PM motor. Intially starting from zero the current rises in an exponential manner typical of an RL circuit.

When the value i=b/K is reached, the rotor begins to accelerate to the rotor. This 'stalling' condition has a duration of f , given by

$$t_s = (L/R)\log(U_oK/(U_oK-Rb)) \tag{17}$$

The angular speed and displacement are

$$\Omega(t') = \Omega_f\left[1 + \frac{\sigma_2}{\sigma_1-\sigma_2}\, e^{\sigma_1 t'} + \frac{\sigma_1}{\sigma_2-\sigma_1}\, e^{\sigma_2 t'}\right] \tag{18}$$

$$\theta(t') = \Omega_f\left[t' + \frac{\sigma_1+\sigma_2}{\sigma_1\sigma_2(\sigma_2-\sigma_1)} + \frac{\sigma_2}{\sigma_1(\sigma_1-\sigma_2)}\, e^{\sigma_1 t'} + \frac{\sigma_1}{\sigma_2(\sigma_2-\sigma_1)}\, e^{\sigma_2 t'}\right] \tag{19}$$

where

$$t' = t - t_s$$

$$\Omega_f = \frac{U_oK - bR}{aR + K^2}$$

$$A = \frac{1}{2}\;\frac{R}{L} + \frac{a}{J}$$

$$W = \frac{aR + K^2}{JL} \quad A^2$$

$$\sigma_1 = -A\sqrt{-W}$$

$$\sigma_2 = -A-\sqrt{-W} \qquad (20)$$

3.3 Deceleration Curve

The voltage applied to the motor must be reversed in order to provide maximum deceleration.

$$-U = K\Omega - Ri - Ldi/dt$$

$$-Ki = a\Omega + b + Jd\,\Omega/dt \qquad (21)$$

If the switching occurs at t_{sw} , so

$$\Omega(t'')_{brake} = A_2 + B_2 e^{\sigma_1 t''} + C_2 e^{\sigma_2 t''} \qquad (22)$$

$$\theta(t'')_{brake} = A_2 t'' + \frac{B_2}{\sigma_1} e^{\sigma_1 t''} + \frac{C_2}{\sigma_2} e^{\sigma_2 t''} \qquad (23)$$

where

$$t = t - t_{sw}$$

$$A_2 = \frac{-KU_o - bR}{aR + K^2}$$

$$B_2 = \frac{-\Omega_{sw}(\sigma_2 + a/J) - b/J + \sigma_2 A_2}{\sigma_1 - \sigma_2}$$

$$C_2 = \frac{-\Omega_{sw}(\sigma_1 + a/J) - b/J + \sigma_1 A_2}{\sigma_2 - \sigma_1} \qquad (24)$$

It will be seen that coefficient B and C depend on and A depends on the basic parameters of a motor

Since in a well desinged machine (ref.1), from (22) the approximate time needed to stop from a speed is given by

$$T = (1/\sigma_1)\ln(-A_2/B_2) \tag{25}$$

and

$$\theta_{brake} = -(1/\sigma_1)(A_2\ln(B_2/-A_2)+A_2) \tag{26}$$

Solving this trajectory, the final conditions on X_1, X_2 and X_3 must be satisfied. This difficulty of solving these equation is to find the switching velocity. Use equation (18),(19),(20),(22),(23) and (24) to solve the moving time to satisfy the final condition (θ_f) by computer. There is no closed form solution to find the moving time, so one can solve this trajectory by writing a program computer.

3.4 Characteristics of Interest

Possible motor characteristics of interest include size, weight, torque, max speed, inertia, motor constant, torque sensitivity, dc resistance, phase resistance (stepper motor), inductance, power rate and etc.

Continuously Variable Parameter Approach

One approach initially considered involved assuming that a motor of arbitrary "size" could be obtained. This requires a functional relationship between the motor "size" and other important parameters. An aproximation to such a fit can be obtained from an empirical curve fitted to manufacturers' data. The scatter on the data was larger than desired and this approach was abandoned temporarily for a catalog approach. In the catalog approach actual motor data is used and only discrete motor sizes are permitted. The continuously variable parameter approach may be of further interest and is presented for completeness.

If we assume that these motor parameter are continuous, we can find the relation between weight with other parameter, such as weight vs torque, weight vs motor constant and etc. by using power law.

$$X_1 = K_1 (X_2)^{K_2}$$

$$\log X_1 = K_2 \log X_2 + \log K_1$$

Note: This is often but not always an appropriate form.

This is the linear equation used in the analysis. For this simulation, design parameters such as motor specifications, structure size will be required. Optimal actuator selection requires that the relationship between several actuator parameters be determined. For example, rotor inertia tends to increase when motor torque capability increases, and weight of the motor tends to increase when motor torque increases. So one can determine an empirical relationship of the motor parameter by a least squares fit of maunfacturer's data to a power law.

Note: The program, for least squares curve fit, is BMDP program. This program was developed by Health Science Computing Facility. Department of Biomathematics. School of Medicine. University of California, Los Angeles.

Figures (2) and (3) show manufacturers' data and the power law fit to that data. Data is from PMI MOTOR, permanent magnet motor series

T01SB	T01MB	T01LB
T03SB	T03MB	T03LB
T06SB	T06MB	T06LB

where

Inertia	has units of	oz-in-sec**2
Rated torque	has units of	oz-in
Armature resistance	has units of	ohm
Rated power	has units of	watts
Motor constant	has units of	v/1000rpm

Figures (4),(5) and (6) shows similar information for INLAND MOTOR, permanent magnet motor series

T-3001	T-3402	T-3207
T-3903	T-2967	NT-2934
T-2955	T-3203	T-2987
NT-2946	NT-2917	NT-2950
T-3208	T-2959	T-2989
NT-2932	T-2938	NT-2921
NT-2970	T-7209	T-7501
T-7202	T-3206	T-9901
T-7266		

where

Peak torque	has units of	kg-m
Rotor inertia	has units of	kg-m**2
Weight	has units of	kg

$$\text{Torque} = 0.17876((\text{Weight})^{1.23272} \qquad \text{R-square} = 0.93798$$

$$\text{JT} = 6.55881\ 10^{-4}((\text{Weight})^{1.57466} \qquad \text{R-square} = 0.85166$$

$$\text{JT} = 0.00529((\text{Torque})^{1.26235} \qquad \text{R-square} = 0.88809$$

(Note R-square is correlation coefficient)

4. Selecting Motors with the Catalog Approach

In the optimization program, we need to select motors given the link masses and movement time for each direction. Total effective load inertia can be found from these given value of parameters. One should note that some effective load inertia also include inertia caused by mass of motors. In this case, procedure of selecting motors should include trial-error routine. We also has this routine in the subroutine of selecting motors.

Use equation (18),(19),(20),(22),(23) and (24) to solve the moving time, and use the least square curve fit to find the approximation relation between $\underline{\bar{\tau}}$, $\underline{\theta}$ and JT

where

$\bar{\tau}$ = nondimensional moving time

θ = $\tau / \sqrt{JT/T_P}$

θ = moving distance (rad)

JT = total inertia (kg-m**2)

T_P = peak torque ((kg-m**2)/sec**2)

Tables (1),(2),(3),(4) and (5) show the approximate equation of $\bar{\tau}$ as the function of θ for 5 values of JT (0.4,0.35,0.04,0.008,0.004). These curves were fit through 51 points with values of from 20 to 10020 rad. From these relations we find the surface $\bar{\tau}$ = f(JT,) for use in our optimization program.

Thus first we find

$$\bar{\tau} = A\theta + B \qquad \text{(JT constant)}$$

Then we use a power law fit for A and B

$$A = a_1*(JT**a_2)$$

$$B = b_1*(JT**b_2)$$

So

$$\bar{\tau} = (a_1*(JT**a_2)* \quad + (b_1*(JT**b_2))$$

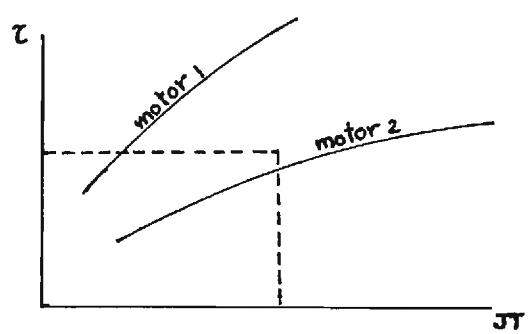
as shown in figures (7),(8),(9) and (10)

Table (6) shown the equations of surfaces. For each motor we can find the relation among (movement time), (movement distance) and JT (total inertia).

$$\bar{\tau} = \{(a*(JT**a_2))*\theta + (b_1*(JT**b_2)\}\cdot\sqrt{JT/T_P}$$

as shown in figures (11),(12),(13),(14) and (15)

Given θ,JT and τ, we can select the motor which most nearly meets the specified JT and τ.



It is not difficult to write computer program for selecting a motor by using this approach.

5. Optimization Program

Figures (16),(17),(18),(19) and (20) is a flowchart of the optimation program. First, input initial values of mass of links ($m\ell_1, m\ell_2, m\ell_3$) and effective radii of rotation (r_1, r_2, r_3). Second, select the motors from these given values of parameters. Third, set the parameters for DSAP and final go to the conjugate gradient subroutine.

(Note: DSAP is the Distributed System Analysis Package And its Application to Modeling Flexible Manipulator. Written by Dr. Wayne J.Book, Mark Majette and Kong Ma.)

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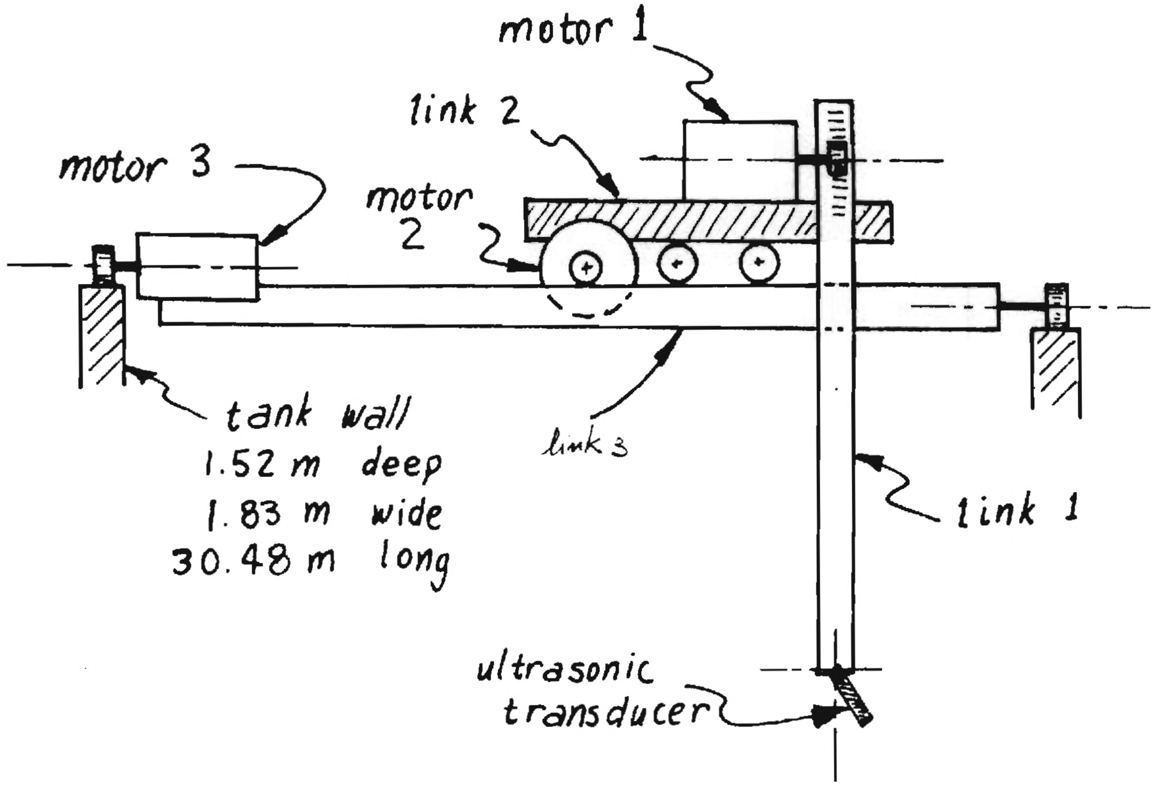


$m_{\ell i}$ = mass of link i

m_{mi} = mass of motor i

J_{mi} = rotor inertia of motor i

r_i = effective radius of the drive of motor i

Design parameters:

$m_{\ell i}$ = specifies stiffness of link with assumed geometry

m_{mi} = specifies motor Torque and J_{mi}

r_i = selected to optimize movement time

Inertia driven by motor 1:

$$J_{T1} = J_{m1} + m_{\ell 1} r_1^2$$

Motor 2:

$$J_{T2} = (m_{\ell 1} + m_{\ell 2} + m_{m1} + m_{m2}) r_2^2 + J_{m2}$$

Motor 3:

$$J_{T3} = (m_{\ell 1} + m_{\ell 2} + m_{\ell 3} + m_{m1} + m_{m2} + m_{m3}) r_3^2 + J_{m3}$$

Figure 1. Schematic of the robot for ultrasonic testing.

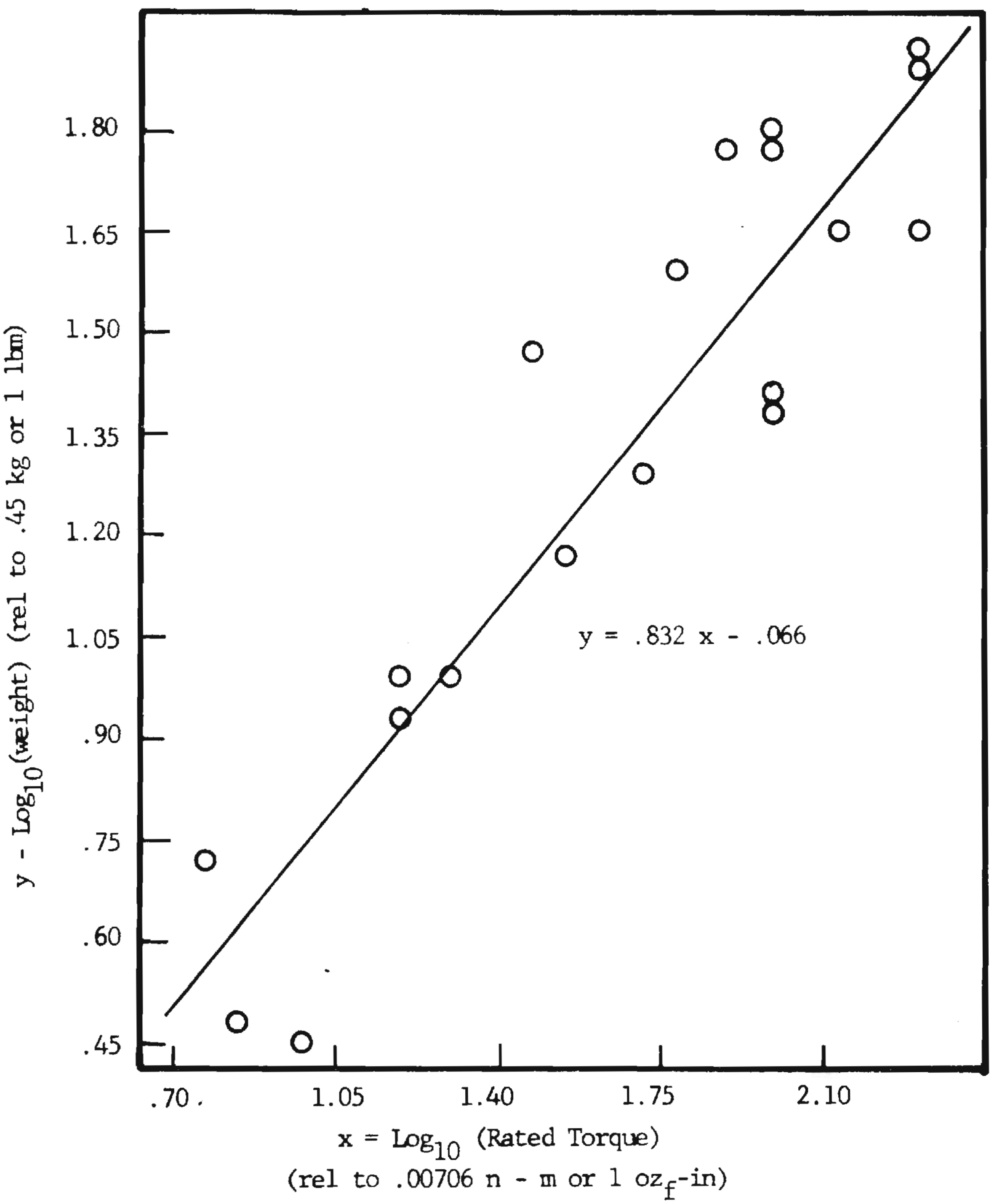


Figure 2. Empirical fit to manufacturer's data.
R = .840 correlation

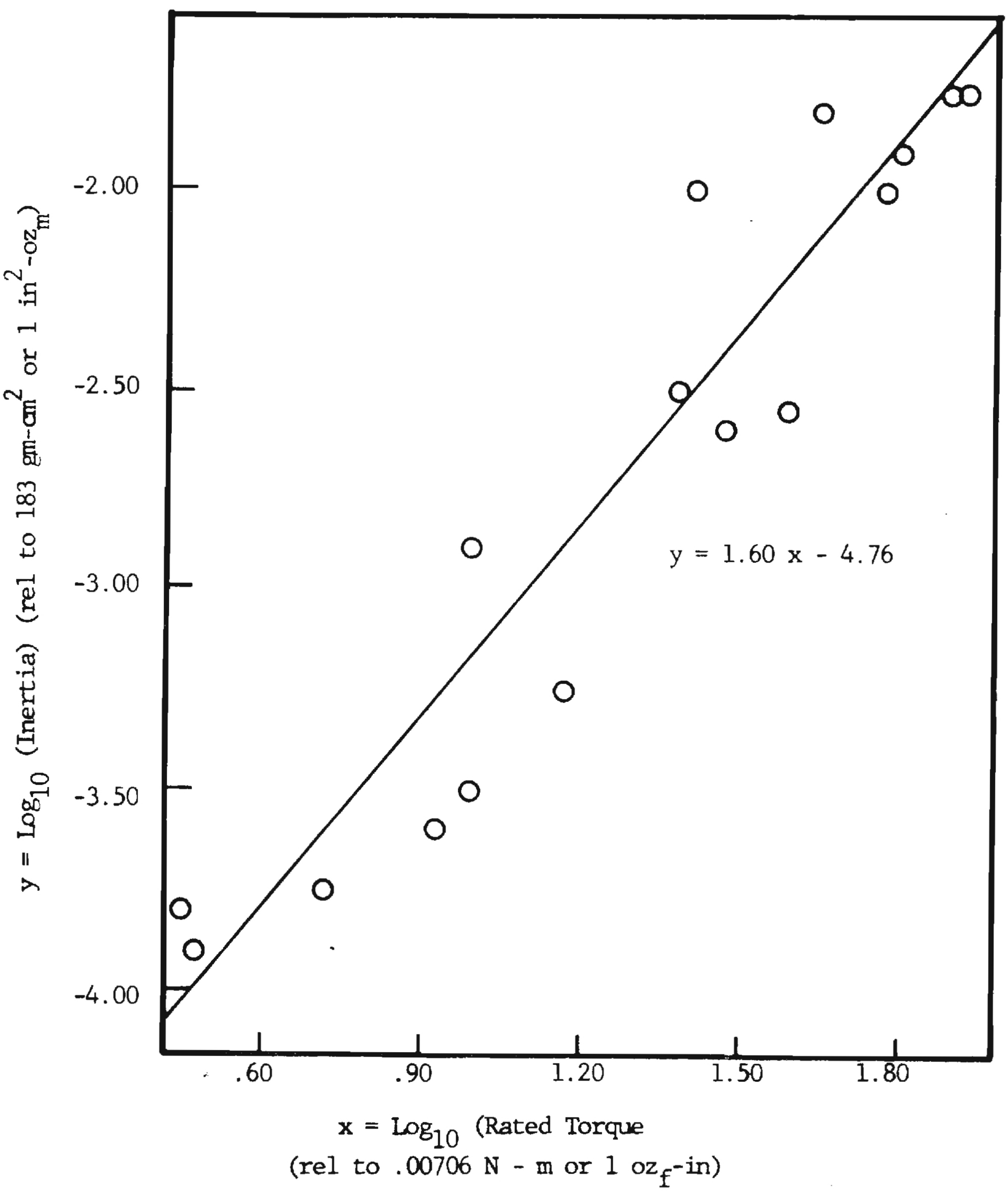


Figure 3. Empirical fit to manufacturers data. R = .900.

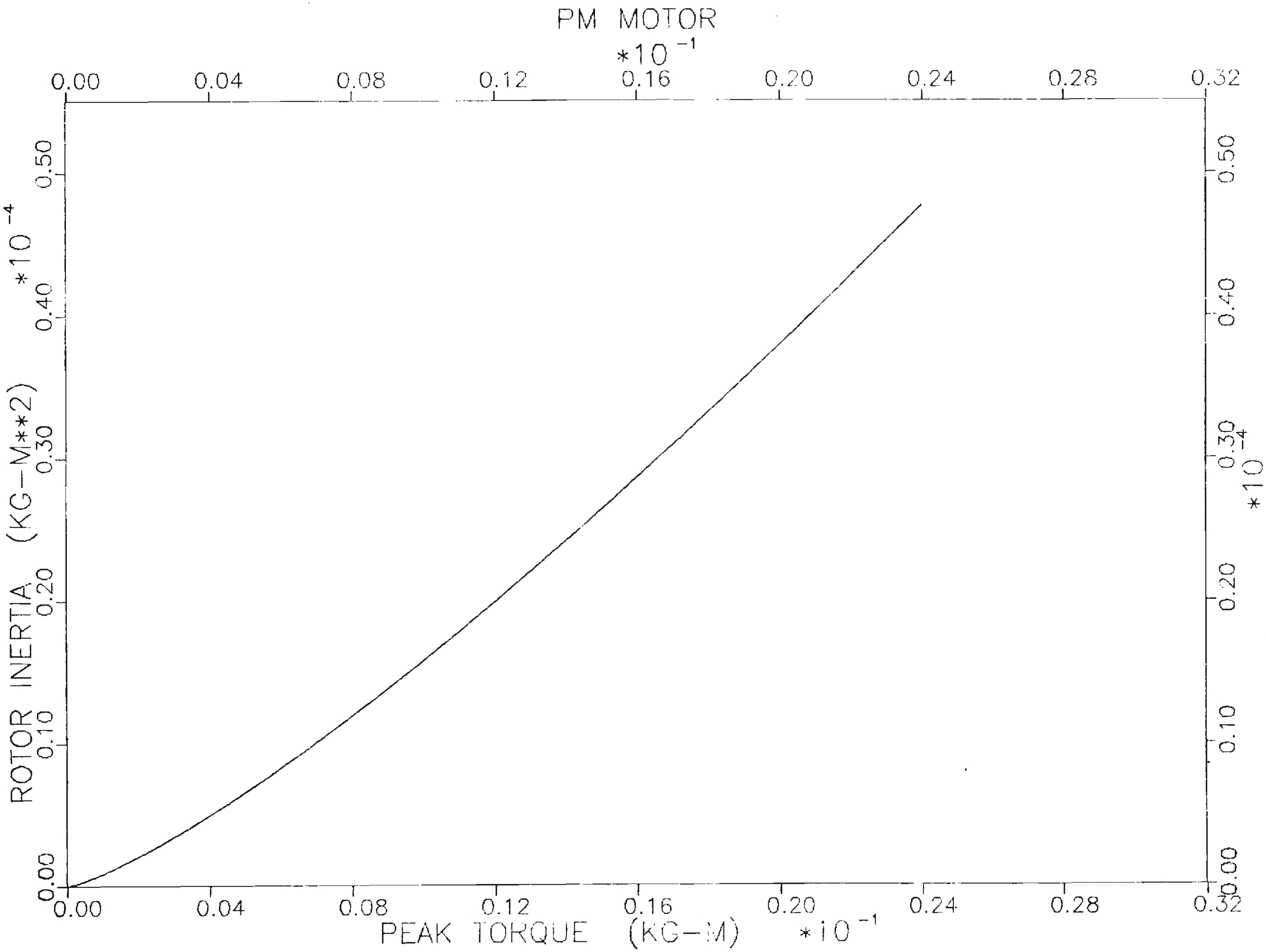

PM MOTOR
*10 −1
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32
ROTOR INERTIA (KG−M**2) *10 −4
0.00
0.10
0.20
0.30
0.40
0.50
PEAK TORQUE (KG−M) *10 −1

FIG 4

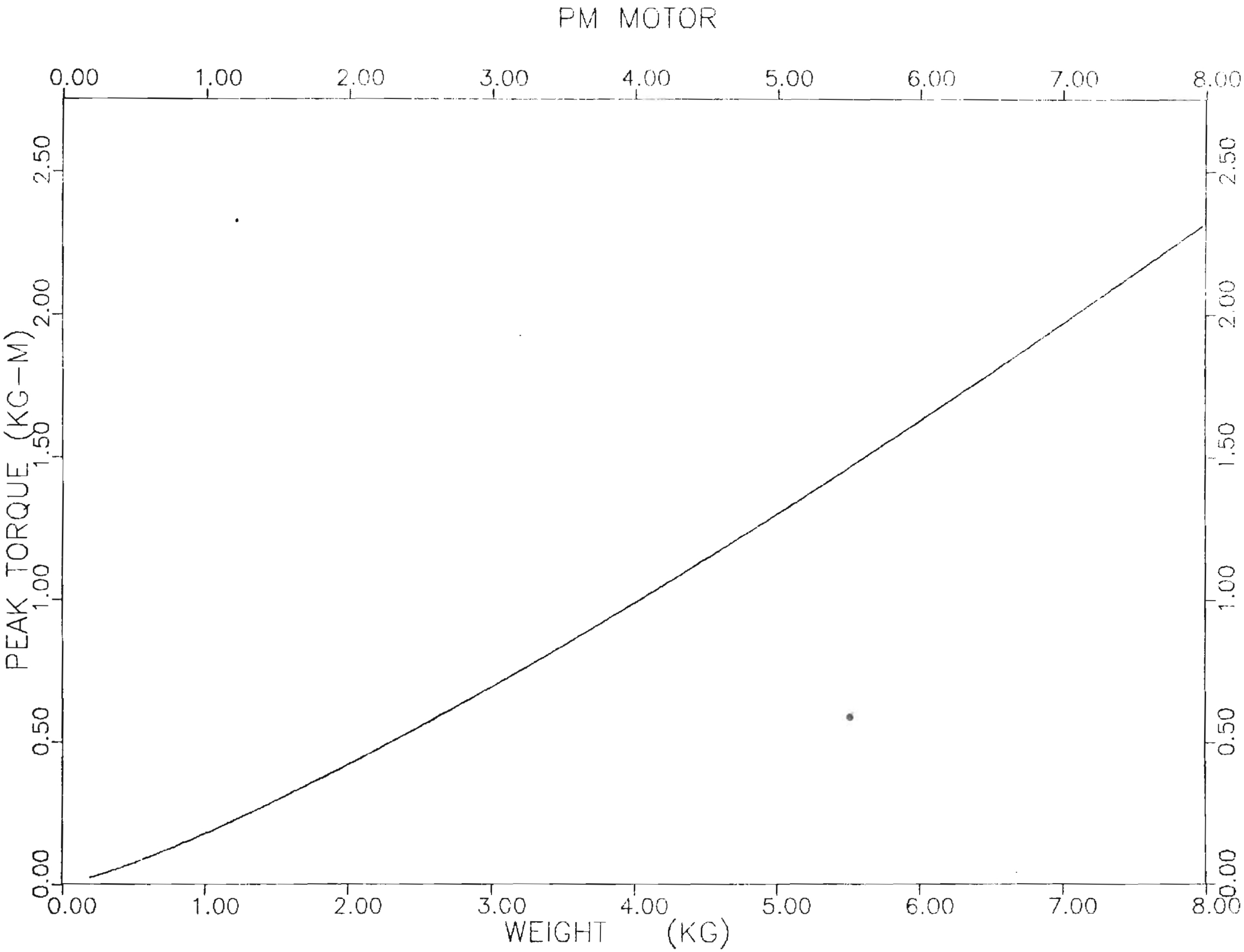

PM MOTOR
0.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
PEAK TORQUE (KG-M)
0.00
0.50
1.00
1.50
2.00
2.50
WEIGHT (KG)

FIG.5

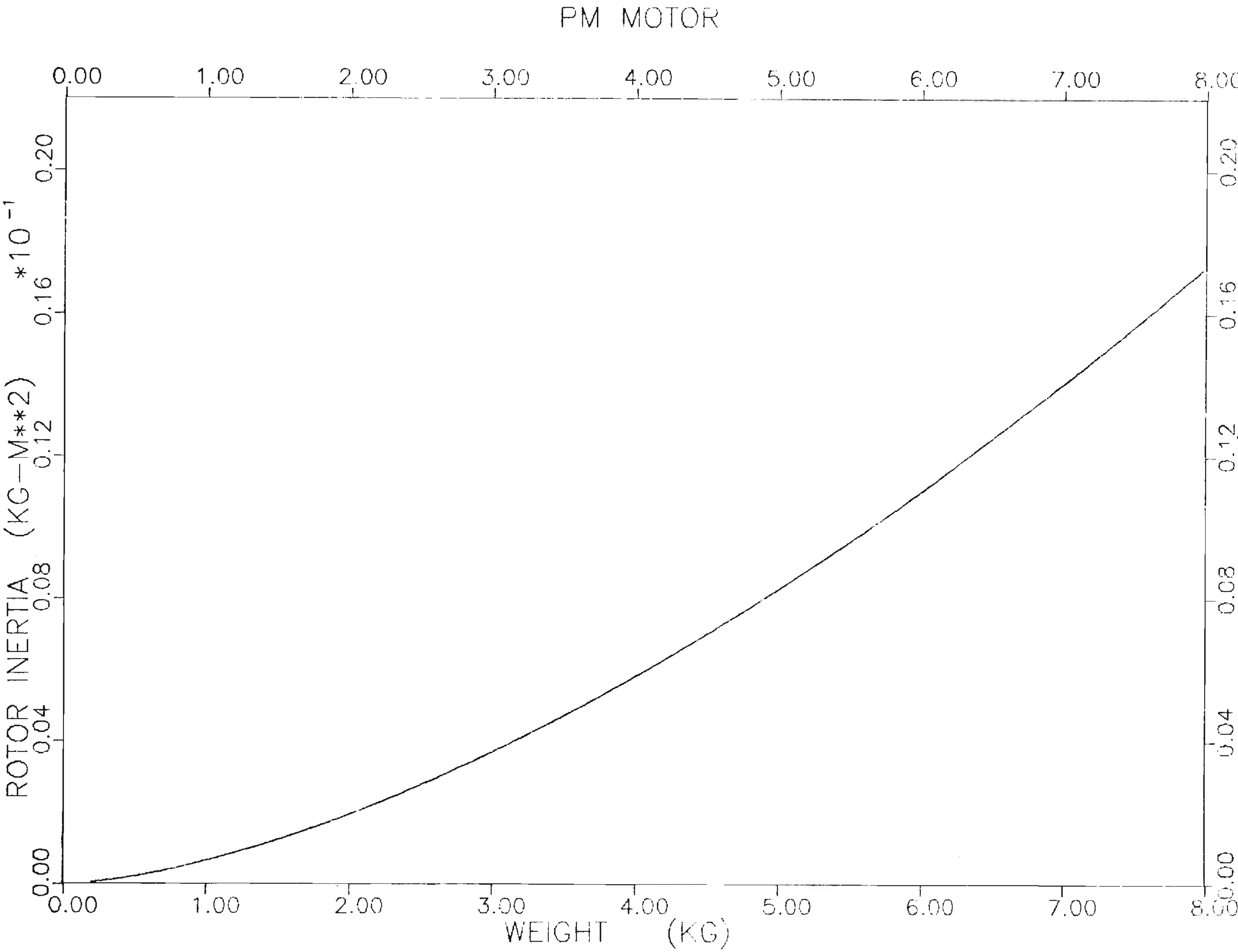

PM MOTOR
ROTOR INERTIA (KG-M**2) *10^-1
WEIGHT (KG)
0.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
0.04
0.08
0.12
0.16
0.20

FIG. 6

TABLE 1. $J_T = 4.0 \times 10^{-1}$ kg-m²

	T_P lb-ft	Weight lb	
T-3001	0.188	0.5	$\bar{\tau} = 0.02087\theta + 88.03663$
T-3402	0.31	0.6	$\bar{\tau} = 0.02079\theta + 90.81546$
T-3207	0.5	0.85	$\bar{\tau} = 0.03879\theta + 278.23985$
T-3903	0.55	0.87	$\bar{\tau} = 0.04244\theta + 72.77784$
T-2967	0.52	1.09	$\bar{\tau} = 0.06219\theta + 498.10136$
NT-2934	0.705	1.5	$\bar{\tau} = 0.10392\theta + 470.92067$
T-2955	0.85	1.5	$\bar{\tau} = 0.12249\theta + 369.37072$
T-3203	1.0	1.6	$\bar{\tau} = 0.15551\theta + 365.79160$
T-2987	1.1	2	$\bar{\tau} = 0.34196\theta + 984.82487$
NT-2946	1.48	2	$\bar{\tau} = 311.61325\theta + 664673.40242$
NT-2917	2	2	$\bar{\tau} = 2.09552\theta + 1376.05000$
T-2950	1.2	2.15	$\bar{\tau} = 0.17084\theta + 202.96458$
T-3208	1.5	2.4	$\bar{\tau} = 26.40623\theta + 33617.65586$
T-2959	1.7	2.5	$\bar{\tau} = 20.99501\theta + 17825.26267$
T-2989	1.73	2.5	$\bar{\tau} = 0.41696\theta + 224.40428$
NT-2932	1.92	2.75	$\bar{\tau} = 0.46831\theta + 135.26694$
T-2938	1.82	3.1	$\bar{\tau} = 355.99116\theta + 582871.66509$
NT-2921	2.4	3.4	$\bar{\tau} = 14.45992\theta + 4832.99776$
NT-2970	2.9	6	$\bar{\tau} = 9.09146\theta + 6148.52729$
T-7209	7.25	7	$\bar{\tau} = 1.46829\theta + 141.54176$
T-7501	6.5	7.5	$\bar{\tau} = 71.18919\theta + 2941.34793$
T-7202	11	10.3	$\bar{\tau} = 2522.05935\theta + 68744.07325$
T-3206	6	11.5	$\bar{\tau} = 62662.22757\theta + 8932854.91172$
T-9901	20.0	15.0	$\bar{\tau} = 1297484.51957\theta + 9451577.89755$
T-7266	19.0	16	$\bar{\tau} = 91674.11381\theta + 830470.24916$

Table 2. $J_T = 0.35$ kg-m^2

	T_p lb-ft	Weight lb	
T-3001	0.138	0.5	$\bar{\tau} = 0.02210\theta + 82.98229$
T-3402	0.31	0.6	$\bar{\tau} = 0.02175\theta + 84.66277$
T-3207	0.5	0.85	$\bar{\tau} = 0.02062\theta + 262.84487$
T-3903	0.55	0.87	$\bar{\tau} = 0.04528\theta + 68.35970$
T-2967	0.52	1.09	$\bar{\tau} = 0.06504\theta + 470.35624$
NT-2934	0.705	1.5	$\bar{\tau} = 0.10977\theta + 444.51476$
T-2955	0.85	1.5	$\bar{\tau} = 0.12994\theta + 348.52279$
T-3203	1.0	1.6	$\bar{\tau} = 0.16542\theta + 344.61385$
T-2987	1.1	2	$\bar{\tau} = 0.36311\theta + 928.57229$
NT-2946	1.48	2	$\bar{\tau} = 331.86511\theta + 625462.01214$
NT-2917	2	2	$\bar{\tau} = 2.23981\theta + 1290.32125$
T-2950	1.2	2.15	$\bar{\tau} = 0.18236\theta + 190.62027$
T-3208	1.5	2.4	$\bar{\tau} = 28.18206\theta + 31578.48118$
T-2959	1.7	2.5	$\bar{\tau} = 22.42617\theta + 16723.81158$
T-2989	1.73	2.5	$\bar{\tau} = 0.44558\theta + 210.81848$
NT-2932	1.92	2.75	$\bar{\tau} = 0.50058\theta + 126.67323$
T-2938	1.82	3.1	$\bar{\tau} = 879.67025\theta + 547848.18024$
NT-2921	2.4	3.4	$\bar{\tau} = 15.45605\theta + 4526.23731$
NT-2970	2.9	6	$\bar{\tau} = 9.71416\theta + 5764.14981$
T-7209	7.25	7	$\bar{\tau} = 1.56962\theta + 132.54170$
T-7501	6.5	7.5	$\bar{\tau} = 76.10421\theta + 2751.68494$
T-7202	11	10.3	$\bar{\tau} = 2696.18082\theta + 64362.31934$
T-3206	6	11.5	$\bar{\tau} = 66985.59160\theta + 8364587.07029$
T-9901	20.0	15.0	$\bar{\tau} = 1387066.14561\theta + 8847153.53214$
T-7266	19.0	16	$\bar{\tau} = 98003.61769\theta + 776600.19206$

TABLE 3

$JT = 0.04\ kg\text{-}m^2$

	T_p lb-ft	Weight lb	
T-3001	0.188	0.5	$\bar{\tau} = 0.06265\theta + 29.93078$
T-3402	0.31	0.6	$\bar{\tau} = 0.06159\theta + 30.67914$
T-3207	0.5	0.85	$\bar{\tau} = 0.10523\theta + 100.58187$
T-3903	0.55	0.87	$\bar{\tau} = 0.13285\theta + 23.77812$
T-2967	0.52	1.09	$\bar{\tau} = 0.16492\theta + 181.25582$
NT-2934	0.705	1.5	$\bar{\tau} = 0.30679\theta + 162.91104$
T-2955	0.85	1.5	$\bar{\tau} = 0.37241\theta + 125.78515$
T-3203	1.0	1.6	$\bar{\tau} = 0.47988\theta + 122.58448$
T-2987	1.1	2	$\bar{\tau} = 1.04520\theta + 333.09844$
NT-2946	1.48	2	$\bar{\tau} = 967.49887\theta + 220484.20898$
NT-2917	2	2	$\bar{\tau} = 6.60932\theta + 444.92992$
T-2950	1.2	2.15	$\bar{\tau} = 0.53637\theta + 66.27243$
T-3208	1.5	2.4	$\bar{\tau} = 82.83755\theta + 10994.83681$
T-2959	1.7	2.5	$\bar{\tau} = 66.13016\theta + 5779.43587$
T-2989	1.73	2.5	$\bar{\tau} = 1.31615\theta + 72.26627$
NT-2932	1.92	2.75	$\bar{\tau} = 1.47991\theta + 43.39203$
T-2938	1.82	3.1	$\bar{\tau} = 1113.2339\theta + 191389.22325$
NT-2921	2.4	3.4	$\bar{\tau} = 45.68970\theta + 1555.34979$
NT-2970	2.9	6	$\bar{\tau} = 28.67514\theta + 1987.23705$
T-7209	7.25	7	$\bar{\tau} = 4.64251\theta + 45.44196$
T-7501	6.5	7.5	$\bar{\tau} = 225.11536\theta + 929.41972$
T-7202	11	10.3	$\bar{\tau} = 7975.36840\theta + 21730.68087$
T-3206	6	11.5	$\bar{\tau} = 198111.85245\theta + 2851557.64074$
T-9901	20.0	15.0	$\bar{\tau} = 4103006.29457\theta + 2883791.95520$
T-7266	19.0	16	$\bar{\tau} = 289898.52686\theta + 260908.32109$

TABLE 4. $JT = 0.8 \times 10^{-2}\ kg \cdot m^2$

	T_p lb-ft	Weight lb	
T-3001	0.138	0.5	$\bar{\tau} = 0.13995\theta + 13.52012$
-3402	0.31	0.6	$\bar{\tau} = 0.13711\theta + 13.87348$
-3207	0.5	0.85	$\bar{\tau} = 0.234260\theta + 45.99873$
T-3903	0.55	0.87	$\bar{\tau} = 0.29699\theta + 10.71838$
-2967	0.52	1.09	$\bar{\tau} = 0.36663\theta + 88.18825$
JT-2934	0.705	1.5	$\bar{\tau} = 0.68494\theta + 73.91360$
T-2955	0.85	1.5	$\bar{\tau} = 0.832060\theta + 56.99189$
T-3203	1.0	1.6	$\bar{\tau} = 1.072480\theta + 55.48526$
T-2987	1.1	2	$\bar{\tau} = 2.33551\theta + 150.89358$
NT-2946	1.48	2	$\bar{\tau} = 2162.504120\theta + 99866.25328$
JT-2917	2	2	$\bar{\tau} = 14.77805\theta + 200.85277$
T-2950	1.2	2.15	$\bar{\tau} = 1.19915\theta + 29.95140$
T-3208	1.5	2.4	$\bar{\tau} = 185.19341\theta + 4974.29466$
T-2959	1.7	2.5	$\bar{\tau} = 147.85609\theta + 2617.82975$
T-2989	1.73	2.5	$\bar{\tau} = 2.942860\theta + 32.63507$
NT-2932	1.92	2.75	$\bar{\tau} = 3.30915\theta + 19.49759$
T-2938	1.82	3.1	$\bar{\tau} = 2488.624160\theta + 86682.79668$
JT-2921	2.4	3.4	$\bar{\tau} = 102.163940\theta + 706.85320$
NT-2970	2.9	6	$\bar{\tau} = 64.11558\theta + 901.07839$
T-7209	7.25	7	$\bar{\tau} = 10.381000\theta + 20.73546$
T-7501	6.5	7.5	$\bar{\tau} = 503.37485\theta + 401.87395$
T-7202	11	10.3	$\bar{\tau} = 798.06319\theta + 214432.02958$ (JT = 4.0)
T-3206	6	11.5	$\bar{\tau} = 442988.37512\theta + 1292592.36736$
T-9901	20.0	15.0	$\bar{\tau} = 410381.32572\theta + 29912683.87852$ (JT = 4.0)
T-7266	19.0	16	$\bar{\tau} = 28993.14763\theta + 2618682.57240$ (JT = 4.0)

TABLE 5 $JT = 0.4 \times 10^{-2}$ kg-m^2

	T_p lb-ft	Weight lb	
T-3001	0.188	0.5	$\bar{\tau} = 0.19770\theta + 9.59578$
T-3402	0.31	0.6	$\bar{\tau} = 0.19288\theta + 9.85783$
T-3207	0.5	0.85	$\bar{\tau} = 0.31120\theta + 32.72825$
T-3903	0.55	0.87	$\bar{\tau} = 0.42000\theta + 7.60795$
T-2967	0.52	1.09	$\bar{\tau} = 0.51834\theta + 59.14404$
NT-2934	0.705	1.5	$\bar{\tau} = 0.96862\theta + 52.42616$
T-2955	0.85	1.5	$\bar{\tau} = 1.17665\theta + 40.58814$
T-3203	1.0	1.6	$\bar{\tau} = 1.51667\theta + 39.49092$
T-2987	1.1	2	$\bar{\tau} = 3.30279\theta + 107.32690$
NT-2946	1.48	2	$\bar{\tau} = 3058.22969\theta + 71016.61574$
NT-2917	2	2	$\bar{\tau} = 20.89930\theta + 142.91432$
T-2950	1.2	2.15	$\bar{\tau} = 1.69583\theta + 21.30472$
T-3208	1.5	2.4	$\bar{\tau} = 261.90252\theta + 2532.30022$
T-2959	1.7	2.5	$\bar{\tau} = 209.10039\theta + 1861.36996$
T-2989	1.73	2.5	$\bar{\tau} = 4.16186\theta + 23.05621$
NT-2932	1.92	2.75	$\bar{\tau} = 4.67984\theta + 13.82834$
T-2938	1.82	3.1	$\bar{\tau} = 3519.41453\theta + 61642.46176$
NT-2921	2.4	3.4	$\bar{\tau} = 144.48056\theta + 485.56805$
NT-2970	2.9	6	$\bar{\tau} = 90.67318\theta + 641.95566$
T-7209	7.25	7	$\bar{\tau} = 14.68093\theta + 15.44481$
T-7501	6.5	7.5	$\bar{\tau} = 711.87708\theta + 295.67582$
T-7202	11	10.3	$\bar{\tau} = 1783.48250\theta + 96899.46046$ (JT = 0.8)
T-3206	6	11.5	$\bar{\tau} = 626481.63541\theta + 1488308.08835$
T-9901	20.0	15.0	$\bar{\tau} = 917456.55977\theta + 13444206.73749$ (JT = 0.8)
T-7266	19.0	16	$\bar{\tau} = 64823.47186\theta + 1177609.34878$ (JT = 0.8)

TABLE 6.

	T_p lb-ft	Weight lb	
T-3001	0.138	0.5	$\bar{\tau} = (0.01323 JT^{-0.48846})\theta + (137.82722 JT^{0.48081})$
T-3402	0.31	0.6	$\bar{\tau} = (0.01312 JT^{-0.48589})\theta + (141.34543 JT^{0.48064})$
T-3207	0.5	0.85	$\bar{\tau} = (0.02481 JT^{-0.46481})\theta + (430.46798 JT^{0.46325})$
T-3903	0.55	0.87	$\bar{\tau} = (0.02685 JT^{-0.49775})\theta + (114.42540 JT^{0.49035})$
T-2967	0.52	1.09	$\bar{\tau} = (0.03990 JT^{-0.45922})\theta + (768.98742 JT^{0.46091})$
NT-2934	0.705	1.5	$\bar{\tau} = (0.06595 JT^{-0.48465})\theta + (735.11568 JT^{0.47590})$
T-2955	0.85	1.5	$\bar{\tau} = (0.0758 JT^{-0.49133})\theta + (577.47417 JT^{0.47590})$
T-3203	1.0	1.6	$\bar{\tau} = (0.09843 JT^{-0.49462})\theta + (572.93433 JT^{0.48333})$
T-2987	1.1	2	$\bar{\tau} = (0.21654 JT^{-0.49251})\theta + (1541.04929 JT^{0.48112})$
NT-2946	1.48	2	$\bar{\tau} = (197.19539 JT^{-0.49597})\theta + (1041848.565 JT^{0.48558})$
NT-2917	2	2	$\bar{\tau} = (1.32644 JT^{-0.49943})\theta + (2162.44092 JT^{0.49197})$
T-2950	1.2	2.15	$\bar{\tau} = (0.10807 JT^{-0.49842})\theta + (318.70264 JT^{0.48959})$
T-3208	1.5	2.4	$\bar{\tau} = (16.70490 JT^{-0.49824})\theta + (52782.53033 JT^{0.48921})$
T-2959	1.7	2.5	$\bar{\tau} = (13.28010 JT^{-0.49913})\theta + (27983.66371 JT^{0.49069})$
T-2989	1.73	2.5	$\bar{\tau} = (0.26372 JT^{-0.49960})\theta + (353.25861 JT^{0.49375})$
NT-2932	1.92	2.75	$\bar{\tau} = (0.29619 JT^{-0.49985})\theta + (213.11702 JT^{0.49522})$
T-2938	1.82	3.1	$\bar{\tau} = (225.22197 JT^{-0.49755})\theta + (914351.6282 JT^{0.48789})$
NT-2921	2.4	3.4	$\bar{\tau} = (9.14559 JT^{-0.49982})\theta + (7637.35284 JT^{0.49615})$
NT-2970	2.9	6	$\bar{\tau} = (5.75049 JT^{-0.49943})\theta + (9645.17810 JT^{0.49084})$
T-7209	7.25	7	$\bar{\tau} = (0.92863 JT^{-0.4997})\theta + (219.48807 JT^{0.48481})$
T-7501	6.5	7.5	$\bar{\tau} = (45.02404 JT^{-0.500})\theta + 4667.96589 JT^{0.50356})$
T-7202	11	10.3	$\bar{\tau} = (1595.47554 JT^{-0.49987})\theta + (108090.7142 JT^{0.49717})$
T-3206	6	11.5	$\bar{\tau} = (39631.32144 JT^{-0.49995})\theta + (12619744.26 JT^{0.42874})$
T-9901	20.0	15.0	$\bar{\tau} = (820622.2230 JT^{-0.49999})\theta + (14958571.68 JT^{0.50828})$
T-7266	19.0	16	$\bar{\tau} = (57982.18762 JT^{-0.49998})\theta + (1312812.636 JT^{0.50101})$

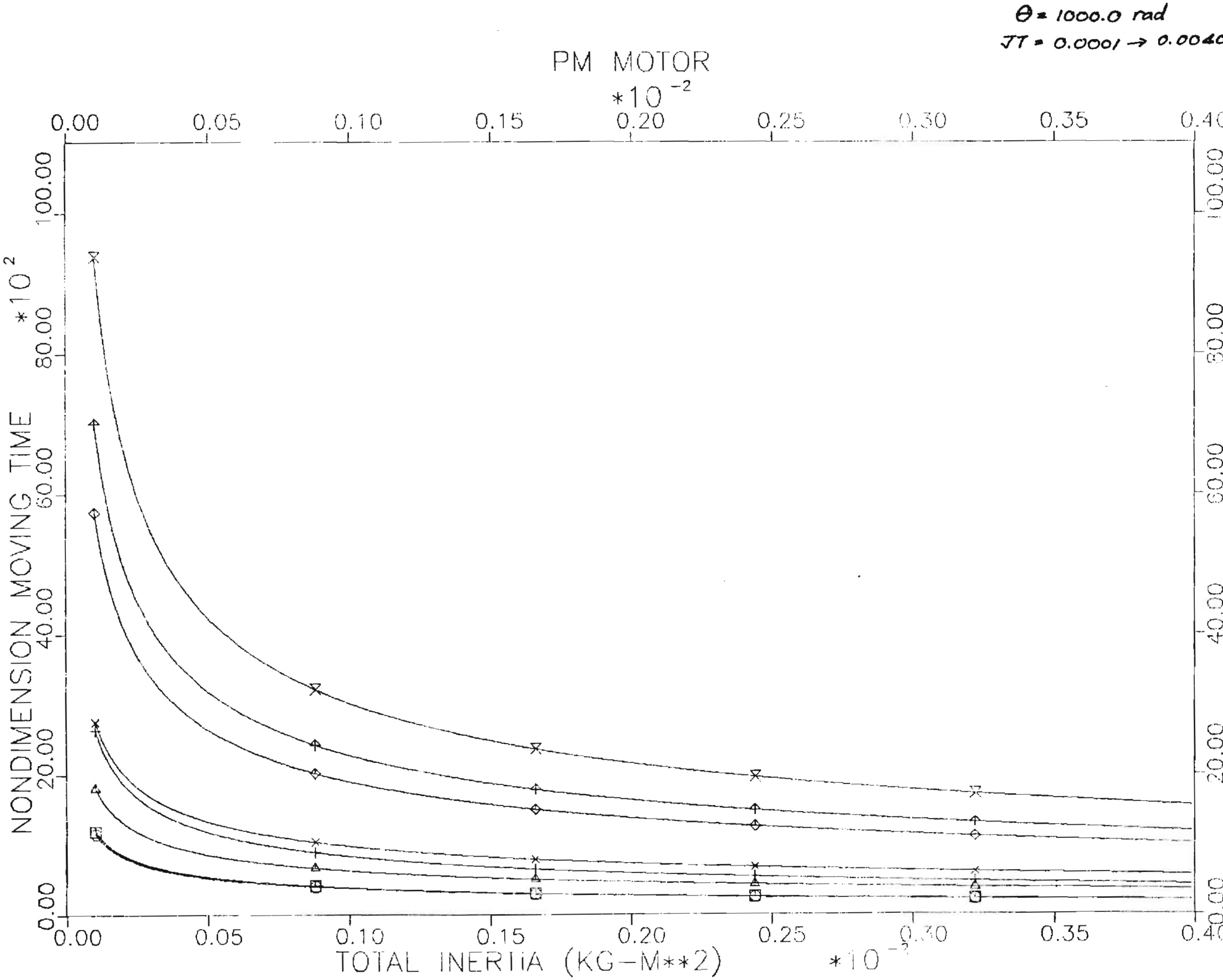

θ = 1000.0 rad
JT = 0.0001 → 0.0040
PM MOTOR
*10^-2
NONDIMENSION MOVING TIME *10^2
TOTAL INERTIA (KG-M**2) *10^-2

FIG. 7

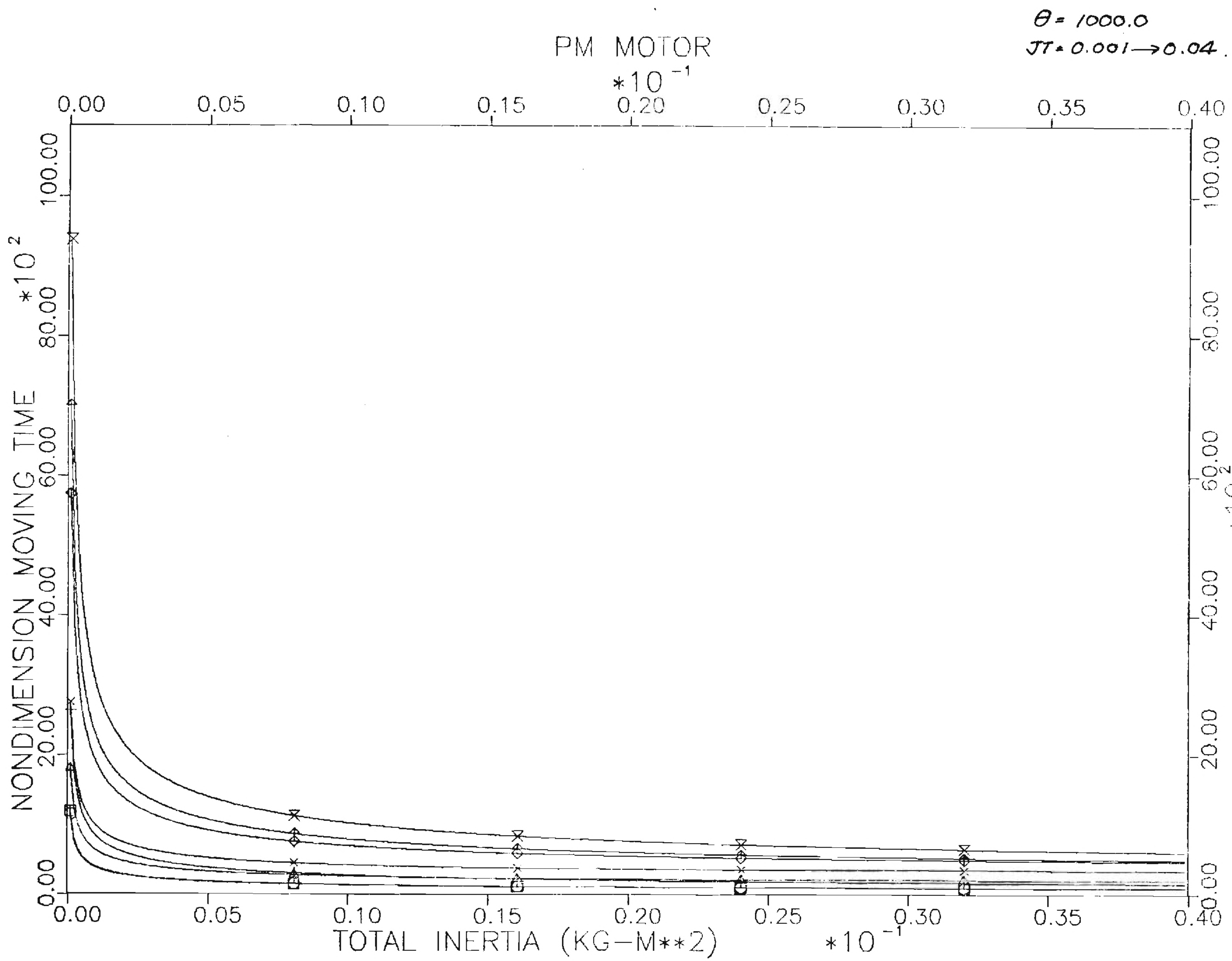
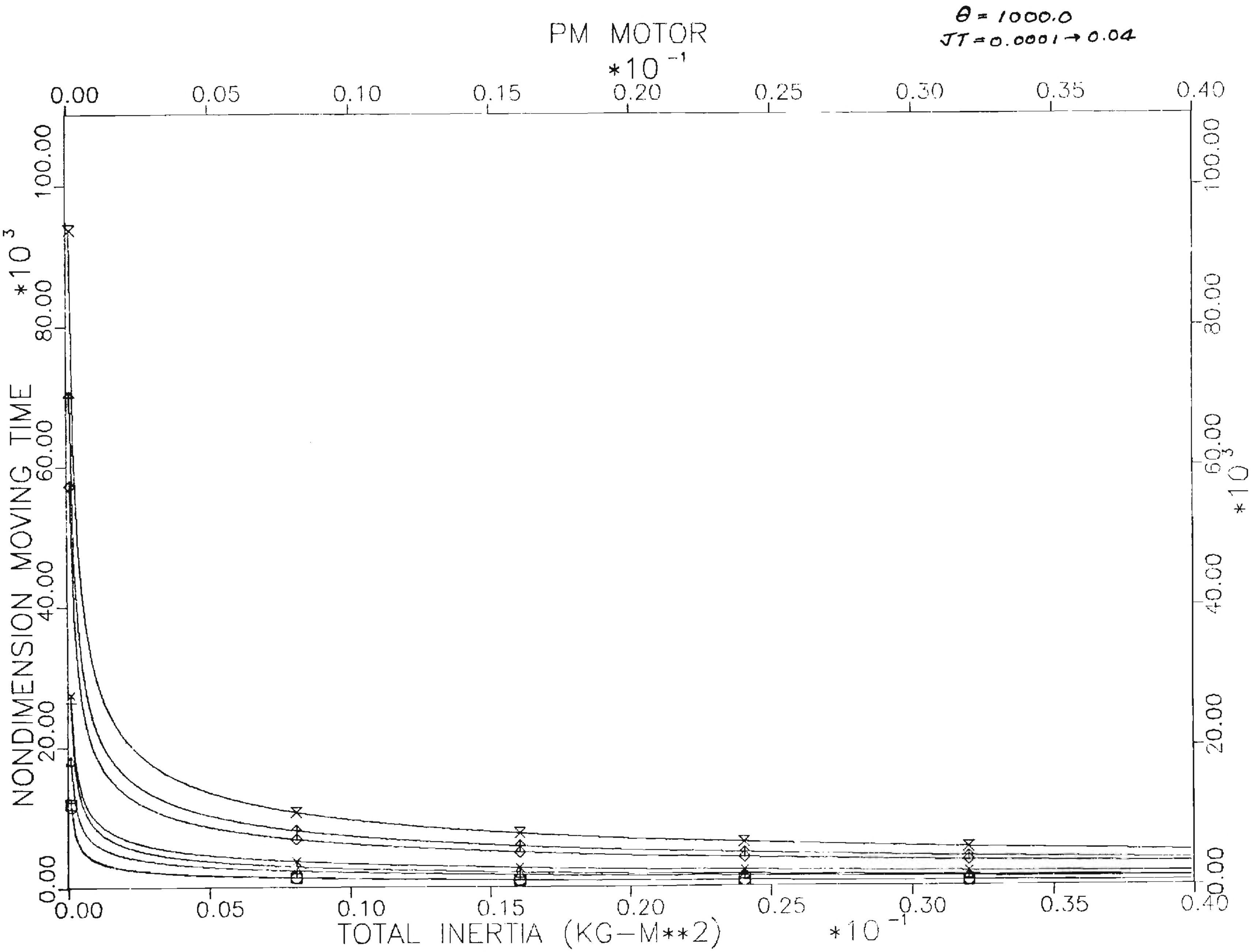

θ = 1000.0
JT = 0.001 → 0.04.
PM MOTOR
*10^-1
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
NONDIMENSION MOVING TIME
*10^2
100.00
80.00
60.00
40.00
20.00
0.00
TOTAL INERTIA (KG-M**2)
*10^-1

FIG 8.



θ = 1000.0
JT = 0.0001 → 0.04
PM MOTOR
*10^-1
0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40
NONDIMENSION MOVING TIME *10^3
0.00 20.00 40.00 60.00 80.00 100.00
TOTAL INERTIA (KG-M**2) *10^-1

FIG. 9

θ = 10000.0 rad

JT = 0.0001 → 0.004

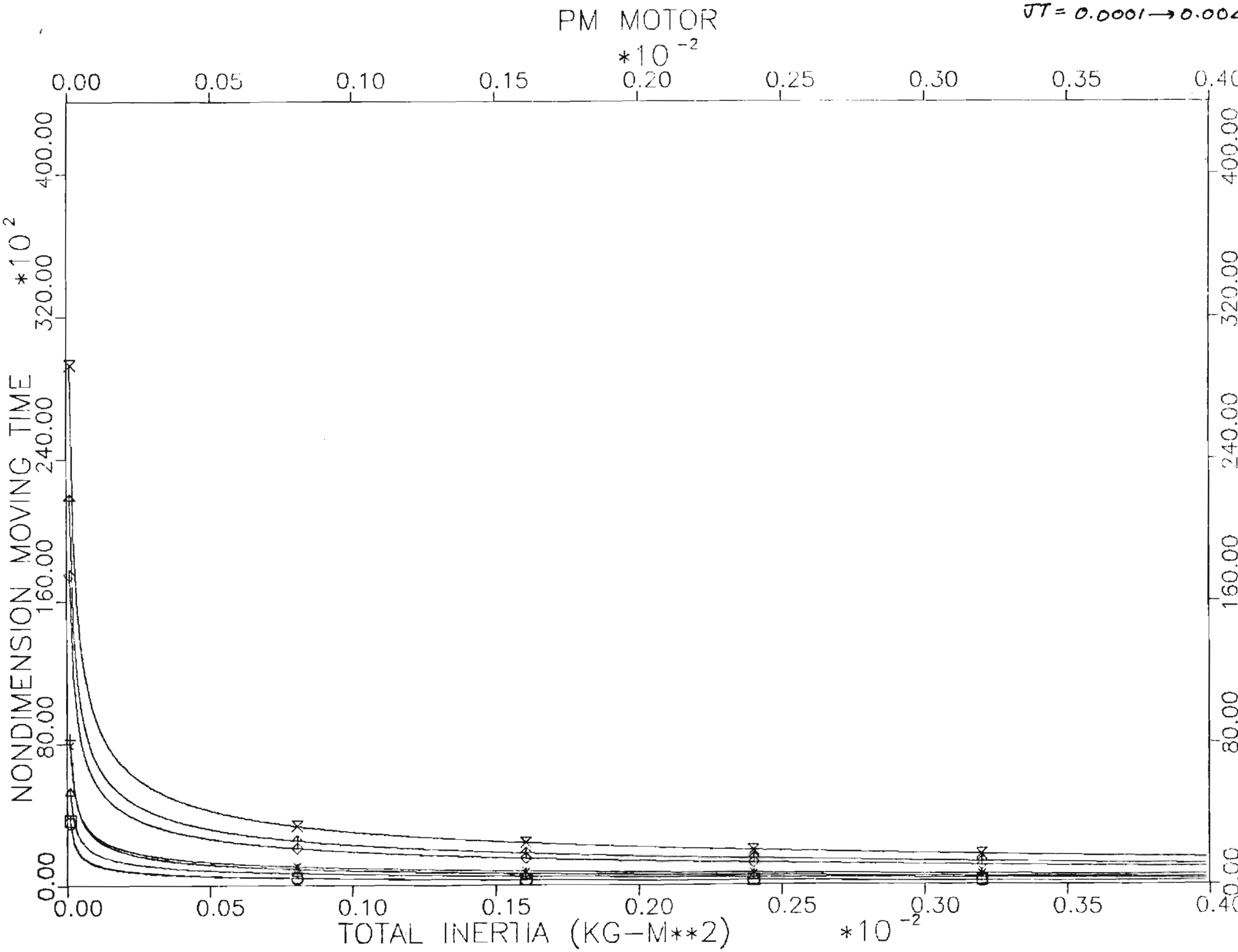


FIG. 10

$\theta = 10000.0$ rad
$JT = 0.0001 \rightarrow 0.004$

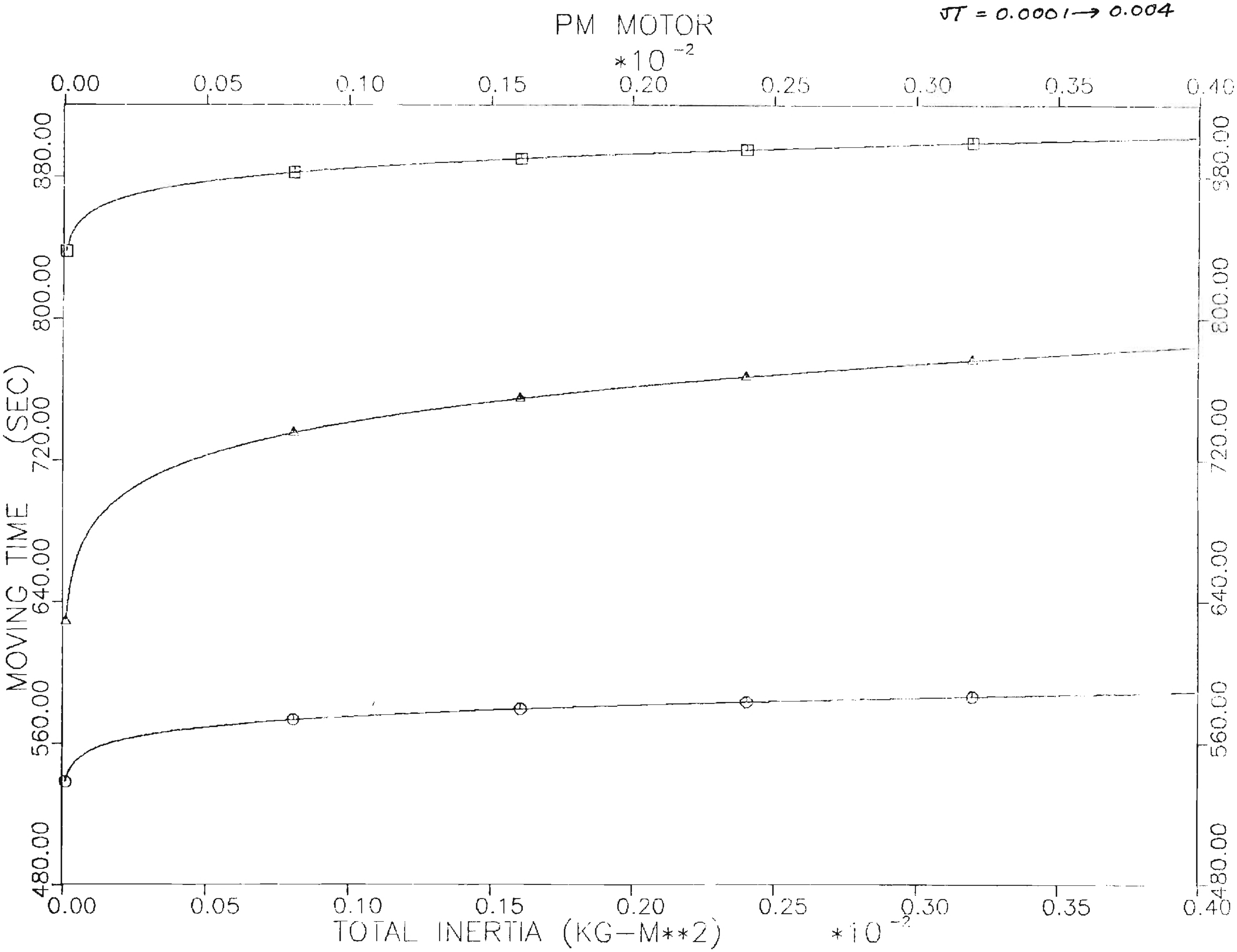


FIG 11

$\theta = 1000.0$ rad
$JT = 0.0001 \rightarrow 0.004$

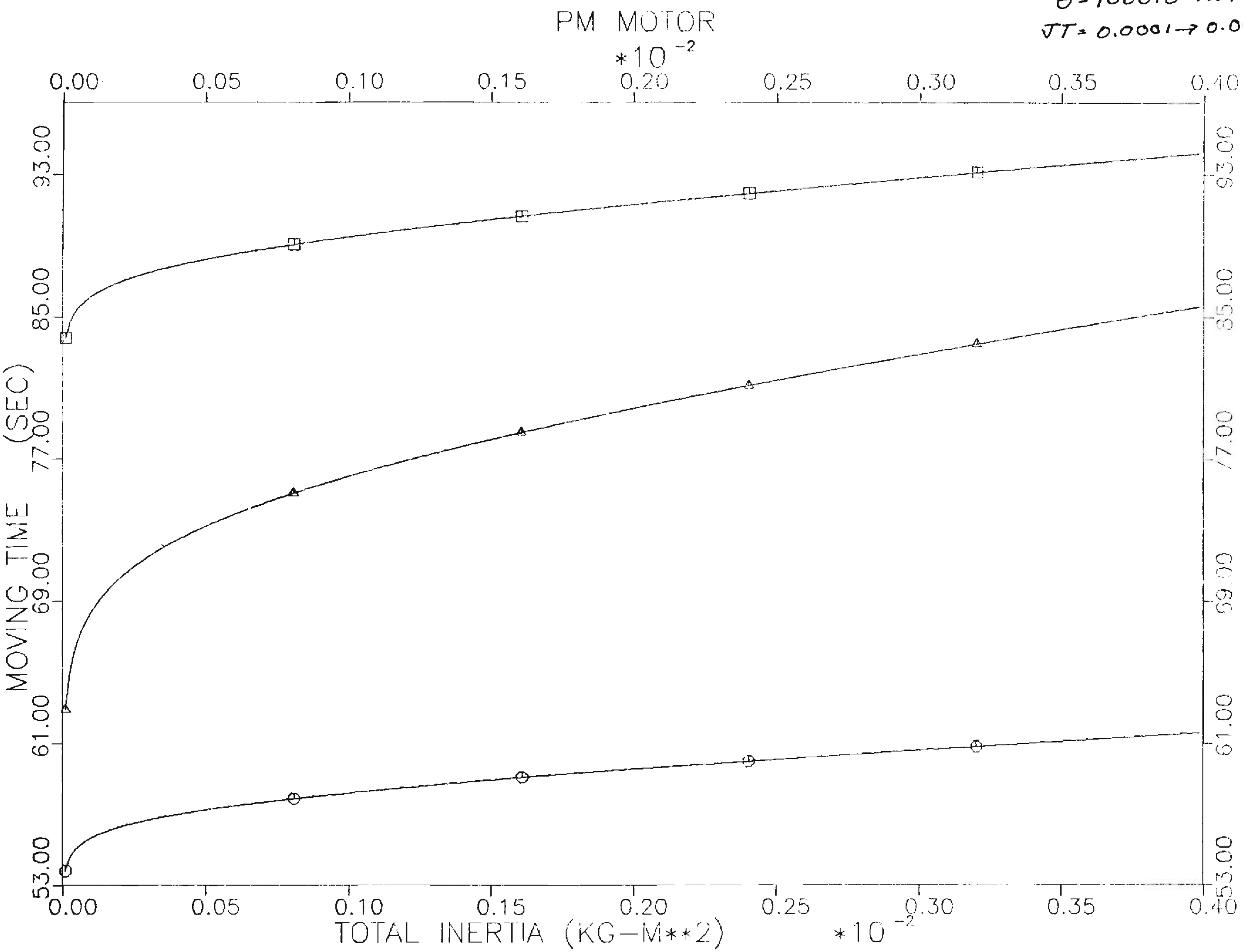


FIG. 12.

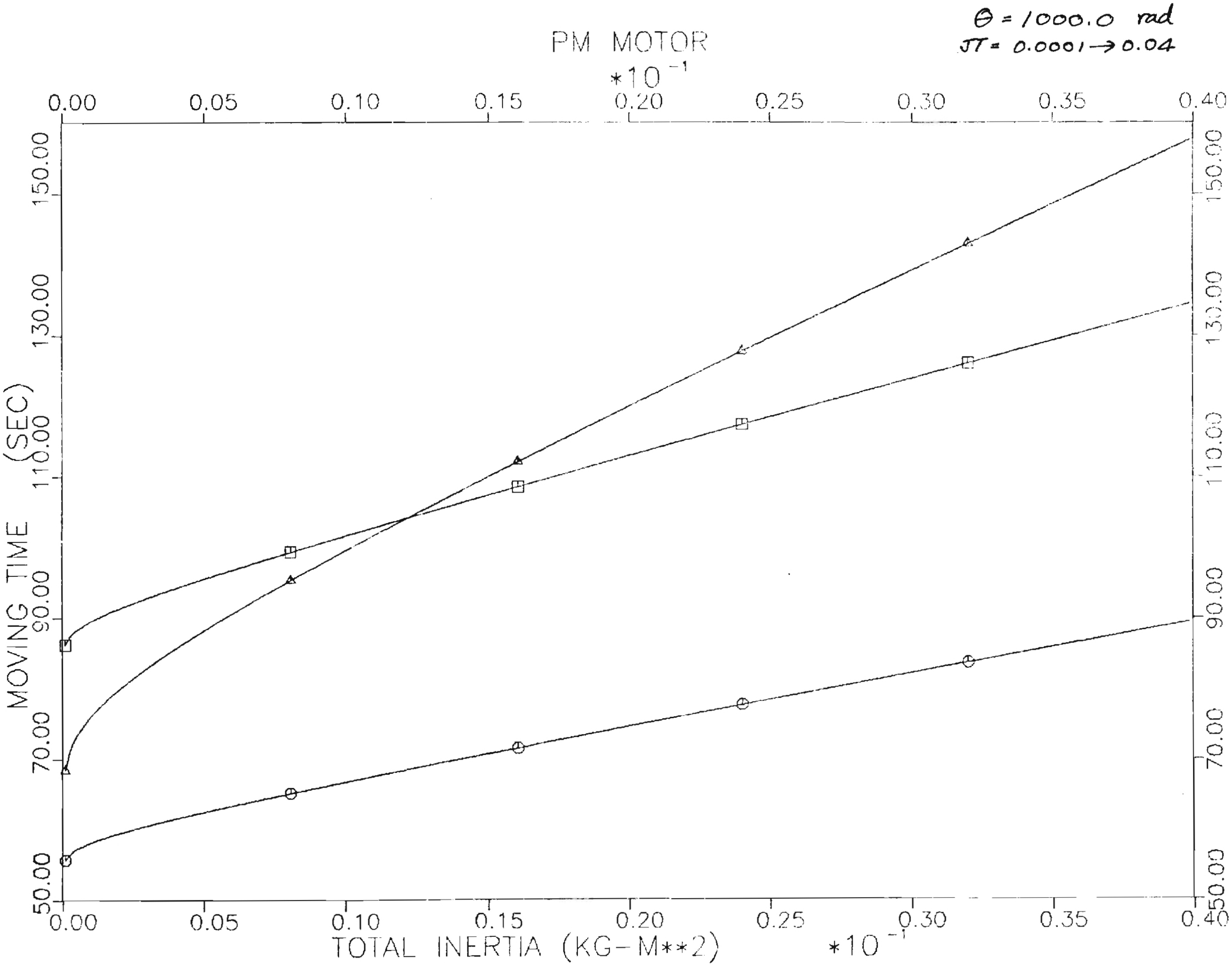
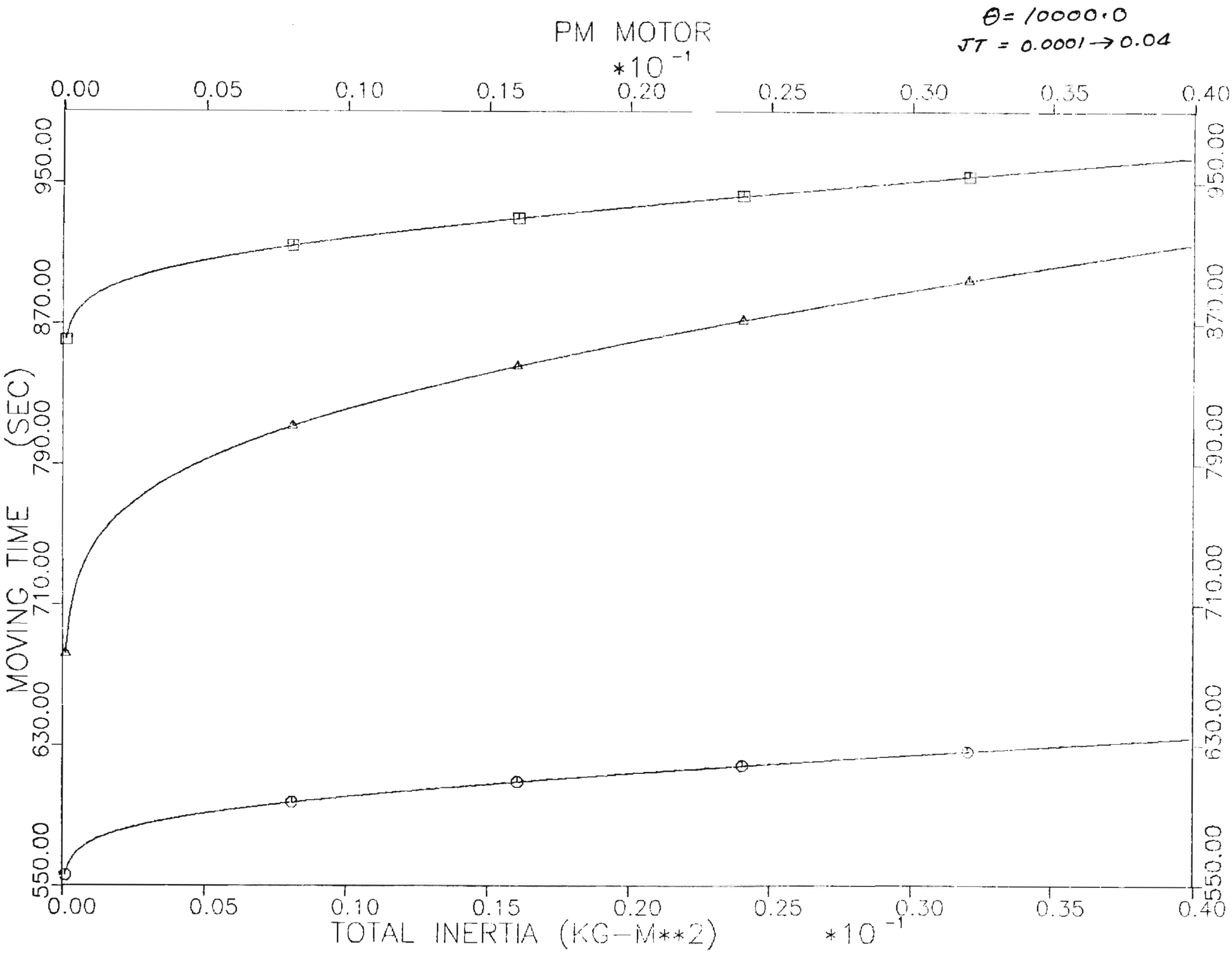

θ = 1000.0 rad
JT = 0.0001 → 0.04
PM MOTOR
*10 -1
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
50.00
70.00
90.00
110.00
130.00
150.00
MOVING TIME (SEC)
TOTAL INERTIA (KG-M**2) *10 -1

FIG. 13



PM MOTOR
θ= 10000.0
JT = 0.0001 → 0.04
*10 -1
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
950.00
870.00
790.00
710.00
630.00
550.00
MOVING TIME (SEC)
TOTAL INERTIA (KG-M**2) *10 -1

FIG 14

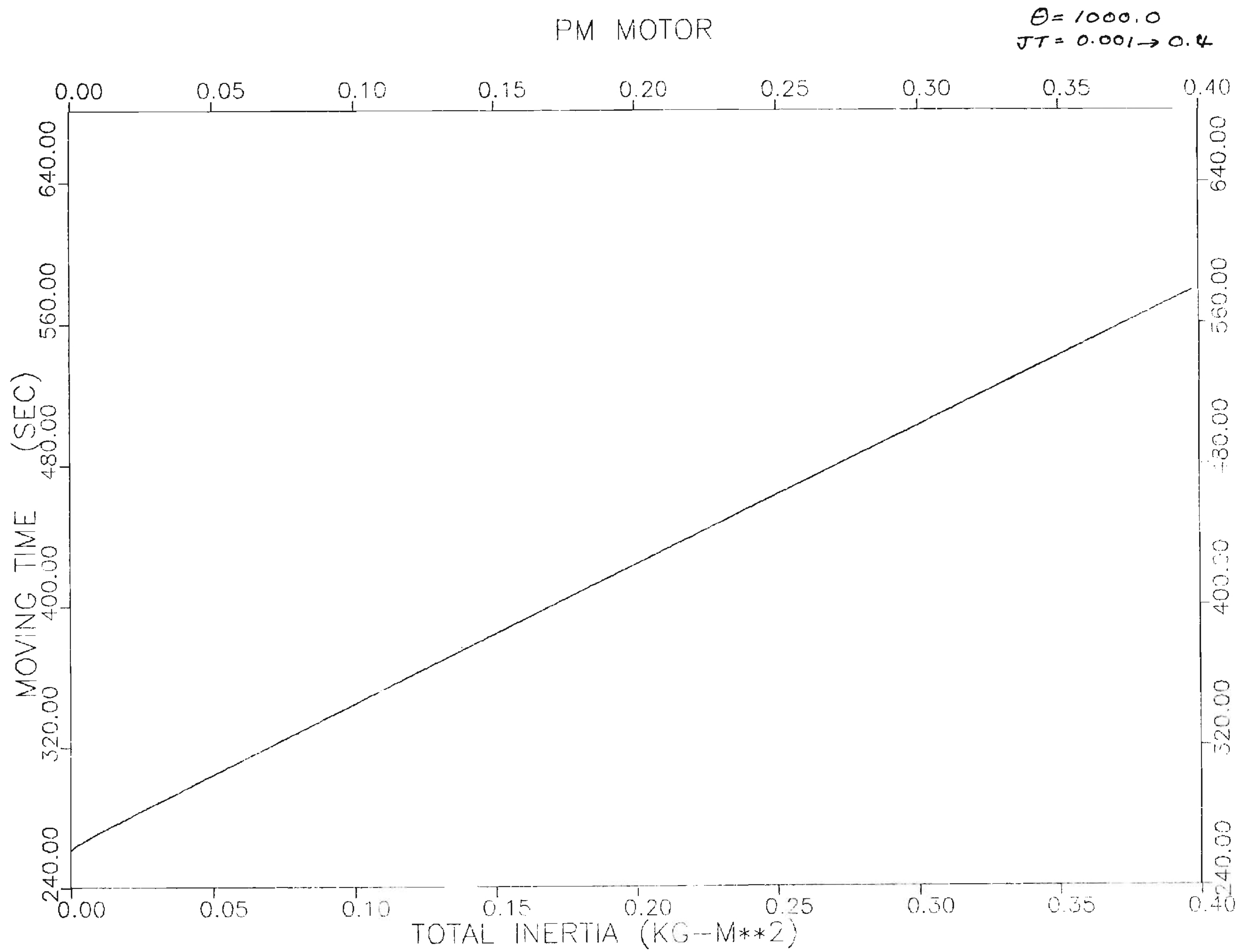

PM MOTOR
θ= 1000.0
JT = 0.001→0.4
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
640.00
560.00
480.00
400.00
320.00
240.00
MOVING TIME (SEC)
TOTAL INERTIA (KG-M**2)

FIG 15

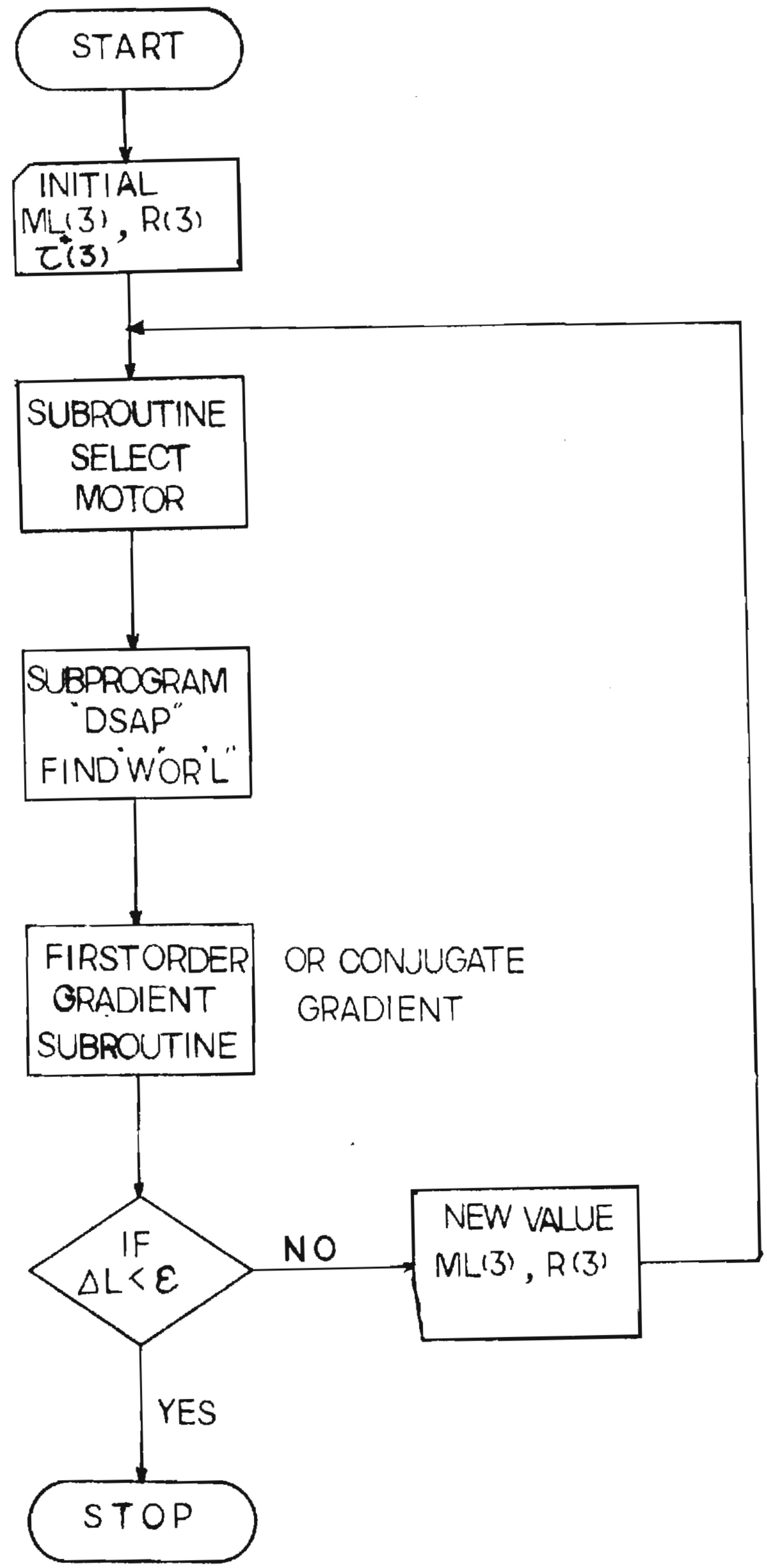


FIG. 16. MAIN PROGRAM

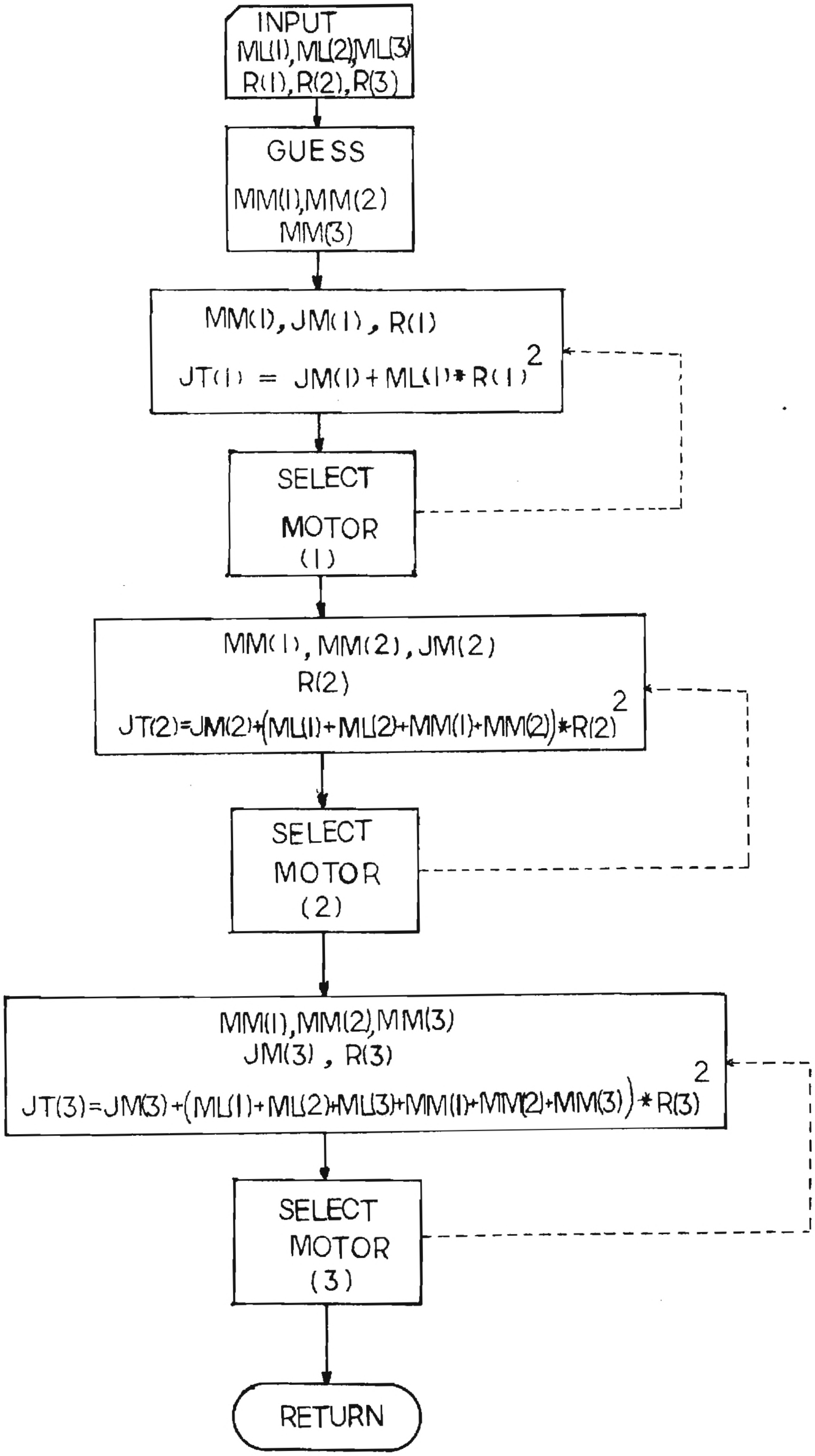

INPUT
ML(1), ML(2), ML(3)
R(1), R(2), R(3)
GUESS
MM(1), MM(2)
MM(3)
MM(1), JM(1), R(1)
JT(1) = JM(1) + ML(1) * R(1)^2
SELECT
MOTOR
(1)
MM(1), MM(2), JM(2)
R(2)
JT(2)=JM(2)+(ML(1)+ML(2)+MM(1)+MM(2))*R(2)^2
SELECT
MOTOR
(2)
MM(1), MM(2), MM(3)
JM(3), R(3)
JT(3)=JM(3)+(ML(1)+ML(2)+ML(3)+MM(1)+MM(2)+MM(3))*R(3)^2
SELECT
MOTOR
(3)
RETURN

FIG 17

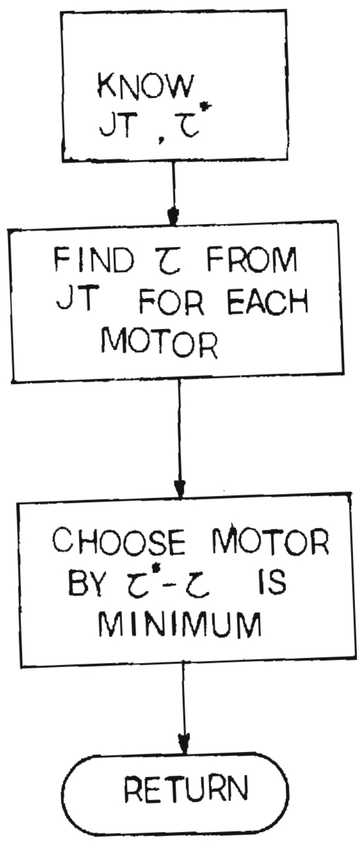


FIG 18 SELECT MOTOR

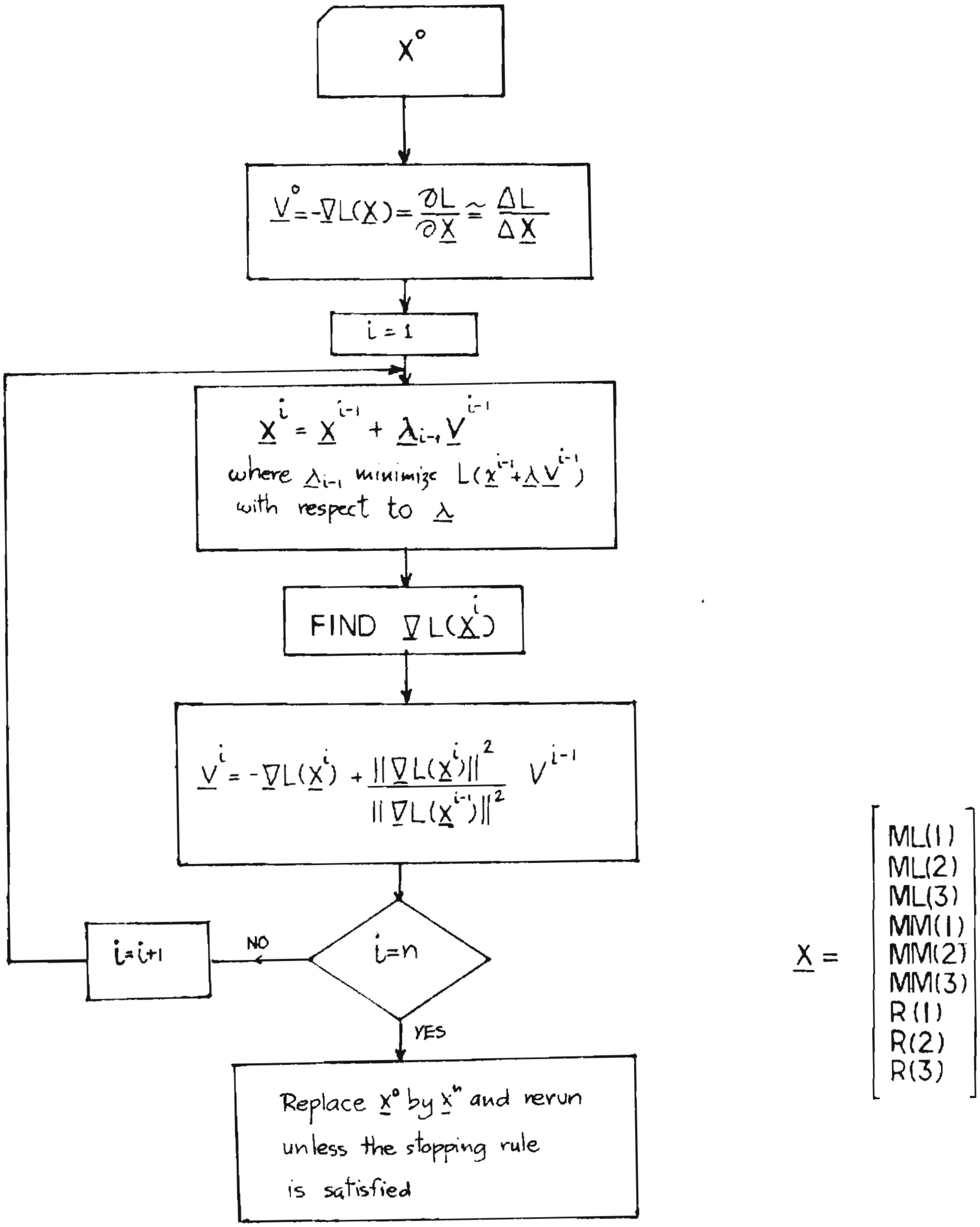
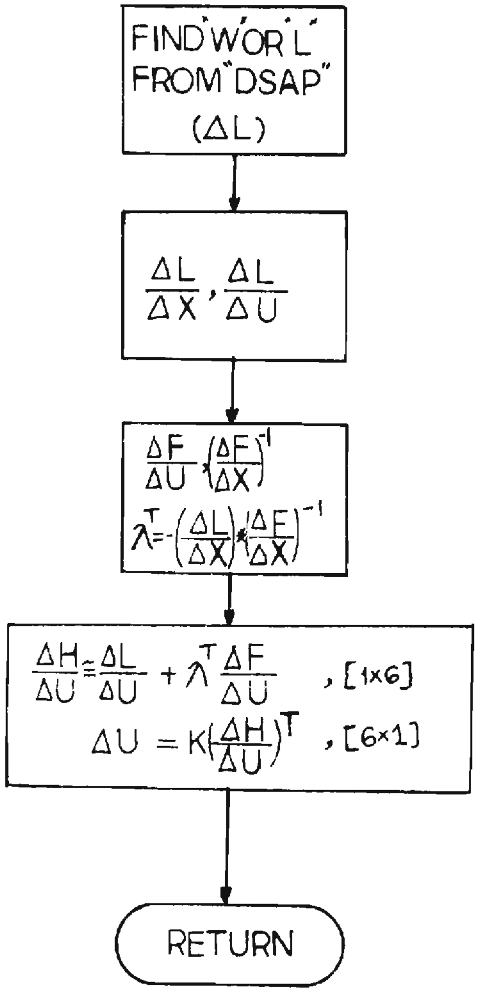


FIG 19 CONJUGATE GRADIENT



$$\underline{X} = \begin{bmatrix} MM(1) \\ MM(2) \\ MM(3) \end{bmatrix} \qquad \underline{U} = \begin{bmatrix} ML(1) \\ ML(2) \\ ML(3) \\ R(1) \\ R(2) \\ R(3) \end{bmatrix}$$

FIG 20 FIRST ORDER GRADIENT

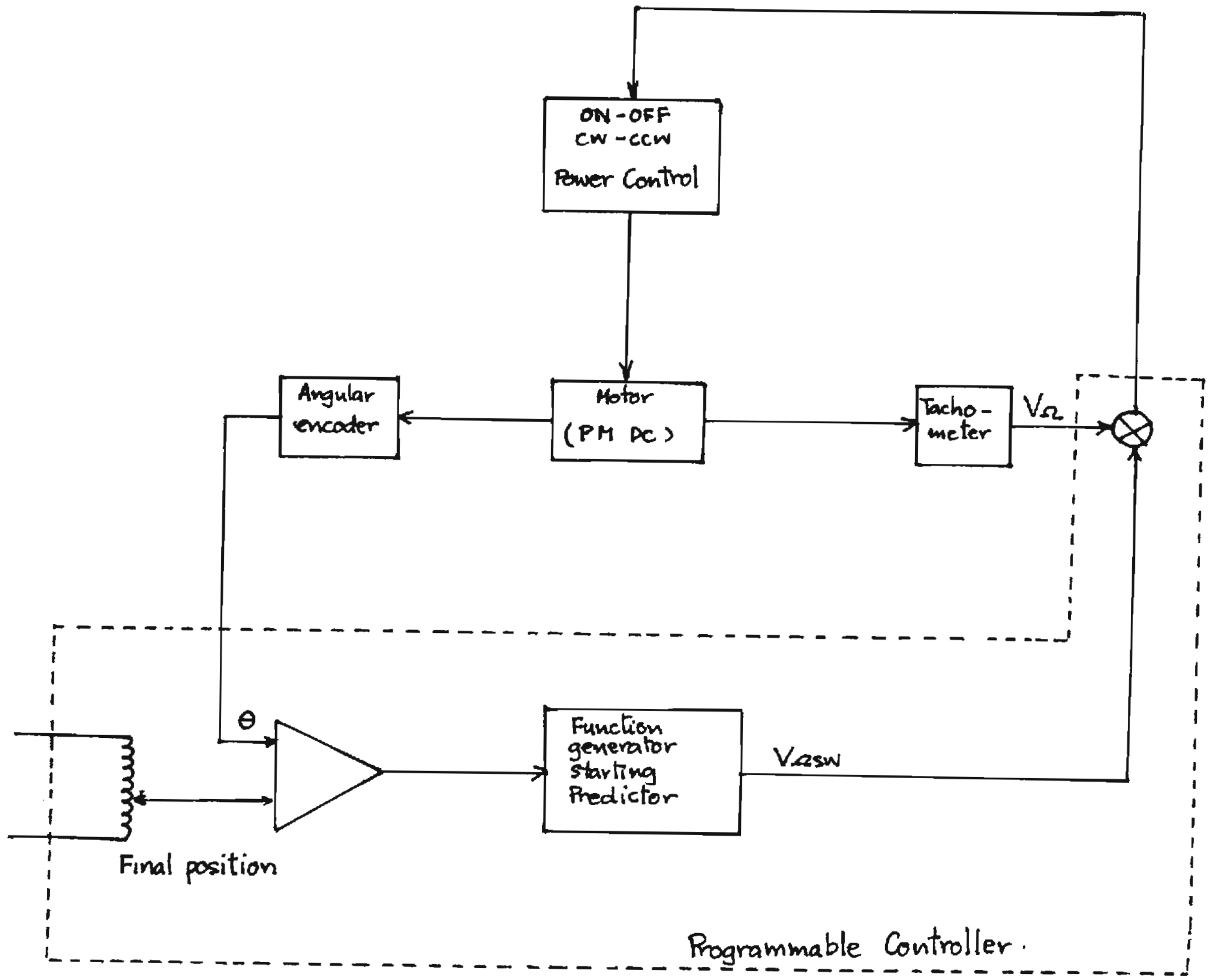

ON-OFF
CW-CCW
Power Control
Angular encoder
Motor
(PM DC)
Tacho-meter
VΩ
θ
Function generator starting predictor
Vasw
Final position
Programmable Controller.

FIG 21

```
      PROGRAM OPT(INPUT,OUTPUT,LINE,TAPE5=INPUT,TAPE6=OUTPUT,
     * TAPE7=LINE)
C
C  MINIMUM-TIME POSITION CONTROL (BANG-BANG CONTROL)
C  THIS PROGRAM SOLVES FOR THE SWITCHING TIME
C   UO = VOLTAGE AT PEAK TORQUE
C   EK = BACK EMF (VOLTAGE/RAD/SEC)
C   AR = AMATURE RESISTANCE (OHM)
C   AL = AMATURE INDUCTANCE (HYS)
C   AIMAX = AMPS AT PEAK TORQUE (AMPS)
C   AKT = TORQUE SENSITIVITY  (KG-M/AMP)
C   AS = INFINITE-IMPEDANCE SOURCE OR DAMPING CONSTANCE (KG-M/SEC)
C   AB = FRICTION TORQUE  (KG-M)
C   AJT = TOTAL INERTIA (KG-M**2)
C   POSIG = ROTATING DISTANCE (RAD)
C   DELG = TESTING CRITERIOR
C   DELT1 = INCREMENT IN TIME
C   TSTART = INITIAL TIME
C   DELTG = INCREMENT IN POSITION
C   FTG = FINAL POSITION
C
C
      REAL UO,EK,AR,AL,AS,AB,AJT,POSIG,DELG,DELT1,TSTART
      REAL SQRT,DELTG,FTG,AIMAX,AKT
C
C
      READ*, POSIG,DELG,DELT1,TSTART
      READ*, DELTG,FTG
      READ*, UO,EK,AR,AL
      READ*,AIMAX,AKT
      READ*,AS,AB,AJT
 5    WRITE(6,13) UO,EK,AR,AL
      WRITE(6,12) AIMAX,AKT
      WRITE(6,14) AS,AB,AJT
      WRITE(6,15) POSIG,DELG,DELT1,TSTART
      VELOF =((UO*EK)-(AB*AR))/((AS*AR)+EK**2)
      AA =(1./2.)*((AR/AL)+(AS/AJT))
      WW =((AS*AR)+EK**2)/(AJT*AL)-AA**2
      IF(WW.LT.0.0) GOTO 2
      WRITE(6,16)
 16   FORMAT ( "  THE ROOTS ARE COMPLEX   ")
      IF(POSIG.GT.FTG) STOP
      GO TO 11
  2   S1 =-AA+SQRT(-WW)
      S2 =-AA-SQRT(-WW)
      AA2=-((EK*UO)+(AB*AR))/((AS*AR)+EK**2)
      SS1=S2/(S1-S2)
      SS2=S1/(S2-S1)
      SS3=(SS2-SS1)/(S1*S2)
      SS4=SS1/S1
      SS5=SS2/S2
      T1=TSTART
```

```
100   S1T1=S1*T1
      S2T1=S2*T1
      IF(ABS(S2T1).GT.50.0) THEN
      EXP2=0.
      ELSE
      EXP2 =EXP(S2T1)
      END IF
      IF (ABS(S1T1).GT.50.0) THEN
      EXP1=0.0
      ELSE
      EXP1 =EXP(S1T1)
      END IF
      VELOT =VELOF*(1.0+SS1*EXP1+SS2*EXP2)
      POSIT =VELOF*(T1+SS3+SS4*EXP1+SS5*EXP2)
      VELOSW=VELOT
      BB2 =(-VELOSW*(S2+(AS/AJT))-AB/AJT+S2*AA2)/(S1-S2)
      TSW =(1./S1)*LOG(-AA2/BB2)
      POSIBR =(-AA2/S1)*(LOG(-BB2/AA2)+1.0)
      POSITO =POSIT+POSIBR
      POS =POSIG-POSITO
      DELPO =ABS(POS)
      IF (DELPO.GT.DELG)THEN
         IF (POSITO.GT.POSIG) GOTO 1000
         T1=T1+DELT1
         GOTO 100
      ELSE
      GOTO 1000
      END IF
1000  TT=T1+TSW
      SQUR=SQRT(AJT/(AKT*9.81**AIMAX))
      TND=TT/SQUR
      WRITE (6,20) VELOSW,TT
      WRITE (6,21) T1,TSW
      WRITE(6,22) POSIG,TND
      WRITE(7,23) POSIG,TND
      IF(POSIG.GT.FTG) STOP
11    POSIG=POSIG+DELTG
      IF(T1.LT.1.0) GOTO 5
      TSTART=AINT(T1)
      GOTO 5
12    FORMAT(2X,6HIMAX= ,F10.6,10X,4HKT= ,F10.6)
13     FORMAT (2X,2HU= ,F5.2,10X,2HK= ,F10.6,10X,3HR =,F5.2,10X,
     * 2HL= ,F10.6)
14    FORMAT (2X,2HA= ,F10.7,5X,2HB= ,F10.6,10X,
     * 7HJTOTAL= ,E13.6)
15    FORMAT (2X,9HPOSITION= ,F10.2,2X,3HRAD,F5.3,15X,F5.3,10X,F10.3)
20    FORMAT (2X,19HSWITCHING VELOCITY= ,F10.3,2X,7HRAD/SEC ,
     * 2X,12HMOVING TIME= ,F10.3,2X,3HSEC)
21    FORMAT (2X,24HTIME BEFORE SWITCHING = ,F7.3,2X,4HSEC ,13X,
     * 18HDECELERATED TIME = ,F7.3)
22    FORMAT(2X,17HMOVING DISTANCE= ,F12.2,1X,3HRAD,3X,
     *  21HNONDIMENSIONAL TIME= ,E18.10//)
23    FORMAT(2X,F12.2,2X,E18.10)
      END
```

```
U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=    .040746          JTOTAL=  .400000E-02
POSITION=     20.00  RAD .100                   .010                  .005
SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=       .699  SEC
TIME BEFORE SWITCHING =     .655  SEC               DECELERATED TIME =   .044
MOVING DISTANCE=         20.00 RAD   NONDIMENSIONAL TIME=    .1060791368E+03

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=    .040746          JTOTAL=  .400000E-02
POSITION=    220.00  RAD .100                   .010                  .005
SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=      6.119  SEC
TIME BEFORE SWITCHING =   6.075  SEC               DECELERATED TIME =   .044
MOVING DISTANCE=        220.00 RAD   NONDIMENSIONAL TIME=    .9282180065E+03

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=    .040746          JTOTAL=  .400000E-02
POSITION=    420.00  RAD .100                   .010                 6.000
SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=     11.534  SEC
TIME BEFORE SWITCHING =  11.490  SEC               DECELERATED TIME =   .044
MOVING DISTANCE=        420.00 RAD   NONDIMENSIONAL TIME=    .1749598357E+04

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=    .040746          JTOTAL=  .400000E-02
POSITION=    620.00  RAD .100                   .010                11.000
SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=     16.954  SEC
TIME BEFORE SWITCHING =  16.910  SEC               DECELERATED TIME =   .044
MOVING DISTANCE=        620.00 RAD   NONDIMENSIONAL TIME=    .2571737137E+04

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=    .040746          JTOTAL=  .400000E-02
POSITION=    820.00  RAD .100                   .010                16.000
SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=     22.374  SEC
TIME BEFORE SWITCHING =  22.330  SEC               DECELERATED TIME =   .044
MOVING DISTANCE=        820.00 RAD   NONDIMENSIONAL TIME=    .3393875918E+04

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=    .040746          JTOTAL=  .400000E-02
POSITION=   1020.00  RAD .100                   .010                22.000
SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=     27.794  SEC
TIME BEFORE SWITCHING =  27.750  SEC               DECELERATED TIME =   .044
MOVING DISTANCE=       1020.00 RAD   NONDIMENSIONAL TIME=    .4216014698E+04
```

```
U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746          JTOTAL=  .400000E-02
POSITION=   1220.00  RAD .100                 .010              27.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=    33.214  SEC
TIME BEFORE SWITCHING =  33.170  SEC              DECELERATED TIME =   .044
MOVING DISTANCE=       1220.00 RAD   NONDIMENSIONAL TIME=    .5038153479E+04

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746          JTOTAL=  .400000E-02
POSITION=   1420.00  RAD .100                 .010              33.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=    38.634  SEC
TIME BEFORE SWITCHING =  38.590  SEC              DECELERATED TIME =   .044
MOVING DISTANCE=       1420.00 RAD   NONDIMENSIONAL TIME=    .5860292260E+04

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746          JTOTAL=  .400000E-02
POSITION=   1620.00  RAD .100                 .010              38.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=    44.054  SEC
TIME BEFORE SWITCHING =  44.010  SEC              DECELERATED TIME =   .044
MOVING DISTANCE=       1620.00 RAD   NONDIMENSIONAL TIME=    .6682431040E+04

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746          JTOTAL=  .400000E-02
POSITION=   1820.00  RAD .100                 .010              44.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=    49.474  SEC
TIME BEFORE SWITCHING =  49.430  SEC              DECELERATED TIME =   .044
MOVING DISTANCE=       1820.00 RAD   NONDIMENSIONAL TIME=    .7504569821E+04

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746          JTOTAL=  .400000E-02
POSITION=   2020.00  RAD .100                 .010              49.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=    54.904  SEC
TIME BEFORE SWITCHING =  54.860  SEC              DECELERATED TIME =   .044
MOVING DISTANCE=       2020.00 RAD   NONDIMENSIONAL TIME=    .8328225463E+04

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746          JTOTAL=  .400000E-02
POSITION=   2220.00  RAD .100                 .010              54.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=    60.324  SEC
```

```
TIME BEFORE SWITCHING =  60.280  SEC               DECELERATED TIME =   .044
MOVING DISTANCE=       2220.00 RAD   NONDIMENSIONAL TIME=    .9150364244E+04

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=    .040746          JTOTAL=  .400000E-02
POSITION=   2420.00  RAD .100                   .010              60.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=    65.744  SEC
TIME BEFORE SWITCHING =  65.700  SEC               DECELERATED TIME =   .044
MOVING DISTANCE=       2420.00 RAD   NONDIMENSIONAL TIME=    .9972503024E+04

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=    .040746          JTOTAL=  .400000E-02
POSITION=   2620.00  RAD .100                   .010              65.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=    71.164  SEC
TIME BEFORE SWITCHING =  71.120  SEC               DECELERATED TIME =   .044
MOVING DISTANCE=       2620.00 RAD   NONDIMENSIONAL TIME=    .1079464180E+05

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=    .040746          JTOTAL=  .400000E-02
POSITION=   2820.00  RAD .100                   .010              71.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=    76.584  SEC
TIME BEFORE SWITCHING =  76.540  SEC               DECELERATED TIME =   .044
MOVING DISTANCE=       2820.00 RAD   NONDIMENSIONAL TIME=    .1161678059E+05

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=    .040746          JTOTAL=  .400000E-02
POSITION=   3020.00  RAD .100                   .010              76.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=    82.004  SEC
TIME BEFORE SWITCHING =  81.960  SEC               DECELERATED TIME =   .044
MOVING DISTANCE=       3020.00 RAD   NONDIMENSIONAL TIME=    .1243891937E+05

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=    .040746          JTOTAL=  .400000E-02
POSITION=   3220.00  RAD .100                   .010              81.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=    87.424  SEC
TIME BEFORE SWITCHING =  87.380  SEC               DECELERATED TIME =   .044
MOVING DISTANCE=       3220.00 RAD   NONDIMENSIONAL TIME=    .1326105815E+05

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=    .040746          JTOTAL=  .400000E-02
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POSITION=   3420.00  RAD .100                    .010              87.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=    92.844  SEC
TIME BEFORE SWITCHING =  92.800  SEC                 DECELERATED TIME =   .044
MOVING DISTANCE=        3420.00 RAD   NONDIMENSIONAL TIME=    .1408319693E+05

U=29.00           K=    .764000            R = 9.40          L=    .032000
IMAX=   3.100000           KT=     .077585
A=  .0006791     B=    .040746            JTOTAL=  .400000E-02
POSITION=   3620.00  RAD .100                    .010              92.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=    98.264  SEC
TIME BEFORE SWITCHING =  98.220  SEC                 DECELERATED TIME =   .044
MOVING DISTANCE=        3620.00 RAD   NONDIMENSIONAL TIME=    .1490533571E+05

U=29.00           K=    .764000            R = 9.40          L=    .032000
IMAX=   3.100000           KT=     .077585
A=  .0006791     B=    .040746            JTOTAL=  .400000E-02
POSITION=   3820.00  RAD .100                    .010              98.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=   103.684  SEC
TIME BEFORE SWITCHING = 103.640  SEC                 DECELERATED TIME =   .044
MOVING DISTANCE=        3820.00 RAD   NONDIMENSIONAL TIME=    .1572747449E+05

U=29.00           K=    .764000            R = 9.40          L=    .032000
IMAX=   3.100000           KT=     .077585
A=  .0006791     B=    .040746            JTOTAL=  .400000E-02
POSITION=   4020.00  RAD .100                    .010             103.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=   109.104  SEC
TIME BEFORE SWITCHING = 109.060  SEC                 DECELERATED TIME =   .044
MOVING DISTANCE=        4020.00 RAD   NONDIMENSIONAL TIME=    .1654961327E+05

U=29.00           K=    .764000            R = 9.40          L=    .032000
IMAX=   3.100000           KT=     .077585
A=  .0006791     B=    .040746            JTOTAL=  .400000E-02
POSITION=   4220.00  RAD .100                    .010             109.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=   114.524  SEC
TIME BEFORE SWITCHING = 114.480  SEC                 DECELERATED TIME =   .044
MOVING DISTANCE=        4220.00 RAD   NONDIMENSIONAL TIME=    .1737175205E+05

U=29.00           K=    .764000            R = 9.40          L=    .032000
IMAX=   3.100000           KT=     .077585
A=  .0006791     B=    .040746            JTOTAL=  .400000E-02
POSITION=   4420.00  RAD .100                    .010             114.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=   119.944  SEC
TIME BEFORE SWITCHING = 119.900  SEC                 DECELERATED TIME =   .044
MOVING DISTANCE=        4420.00 RAD   NONDIMENSIONAL TIME=    .1819389083E+05

U=29.00           K=    .764000            R = 9.40          L=    .032000
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IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746            JTOTAL=   .400000E-02
POSITION=   4620.00  RAD .100                     .010                119.000
SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=   125.364  SEC
TIME BEFORE SWITCHING = 125.320  SEC                  DECELERATED TIME =   .044
MOVING DISTANCE=       4620.00 RAD   NONDIMENSIONAL TIME=    .1901602961E+05

U=29.00          K=    .764000           R = 9.40          L=    .032000
IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746            JTOTAL=   .400000E-02
POSITION=   4820.00  RAD .100                     .010                125.000
SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=   130.784  SEC
TIME BEFORE SWITCHING = 130.740  SEC                  DECELERATED TIME =   .044
MOVING DISTANCE=       4820.00 RAD   NONDIMENSIONAL TIME=    .1983816839E+05

U=29.00          K=    .764000           R = 9.40          L=    .032000
IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746            JTOTAL=   .400000E-02
POSITION=   5020.00  RAD .100                     .010                130.000
SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=   136.204  SEC
TIME BEFORE SWITCHING = 136.160  SEC                  DECELERATED TIME =   .044
MOVING DISTANCE=       5020.00 RAD   NONDIMENSIONAL TIME=    .2066030717E+05

U=29.00          K=    .764000           R = 9.40          L=    .032000
IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746            JTOTAL=   .400000E-02
POSITION=   5220.00  RAD .100                     .010                136.000
SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=   141.624  SEC
TIME BEFORE SWITCHING = 141.580  SEC                  DECELERATED TIME =   .044
MOVING DISTANCE=       5220.00 RAD   NONDIMENSIONAL TIME=    .2148244595E+05

U=29.00          K=    .764000           R = 9.40          L=    .032000
IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746            JTOTAL=   .400000E-02
POSITION=   5420.00  RAD .100                     .010                141.000
SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=   147.044  SEC
TIME BEFORE SWITCHING = 147.000  SEC                  DECELERATED TIME =   .044
MOVING DISTANCE=       5420.00 RAD   NONDIMENSIONAL TIME=    .2230458473E+05

U=29.00          K=    .764000           R = 9.40          L=    .032000
IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746            JTOTAL=   .400000E-02
POSITION=   5620.00  RAD .100                     .010                147.000
SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=   152.464  SEC
TIME BEFORE SWITCHING = 152.420  SEC                  DECELERATED TIME =   .044
MOVING DISTANCE=       5620.00 RAD   NONDIMENSIONAL TIME=    .2312672351E+05
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U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=     .040746          JTOTAL=  .400000E-02
POSITION=   5820.00  RAD .100                 .010              152.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=   157.884  SEC
TIME BEFORE SWITCHING = 157.840  SEC                DECELERATED TIME =   .044
MOVING DISTANCE=       5820.00 RAD   NONDIMENSIONAL TIME=    .2394886230E+05

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=     .040746          JTOTAL=  .400000E-02
POSITION=   6020.00  RAD .100                 .010              157.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=   163.304  SEC
TIME BEFORE SWITCHING = 163.260  SEC                DECELERATED TIME =   .044
MOVING DISTANCE=       6020.00 RAD   NONDIMENSIONAL TIME=    .2477100108E+05

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=     .040746          JTOTAL=  .400000E-02
POSITION=   6220.00  RAD .100                 .010              163.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=   168.724  SEC
TIME BEFORE SWITCHING = 168.680  SEC                DECELERATED TIME =   .044
MOVING DISTANCE=       6220.00 RAD   NONDIMENSIONAL TIME=    .2559313986E+05

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=     .040746          JTOTAL=  .400000E-02
POSITION=   6420.00  RAD .100                 .010              168.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=   174.144  SEC
TIME BEFORE SWITCHING = 174.100  SEC                DECELERATED TIME =   .044
MOVING DISTANCE=       6420.00 RAD   NONDIMENSIONAL TIME=    .2641527864E+05

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=     .040746          JTOTAL=  .400000E-02
POSITION=   6620.00  RAD .100                 .010              174.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=   179.564  SEC
TIME BEFORE SWITCHING = 179.520  SEC                DECELERATED TIME =   .044
MOVING DISTANCE=       6620.00 RAD   NONDIMENSIONAL TIME=    .2723741742E+05

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=   3.100000          KT=      .077585
A=  .0006791      B=     .040746          JTOTAL=  .400000E-02
POSITION=   6820.00  RAD .100                 .010              179.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=   184.984  SEC
TIME BEFORE SWITCHING = 184.940  SEC                DECELERATED TIME =   .044
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MOVING DISTANCE=      6820.00 RAD   NONDIMENSIONAL TIME=    .2805955620E+05

U=29.00          K=    .764000          R = 9.40          L=   .032000
IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746          JTOTAL=  .400000E-02
POSITION=   7020.00  RAD .100                    .010              184.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=   190.404  SEC
TIME BEFORE SWITCHING = 190.360  SEC               DECELERATED TIME =   .044
MOVING DISTANCE=      7020.00 RAD   NONDIMENSIONAL TIME=    .2888169498E+05

U=29.00          K=    .764000          R = 9.40          L=   .032000
IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746          JTOTAL=  .400000E-02
POSITION=   7220.00  RAD .100                    .010              190.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=   195.824  SEC
TIME BEFORE SWITCHING = 195.780  SEC               DECELERATED TIME =   .044
MOVING DISTANCE=      7220.00 RAD   NONDIMENSIONAL TIME=    .2970383376E+05

U=29.00          K=    .764000          R = 9.40          L=   .032000
IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746          JTOTAL=  .400000E-02
POSITION=   7420.00  RAD .100                    .010              195.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=   201.244  SEC
TIME BEFORE SWITCHING = 201.200  SEC               DECELERATED TIME =   .044
MOVING DISTANCE=      7420.00 RAD   NONDIMENSIONAL TIME=    .3052597254E+05

U=29.00          K=    .764000          R = 9.40          L=   .032000
IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746          JTOTAL=  .400000E-02
POSITION=   7620.00  RAD .100                    .010              201.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=   206.664  SEC
TIME BEFORE SWITCHING = 206.620  SEC               DECELERATED TIME =   .044
MOVING DISTANCE=      7620.00 RAD   NONDIMENSIONAL TIME=    .3134811132E+05

U=29.00          K=    .764000          R = 9.40          L=   .032000
IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746          JTOTAL=  .400000E-02
POSITION=   7820.00  RAD .100                    .010              206.000
SWITCHING VELOCITY=    36.898  RAD/SEC  MOVING TIME=   212.084  SEC
TIME BEFORE SWITCHING = 212.040  SEC               DECELERATED TIME =   .044
MOVING DISTANCE=      7820.00 RAD   NONDIMENSIONAL TIME=    .3217025010E+05

U=29.00          K=    .764000          R = 9.40          L=   .032000
IMAX=   3.100000          KT=     .077585
A=  .0006791     B=    .040746          JTOTAL=  .400000E-02
POSITION=   8020.00  RAD .100                    .010              212.000
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SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=   217.504  SEC
TIME BEFORE SWITCHING = 217.460  SEC                DECELERATED TIME =   .044
MOVING DISTANCE=       8020.00 RAD   NONDIMENSIONAL TIME=    .3299238888E+05

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=    3.100000          KT=      .077585
A=  .0006791     B=    .040746          JTOTAL=  .400000E-02
POSITION=   8220.00  RAD .100                  .010                217.000
SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=   222.924  SEC
TIME BEFORE SWITCHING = 222.880  SEC                DECELERATED TIME =   .044
MOVING DISTANCE=       8220.00 RAD   NONDIMENSIONAL TIME=    .3381452766E+05

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=    3.100000          KT=      .077585
A=  .0006791     B=    .040746          JTOTAL=  .400000E-02
POSITION=   8420.00  RAD .100                  .010                222.000
SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=   228.344  SEC
TIME BEFORE SWITCHING = 228.300  SEC                DECELERATED TIME =   .044
MOVING DISTANCE=       8420.00 RAD   NONDIMENSIONAL TIME=    .3463666644E+05

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=    3.100000          KT=      .077585
A=  .0006791     B=    .040746          JTOTAL=  .400000E-02
POSITION=   8620.00  RAD .100                  .010                228.000
SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=   233.764  SEC
TIME BEFORE SWITCHING = 233.720  SEC                DECELERATED TIME =   .044
MOVING DISTANCE=       8620.00 RAD   NONDIMENSIONAL TIME=    .3545880522E+05

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=    3.100000          KT=      .077585
A=  .0006791     B=    .040746          JTOTAL=  .400000E-02
POSITION=   8820.00  RAD .100                  .010                233.000
SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=   239.184  SEC
TIME BEFORE SWITCHING = 239.140  SEC                DECELERATED TIME =   .044
MOVING DISTANCE=       8820.00 RAD   NONDIMENSIONAL TIME=    .3628094400E+05

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=    3.100000          KT=      .077585
A=  .0006791     B=    .040746          JTOTAL=  .400000E-02
POSITION=   9020.00  RAD .100                  .010                239.000
SWITCHING VELOCITY=     36.898  RAD/SEC  MOVING TIME=   244.614  SEC
TIME BEFORE SWITCHING = 244.570  SEC                DECELERATED TIME =   .044
MOVING DISTANCE=       9020.00 RAD   NONDIMENSIONAL TIME=    .3710459965E+05

U=29.00          K=    .764000          R = 9.40          L=    .032000
IMAX=    3.100000          KT=      .077585
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