A GENERAL APPROACH TO NETWORK EQUIVALENTS FOR ON-LINE POWER SYSTEM SECURITY ASSESSMENT

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A GENERAL APPROACH TO NETWORK EQUIVALENTS

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Dedicated to

my family

and especially to my

daughter, Eleftheria

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SUMMARY

This thesis addresses the problem of determining the static equivalent model of an electric power system connected to several external systems. Equivalent models are important for contingency analysis in the process of security assessment. The problem is to find an equivalent representation of the external system which will reproduce the actual power flows for a set of postulated outages with a guaranteed level of accuracy.

Emphasis is placed on obtaining the equivalent model by using information from the internal system only because of limited exchange of information between the neighboring companies.

A procedure is developed which yields the optimal equivalent model over a set of postulated outages. The problem is formulated as an optimization problem. The unknowns are the parameter values of the fictitious branches among the boundary busses. The connectivity among the boundary busses should be constraint so that the admittance matrix of the equivalent model is a sparse matrix.

The classical Norton-type equivalent is treated as an a priori information assuming that the topology and the parameter values of the external system might be available. Mainly, two problems related to the Norton equivalent model are investigated: the sparsity of the equivalent admittance matrix and the sensitivity of the equivalent model to changes in the external system. Two elimination procedures are developed so that the equivalent admittance matrix preserves its

sparse structure. A detection scheme to detect topological changes in the external system is also developed.

If the Norton-type equivalent model does not satisfy the accuracy requirements of the equivalencing problem, one has to solve an optimization problem. This optimization problem is in the form of minimizing a performance index subject to a set of linear constraints. It has been observed that in many cases it is not necessary to solve the optimization problem in its entirety. First, the model obtained from the solution of the unconstrained problem should be tested. If this test is successful, the model is satisfactory. If not, the optimization problem needs to be solved in its entirety. The quadratic programming approach was chosen as the method to solve the constrained problem.

The procedure of this thesis has been implemented and tested. Several simulation results are included.

In summary, this thesis indicates that data on actual system outages can be effectively and directly used to obtain external system equivalents. The resulting scheme of this thesis yields an equivalent representation of the external system with guaranteed accuracy in predicting the effect of postulated system outages for on-line steady state security assessment.

CHAPTER I

INTRODUCTION

1.1 General

This thesis addresses the problem of the static equivalent model of an electric power system connected to several external systems as it relates to the contingency evaluation problem in the process of security assessment. Its objective is the development of a systematic procedure which yields an equivalent representation of the external system with guaranteed accuracy in predicting the effects of postulated outages for on-line steady-state security assessment.

Over the last decade much importance has been given to the security assessment of electric power systems. Advanced techniques from different areas as control theory, pattern recognition, etc., have been introduced in power system analysis. All these techniques aim at helping the operator to assure electric power service under all conditions of operation. The system's operator is concerned with various inequality constraints (frequency drop, overloading of lines, etc.) and with equality constraints (generation meets the demand)[2-4].

Based on these equality and inequality constraints, it is possible to classify the operating conditions of the system which might prevail into three basic states [25]:

1. Normal State
2. Emergency State

3. Restorative State

In the Normal State all equality and inequality constraints are satisfied. In the Emergency State some of the inequality constraints are violated. In the Restorative State some of the equality constraints are violated.

Figure 1.1 shows the several operating states and the associated control strategies. A brief description of the control strategies follows.

If the system is in the emergency state, the operator should try to maintain the generation vs load balance without any further frequency drop. The control action in this case, referred to as emergency control action, consists of a set of strategies for dropping generation and/or load for every possible major fault. The result of the emergency control is to bring the system to the restorative state. Further control action is needed, known as restorative control action, to bring the system from the Restorative State to the Normal State.

The Normal State can be decomposed into two states:

(1) Secure Normal State

(2) Insecure Normal State

If the system is in the Secure Normal State, single system contingencies such as a loss of transmission line or a generator does not cause departure from the Normal State. If the system is in the Insecure Normal State, single system contingencies may cause departure from the Normal State to the Emergency State.

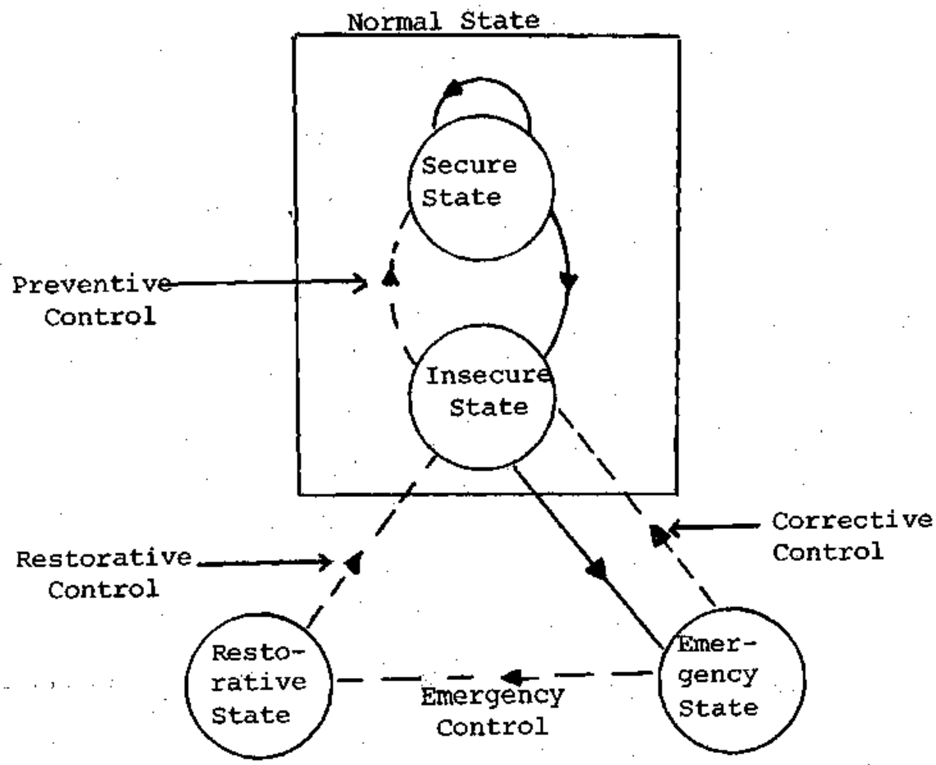

Figure 1.1. Power System Operating States with Associated State Transitions [2].

The primary concern of the system operator is to keep the operating condition of the power system in the Normal State to ensure service continuity at standard frequency and voltage. The operator should continuously test the capability of the power system to withstand postulated next contingencies. This testing is referred to as contingency analysis. The contingency analysis involves two steps.

(1) Computation of a load flow solution of the present operating condition of the system. Load-flow solution means solution of the power flow equations for the voltage magnitudes and voltage phase angles of the busses of the system. This requires application of the classical load-flow methods [11-14] which utilize short-term bus load forecasting or application of more advanced techniques such as estimation techniques [5-10] which utilize on-line information.

(2) Computation of the load-flow solution of the system for the various single line or generator outages.

Based on the contingency analysis, security indices are computed. The security indices show how "secure" the system is under the present operating conditions and indicates if the system is in Secure Normal State or in Insecure Normal State [26]. If the system is in the Secure Normal State, no control action is needed. If the system is in the Insecure Normal State, preventive control action should be taken to bring the system back in the Secure Normal State in the most economical way. Examples of preventive control action are:

(a) Shifting of generation schedules

(b) Switching operations

(c) Start-up of units

(d) Changing of the scheduled exchange of power with the neighboring companies.

The role of the static equivalent model in the security assessment of power systems is discussed next.

In recent years the number of interconnections between neighboring companies has been increased. Power companies do not operate independently of each other as was the common practice in the past. Capital savings which are achieved by reducing spinning reserve requirements or by reducing capacity requirements force the individual companies to become parts of a power pool. Therefore, the operating conditions and performance of each company becomes dependent on the operating conditions of the neighboring companies.

The interconnections with the neighboring companies considerably influence the redistribution of network power flows and voltage levels after outages take place in the particular company. Therefore in performing contingency analysis, the knowledge of the precontingency load flow solution of the entire area is required. This requires complete exchange of information between the neighboring companies. This is impractical and difficult to achieve at present because of storage and time limitations of todays computers. In order for a particular company to perform the contingency analysis an equivalent representation of the external (neighboring) systems is needed. An equivalent representation is a mathematical model which represents the unobservable part of the system in the process of contingency analysis. In some cases, this representation is exact. In the cases under study, this equivalent representation is only an approximation.

The existing approaches to obtain the equivalent representation of the external system can be classified into two categories.

(1) Norton-type equivalents: To obtain a Norton-type equivalent, knowledge of the topology and network parameters of the external system is necessary. The model is obtained by linear reduction of the external network to the boundaries of the internal system.

(2) On-line type equivalents: On-line type equivalents assume no knowledge about the external system and they use information from the internal system only to obtain the equivalent representation of the external system.

In this thesis emphasis is placed on obtaining the equivalent model by utilizing information from the internal system only because of limited exchange of information between neighboring companies. The assumption that the topology and the parameter values of the external system are available is valid for planning purposes but in most cases is unrealistic for on-line operation.

It should be emphasized that equivalence techniques are applied also for planning purposes but for different reasons than for on-line security assessment. In planning the primary purpose of the network reduction is to avoid the computational burden of solving the load-flow for the entire area.

The next section reviews the available methods to obtain the equivalent representation of the external system.

1.2 Historical Background

In any network equivalencing problem the overall area is divided into an internal system and an external system as it is shown in Figure 1.2. In stricter terms, the internal system consists of the observable part of the overall system as obtained from on-line measurements and bus load forecasts and estimates. Some busses of the internal system are connected to the external system. These busses are called boundary busses.

In most of the approaches given in the literature, the following steps are taken:

(a) Define the boundaries of the internal system.

(b) Reduce by means of Norton equivalent the external system to the boundary.

(c) Classify the boundary busses as generation busses or as load busses.

In reference 15 the internal system is augmented by a buffer zone as it is shown in Figure 1.3. This buffer zone includes:

(1) Busses of the external system critical to the accuracy of the equivalent.

(2) Components of the external system of which the operational limitations may be violated due to disturbances in the internal system (weak links).

(3) Generation busses of the external system which control the operating conditions in the internal system (controlling busses).

The weak links are found by imposing extreme stressing conditions

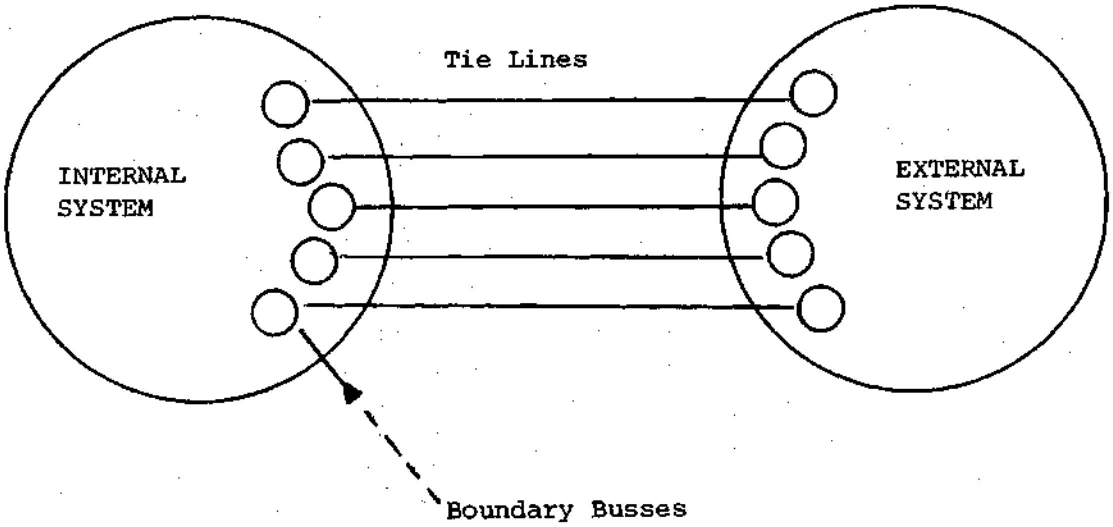

Figure 1.2. System Decomposition with Associated Term Definitions.

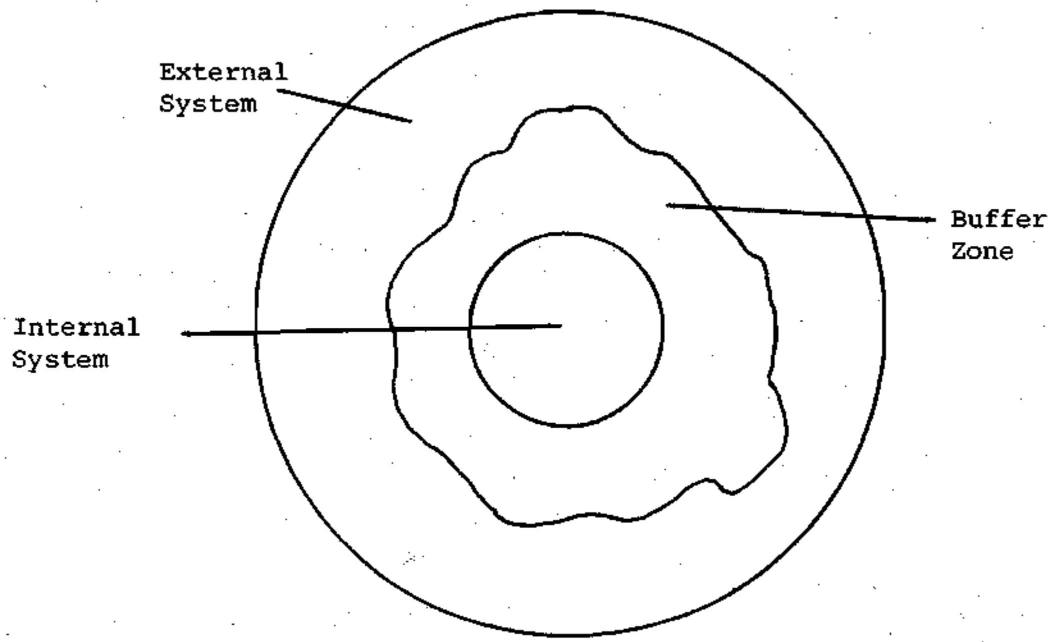

Figure 1.3. Augmentation of Internal System by Means of a Buffer Zone [15].

in the internal system. Since an unobservable part of the external system is included in the equivalent, several simplifications and assumptions are needed which jeopardize the accuracy of the equivalent system.

In reference 16 some busses of the external system are included in order to preserve sparsity in the equivalent representation. Simulation studies on power systems, however, have shown that the problem of sparse structure is not so crucial. Even if the number of the equivalent branches is extremely large, a portion of them may be eliminated by using a technique proposed in reference 17 or by a simpler method as we will propose later without significant sacrifice of the accuracy.

In reference 17 the boundary busses are assumed to be load busses; therefore, many of the equivalent branches between the boundary busses are eliminated by using, as criterion, the ratio

$$\frac{Z_{E,ij}}{Z_{T,ij}}$$

where $Z_{E,ij}$ is the impedance of the equivalent branch ij and $Z_{T,ij}$ is the corresponding transfer impedance given by the rest of the network. If

$$\frac{Z_{E,ij}}{Z_{T,ij}} > C \qquad (1)$$

where C is a predetermined value, the branch ij is eliminated from the equivalent representation.

In references 18 and 19 two approaches are suggested. The first is a Norton-type equivalent where the boundary busses are treated as generation busses. The other is based on DC approximation of the external system. If

$\underline{P}_B \triangleq$ vector of real power flows from the boundary busses to the busses of the external system

$\underline{P}_E \triangleq$ vector of real power injections at the busses of the external system

$\underline{\theta}_E \triangleq$ vector of voltage phase angles of the busses of the external system

$\underline{\theta}_B \triangleq$ vector of voltage phase angles of the boundary busses

then by using DC analysis, one can write:

$$\begin{bmatrix} \underline{P}_E \\ \underline{P}_B \end{bmatrix} = \begin{bmatrix} K_{EE} & K_{EB} \\ K_{BE} & K_{BB} \end{bmatrix} \begin{bmatrix} \underline{\theta}_E \\ \underline{\theta}_B \end{bmatrix} \tag{2}$$

where the matrices K_{EE}, K_{EB}, K_{BE}, K_{BB} are known matrices. Elimination of the vector $\underline{\theta}_E$ yields

$$\underline{P}_B = K_{BE}K_{EE}^{-1}\underline{P}_E + (K_{BB} - K_{BE}K_{EE}^{-1}K_{EB})\underline{\theta}_B$$

$$= H\underline{P}_E + G\underline{\theta}_B \tag{3}$$

where

$$H = K_{BE} K_{EE}^{-1}$$

$$G = K_{BB} - K_{BE} K_{EE}^{-1} K_{EB}$$

Since $\underline{P}_B$ and $\underline{\theta}_B$ are known, the vector

$$H\underline{P}_E = \underline{P}_B - G\underline{\theta}_B \tag{4}$$

is also known and it is assumed to be constant when a contingency occurs.

If $\underline{V}$ and $\underline{\theta}$ are the vectors of voltage magnitudes and voltage phase angles of the busses of the study area, these vectors can be decomposed as

$$\underline{V} = \begin{bmatrix} \underline{V}_I \\ \underline{V}_B \end{bmatrix} \quad , \quad \underline{\theta} = \begin{bmatrix} \underline{\theta}_I \\ \underline{\theta}_B \end{bmatrix}$$

where the subscripts I, B refer to internal and boundary busses respectively.

Then the real and reactive injections in the internal system are:

$$\underline{P}_I = \underline{P}_I(\underline{\theta},\underline{V}) \tag{5}$$

$$\underline{Q}_I = \underline{Q}_I(\underline{\theta},\underline{V}) \tag{6}$$

and

$$\underline{P}_B = \underline{P}_B(\underline{\theta},\underline{V}) \tag{7}$$

or by taking into consideration the linear approximation for the external system given by equation (3), equation (7) becomes:

$$\underline{P}_B(\underline{\theta},\underline{V}) + H\underline{P}_E + G\underline{\theta}_B = \underline{0}$$

or

$$-H\underline{P}_E = \underline{P}_B(\underline{\theta},\underline{V}) + G\underline{\theta}_B \tag{8}$$

Equations (5), (6), and (8) are the load flow equations. The boundary busses are assumed to be generation busses.

In reference 20 a model based on DC analysis is suggested. Deviations from the operating point are used to provide the necessary information for the equivalent representation. The statement of the method is:

The system between the boundary busses is modeled as

$$\underline{Z} = H\underline{u} + \underline{v} \tag{9}$$

where

$\underline{Z} \triangleq$ is the vector of the phase angles of the boundary busses

$\underline{H} \triangleq$ is the unknown boundary impedance matrix

$\underline{u} \triangleq$ is the vector of real powers which depend upon the topology and real injections of the internal system

$\underline{v} \triangleq$ is the vector which depends upon the topology and real injections of the external system.

If

$$\underline{Z}(n) = \underline{Z}(t_{n+1}) - \underline{Z}(t_n) \tag{10}$$

$$\underline{u}(n) = \underline{u}(t_{n+1}) - \underline{u}(t_n) \tag{11}$$

$$\underline{v}(n) = \underline{v}(t_{n+1}) - \underline{v}(t_n) \tag{12}$$

then

$$\underline{Z}(n) = H\underline{u}(n) + \underline{v}(n) \tag{13}$$

It is assumed that $\underline{v}(n)$ has zero expected value and covariance matrix $E\{\underline{v}(n_1)\underline{v}^T(n_2)\} = R \cdot \delta(n_1 - n_2)$. Furthermore, $\underline{u}(n)$ and $\underline{v}(n)$ are uncorrelated. The problem is to estimate H and R by using $\underline{Z}(n)$, $n = 1, \ldots, N$ where N is the number of observations.

Least squares estimation yields:

$$\hat{H} = [\sum_{n=1}^{N} \underline{Z}(n)\underline{u}^T(n)][\sum_{n=1}^{N} \underline{u}(n)\underline{u}^T(n)]^{-1} \tag{14}$$

$$\hat{R} = \frac{1}{N} \sum_{n=1}^{N} (\underline{Z}(n) - H\underline{u}(n))(\underline{Z}(n) - H\underline{u}(n))^T \tag{15}$$

provided that the inverse of

$$\sum_{n=1}^{N} \underline{u}(n)\underline{u}^T(n)$$

exists.

Objections to this approach are:

(1) Since the entire system for a time period is moving in the same direction, $\underline{v}(n)$ has an expected value different than zero.

(2) $\underline{u}(n)$, $\underline{v}(n)$ are not uncorrelated since both depend upon the power injections.

(3) The accuracy of the DC model does not suffice for this problem.

In reference 21 information from outages in the internal system are used to obtain the equivalent system. If $\underline{Z}^1$ and $\underline{Z}^2$ are pre and post-outage internal system measurement vectors, then

$$\underline{Z}^1 = \underline{h}^1(\underline{x}^1) + \underline{v}^1 \tag{16}$$

$$\underline{Z}^2 = \underline{h}^2(\underline{x}^2) + \underline{v}^2 \tag{17}$$

where $\underline{x}^1$ and $\underline{x}^2$ are the pre- and post outage state vectors, $\underline{v}^1$ and $\underline{v}^2$ are measurement error vectors with zero mean and covariances R_1, R_2. It is assumed that the boundary busses have been classified as load or as generation busses. For contingency analysis, the real power and voltage magnitudes at generation busses are assumed to be constant. All these quantities define the vector $\underline{C}$. If $\underline{C}^1$ and $\underline{C}^2$ denote the pre- and post-outage cases, then

$$\underline{C}^1 = \underline{C}^2 + \underline{v}^3 \tag{18}$$

where $\underline{v}^3$ is a random vector of zero mean and covariance R_3, and

$$\underline{C}^1 = \underline{F}^1(\underline{x}^1,\underline{p}) \tag{19}$$

$$\underline{C}^2 = \underline{F}^2(\underline{x}^2,\underline{p}) \tag{20}$$

$\underline{p}$ is the vector of external network equivalent parameters, with initial value $\underline{p}^0$ and a priori covariance error matrix M_0.

Equations (18), (19), and (20) are combined to give:

$$\underline{z}^3 = \underline{0} = \underline{F}^2(\underline{x}^2,\underline{p}) - \underline{F}^1(\underline{x}^1,\underline{p}) + \underline{v}^3 \triangleq \underline{g}(\underline{x}^1,\underline{x}^2,\underline{p}) + \underline{v}^3$$

The optimum $\hat{\underline{x}}^1$, $\hat{\underline{x}}^2$, and $\hat{\underline{p}}$ are those which minimize the index

$$J = (\underline{p}-\underline{p}^0)^T M_0^{-1}(\underline{p}-\underline{p}^0) + (\underline{Z}^1-\underline{h}^1(\underline{x}^1))^T R^{-1}(\underline{Z}^1-\underline{h}^1(\underline{x}^1))$$

$$+ (\underline{Z}^2-\underline{h}^2(\underline{x}^2))^T R_2^{-1}(\underline{Z}^2-\underline{h}^2(\underline{x}^2))$$

$$+ (\underline{Z}^3-\underline{g}(\underline{x}^1,\underline{x}^2,\underline{p}))^T R_3^{-1}(\underline{Z}^3-\underline{g}(\underline{x}^1,\underline{x}^2,\underline{p}).$$

In reference 27 the equivalent representation of the external system is obtained by using real-time information on the voltage magnitude and angle and the real and reactive power at boundary busses. The basis of the method is the decoupled form of the Jacobian equations for power systems. The following model between the boundary busses is proposed:

$$\frac{\underline{\Delta P_T}}{\underline{V_b}} = B'_{TT}\,\underline{\Delta\delta_b} \tag{21}$$

where

$\underline{\Delta P_T} \triangleq$ difference of the vector of the net tie line flows into the external system between a past time instant and the present

$\underline{\Delta\delta_b} \triangleq$ difference of the vector of the voltage phase angles of the boundary busses between a past time instant and the present.

$\underline{V_b} \triangleq$ vector of voltage magnitudes of the boundary busses

$B'_{TT} \triangleq$ unknown admittance matrix.

The unknown matrix B'_{TT} is computed from a given sequence of r measurements:

$$\{\underline{\Delta P}_T(1),\ \underline{\Delta P}_T(2),\ .\ .\ .\ ,\underline{\Delta P}_T(r)\}$$

so that the objective function

$$J = \sum_{i=1}^{r} [\underline{\Delta P}_T(i)-B'_{TT}\underline{\Delta\delta}_b(i)]^T[\underline{\Delta P}_T(i)-B'_{TT}\underline{\Delta\delta}_b(i)]$$

is minimized.

In summary the available methods do not optimize the equivalent representation of the external system; therefore, the available methods do not guarantee accuracy in predicting the effects of postulated outages for on-line steady-state security assessment.

1.3 Outline of Thesis

In Chapter II, the objectives of the equivalent problem are presented. The equivalent problem is formulated as an optimization problem. The decision variables, objective function, and constraints of the problem are discussed. A general solution to the problem is presented.

Chapter III deals with the Norton-type equivalent. The limitations and the problems associated with the Norton-type equivalent model are discussed. Mainly, two problems are investigated: the sparsity of the admittance matrix of the equivalent model and the effect on the equivalent model when changes in the external system take place. The contributions of this dissertation in Chapter III includes:

(1) Two elimination schemes so that the admittance matrix of the equivalent model preserves its sparsity.

(2) A detection scheme to detect topological changes in the external system.

Chapter IV deals with the solution of the optimization problem. Equivalent models are derived by utilizing information from the internal system only. The contributions in Chapter IV include:

(1) An equivalent model based on least square fitting.

(2) An equivalent model based on quadratic programming.

Chapter V deals with computational aspects and presents several simulation results.

One appendix is provided. This appendix provides a brief discussion of the quadratic and linear programming.

CHAPTER II

MATHEMATICAL FORMULATION OF THE PROBLEM

2.1 Objectives

The objective of this thesis is the estimation of the equivalent representation of a power system which is connected to a number of external power systems. The equivalent model should satisfy the following requirements:

(1) It should be accurate, in the sense, that it can yield voltage levels and power flows which are very close to the actual values for a set of postulated conditions.

(2) Changes in the external system should be easily handled. Two kinds of changes in the external system may take place:

(a) Transmission line outages

(b) Generator outages

With regard to the second requirement, three cases may be distinguished.

(1) The equivalent model is insensitive to the change, whereby, it is not necessary to modify the equivalent model.

(2) The equivalent model is sensitive to a known change. In this case, the equivalent model should be updated.

(3) The equivalent is sensitive to an unknown change whereby the change must first be detected and then the equivalent model should be updated.

2.2 Basic Assumptions

This research assumes that the topology and the parameter values of the internal system are known. Also, it is assumed that the internal system is observable to a state estimator. This insures that the present operating condition of the internal system is always available.

The equivalent model is designed so that the boundary busses behave as load busses, i.e. the real and reactive injections at the boundary busses should remain constant before and after each of the N postulated outages. Real error at each bus is defined as the absolute value of the difference between the pre- and post-outage real injections, if the true pre- and post-outage conditions were used to compute these real injections. Similarly, reactive error at each bus is defined as the absolute value of the difference between the pre- and post-outage reactive injections if the true pre- and post-outage conditions were used to compute these reactive injections.

In the ideal case the equivalent model should give zero values for all these errors. In practice, however, this is not feasible. The equivalent model will be determined to satisfy the following inequalities.

$$\text{Total error} < S_1 \tag{22}$$

$$\text{Maximum real error} < S_2 \tag{23}$$

$$\text{Maximum reactive error} < S_3 \tag{24}$$

S_1, S_2, and S_3 are specified tolerances.

This thesis describes a procedure which determines the optimal equivalent representation of the external system for the following cases.

(a) The topology and the parameter values of the external network are well defined.

(b) Information about the external system is not available.

In some cases an equivalent model, such as the Norton equivalent model, is available. The model should be tested to see if it satisfies the inequalities (22), (23), (24). If these inequalities are satisfied, the model is sufficient. If the inequalities are not satisfied or such equivalent model is not available, the problem described by the inequalities (22), (23), (24) is relaxed by the inequality (22) and the model is obtained by solving the following optimization problem.

$$\text{Minimize: Total error} \tag{25}$$

subject to the constraints:

$$\text{Maximum real error} < S_2 \tag{26}$$

$$\text{Maximum reactive error} < S_3 \tag{27}$$

In the following sections, the problem described by the relations (25), (26), and (27) is formulated as an optimation problem.

Minimize:

$$J = \underline{g}^T(u)\underline{g}(\underline{u})$$

subject to the constraints:

$$\underline{F}_N(\underline{u}) \le 0$$

$\underline{u}$ is the vector of the decision variables. The following sections are devoted in the description and interpretation of the various elements of the optimization problem.

2.3 Decision Variables

The equivalent model of the external system consists of fictitious network branches. The conductances and susceptances of these fictitious branches are the unknown variables. The fictitious branches may be lines between the boundary busses, and capacitors or reactors at the boundary busses. The vector of the decision variables is denoted by $\underline{u}$. This unknown vector $\underline{u}$ will be determined from the solution of the problem.

2.4 The Objective Function

The equivalencing technique of this thesis assumes the boundary busses behave as load busses, i.e. the equivalent real and reactive injections on the boundary busses remain constant before and after the k^{th} outage.

We denote by $\underline{x}$ the vector of the complex bus voltages of the internal system. $\underline{x}$ is a known vector because it has been assumed that the internal system is observable to a state estimator. Let the vectors $\underline{x}^{1k}$ and $\underline{x}^{2k}$ denote the pre- and post-outage vector of the complex bus voltages, respectively. Let $\underline{I}^{1k}$, $\underline{I}^{2k}$ be the pre- and post-outage vectors of the injections on the boundary busses.

$$\underline{I}^{1k} = \underline{I}^{1k}(\underline{x}^{1k},\underline{u}) \tag{28}$$

$$\underline{I}^{2k} = \underline{I}^{2k}(\underline{x}^{1k},\underline{u}) \tag{29}$$

If N is the number of postulated outages in the internal system, the objective of the problem is to find a vector $\underline{u}$ such that:

$$\underline{I}^{1k} = \underline{I}^{2k} \quad ; \quad k = 1, \ . \ . \ . \ ,N$$

This is equivalent to:

Find $\underline{u}$ such that

$$\underline{g}_k(\underline{x}^{1k},\underline{x}^{2k},\underline{u}) = \underline{I}^{1k}(\underline{x}^{1k},\underline{u}) - \underline{I}^{2k}(\underline{x}^{2k},\underline{u}) = \underline{0} \tag{30}$$

$$\text{for } k = 1, \ . \ . \ . \ ,N$$

If:

$$\underline{g} = \begin{vmatrix} \underline{g}_1 \\ \cdot \\ \cdot \\ \cdot \\ \underline{g}_k \\ \cdot \\ \cdot \\ \cdot \\ \underline{g}_N \end{vmatrix} \qquad \underline{x}^1 = \begin{vmatrix} \underline{x}^{11} \\ \cdot \\ \cdot \\ \cdot \\ \underline{x}^{1k} \\ \cdot \\ \cdot \\ \cdot \\ \underline{x}^{1N} \end{vmatrix} \qquad \underline{x}^2 = \begin{vmatrix} \underline{x}^{21} \\ \cdot \\ \cdot \\ \cdot \\ \underline{x}^{2k} \\ \cdot \\ \cdot \\ \cdot \\ \underline{x}^{2N} \end{vmatrix}$$

the objective of the problem is to find $\underline{u}$ such that:

$$\underline{g}(\underline{x}^1, \underline{x}^2, \underline{u}) = \underline{0} \tag{31}$$

The dimension of $\underline{g}$ is 2xNxb if b is the number of the boundary busses. The dimension of the vector $\underline{u}$ determines the sparsity of the admittance matrix of the equivalent model. It is desirable to keep the dimension of the vector $\underline{u}$ as small as possible so that the equivalent admittance matrix is a sparse matrix. Therefore, equation (31) is an over-determined set of equations and, in general, there is not a solution which satisfies these equations. Hence, we seek a solution which will minimize the following defined error:

$$J = \underline{g}^T(\underline{x}^1, \underline{x}^2, \underline{u})\underline{g}(\underline{x}^1, \underline{x}^2, \underline{u}) \tag{32}$$

This is a measure of the total error as a function of the decision variables and defines the performance of the equivalent model. If

P_j^{1k}, P_j^{2k} $\underline{\Delta}$ real injected power at the j^{th} bus before and after the k^{th} outage

Q_j^{1k}, Q_j^{2k} $\underline{\Delta}$ reactive injected power at the j^{th} bus before and after the k^{th} outage

then the vectors $\underline{I}^{1k}$, $\underline{I}^{2k}$ defined earlier can be written in the form

$$\underline{I}^{1k} = \left| \begin{matrix} P_1^{1k} \\ Q_1^{1k} \\ \cdot \\ \cdot \\ \cdot \\ P_j^{1k} \\ Q_j^{1k} \\ \cdot \\ \cdot \\ \cdot \\ P_b^{1k} \\ Q_b^{1k} \end{matrix} \right| , \quad \underline{I}^{2k} = \left| \begin{matrix} P_1^{2k} \\ Q_1^{2k} \\ \cdot \\ \cdot \\ \cdot \\ P_j^{2k} \\ Q_j^{2k} \\ \cdot \\ \cdot \\ \cdot \\ P_b^{2k} \\ P_b^{2k} \end{matrix} \right|$$

We define by S the average error over the set of N outages and set of b boundary busses.

$$S = \frac{1}{N \cdot b} J = \frac{1}{N \cdot b} \sum_{k=1}^{N} (\underline{I}^{1k}(\underline{x}^{1k},\underline{u}) - \underline{I}^{2k}(\underline{x}^{2k},\underline{u}))^T (\underline{I}^{1k}(\underline{x}^{1k},\underline{u}) - \underline{I}^{2k}(\underline{x}^{2k},\underline{u}))$$

$$= \frac{1}{N \cdot b} \sum_{k=1}^{N} \sum_{j=1}^{b} \{(P_j^{1k} - P_j^{2k})^2 + (Q_j^{1k} - Q_j^{2k})^2\} \tag{33}$$

If an equivalent model was available, this model would be satisfactory if

$$S < \overline{S}$$

where $\overline{S}$ is a predetermined value.

2.5 Constraints

The objective function measures the total error of the real and reactive injections at the boundary busses over the set of N postulated outages. However, the maximum observed local error is, also, of great importance. The constraints which will be discussed in this section deal with the maximum real and reactive error.

Random changes take place in the external system. Depending on the location and the size of this change; it may or may not have an effect on the equivalent representation. It is expected that outages of transmission lines far away from the boundaries do not affect considerably the equivalent model.

It is assumed that the set of N postulated outages consists of N_1 outages with a nominal topology of the external system and N_2 outages while single outages in the external system took place. Furthermore,

$$N_2 = N_{2,1} + N_{2,2} + \ldots + N_{2,\ell} + \ldots + N_{2,L} \tag{34}$$

where $N_{2,\ell}$ is the number of switching operations in the internal system with the ℓ^{th} branch of the external system out of operation.

Therefore, the vector $\underline{g}$, defined by equation (31), can be decomposed as:

$$\underline{g} = \begin{bmatrix} \underline{g}_1 \\ \vdots \\ \underline{g}_K \\ \vdots \\ \underline{g}_N \end{bmatrix} = \begin{bmatrix} \underline{g}_{N_1} \\ \underline{g}_{N_{2,1}} \\ \vdots \\ \underline{g}_{N_{2,\ell}} \\ \vdots \\ \underline{g}_{N_{2,L}} \end{bmatrix} \tag{35}$$

Next, we define the following accuracy indices:

$$MP = \text{Maximum } \left|P_j^{1k} - P_j^{2k}\right|$$

$$k = 1, \ldots, N_1$$

$$j = 1, \ldots, b \tag{36}$$

$$MQ = \text{Maximum } \left|Q_j^{1k} - Q_j^{2k}\right|$$

$$k = 1, \ldots, N_1$$

$$j = 1, \ldots, b \tag{37}$$

and

$$(MP)_\ell = \text{Maximum } \left|P_j^{1k} - P_j^{2k}\right|$$

$$k = 1, \ldots, N_{2,\ell}$$

$$j = 1, \ldots, b \tag{38}$$

$$(MQ)_\ell = \text{Maximum } \left|Q_j^{1k} - Q_j^{2k}\right|$$

$$k = 1, \ldots, N_{2,\ell}$$

$$j = 1, \ldots, b \tag{39}$$

for $\ell = 1, \ldots, L$.

The equivalent model is further constrained to the following inequalities:

$$MP \leq \overline{MP} \tag{40}$$

$$MQ \leq \overline{MQ} \tag{41}$$

$$(MP)_\ell \leq C_1 \overline{MP} \tag{42}$$

$$(MQ)_\ell \leq C_2 \overline{MQ} \tag{43}$$

for $\ell = 1, \ldots, L$.

The values of $\overline{MP}$, $\overline{MQ}$ are predetermined maximum allowable local errors according to the requirements and applications of the equivalent model. Generally, better accuracy is needed for on-line operations than for planning purposes. C_1, C_2 are positive numbers which correspond to the specified accuracy tolerance.

The differences $(P_j^{1k} - P_j^{2k})$, $(Q_j^{1k} - Q_j^{2k})$ for $k = 1, \ldots, N_1$ are elements of the vector $\underline{g}_{N_1}$. Therefore, the inequalities (40) and (41) are satisfied if:

$$|g_i| < \overline{MP}$$

for all the rows of $\underline{g}_{N_1}$ which correspond to the real power error and

$$|g_i| < \overline{MQ}$$

for all the rows of $\underline{g}_{N_1}$ which correspond to the reactive error.

The inequalities (40) and (41) are equivalent to

$$\underline{F}_{N_1}(\underline{u}) = |\underline{g}_{N_1}(\underline{x}^1, \underline{x}^2, \underline{u})| - \overline{\underline{g}}_{N_1} \leq \underline{0} \tag{44}$$

where

$$\bar{g}_{N_{1,i}} = \begin{cases} \overline{MP} & \text{if the } i^{th} \text{ row of } \underline{g}_{N_1} \text{ corresponds to} \\ & \text{real power error} \\ \overline{MQ} & \text{if the } i^{th} \text{ row of } \underline{g}_{N_1} \text{ corresponds to} \\ & \text{reactive power error} \end{cases}$$

Following the above thinking, inequalities (42) and (43) are equivalent to:

$$\underline{F}_{N_{2,\ell}}(\underline{u}) = |\underline{g}_{N_{2,\ell}}(\underline{x}^1,\underline{x}^2,\underline{u})| - \bar{\underline{g}}_\ell \leq \underline{0} \tag{45}$$

$$\text{for } \ell = 1, \ldots, L$$

where

$$\bar{g}_{\ell,i} = \begin{cases} C_1\, \overline{MP} & \text{if the } i^{th} \text{ row of } \underline{g}_{N_{2,\ell}} \text{ corresponds to} \\ & \text{real power} \\ C_2\, \overline{MQ} & \text{if the } i^{th} \text{ row of } \underline{g}_{N_{2,\ell}} \text{ corresponds to} \\ & \text{reactive power error} \end{cases}$$

The inequalities (44) and (45) are the constraints of the problem. These can be combined as:

$$\underline{F}_N(\underline{u}) = |\underline{g}(\underline{x}^1,\underline{x}^2,\underline{u})| - \bar{\underline{g}}_N \leq \underline{0} \tag{46}$$

where:

$$\underline{\bar{g}}_N = \begin{vmatrix} \underline{\bar{g}}_{N_1} \\ \underline{\bar{g}}_1 \\ \cdot \\ \cdot \\ \cdot \\ \underline{\bar{g}}_\ell \\ \cdot \\ \cdot \\ \cdot \\ \underline{\bar{g}}_L \end{vmatrix} \tag{47}$$

2.6 General Solution to the Problem

The equivalent problem has been formulated as an optimization problem. Minimize:

$$J = \underline{g}^T(\underline{u})\underline{g}(\underline{u}) \tag{48}$$

subject to the constraints:

$$\underline{F}_N(\underline{u}) \leq \underline{0} \tag{49}$$

The dimensionality of the problem is huge. It has been observed that in many cases it is not necessary to solve this large optimization problem. A simpler model may yield a solution which satisfies the constraints (49). Therefore, it pays back if prior to the solution of the general problem, simpler models are tested. In this line of thinking we have developed a procedure which is depicted in Figure 2.1. This procedure yields the equivalent model which satisfies inequalities (49) assuming that there is such a model.

A brief description of the procedure follows. If the topology and the parameter values of the external system are available, the Norton equivalent model can be calculated. This model should be tested if it satisfies inequalities (49). The performance index S should be calculated, also. If this performance index S is less than a predetermined value $\overline{S}$, the Norton equivalent model is used as the equivalent representation of the external system and the solution of the problem is avoided. If the Norton equivalent model does not satisfy the above requirements, the problem described by the relations (48) and (49) should be solved. The following applies, also, to the case where the topology and the parameter values of the external system are not available. Before one has to solve the constrained problem, the unconstrained problem should be considered. The model obtained from the solution of the unconstrained problem should be tested if it satisfies the inequalities (49). If these inequalities are satisfied, this model is used as the equivalent representation of the external system. If the model does not satisfy the inequalities (49), the entire problem should be considered. Solution to the problem exists assuming that the solution space described by the inequalities (49) is feasible.

2.7 Summary

In this chapter, the equivalent problem has been formulated as

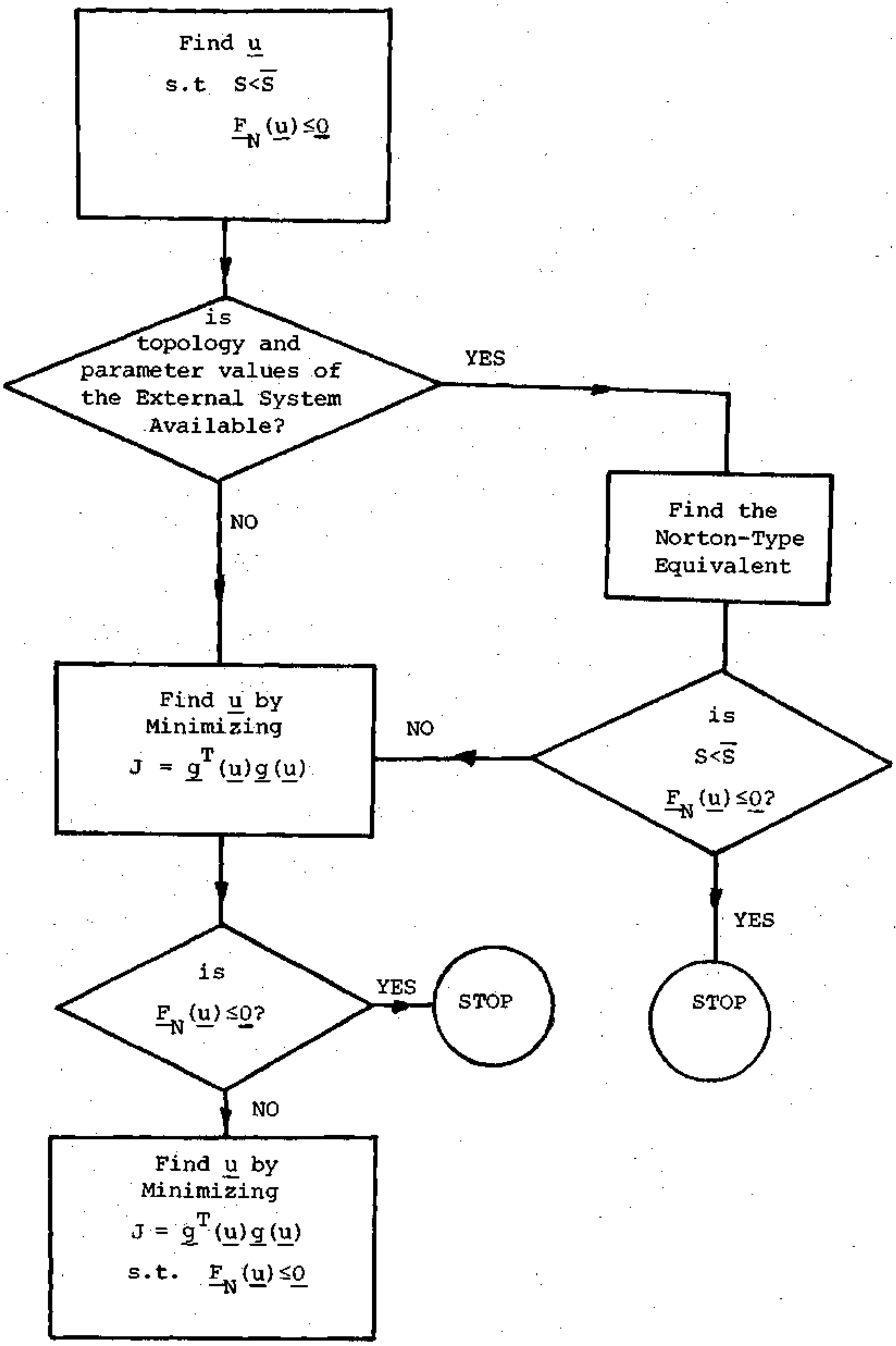

Figure 2.1. General Solution to the Equivalencing Problem.

an optimization problem. The decision variables, objective function, and constraints of the problem were discussed. Finally, a general solution to the problem was given.

CHAPTER III

EQUIVALENCE BY REDUCTION TO THE NORTON EQUIVALENT

3.1 Introduction

The Norton equivalent has a long history of application as the model to represent the external network. It was introduced, by Ward [1] in 1949, because of limitations imposed by the number of analyzer circuits. Today the analyzer power-flow studies have been substituted by digital computer power flow studies. However, the difficulties in performing the contingency analysis remain the same.

The Norton type equivalent assumes that the topology and the parameter values of the external system are known. The loading conditions of the external system are not available and the internal system is assumed to be observable to a state estimator. The next section is a review of the Norton equivalencing theory.

3.2 General Norton Equivalencing Theory

The entire system nodal matrix equation is:

$$Y\underline{V} = \underline{S} \tag{50}$$

where $\underline{S}$, $\underline{V}$ are the vectors of injected complex bus currents and complex bus voltages, respectively, and Y is the nodal admittance matrix. The transmission line model is assumed to be the pi equivalent circuit as it is shown in Figure 3.1.

$y_{k\ell}$ is the complex admittance of the transmission line which connects the k^{th} bus with the ℓ^{th} bus and it is defined as:

$$y_{k\ell} = G_{k\ell} + jB_{k\ell} \tag{51}$$

where $G_{k\ell}$ is the conductance of the line and $B_{k\ell}$ is the susceptance of the line.

$y_{SH,k\ell}$ is the complex shunt admittance of the line. According to the above notation, the nodal admittance matrix is defined as:

$$Y_{k\ell} = \begin{cases} -y_{k\ell} & \text{if } k \neq \ell \\ \sum\limits_{\ell \in a(k)} y_{k\ell} + \sum\limits_{\ell \in a(k)} y_{SH,k\ell} & \text{if } k = \ell \end{cases}$$

a(k) is the set of busses connected to the k^{th} bus.

The vectors $\underline{S}$, $\underline{V}$ and the matrix $\underline{Y}$ are decomposed as follows:

$$\underline{S} = \begin{bmatrix} \underline{S}_E \\ \underline{S}_B \\ \underline{S}_I \end{bmatrix}, \quad V = \begin{bmatrix} \underline{V}_E \\ \underline{V}_B \\ \underline{V}_I \end{bmatrix}, \quad Y = \begin{bmatrix} Y_{EE} & Y_{EB} & 0 \\ Y_{BE} & Y_{BB} & Y_{BI} \\ 0 & Y_{IB} & Y_{II} \end{bmatrix} \tag{52}$$

where the subscripts E, B and I refer to external, boundary, and

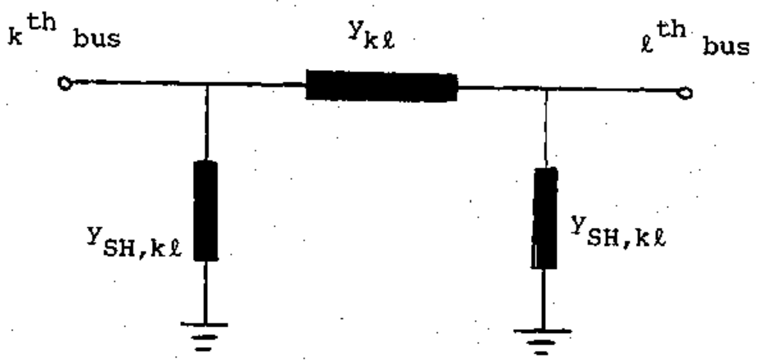

Figure 3.1. Transmission Line Model.

internal systems, respectively. The two zero submatrices indicate that internal and external busses are not connected to one another. According to the above decomposition, equation (50) becomes:

$$\begin{bmatrix} Y_{EE} & Y_{EB} & 0 \\ Y_{BE} & Y_{BB} & Y_{BI} \\ 0 & Y_{IB} & Y_{II} \end{bmatrix} \begin{bmatrix} \underline{V}_E \\ \underline{V}_B \\ \underline{V}_I \end{bmatrix} = \begin{bmatrix} \underline{S}_E \\ \underline{S}_B \\ \underline{S}_I \end{bmatrix} \tag{53}$$

Elimination of the vector $\underline{V}_E$ from equation (53) yields:

$$\begin{bmatrix} Y_{BB}-Y_{BE}Y_{EE}^{-1}Y_{EB} & Y_{BI} \\ Y_{IB} & Y_{II} \end{bmatrix} \begin{bmatrix} \underline{V}_B \\ \underline{V}_I \end{bmatrix} = \begin{bmatrix} \underline{S}_B-Y_{BE}Y_{EE}^{-1}\underline{S}_E \\ \underline{S}_I \end{bmatrix} \tag{54}$$

The matrix $-Y_{BE}Y_{EE}^{-1}Y_{EB}$ represents equivalent network interconnections between the boundary busses because of the linear reduction of the external system to the boundaries of the internal system. The vector $-Y_{BE}Y_{EE}^{-1}\underline{S}_E$ represents equivalent current injections at the boundary busses. The elements of the matrix $Y_{BB}-Y_{BE}Y_{EE}^{-1}Y_{EB}$ correspond to transmission lines connecting the boundary busses. As such, the individual line constants can be represented as a vector $\underline{u}$. The matrix

$$\begin{bmatrix} Y_{BB}-Y_{BE}Y_{EE}^{-1}Y_{EB} & Y_{BI} \\ Y_{IB} & Y_{II} \end{bmatrix}$$

is known as the equivalent matrix and it is denoted by Y_{eq}.

If the first row of the matrix Y is premultiplied by the matrix Y_{EE}^{-1}, then the matrix Y becomes:

$$Y_1 = \left|\begin{array}{c:c:c} I & Y_{EE}^{-1}Y_{EB} & 0 \\ \hdashline Y_{BE} & Y_{BB} & Y_{BI} \\ \hdashline 0 & Y_{IB} & Y_{II} \end{array}\right| \tag{55}$$

where I is the identity matrix.

If the first row of the matrix Y_1 is premultiplied by the matrix Y_{BE} and the resultant row is subtracted from the second row, then Y_1 becomes:

$$Y_2 = \left|\begin{array}{c:c:c} I & Y_{EE}^{-1}Y_{EB} & 0 \\ \hdashline 0 & Y_{BB}-Y_{BE}Y_{EE}^{-1}Y_{EB} & Y_{BI} \\ \hdashline 0 & Y_{IB} & Y_{II} \end{array}\right| \tag{56}$$

Note that the lower part of the matrix Y_2 is the equivalent admittance matrix Y_{eq}. Therefore, the equivalent admittance matrix is obtained by gaussian elimination of the rows of the admittance matrix Y which corresponds to the busses of the external system. Direct inversion of the matrix Y_{EE}^{-1} is thus avoided.

Since the internal system is observable, by definition, the complex voltages of the busses of the internal system are known. Using equation (54), the equivalent complex current injections can be evaluated. If these injections were constant before and after an outage in the internal system, then contingency analysis could be performed exactly. However, these current injections do not remain constant and it is common practice in performing contingency analysis to classify the busses as generation busses denoted as (P,V) busses, or as load busses denoted as (P,Q) busses. Generation busses are defined as those for which the real power injection P and the voltage magnitude V remain constant before and after outages take place in the system. Load busses are defined as those for which the real power injection P and the reactive power injection Q remain constant before and after outages take place. One of the busses in the system is classified as slack bus and for this bus, the voltage phase angle is arbitrarily set to be zero.

Therefore, the load flow equations include:

(1) Two equations for each load bus; one for the real injection and one for the reactive injection.

(2) One equation for each generation bus for the real injection.

From the references cited in Chapter I, it is clear that the classification of the boundary busses as (P,V) busses or as (P,Q) busses is dependent on the particular system and the set of postulated outages. In our research, both assumptions were investigated and for our test systems both assumptions gave similar results.

In our research, the equivalent is designed so that the boundary busses behave as (P,Q) busses. To calculate the Norton equivalent sparsity techniques have been implemented. The computational aspects for the Norton equivalent are given in Section 5.2.1. The conductances and susceptances of the fictitious lines created between the boundary busses are the elements of the vector $\underline{u}$.

After the equivalent model is found by computing the matrix Y_{eq}, this model is tested if it satisfies the requirements of the problem. For the set of N postulated outages, the performance index S is computed.

If

$$S < \overline{S} \tag{57}$$

and

$$\underline{F}_N(\underline{u}) \leq \underline{0} \quad , \tag{58}$$

the Norton-type equivalent is sufficient. If this set of inequalities is not satisfied, the Norton-type equivalent is rejected and another equivalent model needs to be derived by following the procedure to be presented in the next chapter. The N postulated outages are either information from real switching operations or information from contingency analysis simulation using the entire area.

3.3 Sparsity of the Equivalent Admittance Matrix

The reduction of the external system to the boundaries of the internal system creates equivalent branches between the boundary busses.

The admittance matrix of the equivalent network (between the boundary busses) is given by:

$$Y_{BB}-Y_{BE}Y_{EE}^{-1}Y_{EB}$$

The matrix Y_{EE} is a sparse matrix. However, its inverse is in general a full matrix. Therefore, reduction of the external system to the boundaries will create a large number of equivalent branches. The number of these branches will depend on the number of connections between the internal and external system. Three cases may be distinguished:

(a) All the busses of the external system are part of one area. If b is the number of the boundary busses, then the maximum number of equivalent branches are:

$$\frac{b(b-1)}{2}$$

(b) The busses of the external system form m isolated areas. Each one of these m areas is connected to b_i boundary busses of the internal system. Then

$$\sum_{i=1}^{m} \frac{b_i(b_i-1)}{2}$$

equivalent branches are created, where:

$$\sum_{i=1}^{m} b_i = b$$

(c) One or more busses of the external system form an isolated area and this area is connected to one boundary bus. In this case only the shunt admittance of this bus changes.

In general, if the internal system is highly interconnected to the external system, the number of the equivalent branches is large and the sparsity of the admittance matrix of the equivalent model is destroyed. This is undesirable for contingency analysis because of time and storage limitations. Some compromise between accuracy and sparsity is necessary. It has been suggested that some busses of the external system should be included in the equivalent model so the admittance matrix of the equivalent system will preserve its sparse structure. The method is based on the ordering schemes for sparse matrices developed by Tinney [22]. In order to include busses of the external system in the study area, systematic exchange of information between neighboring companies is required. If this information is not available, assumptions should be made about the loading conditions of the external system. These assumptions usually jeopardize the accuracy of the equivalent model.

This thesis reports another method. A large number of the equivalent branches are eliminated according to some criterion. This practice preserves both the sparse structure of the equivalent admittance matrix and the accuracy of the model. Two elimination schemes of branches of the equivalent model were examined; the two schemes are discussed next.

First Elimination Scheme

We define as $(\Gamma P)_j$ the average of magnitude of real power for the j^{th} equivalent branch. Similarly, we define as $(\Gamma Q)_j$ the average of magnitude of reactive power. The indices $(\Gamma P)_j$, $(\Gamma Q)_j$ given by

$$(\Gamma P)_j = \frac{1}{N} \sum_{k=1}^{N} |P_j^{2k}| \tag{59}$$

$$(\Gamma Q)_j = \frac{1}{N} \sum_{k=1}^{N} |Q_j^{2k}| \tag{60}$$

are computed for every equivalent branch. P_j^{2k}, Q_j^{2k} are the real and reactive flows in the j^{th} branch after the k^{th} outage. If for the j^{th} branch,

$$(\Gamma P)_j < \overline{\Gamma P} \tag{61}$$

$$(\Gamma Q)_j < \overline{\Gamma Q} \tag{62}$$

then, the j^{th} branch is eliminated from the equivalent representation. For various values of $\overline{\Gamma P}$, $\overline{\Gamma Q}$ the accuracy indices S, MP, MQ are calculated. The values of $\overline{\Gamma P}$, $\overline{\Gamma Q}$ are selected so that sparsity requirements and accuracy specifications are satisfied.

Second Elimination Scheme

If for the j^{th} branch the conductance and susceptance satisfy the inequalities

$$|G_j| < \overline{G} \tag{63}$$

$$|B_j| < \overline{B} \tag{64}$$

the j^{th} branch is eliminated. For various values of $\overline{G}$, $\overline{B}$ the accuracy indices S, MP, MQ are calculated. The values of $\overline{G}$, $\overline{B}$ are selected so that sparsity requirements and accuracy specifications are satisfied.

3.4 Updating of the Norton Equivalent

If major topological changes take place in the external system, the admittance matrix of the equivalent should be updated. A systematic procedure is needed to determine which transmission lines of the external system have significant effect on the magnitudes of the conductances and susceptances of the equivalent branches. It is expected that outages of transmission lines far away from the boundary busses will have insignificant effect on the equivalent model. The transmission lines of the external system can be classified into two categories:

(1) Lines with significant effect on the equivalent model.

(2) Lines with no significant effect on the equivalent model.

The classification is based on the accuracy indices $(S)_j$. $(MP)_j$, and $(MQ)_j$ which have been defined. For the j^{th} line of the external system, the accuracy indices $(S)_j$, $(MP)_j$, and $(MQ)_j$ are calculated.

If

$$\frac{(S)_j}{\overline{S}} - 1 < C_1$$

$$\frac{(MP)_j}{\overline{MP}} - 1 < C_2$$

and

$$\frac{(MQ)_j}{\overline{MQ}} - 1 < C_3 \tag{65}$$

the j^{th} line belongs to the second category; else it belongs to the first. If one line of the external system goes out of operation and it belongs to the second group, the equivalent model does not need any modification. If the line belongs to the first group, the admittance matrix of the equivalent model should be updated.

The updating can be achieved with minimal computational effort by using the well known matrix inversion lemma. Assume that the line which connects the i^{th} and j^{th} busses of the external system is tripped out. Then the nodal admittance matrix becomes:

$$Y'_{EE} = Y_{EE} - y_{ij}\underline{e}_{ij}\underline{e}_{ij}^T \tag{66}$$

where y_{ij} is the complex admittance of the line ij and,

$$\underline{e}_{ij} = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ -1 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \begin{array}{l} \\ \\ \\ \\ \leftarrow i^{th} \text{ entry} \\ \\ \\ \\ \leftarrow j^{th} \text{ entry} \\ \\ \\ \\ \\ \end{array} \tag{67}$$

By applying the matrix inversion lemma:

$$(Y'_{EE})^{-1} = Y_{EE}^{-1} + \frac{y_{ij}}{q_{ij}} Y_{EE}^{-1}\underline{e}_{ij}\underline{e}_{ij}^{T}Y_{EE}^{-1} \qquad (68)$$

where

$$q_{ij} = 1 + y_{ij}\underline{e}_{ij}^{T}Y_{EE}^{-1}\underline{e}_{ij} \qquad (69)$$

The updated admittance matrix of the equivalent model is given by:

$$Y'_{eq} = \begin{bmatrix} Y_{BB}-Y_{BE}(Y'_{EE})^{-1}Y_{EB} & Y_{BI} \\ Y_{IB} & Y_{II} \end{bmatrix} =$$

$$= Y_{eq} + \begin{bmatrix} -\frac{y_{ij}}{q_{ij}} Y_{BE}Y_{EE}^{-1}\underline{e}_{ij}\underline{e}_{ij}^{T}Y_{EE}^{-1}Y_{EB} & 0 \\ 0 & 0 \end{bmatrix} \qquad (68)$$

Further, define

$$\underline{D}_{ij} \triangleq Y_{BE}Y_{EE}^{-1}\underline{e}_{ij} \qquad (69)$$

Then,

$$Y'_{eq} = Y_{eq} + \begin{vmatrix} -\frac{y_{ij}}{q_{ij}} \underline{D}_{ij}\underline{D}_{ij}^{T} & 0 \\ 0 & 0 \end{vmatrix} \qquad (70)$$

The above matrix equation indicates that the equivalent admittance matrix changes by a matrix

$$-\frac{y_{ij}}{q_{ij}} \underline{D}_{ij}\underline{D}_{ij}^{T}$$

It should be noted that $\underline{D}_{ij}$ is the difference between the i^{th} and the j^{th} column of the matrix $Y_{BE}Y_{EE}^{-1}$.

3.5 Detection of External System Outages

Because of limitations in the exchange of information between neighboring companies, outages of major lines in the external system may be unknown to the operators of the internal system. Thus, the need to detect unreported topological changes in the external system is created. A detection scheme has been developed which takes advantage of the well known DC load flow equations. These equations are approximate. For detection purposes, however, they are adequate. Furthermore, it is assumed that:

(1) The susceptances of the transmission lines are larger than the conductances, i.e.

$$G_{ij} + jB_{ij} \cong jB_{ij}$$

(2) Voltage magnitudes are constant and equal to the nominal value, i.e.

$$V_i = 1 \text{ p.u.}$$

for all busses.

(3) The voltage phase angle difference across a line is small, i.e.

$$\sin(\Theta_i - \Theta_j) \cong \Theta_i - \Theta_j$$

The above realistic assumptions yield the so called DC model. Then, then entire area system equations are:

$$\underline{P} = A\underline{\Theta} \tag{71}$$

where $\underline{P}$ is the vector of real injections and $\underline{\Theta}$ is the vector of the voltage phase angles, and

$$A_{ij} = \begin{cases} -B_{ij} & \text{if } i \neq j \\ \sum\limits_{j \in a(i)} B_{ij} & \text{if } i = j \end{cases}$$

Equation (71) can be written as:

$$\begin{vmatrix} A_{EE} & A_{ER} \\ A_{RE} & A_{RR} \end{vmatrix} \begin{vmatrix} \underline{\Theta}_E \\ \underline{\Theta}_R \end{vmatrix} = \begin{vmatrix} \underline{P}_E \\ \underline{P}_R \end{vmatrix} \tag{72}$$

where the subscripts E and R refer to external and reduced systems, respectively. Elimination of the vector $\underline{\Theta}_E$ yields:

$$A_{eq}\underline{\theta}_R = \underline{P}_{eq} \tag{73}$$

where:

$$A_{eq} = A_{RR} - A_{RE}A_{EE}^{-1}A_{ER} \tag{74}$$

$$\underline{P}_{eq} = \underline{P}_R - A_{RE}A_{EE}^{-1}\underline{P}_E \tag{75}$$

The detection scheme is based on the following proposition:

Proposition: Following the outage of a transmission line which connects the i^{th} and j^{th} busses of the external system, the vector of the voltage phase angles of the reduced system changes by a vector $\underline{\Delta\Theta}_R$ such that:

$$A_{eq}\underline{\Delta\Theta}_R = \lambda\underline{C}_{ij} \tag{76}$$

where λ is a constant and $\underline{C}_{ij}$ is a vector which completely characterizes the line outage. The vector $\underline{C}_{ij}$ is defined as:

$$\underline{C}_{ij} = A_{RE}A_{EE}^{-1}\underline{e}_{ij} \tag{77}$$

where

$$\underline{e}_{ij} = \begin{vmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ -1 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{vmatrix} \begin{array}{l} \\ \\ \\ \\ \leftarrow i^{th} \text{ entry} \\ \\ \\ \\ \leftarrow j^{th} \text{ entry} \\ \\ \\ \\ \\ \end{array}$$

<u>Proof</u>: Following the outage of a transmission line which connects the i^{th} and j^{th} bus of the external system, the matrix A_{EE} is modified as:

$$A'_{EE} = A_{EE} - B_{ij}\underline{e}_{ij}\underline{e}_{ij}^T$$

where B_{ij} is the susceptance of the line. By applying the matrix inversion lemma one obtains:

$$(A'_{EE})^{-1} = (A_{EE} - B_{ij}\underline{e}_{ij}\underline{e}_{ij}^T)^{-1} = A_{EE}^{-1} - \frac{B_{ij}}{D_{ij}} A_{EE}^{-1}\underline{e}_{ij}\underline{e}_{ij}^T A_{EE}^{-1}$$

where

$$D_{ij} = 1 + B_{ij}\underline{e}_{ij}^T A_{EE}^{-1}\underline{e}_{ij} \quad \text{(scalar)} \tag{78}$$

Let us define as:

$$\ell_{ij} \triangleq \frac{B_{ij}}{D_{ij}} \tag{79}$$

and

$$\underline{\beta} = A_{EE}^{-1} \underline{e}_{ij} \tag{80}$$

then

$$(A'_{EE})^{-1} = A_{EE}^{-1} - \ell_{ij} \underline{\beta}\,\underline{\beta}^T \tag{81}$$

The outage changes the matrix of the equivalent model to:

$$\begin{aligned} A'_{eq} &= A_{RR} - A_{RE}(A_{EE}^{-1} - \ell_{ij} \underline{\beta}\,\underline{\beta}^T) A_{ER} \\ &= A_{eq} + \ell_{ij} A_{RE} \underline{\beta}\,\underline{\beta}^T A_{ER} \\ &= A_{eq} + \ell_{ij} \underline{C}_{ij} \underline{C}_{ij}^T \end{aligned} \tag{82}$$

where the vector $\underline{C}_{ij}$ has been defined earlier. The modified real injection vector is:

$$\begin{aligned} \underline{P}'_{eq} &= \underline{P}_R - A_{RE}(A_{EE}^{-1} - \ell_{ij} A_{EE}^{-1} \underline{e}_{ij} \underline{e}_{ij}^T A_{EE}^{-1}) \underline{P}_E \\ &= \underline{P}_R - A_{RE} A_{EE}^{-1} \underline{P}_E + \ell_{ij} A_{RE} A_{EE}^{-1} \underline{e}_{ij} \underline{e}_{ij}^T A_{EE}^{-1} \underline{P}_E \end{aligned} \tag{83}$$

If

$$\mu \triangleq \underline{e}_{ij}^{T} A_{EE}^{-1} \underline{P}_{E} \tag{84}$$

then

$$P'_{eq} = P_{eq} + \mu \ell_{ij} \underline{C}_{ij} \tag{85}$$

The voltage phase angles of the busses in the internal system, after the outage in the external system, change from $\underline{\Theta}_R$ to $\underline{\Theta}_R + \underline{\Delta\Theta}_R$:

$$A'_{eq}(\underline{\Theta}_R + \underline{\Delta\Theta}_R) = \underline{P}'_{eq}$$

By substituting the expressions for A'_{eq}, P'_{eq} from equations (82) and (85), one obtains:

$$(A_{eq} + \ell_{ij}\underline{C}_{ij}\underline{C}_{ij}^{T})(\underline{\Theta}_R + \underline{\Delta\Theta}_R) = \underline{P}_{eq} + \mu \ell_{ij} \underline{C}_{ij}$$

or

$$A_{eq}\underline{\Theta}_R + A_{eq}\underline{\Delta\Theta}_R + \ell_{ij}\underline{C}_{ij}\underline{C}_{ij}^{T}(\underline{\Theta}_R + \underline{\Delta\Theta}_R) = \underline{P}_{eq} + \mu \ell_{ij} \underline{C}_{ij}$$

Since $A_{eq}\underline{\Theta}_R = \underline{P}_{eq}$ and $\underline{C}_{ij}^{T}(\underline{\Theta}_R + \underline{\Delta\Theta}_R)$ is a scalar, the above expression is simplified to:

$$A_{eq}\underline{\Delta\Theta}_R = \ell_{ij}\{\mu - \underline{C}_{ij}^{T}(\underline{\Theta}_R + \underline{\Delta\Theta}_R)\}\underline{C}_{ij}$$

or

$$A_{eq}\underline{\Delta\Theta}_R = \lambda\,\underline{C}_{ij}$$

where

$$\lambda = \ell_{ij}\{\mu - \underline{C}_{ij}^T(\underline{\Theta}_R + \underline{\Delta\Theta}_R)\} \qquad \text{Q.E.D.}$$

Since the study area is observable to a state estimator, the vectors $\underline{\Theta}_R$ and $\underline{\Theta}_R + \underline{\Delta\Theta}_R$, i.e. the voltage phase angles of the busses of the internal system before and after the outage in the external system, are known. The vector:

$$\underline{d} \triangleq A_{eq}\underline{\Delta\Theta}_R \tag{86}$$

is also a known vector. Let $\underline{d}_N$ be the normalized vector of $\underline{d}$

$$\underline{d}_N = \frac{\underline{d}}{||\underline{d}||} \tag{87}$$

and $\underline{C}_{ij,N}$ the normalized vector of $\underline{C}_{ij}$

$$\underline{C}_{ij,N} = \frac{\underline{C}_{ij}}{||\underline{C}_{ij}||} \tag{88}$$

Then, if

$$\underline{d} = \lambda\,\underline{C}_{ij}$$

$$\underline{d}_N \cdot \underline{C}_{ij,N} = \frac{1}{||\underline{d}||\ ||\underline{C}_{ij}||}\,\underline{d} \cdot \underline{C}_{ij}$$

$$= \frac{\lambda}{||d|| \; ||C_{ij}||} \underline{C}_{ij} \cdot \underline{C}_{ij}$$

$$= \frac{\lambda}{|\lambda| \; ||C_{ij}|| \; ||C_{ij}||} ||C_{ij}||^2$$

$$= \text{sign}(\lambda) 1$$

Therefore,

$$|\underline{d}_N \cdot \underline{C}_{ij,N}| = 1 \tag{89}$$

The above developments form the base for the detection scheme which is described next.

For all the lines of the external system which have significant effect on the Norton equivalent model the normalized vectors $\underline{C}_{ij,N}$ are precomputed. When the state estimator detects a sudden change in the phase voltage angles of the boundary busses, the vector $\underline{d}_N$ and the inner products $\underline{d}_N \cdot \underline{C}_{ij,N}$ are computed. For the dropped line the absolute value of the inner product (89) will assume a value close to one. Since the detection scheme is based on DC analysis it is expected that this absolute value will not be exactly equal to one but very close to one.

It has been observed that the transmission lines of the external system form a set of groups. The lines of the ℓ^{th} group have the same normalized vector and this is denoted by $\underline{C}_N^{\ell}$. Therefore, the detection scheme detects the group in which the line in outage belongs. For the i^{th} line belonging the the ℓ^{th} group, there is a normalized constant

K_i^ℓ such that:

$$\underline{C}_i = K_i^\ell \underline{C}_N^\ell \tag{90}$$

From equation (82) it can be seen that with the i^{th} line out of operation, the equivalent matrix A_{eq} is modified by a matrix.

$$(K_i^\ell)^2 \ell_i \underline{C}_N^\ell (\underline{C}_N^\ell)^T$$

The constant ℓ_i has been defined. For two lines i and j belonging to the same group and connected in series the following relationships holds

$$(K_i^\ell)^2 \ell_i = (K_j^\ell)^2 \ell_j \tag{91}$$

Therefore, if all the lines of the same group are in series, then these lines have the same effect on the equivalent model. If the line in outage belongs to such a group, by detecting the group, using the detection scheme described earlier, the equivalent model can be updated appropriately.

In the general case, more information is needed to identify which line of the particular group detected by the detection scheme is out of operation.

How to detect the line in outage after detecting the group to which the line belongs is discussed next. It is assumed that the real base load condition for the external system is available and this is denoted by the vector $\underline{a}_E$. Proceeding as earlier, it is easy to prove that

$$A_{eq}\underline{\Delta\Theta}_R = K_i^\ell \ell_i \{\beta_i - K_i^\ell (\underline{C}_N^\ell)^T (\underline{\Theta}_R + \underline{\Delta\Theta}_R)\}\underline{C}_N^\ell$$
$$= \Lambda\, \underline{C}_N^\ell \tag{92}$$

The constant β_i is defined as:

$$\beta_i = \underline{e}_i^T A_{EE}^{-1} \underline{a}_E \tag{93}$$

and this is a known number.

Since the vectors $A_{eq}\underline{\Delta\Theta}_R$ and $\underline{C}_N^\ell$ are known vectors, the constant Λ can be computed. For each line belonging to the same group which has been found by the detection scheme, the constants

$$\Lambda_i = K_i^\ell \ell_i \{\beta_i - K_i^\ell (\underline{C}_N^\ell)^T (\underline{\Theta}_R + \underline{\Delta\Theta}_R)\} \tag{94}$$

are computed.

The line in outage gives the smallest value of the $|\Lambda - \Lambda_i|$. An illustrative example for the detection scheme is given in Section 5.1.3

CHAPTER IV

ON-LINE EQUIVALENTS

4.1 General

AC power flow equations used in contingency analysis are non-linear. Switching and/or transformer tap-changing operations complicate; furthermore, the analysis of power systems. These nonlinearities should be accounted for by the equivalent model. The Norton-type equivalents are based on engineering insight; whereby, linear reduction is used to obtain equivalents for a non-linear set of equations. Normally, these equivalents give good results in most cases. But there are cases of serious discrepancies ranging from failure of the load flow algorithm to converge to cases of highly erroneous answers. If the external system consists of many busses, the Norton-type equivalent model requires a large amount of data to be processed. Another disadvantage of the Norton type equivalent model is that the classification of the boundary busses as generation busses or as load busses is system dependent. These shortcomings, together with the uncertainty with regard to the topology and the parameter values of the external system because of limited exchange of information between neighboring companies, have increased in recent years the interest for on-line type equivalents [28].

In this chapter, on-line type equivalents are derived by using information from the internal system only. Switching operations, together with on-line state estimation, are the main sources of information to obtain the equivalent model. In the next section the formulation of the problem for on-line equivalents is presented.

4.2 Formulation of the Problem for On-Line Equivalents

In this section the mathematical formulation presented in Chapter II is stated as it is applied for on-line equivalents.

For on-line equivalents it is assumed that no information from the external system is available. An equivalent representation of the external system needs to be obtained by using information from the internal system only.

Figure 4.1 shows a set of boundary busses. The dotted lines indicate equivalent branches between the boundary busses.

Let a(i) be the set of busses of the internal system connected to the i^{th} boundary bus and b(i) the set of the boundary busses connected to the i^{th} boundary bus. The real and reactive injection at the i^{th} boundary bus is given by:

$$P_i = V_i^2\{\sum_{j\epsilon b(i)} G_{ij}\} - V_i \sum_{j\epsilon b(i)} V_j\{G_{ij}\cos\Theta_{ij} + B_{ij}\sin\Theta_{ij}\}$$

$$+ \sum_{j\epsilon\alpha(i)} P_{ij} \tag{95}$$

$$Q_i = -V_i^2\{B_{SHUNT,i} + \sum_{j\epsilon b(i)} B_{ij}\} - V_i \sum_{j\epsilon b(i)} V_j\{G_{ij}\sin\Theta_{ij}$$

$$-B_{ij}\cos\Theta_{ij}\} + \sum_{j\epsilon\alpha(i)} Q_{ij} \tag{96}$$

where:

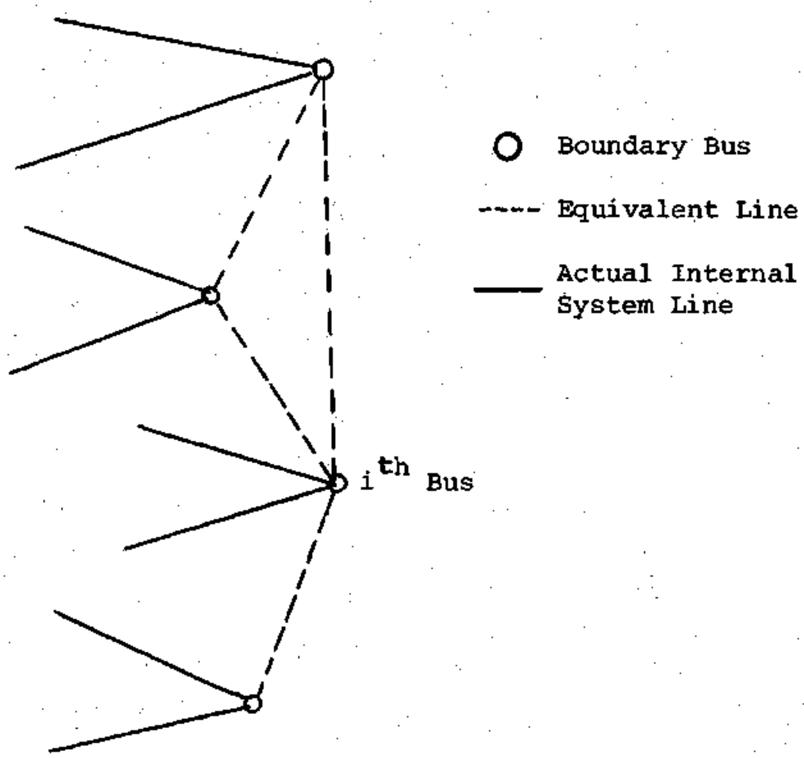

Figure 4.1. Boundary Bus Interconnections.

V_i = voltage magnitude of the i^{th} bus

Θ_{ij} = voltage phase angle difference across the line ij

G_{ij} = conductance of line ij

B_{ij} = susceptance of line ij

$B_{SHUNT,i}$ = shunt susceptance at the i^{th} bus

P_{ij} = real flow from the i^{th} bus to the j^{th} bus

Q_{ij} = reactive flow from the i^{th} bus to the j^{th} bus.

G_{ij}, B_{ij}, $B_{SHUNT,i}$ are components of the vector $\underline{u}$.

Using equations (95), (96), the vectors of real and reactive injections at the boundary busses before and after the k^{th} outage can be expressed as:

$$\underline{I}^{1k} = A(\underline{x}^{1k})\underline{u} + \underline{T}^{1k} \tag{97}$$

$$\underline{I}^{2k} = A(\underline{x}^{2k})\underline{u} + \underline{T}^{2k} \tag{98}$$

where:

$\underline{T}^{1k}, \underline{T}^{2k}$ $\triangleq$ vectors of pre- and post-outage power flows from the boundary busses to the internal system. These are known quantities.

$A(\underline{x}^{1k}), A(\underline{x}^{2k})$ $\triangleq$ matrices which are strictly dependent on $\underline{x}^{1k}$ and $\underline{x}^{2k}$.

The difference between the pre- and post-outage injections at the boundary busses becomes

$$\underline{g}_k = \underline{I}^{1k} - \underline{I}^{2k}$$

$$= [A(\underline{x}^{1k}) - A(\underline{x}^{2k})]\underline{u} + \underline{T}^{1k} - \underline{T}^{2k}$$

$$= H^k\underline{u} + \underline{M}^k \quad ; \quad k=1, \ldots, N \tag{99}$$

Since the internal system is observable to a state estimator, the matrix H^k and the vector $\underline{M}^k$ are known quantities.

The objective function of the equivalent problem becomes:

$$J = \sum_{k=1}^{N} \underline{g}_k^T \underline{g}_k$$

$$= \sum_{k=1}^{N} (H^k\underline{u} + \underline{M}^k)^T(H^k\underline{u} + \underline{M}^k) \tag{100}$$

Next the constraints of the problem are presented. Let us define as:

$$\underline{g}_{N_1} = H_{N_1}\underline{u} + \underline{M}_{N_1} \tag{101}$$

$$\underline{g}_{N_{2,\ell}} = H_{N_{2,\ell}}\underline{u} + \underline{M}_{N_{2,\ell}} \quad ; \quad \ell=1, \ldots, L \tag{102}$$

where:

$$H_{N_1} = \begin{vmatrix} H^1 \\ \cdot \\ \cdot \\ \cdot \\ H^k \\ \cdot \\ \cdot \\ \cdot \\ H^{N_1} \end{vmatrix} \qquad \underline{M}_{N_1} = \begin{vmatrix} \underline{M}^1 \\ \cdot \\ \cdot \\ \cdot \\ \underline{M}^k \\ \cdot \\ \cdot \\ \cdot \\ \underline{M}^{N_1} \end{vmatrix} \tag{103}$$

and

$$H_{N_{2,\ell}} = \begin{vmatrix} H^1 \\ \cdot \\ \cdot \\ \cdot \\ H^k \\ \cdot \\ \cdot \\ \cdot \\ H^{N_{2,\ell}} \end{vmatrix} \qquad \underline{M}_{N_{2,\ell}} = \begin{vmatrix} \underline{M}^1 \\ \cdot \\ \cdot \\ \cdot \\ \underline{M}^k \\ \cdot \\ \cdot \\ \cdot \\ \underline{M}^{N_{2,\ell}} \end{vmatrix} \tag{104}$$

Then the constraints of the problem

$$\underline{F}_N(\underline{u}) = |\underline{g}(\underline{x}^1, \underline{x}^2, \underline{u})| - \bar{\underline{g}}_N \le \underline{0}$$

becomes

$$\underline{F}_N(\underline{u}) = |H_N\underline{u} + \underline{M}_N| - \bar{\underline{g}}_N \le \underline{0} \tag{105}$$

where:

$$H_N = \begin{vmatrix} H_{N_1} \\ H_{N_{2,1}} \\ \cdot \\ \cdot \\ \cdot \\ H_{N_{2,\ell}} \\ \cdot \\ \cdot \\ \cdot \\ H_{N_{2,L}} \end{vmatrix} \qquad \underline{M}_N = \begin{vmatrix} \underline{M}_{N_1} \\ \underline{M}_{N_{2,1}} \\ \cdot \\ \cdot \\ \cdot \\ \underline{M}_{N_{2,\ell}} \\ \cdot \\ \cdot \\ \cdot \\ \underline{M}_{N_{2,L}} \end{vmatrix} \tag{106}$$

In summary, the problem is:

Minimize:

$$J = \sum_{k=1}^{N} (H^k\underline{u} + \underline{M}^k)^T (H^k\underline{u} + \underline{M}^k) \tag{107}$$

subject to the constraints:

$$\underline{F}_N(\underline{u}) = |H_N\underline{u} + \underline{M}_N| - \overline{\underline{g}}_N \le \underline{0} \tag{108}$$

For each outage there are 2b inequality constraints; therefore, there are 2xbxN inequalities to be satisfied. The method to solve the optimization problem described by the equations (107) and (108) is discussed in the next sections.

4.3 Unconstrained Problem

If the internal system is highly interconnected to the external system and the number of switching operations under consideration is large, the number of the inequality constraints to be satisfied is high. Therefore, before the problem is solved in its entirety, the model obtained by solving the unconstrained problem should be tested. In this section the method of solving the unconstrained problem is derived and the various aspects of this model are discussed.

4.3.1 Equivalent Model

The objective function is a quadratic function of the unknown vector $\underline{u}$. This quadratic function takes its minimum when:

$$\frac{\partial J}{\partial \underline{u}} = \underline{0} \tag{109}$$

The optimality condition becomes:

$$\sum_{k=1}^{N} (H^k)^T (H^k \underline{u} + \underline{M}^k) = \underline{0} \tag{110}$$

and the optimal solution is given by

$$\hat{u} = -[\sum_{k=1}^{N} (H^k)^T H^k]^{-1} [\sum_{k=1}^{N} (H^k)^T \underline{M}^k] \tag{111}$$

Therefore, the solution is obtained in one iteration and the solution exists assuming that the matrix

$$\sum_{k=1}^{N} (H^k)^T H^k$$

is a nonsingular matrix.

If ℓ_1 is the number of the equivalent branches and ℓ_2 is the number of the fictitious capacitors or reactors at the boundary busses, the solution of the unconstrained problem requires the inversion of a matrix of dimension $(2x\ell_1+\ell_2) \times (2x\ell_1+\ell_2)$. The computer storage requirements and the computational time are dependent upon the values of ℓ_1, ℓ_2. In general, using sparsity techniques both the storage requirements and the computational time are moderate and the method is suitable for on-line operation. The computational aspects to derive the solution of the unconstrained problem are given in Section 5.2.2.

The value of the quadratic function at the optimum is given by

$$\hat{J} = \sum_{k=1}^{N} (\underline{M}^k)^T \underline{M}^k - [\sum_{k=1}^{N} (H^k)^T \underline{M}^k]^T [\sum_{k=1}^{N} (\underline{H}^k)^T \underline{H}^k]^{-1} [\sum_{k=1}^{N} (H^k)^T \underline{M}^k] \tag{112}$$

$\hat{J}$ gives the total error. Note that

$$\hat{J} = NxbxS \tag{113}$$

where the index S has been defined.

The equivalent model obtained from equation (111) should be tested if it satisfies the inequality constraints:

$$\underline{F}_N(\underline{u}) \leq \underline{0}$$

If the inequalities of the problem are satisfied, this equivalent model is acceptable. In this case, since the solution of the unconstrained problem also satisfies the inequality constraints, the equivalent model is optimal.

4.3.2 Sparsity of the Admittance Matrix

The vector $\underline{u}$ determines the connectivity among the boundary busses and the dimension of the vector $\underline{u}$ determines the sparsity of the admittance matrix of the equivalent model. Complete connectivity among all the boundary busses will result in poor sparsity of the admittance matrix if the number of the boundary busses is large. With complete connectivity however, the number of independent variables is the maximum and the unconstrained problem will obtain its optimum solution. In this case, the objective function J obtains its minimum value, J^*. By eliminating some of the equivalent branches, i.e., by reducing the number of the independent variables, the admittance matrix of the equivalent model becomes a sparse matrix but the value of the performance index J becomes greater than the value of J^*.

This is in contrast to the observations made with regard to the sparsity of the admittance matrix of the Norton equivalent where by eliminating some of the equivalent branches, the performance index was improved. This is due to the fact that the Norton equivalent is based on engineering intuition rather than on mathematical analysis.

To solve the unconstrained problem, a connectivity criterion among boundary busses is needed such that the equivalent representation is optimal. If ℓ equivalent branches are retained, where $\ell < \frac{b(b-1)}{2}$, then there are:

$$\binom{\frac{b(b-1)}{2}}{\ell}$$

possible connectivities. It is impractical to examine all these cases to determine the optimal connectivity.

A systematic procedure to define the connectivity of the equivalent model so that it will compromise between the sparsity of the admittance matrix and the performance index is as follows:

<u>First Step</u>: An initial connectivity between the boundary busses is assumed. This is a-priori information which can be based either on equivalencing techniques using the Norton-type equivalent or on past experience of the particular system. This connectivity should contain sufficient number of equivalent branches. The vector $\underline{u}$ which corresponds to the initial connectivity is denoted by $\underline{u}_o$. The dimension of the vector $\underline{u}$ is restricted by the computer storage requirements to invert the matrix

$$\sum_{k=1}^{N} (H^k)^T H^k \quad .$$

<u>Second Step</u>: Using the connectivity defined in the first step, equation (111) is applied to determine the values of the conductances and susceptances of the equivalent branches. The optimum vector $\underline{u}$ which corresponds to this connectivity is denoted by $\hat{\underline{u}}_o$.

<u>Third Step</u>: The vector $\underline{\hat{u}}_o$ is used to compute for each equivalent branch the indices

$$(\Gamma P)_j = \frac{1}{N} \sum_{k=1}^{N} |P_j^{2k}| \tag{114}$$

$$(\Gamma Q)_j = \frac{1}{N} \sum_{k=1}^{N} |Q_j^{2k}| \tag{115}$$

where P_j^{2k}, Q_j^{2k} have been defined. If for the j^{th} branch:

$$(\Gamma P)_j < \overline{\Gamma P}$$

$$(\Gamma Q)_j < \overline{\Gamma Q}$$

then the j^{th} branch is eliminated from the equivalent representation. For various selections of $\overline{\Gamma P}$, $\overline{\Gamma Q}$ the indices S, MP, MQ are calculated. The connectivity which satisfies the accuracy requirements and preserves the sparse structure of the admittance matrix is selected. The vector $\underline{u}$ which corresponds to this connectivity is denoted by $\underline{u}_1$.

<u>Fourth Step</u>: Shunt terms are included at the boundary busses. For the connectivity selected from the third step, the conductances and susceptances of the equivalent branches are computed by using equation (111).

Step four gives the equivalent representation of the external system. Note that by including shunt terms in the fourth step at the boundary busses the sparsity of the admittance matrix is not affected.

4.4 Constrained Problem

If all the equivalent models examined so far are unable to satisfy the requirements of the problem, one has to solve the problem in its entirety. In this section the method to solve the constrained problem is discussed.

In general, there are two approaches to solve the constrained optimization problem.

(1) Using penalty function methods

(2) Using quadratic programming

The penalty function methods transform the constrained problem into an unconstrained problem. A number of methods can be applied to solve the unconstrained problem. Convergence of these methods becomes dependent on the selection of the penalty factors. Early attempts to solve the constrained problem using penalty function methods were unsuccessful because of convergence difficulties.

Quadratic programming was chosen as the method to solve the problem. It guarantees that the optimal solution is obtained in a finite number of steps assuming that such solution exists.

4.4.1 Equivalent Model

The general statement of a quadratic problem is:

Minimize the quadratic function:

$$x_o = \underline{c}^T\underline{u} + \underline{u}^T D\underline{u} \tag{116}$$

subject to a linear system of constraints

$$A\underline{u} \leq \underline{P}_o \tag{117}$$

where:

$\underline{c}$ = an n-vector of constraints

$\underline{P}_o$ = an m-vector of constraints

D = an nxn matrix

A = an mxn matrix

$\underline{u}$ = an n-vector of unknowns

The matrix D is assumed to be positive semi-definite. This assures that the quadratic function x_o is convex in $\underline{u}$ and since the constraints are linear, the solution space is convex too. Therefore, if a minimum is found, it is a global minimum.

Next, the equivalent problem is formulated as a quadratic programming problem. The objective function of the problem is:

$$J = \sum_{k=1}^{N} (H^k\underline{u} + \underline{M}^k)^T(H^k\underline{u} + \underline{M}^k)$$

$$= \sum_{k=1}^{N} (\underline{M}^k)^T\underline{M}^k + \underline{u}^T \sum_{k=1}^{N} (H^k)^T\underline{M}^k + \{\sum_{k=1}^{N} (\underline{M}^k)^T H^k\}\underline{u}$$

$$+ \underline{u}^T\{\sum_{k=1}^{N} (H^k)^T H^k\}\underline{u}$$

$$= \sum_{k=1}^{N} (\underline{M}^k)^T\underline{M}^k + 2\{\sum_{k=1}^{N} (H^k)^T\underline{M}^k\}^T\underline{u} + \underline{u}^T\{\sum_{k=1}^{N} (H^k)^T H^k\}\underline{u} \tag{118}$$

Since

$$\sum_{k=1}^{N} (\underline{M}^k)^T \underline{M}^k$$

is a constant, the equivalent problem described by equations (107) and (108) can be restated as:

Minimize:

$$x_o = 2\{\sum_{k=1}^{N} (H^k)^T \underline{M}^k\}^T \underline{u} + \underline{u}^T \{\sum_{k=1}^{N} (H^k)^T H^k\} \underline{u} \tag{119}$$

subject to the constraints:

$$H_N \underline{u} \le \bar{\underline{g}}_N - \underline{M}_N \tag{120}$$

$$-H_N \underline{u} \le \bar{\underline{g}}_N + \underline{M}_N \tag{121}$$

The problem described by equations (119), (120), and (121) is in the form:

Minimize:

$$x_o = \underline{c}^T \underline{u} + \underline{u}^T D \underline{u} \tag{122}$$

subject to:

$$A\underline{u} \le \underline{P}_o \tag{123}$$

where:

$$\underline{c} = 2 \sum_{k=1}^{N} (H^k)^T \underline{M}^k \tag{124}$$

$$D = \sum_{k=1}^{N} (H^k)^T H^k \tag{125}$$

$$A = \begin{bmatrix} H_N \\ -H_N \end{bmatrix} \tag{126}$$

$$\underline{P}_o = \begin{bmatrix} \bar{\underline{g}}_N - \underline{M}_N \\ \bar{\underline{g}}_N + \underline{M}_N \end{bmatrix} \tag{127}$$

The vector $\underline{u}$ is of dimension $n = 2\ell_1 + \ell_2$ where ℓ_1, ℓ_2 as defined earlier. The vector $\underline{c}$ is of dimension m = 4xNxb. The defined matrix D is a positive semi-definite matrix.

The equivalent model $\underline{u}$ is obtained by solving the quadratic programming problem described by the equations (119), (120), and (121).

The quadratic programming problem is solved by direct application of the Kuhn-Tucker conditions. The Kuhn-Tucker conditions reduce the quadratic problem to a linear programming problem. The problem becomes:

Find an n-vector, $\underline{u}$, and n-vector, $\underline{v}$, an m-vector, $\underline{\lambda}$, and an m-vector, $\underline{S}$, such that:

$$-2D\underline{u} - A^T\underline{\lambda} + \underline{v} = \underline{c} \tag{128}$$

$$A\underline{u} + \underline{S} = \underline{P}_o \tag{129}$$

$$\underline{v}^T\underline{u} = 0 \tag{130}$$

$$\underline{\lambda}^T\underline{S} = 0 \tag{131}$$

$$\underline{\lambda} \geq 0,\ \underline{u} \geq 0,\ \underline{v} \geq 0,\ \underline{S} \geq 0$$

The above conditions may be combined as:

$$\left|\begin{array}{c:c:c:c} -2D & -A^T & I & 0 \\ \hdashline A & 0 & 0 & I \end{array}\right| \quad \left|\begin{array}{c} \underline{u} \\ \underline{\lambda} \\ \underline{v} \\ \underline{S} \end{array}\right| = \left|\begin{array}{c} \underline{c} \\ \hdashline \underline{P}_o \end{array}\right|$$

$$\underline{v}^T\underline{u} = 0$$

$$\underline{\lambda}^T\underline{S} = 0$$

$$\underline{\lambda} \geq \underline{0},\ \underline{u} \geq \underline{0},\ \underline{S} \geq \underline{0},\ \underline{v} \geq \underline{0}$$

Therefore, the quadratic programming is equivalent to finding the solution to a set of linear equations. This solution should satisfy the additional constraints $\underline{v}^T\underline{u} = 0$, $\underline{\lambda}^T\underline{S} = 0$.

The solution to the above problem is obtained by using a modification of the revised simplex method. Details are given in Appendix 1.

Solution to the problem exists assuming that the constraints:

$$A\underline{u} \leq \underline{P}_o$$

define a non empty space.

The sparsity of the admittance matrix of the equivalent model is discussed next. The constrained problem is considered after it has been concluded that the equivalent model based on the solution of the unconstrained problem does not satisfy the inequality (108). This model should include the maximum possible number of branches between the boundary busses. This maximum number of branches is specified by the requirement that the admittance matrix should preserve its sparsity. The connectivity determines the variables for the constrained problem also. Note that the performance index of the model based on the solution of the unconstrained problem is the lower limit of the performance index of the model obtained by solving the constrained problem.

The computational aspects to solve the constrained problem are discussed in Section 5.2.3.

4.5 Updating of the Equivalent Model

During the daily operation of a power system, several switching operations are performed by the operator. Transmission lines are taken out of operation during the base load period so that the system remains stable and these lines are inserted back in the system during the peak load period. Furthermore, forced outages may take place in the internal system. Therefore, switching operations and/or forced outages are available on a daily basis and they define the set of N postulated outages.

To update the equivalent model the procedure outlined in the general solution is applicable. For the set of the N postulated outages, the inequality constraints $\underline{F}_N(\underline{u})$ are formed. First, the existing equivalent model should be investigated if it satisfies the specifications of the problem. The performance index S is computed. If

$$S < \overline{S}$$

and

$$\underline{F}_N(\underline{u}) \leq \underline{0}$$

there is no need to update the equivalent model. If the above requirements are not satisfied the equivalent model should be updated. If the model obtained by solving the unconstrained problem satisfies the inequality constraints the model is sufficient. If not one has to solve the constrained problem. The connectivity of the existing equivalent model is used when the updated values of the conductances and susceptances of the equivalent branches are computed. The connectivity of the equivalent model should be investigated again if major outages have been taking place in the external system.

4.6 Summary

In this chapter the solution to the optimization method has been presented. First, the model obtained by solving the unconstrained problem is presented. A systematic procedure to define the connectivity of the equivalent model is developed. Next the optimization problem is formulated as a quadratic programming problem. Finally, the method to solve the quadratic programming problem is presented.

CHAPTER V

TEST CASES AND COMPUTATIONAL ASPECTS

5.1 Test Cases

Five examples are presented in this section. The first deals with the Norton Equivalent. The second is an application of the elimination schemes derived in Section 3.3. The third is an application of the detection scheme derived in Section 3.5. The fourth deals with the unconstrained problem. The fifth deals with the constrained problem.

5.1.1 Example for the Norton Equivalent

This example deals with the Norton equivalent. The 30 bus system shown in Figure 5.1 is the entire system. This is an IEEE test system. The dotted line separates the internal system from the external system. The busses 8, 25, and 30 are the boundary busses. The Norton Equivalent is shown in Figure 5.2. Three equivalent branches are created between the three boundary busses and these branches are denoted by the dotted lines. Five outages were considered in the internal system. The post-outage conditions were obtained by performing the load-flow analysis with the entire area. The Norton Equivalent was tested for the set of these five outages. The indices S, MP, MQ were computed. These indices are given below.

$$S = 7.24 \quad (\text{MVA})^2$$

$$MP = .883 \quad (\text{MW})$$

$$MQ = 8.541 \quad (\text{MVAR})$$

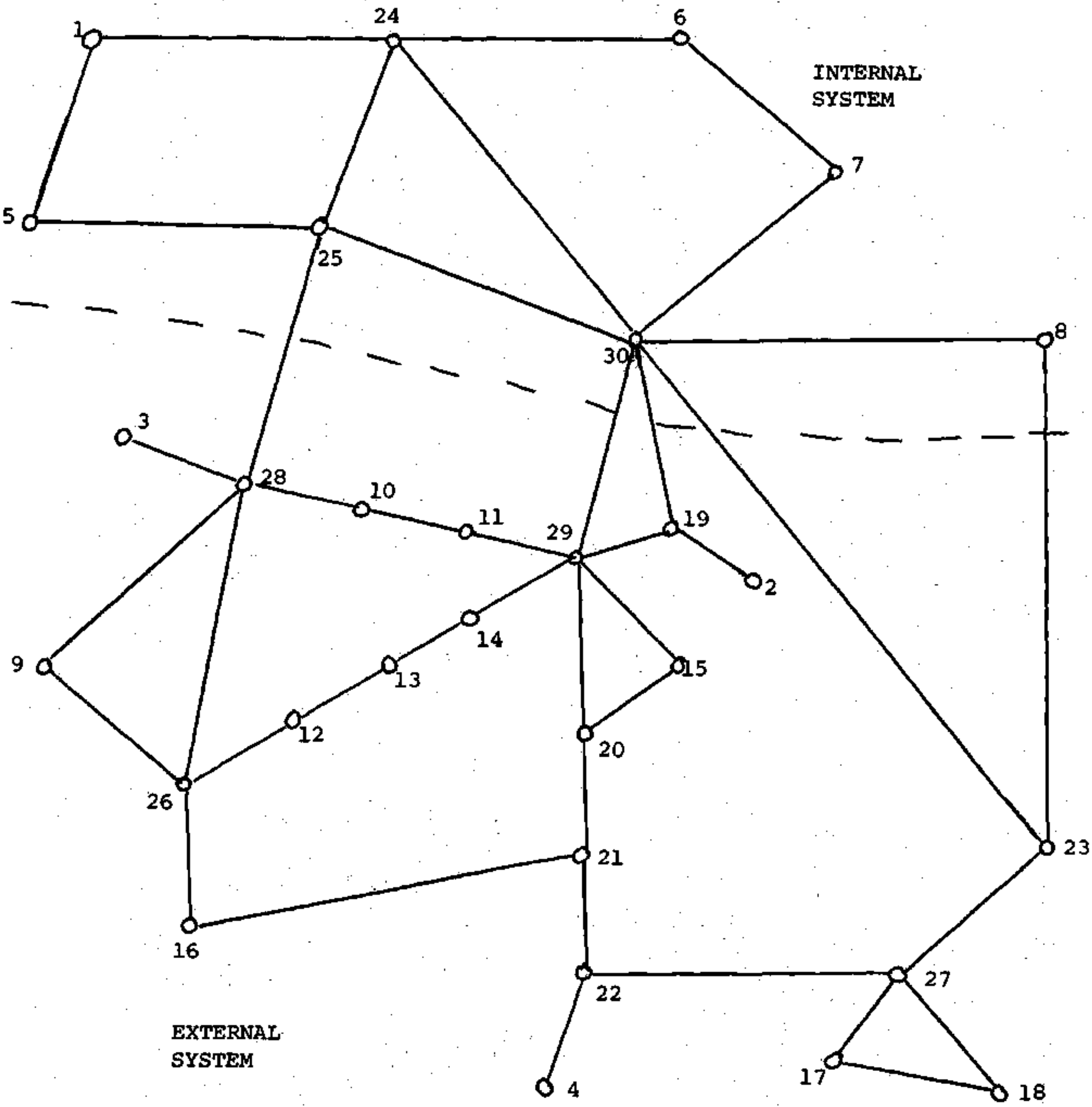

Figure 5.1. Topology of the Internal and External System. (Example 5.1.1)

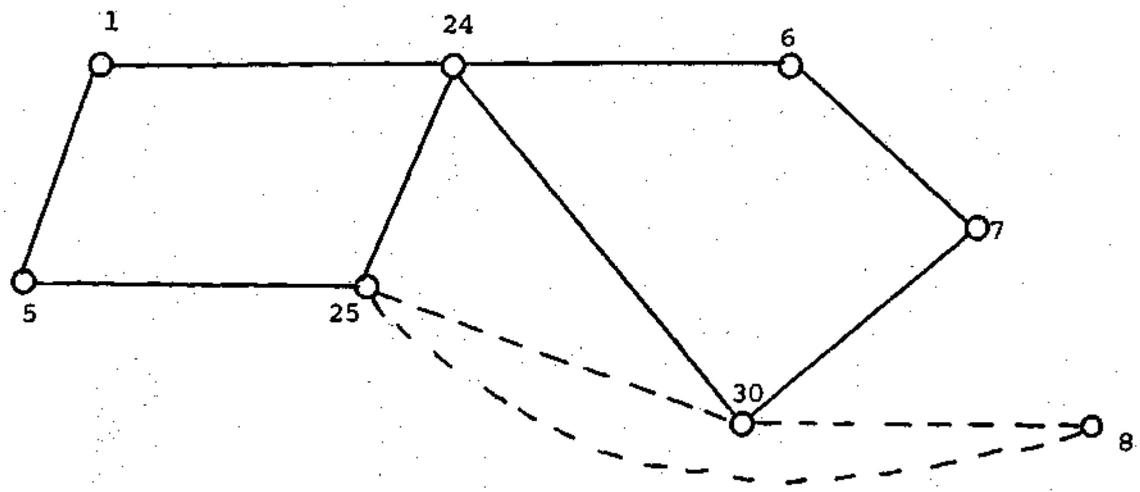

Figure 5.2. Topology of the Norton Equivalent Model of the Internal System. (Example 5.1.1)

If these values of S, MP, MQ are less than the specified tolerances of the problem, the Norton equivalent model is satisfactory. The index S gives the total error. The average difference of the real and reactive injections before and after the five outages is 2.69 (MVA). ($\approx \sqrt{S}$). From the values of MP and MQ it is concluded, for this particular example, that the assumption that the real injections remain constant is more valid than the assumption that the real injections remain constant. This was true for all the other systems and sets of outages we consider in this research.

5.1.2 Example on the Sparsity of the Admittance Matrix of the Norton Equivalent

The two elimination schemes presented in Section 3.3 were tested with a 444 bus system. This is part of the Bonneville Power Administration (BPA) system. The internal system includes 87 busses with 31 boundary busses. The set of postulated outages includes 29 outages in the internal system. The results of these outages were obtained by performing load-flow analysis with the 444 bus system. The Norton equivalent model was obtained with the algorithm we present in Section 5.2.1. 437 equivalent branches were created between the boundary busses. Obviously, the admittance matrix of the equivalent model is not a sparse matrix. The two elimination schemes derived in Section 3.3 were applied to define a connectivity which satisfies both the sparsity requirements and accuracy tolerances. For various selections of $\overline{\Gamma P}$, $\overline{\Gamma Q}$ and $\overline{G}$, $\overline{B}$ the accuracy indices S, MP, MQ were computed and the results are summarized in Tables 1 and 2. The results are shown also in Figures 5.3, 5.4, and 5.5. By examination of the Tables 1 and 2, we conclude that:

Table 1. Performance of the First Elimination Scheme. (Example 5.1.2)

$\overline{\Gamma P}$ (MW)	$\overline{\Gamma Q}$ (MVAR)	No. of Branches Between Boundary Busses	S $(MVA)^2$	MP (MW)	MQ (MVAR)
Initial Connectivity		437	753.34	114.95	354.39
1.	1.	130	726.31	113.21	328.63
2.	2.	109	716.69	111.54	322.84
3.	2.	97	704.55	111.54	306.23
3.	3.	92	675.44	111.54	246.28
4.	3.	83	674.09	111.54	246.28
5.	5.	74	665.51	105.41	246.28
8.	8.	72	665.97	105.41	246.28
10.	10.	66	653.12	151.78	186.37
12.	12.	63	650.59	190.93	186.37
15.	15.	59	648.68	190.93	186.37
18.	18.	57	650.31	190.93	186.37
20.	20.	54	639.37	190.93	184.80
23.	23.	48	886.53	230.28	186.63
25.	25.	47	1046.88	365.25	186.63

Table 2. Performance of the Second Elimination Scheme. (Example 5.1.2)

$\overline{G}$ (p.u)	$\overline{B}$ (p.u)	No. of Branches Between Boundary Busses	S $(MVA)^2$	MP (MW)	MQ (MVAR)
Initial Connectivity		437	753.34	114.95	354.39
.05	.05	135	740.32	113.21	345.39
.1	.1	119	720.14	111.54	322.84
.15	.15	114	719.51	111.54	322.84
.1	.2	104	707.52	111.54	306.23
.2	.2	101	706.06	111.54	306.23
.2	.3	90	688.13	111.54	279.78
.35	.35	88	688.48	111.54	279.78
.4	.4	85	673.86	111.54	246.28
.4	.5	82	669.28	111.54	246.28
.5	.5	81	601.18	105.41	246.28
.6	.6	76	637.41	125.54	195.92
.7	.7	71	642.84	125.54	192.87
.9	.9	68	640.96	190.93	192.87
1.1	1.	66	1118.12	316.47	229.33
.01	.01	184	751.71	114.68	354.15

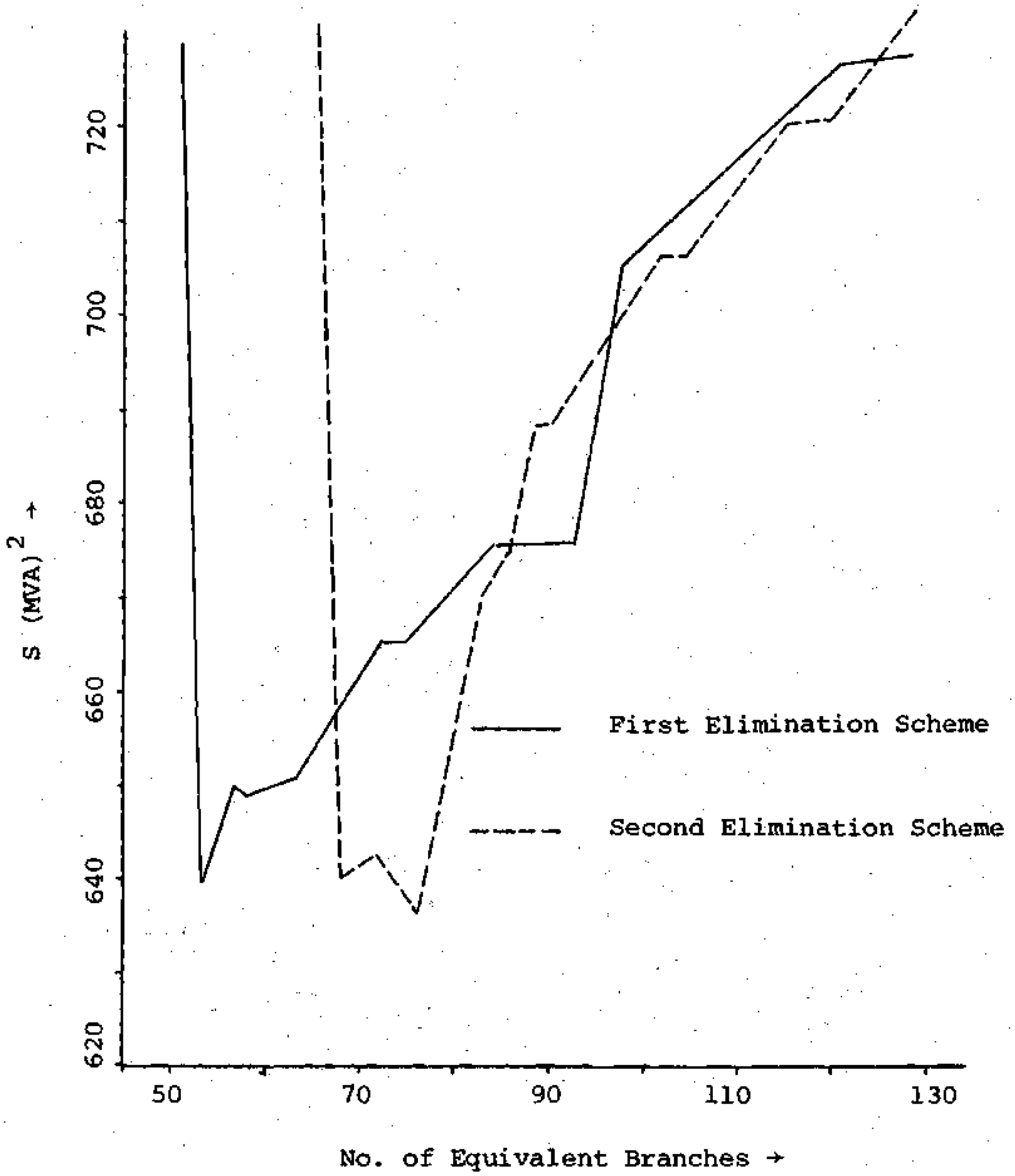

Figure 5.3. The Performance Index S as a Function of the Retained Equivalent Branches with the First and Second Elimination Scheme.

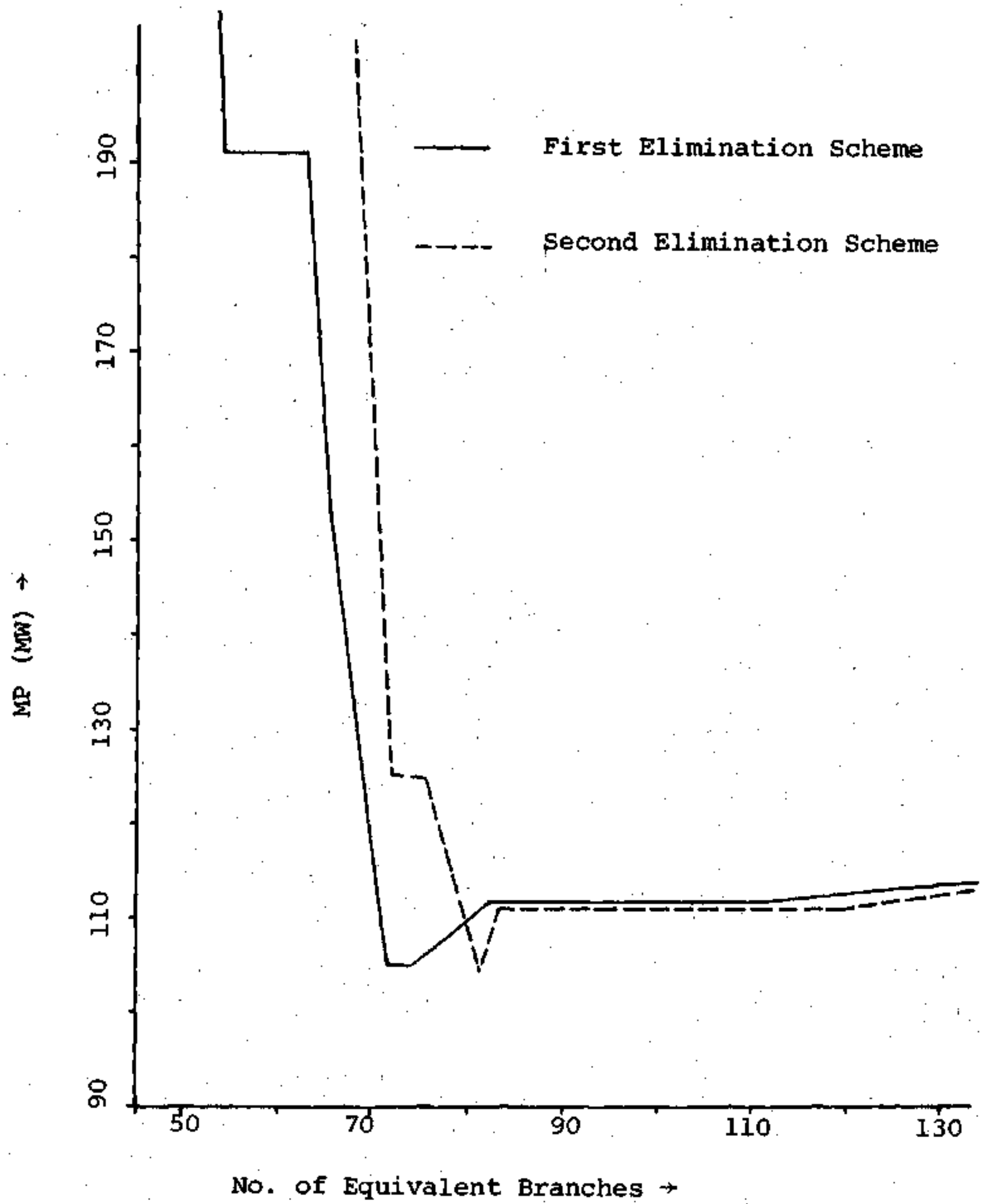

Figure 5.4. The Performance Index MP as a Function of the Retained Equivalent Branches with the First and Second Elimination Scheme.

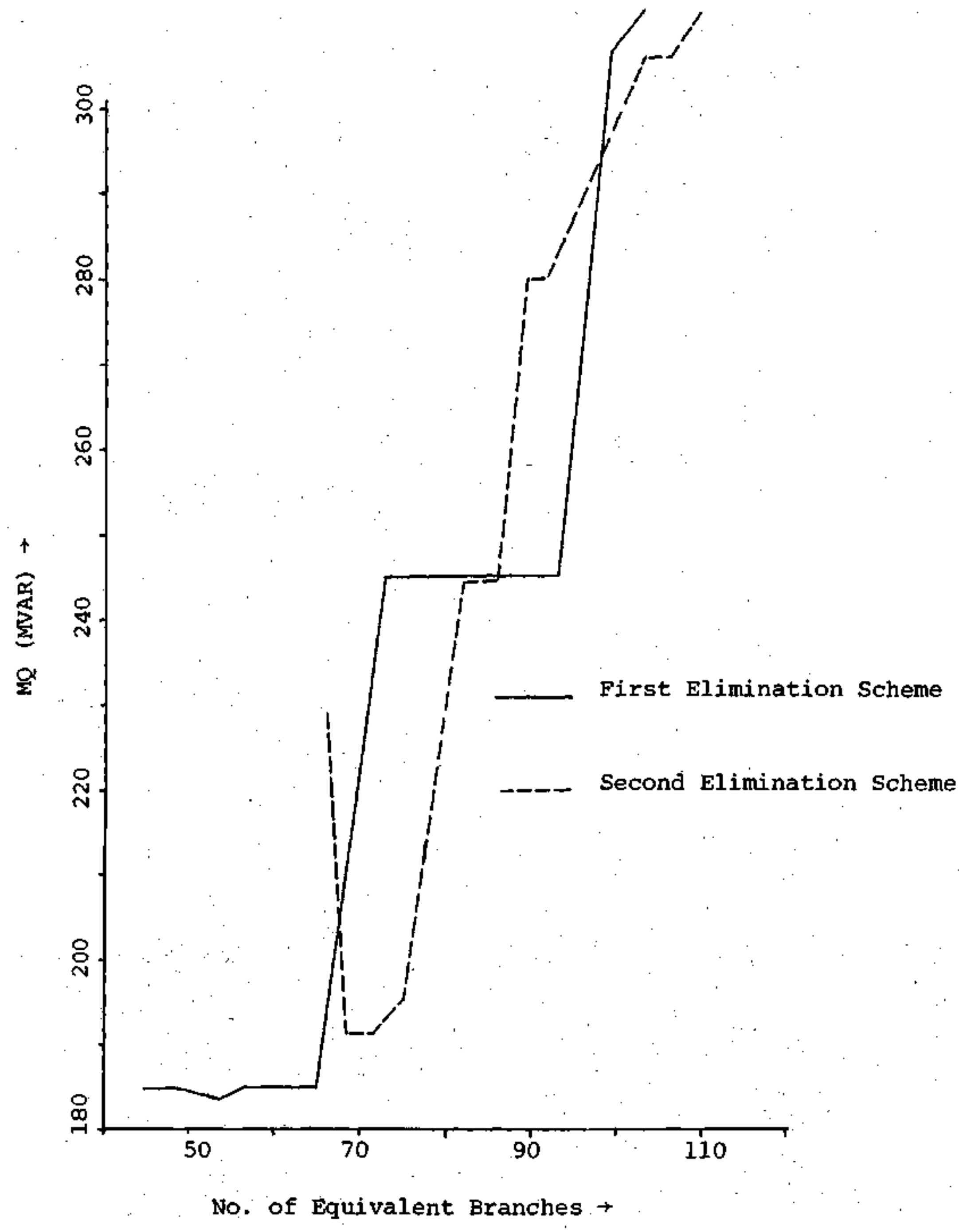

Figure 5.5. The Performance Index MQ as a Function of the Retained Equivalent Branches with the First and Second Elimination Scheme.

(1) The performance of the Norton equivalent model is improved by eliminating some of the equivalent branches.

(2) The first elimination scheme is more effective than the second elimination scheme. Using the first elimination scheme less equivalent branches between the boundary busses are required to succeed the minimum values of the indices S, MP, MQ. This was expected since the first elimination scheme takes into consideration not only the magnitudes of the conductances and susceptances of the equivalent branches but, also, the operating conditions of the system.

(3) The index MP is less sensitive to the number of retained equivalent branches than the index MQ.

The above conclusions cannot be generalized for every power system. Similar investigations should be performed with the particular system and set of postulated outages to define the optimum values of $\overline{\Gamma P}$, $\overline{\Gamma Q}$ or $\bar{G}$, $\bar{B}$. These values will be dependent on the loading condition of the system and the postulated outages.

5.1.3 Example for the Detection Scheme

The detection scheme derived in Section 3.5 was tested with a 30 bus system shown in Figure 5.6. The dotted line separates the internal system from the external system. The busses 6, 16, 24, 25, 26, and 28 are the boundary busses. Since the topology and the parameter values of the external system are assumed to be known, the vectors $\underline{c}_{ij}$, defined by Equation (77), can be computed for all the transmission lines of the external system. From the vectors $\underline{c}_{ij}$ the normalized vectors $\underline{c}_{ij,N}$ were computed. According to these normalized vectors the transmission lines of the external system form six groups. All the transmission lines

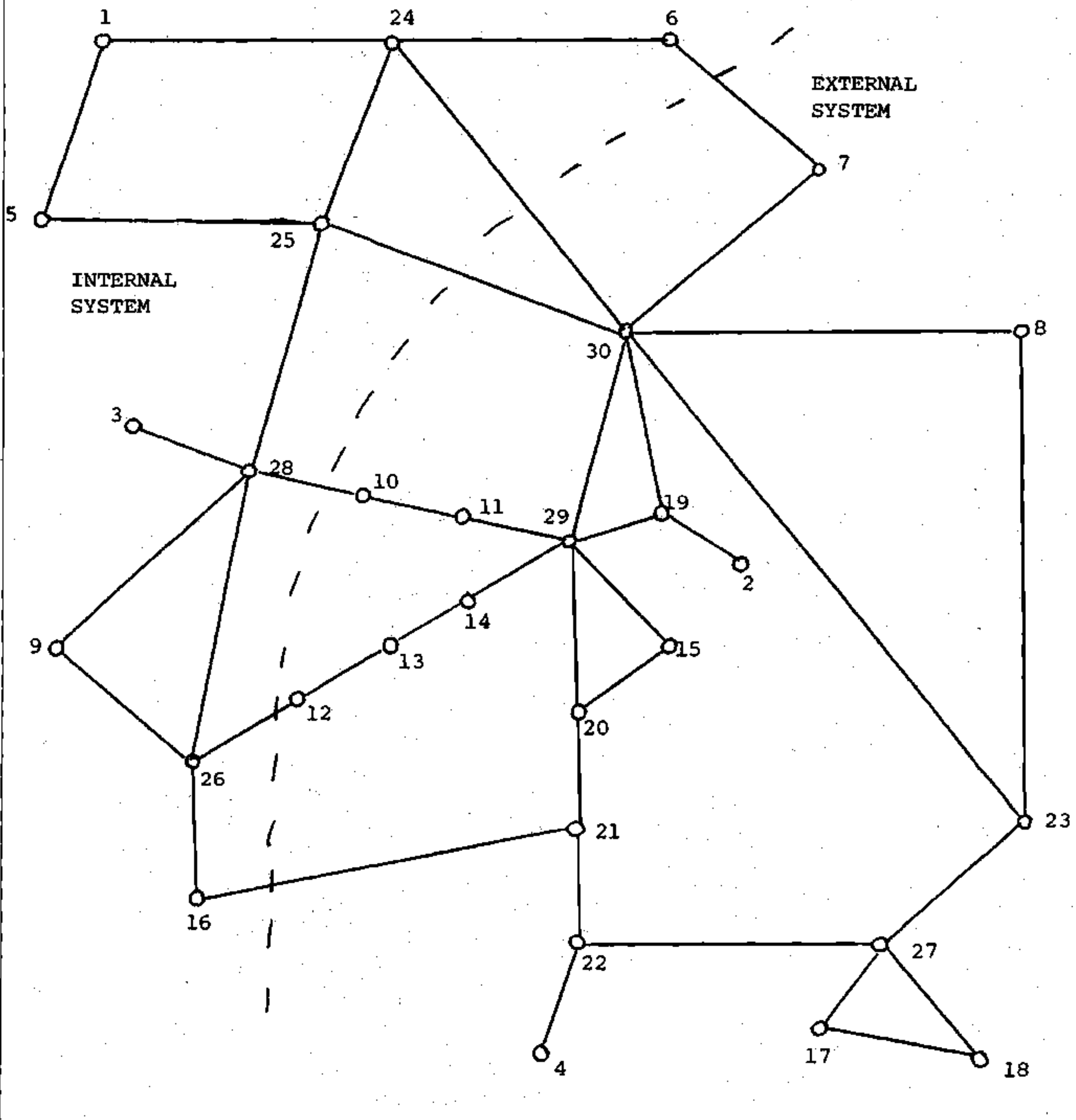

Figure 5.6. Topology of the Internal and External System.
(Example 5.1.3)

belonging to the ℓ^{th} group, where $\ell=1,\ldots,6$, have the same normalized vector c_N^ℓ. For the i^{th} line belonging to the ℓ^{th} group there is a normalized constant K_i^ℓ such that

$$\underline{c}_i = K_i^\ell \underline{c}_N^\ell$$

From Equations (82) and (90), it is obvious that with the i^{th} line of the ℓ^{th} group out of operation, the matrix A_{eq} is modified to

$$A'_{eq} = A_{eq} + (K_i^\ell)^2 \ell_i (\underline{c}_N^\ell)(\underline{c}_N^\ell)^T$$

The constant ℓ_i was defined in Section 3.5 by Equation (79). We denote by $c_N^\ell(i)$ the element of the vector $\underline{c}_N^\ell$ which corresponds to the i^{th} boundary bus.

The groups and the normalized vectors are summarized in Table 3. The constants K_i, ℓ_i, $(K_i^\ell)^2 \ell_i$ are summarized in Table 4.

As it was expected the lines (2-19), (4-22), (17-18), (17-27), and (18-27) do not have any effect on the equivalent model. Examination of the groups reveals that either all the lines of the group are in series or some of the lines of the group form a closed loop and this loop is in series with the rest of the lines of the same group. Notice also that lines in series belong to the same group and they have the same coefficient $(K_i^\ell)^2 \ell_i$.

All the lines of the groups 3, 4, and 5 are in series. If the line in outage belongs to any one of these groups, the Norton equivalent model can be updated simply by detecting the group to which this line

Table 3. Groups and Normalized Vectors for Example 5.1.3.

Group		1	2	3	4	5	6
Lines Belonging to the Group		21-22	20-21	7-30	11-29	14-29	29-30
		22-27	15-29		11-10	12-13	19-29
		23-27	20-29			13-14	19-30
		8-23	15-20				
		23-30					
		8-30					
Normalized Vector	$C_N^\ell(6)$	.12664	.05879	-.79628	-.07738	-.07303	.14899
	$C_N^\ell(16)$	-.73753	-.89028	.02903	-.16910	-.15958	-.31386
	$C_N^\ell(24)$	.14574	.06766	.13487	-.08905	-.08404	.17147
	$C_N^\ell(25)$	.63551	.29506	.58812	-.38834	-.36648	.74772
	$C_N^\ell(26)$	-.07252	.19953	.01883	-.15992	.88677	-.32108
	$C_N^\ell(28)$	-.09785	.26922	.02542	.88379	-.20362	-.43323

Table 4. Parameters k_i^ℓ, ℓ_i, $(k_i^\ell)^2\ell_i$ For Example 5.1.3.

Group	Line	k_i^ℓ	ℓ_i	$(k_i^\ell)^2\ell_i$
1	21-22	.24295	7.11173	.41976
	22-27	.14781	19.21464	.41976
	23-27	.22009	8.66600	.41976
	8-23	.02396	6.74802	.00387
	23-30	.02890	68.25636	.057
	8-30	.00494	158.58120	.00387
2	20-21	.30499	11.17202	1.0392
	15-29	-.06657	31.83888	.14109
	20-29	-.08798	8.35706	.06468
	15-20	-.02141	307.54693	.14109
3	7-30	.44138	31.37112	1.
4	11-29	.15493	75.34560	1.808
	11-10	-.36502	13.57483	1.808
5	14-29	.31501	14.26719	1.41575
	12-13	-.20187	34.73844	1.41575
	13-14	-.10671	124.32689	1.41575
6	29-30	.38871	2.23631	.33789
	19-29	.13446	39.95601	.72238
	19-30	.25425	11.17481	.72238

Table 5. Sample Output of the Detection Scheme. (Example 5.1.3)

Line in outage / $\lvert \underline{d}_N \cdot \underline{c}_N^{\ell} \rvert$	23-30	20-21	7-30	11-29	14-29	19-30	15-29	29-30
Group 1	.96990	.82891	.24858	.24075	.26048	.79422	.81208	.77735
Group 2	.77738	.98834	.11265	.23849	.18260	.31036	.95658	.28855
Group 3	.17426	.13667	.99924	.18117	.22509	.29872	.11798	.30484
Group 4	.29751	.11839	.14844	.99481	.12099	.67465	.06187	.70597
Group 5	.11483	.23985	.15470	.06201	.96016	.36336	.27458	.33568
Group 6	.81048	.36515	.29528	.63739	.61336	.99348	.36954	.98776
Group to Which the Line Belongs	1	2	3	4	5	6	2	6

belongs.

Several examples of detecting the group to which the line in outage belongs are presented below. In our simulation with a line in the external system out of operation, the pre- and post-outage vectors of phase voltage angles of the internal system were obtained by solving the load flow for the 30 bus system. This is so because the internal system is assumed to be observable. From the pre- and post-outage vectors of the phase voltage angles, the vector $\underline{d}$, defined by equation (86), and the normalized vector $\underline{d}_N$ were computed. Finally, for each group the $|\underline{d}_N \cdot \underline{c}_N^{\ell}|$ was computed. Based on the values of $|\underline{d}_N \cdot \underline{c}_N^{\ell}|$, the group to which the line in outage belongs was detected. Some of the results are summarized in Table 5.

From Table 5 it can be seen that the value of $|\underline{d}_N \cdot \underline{c}_N^{\ell}|$ for the line outage is not exactly equal to one. This was expected since the detection scheme is based on DC analysis. The detection scheme, however, safely identifies the group to which the line in outage belongs.

5.1.4. Example for the Unconstrained Problem

The model obtained by solving the unconstrained problem was tested with the 444 bus system described in Section 5.1.2. The set of postulated outages includes 29 outages in the internal system (N=29).

The equivalent model was obtained by following the procedure outlined in Section 4.3.2.

Step One: The initial connectivity between the boundary busses is obtained by the first elimination scheme developed for the Norton equivalent in Section 3.3. This connectivity consists of 92 branches between the 31 boundary busses. In Section 5.1.2 the values of the performance indices S, MP, MQ for the set of 29 outages were given and are cited again:

$$S = 675.44 \text{ (MVA)}^2$$
$$MP = 111.54 \text{ (MW)}$$
$$MQ = 246.28 \text{ (MVAR)}$$

Step Two: For the connectivity obtained in Step One, equation (111) was applied to determine the conductances and susceptances of the 92 equivalent branches ($\ell_1 = 92$, $\ell_2 = 0$). This model was tested for the set of 29 outages and the results are given below:

$$S = 231.53 \text{ (MVA)}^2$$
$$MP = 67.59 \text{ (MW)}$$
$$MQ = 109.49 \text{ (MVAR)}$$

Step Three: The model obtained in Step Two was used to compute for each equivalent branch the indices $(\Gamma P)_j$ and $(\Gamma Q)_j$ defined by equations (114) and (115), respectively. For various selections of $\overline{\Gamma P}$, $\overline{\Gamma Q}$ the performance indices S, MP, MQ were computed and the results are summarized in Table 6. From Table 6 it can be seen that all the selections of $\overline{\Gamma P}$, $\overline{\Gamma Q}$ except the last one gave equivalent models whose performances are almost the same with the performance of the model obtained by solving the unconstrained problem. Based on this elimination procedure, the connectivity consisted of 85 branches between the boundary busses was selected as the connectivity for the equivalent model.

Step Four: For the connectivity obtained in Step Three, the model by solving the unconstrained problem was computed. Using equation (111), the conductances and susceptances of the 85 equivalent branches were computed ($\ell_1 = 85$, $\ell_2 = 0$). This model was tested for the set of 29 outages and the results are given below.

$$S = 234.176 \ (\text{MVA})^2$$
$$MP = 67.70 \ (\text{MW})$$
$$MQ = 109.5 \ (\text{MVAR})$$

Finally, shunt terms were included on 17 boundary busses and equation (111) was applied to determine the conductances and susceptances of the 85 equivalent branches and the susceptances of the 17 shunt terms ($\ell_1 = 85$, $\ell_2 = 0$). This final equivalent model was tested and the results for the 29 outages are given below.

$$S = 184.12 \ (\text{MVA})^2$$
$$MP = 65.82 \ (\text{MW})$$
$$MQ = 106.22 \ (\text{MVAR})$$

The results of this example are summarized in Table 7.

From Table 7, we can conclude that:

(1) The performance of the models obtained by solving the unconstrained problem is superior to the performance of the Norton-type equivalent.

(2) By increasing the number of the equivalent branches, the

Table 6. Results of Elimination Procedure for Example 5.1.4.

$\overline{\Gamma P}$ (MW)	$\overline{\Gamma Q}$ (MVAR)	No. of Branches Between Boundary Busses	S $(MVA)^2$	MP (MW)	MQ (MVAR)
Initial Connectivity		92	231.53	67.59	109.49
5.	5.	88	235.8	67.59	109.49
7.	5.	87	236.33	67.59	109.49
8.	8.	85	237.06	67.59	109.49
10.	9.	83	259.10	164.73	109.49

Table 7. Simulation Results for Example 5.1.4.

Performance Indices \ Equivalent Model	Norton Equivalent 92 Branches	Model by Solving the Unconstrained Problem 92 Branches No Shunt Terms	Model by Solving the Unconstrained Problem 85 Branches No Shunt Terms	Model by Solving the Unconstrained Problem 85 Branches With Shunt Terms
S $(MVA)^2$	675.11	231.53	234.17	184.12
MP (MW)	111.54	67.59	67.70	65.82
MQ (MVAR)	246.28	109.49	109.50	106.22

performance index S is improved. This was expected since by increasing the number of equivalent branches the number of independent variables is increased.

(3) By including shunt terms in the equivalent model, the performance index S is improved. Note that these shunt terms do not affect the sparse structure of the equivalent matrix.

5.1.5 Example for the Constrained Problem

This example demonstrates the feasibility of obtaining an equivalent model by solving a quadratic programming problem as it was presented in Section 4.4. This example will serve also as a comparison between the Norton-type equivalent model, the model obtained by solving the unconstrained problem, and the model obtained by solving the constrained problem. The entire system consists of 30 busses and is shown in Figure 5.1. The dotted line separates the internal system from the external system. The busses 8, 25, 30 are the boundary busses (b=3). The set of postulated outages consists of five outages in the internal system (N=5).

The Norton-type equivalent model was computed in Section 5.1.1. Three equivalent branches are created between the boundary busses. In Section 5.1.1, the performance indices S, MP, MQ for the set of five outages were computed and are cited again:

$$S = 7.24 \quad (MVA)^2$$

$$MP = .833 \ (MW)$$

$$MQ = 8.541 \ (MVAR)$$

The on-line equivalent models were obtained by solving the unconstrained and the constrained problem and assuming only two lines between the boundary busses, one line between the busses 25 and 30 and one line between busses 8 and 30 ($\ell_1 = 2$, $\ell_2 = 0$).

The model obtained by solving the unconstrained problem (Section 4.3) was tested for the set of five postulated outages. The performance indices S, MP, MQ are given below.

$$S = 4.43 \quad (\text{MVA})^2$$

$$MP = .624 \ (\text{MW})$$

$$MQ = 4.595 \ (\text{MVAR})$$

The constrained problem (Section 4.4) was solved under two conditions of constraints:

(a) Maximum allowable real error

$\overline{MP} = 4.5$ (MW)

Maximum allowable reactive error

$\overline{MQ} = 4.5$ (MVAR)

This model was tested for the set of the five postulated outages and the results are given below.

$$S = 4.45 \quad (\text{MVA})^2$$

$$MP = .603 \ (\text{MW})$$

$$MQ = 4.5 \quad (\text{MVAR})$$

(b) Maximum allowable real error

$\overline{MP} = 4.25$ (MW)

Maximum allowable reactive error

$\overline{MQ} = 4.25$ (MVAR)

The obtained model was tested for the same set of five postulated outages. The performance indices are given below.

$$S = 6.19 \quad (MVA)^2$$

$$MP = 3.45 \quad (MW)$$

$$MQ = 4.25 \quad (MVAR)$$

The results of this example are summarized in Table 8.

Table 8 leads to several conclusions:

(1) The performances of the models obtained by solving the unconstrained and the constrained problem are superior to the performance of the Norton-type equivalent model.

(2) The performances of the models obtained by solving the unconstrained and the constrained problem can be improved by including more fictitious branches in the equivalent model ($\ell_1 = 3$, $\ell_2 = 3$).

(3) As it was expected, the performance index S takes it minimum value for the model obtained from the solution of the unconstrained problem. As the constraints of the problem become tighter, the value of the performance index S becomes larger.

Table 8. Simulation Results for Example 5.1.5.

Equivalent Model / Performance Indices	Norton Equivalent 3 Branches	Model by Solving the Unconstrained Problem 2 Branches	Model by Solving the Constrained Problem $\overline{MP}$ = 4.5 (MW) $\overline{MQ}$ = 4.5 (MVAR) 2 Branches	Model by Solving the Constrained Problem $\overline{MP}$ = 4.25 (MW) $\overline{MQ}$ = 4.25 (MVAR) 2 Branches
S $(MVA)^2$	7.24	4.43	4.45	6.19
MP (MW)	.833	.624	.603	3.45
MQ (MVAR)	8.541	4.595	4.5	4.25

5.2 Computational Aspects

In this section the computational aspects of the Norton equivalent model, the equivalent model by solving the unconstrained problem and the equivalent model by solving the constrained problem, are presented.

The basic computational problem of the Norton equivalent and the unconstrained problem is the triangular decomposition of a sparse matrix by Gaussian elimination. Triangular decomposition is a very effective scheme for computing solutions of large sparse systems of linear equations. Basically, we seek the solution of the equation:

$$A\underline{x} = \underline{b} \tag{132}$$

where A is a nonsingular matrix, $\underline{x}$ is the unknown vector, and $\underline{b}$ is a known vector. The triangular decomposition consists of decomposing the matrix A as:

$$A = LU \tag{133}$$

where L is a lower triangular matrix, and U is an upper triangular matrix.

The unknown vector $\underline{x}$ is computed in two steps. First, solve the set of equations:

$$L\underline{y} = \underline{b} \tag{134}$$

for the vector $\underline{y}$. This operation is known as forward substitution. Then solve the set of equations:

$$U\underline{x} = \underline{y} \tag{135}$$

for the vector $\underline{x}$. This operation is known as back substitution.

The decomposition of the matrix A as a product of a lower and upper triangular matrix is accomplished in on step by Gaussian elimination. The elements of the matrices L and U are stored in a table, called the table of factors.

Because of savings in operations and computer memory, it is desirable to process and store only the non-zero elements of the table of factors. The number of the non-zero elements in the table of factors depends on the order which rows are processed. Several ordering schemes have been developed so that the elimination process yields the least possible non-zero elements in the table of factors.

The arithmetic operations required to compute the table of factors for an n^{th} order system are as follows:

$$\text{divisions} = n$$

$$\text{multiplications} = \sum_{i=1}^{n-1} r_i$$

$$\text{multiplications - additions} = \sum_{i=1}^{n-1} r_i^2$$

where r_i is the number of non-zero elements to the right of the diagonal in row i in the table of factors. The number r_i depends on the selected ordering scheme.

The forward and back substitution requires:

$$\text{multiplications} = n$$

$$\text{additions} = n$$

$$\text{multiplications - additions} = 2 \sum_{i=1}^{n-1} r_i$$

The storage requirements and arithmetic operations for the Norton-type equivalent model, the model obtained by the solution of the unconstrained problem and the model obtained by the solution of the constrained problem to be presented next are not optimal. Both the storage requirements and execution time can be further improved. However, the procedure we developed to define the equivalent representation of the external system does not require frequent updating of the parameters of the equivalent model. The algorithm needs to be executed once or twice a day depending on the number of the switching operations. Note also that if the existing equivalent model satisfies the set of inequality constraints defined by the new set of switching operations, there is no need to update the parameter values of the equivalent model. Therefore, the algorithm is not restricted by storage and execution time limitations.

The computational aspects for the Norton equivalent, the unconstrained problem and the constrained problem are discussed next.

5.2.1 Norton Equivalent

The admittance matrix of the entire area is of the form:

$$Y = \begin{vmatrix} Y_{EE} & Y_{EB} & 0 \\ Y_{BE} & Y_{BB} & Y_{BI} \\ 0 & Y_{IB} & Y_{II} \end{vmatrix} \tag{136}$$

If the matrix Y_{EE} can be decomposed as a product of a lower and upper triangular matrix

$$Y_{EE} = LU \tag{137}$$

then the computation of the Norton equivalent model involves the transformation of the admittance matrix Y into the form:

$$\begin{vmatrix} U & L^{-1}Y_{EB} & 0 \\ 0 & Y_{BB}-Y_{BE}Y_{EE}^{-1}Y_{EB} & Y_{BI} \\ 0 & Y_{IB} & Y_{II} \end{vmatrix}$$

Obviously, this transformation is obtained by performing the triangular decomposition of the rows of the matrix Y which corresponds to the busses of the external system.

The rows of the matrix Y which correspond to the busses of the external system and to the boundary busses are ordered according to the non-zero off-diagonal terms before elimination. Rows with the least off-diagonal terms are numbered first and those with the most terms last.

Only the non-zero elements of the admittance matrix of the entire area and the non-zero elements of the table of factors are stored.

The arithmetic operations required to compute the Norton equivalent are as follows:

$$\text{divisions} = \eta_E$$

$$\text{multiplications} = \sum_{i=1}^{\eta_E} r_i$$

$$\text{multiplication - additions} = \sum_{i=1}^{\eta_E} r_i^2$$

where η_E is the number of the busses of the external system and r_i as defined earlier.

5.2.2 Unconstrained Problem

The solution of the unconstrained problem is given by:

$$\hat{u} = -\left(\sum_{k=1}^{N} (H^k)^T H^k\right)^{-1} \left(\sum_{k=1}^{N} (H^k)^T \underline{M}^k\right) \tag{138}$$

The dimension of the matrix H^k is $(2xb) \times (2\cdot\ell_1 + \ell_2)$ and the number of non-zero elements is $8\cdot\ell_1 + \ell_2$.

The arithmetic operations required to compute the vector

$$\sum_{k=1}^{N} (H^k)^T \underline{M}^k$$

are:

$$\text{multiplications - additions} = (8\cdot\ell_1+\ell_2) \times N$$

The matrix

$$\sum_{k=1}^{N} (H^k)^T H^k$$

is of dimension $(2\cdot\ell_1+\ell_2) \times (2\cdot\ell_1+\ell_2)$ and the number of non-zero off-diagonal elements are:

$$\sum_{k=1}^{b} \{(2\cdot a_k)^2 + 4\cdot\mu_k\cdot a_k\} - 6\cdot\ell_1$$

where a_k is the number of equivalent branches connected to the k^{th} boundary bus, and:

$$\mu_k = \begin{cases} 1 & \text{if shunt term is included at the } k^{th} \text{ bus} \\ 0 & \text{elsewhere} \end{cases}$$

where,

$$\sum_{k=1}^{b} \mu_k = \ell_2 \quad .$$

It can be proven that the number of arithmetic operations required to compute the matrix

$$\sum_{k=1}^{N} (H^k)^T H^k$$

are

$$\text{multiplications - additions:}\quad N\{\sum_{k=1}^{b}(8a_k^2 + 4\mu_k a_k) + \ell_2\}$$

The storage requirements and the arithmetic operations to compute the table of factors for the matrix

$$\sum_{k=1}^{N}(H^k)^T H^k$$

have been discussed earlier. The vector $\underline{u}$ is computed by forward and back substitution of the vector

$$\sum_{k=1}^{N}(H^k)^T \underline{M}^k$$

5.2.3 Constrained Problem

The constrained problem is in the form:

Minimize:

$$x_o = \underline{c}^T\underline{u} + \underline{u}^T D\underline{u} \tag{139}$$

subject to the constraints:

$$A\underline{u} \leq \underline{P}_o \tag{140}$$

where

$$\underline{c} = 2\sum_{k=1}^{N}(H^k)^T \underline{M}^k \tag{141}$$

$$D = \sum_{k=1}^{N}(H^k)^T H^k \tag{142}$$

$$A = \begin{bmatrix} H^1 \\ \vdots \\ H^k \\ \vdots \\ H^N \\ -H^1 \\ \vdots \\ -H^k \\ \vdots \\ -H^N \end{bmatrix} \quad ; \quad \underline{P}_o = \begin{bmatrix} \bar{\underline{g}}_1 - \underline{M}^1 \\ \vdots \\ \bar{\underline{g}}_k - \underline{M}^k \\ \vdots \\ \bar{\underline{g}}_N - \underline{M}^N \\ \bar{\underline{g}}_1 + \underline{M}^1 \\ \vdots \\ \bar{\underline{g}}_k + \underline{M}^k \\ \vdots \\ \bar{\underline{g}}_N + \underline{M}^N \end{bmatrix} \tag{143}$$

$\underline{c}$ = an n-vector

$\underline{P}_o$ = an m-vector

D = an nxn matrix

A = an mxn matrix

$\underline{u}$ = an n-vector

where:

$$n = 2\ell_1 + \ell_2 \quad \text{and} \quad m = 4\text{x}b\text{x}N$$

The storage requirements and the arithmetic operations required to compute the vector $\underline{c}$ and the matrix D were discussed in Section

5.2.2 for the unconstrained problem.

The number of non-zero elements of the matrix A is:

$$2(8\cdot\ell_1 + \ell_2)N \quad .$$

The iterative scheme to solve the quadratic programming is discussed in Appendix 1. In each iteration the basic matrix B should be updated using the formula

$$B^{-1}_{NEXT} = EB^{-1}_{CURRENT}$$

The matrix E is defined in Appendix 1. The basic matrix B is of dimension $(n+m)x(n+m)$.

In every iteration there are:

basic variables:	m+n
non-basic variables:	2n+m

Let us define as σ the number of non-zero elements in the matrix $B^{-1}_{CURRENT}$ and as k_1 the number of artificial variables R_i (see Appendix 1 for the definition of R_i) remained in the current iteration as basic variables.

The arithmetic operations required to define the entering variable in every iteration are:

$$\text{multiplications - additions:} \quad 2(\sigma + k_1)k_1$$

The arithmetic operation required to define the leaving variable in every iteration are:

multiplications - additions:	σ
maximum possible number of divisions:	$m+n$

After the entering and leaving variables are defined, the inverse of the basic matrix is updated for the next iteration. The initial value of k_1 is n. The solution to the quadratic programming is obtained when k_1 becomes zero.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

This thesis has presented a general approach to the static equivalent problem as it relates to the security monitoring of power systems. The problem is to define an equivalent representation of the external system which will reproduce the true results for a set of postulated outages with a guaranteed level of accuracy.

Emphasis is placed on obtaining the equivalent model by utilizing information from the internal system only. In particular, this thesis indicates that data on actual system outages can be effectively and directly used to obtain external system equivalents.

The equivalencing problem is formulated as an optimization problem. The unknowns of the problem are the parameter values of the fictitious branches among the boundary busses. The connectivity among the boundary busses should guarantee that the admittance matrix of the equivalent model is a sparse matrix.

The Norton type equivalent model is treated as an a priori information assuming that the topology and the parameter values of the external system might be available. In this case, the Norton-type equivalent model can be computed and then tested if it satisfies the accuracy requirements of the equivalencing problem. If the internal system is highly interconnected to the external system, the linear reduction of the external system to the boundaries of the internal system creates many equivalent branches among the boundary busses. This results in poor sparse structure of the equivalent admittance matrix which is undesirable for contingency analysis. In our research we found that by eliminating some of the equivalent branches, the performance of the retained model may be superior to the performance of the Norton-type equivalent. Two such elimination procedures have been developed and tested successfully. A simple method to update the Norton equivalent model when outages take place in the external system is presented. A detection scheme to detect unreported topological changes in the external system is also derived. This detection scheme is based on DC analysis and it has been successfully tested.

If the Norton-type equivalent model does not satisfy the accuracy requirements of the equivalencing problem, one has to solve the optimization problem. It has been observed that in many cases it is not necessary to solve the optimization problem in its entirety. First, the model obtained from the solution of the unconstrained problem should be tested. A systematic procedure to define the connectivity of the equivalent model is developed. The solution to the unconstrained problem is obtained in one iteration. Sparsity techniques have been implemented.

There is no guarantee, however, that the solution to the unconstrained problem will satisfy the inequality constraints of the optimization problem. There will be cases when the optimization problem needs to be solved in its entirety. The quadratic programming approach was chosen as the method to solve the constrained problem. The quadratic programming guarantees that a solution is obtained in a finite number of steps assuming that the constraints define a non-empty space.

The equivalent model is updated when a new set of postulated outages is created from recent switching operations. The existing equivalent model should be tested first if it satisfies the new set of constraints. If these constraints are satisfied, there is no need to update the equivalent model. If these constraints are not satisfied, one has to solve the optimization problem.

It should be noted that in our procedure to define the external system equivalent, the equivalent model is not updated in a constant fashion. It is expected that the equivalent model will be updated once or twice everyday depending on the number of switching operations. Thus our method is not restricted by storage and time limitations.

To summarize, the main advantages of our procedure are:

(1) A small amount of data needs to be processed.

(2) The accuracy of the equivalent model is controllable.

(3) The equivalent model is updated once or twice everyday.

6.2 Recommendations

The procedure we presented in this thesis to compute external network equivalents requires on-line information from the internal system only. Information from off-line studies can be easily combined with on-line information to be used as input to the algorithm. It is our belief that familiarity with the internal system of interesting is a strong factor for the successful application of the algorithm.

We feel that the following two minor modifications of our procedure are worthwhile of further investigation:

(1) Investigation of the possibility to decompose the set of outages in several subsets. Then for each subset of outages our procedure can apply to define the corresponding equivalent model.

(2) Investigation of the possibility to decompose the boundary busses to several subsets. For each subset the connectivity and the parameter values of the equivalent branches are defined by straightforward application of our algorithm. When a new set of outages is created, each equivalent model among the boundary busses of the same set is tested separately. Only those equivalent models need to be updated for which the corresponding set of constraints are violated.

APPENDIX A

This appendix presents the formulation of the quadratic and linear programming. The quadratic and the linear programming are discussed extensively in [29-32].

Quadratic Programming

The quadratic program is defined as the problem of finding the minimum of the quadratic objective function

$$x_o = \underline{c}^T\underline{u} + \underline{u}^T D\underline{u} \tag{A1}$$

subject to the following set of linear constraints

$$A\underline{u} \le \underline{P}_o \tag{A2}$$

where

$\underline{c}$ = an n-vector of constants

D = an nxn matrix

A = an mxm matrix

$\underline{P}_o$ = an m-vector of constants

$\underline{u}$ = an n-vector of unknowns

It is assumed that the matrix D is at least a positive semi-definite matrix.

The Kuhn-Tucker conditions for the solution of the above problem can be stated as:

$$2D\underline{u} + A^T\underline{\lambda} - \underline{v} = -\underline{c} \tag{A3}$$

$$A\underline{u} + \underline{S} = \underline{P}_o \tag{A4}$$

$$\underline{v}^T\underline{u} = 0 \tag{A5}$$

$$\underline{\lambda}^T\underline{S} = 0 \tag{A6}$$

$$\underline{x} \geq 0,\ \underline{v} \geq 0,\ \underline{S} \geq 0 \tag{A7}$$

The problem is thus equivalent to finding the solution to a set of linear equations (A3) and (A4) which also satisfies the conditions (A5) and (A6). Without loss of generality it is assumed that the elements of the vector $\underline{P}_o$ are non-negative. Solution to the above problem is obtained by solving the problem:

Minimize:

$$\rho_o = \sum_{i=1}^{n} \rho_i \tag{A8}$$

subject to:

$$2D\underline{u} + A^T\underline{\lambda} - \underline{v} + \underline{R} = -\underline{c} \tag{A9}$$

$$A\underline{u} + \underline{S} = \underline{P}_o \tag{A10}$$

$$\underline{v}^T\underline{u} = 0 \tag{A11}$$

$$\underline{\lambda}^T\underline{S} = 0 \tag{A12}$$

$$\underline{x} \geq 0,\ \underline{v} \geq 0,\ \underline{S} \geq 0$$

where:

$$R_i = \begin{cases} -\rho_i & \text{if } c_i > 0 \\ \rho_i & \text{if } c_i \leq 0 \end{cases}$$

$$\rho_i \geq 0 \qquad i = 1, \ldots, n$$

The problem described by the conditions (A8), (A9), and (A10) can be solved using the simplex method. The non-linearities described by the conditions (A11) and (A12) can be taken into consideration by a simple procedure within the simplex algorithm. If solution to the problem exists, the simplex method terminates with the sum of the artificial variables ρ_i equal to zero. A brief description of the simplex method is given next.

The Simplex Method

The simplex method is a systematic procedure for solving the linear programming problem:

Minimize:

$$z = \underline{c}^T\underline{x} \tag{A13}$$

subject to:

$$A\underline{x} = \underline{b} \tag{A14}$$

where A is an mxn matrix, $\underline{c}$, $\underline{x}$ are n-vectors, and $\underline{b}$ is an m-vector. The j^{th} column of A is denoted by $\underline{a}_j$, j=1, ... ,n. It is assumed that m of the n vector a_i are independent and form a matrix B. This matrix is a basis for E^m and is referred to as a basic matrix. Therefore, any column of the matrix A can be expressed as a linear combination of the column of B, i.e.,

$$\underline{a}_j = \sum_{i=1}^{m} y_{j,i}\underline{a}_i$$

where:

$$a_i \in B$$

$$a_j \notin B$$

It is assumed without loss of generality that the matrix B consists of the first m columns of A. i.e.,

$$A = (B,N) \tag{A15}$$

Let us define as

$$\underline{y}_j = \begin{vmatrix} y_{j,1} \\ \cdot \\ \cdot \\ \cdot \\ y_{j,i} \\ \cdot \\ \cdot \\ \cdot \\ y_{j,m} \end{vmatrix}$$

Then

$$\underline{a}_j = B\underline{y}_j$$

or

$$\underline{y}_j = B^{-1}\underline{a}_j \tag{A16}$$

If the vector $\underline{x}$ is partitioned as

$$\underline{x} = \begin{bmatrix} \underline{x}_B \\ \underline{x}_N \end{bmatrix} \tag{A17}$$

Then

$$A\underline{x} = (B,N)\begin{bmatrix} \underline{x}_B \\ \underline{x}_N \end{bmatrix} = \underline{b}$$

or

$$B\underline{x}_B + N\underline{x}_N = \underline{b}$$

A basic solution corresponding to the basis B is obtained by setting $\underline{x}_N=0$. Therefore,

$$B\underline{x}_B = \underline{b}$$

or

$$\underline{x}_B = B^{-1}\underline{b}$$

$\underline{x}_B$ is a basic solution to Ax=b. The variables in $\underline{x}_B$ are called basic variables. The variables in $\underline{x}_N$ are called non-basic variables.

If

$$\underline{c} = \begin{bmatrix} \underline{c}_B \\ \underline{c}_N \end{bmatrix}$$

then for each vector $\underline{y}_j$, there is a scalar z_i defined as

$$z_j = \underline{c}_B^T\underline{y}_j$$

Given the current basic solution, the simplex method proceeds by exchanging a basic variable for a non-basic variable in a way that decreases the objective function. The selection of the entering and leaving variables in a new basic solution is based on the optimality and feasibility conditions which are described next.

The optimality condition determines the entering variables. The optimality condition states that any non-basic variable $\underline{a}_j$ is a promising candidate for entering the solution provided that $(z_j-c_j) > 0$. When $(z_j-c_j) \leq 0$ for all the non-basic vectors, $\underline{a}_j$, the current solution, is optimal. As a general rule the non-basic variable with the larger positive number (z_j-c_j) is selected as the entering variable.

The feasibility condition determines the leaving variable. The leaving variable is selected such that the elements of the new basic solution will be non-negative. If $\underline{a}_j$ is the entering variable then

$$B\underline{y}_j = \underline{a}_j$$

Let θ be any real number. Then,

$$\theta B\underline{y}_j = \theta\underline{a}_j$$

Since

$$B\underline{x}_\theta = \underline{b}$$

$$B(\underline{x}_B - \theta\underline{y}_j) + \theta\underline{a}_j = \underline{b}$$

This equation indicates that the (m+1) vector

$$\underline{x}^1 = \begin{pmatrix} \underline{x}_B - \theta \underline{y}_j \\ \theta \end{pmatrix}$$

is a feasible solution to the given linear programming problem for appropriate values of the variable θ. The new solution will be a basic solution if θ is so selected that one of the old basic variables becomes zero and the new elements of x remain non-negative. The above requirements are met by the following selection of θ:

$$\begin{aligned} \theta &= \min_k \{ \frac{x_k}{y_{j,k}}, \quad y_{j,k} > 0\} \\ &= \min_k \{ \frac{(B^{-1}b)_k}{y_{j,k}}, \quad y_{j,k} > 0\} \\ &= \frac{(B^{-1}b)_r}{y_{j,r}}, \quad y_{j,r} > 0 \end{aligned} \tag{A18}$$

where x_k is the k^{th} element of the vector $\underline{x}$ and $y_{j,k}$ is the k^{th} element of the vector y_j. The variable x_r is the leaving variable.

After the entering and leaving variables have been selected, the matrix B is updated and the process is repeated. Note, that each iteration requires the inversion of the current basis matrix B.

The Revised Simplex Method eliminates the need of inverting the matrix B at each iteration. Let, $B_{CURRENT}$ and B_{NEXT} be the basic matrices of the current and next iterations, respectively. The Revised Simplex Method computes the B^{-1}_{NEXT} by using the formula:

$$B^{-1}_{NEXT} = EB^{-1}_{CURRENT} \tag{A19}$$

where:

$$E = (e_1, e_2, \; . \; . \; . \; , e_{r-1}, \xi, e_{r+1}, \; . \; . \; . \; , e_m) \tag{A20}$$

e_i is a unit vector with the unit element at the i^{th} place, r is the leaving variable and

$$\xi = \begin{bmatrix} -a_{j,1}/a_{r,j} \\ -a_{j,2}/a_{r,j} \\ \cdot \\ \cdot \\ \cdot \\ 1/a_{r,j} \\ \cdot \\ \cdot \\ \cdot \\ -a_{j,m}/a_{r,j} \end{bmatrix} \tag{A21}$$

where j is the entering variable. The Revised Simplex Method yields savings in computational time and computer storage.

Solution to the Quadratic Programming Problem

The problem described by equations (A8), (A9), (A10), (A11) and (A12) can be solved by a simple modification of the Simplex Method we presented earlier. In the selection of the vectors to enter the basis, the following modifications are needed so that the nonlinearities

$$\underline{\lambda}^T \underline{S} = 0$$

and

$$\underline{v}^T \underline{u} = 0$$

are satisfied.

1. If a variable u_j is currently a basic variable at a positive level, do not consider v_j as a candidate for entering the basic solution; if u_j is currently a basic variable at zero level, v_j may enter the basis only if u_j remains at zero level.

2. If a variable λ_j is currently a basic variable at a positive level do not consider S_j as a candidate for entering the basic solution; if λ_j is currently a basic variable at zero level, S_j may enter the basis only if λ_j remains at zero level.

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