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Principal Investigator Dr. Stuart J. Deutsh

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Quarterly Report #4

April 1, 1976 - June 30, 1976

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Stochastic Modeling and Analysis of Crime

April 1, 1976 - June 30, 1976

(Project Director/Principal Investigator: Dr. Stuart Jay Deutsch)

This report summarizes the progress on the Stochastic Modeling and Analysis of Crime Grant through the fourth quarter of this effort.

Technical Note #14, "Estimation of Shifts in Stochastic Models of Crime Occurrence", describes the methodological procedures for detecting a shift in reported crime occurrences from time series or empirical-stochastic models of crime occurrence. Here a shift parameter is embedded into the multiplicative autoregressive moving average model forms previously used to describe crime occurrence for each of the seven FBI index crimes. Examples of the application of these statistical methods to forcible rape data in Los Angeles and Atlanta are also presented.

In earlier technical notes a network flow model was constructed to predict criminal displacement and deterrence. A fundamental data set used in this model is the observed flow patterns in prior time periods between all pairs of geographic zones or precincts. In a metropolitan area, then, which would typically have in excess of one hundred zones or precincts would result in a flow matrix in excess of one hundred by one hundred. The larger the dimensions of the matrix, the longer the model computation time. Further, the flow matrix could often be substantially sparse. That is, there has been a zero flow between many pairings.

To reduce model execution time and computational difficulties due to sparseness, a data aggregation procedure has been developed. Technical Note #15 describes this data aggregation procedure which reduces the dimensionality of the flow matrix and eliminates matrix sparseness while maintaining information content.

Technical Note #16 contains the full documentation of software developed for the network flow models, both linear and nonlinear, for modeling and forecasting crime displacement. Included is a brief description of each program, its required inputs, its outputs, and how to execute it as well as sample runs.

A multivariate time series model for Robbery is documented in Technical Note #17. In previous technical notes univariate time series models were developed for each index crime occurrence in ten major cities. These models were built using only the information contained in the time series data for a given index crime within a given city. The purpose of this preliminary multivariate effort is to allow the modeling procedure to utilize not only the information contained within, but also, the information in these crime statistics contained between cities for a given index crime. That is, the multivariate approach allows for the description of potential similarities in crime occurrence between cities, namely the dynamics of delays. Thus, information of a leading indicator nature between all pairs of "n" cities are utilized. This note presents the methodological modeling procedures and an example utilizing robbery data for the cities of St. Louis, Portland, Los Angeles, Kansas City, Atlanta, Boston, and Denver. Forecasts of future robbery occurrences are also developed from the multivariate time series model.

Technical Note #18 introduces modeling aspects for an extension of the basic network flow model for predicting criminal displacement. The specific extensions discussed deal with multiple offenses, that is, in previous notes, it was explicitly assumed that all displacement involved only the commission of a single crime type. The issue of multiple offenses during the same time period are discussed in this note.

An example of building and forecasting the linear and nonlinear objective function (in the flow) displacement models is presented in Technical Note #19. Here a sample set of data for the City of Atlanta is used. The data aggregation procedure developed in Technical Note #15 is applied with the resulting flow matrix utilized in developing the displacement models. Forecast for both models are presented.

Technical Note #20 contains an initial evaluation of the effect of the Massachusetts' Gun Control Law on gun-related crimes in the City of Boston. Utilizing the methodology described in Technical Note #14, a statistical analysis of shifts in homicide, gun assault and armed robbery is conducted. The evaluation covers the period prior to enactment of the Law in April, 1975 through October, 1975. Suggestions for further evaluation of the impact of the Law on the police component are described.

TECHNICAL NOTE #14

Estimation of Shifts in Stochastic Models of Crime Occurrence

Introduction

In recent years, there has been an increasing usage of statistical methodology to analyze law enforcement problems. One of these techniques has met with considerable success in modeling the monthly crime occurrences as tabulated in the Uniform Crime Reports[2,3,4]. Here, multiplicative empirical-stochastic models of order $(p,d,q) \times (P,D,Q)_S$, as proposed by Box and Jenkins[1], were employed. In this work, each of the seven index crimes across ten different cities was shown to be represented by the same form of model. These models by themselves are useful in future planning via their forecasting. In addition, they form a starting point for a quantitative evaluation mechanism. It is the purpose of this paper to explore their role in an evaluation framework.

In monitoring industrial processes for changes in level, the industrial decision maker has available the control chart, which gives a pictorial representation of the past history and current status of the process level. When changes in the underlying process occur, the control chart illustrates this. The need for the adaptation of the control chart concept to the law enforcement scenario is easily visualized. When policy makers commence a program that may alter the level of different types of crime, they need to have an on going means of evaluating the merits and demermits of these programs. However, the statistical methodology underlying control charts is not directly applicable to detecting shifts in time series data since the monthly occurrences of a particular type of crime have been shown to be correlated[2,3,4].

In the first section of the paper, salient aspects of the methodological considerations for including a shift parameter in the multiplicative empirical-stochastic model forms and its statistical estimation are reviewed from Technical Note #10. In order to make the methodology viable, the second section of this paper describes the computer inplementation of this procedure. Since it is desirable for the policy maker to receive information regarding program effectiveness as quickly as possible after the commencement of such a program, the sensitivity of the underlying statistical methodology for various data structures is analyzed in the third section. In the last section of the paper, two actual data bases, one representing Los Angeles rape and a second representing the corresponding series for Atlanta, are employed as examples.

Theoretical Framework

For a temporal sequence of crime occurrences (Z_t) for a given index crime, the general form of the multiplicative model of order (p,d,q) x $(P,D,Q)_S$ is given by

$$\phi_p(B)\ \Phi_P\ (B^S)\ \nabla^d\ \nabla_S^D\ Z_t = \theta_q(B)\ \Theta_Q\ (B^S)\ a_t, \qquad (1)$$

where $\phi_p(B)$ and $\Phi_p(B^S)$ are the nonseasonal and seasonal autoregressive operators, $\theta_q(B)$ and $\Theta_Q(B^S)$ are the nonseasonal and seasonal moving average operators, ∇^d and ∇_S^D are nonstationary and seasonal differencing operators and S is the seasonal lag. For example, the multiplicative model of order (0,1,1) x $(0,1,1)_{12}$ is explicitly written as

$$Z_t - Z_{t-1} - Z_{t-12} + Z_{t-13} = a_t - \theta\ a_{t-1} - \Theta\ a_{t-12} + \theta\ \Theta\ a_{t-13}. \qquad (2)$$

When there is no seasonal component (P=0, D=0, and Q=0), the multiplicative model reduces to the ARIMA model of order (p,d,q), which is given by

$$\phi_p(B)\ \nabla^d\ Z_t = \theta_q(B)\ a_t. \qquad (3)$$

Thus, (0,1,1) models are of the form

$$Z_t = Z_{t-1} - \theta\ a_{t-1} + a_t. \qquad (4)$$

In the previously cited modeling of the seven index crimes, each crime was shown to be represented by a $(0,1,1) \times (P,D,Q)_S$ form. When evaluating a change in the process level, a policy maker would like his procedures to give minimal real time delay between the time frame in which the process shifted to the time frame of shift detection, assuming a shift occurred. Therefore, the within component (the (0,1,1) segment) of these models are of primary focus in our analysis. If P,D, and Q are not zero, then this seasonal component will be used to transform the original crime data Z_t to W_t. That is,

$$\left.\begin{aligned} &\frac{\Phi_P(B^S)\,\nabla_S^D}{\Theta_Q(B^S)}\, Z_t = W_t \\ &\text{where} \\ &W_t = \frac{(1-\theta B)}{(1-B)}\, a_t, \end{aligned}\right\} \quad (5)$$

since p=0, d=1, and q=1. This transformation ensures early detection from the (0,1,1) within component structure. Box and Tiao [1] have developed the methodology for this specific model form. Glass et ali [5] have recently presented the methodology for other model forms.

In our current problem setting, decision makers are presented with a total of $N = n_1 + n_2$ observations, where the first n_1 observations occur prior to an intervention effect, A, while the second set of n_2 observations occur after A. These $n_1 + n_2$ observations are denoted by $Z_1, \ldots, Z_{n_1}, Z_{n_1+1}, \ldots, Z_{n_1+n_2}$. It is assumed that all N observations emanate from an ARIMA (0,1,1) model, which can be expressed in random shock form as :

$$\left.\begin{aligned} &Z_1 = L + a_1 \\ &Z_t = L + (1-\theta_1)\sum_{j=1}^{t-1} a_{t-j} + a_t, \quad t = 2, \ldots, n_1 \end{aligned}\right\} \quad (6)$$

while

$$Z_t = L + \delta + (1-\theta_1) \sum_{j=1}^{t-1} a_{t-j} + a_t, \quad t = n_1 + 1, \ldots, n_1 + n_2, \tag{7}$$

where the parameter θ_1 is known to a sufficient approximation. This is not an unreasonable assumption when the data base if fairly large. In Technical Note #10, a parameter γ_0 was used in place of θ_1, where $\gamma_0 = 1-\theta_1$. The only unknown parameters in equations (6) and (7) are L, the true level of the process at t=1, and δ, the shift accompanying the intervention effect.

In order to make statistical inferences about L and δ, one must first transform the original observations, Z_t, into Y_t's. This is accomplished by the following transformation:

$$\left.\begin{aligned} Y_1 &= Z_1 \\ Y_t &= Z_t - (1-\theta_1) \sum_{j=0}^{t-2} \theta_1^j Z_{t-1-j}, \quad t = 2, \ldots, n_1 + n_2. \end{aligned}\right\} \tag{8}$$

Equivalently, one could also use the following recursive relationship:

$$\left.\begin{aligned} Y_1 &= Z_1 \\ Y_t &= (Z_t - Z_{t-1}) + \theta_1 Y_{t-1}, \quad t = 2, \ldots, n_1 + n_2. \end{aligned}\right\} \tag{9}$$

One then employs the concepts of statistical linear models to obtain the following estimates of L and δ:

$$\hat{L} = \frac{1-\theta_1}{1-\theta_1^{2n_1}} \left[\sum_{j=1}^{n_1} \theta_1^{j-1} Z_j + \theta_1^{n_1} \sum_{j=1}^{n_1} \theta_1^{n_1-j} Z_j \right] \tag{10}$$

and

$$\hat{\delta} = \frac{1-\theta_1}{1-\theta_1^{2n_2}} \left[\sum_{j=1}^{n_2} \theta_1^{j-1} Z_{n_1+j} + \theta_1^{n_2} \sum_{j=1}^{n_2} \theta_1^{n_2-j} Z_{n_1+j} \right]$$

$$- \frac{1-\theta_1}{1-\theta_1^{2n_1}} \left[\sum_{j=1}^{n_1} \theta_1^{n_1-j} Z_j + \theta_1^{n_1} \sum_{j=1}^{n_1} \theta_1^{j-1} Z_j \right] \tag{11}$$

In order to make additional statistical inferences, one must assume that the a_t's of equations (6) and (7) are NID$(0,\sigma_a^2)$, where σ_a^2 is an unknown parameter representing the variation of the residual a_t's. An estimate of σ_a^2 is provided by

$$\hat{\sigma}_a^2 = \frac{1}{n_1+n_2-2}\left\{\sum_{j=1}^{n_1}\left[Y_j - \hat{L}\,\theta_1^{j-1}\right]^2 + \sum_{j=n_1+1}^{n_1+n_2}\left[Y_j - \hat{L}\,\theta_1^{j-1} - \hat{\delta}\,\theta_1^{j-n_1-1}\right]^2\right\} \tag{12}$$

Although the point estimate of δ, given by equation (11), provides some indication of the magnitude of δ, additional flexibility above and beyond the point estimate is needed to allow the decision maker to test $H_0:\delta=0$ vs. $H_1:\delta\neq 0$. If the null hypothesis of no shift is true, then

$$\hat{\delta}\,/\sqrt{c_{22}\,\hat{\sigma}_a^2} \sim t_{n_1+n_2-2}, \tag{13}$$

where

$$c_{22} = \frac{(1-\theta_1)\,(1+\theta_1)\left[1-\theta_1^{2(n_1+n_2)}\right]}{\left[1-\theta_1^{2n_1}\right]\left[1-\theta_1^{2n_2}\right]}$$

Our decision rule is to reject $H_0:\delta=0$ whenever

$$\left|\hat{\delta}\,/\sqrt{c_{22}\,\hat{\sigma}_a^2}\right| > t_{\alpha/2,n_1+n_2-2}. \tag{14}$$

By making use of the distributional property of $\hat{\delta}$, a confidence interval estimate for δ can also be provided:

$$\hat{\delta} \pm t_{\alpha/2,n_1+n_2-2}\,\hat{\sigma}_a\sqrt{\frac{\left[1-\theta_1^{2(n_1+n_2)}\right](1-\theta_1)\,(1+\theta_1)}{\left[1-\theta_1^{2n_1}\right]\left[1-\theta_1^{2n_2}\right]}} \tag{15}$$

This confidence interval estimate of δ is extremely useful when one rejects the null hypothesis of no shift. The confidence interval is of the form (c_1,c_2), where $c_1 < c_2$. If both c_1 and c_2 are positive, then the decision maker can be quite sure that there has been a positive shift in the level of the series such as would accompany a change in reporting attitude. The commencement of a crime reduction program should be reflected by a confidence interval where both c_1 and c_2 are negative. A confidence interval in which $c_1 < 0$ while $c_2 > 0$ is indicative of no shift in the series level. That is, even if there was a shift in the level of the series, it was not enough to be statistically significant at the α level. Furthermore, an interval from a large negative value of c_1 to a small positive one for c_2 indicates that if the shift is positive, its magnitude is probably small.

Computer Implementation

In order to make this methodology viable, the computer program SHIFT was written to perform the needed calculations for making inferences about L and δ. As a first step, one must establish a data file which contains N observations and specify both n_1 and n_2. One must also read in θ_1, the value of the moving average parameter. SHIFT then makes use of equation (9) and transforms the original Z_t's into Y_t's. Because the Y_t's have no intrinsic value, the program does not print them out. However, these values are stored. The program then calculates and prints out $\hat{L}$ and $\hat{\delta}$, using equations (10) and (11). The actual output format used is "L HAT =" and "DELTA HAT =". It should be pointed out that one is primarily interested in $\hat{\delta}$, the estimated shift, with only secondary interest in $\hat{L}$, the estimated level of the series at the first observation. Another preliminary step involves obtaining the estimate of σ_a^2, which is performed using equation (12). The output reads "SIGMA HAT SQ=". A test of the null hypothesis that δ=0 is performed using equation (13), where the output format reads "T=". Because the choice of α, the probability of Type I error, depends upon the decision maker, program SHIFT was designed to print out the significance level of the test. Recall that the significance level is the probability of obtaining a result as extreme as or more extreme than the observed result under the given hypothesis of no shift. An extreme result implies disagreement with the null hypothesis. Thus, larger significance levels indicate agreement of the data with the hypothesis. The calculation of the significance level is accomplished by using the MDTD program of the International Mathematics and Statistics Library (IMSL). The input to this program merely consists of the t value determined from equation (13) together with its $n_1 + n_2 - 2$ degrees of freedom. The output format is "SIG LEVEL =". When presented with the value of the significance level, the decision maker then determines whether the data support the hypothesis. The confidence interval estimate of δ is obtained using equation (15). The print out reads "CONF INT = (,)". To calculate the confidence interval, the MDSTI program of the IMSL library was called upon to determine $t_{\alpha/2, n_1+n_2-2}$ once the confidence coefficient of $1-\alpha$ has been selected by the decision maker. Currently, a confidence coefficient of 0.95 is being used.

Table 1 provides a summary of the required input, calculations performed, and output format to program SHIFT. The decision maker would focus his attention on "DELTA HAT =", "SIG LEVEL =" and "CONF INT = (,)". First, "DELTA HAT =" provides some indication about the magnitude of the true process shift δ. Second, "SIG LEVEL =" determines whether a non-zero point estimate of δ was primarily due to statistical variation or could have indeed been caused by a true process shift. Small significance levels reflect that there is strong reason to believe a shift has occurred, where small is usally taken to be 0.05 or less. Finally, the "CONF INT = (,)" statement provides the decision maker with information about the direction of the shift.

Although the interpretation of the computer output from program SHIFT is a prerequisite for evaluation purposes, the decision maker also needs some reassurance

Table 1. Summary of Computer Program SHIFT

Input	Calculations Performed	Output
Data Base	Obtain Y_t's (Equation (9))	
θ_1	Obtain $\hat{L}$ (Equation (10))	L HAT =
	Obtain $\hat{\delta}$ (Equation (11))	DELTA HAT =
	Obtain $\hat{\sigma}_a^2$ (Equation (12))	SIGMA HAT SQ =
	Test H_0 : $\delta=0$ (Equation (13))	T =
	Significance level for above H_0	SIG LEVEL =
α	Confidence interval for δ (Equation (15))	CONF INT = (,)

that the underlying statistical procedures are reliable. That is, suppose there has been only a small shift in the series level and a relatively small number of data points after its occurrence. Can the statistical procedure pick up this shift? What effect does the value of θ_1 have on the estimation procedure? The next section explores this and other questions.

Sensitivity Analysis

To determine the methodological sensitivity on which program SHIFT is based to changes in the underlying process parameters, 131 values of white noise or a_t's were generated. In turn, these were used to generate 131 observations, $Z_1, \ldots, Z_{131}$, from a ARIMA (0,1,1) process. The first 120 observations were generated in accordance with equation (6) while the last 11 observations were obtained using equation (7), which takes into account that there is a shift in the process after the 120th observation. The Z_t's were then transformed into Y_t's using equation (9). Since we are primarily concerned with the estimation of the shift and not with estimating the level of the series at the first observation, L was arbitrarily chosen to be 100 and L retained this value throughout the entire sensitivity analysis. It was felt that the value of L does play a role in the estimation of δ, but only indirectly, viz., the size of δ relative to L.

In the cited modeling effort [2,3,4], it was found that all internal crime analysis units maintained five to ten years of monthly crime occurrence data. To conform to this, n_1 was chosen to equal 60 and 120. This represents five and ten years of monthly data, respectively. These two values of n_1 also permit us to measure the effect of n_1 on the estimation procedure. The first 120 observations were segmented into two blocks. Block 1 consisted of observation $Z_1, \ldots, Z_{60}$ while observations $Z_{61}, \ldots, Z_{120}$ comprised block 2. Whenever $n_1 = 120$, both blocks 1 and 2 were used, while only block 2 was used for $n_1 = 60$.

Examination of the estimation formulas used in computer program SHIFT suggests that the estimation procedure is also dependent upon n_2, the number of observations after the shift. However, since one is interested in detecting a shift as soon as it occurs, the magnitude of n_2 was kept relatively small by letting n_2=1,3,5,7,9, and 11. To gain further insight into the estimation procedure for small n_2, let us examine the estimation equations when n_2=1. We see that the estimate of L, given by equation (10), does not change since $\hat{L}$ is determined solely from the first n_1 observations. However, the estimate of δ, as given by equation (11), reduces to the following:

$$\hat{\delta} = Z_{n_1+1} - \frac{1-\theta_1}{1-\theta_1^{2n_1}} \left[\sum_{j=1}^{n_1} \theta_1^{n_1-j} Z_j + \theta_1^{n_1} \sum_{j=1}^{n_1} \theta_1^{j-1} Z_j \right], \qquad (16)$$

which is merely the first observation after the shift less a weighted average of the first n_1 observations. Furthermore, the estimate of σ_a^2 is based solely on the first n_1 observations since equation (12) reduces to

$$\hat{\sigma}_a^2 = \sum_{j=1}^{n_1} \left[Y_j - \hat{L}\, \theta_1^{j-1} \right]^2 \Big/ (n_1 + n_2 - 2).$$

Another parameter that was altered in the sensitivity analysis was θ_1, the moving average parameter. The ARIMA (0,1,1) model can be written in difference equation form as

$$Z_t - Z_{t-1} = -\theta_1 a_{t-1} + a_t.$$

If we substitute S_t for $(Z_t - Z_{t-1})$, which is equivalent to taking the first difference, the above equation reduces to the familiar first-order moving average process:

$$S_t = -\theta_1 a_{t-1} + a_t.$$

Recall that for this differenced model, the first-order auto-correlation between S_t and S_{t-1} is easily shown to be

$$\rho_1 = -\theta_1 \;/\; (1+\theta_1^2),$$

while for all higher-order auto-correlations

$$\rho_j = 0, \; j > 1.$$

Thus, its memory is only one period long or the persistence of the process on one or the other side of the mean is short-lived. However, the appearance of the series is also determined by the sign of θ_1. If θ_1 is negative, then ρ_1 is positive and the series is relatively smooth with a tendency for low observations to be immediately followed by low observations and high observations by high observations. If θ_1 is positive however, then ρ_1 is negative and the series has a choppy appearance. In this instance, it is expected that the estimation of δ is less sensitive since the inherent variation of the series obscures any shifts that may have occurred. To investigate the sensitivity of the statistical procedures to changes in θ_1, θ_1 was allowed to equal -0.8, -0.3, +0.3, and +0.8. The corresponding values of ρ_1 are 0.49, 0.28, -0.28, and -0.49. Thus, we would expect that when $\theta_1 = +0.8$ the estimation procedure would not perform as well as for the other three values of θ_1.

The final parameter that was investigated in the estimation procedure was the magnitude of the true shift itself. Of course, the magnitude of the shift must take into account the true level of the series at the first observation. Since L=100, it was decided to let δ=2,8,16, and 50. Thus, δ ranged from 2% to 50% of the initial level of the series. It was expected that the estimation procedure would be more precise for the larger values of δ.

A summary of the parameters to be varied in the sensitivity analysis is shown in Table 2. There were 192 different combinations of parameter values that were investigated. For each of these runs computer ouput similar to that of Table 1 was obtained with one minor exception. Namely, the T-value and significance level were

Table 2. Parameter Values Used in the Sensitivity Analysis

PARAMETER VALUES	
θ_1	-0.8, -0.3, +0.3, +0.8
δ	2,8,16,50
n_1	60, 120
n_2	1,3,5,7,9,11

also obtained for testing $H_0:\delta=\delta_0$ vs. $H_1:\delta\neq\delta_0$, where $\delta_0=2,8,16$, and 50. Because of space limitations, not all runs are listed in Table 3. The table does contain the most critical cases in which δ is small.

All of the output shown in Table 3 is for $\delta=2$. The first tabeled case considered $\theta_1= -0.8$, in which case the series is relatively smooth. In this instance, the significance level of the T-test always differed by less than 0.02 for any pair where $n_1=60$ and $n_2=120$. Thus, the policy maker can be reasonably sure of the procedure when he has at least five years of monthly data. Another point to be drawn from Table 3 is that, even though the significance level is relatively high (0.682) for $n_2=1$, it increases to 0.977 for $n_2=11$. For the next segment of cases ($\theta_1= -0.3$) where the autocorrelation between successive differenced observations is still positive (0.28), we see that this increase in θ_1 results in an overall lessening of the significance level for the larger values of n_2 as compared to $\theta_1= -0.8$.

Because of the roughening of the series for positive θ_1, the significance levels were smaller here. The surprising result, however, is that for both of these values, the significance levels were much higher for the smaller values of n_2 than for the larger values. An explanation of this is that, for the larger values of n_2, the choppy behavior of the series after the shift actually tends to obscure the shift. It is easier to detect the shift when the data base after program commencement is 3 or less which matches the desire to evaluate the occurrence of a real shift with minimal time delay.

Although Table 3 does not contain information about $\hat{L}$, the computer output revealed that, regardless of the other parameter values, $\hat{L}$ ranged from 99.398 to 101.391, which closely agrees with the true value of 100. Furthermore, in testing $H_0:\delta=0$, the maximum value of the significance level for all 192 runs was 0.034 with most of the significance levels being less than 0.001. Thus, the procedure is capable of detecting even small shifts with a high degree of accuracy.

Table 3. Summary of Sensitivity Analysis Output

θ_1	n_1	n_2	δ	$\hat{\delta}$	T	SIG. LEVEL
-0.8	60	1	2	2.452	0.412	0.682
-0.8	120	1	2	2.452	0.430	0.668
-0.8	60	3	2	2.289	0.377	0.708
-0.8	120	3	2	2.289	0.393	0.695
-0.8	60	5	2	2.240	0.346	0.730
-0.8	120	5	2	2.240	0.360	0.719
-0.8	60	7	2	2.075	0.112	0.911
-0.8	120	7	2	2.075	0.116	0.908
-0.8	60	9	2	2.111	0.168	0.867
-0.8	120	9	2	2.111	0.174	0.862
-0.8	60	11	2	2.019	0.029	0.977
-0.8	120	11	2	2.019	0.030	0.976
-0.3	60	1	2	2.452	0.412	0.682
-0.3	120	1	2	2.452	0.430	0.668
-0.3	60	3	2	2.280	0.267	0.790
-0.3	120	3	2	2.280	0.279	0.781
-0.3	60	5	2	2.268	0.257	0.798
-0.3	120	5	2	2.268	0.268	0.789
-0.3	60	7	2	2.267	0.257	0.798
-0.3	120	7	2	2.267	0.267	0.790
-0.3	60	9	2	2.267	0.257	0.798
-0.3	120	9	2	2.267	0.267	0.790
-0.3	60	11	2	2.267	0.255	0.800
-0.3	120	11	2	2.267	0.265	0.791
+0.3	60	1	2	2.452	0.412	0.682
+0.3	120	1	2	2.452	0.431	0.667
+0.3	60	3	2	2.758	0.725	0.471
+0.3	120	3	2	2.758	0.759	0.449
+0.3	60	5	2	2.783	0.754	0.453
+0.3	120	5	2	2.783	0.787	0.433
+0.3	60	7	2	2.783	0.758	0.451
+0.3	120	7	2	2.783	0.788	0.432
+0.3	60	9	2	2.783	0.758	0.451
+0.3	120	9	2	2.783	0.788	0.432
+0.3	60	11	2	2.783	0.751	0.455
+0.3	120	11	2	2.783	0.783	0.435
+0.8	60	1	2	2.452	0.414	0.680
+0.8	120	1	2	2.452	0.439	0.662
+0.8	60	3	2	2.971	1.290	0.202
+0.8	120	3	2	2.971	1.358	0.177
+0.8	60	5	2	3.121	1.660	0.102
+0.8	120	5	2	3.121	1.736	0.085
+0.8	60	7	2	2.958	1.469	0.147
+0.8	120	7	2	2.958	1.534	0.127
+0.8	60	9	2	3.070	1.668	0.100
+0.8	120	9	2	3.070	1.739	0.084
+0.8	60	11	2	2.996	1.541	0.128
+0.8	120	11	2	2.996	1.614	0.109

It now remains to implement the methodology and the accompanying computer program in a real world scenario.

Analysis of Forcible Rape

In this section, two examples are presented of the earlier discussed methods for determining a statistically significant process shift. The example data represent the monthly occurrences of forcible rape in both Los Angeles and Atlanta from January of 1966 to March of 1973. Figures 1 and 2 display a segment of these time series. For each of these series, a (0,1,1) model was shown to be an adequate representation with $\theta=0.719$ for Los Angeles while $\theta=0.640$ for Atlanta [4].

For our problem setting, we are standing at March of 1973 with 87 months of past information about the process. In each of these analyses, we will compute the T value and significance level for testing $H_0: \delta=0$ since we currently have no information to discredit this null hypothesis. That is, we will test that the process level will be unchanged in the future from what we have observed in the past.

Table 4 summarizes the analysis for Los Angeles and Atlanta. We see from the table that, when information of April's occurrence of forcible rape is available for both cities, neither shift was statistically different from zero at the 95% level since zero is contained in the corresponding confidence intervals, although δ was estimated to be nonzero for both cities. Further, we see that the estimated value of the shift for Los Angeles and Atlanta would be significant for type I values of 0.400 and 0.231, respectively. However, if we are standing at March of 1973 and we have information for the months of April and May ($n_2=2$), we see that a statistically significant shift has occurred in the forcible rape incidence for Type I error values less than 0.054 and 0.001 for Los Angeles and Atlanta, respectively.

If this procedure were implemented in the field, a Type I error would be set prior to analysis. With each verification of the lack of occurrence of a statistically significant shift, the data element contained in n_2 consistent with the

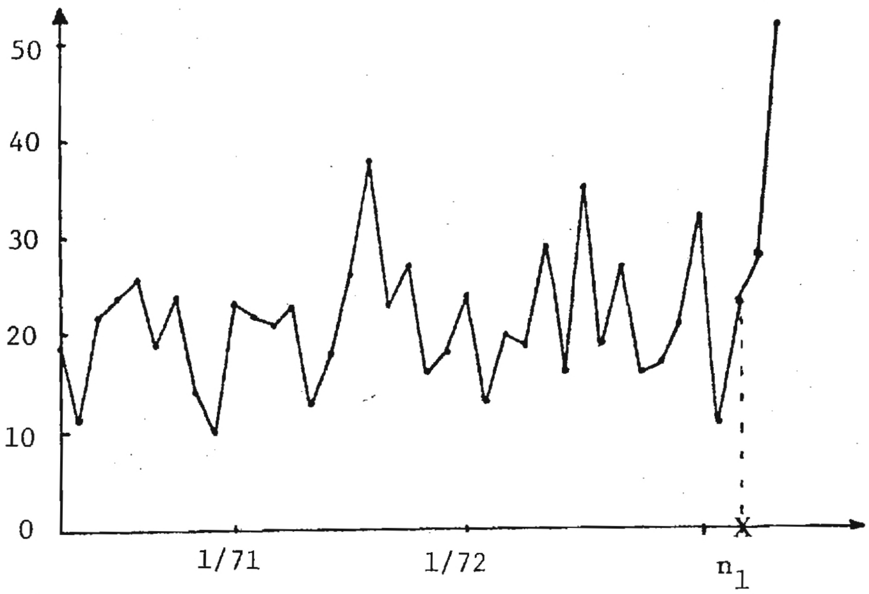

Figure 1. Forcible Rape for Atlanta

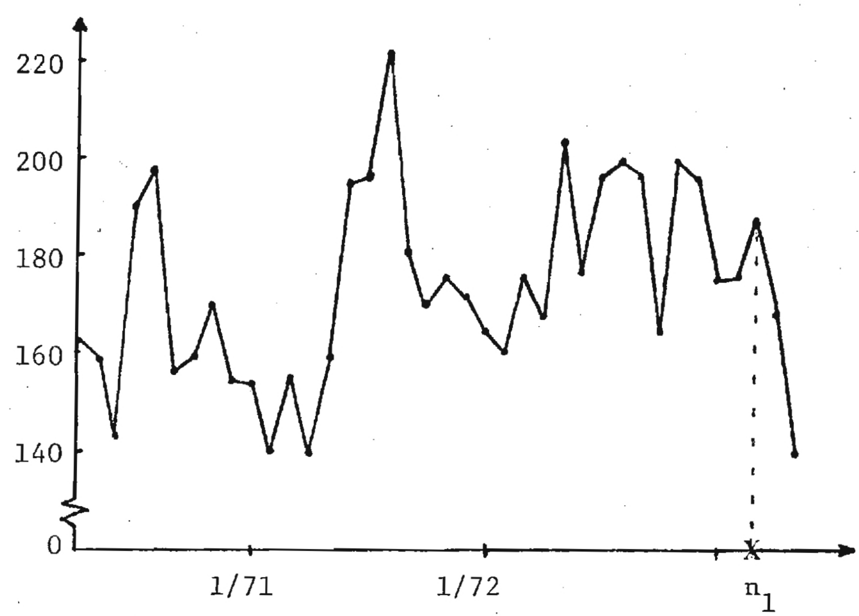

Figure 2. Forcible Rape for Los Angeles

Table 4. Results for Los Angeles and Atlanta

Los Angeles θ=0.719					
n_1	n_2	$\hat{\delta}$	T	SIG. LEVEL	95% CONF. INT.
87	1	-15.593	-0.843	0.401	(-52.345,21.159)
87	2	-29.338	-1.948	0.054	(-59.279, 0.603)
88	1	-40.212	-2.179	0.032	(-76.896,-3.523)
Atlanta θ=0.640					
87	1	7.151	1.203	0.232	(-4.663,18.966)
87	2	18.048	3.405	0.001	(7.511,28.584)
88	1	28.577	4.796	0.000	(16.734,40.420)

past process data elements contained in n_1 would be added to those contained in n_1 and the analysis repeated. This analysis is also contained in Table 4. If the Type I error were fixed initially to a value of 0.05, the analysis in March for both cities would not indicate a significant shift. However, if we move to April (n_1=88) and have information on May's incidence, both Los Angeles and Atlanta are seen to have a significant shift, viz., an undesirable increase for Atlanta and a desirable decrease for Los Angeles. Lastly, it should be noted that when n_1=87 and n_2=2, the significance levels are larger than indicated for the parallel cases when n_1=88 and n_2=1. This occurs because in the former cases one observation is in control and one observation is out of control in the sets where n_2=2.

Conclusions

A quantitative evaluation procedure for analysis of crime occurrence has been specified. Multiplicative autoregressive moving average models with an imbedded shift parameter, to capture potential changes in future crime occurrence, are utilized after being transformed to a linear model representation. A sensitivity analysis of the procedure was exhibited, which illustrates the ability of the procedure to efficiently detect small shifts. Two examples of the evaluation procedure applied to uniform crime report data of forcible rape in Los Angeles and Atlanta were analyzed.

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TECHNICAL NOTE #15

A Data Aggregation Procedure for the
Network Flow Model of Displacement

Introduction

In previous Technical Notes [2 , 3], a network flow model was constructed and demonstrated in order that criminal displacement and deterrence be predicted. Specifically, the model requires historical information regarding criminal (crime) movement in a pre-specified region relative to a chain of discrete time periods. The essense of the model is a tuning scheme which considers successive transportation models the solutions to which agree closely with the observed flow (displacement) in the periods. When the model is "tuned", point forecasts can be made relative to future displacement and deterrence in the region.

A fundamental data set required in the tuning algorithm [1], is the observed flow pattern in prior time periods. Given by matrices F_t, particular flow values f_{ij}^t specify the level of observed flow in time period t between zones i and j where i and j may be identical. Morever, it may be that the data base around which F_t are constructed, involves a large number of zones the result of which is a series of large arrays. A size for each array of 100 x 100 would not be atypical for a region the size say of Atlanta, Georgia where each zone, might represent a police precinct. Additionally, many values f_{ij}^t would be zero. creating substantive sparseness in matrices F_t. This leads directly to a computational issue and is the subject of the current technical note. Accordingly, we develop and demonstrate a data aggregation procedure which in effect reduces the size of arrays F_t by consolidating zones.

The composition of this technical note involves a discussion of the concept and basic notions of an aggregation procedure followed by a computational statement of the scheme. The algorithm is demonstrated by considering a small hypothetical problem; however, the application of the procedure is reported relative to a real data set. In addition, some relevant properties and modifications of the procedure are considered.

The Development of an Aggregation Algorithm

The basic notion upon which the aggregation procedure is developed is that if there is no flow between two contiguous zones then the two zones can be consolidated creating a "new" zone without loss of the displacement or deterrence information into or out of the original zones. Of course such flow will be defined over a new zone of less resolution relative to the entire region. This is obvious if one examines the extremes of any aggregation attempt. Suppose there is some non-zero flow into or out of every zone. Under such a case, no aggregation is made which is rational since displacement (and/or deterrence) from or to any zone is occurring and any aggregation or consolidation would destroy at least a portion of such displacement. On the other hand, suppose there is no criminal displacement at all. Given by $f_{ij}^{t} = 0; \forall i,j,t$, the final aggregation would result with a single zone which corresponds to the entire region. The loss of zone to zone resolution is irrelevant since there is no displacement anyway. Of course in practice it would be rare for either of these extreme views to be representative.

In determining a final aggregation relative to a set of time periods in which historical (observed displacement) data is collected, it is evident that data in all periods must be considered simultaneously. That is, one must construct from the arrays F_t, $t=1,2,\ldots,T$ a new matrix $\hat{F}$ such that:

$$\hat{f}_{ij} = \sum_{t} f_{ij}^{t} \;, \;\forall i,j$$

This is done in order to avoid an occurrence of zone consolidation suitable in some periods but not suitable in others where there is displacement between previously consolidated zones. Clearly, an aggregation constructed $\hat{F}$ is sufficient since array $\hat{f}_{ij} = \hat{f}_{ji} = 0$ implies $f^t_{ij} = f^t_{ij} = 0$ for all t.

The process of the aggregation scheme is an ituitive one where a given zone is considered for possible consolidation with another, adjacent zone creating a new zone which is the composite of the two. Clearly, each time a pair of zones are combined, the dimension of $\hat{F}$ is reduced by one. Note that two regions are considered adjacent if any portion of the boundary of one zone is shared by the other. Zones sharing only a point are not considered to be adjacent however. For completeness, all zones are adjacent to themselves. The information concerning adjacency of zones is given in an adjacency matrix A such that a_{ij} is one of i and j are adjacent zones and zero otherwise. Note that A is symmetric.

Once two zones are found which can be consolidated, the aggregation should be made immediately relative to the update of $\hat{F}$ as well as to A. If such an update is not made, the nature of certain displacement patterns may be destroyed. For example, consider adjacent zones a,b, and c. Let $\hat{f}_{ab}=\hat{f}_{ba}=\hat{f}_{ac}=\hat{f}_{ca}=0$. If one attempts to consolidate a and b and the matrix $\hat{F}$ is not updated accordingly, then it is logical that one would attempt to consolidate a and c yet if $\hat{f}_{bc} \neq 0$ the consolidated, new zone consisting of original zones a,b, and c would not show the zone to zone displacement between c and b. Caused by the non-transitive nature of inter-zone displacement, such a phenomena would not occur if the updated flow matrix were constructed after consolidation of a and b (or a and c).

An obvious result of the manner in which successive $\hat{F}$ are updated is that total displacement is conserved throughout the aggregation process. That is, for any array $\hat{F}$, $\sum_i \sum_j \hat{f}_{ij}$ is constant. In addition, it should be clear that the final aggregation is not unique both in terms of the number of zones in a final aggregation as well as in terms of the composition of such zones. This is demonstrated in the subsequent sample problem. Following however, we present a step-by-step computational procedure for the aggregation process.

Step 1. Initialization

1.1 Construct the adjacency matrix A such that;

$$a_{ij} = \begin{cases} 1, & \text{if zone } i \text{ is adjacent to zone } j \\ 0, & \text{otherwise} \end{cases}$$

1.2 Initialize the flow matrix $\hat{F}$ such that

$$\hat{f}_{ij} = \sum_{t=1}^{T} f^t_{ij} \; ; \forall i,j$$

Step 2. Attempt to combine adjacent zones

2.1 For a given zone i* search $\hat{f}_{i^*j}$ for all j until $f_{i^*j^*}$ is reached such that $\hat{f}_{i^*j^*} = \hat{f}_{j^*i^*}=0$. If $a_{i^*j^*}=1$ proceed to Step 3. If $a_{i^*j^*}=0$ continue the search in row i*.

2.2 If no zone can be combined with the current zone i* select a new zone, call it i* and return to 2.1. If all zones have been considered and no further aggregation is possible, terminate the process.

Step 3. Update the flow and adjacency matrices relative to the current aggregation.

3.1 For given zones i* and j* chosen in Step 2, create a new zone and lable it ϕ where ϕ=min (i*,j*). Update $\hat{F}$ such that

$$\hat{f}_{i\phi} = \hat{f}_{ik} + \hat{f}_{i\ell}; \forall i \text{ such that } k=i^* \text{ and } \ell=j^*$$

and

$$\hat{f}_{\phi j} = \hat{f}_{kj} + \hat{f}_{\ell j}; \forall j \text{ such that } k=i^* \text{ and } \ell=j^*.$$

Delete row and column j* in $\hat{F}$, where zone j* was just combined with ϕ.

3.2 Update the adjacency matrix such that

$$a_{i\phi} = \max (a_{ik}, a_{i\ell}); \forall i \text{ where } k=i^* \text{ and } \ell=j^*$$

and delete all a_{ij^*} where j* is combined with ϕ. Recall $a_{mn}=a_{nm}$ for all m and n. Return to Step 2.

Sample Problem

Suppose we consider a small hypothetical problem involving six zones. Further, let the zones be oriented relative to each other and the entire region by the schematic in Figure 1. In order to facilitate the discussion let observed displacement between zones be depicted by the arrows on the diagram. The numerical flow values are given in Table 1a where we assume that the values are already initialized in $\hat{F}$. For example, the total displacement in all time periods, from zone 1 to zone 5 is two.

If we begin arbitrarily with zone 1 and attempt to consolidate, the first candidate for such consolidation is zone 3 since $\hat{f}_{13}=\hat{f}_{31}=0$. It can be seen however that zones 1 and 3 are not adjacent since $a_{13}=0$. Note that A is given initially in Table 1b. The next candidate is zone 4 which is adjacent to zone 1 and the two can be combined. A new zone is constructed and labeled zone 1. The flow matrix is updated and given in Table 1c along with the corresponding adjacency matrix in 1d. The procedure continues such that the only remaining consolidation involves zones 5 and 6. The new, final flow matrix is constructed and given in Table 1e along with the final adjacency matrix in 1f. It can easily be verified that no further aggregration can be made. The final regional layout would appear as in Figure 2 as a result of the aggregation.

It is of interest to note that if one begins the aggregation process with zone 2 as the initial zone, a different final aggregation results as

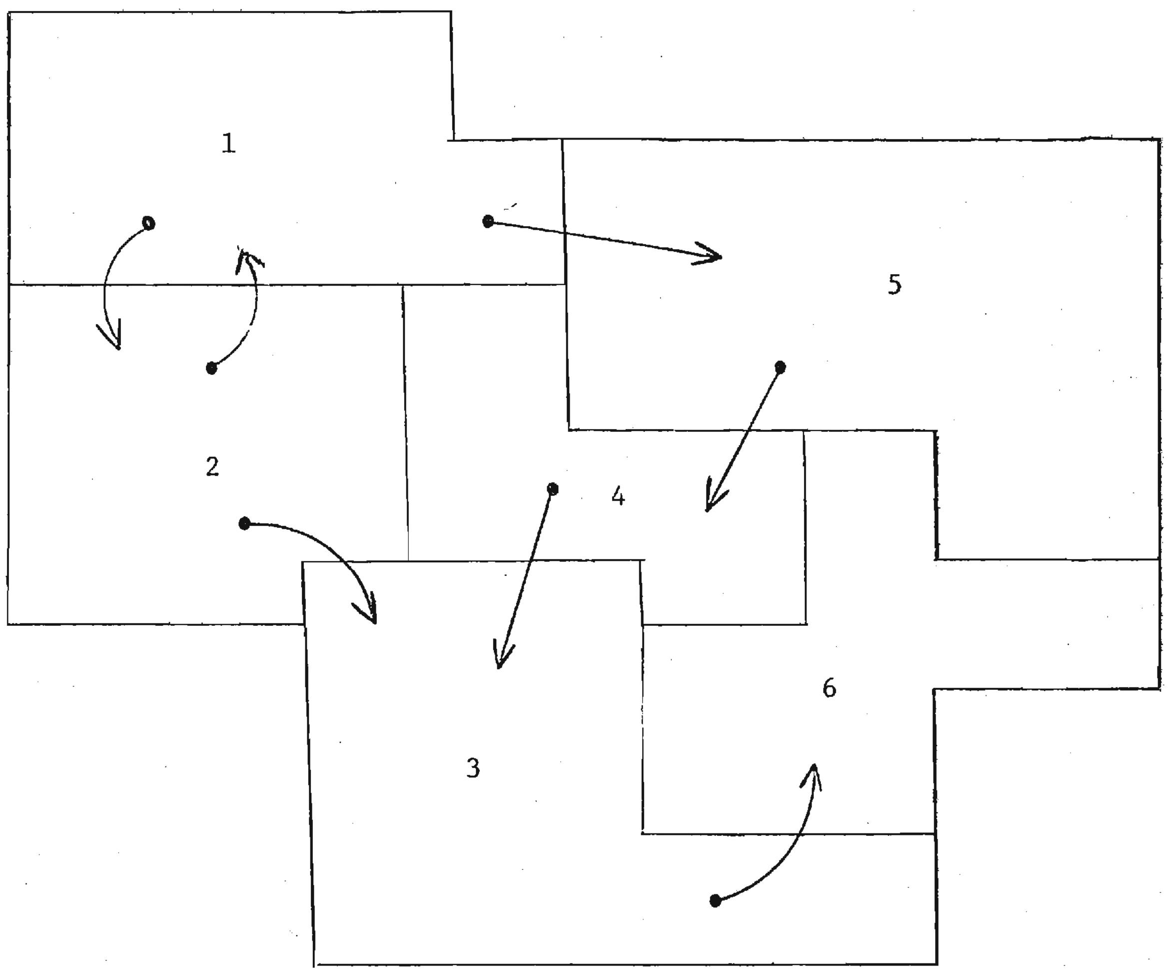


Figure 1. Original Layout of Region for Sample Problem

Table 1. Flow and adjacency Matrices Relative to Successive Aggregations for Sample Problem

$$\hat{F} = \begin{array}{cc} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 0 & 3 & 0 & 0 & 2 & 0 \\ 2 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(a)

$$A = \begin{array}{c|ccccc} 2 & 1 & & & & \\ 3 & 0 & 1 & & & \\ 4 & 1 & 1 & 1 & & \\ 5 & 1 & 0 & 0 & 1 & \\ 6 & 0 & 0 & 1 & 1 & 1 \\ \hline & 1 & 2 & 3 & 4 & 5 \end{array}$$

(b)

$$\begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 5 & 6 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 0 & 3 & 2 & 2 & 0 \\ 2 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(c)

$$\begin{array}{c|cccc} 2 & 1 & & & \\ 3 & 1 & 1 & & \\ 5 & 1 & 0 & 0 & \\ 6 & 1 & 0 & 1 & 1 \\ \hline & 1 & 2 & 3 & 5 \end{array}$$

(d)

$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 5 \end{array} & \begin{bmatrix} 0 & 3 & 2 & 2 \\ 2 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \\ 3 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(e)

$$\begin{array}{c|ccc} 2 & 1 & & \\ 3 & 1 & 1 & \\ 5 & 1 & 0 & 1 \\ \hline & 1 & 2 & 3 \end{array}$$

(f)

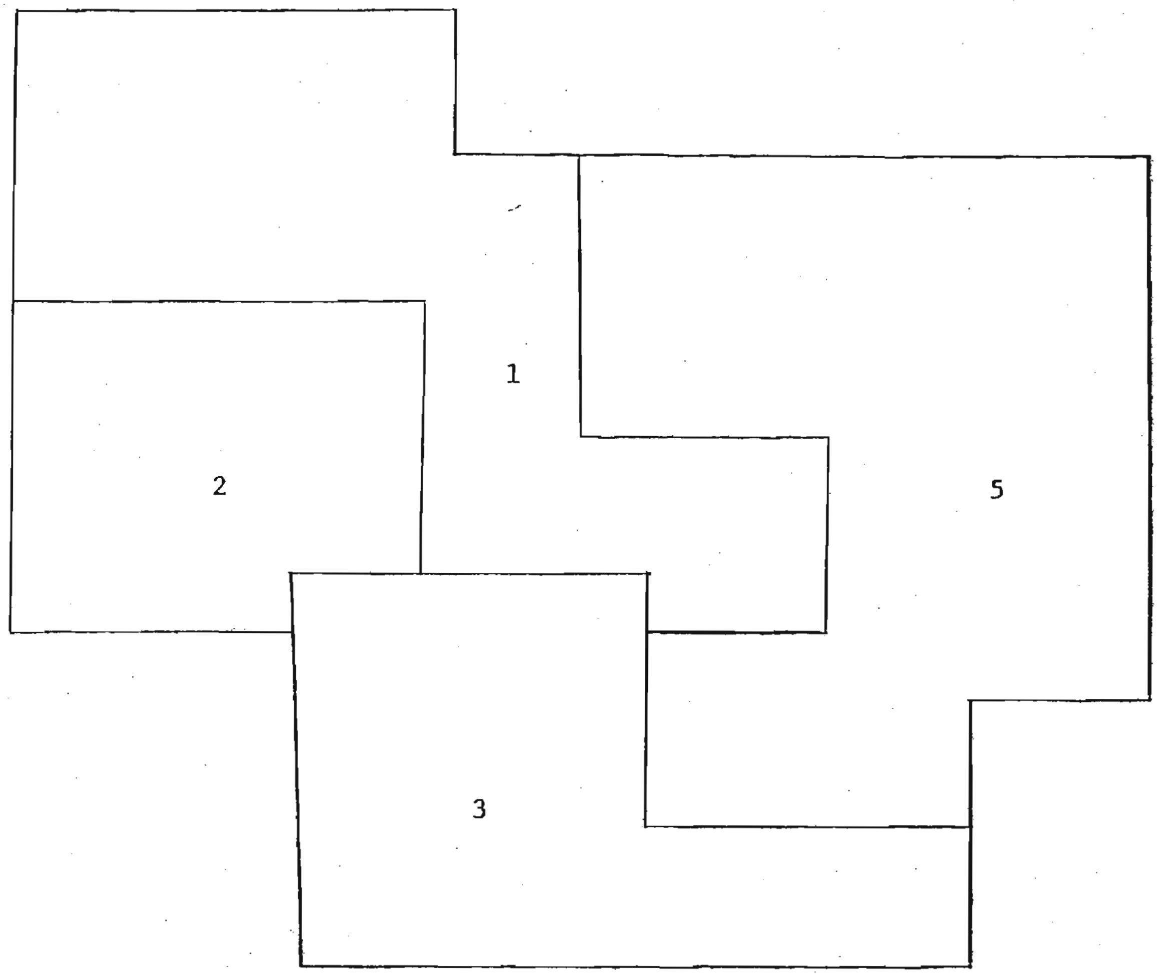


Figure 2. Final Aggregation Relative to Starting Point Zone 1.

shown in Figure 3. Note the conservation of displacement flow in both aggregations.

Modifications to the Procedure and Experience with a Real Example

Relative to the form just presented and demonstrated, the aggregation algorithm will converge to some final arrangement of zones given a set of flows and adjacency indices. It can occur however, that the final layout possesses characteristics while "correct" in the algorithmic sense, are not geographically appealing or even administratively functional. Consider a five zone initial layout given in Figure 4(a). Suppose a final aggregation results as in 4(b). The resulting three zone aggregation may not be a workable geographical layout yet as long as a_{51} and a_{53} exist at non-zero values, such a final configuration could arise (depending upon $\hat{f}_{5j}$ and/or $\hat{f}_{j5}$). To prevent such an occurrence, we seperate the notion of adjacency into two classes: strong and weak.

Two zones are said to be adjacent in the strong sense if a substantial portion of their joint boundary is contiguous. The zones would be only weakly adjacent otherwise. The measure of "substantial" relative to contiquity is system or user-dependent and one would specify some rule (eg. ratio of contiguity to entire boundary length) for such a measurement. Zones which are adjacent in the weak sense are given a_{ij} values of zero. Relative to the illustration in Figure 4, zones one and three exhibit weak adjacency and the layout in Figure 4(b) would not arise.

Another issue that may be of concern relative to a final aggregation, occurs when an "island" zone is formed. Caused by a particular mix of a_{ij} and $\hat{f}_{ij}$ values, an aggregation may arise such as that in Figure 5 where one zone in the aggregation is completely surrounded by another. Relative to a real data set upon which the

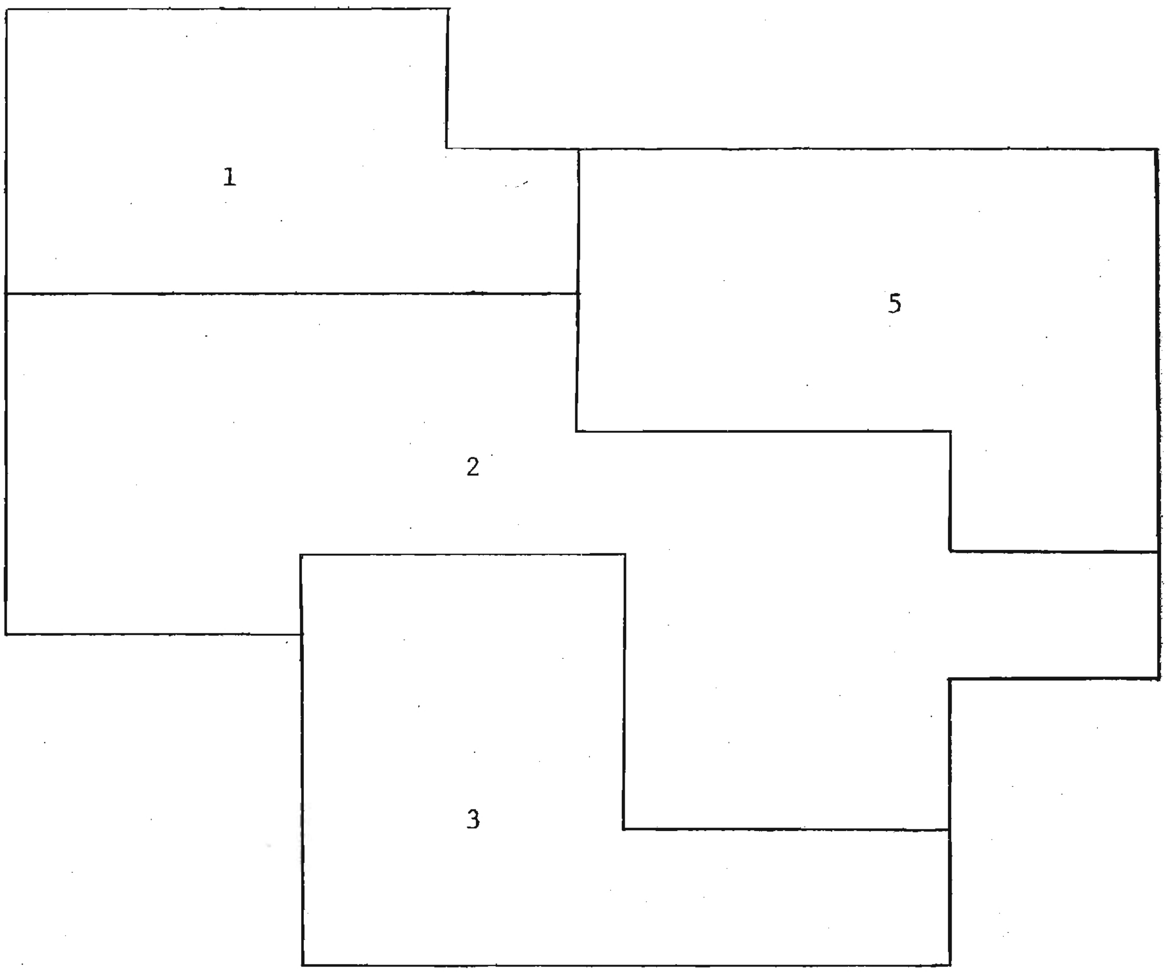
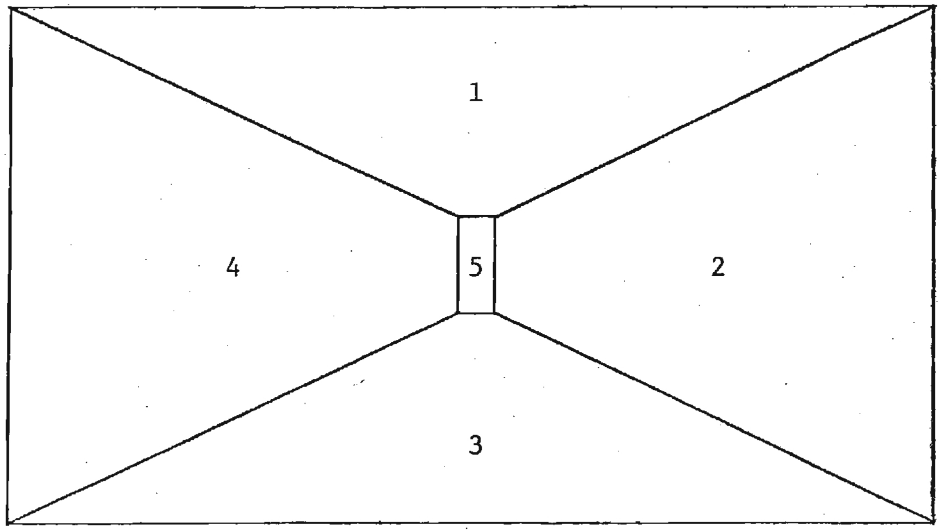
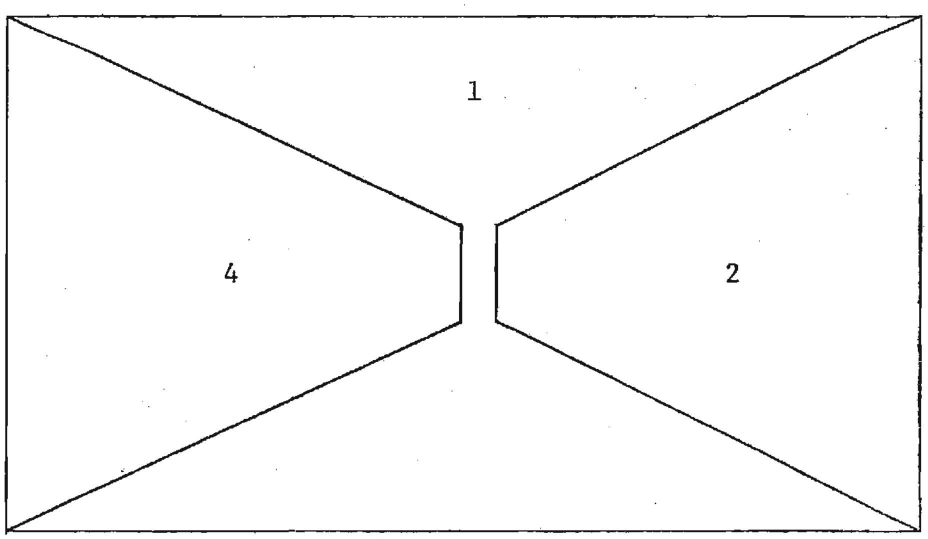


Figure 3. Final Aggregation Relative to Starting Point Zone 2.



(a) Original Layout



(b) Final Aggregation

Figure 4. Example of the Effect of Weak Adjacency

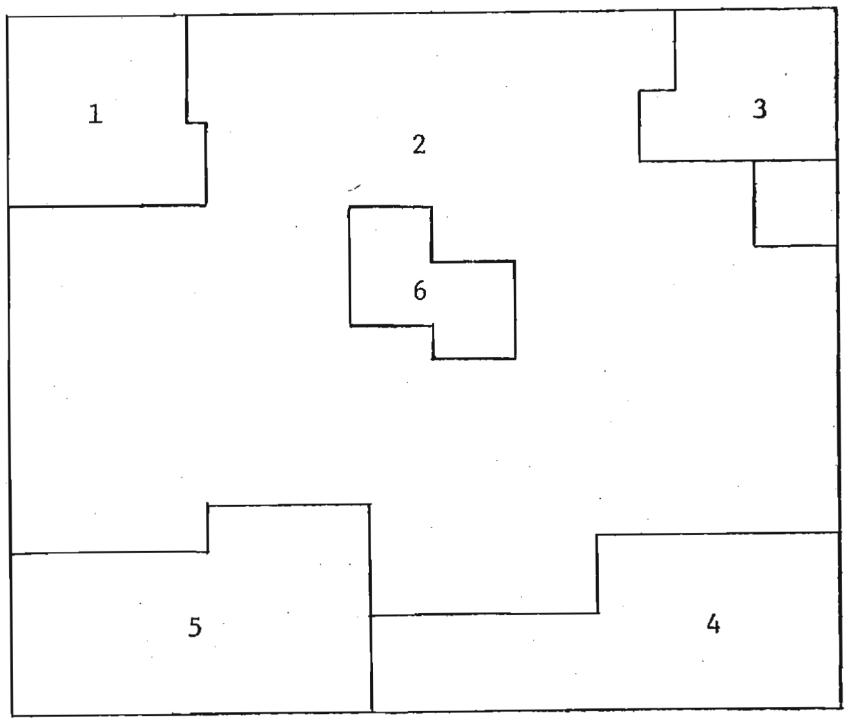


Figure 5. Creation of an Island Zone (Zone 6) in a Final Aggregation

aggregation algorithm was tested, such an aggregation occurred. Assuming that under certain conditions, an island zone is not admissable, a manual updating is probably most expedient for "modifying" a final aggregation.

The aggregation procedure along with the weak and strong adjacency as well as the manual updating capability, was used on a sample set of real data. The data were collected relative to seven crime indices for 1974 in Fulton County, Atlanta, Georgia. In Figure 6, a layout of Fulton County is given showing police precinct sub-divisions. An aggregation was determined which resulted in a consolidation to fourteen zones. The final aggregation is depicted in Figure 7. It is of interest to note that in arriving at the final layout in Figure 7, a number of different starting points (zones) were attempted. In addition, weak adjacency was created for a few zones and manual updates were affected in order to remove island zone formations. Moreover, the final, fourteen-zone aggregation was used in the network flow model [2] and the results are reported in a subsequent technical note.

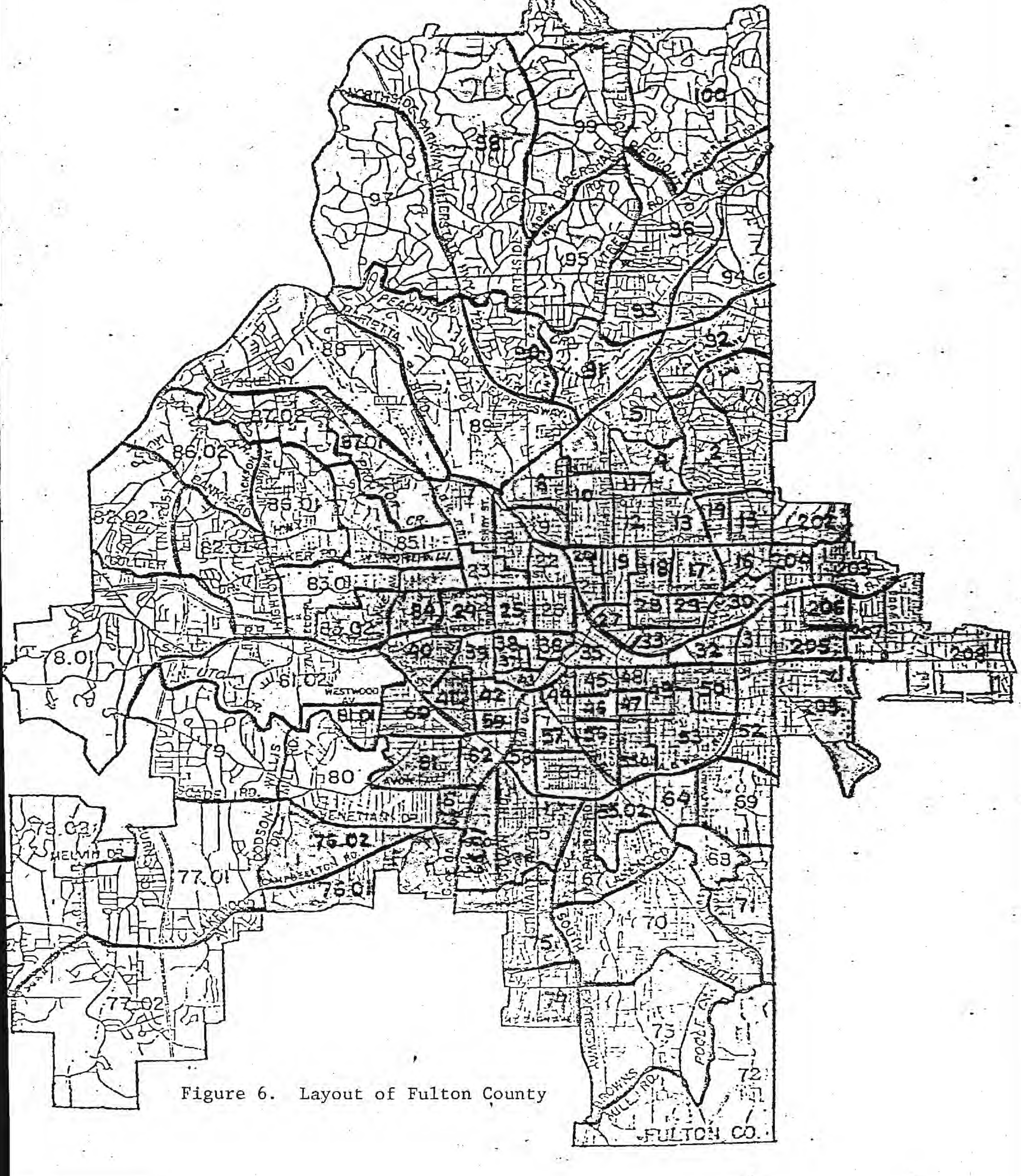


Figure 6. Layout of Fulton County

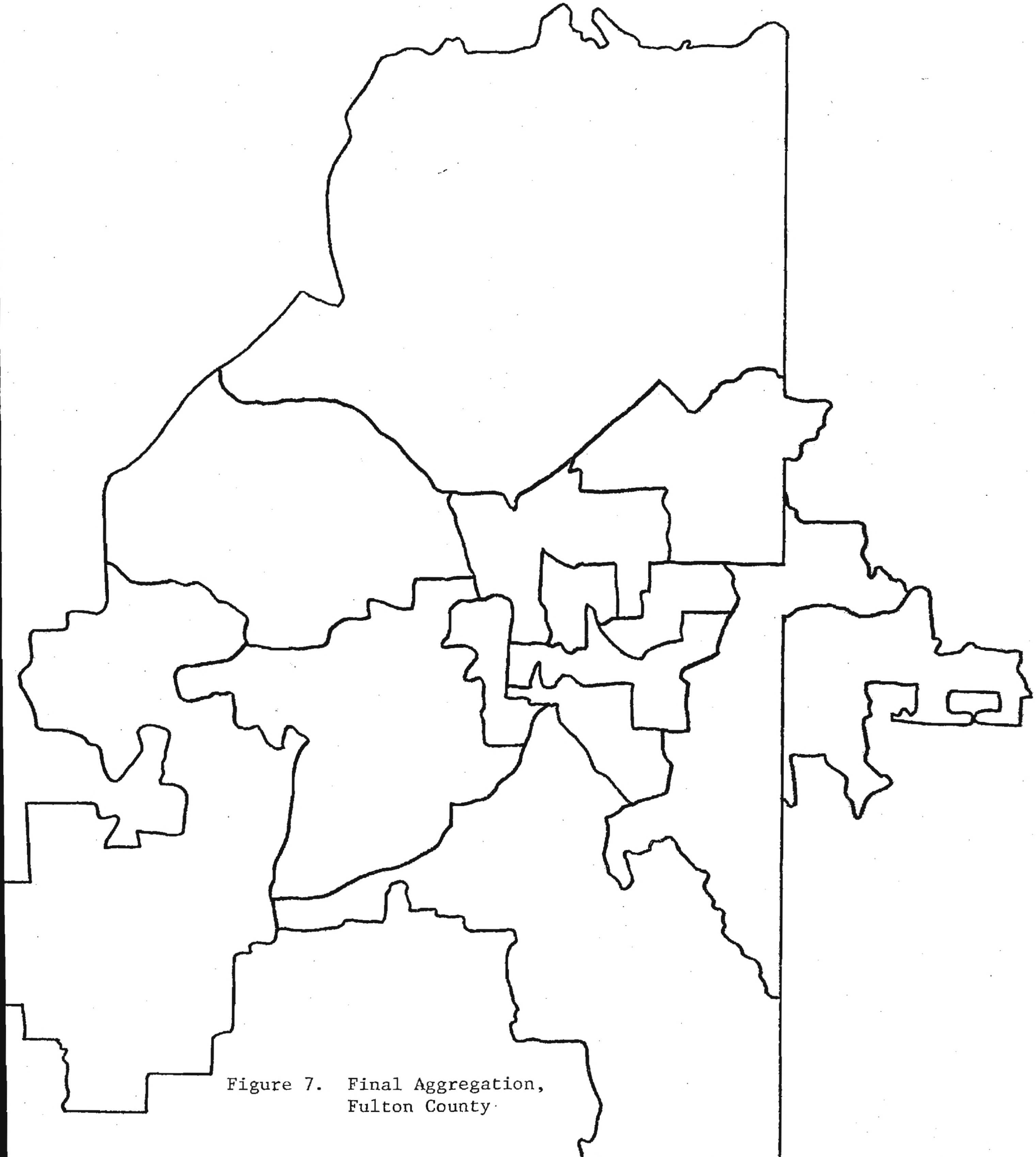


Figure 7. Final Aggregation,
Fulton County

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TECHNICAL NOTE #16

Documentation of Software for Network Flow Models in Crime Displacement

Introduction

Several computer programs have been developed as part of the crime displacement task. Each program is available on the CDC Cyber 74 in both symbolic (with a prefix of "S") and executable (with a prefix of "X") form.

In this technical note we document each of those programs. We include a brief description of each program, its required inputs, its outputs, how to execute it as well as sample runs. We also document each data set available.

Programs Available

A. AGGRE (XAGGRE, SAGGRE)

This program aggregates a 115 by 115 matrix of flows randomly into a minimum size matrix without losing any off diagonal flow. The program orders the districts randomly, and then aggregates them starting from the first district and comparing it with second, third, etc., until the 115th district is compared with the first district. Anytime an aggregation takes place, the aggregated district is compared to all the other districts including the ones that are checked before. This is necessary since every time an aggregation occurs, the adjacency matrix of this new aggregated district will also change.

The criteria for aggregating district i into district j is

a. There must be no flow from i to j or j to i.

b. Districts i and j must be adjacent to each other.

Adjacency is given by a 115 X 115 matrix that contains a 1 in the (i,j) location if districts i and j are adjacent, and 0 otherwise. Every time an aggregation takes place, the adjacency matrix is aggregated as well.

Since the initial ordering of the districts are random, the aggregation is also random.

B. AGGFLO (XAGGFLO, SAGGFLO)

This program is capable of producing the data that will be used in the linear and nonlinear transportation models. After the original map with 115 districts is reduced to 14 aggregated districts. This program takes the numbers of the districts in each aggregated zone and computes the actual flow between those aggregated districts for any specified amount of time. As an example, in our later analyses we used the 9 months data and the last 3 months data so that the optimal parameters to be obtained from the 9 month data could be used for the forecasting of the last 3 months flow. This program is also capable of producing the aggregated flows for any month or any combination of months.

C. LINEAR (XLINEAR, SLINEAR)

This is the linear transportation model. The total cost is given by

$$\text{Total Cost} = \sum_{i,j}^{14} c_{ij} x_{ij}$$

The cost coefficients c_{ij} are obtained from

$$c_{ij} = \exp(\alpha *(p_j - p_i)) + \exp(\beta * d_{ij}^2) + \exp(\gamma *(p_j - p_i)* d_{ij}^2)$$

where α, β and γ are the unknown parameters to be determined, p_j is tha ratio of police units per unit area at district j and d_{ij} is the distance between district i and district j.

The search routine operates as follows: For initial starting parameters α, β and γ, c_{ij}'s are calculated. For given values of supply and demand at each district, and cost per unit flow c_{ij}, the transportation problem is solved. The sum of squares is obtained from

$$SSQ = \sum_{i,j}^{14} (x_{ij} - f_{ij})^2$$

where x_{ij} is the solution of the transportation problem corresponding to the current set of c_{ij}'s, and the f_{ij}'s are the actual flows obtained from the program AGGFLO.

Starting from any parameter, say β, the new cost coefficients c_{ij}'s are obtained by replacing β by $\beta + \Delta$. Again the transportation problem is solved and a new sum of squares is obtained. This sum of squares is compared with the current best sum of squares so far. Depending upon the comparison, either the β is increased again by Δ (if the new solution is better) or β is reduced by $\Delta/2$ (if the new solution is not better). After a parameter is completely searched, the next parameter is searched for an improvement in the sum of squares. This iterative procedure continues until no better sum of squares can be obtained within the specified limits of α, β, γ and ε (the minimum step size). At this point the best solution is printed out with the optimum parameters α, β and γ.

D. NONLIN (XNONLIN, SNONLIN)

The searching routine of this program is similar to LINEAR but the problem solved at every iteration is a nonlinear transportation problem compared to the linear transportation algorithm. It uses three subroutines for calculating the optimal solution for the nonlinear transportation problem with the objective function being given by

$$\text{Total Cost} = \sum_{ij}^{14} c_{ij} x_{ij}^2 .$$

The cost coefficients are given by

$$c_{ij} = \alpha \frac{p_j}{p_i} + \beta d_{ij}^2 + \gamma \frac{p_j}{p_i} d_{ij}^2$$

where p_i, p_j, d_{ij}, α,β and γ are as defined in the previous section.

The three subroutines are based on T.C. Hu's nonlinear transportation algorithm for convex costs. Each single flow is assigned to a certain arc from the solution of a shortest path problem. Every time a single flow is assigned the costs on the arcs are updated as well as the upper and lower capacities of the arcs. Therefore, for a transportation problem of 450 flows, 450 shortest path problems are solved for every change in the parameters α, β and γ.

A. AGGRE

This program does not need any input from the teletype. It utilizes Tape 40 thru Tape 45 and Tape 51 to generate the aggregation. As explained before Tapes 40 through 45 contain the actual Atlanta crime data. Tape 51 contains the adjacency data for Atlanta.

B. AGGFLO

This program must be supplied externally (from the teletype) the parameter IBULL. The actual data between the months 1 thru IBULL will be obtained for the aggregated map. For example, IBULL = 9 will produce the flow matrix of the first nine months for the aggregated map.

The other data is supplied to this program from the actual Atlanta crime data (Tape 40,...Tape 45).

C. LINEAR

This program requires the following external input in the given order.

Card 1

IMAXT,M,N

Where IMAXT: Number of periods in the model. (It was one for most of our examples).

M: Numbers of districts (demand points)

N: Numbers of districts (Supply points)

Card 2

α, β, γ, and Δ

where α is the first parameter being searched.

β is the second parameter being searched.

γ is the third parameter being searched.

Δ is the step size.

Card 3

INDEX, IFLAG, ISEARC, KFLAG, DEPS

where

INDEX: parameter indicator.

1 parameter α is under consideration.

2 parameter β is under consideration.

3 parameter γ is under consideration.

IFLAG: In which direction of a given parameter we are searching.

1 we are searching in the direction of the positive side.

2 we are searching in the direction of the negative side.

KFLAG: Are we in the expansion or contraction stage?

1 Expansion by size of Δ.

2 Contraction by size of $\Delta/2$.

ISEARCH: Sides of the parameter being searched.

1 Only one side is searched.

2 Both sides are being searched.

DEPS: Criteria for terminating the search in that given direction for the given parameter. If $\Delta/2$ < DEPS the other direction of that parameter will be searched. If both directions are searched, then the next parameter will be searched.

D. NONLIN

The following information should be supplied to this program externally.

Card 1: IMAXT, MZM, M, N, KWRI, LMKO

where IMAXT the same as in LINEAR.

MZM: Number of nodes in the network created from the M X N transportation problem.

M: Number of supply points in the transportation problem.

N: Number of demand points in the transportation problem.

KWRI: Initial node in the network obtained from the transportation problem.

LMKO: Final node in the network obtained from the transportation problem.

Card 2: $\alpha,\beta,\gamma,\Delta$

Explanation: Same as in LINEAR.

Card 3: INDEX, IFLAG, ISEARC, KFLAG, DEPS

Explanation: Same as in LINEAR.

As in LINEAR, after the optimal α,β, and γ are determined, the following card should be inputed to either terminate or rerun the program.

Card 4: MORER

Explanation: Same as in LINEAR.

After the above information is given externally to the program, it will read Tape 28 and Tape 29, compute the c_{ij}'s and perform the search for all the parameters until the optimal α, β and γ are obtained. After the final iteration and output is obtained the following parameters must be inputed externally:

Card 4: MORER

MORER: Do we need another run with different initial points or not?

1 = yes

0 = no

MORER = 1 will take the control to the very beginning of the program to the first read. Therefore the following are the order of cards to be read in by the program.

1 IMAXT, M, N

2 $\alpha,\beta,\gamma,\Delta$

3 INDEX, IFLAG, ISEARC, KFLAG, DEPS

A. AGGRE

The output will contain the list of the random ordering of the districts. It will also print the number of diagonal and off diagonal flows. The next output is the complete list of the aggregation produced in the format.

(District no.) into (District no.)

After the final aggregation, the heading FINAL REDUCED TABLEAU and the flow in this aggregated matrix are printed. Every run of this program produced 25 such random aggregations.

B. AGGFLO

This program first prints the wild data cards, if there are any, and then prints the aggregated flow matrix depending upon the input IBULL as described in section 2.

C. LINEAR

The program prints the heading, ITERATION, ALPHA, BETA, GAMMA, SUMSQR and then prints the iteration number, the current values of the parameters and sum of squares at each iteration. After the search is complete the optimal vlues of the parameters and corresponding flow matrix are printed after the heading OPTIMAL SOLUTION.

D. NONLIN

The format of the output is exactly the same as LINEAR.

Sample Runs of Each Program

BASIC SOLUTION

5	4	25	0	0
8	0	0	29	0
50	0	0	0	0
69	0	0	0	0
0	26	0	0	25

MSS - ERROR= 9414

BASIC SOLUTION

4	5	25	0	0
8	0	0	29	0
0	25	0	0	25
69	0	0	0	0
51	0	0	0	0

MSS - ERROR= 15782

BASIC SOLUTION

4	30	0	0	0
8	0	0	29	0
0	0	25	0	25
69	0	0	0	0
51	0	0	0	0

MSS - ERROR= 16032

BASIC SOLUTION

1	8	25	0	0
12	0	0	0	25
50	0	0	0	0
69	0	0	0	0
0	22	0	29	0

MSS - ERROR= 9044

BASIC SOLUTION

1	30	3	0	0
12	0	0	0	25
50	0	0	0	0
69	0	0	0	0
0	0	22	29	0

MSS - ERROR= 9264

BASIC SOLUTION

0	9	25	0	0
12	0	0	0	25
0	21	0	29	0
69	0	0	0	0
51	0	0	0	0

MSS - ERROR= 15694

⋮

A. AGGRE

```
690 380 661 900 790 500 800 250 2030 831 80 420 60 762 872
980 761 640 2020 630 150 950 300 910 90 750 10 740 610 880
850 2080 140 440 160 812 400 832 940 700 360 580 620 2070
220 771 871 670 2090 460 110 70 840 890 190 50 2040 781
821 180 520 990 570 130 100 772 2060 861 811 310 470 200
2050 1000 410 730 390 970 330 370 600 320 120 430 930 862
280 350 20 551 450 960 782 680 530 650 662 230 552 210 590
170 260 40 710 560 822 480 920 490 720 270 240 290 2010
334 172
 640 INTO  690
 520 INTO  690
 500 INTO  690
2090 INTO  690
 310 INTO  690
 300 INTO  690
 160 INTO  690
 150 INTO  690
  10 INTO  690
 140 INTO  690
 130 INTO  690
 180 INTO  690
 320 INTO  690
 280 INTO  690
  20 INTO  690
 680 INTO  690
 700 INTO  690
 750 INTO  690
 661 INTO  690
 740 INTO  690
 670 INTO  690
 530 INTO  690
 650 INTO  690
 630 INTO  690
 580 INTO  690
 420 INTO  690
 410 INTO  690
 400 INTO  690
 812 INTO  690
 831 INTO  690
 832 INTO  690
 811 INTO  690
 430 INTO  690
 370 INTO  690
 662 INTO  690
  40 INTO  690
 560 INTO  690
 460 INTO  690
 470 INTO  690
 920 INTO  690
 910 INTO  690
 900 INTO  690
 980 INTO  690
 940 INTO  690
 970 INTO  690
 930 INTO  690
```

360	INTO	380
390	INTO	380
350	INTO	380
330	INTO	380
260	INTO	380
480	INTO	380
490	INTO	380
290	INTO	380
800	INTO	790
762	INTO	790
761	INTO	790
610	INTO	790
620	INTO	790
230	INTO	250
80	INTO	250
90	INTO	250
60	INTO	250
70	INTO	250
871	INTO	250
50	INTO	250
100	INTO	250
110	INTO	250
200	INTO	250
190	INTO	250
120	INTO	250
270	INTO	250
2020	INTO	2030
2080	INTO	2030
2070	INTO	2030
2040	INTO	2030
2060	INTO	2030
880	INTO	872
850	INTO	872
840	INTO	872
890	INTO	872
862	INTO	872
821	INTO	872
822	INTO	872
240	INTO	872
990	INTO	950
1000	INTO	950
960	INTO	950
570	INTO	440
450	INTO	440
210	INTO	220
772	INTO	771
782	INTO	771
720	INTO	730

FINAL REDUCED TABLEAU

52	13	4	7	0	5	2	6	1	0	0	2	0	0	1	2	0	1	0	0	1
5	15	0	1	2	1	0	1	0	0	0	0	0	1	1	1	0	1	1	0	0
6	1	6	4	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
5	4	0	14	0	6	0	0	2	0	1	1	0	0	0	0	0	0	0	0	0
4	2	0	1	15	0	0	0	1	0	0	0	1	1	0	1	0	0	0	0	0
4	1	0	3	0	15	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	2	0	1	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	3	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
1	0	1	1	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	1	0	1	1	1	0	6	0	0	0	0	2	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0
3	0	1	0	0	1	0	0	0	0	0	0	0	6	0	0	0	0	0	0	0
0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	7	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	5	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0
3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

```
? 9
 350 3 203
 877 7 871
 208 6 2080
 771 3 1030
 7601 11 761
 850 10 8321
 460 10 3101
 920 5 2
 350 8 2381
 661 3 660
   1   1   0   0   2   0   0   0   2   0   1   0   2   0
   0  19   1   3   1   1   1   0   1   1   0   1   0   1
   0   1   2   2   1   0   0   0   1   1   0   0   0   0
   1   1   1   9   2   1   0   0   0   2   0   1   0   1
   1   1   1   3  34   3   1   0   0   0   1   0   0   1
   0   3   1   1   0  23   2   1   0   0   0   4   1   0
   0   0   0   1   0   0  25   0   0   0   1   1   0   0
   0   0   0   1   0   2   0   2   2   0   0   0   0   0
   1   1   2   5   0   1   0   0  11   0   0   0   1   2
   0   0   0   1   1   0   0   0   0   2   0   1   1   0
   0   2   0   0   2   1   2   0   0   0  13   1   1   1
   0   0   0   2   4   0   2   1   3   1   6  15   2   1
   0   0   1   1   0   1   1   0   0   0   0   1   6   1
   0   1   0   2   0   0   1   0   2   1   0   0   0  13
      2.757 CP SECONDS EXECUTION TIME
/
```

C. LINEAR


```
 ENTER IMAXT,M,N
? 1,14,14
 ENTER ALPHA,BETA,GAMMA,DELTA
? .55,.7,-.15,.3
 ENTER INDEX,IFLAG,ISEARC,KFLAG,DEPS
? 3,1,1,2,.1
```

ITERATION	ALPHA	BETA	GAMMA	SUM OF SQRS
1	.5500	.7000	-.1500	1712.
2	.5500	.7000	.1500	950.
3	.5500	.7000	.4500	1320.
4	.5500	.7000	.3000	950.
5	.5500	.7000	.2250	968.
6	.5500	.7000	-.1500	1712.
7	.5500	.7000	0.0000	950.
8	.5500	.7000	.0750	942.
9	.5500	.7000	.3750	1228.
10	.5500	.7000	.2250	968.
11	.5500	.7000	.1500	950.
12	.5500	.7000	-.2250	1362.
13	.5500	.7000	-.0750	950.
14	.5500	.7000	.0000	950.
15	.8500	.7000	.0750	1228.
16	.7000	.7000	.0750	1228.
17	.6250	.7000	.0750	1228.
18	.2500	.7000	.0750	906.
19	.5500	.7000	.0750	942.
20	.4000	.7000	.0750	1644.
21	.3250	.7000	.0750	1568.
22	-.0500	.7000	.0750	1908.
23	.1000	.7000	.0750	1174.
24	.1750	.7000	.0750	1208.
25	.2500	1.0000	.0750	1344.
26	.2500	.8500	.0750	946.
27	.2500	.7750	.0750	946.
28	.2500	.4000	.0750	1264.
29	.2500	.5500	.0750	1576.
30	.2500	.6250	.0750	1568.
31	.2500	.7000	.3750	1392.
32	.2500	.7000	.2250	1404.
33	.2500	.7000	.1500	1568.
34	.2500	.7000	-.2250	1416.
35	.2500	.7000	-.0750	978.
36	.2500	.7000	.0000	972.
37	.5500	.7000	.0750	942.
38	.4000	.7000	.0750	1644.
39	.3250	.7000	.0750	1568.
40	-.0500	.7000	.0750	1908.
41	.1000	.7000	.0750	1174.
42	.1750	.7000	.0750	1208.
43	.2500	1.0000	.0750	1344.
44	.2500	.8500	.0750	946.
45	.2500	.7750	.0750	946.
46	.2500	.4000	.0750	1264.
47	.2500	.5500	.0750	1576.
48	.2500	.6250	.0750	1568.
49	.2500	.7000	.3750	1392.
50	.2500	.7000	.2250	1404.
51	.2500	.7000	.1500	1568.
52	.2500	.7000	-.2250	1416.
53	.2500	.7000	-.0750	978.
54	.2500	.7000	.0000	972.

```
        OPTIMAL SOLUTION
ALPHA=  .2500000
BETA=  .7000000
GAMMA=  .0750000
SUM OF SQR=        906.
```

```
        COST MATRIX

3. 547. 1.571034255E+9 73149. 1.571034269E+9
73320. 309846. 21. 74. 6019. 213221. 28366.
480. 2745.
545. 3. 114. 14. 148888. 73196. 7230015.
1636. 34. 49. 1203. 1394. 676. 283.
1.571034257E+9 116. 3. 20. 5303. 3746533.
1.1251306337E+10 80762908. 20605. 772.
86096. 1031989. 402723. 3192490.
73131. 13. 17. 3. 35. 203. 24540. 1164.
17. 9. 22. 34. 9. 39.
1.571034254E+9 148887. 5298. 37. 3. 35.
1301. 73150. 991. 65. 40. 107. 366. 2517.
73130. 73130. 3744318. 200. 30. 3. 3.
29. 27. 24. 10. 10. 7. 27.
309821. 7230003. 1.1251305463E+10 24540.
1298. 5. 3. 32. 213. 488. 67. 37. 17.
312.
17. 1626. 80760618. 1161. 73131. 30. 30.
3. 7. 58. 54. 23. 7. 20.
52. 23. 18840. 12. 930. 24. 199. 4. 3.
5. 9. 7. 4. 8.
2902. 23. 325. 2. 34. 16. 391. 43. 3.
3. 4. 5. 5. 5.
18840. 360. 6780. 6. 11. 3. 29. 29. 4.
3. 3. 3. 3. 4.
2902. 360. 36714. 11. 34. 2. 13. 8. 3.
3. 3. 3. 2. 4.
325. 544. 309821. 3. 294. 4. 9. 3. 3.
6. 5. 4. 3. 7.
199. 47. 18840. 8. 294. 8. 79. 4. 2. 2.
3. 3. 2. 3.
```

```
        COMPUTED FLOW MATRIX

  4   0   0   0   0   0   0   1   0   0   0   0   0   0
  0  28   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   9   0   0   0   0   0   0   0   0   0   0   0
  0   1   1  17   0   0   0   0   0   0   0   0   0   0
  0   0   0   0  40   0   0   0   0   0   0   0   0   0
  0   0   0   0   0  18   8   0   0   0   0   0   0   0
  0   0   0   0   0   0  17   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   3   0   0   0   0   3   0
  0   0   0   0   0   0   0   0  19   4   0   0   0   0
  0   0   0   7   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   1   0   0   0   0   0  16   0   0   4
  0   0   0   0   0   6   0   0   0   0   1  18   8   0
  0   0   0   6   0   0   0   0   0   0   0   4   0   0
  0   0   0   0   0   0   0   0   0   3   0   0   0  12
DO YOU WANT A NEW RUN 1 YES,0 NO
? 0
   16.098 CP SECONDS EXECUTION TIME
/
```

D. NONLIN

```
  ENTER IMAXT,MZM,M,N,KWRI,LMKO
?  1,30,14,14,1,30
  ENTER ALPHA,BETA,GAMMA,DELTA
?  7.,3.,5.,4.
  ENTER INDEX,IFLAG,ISEARC,KFLAG,DEPS
?  1,1,1,2,1.
  ITERATION        ALPHA       BETA       GAMMA       SUM OF SQRS

          1        7.0000     3.0000     5.0000      828.
          2       11.0000     3.0000     5.0000      830.
          3        9.0000     3.0000     5.0000      828.
          4        8.0000     3.0000     5.0000      828.
          5        7.5000     3.0000     5.0000      828.
          6        3.0000     3.0000     5.0000      828.
          7        5.0000     3.0000     5.0000      828.
          8        6.0000     3.0000     5.0000      828.
          9        6.5000     3.0000     5.0000      828.
         10        7.0000     7.0000     5.0000      802.
         11        7.0000    11.0000     5.0000      792.
         12        7.0000    15.0000     5.0000      808.
         13        7.0000    13.0000     5.0000      810.
         14        7.0000    12.0000     5.0000      810.
         15        7.0000    11.5000     5.0000      792.
         16        7.0000     7.0000     5.0000      802.
         17        7.0000     9.0000     5.0000      792.
         18        7.0000    10.0000     5.0000      792.
         19        7.0000    10.5000     5.0000      810.
         20        7.0000    11.0000     9.0000      820.
         21        7.0000    11.0000     7.0000      808.
         22        7.0000    11.0000     6.0000      810.
         23        7.0000    11.0000     5.5000      792.
         24        7.0000    11.0000     1.0000      792.
         25        7.0000    11.0000     3.0000      792.
         26        7.0000    11.0000     4.0000      792.
         27        7.0000    11.0000     4.5000      792.
         28       11.0000    11.0000     5.0000      792.
         29        9.0000    11.0000     5.0000      792.
         30        8.0000    11.0000     5.0000      792.
         31        7.5000    11.0000     5.0000      808.
         32        3.0000    11.0000     5.0000      810.
         33        5.0000    11.0000     5.0000      810.
         34        6.0000    11.0000     5.0000      808.
         35        6.5000    11.0000     5.0000      792.

                OPTIMAL   SOLUTION

 ALPHA= 7.00000000
 BETA=11.0000000
 GAMMA= 5.0000000
 SUM OF SQRS=       792.
```

COMPUTED FLOW MATRIX

4	1	0	0	0	0	0	0	0	0	0	0	0	0
0	28	0	0	0	0	0	0	0	0	0	0	0	0
0	0	9	0	0	0	0	0	0	0	0	0	0	0
0	0	0	19	0	0	0	0	0	0	0	0	0	0
0	0	0	0	40	0	0	0	0	0	0	0	0	0
0	0	0	1	0	23	2	0	0	0	0	0	0	0
0	0	0	0	0	0	17	0	0	0	0	0	0	0
0	0	0	0	0	0	1	4	0	0	0	0	1	0
0	0	0	1	0	0	1	0	18	1	0	0	1	1
0	0	1	3	0	0	0	0	0	3	0	0	0	0
0	0	0	2	0	0	1	0	0	1	16	0	0	1
0	0	0	1	1	1	1	0	1	2	1	22	2	1
0	0	0	2	0	0	1	0	0	0	0	0	7	0
0	0	0	1	0	0	1	0	0	0	0	0	0	13

APPENDIX A

Listing of Each Program

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,
     *TAPE40,TAPE41,TAPE42,TAPE43,TAPE44,TAPE45,TAPE51)
      DIMENSION IX(1800,3),ICODE(115),ITAB(115,115)
      DIMENSION IRED(115,115)
      DIMENSION IICODE(115),ILIST(115)
      DIMENSION KKROW(115),KKCOL(115),LLROW(115),LLCOL(115)
      DIMENSION KINCOL(115),LINCOL(115),INCID(115,115)
      DIMENSION LINROW(115),KINROW(115),INULL(20)
      DATA ICODE/1,2,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,
     *19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,35,36,
     *37,38,39,40,41,42,43,44,45,46,47,48,49,50,52,53,551,
     *552,56,57,58,59,60,61,62,63,64,65,661,662,67,68,69,
     *70,71,72,73,74,75,761,762,771,772,781,782,79,80,
     *811,812,821,822,831,832,84,85,861,862,871,872,
     *88,89,90,91,92,93,94,95,96,97,98,99,100,201,202,203,
     *204,205,206,207,208,209/
      YZZ=TIME(AAAAA)
      ISEED=YZZ*100000
      CALL RANSET(ISEED)
      DO 5683 I=1,115
5683  IICODE(I)=ICODE(I)
      REWIND 40
      REWIND 41
      IPTY=115
      IS=0
      REWIND 42
      REWIND 51
      REWIND 43
      REWIND 44
      REWIND 45
      READ(40,*)KOP
      DO 56 JK1=1,KOP
      READ(40,7)(IX(JK1,M),M=1,3)
      GO TO 56
124   FORMAT(12H CRIME TYPE=,I2//)
  123 WRITE(6,709)(IX(K,J),J=1,3)
709   FORMAT(18X,I2,7X,I4,11X,I4)
7     FORMAT(I2,6X,I4,30X,I4)
56    CONTINUE
      READ(41,*)KOP1
      DO 446 JK2=1,KOP1
      IT1=JK2+KOP
      READ(41,7)(IX(IT1,M),M=1,3)
446   CONTINUE
      READ(42,*)KOP2
      DO 447 JK3=1,KOP2
      IT2=IT1+JK3
      READ(42,7)(IX(IT2,M),M=1,3)
447   CONTINUE
      READ(43,*)KOP
      DO 448 JK4=1,KOP
      IT3=IT2+JK4
      READ(43,7)(IX(IT3,M),M=1,3)
448   CONTINUE
      READ(44,*)KOP
      DO 449 JK5=1,KOP
      IT4=IT3+JK5
      READ(44,7)(IX(IT4,M),M=1,3)
```

```
449     CONTINUE
        READ(45,*)KOP
        DO 450 JK6=1,KOP
        IT5=IT4+JK6
        READ(45,771)IX(IT5,1),IX(IT5,2),IX(IT5,3)
771     FORMAT(6X,I2,I4,19X,I4,12X,3X)
772     FORMAT(I2,6X,I4,23X,I4)
450     CONTINUE
5684    DO 5685 I=1,115
5685    ICODE(I)=IICODE(I)
        KOP=IT5
        DO 777 JKL=1,115
        IF(ICODE(JKL).LT.220)ICODE(JKL)=ICODE(JKL)*10
777     CONTINUE
        GO TO 13377
13378   DO 40 I=1,115
        KINROW(I)=ICODE(ILIST(I))
        DO 40 KL=1,115
        INCID(I,KL)=0
40      ITAB(I,KL)=0
        DO 8800 I=1,115
8800    ICODE(I)=KINROW(I)
        DO 1 I=1,KOP
        ICOL=0
        IROW=0
        DO 5 K=1,115
        IF(IX(I,2).NE.ICODE(K))GO TO 4
        ICOL=K
4       IF(IX(I,3).NE.ICODE(K))GO TO 5
        IROW=K
5       CONTINUE
        IF(ICOL.EQ.0.OR.IROW.EQ.0)GO TO 1
        ITAB(IROW,ICOL)=ITAB(IROW,ICOL)+1
1       CONTINUE
        KSUMK=0
        KKSUMK=0
        GO TO 13379
13377   DO 13349 I=1,115
1351    R=RANF(ISEED)
        M=I-1
        K=115*R+1
        IF(I.EQ.1)GO TO 1362
        DO 13350 L=1,M
        IF(K.EQ.ILIST(L))GO TO 1351
13350     CONTINUE
1362     ILIST(I)=K
13349    CONTINUE
        GO TO 13378
13379   DO 7216 I=1,115
        READ(51,*)N,(INULL(K),K=1,N)
        DO 7217 L=1,N
        DO 7218 J=1,115
        IF(INULL(L).NE.ILIST(J))GO TO 7218
        KOK=J
        DO 7219 LLM=1,115
        IF(ILIST(LLM).NE.I)GO TO 7219
        INCID(LLM,KOK)=1
7219     CONTINUE
7218     CONTINUE
7217     CONTINUE
```

```
7216  CONTINUE
      WRITE(6,*)(ICODE(I),I=1,115)
      DO 678 IH=1,115
      KKSUMK=KKSUMK+ITAB(IH,IH)
      DO 678 IU=1,115
      KSUMK=KSUMK+ITAB(IH,IU)
      IRED(IH,IU)=ITAB(IH,IU)
678   CONTINUE
      WRITE(6,*)KSUMK,KKSUMK
      ICURR=115
      IZCURR=ICURR-1
      KMAX=2
693   DO 679 K=1,ICURR
      DO 679 L=1,ICURR
      IF(K.EQ.L)GO TO 679
      IF(IRED(K,L).NE.0.OR.IRED(L,K).NE.0)GO TO 679
      IDD=0
      DO 4251 JHG=1,ICURR
      IF(IRED(L,JHG).EQ.0)GO TO 4251
      IDD=IDD+1
4251   CONTINUE
      IF(IDD.GT.IRTY)GO TO 679
      IF(INCID(K,L).NE.1)GO TO 679
      KK=K
      LL=L
      GO TO 694
679   CONTINUE
      GO TO 697
694   WRITE(6,116)ICODE(LL),ICODE(KK)
116   FORMAT(1H ,I4,5H INTO,I5)
      DO 680 MM=1,ICURR
      KKROW(MM)=IRED(KK,MM)
      KKCOL(MM)=IRED(MM,KK)
      LLROW(MM)=IRED(LL,MM)
      KINROW(MM)=INCID(KK,MM)
      KINCOL(MM)=INCID(MM,KK)
      LINROW(MM)=INCID(LL,MM)
      LINCOL(MM)=INCID(MM,LL)
      LLCOL(MM)=IRED(MM,LL)
680   CONTINUE
      DO 1349 MM=1,ICURR
      IRED(KK,MM)=KKROW(MM)+LLROW(MM)
      INCID(KK,MM)=0
      IF(KINROW(MM).EQ.1.OR.LINROW(MM).EQ.1)INCID(KK,MM)=1
      INCID(MM,KK)=0
      IF(KINCOL(MM).EQ.1.OR.LINCOL(MM).EQ.1)INCID(MM,KK)=1
1349   IRED(MM,KK)=KKCOL(MM)+LLCOL(MM)
      INCID(KK,KK)=0
      IRED(KK,KK)=KKROW(KK)+LLROW(LL)
      JCURR=ICURR-1
      DO 681 MM=LL,JCURR
      ICODE(MM)=ICODE(MM+1)
      DO 681 JT=1,ICURR
      IRED(MM,JT)=IRED(MM+1,JT)
      INCID(MM,JT)=INCID(MM+1,JT)
681   CONTINUE
      DO 6681 MM=LL,JCURR
      DO 6681 JT=1,JCURR
      INCID(JT,MM)=INCID(JT,MM+1)
6681   IRED(JT,MM)=IRED(JT,MM+1)
```

```
199     FORMAT(18H REMOVED DISTRICT=,I3,5H INTO,I3)
        ICURR=ICURR-1
        IZCURR=ICURR-1
        GO TO 693
697     WRITE(6,198)
198     FORMAT(22H FINAL REDUCED TABLEAU//)
        DO 942 MML=1,ICURR
        WRITE(6,*)(IRED(MML,LS),LS=1,ICURR)
942     CONTINUE
2       CONTINUE
        ISEED=10000000*RANF(ISEED)
        REWIND 51
        IS=IS+1
        CALL RANSET(ISEED)
        IF(IS.NE.25)GO TO 5684
        STOP
        END
/
```

B. AGGFLO

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE40,
     *TAPE41,TAPE42,TAPE43,TAPE44,TAPE45)
      DIMENSION I1(13),I2(8),I3(5),I4(11),I5(13),I6(15),I7(5),
     *I8(8),I9(10),I10(4),I11(7),I12(8),I13(4),I14(4),
     *IX(1800,3),IFLOW(14,14)
      DATA I1/880,890,900,910,920,930,940,950,960,970,980,
     *990,1000/
      DATA I2/821,822,831,850,861,862,871,872/
      DATA I3/771,772,781,782,790/
      DATA I4/400,410,600,610,620,762,800,811,812,832,840/
      DATA I5/570,580,630,650,661,662,670,700,720,730,740,750,
     *761/
      DATA I6/160,300,310,320,500,520,530,552,640,680,690,
     *710,2020,2030,2040/
      DATA I7/2050,2060,2070,2080,2090/
      DATA I8/10,20,40,50,130,140,150,2010/
      DATA I9/60,70,80,90,100,110,120,190,230,250/
      DATA I10/240,390,420,590/
      DATA I11/370,430,440,450,460,551,560/
      DATA I12/290,330,350,360,380,470,480,490/
      DATA I13/170,180,270,280/
      DATA I14/200,210,220,260/
      REWIND 40
      REWIND 41
      REWIND 42
      REWIND 43
      REWIND 44
      READ(6,*)IBULL
      REWIND 45
7     FORMAT(8X,I4,6X,I2,22X,I4)
      DO 19 I=1,14
      DO 19 J=1,14
19    IFLOW(I,J)=0
      KOP=1
      READ(40,*)KOPEK
      DO 56 JK1=1,KOPEK
      READ(40,7)(IX(KOP,M),M=1,3)
      IF(IX(KOP,2).EQ.IBULL)GO TO 56
      KOP=KOP+1
56    CONTINUE
      IT1=KOP+1
      READ(41,*)KOP1
      DO 446 JK2=1,KOP1
      READ(41,7)(IX(IT1,M),M=1,3)
      IF(IX(IT1,2).EQ.IBULL)GO TO 446
      IT1=IT1+1
446   CONTINUE
      IT2=IT1+1
      READ(42,*)KOP2
      DO 447 JK3=1,KOP2
      READ(42,7)(IX(IT2,M),M=1,3)
      IF(IX(IT2,2).EQ.IBULL)GO TO 447
      IT2=IT2+1
447   CONTINUE
      IT3=IT2+1
      READ(43,*)KOP
      DO 448 JK4=1,KOP
      READ(43,7)(IX(IT3,M),M=1,3)
      IF(IX(IT3,2).EQ.IBULL)GO TO 448
      IT3=IT3+1
448   CONTINUE
```

```
771     FORMAT(8X,I4,19X,I4,12X,I2)
        IT4=IT3+1
        READ(44,*)KOP
        DO 449 JK5=1,KOP
        READ(44,7)(IX(IT4,M),M=1,3)
        IF(IX(IT4,2).EQ.IBULL)GO TO 449
        IT4=IT4+1
449     CONTINUE
        IT5=IT4+1
        READ(45,*)KOP
        DO 450 JK6=1,KOP
        READ(45,771)IX(IT5,1),IX(IT5,3),IX(IT5,2)
        IF(IX(IT5,2).EQ.IBULL)GO TO 450
        IT5=IT5+1
450     CONTINUE
        K=1
        DO 1 I=1,IT5
        IF(IX(I,1).EQ.0.OR.IX(I,3).EQ.0)GO TO 1
        IF(IX(I,3).EQ.9999.OR.IX(I,1).EQ.9999)GO TO 1
102     DO 2 J=1,13
        IF(IX(I,K).NE.I1(J))GO TO 2
        IROW=1
        GO TO 100
2       CONTINUE
        DO 3 J=1,8
        IF(IX(I,K).NE.I2(J))GO TO 3
        IROW=2
        GO TO 100
3       CONTINUE
        DO 4 J=1,5
        IF(IX(I,K).NE.I3(J))GO TO 4
        IROW=3
        GO TO 100
4       CONTINUE
        DO 5 J=1,11
        IF(IX(I,K).NE.I4(J))GO TO 5
        IROW=4
        GO TO 100
5       CONTINUE
        DO 6 J=1,13
        IF(IX(I,K).NE.I5(J))GO TO 6
        IROW=5
        GO TO 100
6       CONTINUE
        DO 8 J=1,15
        IF(IX(I,K).NE.I6(J))GO TO 8
        IROW=6
        GO TO 100
8       CONTINUE
        DO 9 J=1,5
        IF(IX(I,K).NE.I7(J))GO TO 9
        IROW=7
        GO TO 100
9       CONTINUE
        DO 10 J=1,8
        IF(IX(I,K).NE.I8(J))GO TO 10
        IROW=8
        GO TO 100
10      CONTINUE
        DO 11 J=1,10
        IF(IX(I,K).NE.I9(J))GO TO 11
        IROW=9
        GO TO 100
```

```
11      CONTINUE
        DO 12 J=1,4
        IF(IX(I,K).NE.I10(J))GO TO 12
        IROW=10
        GO TO 100
12      CONTINUE
        DO 13 J=1,7
        IF(IX(I,K).NE.I11(J))GO TO 13
        IROW=11
        GO TO 100
13      CONTINUE
        DO 14 J=1,8
        IF(IX(I,K).NE.I12(J))GO TO 14
        IROW=12
        GO TO 100
14      CONTINUE
        DO 15 J=1,4
        IF(IX(I,K).NE.I13(J))GO TO 15
        IROW=13
        GO TO 100
15      CONTINUE
        DO 16 J=1,4
        IF(IX(I,K).NE.I14(J))GO TO 16
        IROW=14
        GO TO 100
16      CONTINUE
        WRITE(6,*)(IX(I,M),M=1,3)
        K=1
        GO TO 1
100     IF(K.EQ.3)GO TO 101
        K=3
        IIROW=IROW
        GO TO 102
101     IICOL=IROW
        IFLOW(IIROW,IICOL)=IFLOW(IIROW,IICOL)+1
        K=1
1       CONTINUE
        WRITE(6,29)((IFLOW(I,J),J=1,14),I=1,14)
29      FORMAT(14I4)
        STOP
        END
/
```

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,
     *TAPE28,TAPE29)
      DIMENSION PARAM(3),ASUM(500),DD(20),EE(50),FF(20,20),
     *CT(20,50),CP(20),R(50),WP(20,50),COST(20,20)
      DIMENSION IGEZ(14,14)
      INTEGER COST,CP,R,WP
      DOUBLE PRECISION CT
      EEPS=-50.
      WRITE(6,501)
501   FORMAT(16H ENTER IMAXT,M,N)
413   READ(5,*)IMAXT,M,N
      WRITE(6,502)
502   FORMAT(29H ENTER ALPHA,BETA,GAMMA,DELTA)
      READ(5,*)(PARAM(I),I=1,3),ADELTA
      WRITE(6,503)
503   FORMAT(36H ENTER INDEX,IFLAG,ISEARC,KFLAG,DEPS)
      READ(5,*)INDEX,IFLAG,ISEARC,KFLAG,DEPS
      DELTA=ADELTA
      IFN=0
      KINDEX=INDEX
      AMIN=9999999999999.
      EPS=10
      AAAA=PARAM(1)
      WRITE(6,9333)
9333   FORMAT(10H ITERATION,10H      ALPHA,10H       BETA,
     *10H      GAMMA,15H     SUM OF SQRS/)
      BBBB=PARAM(2)
      CCCC=PARAM(3)
      ITER=1
98    AAA=PARAM(1)
      BBB=PARAM(2)
      CCC=PARAM(3)
   99 AA=PARAM(1)
      BB=PARAM(2)
      CC=PARAM(3)
      REWIND 28
      REWIND 29
      ITIME=1
      SUM=0.
103   READ(28,*)(CP(I),I=1,N)
      READ(28,*)(R(J),J=1,M)
      READ(28,*)((COST(I,J),J=1,N),I=1,M)
      READ(29,*)(DD(I),I=1,M)
      READ(29,*)(EE(J),J=1,M)
      READ(29,*)((FF(I,J),J=1,N),I=1,M)
481   DO 10011 I=1,N
      DO 10011 J=1,M
      BOK1=AA*(EE(J)-EE(I))
      BOK2=BB*FF(I,J)**2
      BOK3=CC*(EE(J)-EE(I))*FF(I,J)**2
      EEEPS=63
      IF(BOK1.GE.EEEPS.OR.BOK2.GE.EEEPS.OR.BOK3.GE.EEEPS)
     *GO TO 443
      IF(BOK1.LE.EEPS.OR.BOK2.LE.EEPS.OR.BOK3.LE.EEPS)
     *GO TO 445
      GO TO 446
443   CT(I,J)=1.E+12
C     IF(COST(I,J).EQ.0)CT(I,J)=1.E+10
      GO TO 10011
```

```
445   CT(I,J)=0.
      IF(BOK1.GT.EEPS)CT(I,J)=EXP(BOK1)
      IF(BOK2.GT.EEPS)CT(I,J)=CT(I,J)+EXP(BOK2)
      IF(BOK3.GT.EEPS)CT(I,J)=CT(I,J)+EXP(BOK3)
      IF(CT(I,J).GE.1.E+12)CT(I,J)=1.E+12
      KEW=CT(I,J)
      CT(I,J)=KEW
C     IF(COST(I,J).EQ.0)CT(I,J)=1.E+10
      GO TO 10011
446   CT(I,J)=EXP(AA*(EE(J)-EE(I)))+EXP(BB*FF(I,J)**2)
     *+EXP(CC*(EE(J)-EE(I))*FF(I,J)**2)
      IF(CT(I,J).GE.1.E+12)CT(I,J)=1.E+12
      KEWQ=CT(I,J)
      CT(I,J)=KEWQ
C     IF(COST(I,J).EQ.0)CT(I,J)=1.E+10
10011 CONTINUE
489   IF(IFN.EQ.1)GO TO 407
1111  CONTINUE
49    CONTINUE
      CALL AOOK(CT,CP,R,M,N,WP)
      DO 2 I=1,N
      DO 2 J=1,M
C     IF(COST(I,J).EQ.0)GO TO 2
      SUM=SUM+(WP(I,J)-COST(I,J))**2
2     CONTINUE
      ITIME=ITIME+1
      IF(ITIME.LE.IMAXT)GO TO 103
      ASUM(ITER)=SUM
      WRITE(6,933)ITER,(PARAM(I),I=1,3),SUM
933   FORMAT(I10,3F10.4,F15.0)
73    FORMAT(15H SUM OF SQUARE=,F16.7)
      IF(ASUM(ITER).LT.AMIN)GO TO 104
      IF(DELTA.LE.DEPS)GO TO 744
      IF(KFLAG.NE.2)GO TO 730
745   IF(IFLAG.EQ.2)GO TO 731
      DELTA=DELTA/2.
      PARAM(KINDEX)=PARAM(KINDEX)-DELTA
      IFLAG=1
      ITER=ITER+1
      KFLAG=1
      GO TO 99
731   DELTA=DELTA/2.
      PARAM(KINDEX)=PARAM(KINDEX)+DELTA
      ITER=ITER+1
      IFLAG=2
      KFLAG=1
      GO TO 99
 730  KFLAG=1
      GO TO 7748
744   IF(ISEARC.EQ.1)GO TO 1288
      KINDEX=KINDEX+1
      PARAM(KINDEX-1)=AAAA
      IF((KINDEX-1).EQ.2)PARAM(KINDEX-1)=BBBB
      IF((KINDEX-1).EQ.3)PARAM(KINDEX-1)=CCCC
      IF(KINDEX.EQ.4)GO TO 4444
      ISEARC=1
      DELTA=ADELTA
      KFLAG=2
      INDEX=KINDEX
      ITER=ITER+1
      IFLAG=1
      PARAM(KINDEX)=PARAM(KINDEX)+DELTA
      GO TO 99
```

```
7748   IF(ISEARC.EQ.2)GO TO 7745
       GO TO 745
7745    CONTINUE
       DELTA=DELTA/2.
       PARAM(KINDEX)=PARAM(KINDEX)+DELTA
       ITER=ITER+1
       IFLAG=2
       KFLAG=1
       GO TO 99
104    AMIN=ASUM(ITER)
       IF(KINDEX.EQ.1)GO TO 8768
       IF(KINDEX.EQ.2)GO TO 8868
       CCCC=PARAM(KINDEX)
       GO TO 8869
8768   AAAA=PARAM(KINDEX)
       GO TO 8869
8868    BBBB=PARAM(KINDEX)
8869    CONTINUE
       ISEARC=1
       DELTA=ADELTA
       ITER=ITER+1
       PARAM(KINDEX)=PARAM(KINDEX)+DELTA
       IFLAG=1
       KFLAG=2
       GO TO 99
1288    IF(KFLAG.EQ.1.AND.IFLAG.EQ.1)GO TO 1289
       IF(KINDEX.EQ.2)GO TO 1291
       IF(KINDEX.EQ.1)GO TO 1290
       PARAM(KINDEX)=CCCC
       GO TO 1292
1290   PARAM(KINDEX)=AAAA
       GO TO 1292
1291   PARAM(KINDEX)=BBBB
1292    DELTA=ADELTA
       PARAM(KINDEX)=PARAM(KINDEX)+DELTA
       KFLAG=2
       IFLAG=1
       ITER=ITER+1
       ISEARC=2
       GO TO 99
1289   IF(KINDEX.EQ.1)GO TO 1294
       IF(KINDEX.EQ.2)GO TO 1295
       PARAM(KINDEX)=CCCC
       GO TO 1296
1294   PARAM(KINDEX)=AAAA
       GO TO 1296
1295   PARAM(KINDEX)=BBBB
1296   DELTA=ADELTA
       PARAM(KINDEX)=PARAM(KINDEX)-DELTA
       KFLAG=2
       IFLAG=2
       ITER=ITER+1
       ISEARC=2
       GO TO 99
746    PARAM(KINDEX)=PARAM(KINDEX)-DELTA
       IFLAG=2
       KFLAG=2
       ISEARC=2
       GO TO 99
```

```
5555  INDEX=1
      DELTA=ADELTA
      AAA=PARAM(1)
      BBB=PARAM(2)
      CCC=PARAM(3)
      PARAM(1)=PARAM(1)+DELTA
      KINDEX=1
      ITER=ITER+1
      IFLAG=1
      KFLAG=2
      ISEARC=1
      GO TO 99
4444  CONTINUE
      IF(ABS(PARAM(1)-AAA).GT..000001)GO TO 5555
      IF(ABS(PARAM(2)-BBB).GT..000001)GO TO 5555
      IF(ABS(PARAM(3)-CCC).GT..000001)GO TO 5555
3333   CONTINUE
55    FORMAT(4H END)
404   FORMAT(14I4)
402   FORMAT(10X,18H ACTUAL FLOW MATRI/)
      AA=AAA
      BB=BBB
      CC=CCC
      WRITE(6,401)AAA,BBB,CCC,AMIN
401   FORMAT(10X,16HOPTIMAL SOLUTION/7H ALPHA=,F10.7/
     *6H BETA=,F10.7/7H GAMMA=,F10.7/12H SUM OF SQR=,F10.0//)
      WRITE(6,406)
406   FORMAT(10X,11HCOST MATRIX/)
      REWIND 28
      REWIND 29
      READ(28,*)(CP(I),I=1,N)
      READ(28,*)(R(J),J=1,N)
      READ(29,*)(DD(I),I=1,M)
      READ(29,*)(EE(J),J=1,N)
      READ(29,*)((FF(I,J),J=1,M),I=1,N)
      IFN=1
      GO TO 481
407   CONTINUE
      DO 409 I=1,N
      WRITE(6,*)(CT(I,J),J=1,M)
409   CONTINUE
      CALL AOOK(CT,CP,R,M,N,WP)
400   FORMAT(14F4.0)
      WRITE(6,403)
403   FORMAT(/8X,20HCOMPUTED FLOW MATRIX/)
      WRITE(6,404)((WP(I,J),J=1,N),I=1,N)
      IFN=0
      WRITE(6,510)
 510  FORMAT(33H DO YOU WANT A NEW RUN 1 YES,0 NO)
      READ(5,*)MORER
      IF(MORER.NE.0)GO TO 413
      STOP
      END
      SUBROUTINE AOOK(CT,CP,R,MMM,NNN,WP)
      DIMENSION CT(20,50),CP(20),R(50),WP(20,50)
      COMMON /A/ ID(320),IU(320),IR(320),IP(320),IC(1000) ,NS,ND
     1,IDA(320)
      DIMENSION LX(25920),IAL(320),IAR(320),ITL(320),ITR(320)
      DIMENSION LA(320)
      INTEGER CP,R,WP
      DOUBLE PRECISION CT
      REAL IC,IP,MCN,ICB,IPD,JEFF
      MBIG=9999999999
```

```
      ND=NNN
      NS=MMM
      N=NS*ND
      NPP=N+NS
      NQ=N+1
      NQQ=N+NS+ND
      NB=NS+ND
      NT=NB+1
      DO 600 I=1,NQQ
  600 LX(I)=0
      DO 731 I=1,NS
  731 LX(N+I)=CP(I)
      DO 732 I=1,ND
  732 LX(NPP+I)=R(I)
      DO 733 I=NT,NQQ
  733 IC(I)=0.
      DO 4526 I=1,NS
      DO 4527 J=1,ND
 4527 IC((I-1)*ND+J)=CT(I,J)
 4526 CONTINUE
      K=0
      DO 3 I=1,NB
      IDA(I)=N+I
      ID(I)=0
      IU(I)=0
    3 IB(I)=0
      DO 4 I=1,NS
      K=N+1
    4 IP(I)=IC(K)
      DO 5 I=1,ND
      K=NPP+I
      J=I+NS
    5 IP(J)=-IC(K)
      ITER=0
      K=0
      IFSG=1
  500 IF(K.EQ.NS) K=0
      IF1=0
      DO 504 KC=1,NS
      K=K+1
      IF(LX(N+K).EQ.0) GO TO 505
      J=(K-1)*ND
      DO 501 L=1,ND
      J=J+1
      IF(LX(NPP+L).EQ.0) GO TO 501
      IF(IF1.EQ.1) GO TO 502
  506 MCN=IC(J)
      IF1=1
      LST=L
      JST=J
      GO TO 501
  502 IF(IC(J).GE.MCN) GO TO 501
      GO TO 506
  501 CONTINUE
      IF(IF1.EQ.0) GO TO 507
      GO TO 503
  505 IF(K.EQ.NS) K=0
  504 CONTINUE
  507 IFSG=2
      K=NS
      GO TO 22
  503 L=LST+NS
      J=JST
      GO TO 55
   22 IF(K.EQ.NS) K=0
      J=ND*K
      IF1=0
      ITER=ITER+1
      DO 190 KC=1,NS
      K=K+1
      DO 191 I=1,ND
      L=NS+I
      J=J+1
      ICB=IC(J)-IP(K)+IP(L)
      IF(ICB.GE.0) GO TO 191
      IF(IF1.EQ.1) GO TO 192
  193 MCN=ICB
      IF1=1
      LST=L
      JST=J
      GO TO 191
  192 IF(ICB.LT.MCN) GO TO 193
  191 CONTINUE
      IF(IF1.EQ.1) GO TO 194
      IF(K.LT.NS) GO TO 190
      K=0
      J=0
  190 CONTINUE
      K=NS
      DO 195 I=1,NS
      J=N+I
      ICB=IC(J)-IP(I)
      IF(ICB.GE.0) GO TO 195
      KQ=I
      GO TO 60
  195 CONTINUE
      GO TO 34
  194 L=LST
      J=JST
   55 CONTINUE
      I=0
      KX=K
    7 I=I+1
      ITL(I)=KX
      KX=ID(KX)
      IF(KX.NE.0) GO TO 7
      KX=L
      M=0
    8 M=M+1
      ITR(M)=KX
      KX=ID(KX)
      IF(KX.NE.0) GO TO 8
      IEP=MBIG
      IF(ITL(I).EQ.ITR(M)) GO TO 9
      I=0
      KX=K
      IB=-1
   10 KY=KX
      KX=ID(KX)
      IB=-IB
      I=I+1
      IF(KX.EQ.0) GO TO 11
      JM=IDA(KY)
      IAL(I)=JM
      IF(IB.LT.0) GO TO 10
      IDT=LX(JM)
```

```
      IF(IDT.GT.IEP) GO TO 10
      IEP=IDT
      IFG=1
      KCN=KX
      LV=JM
      GO TO 10
   11 IAL(I)=N+KY
      IF(KY.GT.NS) GO TO 12
      IDT=LX(N+KY)
      IF(IDT.GT.IEP) GO TO 12
      IEP=IDT
      KCN=KY
      LV=N+KY
      IFG=2
   12 M=0
      KX=L
      IB=-1
   13 KY=KX
      KX=ID(KY)
      IB=-IB
      M=M+1
      IF(KX.EQ.0) GO TO 14
      JM=IDA(KY)
      IAR(M)=JM
      IF(IB.LT.0) GO TO 13
      IDT=LX(JM)
      IF(IDT.GT.IEP) GO TO 13
      IEP=IDT
      IFG=3
      KCN=KX
      LV=JM
      GO TO 13
   14 IAR(M)=N+KY
      IF(KY.LE.NS) GO TO 16
      IDT=LX(N+KY)
      IF(IDT.GT.IEP) GO TO 16
      IEP=IDT
      KCN=KY
      LV=N+KY
      IFG=4
   16 LX(J)=LX(J)+IEP
      IB=1
      DO 17 IG=1,I
      IB=-IB
      JB=IAL(IG)
   17 LX(JB)=LX(JB)+IEP*IB
      IB=1
      DO 18 IG=1,M
      IB=-IB
      JB=IAR(IG)
   18 LX(JB)=LX(JB)+IEP*IB
      JT=J
      IF(IFG.EQ.1.OR.IFG.EQ.2) GO TO 19
      IF(IFG.EQ.3.OR.IFG.EQ.4) GO TO 20
   19 CALL GRFT(ITR(1),ITL(1),KCN,KYZ,JT)
      IM=ITL(1)
      IZ=ITR(1)
      IF(IFG.EQ.1) GO TO 21
      IF(I.EQ.1) GO TO 21
      JX=KCN
      JY=ITL(I-1)
      KU=IU(JY)
```

```
      ID(JX)=JY
      IDA(JX)=JT
      IU(JY)=JX
      IR(JX)=KU
      GO TO 21
   20 CALL GRFT(ITL(1),ITR(1),KCN,KYZ,JT)
      IM=ITR(1)
      IZ=ITL(1)
      IF(IFG.EQ.3) GO TO 21
      IF(M.EQ.1) GO TO 21
      JX=KCN
      JY=ITR(M-1)
      KU=IU(JY)
      ID(JX)=JY
      IDA(JX)=JT
      IU(JY)=JX
      IR(JX)=KU
   21 IF(IM.GT.NS) GO TO 80
      IPD=-IP(IM)+IC(J)+IP(IZ)
      GO TO 81
   80 IPD=-IP(IM)+IP(IZ)-IC(J)
   81 IP(IM)=IP(IM)+IPD
      CALL PIRT(IM,IPD)
      IF(IFSG.EQ.1) GO TO 500
      IF(IFSG.EQ.2) GO TO 22
    9 I1=I
      M1=M
   23 I1=I1-1
      M1=M1-1
      IF(I1.EQ.0.OR.M1.EQ.0) GO TO 40
      IF(ITL(I1).EQ.ITR(M1)) GO TO 23
   40 KJN=ITL(I1+1)
      I=0
      KX=K
      IB=-1
      IF(ID(KX).EQ.0) GO TO 25
   24 KY=KX
      KX=ID(KY)
      IB=-IB
      IF(KY.EQ.KJN) GO TO 25
      I=I+1
      JM=IDA(KY)
      IAL(I)=JM
      IF(IB.LT.0) GO TO 24
      IDT=LX(JM)
      IF(IDT.GT.IEP) GO TO 24
      IEP=IDT
      IFG=1
      KCN=KX
      LV=JM
      GO TO 24
   25 M=0
      KX=L
      IB=-1
      IF(ID(KX).EQ.0) GO TO 27
   26 KY=KX
      KX=ID(KY)
      IB=-IB
      IF(KY.EQ.KJN) GO TO 27
      M=M+1
      JM=IDA(KY)
      IAR(M)=JM
      IF(IB.LT.0) GO TO 26
      IDT=LX(JM)
      IF(IDT.GT.IEP) GO TO 26
      IEP=IDT
      IFG=2
      KCN=KX
      LV=JM
      GO TO 26
   27 LX(J)=LX(J)+IEP
      IB=1
      IF(I.EQ.0) GO TO 42
      DO 28 IG=1,I
      IB=-IB
      JB=IAL(IG)
   28 LX(JB)=LX(JB)+IEP*IB
   42 IB=1
      IF(M.EQ.0) GO TO 43
      DO 29 IG=1,M
      IB=-IB
      JB=IAR(IG)
   29 LX(JB)=LX(JB)+IEP*IB
   43 JT=J
      IF(IFG.EQ.1) GO TO 31
      IF(IFG.EQ.2) GO TO 32
   31 CALL GRFT(ITR(1),ITL(1),KCN,KYZ,J
      IM=ITL(1)
      IZ=ITR(1)
      GO TO 33
   32 CALL GRFT(ITL(1),ITR(1),KCN,KYZ,J
      IM=ITR(1)
      IZ=ITL(1)
   33 IF(IM.GT.NS) GO TO 82
      IPD=-IP(IM)+IC(J)+IP(IZ)
      GO TO 83
   82 IPD=-IP(IM)-IC(J)+IP(IZ)
   83 IP(IM)=IP(IM)+IPD
      CALL PIRT(IM,IPD)
      IF(IFSG.EQ.1) GO TO 500
      IF(IFSG.EQ.2) GO TO 22
   60 I=0
      IEP=MBIG
      KX=KQ
      IB=-1
   61 I=I+1
      KY=KX
      ITL(I)=KY
      KX=ID(KX)
      IB=-IB
      IF(KX.EQ.0) GO TO 62
      JM=IDA(KY)
      IAL(I)=JM
      IF(IB.LT.0) GO TO 61
      IDT=LX(JM)
      IF(IDT.GT.IEP) GO TO 61
      IEP=IDT
      IFG=1
      KCN=KX
      LV=JM
      GO TO 61
```

```
 62 IAL(I)=N+KY
    IF(KY.GT.NS) GO TO 63
    IDT=LX(N+KY)
    IF(IDT.GT.IEP) GO TO 63
    IEP=IDT
    KCN=KY
    LV=N+KY
    IFG=2
 63 LX(J)=LX(J)+IEP
    IB=1
    DO 64 L=1,I
    IB=-IB
    IG=IAL(L)
 64 LX(IG)=LX(IG)+IEP*IB
    KX=KQ
    KD=ID(KX)
    KDA=IDA(KX)
    IDA(KX)=N+KX
    ID(KX)=0
    KR=IR(KX)
    IR(KX)=0
    IF(IU(KD).EQ.KX) GO TO 65
    KB=IU(KD)
 67 IF(IR(KB).EQ.KX) GO TO 66
    KB=IR(KB)
    GO TO 67
 65 IU(KD)=KR
    GO TO 68
 66 IR(KB)=KR
 68 JT=KDA
    IF(KCN.EQ.KD) GO TO 70
    CALL GRFT(KX,KD,KCN,KYZ,JT)
 70 IM=KX
    IPD=-IP(IM)+IC(J)
    IP(IM)=IP(IM)+IPD
    IF(IFG.EQ.1) GO TO 69
    JX=KCN
    JY=ITL(I-1)
    KU=IU(JY)
    ID(JX)=JY
    IDA(JX)=JT
    IU(JY)=JX
    IR(JX)=KU
 69 CALL PIRT(IM,IPD)
    IF(IFSG.EQ.1) GO TO 500
    IF(IFSG.EQ.2) GO TO 22
 34 CONTINUE
    JEFF=0
    DO 150 I=1,NQQ
150 JEFF=JEFF+LX(I)*IC(I)
    DO 734 I=1,NS
    DO 735 J=1,ND
735 WP(I,J)=LX((I-1)*ND+J)
734 CONTINUE
    RETURN
    END
    SUBROUTINE GRFT(N,IS,NC,K,JT)
    COMMON /A/ ID(320),IU(320),IR(320),IP(320),IC(1000) ,NS,ND
   1,IDA(320)
    REAL IC,IP,MCN,ICB,IPD,JEFF
    K=1
```

```
      L=N
      M=IS
      MR=IP(M)
      LU=IU(L)
      KD=ID(M)
      KDA=IDA(M)
      IDA(M)=JT
      ID(M)=L
      IU(L)=M
      IR(M)=LU
      IF(M.EQ.NC) GO TO 7
    1 L=M
      M=KD
      IF(M.EQ.0) GO TO 7
      IF(IU(M).EQ.L) GO TO 2
      MZ=IU(M)
    3 IF(IR(MZ).EQ.L) GO TO 4
      MZ=IR(MZ)
      GO TO 3
    2 IU(M)=MR
      GO TO 5
    4 IR(MZ)=MR
    5 IF(M.EQ.NC) GO TO 7
      MR=IR(M)
      KD=ID(M)
      JT=IDA(M)
      IDA(M)=KDA
      KDA=JT
      ID(M)=L
      IZ=IU(L)
      IU(L)=M
      IR(M)=IZ
      K=K+1
      GO TO 1
    7 JT=KDA
      RETURN
      END
      SUBROUTINE PIRT(I,IPD)
      COMMON /A/ ID(320),IU(320),IR(320),IP(320),IC(1000) ,NS,ND
     1,IDA(320)
      REAL IC,IP,MCN,ICB,IPD,JEFF
      J=I
      IF(IU(J).EQ.0) GO TO 1
    2 L=IU(J)
   10 IP(L)=IP(L)+IPD
    4 IF(IU(L).EQ.0) GO TO 7
      J=L
      GO TO 2
    7 IF(IR(L).EQ.0) GO TO 8
      L=IR(L)
      GO TO 10
    8 IF(J.EQ.I) GO TO 1
      IF(IR(J).EQ.0) GO TO 9
      L=IR(J)
      J=ID(J)
      GO TO 10
    9 J=ID(J)
      GO TO 8
    1 RETURN
      END
```

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,
     *TAPE28,TAPE29)
      DIMENSION PARAM(3),ASUM(500),DD(30),EE(50),FF(30,30),
     *CT(30,30),CP(30),R(50),WP(30,30),COST(30,30)
      DIMENSION U(30,30)
      INTEGER CT,U,COST,CP,R,WP
413   CONTINUE
      WRITE(6,500)
500   FORMAT(30H ENTER IMAXT,MZM,M,N,KWRI,LMKO)
      READ(5,*)IMAXT,MZM,M,N,KWRI,LMKO
      WRITE(6,501)
501   FORMAT(29H ENTER ALPHA,BETA,GAMMA,DELTA)
      READ(5,*)(PARAM(I),I=1,3),ADELTA
      WRITE(6,502)
502   FORMAT(36H ENTER INDEX,IFLAG,ISEARC,KFLAG,DEPS)
      READ(5,*)INDEX,IFLAG,ISEARC,KFLAG,DEPS
      DELTA=ADELTA
      IFN=0
      KINDEX=INDEX
      AMIN=9999999999999.
      EPS=10
      AAAA=PARAM(1)
      WRITE(6,9333)
9333   FORMAT(10H ITERATION,10H     ALPHA,10H      BETA,
     *10H     GAMMA,15H     SUM OF SQRS/)
      BBBB=PARAM(2)
      CCCC=PARAM(3)
      ITER=1
98    AAA=PARAM(1)
      BBB=PARAM(2)
      CCC=PARAM(3)
   99 AA=PARAM(1)
      BB=PARAM(2)
      CC=PARAM(3)
      REWIND 28
      REWIND 29
      ITIME=1
      SUM=0.
103   READ(28,*)(CP(I),I=1,N)
      READ(28,*)(R(J),J=1,M)
      READ(28,*)((COST(I,J),J=1,N),I=1,M)
      READ(29,*)(DD(I),I=1,M)
      READ(29,*)(EE(J),J=1,M)
      READ(29,*)((FF(I,J),J=1,N),I=1,M)
      NKL=MZM/2
481   DO 447 I=1,MZM
      DO 447 J=1,MZM
      U(I,J)=0
447   CT(I,J)=99999
      DO 10011 I=2,NKL
      CT(1,I)=0
      CT(NKL-1+I,MZM)=0
      U(1,I)=CP(I-1)
      U(NKL-1+I,MZM)=R(I-1)
10011 CONTINUE
      NKLL=MZM-1
      NKLLL=NKL+1
      DO 445 I=2,NKL
      DO 445 J=NKLLL,NKLL
```

```
      U(I,J)=999
      CT(I,J)=AA*FF(J-NKL)/EE(I-1)+BB*FF(I-1,J-NKL)**2
     *+CC*(FF(I-1,J-NKL)**2)*EE(J-NKL)/EE(I-1)
      IF((I-1).EQ.(J-NKL))CT(I,J)=0
445   CONTINUE
      U(1,MZM)=0
      CT(1,MZM)=99999
489   IF(IFN.EQ.1)GO TO 407
10012 FORMAT(14I10)
      CALL AOOK(CT,U,WP,MZM,KWRI,LMKO)
482   CONTINUE
93    FORMAT(3F10.4)
40004 FORMAT(14I4)
705   FORMAT(12H FLOW MATRIX//)
779   FORMAT(3X,5I5)
      DO 2 I=1,N
      DO 2 J=1,M
C     IF(COST(I,J).EQ.0)GO TO 2
      SUM=SUM+(WP(I+1,J+NKL)-COST(I,J))**2
2     CONTINUE
483   CONTINUE
  377 CONTINUE
      ITIME=ITIME+1
      IF(ITIME.LE.IMAXT)GO TO 103
      ASUM(ITER)=SUM
      WRITE(6,933)ITER,(PARAM(I),I=1,3),SUM
933   FORMAT(I10,3F10.4,F15.0)
73    FORMAT(15H SUM OF SQUARE=,F16.7)
      IF(ASUM(ITER).LT.AMIN)GO TO 104
      IF(DELTA.LE.DEPS)GO TO 744
      IF(KFLAG.NE.2)GO TO 730
745   IF(IFLAG.EQ.2)GO TO 731
      DELTA=DELTA/2.
      PARAM(KINDEX)=PARAM(KINDEX)-DELTA
      IFLAG=1
      ITER=ITER+1
      KFLAG=1
      GO TO 99
731   DELTA=DELTA/2.
      PARAM(KINDEX)=PARAM(KINDEX)+DELTA
      ITER=ITER+1
      IFLAG=2
      KFLAG=1
      GO TO 99
 730  KFLAG=1
      GO TO 7748
744   IF(ISEARC.EQ.1)GO TO 1288
      KINDEX=KINDEX+1
      PARAM(KINDEX-1)=AAAA
      IF((KINDEX-1).EQ.2)PARAM(KINDEX-1)=BBBB
      IF((KINDEX-1).EQ.3)PARAM(KINDEX-1)=CCCC
      IF(KINDEX.EQ.4)GO TO 4444
      ISEARC=1
      DELTA=ADELTA
      KFLAG=2
      INDEX=KINDEX
      ITER=ITER+1
      IFLAG=1
      PARAM(KINDEX)=PARAM(KINDEX)+DELTA
      GO TO 99
```

```
7748  IF(ISEARC.EQ.2)GO TO 7745
      GO TO 745
7745   CONTINUE
      DELTA=DELTA/2.
      PARAM(KINDEX)=PARAM(KINDEX)+DELTA
      ITER=ITER+1
      IFLAG=2
      KFLAG=1
      GO TO 99
104   AMIN=ASUM(ITER)
      IF(KINDEX.EQ.1)GO TO 8768
      IF(KINDEX.EQ.2)GO TO 8868
      CCCC=PARAM(KINDEX)
      GO TO 8869
8768  AAAA=PARAM(KINDEX)
      GO TO 8869
8868   BBBB=PARAM(KINDEX)
8869   CONTINUE
      ISEARC=1
      DELTA=ADELTA
      ITER=ITER+1
      PARAM(KINDEX)=PARAM(KINDEX)+DELTA
      IFLAG=1
      KFLAG=2
      GO TO 99
1288   IF(KFLAG.EQ.1.AND.IFLAG.EQ.1)GO TO 1289
      IF(KINDEX.EQ.2)GO TO 1291
      IF(KINDEX.EQ.1)GO TO 1290
      PARAM(KINDEX)=CCCC
      GO TO 1292
1290  PARAM(KINDEX)=AAAA
      GO TO 1292
1291  PARAM(KINDEX)=BBBB
1292   DELTA=ADELTA
      PARAM(KINDEX)=PARAM(KINDEX)+DELTA
      KFLAG=2
      IFLAG=1
      ITER=ITER+1
      ISEARC=2
      GO TO 99
1289  IF(KINDEX.EQ.1)GO TO 1294
      IF(KINDEX.EQ.2)GO TO 1295
      PARAM(KINDEX)=CCCC
      GO TO 1296
1294  PARAM(KINDEX)=AAAA
      GO TO 1296
1295  PARAM(KINDEX)=BBBB
1296  DELTA=ADELTA
      PARAM(KINDEX)=PARAM(KINDEX)-DELTA
      KFLAG=2
      IFLAG=2
      ITER=ITER+1
      ISEARC=2
      GO TO 99
746   PARAM(KINDEX)=PARAM(KINDEX)-DELTA
      IFLAG=2
      KFLAG=2
      ISEARC=2
      GO TO 99
5555  INDEX=1
```

```
      DELTA=ADELTA
      AAA=PARAM(1)
      BBB=PARAM(2)
      CCC=PARAM(3)
      PARAM(1)=PARAM(1)+DELTA
      KINDEX=1
      ITER=ITER+1
      IFLAG=1
      KFLAG=2
      ISEARC=1
      GO TO 99
4444  CONTINUE
      IF(ABS(PARAM(1)-AAA).GT..000001)GO TO 5555
      IF(ABS(PARAM(2)-BBB).GT..000001)GO TO 5555
      IF(ABS(PARAM(3)-CCC).GT..000001)GO TO 5555
3333   CONTINUE
55    FORMAT(4H END)
404   FORMAT(14I4)
402   FORMAT(10X,18H ACTUAL FLOW MATRI/)
      AA=AAA
      BB=BBB
      CC=CCC
      WRITE(6,401)AAA,BBB,CCC,AMIN
401   FORMAT(10X,16HOPTIMAL SOLUTION/7H ALPHA=,F10.7/
     *6H BETA=,F10.7/7H GAMMA=,F10.7/12H SUM OF SQR=,F10.0//)
      WRITE(6,406)
406   FORMAT(10X,11HCOST MATRIX/)
      REWIND 28
      REWIND 29
      READ(28,*)(CP(I),I=1,N)
      READ(28,*)(R(J),J=1,N)
      READ(29,*)(DD(I),I=1,M)
      READ(29,*)(EE(J),J=1,N)
      READ(29,*)((FF(I,J),J=1,M),I=1,N)
      IFN=1
      GO TO 481
407   CONTINUE
      DO 409 I=2,15
      WRITE(6,*)(CT(I,J),J=16,29)
409   CONTINUE
      CALL AOOK(CT,U,WP,MZM,KWRI,LMKO)
400   FORMAT(14F4.0)
      WRITE(6,403)
403   FORMAT(/8X,20HCOMPUTED FLOW MATRIX/)
      WRITE(6,404)((WP(I,J),J=NKLLL,NKLL),I=2,NKL)
      IFN=0
      WRITE(6,508)
508   FORMAT(35H DO YOU NEED ANOTHER RUN 1 YES 0 NO)
      READ(48,*)MORER
      IF(MORER.NE.0)GO TO 413
      STOP
      END
      SUBROUTINE AOOK(IDIST,U,X,N,ISOUR,ISINK)
      DIMENSION IW(30,2)
      DIMENSION U(30,30),X(30,30),IDIST(30,30),DUMMY(30,30)
      INTEGER U,X,DUMMY
      ICOUNT=0
      ISUM=0
      DO 2 I=1,N
2     ISUM=ISUM+U(1,I)
6876  FORMAT(5X,I5)
```

```
5       DO 1 I=1,N
        DO 1 J=1,N
        DUMMY(I,J)=IDIST(I,J)
1       X(I,J)=0
44      CONTINUE
        CALL SPATH(IDIST,IW,N)
        CALL COSTT(ISINK,ISOUR,U,N,DUMMY,X,IW,IDIST)
        ICOUNT=ICOUNT+1
        IF(ICOUNT.EQ.ISUM)GO TO 6
        GO TO 44
6       SUM=0.
        DO 7 I=1,N
        DO 7 J=1,N
7       SUM=SUM+DUMMY(I,J)*X(I,J)**2
        RETURN
        END
        SUBROUTINE SPATH(IDIST,IW,M)
        DIMENSION IDIST(30,30),IW(30,2)
        IW(1,1)=0
        IW(1,2)=99999
        DO 8 I=2,M
        IW(I,2)=1
      8 IW(I,1)=99999
7       DO 1 I=1,M
        DO 2 J=1,M
        IF(IDIST(I,J).EQ.99999)GO TO 2
        IF((IW(J,1)-IW(I,1))-IDIST(I,J))2,2,6
6       IW(J,1)=IW(I,1)+IDIST(I,J)
        IW(J,2)=I
        GO TO 7
2       CONTINUE
1       CONTINUE
        RETURN
        END
        SUBROUTINE COSTT(ISINK,ISOUR,U,N,DUMMY,X,IW,IDIST)
        DIMENSION DUMMY(30,30),IW(30,2)
        DIMENSION U(30,30),X(30,30),IDIST(30,30)
        INTEGER U,X,DUMMY
        KKK=ISINK
      3 JNODE=IW(KKK,2)
        IF(JNODE.LT.KKK)GO TO 99
        X(KKK,JNODE)=X(KKK,JNODE)-1
        U(JNODE,KKK)=U(JNODE,KKK)-1
        U(KKK,JNODE)=U(KKK,JNODE)+1
        IDIST(KKK,JNODE)=DUMMY(KKK,JNODE)*((X(KKK,JNODE)+1)**2
       *-X(KKK,JNODE)**2)
        IDIST(JNODE,KKK)=DUMMY(KKK,JNODE)*((X(KKK,JNODE)-1)**2
       *-X(KKK,JNODE)**2)
        IF(U(JNODE,KKK).EQ.0)IDIST(JNODE,KKK)=99999
        GO TO 100
99      U(JNODE,KKK)=U(JNODE,KKK)-1
        X(JNODE,KKK)=X(JNODE,KKK)+1
        U(KKK,JNODE)=U(KKK,JNODE)+1
        IDIST(KKK,JNODE)=DUMMY(JNODE,KKK)*((X(JNODE,KKK)-1)**2
       *-X(JNODE,KKK)**2)
        IDIST(JNODE,KKK)=DUMMY(JNODE,KKK)*((X(JNODE,KKK)+1)**2
       *-X(JNODE,KKK)**2)
        IF(U(JNODE,KKK).EQ.0)IDIST(JNODE,KKK)=99999
100     IF(JNODE.EQ.ISOUR)GO TO 6
        KKK=JNODE
        GO TO 3
6       CONTINUE
676     FORMAT(3H OK)
        RETURN
        END
/
```

TECHNICAL NOTE #17

Multivariate Time Series Model for Robbery

Introduction

In previous technical notes we have described and built univariate time series models for each index crime in ten major cities [1]. These models were then used to predict future occurrences of an index crime for a city of interest. The univariate nature of these models necessitates that forecasts of an index crime for a city be a function of previous observations of the crime within that city. In other words, the information used in this univariate forecasting system comes entirely from the series itself. For instance, the possible information that burglary in Los Angeles might provide towards describing burglary in Portland is not considered here.

This univariate forecasting system then, behaves as if crime in each city acts independently of crime in every other city. On an individual city level, where interest in crime intensities is focused entirely on the situation within the city, this univariate forecasting system is quite appropriate. If more information is desired by a city on the nature of its crime intensity relative to other comparable cities, the normative forecasting scheme described in [2] can be used.

On the national level, however, there is interest in a forecasting system that recognizes the interdependencies between crimes in various cities in the nation. It seems quite probable that relationships do exist between the level of an index crime in a given city and the level of that crime in other cities. Not only is the specific nature of these relationships of importance to national decision makers, but also the resulting forecasting system will provide better forecasts since the model makes use of any between cities multivariate information.

The purpose of this technical note is to describe a multivariate forecasting system. First we will explain the specific nature of interrelationships between time series. With that as a basis we will introduce the model specification and the model building procedure for the triangular two-sided moving average multivariate time series model. As an example, a multivariate forecasting system for robbery is constructed for the following cities: St. Louis, Portland, Los Angeles, Kansas City, Atlanta, Boston, and Denver. The forecasts from this multivariate forecasting system is then compared to the forecasts from the univariate models for robbery in the same cities.

Structure of the "Forecasting System"

Before we present the mechanics of the multivariate time series modeling procedure, it will be instructive to elaborate on the nature of the possible relationships between two (or more) time series. Consider two distinct time series $Y_1(T)$ and $Y_2(T)$, where T runs from 1 to N. The only restrictions we must place upon these two series are that they share the same time axis, i.e., $Y_1(T_1)$ and $Y_2(T_2)$ are observed simultaneously whenever $T_1 = T_2$ and that they represent deviations from appropriate mean values. Furthermore, assume we are interested in modeling and subsequently forecasting the $Y_2(T)$ series.

Fig. 1 represents the univariate forecasting system. Here we have used the model form expressing an observation as a linear combination of all previous random errors.

$$Y(T) = \psi(B)\ A(T)$$

where

$$\psi(B) = 1 + \psi_1 B + \psi_2 B^2 + \ . \ . \ .$$

and A(T) is white noise. Now suppose that there is interdependency between Y_1 and Y_2, that is, knowledge of the $Y_1(T)$ series can help us predict or explain the Y_2 series. Figure 2 shows a bivariate system where $Y_1(T)$ is a linear function of $Y_2(T)$.

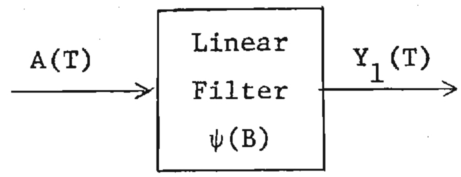

Figure 1.

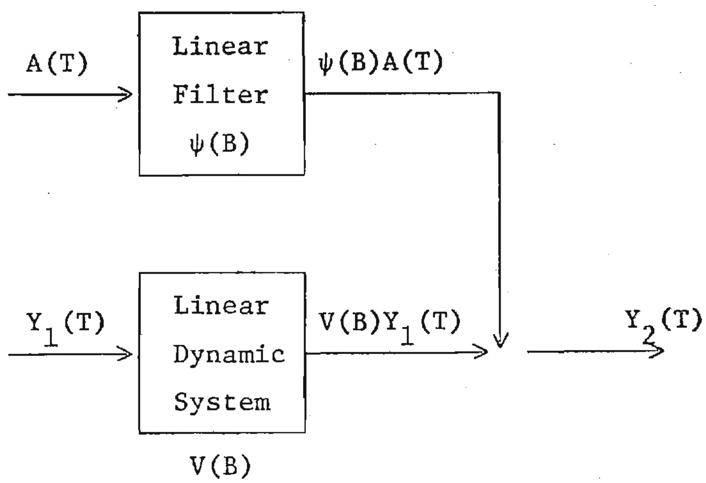

Figure 2.

$$Y_2(T) = V(B)\ Y_1(T) + \psi(B)\ A(T)$$

where

$$V(B) = \nu_0 + \nu_1 B + \nu_2 B^2 + \ldots$$

More simply in this model, $Y_1(T)$ can be expressed as a linear combination of past pure error terms plus a linear combination of past and concurrent observations of the $Y_1(T)$ series.

We call V(B) a transfer function because it describes how the input series, $Y_1(T)$, is transfered into a component of the output, $Y_2(T)$ series. Note that as a special case if $\nu_0 = \nu_1 = \nu_2 = ,\ldots = 0$, then this model collapses to the model of Figure 1, the general univariate time series model. Note also that in predicting the Y_2 series, only past observation of $Y_1(T)$ can be employed. Hence for $Y_1(T)$ to be useful in modeling $Y_2(T)$, observations of the Y_1 series must be related to future observations of the Y_2 series. We say that Y_1 must be a leading indicator of the Y_2 series. If the relationship extended the other way, that is, if observations of the Y_1 series were related to past observations of the Y_2 series, we would say that Y_1 lags Y_2. In this case Y_1 would not be useful in predicting Y_2. However if Y_1 lags Y_2, then Y_2 necessarily leads Y_1 and Y_2 could be used to model Y_1.

Suppose that instead of centering our interest on one particular series, we have "n" series of concern, and we wish to develop a forecasting system for each and every one of them. This brings us to the question of modeling "n" series simultaneously. Figure 3 shows the natural extension of our transfer function model to "n" series. This type of model could be built "n" times, each time changing the position of the output series and an input series. This approach is a natural one but not very efficient. A much better approach is represented in Figure 4. This model form simultaneously explains all interrelationships between variables and produces forecasts for each of the "n" series. The specifics of this model will be given in the next sections.

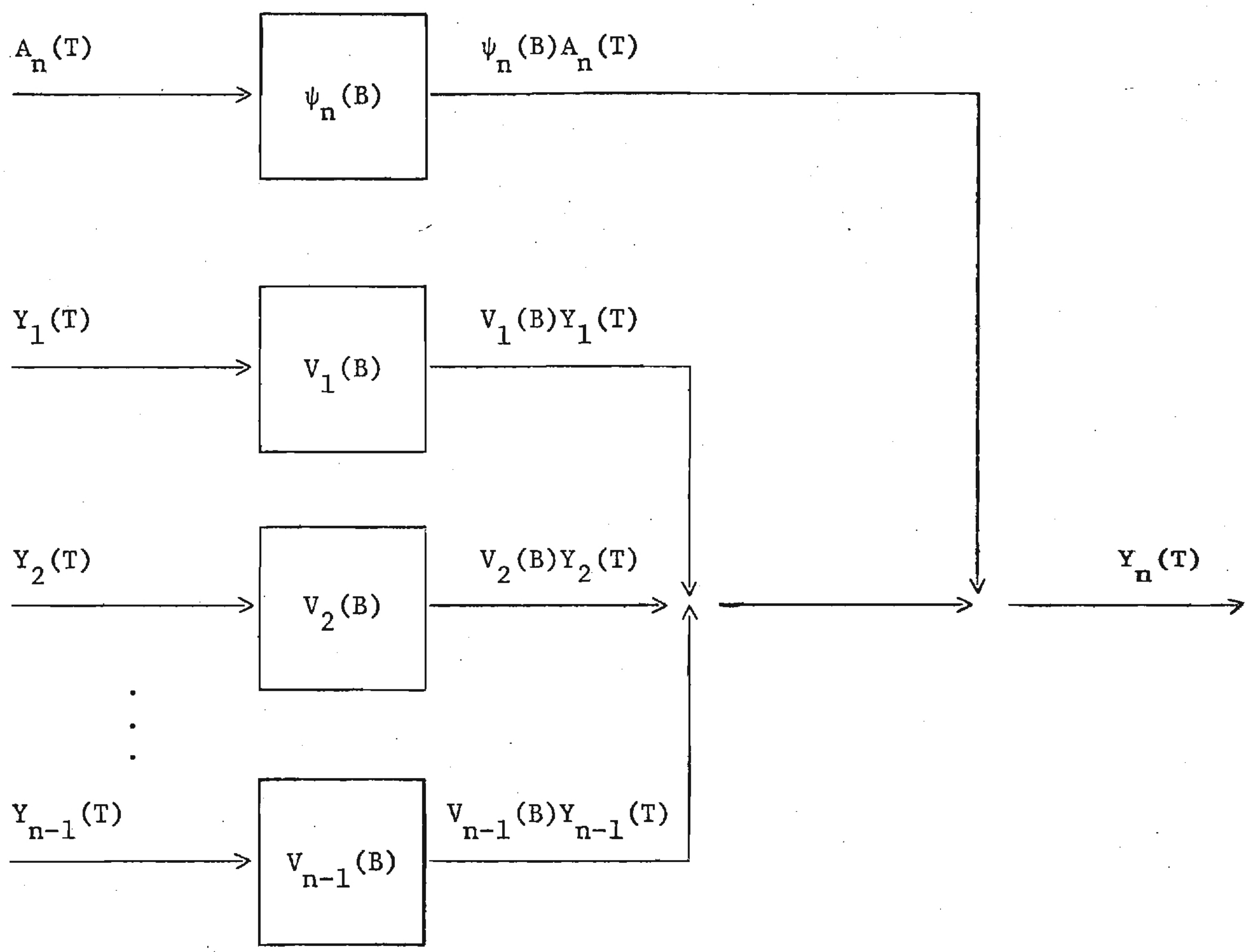
$A_n(T)$
$\psi_n(B)$
$\psi_n(B)A_n(T)$
$Y_1(T)$
$V_1(B)$
$V_1(B)Y_1(T)$
$Y_2(T)$
$V_2(B)$
$V_2(B)Y_2(T)$
$Y_n(T)$
$Y_{n-1}(T)$
$V_{n-1}(B)$
$V_{n-1}(B)Y_{n-1}(T)$

Figure 3.

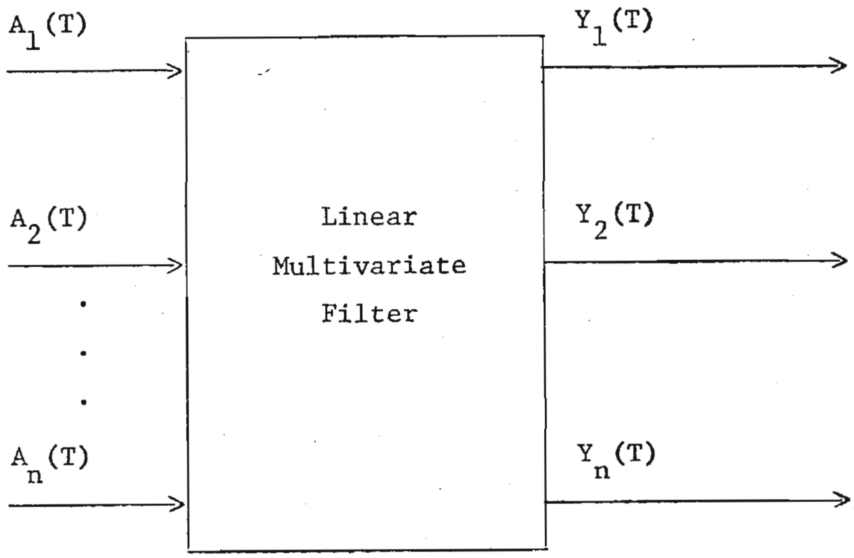

Figure 4.

<u>Specification of Interrelationships Between Time Series - Cross Correlation Function</u>

Let us return to the model of Figure 2, $Y_2(T)$ modeled as a function of $Y_1(T)$. As was mentioned before, this is a useful model if and only if Y_1 is a leading indicator of $Y_2(T)$. The cross-correlation function between $Y_1(T)$ and $Y_2(T)$ is used to determine the nature of the relationship of Y_1 to Y_2 and furthermore to specify the form of the impulse response function, V(B).

The cross-covariance function between Y_1 and Y_2 at lag L is defined as

$$\gamma_{Y_1Y_2}(L) = E\left\{(Y_1(T) - EY_1) \cdot (Y_2(T+L) - EY_2)\right\}$$

or since $EY_1 = EY_2 = 0$

$$\gamma_{Y_1Y_2}(L) = E\Big(Y_1(T) \cdot Y_2(T+L)\Big)$$

The sample cross-covariance function is usually calculated as

$$\hat{\gamma}_{Y_1Y_2}(L) = \frac{\sum_{t=1}^{N-L} (Y_1(t) - \bar{Y}_1)\ (Y_2(t+L) - \bar{Y}_2)}{(N-L)}$$

Now what is the connection between the cross-covariance function and the form of V(B). Recall,

$$Y_2(T) = (\nu_0 + \nu_1 B + \nu_2 B^2 + ,\ldots)\ Y_1(T) + \psi(B)\ A(T)$$

multiplying both sides by $Y_1(T-K)$ we get

$$Y_1(T-K)\ Y_2(T) = \nu_0\ Y_1\ (T-K)\ Y_1(T) + \nu_1 Y_1(T-K)\ Y_1(T-1)$$

$$+,\ldots\ldots\ldots\ldots\ldots + \psi(B) Y_1(T-K) A(T)$$

Now if we further assume $Y_1(T-K)$ is uncorrelated with A(T) for all K we get upon taking expected values,

$$\gamma_{Y_1Y_2}(K) = \nu_0\ \gamma_{Y_1Y_1}(K) + \nu_1\gamma_{Y_1Y_1}(K-1) + \ldots. + \nu_{K-1}\ \gamma_{Y_1Y_1}(1)$$

$$+ \nu_K\gamma_{Y_1Y_1}(0) + \nu_{K+1}\gamma_{Y_1Y_1}(1) + \ldots$$

Now if the Y_1 series happens to be white noise then,

$$\gamma_{Y_1Y_1}(K) = 0$$

$$\text{for } K \neq 0$$

and thus,

$$\gamma_{Y_1Y_2}(K) = \nu_K \gamma_{Y_1Y_1}(0)$$

or

$$\nu_K = \frac{\gamma_{Y_1Y_2}(K)}{\gamma_{Y_1Y_1}(0)} = \frac{\gamma_{Y_1Y_2}(K)}{\sigma^2_{Y_1}}$$

Now defining the auto-correlation of Y_1 to Y_2 at lag L as

$$\rho_{Y_1Y_2}(K) = \frac{\gamma_{Y_1Y_2}(L)}{\sigma_{Y_1}\sigma_{Y_2}}$$

we get

$$\nu_K = \frac{\sigma_{Y_2}}{\sigma_{Y_1}} \rho_{Y_1Y_2}(K) .$$

Hence the sample cross-correlation function directly measures the relationships between 2 series. If $\rho_{Y_1Y_2}(L)$ is large in magnitude for some positive L this implies Y_1 leads Y_2 at lag L, that is, $Y_1(T)$ can help predict $Y_2(T+L)$. Alternatively if $\rho_{Y_1Y_2}(L)$ is large in magnitude for an L less than zero then Y_1 lags Y_2 and is of no use in predicting Y_2, but consequently Y_2 leads Y_1 and Y_2 should be used in a model for Y_1. On the other hand, if $\rho_{Y_1Y_1}(L)$ is zero for all L (or small in a statistical sense) then ν_K is zero for all K and series Y_1 is unrelated to Y_2. In this case Y_1 and Y_2 are said to be independent and a model for Y_2 will not contain terms involving Y_1.

To illustrate the use of the sample cross-correlation function to quantify the relationships between 2 series, consider $Z_1(T)$, Robbery in Dallas and $Z_2(T)$, Robbery in St. Louis. Because these series are known to be nonstationary, the transformations $Y_1(T) = (1-B)(1-B^{12})\ Z_1(T)$ and $Y_2(T) = (1-B)(1-B^{12})\ Z_2(T)$ were undertaken. The sample cross-correlation function between $Y_1(T)$ and $Y_2(T)$ is given in Table 1.

From Table 1 we can conclude that Robbery in Dallas is a leading indicator of Robbery in St. Louis, and the lag is approximatly 6 months. We may now safely say that a model for forecasting robbery in St. Louis that makes use of the information between the Dallas and St. Louis Series will provide better forecasts than a univariate model of St. Louis alone.

Methodology of the TTSMA Model

The triangular two-sided moving average (TTSMA) model is designed to facilitate the modeling of "n" separate but statistically linked time series. If one were to attempt to specify the model form of Figure 3 directly for all "n" series, one would find the task complicated and lengthy. The TTSMA model provides the following advantages:

1. It facilitates model specification by concentrating on one auto-correlation or one cross-correlation function at a time.
2. Efficient parameter estimates can be obtained by successive least squares estimation.
3. Diagnostic checking is simple and model inadequacies lead to a usually better updated model.
4. The TTSMA model is incremental, that is, adding another time series to the model does not change the model form of those already in the model.

The disadvantage of the TTSMA model is that it can not be directly used to provide forecasts. The model must first be transformed to canonical form before forecasts can be derived.

L	$\hat{\rho}_{Y_1Y_2}(L)$	$\frac{\hat{\rho}_{Y_1Y_2}(L)}{\hat{\sigma}_{\hat{\rho}}}$	L	$\hat{\rho}_{Y_1Y_2}(L)$	$\frac{\hat{\rho}_{Y_1Y_2}(L)}{\hat{\sigma}_{\hat{\rho}}}$
-15	.074	.272	1	-.078	.288
-14	-.127	-.497	2	.068	.245
-13	.179	.695	3	-.152	-.579
-12	-.183	-.703	4	.273	1.283
-11	.087	.346	5	-.445	-3.246
-10	.012	.049	6	.546	4.881
- 9	-.010	-.041	7	-.502	-3.867
- 8	-.057	-.262	8	.271	1.963
- 7	.146	.868	9	-.057	-.262
- 6	-.111	-.685	10	-.146	-.702
- 5	.066	.339	11	.358	2.045
- 4	-.008	-.039	12	-.169	-1.595
- 3	-.056	-.281	13	-.064	-.333
- 2	.128	.620	14	.051	+.185
- 1	-.225	-.940	15	.049	.162
0	.190	.729			

Table 1. Sample Cross-correlation Between $Y_1(T)$ and $Y_2(T)$

The general form of the TTSMA model is as follows:

$$\begin{bmatrix} Y_1(T) \\ Y_2(T) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ Y_n(T) \end{bmatrix} = \begin{bmatrix} \psi_{11}(B) & 0 & \cdots & 0 \\ \psi_{21}(B) & \psi_{22}(B) & 0 \cdots & 0 \\ & & & \vdots \\ & & & 0 \\ \psi_{n1}(B) & \psi_{n2}(B) & \cdots & \psi_{nn}(B) \end{bmatrix} \begin{bmatrix} e_1(T) \\ e_2(T) \\ e_3(T) \\ \cdot \\ \cdot \\ e_n(T) \end{bmatrix}$$

where : $Y_i(T)$ is a stationary time series with mean zero

$$\psi_{ij}(B) = \ldots \psi_{ij}(-2)B^{-2} + \psi_{ij}(-1)B^{-1} + \psi_{ij}(0) + \psi_{ij}(1)B^{1} + \ldots$$

$$|\psi_{ij}(K)| < 1$$

and $e_i(T)$ is uncorrelated white noise with $e_i(T)$ and $e_j(T)$ being uncross-correlated at all lags. Thus, triangular refers to the lower triangular form of the ψ matrix. Two-sided refers to the form of $\psi_{ij}(B)$, which includes backshift operators with both positive and negative exponents. Here negative lags imply a shift forward in time, and hence a model in this form is not directly usable for prediction. Moving average refers to the general moving average form taken by the model. Because the model only allows for moving average representations, an autoregressive type system must be approximated by a finite number of terms in a TTSMA model.

The specific steps involved in constructing a TTSMA model involves the three interactive phases; model specification, parameter estimation and diagnostic checking, that are used in constructing a Box-Jenkins univariate time series model. The key to the TTSMA procedure however, is that these three phases are undertaken <u>each</u> time a time series is added to the model. So for an "n" time series model, the 3 phase procedure will be inacted "n" times. Figure 5 shows a flow diagram for the general TTSMA procedure.

A second important aspect of the TTSMA procedure is that at any stage of the

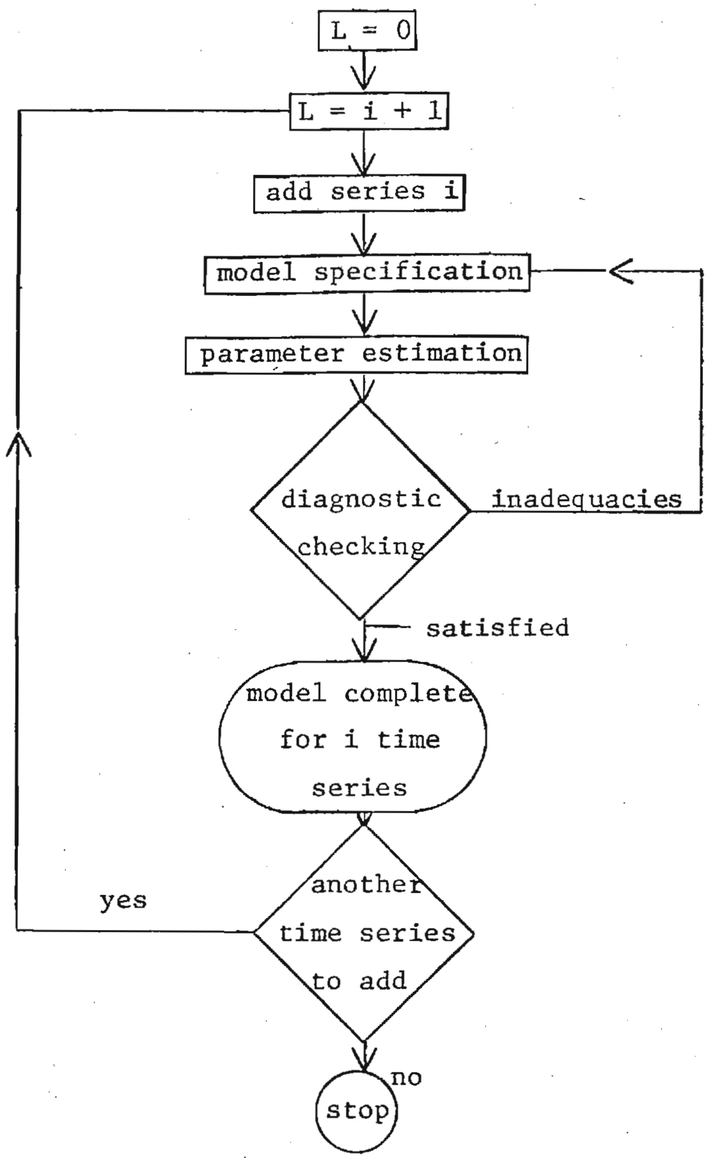

Figure 5. Flow Diagram for TTSMA Modeling

modeling procedure, i.e., after any time series has been successfully added to the model, the TTSMA model can stand by itself. For example a TTSMA model for 3 time series also necessarily includes a model for 2 time series, which in turn includes a univariate time series model. As we shall see when the specifics of the methodology are given, the order of entering the time series determines what specific models are included in the total "n" series model.

Step-by-Step Procedure for Building a TTSMA Model

The flow diagram of Figure 5 gives only the most basic structure of the TTSMA modeling procedure. To illustrate the details of this model-building methodology, the step-by-step procedure for adding the first and $I^{\underline{th}}$ series is presented.

Adding the First Series to the Model

From the ψ matrix previously described, we see that the model for the first time series is

$$Y_1(T) = \psi_{11}(B)\ e_1(T) .$$

In this case there is no need for $\psi_{11}(B)$ to contain terms with negative lags and $\psi_{11}(0) \equiv 1$, exactly parrelleling a general univariate time series model. We have

$$Y_1(T) = e(T) + \psi_{11}(1)\ e(T-1) + \psi_{11}(2)\ e(T-2) + \ldots$$

Model Specification

From the Box-Jenkins univariate modeling approach the specification of a moving-average model involves examining the sample auto-correlation function and adding terms to the model to explain each significant auto-correlation. It can be shown that there is a one to one relationship between terms in a moving average model and the resulting auto-correlation function. If all sample auto-correlations are insignificant, we set all $\psi_{11}(L) = 0$ $L = 1,2,...$ and our model becomes;

$$Y_1(T) = e_1(T),$$

the model for white noise. If one or more auto-correlations are significant our model is of the form,

$$Y_1(T) = e_1(T) + \psi_{11}(L_1)\ e(T-L_1) + \psi_{11}(L_2)\ e(T-L_2),$$

in order to explain two significant auto-correlations at lags L_1 and L_2 respectively.

Parameter Estimation

The maximum likelihood estimates for the parameters included in $\psi_{11}(B)$ are those parameter values that minimize

$$\sum_{T=1}^{N} \hat{e}_1(T)^2.$$

Where the $\hat{e}(T)$ series is calculated recursively as

$$\hat{e}_1(T) = Y_1(T) - \sum_{L=1}^{\infty} \psi_{11}(L)\ \hat{e}_1(T-L),\ T = 1,2,...N.$$

It is understood that if a parameter is not included in the model, then it is defined to be zero. For an MA(2) model then,

$$\hat{e}_1(T) = Y_1(T) - \psi_{11}(1)\ \hat{e}_1(T-1)$$
$$- \psi_{11}(2)\ \hat{e}_1(T-2)$$

and $\hat{\psi}_{11}(1)$, $\hat{\psi}_{11}(2)$ are the values that minimize

$$\sum_{T=1}^{N} e_1(T)^2.$$

Since the estimation of the parameters in a moving average model is a non-linear estimation problem, there is no closed form expression for $\hat{\psi}_{ij}(L)$.

Diagnostic Checking

For the TTSMA model for the first series, the only assumption one must investigate is that the $e_1(T)$ series is a realization of an idealized white noise series. To check this, the sample auto-correlation function of the $\hat{e}_1(T)$ series is used in order to test the hypothesis that all

$$\rho_{\hat{e}_1\hat{e}_1}(L) = 0 \quad L = 1,2, \ldots$$

If there exists a significant auto-correlation at any lag, a term for that lag is added to the model and we reestimate the parameters. This process continues until all sample auto-correlations can be considered to be zero, while at the same time each parameter added to the model must significantly reduce the residual sum of squares.

Once we have finished the three phase procedure for the first series in our TTSMA model, we are left with a completed univariate time series model. The model form and parameter values of $\psi_{11}(B)$ are specified and will not change as we add more series to the model. Also the $e_1(T)$ stream is estimated by the $\hat{e}_1(T)$ series and it also is determined for the remainder of the total model building scheme.

Adding the $I^{\underline{th}}$ Time Series

Now we shall describe the procedures involved in adding time series I to a TTSMA model already constructed for the first I-1 series. Because I-1 series are in the model we have $\hat{\psi}_{11}(B)$, $\hat{\psi}_{21}(B)$, $\hat{\psi}_{22}(B)$, . . . $\hat{\psi}_{I-1,1}(B)$, $\hat{\psi}_{I-1,2}(B)$, . . . $\hat{\psi}_{I-1,I-1}(B)$, and $\hat{e}_1(T)$, $\hat{e}_2(T)$, . . . $\hat{e}_{I-1}(T)$ resulting from an I-1 dimensional TTSMA model. The TTSMA assumptions and the modeling procedures ensure that the parameters already included in the model are statistically significant and that $\hat{e}_1(T)$, . . . $\hat{e}_{I-1}(T)$ are realizations of uncross-correlated white noise series.

Model Specification

From the ψ matrix previously described, we see the model for $Y_I(T)$ is

$$Y_I(T) = \psi_{I1}(B)\ e_1(T) + \psi_{I2}(B)\ e_2(T) + \ldots + \psi_{II}(B)\ e_I(T).$$

To determine which terms to include in this stage of the TTSMA model we follow a generalized version of the model specification procedure for a univariate time series model. The I-1 sample cross-correlations are calculated between $\hat{e}_j(T)$ $j = 1, 2, \ldots$ I-1 and $Y_I(T)$. These cross-correlation functions are examined for significant terms. For each $\hat{\rho}_{\hat{e}_j Y_I}(L)$ that is significant, the term $\psi_{Ij}(L)$ is added to the model. Once this is done for all I-1 cross-correlation functions, the following transformation is made:

$$W(T) = Y_I(T) - \hat{\psi}_{I1}(B)\ \hat{e}_1(T) - \hat{\psi}_{I2}(B)\ \hat{e}_2(T) - \ldots \hat{\psi}_{I,I-1}(B)\ \hat{e}_{I-1}(T)$$

where $\psi_{IJ}(B)$ contains the terms and initial parameter estimates just obtained. From the TTSMA model we know

$$W(T) = \psi_{II}(B)\ e_I(T).$$

Thus, the last part of the model specification phase involves examining the sample auto-correlation function for the W(T) series and from it specifying the form and initial parameter estimates of $\psi_{II}(B)$.

Parameter Estimation

With the form of $\psi_{IJ}(B)$ J=1, 2, . . . I specified we proceded to estimate the paramters in this stage of the model. It can be shown that parameter values in $\psi_{IJ}(B)$ J = 1, 2, . . . I that minimize

$$\sum_{T=1}^{N} \hat{e}_I^{\,2}(T)$$

are maximum likelihood estimates.

Diagnostic Checking

The TTSMA assumptions are that $e_I(T)$ is white noise and that $e_I(T)$ and $e_j(T)$ $j = 1, 2, \ldots$ I,1 are uncross-correlated. Our diagnostic checking procedure starts by examining the sample cross-correlations between $\hat{e}_{I-1}(T)$, $\hat{e}_{I-2}(T)$, . . . $\hat{e}_1(T)$ and $\hat{e}_I(T)$ and the sample auto-correlation function for $\hat{e}_I(T)$. If any significant correlations exist, a term is added to the model to account for it, and if any parameters already included in the model prove statistically insignificant they are removed. If changes have been made in the model form, parameters are reestimated, if not the $I^{\underline{th}}$ series has been successfully included in the model.

A step-by-step summary of the TTSMA procedure follows:

<u>Adding the $I^{\underline{th}}$ Time Series</u>

A. Model Specification

1. Calculate $\hat{\rho}_{\hat{e}_J Y_I}(L)$ J = 1, 2, . . . I-1

2. Add the term $\psi_{IJ}(L)$ for each significant $\hat{\rho}_{\hat{e}_J Y_I}(L)$. Initially estimate

$$\hat{\psi}_{IJ}(L) = \frac{\hat{\sigma}_{Y_I}}{\hat{\sigma}_{\hat{e}_J}} \hat{\rho}_{\hat{e}_J Y_I}(L)$$

3. Construct $W(T) = Y_I(T) - \sum_{J=1}^{I-1} \hat{\psi}_{IJ}(B)\, \hat{e}_J(T)$

4. Calculate $\hat{\rho}_{WW}(L)$

5. Add the term $\psi_{II}(L)$ for each significant $\hat{\rho}_{WW}(L)$ initially estimate

$$\hat{\psi}_{II}(L) = \hat{\rho}_{WW}(L)$$

B. Parameter Estimation - Find parameter values of those parameters in the model that minimize

$$\sum_{T=1}^{N} \hat{e}_I^{\,2}(T) \quad \text{where} \quad \hat{e}_I(T) = \hat{W}(T) - \sum_{L=1}^{\infty} \hat{\psi}_{II}(L)\, \hat{e}_I(T-L)$$

C. Diagnostic Checking

1. Construct $\hat{\rho}_{\hat{e}_J \hat{e}_I}(L)$ the sample cross-correlation function for $e_J e_I$ J = 1, 2, . . . I-1.

2. Construct $\hat{\rho}_{\hat{e}_I \hat{e}_I}(L)$ the sample auto-correlation function for $\hat{e}_I(T)$.

3. Add the terms $\psi_{IJ}(L)$ to explain significant cross or auto-correlations $\hat{\rho}_{\hat{e}_J \hat{e}_I}(L)$ $J = 1, 2, \ldots I$.

4. Delete any $\psi_{IJ}(L)$ from the model if the hypothesis $\psi_{IJ}(L) = 0$ can not be rejected.

5. If the model has been updated, reestimate parameters, otherwise stop.

Data Base

As an example of a multivariate time series forecasting system we will build a TTSMA model for robbery occurrence in 7 cities. The cities are: St. Louis, Portland, Los Angeles, Kansas City, Atlanta, Boston and Denver. A fuller description of these time series is contained in [3]. The univariate modeling of these robbery series is contained in [1]. The univariate model best describing robbery is a $(0,1,1) \times (0,1,1_{12})$ or

$$(1-B)\ (1-B^{12})\ Y(T) = (1-\theta_1 B)\ (1-\Theta_{12}B^{12})\ A(T)\ .$$

From our example we wish to draw some conclusions about the relative strength of 7 univariate models compared to the 7-variate TTSMA model. In other words, is there enough between cities information to make the multivariate model a better predictor of robbery in a given city.

Building a TTSMA Model for Forecasting Robbery

To gain additional insight into the TTSMA model and how it is built, the step-by-step procedure as previously described, will be presented in detail for the first 3 data series; St. Louis, Portland, and Los Angeles.

The model for St. Louis Robbery, the first series entered into the model has the general form

$$Y_1(T) = e_1(T) + \psi_{11}(1)\ e_1(T-1) + \psi_{11}(2)\ e_1(T-2) + \ldots$$

The first step in the model specification phase is to calculate the sample auto-correlation function for the transformed series. Because Robbery is a nonstationary series, a transformation from Z(T), the original series to a stationary representation Y(T) was made,

$$Y(T) = (1-B)\ (1-B^{12})\quad Z(T) - \mu\ .$$

The original series runs from Jan., 1966 to October, 1974 and contains 118 observations. Due to starting value considerations with this transformation, after differencing we are left with 105 observations to model.

In this step, the model for St. Louis is to be constructed from the general moving average model form. Since we already know that this univariate series can be best described by the seasonal $(0,1,1) \times (0,1,1)_{12}$ model, we can use this information to save some identification work in constructing this first stage of the TTSMA model. The univariate seasonal model form is:

$$(1-B)\ (1-B^{12})\ Z_1(T) = (1-\theta_1 B)\ (1-\Theta_{12}B^{12})\ A(T).$$

But since

$$Y_1(T) = (1-B)\ (1-B^{12})\ Z_1(T) - \mu$$

we get

$$Y_1(T) + \mu = (1-\theta_1 B - \Theta_{12}B^{12} + \theta_1\Theta_{12}B^{13})\ A(T),$$

as a representation of the seasonal univariate model. To duplicate this model form with a TTSMA model representation we have, TTSMA

$$Y_1(T) = (1 + \psi_{11}(1)\ B + \psi_{11}(12)\ B^{12} + \psi_{11}(13)\ B^{13})\ A(T).$$

Here we note some differences between the two models. First, the TTSMA procedure always works with centered data, i.e. data that has mean zero. Box-Jenkins univariate models for nonstationary series are built directly with the differenced data. The effect of the parameter μ is usually small. It will, however, prohibit direct numerical comparison of the two models. A second difference in the two models is that the Box-Jenkins univariate model has explicit terms to model seasonal components, whereas TTSMA must handle seasonal relationships with individual ψ parameters at large lags. For the models at hand to be exactly equivalent one would have to have:

$$\mu = 0$$

$$\psi_{11}(B) = -\theta_1$$

$$\psi_{11}(12) = -\Theta_{12}$$

$$\psi_{11}(13) = \theta_1\Theta_{12}.$$

The Box-Jenkins seasonal model is a two parameter model that contains an implicit parameter at lag 13. To duplicate the effect of this seasonal model, three TTSMA parameters must be used. It is a disadvantage of the TTSMA model that it needs three parameters instead of two, but it has the advantage that $\psi_{11}(13)$ is not constrained to equal $\psi_{11}(1)$ times $\psi_{11}(12)$.

Parameter estimation using Marquardts' compromise algorithm gave the following least squares estimates:

TTSMA	Box-Jenkins
$\hat{\psi}_{11}(1) = -0.5256$	$\hat{\theta}_1 = 0.5568$
$\hat{\psi}_{11}(12) = -0.7558$	$\hat{\Theta}_{12} = 0.7133$
$\hat{\psi}_{11}(13) = \ \ 0.4132$	
$\sum_{T=1}^{105} \hat{e}_1^{\,2}(T) = 173190.2$	$\hat{\sigma}^2_{e_1} = \dfrac{\sum_T e_1^{\,2}(T) - \dfrac{(\sum_T e_1(T))^2}{105}}{103} = 1683.9$

$$\hat{\sigma}_{e_1}^{\ 2} = \frac{173190.2 - \dfrac{.9542^2}{105}}{(105 - 3)} = 1697.9 \ .$$

Note that the Box-Jenkins univariate model has a lower mean square for error. This is because the extra TTSMA parameter, $\psi_{11}(13) = 0.4132$, is sufficiently close to the product of $\hat{\psi}_{11}(1)$ and $\hat{\psi}_{12}(12)$,

$$(-0.5256)\ (-0.7558) = 0.3972 \approx 0.4132$$

Adding the Portland Series

From the sample cross-correlation function between $\hat{e}_1(T)$ and $Y_2(T)$ in Table 2, a significant correlation is noted at lag 13. The W series is then calculated as

$$W(T) = Y_2(T) - \hat{\psi}_{21}(13)\ \hat{e}_1(T-13)$$

where

$$\hat{\psi}_{21}(13) = \frac{\hat{\sigma}_{Y_2}}{\hat{\sigma}_{e_1}}\ \hat{\rho}_{e_1 Y_2}(13) = \frac{31.9}{40.8}\ (0.288) = 0.225$$

The sample auto-correlation function for the W series exhibits significant correlations at lags 1 and 12. This ends the model specification phase of the Portland stage of the TTSMA model building procedure. The proposed model is

$$Y_2(T) = \hat{\psi}_{21}(13)\ \hat{e}_1(T-13) + \hat{e}_2(T) + \hat{\psi}_{22}(1)\ \hat{e}_2(T-1) + \hat{\psi}_{22}(12)\ \hat{e}_2(T-12).$$

Parameter estimation yields the following results:

$$\hat{\psi}_{21}(13) = 0.08 \qquad \hat{\psi}_{22}(1) = -0.53$$

$$\hat{\psi}_{22}(12) = -0.48 \qquad \sum_{T=1}^{105} \hat{e}_2^{\,2}(T) = 72079.26$$

In proceeding to the diagnostic checking phase, we note that all three parameters are significant. Exmaining the sample correlation functions between $\hat{e}_1(T)$ and $\hat{e}_2(T)$ and between $\hat{e}_2(T)$ and $\hat{e}_2(T)$, however points out a significant auto-correlation at lag 3 in the $\hat{e}_2(T)$ series. This leads us to the updated model,

$$Y_2(T) = \psi_{21}(13)\ \hat{e}_1(T-13) + \hat{e}_2(T) + \psi_{11}(1)\ \hat{e}_2(T-1) +$$

$$\psi_{11}(3)\ \hat{e}_2(T-3) + \psi_{11}(12)\ \hat{e}_2(T-12).$$

L	$\hat{\rho}_{\hat{e}_1 Y_2}(L)$	$\frac{\hat{\rho}_{\hat{e}_1 Y_2}(L)}{\hat{\sigma}_{\hat{\rho}}}$	L	$\hat{\rho}_{\hat{e}_1 Y_2}(L)$	$\frac{\hat{\rho}_{\hat{e}_1 Y_2}(L)}{\hat{\sigma}_{\hat{\rho}}}$
-15	.112	1.063	1	-.093	-.952
-14	.053	.507	2	-.031	-.311
-13	-.044	-.470	3	.150	1.518
-12	.030	.292	4	-.083	-.838
-11	.120	1.160	5	-.015	-.155
-10	.034	.331	6	.007	.067
- 9	-.140	-1.368	7	.077	.764
- 8	.119	1.175	8	-.030	-.294
- 7	-.044	-.433	9	-.018	-.172
- 6	.039	.388	10	-.068	-.662
- 5	-.090	-.901	11	-.055	-.529
- 4	.118	1.181	12	.088	.850
- 3	-.054	-.543	13	.288	2.762
- 2	-.042	-.427	14	-.165	-1.577
- 1	.062	.634	15	.065	.619
0	-.115	-1.115			

Table 2. Sample Cross-correlation Between $\hat{e}_1(T)$ and $Y_2(T)$

Estimating the parameters in this model yields;

$$\hat{\psi}_{21}(12) = -0.054$$

$$\hat{\psi}_{22}(1) = -0.4159$$

$$\hat{\psi}_{22}(3) = -0.3178$$

$$\hat{\psi}_{22}(12) = -0.7608 .$$

The $\psi_{21}(12)$ parameter proved insignificant as its approximate 95% confidence interval on a linear hypothesis is (-0.129, 0.021). Dropping the insignificant parameter we fit the remaining three parameter model and obtain;

$$\hat{\psi}_{22}(1) = -0.3663$$

$$\hat{\psi}_{22}(3) = -0.2615$$

$$\hat{\psi}_{22}(12) = -0.6123$$

$$\Sigma\hat{e}_2^{\,2}(T) = 66862.43 .$$

For this fit, all parameters are significant and the sample residual correlations are acceptable.

Adding the Los Angeles Series

The sample cross-correlation function between $\hat{e}_1(T)$ and $Y_3(T)$ and between $\hat{e}_2(T)$ any $Y_3(T)$ exhibit significant correlations;

$$\hat{\rho}_{\hat{e}_1Y_3}(11) = .268$$

$$\hat{\rho}_{\hat{e}_2Y_3}(-5) = -.230$$

$$\hat{\rho}_{\hat{e}_2Y_3}(7) = .222$$

$$\hat{\rho}_{\hat{e}_2Y_3}(9) = -.231$$

$$\hat{\sigma}_{e_1} = 40.8$$

$$\hat{\sigma}_{e_2} = 25.3 .$$

Initial parameter estimates for the ψ parameters are,

$$\hat{\psi}_{31}(11) = \frac{\hat{\sigma}_{Y_3}}{\hat{\sigma}_{e_1}} \hat{\rho}_{\hat{e}_1 Y_3}(11) = \frac{98.8}{40.8} (0.268) = 0.649$$

$$\hat{\psi}_{32}(-5) = \frac{98.8}{25.3} (-0.23) = -0.898$$

$$\hat{\psi}_{32}(7) = \frac{98.8}{25.3} (+0.222) = 0.867$$

$$\hat{\psi}_{32}(9) = \frac{98.8}{25.3} (-0.231) = -0.902$$

The W(T) series is calculated as

$$W(T) = Y_3(T) - \hat{\psi}_{31}(11)\, \hat{e}_1(T-11) - \hat{\psi}_{32}(-5)\, \hat{e}_2(T+5) -$$
$$\hat{\psi}_{32}(7)\, \hat{e}_2(T-7) - \hat{\psi}_{32}(9)\, \hat{e}_2(T-9)$$

The sample auto-correlation function for the W(T) series shows significant values at lags 1,7 and 12. The model containing terms, $\psi_{31}(11)$, $\psi_{32}(-5)$, $\psi_{32}(7)$, $\psi_{32}(11)$, $\psi_{33}(1)$, $\psi_{33}(7)$, and $\psi_{33}(12)$ proved to contain an over-supply of parameters. Only two parameters, $\psi_{31}(11)$ and $\psi_{33}(12)$ proved significant. Parameter estimates obtained for the two parameter model are;

$$\hat{\psi}_{31}(11) = 0.509$$

$$\hat{\psi}_{33}(12) = -0.7105$$

$$\Sigma \hat{e}_3(T)^2 = 642202.9 \ .$$

As expected both parameters proved significant, however, a significant auto-correlations, $\hat{\rho}_{33}(2) = -0.250$, appeared which dictates the three parameter model,

$$Y_3(T) = \psi_{31}(11)\, \hat{e}_1(T-11) + e_3(T) + \psi_{33}(2)\, e_3(T-2) + \psi_{33}(12)\, e_3(T-12).$$

Parameter estimates for this model are;

$$\hat{\psi}_{31}(11) = .5606$$

$$\hat{\psi}_{33}(2) = -.1105$$

$$\hat{\psi}_{33}(12) = -.6984$$

$$\Sigma\hat{e}_3(T)^2 = 622710.2$$

We now note a significant correlation $\hat{\rho}_{31}(2) = 0.225$. Thus, the parameter, $\psi_{31}(2)$ was added to the model and all parameters simultaneously estimated:

$$\hat{\psi}_{31}(2) = .3026$$

$$\hat{\psi}_{31}(11) = .5642$$

$$\hat{\psi}_{33}(2) = -.0855$$

$$\hat{\psi}_{33}(12) = -.7421$$

$$\Sigma\hat{e}_3^2(T) = 595215.8$$

The sample auto-correlation function for this fitted model looks adequate. Further, the marginal significance of the $\psi_{31}(2)$ parameter was calculated and proved significant at the 95% level.

The remaining 4 series were entered into the TTSMA model in a manner paralleling that just illustrated. At each stage, the 3 phases of time series model building were repeated until an adequate model was decided upon. The final 7 stage model follows:

I	J	L	$\psi_{IJ}(L)$
1	1	0	1.0000
1	1	1	-0.5258
1	1	12	-0.7558
1	1	13	0.4133
2	2	0	1.0000

I	J	L	$\psi_{IJ}(L)$
2	2	1	-0.3663
2	2	3	-0.2615
2	2	12	-0.6123
3	1	2	0.3026
3	1	11	0.5642
3	3	0	1.0000
3	3	2	-0.0855
3	3	12	-0.7421
4	1	14	0.1517
4	2	2	-0.2311
4	3	12	-0.0909
4	4	0	1.0000
4	4	1	-0.4367
4	4	12	-0.6474
5	3	-2	0.1275
5	3	0	0.1299
5	3	8	-0.1477
5	5	0	1.0000
5	5	1	-0.2984
5	5	7	0.2890
6	5	7	0.5023
6	6	0	1.0000
6	6	1	-0.5679
6	6	7	-0.2055
6	6	10	-0.2938
7	2	0	0.2639
7	6	-4	0.0844
7	6	13	0.1605

I	J	L	$\psi_{IJ}(L)$
7	7	0	1.0000
7	7	1	-0.3001
7	7	12	-0.8526

Canonical Representation of the TTSMA Model

Before we can proceed to develop the forecasting function for the TTSMA model of robbery, we must first transform the ψ matrix into canonical form. The Canonical model is an equivalent representation of the TTSMA model in that the correlative structure of the TTSMA model is duplicated exactly by the Canonical model. The two models then, express the same mathematical relationships. The TTSMA form was seen to have advantages during the model building procedure, and the Canonical form has special application towards developing the forecasting function.

The nature of the transformation of the TTSMA model to canonical form is rather complicated. The procedure consists of 1) Removal of the poles inside the unit circle and 2) removal of the zeros inside the unit circle. When there are no poles of $\psi(B)$ inside the unit circle indicates "stationarity" and when there are no zeros of $\det[\psi(B)]$ indicates invertibility. Details of the transformation can be found in [4]. The model in canonical form will have the property that all terms $\psi_{ij}(B)$ will have positive exponents. Hence $Y_i(T)$ is expressed as a function of current and previous random shocks, random shocks not only from series i, but from all series in the model. The ψ matrix then, will no longer be triangular.

For our example, the canonical representation of the TTSMA model follows:

I	J	L	$\hat{\psi}_{IJ}(L)$
1	1	0	1.0000
1	1	1	-0.5258

I	J	L	$\hat{\psi}_{IJ}(L)$
1	1	12	-0.7558
1	1	13	0.4133
2	2	0	1.0000
2	2	1	-0.3663
2	2	3	-0.2615
2	2	12	-0.6123
3	1	2	0.3026
3	1	11	0.5642
3	3	0	1.0000
3	3	2	-0.1589
3	3	12	-0.7421
3	5	1	0.1455
3	5	2	0.5228
3	5	13	-0.1080
3	5	14	-0.3879
4	1	14	0.1517
4	2	2	-0.2311
4	4	0	1.0000
4	4	1	-0.4367
4	4	12	-0.6474
5	3	8	-0.1477
5	5	0	1.0000
5	5	1	-0.2580
5	5	7	0.2681
6	5	7	0.4660
6	6	0	1.0000
6	6	1	-0.5758
6	6	7	-0.2055
6	6	10	-0.2937
6	7	4	0.2380
6	7	5	-0.1621
7	2	12	0.2191
7	6	13	0.1605
7	7	0	1.0000
7	7	1	-0.2922
7	7	12	-0.8301

Development of the Minimum Mean Square Error Forecasts

The derivation of the forecasting function for the canonical representation of the TTSMA model follows closely the univariate forecasts derivation contained in [5]. It again will be shown that the minimum means square error forecast for the observation of Y_i at time T + ℓ where T is the present is simply the codit-ional expectation of $Y_i(T + \ell)$ and time T.

Let the canonical model for Y(T) be represented as

$$Y(T) = \psi^*(B)\ b(T).$$

Also let $\psi^*(B)$ have the convergent Taylor series representation,

$$\psi^*(B) = \sum_{K=0}^{\infty} \psi^*(K)\ B^K .$$

Now suppose at origin T, we wish to forecast $Y(T + \ell)$, and furthermore we wish our forecast to be a linear function of current and previous observations Y(T), Y(T - 1), Y(T - 2), . . . It follows that our forecast $\hat{Y}_T(\ell)$ can also be expressed as a linear function of current and previous innovations b(T), b(T - 1), ...

Suppose then, that the best forecast is given by,

$$\hat{Y}_T(\ell) = \hat{\psi}_\ell(0)\ b(T) + \hat{\psi}_\ell(1)\ b(T-1) + \hat{\psi}_\ell(2)\ b(T-2) + \ldots = \sum_{K=0}^{\infty} \hat{\psi}_\ell(K)\ b(T-K),$$

where the $\hat{\psi}_\ell(K)$ matrices are to be determined. The ℓ step-ahead forecast error is then

$$\hat{e}_T(\ell) = Y(T+\ell) - \hat{Y}_T(\ell) = \sum_{K=0}^{\ell-1} \psi^*(K)\ b(T+\ell-K) + \sum_{K=0}^{\infty} (\psi^*(\ell+K) - \hat{\psi}_\ell(K)\ )\ b(T-K)$$

and its variance covariance matrix is

$$E[\hat{e}_T(\ell)\ \hat{e}_T(\ell)'\ ] = A_1 + A_2$$

where

$$A_1 = \psi^*(0)\ \Sigma_b\ \psi^*(0)' + \psi^*(1)\ \Sigma_b\ \psi^*(1)' + \ldots + \psi^*(\ell-1)\ \Sigma_b\ \psi^*(\ell-1)'$$

$$A_2 = \sum_{K=0}^{\infty} [\psi^*(\ell+K) - \hat{\psi}_\ell(K)\ ]\ \Sigma_b\ [\psi^*(\ell+K) - \hat{\psi}_\ell(K)\ ]'$$

Here Σ_b is the variance covariance matrix for the b(T) series.

Note that A_1 is not a function of $\hat{\psi}_\ell$ and that the variance of any linear function of forecast errors $h'\hat{e}_t(\ell)$ is minimized if $A_2 \equiv 0$. Therefore our minimum mean square error forecasts must have

$$\hat{\psi}_\ell(K) = \psi^*(\ell+K)\ .$$

Hence, the forecasting function for $\hat{Y}_T(\ell)$ in terms of innovations is given by,

$$\hat{Y}_T(\ell) = \sum_{K=\ell}^{\infty} \psi^*(K)\ b(T+\ell-K) = E[Y(T+\ell)\ |\ Y(T),\ Y(T-1),\ \ldots]\ .$$

Thus, the minimum mean square error forecasts at time T is simply the conditional expectations of $Y(T+\ell)$ at time T.

The ℓ-step ahead forecast error is given by

$$\hat{e}_T(\ell) = \sum_{K=0}^{\ell-1} \psi^*(K)\ b(T+\ell-K)$$

and since $E[\hat{e}_T(\ell) = 0]$, i.e. the forecast is unbiased, we have

$$\text{Var}\ [\hat{e}_T(\ell)] = A_1 = \sum_{K=0}^{\ell-1} \psi^*(K)\ \Sigma_b \psi^*(K)'$$

As in the univariate case, the ℓ-step ahead forecasts can be expressed as a function of previous observations. By inverting the cononical representation we get

$$b(T) = [\psi^*(B)\ ]^{-1}\ Y(T)\ .$$

If we let $\Pi^*(B) = [\psi^*(B)]^{-1}$ and take the Taylor Series expansion,

$$\Pi^*(B) = \sum_{K=0}^{\infty} \Pi^*(K)B^K$$

since $\Pi^*(0) = I$ we get

$$\hat{Y}(T) = \Pi^*(1)\ \hat{Y}(T-1) + \Pi^*(2)\ Y(T-2) + \ldots + b(T)\ .$$

Now using the conditional expectation concept we obtain an expression for the ℓ-step ahead forecast of $Y(T+\ell)$ in terms of previous observations and forecasts

$$\hat{Y}_T(\ell) = \Pi^*(1)\ \hat{Y}_T(\ell-1) + \Pi^*(2)\ \hat{Y}_T(\ell-2) + \ldots + \Pi^*(\ell-1)\ \hat{Y}_T(1) + \Pi^*(\ell)\ Y(T) +$$

$$\Pi^*(\ell+1)\ Y(T-1) + \Pi^*(\ell+2)\ Y(T-2) + \ldots$$

Robbery Forecasts from the TTSMA Model

Forecasts from the multivariate model are presented in Table 3. Included are the corresponding forecasts from the univariate model and the actual observations. The base point for these forecasts is October, 1975, and hence the forecasts at lag 1 is for November, 1975. Table 4 gives the estimated forecasting variance of the one-step ahead forecast errors. The variance of the one-step ahead forecast errors directly measures how well the model under consideration has fit the data. A smaller error variance implies that more of original series variation has been explained, and hence the forecasted values can be expected to be closer to the actual realizations. Examination of Table 4 will tell if between cities information, as utilized by the TTSMA model, can help predict robbery in the seven cities.

The values of Table 4 seem to indicate very little difference between the 7 univariate models and the 7-variate TTSMA model in their respective ability to model robbery. Indeed, F tests for equality of variances fail to reject the hypothesis that the variances of the forecast errors are equal.

The differences that do exist can be related to the canonical ψ matrix. Examination of the Canonically reduced ψ matrix reveals that both St. Louis and Portland are represented as univariate models. This means that knowledge of

City	Lag	Univariate	TTSMA	Actual
St. Louis	1	571.7	570.8	572
	2	603.2	599.9	608
	3	532.4	535.8	445
	4	464.9	470.4	436
	5	480.7	486.2	433
Portland	1	162.9	144.7	157
	2	156.3	147.3	196
	3	134.0	116.5	172
	4	141.6	122.3	152
	5	149.6	124.3	169
	6	121.3	95.9	120
Los Angeles	1	1270.2	1280.9	1267
	2	1388.2	1331.6	1333
	3	1309.8	1206.5	1327
	4	1226.6	1145.2	1279
	5	1283.9	1100.6	1222
	6	1217.0	1059.2	1145
Kansas City	1	255.8	298.3	251
	2	262.8	284.0	280
	3	242.8	270.8	221
	4	233.6	263.9	191
	5	248.2	282.9	*
Atlanta	1	327.4	399.9	292
	2	380.7	486.1	*
	3	349.0	343.8	*
	4	267.4	281.3	*
	5	283.3	315.0	*
Boston	1	716.6	670.4	722
	2	721.2	608.6	658
	3	732.3	760.1	608
	4	674.2	633.7	546
	5	677.7	631.4	459

City	Lag	Univariate	TTSMA	Actual
Denver	1	230.2	237.3	198
	2	248.6	247.9	218
	3	232.9	214.3	170
	4	220.9	249.5	144
	5	222.7	244.2	153
	6	206.6	231.3	142

* not available

Table 3. Robbery Forecasts and Realizations

City	Univariate	TTSMA
St. Louis	1683.9	1697.9
Portland	717.4	655.5
Los Angeles	6085.1	5467.8
Kansas City	1129.8	1160.5
Atlanta	1453.6	1545.3
Boston	2839.6	2466.2
Denver	764.0	794.8

Table 4. Estimated Variance of 1-step Ahead Forecast Errors

robbery in other cities did not help in predicting robbery in these two cities. The error variance for these two cities are then approximately equal for the TTSMA model and the univariate Box-Jenkins model. The numerical differences between the two results from the inability of a TTSMA model to exactly duplicate a Box-Jenkins seasonal model.

The Canonical model for Los Angeles on the other hand, has 9 parameters and cross-terms both with St. Louis and Atlanta. From this it is concluded that between cities information is important in the modeling of Los Angeles robbery and hence the forecast error variance for the TTSMA model is smaller than the univariate model's variance. The same observations can be made regarding the Boston series. Seven parameters and cross-terms with both Atlanta and Denver lead to a smaller error variance.

It should be noted that the Canonical ψ matrix, the forecast error variances and the forecasts themselves are all extremely sensitive to the decisions made during the TTSMA model building procedure. The order of entry of the time series into the model, although theoretically of no importance, in practice turns out to be a significant factor in the final form of the original ψ matrix. The experience gained while building the multivariate robbery model has pointed out that decisions that often at the time seem somewhat insignificant, have profound effects on the resulting form of the model. The decision to include a marginal parameter in an early stage of the modeling procedure, for instance, can lead to a vastly different model. Combinations of parameters that have only minimal effect individually can cause extremely significant reductions in $\Sigma\hat{e}^2(T)$ at some stage in the model. This reduction in sum of squares is somewhat artificial in that the parameters are not describing a correlative relationship, but are often <u>creating</u> significant correlations in the $\hat{e}(T)$ streams. In this instance the estimated parameter values often approach unity in absolute value, a condition for stationarity of the model. The calculated $\hat{e}(T)$ streams and the resulting forecasts in this case can "explode" and produce nonsensical results.

Conclusions

In this technical note, the concept of a multivariate forecasting system was presented. The basis for a multivariate system is the incorporation of information that exists between pairs of series into a model that will provide forecasts better than those that ignore this information. The methodology for a particular multivariate time series model was presented and applied to robbery data for seven cities. It was seen that some between cities informations did exist, and upon comparing forecast error variances from the TTSMA model with those of the Box-Jenkins univariate models, it was seen that the multivariate model led to improved estimation for those cities which exhibited significant between series relationships.

This technical note has also pointed out some of the difficulties involved in applying the TTSMA model. Start-up problems, round off errors, and certain inadaquacies of the procedure itself have made it difficult to effectively evaluate the multivariate modeling of robbery.

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TECHNICAL NOTE #18

Aspects of a Multiple Offense Model

Introduction

The purpose of this technical note is to document potential modeling aspects for an extension to the basic network flow model for predicting of criminal displacement. The latter model has been developed, analyzed, and reported in numerous, prior technical notes, e.g. [2, 3, 4]. Fundamentally, the extension to which we refer involves the notion of multiple offenses or multiple crime type occurrences. Following, we address this problem from a modeling standpoint and discuss the motivation for particular constructions. In addition, the effect of cost functions and problem solution relative to contemporary methodologies is briefly discussed.

Recall that in previous technical notes, the phenomena of criminal (crime) displacement was modeled using some classic notions from network flow theory. A series of transportation problems were solved successively such that a solution (feasible flow pattern) provided a set of crime displacement patterns in a given time period. Throughout the previous work, it was explicitly assumed that all displacement involved only the commission of a single crime type. The issue of multiple commissions (e.g. assault and homicide) during the same time period was not addressed. Independence of crime types was assumed in which case, a segregation of the crime-type displacement analyses can be justified. Suppose, however, that some crime types are not independent. One can easily comprehend situations where the commission of one crime (of a given index) might foster the simultaneous commission of another crime type. If displacement occurred relative to the former crime, it must also occur relative to the coincident, latter crime.

Using the same ideas developed previously, let us denote the displacement of crime type k from zone i to zone j in time period t to be x^t_{ijk}. Recall that the network flow structure developed earlier is dynamic, hence the superscript denoting discrete time. Just as in the single crime-type model where one can depict the displacement from i to j by a directed arc between points i and j, we can create a similar depiction for the multiple offense model. In Figure 1a the single crime flow depiction is given while in Figure 1b that for the possible commission of K crime types is depicted. Clearly, the structure in (1b) is a generalization of that in (1a) where K is one. Note also that we choose to depict the multiple crime "flow" by K distinct edges. There is no requirement in doing this since all crimes would displace or "flow" along the arc given uniquely by the pair (i,j).

As mentioned at the outset, the original network model for displacement considered a series of transportation models. Although not referred to as such, the transportation problems could have been described as single commodity problems. Such a descriptor would obviously arise from the single crime (commodity) aspect of the system for which the structure is a model. The natural extension then, when multiple offenses are examined would result in a structure similar to a so-called multi-commodity transportation problem. We hasten to point out that our problem is not, strictly speaking, equivalent to the classic multi-commodity flow problem. This should be made clear subsequently.

A Model for the Case of Multiple Offenses

For convenience, let us disregard the multi-period nature of the general model of displacement. As such, we can specify the classic least cost, multi-commodity transportation problem as follows:

$$\min \sum_{k=1}^{k} \sum_{j=1}^{n} \sum_{i=1}^{m} c_{ijk} x_{ijk}$$

$$\text{s.t.} \sum_{j=1}^{n} x_{ijk} = a_{ik} \; ; \; \forall_{i,k}$$

$$\sum_{i=1}^{m} x_{ijk} = b_{jk} \; ; \; \forall_{j,k}$$

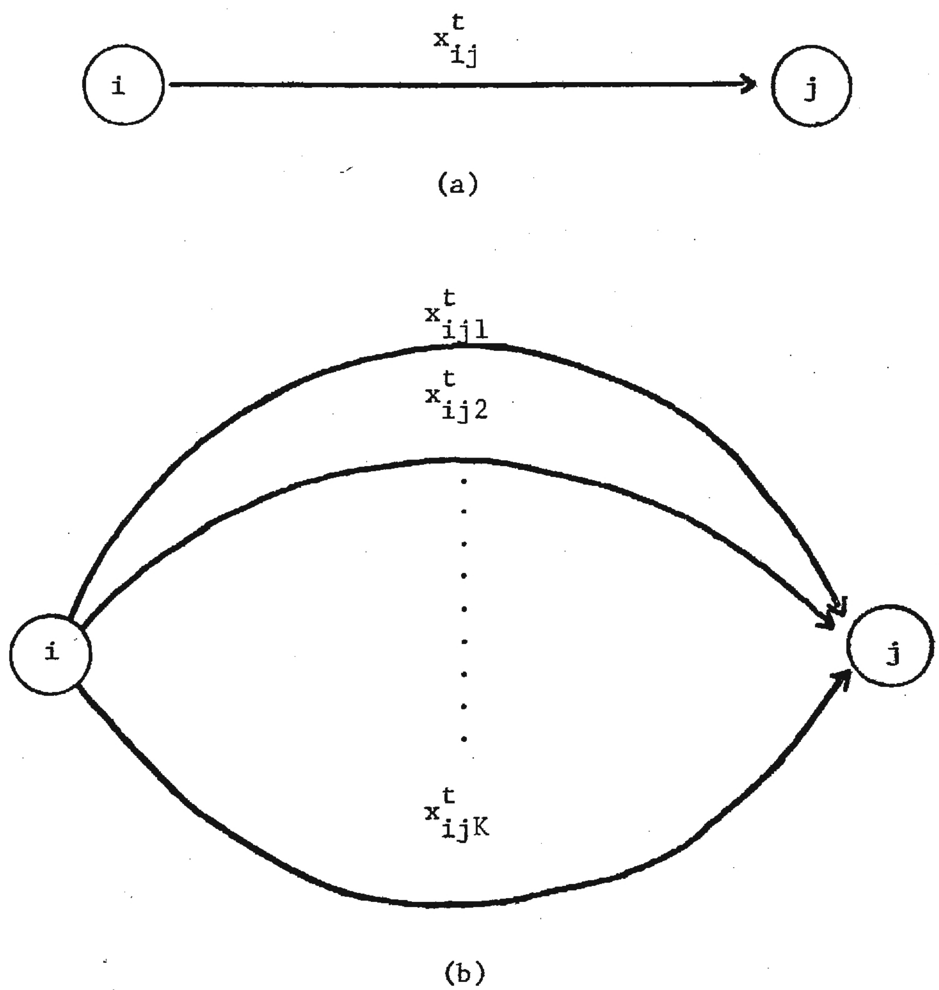

Figure 1. Depiction of Single and Multi-Crime Displacement (and/or Deterrence) Variables

$$\sum_{k=1}^{k} x_{ijk} \leq u_{ij} \quad ;\forall_{i,j}$$

$$x_{ijk} \geq 0 \quad ;\forall_{i,j,k}$$

Note that c_{ijk}, a_{ik}, and b_{jk} are the per unit cost of flow of commodity k from i to j, the "availability" of commodity k at zone i and the "demand" of k at j respectively. The parameter u_{ij} expresses a capacity restriction or upper bound on flow along the directed arc (i,j). Generally, we are not concerned with such a restriction in our problem.

Assuming the applicability of the above model to the multiple offense problem, the entire network tuning algorithm [2] developed earlier would proceed in the usual manner. We do not re-specify that process since its documentation is complete in prior notes. Rather, we proceed with some thoughts as to how our problem departs from the class multi-commodity problem as modeled above.

In the usual multi-commodity transportation problem (itself a special version of the general multi-commodity flow problem) it is generally assumed that a set of least cost flows for K commodities be determined subject only to the constraint of capacity restrictions and availability and demand levels. If the flow patterns are feasible, it is of little consequence that commodities k' and k" say, flow coincidently between points i and j. The corresponding $x_{ijk'}$ and $x_{ijk''}$ are independent in the sense that some perturbation to one of the variables would not necessarily affect the other. This is not the case when we let k' and k" be crime types which are committed simultaneously.

Suppose $f_{ijk'}$ is the observed displacement between i and j of crime type (index) k'. Further, let $f_{ijk''}$ be the observed flow for crime type k". Suppose that the criminals commiting k' also commit a subset of the crimes of type k". Relative to the tuning algorithm, the objective would be to find those computed flows, say $x^*_{ijk'}$ and $x^*_{ijk''}$ which are in close agreement with $f_{ijk'}$ and $f_{ijk''}$. The difficulty however, is that determination of certain flows of type k' is related to concommittant changes in flow for type k". Solution of the model given above makes no such demand. The outcome may be that a tuned model results which is correct in the mathematical sense as defined in previous developments [2] yet which does not allow predictions of displacement that make sense in terms of multiple crime commission.

Consider a relaxation of the above aspect in which we assume an accurate matching of the classic model with the multi-crime system. Even then, it is likely that the use of contemporary multi-commodity flow algorithms will not hold much prospect for efficient analyses of problems the magnitude of which we counter. The lack of analagous structural properties relative to the single commodity case (unimodularity, etc.) make the multi-commodity problem computationally untenable.

Another aspect which may contribute to the complexity of a multi-commodity model of displacement would be the cost function construction. While the costs c_{ijk} are, as in the single commodity case, only driving mechanisms in the tuning process their functional representation should be based upon some rational system relationships, e.g. police concentration, socio-economic levels and so forth. This is not a simple resolution in the single crime version and may be compounded in the multi-commodity extension especially in light of the notion of hypothesized crime type dependence. An optimistic view however might conjecture that if there is a multiple crime commission, then the conditions contributing to the occurrence of one crime type might be similar to that of another (committed simultaneously). Otherwise, there commission would not likely have been simultaneous.

A Generalization and Summary

The notion of multiple crime commission as discussed above was based upon the aspect of commission simultaneity. That is, it was assumed that the perpetuation of one crime fostured the coincident commission of another. Certainly, one can visulize those circumstances where such a pattern is relevant. It may be however, that simultaneity of commission in this sense does not hold, yet there is a multiple commission due to one crime's influence upon another, the latter taking place in the same time period but not explicitly coincident with the former incident. Clearly, if there is no such influence then the crimes would be independent and their analyses would take the form of the single commodity modeling. Otherwise, a multi-commodity like condition would arise.

The notion of non-simultaneous, yet related multiple commissions gains credence in light of results such as those regarding crime switch analyses discussed in [1]. Suggested are estimates of the likelihoods of certain crime types being committed given that they are preceded by a particular, other crime type. Regardless, the relaxation of the coincident crime commission assumption would appear to pose no difficulty in itself relative to the multi-commodity construction discussed in this note.

Conclusion

Summarizing, it was the purpose of this technical note to expose the existence of multiple crime type commissions and the necessity to model such occurrences with alternative structures than those developed earlier. Fundamentally, the difference lies in the transportation structures which are used to model potential displacement. The analysis and solution of such models for the multi-commodity case are substantially more difficult if not computationally untenable relative to the single commodity models considered thus far. Substantial investigation would be required in order to treat effectively, in a multi-commodity framework, the multiple commission problem. Such an appraisal is made from the view of the inherent problem size created in the real displacement model. Further, various properties of the multi-commodity problem create a condition whereby methodological capabilities are largely theoretical.

References

1. Blumstein, A., and R. Larson, "Models of a Total Criminal Justice System", Operations Research, 17, 2, 1969.

2. Deutsch, S.J., J.J. Jarvis, and R.G. Parker, "Aspects of the Single Period Structure and a Tuning Procedure for the Overall Network Model", Technical Note #7, Dept. of Justice - L.E.A.A., Grant #75NI-99-0091, 1976.

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4. Deutsch, S.J., J.J. Jarvis, and R.G. Parker, "A Nonlinear Transportation Model for Predicting Displacement", Technical Note #13, Dept. of Justice - L.E.A.A., Grant #75NI-99-0091, 1976.

TECHNICAL NOTE #19

Application of the Linear and Nonlinear Models to Predict Criminal Displacement From a Sample Set of Data for the City of Atlanta

Introduction

In Technical Note #2 we developed a network flow model for predicting criminal displacement and deterrence. In Technical Note #12 this general model was specialized to the case of a linear objective function (in the flows). In Technical Note #13 an algorithm was presented for the case of a nonlinear objective function (in the flows). In these technical notes we also presented an example of the application of each procedure to find the "optimal" set of parameters in each respective model to obtain the "best" least squares fit to historical data.

In the current technical note we shall present examples of the application of each of the fitted models in the prediction of criminal displacement for a sample set of data for the city of Atlanta. We shall also raise certain issues relative to the confidence that one might expect in the results of the predictions.

Developing the Input Data for the Models

The predicting models require three types of input information:

1. Historical flows representing criminal displacement by aggregated district,
2. Demographic, law enforcement and other relevant data associated with the historical flows, and
3. Future projections of supply (criminals residing in each district) and demand (crimes committed in each district) information.

The city of Atlanta, because of its proximity, was selected as a test site for the displacement prediction methods. The Crime Analysis Team of the city made available sample data for the seven crime types for the twelve months of 1974. While this data set did not represent the complete enumeration of all crimes occuring during that year it did provide a basis for testing the model. The CAT also supplied estimates of average manpower allocation by district during that period.

To test the predicting models it was decided to use the data for the first nine months of 1974 to "tune" each model. Each "tuned" model was then employed to predict criminal displacement during the last three months of 1974. In this manner comparisons between actual and predicted three month displacement data could indicate how well the predicting models had done.

Combined twelve month data for crime types 1 through 6 were supplied to the aggregation procedure described in Technical Note #14. The resulting aggregations resulted in fourteen districts and is presented in Figure 1.

An aggregated nine month flow matrix was developed from the original data and the map of Figure 1. This matrix is presented in Table 1. Interpreting that table we see that 29 crimes were committed in district 5 during the first nine months of 1974 by criminals whose residences were listed in district 5. Also, 3 crimes were committed in district 12 by criminals with residences in district 6.

Using the aggregated map of Figure 1, a three month flow matrix was also developed from the original data. This matrix is presented in Table 2. Only the row and column totals for the three month matrix were supplied to the predicting procedures.

A distance matrix was computed from the aggregated map of Figure 1. This distance matrix is presented in Table 3. The numbers in that table are measured in terms of transformed units but are reasonably measured to scale. No entries are given below the main diagonal as the matrix is assumed to be symmetric.

Finally, manpower estimates by original district, were supplied by the CAT for 1974. These estimates were aggregated according to the map of Figure 1. Next

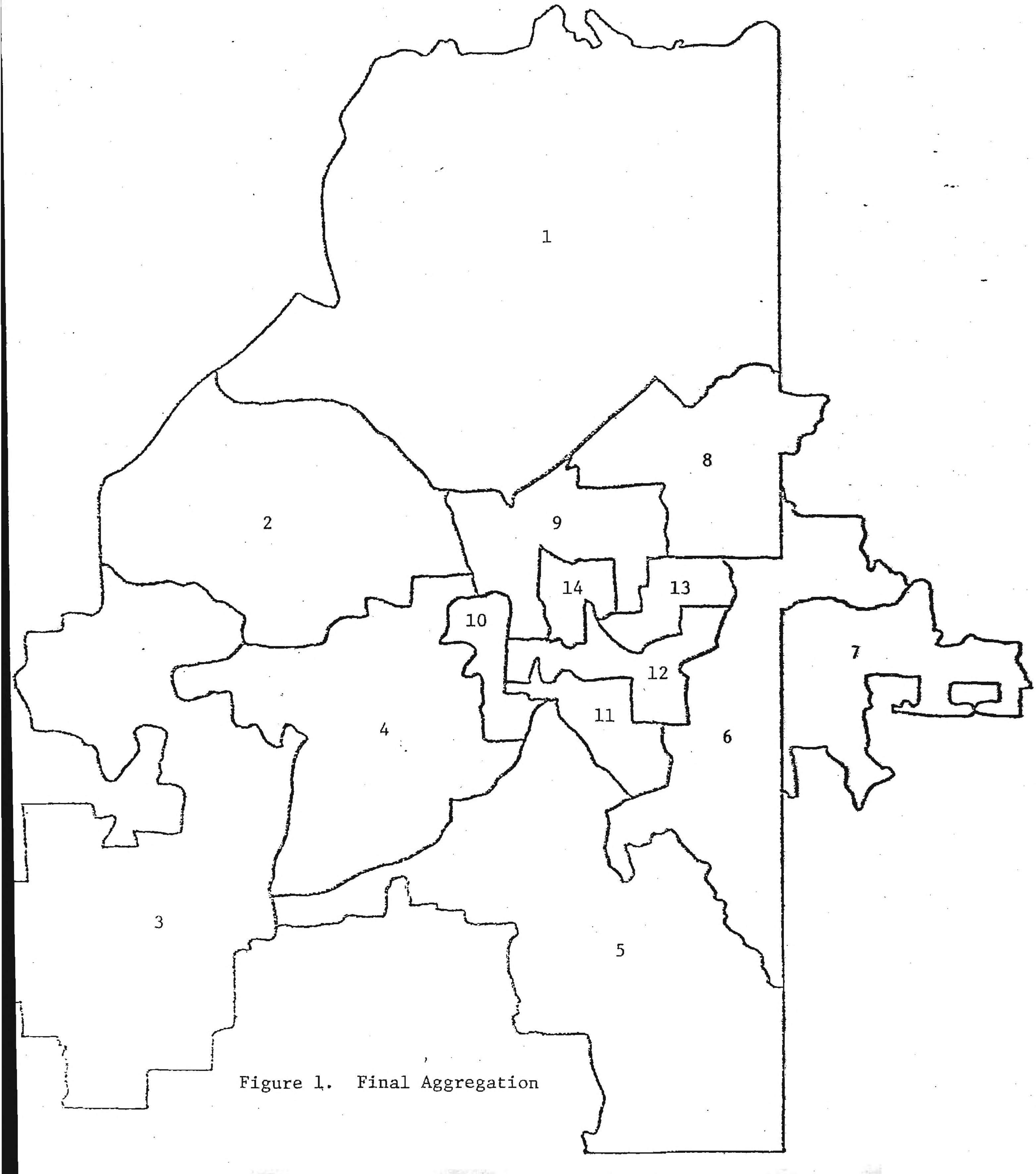

Figure 1. Final Aggregation

Table 1. Sample Data, f_{ij}'s, for Crime Types

1 Through 6 for the First Nine Months of 1974

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Row Totals
1	1	1			1				1				1		5
2		18	1	2		1	3		1			1		1	28
3		2	3	2	1				1						9
4	1	2	1	8	1	2				2		1		1	19
5	1		1	4	29	2	1			1				1	40
6		3	1			17	1					3	1		26
7				1			14				1	1			17
8				1		2		2	1						6
9	1	1	2	5	1		1		11					1	23
10				2	1				1	2		1			7
11		1			3		2				12	1	1	1	21
12				2	4		2	1	2	1	4	13	3	1	33
13			1	1			1					1	5	1	10
14		1		2				1	1	1				9	15
Column Totals	4	29	10	30	41	24	25	4	19	7	17	22	11	16	259

TOTAL Sum of Squares = 2503

Table 2. Sample Data, f_{ij}'s, for Crime Types
1 Through 6 for the Last Three Months of 1974

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Row Totals
1	1				1				1		1		1		5
2		5	1	1	1					1					9
3										1					1
4				1	2									1	4
5		1			7	1					1				10
6				1		7	1	1				1			11
7							12								12
8									1						1
9				3		1			1				1	1	7
10													1		1
11		1				1					3				5
12									1		3	5			9
13						1							3		4
14							1		1					6	8
Column Totals	1	7	1	6	11	11	14	1	5	2	8	6	6	8	87

TOTAL Sum of Squares = 399

Table 3. The Distance Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.	3.	5.5	4.	5.5	4.	4.25	2.	2.375	3.375	3.75	3.375	2.875	2.75
2		0.	2.6	1.875	4.125	4.	4.75	3.25	2.125	2.125	2.9	2.9	3.	2.35
3			0.	2.	3.5	4.65	5.75	5.1	3.75	2.875	3.55	3.875	4.25	3.75
4				0.	2.25	2.75	3.8	3.175	1.875	.9	1.625	1.875	1.25	1.75
5					0.	2.2	3.2	4.	3.125	2.25	1.87	2.25	2.85	2.85
6						0.	1.125	2.18	2.125	2.	1.125	1.05	1.25	1.75
7							0.	2.2	2.75	2.92	2.2	1.92	1.79	2.5
8								0.	1.375	2.32	2.2	1.75	1.2	1.375
9									0.	1.05	1.375	1.125	.92	.375
10										0.	.875	1.	1.375	.875
11											0.	.5	1.	1.
12												0.	.56	.74
13													0.	.92
14														0.

an estimate of the area of each district was computed for the map of Figure 1. These areas were used to normalize the manpower estimates supplied by the CAT. Table 4 presents the resulting values of manpower per unit area measured in transformed units.

"Tuning" the Linear Model

The data of Tables 1, 3 and 4 were subjected to the linear model to obtain the best choice of parameters. The objective was

$$\sum_i \sum_j c_{ij} x_{ij}$$

where

$$c_{ij} = e^{\alpha(p_j - p_i)} + e^{\beta d_{ij}^2} + e^{\gamma(p_j - p_i) d_{ij}^2}.$$

Table 5 summarizes the one-at-a-time search for the optimal choice of the parameters α, β and γ. An optimal set of parameters for the linear model is given by

$$\alpha = .25$$
$$\beta = .70$$
$$\gamma = .075$$

with a total error sum of squares of

$$\sum_i \sum_j (x_{ij} - p_{ij})^2 = 906.$$

Table 6 presents the predicted nine month flows based on the above parameters.

"Tuning" the Nonlinear Model

The same input data of tables 1, 3 and 4 were supplied to the nonlinear model. The objective for that model was selected to be

$$\sum_i \sum_j c_{ij} x_{ij}^2$$

where

$$c_{ij} = \alpha \frac{p_j}{p_i} + \beta d_{ij}^2 + \gamma \frac{p_j}{p_i} d_{ij}^2.$$

Table 4. Manpower Allocations per Unit Area

District	Manpower/Unit Area
1	1.25395
2	2.14675
3	.86486
4	3.61026
5	2.42562
6	5.61404
7	3.59552
8	4.83019
9	7.95032
10	10.66666
11	12.80001
12	13.12831
13	9.30903
14	15.05900

Table 5. Results of an Application of the Linear Model on the Nine Month Data of Table 1.

Iteration	Alpha	Beta	Gamma	Error Sum of Squares
1	.5500	.7000	-.1500	1712.
2	.5500	.7000	.1500	950.
3	.5500	.7000	.4500	1320.
4	.5500	.7000	.3000	950.
5	.5500	.7000	.2250	968.
6	.5500	.7000	-.1500	1712.
7	.5500	.7000	0.0000	950.
8	.5500	.7000	.0750	942.
9	.5500	.7000	.3750	1228.
10	.5500	.7000	.2250	968.
11	.5500	.7000	.1500	950.
12	.5500	.7000	-.2250	1362.
13	.5500	.7000	-.0750	950.
14	.5500	.7000	.0000	950.
15	.8500	.7000	.0750	1228.
16	.7000	.7000	.0750	1228.
17	.6250	.7000	.0750	1228.
18	.2500	.7000	.0750	906. <——— Optimal
19	.5500	.7000	.0750	942.
20	.4000	.7000	.0750	1644.
21	.3250	.7000	.0750	1568.
22	-.0500	.7000	.0750	1908.
23	.1000	.7000	.0750	1174.
24	.1750	.7000	.0750	1208.
25	.2500	1.0000	.0750	1344.
26	.2500	.8500	.0750	946.
27	.2500	.7750	.0750	946.
28	.2500	.4000	.0750	1264.
29	.2500	.5500	.0750	1576.
30	.2500	.6250	.0750	1568.
31	.2500	.7000	.3750	1392.
32	.2500	.7000	.2250	1404.
33	.2500	.7000	.1500	1568.
34	.2500	.7000	-.2250	1416.
35	.2500	.7000	-.0750	978.
36	.2500	.7000	.0000	972.

Table 5. (Continued)

Iteration	Alpha	Beta	Gamma	Error Sum of Squares
37	.5500	.7000	.0750	942.
38	.4000	.7000	.0750	1644.
39	.3250	.7000	.0750	1568.
40	-.0500	.7000	.0750	1908.
41	.1000	.7000	.0750	1174.
42	.1750	.7000	.0750	1208.
43	.2500	1.0000	.0750	1344.
44	.2500	.8500	.0750	946.
45	.2500	.7750	.0750	946.
46	.2500	.4000	.0750	1264.
47	.2500	.5500	.0750	1576.
48	.2500	.6250	.0750	1568.
49	.2500	.7000	.3750	1392.
50	.2500	.7000	.2250	1404.
51	.2500	.7000	.1500	1568.
52	.2500	.7000	-.2250	1416.
53	.2500	.7000	-.0750	978.
54	.2500	.7000	.0000	972.

Optimal Solution

Alpha = .2500000

Beta = .7000000

Gamma = .0750000

Table 6. Predicted Flows, x_{ij}'s, from the Linear Model for the Nine Month Data of Table 1.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Row Totals
1	4							1							5
2		28													28
3			9												9
4		1	1	17											19
5					40										40
6						18	8								26
7							17								17
8								3					3		6
9									19	4					23
10				7											7
11					1						16			4	21
12						6					1	18	8		33
13				6								4			10
14										3				12	15
Column Totals	4	29	10	30	41	24	25	4	19	7	17	22	11	16	259

$\alpha = 0.25$

$\beta = 0.70$

$\gamma = 0.075$

Error Sum of Squares = 906

The results of the one-at-a-time search on the parameters of the nonlinear model are given in Table 7. An optimal set of parameters is given by

$$\alpha = 7$$
$$\beta = 11$$
$$\gamma = 5$$

with a total error sum of squares of

$$\sum_i \sum_j (x_{ij} - f_{ij})^2 = 792.$$

The nine month predicted flows of the nonlinear model for the above parameters are given in Table 8.

Comparing Tables 1, 6 and 8 we see that both models did a reasonable job of predicting the intra-district flows while the nonlinear model appeared to do a slightly better job of predicting the observed displacement.

Employing the "Tuned" Models to Predict the Three Month Flow Matrix

Both models were utilized, with their respective optimal set of parameters, to predict the three month flow matrix corresponding to the last quarter of 1974. The input to the models were only the row and column totals of Table 2, together with the data of Tables 3 and 4.

The information indicated by the row and column totals of Table 2 would normally be derived from other forecast models for the total crimes expected to be committed in a district and from projections of criminal residence data.

The optimal predictions based on the linear model are given in Table 9 while those based on the nonlinear model are given in Table 10. Total error sum of squares are computed in each table based on the actual three month data. Again we see that while the linear model performed a reasonably good job of prediction, the nonlinear model seemed to perform a slightly better job of predicting displacement (i.e. the off-diagonal flows).

Table 7. Results of an Application of the Nonlinear Model on the Nine Month Data of Table 1.

Iteration	Alpha	Beta	Gamma	Error Sum of Squares
1	7.0000	3.0000	5.0000	828.
2	11.0000	3.0000	5.0000	830.
3	9.0000	3.0000	5.0000	828.
4	8.0000	3.0000	5.0000	828.
5	7.5000	3.0000	5.0000	828.
6	3.0000	3.0000	5.0000	828.
7	5.0000	3.0000	5.0000	828.
8	6.0000	3.0000	5.0000	828.
9	6.5000	3.0000	5.0000	828.
10	7.0000	7.0000	5.0000	802.
11	7.0000	11.0000	5.0000	792. ← Optimal
12	7.0000	15.0000	5.0000	808.
13	7.0000	13.0000	5.0000	810.
14	7.0000	12.0000	5.0000	810.
15	7.0000	11.5000	5.0000	792.
16	7.0000	7.0000	5.0000	802.
17	7.0000	9.0000	5.0000	792.
18	7.0000	10.0000	5.0000	792.
19	7.0000	10.5000	5.0000	810.
20	7.0000	11.0000	9.0000	820.
21	7.0000	11.0000	7.0000	808.
22	7.0000	11.0000	6.0000	810.
23	7.0000	11.0000	5.5000	792.
24	7.0000	11.0000	1.0000	792.
25	7.0000	11.0000	3.0000	792.
26	7.0000	11.0000	4.0000	792.
27	7.0000	11.0000	4.5000	792.
28	11.0000	11.0000	5.0000	792.
29	9.0000	11.0000	5.0000	792.
30	8.0000	11.0000	5.0000	792.
31	7.5000	11.0000	5.0000	808.
32	3.0000	11.0000	5.0000	810.
33	6.0000	11.0000	5.0000	810.
34	6.0000	11.0000	5.0000	808.
35	6.5000	11.0000	5.0000	792.

Table 7. (Continued)

Optimal Solution

Alpha = 7.00000000

Beta = 11.0000000

Gamma = 5.0000000

Table 8. Predicted Flows, x_{ij}'s, from the Nonlinear Model for the Nine Month Data of Table 1.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Row Totals
1	4	1													5
2		28													28
3			9												9
4				19											19
5					40										40
6				1		23	2								26
7							17								17
8							1	4					1		6
9							1		18	1			1	1	23
10			1	3						3					7
11				2			1			1	16			1	21
12				1	1	1	1		1	2	1	22	2	1	33
13				2			1						7		10
14				1			1							13	15
Column Totals	4	29	10	30	41	24	25	4	19	7	17	22	11	16	259

$\alpha = 7$

$\beta = 11$

$\gamma = 5$

Error Sum of Squares = 792

Table 9. Predicted Flows, x_{ij}'s, from the Linear Model for the Three Month Data of Table 6.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Row Totals
1	1							1	3						5
2		7		2											9
3			1												1
4				4											4
5					10										10
6						9	2								11
7							12								12
8													1		1
9									2	1			4		7
10										1					1
11					1						4				5
12						2					4	2	1		9
13												4			4
14														8	8
Column Totals	1	7	1	6	11	11	14	1	5	2	8	6	6	8	87

$\alpha = 0.25$

$\beta = 0.70$

$\gamma = 0.075$

Error Sum of Squares = 130

Table 10. Predicted Flows, x_{ij}'s, from the Nonlinear Model for the Three Month Data of Table 6.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Row Totals
1	1	1		1				1	1						5
2		6		1					1	1					9
3			1												1
4				4											4
5					10										10
6						10	1								11
7							12								12
8							1								1
9									3	1	1		1	1	7
10					1										1
11											5				5
12						1					1	6	1		9
13													4		4
14											1			7	8
Column Totals	1	7	1	6	11	11	14	1	5	2	8	6	6	8	87

$\alpha = 7$

$\beta = 11$

$\gamma = 5$

Error Sum of Squares = 90

Issues Relative to Accuracy of the Models

We have indicated that both models seem to perform a good job in predicting actual displacement, with the nonlinear model performing a slightly better job than the linear model. The question arises as to how good a job is each model actually doing? Specifically, what level of confidence do we have in the predictions obtained from the two models?

Currently no satisfactory method is available for establishing confidence intervals for the forecasts obtained through the models. This aspect is being pursued by the project team with the hope of a successful resolution of the question in the near future.

Confounding the question of confidence in the projections is the concept of effect of resolution in the district size. If we examine closely Tables 2 and 10 we see that those locations in the respective matrices where the off-diagonal entries are radically different the associated districts are also adjacent. For example, this is the case with the (9,4) cell and the (12,11) cell. If we had decreased the resolution in the aggregated district map or produced another, possibly more natural aggregation the strength of the prediction might well have substantially increased. This additional aspect is also currently under consideration.

Conclusion

This technical note has demonstrated that both the linear model and the nonlinear model can be successfully employed in predicting criminal displacement.

Beyond goodness-of-fit issues the next major efforts include more comprehensive application to other real data sets, possibly from some other selected cities. Also significant effort should be devoted to including the decision making aspect into the models premitting law enforcement agencies to utilize the models in manpower and other resource allocation considerations.

References

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TECHNICAL NOTE #20

The Effect of Massachusetts' Gun Control Law on Gun-Related Crimes in the City of Boston

Introduction

Since eighty-seven percent of the firearms used in crimes committed in Massachusetts are purchased outside of the state, the legislature of this state has made a serious effort to curb violent crimes involving firearms by imposing a mandatory one year minimum sentence for anyone convicted of carrying a firearm without an appropriate license. The purpose of this technical note is to measure the deterrent effect of this law. Thus, the gun-related offenses of homicide, assault with a gun, and armed robbery for the city of Boston will be examined for shifts or changes in their levels in time periods prior to, concurrent with and after the enactment of this law.

To provide the necessary background for this evaluation study, the first three sections of this technical note describe the Massachusetts' Gun Law, the accumulation of the data base for the gun-related crimes of homicide, armed robbery, and assault with a gun, and the postulated impact of the new law on these three crimes. In the next section, the methodological considerations of employing multiplicative empirical-stochastic models [1,2,3] with an embedded shift parameter are briefly described. The last section contains the complete statistical analysis of changes in the occurrence of gun-related crime indices. Here, the deterrent impact of the law is measured by evaluating the changes in the occurrence of homicide, assault with a gun, and armed robbery.

Summary of Massachusetts' Gun Control Law

In April of 1975, the State of Massachusetts formally put into operation a gun control law which mandates a one year minimum sentence upon conviction of carrying a firearm without a special license. The consequences of this law merit serious study for at least two reasons. First, this state-level attempt to curb firearm violence represents a substantial variation from present and prior policy not only in Massachusetts but in the entire United States. Does this altered policy have a deterrent effect? Secondly, prior to the commencement of this law, there was virtually no limit on judicial discretion in providing minimum sentences. What is the effect of this increased pressure on the prosecuting and judicial elements of the criminal justice system?

Although Massachusetts' law on the carrying and ownership of firearms is multifaceted, it can be summarized as follows:

A. A Firearms Owner Identification (F.O.I.) card is required in order to own or possess either a firearm or ammunition. This card can only be issued to non-aliens over eighteen years old who have never been convicted of a felony or hospitalized for drug addiction, drunkeness, or mental illness. The unusual aspect of this facet of the law is that only about 40% of the states require prospective firearms purchasers to prove in advance of acquiring a gun that they have not been excluded according to the above criteria.

B. In addition to satisfying the criteria mentioned in part (A), a prospective handgun purchaser in Massachusetts must also satisfy police of their need to own the handgun, whereupon the police may issue a special license if they are satisfied that such a need exists. However, the police are not required to issue the license even if need of ownership has been established. Although these two facets of the law attempt to curb the availability of firearms, they do not prohibit the importation of firearms from contiguous states and their illegal possession.

C. While the first two facets of the law are directed towards curbing the availability of firearms (including handguns), the third facet is concerned with the carrying of firearms. Although carrying a firearm in most other states is also a criminal offense, the unique feature of Massachusetts' law is the mandatory one-year minimum sentence upon conviction of carrying a handgun without a license to carry or purchase or carrying a rifle or shotgun without a F.O.I. card. Prior to the enactment of the new law, there was virtually no limit on judicial discretion in providing minimum sentences. Under the new law, sentences cannot be suspended and parole cannot be granted until at least one year has been served in jail.

Although the mandatory jail sentencing does remove most judicial leeway in sentencing a defendant, the defendant can still escape the mandatory one-year sentence via three avenues. First, if a person is apprehended with a firearm on his person, the police can file a charge of merely possessing an unlicensed gun in contrast to carrying an unlicensed gun. The possession violation does not carry a mandatory minimum penalty. Secondly, the prosecutor can also press for the lesser violation of possession, regardless of the initial police charge. Thus, the prosecutor still retains the plea bargain option and all its ramifications. Finally, the judge or jury can always find the defendant guilty of the lesser charge. To quote Zimring [4],"the one-year minimum will only invoke mandatory one-year jail terms for carrying firearms without a license to the extent that police, prosecutors, and judges want it to produce such results. If there is strong resistance from any single link in this chain, the mandatory minimum can be avoided".

Although the impact of the new law on the prosecuting and judicial elements of the criminal justice system is uncertain, Zimring hypothesizes that, while the number of jury trials for carrying violations will increase, the number of prosecutions and convictions will decrease. The type of defendant will also change in that he will have a prior criminal record involving violent crimes committed with a gun. Furthermore, the new law may lead to more jail sentences of duration less than one year since more defendants will be charged with the lesser possession violation. This aspect of the new law should also influence the crime reports by increasing the number of possession violations and simultaneously decreasing the number of carrying violations. Although these facets of the law's impact merit investigation, this research is specifically concerned with the deterrence properties, if any, that the new law may have on the commission of certain gun-related crimes.

To measure the effectiveness of the new law as a deterrent to carrying guns and the commission of gun-related crimes, the offenses of homicide, assault with a gun, and armed robbery will be examined for a change in their occurrence levels prior to an after the enactment of the law. Also, because of the localized nature of crime and the criminal justice system and the concentration of crime in bigger cities, the City of Boston will be used as the evaluation site. The data base describing the offenses of homicide, assault with a gun, and armed robbery for the city of Boston is presented in the next section.

Data Base Description

As mentioned in the previous section, the impact of the gun control law was to be examined by investigating three particular types of crime, viz., assault with a gun, armed robbery, and homicide. The purpose of this section is to describe the data base used in the analysis.

The City of Boston forwards to the Uniform Crime Reporting Section of the Federal Bureau of Investigation a form listing the monthly offenses known to police of seven particular types of crime. Certain of these offenses, such as homicide, armed robbery, and assault with a gun, involve the use of firearms. Thus, it is the information concerning these crimes that will be extracted from the reporting forms. However, these reporting forms have undergone some modification over the past ten years, and, as a result, adjustments will have to be made to maintain some type of constancy for the three crimes under study. The time period of interest is from January, 1966 to October, 1975. Although it would be desirable to have months subsequent to October available for analysis, the information for these months is not readily available at this time. Thus, this report is a preliminary one with a six month horizon date since the April, 1975 official enactment of the gun control law. However, this six-month period should be sufficient to detect any immediate or short-term effects of the law as well as to lay a foundation for detecting long-term effects.

In investigating the monthly offense reports from January, 1966 through October, 1975, one sees that there has always been a separate classification for assault with a gun. Thus, no adjustment is needed in the data base listing this particular type of crime. However, it should be noted that commencing with January, 1974 the classification was changed to read "assault with a firearm". A listing of the month offenses for assault with a gun is given in Table 1. The acronym for this data base will be BAG (Boston Assault with a Gun).

In the period of interest, the monthly offense reports have listed murder and nonnegligent manslaughter as a seperate classification. Thus, again no adjustment is needed in the data base describing this type of crime. However, it should be recognized that the murder and nonnegligent manslaughter classification also includes those homicides that resulted from knives or cutting instruments as well as other dangerous weapons. Since the criminal homicides resulting from these other means usually represent a proportionately small amount of the total as compared to homicides with firearms, no attempt was made to adjust for nonfirearm homicides. Furthermore, such an adjustment would have been impossible using the current reporting forms. Table 2 is a listing of the monthly offenses

Table 1. Monthly Offenses of Assault with a Gun for Boston (BAG)

	'66	'67	'68	'69	'70	'71	'72	'73	'74	'75
J	13	21	29	21	26	33	42	29	45	53
F	3	16	22	28	28	39	38	40	40	53
M	16	12	24	28	37	35	34	55	47	29
A	16	15	29	21	32	27	32	43	48	33
M	12	28	33	23	34	36	38	38	61	51
J	7	10	33	27	17	33	48	55	52	40
J	21	21	27	31	40	49	41	37	46	50
A	20	21	30	25	36	50	48	54	53	40
S	20	23	28	40	26	63	61	61	53	47
O	16	33	24	31	37	53	34	47	68	52
N	27	33	31	33	38	34	43	55	56	
D	17	31	19	23	36	53	21	41	57	

Table 2. Monthly Offenses of Homicide for Boston (BOH)

	'66	'67	'68	'69	'70	'71	'72	'73	'74	'75
J	1	6	5	8	3	8	12	12	11	12
F	2	9	4	4	2	9	4	7	11	6
M	3	6	7	10	2	10	5	9	12	12
A	2	4	6	4	10	7	6	6	12	10
M	3	5	8	11	10	8	16	9	11	7
J	4	6	12	9	5	4	3	17	4	10
J	6	7	6	7	11	10	8	10	15	12
A	5	6	7	14	14	10	10	12	11	11
S	11	5	4	7	21	11	9	9	6	11
O	7	3	12	5	12	11	11	14	11	8
N	8	7	17	5	9	16	13	14	14	
D	6	7	14	7	15	12	7	16	16	

for murder and nonnegligent manslaughter. For conciseness, this type of crime will be designated homicide. The acronym for this data base will be BOH (Boston Homicide).

The one type of crime that did require an adjustment was armed robbery. From January, 1966 through December, 1973, the monthly forms contained an armed robbery classification. It should be noted that this classification was used when the robber was armed with any weapon, not merely a firearm. Commencing with January, 1974, the reporting form was altered by dividing the armed robbery classification into three components: robbery with a firearm, robbery with a knife or cutting instrument, and robbery with some other dangerous weapon. To maintain some constancy in the data base for this time period, it was necessary to combine the three seperate armed robbery classifications and to let them be considered as a single entity designated as armed robbery. This approach was adopted since it was impossible to extract those armed robbery offenses prior to January, 1974 which were committed exclusively with a firearm from other types of armed robbery. A listing of armed robbery since January, 1966 is presented in Table 3. This data base will henceforth be referred to as BAR (Boston Armed Robbery).

Thus, in order to analyze the impact of Boston's gun control law, which officially became effective in April, 1975, there are available 118 months of data for each of three types of crime (assault with a gun, homicide, and armed robbery) where six of these data points have been recorded after April, 1975. Although the official enactment date of the gun control law was April, 1975, it should be noted that several of the associated intervention programs commenced several months prior to April. For example, the news media continually publicized the impending law with increasing dosage as April drew near. Thus, if the gun control law did have an impact, it may have started showing up prior to April.

Table 3. Monthly Offenses of Armed Robbery for Boston (BAR)

	'66	'67	'68	'69	'70	'71	'72	'73	'74	'75
J	41	50	74	158	99	136	238	298	327	500
F	39	59	62	124	107	161	213	273	324	451
M	50	63	55	140	112	171	257	312	285	375
A	40	32	84	109	90	149	293	249	243	372
M	43	39	94	114	98	184	212	286	241	302
J	38	47	70	77	125	155	246	279	287	316
J	44	53	108	120	155	276	353	309	355	398
A	35	60	139	133	190	224	339	401	460	394
S	39	57	120	110	236	213	308	309	364	431
O	35	52	97	92	189	279	247	328	487	431
N	29	70	126	97	174	268	257	353	452	
D	49	90	149	78	178	287	322	354	391	

Postulated Impact of the Law

Broadly speaking, the Massachusetts' Gun Control Law, would a priori be expected to make an impact upon two major components of the Criminal Justice System: the police and the courts. It is with respect to the police component alone that this technical note is addressed, in particular, the reported incidence occurrence of homicide, assault with a gun, and armed robbery. If the law has a desirable impact in the police component, one would anticipate the levels for these gun related crimes to decrease because of its general deterrence property.

The manifestations of such changes in reported crimes in these categories are not expected to be identical. That is, changes in the reported rates after "implementation" of the Law, are expected to occur not only in terms of absolute shifts, but also with regards to differring dynamics and delays. Let us consider the "implementation" of the law to start at some time point prior to formal enactment (for example to include preliminary publicity) to be considered as a change in the environment in which crimes are committed. The patterns of potential incremental changes in reported rates in sequential months will encompass the dynamics or transmittal of the effect of the change in the environment on the future reported rate. Lastly, any potential change in future reported rates need not be necessarily observed on the future observed rates instantaneously, but rather after a period of time or delay (a delay in general being greater than or equal to zero, with a delay of zero corresponding to instantaneous impact).

For the three gun related crime categories we would a priori expect different types of measurable impact with respect to the dynamics and delays of a change or shift. That is, with respect to homicide (particularly residential homicide) one would expect a long delay perhaps of the order of magnitude of years before any measurable shift would occur, if, in fact, a decrease in this incidence would occur at all. For example, there is already within Massachusetts a pool of guns owned by residents each representing a potential weapon for a future homicide. With tighter restrictions on licensing, the future pool of such guns would decrease. This in turn might manifest itself in terms of an observable decrease in future homicide occurrences.

On the other hand a shift over a shorter horizon or smaller delay might be expected for the cases of assault with a gun and armed robbery, particularly with respect to the former. One reasonable scenario here would revolve not only around gun availability but also around the minimum jail sentence. Given that the criminal is a rational decision-maker, he perhaps would vie for other weapons in lieu of guns.

The following analysis looks at the crime related data for homicide, gun assault, and armed robbery. It should be noted that this analysis does not address the latter point. That is, it does not look at all assaults versus gun assaults to see if the law merely deters gun assaults by effecting an increase in other assaults.

Methodology Overview

Previous Technical Notes [2, 3] have pointed out the success of the multiplicative empirical-stochastic models of order $(p,d,q) \times (P,D,Q)_S$ in modeling the monthly crime occurrences as tabulated in the Uniform Crime Reports. This section commences with a brief overview of these model forms. In this section, we will also review the highlights of Technical Notes #10 and #14 which addressed the methodological approach needed to detect shifts in a (0,1,1) model form.

For a temporal sequence of crime occurrences (Z_t) for a given index crime, the general form of the multiplicative model of order $(p,d,q) \times (P,D,Q)_S$ is given by

$$\phi_p(B)\ \Phi_P\ (B^S)\ \nabla^d\ \nabla_S^D\ Z_t = \theta_q(B)\ \Theta_Q(B^S)\ a_t, \tag{1}$$

where $\phi_p(B)$ and $\Phi_P(B)$ are the nonseasonal and seasonal autoregressive operators, $\theta_q(B)$ and $\Theta_Q(B^S)$ are the nonseasonal and seasonal moving average operators, ∇^d and ∇_S^D are nonstationary and seasonal differencing operators and S is the seasonal lag. For example, the multiplicative model of order $(0,1,1) \times (0,1,1)_{12}$ is explicitly written as

$$Z_t - Z_{t-1} - Z_{t-12} + Z_{t-13} = a_t - \theta\ a_{t-1} - \Theta\ a_{t-12} + \theta\ \Theta\ a_{t-13}\ . \tag{2}$$

When there is no seasonal component, (P=0, D=0, and Q=0), the multiplicative model reduces to the ARIMA model of order (p,d,q), which is given by

$$\phi_p(B)\ \nabla^d\ Z_t = \theta_q(B)\ a_t\ . \tag{3}$$

Thus, (0,1,1) models are of the form

$$Z_t = Z_{t-1} - \theta\ a_{t-1} + a_t\ . \tag{4}$$

In the previously cited modeling of the seven index crimes, each crime was shown to be represented by a $(0,1,1) \times (0,1,1)_{12}$ form. Since P,D, and Q are not zero, then this seasonal component will be used to transform the original crime data Z_t to W_t where W_t is described by a (0,1,1) integrated moving average model. That is,

$$\left.\begin{aligned} &\frac{\Phi_P(B^S)\ \nabla_S^D}{\Theta_Q(B^S)}\ Z_t = W_t \\ &\text{where} \\ &W_t = \frac{(1-\theta\ B)}{(1-B)}\ a_t \end{aligned}\right\} \tag{5}$$

For a $(0,1,1) \times (0,1,1)_{12}$ model, equation (5) becomes

$$\frac{(1-B^{12})}{(1-\Theta_1 B^{12})} Z_t = W_t \tag{6}$$

where $W_t = [(1-\theta_1 B) / (1-B)]\ a_t$. By using long-division, we see that equation (6) can be written

$$W_t = Z_t - Z_{t-12} + \Theta_1 Z_{t-12} - \Theta_1 Z_{t-24} + \Theta_1^2 Z_{t-24} - \Theta_1^2 Z_{t-36} + \Theta_1^3 Z_{t-36} + \ldots, \tag{7}$$

which is an infinite series. Because W_t depends on infinitely many Z_t's, this poses several startup problems in applying the transformation. For example,

$$W_1 = Z_1 - Z_{-11} + \Theta_1 Z_{-11} - \Theta_1 Z_{-23} + \ldots,$$

yet we do not have any observations prior to Z_1. There are several viable alternatives to overcome this difficulty, of which we shall consider two. In both alternatives, we let $Z_t = 0$ for $t \leq 0$. Thus, from equation (7), we have that $W_1 = Z_1$, . . . , $W_{12} = Z_{12}$, $W_{13} = Z_{13} - Z_1 + \Theta_1 Z_1 = Z_{13} - Z_1 + \Theta_1 W_1$, . . . , $W_{24} = Z_{24} - Z_{12} + \Theta_1 Z_{12} = Z_{24} - Z_{12} + \Theta_1 W_{12}$, etc. In general,

$$W_t = Z_t - Z_{t-12} + \Theta_1 W_{t-12}$$

In the first alternative, we let $W_1 = Z_1$, . . . , $W_{12} = Z_{12}$ and then obtain successive values of W_t by making use of the above equation. Thus, $W_1 = Z_1$, . . . , $W_{12} = Z_{12}$, $W_{13} = Z_{13} - Z_1 + \Theta_1 W_1$, etc. The W_t's obtained in this fashion will be denoted Transformation 1. In the second alternative, we let W_1 through W_{12} inclusive equal to zero. In this case, $W_{13} = Z_{13} - Z_1$, . . . , $W_{24} = Z_{24} - Z_{12}$, $W_{25} = Z_{25} - Z_{13} + \Theta_1 W_{13}$, etc. These W_t's will be denoted Transformation 2. In both transformations one discards W_1 through W_{12} , inclusive, since a twelfth order difference was used.

When evaluating a change in the process level, a policy maker would like his procedures to yield minimal real time delay between the time frame in which the process shifted to the time frame of shift detection, assuming a shift occurred. The transformation, presented in equation (6), ensures early detection from the (0,1,1) within component structure. Therefore, the within component (the (0,1,1) segment) of these models is of primary focus in our analysis. Let us now review the methodology needed for detecting a shift in a (0,1,1) model.

In our current problem setting, decision makers are presented with a total of $N = n_1 + n_2$ observations, where the first n_1 observations occur prior to an intervention effect, A, while the second set of n_2 observations occur after A. These $n_1 + n_2$ observations are denoted by $Z_1, \ldots, Z_{n_1}, Z_{n_1+1}, \ldots, Z_{n_1+n_2}$. It is assumed that all N observations emanate from an ARIMA (0,1,1) model, which can be expressed in random shock form as:

$$\left.\begin{aligned} Z_1 &= L + a_1 \\ Z_t &= L + (1-\theta_1)\sum_{j=1}^{t-1} a_{t-j} + a_t, \quad t = 2, \ldots, n_1 \end{aligned}\right\} \quad (8)$$

while

$$Z_t = L + \delta + (1-\theta_1)\sum_{j=1}^{t-1} a_{t-j} + a_t, \quad t = n_1 + 1, \ldots, n_1 + n_2, \qquad (9)$$

where the parameter θ_1 is known to a sufficient approximation. This is not an unreasonable assumption when the data base is fairly large. The only unknown parameters in equations (8) and (9) are L, the true level of the process at $t = 1$, and δ, the shift accompanying the intervention effect.

In Technical Note #10, it was shown that point estimates of L and δ are given by:

$$\hat{L} = \frac{1-\theta_1}{1-\theta_1^{2n_1}} \left[\sum_{j=1}^{n_1} \theta_1^{j-1} Z_j + \theta_1^{n_1} \sum_{j=1}^{n_1} \theta_1^{n_1-j} Z_j \right] \tag{10}$$

and

$$\hat{\delta} = \frac{1-\theta_1}{1-\theta_1^{2n_2}} \left[\sum_{j=1}^{n_2} \theta_1^{j-1} Z_{n_1+j} + \theta_1^{n_2} \sum_{j=1}^{n_2} \theta_1^{n_2-j} Z_{n_1+j} \right]$$

$$- \frac{1-\theta_1}{1-\theta_1^{2n_1}} \left[\sum_{j=1}^{n_1} \theta_1^{n_1-j} Z_j + \theta_1^{n_1} \sum_{j=1}^{n_1} \theta_1^{j-1} Z_j \right] \tag{11}$$

In order to make additional statistical inferences, one must assume that the a_t's of equations (8) and (9) are NID$(0,\sigma_a^2)$, where σ_a^2 is an unknown parameter representing the variation of the residual a_t's. An estimate of σ_a^2 is provided by

$$\hat{\sigma}_a^2 = \frac{1}{n_1+n_2-2} \left\{ \sum_{j=1}^{n_1} \left[Y_j - \hat{L}\,\theta_1^{j-1}\right]^2 + \sum_{j=n_1+1}^{n_1+n_2} \left[Y_j - \hat{L}\,\theta_1^{j-1} - \hat{\delta}\,\theta_1^{j-n_1-1}\right]^2 \right\} \tag{12}$$

Although the point estimate of δ, given by equation (11), provides some indication of the magnitude of δ, additional flexibility above and beyond the point estimate is needed to allow the decision maker to test $H_0:\delta=0$ vs. $H_1:\delta\neq0$. If the null hypothesis of no shift is true, then

$$\hat{\delta} \Big/ \sqrt{c_{22}\,\hat{\sigma}_a^2} \sim t_{n_1+n_2-2}, \tag{13}$$

where

$$c_{22} = \frac{(1-\theta_1)\,(1+\theta_1)\left[1-\theta_1^{2(n_1+n_2)}\right]}{\left[1-\theta_1^{2n_1}\right]\left[1-\theta_1^{2n_2}\right]}$$

Our decision rule is to reject $H_0:\delta=0$ whenever

$$\left|\hat{\delta} \Big/ \sqrt{c_{22}\,\hat{\sigma}_a^2}\right| > t_{\alpha/2,n_1+n_2-2}. \tag{14}$$

By making use of the distributional property of $\hat{\delta}$, a confidence interval estimate for δ can also be provided:

$$\hat{\delta} \pm t_{\alpha/2, n_1+n_2-2}\, \hat{\sigma}_a \sqrt{\frac{\left[1-\theta_1^{\,2(n_1+n_2)}\right](1-\theta_1)(1+\theta_1)}{\left[1-\theta_1^{\,2n_1}\right]\left[1-\theta_1^{\,2n_2}\right]}} \qquad (15)$$

This confidence interval estimate of δ is extremely useful when one rejects the null hypothesis of no shift. The confidence interval is of the form (c_1,c_2), where $c_1 < c_2$. If both c_1 and c_2 are positive, then the decision maker can be quite sure that there has been a positive shift in the level of the series such as would accompany a change in reporting attitude. The commencement of a crime reduction program should be reflected by a confidence interval where both c_1 and c_2 are negative. A confidence interval in which $c_1 < 0$ while $c_2 > 0$ is indicative of no shift in the series level. That is, even if there was a shift in the level of the series, it was not enough to be statistically significant at the α level. Furthermore, an interval from a large negative value of c_1 to a small positive one for c_2 indicates that if the shift is positive, its magnitude is probably small.

In order to make this methodology viable, the computer program SHIFT was written to perform the calculations needed for making inferences about L and δ. The details of this are presented in Technical Note #14.

In summary, the methodology consists of transforming out the $(P,D,Q)_S$ component of the multiplicative model, leaving only the (p,d,q) component. The methodology presented in Technical Note #10 is then used for detecting a shift in this (p,d,q) component. The next section focuses on the implementation of this methodology by analyzing homicide (BOH), assault with a gun (BAG), and armed robbery (BAR) for the City of Boston.

Analysis

In this section, three examples are presented of the earlier discussed methods for determining a statistically significant process shift. The example data represents the monthly occurrences of homicide, assault with a gun, and armed robbery for the City of Boston from January, 1966 through October, 1975. Figures 1, 2, and 3 display a segment of these series.

As a first step in analyzing these three crimes for a shift in their levels of occurrence, it was necessary to identify the models of the underlying processes and to estimate the models' parameters. This was accomplished by using identification and estimation procedures cited in previous technical notes. The results are shown in Table 4.

To explore the results presented in Table 4 in more detail, let us specifically consider the BAG data file presented in Table 1. The identification procedure suggested that a $(0,1,1) \times (0,1,1)_{12}$ model was appropriate in describing Boston's monthly occurrences of assault with a gun, while the estimation procedure revealed that $\Theta_1 = 0.7751$ and $\theta_1 = 0.8267$. Since neither confidence interval for each of these parameters contains the value zero, both of these parameters are retained in the model. These results agree with the results presented in Technical Note #9 for the more general crime of assault in that a $(0,1,1) \times (0,1,1)_{12}$ model was also appropriate there. By using the value of $\Theta_1 = 0.7751$, one then obtains two new data files: BAGS1 and BAGS2. Both of these data files contain only 106 data points (as opposed to the original 118 data points) and were obtained by using the following recursive relationship:

$$W_t = Z_t - Z_{t-12} + \Theta_1 W_{t-12}, \qquad (16)$$

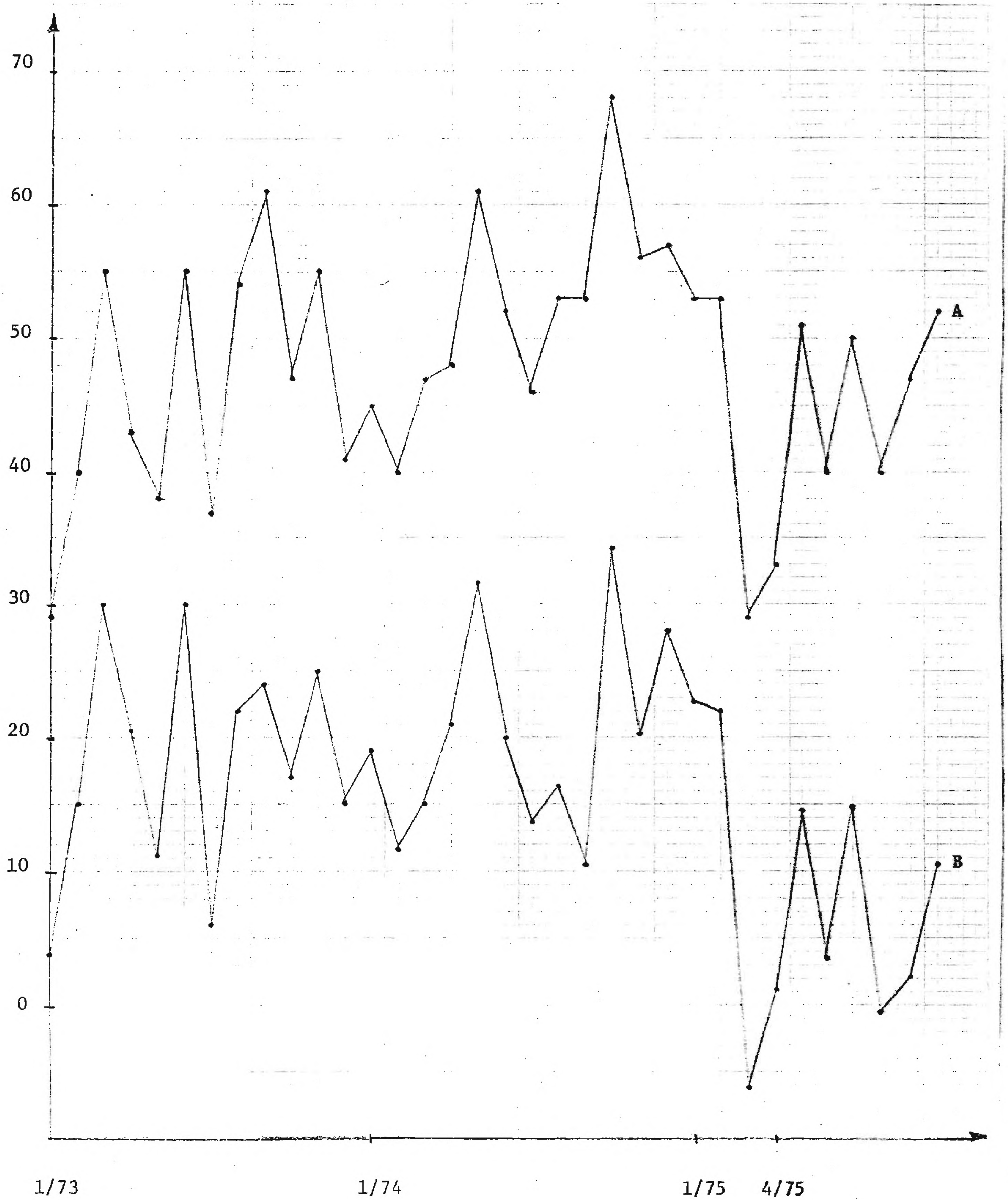

Figure 1. Boston's monthly occurrences of assault with a gun (series A) and the seasonally adjusted data (series B).

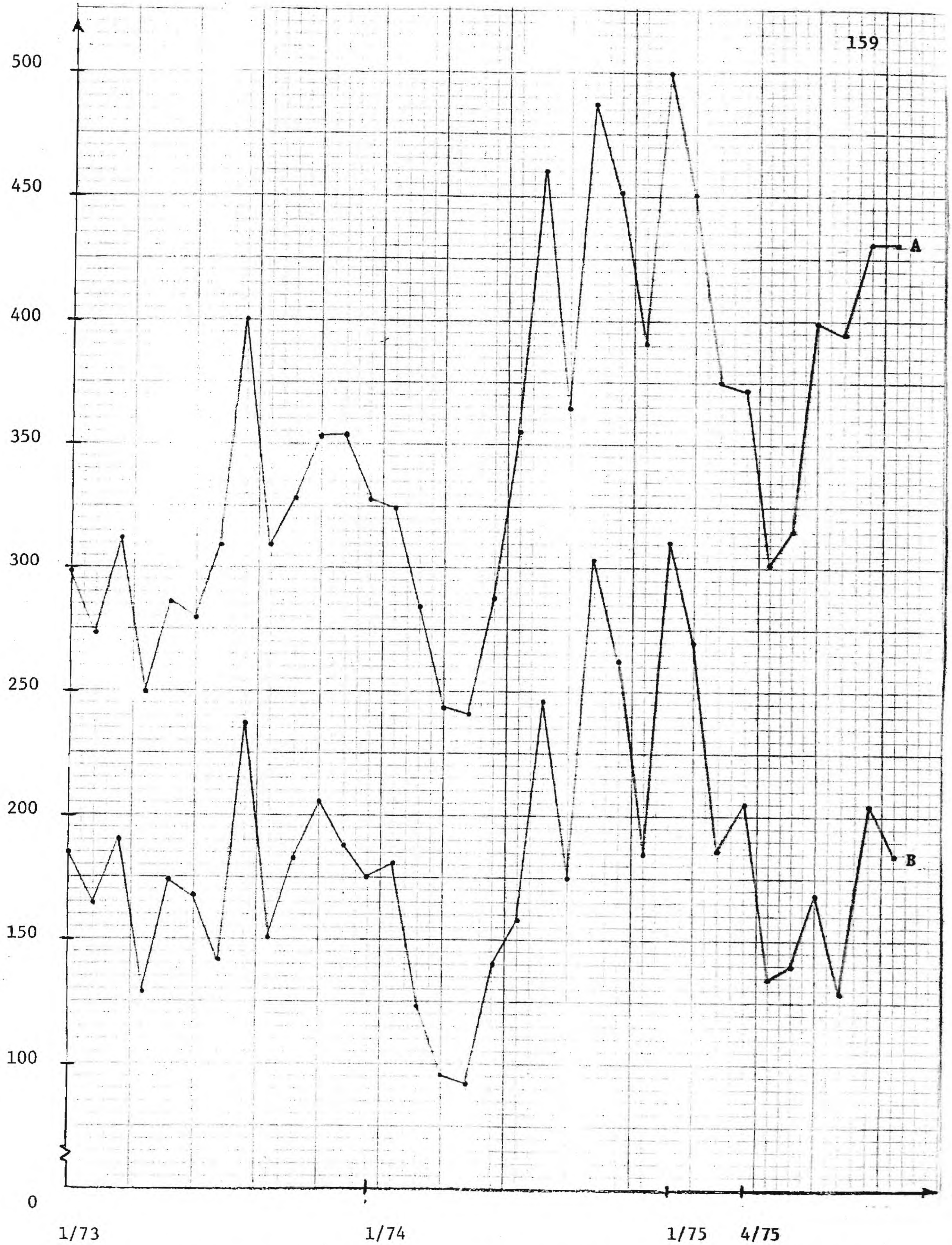

Figure 2. Boston's monthly occurrences of armed robberies (series A) and the seasonally adjusted data (series B).

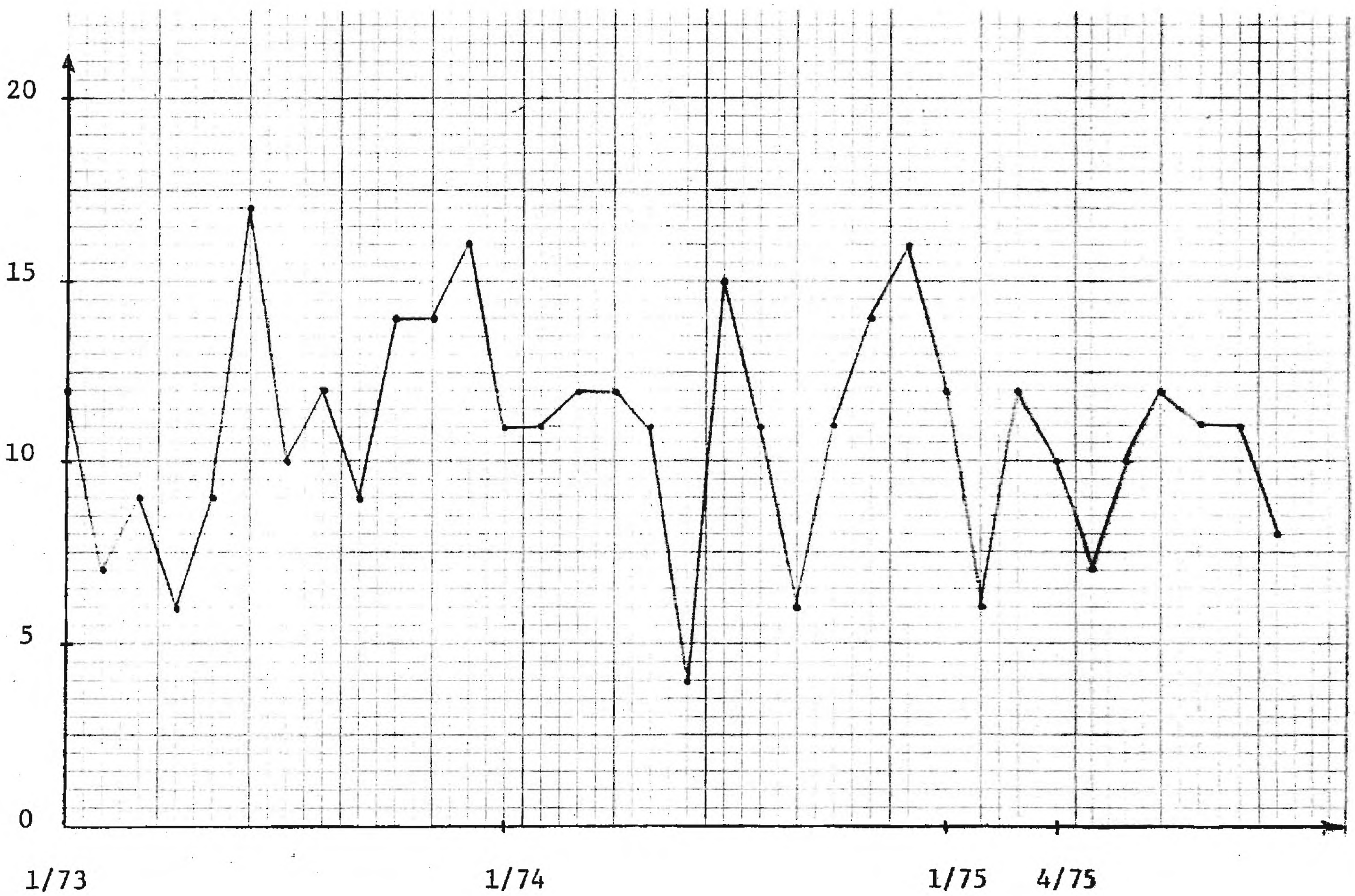

Figure 3. Boston's monthly occurrences of homicide
(no seasonal component).

Table 4. Properties of Data Bases Used in the Analysis

Code Name	Parameter Values θ_1	Θ_1	Residual Variance
ASSAULT*	0.56	0.63	
BAG	0.8267	0.7751	79.79
BAGS1	0.8065	-	73.02
BAGS2	0.8357	-	79.68
ROBBERY*	0.46	0.73	
BAR	0.5128	0.7905	1439
BARS1	0.4996	-	1469
BARS2	0.4986	-	1474
HOMICIDE*	0.81	-	

* Values obtained from Technical Note #9.

where $\theta_1 = 0.7751$. The 106 values of W_t contained in BAGS1 were obtained using Transformation 1 while those in BAGS2 were obtained using Transformation 2 (refer to section on methodology overview). Since the W_t's represent the (0,1,1) within component structure of the process, it is these data points to which the shift detection methodology of Technical Note #10 will be applied. Because of the startup problems mentioned earlier, the estimation procedure was used to estimate the first-order moving average parameter of both BAGS1 and BAGS2. For BAGS1, $\theta_1 = 0.8065$ while for BAGS2, $\theta_1 = 0.8357$. Thus, even though the W_t's differ for BAGS1 and BAGS2, the values of their moving average parameters are nearly identical. We also see from Table 4 that $Var(a_t)$, the residual variance, are all in the same neighborhood. For these reasons, the shift detection methodology will be directed solely to BAGS1. Since similar statements are applicable to the BAR data base only the BARS1 data base will be used to detect a shift in armed robbery. Because the occurrence of homicide does not follow a seasonal pattern, we need an estimate only of the moving average parameter, and this was equal to 0.81 (refer to Technical Note #9).

We will first consider the crimes of assault with a gun and armed robbery. For our problem setting, we originally had 118 data points for each of these crimes, and this covers the period from January, 1966 through October, 1975. The segment from January, 1973 through October, 1975 is presented in Figures 1 and 2. Since a seasonal differencing operator was applied to the original data (Z_t), this leaves us with 106 data points (W_t) comprising the nonseasonal component. If we can assume that the W_t's correspond to the Z_t's, then this data extends from January, 1967 through October, 1975. The latter thirty-four months of these nonseasonal components is depicted in the lower portions of Figures 1 and 2. It is these nonseasonal components that will be examined for a statistically significant process shift. We will be testing $H_0: \delta = 0$ since we have no prior information regarding the magnitude of δ. That is, we wish to test that the process level will be unchanged in the future from what we have observed in the past.

Table 5 summarizes the analysis for assault with a gun. We will see that the analysis and subsequent conclusions are contingent upon our reference point. If we are standing at October, 1974 ($n_1 = 94$) and have information regarding only November, 1974 ($n_2 = 1$), we have strong reason to believe that no shift has occurred because the significance level is approximately one. If we continue to remain at October and look forward two, three, even four months, we still must conclude that the shift, if any, was not statistically different than zero. The conclusion remains unchanged even when we look forward through October, 1975 ($n_2 = 12$). However, the estimates of δ as well as the significance levels for this reference point of $n_1 = 94$ decrease as the number of months we look ahead increases. This phenomenon occurs because even though the process does remain relatively stable through February, 1975 ($n_2 = 4$), gun assaults decreased dramatically commencing March, 1975 and remained at a lower level. Thus, for $n_1 = 94$ and $n_2 = 5, \ldots, 12$, we conclude that the process level has not shifted because we are mixing the pre-and post-shift observations. However, this is not the manner in which we advocate using the shift methodology. The larger values of n_2 were used only to indicate the long-term behavior of any shift. Since we did not reject $H_0{:}\delta=0$ for $n_1 = 94$ and $n_2 = 1$, the data element in n_2 consistent with the past process data elements contained in n_1 would be added to those contained in n_1 and the analysis repeated. For both $n_1 = 95$, $n_2 = 1$ and $n_1 = 96$, $n_2 = 1$, we again conclude that there has been no change in the process level. Note that for the $n_1 = 96$ reference point, we again experience the phenomenon of decreasing estimates of δ and decreasing significance levels as n_2 increases. This is especially evident as n_2 increases from 2 to 3. Again, this suggests that the process level may be changing between February and March of 1975. This is substantiated when $n_1 = 97$ and $n_2 = 2$. Since we conclude that there has

Table 5. Shift Detection Results for Assault with a Gun.

n_1	n_2	$\hat{\delta}$	T	SIG. LEVEL	95% CONF. INT.
94	1	0.04	0.00	0.997	(-16.13, 17.10)
	2	3.79	0.60	0.550	(- 8.74, 16.31)
	3	3.30	0.59	0.557	(- 7.80, 14.40)
	4	2.93	0.57	0.576	(- 7.44, 13.29)
	.	.	.	.	.
	.	.	.	.	.
	.	.	.	.	.
	12	-5.13	-1.03	0.307	(-15.05, 4.79)
96	1	0.94	0.12	0.908	(-15.13, 17.00)
	2	0.61	0.10	0.922	(-11.80, 13.03)
	3	-8.38	-1.44	0.153	(-19.95, 3.18)
	4	-10.84	-1.98	0.051	(-21.71, 0.04)
	5	-9.90	-1.88	0.063	(-20.34, 0.54)
	6	-10.79	-2.10	0.038	(-20.98, -0.61)
	.	.	.	.	.
	.	.	.	.	.
	.	.	.	.	.
	10	-11.23	-2.28	0.025	(-21.00, -1.45)
97	1	0.10	0.01	0.990	(-15.88, 16.08)
	2	-13.74	-2.13	0.036	(-26.56, -0.92)
98	1	-28.21	-3.52	0.001	(-44.11,-12.31)
	2	-24.56	-3.96	0.000	(-36.88,-12.24)
	.	.	.	.	.
	.	.	.	.	.
	.	.	.	.	.
	8	-18.01	-3.76	0.000	(-27.50, -8.52)

been no shift in the process for $n_1 = 97$, $n_2 = 1$, we update n_1 by one. When we are standing at February, 1975 ($n_1 = 98$) and have information regarding March ($n_2 = 1$), we see that a statistically significant shift has occurred. The small significance level (0.001) strongly supports this conlusion. Furthermore, when we continue to remain at February and sequentially update n_2, we see that the significance level remains at exceptionally small values (0.001 or less). Thus, commencing in March of 1975, the process level shifted dramatically and it continued to remain at this level, at least through October, 1975. That the level of gun assaults significantly decreased one month prior to April (the official enactment date of the law) is not too surprising in view of the early publicity provided by the news media.

Let us now turn our attention to armed robbery and again use October, 1974 ($n_1 = 94$) as our first reference point. Table 6 summarizes the shift detection results for this crime. If we have information regarding only November, 1974, ($n_2 = 1$), we would conclude that the process level has not changed. As a matter of fact, we would cling to this conclusion at least for five additional months, i.e., $n_2 = 5$. Although this suggests relative stability within the October, 1974 through March, 1975 time frame, subsequent analysis obtained by updating n_1 revealed that a statistically significant decrease occurred at the 0.061 level when $n_1 = 95$, $n_2 = 1$, while a statistically significant increase occurred at the 0.012 level when $n_1 = 96$ and $n_2 = 1$. Another statistically significant decrease occurred when $n_1 = 98$, $n_2 = 1$. However, because the data element contained in n_2 is not consistent with the n_1 data elements for this time frame, one may retain the relative stability conclusion that was proposed when $n_1 = 94$, $n_2 = 5$. Let us now update our reference point to February, 1975 ($n_1 = 98$) and look one month ahead. We conclude there has been a process shift at the 0.033 significance level; viz., a desirable decrease. By incrementing n_2 for this reference point, we see the decreasing significance levels and continued, relatively large negative

Table 6. Shift Detection Results for Armed Robbery

n_1	n_2	$\hat{\delta}$	T	SIG. LEVEL	95% CONF. INT.
94	1	17.71	0.49	0.622	(-53.48, 88.90)
	2	-12.95	-0.40	0.691	(-77.50, 51.61)
	3	5.32	0.16	0.871	(-59.51, 70.15)
	4	5.70	0.18	0.860	(-58.41, 69.80)
	5	1.91	0.06	0.954	(-63.30, 67.12)
98	1	-80.36	-2.17	0.033	(-153.86, -6.86)
	2	-73.28	-2.22	0.029	(-138.74, -7.82)
	3	-85.38	-2.63	0.010	(-149.84,-20.92)
	4	-87.73	-2.72	0.008	(-151.62,-23.83)
	5	-86.96	-2.72	0.008	(-150.50,-23.42)
	6	-87.69	-2.74	0.007	(-151.10,-24.28)
	7	-86.98	-2.70	0.008	(-150.90,-23.06)
	8	-86.93	-2.71	0.008	(-150.55,-23.30)
99	1	-22.49	-0.60	0.553	(-97.36, 52.38)
	2	-50.73	-1.49	0.140	(-118.29, 16.84)
100	1	-81.83	-2.18	0.032	(-156.45, -7.22)
	2	-79.71	-2.38	0.019	(-146.11,-13.31)
	⋮	⋮	⋮	⋮	⋮
	6	-72.91	-2.25	0.027	(-137.20, -8.62)
101	1	-35.60	-0.93	0.355	(-111.59, 40.39)
	2	-24.14	-0.71	0.481	(-91.91, 43.63)
	3	-29.44	-0.89	0.378	(-95.41, 36.52)
	4	-23.52	-0.71	0.482	(-89.63, 42.59)
	5	-23.05	-0.70	0.488	(-88.76, 42.65)

estimates of the process shift. This suggests a significant decrease in the level of the process and its maintenance at this reduced level. When we change our reference point to April, 1975 ($n_1 = 100$), we again see that all the significance levels are less than 0.05 for every possible value of n_2. The results for the February and April reference points indicate a substantial decrease in the number of armed robberies commencing with February, 1975 and continuing with April, 1975 through October, 1975. Again, the gun control law has left its impact.

A plot of the monthly occurrences of homicide from January, 1973 through October, 1974 is shown in Figure 3. Since there is no seasonal component in homicidal occurrences, we retain the 118 observations given in Table 2 and note that April, 1975 corresponds to $n_1 = 112$. Table 7 summarizes the results of detecting a shift for homicide.

When we are standing at December of 1974 with 108 months of past information about the process and information for any subsequent month(s) up to $n_2 = 10$, the analysis revealed that at no time was the significance level below the 0.460 level. As a matter of fact, for all subsequent values of n_1 (up to 117), there were only two instances in which the significance level was less than 0.30. For $n_1 = 109$, $n_2 = 1$, the significance level was 0.125; for $n_1 = 109$, $n_2 = 4$, the significance level was 0.203. Thus, it is reasonable to conclude that there has not been a shift in the monthly occurrences of homicide from at least December, 1974 through October, 1975. If the new law has had an impact on homicide, the results are not statistically significant at the usual levels of significance.

Thus, the gun control law has affected armed robbery and gun assault while any effect it may have on homicide has not become apparent. Because of the large proportion of residential homicides, any future impact of gun control on homicide in general may not show up for several years, if ever.

Table 7. Shift Detection Results for Homicide

n_1	n_2	$\hat{\delta}$	T	SIG. LEVEL	95% CONF. INT.
108	1	1.50	0.40	0.690	(-5.93, 8.94)
	2	-1.92	-0.66	0.513	(-7.73, 3.89)
	3	-1.11	-0.43	0.670	(-6.27, 4.05)
	4	-1.19	-0.49	0.626	(-6.01, 3.63)
	5	-1.73	-0.74	0.460	(-6.37, 2.90)
	6	-1.53	-0.67	0.502	(-6.04, 2.98)
	7	-1.20	-0.53	0.592	(-5.64, 3.23)
	8	-1.16	-0.53	0.601	(-5.54, 3.22)
	9	-1.13	-0.52	0.605	(-5.46, 3.20)
	10	-1.27	-0.59	0.559	(-5.58, 3.03)
109	1	-5.78	-1.55	0.125	(-13.19, 1.62)
110	1	1.32	0.35	0.727	(-6.14, 8.77)
111	1	-0.93	-0.25	0.804	(-8.36, 6.49)
112	1	-3.76	-1.01	0.316	(-11.15, 3.63)
113	1	0.96	0.26	0.798	(-6.43, 8.35)
114	1	2.78	0.75	0.456	(-4.58,10.13)
115	1	0.25	0.07	0.947	(-7.09, 7.59)
116	1	0.20	0.05	0.957	(-7.11, 7.51)
117	1	-2.84	-0.77	0.442	(-10.12, 4.44)

Conclusions

A statistical evaluation of the Massachusetts' Gun Control Law has been conducted. The evaluation covered the time period prior to enactment of the law and included a six month horizon after enactment of the law. This evaluation centered on the laws' potential impact on the police component of the Massachusetts' Criminal Justice System, in particular, the occurrences of homicide, assault with a gun, and armed robbery in Boston.

The analysis, which employed empirical-stochastic models with an embedded shift parameter, has indicated a statistically significant decrease in both armed robbery and assault with a gun in this time period. No statistically significant changes in the homicide rate, however, were observed. Further, the specific time points in which these decreases were noted strongly suggests their probable direct association with the introduction and enactment of this law.

It should be noted that this work is not intended to be the total evaluation of the Gun Control Law. The methods and results, however, of this initial evaluation would suggest its use in continual monitoring. For example, in monitoring over longer horizons, any real changes observed here in armed robbery and assault with a gun might dissipate with the occurrence level returning to the pre-gun law level or might, in fact, more desirably perpetrate at its newly observed lower level. In addition, it is expected that, even though the homicide rate is not now effected by the law, a longer horizon is needed to allow for observation of any potential real-time delayed impact. Further, this initial evaluation should be expanded to address non-gun related crimes to assess whether the law is, in addition to deterring gun-related crimes, displacing previously potential gun related incidences to comparable non-gun related incidences.

The initial evaluation, nonetheless, has illustrated, with real data, the feasibility of statistically quantifying in an evaluation setting changes in the overall criminal justice environment.

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Quarterly Report #5

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Title: Stochastic Modeling and Analysis of Crime

Project Director/Principal Investigator:

Quarterly Report #5

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Project Director/Principal Investigator:

Dr. Stuart Jay Deutsch
October 31, 1976

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Stochastic Modeling and Analysis of Crime

July 1, 1976 - September 30, 1976

(Project Director/Principal Investigator: Dr. Stuart Jay Deutsch)

This quarterly report is intended to summarize the progress and events that have occurred during the fifth quarter of Grant 75NI-99-0091.

Technical Note #21, "Preliminary Between Cities Comparison of Crime Models", describes the procedures for comparing the univariate time series models developed earlier for each of the index crimes. Joint confidence intervals are developed for six of the eight index crime rates which resulted in three parameter models. Pictorial representation of the joint confidence intervals are presented for Atlanta, Boston, Cleveland, Kansas City, Los Angeles, Portland, and St. Louis for the following crime rate models; all crime, vehicle theft, assault, larceny, robbery, and burglary. Univariate confidence intervals are also summarized for the crime rate models of homicide and forcible rape for all cities understudy.

This note represents the first report on our ongoing effort of identifying differences between crime rates in cities. Currently, we are reviewing previously reported literature on socio-economic and demographic variables and their impact on crime rates. Simultaneously we are accumulating this type of data for subsequent statistical analysis and correspondence to the observed differences in the parameter values of our crime models.

In the next Technical Note #22 titled, "A Negative Cycle Approach for the Non-linear Transportation Model of Criminal Displacement and Deterrence", the computational efficiency of the displacement and deterrence model is addressed. A refinement which results in an order of magnitude saving in the computational time is presented here. In previous technical notes using the displacement and deterrence model on sample data the computational times observed necessitated modification to the algorithm prior to using the model on a full set of data. With this modification using a negative cycle approach we are now in a reasonable position to apply the model to a full data set.

During this quarter we have expanded further effort in accumulating a data base for this model. We have identified the availability of the needed data in Kansas City. However, "constraints" on manpower in the data section in Kansas City does not allow transfer until next spring. In addition, we have received approximately 150,000 records from Atlanta for one calender year. We are currently encoding this information to assess its applicability. We feel however, that this full data set will be useful, particularly in addressing the displacement capabilities of our current model.

Technical Note #23 titled, "An Overview of the Development of a Criminal Displacement and Deterrence Model", summarizes the progress to date, future considerations, and work to be done on the displacement and deterrence model.

On August 24, 1976, a site visit was conducted. Here, we presented the work completed during the current quarter and the previous four quarters of our effort. The full site evaluation is contained in TA#184, "Stochastic Modeling and Analysis of Crime", prepared by J. Thomas McEwen, Ph.D., PRC Public Management Services, Inc.

TECHNICAL NOTE #21

Preliminary Between Cities Comparison of Crime Models

Introduction

In earlier notes, univariate time series models were developed for the part I, F.B.I. Index crimes [1,2,3]. One result of these modeling efforts was that each index crime was described by a single model form. Therefore, any differences in the characteristics of the $j^{\underline{th}}$ crime type between cities is described by differences in the parameter value(s) associated with the models. In six of the eight crime types; all crime, vehicle theft, assault, larceny, robbery, and burglary, resulted in a three parameter model. For homicide and forcible rape the resulting models were two parameter models. In each of these eight categories, either the variance of the observations or the variance of the model's residuals must be specified along with the remaining (p-1) autoregressive or moving average parameters.

Inference about similarities or differences in city crime rates are dependent upon the joint confidence interval of the parameters within the $j^{\underline{th}}$ crime category. The purpose of this technical note is to develop joint confidence intervals for the crime rate models so that preliminary comparative analysis can be made. In the first section, the methodological procedure for developing joint confidence intervals is briefly described. Following this development, the joint intervals for the six crime types listed earlier for the cities of Atlanta, Boston, Cleveland, Kansas City, Los Angeles, Portland, and St. Louis are constructed. A similar comparison for the crime types of homicide and forcible rape based upon marginal confidence intervals is then presented. Lastly, a discussion of these preliminary results and further consideration and future tasks are summarized.

Joint Confidence Intervals

In this section we proceed to develop the joint confidence interval specification via the likelihood function. To start, recall that our basic assumption regarding the residuals of the univariate models is that $a_t \sim$ NID $(0,\sigma^2)$. Since a set of a_t's, t=1,2,...N are independent, and each is normally distributed, then for any of our models;

$$P(a_i|\underline{\phi},\underline{\Phi},\underline{\theta},\underline{\Theta}) = \frac{1}{\sqrt{2\pi}}\sigma\ e^{-1/2a_i^2/\sigma^2}$$

$$P(a_1,a_2,\ldots,a_n|\underline{\phi},\underline{\Phi},\underline{\theta},\underline{\Theta}) = \left(\frac{1}{\sqrt{2\pi}\ \sigma}\right)^n \left(e^{-\frac{1}{2\sigma^2}2a_1^2}\right) \cdots\cdots \left(e^{-\frac{1}{2\sigma^2}2a_n^2}\right) \quad \text{or}$$

$$P(\underline{a_t}|\underline{\phi}\ \underline{\Phi}\ \underline{\theta}\ \underline{\Theta}) = \left(\frac{1}{\sqrt{2\pi}\ \sigma}\right)^n e^{-\frac{1}{2\sigma^2}\sum_{t=1}^{n} a_t^2}$$

From a tentative model form, the a_t's are related to the observations Z_t,

$$a_t = Z_t - f(\underline{Z_t},\underline{\phi}\ \underline{\Phi}\ \underline{\theta}\ \underline{\Theta}).$$

Thus, we can replace $\sum_{t=1}^{n} a_t^2$ by,

$$\sum_{t=1}^{n}\left(Z_t - f(\underline{Z_t},\underline{\phi},\underline{\Phi},\underline{\theta},\underline{\Theta})\right)^2.$$

The likelihood function for the joint distribution of the residuals is then,

$$\ell(\underline{\phi},\underline{\Phi},\underline{\theta},\underline{\Theta}|\underline{Z_t}) \propto \sigma^{-n}e^{-\frac{1}{2\sigma^2}S(\underline{\phi},\underline{\Phi},\underline{\theta},\underline{\Theta})},$$

where $S(\underline{\phi},\underline{\Phi},\underline{\theta},\underline{\Theta})$ is called the sum of squares function. It should be noted that given the observations, $\underline{Z_t}$, this functions value is only dependent upon the choice of numerical values of $\underline{\phi},\underline{\Phi},\underline{\theta}$, and $\underline{\Theta}$. The problem of estimation is that of selecting the appropriate combination of these parameter values to maximize the likelihood function. Further, it should be noted that the estimate of σ^2 is also determined from the choice of these autoregressive and moving average parameters, since it is the variance of the models residuals. Thus, upon taking natural logs,

$$L(\underline{\phi},\underline{\Phi},\underline{\theta},\underline{\Theta}|\underline{Z}_t) \propto S(\underline{\phi},\underline{\Phi},\underline{\theta},\underline{\Theta}).$$

The sum of squares function can be broken down to two components. For notational convenience, let us simply denote all parameters by $\underline{\beta}$ and $f(\underline{Z}_t,\underline{\phi},\underline{\Phi},\underline{\theta},\underline{\Theta})$ by $\underline{X}\ \underline{\beta}$. Then,

$$S(\underline{\beta}) = (\underline{Z}_t-\underline{X}\ \underline{\beta})^1(\underline{Z}_t-\underline{X}\ \underline{\beta}) = (\underline{Z}_t-\hat{\underline{Z}}_t)^1(\underline{Z}_t-\hat{\underline{Z}}_t) + (\hat{\underline{Z}}_t-\underline{X}\ \underline{\beta})^1(\hat{\underline{Z}}_t-\underline{X}\ \underline{\beta}).$$

The first term is the sum of squares due to the error and the second term, the sum of squares due to regression. It can be shown that given a set of N observations, $S(\underline{\beta})$ has a Chi-Square distribution with N degrees of freedom,

$$S(\underline{\beta}) \sim \sigma^2\ \chi^2_N.$$

Furthermore, the two components on the right hand side can be expressed as,

$$(\hat{\underline{Z}}_t-\underline{X}\ \underline{\beta})^1(\hat{\underline{Z}}_t-\underline{X}\ \underline{\beta}) \sim \sigma^2\ \chi^2_K$$

$$(\underline{Z}_t-\hat{\underline{Z}}_t)^1(\underline{Z}_t-\hat{\underline{Z}}_t) \sim \sigma^2\ \chi^2_{N-K},$$

where K is the number of parameters in $\underline{\beta}$ and where these two Chi-Squares are independent.

Rewriting $S(\underline{\beta})$ in more convenient terms, we have

$$S(\underline{\beta}) = S(\hat{\underline{\beta}}) + (\underline{\beta}-\hat{\underline{\beta}})^1\underline{X}^1\underline{X}\ (\underline{\beta}-\hat{\underline{\beta}}).$$

Note, $S(\underline{\beta})$ is quadratic in $\underline{\beta}$, $S(\hat{\beta})$ is a constant and $(\underline{\beta}-\hat{\underline{\beta}})^1\underline{X}^1\underline{X}(\underline{\beta}-\hat{\underline{\beta}})$ is also quadratic. This last term, we will for simplicity, label $Q(\underline{\beta})$. Thus, $Q(\underline{\beta})$

defines the equation of an ellipse if K=2, an ellipsoid if K=3, and an hyper-ellipsoid if K>3. The ratio of $Q(\underline{\beta})$ to $S(\hat{\underline{\beta}})$ each standardized by their associated degrees of freedom is distributed F,

$$\frac{Q(\underline{\beta})/K}{S(\hat{\underline{\beta}})/N\text{-}K} \sim F_{K,N-K} .$$

Pictorially the sum of squares function can be easily described by contours of constant sum of squares. The entire surface of this function for a two parameter model is contained in Figure 1. Here we see that the minimum value of this function $S(\hat{\underline{\beta}})$ occurs at the least squares values of the parameters $\hat{\beta}_1$ and $\hat{\beta}_2$. Further, if we move in any direction on the β_1, β_2 plane away from the point $\hat{\beta}_1$, $\hat{\beta}_2$ the value of the sum of squares function is greater than $S(\hat{\beta})$. The increase in the value above the minimum value $S(\hat{\beta})$ is dependent upon the distance of $(\beta-\hat{\beta})$ as contained in the $Q(\underline{\beta})$ expression.

For each value of $S(\underline{\beta}) > S(\hat{\underline{\beta}})$ a plane parallel to the β_1, β_2 plane through the sum of square function produces a contour in the form of an ellipse or an approximate ellipse. This contour will be perfectly elliptical when the model is linear in the parameters and approximately elliptical when the model is nonlinear in the parameters. The degree of elliptical approximation being dependent on the degree of nonlinearity.

The larger the difference between $\underline{\beta}$ and $\hat{\underline{\beta}}$, the larger the area of the contour and therefore, the larger the confidence that the values $\underline{\beta}$ will be contained in the contour. The exact association of this degree of confidence to a value of the sum of squares function which results in a given size contour is determined by fixing the type I or α error in the selection of a value of F, $F_{K,N-K,1-\alpha}$. Thus,

$$S_{critical} = S(\hat{\underline{\beta}}) \left[1+ \frac{K}{N-K} F_{K,N-K,1-\alpha} \right] ,$$

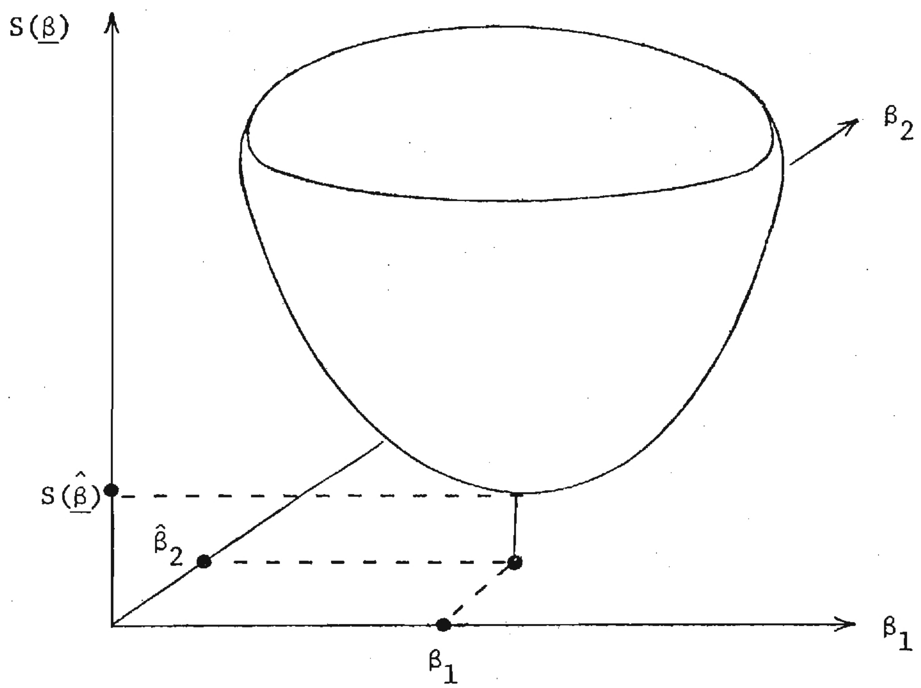

Figure 1. Sum of Squares Function for Two Parameter Model

gives the value of the sum of squares function that corresponds to the area in which our faith is $(1-\alpha)$ 100% that the parameters $\underline{\beta}$ are contained within.

Construction of Joint Confidence Intervals

The previously described method of developing joint confidence intervals was employed for the following crime types; all crime, vehicle theft, assault, larceny, robbery, and burglary. It should be noted that in each of these crime types the models contain three independent parameters, one of which is the residual variance (or alternately, the variance of the observations). As seen in the development of the joint confidence intervals through the sum of squares function, the entire surface is generated by different combinations of the (p-1) autoregressive and moving average parameters alone, with the p^{th} parameter, σ^2, being fixed by the choice of the estimates of the previous (p-1) parameters.

Thus, all three parameter crime models give rise to a sum of squares function of the same dimension as illustrated in Figure 1. For each of these surfaces the sum of square value corresponding to the ninety-five percent confidence interval was computed. Figures 2 through 7 contain the ninety-five percent confidence intervals for all crime, vehicle theft, assault, larceny, robbery, and burglary, respectively. In these figures, each associated with a given crime type are overlays of the comparable confidence intervals for the cities of Atlanta, Boston, Cleveland, Kansas City, Los Angeles, Portland, and St. Louis.

Marginal Intervals of Homicide and Forcible Rape

For the crime types of homicide and forcible rape, the resulting models contained two parameters. As in the case of three parameter models, the confidence interval of the parameters is reduced since σ^2 is fixed by the estimate of the moving average parameter. The confidence intervals for the moving average parameter for the crime types of homicide and forcible rape are contained in Figures 8 and 9, respectively.

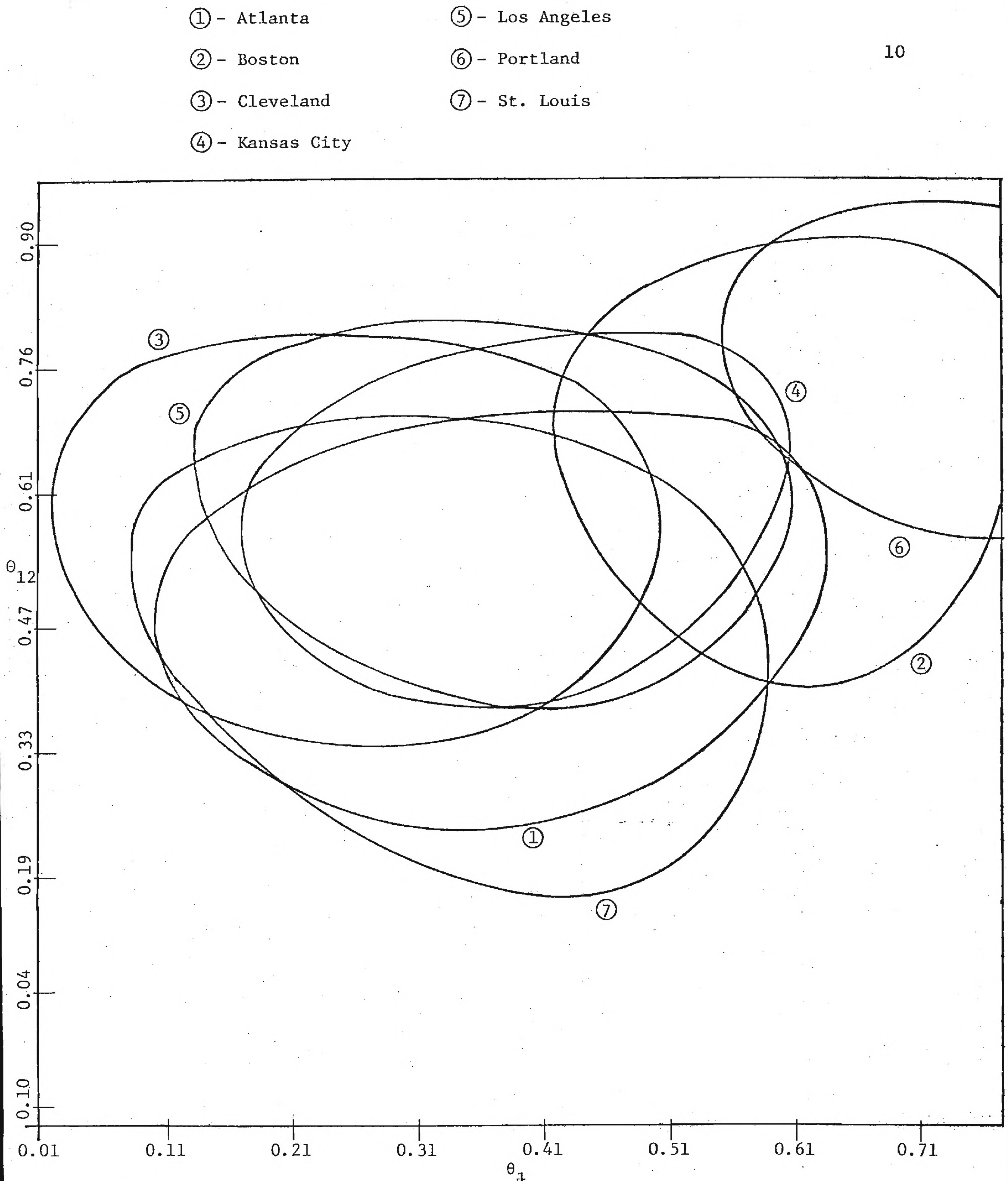

Figure 2. 95% Confidence Intervals for All Crime Totals.

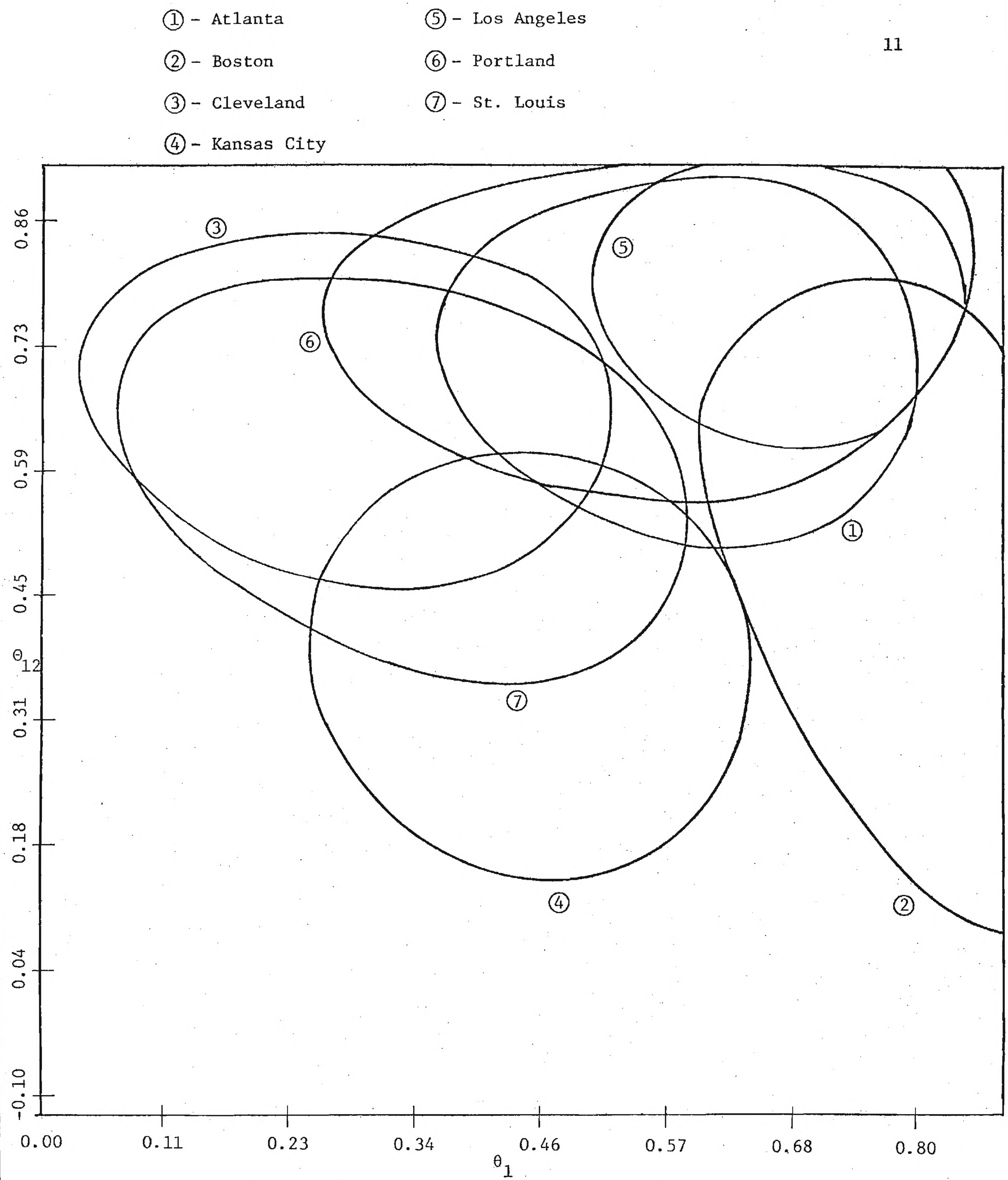

Figure 3. 95% Confidence Intervals for Vehicle Theft.

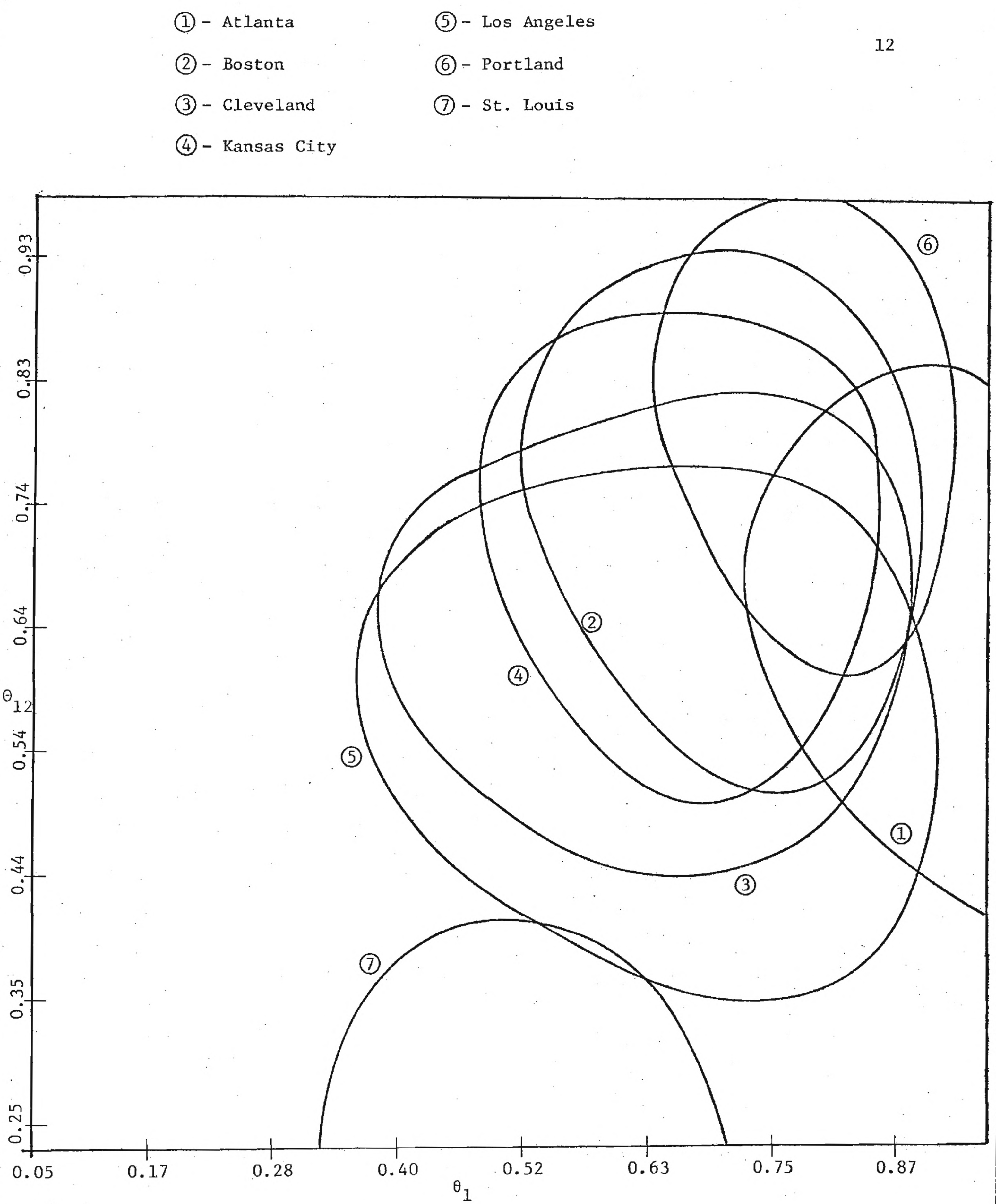

Figure 4. 95% Confidence Intervals for Assault.

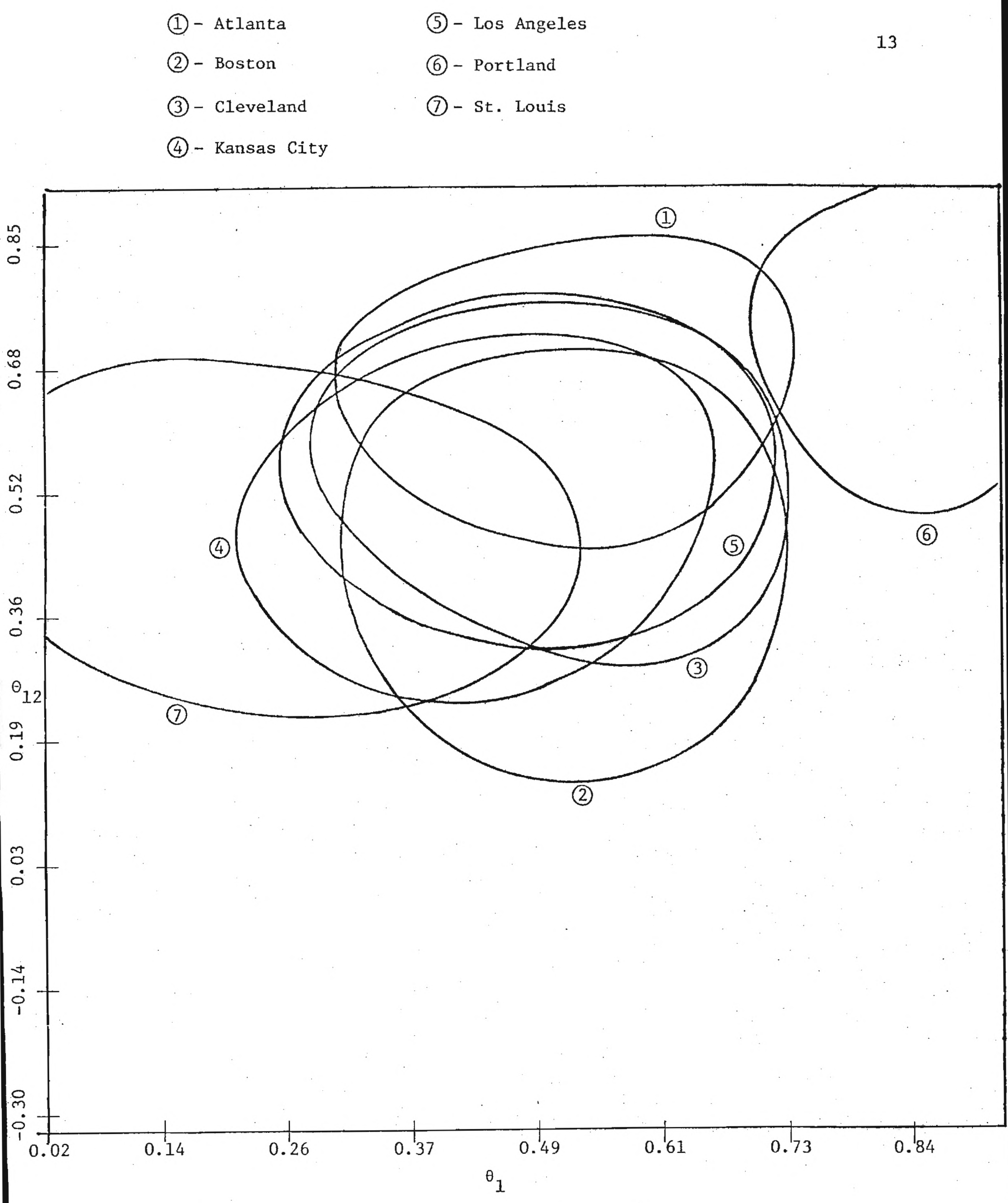

Figure 5. 95% Confidence Intervals for Larceny.

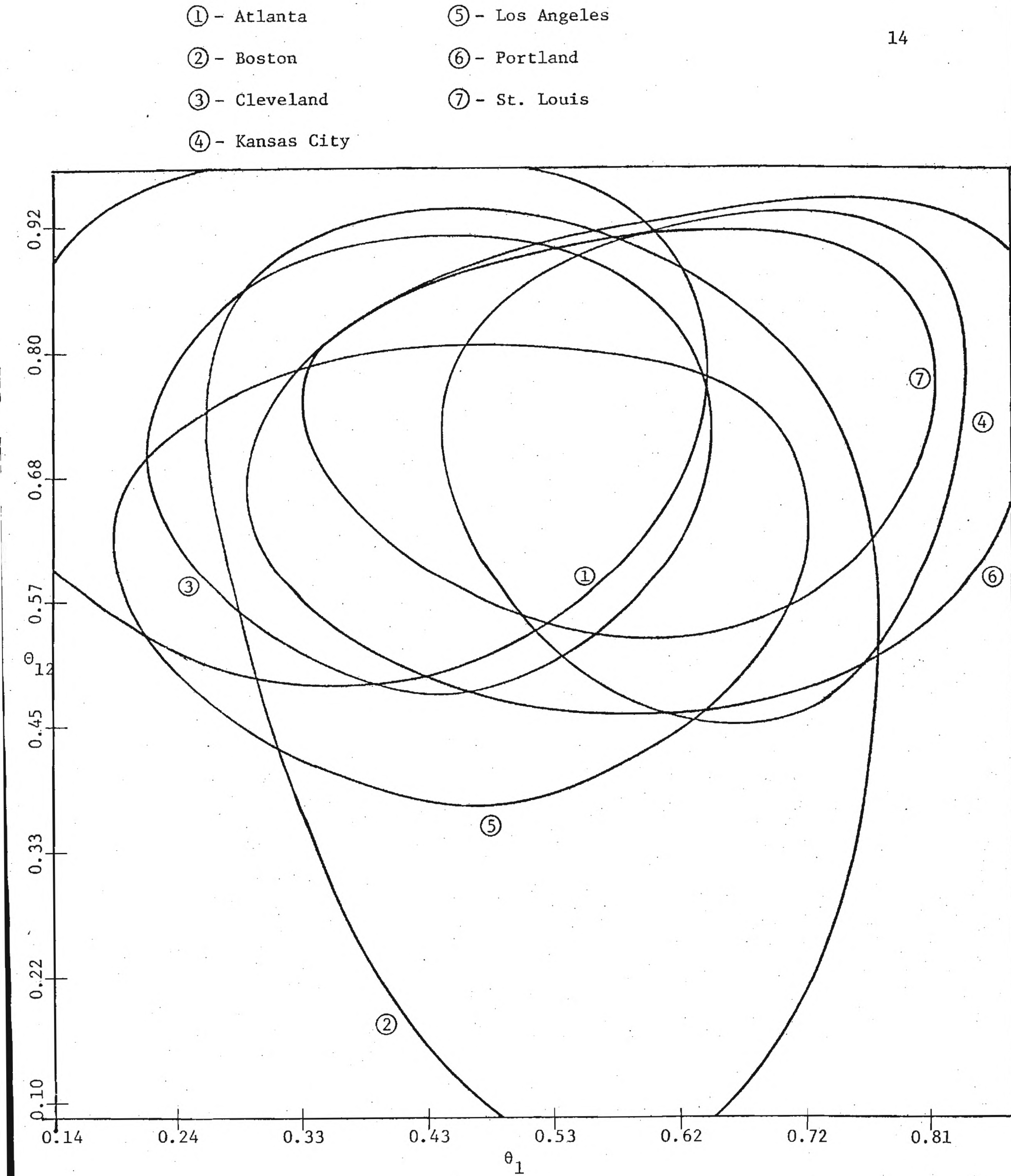

Figure 6. 95% Confidence Intervals for Robbery.

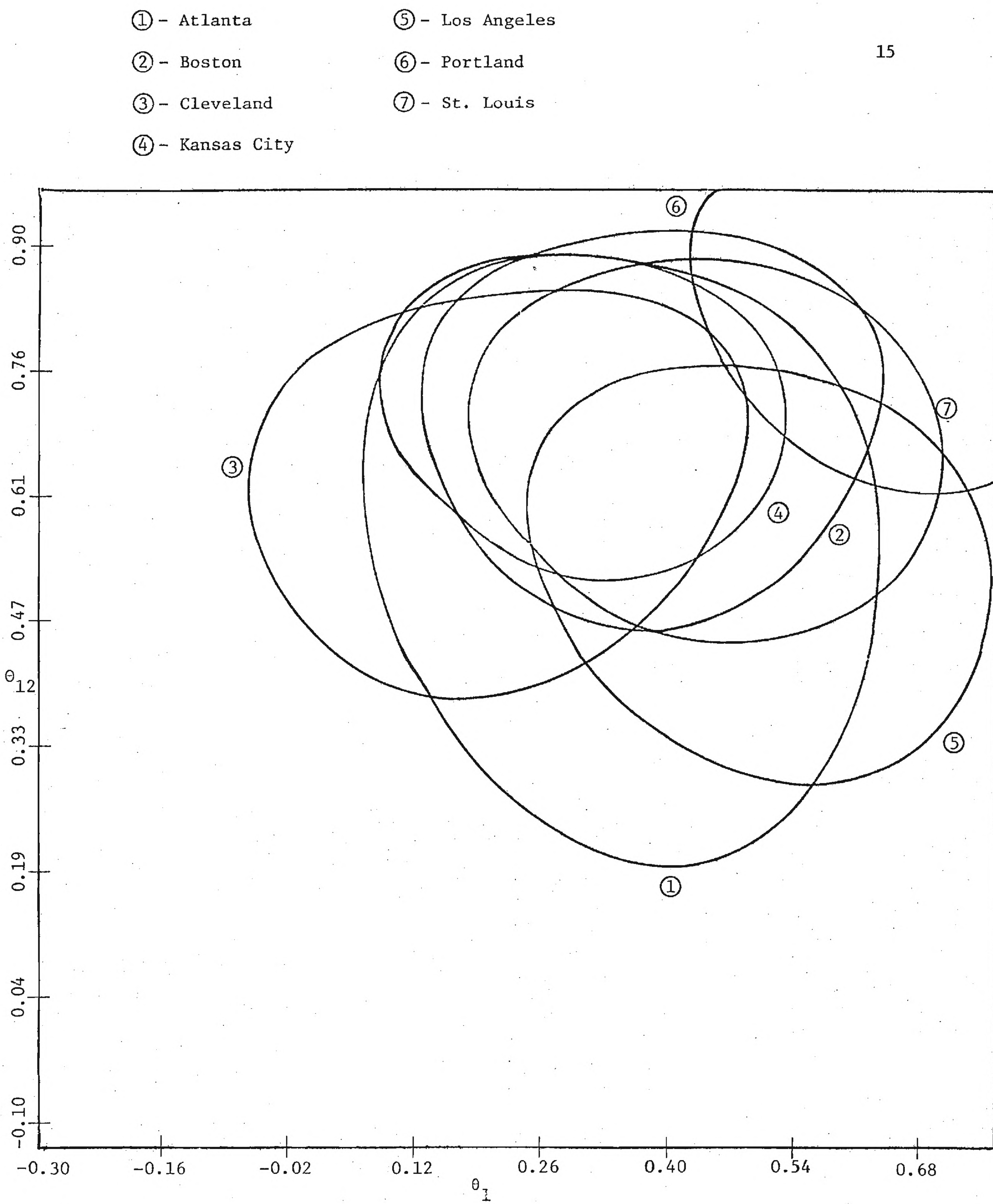

Figure 7. 95% Confidence Intervals for Burglary.

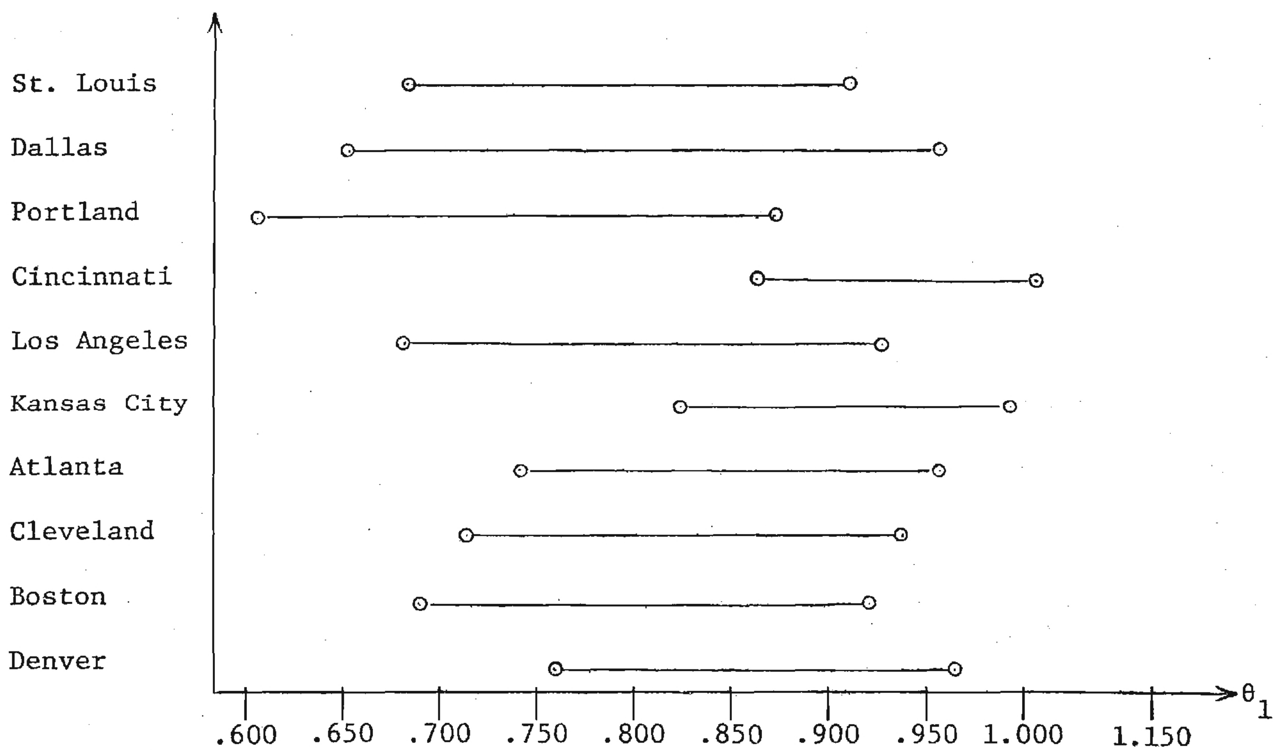

Figure 8. Confidence Intervals for Homicide

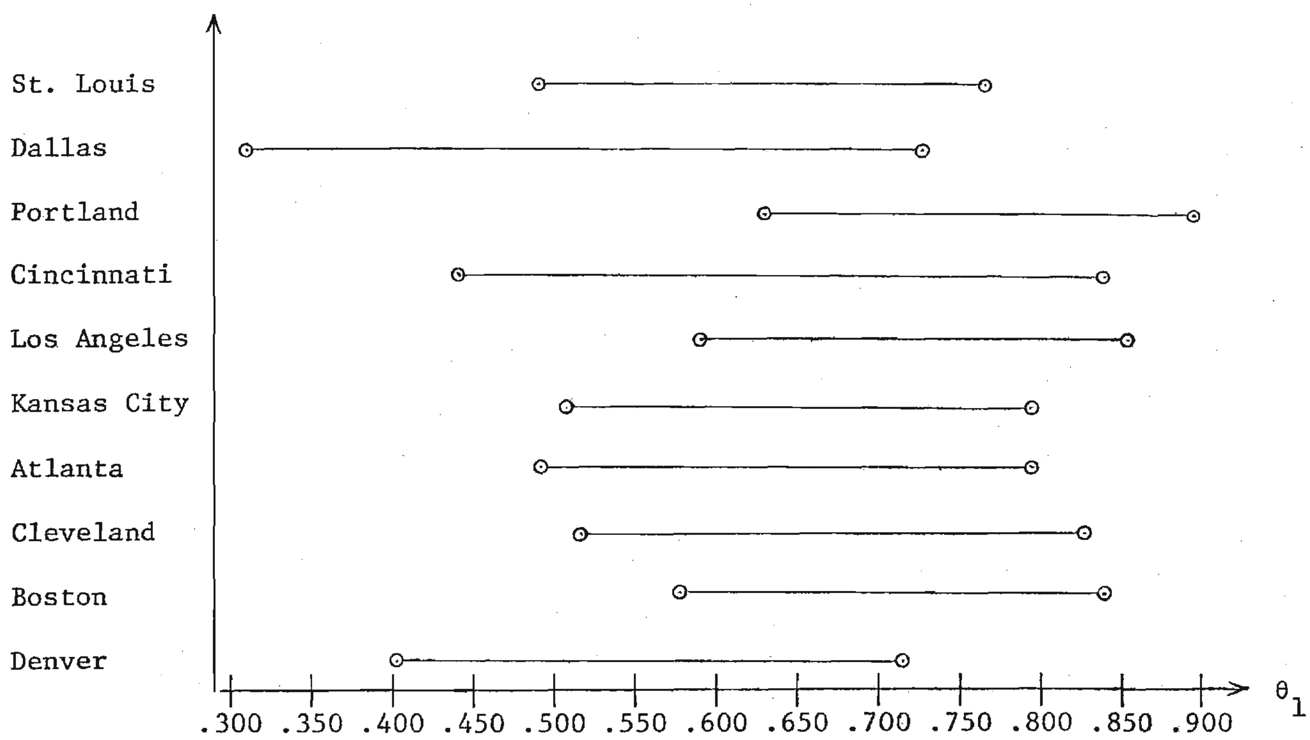

Figure 9. Confidence Intervals for Forcible Rape

Preliminary Discussion of Results

With the joint confidence intervals, direct comparison of the crime rate characteristics between cities for a given crime type can be made. That is, for a given crime type if the point estimates of the $j^{\underline{th}}$ city does not fall in the $(1-\alpha)$ 100% confidence interval for the $i^{\underline{th}}$ city, $i \neq j$, $j=1,2,\ldots n$, $i=1,2,\ldots,n$ where n is the number of cities in the comparison set then the characteristics of the crime rates are statistically different.

For example, let us look at Robbery in the seven city set illustrated in Figure 6. The pairwise comparisons indicate that, at the 95% level of significance;

- Boston is Different from Atlanta
- Kansas City is different than Atlanta
- Portland is different than Atlanta
- Kansas City is different than Cleveland
- Kansas City is different than Los Angeles,

with all other paired comparisons being statistically identical.

Similar comparisons of the joint intervals for larceny from Figure 5 indicate that at the 95% level of significance;

- Boston is different than Atlanta
- Portland is different than Atlanta
- St. Louis is different than Atlanta
- Portland is different than Boston
- St. Louis is different than Boston
- Portland is different than Cleveland
- St. Louis is different than Cleveland
- Portland is different than Kansas City
- Portland is different than Los Angeles
- St. Louis is different than Portland.

The cross tabulation between the above sets of statistically significant differences within Robbery and Larceny raises several interesting questions.

Note that of the five statistically significant paired comparisons within the crime type of robbery, two elements; Boston vs. Atlanta and Portland vs. Atlanta are also statistically different in the larceny comparison. Yet the statistically significant difference in robbery noted for Kansas City vs. Atlanta or Cleveland or Los Angeles is statistically identical for the crime type of larceny. Also to be noted, is that several cross-comparisons that are statistically significant within the larceny comparisons are statistically identical within the robbery comparisons.

As stated in the introduction, this note is a preliminary step in the identification and reconciliation of differences in crime rates. Our hypothesis is that different levels of select socio-economic and demographic variables will influence the crime rate. Currently, we are doing a more substantive statistical analysis while reviewing previously reported studies and results in this area. We plan to accumulate potential socio-economic/demographic data sets to relate such variables to the parameters of the time series models so that the models will have the self-contained capabilities of discrimination between crime rates. These studies will be reported in subsequent technical notes.

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A Negative Cycle Approach for the Nonlinear Transportation Model of Criminal Displacement and Deterrence

Introduction

In Technical Notes [1,2,3] we have discussed linear and nonlinear transportation models for describing the flows in the criminal displacement model. Sample computational experience has indicated that the shortest path approach described by T.C. Hu [5] may prove to be time consuming for reasonable sized problems.

In this technical note we briefly discuss another method due to Klein [6] for solving nonlinear (convex) transportation problems associated with the displacement model. The approach described herein has the advantage of utilizing the (very fast) linear cost transportation method to obtain a "good" starting solution; thereby, reducing the total computational effort.

The present approach requires the location of "negative cycles" in a network. A "negative cycle" is a cycle (closed loop) around which the sum of costs is negative. We shall, later in this technical memo, discuss various methods for locating negative cycles.

For clarity of presentation we shall assume that the nonlinear transportation problem is of the form:

$$\text{Minimize} \quad \sum_{i=1}^{m} \sum_{j=1}^{m} k_{ij}(x_{ij})$$

subject to

$$\begin{aligned} &\sum_{j=1}^{m} x_{ij} = a_i \qquad i=1, \ldots, m \\ &\sum_{i=1}^{m} x_{ij} = b_j \qquad j=1, \ldots, m \\ &x_{ij} \geq 0 \qquad i,j = 1, \ldots, m \end{aligned} \tag{1}$$

where $k_{ij}(x_{ij})$ is a convex function of the single variable x_{ij}. Note that if $k_{ij}(x_{ij})$ is convex then,

$$k_{ij}(x_{ij}+1) - k_{ij}(x_{ij}) \geq k_{ij}(x_{ij}) - k_{ij}(x_{ij}-1) .$$

We shall refer to the left-hand side of the above inequality as the up-cost, $u_{ij}(x_{ij})$, and the right-hand side as the down-cost, $d_{ij}(x_{ij})$ associated with displacement flow x_{ij}.

The Nonlinear Transportation Procedure

In the negative cycle approach to solving the nonlinear cost displacement models we begin with any feasible displacement pattern satisfying the zone (node) labels. There are simple procedures for accomplishing this, as we shall see later in this note.

Once a feasible displacement pattern is obtained, up-costs and down-costs are computed as follows:

$$u_{ij}(x_{ij}) = k_{ij}(x_{ij}+1) - k_{ij}(x_{ij})$$

$$d_{ij}(x_{ij}) = \begin{cases} \infty & \text{if } x_{ij} = 0 \\ k_{ij}(x_{ij}-1) - k_{ij}(x_{ij}) & \text{if } x_{ij} > 0. \end{cases}$$

Here, $u_{ij}(x_{ij})$ represents the incremental cost of increasing the displacement (flow) from zone i to zone j by one unit; $d_{ij}(x_{ij})$ represents the incremental cost of decreasing x_{ij} by one unit.

With the up-costs and down-costs defined, we look for a set of flows to change so that a new feasible displacement pattern results and the total cost (disutility) is reduced). This requires changing flows around a negative cycle by the same amount. Unless special knowledge (in addition to convexity) of the cost function is at hand, it is best to change flows around the cycle by a single unit; some flows go up and others go down in such a way that the constraint equations (1) remain satisfied. Given the cycle, this is very easy to accomplish.

Once a negative cycle has been located and flows changed we are ready to repeat the procedure with a recomputation of up-costs and down-costs. The procedure is continued until such time as no more negative (improving) cycles exist in the network.

An Improved Starting Solution Based on the Linear Cost Transportation Method

Obviously, the "better" the starting solution, the more rapidly the negative cycles approach will converge to the optimal solution, as it must for convex cost functions. Since the linear cost transportation method is so fast, we may utilize it to obtain a reasonably good starting solution for the negative cycles, nonlinear method. It is best to utilize a cost function designed especially for the linear model when obtaining the initial solution.

Once the linear transportation method has been applied to obtain an initial feasible displacement pattern we simply apply the nonlinear transportation method from this point utilizing its corresponding up-costs and down-costs until optimality is achieved. Experience indicates that, using the starting displacement pattern specified by the linear transportation method, we proceed more rapidly to the (optimal) nonlinear displacement pattern specified by the nonlinear transportation method.

The Comparison of Running Times for Various Negative Cycle Location Algorithms

The nonlinear criminal displacement model involves examining a network for the existence of negative cycles. Numerous techniques are available for the search including a direct search method and various shortest path algorithms. It should be noted that our test problems were run on a 30 node network.

Our first attempt was a method which terminated after locating 26 negative cycles in 320 sec. The 26 cycles were inadequate to complete the evaluation of a single set of weighting values (α,β,γ).

The negative cycle location routine was reprogrammed using Florian's Direct Search Method (DSM) [4]. Florian's algorithm completed 11 iterations in 512 sec. which involved the evaluation of 11 sets of weighting factors and the discovery and subsequent tracing of 158 negative cycles. The 11 iterations did not provide a complete solution to the problem. The difficulty with Florian's algorithm as noted by Yen [7] is the upper bound on the number of additions and comparisons required to detect the existence of a negative cycle. Yen illustrates that the DSM may require as many as $\Sigma_{i=1}^{N-1} \binom{N-1}{i} . i!$ Additions and comparisons to locate a negative cycle (where N = number of nodes in the network).

Yen suggests a dynamic programming algorithm for the shortest path problem which can be employed to locate negative cycles [8]. This shortest path algorithm has an upper bound of $N^3/2$. If the algorithm does not cover in N iterations it is because there exists at least a shortest route from some (i) to (N) that has more than N-1 arcs (i.e., a negative cycle). The cycle can be located by tracing the optimal paths at the Nth iteration.

Yen's algorithm solved the example problem for the criminal displacement model in 42 sec. The linear model was solved in 5 sec. with an objective function value of 1218. The nonlinear model required 37 sec. for solution and had an objective function of 792. The complete solution to the nonlinear model required 39 iterations (.95 sec/iter.) and the location of 50 negative cycles. The efficiency of Yen's algorithm can be seen in that cycles of "large" negative value are located first. Whereas, Yen's algorithm solved the nonlinear model (39 iterations) by locating 50 negative cycles, Florian's DSM performed only 11 iterations while locating 158 negative cycles.

A worst case example problem for the nonlinear model was formulated and solved using Yen's algorithm in 72 sec. This problem involved 49 iterations

(1.47 sec/iter.) and the location of 117 negative cycles. A summary of the 3 methods employed on the original 30 node network problem is shown in Table 1.

TABLE 1

Algorithm	Time (Sec.)	Iterations	Time/iter.	#Cycles
Simple	320	0	∞	26
Florian DSM	512	11	46.54	158
Yen	37	39	.95	50

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TECHNICAL NOTE #23

An Overview of the Development of a Criminal Displacement and Deterrence Model

I - Introduction

Over the past year a number of Technical reports [1, 3, 4, 5, 6, 7, 8] have been developed which describe a displacement and deterrence model based on optimization methodology. In this memorandum we shall review the current state of development of that model. Through the memorandum we hope to bring into sharper focus the role of the displacement model in the evaluation of criminal justice projects and programs.

In addition to reviewing the development of the displacement model and how it may be used, we shall also discuss how data may be obtained for the model and also what near term, future efforts remain for extension and refinement of the capabilities of the displacement model.

What Do We Mean By Displacement?

The current public perception is that crime is moving from the inner city out into the suburbs. Just what, exactly, does this mean? At one level this statement may simply mean that crime is increasing in the surburbs at a faster rate than in the inner city. It is, however, likely that a deeper sense of public perception exists: that increasing numbers of criminals are moving out of the inner city into the suburbs to commit their crimes. It is to this second level of concern that the current displacement model is directed.

To better understand the displacement model, consider a metropolitan area. Suppose we consider the metropolitan area to be divided into a number of zones. The zones may be defined by natural boundries, economic conditions or any community cohesiveness factors.

If a criminal who resides in zone i also commits a crime in zone i we shall not regard this as displacement. However, if a criminal who resides in zone i travels to zone j to commit a crime that we shall regard this as displacement. Thus, the current model adopts a physical view of displacement.

Obviously, due to our point of reference, the individual zone, the concept of displacement is a relative term. Clearly, if our definition of "zone" is small enough then displacement would always take place when crimes are committed; whereas if our definition of "zone" is large enough then no displacement ever occurs. Much care must be placed on a proper definition of "zone" in order that reasonable conclusions regarding displacement can be made.

What Is The Distinction Between Displacement And Deterrence?

As we have indicated, displacement is a term used to classify a specific action of a criminal, viz. the traveling from one zone to another to commit a crime. Deterrence represents the action of some outside force, the police, community, etc., which causes a criminal to not commit a crime which, in the absence of that force, he would most surely have committed.

With this view, a clearly identifiable community goal would be the proper application of its total resources in much a way as to maximize deterrence, i.e., reduce crimes committed. A natural question is how does the concept of displacement integrate into the concept of deterrence.

A Potential Community Goal: Minimize Displacement

It is often said that in order to effectively reduce crime it must first be contained. Applying this concept of containment to displacement, programs and projects might be evaluated on the basis of their ability to affect reductions in crime displacement. Minimizing displacement is equivalent to containing the criminal.

In subsequent sections of this memorandum we shall detail the displacement model as well as its use and data requirements. Emphasis will also be directed toward the important relationship between the displacement model and the univariate models which have been utilized in other tasks of the contract.

II - Comparison with Regression

In a forecasting mode the displacement model performs in essentially the same manner as a regression model. The fundamental difference is that regression models require a fundamental set of closed form equations which relate the forecasted values over each time period; whereas, the displacement model developed to date utilizes an optimization process to establish the forecasts. We shall briefly review the steps in the application of each method to provide a more direct comparison of the two.

The fundamental steps in the application of a regression technique to obtain forecasts of criminal displacement are as follows:

1a. Obtain historical data on criminal displacement (this requires crimes reported and cleared by arrests). This will become the dependent variable in the regression models.

b. Obtain historical data on socio-economic, demographic and police resource data to be used as independent variables in the regression models.

2. Establish appropriate forms for the regression models, relating the independent variables to the dependent variables. Specifically, determine the form of the regression equations.

3. Apply a least squares method to fit the regression models to the historical data. The results will be the attainment of the regression coefficients.

4. Apply future forecasts of the independent variables to the regression models to obtain the future forecasts. This step corresponds to using the future values of the independent variables in the regression equations to obtain the required estimates.

The steps in the application of the displacement model in a forecasting mode are similar.

1a. Obtain historical data on criminal displacement.

b. Obtain historical data on socio-economic, demographic and police resource data.

This step is identical to that of the regression model.

2. Establish appropriate forms of the criterion function in the displacement model. Unlike the regression models, equations for direct relationships between displacement and the independent variables are not established. Rather, equations relating the attractiveness values to the independent variables are established.

3. Apply a search procedure to determine the values of the parameters of the criterion function which provides the "best" (in a least squares sense) fit to the historical data. This step is similar, although the exact execution of the step is different from the corresponding step in the regression model.

4a. Apply the univariate models to obtain crime occurrence and criminal activity projections.

b. Apply future forecasts of the independent variables and the forecasts from step 4(a) to the displacement model to obtain the future forecasts of displacement.

The fundamental difference in this step is that instead of using equations in a closed form manner, the displacement model utilizes an optimization procedure.

As can be seen, the regression method and the displacement procedure used in a forecasting mode are very similar in their application. Two important differences should be discussed. First, the displacement model includes the effect of interactions and interrelationships in the displacement data. Individual regression models preclude any capability of handling interaction effects. Second, the regression equations attempt to relate displacement directly to the independent variables. These relationships are complex and as such, are difficult to understand. On the other hand, the displacement model attempts to relate the relative attractiveness of zones to the independent variables. While this relationship is also difficult to understand it is one step less removed than that of regression. Also, displacement projections are more readily interpretible when accompanied by attractiveness values.

III - The Displacement and Deterrence Model

In this section of the memorandum, we address the fundamental structure of a quantitative model for displacement and deterrence. Further, the organization of this section is such that we will consider displacement appart from deterrence. We create such a format in order to make clear a basic difference which exists in the way the two aspects are analyzed.

The notion of mathematical modeling, in general, is an important one in quantitative evaluation. Generally speaking, a given system (marketing, inventory, criminal displacement, etc.) is represented symbolically such that variables and parameters are combined into appropriate mathematical functions which reflect, analytically, the specific system's attributes. The model, in essense, mimics the system and a specific outcome relative to the "real" system can be analyzed in the form of solutions to its mathematical representation. Obviously, the decision making power of a model is only as good as its construction. Following, we consider a model for criminal displacement and look specifically as what its manipulation involves and what its solution means.

Network Flows and the Basic Model of Crime Displacement

Earlier, a definition for displacement was suggested which referred to the physical transport of a crime from one "zone" to another "zone" in some metropolitan region. That is, if a crime is committed in a given zone by a perpetrator who resides in another zone, then displacement of that crime has resulted. If the offender resides in the zone of commission, no displacement has taken place. Let us symbolize such a displacement system by using some elementary aspects of network models.

Given a metropolitan region and some specification of zones, suppose we let each zone be a node or point. Further, let us assume that each criminal (or potential, crime perpetrator) in any zone can move to any other zone to commit a crime.

Obviously, some zone to zone movement may be more intuitive than other, but we make no subjective specifications at this point. Then, in order to depict the movement from zone of residence to zone of crime commission, we can connect every pair of nodes (zones) by an arc (arrow). Each arc is directed or stated alternately, is pointing from one node to another node, in order to depict that movement of a crime is "from" some point "to" another. Consider Figure 1, where a four zone (4 node) metropolitan area is characterized. Note in the figure that every node has an arc directed from itself to itself. This depiction takes into account the instance where a crime is committed in the same zone as that of residence. Recall that such a depiction implies that no displacement has occurred.

The structure given in Figure 1 is, at least structurally, feasible. However, we can create a more manageable representation by transforming the information in the figure to that of Figure 2 where a different network structure appears. In the network model in Figure 2, we create two columns of nodes, one depicting residence zones and another for crime location zones. Note that the residence nodes have arcs depicted away from them and the crime location nodes, arcs directed into them. Every arc in Figure 1 and hence, every displacement (or lack thereof) possibility is depicted in Figure 2. Although we make some modifications shortly, it is this new structure in Figure 2 that we adopt as the underlying network model for displacement.

The structure in Figure 2 is nothing more than a schematic for crime displacement. At this point, it may not be evident how the structure can be used in an analysis mode. Presently, we develop the notion of such a capability. The network depicted in Figure 2 is representative of a rather general and well-known class of network structures referred to as transportation problems. Briefly, the transportation problem specifies that quantities of some type of good be shipped (transported)

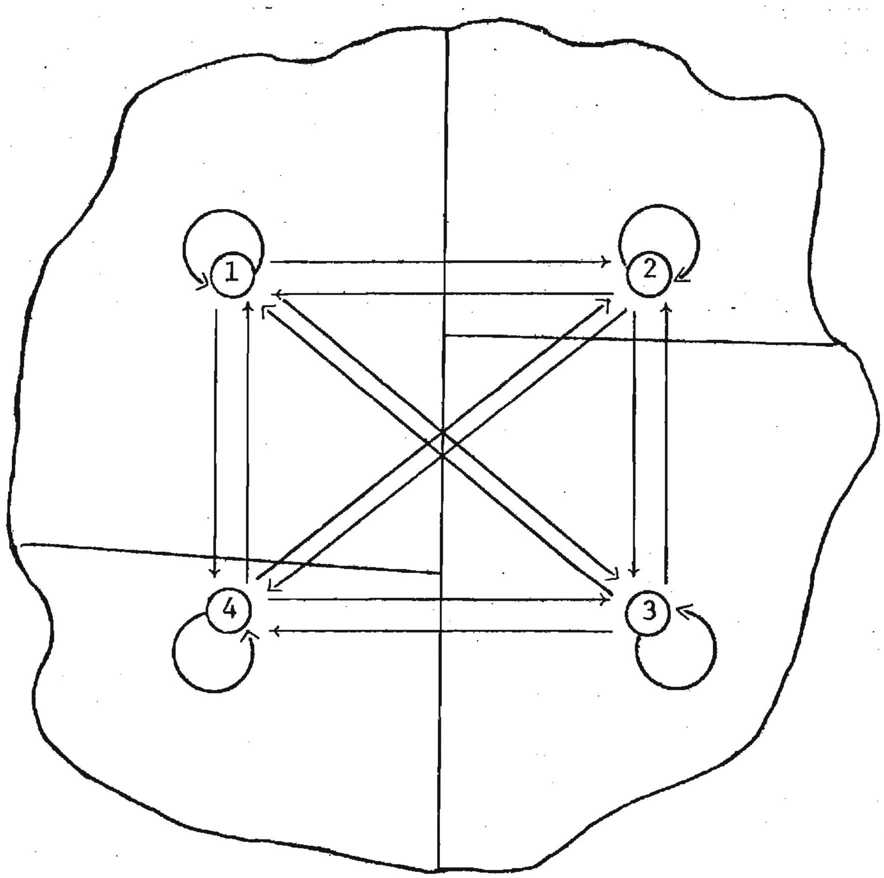

Figure 1. Simple Network Depiction of a Structure for Modeling Displacement

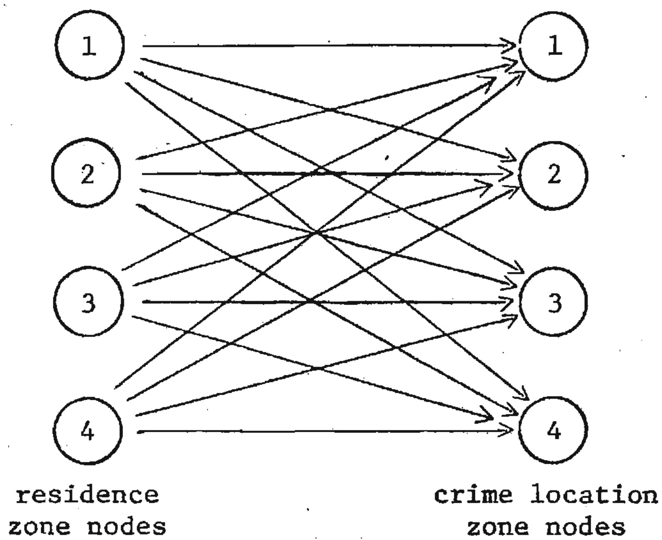

Figure 2. A Transportation Format for the Basic Network Model of Displacement

from some known destinations to a set of known demand points. A typical example would be the shipment of a commodity from a group of seaports to a group of inland bases. It is assumed that there is a unit cost of travel between every port and every base and that the objective is to satisfy the base demands with least cost shipping patterns. Alternately, we are seeking a set of "flow" patterns from ports to bases which minimize total cost. Our crime displacement problem can be considered a form of transportation model.

Suppose we consider every node of criminal residence to be a seaport in the illustration above. Further, let every zone of crime commission be an inland base. Note that while the earlier example suggested distinct supply (ports) and destination (bases) nodes, our displacement model does not. That is, a residence node is also a potential crime commission node. This creates no difficulty in the ensuing analytical model. In any event, we are seeking a flow pattern from zone to zone where our "commodity" is a crime.

Given that the displacement model takes on the characteristics of the classic transportation model, it remains that two items be considered before the direct analogy is complete. First, it is obvious that in a transportation problem there must be some specified level of availability of commodity at each of the shipment nodes. Similarly, every demand node must possess some requirement level. In our problem of crime displacement these requirement and demand specifications (which we will call node lables) take the form of potential crimes in a given zone i and forecasted crime occurrences in a zone j. These labels are considered input to the displacement model and are generated from the univariate time-series models.

The second point that we must consider is the notion of the cost of criminal (or crime) flow from zone to zone. Recall that in the literal transportation system, there exists a unit transportation cost say c_{ij} for all pairs of nodes i and j. Such a parameter was, in fact, a cost which would be assessed each unit transported (fuel cost, labor cost, etc.). Does such a cost make sense in the displacement model? In the literal sense, the answer would be negative yet there would be some measure of taxation or retardation (or its opposite:attraction) relative to a criminal's ability to move from i to j. If we view "cost" in a general sense then one might refer to c_{ij} values in the displacement model as resistence coefficients. For the present, we adopt such a convention. It is noted however, that the formulation of these cost indice or measures is a crucial step in the construction and manipulation of our model. Consequently, we return to this point later in the memo.

Using the Basic Network Flow Model

In the current section, we consider how the classic transportation model can be used in the displacement format. Recall that we specify particular input data in the form of crime commission numbers: potential crimes to be committed by residents of each zone and committed crimes at each zone. Further, assume that the resistant costs between each pair of zones i and j are given by parameters, c_{ij}. The idea then is to "solve" the corresponding transportation model and in such a solution, interpret the corresponding implications relative to displacement. We know we can easily solve transportation like-problems in that numerous well defined and efficient mathematical techniques exist.

In order to illustrate the use of the transportation model as a vehicle in displacement analysis, consider a simple problem. Suppose we let a metro area be divided into three zones. Suppose the zones are created subject to natural boundaries and let the corresponding transportation depiction be given by the structure in Figure 3. Further, suppose the specified crime lables for both residence zones and crime commission zones, be given next to the respective nodes in the figure. For example, it is forecast that ten crimes will be committed by residents of zone 1 and that zone 3 will be the location of 15 crime commissions.

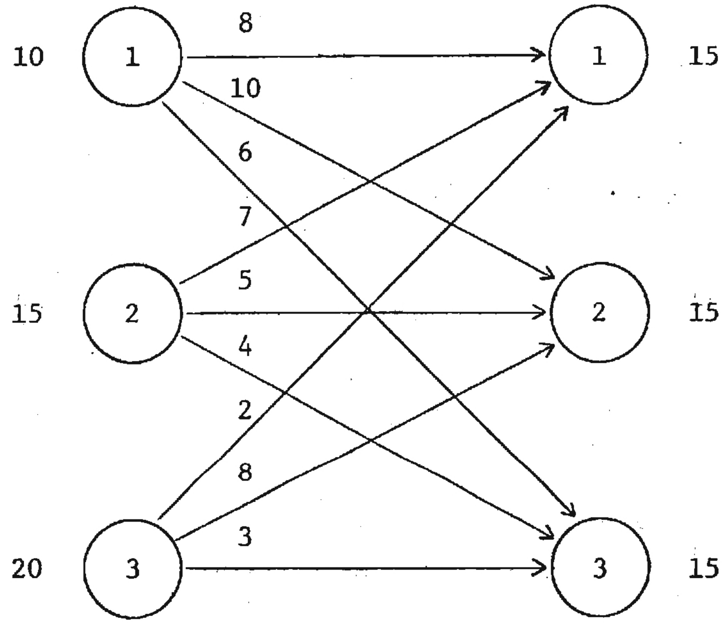

Figure 3. Sample Problem Transportation Formulation

The resistance costs are given on each arc. We accept these cost numbers for now, but realize that they are representative of, indeed calculated from, some relevant socio-economic/demographic factors pertinent to the system or region under investigation.

Suppose we seek a displacement flow pattern that minimizes displacement cost and let us depict by x_{ij} the unknown value of crime movement from i to j. That is, we are after values of x_{ij} for all pairs i and j which minimize (generally, optimize) the displacement cost. It can be easily verified (using a well known technique) that the best (optimal) set of values for x_{ij} are those given in Table 1. Relative to the network depiction, these flow values can be depicted in network form as in Figure 4. Note that all criminal residents of zone 1 commit their crimes in zone 3. Similarly, all potential crime perpetrators in zone 2 commit their crimes in zone 2. Finally, perpetrators in residence zone 3 commit a portion of the crimes in zone 1 and a portion in zone 3. Note that the flow patterns are feasible in the example in that no more crimes are committed by residents than were forecasted and that no more were recorded in any zone than were predicted.

So then, what does a set of computed crime displacement patterns really mean? Without much analysis, it is clear that, alone, the solution of a given crime displacement-type transportation problem provides little in the way of useful information regarding current displacement and in particular, future displacement. Note in the above illustration a given set of flows were determined based upon a set of c_{ij} values. Given an alternate set of such values, a new and possibly very different set of flows might result. What then is the value of solving the transportation problem in the first instance? In order to answer this question, we introduce the notion of tuning relative to the successive solution of iteratively created transportation problems and historical data. It is this tuning process that provides the key to the actual "model" of crime displacement.

Table 1. Summary of Computed Flows for Sample Problems

Displacement Variable	Value of Flow
x_{11}	0
x_{12}	0
x_{13}	10
x_{21}	0
x_{22}	15
x_{23}	0
x_{31}	15
x_{32}	0
x_{33}	5

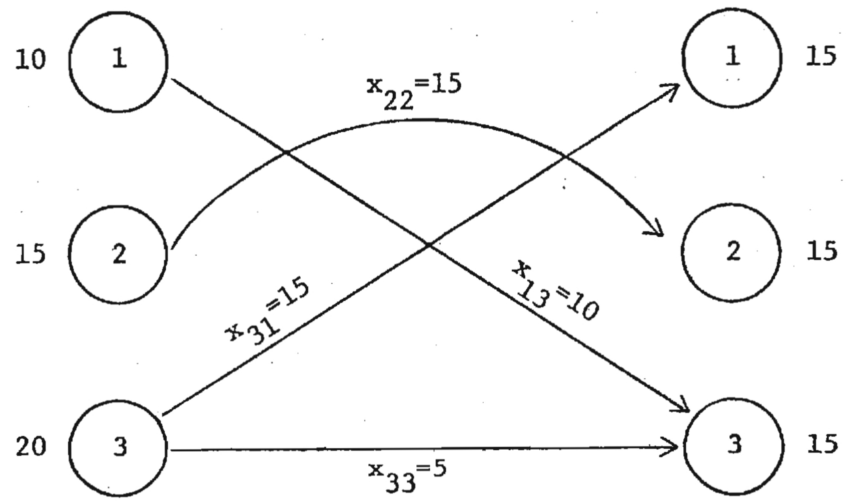

Figure 4. Network Depiction of Computed Flows From Sample Problems.

Consider that we have a transportation model like that in the earlier illustration. Moreover, suppose we have arrest information relative to zone of residence and zone of crime commission. Let these historical data values be given by f_{ij} for all pairs i and j. Again considering the simple illustration, suppose f_{13} is 10, f_{22} is 15, f_{31} is 15 and f_{33} is 5. All other f_{ij} are recorded as zero. Given such a set of observed flow data, we would see that the computed flows x_{ij} agree exactly with the historical values of f_{ij}. We would then say that the transportation model was tuned exactly in that its solution matched precisely the values of real occurrence. Obviously, this sort of exact match is rare such that in the real application of the model we must affect an updating scheme which iteratively improves the match between x_{ij} and f_{ij}.

How can a match or comparison between values x_{ij} and f_{ij} be improved? In fact, how can it be altered at all? For a given set of f_{ij} values, the only way the match between x_{ij} and f_{ij} can be improved is to change values x_{ij}. This implies the solution to new transportation problems and further, that new costs c_{ij} need to be obtained. That is, the only way a new set of computed flows can be acheived is to alter the driving mechanism of the transportation model, namely the cost or resistance coefficients. A procedure is suggested whereby, this alteration or perturbation can be made in an orderly manner until a point is reached where particular costs say c_{ij} result which in turn, create values x_{ij} which match closely those values, f_{ij}. We suggest that a match is "close" when the value of $\sum_i \sum_j (x_{ij}-f_{ij})^2$ is within some tolerable range. The result is a set of computed flows or to be precise, a set of costs, c_{ij} (which create flows) which are "tuned" to the historical data.

Cost Coefficient Considerations

It is a simple matter to remark that the tuning process involves the iterative determination of successive values of c_{ij}. It turns out that a non-trivial computational effort accompanies such attainment. More importantly, the functional construction for c_{ij} values can be an elusive consideration in itself. By functional generation, we refer to the mechanism by which costs of crime movement are created. Specifically, we must assume the existence of some function for c_{ij} which combines various system ingredients and considerations. In addition we must determine the form in which those parametric attributes are combined.

It is obvious that the construction of any function of crime movement cost is a complex issue. For example, what parameters are important as reflectors of crime transport? Clearly, a list could be created the length of which would be substantial. It is sufficient to remark that in subsequent work we address the problem of investigating those parameters relative to displacement and/or deterrence. To the extent that interfaces exist with the current research to date, care will be taken to make use of previous studies in the area.

While the problem of cost function composition is important, the task of determining the form of the function, in general, may be even more crucial. Recall that the ultimate analysis with the displacement model is to achieve a set of displacement patterns which are affected by alternatives in cost coefficients. In short, successive values of these coefficients are calculated until a tuned model is achieved. To a degree, the ease of generation of values c_{ij} depends on the structure of the generating function for each c_{ij}. For example, given a set of system parameters say A, B, and C how do they combine mathematically such that successive creations of c_{ij} are achieved expediently and in a meaningful way? Is the general function of the form ABC? Is it AB/C? Perhaps $Be^{A}+C$ or even $C [\log (A/B)]$? This is an important point for as we shall discuss subsequently, if such a functional form is not constructed properly, the application of the displacement model can be weakened if not made entirely insensitive.

Temporal Aspects of the Model

To this point, our concern has been limited to spatial constructions; however, the final point to be covered in the basic displacement model description is that regarding temporal or time related aspects.

Since the overall objective of the model is its use in a forecasting or predictive mode , it follows that substantial care must be taken to assure that an appropriate amount of historical data be considered. We recall that the overall process is one of tuning a given model to past occurrence in the real system around which the model is constructed. We have neglected, for the most part, any notion of time variance in the system. Earlier discussion about the residence zone, crime commission zone construction was taken in something of a static sense in that a single period (month, quarter, year, etc.) was assumed. Certainly, any procedure which is aimed ultimately at the creation of a forecasting capability would necessitate a dynamic construction or one where multiple period aspects of historical occurrence were considered. Hence, if we allow that developments up to this point have involved a single period structure, then we must generalize to the point of creating a multi-period model of displacement.

Consider Figure 5 such that multiples of the single period model of Figure 2 are simply combined. Certainly, it may occur that the gain in forecasting accuracy created by the multi-period structure may be offset by the manipulative burden created by the larger network structure. Consider the following computational refinement to the multi-period model.

Suppose we consider the existence of a so-called "population" of criminal or potential crime-committing elements. Further, suppose at the beginning of every period this population is decomposed or "sorted" out into residence zones

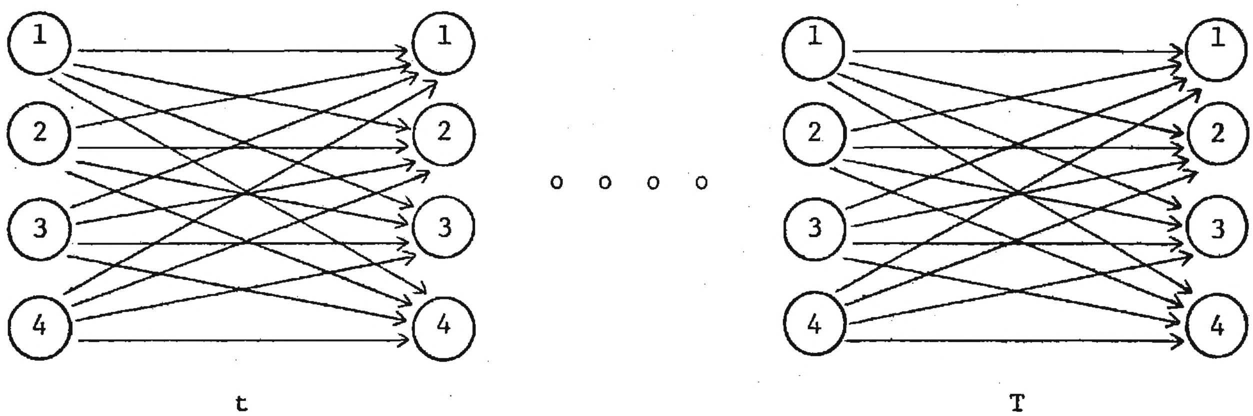

Figure 5. Basic, Multi-Period Displacement Model

(residence zone lables). Also, consider that all crimes that are committed in each zone are re-consolidated into the population node at the conclusion of the period after which they are re-sorted into the residence zones for the subsequent period and so forth. Augumenting the structure in Figure 5 with a population node would result in a network like that depicted in Figure 6.

While it may appear that the simple change in the structure of Figure 5 is only a cosmetic one, it results that the new structure in Figure 6 can be decomposed into a set of single period structures (transportation problems) which can be analyzed individually yet which can be re-combined to take advantage of the necessitated, multi-period aspects of the model for forecasting purposes. The result is a substantial reduction in the required effort by the mathematical technique used in the computation of displacement flow patterns. In effect, we simply solve a series of single period problems which is substantially easier than solving a single, multi-period problem. The details of the decomposition property and proof of its validity appears in a prior Technical Note [2].

The notion of tuning in the multi-period context is a simple extension of the single period version. If the system involves T periods each given by index t where t = 1, 2, . . . , T, then we are interested in finding those flow patterns x_{ij}^t which match closely, the observed flows f_{ij}^t, where all periods are considered simultaneously. Hence, we generalize the function specified earlier such that we now seek to minimize $\sum_t \sum_i \sum_j (x_{ij}^t - f_{ij}^t)^2$. At its minimum (or tolerable value, those c_{ij}^t yielding x_{ij}^t are used to compute values x_{ij}^{T+1} which provide forecasts of displacement.

Aspects of Modeling for Deterrence

As suggested at the beginning of this section, we have separated the notion of crime deterrence from that of displacement. The motivation for such organization

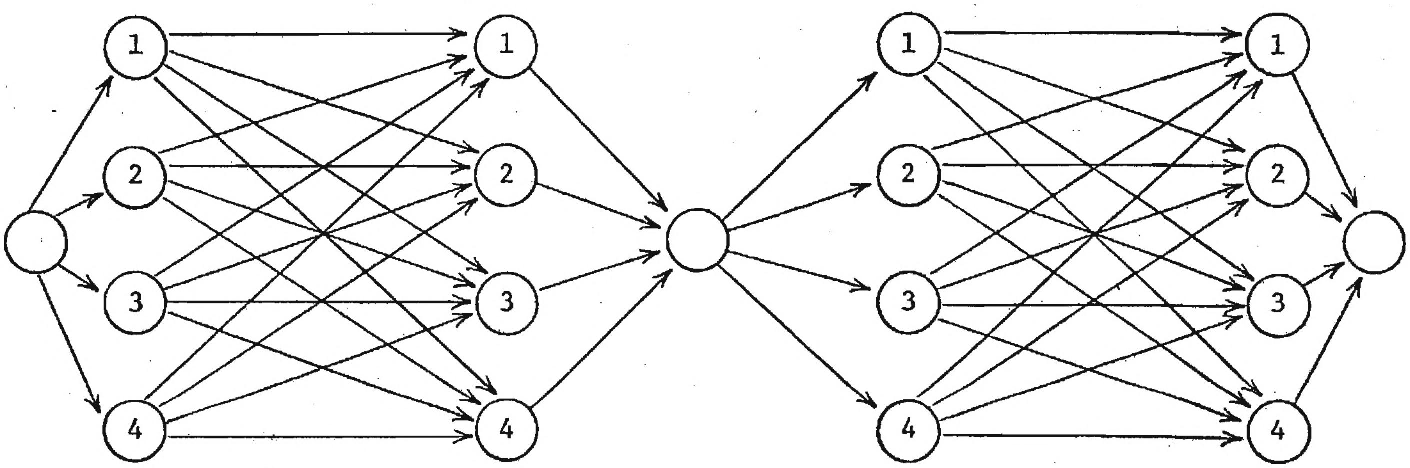

Figure 6. Complete, Multi-Period Network Model (for two periods).

lies primarily in the ease with which displacement can be modeled and facilitated using existing data bases. A similar facilitation does not seem to exist for deterrence. Following, we consider the structural augmentations involved in including deterrence aspects in our model. In subsequent section, we consider the question of data requirements surrounding deterrence analysis.

Consider again Figure 6. The arcs from residence zone i to crime location zone j depict displacement when $i \neq j$ and no displacement if $i = j$. In a sense, one might submit that flow from some zone i to j where $i = j$ (flow from a node to itself) is deterrence in that criminals are "deterred" or prohibited from flowing out of the zone. In truth, there is no deterrence in the sense of crime reduction or abatement but rather containment in that crime is not moving from zone to zone. While such containment is, in itself desirable, it should not be confused with deterrence.

In order for deterrence to be detected, there must be a measure of crime reduction or in the sense of the model depicted in Figure 6, a flow from residence node back to the population. In other words, from some number of potential crimes at zone i, only a portion occur while the remainder do not (are deterred) and as such return to the population node to be re-sorted for the subsequent time period. We can augment the structure in Figure 6 such that Figure 7 results. Note that a dummy crime location zone is added such that any flow into the node, in essence, reflects no crime occurrence during the specific time period. A similar modeling augmentation could as easily have been to create an additional arc directed from every location node directly to the population node. The above update to the displacement model provides an obvious structural characterization of the added aspect of deterrence. The tuning process would then proceed as before such that a set of displacement <u>and</u> deterrence flows would be sought which matched historical values. There is a difficulty however. Recall that in the displacement model, histor-

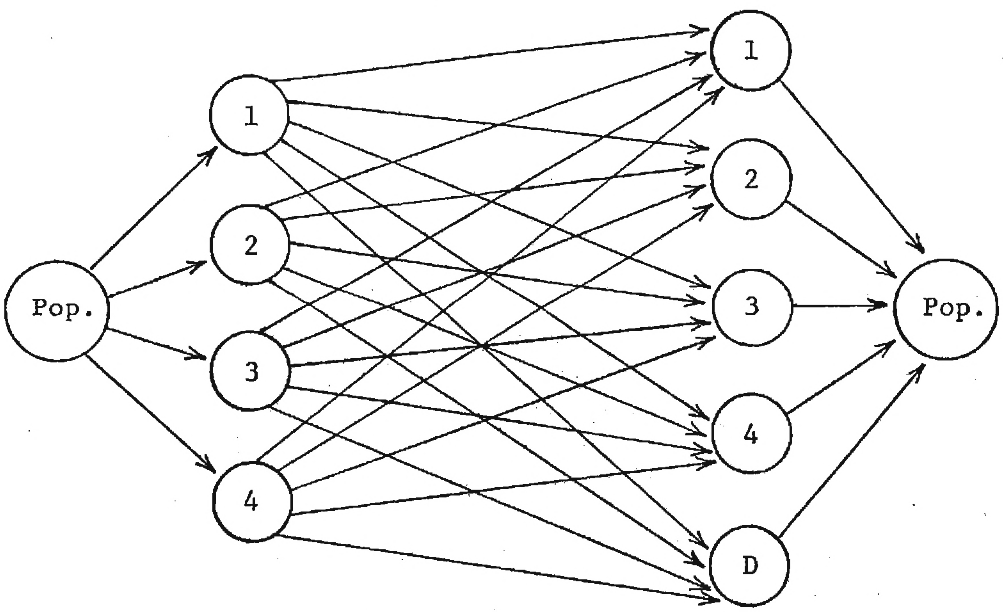

Figure 7. The Basic Model Augmented with Deterrence Arcs (Node D is a dummy node).

ical values of displacement could be readily obtained - generally from police arrest records. There is no such method of obtaining a deterrence record by definition such information would not be in "arrest" records. In fact, there is no way, analogous to the case of displacement, to tune a model of deterrence.

At present, questions of pure deterrence modeling in the context of the displacement model structure is not resolved. To the extent, however, that such issues as containment and displacement pattern changes provide a link with real deterrence, the current model is a viable vehicle. These avenues are being pursued presently.

IV - Developing the Data for the Model

Displacement Model in a Predictive Mode

In order to establish the values of the parameters of the displacement model it will be necessary to obtain zone of criminal residence corresponding to each reported crime. Obviously, this reduces the available data from reported crimes to crimes cleared by arrest. This information, as well as period of occurrence, appears on the criminal arrest record and is, therefore, obtainable.

If the model is validated at the level of reported crimes cleared by arrest then how can the model be used to portray criminal patterns at the level of reported crimes data (which will be more abundent)? Suppose we make the assumption that the probability of arrest associated with a particular crime does not vary with zone of criminal residence. Then we may conclude that the same proportions of displacement will occur at the level of reported crimes as will occur at the level of reported crimes cleared by arrest. Thus, the model can be validated using arrest data and subsequently factored up to the level of reported crimes.

There is no current method of validating the foregoing assumption. However, it appears to be the only reasonable assumption which can be made in this respect. We shall be employing this assumption in the displacement model.

The basic data for the displacement model will be arrest records. From these records we will be able to obtain, for each period and crime type or combination of types, the number of crimes committed in zone j by criminals residing in zone i. This data constitutes the basic flow, f_{ij}, or dependent variable matrix. In addition, the displacement model requires socio-economic, demographic and police resource data as independent variables for the criterion function.

Clearly, careful attention is required to establish the set of essential independent variables for the displacement model. As with any regression model the greater the number of independent variables used in the displacement the more accurate the model will be in predicting actual displacement. However, as with regression models, the increased number of independent variables brings with it a very rapid increase in the computational complexity of establishing the parameter values of the criterion function.

It is anticipated that the displacement model will utilize much of the same socio-economic/demographic data, albeit at a more detailed level, as the univariate models. Thus, during the execution of the overall contract task of establishing the needs of socio economic/demographic data for the time series models , consideration will be given to similar needs of the displacement model.

On the surface, it would appear that the data requirements for deterrence modeling would be similar to those for displacement. Indeed, there is much similarity in terms of such aspects as the identification of independent variables and the ascertainment of relevant socio-economic/demographic parameters. There is a major difference however, and unfortunately, one which leaves explicit deterrence modeling unrefined at this stage. Recall, from the previous section that the basis of the network tuning model requires the existence of historical data. Arising from arrest records such data is easy to obtain in the case of displacement. For crimes "deterred", there is no such record and hence, no analogous method for "tuning" for deterrence. We return to this point in a later section dealing with future work.

V - Applying the Model to the Data - In a Predictive Mode

Specifying the Form of the Criterion Function

The first step in the application of the model to the data is the development of an appropriate form of the criterion function. The function may be linear or non-linear and should reflect, in a reasonable way, the nature of the association of the particular dependent variable (or combination of dependent variables) flow.

As with regression models there are no set rules for specifying the form of the criterion function. However, unlike regression, the presence of non-linear functions in the displacement model provides no serious difficulty in the computation of the parameter values. For clarity of discussion in this section we shall assume that a reasonable number of different types of independent variables, X_1, X_2, . . . X_m, have been collected and are utilized in a specified functional form to produce the attractiveness coefficients, c_{ij}, of the criterion function. That is,

$$c^t_{ij} = f(X^t_1, \ldots, X^t_m; p_1, \ldots, p_n) \tag{1}$$

where X^t_1, . . . , X^t_m are the independent variables and p_1, . . . , p_n are the parameters. Each variable may give the condition of a particular zone (such as income per capita), or it may relate conditions between two zones (degree of difficulty of crossing a natural boundary or distance).

Developing the Parameter Values of the Criterion Function

Once the functional form of the criterion function has been selected we may proceed to develop the values of the parameters in the function. This is accomplished through the use of optimization methods.

A search is conducted in the parameter space for the choice of parameters that minimizes the sum of squared deviation between actual and predicted displacement over all periods using the available actual data. Recall that functionally, we may write the following:

$$\underset{p_1,\ldots,p_n}{\text{Minimize}} \quad \sum_{t=1}^{T} \sum_{i,j} (x_{ij}^t - f_{ij}^t)^2 \tag{2}$$

where x_{ij}^t is given by the solution of

$$\text{Minimize} \quad \sum_{i,j} k_{ij}^t (x_{ij}^t) \tag{3}$$

$$\text{Subject to} \quad \sum_j x_{ij}^t = a_i^t$$

$$\sum_i x_{ij}^t = b_i^t$$

$$x_{ij}^t \geq 0$$

Here, b_{ij}^t are the number of crimes committed in zone j during period t and a_j^t are the number of crimes which will be committed by criminals residing in zone i.

To accomplish the above search we proceed as follows. Select an initial set of reasonable parameter values to begin the search. Utilizing these values and the given values of the independent variable, determine the criterion coefficients from (1). Using the coefficients determined in (1) solve the transportation problem defined by (3). Utilizing the x_{ij}^t's determined in (3) we evaluate the functional specified by (2). If the value of the functional just determined is the smallest so far obtained we retain the current set of parameter values; otherwise, the current set of parameter values is discarded. The entire process is repeated using a different set of parameter values until a point is reached where we are reasonably certain that no further improvement can be obtained.

There are a number of systematic search procedures including one-at-a-time search, simplex search, gradient search and others. Depending on the nature of the criterion function some of these search procedures will be more appropriate than others.

Also, different methods will be appropriate for solving the transportation problems depending on the nature of the transportation functional $k_{ij}^t(x_{ij}^t)$. Note that this function is completely different from the basic criterion function. The general form of the transportation function is

$$k_{ij}^t(x_{ij}^t) = k_{ij}^t(x_{ij}^t \; ; \; c_{ij}^t)$$

where c_{ij}^t are considered parameters. It is invisioned that this function will probably take one of several simplified forms. Two possibilities are

1. $k_{ij}^t(x_{ij}^t) = c_{ij}^t x_{ij}^t$

and

2. $k_{ij}^t(x_{ij}^t) = c_{ij}^t \cdot (x_{ij}^t)^2$.

Both of these forms have provided reasonable estimates on sample data.

Obtaining the Projections from the Univariate Models: The Node Labels

As has been indicated before, the displacement model is designed to disaggregate zonal projections of reported crimes into crimes committed by zone of commission and zone of offender. As such, the displacement model, in a prediction mode, requires forecasts of reported crimes by zone. This can be accomplished by use of the univariate time series models developed in earlier tasks.

The same data used to establish the parameters of the displacement model would also be utilized to develop the parameters for the univariate time series models. Once the univariate models are "tuned" they can be used to develop the forecasts of crime occurrence by zone. In order to employ the displacement model for projections there remains only the determination of estimates of criminal activity by zone of residence. Again, the univariate time series models can be used for this purpose. However, since only crimes cleared by arrest constitute the data base for criminal activity by residence, the univariate models will be forecasting lower totals than criminal activity corresponding to reported crimes. Fortunately, the basic assumption made earlier in this report can be used to facilitate the factoring up of projected criminal activity data by zone of residence.

Applying the Model to the Projections

Once the univariate models have been used to project reported crimes by zone of occurrence and criminal activity by zone of residence the displacement model may be employed to obtain the projected levels of displacement.

Future forecasts of the independent variables are used to obtain the coefficients in the criterion function for each future time period. Given these coefficients and the projections from the univariate time series models each future period's forecasts are obtained by applying the transportation model. This process is repeated for each future time period until all projections have been obtained.

Confidence Intervals

A natural question, and one which still essentially remains unanswered, is the question regarding the establishment of confidence intervals for the forecasts from the displacement models. To date no definitive answers to this question have been obtained. Not surprisingly though, the difficulty lies in being unable to establish variances for the forecast values.

There is an empirical approach which can be used to establish confidence intervals. This approach builds up the distributions of forecasts through Monte-Carlo simulation. Since the univariate models were used to establish levels of crime occurrence and criminal activity, variances of these estimates are available. Making some distribution assumption (normal, for example) it would be possible to draw sample forecasts of crime occurrences and criminal activity levels. These sample values would be used to obtain estimates from the displacement model. Repeating the process many times, we would produce distributions of criminal displacement between zones. Utilizing these distributions we could obtain confidence interval estimates.

It also may be possible to establish variances of the displacement forecasts directly from the variances of the crime occurrence and criminal activity forecasts by examining the structure of the transportation model. However, this has not been pursued and success in this regard is uncertain due to the complicating mechanism of the objective function.

VI - Computational Experience

In this section, we briefly document the computational experience gained to date relative to the displacement model. Of particular interest is the computational experience on the non-linear model since its characteristics necessarily, suggest greater solution effort than for its linear counterpart.

The following experience is based on sample data. Specifically, a set of data was used which was a sample from an actual crime data base recorded in Atlanta, Georgia. The data was compressed into a single period and further, was aggregated into fourteen zones. The resultant transportation model was then of size 14 x 14. The cost coefficients in the model, c_{ij} were functions of three hypothesized independent variables each weighted by a parameter α, β, and γ. This cost function was discussed earlier.

Relative to the non-linear model, complete solution required 37 seconds on a Cyber 74 facility. The value of the sum of squares of differences between computed flows and historical flows was 792. Using the linear model, the computational requirement was only 5 seconds, but, the final sum of squares value was 1218. In both cases, the search process employed was a simple one-at-a-time or successive variable variation technique. To date, no other search procedure has been considered.

It should also be remarked that other algorithms for solving the models (both non-linear and linear) have been investigated. The results for the algorithm yielding the results suggested above [9], has proven superior thus far. At any rate, the extension of the algorithm to a multiple period model involving a complete set of real data is now being investigated. The computational results reported above and gained thus far in this research would appear encouraging relative to the model's application on realistic data sets.

VII - Future Efforts

In this section, we describe the current status of the research dealing with displacement and deterrence modeling after which we discuss briefly those areas where further work appears both necessary and most fruitful.

Current Status of the Displacement and Deterrence Model

Presently, a model for criminal displacement has been constructed and validated on a sample set of real data. In the working mode, the model is functional to the extent that a set of tuned network flows can be computed which in turn, provide the vehicle for prediction of future displacement patterns. Necessary input to the model include parameters such as historical or observed flows, crime zone-node lables which are generated from the univariant time series models, and various socio-economic/demographic data used in the construction of cost functions, the latter being used to affect directly, the tuning process. The overall model has not as of yet been tested on a large scale set of data over multiple periods and in addition, the notion of explicit deterrence modeling has not been addressed.

Refining Deterrence

At the outset, reference has been made to a model of both deterrence and displacement. We have shown how the model of displacement can be identified and have devoted a major share of this memo to such description. Caution should be taken however, in considering true deterrence aspects. Recall that the nature of the tuning process upon which the displacement model is based, requires historical data of displacement. No such data exists relative to deterrence. In fact, data used in displacement modeling arises from arrest records. By contrast, deterrence "data" would have no such basis. The end result is that there appears to be no easy method of "tuning" for deterrence. Note that we consider here, explicit zone-to-zone deterrence. An aggregation or overall deterrence measure could be hypothesized as was suggested in Section III.

A major aspect remains with regard to the modeling of explicit crime deterrence. Presently, it would appear that alternate modeling strategies may be necessary, both from the structural view as well as with reference to parametric requirements. It may be that separate though not necessarily independent models for displacement and deterrence will have to be considered.

Development of Data Bases

Currently, the manipulation and validation of the displacement model has involved elementary data collection and retrieval formats. In order to enhance user ease and practicality of the model, an improved data management scheme must be addressed. To this point in the research such a project has not been necessary since the primary thrust has been concerned with modeling and validation.

An effective data storage and retrieval scheme would involve management of basic information for the model's tuning aspects such as crime location and location of the offender's residence. In addition, the entire realm of socio-economic/demographic data collection and storage appears to be a major consideration in the ensuing research, especially to the extent that refinements of cost functions are considered.

Refinement of Cost Functions

The construction of suitable cost functions, which provide the driving mechanism in the tuning algorithm, will receive substantial attention in future work. Thus far, cost functions have been constructed using only heuristic notions and as such, no real effort has been made in seeking more "appropriate" or "meaningful" functions which relate zone to zone displacement. Two issues arise here. First, the notion regarding the type of variables and parameters that might comprise a suitable cost function must be considered in detail. An effort is currently underway to examine the work of other investigators in this area. Secondly, the form of the cost function can be important both from the view of its ability to generate suitable cost coefficients as well as its influence on computational efforts in the overall tuning process. While this second aspect of function refinement is more methodological than the first, its analysis is important relative to the overall displacement model from the viewpoint of user capability.

Conduct Selected Applications of the Model to Appropriate Data

Naturally, it will be worthwhile to apply the models which have been developed thus far to data from other metropolitan areas. Not only will such "testing" provide stronger validation confidence, but as important, insight into peculiarities and/or similarities in displacement patterns among other regions can be observed.

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E-24-641

GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GEORGIA 30332

OFFICE OF
THE DIRECTOR OF
FINANCIAL AFFAIRS

February 22, 1978

U. S. Department of
Justice, LEAA
Office of the Controller
Control Desk
Washington, D. C. 20531

Gentlemen:

Enclosed is the Final Financial Status Report for Grant Number 75-NI-99-0091 covering the period October 1, 1977 through October 31, 1977.

If you have questions or desire additional information, please let us know.

Sincerely,

David V. Welch, Manager
Grants & Contracts Acctg.

DVW/bs
Enclosure
cc: Dr. R. N. Lehrer
Dr. S. J. Deutsch
Mr. E. E. Renfro
Mr. A. H. Becker
File E-24-641

U. S. DEPARTMENT OF JUSTICE

LAW ENFORCEMENT ASSISTANCE ADMINISTRATION

H-1 Report
OMB NO. 43-R0532
Expires: 6/30/76

FINANCIAL STATUS REPORT

1. Federal Agency and Organizational Element: U. S. Department of Justice, LEAA

2. Federal Grant No. or Other Identifying No.: 75-NI-99-0091

3. Name and Address of Grantee Organization: Georgia Institute of Technology, Atlanta, Georgia 30332

4. Employer Identification No.: 586002023

5. Grantee Account No. or Identifying No.: E-24-641

6. Final Report: [X] Yes [ ] No

7. Basis of Report: [X] Cash [ ] Accrued Expenditures

8. Project Period (Month, Day, Year): FROM 6 | 15 | 75 TO 10 | 31 | 77

9. Report Period (Month, Day, Year): FROM 10 | 1 | 77 TO 10 | 31 | 77

10. STATUS OF FUNDS — PROGRAMS — FUNCTIONS — ACTIVITIES

	(1)	(2)	(3)	(4)	(5)	(6)	TOTAL
a. Total outlays previously reported (Line 10e from previous report)							194,185.34
b. Total program outlays this period							10,795.91
c. Less: Program income credits							-0-
d. Net program outlays this period (Line b minus Line c)							10,795.91
e. Total program outlays to date (Sum of Lines a and d)							204,981.25
f. Less: Non-Federal share of program outlays							-0-
g. Total Federal share of program outlays (Line e minus Line f)							204,981.25
h. Total unpaid obligations							-0-
i. Less: Non-Federal share of unpaid obligations							-0-
j. Federal share of unpaid obligations (Line h minus Line i)							-0-
k. Total Federal share of outlays and unpaid obligations (Line g plus Line j)							204,981.25
l. Total Federal funds authorized							204,990.00
m. Unobligated balance of Federal funds (Line l minus Line k)							8.75

11. Indirect Expense: a. Type of rate (Mark box)
[ ] Provisional [ ] Final
[X] Predetermined [ ] Fixed

b. Rate: 68%
c. Base: 5,089.36
d. Total amount: 3,460.76
e. Federal share: 3,460.75

12. REMARKS (Attach additional sheets if necessary)

Dr. S. J. Deutsch
Project Director

13. CERTIFICATION — I certify that to the best of my knowledge and belief this report is correct and complete and that all outlays and unpaid obligations are for the purposes set forth in the grant award documents.

Name: David Welch, Cont.
Title: Mgr. Grants Acctg.
TELEPHONE: Area Code 404 | Number 894 | Ext. 4624
Signature of Authorized Certifying Official:
Date Report Is Submitted: February 14, 1978

LEAA FORM 7160/1 (8-73)

REPLACES LEAA/OLEP 153 AND LEAA/OLEP 155 WHICH ARE OBSOLETE.