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EXPERIMENTS IN OPTIMAL CONTOL OF A FLEXIBLE ARM

Gordon G. Hastings, and Wayne J. Book
Department of Mechanical Engineering
Georgia Institute of Engineering, Atlanta, Ga. 30308

INTRODUCTION

Research in the control of flexible arms may offer long term solutions to factory automation problems in applications where high performance, long reach, or mobility is required. This paper discusses the reconstruction of modal quantities from strain gage measurements, and the sensitivity of an optimal controller to variations in payload mass based on analytical models and experiment for a single-link arm.

EXPERIMENTAL SETUP

The setup is a complete laboratory for examining the control of flexible arms with frequencies as high as 100 Hz. The system consists of a flexible arm with payload, DC torque motor with servo-amp, A/D and D/A conversion for measurement sampling, signal conditioning, and 16 bit computer system for implementation of control algorithms. The control computer is equipped with floating point hardware, 64 megabyte hard storage, 24 channels of A/D conversion, and 2 channels of D/A conversion. A typical value for 32 bit floating point multiplication is 17 microseconds. The physical configuration of the flexible arm, torque motor, and sensors is represented in figure no. 1. Figure no. 2 is a block diagram of the system components.

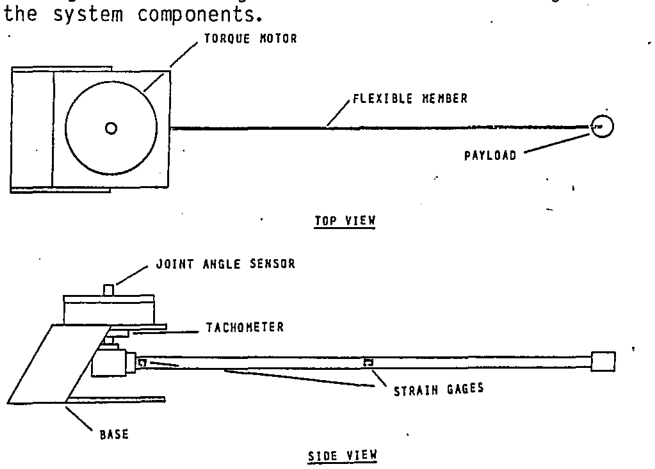
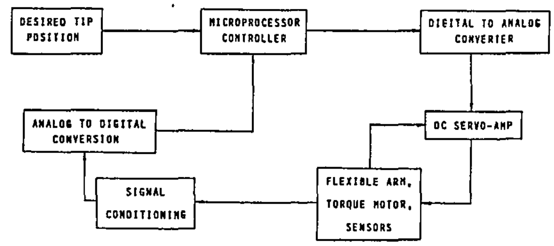


Figure no. 1

The arm is a four foot long rectangular aluminum beam, the first two clamped-free vibration modes were found to be 2.0,and 13.5 hz when mounted in the experimental apparatus with payload.

DYNAMIC MODELING

The first step in the design of controllers for a flexible arm is to construct an analytical model of the physical system. The model must include the major features of the real system, yet still lend itself to available analysis tools. A truncated series of assumed modes was selected, with the first mode being a rigid body rotation. Two additional flexible modes corresponding to clamped-free beam vibrations complete the series.



System Block Diagram

Figure no. 2

LaGrange's equations are formulated for the three mode series after normalizing the flexible modes. The resulting dynamic equations are then linearized by assuming small motions, and neglecting terms of higher order than one. The equations can then be organized into a sixth order state space model of the following form:

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{z}} \end{bmatrix} = \begin{bmatrix} \Phi_{aa} & \Phi_{ab} \\ \Phi_{ba} & \Phi_{bb} \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{z} \end{bmatrix} + \begin{bmatrix} \tilde{\Gamma}_a \\ \Gamma_b \end{bmatrix} u \qquad 1.$$

$$\tilde{x} = \begin{bmatrix} \theta \\ q_1 \\ q_2 \\ \dot{\theta} \end{bmatrix} \qquad \tilde{z} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \qquad 2.$$

$\tilde{x}$ - measured state vector
$\tilde{z}$ - unmeasured state vector
$\tilde{\Gamma}$ - input vector
q - flexible mode
θ - joint angle
$\dot{}$ - time derivative
u - control torque

A detailed description of the modeling procedure may be found in [1],[2].

CONTROL SYSTEM DESIGN

Rapid response of the flexible arms payload to commanded positions requires high rates and torques which tend to excite the flexible modes. The design of a controller must compensate for the flexibility, or accept limited performance. Combining the state-space model with modern control techniques provides a method for specifiying control laws which optimize functions of the systems states. Deterministic and stochastic controllers have been designed for the experimental system, but the initial experiments implemented a deterministic controller.

A control law was selected which satisfied a standard formulation of a linear quadratic continuous transient regulator problem. The steady state solution was computed using subroutines in the ORACLS [3] software package. The basic problem was modified so that the closed loop poles of the system could be specified with an arbitrary degree of stability.

MODAL RECONSTRUCTION

The modern control system employed requires the entire state vector be identified for the control law. Direct measurement of the modal quantities is not possible, only measurement of variables which are functions of linear combinations of modal quantities. Two types of measurement are currently recieving attention, optical measurement of deflections, and strain measurement. The measurement selected for the initial experiments is strain, as this provides a simple, low cost method of collecting the necessary data. The basic approach can be readily adapted to optical measurement of deflection, and future experiments are planned to compare the methods.

Equation 3 presents the basic relationship between the flexible modes and the measured strain data. Since we are interested in reconstructing two separate modes, two strain measurements are made, one from the base of the beam and one from the mid-point. Four active gages are used in a full bridge at each measuring point. This implementation compensates for torsional,axial, and tranverse strains that would otherwise reduce disturbance rejection.

$$\begin{bmatrix} \varepsilon(y_1) \\ \varepsilon(y_2) \end{bmatrix} = \begin{bmatrix} c_{11}\frac{\partial^2 \phi_1(y_1)}{\partial y^2} & c_{12}\frac{\partial^2 \phi_2(y_1)}{\partial y^2} \\ c_{21}\frac{\partial^2 \phi_1(y_2)}{\partial y^2} & c_{22}\frac{\partial^2 \phi_2(y_2)}{\partial y^2} \end{bmatrix} \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} \quad 3.1$$

$$\bar{\xi} = A^{-1}\bar{\varepsilon} \quad 3.2$$

$\varepsilon(y)$ - indicates strain at axial distance y

$\phi(y)$ - spatial mode functions

$\xi(t)$ - time dependent modal amplitude

The coefficients for the reconstruction can be determined by inserting the asumed mode functions into equation 3. The beam was harmonically excited at the frequency of the mode being considered with a unit tip deflection amplitude, and the strain responses monitored. The experiments agreeded well with analytical modeling, resulting in a nearly orthogonal relationship between the modes providing 6 to 8 decibels of rejection between the reconstructed modes.

MASS SENSITIVITY

The model used for developing the control law was based on a set of assumed modes, manipulators often transport payloads of different masses, therefore, the effect of varying the payload was investigated. The sensitivity of the closed loop poles to variations in payload mass is depicted in figure 3. The locus was determined from a linear model. The mass was varied from the design value, then the dynamic and input matrix was recalculated for each mass point. The optimal gains determined for the design condition were retained, and the closed loop poles determined. The measured time response of the system is shown in figure 4 for no payload, design payload, and four times the design payload using the same optimal gains.

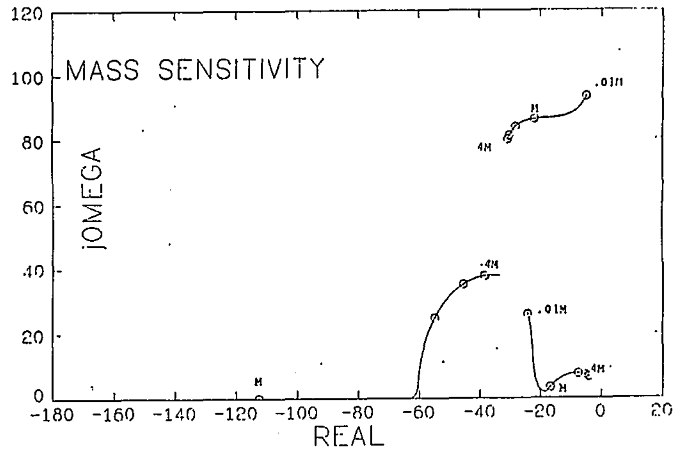


Figure no. 3

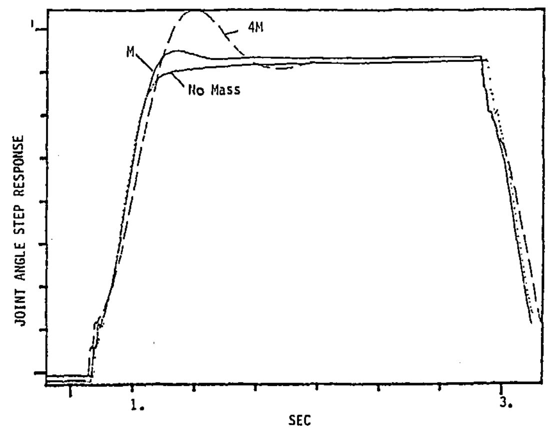


Figure no. 4

CONCLUSIONS

The initial experiments have successfully demonstrated the use of strain measurements to reconstruct flexible modes. Although it is difficult to correlate the root locus with the time response plots both depict moderate sensitivity to large variations in payload mass without sudden transitions to instability.

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