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Discrete Switching Vibration Suppression for Flexible Systems with Redundant Actuation

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Abstract—**If the modal response for a single degree of freedom flexible system is known, a command generation architecture can be determined which schedules on/off actuator effort such that the resulting motion will have zero vibration. If the system possesses redundant on/off actuation, the number of possible zero vibration commands increases. Of particular interest is the command that has the minimum number of actuator changes in state. This paper presents how to determine this command and applies it in simulation to a flexible actuator inspired by human muscle.**

Keywords- Flexible Manipulators and Structures; Biomechatronics; Motion Vibration and Noise Control.

I. INTRODUCTION

As technology becomes increasingly democratized, devices that perform automatically generated motion are becoming increasingly common, and are being manufactured in much larger numbers. Because of weight, material cost, and aesthetic appeal, often times it is desirable for these devices to have structural members that are thinner those of traditional industrial robots. Because these slender members are inherently less rigid, they are susceptible to oscillation during motion, which can severely degrade performance. One such device is a cellular actuator [1] inspired by human muscle fibers, in which piezoelectric stacks drive a flexural mechanism with a very thin cross section (0.1mm). Each cellular actuator has six independently addressable on/off actuators in series that produce motion in the same direction. Hence, the cellular actuator possesses redundant actuation.

During recent decades, several successful analog and discrete methods of suppressing vibration for automated systems have been developed, employing feedback, feed-forward, and command shaping techniques. When the natural frequencies of the first few modes of a lightly damped flexible system are known, the input shaping approach, described by Singer and Seering in [2] is an effective method of controlling vibration for both open loop and closed loop point to point motion. This approach convolves the desired input with sets of impulses corresponding to each natural frequency that effectively place zeros near the flexible poles of the system. Additional impulses added to the convolution sequence increase robustness to frequency and damping parameter variation at the expense of system rise time. In [3], Pao presents a method that applies the input shaping concept to systems with multiple inputs and shows how multiple actuators can work together to suppress vibration with different input shapers applied to each input. This method results in faster motion than a global input shaper applied identically to all inputs. Pao considers the effect of each actuator in the system as well as the complete system model and solves for separate shapers for each actuator. Each shaper alone would not cancel vibration, but acting in conjunction they eliminate residual vibration. In [2] the amplitudes are known a priori, but in [3] the amplitudes are solved for by the algorithm that constructs the shapers. Lim, Stevens and How in [4] frame the selection of input shapers for a multi-input system as a quasi-convex optimization problem. Residual vibration, difference from unity DC gain, amount of actuation, and amplitude of transient response are incorporated as optimization constraints. The resulting shapers are applied to a flexible two degree of freedom manipulator.

References [2], [3], and [4] apply to systems that have a continuously variable input to each actuator, but in the case of the cellular actuator it is advantageous to take a discrete approach, where the command to each first layer unit is simply on or off at any given time. Analog commands require linear amplifiers, which are costly, bulky and generate significant waste heat. Linear amplifiers are also vulnerable to noise present in the system. Analog voltage commands also require a digital to analog converter, increasing pin count on the processor and greatly complicating the interface. Since electronics that drive switching voltage signals are much simpler than those that impose linear commands, often times a digital implementation requires a smaller circuit board footprint. By operating strictly in the on or off state, no compensation is necessary for actuator hysteresis.

This paper proposes a command generation methodology that will suppress vibration for systems with a redundant number of on-off actuators and applies it in simulation to a simplified model of the cellular actuator.

II. BIOLOGICALLY INSPIRED CELLULAR ACTUATORS

A. Biologically Inspired Actuation

When designing an automatic device that performs some kind of motion, it is helpful to learn from biology, since biological systems have been performing efficient, stable, motion for thousands of years. Human muscle creates motion by contracting and relaxing muscle fibers (sarcomeres). Muscle is a system with redundant actuation, since many muscle fibers actuate in a single direction. Like muscle fibers, the cellular actuator generates motion by deforming its own structure and produces an output that is the net sum of several on/off actuators. Actuators of this type are useful in situations where zero backlash and fast response is required, such as a pan-tilt mechanism for a high speed camera.

B. Actuator Construction

The actuation effort is provided by a lead zirconate titanate (PZT) piezoelectric stack, which increases in length when a voltage is applied. A rhomboidal structure surrounds the PZT stack. The displacement of the mating surface of the rhomboidal structure perpendicular to the PZT motion will be amplified if the angle relative to the PZT axis is less than 45 degrees, as described in [1] and [5]. These PZT stacks with rhomboids that amplify strain in a perpendicular direction are commercially available from the Cedrat Corporation [6]. In order to increase the overall displacement, six Cedrat piezoelectric stack actuators with strain amplifiers are connected in series, forming the "first layer." An additional "second layer" amplifies strain further, and its displacement is in the mutually perpendicular direction. The complete actuator is shown in Fig. 1. With the second layer in place, the complete actuator can achieve free displacement at the output of up to 21% of its resting length when the maximum allowable voltage is applied to each of the first layer actuators.

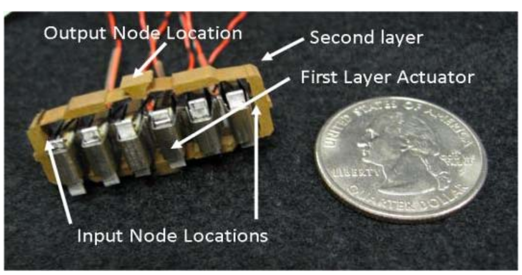


Fig. 1. Photo of Piezoelectric Cellular Actuator

Fig. 2 shows a schematic of the piezoelectric cellular actuator system. A desired position is provided to the command scheduler, that determines times at which to turn the piezoelectric stacks on and off in such a way so as not excite the vibrational modes of the system. These digital signals are fed to a switching network, which provides voltage to the first layer actuators at the times selected by the command scheduler. The small contraction of the first layer units produces a scaled-up extension of the second layer output.

The PZT stack actuators contained in the first layer units are capable of receiving an analog voltage command, but, for the reasons mentioned earlier, there is much to be gained by operating each stack in an on/off configuration only. This discrete command configuration allows the actuator to take 5 intermediate positions between the extremes of its operating

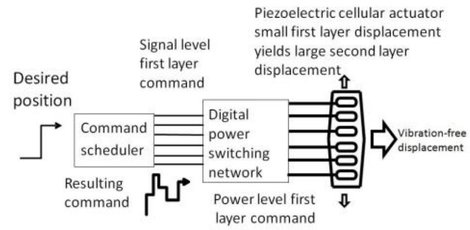


Fig 2. Schematic of Piezoelectric Cellular Actuator with Discrete Switching Command to Suppress Vibration

range, corresponding to the number of first layer actuators turned on. This may not seem adequate for positioning applications, but when several cellular actuators are cascaded in series and parallel combinations, similar to biological muscle fibers, the output of the set of all possible discrete inputs will be not be discernable from that of a continuously variable input applied to the same system.

C. Finite Element Modeling of the Actuator

The resonant modes shapes and frequencies were determined for an unloaded piezoelectric cellular actuator fixed to a rigid surface using COSMOSWORKS finite element software. An unloaded actuator is used merely as an example. If the actuator is loaded, the system dynamics of the load must be included in the model. COSMOSWORKS reported the first five frequencies and mode shapes, which are shown in Table I. The modal displacements at the input and output nodes of the mesh, $\Phi_{i,\,input}$ $\Phi_{i,output}$ are also reported. The transfer function from force to displacement of a flexible system such as the cellular actuator, that has no rigid body displacement, can be represented by (1).

$$G(s) = \sum_{i=1}^{N} \frac{\Phi_{i,input}\ \Phi_{i,output}}{s^2 + 2\zeta_i \omega_i + \omega_i^2} \qquad (1)$$

N is the total number of modes considered; in the case of the cellular actuator, $N = 5$. As a preliminary simplifying assumption, uniform damping of $\zeta_i = 0.01 (i = 1 \ldots 5)$ was assumed for all modes. This is meant merely to demonstrate the method, since the structure is lightly damped. This analysis will need to be refined for any experimental system, since higher modes decay faster.

TABLE I. CELLULAR ACTUATOR MODES

Mode	Frequency [Hz]	Description
1	28.41	rocking
2	42.76	extensional
3	64.36	torsional
4	214.4	extensional
5	249.7	rippling

Because of the geometry of the input and output nodes, the most significant modes are second and fourth modes shown in Fig. 3(a) and (b). The Bode plot of $G(s)$ shown in Fig. 3(c) shows only two peaks, one each from mode 2 and mode 4. Slight assembly asymmetries do not change this significantly.

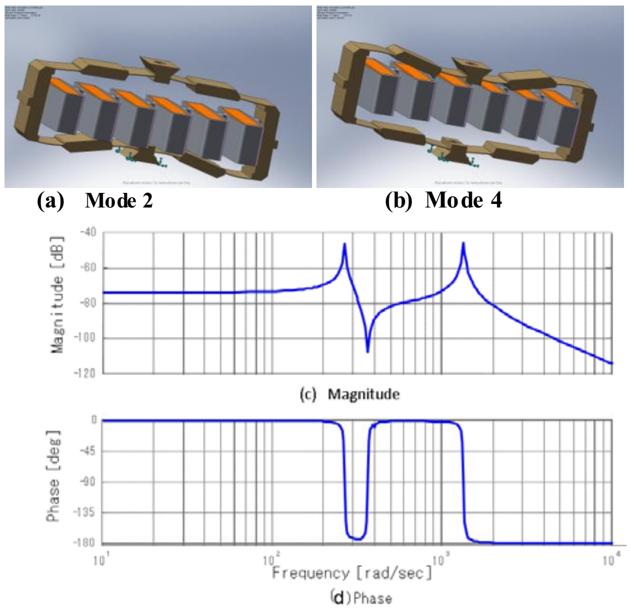


Fig. 3 Modal Response of Piezoelectric Cellular Actuator from FEA

III. Discrete Switching Vibration Suppression

A. Background

There is considerable published research in the literature describing how to choose on/off commands to reach a desired position while suppressing vibration by varying the times at which an actuator is switched on and off [7], [8], [9] and [10]. The effect of each impulse can be represented by phasor addition. For a single mode, if the amplitudes of the individual phasors are known, one can solve for the phase angles between the phasors that will cause the sum of their real and imaginary parts to be zero [11],[12]. This method can be extended to suppress multiple modes of vibration, but the phase angles must be solved numerically. Fig. 4 shows a vector diagram for such an impulse sequence that excites two modes of vibration. The solid arrows show the impulses relative to the lower frequency, and the dashed arrows relative to the higher frequency. One cycle from 0 to 2π represents one period of the lower frequency. The second mode (dashed arrows) have the same amplitudes, but because of the difference in frequency, in general they will not be at the same locations on the diagram. Their location is related to that of the first mode (solid arrows) by the relationship

$$\varphi_{i,2} = \left(\frac{f_2}{f_1}\right)\varphi_{i,1}, \tag{2}$$

where $\varphi_{j,k}$ is the phase angle of impulse i for vibrational mode k. By solving for the phase angles that cause the vector sum of the impulses $A_0 \ldots A_m$ to be zero for all n frequencies represented, a command can be synthesized that will have zero residual oscillation at these frequencies. These techniques are often applied to flexible spacecraft, as in [7]. Additional constraints can be added to the zero vibration and desired position constraints to limit the amount of fuel used or increase robustness, as in [8] and [9].

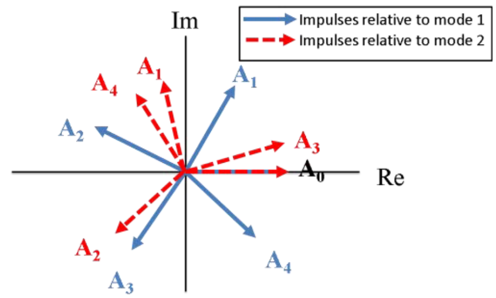


Fig. 4 Vector Diagram for Two Frequencies

These applications have a single actuator per degree of freedom and therefore each impulse amplitude is restricted to a value of +1 or -1 [8],[9],[10],[11],[12]. Convolving these amplitudes with a step function of amplitude equal to the maximum allowable actuator effort generates a command made up of an alternating series of positive and negative pulses with switching times set by the phase angles that satisfy a zero resultant on the vector diagram for all frequencies.

The on/off control methods used in [8], [9] ,[10], and [11] can be implemented on the piezoelectric cellular actuator by treating several actuators as a unit. Actuators can be switched on and off in unison according to the schedule calculated for a system with a single actuator. We call this all on/ all off control. The resulting motion will have identical vibration suppression properties to the single actuator case, assuming actuation is fast enough relative to the significant vibrational modes of the system such to neglect small delays in actuation. This is the case for the cellular actuator, because the first layer actuators have a much higher resonant frequency than first few modes of the system.

B. Motivation and Idea

When multiple discrete inputs are available for a single degree of freedom, all on/all off control is actually a subset of a larger set of vibration suppression commands, which we call discrete switching vibration suppression (DSVS). It is not strictly necessary to switch all first layer actuators required to reach the goal in unison in order to suppress vibration. In fact, there are many combinations that can yield a satisfactory result. In general, for DSVS, the amplitudes of the impulses on the vector diagram can take any integer value from one to the total number of discrete inputs. For the cellular actuator, this is six. For the piezoelectric cellular actuator, turning first layer actuators on corresponds to an impulse amplitude of positive sign and turning an actuator off can be interpreted an amplitude with a negative sign. Combining the positive and negative impulses on the vector diagram, we can solve for the phases that will result in zero vibration. This is somewhere between the analog input shaping case and the on/off control case.

There are numerous other subsets to DSVS, illustrated in Figure 5. Of particular interest is the case where the set of inputs experiences the minimum number of changes in state, or minimum switching discrete switching vibration suppression (MSDSVS). For a goal position of a single actuator on, all on/all off control is a minimum switching solution, but when the goal is >1, often times a satisfactory motion can be accomplished with fewer switches. MSDSVS has several advantages, providing a satisfactory solution can be obtained. It reduces the amount of heat generated by the switching transistors for a given move. It reduces the number and amplitude of loading/unloading cycles on the first and second layers, which could result in longer actuator life. This can be further exploited by imposing an algorithm that distributes the control effort evenly over the first layer actuators over all moves in time. In general, for actuators of this type, the number of changes of state corresponds to the control effort, so MSDSVS is a form of energy saving control.

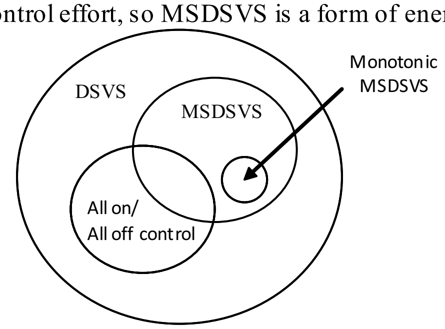


Fig. 5. Venn Diagram of DSVS Methods

One of the drawbacks of all on/all off control is that the negative amplitude impulses tend to amplify unmodeled high frequencies [12]. Since MSDSVS could posses fewer than all on/ all off control, it amplifies higher frequencies to a lesser extent MSDSVS has a smaller overall control effort than all on/ all off control, so it usually takes longer to complete the move. In certain cases, the minimum switching discrete control solution is monotonic in nature, resembling a staircase. This is desirable because no additional switches are necessary to reduce vibration beyond those required to reach the goal position and the oscillation due to unmodeled resonances will not be amplified. It is not always possible to calculate a solution of this type. The ability to calculate a monotonic solution depends on the number of actuators, the final position, the number of frequencies to be controlled, and the relative spacing of these frequencies.

C. Formulation and Search Algorithm

In order to have motion without vibration, each mode contributes two constraint equations that must be satisfied: the real components on the vector diagram must sum to zero, and so must the imaginary components. The desired number of actuators on, y_g, contributes an additional constraint. Since all A_i must have integer values, in general they cannot be used as unknowns. Therefore, we must have $2n + 1$ unknown phases, φ_i, to solve the system of equations. Each phase requires an impulse, so $2n + 1$ impulses are necessary. In certain cases, a solution is returned where the phase between two impulses is zero, and they can be consolidated into a single impulse whose amplitude is their sum. We refer to this phenomenon as "phase cancellation."

Consider controlling a single mode of vibration with damping neglected. This requires three impulses, A_0, A_1 and A_2. After impulse A_2 is applied there should be no vibration. The impulse amplitudes must be integer values chosen such that

$$\sum_{i=0}^{2n} A_i = y_g, \tag{3}$$

where y_g is the desired position in terms of number of actuators on, and at any intermediate point in the summation

$$0 \leq \sum_{j=0}^{i} A_j \leq r. \tag{4}$$

where r is the total number of redundant actuators and $i = [0 \ldots 2n]$. Once an allowable set of impulse amplitudes is obtained, the phases φ_1 and φ_2 can be found that will result in a vector sum of zero according to the relationship in (5) and (6).

$$\varphi_1 = \frac{A_2^2 - A_0^2 - A_1^2}{2A_0 A_1} \tag{5}$$

$$\varphi_2 = \left(\frac{3}{2} - \frac{sgn(A_2)}{2}\right)\pi - \tan^{-1}\left(\frac{A_1 \sin\varphi_1}{A_0 + A_1 \sin\varphi_1}\right) \tag{6}$$

For a single mode of vibration, because of the analytical expressions (5) and (6), and the fact that there are only 2 unknowns, it is a simple matter to calculate all possible solutions. However, to suppress multiple modes, unless the natural frequencies considered are integer multiples of one another, the vibration in the various modes will couple, and the phases of the impulses cannot be computed analytically. To find a command to suppress multiple modes of vibration, a set of $2n$+1 amplitudes must be chosen that satisfy (3) and (4). The real and imaginary components of these amplitudes on the vector diagram lead to the equations in (7). These can be solved numerically for $\varphi_1 \ldots \varphi_m$.

$$\begin{gathered}
A_0 + A_1\cos\varphi_1 + \cdots + A_m\cos\varphi_m = 0 \\
A_1\sin\varphi_1 + \cdots + A_m\sin\varphi_m = 0 \\
A_0 + A_1\cos\left(\frac{f_2}{f_1}\varphi_1\right) + \cdots + A_m\cos\left(\frac{f_2}{f_1}\varphi_m\right) = 0 \\
A_1\sin\left(\frac{f_2}{f_1}\varphi_1\right) + \cdots + A_m\sin\left(\frac{f_2}{f_1}\varphi_m\right) = 0 \\
\vdots \\
A_0 + A_1\cos\left(\frac{f_n}{f_1}\varphi_1\right) + \cdots + A_m\cos\left(\frac{f_n}{f_1}\varphi_m\right) = 0 \\
A_1\sin\left(\frac{f_n}{f_1}\varphi_1\right) + \cdots + A_m\sin\left(\frac{f_n}{f_1}\varphi_m\right) = 0
\end{gathered} \tag{7}$$

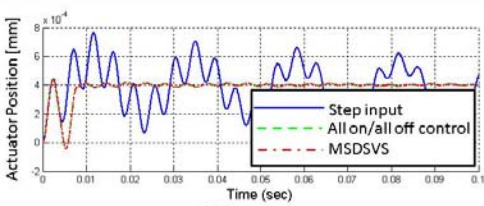
Since it is impractical to run a numerical solver algorithm for every single possible combination, a minimum switching candidate must be identified before the numerical solver is run. In order to accomplish this, we begin by choosing a vector $\boldsymbol{A} = [A_0 \quad A_1 \quad \ldots \quad A_m] = [1 \quad 1 \quad \ldots \quad 1]_{2n+1}$. We then modify the values in $\boldsymbol{A}$ by subtracting 2 (thereby converting 1s to -1s) and/or adding 1 to individual elements such that (3) and (4) remain satisfied. This insures that all impulses will have the minimum amplitude possible.

If the numerical solver successfully determines a solution for this set of amplitudes, it is chosen as the minimum switching solution. If the solver is unsuccessful, the next most optimal $\boldsymbol{A}$ is tried and so on until the solver returns a minimum switching solution. If a sufficient number of iterations elapse, the number of switches will be the same as for all on/all off control and there is no point in going further. In cases where there should be phase cancellation, the solver may not exactly overlay two impulses. The minimum switching solution is then run through an algorithm that detects impulses that are very close together (subject to some threshold) and consolidates them.

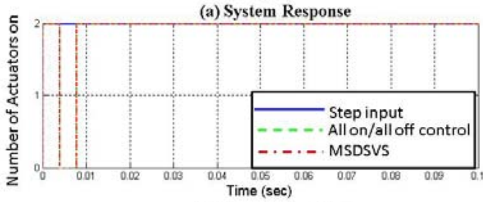
IV. Simulation Results

Our aim in the rest of this paper is to determine a discrete minimum switching command that will suppress oscillation at the frequencies of the two modes of the piezoelectric cellular actuator shown in Fig 3. In this case, $r = 6$ and $n = 2$.

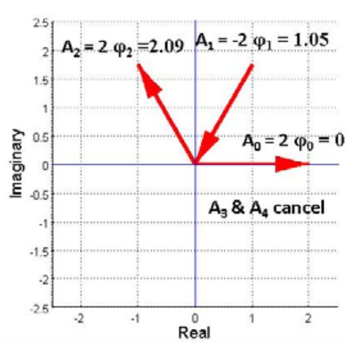
Simulations have been conducted for the command shaping approach described above. *G(s)* is constructed for modes 2 and 4 from Table I. The response of *G(s)* is then simulated for a step input, all on/all off control, and MSDSVS. Fig. 6 shows the response, command and MSDSVS vector diagram for a goal position of 2 actuators on. In this case, the MSDSVS command is identical to all on/all off control. Since the second frequency we want to suppress is a multiple of the first, in this special case the solver eliminates the fourth and fifth impulses.



(a) System Response



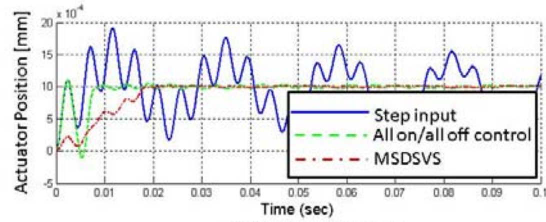
(b) Command to System



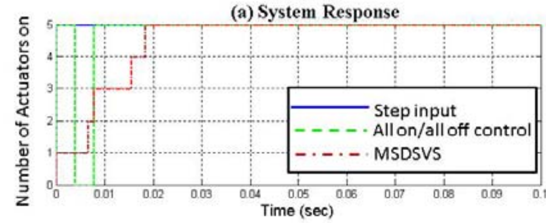
(c) Vector Diagram of MSDSVS Command

Fig. 6 Cellular Actuator Vibration Suppression $y_g = 2$

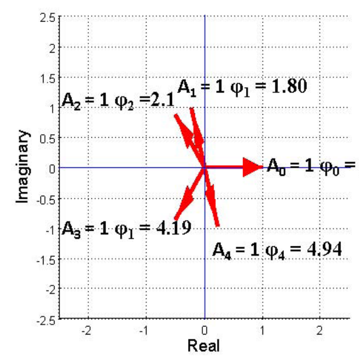
Fig. 7 shows the response, command and phase for a goal position of 5 actuators on. In this case, a monotonically increasing command is calculated for MSDSVS. Fig. 8 shows the response, command, and vector diagram for a goal position of 6 actuators on. In this case, a monotonic solution for MSDSVS is not possible, but the number of changes of state of the first layer actuators is still less than that required for all on/all off control.



(a) System Response

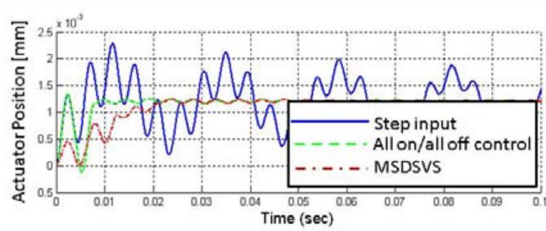


(b) Command to System

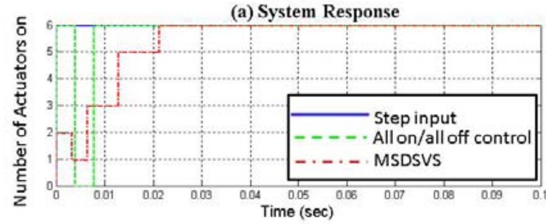


(c) Vector Diagram of MSDSVS Command

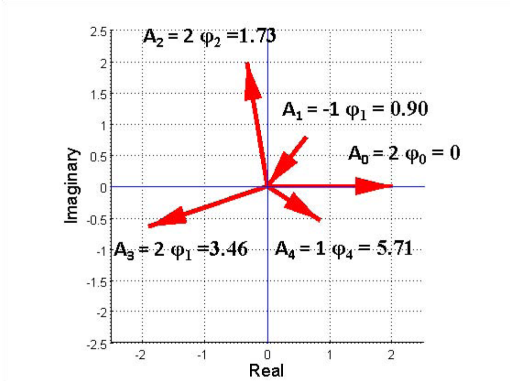
Fig. 7 Cellular Actuator Vibration Suppression, $y_g = 5$



(a) System Response



(b) Command to System



(c) Vector Diagram of MSDSVS Command

Fig. 8 Cellular Actuator Vibration Suppression, $y_g = 6$

In figures 7 and 8, the MSDSVS solution has a longer rise time than the all on/all off solution. This is merely because the MSDSVS solution has less control effort. However, it should be noted that the response to the MSDSVS command reaches steady state within one period of the mode of vibration. If the MSDSVS solution has a rise time that is too large for the application, other DSVS solutions could be examined that are not minimum switching solutions, but require less control effort than all on/all off control. The existence of various DSVS solutions gives the designer flexibility with regard to the tradeoff between control effort and speed of response. The individual impulse amplitudes are smaller than those required for all on/all off control. This results in smaller transients, which may be beneficial if the cellular actuator is moving shock-sensitive components.

Table II shows the number of switches and settling time, t_s, required for each of the three cases illustrated above. Notice that as goal increases, a zero vibration solution can be constructed that requires fewer changes in state than all on/all off control.

TABLE II COMPARISON OF ALL ON /ALL OFF CONTROL AND MSDSVS

y_g	All on/all off control		MSDSVS	
	# switches	t_s [ms]	# switches	t_s [ms]
2	6	7.75	6	7.75
5	15	7.75	5	18.4
6	18	7.75	8	21.2

V. CONCLUSION AND FUTURE WORK

This paper has presented a method to determine a command profile for a piezoelectrically driven cellular actuator with redundant discrete actuation. Several solutions exist, but the minimum switching solution presents several advantages, most notably energy saving and reduced amplification of unmodeled high frequency modes. Simulation results indicate that solutions other than all on / all off control can successfully produce the desired displacement without oscillation. Future work could include experimental verification of the command profile and adding additional constraints to the command generation process to increase the robustness to modeling errors. This amounts to extending DSVS by introducing additional impulses, along the same lines as robust input shaping techniques.

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