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UNSTEADY FLOW OF AQUEOUS SOLUTION OF LONG-CHAIN POLYMERS IN PIPE NETWORKS

A THESIS

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LONG-CHAIN POLYMERS IN PIPE NETWORKS

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SUMMARY

The addition of certain foreign materials to a fluid system results in a decrease in the frictional drag and hence increased velocities. The possibility of using friction-reducing additives to temporarily improve water distribution systems during emergencies is investigated mathematically. The primary concern is the prediction of the unsteady flow conditions which result from the local injection of a long-chain polymer into a pipe network during emergency flow conditions. A computer program is developed to solve the differential equations associated with unsteady network flow resulting from such local injections. The program is then used to show that an existing inadequate water distribution system for fire fighting can be made adequate quickly enough by a one-point injection of a friction-reducing long-chain polymer.

CHAPTER I

INTRODUCTION

The addition of certain foreign materials can significantly reduce the friction drag in turbulent flow. Widely differing materials such as sand, neutrally buoyant particles, and wood fibers have been observed to reduce friction drag in aqueous mixtures. However, the most effective drag-reducing materials appear to be certain long-chain polymers in solution.

The first study of the effect of polymer additives was made in 1948 by Toms [1, 12][1]. He reported a "hitherto unknown feature of the relation between polymer concentration and rate of flow at constant pressure gradient." In recognition of his work, the phenomenon has been called the "Toms Effect."

Most of the work to date has dealt with the effects of polymer additives on steady-state motion. These studies have been made with rotating disks, pipe flow with constant head differences, submerged bodies, and flow through porous media. Rotating disk studies [3, 4, 5] were usually made with premixed solutions. Pipe flow [1, 2, 3, 5, 6, 7, 8, 9, 10] has been studied using various types and concentrations of polymers with either premixed solutions or with various methods of injection. Studies with bodies moving through dilute polymer solutions

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[1] Numbers in brackets refer to similarly numbered references in the bibliography.

[3, 11] were made to investigate the usefulness of polymers in reducing the drag on ships and subsurface marine vehicles. The flow of polymer solutions through porous media [12] has also been studied.

The work of unsteady or time-varying effects seems to have been limited to the observation of some of the polymers as they deteriorated with time [3, 9, 12]. In general, an unsteady state results when a polymer is introduced locally into a moving single-phase liquid system. The reduced frictional drag in that part of the system causes the entire flow to accelerate. The system contains in fact two different fluids. This transient condition persists even after the additive has come in contact with the entire system. Eventually, of course, a new steady-state condition exists.

The purpose of this study is to develop a mathematical method to predict the transient flow patterns in a pipe network subject to locally introduced polymer additives. There are analytical methods for determining the steady-state conditions with or without additives. Single-phase transient conditions are more difficult to predict. No reference has been found in the literature dealing with two-phase transient flow.

One possible use for the results of this study lies in the field of water distribution. Previous studies [6, 9] have shown that many of the polymers are not harmful to plant or animal life. These polymers may be used to increase the capacity of water-distribution systems for fire fighting. The use of polymers may be economical only during periods of high demand. Still it would be necessary to know the time required for the transient effects of the polymer to become noticeable to the system after injection at one or more points of a system.

Thus, a method for predicting the short-term time-dependent effects of polymer additives is needed. The methods of solution for pipe-flow transients developed herein are computer orientated since classical methods of solution of the differential equations are not applicable.

CHAPTER II

LONG-CHAIN POLYMERS

A review of the literature in the field of friction-reducing additives may be helpful to the reader in understanding the nature of the phenomenon.

Background

In 1948 Toms [1] reported that the addition of polymethylmethacrylate to the chlorobenzine resulted in a reduction of pipe-friction drag. This accidental discovery became the basis for the later studies, although several years passed before these subsequent studies were started.

The oil industry became interested in possible uses of polymers. Dodge and Metzner [3] used sodium carboxymethylcellulose in their studies with pipeflow of oil and oil recovery materials. Their results have applied commercially in oil fields. Since then, aqueous solutions have been studied by many investigators.

Most of the early investigators attributed the friction-reducing phenomenon to "non-Newtonian" properties. However, work done at the Naval Ordinance Test Station [3] showed that the turbulent friction-reduction effect can be observed at polymer concentrations at which the solutions are Newtonian by conventional viscometry.

Explanations of the "Toms Effect"

Four types of explanation have been proposed for the "Toms Effect." To date, none of these have been confirmed. These explanations were as follows:

(1) "Effective" slip is induced by an abnormally mobile, oriented layer of macromolecules (very large molecules with molecular weights of the order of a million grams per mole) near the pipe wall.

(2) The polymers delay the laminar to turbulent transition in the boundary layer by damping of disturbances which results in reduction of turbulent energy production.

(3) The macromolecules elongate in the direction of flow under shear ("anisotropic viscosity") and thereby impede the transverse transport of momentum and thus reduce the turbulent shear stresses and hence reduce drag.

(4) The most popular explanation in current literature is that of visco-elasticity. Elastic interactions between macromolecules and turbulence result in the reduction of turbulent energy production and energy dissipation, and hence reduced friction losses.

Parameters

Much of the early work was done with turbulent flow between flat plates and flow about rotating discs. Hoyt and Fabula [3] reported that in their studies with rotating discs the three most significant parameters affecting the ability of a polymer to reduce the turbulent frictional resistance of a fluid were linearity, molecular weight, and solubility.

The most effective polymers are "long-chain" molecules having an essentially unbranched structure. The exact configuration for these macromolecules is poorly understood, but it is thought that the length to diameter ratios may be as high as 165,000 for poly(ethylene oxide) with a molecular weight of approximately six million grams per mole and may be as low as 350 for guar gum. These ratios depend on the helix model and the molecular chain flexibility. The more flexibile molecules tend to be random coiling and hence have lower length to diameter ratios than would otherwise be expected.

Typical effects of molecular weight are demonstrated in Figure 1. Accordingly, higher molecular weight polymers are more effective in reducing drag. However, this is not always the case, and Table 1 indicates that poly(ethylene oxide) is about 65 times more effective than the heavier gum karaya molecule on a weight basis. Table 1 also indicates that polymers with higher solubilities are more effective than similar polymers with lower solubilities. Figure 2 shows the friction-reducing capacity of a polymer as a function of the concentration which may be dependent on solubility.

Experiments Conducted at the Georgia Institute of Technology

A series of experiments were conducted in the Hydraulics Laboratory of the School of Civil Engineering, the Georgia Institute of Technology. The polymer used was Polyhall 295 which is an anionic copolymer of polyacrylamide, polyacrylic acid, and polysodium styrene sulphomate. The molecular weight of this nonrandom-coiling polymer is

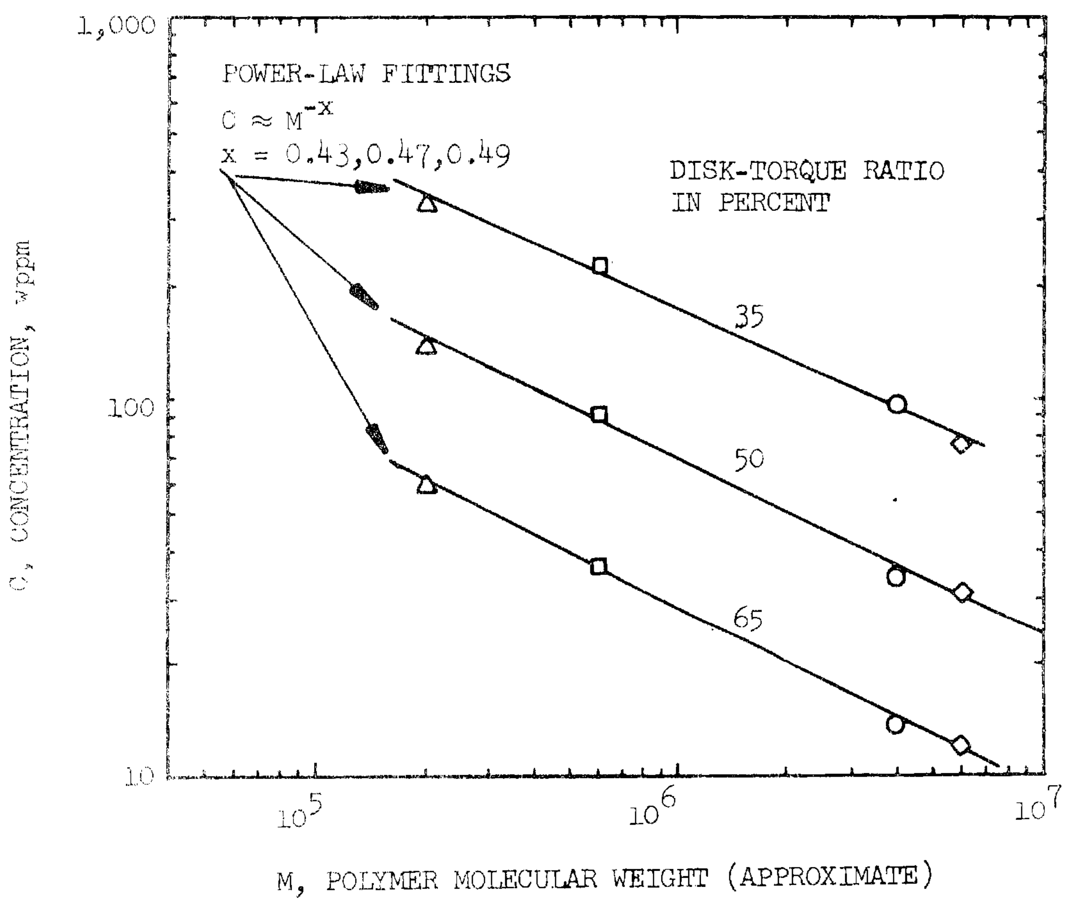

Figure 1. Dependence of Required Concentrations for Various Disk Torque Ratios on Molecular Weight of Poly (Ethylene oxide). (After Hoyt and Fabula [3])

approximately 3.8×10^6 grams per mole.

J. B. Jackson [12] conducted a series of experiments as a graduate research project in 1967 to investigate the effects of polymer additives. The series dealt with laminar flow of polymer solutions through a sand bed. In cases where he found the apparent viscosity of the solution to be very close to that of water, the discharge was actually decreased appreciably. The result was a decrease in the permeability with in-

Table 5. Comparison of Friction-Reduction Effectiveness of Water-soluble Polymer Additives Measured with Rotating-Disk Facility. (After Hoyt and Fabula [5])

Additive	C_R [a]	$M \times 10^{-6}$ [b]	Notable Characteristics
Guar gum, w, x (J-2FP) [c]	60	0.2	Straight chain molecule with single-membered side branches
Locust bean gum, m	260 (260) [d]	0.31	Similar to guar but with fewer side branches, causing reduced solubility and less hydrogen bonding
Carrageenan or Irish moss, m (Stamere NK)	650 (420)	0.1 - 0.8	Strongly charged anionic polyelectrolyte
Gum karaya, m	780	9.5	Highly branched molecule; relatively insoluble; acidic
Gum arabic, b	Ineff.	0.24 - 1	Highly branched molecule
Amylose, s (Superlose)	Ineff.	>0.15	Linear chain molecule; retrogrades rapidly
Amylopectin, s (Ramalin G)	Ineff.	1.2	Highly branched molecule
Hydroxyethyl cellulose, u			
(Cellosize QP-15000)	220	...	Nonionic; formed by additive of ethylene oxide to cellulose; has side branches of various lengths
(Cellosize QP-30000)	220	...	
(Cellosize QP-52000)	160	...	
Sodium carboxymethyl-cellulose, h (CMC 7HSP)	400	0.2 - 0.7	....

Table I continued

Additive	C_R [a]	$M \times 10^{-6}$ [b]	Notable Characteristics
Poly(ethylene oxide), u			
(Polyox WSR-35)	70	0.2	Very water soluble; no biological
(Polyox WSR-205)	44	0.6	oxygen demand; apparently an un-
(Polyox WSR-301)	17	4	branched molecule with unusual
(Polyox coagulant)	12	>5	affinity for water
Polyacrylamide, d			
(Separan NP10)	26	1	Nonionic
(Separan NP20)	25	2	Nonionic
(Separan AP30)	29	2 - 3	Anionic
Polyhall-27, s	130	...	....
Poly vinylpyrrolidone, f			
(K30)	Ineff.	0.04	....
(K90)	2900	0.36	....
Polyvinyl alcohol, e			
(Elvanol 51-05)	Ineff.	0.032	....
(Elvanol 72-60)	Ineff.	0.17 - 0.22	....
Silicone, u (L-531)	Ineff.	...	....
Polyacrylic acid, g			
(Goodrite 773x020 B-3)	Ineff.	0.006	....
(Goodrite K-702)	Ineff.	0.090	....
(Goodrite K-714)	Ineff.	0.2 - 0.25	....

Table I. Continued

Additive	C_R[a]	$M \times 10^{-6}$[b]	Notable Characteristics
Carboxy vinyl polymer, g (Carbopol 941)	Ineff.	...	Inconclusive test due to precipitation upon dilution

[a]C_R = concentration required (in weight parts per million) for 35% disk-torque reduction at 40 rev/sec with lake water as the solvent.

[b]M = approximate molecular weight of the polymer according to the literature.

[c]The source of each polymer for this work is indicated by the letter after its name: b = Braun Div., Van Waters and Rogers, Inc.; d = Dow Chemical Co.; e = E. I. Dupont; f = General Aniline and Film Corp.; g = B. F. Goodrich Chemical Co.; h = Hercules Powder Co.; m = Meer Corp.; s = Stein, Hall and Co.; u = Union Carbide Chemicals Co.; w = Westco Research.

[d]C_R values in parenthesis are for solutions given heat treatment to increase polymer solubility.

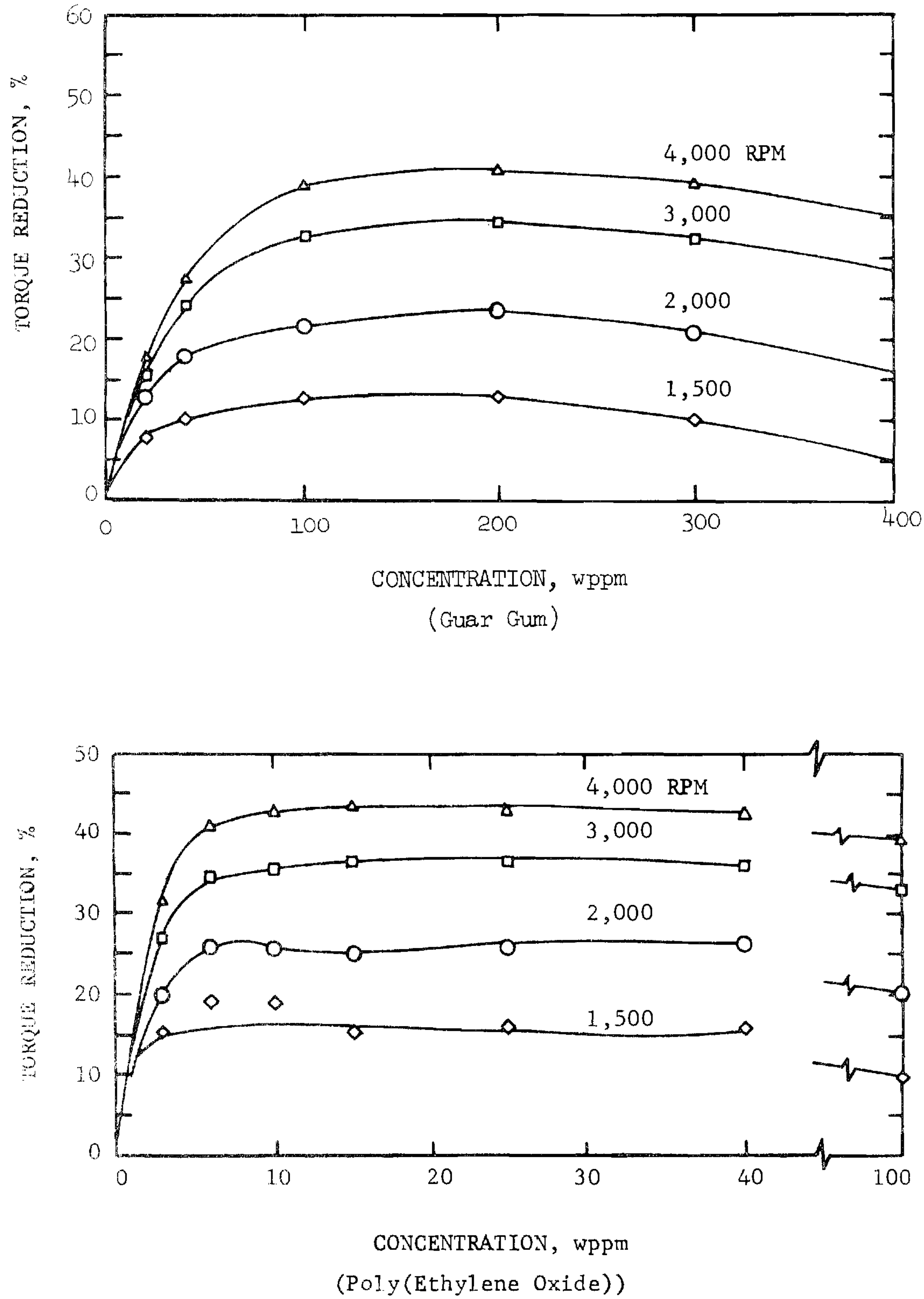

Figure 2. 7.6-cm Disk Torque Reduction Versus Polymer Concentration. (After Hoyt and Fabula [3]

creasing concentration (See Figure 3). He attributed this to a bridging of the long-chain polymer across the voids in the media which effectively reduced the volume of the voids. This series of experiments indicated that the polymer additive increases friction drag in laminar flow through porous media.

J. B. Jackson's other experiments dealt with turbulent pipe flow. Centrifugal pumps were employed in a recirculating system. Polymer solutions were passed through a two-inch nominal diameter test section of galvanized pipe. He reported a 68 percent reduction in friction drag when a 300 wppm (parts per million by weight) solution was used (See Figure 4). He also reported that immediately upon the addition of the polymer, there was an even higher reduction which, however, lasted for only a short time (See Figure 5). It was reasoned that this degradation was due to physical scisson of the polymer in the pump. This reasoning was reinforced by observations of slower rates of degradation which occurred when a pump with larger passages was used (See Figure 4). The fact that the polymer's friction-reducing properties were diminished after repeated passes through a pump seems to indicate that the long molecular chains are broken mechanically by the pump's impeller.

In an undergraduate research project conducted by P. H. Flowers and H. C. Jackson [6] in 1968, time-varying flows resulting from the local injection of concentrated polymer solution into a pipe-flow system were investigated experimentally. Figure 6 shows the increase in mean velocity when a polymer solution was injected at the upstream end of a two-inch nominal diameter test section of pipe (Same as J. B. Jackson's

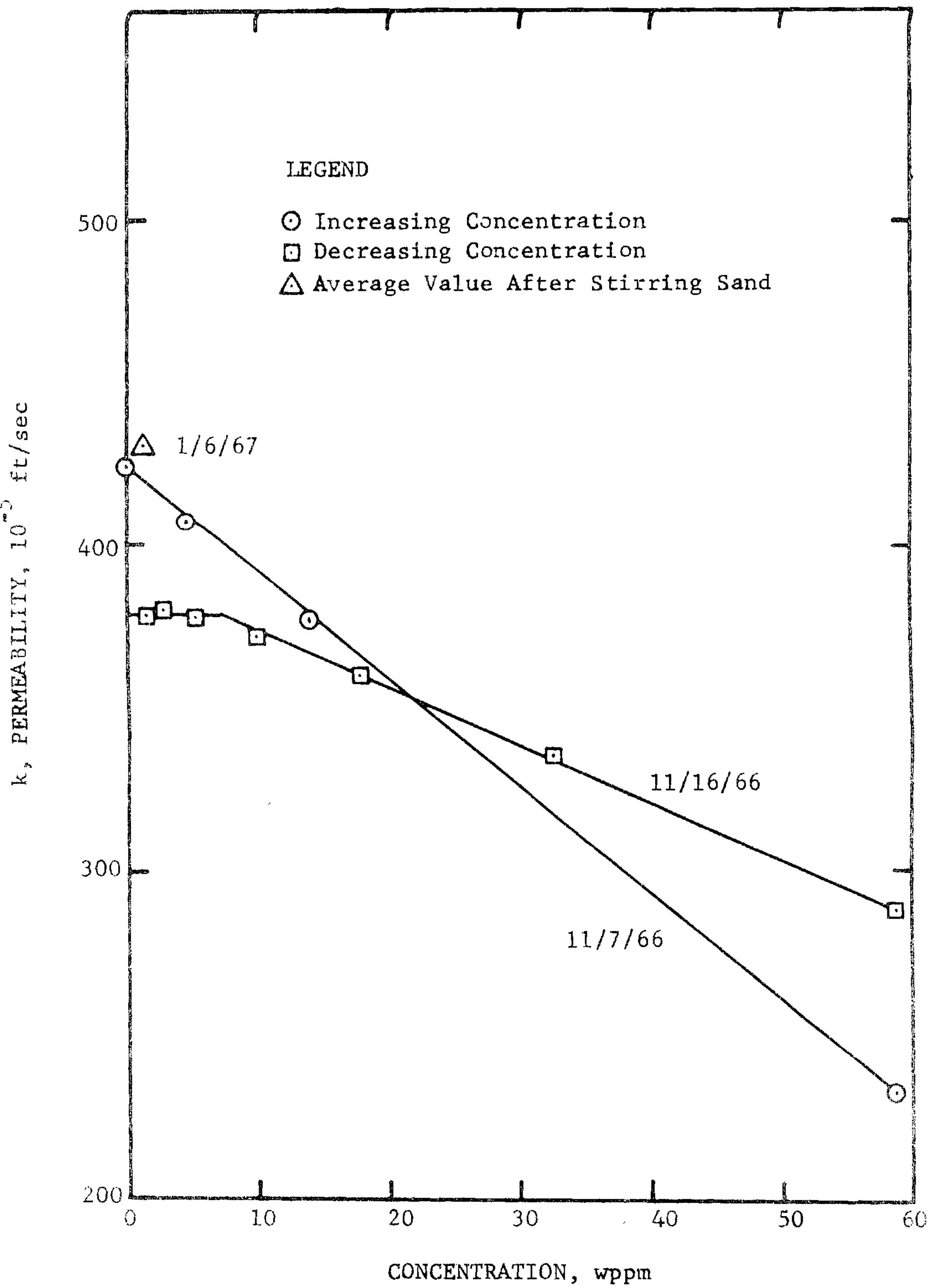

Figure 3. Dependence of Permeability in Sand on Polymer Concentration (After J. B. Jackson [12])

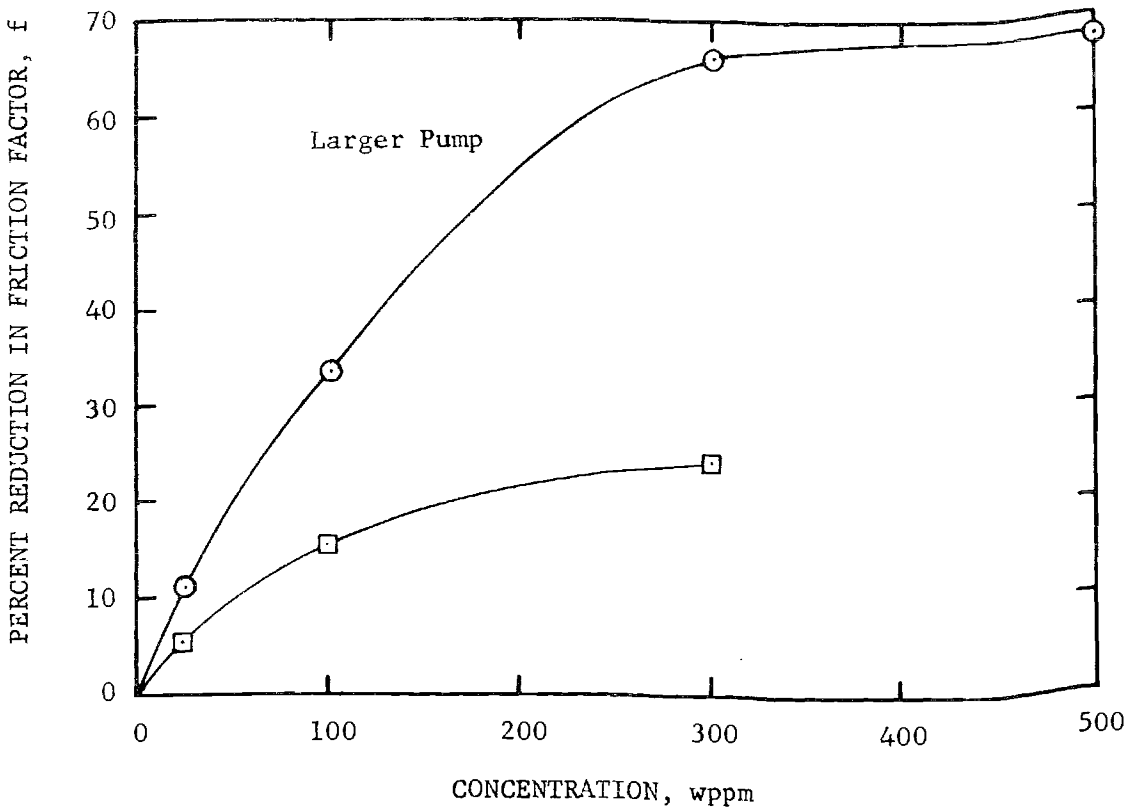

Figure 4. Effects of Polymer Concentration on Recirculating Pump Systems. (After J. B. Jackson [12])

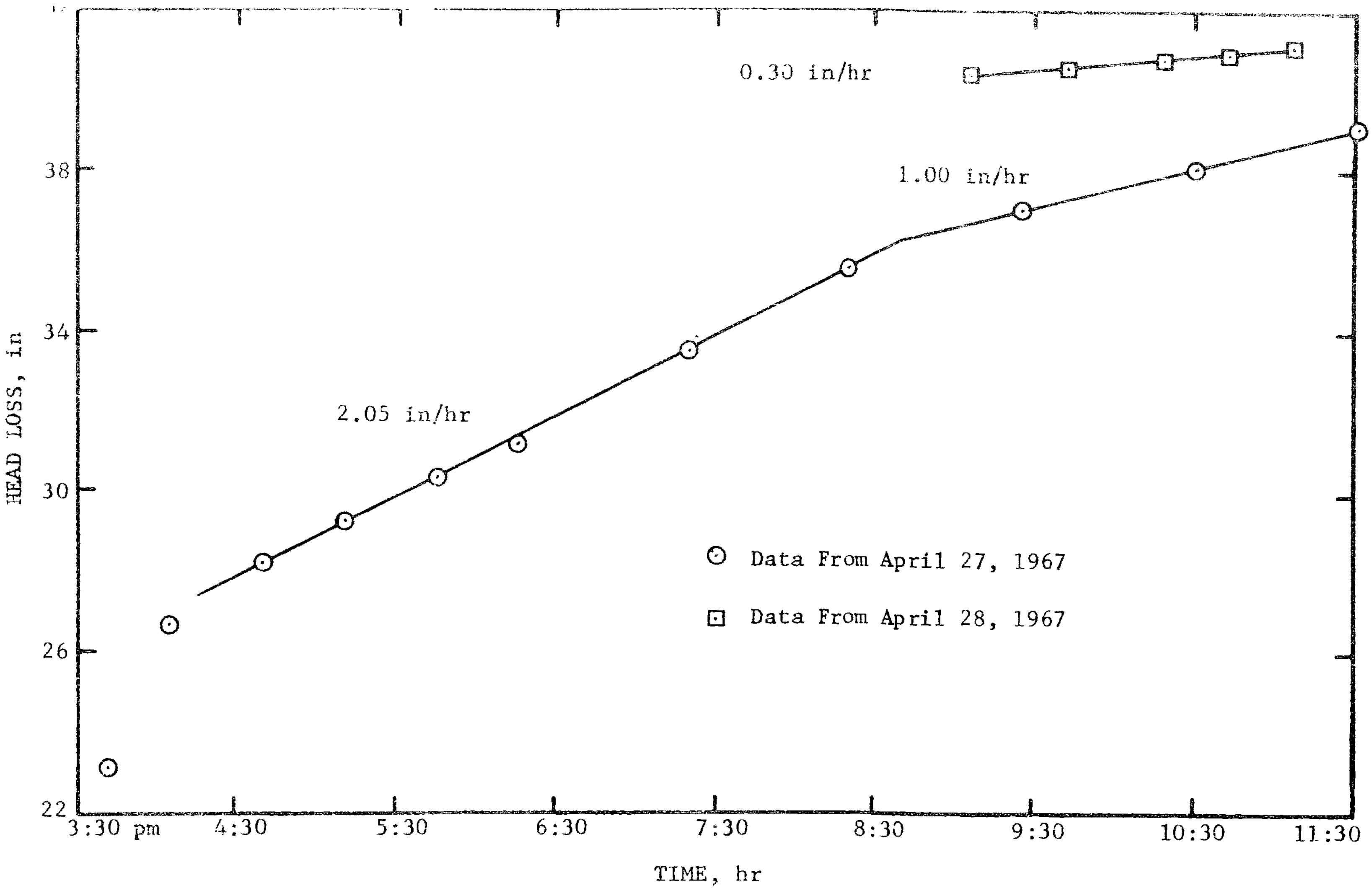

Figure 5. Typical Head Loss Versus Time Plot For Pump System (Concentration 100 wppm). (After J. B. Jackson [12])

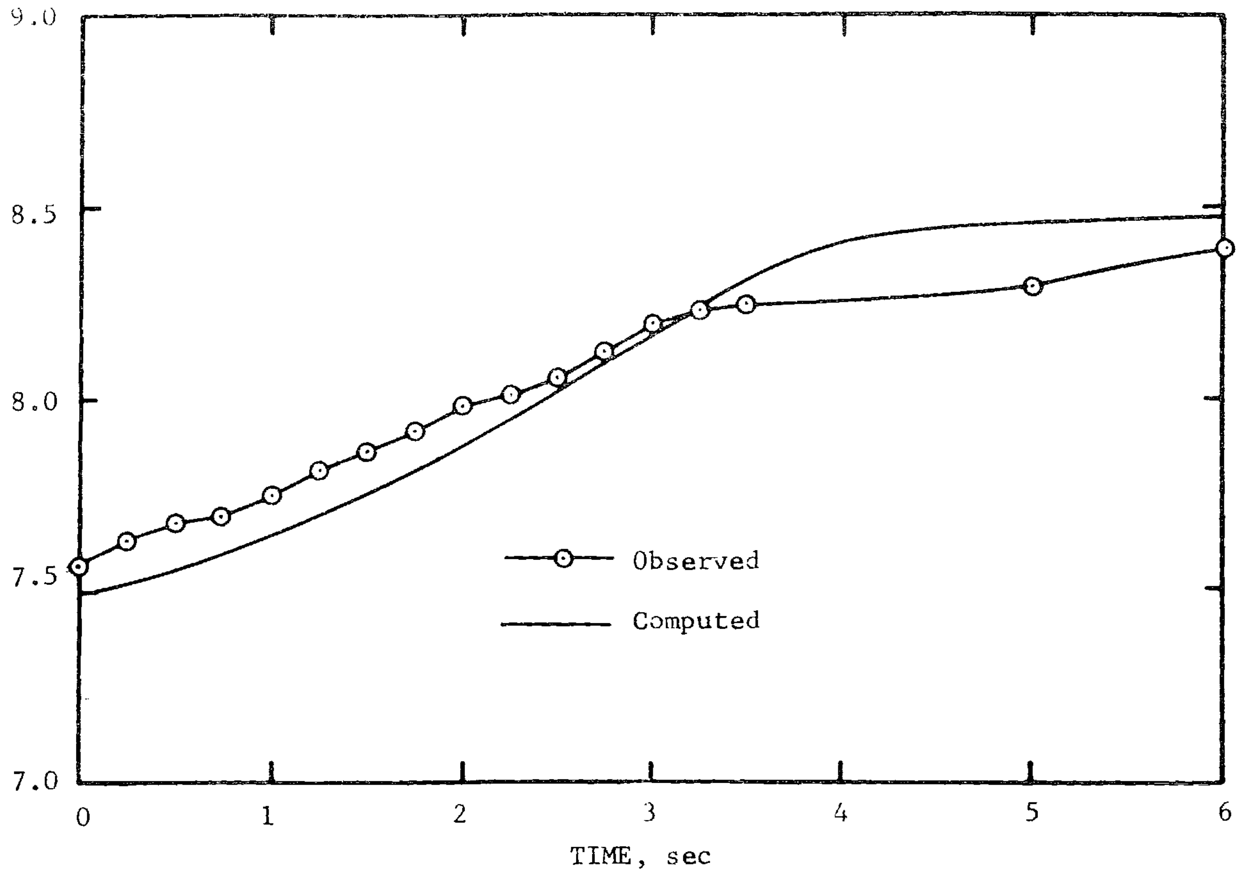

Figure 6. Time Variation of Velocity After Injection of Polymer (Concentration 100 wppm). (After Flowers and Jackson [6])

pipe). This curve compares favorably with the computer-predicted curve which will be discussed in Chapter IV.

The knowledge gained from the studies at Georgia Tech also indicated that care should be taken in preparing the aqueous solutions. Polymers in powder form may not mix readily with water due to the formation of globules with tough skins. Also the macromolecules tended to settle in aqueous solutions when left at rest.

Summary of Polymer Properties

The general properties of friction-reducing polymers may be summarized as follows.

(1) Most "effective" polymers have high molecular weights.

(2) Effective polymers usually have high length to diameter ratios.

(3) The effects of polymer additives were most noticeable when injected in the laminar boundary layer [2, 8].

(4) Solubility played an important role in a polymer's effectiveness.

(5) Pipe friction reductions of as high as 68 percent were reported by J. B. Jackson in his studies with Polyhall 295.

(6) Polymer additives increased resistance and reduced the laminar flow through porous media.

(7) Polymer chains may be destroyed by mechanical action.

(8) The polymer solution coated the pipe walls and pro-its effectiveness after injection had ceased during unsteady flow tests [6].

(9) Care must be taken when mixing powdered polymer to form solutions.

(10) There is an optimum concentration of polymers above which additional polymer produces little further reduction in pipe friction [2, 12].

(11) There may be a Reynolds Number above the laminar-turbulent transition range below which no friction reduction occurs [2].

CHAPTER III

STEADY FLOW OF POLYMER SOLUTIONS IN A SINGLE PIPE

The Darcy-Weisbach friction factor, f, will be used as the basis for all discussions. For the purposes of this study, the friction factor is defined as

$$f = 2gD\Delta H/LV^2 \tag{1}$$

where

g = gravitational constant (32.17 ft/sec/sec),

D = pipe diameter (ft),

ΔH = head loss along test section (ft-lb/lb),

L = length of test section (ft), and

V = mean velocity (ft/sec).

If f_B is defined as the friction factor in water, and f_A is defined as the friction factor in a polymer solution, then the percent reduction of friction drag, R, may be defined as

$$R = \left(\frac{f_B - f_A}{f_B}\right) \times 100 \tag{2}$$

If a single pipe connecting two reservoirs with a constant difference in surface elevation is used (See Figure 7), then the head loss, diameter, and length do not change when the fluid is changed from water to a polymer solution. Thus, it is apparent that at steady state

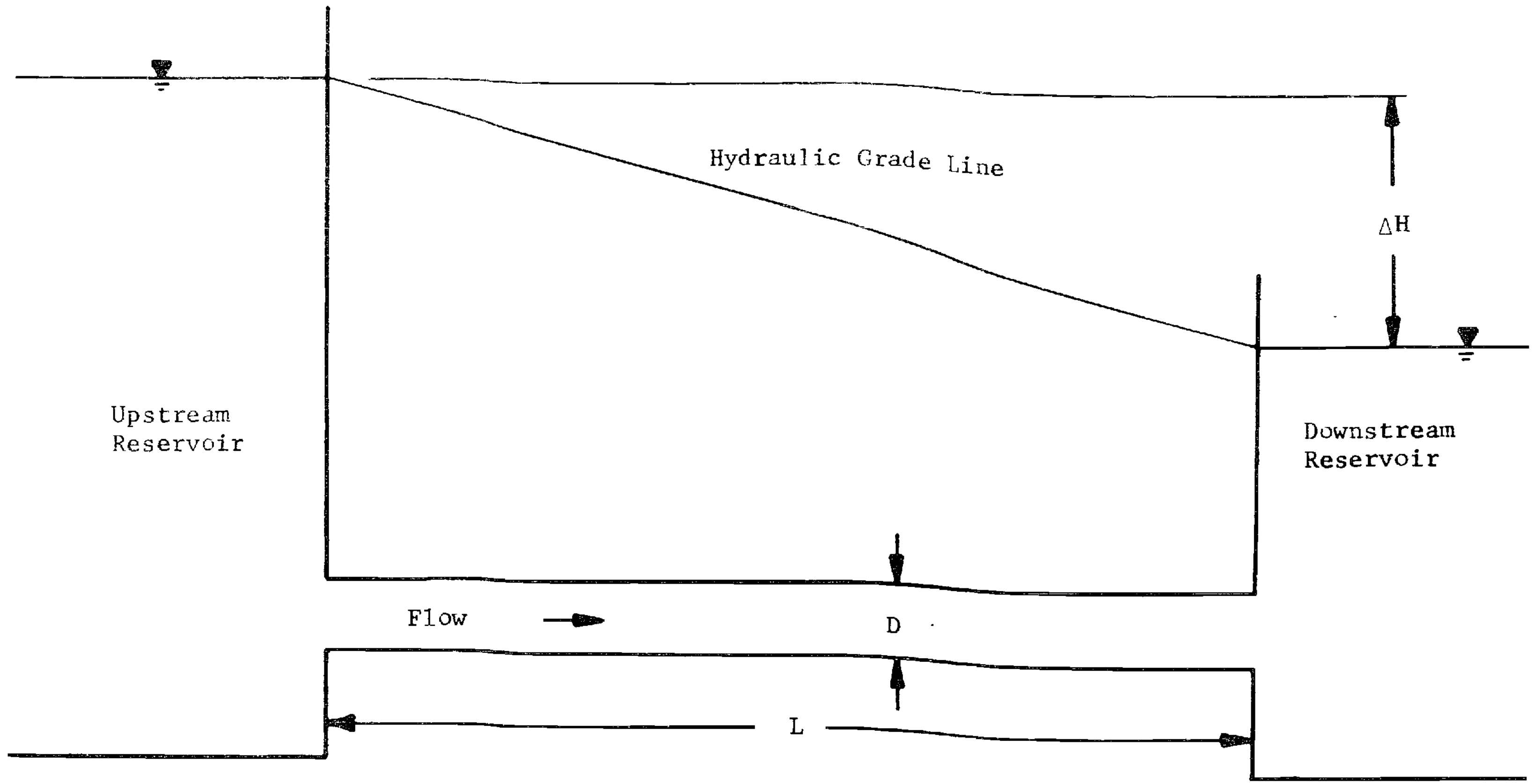

Figure 7. Diagram of Single-Pipe System.

$$f_B V_B^2 = f_A V_A^2 \quad (3)$$

where

V_B = velocity without polymer, and

V_A = velocity with polymer.

Algebraic manipulation of equations 2 and 3 yields

$$V_A = \frac{V_B}{1 - R/100} \quad (4)$$

Figure 8 shows velocity increase versus percent reduction in friction drag as computed from equation 4. Accordingly, a 75 percent reduction in friction drag will double the velocity. Since the manufacturer's claim was an 82 percent reduction [11], a 75 percent reduction using Polyhall 295 would not be unreasonable. A reduction of 50 percent will cause the original velocity to be increased by 1.414.

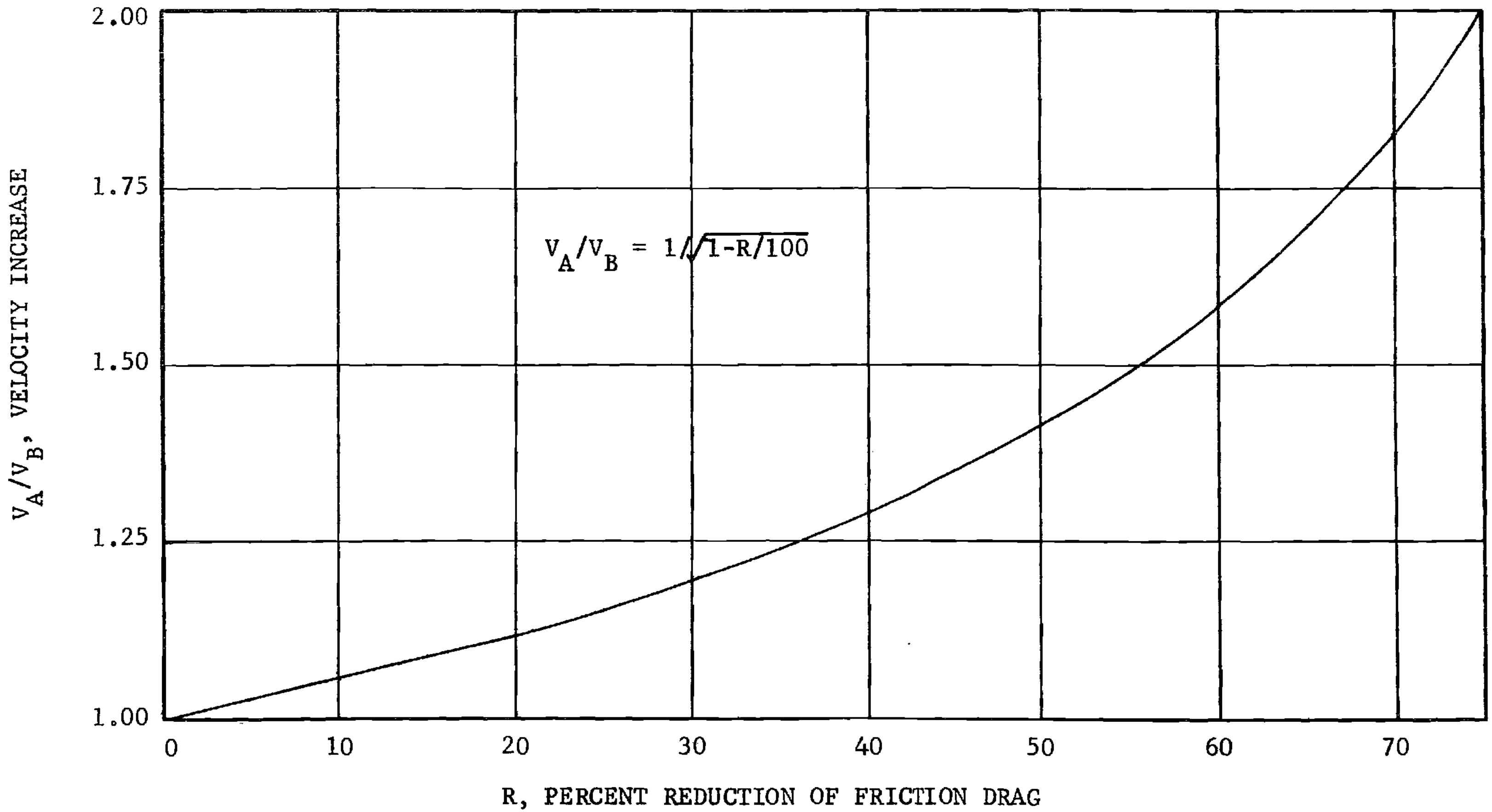

Figure 8. Graphical Representation of Equation 4 Showing Velocity Increase as a Function of R.

CHAPTER IV

UNSTEADY POLYMER FLOW IN A SINGLE PIPE

The simplest example of unsteady pipe flow resulting from friction-reducing additives is a system composed of a single pipe connecting two constant elevation reservoirs (See Figure 9). Before time, T = 0, there is steady-state flow. At time, T > 0, an additive is introduced at a constant rate at the upstream reservoir. All fluid leaving the upstream reservoir is assumed to be a homogeneous mixture or solution. Since the additive reduces friction, the flow accelerates to a new steady state.

Mathematical Model

Prediction of the unsteady flow in the above example can be accomplished by means of a suitably constructed mathematical model. It is convenient to make the following reasonable assumptions:

(1) The local velocity is the average velocity (Q/A).

(2) The flow is fully turbulent, and the friction factor does not vary with small changes in velocity.

(3) The fluid is incompressible.

(4) The density of the mixture is the same as that of water.

(5) Minor losses can be neglected.

(6) An immiscible interface exists between the water and the mixture.

(7) The interface is normal to the direction of flow and moves

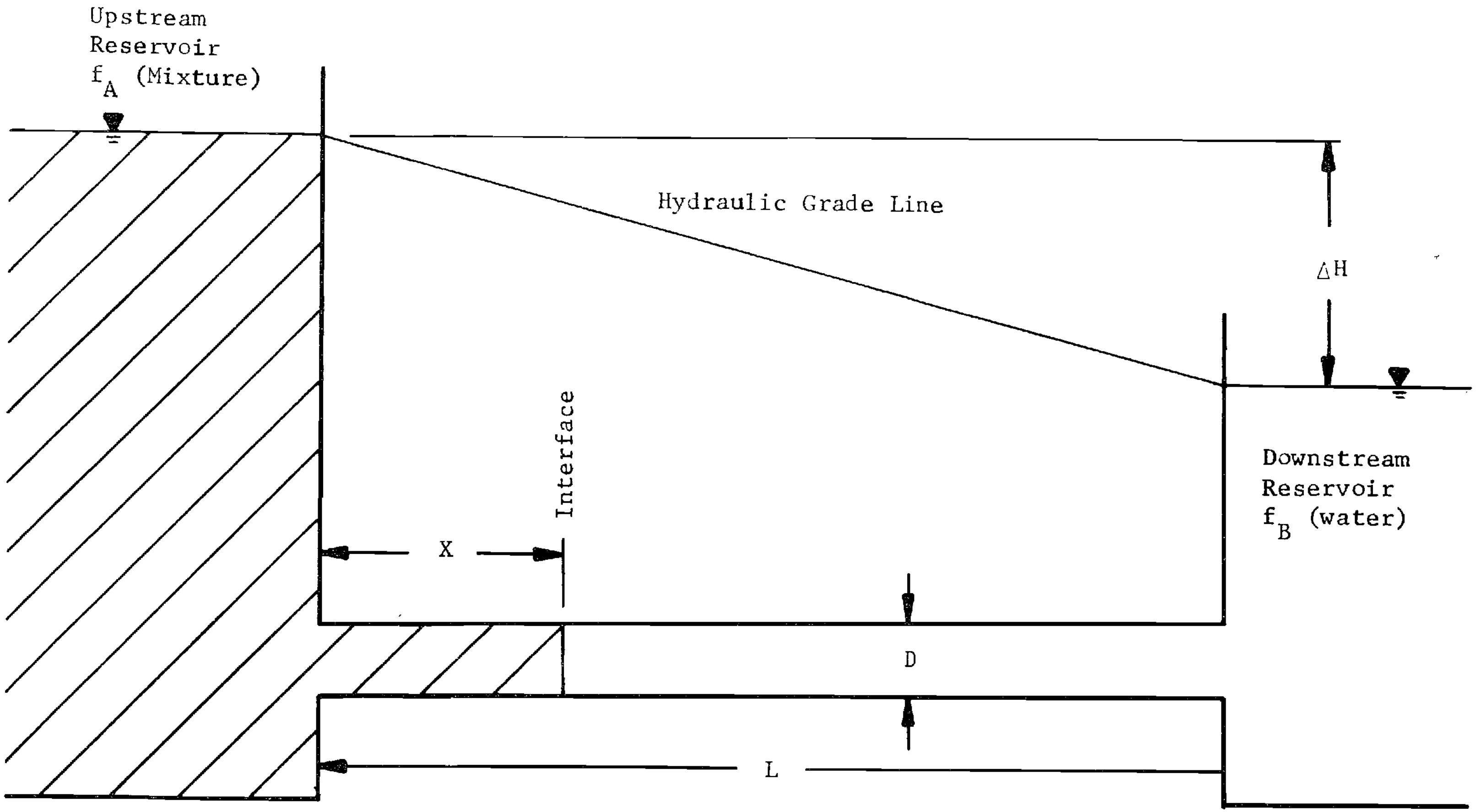

Figure 9. Diagram of Single-Pipe System With Unsteady Flow.

with the average velocity, V.

(8) The local friction factor is changed instantly when the interface passes.

(9) The flow parameters may be related by the Darcy-Weisbach energy equation.

The equation (steady-state) at time, T = 0, is

$$\Delta H = f_B \frac{L}{2gD} \left(\frac{dX}{dt}\right)^2 \qquad (5)$$

where

ΔH = difference in reservoir elevations (ft),

f_B = friction factor before the interface passes,

L = length of pipe (ft),

D = diameter of pipe (ft),

$\frac{dX}{dt}$ = V, velocity (ft/sec), and

g = gravitational constant (32.17 ft/sec/sec).

The term $\frac{dX}{dt}$ is used in equation 5 since X will later be used to denote the variable distance in feet from the upstream reservoir to the interface, and t will be used to denote an increment of time.

At T > 0, the velocity will become time dependent (hence $\frac{dX}{dt}$). The unsteady Darcy-Weisbach energy equation is

$$\Delta H = \frac{\left[f_A X + f_B (L - X)\right]}{2gD} \left(\frac{dX}{dt}\right)^2 + \frac{L}{g}\left(\frac{d^2X}{dt^2}\right) \qquad (6)$$

where

f_A = friction factor after interface passes, and

$\frac{d^2X}{dt^2}$ = acceleration (ft/sec/sec).

In effect, the pipe is treated as two pipes in series with different friction factors and with time-varying lengths. Since the fluid is accelerating, there must be an inertia term. This term is $\frac{L}{g}\left(\frac{d^2X}{dt^2}\right)$, and it represents the inertia force after an appropriate conversion for the energy equation.

Thus, the interface moves from the upstream reservoir to the downstream reservoir in an accelerating flow. However, the new steady state is not established until some time after the interface has reached the downstream reservoir. In fact, the acceleration is greatest at the instant when the interface reaches the downstream reservoir, and mathematical continuity requires that a positive acceleration be maintained. Equation 6 applies when $0 \leq x \leq L$; however, when x = L, we have

$$\Delta H = f_A \frac{L}{2gD}\left(\frac{dX}{dt}\right)^2 + \frac{L}{g}\left(\frac{d^2X}{dt^2}\right) \tag{7}$$

Solution

Classical solutions for these second-order differential equations are difficult to obtain [13]. Therefore, the techniques of numerical analysis and the use of the digital computer have been chosen. The Runge-Kutta method (See Appendix 1) has been applied to equations 6 and 7.

The standard form for the Runge-Kutta solution of second-order equations is

$$y'' = \emptyset\ (x,y,y')$$

where $\emptyset$ represents "a function of." Equation 6 may now be expressed as

$$\frac{d^2X}{dt^2} = C_3 \left[\Delta H - C_1 X \left(\frac{dX}{dt}\right)^2 - C_2 \left(\frac{dX}{dt}\right)^2\right] \qquad (8)$$

where

$C_1 = (f_A - f_B)/2gD$

$C_2 = f_B L/2gD$, and

$C_3 = g/L$.

Obviously, equation 8 is

$$y'' = \emptyset\ (y,y')$$

which is a special case of the standard Runge-Kutta equation in which the independent variable, t, does not appear.

The initial values for the Runge-Kutta solution are the original steady-state solution. They are as follows:

$T = 0$,

$X = 0$,

$\frac{dX}{dt} = 2gD\Delta H/f_B L$, and

$\frac{d^2X}{dt^2} = 0$.

The Runge-Kutta "dummy" variables may be found on lines B043 to B048 of the computer program on page 100, Appendix 2.

As time is incremented, new values for the flow parameters are calculated until $X \geq L$. Equation 7, which then prevails, may be expressed as

$$\frac{d^2X}{dt^2} = C_1 - C_2 \left(\frac{dX}{dt}\right)^2 \qquad (9)$$

where

$C_1 = \Delta Hg/L$, and

$C_2 = f_A/2gD$.

Equation 9 is of the form

$$y'' = \emptyset\ (y')$$

which may have classical solutions. In this study, the Runge-Kutta method will still be used with the initial values taken from the last iteration of the solution of equation 8. Again, the Runge-Kutta "dummy" variables may be found on lines B058 to B061 of the computer program on page 100, Appendix 2.

Example Problem

The computer program in Appendix 2 has been used to solve the single-pipe problem. This program and all others developed herein are written in FORTRAN V for use on the UNIVAC 1108 of the Rich Electronic Computer Center of the Georgia Institute of Technology. The input data appear in Table 2, and a modulated printout of the results appears in Figure 10. The column labeled "HEAD" in Figure 10 is used as a check on the accuracy of the computation. With an unmodulated printout, the value of HEAD will change at the instant that the interface reaches the downstream end of the pipe. The results have been plotted on Figures 11,

RUNGE-KUTTA SOLUTION FOR SINGLE PIPE

TIME (SEC)	X (FT)	VELOCITY (FT/SEC)	ACCELERATION (FT/SEC/SEC)	HEAD (FT)
.000	.00000	17.93669	.00000	10.00000
1.000	17.98099	18.06628	.24679	10.00000
2.000	36.20444	18.41285	.44065	10.00000
3.000	54.86686	18.94080	.61390	10.00000
4.000	74.14338	19.64146	.78956	10.00000
5.000	94.21136	20.52759	.98838	10.00000
6.000	100.00000	21.51551	.90276	10.00000
7.000	100.00000	22.32658	.72487	10.00000
8.000	100.00000	22.97553	.57788	10.00000
9.000	100.00000	23.49138	.45803	10.00000
10.000	100.00000	23.89930	.36137	10.00000
11.000	100.00000	24.22056	.28407	10.00000
12.000	100.00000	24.47274	.22277	10.00000
13.000	100.00000	24.67019	.17416	10.00000
14.000	100.00000	24.82449	.13597	10.00000
15.000	100.00000	24.94488	.10602	10.00000
16.000	100.00000	25.03869	.08257	10.00000
17.000	100.00000	25.11172	.06426	10.00000
18.000	100.00000	25.16854	.04997	10.00000
19.000	100.00000	25.21272	.03884	10.00000

Figure 10. Computer Printout of Solution to Single-Pipe Problem.

RUNGE-KUTTA SOLUTION FOR SINGLE PIPE

TIME (SEC)	X (FT)	VELOCITY (FT/SEC)	ACCELERATION (FT/SEC/SEC)	HEAD (FT)
20.000	100.00000	25.24705	.03018	10.00000
21.000	100.00000	25.27372	.02345	10.00000
22.000	100.00000	25.29444	.01821	10.00000
23.000	100.00000	25.31052	.01414	10.00000
24.000	100.00000	25.32301	.01098	10.00000
25.000	100.00000	25.33271	.00852	10.00000
26.000	100.00000	25.34023	.00671	10.00000
27.000	100.00000	25.34607	.00513	10.00000
28.000	100.00000	25.35061	.00398	10.00000
29.000	100.00000	25.35412	.00309	10.00000
30.000	100.00000	25.35685	.00240	10.00000
31.000	100.00000	25.35897	.00186	10.00000
32.000	100.00000	25.36062	.00145	10.00000
33.000	100.00000	25.36189	.00112	10.00000
34.000	100.00000	25.36288	.00087	10.00000
35.000	100.00000	25.36365	.00078	10.00000
36.000	100.00000	25.36425	.00052	10.00000
37.000	100.00000	25.36471	.00041	10.00000
38.000	100.00000	25.36507	.00032	10.00000
39.000	100.00000	25.36535	.00025	10.00000

Figure 10. Continued

12, and 13 with the aid of a CALCOMP plotter. The time required to run the program on the UNIVAC 1108 was approximately six seconds.

Table 2. Summary of Data for Single Pipe

Darcy Friction Factor After Interface, f_A, ...	0.01
Darcy Friction Factor Before Interface, f_B, ...	0.02
Pipe Diameter in Feet, D, ...	1.0
Difference in Reservoir Elevations in Feet, ΔH, ...	10.0
Pipe Length in Feet, L, ...	100.0
Time Increment in Seconds, t, ...	0.1
Time of Study in Seconds, T, ...	39.9

The acceleration versus time plot in Figure 11 shows that there is a rapid increase in acceleration until the interface reaches the downstream reservoir (See position versus time plot in Figure 13). The maximum acceleration in the example is about one ft/sec/sec. When X = L, the driving force ceases, but the fluid continues to accelerate with exponentially decreasing values for the acceleration function. The flow approaches a new steady state gradually.

The velocity versus time plot in Figure 12 shows a smooth "S"-shaped curve from one steady state to another. As expected, the steepest portion of the curve (at the inflection point) occurs when the interface reaches the end of the pipe and the acceleration is largest. The maximum velocity of 25.4 ft/sec may be predicted also from equation 4 knowing the original velocity and the friction factors.

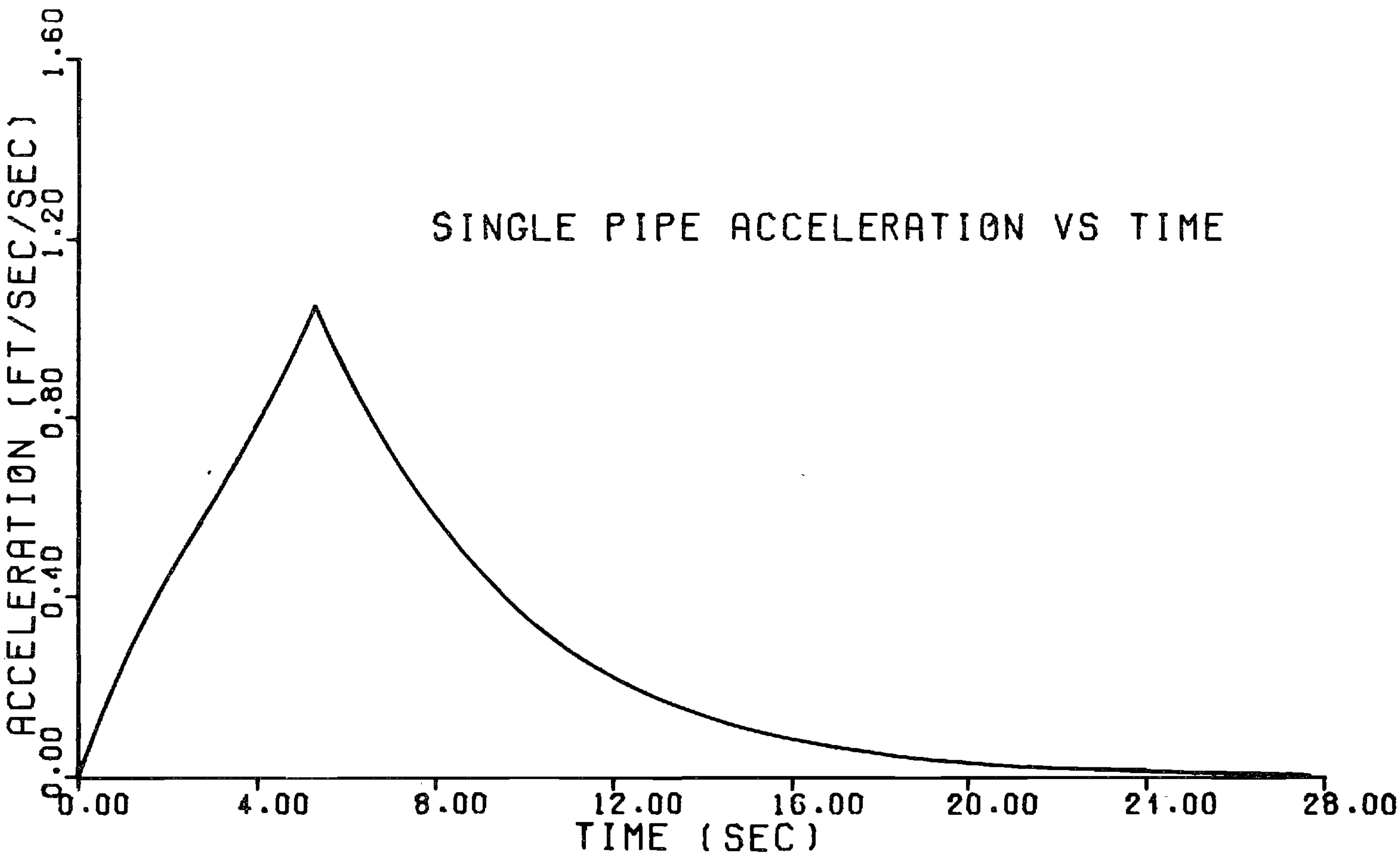

Figure 11. Acceleration Versus Time Plot for Single-Pipe Problem.

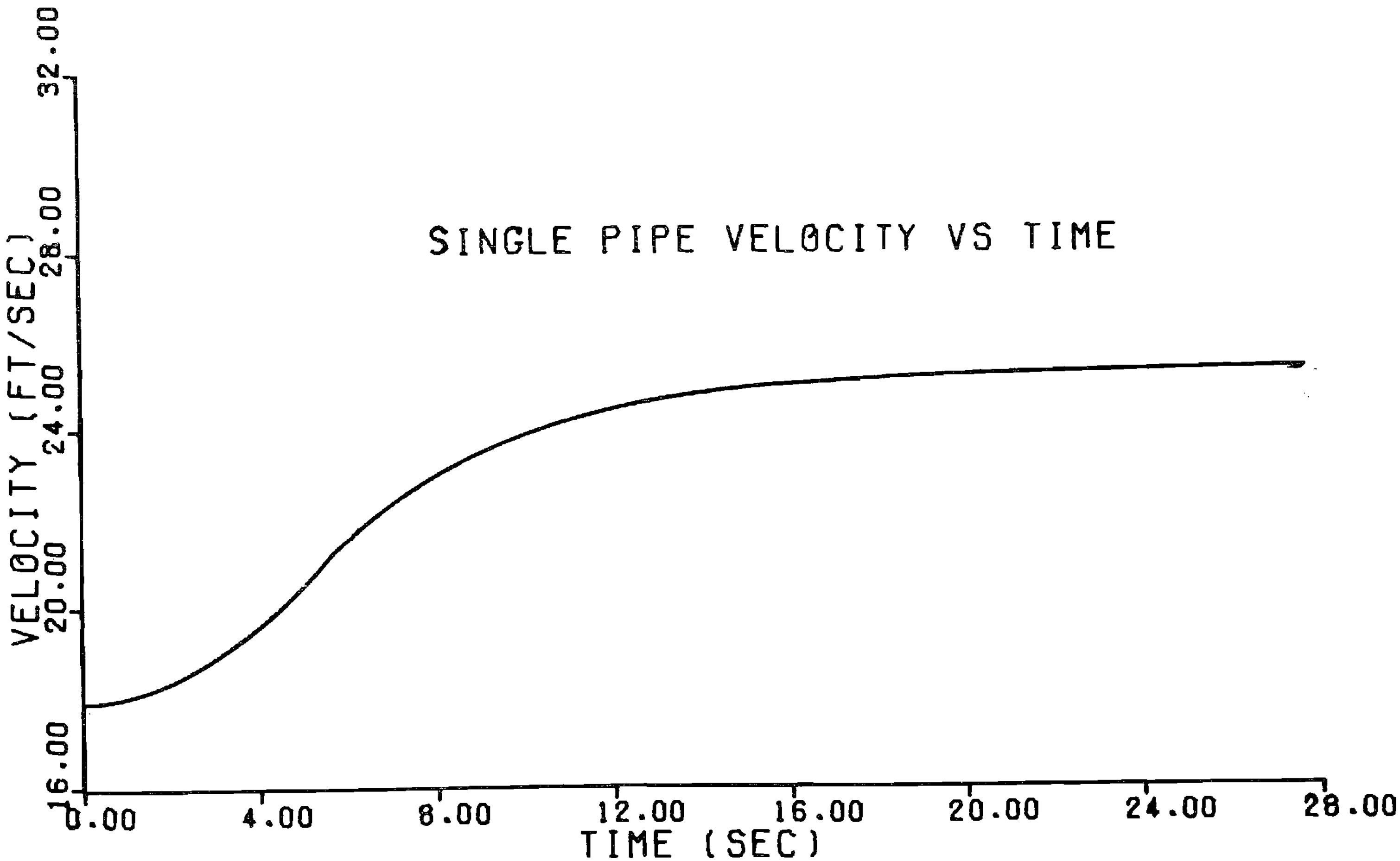

Figure 12. Velocity Versus Time Plot for Single-Pipe Problem.

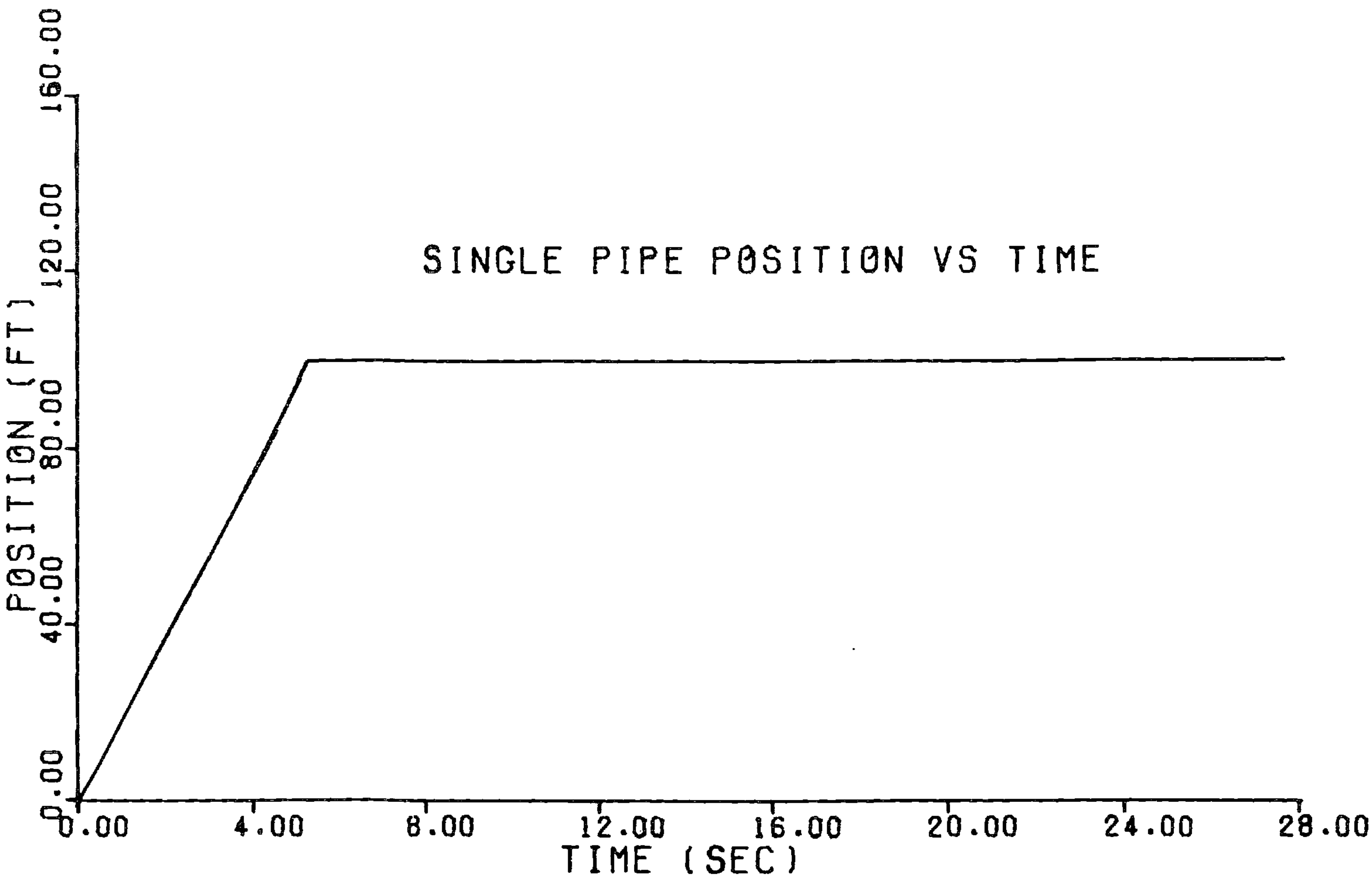

Figure 13. Interface Position Versus Time Plot for Single-Pipe Problem.

Comparison With Experiment

As reported in Chapter II, the unsteady flow characteristics of water passing through a single pipe connecting two constant elevation reservoirs with a polymer injection system at the upstream reservoir were studied experimentally. The experimental results were compared with the computer-predicted results in Figure 6. It should be noted that the data for the above example problem and the data for the experiment were not the same. The curves have similar "S" shapes for the velocity variation with time. However, the experimental curve seemed to indicate that there may not have been an instantaneous reduction in friction factor as the interface passed. This may also have been because of the injection mechanism and because of dispersion of the polymer at the interface. This would result in a more gradual reduction of friction factor due to lower polymer concentration in front of the interface. The time delay was so small that the experimental system appears only slightly more sluggish than the computer solution. Thus the assumption of an immiscible interface is not unreasonable.

CHAPTER V

STEADY FLOW IN PIPE NETWORKS

The problem of predicting steady flow patterns in pipe networks has been treated in various ways. Pertinent parameters such as pipe lengths, pipe diameters, junction elevations, reservoir elevations, and pump characteristics are usually known with reasonable accuracy. The frictional coefficients of the pipes, however, are seldom known within five percent of the actual values. The results of a network analysis can be no better, obviously, than an approximation of the actual conditions.

The flow patterns in complex networks are impossible to predict by inspection. Therefore, iterative schemes of solution have been devised in order to approach the correct flow patterns. Since flows must satisfy the basic relations of continuity and energy, the following criteria must be met:

(1) The flow into any junction must equal the flow out of it.

(2) The flow in each pipe must satisfy the appropriate pipe-friction formula.

(3) The algebraic sum of the head losses around any closed pipe loop must be zero.

Hardy Cross Method

One of the best known iterative solutions for pipe-network problems is the Hardy Cross Method. This method for steady network flow is based on the minimum energy concept and utilizes redistributed flows in successive approximations.

In this study, pipe-friction calculations are based on the Darcy-Weisbach relationship, equation 1. The steps in applying the Hardy Cross method are as follows:

(1) Assume the most reasonable distribution of flows which satisfies criterion 1 above.

(2) Write criterion 2 for each pipe

$$\Delta H = KQ^2 \tag{10}$$

where

ΔH = head loss along pipe (ft-lb/lb), and

$K = 8fL/\pi^2 gD^5$.

(3) Compute the algebraic sum of the head losses (criterion 3) around each loop (Losses from clockwise flows are positive, counterclockwise negative).

(4) Adjust flow in each loop by ΔQ in order to balance the heads and satisfy $\Sigma KQ^2 = 0$. Thus for any pipe in the system

$$Q = Q_o + \Delta Q \tag{11}$$

where Q is the corrected discharge, and Q_o is the previous discharge (assumed or computed). ΔQ is approximated by

$$\Delta Q = \frac{-\Sigma KQ_o^2}{\Sigma |2KQ_o|} \tag{12}$$

(5) Since the flows in the loops will be unbalanced initially, the process is repeated using the corrected discharges until the system is satisfactorily balanced.

An attractive feature of the Hardy Cross method is reportedly that in spite of errors in judgement of initial flow distribution, the solution will converge rapidly.

Computer Solution by Iteration

The iterative methods for solving pipe-network problems lend themselves well to the use of digital computers. Streeter [14] devised a computer program which was based on a method similar to the Hardy Cross method. The major difference between his solution method and the Hardy Cross method was that junction heads were assumed, rather than discharges. Streeter's method was modified for this study and will be presented below.

Streeter's method allows the analysis of networks containing pumping stations, reservoirs, and constant flow outlets. With present-generation computers (32 K or larger core storage), networks containing a large number of the above features may be analyzed.

For digital computation Streeter developed an indexing system comprised of a single array of numbers in the input data which described the network completely. Thus, the computer program remained simple and short. Systems were solved for the various boundary conditions imposed.

In this study, computer solutions required the use of an indexing system which describes the network elements, and Streeter's method was adapted. For the purpose of illustration, Figure 14 shows a network

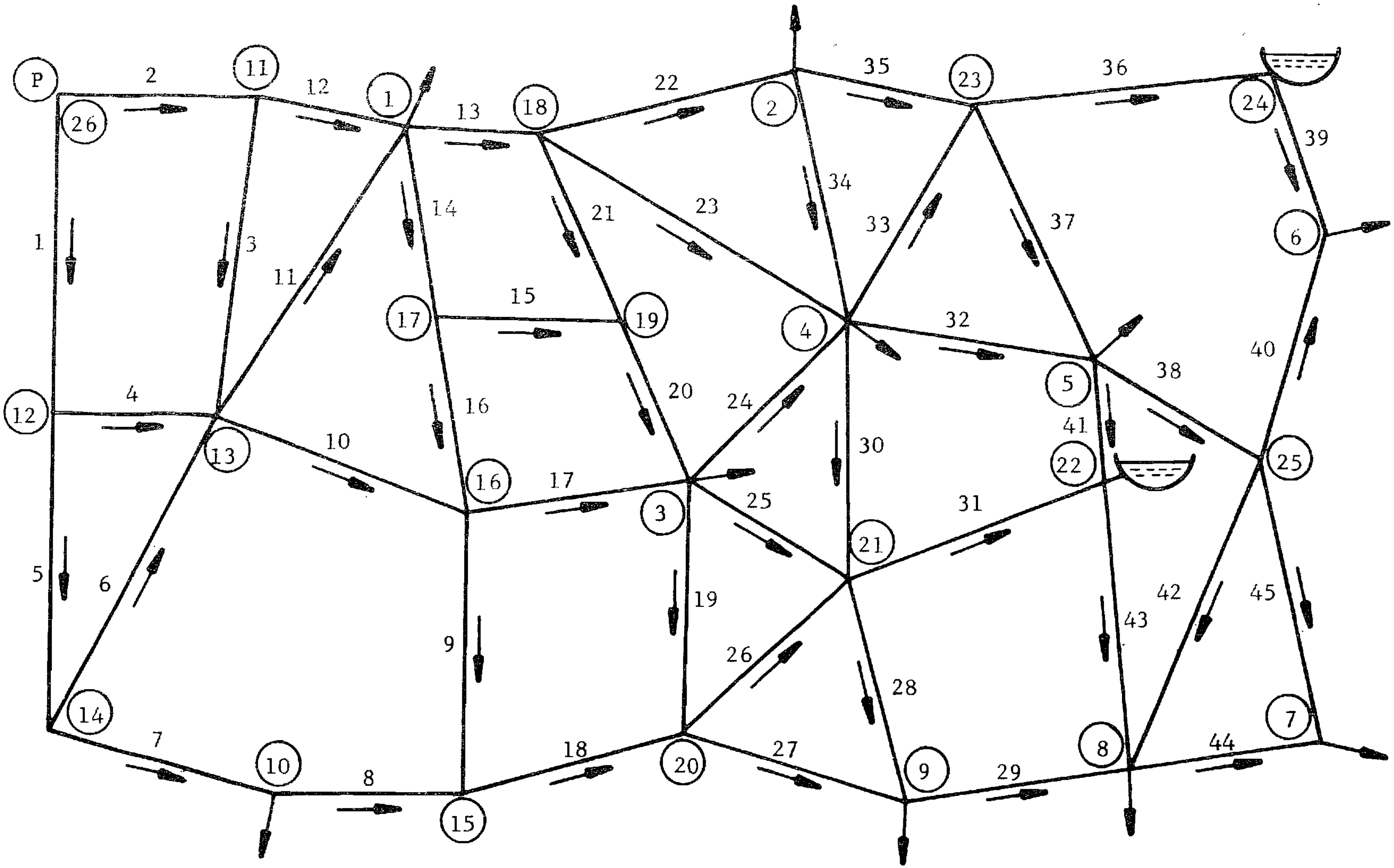

Figure 14. Diagram of Network with 45 Pipes.

(also adapted from Streeter) having a pumping station at junction 26, reservoirs at junctions 22 and 24, and outlets at junctions 1 to 10.

Generally, the indexing system lists first all junctions with one pipe in any order, followed by all junctions with two pipes, then all junctions with three pipes, etc., until all junctions are listed. In the network each pipe is numbered, and arbitrary flow directions are assumed. Each junction is also numbered (for economy of computations, outlet junctions should have the lowest numbers). For clarity, the listing below corresponds to any array typical for a junction, and consecutive numbers describe the following items:

(1) junction number

(2) type of junction (0 for ordinary, 1 for outlet, 2 for pumping station, and 3 for reservoir),

(3) pipe number of pipe entering junction,

(4) junction at other end of pipe given by (3),

(5) positive flow direction is designated by 1 if into junction, and by 2 if out of junction.

(6), (7), and (8) are the same as (3), (4), and (5) for another pipe entering the junction. For example, junction 20 is a two-pipe junction and is described by eight numbers: 20, 2, 1, 12, 2, 2, 11, 2; and junction 3 is a five-pipe junction and is described by seventeen numbers: 3, 1, 20, 19, 1, 17, 16, 1, 19, 20, 2, 25, 21, 2, 24, 4, 2. These series of numbers are combined to form the X-array.

Another series of numbers, the N-array, specifies the number of junctions having a specified number of entering pipes. For example, N = 0, 5, 9, 8, 3, 1 indicates that there are no one-pipe junctions, five two-pipe junctions, nine three-pipe junctions, etc. Accordingly, the number starting a description of junctions having three pipes is I(41), since 0 x 5 + 5 x 8 = 40 places are reserved for the one-pipe and the two-pipe junctions.

An additional indexing system was used by Streeter and is also used here in calculating flows through each pipe after the heads at each junction have been calculated. Called the XX-array, it consists in order of pipe number, upstream junction, and downstream junction for each pipe of the network.

In the type of network under consideration, reservoir elevations are given, pumping station head-discharge curves are presumably known, flows out of outlets are specified, as are the pipe properties (length, diameter, and friction factor). The Darcy-Weisbach friction relation is used here. For each pipe equation 1 may be rewritten as

$$Q = RN\sqrt{\Delta H} \tag{13}$$

where

$$RN = \sqrt{1/K} = \left(\pi^2 gD^5/8fL\right)^{\frac{1}{2}}.$$

First, an estimate of the head (elevation of hydraulic grade line) at each junction is made, and the values are placed in the HH-array. By successive approximations, using equation 13, a correction, DH, is applied in turn to each junction head until steady-state relations are established to a prescribed degree of accuracy.

In a manner suitable for computer application, equation 13 may be written

$$QQ(Y) = RN(Y)\ (HH(Z) - HH(E))^{\frac{1}{2}},\ HH(Z) > HH(E). \tag{14}$$

In equation 14, E is the junction number under consideration, and Z is the junction on the other end of pipe Y. Linearization is accomplished by replacing HH(E) by HH(E) + DH, where HH(E) is the previously determined head at junction E and DH is the correction to HH(E) required to satisfy continuity.

$$QQ(Y) \approx RN(Y)\ (HH(Z) - HH(E))^{\frac{1}{2}} \left(1 - \frac{\frac{1}{2}\ DH}{HH(Z) - HH(E)}\right) \tag{15}$$

$$= A - C\ DH$$

where A and C are known constants given by

$$A = RN(Y)\ (HH(Z) - HH(E))^{\frac{1}{2}},\ \text{and} \tag{16}$$

$$C = \tfrac{1}{2}\ RN(Y)\ (HH(Z) - HH(E))^{-\frac{1}{2}}. \tag{17}$$

In instances when HH(E) > HH(Z),

$$-QQ(Y) = A - C\ DH,\ \text{and} \tag{18}$$

$$A = -RN(Y)\ (HH(E) - HH(Z)^{\frac{1}{2}} \tag{19}$$

and

$$C = \tfrac{1}{2}\ RN(Y)\ (HH(E) - HH(Z))^{-\frac{1}{2}}. \tag{20}$$

For an ordinary junction, continuity requires that the net flows into the junction must be zero. Thus,

$$\Sigma A - \Sigma C\ DH = 0, \tag{21}$$

and

$$DH = \frac{\Sigma A}{\Sigma C} \tag{22}$$

is the head correction to balance inflows into E.

For a junction with known outflow, QVV(E), continuity requires that

$$\Sigma A - \Sigma C\ DH - QVV(E) = 0, \tag{23}$$

and

$$DH = \frac{\Sigma A - QVV(E)}{\Sigma C} \tag{24}$$

is the appropriate head correction at E.

For a junction containing a pumping station, the head-discharge curve is required data. Here, Streeter's procedure put it in tabular form. Starting at head HOO(E), the discharge is listed for this head, followed by the discharge for each additional head increment, DDH, (i.e., QP(E,*) = 100.0, 95.0, 90.0, etc.). A parabola is computed through the data points, approximating the head-discharge relationship for HH(E). Thus,

$$Q_{pump} = C_1 + C_2\ HH(E) + C_3\ HH(E)^2 \tag{25}$$

which is approximated by

$$Q_{pump} \approx C_1 + C_2 \ HH(E) + C_3 \ HH(E)^2 + DH \ [C_2 + 2C_3 \ HH(E)] \tag{26}$$

after linearization and substitution. As before, the head correction can be determined from

$$\Sigma A - DH \ \Sigma C + C_1 + C_2 \ HH(E) + C_3 \ HH(E)^2 + DH \ [C_2 + 2C_3 \ HH(E)]. \tag{27}$$

Heads at junctions containing a reservoir do not need to be calculated. By use of the index array, X, each junction head is adjusted. In the program, for a reservoir the constant A in equation 15 is set equal to zero. The sum of the absolute values of each head correction for all junctions is compared with the desired degree of accuracy. The junctions are adjusted repeatedly, and new comparisons are made. Finally, discharges are computed by equation 13.

Computer Program

A program using FORTRAN V suitable for the UNIVAC 1108 was written and appears on pages 106 to 113 of Appendix 3. This program consists of a group of subroutines which were also used in the unsteady network flow program to be discussed in Chapter VI.

Each of the steady-state subroutines contain lists which explain the symbols used. On occasion the reader will also be referred to the BLOCK DATA subroutine which is used to insert data into the computer. BLOCK DATA appears on page 123 of Appendix 3. Following the list of symbols are the declaration statements and the command statements.

The first steady-state subroutine, called STEADY, is the main subroutine. In it, the heads and discharges are determined by iteration and the results are printed. Comments in the listing are provided to facilitate the reader's understanding of the subroutine.

The next subroutine, called PRT, is used to output the results while iteration is in process. PRT prints the sum of the head corrections, and the heads at selected junctions using a modulation constant with the number of iterations.

The third subroutine, called CND, is used to determine the values of A and C at each junction with proper regard to junction type.

The last subroutine, NCD, is used to correct the head at each junction. The NCD subroutine is divided into segments to deal with each type of junction. The constants for the pump parabola are shown on lines E041 to E043 of NCD.

Example Problem

The network shown in Figure 14 has been analyzed with the aid of the UNIVAC 1108 computer. Although this network will later be used in an unsteady analysis, only the pertinent parts of BLOCK DATA appear in Figure 15. The reader should refer to the BLOCK DATA on page 122 of Appendix 3 for a list of the symbols used in Figure 15. The printed results appear in Figure 16. The column headings labeled HJxx refer to the junctions selected for monitoring. The discharges and junction heads are given below the iteration printout in the order of the pipe or junction number. The computer time required for this example was nine seconds.

```
      DATA (X(I),I=1,40 )/10,1,7,14,1,6,15,2,
     1                    24,3,30,2[illegible],1,[illegible],6,2,
     2                    6,1,39,2[illegible],1,4[illegible],25,1,
     3                    7,1,44,8,1,4[illegible],2[illegible],1,
     4                    2[illegible],2,2,11,2,1,12,2,
      DATA(X(I),I=41,13[illegible] )/11,0,2,[illegible],1,[illegible],10,2,12,1,2,
     1                    12,0,1,26,1,4,15,2,5,14,2,
     2                    14,0,5,12,1,6,13,2,7,10,2,
     3                    15,0,8,10,1,7,16,1,18,20,2,
     4                    17,0,14,1,1,15,14,2,16,16,2,
     5                    19,0,21,1[illegible],1,15,17,1,20,3,2,
     6                    9,1,27,20,1,28,21,1,29,3,2,
     7                    2,1,22,15,1,34,4,2,35,23,2,
     8                    22,3,41,5,1,31,21,1,43,8,2,
      DATA(X(I),I=140,251)/1,1,12,11,1,11,13,1,14,17,2,13,18,2,
     1                    16,0,10,13,1,16,17,1,9,15,2,17,3,2,
     2                    20,0,19,3,1,18,15,1,26,21,2,27,9(2,
     3                    18,0,13,1,1,21,19,2,23,4,2,22,2,2,
     4                    23,0,35,2,1,33,4,1,36,24,2,37,5,2,
     5                    5,1,37,23,1,32,4,1,41,22,2,38,25(2,
     6                    8,1,29,9,1,43,22,1,42,25,1,44,7,2,
     7                    25,0,38,5,1,42,8,2,45,7,2,40,6,2(
```

Figure 15. Data for Steady Network Analysis.

```
 DATA(X(I),I=252,302)/13,0,4,12,1,3,11,1,0,14,1,11,1,2,10,16,2,
1                     3,1,17,16,1,20,19,1,19,20,2,24,4(2,25,21,2,
2                     21,0,25,3,1,26,20,1,30,4,1,31,22(2,28,9,2,
 DATA(X(I),I=303,322)/4,1,23,12,1,34,2,1,24,3,1,30,21,2,32,5,2,33,
1                     23,2/
 DATA (XX(I), I=1,120)/1,26,12, 2,26,11, 3,11,13, 4,12,13,
1                      5,12,14, 6,14,13, 7,14,10, 8,10,15,
2                      9,16,15, 10,13,16, 11,13,1, 12,11,1,
3                      13,1,18, 14,1,17, 15,17,19, 16,17,16,
4                      17,16,9, 18,15,20, 19,3,20, 20(19,3,
5                      21,18,19, 22,18,2, 23,19,4, 24,3,4,
6                      25,3,21, 26,20,21, 27,20,9, 28,21,9,
7                      29,9,8, 30,4,21, 31,21,22, 32,4,5,
8                      33,4,23, 34,2,4, 35,2,23, 36,23,24,
9                      37,23,5, 38,5,25, 39,24,6, 40,25,6,
 DATA(XX(I),I=121,135)/41,5,24, 42,25,8, 43,22,8, 44,8,7,
1                      45,25,7/
 DATA JU/26/
 DATA JP/45/
 DATA JV/10/
 DATA (N(I), I=1,6)/0,5,9,8,3,1/
 DATA NI/6/
 DATA (XL(I),I=1,45)/1000.0,900.0,400.0,1100.0,1000.0,500.0,
1                     1500.0,800.0,600.0,450.0,550.0,1000.0,
```

Figure 15. Continued

```
2                        950.0,5[illegible],4[illegible],6[illegible],8[illegible],100[illegible],
3                        650.0,5[illegible],5[illegible],11[illegible],1[illegible],45[illegible],
4                        95[illegible],45[illegible],11[illegible],[illegible],7[illegible],5[illegible],
5                        1000.0,6[illegible],11[illegible],40[illegible],60[illegible],45[illegible],
6                        500.0,55[illegible],10[illegible],600.0,12[illegible],5[illegible],
7                        1000.0,[illegible],45[illegible]/
  DATA (D(I),I=1,45)/1.0,1.0,0.5,1.0,1.0,0.5,1.0,1.0,0.5,0.5,
1                      0.5,1.0,1.0,0.5,0.5,0.5,0.5,1.0,0.5,0.5,
2                      0.5,1.0,1.0,0.5,1.0,0.5,1.0,0.5,0.5,0.5,
3                      1.0,0.5,1.0,0.5,0.5,1.0,0.5,0.5,1.0,0.5,
4                      1.0,0.5,1.0,0.5,0.5/
  DATA (F(I), I=1,45)/45*0.02/
  DATA (HH(I), I=1,26)/26*530.0/
  DATA (QVV(I), I=1,10)/3*0.1,2*0.2,5*0.05/
  DATA HOO(26)/440.0/
  DATA DDH/10.0/
  DATA (QP(26,I), I=1,12)/100.0,95.0,90.0,84.0,76.0,68.0,60.0,
1                            50.0,39.0,26.0,7.0,-30.0/
  DATA III/1000/
  DATA MODPR/3/
  DATA JHNUM/10/
  DATA (JH(I), I=1,10)/1,3,4,[illegible],13,1[illegible],18,21,25,26/
  DATA TOL/0.01/
```

Figure 15. Continued

NETWORK ANALYSIS FOR POLYMER

ADJ	TL	HJ 1	HJ 3	HJ 4	HJ 5	HJ13	HJ15	HJ18	HJ21	HJ25	HJ26
0	336.24	658.86	529.53	530.67	529.09	559.45	529.90	537.31	529.45	529.73	562.50
3	296.06	552.97	527.82	526.97	529.37	548.84	529.26	525.09	528.64	529.79	545.94
6	157.82	540.96	533.34	532.35	530.04	541.60	529.62	540.09	531.93	529.92	548.15
9	53.59	538.67	531.01	531.31	530.04	541.66	534.80	533.07	530.90	530.11	551.23
12	17.62	538.27	531.96	531.99	530.09	542.20	535.33	534.07	531.64	530.04	549.94
15	4.66	539.23	532.51	532.44	530.14	542.74	536.51	534.53	532.08	530.08	549.96
18	1.59	539.29	532.88	532.61	530.16	542.99	536.93	534.65	532.35	530.09	549.99
21	1.16	539.46	533.06	532.74	530.17	543.18	537.25	534.82	532.50	530.09	550.02
24	.87	539.60	533.19	532.84	530.18	543.33	537.48	534.95	532.60	530.10	550.04
27	.64	539.71	533.28	532.91	530.19	543.45	537.66	535.06	532.68	530.11	550.06
30	.47	539.80	533.35	532.96	530.19	543.53	537.78	535.13	532.73	530.11	550.07
33	.34	539.86	533.40	533.00	530.19	543.59	537.87	535.19	532.77	530.12	550.09
36	.25	539.90	533.44	533.03	530.20	543.64	537.94	535.23	532.80	530.12	550.09
39	.18	539.93	533.46	533.05	530.20	543.67	537.99	535.25	532.82	530.12	550.10
42	.13	539.95	533.48	533.06	530.20	543.69	538.02	535.27	532.84	530.12	550.10
45	.09	539.97	533.50	533.07	530.20	543.71	538.05	535.29	532.85	530.12	550.11
48	.07	539.98	533.51	533.08	530.20	543.72	538.06	535.30	532.86	530.12	550.11
51	.05	539.99	533.51	533.09	530.20	543.73	538.08	535.31	532.86	530.12	550.11
54	.04	540.00	533.52	533.09	530.20	543.74	538.09	535.31	532.87	530.12	550.11
57	.03	540.00	533.52	533.09	530.20	543.74	538.09	535.32	532.87	530.12	550.11
60	.02	540.00	533.53	533.09	530.20	543.75	538.10	535.32	532.87	530.12	550.11
63	.01	540.01	533.53	533.10	530.20	543.75	538.10	535.32	532.87	530.12	550.11
66	.01	540.01	533.53	533.10	530.20	543.75	538.11	535.32	532.87	530.12	550.11

H= 540.008 534.558 533.529 533.097 530.204 530.002 530.123 530.133 534.970 539.806 544.723 544.842 543.752
543.152 538.106 538.289 538.144 535.324 535.507 535.688 532.875 530.000 531.077 530.000 530.125 550.113

Q= 3.234 3.447 .388 1.402 1.831 -.273 2.104 2.053 .138 .868 .650 3.059 3.128 .481 .603
-.123 .607 2.190 -.454 .452 -.150 1.176 2.102 .244 1.169 .623 1.114 -.431 .632 .166
2.389 .547 1.909 .476 .600 2.180 .329 .095 -.063 .113 .581 -.033 -.514 .035 .015

Figure 16. Results of Steady Network Analysis.

CHAPTER VI

UNSTEADY POLYMER FLOW IN A PIPE NETWORK

The injection of a polymer additive at one or more points in a steady flow pipe network results in an unsteady flow condition. From each point of injection an interface moves down the pipe in the original flow direction. The behavior of the interface in each pipe affected by the polymer is similar to that of the interface in the single-pipe problem discussed in Chapter IV except that the head difference between the ends of the pipe is not constant. The reduction of friction in the affected pipes will cause accelerating flows. Since more water passes through the other pipes of the network, the head losses in these other pipes must increase if they do not as yet benefit from the friction-reducing properties of the polymer. Hence, the heads at the junctions are time dependent.

When an interface reaches a junction, the polymer solution flows into all of the outflowing pipes. Since there may be other pipes entering the junction, the question of dilution arises when one or more of these pipes has not been affected by the polymer. For simplicity, it was assumed that this dilution causes insignificant changes in the friction-reducing properties. This assumption is reasonable if the dilution is not too great (See Figure 2). If the dilution is very great, a different percent reduction in friction drag, R, may be specified for the pipes.

Unsteady flow conditions will persist until some time after the interfaces have reached every possible point in the network. Depending on the injection point, some pipes of the network may never be affected since an interface cannot move upstream.

Method of Solution

The movements of the interfaces through a pipe network is junction oriented. That is to say, an interface will not be present in a pipe until its upstream junction is affected by the polymer. This can occur in two ways: the junction may be a point of injection, or the interface in a preceding pipe may reach the junction. Thus, a given junction is either reached by the polymer or it is not. This true or false condition is handled on the computer with a logical variable for each junction.

The unsteady-state condition may also be considered a series of steady states in a network composed of many single pipes. Thus, the method of Chapter V is used to solve the original steady-state problem, and then the Runge-Kutta method of Chapter IV is used to solve the unsteady-state problem in each pipe. At specified time intervals, the steady-state method of Chapter V is used to correct possible mathematical instabilities of the unsteady solution method. Instabilities may result from the variable head differences along the pipes. During the network solution, the interface positions must be carefully monitored so that branching conditions may be handled properly.

The steps used in the unsteady network solution are as follows:

(1) Solve for original steady-state heads and discharges by the method of Chapter V.

(2) Specify points of polymer injection at which interfaces will start to move in each pipe, flowing away from these points.

(3) Use the Runge-Kutta method to investigate the unsteady flow in each pipe. If there is an interface in the pipe, use the reduced friction factor behind the interface and the original friction factor in front of the interface. If the interface is not in the pipe, and if no polymer is in that pipe, use the original friction factor on both sides of an imaginary interface. The position of this imaginary interface is reset to zero after each increment of time until the interface arrives at the upstream junction of the pipe.

(4) The Runge-Kutta solution is applied for a specified number of time increments. Then, the steady solution method is used as an intermediate solution in order to avoid instabilities which might be caused by the use of wrong head differences in the unsteady single-pipe solutions.

(5) The above process is repeated until a new steady state is achieved.

Since the friction factors in all pipes are dependent upon the locations of the interfaces, an apparent friction factor, f, was used in the series of intermediate steady-state solutions. This f is the average friction factor in a pipe, and is given by

$$F = [f_A X + f_B (L - X)]/L. \qquad (28)$$

The assumption was made that the acceleration head, $L/g \left(\frac{d^2X}{dt^2}\right)$, is small, and that it probably could be included in the friction factor as the acceleration head is a resistance term. Thus, f is defined as

$$f = \frac{2gD}{L}\left[\Delta H - L/g\left(\frac{d^2X}{dt^2}\right)\right] / \left(\frac{dX}{dt}\right)^2 . \qquad (29)$$

Experience has shown, however, that this assumption may lead to mathematical instabilities, and the final steady-state apparent friction factor was not always equal to the reduced friction factor. Thus, the friction factor, f, as defined by equation 28 has been used.

Computer Program

The complete unsteady network program is presented in Appendix 3. Again, the lists of symbols and the comment statements in the program should prove useful to the reader. As previously stated, a commentary on the necessary data appears in BLOCK DATA at the end of the program.

The first section of the program, called MAIN, on page 103 is the main program which in turn call the various subroutines. First, it calls STEADY (described in Chapter V). Then, it determines the velocities in the pipes (absolute values of velocities are used as flow direction is defined by the sign of the discharge. PLOTT is called on a modulated basis to store the time, velocity, acceleration, position, and head values in the arrays which are to be plotted later.

Next the main unsteady subroutine, RUNGE, is called for each pipe. The input data in the order of listing on lines A073 and A074 of MAIN are pipe diameter, pipe length, polymer friction factor, water friction factor, apparent friction factor (the only tie between steady and unsteady subroutines), interface position, velocity, acceleration, logical variable for assumed upstream junction (TRUE if interface has reached this junction), logical variable for assumed downstream junction, head at assumed upstream junction, head at assumed downstream junction, and time increment. In RUNGE (See Appendix 3, p. 113) the proper flow direction is determined by the heads at the pipe ends. Then, the proper calling procedure for KUTTA (Same as subroutine used to solve differential equations for single pipe in Chapter IV) is determined on the basis of whether or not an interface has reached the actual upstream junction of a given pipe. If an interface has reached the upstream junction, then the calling procedure is pipe diameter, pipe length, polymer friction factor, water friction factor, time increment, interface position, velocity, acceleration, and positive head difference. If an interface has not reached the upstream junction, then the water friction factor is used in place of the polymer friction factor, as well as in its normal position. As indicated earlier, the interface position is reset to zero after the return to RUNGE.

KUTTA applies the Runge-Kutta method for one time increment each time that it is called. After each call to KUTTA, a check is made in RUNGE to see if the interface has reached the end of the pipe. If so, the downstream junction logical variable is made TRUE, and the position is set equal to the pipe length, L. The apparent friction factor is then calculated by equation 28.

After returning to MAIN, time is incremented, and the unsteady results are printed. STEADY is called again to determine the intermediate heads and discharges. The velocity in each pipe is then corrected to agree with the steady-solution discharge. New unsteady values are stored for plotting, and the unsteady solution begins again.

When the time limit has been exceeded, the program finishes by calling PLOTF to plot the results from selected pipes and junctions on a CALCOMP plotter.

Example Problem

The simplest example of an unsteady network problem is that of parallel pipes. Figure 17 illustrates a four-pipe network connecting two constant elevation reservoirs. The data for this system may be found in the BLOCK DATA on page 122 of Appendix 3.

At time, T > 0, the polymer is introduced at the upstream reservoir (junction 1), and an interface begins moving down pipe 1. The interface branches at junction 2. Since pipe 2 is shorter than pipe 3 and both have the same diameter and friction factor, the interface in pipe 2 reaches junction 3 first, and an interface then moves down pipe 4. The interface in pipe 3 reaches junction 3 before the interface in pipe 4 reaches junction 4 (the downstream reservoir) due to the relative resistances of the pipes.

Plots of the unsteady flow parameters (position, velocity, and acceleration) in each of the pipes were made, as were the time variation of the heads at the junctions. These plots do not show the same "S"-shaped curves which resulted in the single-pipe problem. This was due to the interaction of the various pipes and also due to the number of points plotted. In a computer run which required 29 seconds on the UNIVAC 1108, time was incremented 3000 times, but only 150 points were

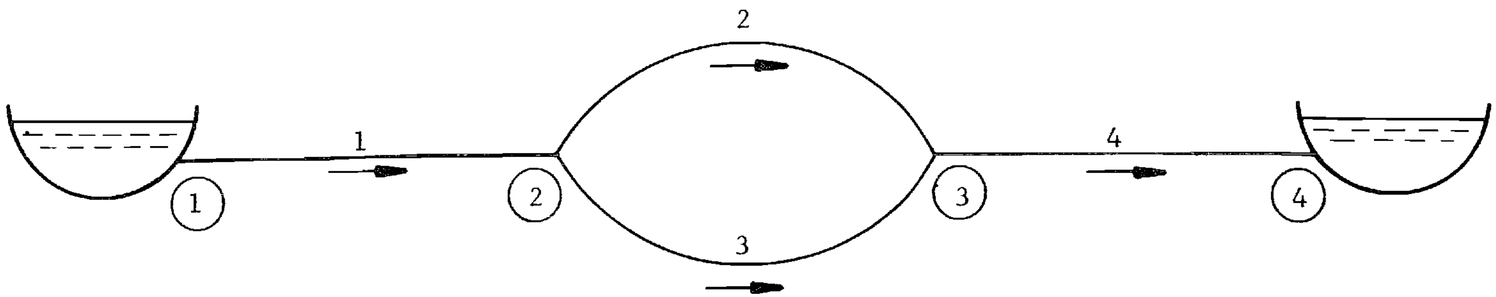

Figure 17. Diagram of Parallel-Pipe Network.

used in the modulated plots. Thus, the curves appear discontinuous, but in reality the changes may not be so abrupt.

Figure 18 shows the unsteady flow characteristics of pipe 1. The acceleration versus time plot is not likely an accurate representation of the physical phenomenon. Computer errors result apparently when the tolerance of the intermediate steady heads (0.01 ft) is of a much greater magnitude than the valves of the acceleration ($\approx 10^{-5}$ ft/sec/sec). The error is most noticeable when the interface is passing through the pipe in question, and the position term must appear in the differential equation. As time goes on, however, the repeated application of the steady solution will decrease the error. In subsequent calculations for a more extensive network, the same head tolerances proved adequate. In general, velocity gradients are necessarily continuous and hence less subject to computer errors. Thus the acceleration versus time plot for pipe 4 in Figure 21 should also apply for pipe 1, since pipes 1 and 4 have equal diameters and are effectively connected in series. Accordingly, there is a fairly constant and small acceleration (effectively zero for the computer solution) at the beginning, followed by a pulse as the interface passes through the parallel pipes, and ending with another constant and small acceleration.

The velocity versus time plot for pipe 1 (See Figure 18) shows the expected "S" curve. It should be noted that this curve is almost identical to the one in Figure 21. Thus the error in the acceleration discussed above had little effect on velocity. The position versus time curve for pipe 1 shows an almost linear increase which is followed by a constant valve after the pipe is completely filled with polymer

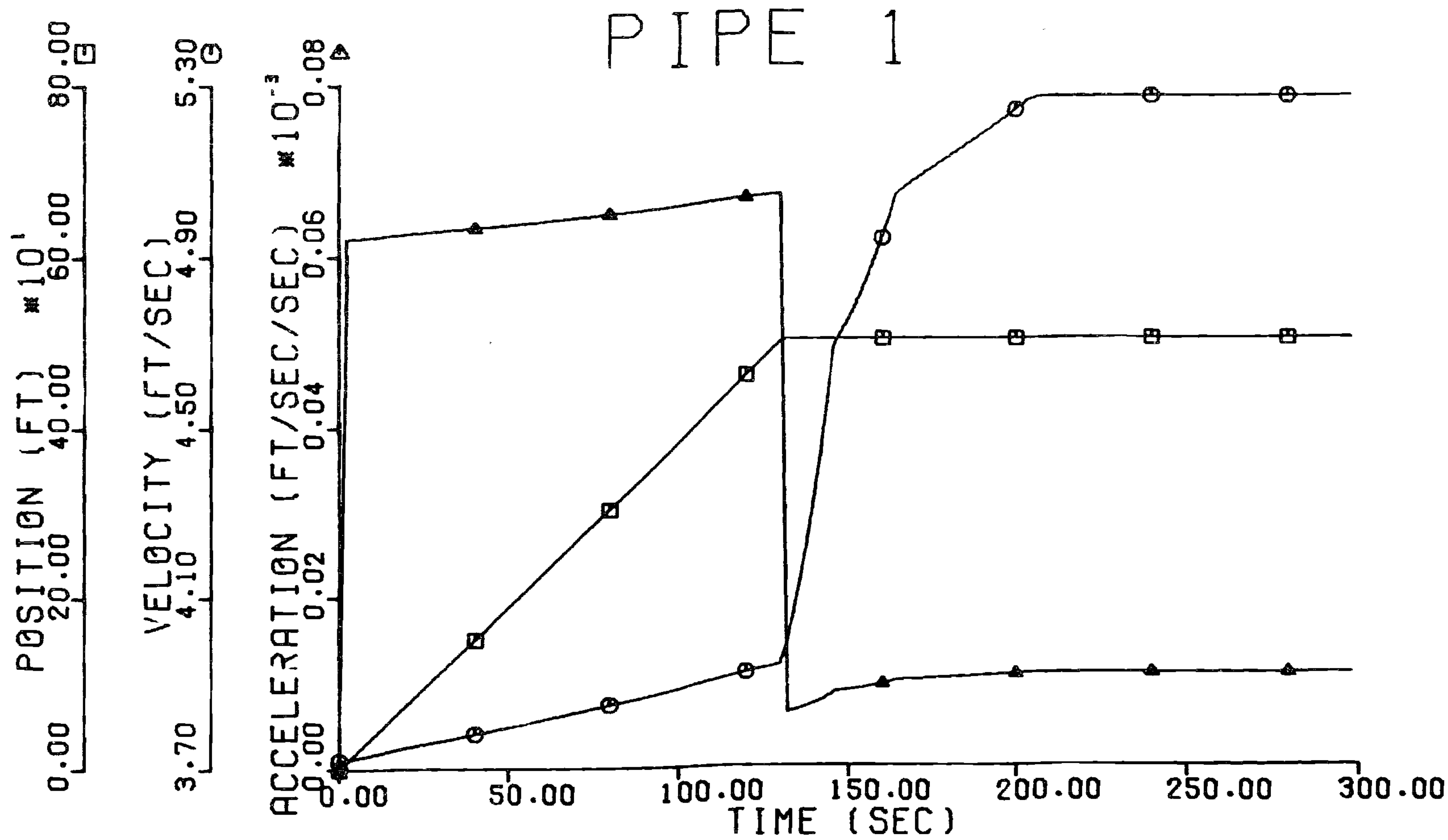

Figure 18. Unsteady Flow Parameters for Pipe 1 of Parallel-Pipe Network.

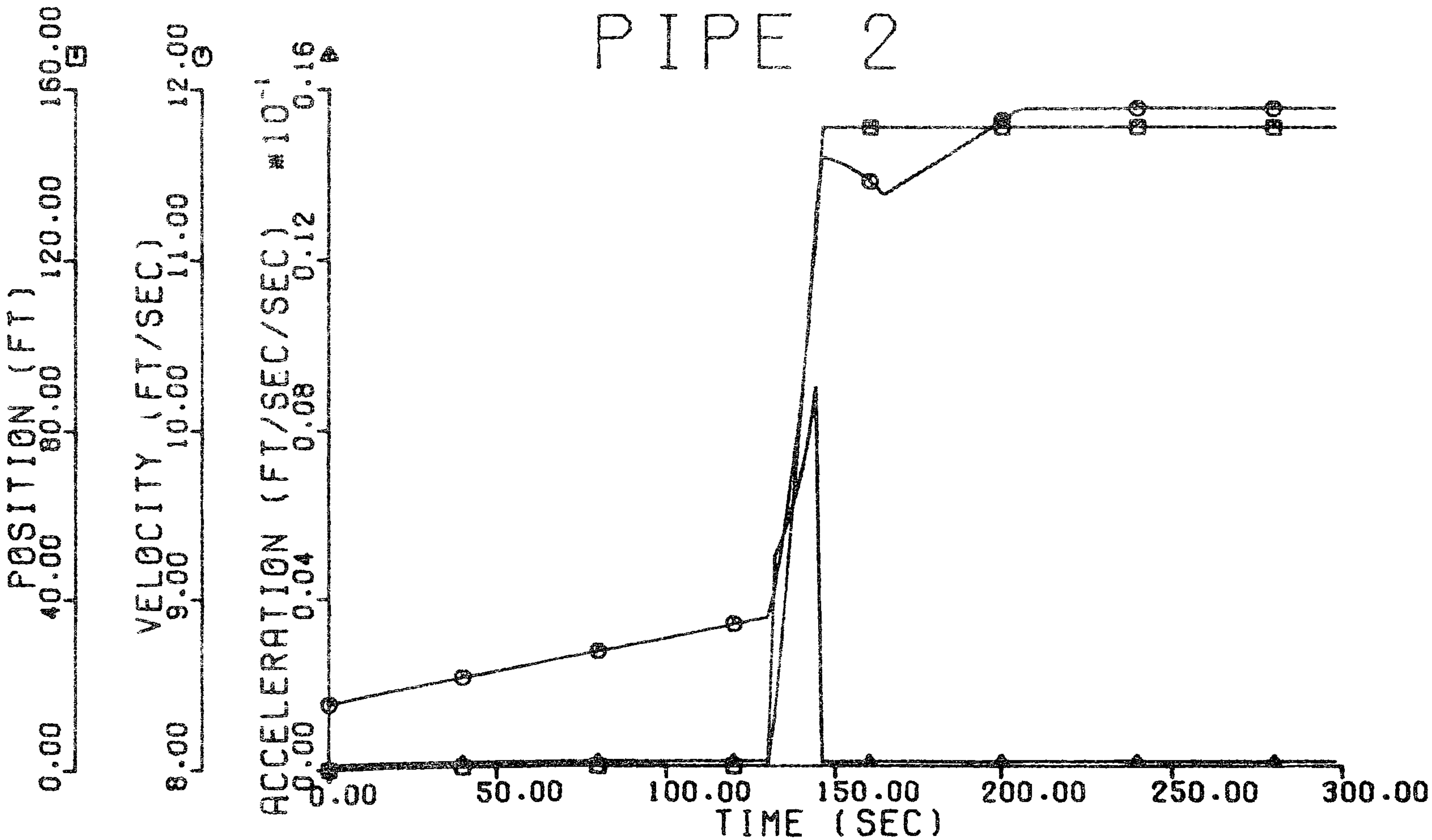

Figure 19. Unsteady Flow Parameters for Pipe 2 of Parallel-Pipe Network.

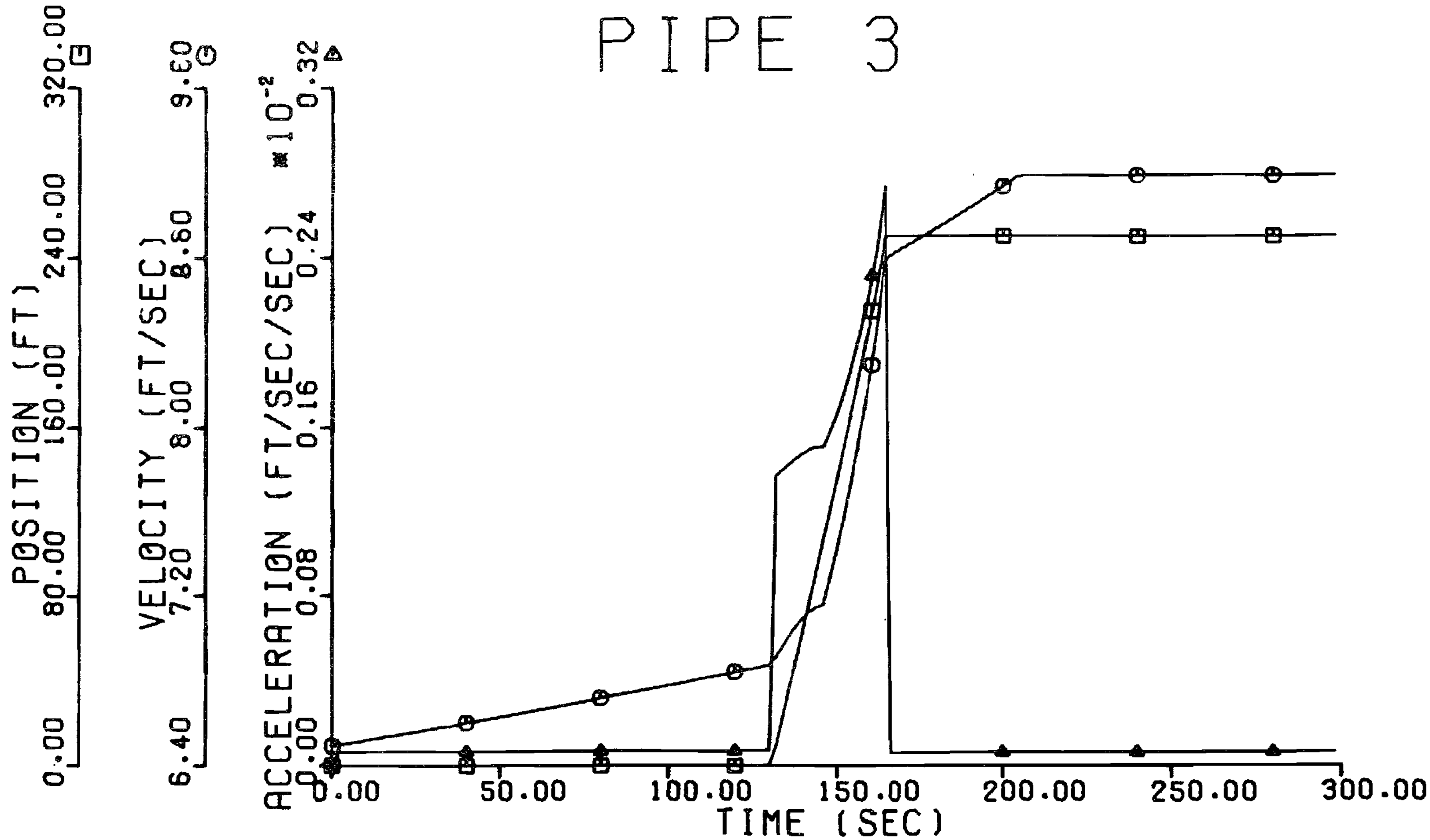

Figure 20. Unsteady Flow Parameters for Pipe 3 of Parallel-Pipe Network.

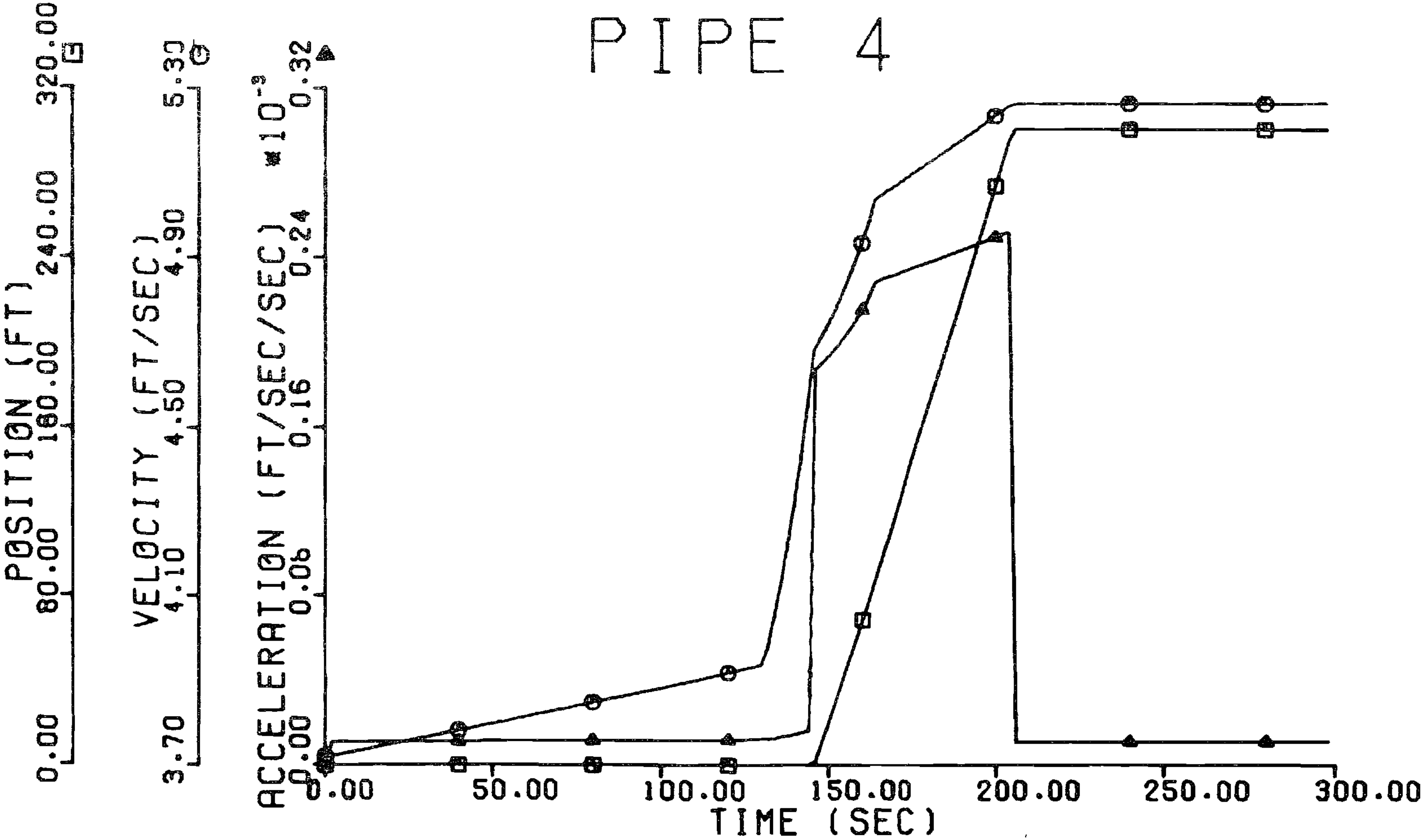

Figure 21. Unsteady Flow Parameters for Pipe 4 of Parallel-Pipe Network.

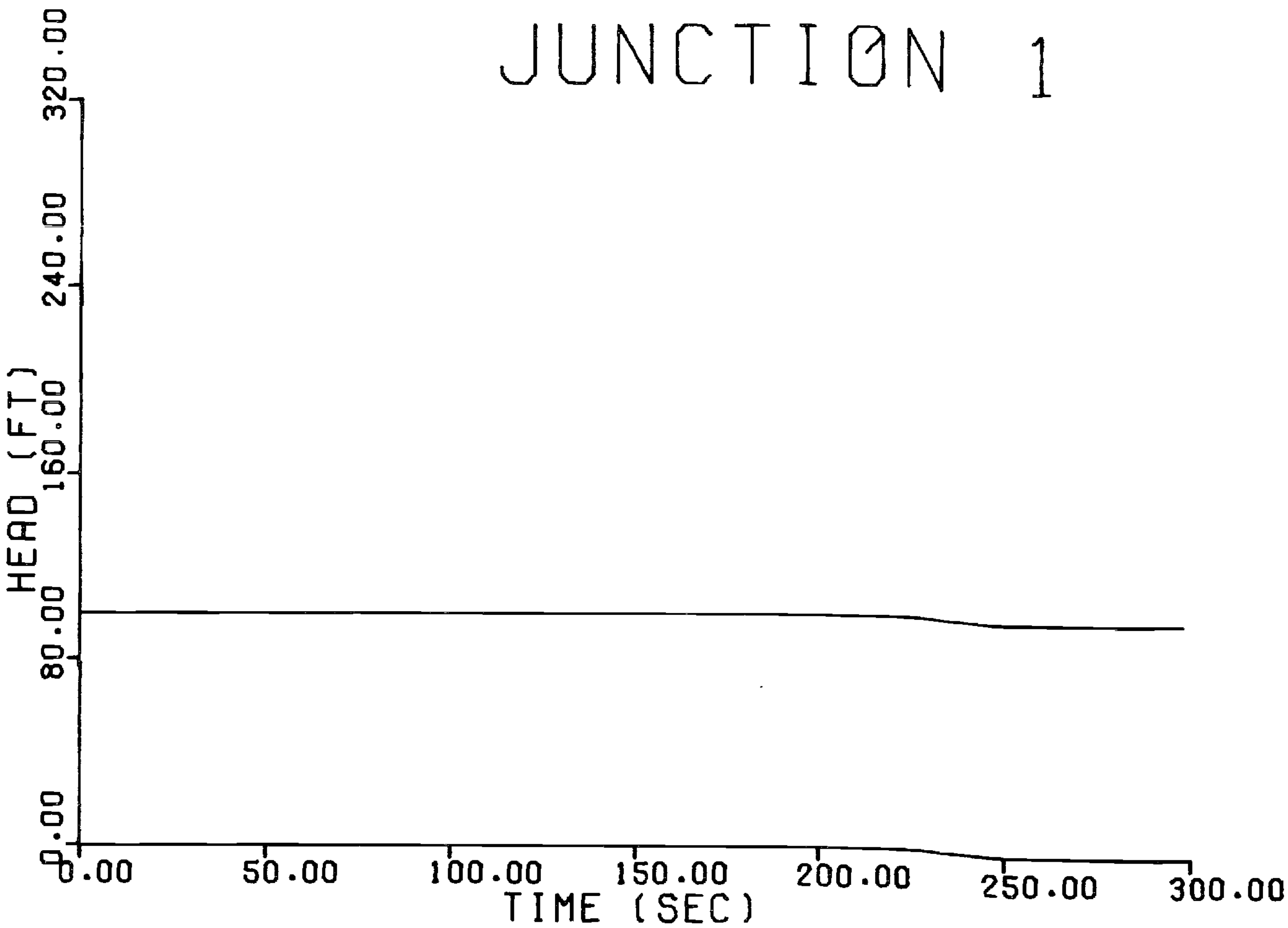

Figure 22. Head Versus Time Plot for Junction 1 of Parallel-Pipe Network.

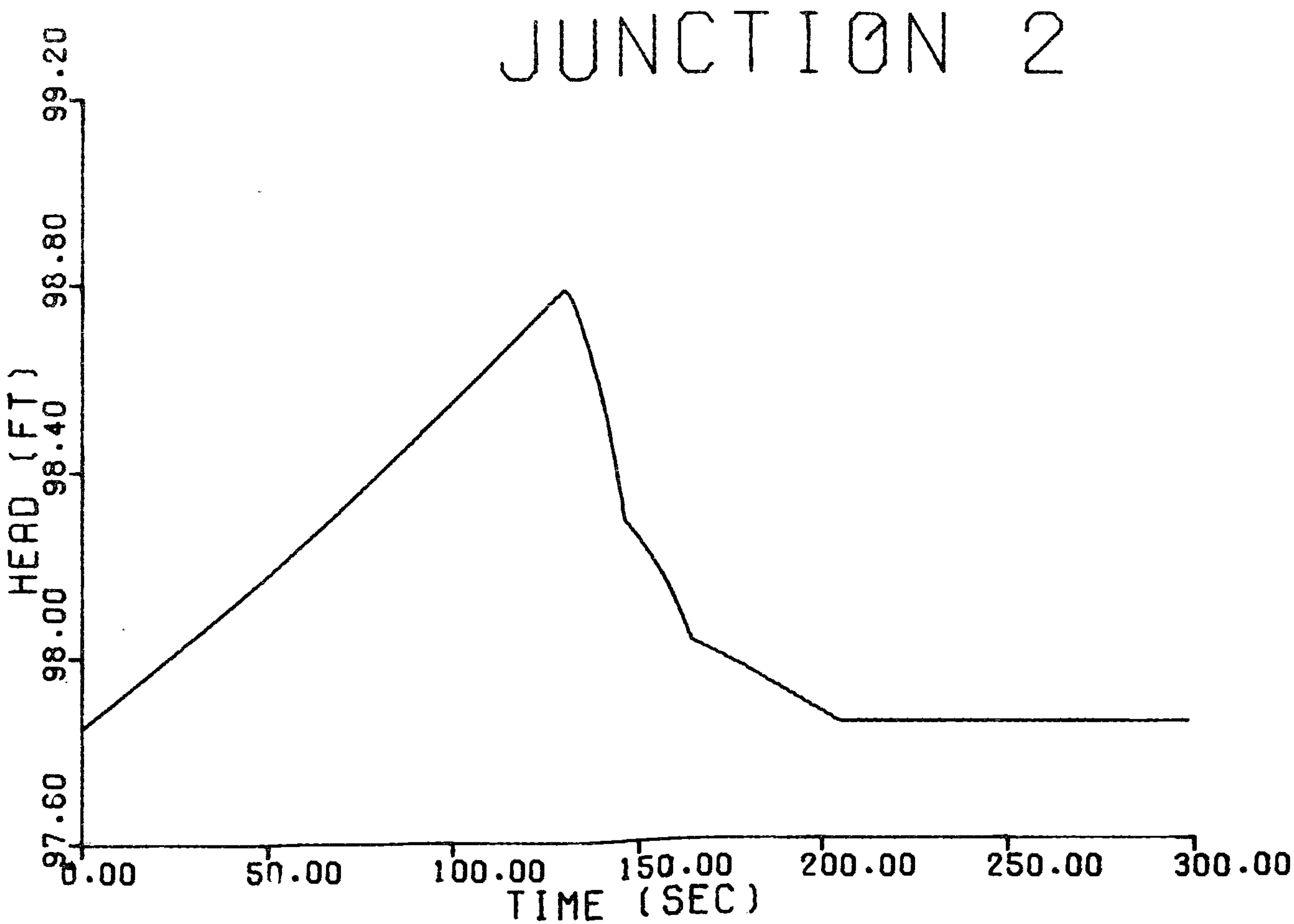

Figure 23. Head Versus Time Plot for Junction 2 of Parallel-Pipe Network.

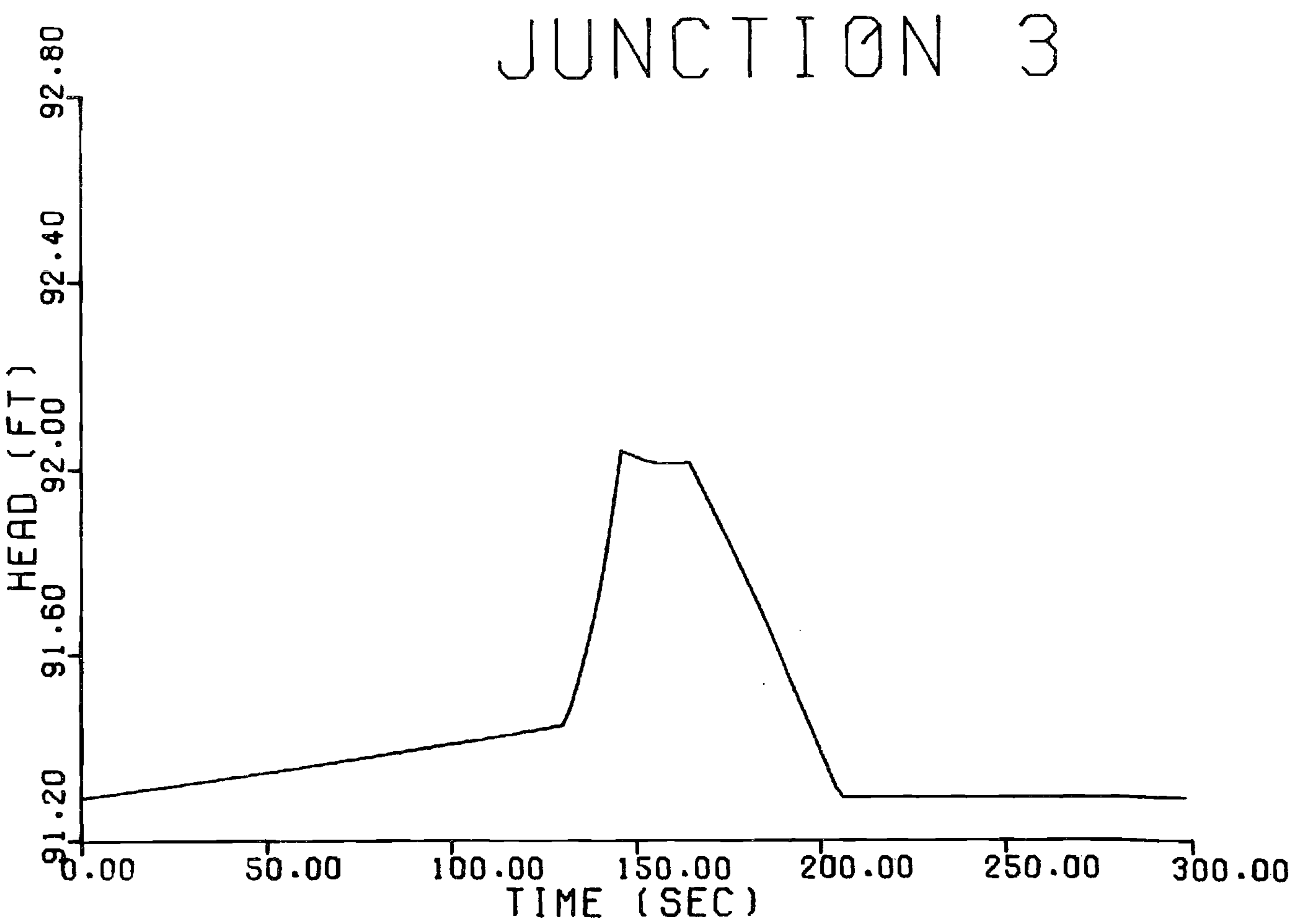

Figure 24. Head Versus Time Plot for Junction 3 of Parallel-Pipe Network.

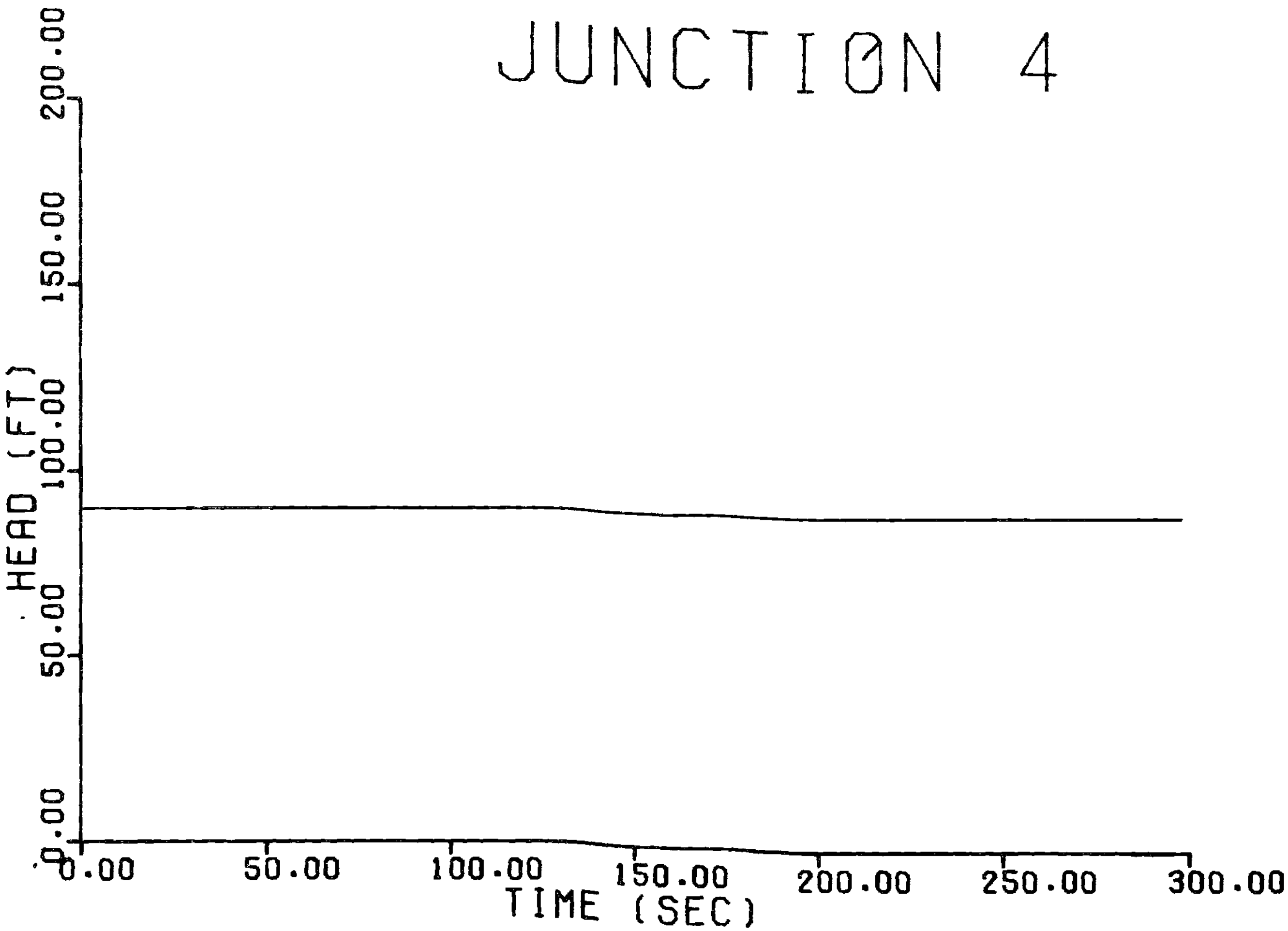

Figure 25. Head Versus Time Plot for Junction 4 of Parallel-Pipe Network.

solution.

Figure 19 shows the results for pipe 2. The events were rather crowded in time. The accelerations were much larger than in pipe 1 (9×10^{-3} ft/sec/sec), but again except for one brief spurt, the accelerations were essentially zero. The velocity curve is more complicated in the case of pipe 2 as it increased gradually until the interface in pipe 1 reached junction 2, then increased rapidly until the interface in pipe 2 reached junction 3. Then, there was a brief drop in velocity while the interface in pipe 3 was completing its journey. When this happened, the velocity increased rapidly until the interface in pipe 4 reached junction 4, which was the downstream reservoir.

The acceleration versus time plot of pipe 3 (See Figure 20) was even more complicated. However, as before, the segments of its curve may be explained by the events of the junctions and interactions of the network. The velocity versus time curve is a complicated "S" curve.

The acceleration versus time plot for pipe 4 (See Figure 21) appears like a pulsation with steps. The velocity and position plots for pipe 4 are similar to those of pipe 1, except for the time delay on the position plot.

The head in junction 1 (See Figure 22) was constant, as was expected with a reservoir. The head in junction 2 showed time dependency with a variation of about one foot. There was an almost linear increase in head until the interface reached junction 2. This increase in head was followed by a stepped decrease back to the original value. This was indicative of the adjustment of the hydraulic grade line as the interface moved downstream between the reservoirs.

The time variation of head at junction 2 may be seen in Figure 23. Figure 24 shows the head curve for junction 3. The curve behavior may be explained in a manner similar to that used for junction 2. However, the curved portion at the top of the pulse showed that the head variations were not linear. The plot for junction 4 (See Figure 25) showed a constant head at the downstream reservoir as expected.

About three and one-half minutes were required to increase the velocities in a 950 foot-long parallel pipe network by a factor of 1.414.

CHAPTER VII

A PRACTICAL EXAMPLE

In order to demonstrate the practical use for the method of unsteady pipe network analysis, the water distribution network of Polyville, a hypothetical city of some 200,000 inhabitants, will be used. Like many another city, Polyville has been growing rapidly and its water distribution system, though once quite adequate, has shown insufficient capacity during critical demand periods. Emergency requirements could barely be met, and the contingency arose that the system might be inadequate under certain circumstances. This caused the authorities to seek a solution which would provide for the basic needs of the city, provide for capabilities to cope with emergencies, and not least prevent fire insurance rates from becoming unreasonably high.

The requirements for discharges and pressures in fire fighting systems are based on population density and on the type of structures involved. The National Board of Fire Underwriters [15] has a graduated scale for required discharges based on population. For Polyville, a city of over 200,000 people, the required discharge is stated as 12,000 gallons per minute with 2,000 to 8,000 gallons per minute for an additional fire. The recommended normal static pressure is 60 to 75 psi. During heavy fire demands, the pressure is permitted to drop. However, when pumpers are included in the fire fighting equipment, the pressures should never fall below 20 psi.

The portion of the Polyville water distribution system under consideration is illustrated in Figure 14. It consists of a pumping station at junction 26, reservoirs at junctions 22 and 24, and outlets at junctions 1 through 10. The head-discharge curve for the pump appears in Figure 26. The heads are based on elevation zero (MSL), but the ground level of Polyville is at elevation 400 feet (MSL). The main pipes are one foot in diameter, and the intermediate pipes are six inches in diameter. The local grid pipes (not illustrated) are four inches in diameter. The pipe lengths vary from 400 feet to 1500 feet.

Normal Steady State

Outlet discharges range normally from 0.05 cfs to 0.2 cfs, and reservoir elevations are at 530 feet. The normal flow pattern is such that the pumping station (H = 550.1 feet) is the highest point on the hydraulic grade lines, and the reservoirs (H = 530.0 feet) are the lowest points (See Figure 16). Thus water flows from the pumping station into the reservoirs during normal operation.

The method of solution used for the steady-state analysis was as presented in Chapter V. The necessary data for BLOCK DATA appears in Figure 15. For convenience, initial heads (HH-array) were all set equal to the reservoir elevations of 530 feet. The maximum number of iterations, III, was set equal to 1000, although a smaller number might have been used. The modulation constant, MODPR, was set equal to three to reduce the printout and to save paper. The head tolerance was set equal to 0.01 since computer time was not critical, and it was thought that this would be more than adequately accurate for any future calculations.

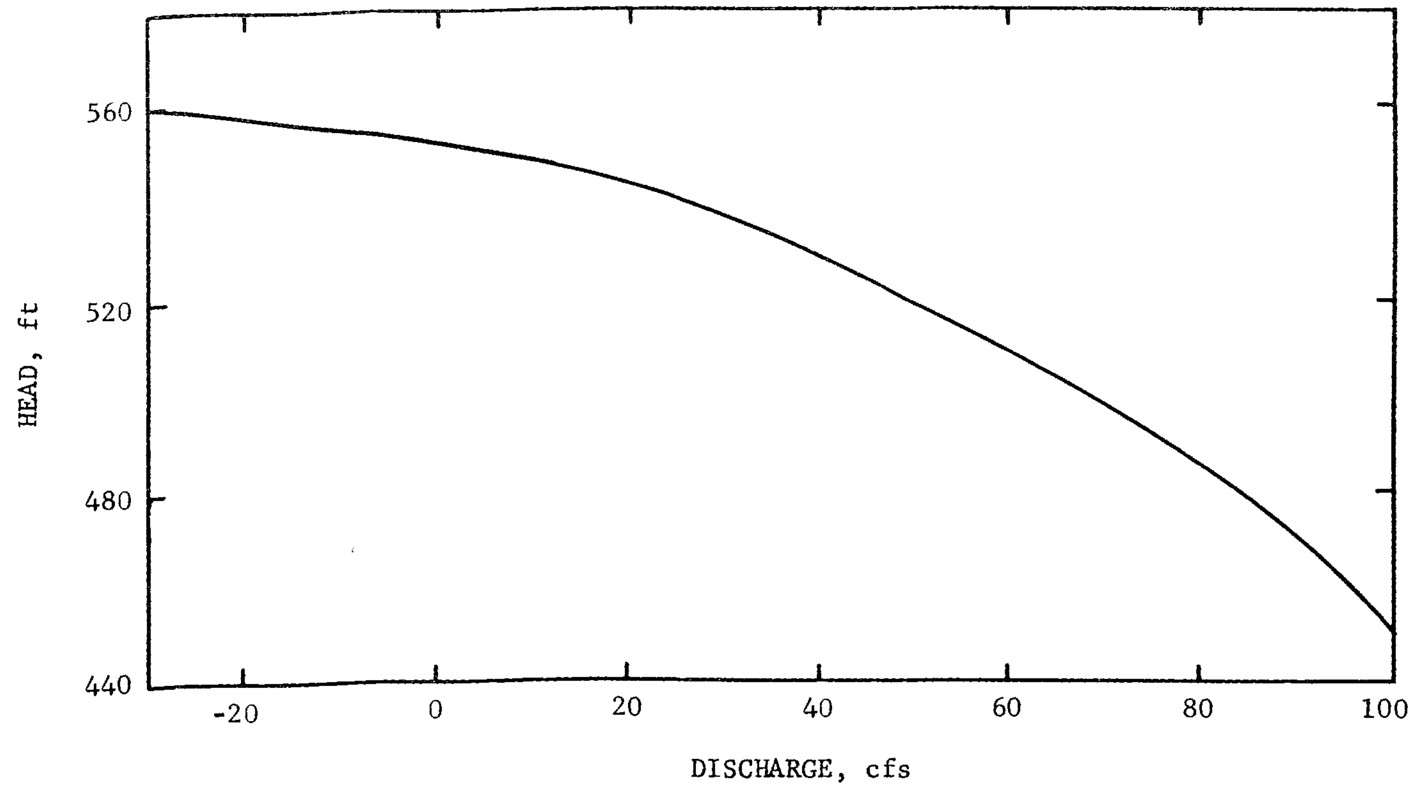

Figure 26. Head-Discharge Curve for Polyville Pumping Station.

The junctions to be monitored were chosen randomly, except for junctions 4, 5, and 26 which will be discussed below. The friction factor was assumed to be equal to 0.02 for all pipes. Steady-state analysis showed that the water pressures in the system varied from 65.0 psi at the pumping station to 56.3 psi at the reservoirs.

Fire Demand

The above steady-state flow demonstrated the adequacy of the Polyville system during normal demand periods. To investigate further, it was assumed that two fires occur near junctions 4 and 5, and equal demands of 20 cfs (9,000 gallons per minute) were required at each of these junctions. Also the normal demands were supplied to the remaining eight outlets. Under these conditions, steady-state analysis (See Figure 27) showed a critical drop in the head at junctions 4 and 5. Accordingly, the head at junction 4 was 470.7 feet (30.6 psi), and the head at junction 5 was 437.9 feet (16.4 psi). The pumping station was again the highest point on the hydraulic grade lines with a head of 548.8 feet, and junction 5 was the lowest point. Thus, water was drawn from the reservoirs, as well as supplied from the pumping station. Since the pressure at junction 5 was less than the required 20 psi, the system was shown inadequate under the usual safety standards.

It was then proposed that the existing system can be rendered adequate temporarily if a polymer additive with the capability to reduce the friction factor by 50 percent was injected at the pumping station. Table 3 shows the heads and discharges at selected points and times according to the unsteady analysis. The data shown in Figure 28 along

NETWORK ANALYSIS FOR POLYMER

ADJ	TL	HJ 1	HJ 3	HJ 4	HJ 5	HJ13	HJ15	HJ18	HJ21	HJ25	HJ26
0	393.65	658.86	529.53	523.23	478.81	559.45	529.90	537.31	529.45	528.69	562.50
3	311.35	517.81	524.58	462.23	437.88	537.55	529.20	481.46	526.99	519.64	545.94
6	59.08	504.45	519.86	460.45	433.65	528.99	528.44	482.00	522.66	517.32	547.63
9	18.95	515.22	519.62	467.38	436.64	532.19	527.17	491.60	522.46	517.24	548.29
12	3.65	516.53	520.07	468.50	437.09	533.15	527.01	493.31	522.69	517.23	548.54
15	2.63	517.36	520.35	469.21	437.34	533.57	527.23	494.38	522.87	517.27	548.62
18	1.74	517.81	520.55	469.64	437.50	533.81	527.45	495.02	523.01	517.31	548.65
21	1.32	518.08	520.71	469.90	437.59	533.97	527.62	495.40	523.13	517.34	548.68
24	1.01	518.31	520.84	470.08	437.65	534.08	527.78	495.66	523.23	517.37	548.70
27	.85	518.51	520.96	470.21	437.70	534.17	527.93	495.84	523.32	517.40	548.71
30	.74	518.67	521.06	470.31	437.74	534.25	528.06	495.99	523.40	517.42	548.72
33	.68	518.79	521.15	470.39	437.76	534.36	528.18	496.10	523.47	517.45	548.73
36	.55	518.89	521.22	470.46	437.79	534.44	528.29	496.19	523.53	517.46	548.74
39	.45	518.98	521.28	470.51	437.81	534.51	528.38	496.27	523.58	517.48	548.75
42	.37	519.05	521.33	470.55	437.82	534.57	528.46	496.33	523.62	517.49	548.76
45	.32	519.11	521.37	470.59	437.83	534.63	528.53	496.38	523.65	517.51	548.77
48	.26	519.16	521.41	470.62	437.85	534.68	528.59	496.42	523.68	517.52	548.78
51	.21	519.20	521.43	470.64	437.85	534.71	528.63	496.45	523.70	517.53	548.78
54	.17	519.23	521.46	470.66	437.86	534.74	528.67	496.48	523.72	517.53	548.79
57	.14	519.25	521.48	470.68	437.87	534.76	528.70	496.50	523.74	517.54	548.79
60	.11	519.27	521.49	470.69	437.87	534.78	528.72	496.52	523.75	517.54	548.79
63	.09	519.29	521.50	470.70	437.88	534.79	528.74	496.54	523.76	517.55	548.79
66	.07	519.30	521.51	470.71	437.88	534.80	528.75	496.55	523.77	517.55	548.80
69	.05	519.31	521.52	470.72	437.88	534.81	528.76	496.56	523.77	517.55	548.80
72	.04	519.32	521.53	470.72	437.88	534.82	528.77	496.56	523.78	517.55	548.80
75	.03	519.33	521.53	470.73	437.89	534.82	528.78	496.57	523.78	517.56	548.80
78	.03	519.33	521.53	470.73	437.89	534.83	528.79	496.57	523.78	517.56	548.80
81	.02	519.34	521.54	470.73	437.89	534.83	528.79	496.58	523.79	517.56	548.80
84	.02	519.34	521.54	470.73	437.89	534.83	528.79	496.58	523.79	517.56	548.80
87	.01	519.34	521.54	470.74	437.89	534.84	528.80	496.58	523.79	517.56	548.80
90	.01	519.34	521.54	470.74	437.89	534.84	528.80	496.58	523.79	517.56	548.80

H= 519.343 495.698 521.542 470.737 437.889 529.298 521.991 527.566 527.074 530.855 534.836 537.755 534.836
534.885 528.799 526.795 519.477 496.585 515.494 527.161 523.789 530.000 501.939 530.000 517.559 548.800

Q= 4.682 5.549 -.004 2.295 2.387 .077 2.309 2.258 -.455 1.053 1.322 5.545 6.895 -.129 .741
-.870 .638 1.803 -.732 -.791 -1.531 1.264 7.162 2.646 -2.167 .682 .389 -.539 -.202 -2.565
-3.511 1.843 -7.502 1.967 -.803 -11.124 2.818 -2.997 1.152 -1.102 -12.342 -1.114 2.198 .832 -.782

Figure 27. Steady Solution for Polyville Network with Fire Demand.

Characteristic	Normal	0 Min	5 Min	10 Min	20 Min	40 Min
Head (ft above MSL) at junction 4	533.097	470.137	475.889	481.190	486.145	489.078
Pressure (psi) in pipes at junction 4	57.62	30.62	32.85	35.15	37.29	38.56
Head (ft above MSL) at junction 5	530.204	437.889	439.505	446.631	448.179	449.070
Pressure (psi) in pipes at junction 5	56.37	16.40	17.10	20.19	20.86	21.24
Head (ft above MSL) at junction 26	550.113	548.800	548.176	547.251	547.624	547.613
Pressure (psi) in pipes at junction 26	64.98	64.42	64.15	63.96	63.91	63.90
Inflow (cfs) at junction 22	-3.484	18.051	17.112	16.466	16.588	16.850
Inflow (cfs) at junction 24	-2.243	12.276	11.632	11.240	10.810	10.528
Inflow (cfs) at junction 26	6.681	10.231	11.808	12.843	13.146	13.171

with the data in Figure 15 was used (The QVV-array in Figure 28 supersedes that in Figure 15). A sample of the unsteady printout appears in Figure 29.

In the program, NUMIT was set equal to one for greater accuracy, but for a long-term analysis it could have been much higher. The time increment, T, was set equal to 5.0 seconds which was large enough to be economical computationally and yet was small enough for the desired accuracy of results. In general, the most critical pipe was the one with the most rapid passage of an interface. As a rule, the time increment multiplied by the velocity should only be a fraction of the pipe length. In this example the largest velocity was about 15 feet per second, and its product with the time increment was 75 feet. This was certainly less than the shortest pipe of 400 feet. The time of study of 2400 seconds (40 minutes) was chosen arbitrarily. A plotter modulation constant of one was then satisfactory since 2400.0/5.0 = 480 was less than the maximum number of 500 points per graph. If a time of 2500.0 seconds had been used, MODPL should have been two or larger. The flow characteristics in nine of the pipes have been plotted in Figures 30 through 38. These nine pipes supplied water to the two critical junctions nearest the fires. The time-varying heads at junctions 4 and 5 have been plotted in Figures 39 and 40.

The plots of the time-varying flow parameters in the pipes supplying the critical junctions 4 and 5 (Figures 30 through 38) illustrate the complexity of the flow patterns. In most cases, a positive acceleration implies an increase in velocity, but in some cases such as pipe 41 (Figure 38), the original flow direction was

```
DATA (QVV(I), I=1,10)/3*0.1,2*2.0,5*0.05/
DATA NUMIT/1/
DATA [illegible]P/12/
DATA T/ 5.0/
DATA TMAX/2400./
DATA (FL(I),I=1,45)/45*0.02/
DATA (FA(I),I=1,45)/45*0.01/
DATA CK(26)/1/
DATA MODPL/1/
DATA NUMPL/9/
DATA (NPLOT(I),I=1,9)/23,24,3[illegible],32,33,34,37,38,41/
DATA NUMPLH/2/
DATA (NHPLOT(I),I=1,3)/4,5,26/
```

Figure 28. Unsteady Data for Polyville Network.

```
TIME= 1085.00

POS=   .10000+04   .90000+03   .40000+03   .11000+04   .10000+04   .50000+03   .15000+04   .57223+03   .00000      .45000+03
       .55000+03   .10000+04   .95000+03   .24162+03   .45000+03   .60000+03   .80000+03   .00000      .00000      .60000+03
       .50000+03   .11000+04   .10000+04   .45000+03   .00000      .00000      .00000      .00000      .00000      .00000
       .00000      .60000+03   .00000      .40000+03   .00000      .00000      .00000      .00000      .00000      .00000
       .00000      .00000      .00000      .00000      .00000
VEL=   .76245+01   .90597+01   .41450-01   .37822+01   .38415+01   .98923+00   .35942+01   .35327+01   .27544+01   .74261+01
       .86505+01   .90725+01   .11061+02   .18699+00   .54142+01   .52229+01   .49576+01   .28409+01   .39900+01   .40612+01
       .94757+01   .23246+01   .11105+02   .16087+02   .29273+01   .37350+01   .90968+00   .28789+01   .50520+00   .11162+02
       .40643+01   .14268+02   .80864+01   .12113+02   .33213+01   .12399+02   .13930+02   .14417+02   .13673+01   .52146+01
       .14811+02   .54125+01   .23017+01   .40452+01   .37906+01
ACC=   .15438-04   .20295-04   .13306-08   .46026-05   .47339-05   .68906-06   .41956-05   .12348-02   .68060-05   .20474-04
       .24706-04   .20329-04   .27381-04   .70643-06   .13269-04   .12584-04   .11647-04   .47260-05   .11211-04   .85458-05
       .27463-04   .18701-05   .27530-04   .54449-04   .49737-05   .10315-04   .58769-06   .72555-05   .35888-06   .85266-04
       .85584-05   .44227-04   .22795-04   .36016-04   .88435-05   .37040-04   .30210-03   .37432-03   .12675-05   .15271-04
       .46867-04   .15916-04   .32724-05   .11402-04   .10513-04
  F=   .10000-01   .10000-01   .10000-01   .10000-01   .10000-01   .10000-01   .10000-01   .12847-01   .20000-01   .10000-01
       .10000-01   .10000-01   .10000-01   .15168-01   .10000-01   .10000-01   .10000-01   .20000-01   .20000-01   .10000-01
       .10000-01   .10000-01   .10000-01   .10000-01   .20000-01   .20000-01   .20000-01   .20000-01   .20000-01   .20000-01
       .20000-01   .10000-01   .20000-01   .10000-01   .20000-01   .20000-01   .20000-01   .20000-01   .20000-01   .20000-01
       .20000-01   .20000-01   .20000-01   .20000-01   .20000-01
  ADJ        TL        HJ 1       HJ 3      HJ 4       HJ 5       HJ13       HJ15       HJ18       HJ21       HJ25       HJ26

     0        .04     523.37     522.34    486.14     448.17     536.16     531.28     505.30     524.87     519.25     547.64
H=  523.367  504.379  522.336  486.137  448.173  529.390  523.267  528.354  528.475  533.285  536.160  538.604  536.160
    536.308  531.289  528.450  523.359  505.303  519.259  528.773  524.866  530.000  508.495  530.000  519.248  547.641

Q=    5.989    7.115    -.004    2.970    3.019     .191    2.828    2.777    -.542    1.458    1.698    7.125    8.687     .037    1.063
     -1.026     .974    2.234    -.784    -.798   -1.861    1.826    8.722    3.159   -2.299     .734     .717    -.565     .100   -2.192
     -3.192    2.801   -6.351    2.378    -.652   -9.738    2.735   -2.831    1.074   -1.024  -11.633   -1.063    1.807     .794    -.744
```

Figure 29. Sample of Printout of Unsteady Solution for Polyville Network.

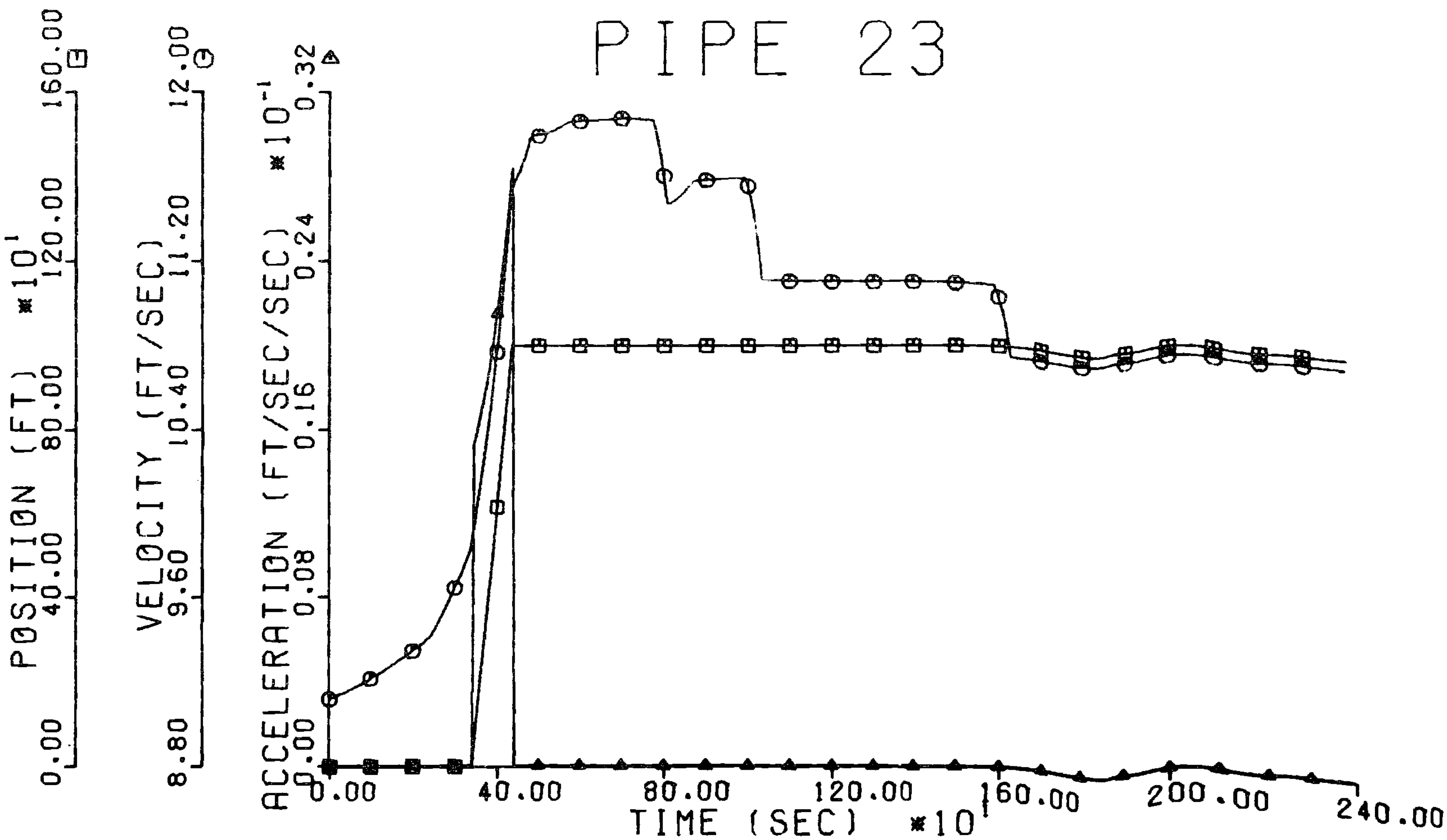

Figure 30. Unsteady Flow Parameters for Pipe 23 of Polyville Network.

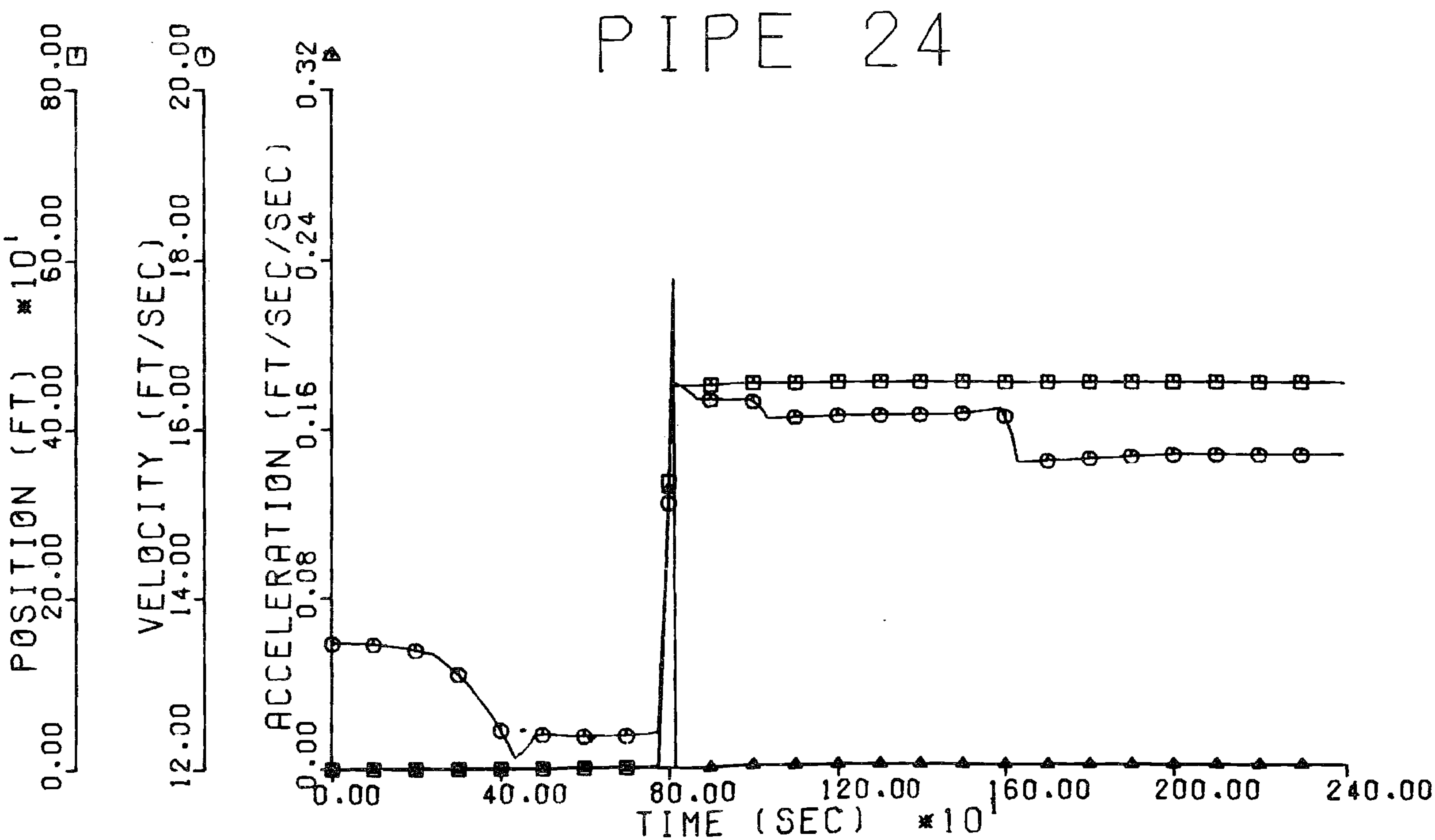

Figure 31. Unsteady Flow Parameters for Pipe 24 of Polyville Network.

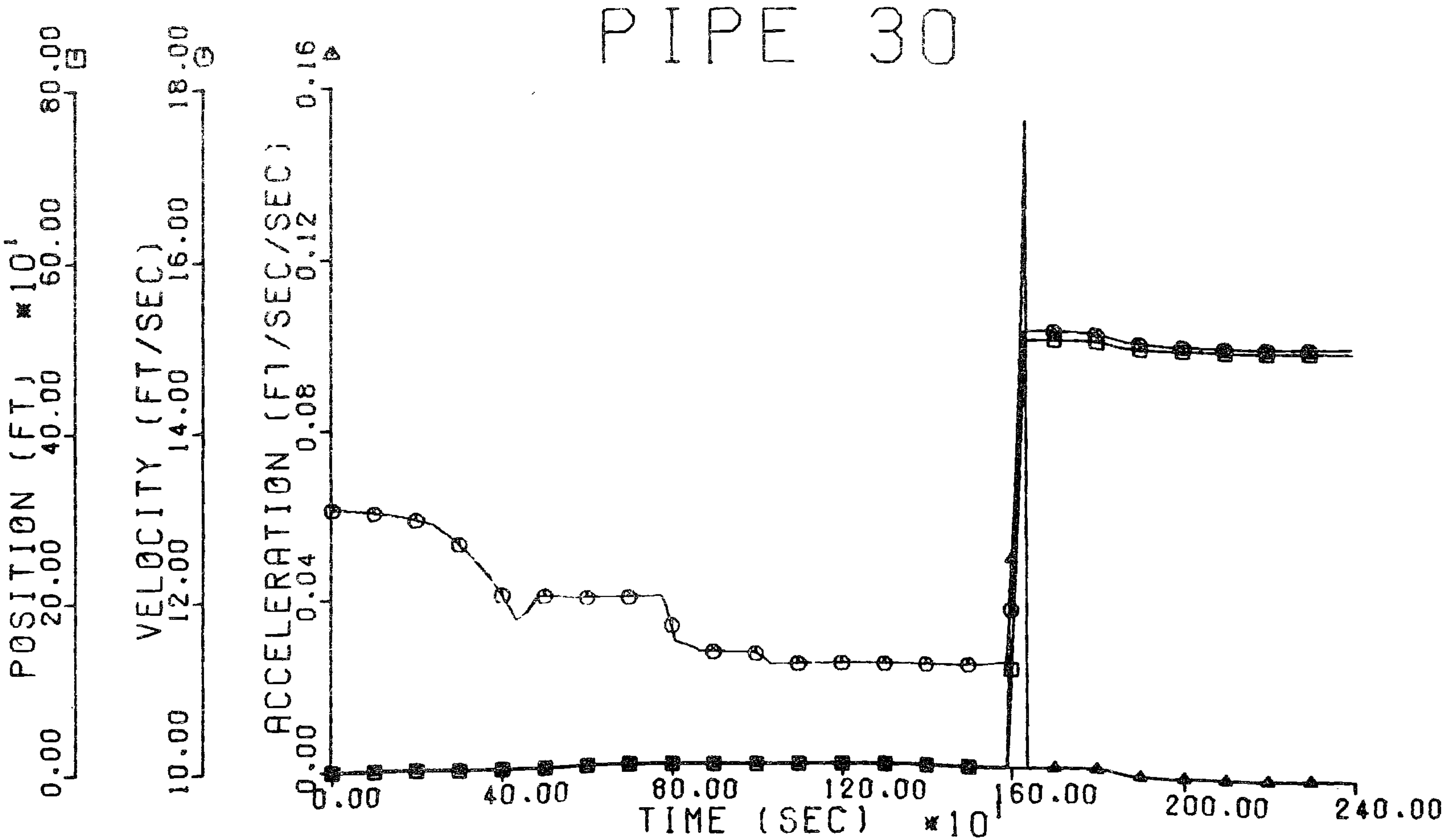

Figure 32. Unsteady Flow Parameters for Pipe 30 of Polyville Network.

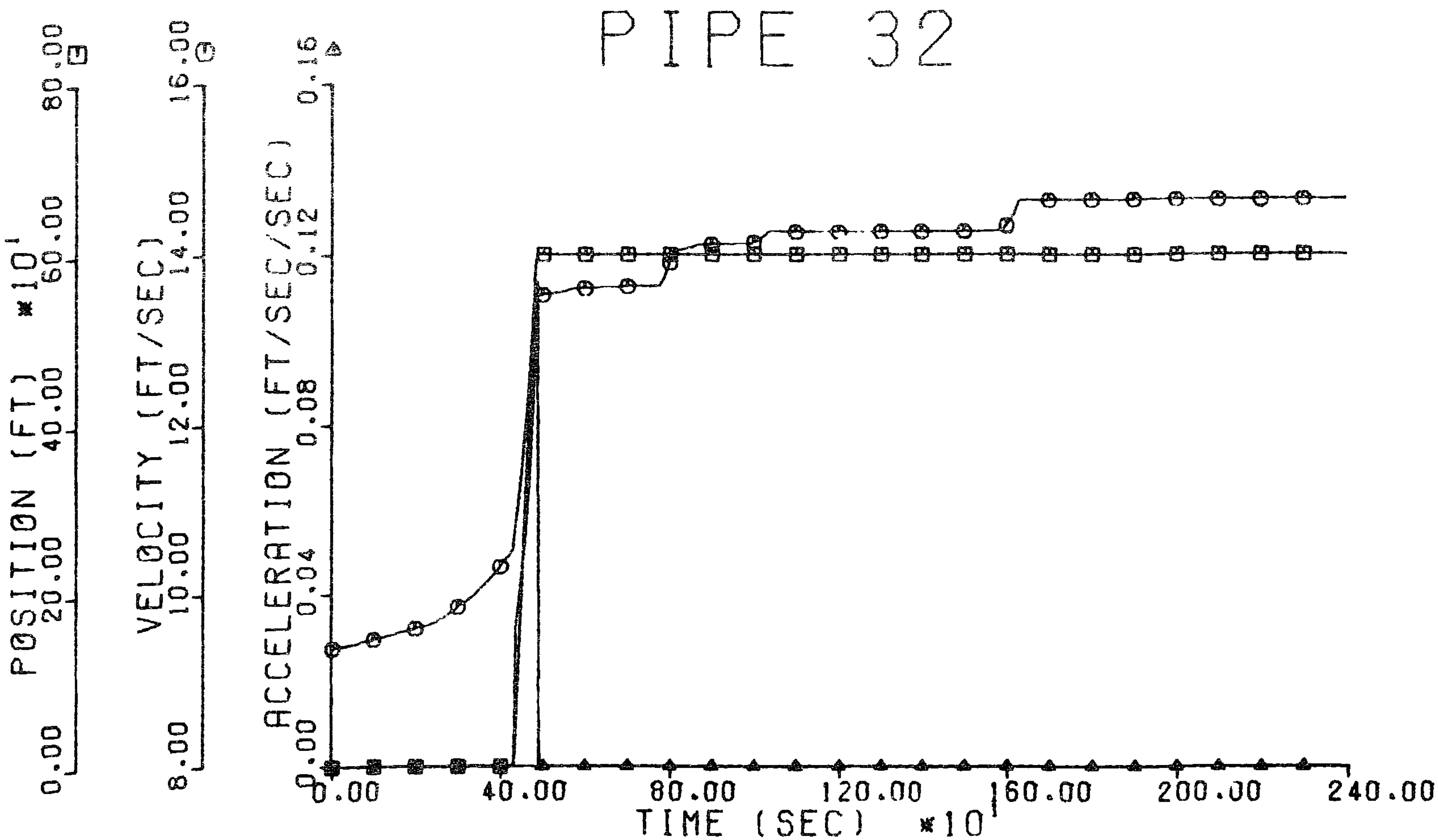

Figure 33. Unsteady Flow Parameters for Pipe 32 of Polyville Network.

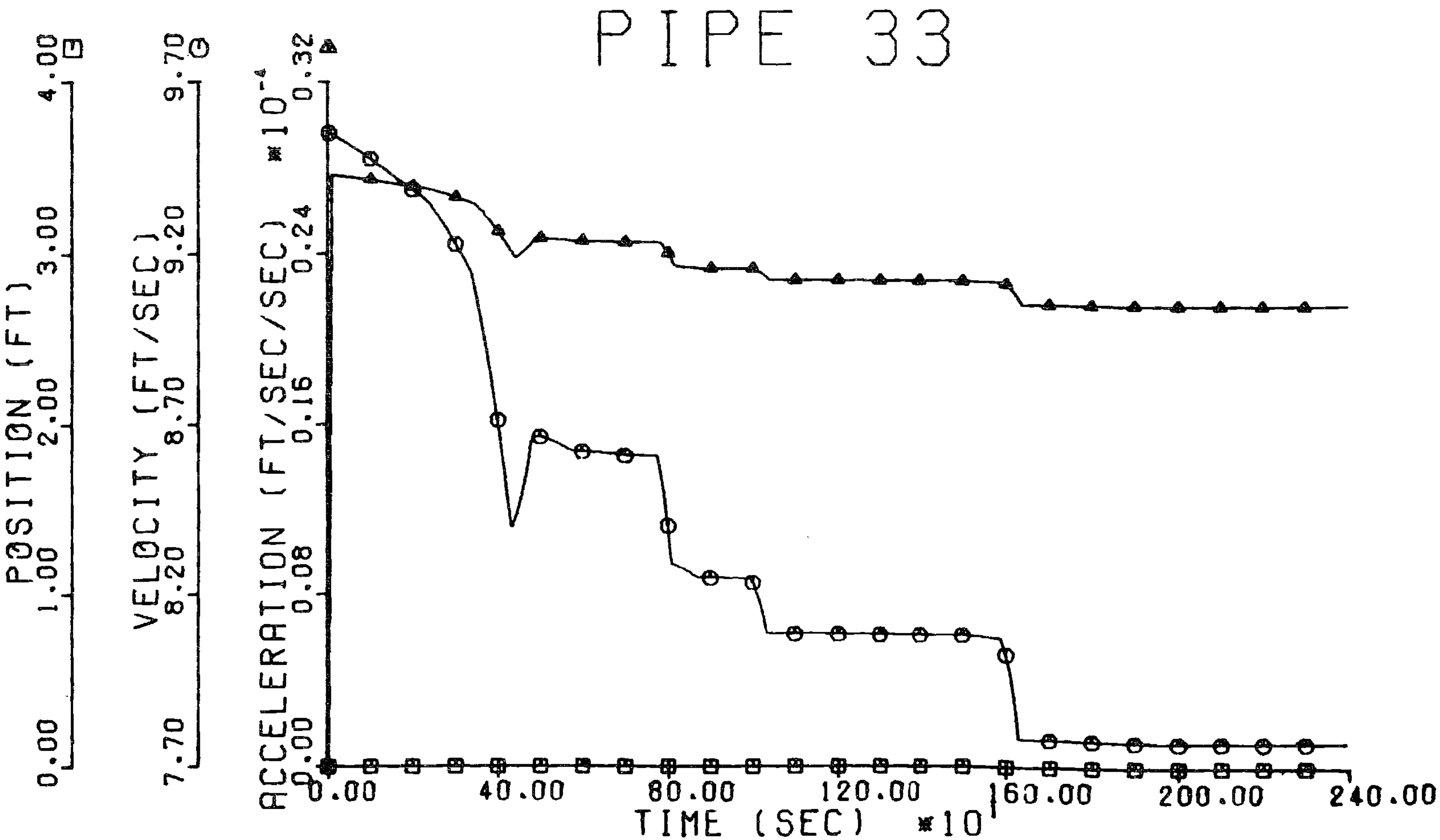

Figure 34. Unsteady Flow Parameters for Pipe 33 of Polyville Network.

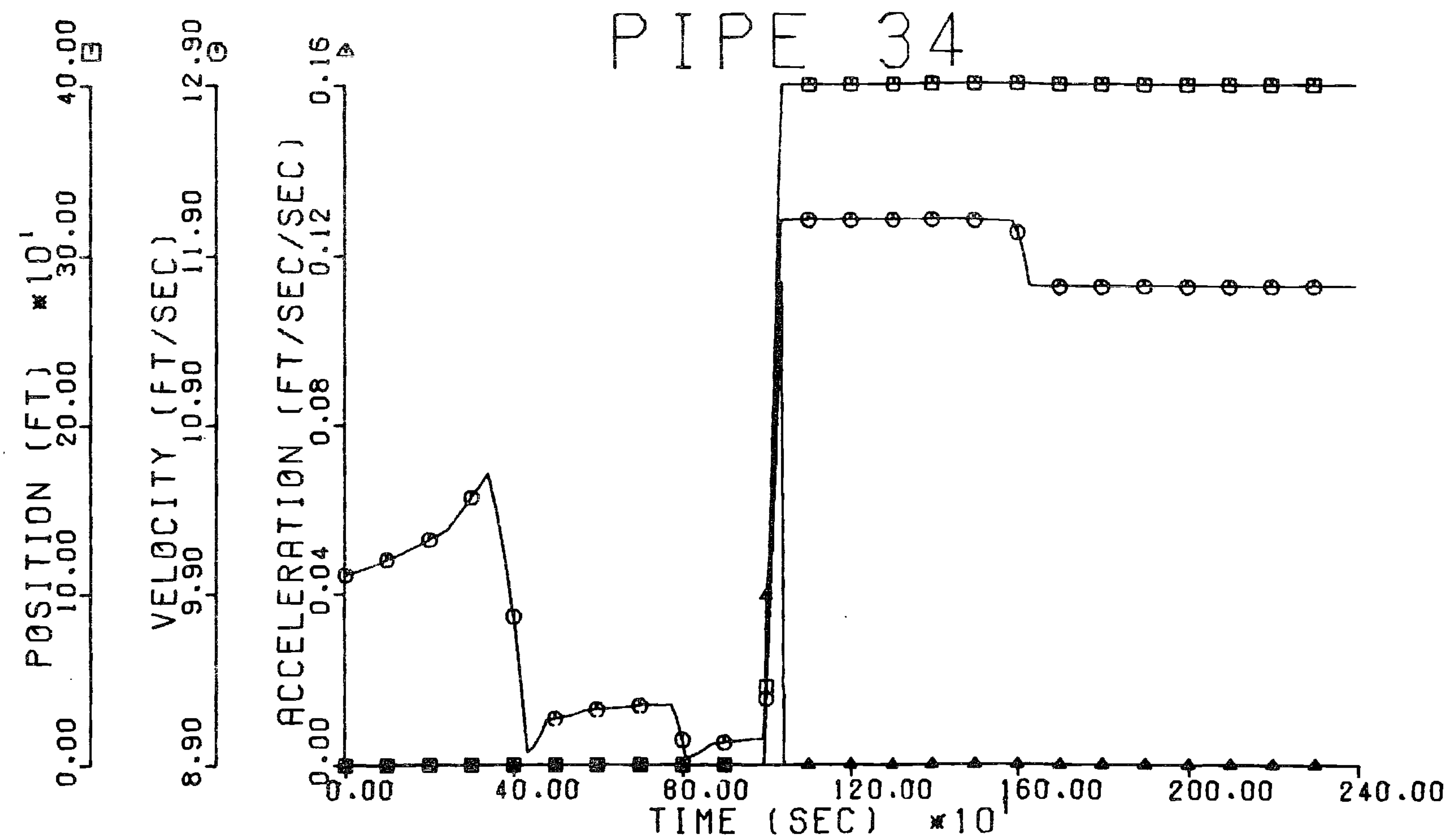

Figure 35. Unsteady Flow Parameters for Pipe 34 of Polyville Network.

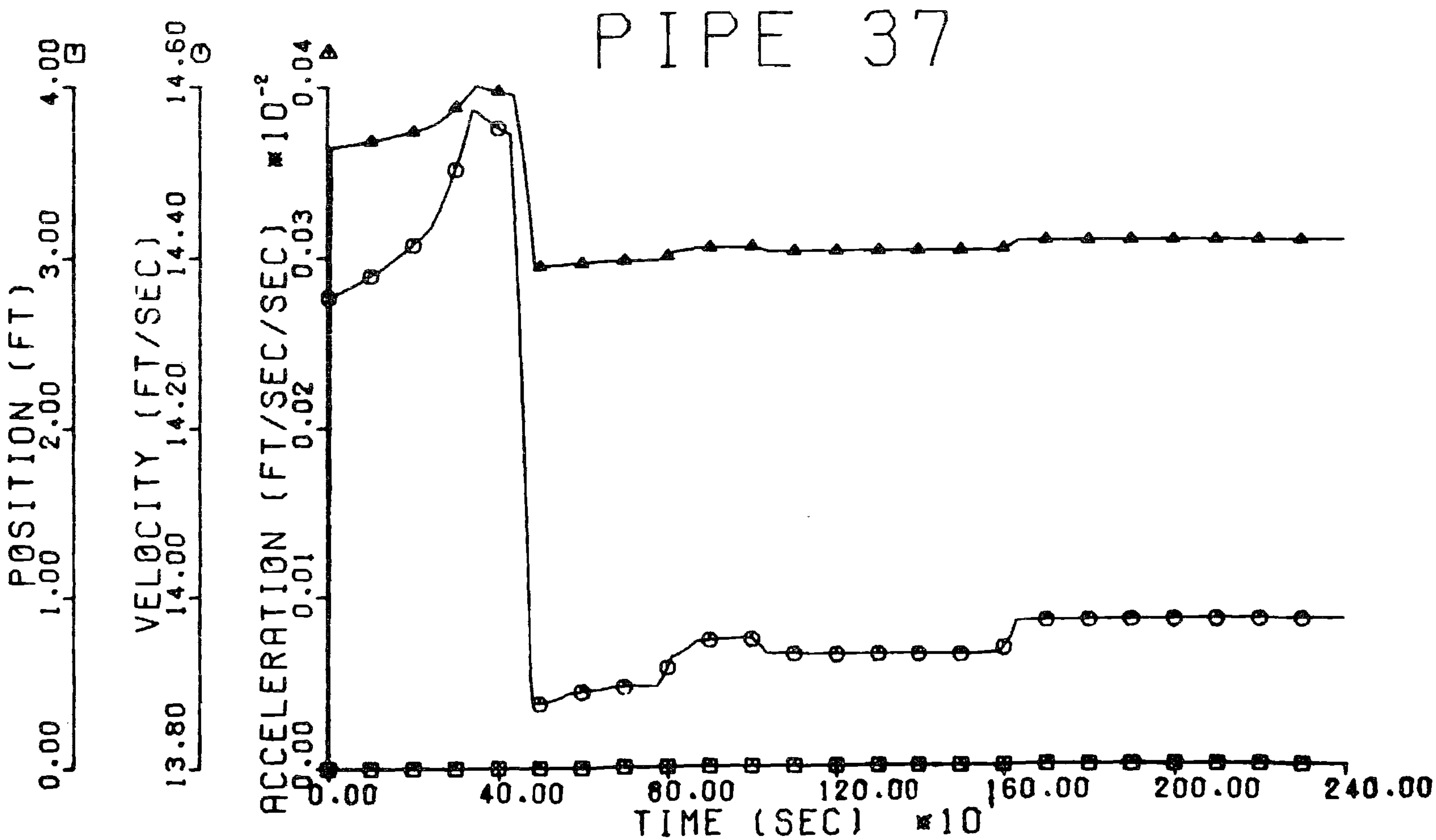

Figure 36. Unsteady Flow Parameters for Pipe 37 of Polyville Network.

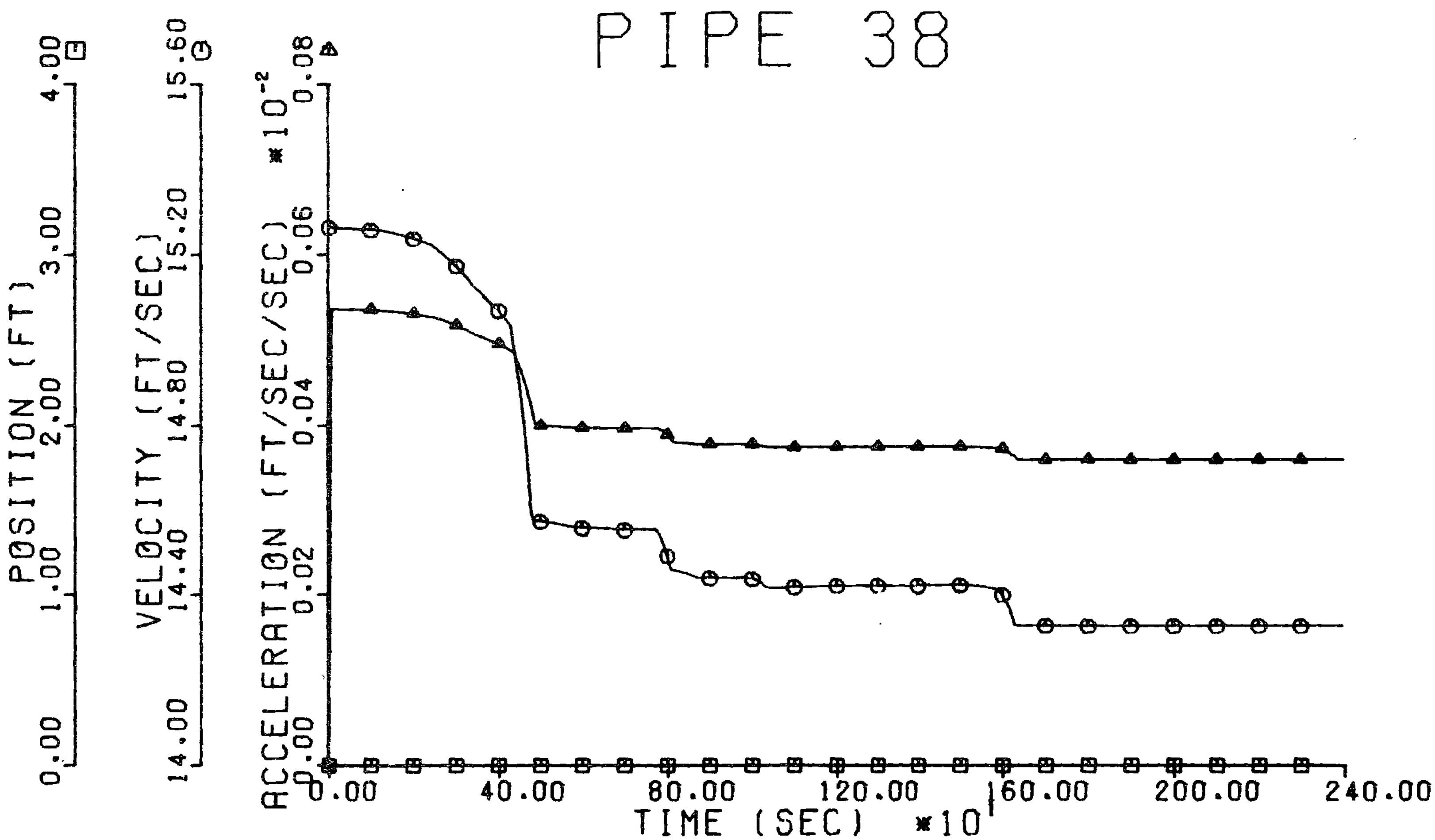

Figure 37. Unsteady Flow Parameters for Pipe 38 of Polyville Network.

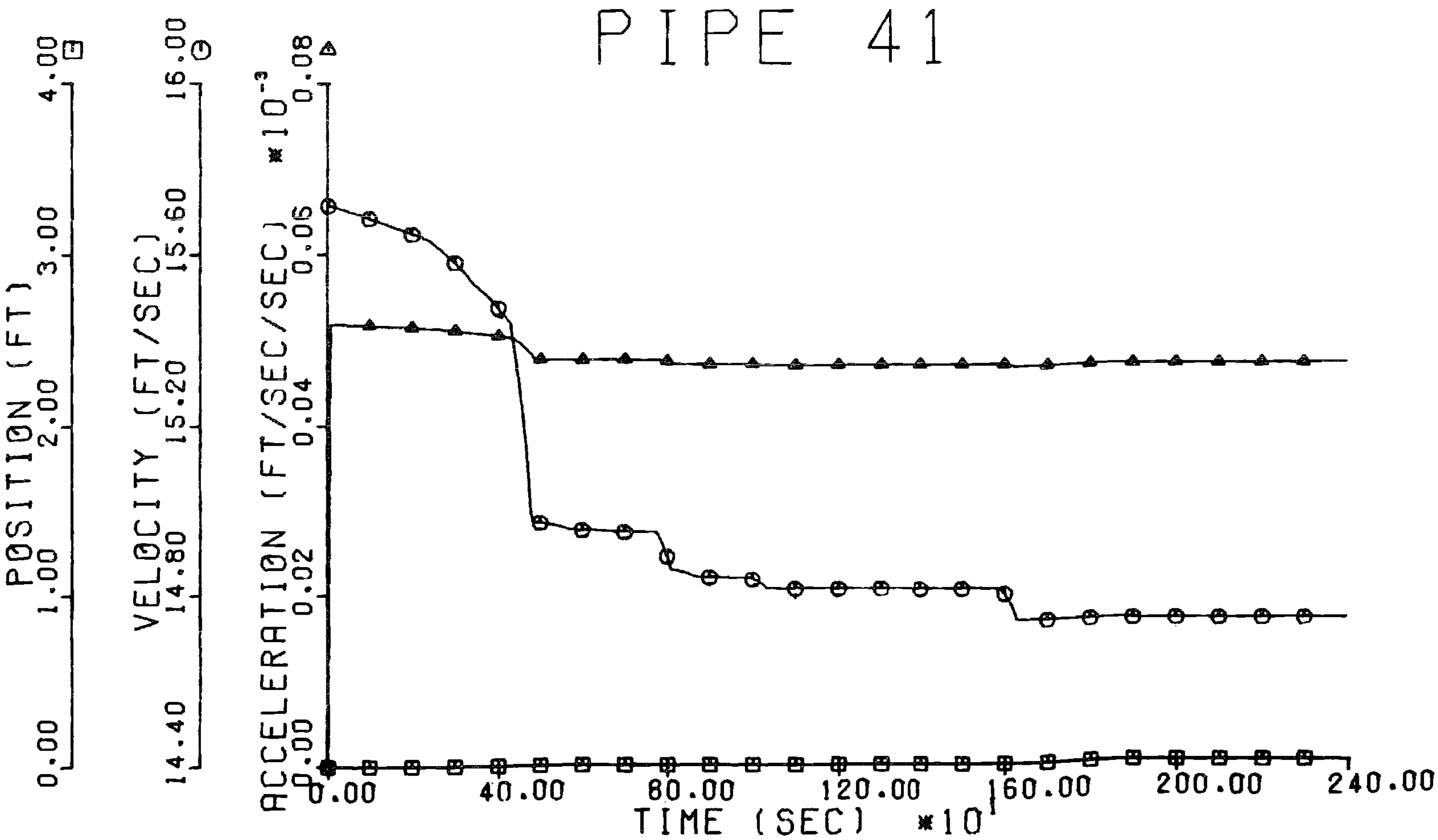

Figure 38. Unsteady Flow Parameters for Pipe 41 of Polyville Network.

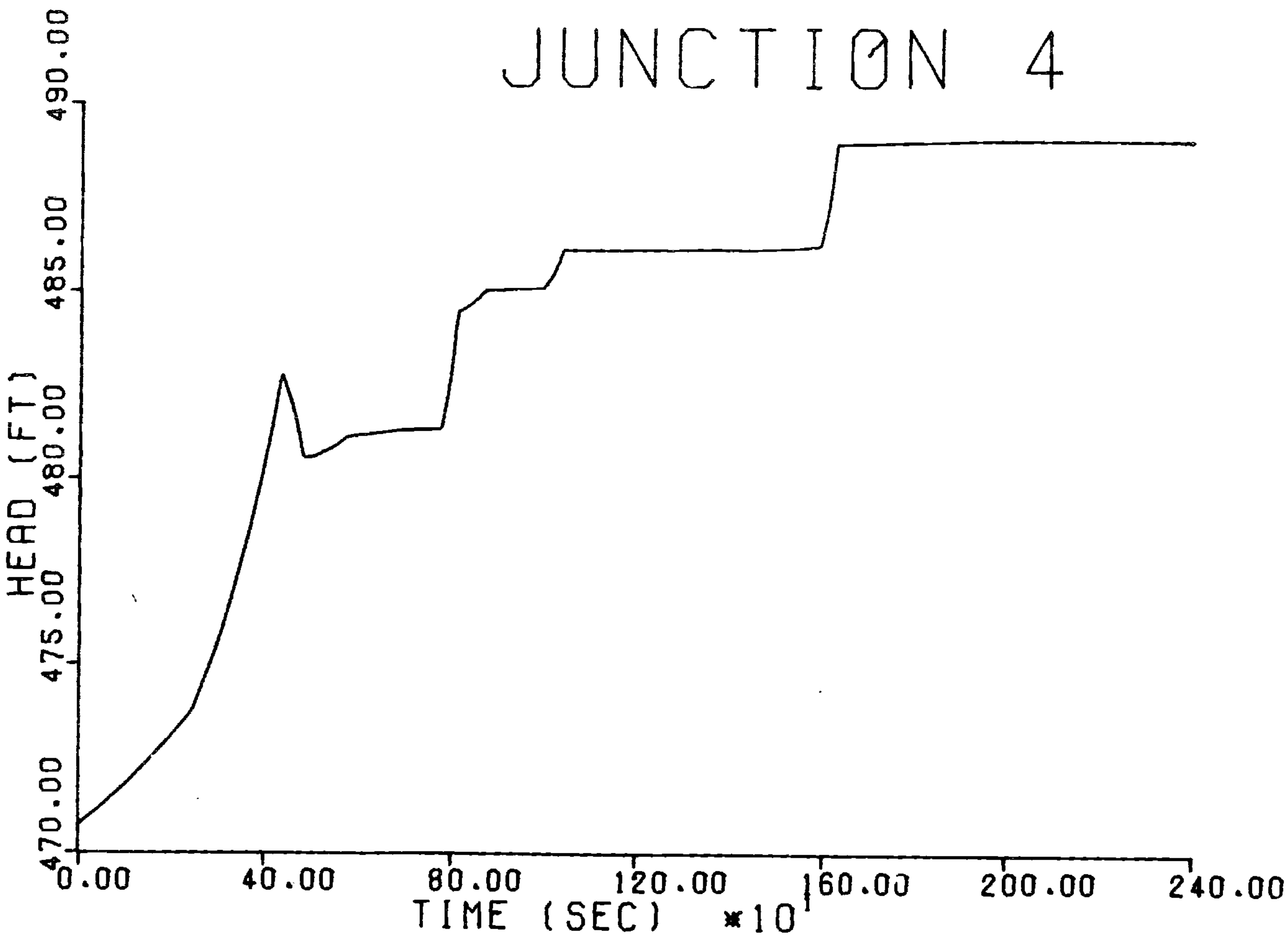

Figure 39. Head Versus Time Plot for Junction 4 of Polyville Network.

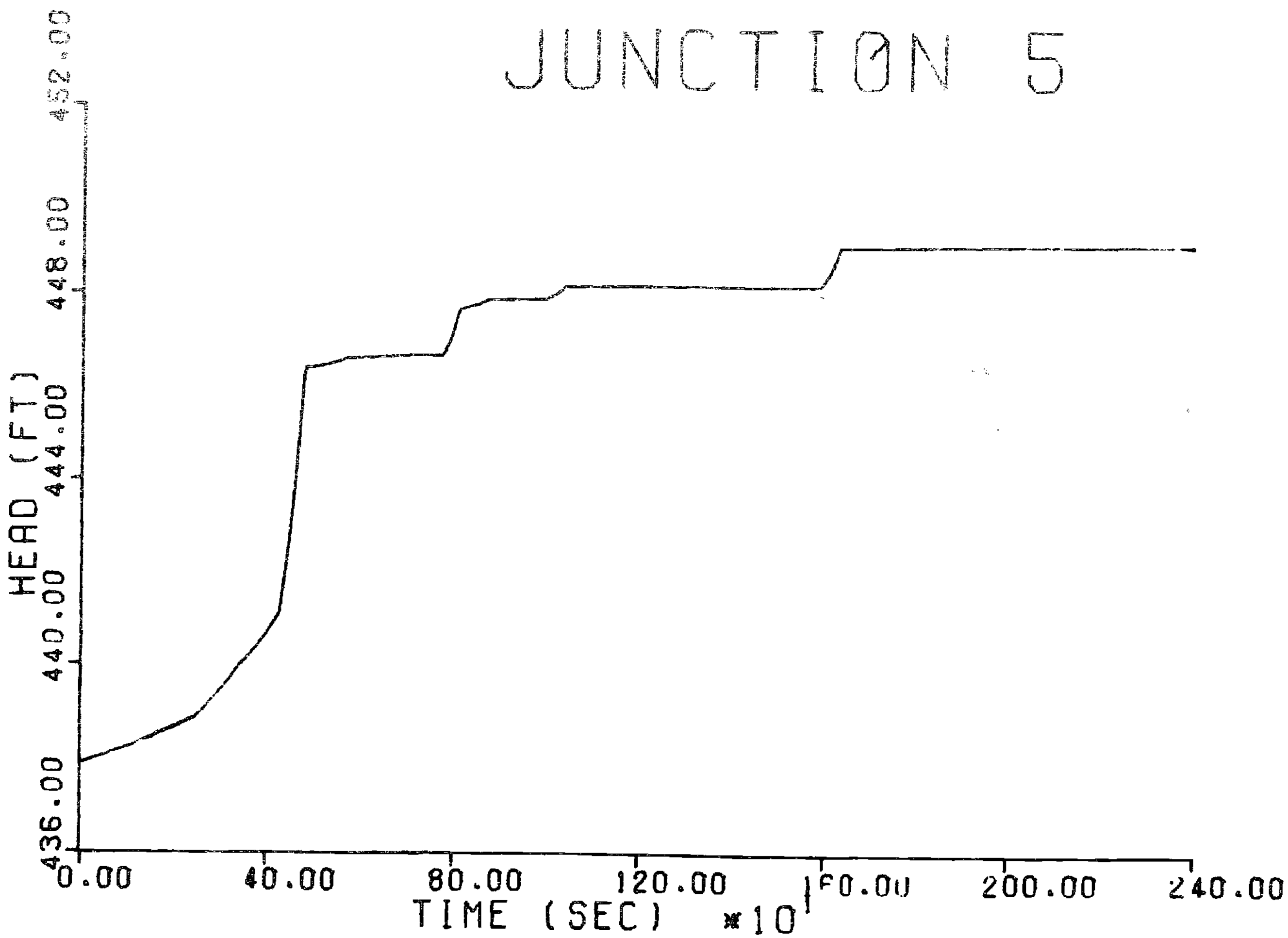

Figure 40. Head Versus Time Plot for Junction 5 of Polyville Network.

incorrectly assumed. The flow in pipe 41 is actually decelerating, but since the absolute value of the velocity was used in the computation, the signs must be determined by inspection. In some of the pipes illustrated, an interface never existed. Examples of this would be the pipes carrying water from the reservoir.

Obviously, the head transients in all pipes interact with one another while the polymer interfaces progress. This progress is dictated by minimum energy considerations. The flow patterns adjust according to the paths of least resistance. It could be visualized at an instant of time that the flows will progress towards the fire at junctions 4 and 5 as if they were moving along a valley formed by the topology of the hydraulic grade lines.

The head versus time plots of junctions 4 and 5 (Figures 39 and 40) showed stepped increases in head with time. The head corresponding to the minimum pressure requirement of 20 psi was 446.2 feet. According to Figure 39 this head was attained after approximately eight minutes.

Heads and discharges for the system at a time 40 minutes after the continuous injection of polymer at junction 26 was started are shown in Figure 41. A comparison with the steady-state analysis for the same flow demands without polymer injection on Figure 27 showed interesting flow adjustments in the network. Actually, the polymer injection resulted in reversal of the flow direction in some of the pipes. The time required to run the computer program was 90 seconds.

Additional improvements in Polyville's water distribution system could be anticipated if polymers had been injected also at the reservoirs. The program could handle polymer injections at any and all junctions.

```
NETWORK ANALYSIS FOR POLYMER

       0        .00   524.06   523.27   489.08   449.07   535.99   530.07   507.05   524.28   519.24   547.61
H=  524.057  506.159  523.272  489.078  449.070  529.389  523.171  528.158  528.151  532.072  536.119  538.388  535.989
    535.952  530.066  528.806  524.052  507.051  520.264  528.304  524.281  530.000  509.731  530.000  519.236  547.613


Q=    6.051    7.120     .201    2.942    3.109    -.095    3.204    3.155    -.510    1.407    1.640    6.919    8.429     .030    1.022
      -.991     .926    2.644    -.980    -.789   -1.810    1.794    8.446    3.070   -2.052    1.053     .612    -.586    -.024   -2.955
     -3.369    2.876   -6.104    2.301    -.608   -9.454    2.743   -2.813    1.074   -1.024  -11.569   -1.052    1.912     .786    -.736
```

Figure 41. Intermediate Steady Solution of Polyville Network After 40 Minutes of Injection

CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

Long-chain polymers are effective additives in reducing frictional resistance in pipe flows. The injection of polymer additives into pipes results in hydraulic transients.

Preliminary studies, both experimental and computational, established the feasibility of using mathematical modeling in solving complex system transients in pipe networks.

In this study, a computer program was developed capable of solving the differential equations associated with unsteady network flows resulting from local injections of friction-reducing additives. The program was used to demonstrate the feasibility of temporarily improving the capacity of existing pipe networks during emergency conditions. The program is able to monitor the hydraulic transients throughout the system and to provide information for the proper operation of an injection system. The mathematical model allows also for convenient access to an existing system in order to experiment with various modifications and schemes of polymer injection which might lead to satisfactory engineering solutions to various contingencies imposed on a system.

The assumption of incompressible fluid flow was mathematically convenient. In relatively long pipes, the accelerating flows might result in water-hammer effects. A further improvement on the method presented here could be the superposition of a water-hammer solution. In the example cited, the flow adjustments resulting from polymer injection were quite reasonable, and some of the sharp changes in gradients may have been the result of machine computation rather than actual physical events.

Although polymeric substances now marketed have been declared safe as dilute additives to water distribution systems, little is known of their effects on biological processes and of their removability in present-day water and waste treatment processes. Its effects on heat and mass transfer are little understood.

The method of improving flows in pipe networks by injection of polymeric substances may also have considerable merit in industrial systems involving heat transfer and chemical reaction processes.

The effects of polymer additives on hydraulic transients, on friction reduction, on heat transfer, and on chemical and biological processes would vary with both the polymer type and with concentration. The program is suitably constructed for generally decelerating flows. An example of this would be the return to normal operation after polymer injection was ceased.

In the proper use of the mathematical model developed for solving hydraulic transients in a water distribution system subjected to injection of friction-reducing long-chain polymers, and in its extension to other systems, the physical properties of the network and of the additives need to be known *a priori*.

APPENDIX 1

DISCUSSION OF THE RUNGE-KUTTA METHOD

The Runge-Kutta method is a numerical method for solving differential equations by approximations over short intervals. It is a one-step method in that only one starting point is required along with the necessary boundary conditions. There are several possible orders of Runge-Kutta solutions. The fourth-order solution for second-order differential equations will be presented here [16]. The fourth-order designation implies that there will be four constants involved.

Given an equation of the form

$$y'' = \emptyset\ (x,y,y')$$

where $\emptyset$ represents "a function of," the next y and y' after the interval of length has been added to the independent variable (x + h) may be expressed by

$$y_{n+1} = y_n + h\left[y'_n - \frac{1}{6}\left(K_1 + K_2 + K_3\right)\right] + 0\left(h^5\right), \text{ and}$$

$$y'_{n+1} = y'_n + \frac{1}{6}\left(K_1 + K_2 + K_3 + K_4\right)$$

where

$$K_1 = h\ \emptyset\left(x_n, y_n, y'_n\right),$$

$$K_2 = h\ \emptyset\left(x_n + \tfrac{1}{2}h, y_n + \frac{h}{2}\ y_n + \frac{h}{8}\ K_1, y'_n + \frac{K_1}{2}\right),$$

$$K_3 = h\ \emptyset\left(x_n + \tfrac{1}{2}h, y_n + \frac{h}{2}\ y'_n + \frac{h}{8}\ K_1, y'_n + \frac{K_2}{2}\right), \text{ and}$$

$$K_4 = h\ \emptyset\left(x_n + h, y_n + h\ y'_n + \frac{h}{2}\ K_3,\ y'_n + K_3\right).$$

Thus knowing x, y, and y' at one initial point, one may determine the functional values for a nearby value of the independent variable, x.

The function may be traced by using the computed functional values to determine the next values.

The term $0(h^5)$ implies that the error is a function of the interval raised to the fifth power. Thus an interval smaller than unity should produce more reliable results than a larger interval. In fact, the use of large intervals may lead to instabilities in the solution.

The Runge-Kutta method is well suited for use with a digital computer, since a digital computer is very efficient at iteration.

APPENDIX 2

FORTRAN V PROGRAM TO SOLVE UNSTEADY SINGLE-PIPE PROBLEMS

```
      GO TO 1
  500 CONTINUE
      CALL PLOT (0.0,0.0,999)
      END

C     SUBROUTINE TO SOLVE DIFFERENTIAL EQUATIONS OF UNSTEADY FLOW BY THE
C     RUNGE-KUTTA METHOD FOR SECOND ORDER EQUATIONS
C
C
C                          LIST OF ARGUMENTS
C
C    ARGUMENT    MEANING                                      UNITS
C
C    D           DIAMETER OF PIPE                             FT
C    XL          LENGTH OF PIPE                               FT
C    FB          FRICTION FACTOR BEFORE INTERFACE             --
C    FA          FRICTION FACTOR AFTER INTERFACE              --
C    T           TIME INCREMENT                               SEC
C    X           POSITION OF INTERFACE                        FT
C    X1          VELOCITY                                     FT/SEC
C    X2          ACCELERATION                                 FT/SEC**2
C    H           DRIVING HEAD                                 FT
C
      SUBROUTINE KUTTA (D,XL,FB,FA,T,X,X1,X2,H)
C
C                  LIST OF VARIABLES AND CONSTANTS
C
C    SYMBOL      MEANING                                      UNITS
C
C    C1          DUMMY FOR COMPUTATION                        SEC/FT
C    C2          DUMMY FOR COMPUTATION                        SEC**2/FT
```

```
C   C3           DUMMY FOR COMPUTATION                          1/SEC**2
C   G            GRAVITATIONAL CONSTANT                         FT/SEC**2
C   K1           RUNGE-KUTTA DUMMY VARIABLE                     FT
C   K2           RUNGE-KUTTA DUMMY VARIABLE                     FT
C   K3           RUNGE-KUTTA DUMMY VARIABLE                     FT
C   K4           RUNGE-KUTTA DUMMY VARIABLE                     FT
C
      REAL K1,K2,K3,K4
      G=32.1725
      IF (XL-X) 100,100,1
C   SEGMENT FOR INTERFACE BETWEEN RESERVOIRS
C   SEGMENT TO DETERMINE DUMMY VARIABLES
    1 C1=(FA-FB)/2.0/G/D
      C2=(FB*XL)/2.0/G/D
      C3=G/XL
C   SEGMENT TO DETERMINE RUNGE-KUTTA DUMMY VARIABLES
      K1=T*C3*(H-C1*X*X1**2-C2*X1**2)
      K2=T*C3*(H-C1*(X+T/2.0*X1+T/8.0*K1)*(X1+K1/2.0)**2-C2*(X1+K1/2.0)*
     1*2)
      K3=T*C3*(H-C1*(X+T/2.0*X1+T/8.0*K1)*(X1+K2/2.0)**2-C2*(X1+K2/2.0)*
     1*2)
      K4=T*C3*(H-C1*(X+T*X1+T/2.0*K3)*(X1+K3)**2-C2*(X1+K3)**2)
C   SEGMENT TO DETERMINE NEW FLOW PARAMETERS
      X=X+T*(X1+1.0/6.0*(K1+K2+K3))
      X1=X1+1.0/6.0*(K1+2.0*K2+2.0*K3+K4)
      X2=C3*(H-C1*X*X1**2-C2*X1**2)
      GO TO 200
C   SEGMENT FOR INTERFACE AT DOWNSTREAM RESERVOIR
C   SEGMENT TO DETERMINE DUMMY VARIABLES
  100 C1=H*G/XL
      C2=FA/2.0/D
C   SEGMENT TO DETERMINE RUNGE-KUTTA DUMMY VARIABLES
      K1=T*(C1-C2*X1**2)
```

```
      K2=T*(C1-C2*(X1+K1/2.0)**2)
      K3=T*(C1-C2*(X1+K2/2.0)**2)
      K4=T*(C1-C2*(X1+K3)**2)
      X=XL
C   SEGMENT TO DETERMINE NEW FLOW PARAMETERS
      X1=X1+(K1+2.0*K2+2.0*K3+K4)/6.0
      X2=C1-C2*X1**2
  200 RETURN
      END
```

APPENDIX 3

FORTRAN V PROGRAM TO SOLVE UNSTEADY NETWORK PROBLEMS

FORTRAN V PROGRAM TO SOLVE UNSTEADY NETWORK PROBLEM

```
C    MAIN PROGRAM
C
C
C                     LIST OF VARIABLES AND CONSTANTS
C
C   SYMBOL      MEANING                                          UNITS
C
C   ACC(I)      ACCELERATION IN PIPE I                           FT/SEC**2
C   CK          SAME AS IN BLOCK DATA                            -
C   D(I)        SAME AS IN BLOCK DATA                            FT
C   F(I)        SAME AS IN BLOCK DATA                            --
C   FA(I)       SAME AS IN BLOCK DATA                            -
C   FB(I)       SAME AS IN BLOCK DATA                            -
C   HH(I)       SAME AS IN BLOCK DATA                            FT
C   I           COUNTER
C   IP(I)       PIPE NUMBER (SHOULD BE EQUAL TO I)               --
C   J           COUNTER
C   JDN(I)      DOWNSTREAM JUNCTION NUMBER OF PIPE I             --
C   JP          SAME AS IN BLOCK DATA                            -
C   JUP(I)      UPSTREAM JUNCTION NUMBER OF PIPE I               --
C   K           DUMMY PIPE NUMBER                                -
C   K1          DUMMY UPSTREAM JUNCTION NUMBER                   --
C   K2          DUMMY DOWNSTREAM JUNCTION NUMBER                 --
C   MM          COUNTER FOR MODULATION OF PRINTING AND PLOTTING  --
C   MODP        SAME AS IN BLOCK DATA                            -
C   MODPL       SAME AS IN BLOCK DATA                            -
C   NUMIT       SAME AS IN BLOCK DATA                            -
C   POS(I)      POSITION OF INTERFACE IN PIPE I                  FT
C   PRONT       LOGICAL VARIABLE WHICH WHEN TRUE CAUSES PRT TO   --
C               BE CALLED
C   QQ(I)       DISCHARGE IN PIPE I                              FT**3/SEC
C   T           SAME AS IN BLOCK DATA                            SEC
C   TIME        ELAPSED TIME OF STUDY                            SEC
```

```
C   TMAX        SAME AS IN BLOCK DATA                          SEC
C   VEL(I)      VELOCITY IN PIPE I                             FT/SEC
C   XL(I)       SAME AS IN BLOCK DATA                          FT
C   XX( )       SAME AS IN BLOCK DATA                          --
C
      COMMON/LABEL1/X(1600),HH(135),XX(420),R(200),QVV(20),HOO(50),DDH,
     1QP(50,20),JU,JP,TL,III,JV,N(9),NI,TOL,G(10)
      COMMON/LABEL2/RN(200),QQ(250),I,M,J2,J3,A,C,PO,E,VV,L,V,Y,Z,DH,PP,
     1C3,C2,C1,MODP,MODPR
      COMMON/LABEL3/F(200),D(200),XL(200),PRONT,TIME,TMAX
      COMMON/LABEL4/FA(200),FB(200),JH(10),JHNUM,NUMIT
      COMMON/LABEL5/IP(200),JUP(200),JDN(200),VEL(200),ACC(200),POS(200)
     1,CK(200),T
      COMMON/LABEL6/NPLOT(10),NHPLOT(10),NUMPL,NUMPLH,MODPL
      LOGICAL PRONT,CK
      INTEGER E,VV,V,Y,Z,X,XX,G,PP,PQ
      DATA G(1)/0/
      PRONT=.TRUE.
      MM=-1
      TIME=0.0
      WRITE (6,2000)
 2000 FORMAT (1H1)
C   SEGMENT TO OBTAIN STEADY SOLUTION
      CALL STEADY
      WRITE (6,2000)
C   SEGMENT TO INITIALIZE UNSTEADY SOLUTION VALUES
      DO 10 I=1,JP
      IP(I)=XX(3*I-2)
      JUP(I)=XX(3*I-1)
      JDN(I)=XX(3*I)
      VEL(I)=ABS(QQ(I)/3.1416/D(I)**2*4.0)
   10 POS(I)=0.0
   11 CONTINUE
```

```
      MM=MM+1
      IF (MOD(MM,MODPL) .EQ. 0) CALL PLOTT (TIME,VEL,ACC,POS,HH)
C   SEGMENT TO ITERATE WITH UNSTEADY SUBROUTINES
      DO 50 J=1,NUMIT
      DO 20 I=1,JP
      K=IP(I)
      K1=JUP(I)
      K2=JDN(I)
C   SEGMENT TO CALL UNSTEADY SUBROUTINES
   20 CALL RUNGE (D(K),XL(K),FA(K),FB(K),F(K),POS(K),VEL(K),ACC(K),
     1CK(K1),CK(K2),HH(K1),HH(K2),I)
      TIME=TIME+T
C   SEGMENT TO MODULATE UNSTEADY RESULTS
      IF (MOD(MM,MODP) .NE. 0) GO TO 50
      WRITE (6,1065) TIME
      WRITE (6,1061) (POS(II), II=1,JP)
      WRITE (6,1062) (VEL(II), II=1,JP)
      WRITE (6,1063) (ACC(II), II=1,JP)
      WRITE (6,1064) (  F(II), II=1,JP)
   50 CONTINUE
 1061 FORMAT (5H POS=,10E12.5/(5X,10E12.5))
 1062 FORMAT (5H VEL=,10E12.5/(5X,10E12.5))
 1063 FORMAT (5H ACC=,10E12.5/(5X,10E12.5))
 1064 FORMAT (5H   F=,10E12.5/(5X,10E12.5))
 1065 FORMAT (6H TIME=,F8.2/)
C   MODULATION STATEMENT FOR PRONT
      IF (MOD(MM,MODP) .EQ. 0) PRONT=.TRUE.
      CALL STEADY
C   SEGMENT TO CORRECT VELOCITIES
      DO 100 I=1,JP
  100 VEL(I)=ABS(QQ(I)/3.1416/D(I)**2*4.0)
      IF (TIME .LT. TMAX) GO TO 11
      IF (NUMPLH .GT. 0 .OR. NUMPL .GT. 0) CALL PLOTF
```

FORTRAN V PROGRAM TO SOLVE UNSTEADY NETWORK PROBLEM

```
      END                                                               A007


      SUBROUTINE STEADY                                                 B001
C   MAIN SUBROUTINE FOR STEADY SOLUTION                                 B002
C                                                                       B003
C                                                                       B004
C                     LIST OF VARIABLES AND CONSTANTS                   B005
C                                                                       B006
C  SYMBOL       MEANING                                    UNITS        B007
C                                                                       B008
C  A            DUMMY FOR COMPUTATION                      FT**3/SEC    B009
C  C            DUMMY FOR COMPUTATION                      FT**2/SEC    B010
C  D(I)         SAME AS IN BLOCK DATA                      FT           B011
C  E            JUNCTION NUMBER FROM X-ARRAY OR UPSTREAM   --           B012
C               JUNCTION NUMBER FROM XX-ARRAY                           B013
C  F(I)         SAME AS IN BLOCK DATA                      -            B014
C  G(I)         SAME AS IN BLOCK DATA                      -            B015
C  HH(I)        SAME AS IN BLOCK DATA                      FT           B016
C  I            COUNTER                                                 B017
C  III          SAME AS IN BLOCK DATA                      -            B018
C  J2           NUMBER OF X-ARRAY VALUES FOR A SINGLE JUNCTION --        B019
C  J3           NUMBER OF X-ARRAY VALUES FOR GROUP OF JUNCTIONS --       B020
C  JJJ          DUMMY VARIABLE                                          B021
C  JP           SAME AS IN BLOCK DATA                      -            B022
C  JXX          DUMMY VARIABLE                                          B023
C  K            COUNTER (POSITION OF JUNCTION NUMBER IN GROUP) --        B024
C  L            COUNTER (PIPES FROM A SINGLE JUNCTION)     --           B025
C  M            COUNTER (NUMBER OF PIPES AT A JUNCTION)    --           B026
C  N(I)         SAME AS IN BLOCK DATA                      -            B027
C  PQ           SUBSCRIPT OF X-ARRAY FOR A JUNCTION NUMBER --           B028
C  PRONT        LOGICAL VARIABLE WHICH WHEN TRUE CAUSES PRT TO --       B029
```

```
C                 BE CALLED
C   QQ(I)         DISCHARGE IN PIPE I                              FT**3/SEC
C   R(I)          DARCY RESISTANCE IN PIPE I                       --
C   RN(I)         SQUARE ROOT OF RECIPROCAL OF R(I)                --
C   TL            SUM OF ABSOLUTE VALUES OF HEAD CORRECTIONS       FT
C   TOL           SAME AS IN BLOCK DATA                            FT
C   V             DUMMY VARIABLE
C.  VV            CODE FOR TYPE OF JUNCTION                        --
C   X( )          SAME AS IN BLOCK DATA                            --
C   XL(I)         SAME AS IN BLOCK DATA                            FT
C   Y             PIPE NUMBER
C   Z             JUNCTION NUMBER AT OTHER END OF PIPE Y OR        --
C                 DOWNSTREAM JUNCTION NUMBER FROM XX-ARRAY
C
      COMMON/LABEL1/X(1600),HH(135),XX(420),R(200),QVV(20),HOO(50),DDH,
     1QP(50,20),JU,JP,TL,III,JV,N(9),NI,TOL,C(10)
      COMMON/LABEL2/RN(200),QQ(250),I,M,J2,J3,A,C,PQ,E,VV,L,V,Y,Z,DH,PP,
     1C3,C2,C1,MODP,MODPR
      COMMON/LABEL3/F(200),D(200),XL(200),PRINT,TIME,TMAX
      INTEGER E,VV,V,Y,Z,X,XX,G,PP,PQ
      LOGICAL PRINT
      TL=100.0
C   SEGMENT TO DETERMINE STARTING SUBSCRIPTS FOR DATA GROUPS
      DO 10 I=2,NI
C   SEGMENT TO DETERMINE RESISTANCE COEFFICIENTS
   10 G(I)=G(I-1) + N(I-1)*(3*I-1)
      DO 15 I=1,JP
      R(I)=F(I)*XL(I)/D(I)/64.34/(3.1416*D(I)**2/4.0)**2
   15 RN(I)=SQRT(1.0/R(I))
   21 CONTINUE
C   SEGMENT TO CORRECT ASSUMED HEADS AND DISCHARGES BY ITERATION
      DO 100 I=0,III
C   EXIT STATEMENT FOR EARLY CONVERGENCE
```

```
      IF (TL .LT. TOL) GO TO 101
      TL=0.0
C   SEGMENT TO STUDY JUNCTIONS IN ASCENDING ORDER OF NUMBER OF PIPES
      DO 80 M=1,NI
      J2=2 + 3*M
      J3=N(M)*J2
      IF (J3 .EQ. 0) GO TO 80
C   SEGMENT TO STUDY EACH JUNCTION IN A GROUP
      DO 30 K=1,J3,J2
      A=0.0
      C=0.0
      PQ=G(M)+K
      E=X(PQ)
      VV=X(PQ+1)
      DO 25 L=1,M
      V=K+3*L
      JJJJ=G(M)
      Y=X(JJJJ+V-1)
      Z=X(JJJJ+V)
   25 CALL CND
   30 CALL NCD
   80 CONTINUE
  100 IF (PRONT) CALL PRT
  101 CONTINUE
C   SEGMENT TO CALCULATE DISCHARGES IN PIPES
      JXX=3*JP
      DO 111 I=1,JXX,3
      Y=XX(I)
      E=XX(I+1)
      Z=XX(I+2)
      IF (HH(E)  -  HH(Z))  110,105,105
C   SEGMENT USED WHEN PROPER DISCHARGE DIRECTION WAS ASSUMED
  105 QQ(Y)=RN(Y)*SQRT(HH(E)-HH(Z))
```

```
      GO TO 111
C  SEGMENT USED WHEN IMPROPER DISCHARGE DIRECTION WAS ASSUMED
  110 QQ(Y)=-RN(Y)*SQRT(HH(Z)-HH(F))
  111 CONTINUE
C  SEGMENT TO PRINT HEADS AT ALL JUNCTIONS
      IF (PRONT) WRITE (6,1010) (HH(I), I=1,JH)
 1010 FORMAT (3H H=,13F9.3/(3X,13F9.3))
C  SEGMENT TO PRINT DISCHARGES IN ALL PIPES
      IF (PRONT) WRITE (6,1011) (QQ(I), I=1,JP)
 1011 FORMAT (//3H Q=,15F8.3/(3X,15F8.3))
      PRONT=.FALSE.
      RETURN
      END


      SUBROUTINE PRT
C   SUBROUTINE TO PRINT MODULATED ITERATIONS OF STEADY SOLUTION
C
C
C                      LIST OF VARIABLES AND CONSTANTS
C
C  SYMBOL      MEANING                                       UNITS
C
C  HH(I)       SAME AS IN BLOCK DATA                         FT
C  HHH(I)      HEAD ARRAY TO BE PRINTED                      FT
C  I           COUNTER (EQUAL TO ITERATION NUMBER)           --
C  J           COUNTER
C  JH(I)       SAME AS IN BLOCK DATA                         -
C  JHNUM       SAME AS IN BLOCK DATA                         -
C  JJJ         DUMMY VARIABLE
C  K           COUNTER
C  MODPR       SAME AS IN BLOCK DATA                         -
```

```
C   C              DUMMY FOR COMPUTATION                                 FT**2/SEC
C   E              JUNCTION NUMBER UNDER CONSIDERATION                   --
C   HH(I)          SAME AS IN BLOCK DATA                                 FT
C   RN(I)          SQUARE ROOT OF RECIPROCAL OF RESISTANCE IN            --
C                  PIPE I
C   VV             CODE FOR TYPE OF JUNCTION                             --
C   Y              PIPE NUMBER
C   Z              JUNCTION NUMBER AT OTHER END OF PIPE Y                --
C
      COMMON/LABEL1/X(1600),HH(135),XX(420),R(200),QVV(20),HOO(50),DDH,
     1QP(50,20),JU,JP,TL,III,JV,N(9),NI,TOL,G(10)
      COMMON/LABEL2/RN(200),QQ(250),I,M,J2,J3,A,C,PQ,E,VV,L,V,Y,Z,DH,PP,
     1C3,C2,C1,MODP,MODPR
      INTEGER E,VV,V,Y,Z,X,XX,G,PP,PQ
      IF (VV .NE. 3) GO TO 300
C  SEGMENT FOR RESERVOIR JUNCTIONS (NO HEAD CORRECTIONS)
      C=1.0
      A=0.0
      GO TO 330
  300 IF (HH(Z)- HH(E)) 310,330,305
C  SEGMENT FOR FLOW FROM Z TO E
  305 A=A+RN(Y)*SQRT(HH(Z)-HH(E))
      C=C+0.5/SQRT(HH(Z)-HH(E))*RN(Y)
      GO TO 330
C  SEGMENT FOR FLOW FROM E TO Z
  310 A=A-RN(Y)*SQRT(HH(E)-HH(Z))
      C=C+0.5/SQRT(HH(E)-HH(Z))*RN(Y)
  330 CONTINUE
      RETURN
      END
```

FORTRAN V PROGRAM TO SOLVE UNSTEADY NETWORK PROBLEM

```
C   FB          FRICTION FACTOR WITHOUT ADDITIVE (BEFORE             --
C               INTERFACE HAS PASSED)
C   F           APPARENT FRICTION FACTOR FOR ENTIRE PIPE             --
C   POS         PORTION OF PIPE AFFECTED BY ADDITIVE                 FT
C   VEL         VELOCITY IN PIPE                                     FT/SEC
C   ACC         ACCELERATION IN PIPE                                 FT/SEC**2
C   CU          LOGICAL VARIABLE WHICH IS TRUE IF THE INTERFACE      --
C               HAS REACHED THE UPSTREAM JUNCTION AND
C               FALSE OTHERWISE
C   CD          LOGICAL VARIABLE WHICH IS TRUE IF THE INTERFACE      --
C               HAS REACHED THE DOWNSTREAM JUNCTION AND
C               FALSE OTHERWISE
C   HU          HEAD AT UPSTREAM JUNCTION                            FT
C   HD          HEAD AT DOWNSTREAM JUNCTION                          FT
C   T           TIME INCREMENT                                       SEC
C
      SUBROUTINE RUNGE (D,XL,FA,FB,F,POS,VEL,ACC,CU,CD,HU,HD,T)
      LOGICAL CU,CD
      IF (POS .LT. 0.0) POS=0.0
      IF (HU-HD) 10,100,100
C   SEGMENT FOR IMPROPERLY ASSUMED FLOW DIRECTION
   10 DHE=HD-HU
      IF (CD) GO TO 50
C   SEGMENT FOR PIPE NOT AFFECTED BY ADDITIVE
      POS =0.0
      CALL KUTTA (D,XL,FB,FB,T,POS,VEL,ACC,DHE)
      POS =0.0
      GO TO 200
C   SEGMENT FOR PIPE AFFECTED BY ADDITIVE
   50 CALL KUTTA (D,XL,FB,FA,T,POS,VEL,ACC,DHE)
      IF (POS .GE. XL) CU=.TRUE.
      IF (POS .GT. XL) POS=XL
      GO TO 200
```

```
C   X1          VELOCITY                                        FT/SEC
C   X2          ACCELERATION                                    FT/SEC**2
C   H           DRIVING HEAD                                    FT
C
      SUBROUTINE KUTTA (D,XL,FB,FA,T,X,X1,X2,H)
C
C                     LIST OF VARIABLES AND CONSTANTS
C
C   SYMBOL      MEANING                                         UNITS
C
C   C1          DUMMY FOR COMPUTATION                           SEC/FT
C   C2          DUMMY FOR COMPUTATION                           SEC**2/FT
C   C3          DUMMY FOR COMPUTATION                           1/SEC**2
C   G           GRAVITATIONAL CONSTANT                          FT/SEC**2
C   K1          RUNGE-KUTTA DUMMY VARIABLE                      FT
C   K2          RUNGE-KUTTA DUMMY VARIABLE                      FT
C   K3          RUNGE-KUTTA DUMMY VARIABLE                      FT
C   K4          RUNGE-KUTTA DUMMY VARIABLE                      FT
C
C
      REAL K1,K2,K3,K4
      G=32.1725
      IF (XL-X) 100,100,1
C   SEGMENT FOR INTERFACE BETWEEN JUNCTIONS
C   SEGMENT TO DETERMINE DUMMY VARIABLES
    1 C1=(FA-FB)/2.0/G/D
      C2=(FB*XL)/2.0/G/D
      C3=G/XL
C   SEGMENT TO DETERMINE RUNGE-KUTTA DUMMY VARIABLES
      K1=T*C3*(H-C1*X*X1**2-C2*X1**2)
      K2=T*C3*(H-C1*(X+T/2.0*X1+T/8.0*K1)*(X1+K1/2.0)**2-C2*(X1+K1/2.0)*
     1*2)
      K3=T*C3*(H-C1*(X+T/2.0*X1+T/8.0*K1)*(X1+K2/2.0)**2-C2*(X1+K2/2.0)*
```

```
     1*2)
      K4=T*C3*(H-C1*(X+T*X1+T/2.0*K3)*(X1+K3)**2-C2*(X1+K3)**2)
C  SEGMENT TO DETERMINE NEW FLOW PARAMETERS
      X=X+T*(X1+1.0/6.0*(K1+K2+K3))
      X1=X1+1.0/6.0*(K1+2.0*K2+2.0*K3+K4)
      X2=C3*(H-C1*X*X1**2-C2*X1**2)
      GO TO 200
C  SEGMENT FOR INTERFACE AT DOWNSTREAM JUNCTION
C  SEGMENT TO DETERMINE DUMMY VARIABLES
  100 C1=H*G/XL
      C2=FA/2.0/D
C  SEGMENT TO DETERMINE RUNGE-KUTTA DUMMY VARIABLES
      K1=T*(C1-C2*X1**2)
      K2=T*(C1-C2*(X1+K1/2.0)**2)
      K3=T*(C1-C2*(X1+K2/2.0)**2)
      K4=T*(C1-C2*(X1+K3)**2)
C  SEGMENT TO DETERMINE NEW FLOW PARAMETERS
      X=XL
      X1=X1+(K1+2.0*K2+2.0*K3+K4)/6.0
      X2=C1-C2*X1**2
  200 RETURN
      END


C   SUBROUTINE TO BUILD ARRAYS FOR PLOTTING
C
C
C                          LIST OF ARGUMENTS
C
C  ARGUMENT   MEANING                                         UNITS
C
C  TIME       ELAPSED TIME OF STUDY                           SEC
```

```
      DO 10 I=1,NUMPL
      J=NPLOT(I)
      VVV(W,I)=VEL(J)
      AAA(W,I)=ACC(J)
   10 PPP(W,I)=POS(J)
   11 CONTINUE
      IF (NUMPLH .EQ. 0) GO TO 21
C     SEGMENT TO STORE PROPER HEAD POINTS
      DO 20 I=1,NUMPLH
      J=NHPLOT(I)
   20 HHH(W,I)=HH(J)
   21 CONTINUF
C     SEGMENT TO STORE TIME POINTS
      TTT(W)=TIME
      RETURN
      END
```

```
      SUBROUTINE PLOTF
C      SUBROUTINE TO PLOT GRAPHS OF FLOW PARAMETERS AS FUNCTIONS OF TIME
C
C
C                         LIST OF VARIABLES AND CONSTANTS
C
C     SYMBOL       MEANING                                         UNITS
C
C     A( )         ACCFLERATION ARRAY FOR PLOTTING ONE GRAPH      FT/SEC**2
C     AAA(J,I)     ACCELERATION POINT J IN PIPE I                 FT/SEC**2
C     H( )         HEAD ARRAY FOR PLOTTING ONE GRAPH              FT
C     HHH(J,I)     HEAD POINT J AT JUNCTION I                     FT
C     I            COUNTER
C     IBUF( )      CALCOMP PLOTTER ARRAY                          -
```

```
C   J            COUNTER
C   NHPLOT(I)    SAME AS IN BLOCK DATA                               -
C   NPLOT(I)     SAME AS IN BLOCK DATA                               -
C   NUMPL        SAME AS IN BLOCK DATA                               -
C   NUMPLH       SAME AS IN BLOCK DATA                               -
C   P( )         POSITION ARRAY FOR PLOTTING ONE GRAPH               FT
C   PPP(J,I)     POSITION POINT J IN PIPE I                          FT
C   TTT( )       TIME ARRAY FOR ALL GRAPHS                           SEC
C   V( )         VELOCITY ARRAY FOR PLOTTING ONE GRAPH               FT/SEC
C   W            NUMBER OF POINTS PER GRAPH                          --
C   VVV(J,I)     VELOCITY POINT J IN PIPE I                          FT/SEC
C   X            DUMMY VARIABLE
C
      COMMON/LABEL6/NPLOT(10),NHPLOT(10),NUMPL,NUMPLH,MODPL
      COMMON/LABEL7/AAA(500,10),VVV(500,10),PPP(500,10),TTT(500),W,
     1HHH(500,10)
      DIMENSION A(502),V(502),P(502),H(502),IBUF(2000)
      INTEGER W
C   SEGMENT TO INITIALIZE PLOTTER SUBROUTINES
      CALL PLOTS (IBUF,2000,3)
      CALL SCALE (TTT,6.0,W,1)
      IF (NUMPL .EQ. 0) GO TO 101
C   SEGMENT TO PLOT PIPE PARAMETER VS TIME GRAPHS
      CALL PLOT (0.0,-30.0,-3)
      CALL PLOT (3.0,2.3,-3)
      DO 50 I=1,NUMPL
C   CALL SUBROUTINE TO DRAW TIME AXIS (X-AXIS)
      CALL AXIS (0.0,0.0,10HTIME (SEC),-10,6.0,0.0,TTT(W+1),TTT(W+2))
C   SEGMENT TO FORM Y-ARRAYS FOR PLOTTING
      DO 10 J=1,W
      A(J)=AAA(J,I)
      V(J)=VVV(J,I)
   10 P(J)=PPP(J,I)
```

```
C   SEGMENT TO SCALE Y-ARRAYS FOR PLOTTING
       CALL SCALE (A,4.0,W,1)
       CALL SCALE (V,4.0,W,1)
       CALL SCALE (P,4.0,W,1)
C   SEGMENT TO DRAW Y-AXES
       CALL AXIS (0.0,0.0,25HACCELERATION (FT/SEC/SEC),25,4.0,90.0,A(W+1)
      1,A(W+2))
       CALL AXIS (-0.75,0.0,17HVELOCITY (FT/SEC),17,4.0,90.0,V(W+1),V(W+2
      1))
       CALL AXIS (-1.5,0.0,13HPOSITION (FT),13,4.0,90.0,P(W+1),P(W+2))
C   SEGMENT TO ANNOTATE Y-AXES WITH PROPER SPECIAL SYMBOLS
       CALL SYMBOL (-1.50,4.2,0.105,0,0.0,-1)
       CALL SYMBOL (-0.75,4.2,0.105,1,0.0,-1)
       CALL SYMBOL ( 0.00,4.2,0.105,2,0.0,-1)
       X=NPLOT(I)
C   SEGMENT TO LABEL GRAPH
       CALL SYMBOL (1.6,4.1,0.315,4HPIPE,0.0,4)
       CALL NUMBER (3.175,4.1,0.315,X,0.0,-1)
C   SEGMENT TO DRAW CURVES
       CALL LINE (TTT,A,W,1,20,2)
       CALL LINE (TTT,V,W,1,20,1)
       CALL LINE (TTT,P,W,1,20,0)
    50 CALL PLOT (11.0,0.0,-3)
   101 CONTINUE
       IF (NUMPLH .EQ. 0) GO TO 201
C   SEGMENT TO PLOT HEAD VS TIME GRAPHS
       CALL PLOT (0.0,-30.0,-3)
       CALL PLOT (2.0,2.3,-3)
       DO 150 I=1,NUMPLH
C   CALL SUBROUTINE TO DRAW TIME AXIS (X-AXIS)
       CALL AXIS (0.0,0.0,10HTIME (SEC),-10,6.0,0.0,TTT(W+1),TTT(W+2))
C   SEGMENT TO FORM AND SCALE HEAD ARRAY
       DO 110 J=1,W
```

```
  110 H(J)=HHH(J,I)
      CALL SCALE (H,4.0,W,1)
C   SEGMENT TO DRAW HEAD AXIS
      CALL AXIS (0.0,0.0, 9HHEAD (FT),9,4.0,90.0,H(W+1),H(W+2))
C   SEGMENT TO LABEL GRAPH
      X=NHPLOT(I)
      CALL SYMBOL (2.268,4.1,0.315,8HJUNCTION,0.0,8)
      CALL NUMBER (5.1,4.1,0.315,X,0.0,-1)
C   CALL SUBROUTINE TO DRAW CURVE
      CALL LINE (TTT,H,W,1,0,0)
  150 CALL PLOT (11.0,0.0,-3)
C   CALL SUBROUTINE TO END PLOT
  201 CALL PLOT (0.0,0.0,999)
      RETURN
      END


      BLOCK DATA
C
C
C                             LIST OF DATA SYMBOLS
C
C
C   SYMBOL       MEANING                                          UNITS
C
C   CK(I)        LOGICAL VARIABLE WHICH IS                        --
C                     0 IF ADDITIVE IS NOT INJECTED AT JUNCTION I
C                     1 IF ADDITIVE IS INJECTED AT JUNCTION I
C   D(I)         DIAMETER OF PIPE I                               FT
C   DDH          INCREMENT OF HEAD FOR HEAD-DISCHARGE CURVES OF   FT
C                PUMPS
C   F(I)         CURRENT APPARENT FRICTION FACTOR IN PIPE I       --
```

```
C   FA(I)         FINAL FRICTION FACTOR OF PIPE I                      --               K016
C   FB(I)         ORIGINAL FRICTION FACTOR IN PIPE I                   --               K017
C   JH( )         ARRAY OF JUNCTION NUMBERS TO BE MONITORED IN         --               K018
C                 STEADY SOLUTION PRINTOUT                                              K019
C   JHNUM         NUMBER OF JUNCTIONS TO BE MONITORED IN STEADY        --               K020
C                 SOLUTION PRINTOUT (MAXIMUM OF 10)                                     K021
C   JP            NUMBER OF PIPES IN SYSTEM                                             K022
C   JU            NUMBER OF JUNCTIONS IN SYSTEM                        --               K023
C   JV            NUMBER OF OUTLET JUNCTIONS                           --               K024
C   HH(I)         ASSUMED HEAD AT JUNCTION I                           FT               K025
C   HOO(I)        BASE HEAD FOR HEAD-DISCHARGE CURVE FOR PUMP AT       FT               K026
C                 JUNCTION I                                                            K027
C   III           MAXIMUM NUMBER OF ITERATIONS IN STEADY SOLUTION      --               K028
C   MODP          MODULATION CONSTANT FOR UNSTEADY SOLUTION            --               K029
C                 PRINTOUT                                                              K030
C   MODPL         MODULATION CONSTANT FOR PLOTTING (SHOULD BE          --               K031
C                 SUCH THAT TOTAL NUMBER OF PLOTTED POINTS IS LESS                      K032
C                 THAN 500 WHERE TMAX/T IS THE NUMBER OF ITERATIONS)                    K033
C   MODPR         MODULATION CONSTANT FOR STEADY SOLUTION PRINTOUT--                    K034
C   N(I)          NUMBER OF JUNCTIONS WITH I PIPES                     --               K035
C   NHPLOT( )     JUNCTION NUMBERS OF PLOTTED JUNCTIONS                --               K036
C   NI            MAXIMUM NUMBER OF PIPES AT A JUNCTION                --               K037
C   NPLOT( )      PIPE NUMBERS OF PLOTTED PIPES                        --               K038
C   NUMIT         NUMBER OF TIMES UNSTEADY SOLUTION IS TO BE           --               K039
C                 OBTAINED PER STEADY SOLUTION                                          K040
C   NUMPL         NUMBER OF PIPE CURVES TO BE PLOTTED                  --               K041
C   NUMPLH        NUMBER OF HEAD CURVES TO BE PLOTTED                  --               K042
C   QP(I,J)       DISCHARGE VALUE NUMBER J FOR HEAD-DISCHARGE          FT**3/SEC        K043
C                 CURVE FOR PUMP AT JUNCTION I                                          K044
C   QVV(I)        OUTFLOW AT JUNCTION I (NOTE THAT OUTLET              FT**3/SEC        K045
C                 JUNCTIONS HAVE LOWEST JUNCTION NUMBERS)                               K046
C   T             TIME INCREMENT FOR UNSTEADY SOLUTION                 SEC              K047
C   TMAX          MAXIMUM TIME OF STUDY                                SEC              K048
```

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