Inventory Control with Risk of Major Supply Chain Disruptions

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To my wife and family

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

SUMMARY

This thesis studies inventory control given the risk of major supply chain disruptions, specifically border closures and congestion. We first investigate an inventory system in which the probability distributions of order leadtimes are dependent on the state of an exogenous Markov process; we will model border disruptions via this exogenous process. We consider stationary, state-dependent basestock policies, which are known to be optimal for the system under study, and develop an expression for the long-run average cost of an arbitrary policy of this form. Restricting our attention to state-invariant basestock policies, we show how to calculate the optimal basestock (or order-up-to) level and long-run average cost. We provide a sufficient condition for the optimality of a state-invariant basestock policy and monotonicity results for the optimal state-invariant order-up-to level with respect to a ratio of the holding and penalty costs, the individual holding and penalty costs, and stochastically larger demand. We finally provide a method to calculate the optimal state-invariant order-up-to level for a special class of exogenous system states.

Motivated by the possibility of port of entry closures in the event of a security incident, we specialize the inventory control model to a two-stage international supply chain. We consider a simple scenario in which a domestic manufacturer orders a single product from a foreign supplier, and the orders must cross an international border that is subject to closure. We first assume that orders accumulate at the border during periods of closure and arrive at the manufacturer without further delay once the border reopens; that is, border congestion has negligible effects. The manufacturer's optimal inventory policy and long-run average cost are analyzed. We prove the optimality of a state-invariant basestock policy in this case and show that the order-up-to level is monotonic in the leadtime from the supplier to the international border. We then conduct a comprehensive numerical study for this scenario, using the procedures developed in the first part of the thesis. The results show that the optimal inventory policy and long-run average cost are much more sensitive to

the expected duration of a disruption than to the occurrence likelihood of a disruption. While the prevention of a disruption is critically important, these results have important implications for business to engage and cooperate with government in disruption management and contingency planning in order to reduce the duration of a closure. Contingency planning for potential border closures can lead to substantial cost savings for supply chain operators, and such planning provides greater benefits when the leadtime from the supplier to the international border is small. The numerical results regarding the impacts on the optimal state-invariant order-up-to level with respect to the leadtime from the supplier to the international border, the holding and penalty cost parameters, and the demand distribution illustrate the theoretical monotonicity results. To conclude this part, we present three modeling extensions that model a positive inland transportation time, a maximum delay at the border, and multiple open border states representing increasing probabilities of closure.

Finally we extend the border closure model to include both border closures and the resulting congestion. We model the border processing system and congestion with a discrete-time, single-server queue with constant deterministic arrival rate and Markov-modulated (but otherwise deterministic) service rate. A key task is the development of the leadtime distribution, which is more complex than in the previous model. We prove by counter-example that the optimal policy for the border closure model with congestion is not state-invariant and observe that the optimal order-up-to levels tend to increase when the border is closed and with the level of congestion. Using value iteration to determine optimal policies for problems in this scenario, we conduct a comprehensive numerical study. Based on the results, we provide managerial and policy insights regarding business operations and the management of the infrastructure utilized by supply chains (e.g. ports of entry). We show that the optimal inventory policy is more reactive than proactive, meaning that the manufacturer tends to only change its order-up-to levels after a border closure has occurred and while congestion remains, rather than in anticipation of a border closure and congestion. We also show that the optimal order-up-to levels and long-run average cost are again much more sensitive to the expected duration of a disruption than to the occurrence likelihood of a disruption, and these quantities increase more than linearly with the utilization of the border queueing system. These results have important implications for business to engage and cooperate with government in contingency planning and disruption management and for business to encourage government investment to improve the processing capabilities of publicly owned and/or operated ports of entry in order to reduce the effects of post-disruption congestion. We show again in this scenario that contingency planning is critically important for a manufacturer facing border closures and congestion, especially in supply chains with small leadtimes from the supplier to the international border. Additionally we observe that the optimal order-up-to levels and long-run average cost exhibit similar characteristics with respect to the leadtime from the supplier to the international border, the holding and penalty cost parameters, and the demand distribution.

CHAPTER I

INTRODUCTION

1.1 Introduction

Businesses operate in uncertain environments and employ a variety of risk management strategies to protect their interests against, or in the event of, adverse situations. One of the most common risk management strategies is supply chain inventory management. Since uncertainty takes many forms, the specific inventory control measures that are implemented resultantly take many forms. This thesis studies inventory control with risk of major supply chain disruptions, specifically border closures and congestion.

Supply chain operators have long understood their vulnerability to minor security breaches, the primary concern being cargo theft, and have relied on basic deterrent measures such as fencing, lighting, closed-circuit TV, and security guards. In the aftermath of the terrorist attacks that occurred in the United States on September 11, 2001, the notion of supply chain security quickly expanded beyond cargo theft and is receiving significantly greater attention by both businesses and government. For example, the US Bureau of Customers and Border Protection (CBP) quickly implemented new supply chain security programs such as the Customs-Trade Partnership Against Terrorism (C-TPAT) and new submission regulations for cargo data such as the so-called "24-Hour Rule" legislated by the US Congress. Most importantly, however, the terrorist attacks highlighted that fact that US transportation systems could and would be severely constrained, possibly to the point of closure, during such events.

In the new era of supply chain security, two challenges face businesses: operating efficiently in an environment with heightened security measures designed to prevent disruptions, and planning systems that function efficiently given possible occurrence of disruptions [33]. This thesis focuses on the latter. Since the late 1980s, widespread adoption of just-in-time (JIT) and lean management principles has resulted in safety stock inventory reductions.

While JIT principles lead to cost reductions under normal operations, they introduce operational fragility that may increase costs substantially when operations are disrupted. Despite the risk of disruptions, lean management principles need not, and should not, be arbitrarily abandoned for reactionary or just-in-case management approaches. Rather, supply chain disruptions should be incorporated into planning models to develop appropriate inventory management policies.

1.1.1 Supply Chain Disruptions

We now discuss various types of supply chain disruptions. Supply chains are complex systems of materials, equipment, people, facilities, transportation lanes, firms, and nations. They span both time and distance, involve large numbers of transactions and decisions, and their success is often difficult to fully quantify, especially when customer service is considered. According to [35], supply chain management is defined to be

> a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize systemwide costs while satisfying service level requirements.

It is clear that managing supply chains is no easy task.

Through globalization and improved communication and transportation capabilities, supply chains now cover larger geographic areas and have more and more direct and indirect stakeholders. Over half of US companies have increased the number countries in which they operate since the late 1980's [11]. This growing complexity of modern supply chains increases the risks of exposure to various types of major disruptions, which we define to be events that severely interrupt the normal course of business. Responding to a disruption often requires altering an established strategy, ranging from high-level decisions such as postponing the release of a new product to operational decisions such as increasing the order amounts for raw materials. While there are many specific examples of supply chain disruptions, most can be classified by the following three categories: economic, demand, and supply. We note that supply chain security disruptions are found in all three categories.

Economic disruptions include unexpected changes to purchasing costs, selling prices, interest rates, currency exchange rates, contract parameters, etc. For example, the annual terrorism insurance premiums paid by Delta Air Lines rose from $2 million prior to September 11, 2001 to $152 million in 2002 [22] affecting net profits. Since the quality of a firm is largely measured by economic performance, economic disruptions are cause for serious concern and attention by top management. We note that the economics disruptions discussed here are ultimately the result of changes in supply and demand of products and services.

For any business, demand for its product is essential. Without demand, there is no basis for its existence. While one generally thinks of a demand disruption as a sudden drop in customer ordering, it can also be a sudden increase. For example, demand for building supply products may spike immediately after a tornado. Decreases in demand cause a firm to hold more inventory than anticipated while increases in demand deplete safety stock inventories and cause stock-outs and backorders.

In this thesis, we focus on a specific type of supply disruption. We classify supply disruptions as either disruptions of supplier availability or disruptions in the transportation of product from supplier to customer. In the first case, when an order is placed to a supplier, the supplier is either able to fill the order or not. A supplier may be unavailable to fill an order for a variety of reasons including equipment failures, damaged facilities, problems procuring necessary raw materials, or rationing its supply among its customers. For example in 2000, Sony was unable to deliver Playstation 2's for the holiday season due to parts shortages from its suppliers [18].

We differentiate disruptions in the transportation of the product from the supplier to customer from this type of disruption, since the supplier is not at fault. The supplier is available to fill orders when they are placed and leadtime delays occur while the orders are in transit. For example, delays at the US-Canadian border after the September 11 terrorist attacks quickly increased from the normal few minutes to an extreme 12 hours [7]. Ford Motor Company was forced to intermittently idle production at five of its assembly plants due to delays at US land borders [30] while Toyota came within hours of halting production at one plant since parts shipped by air from Germany were delayed due to the grounding of all US air traffic [33].

These two types of supply disruptions are modeled differently as well. For example assume that a basestock policy is implemented, such that when the system inventory level decreases below the basestock level, an order is placed to increase the system inventory level up to the basestock level. If the supplier is unavailable and an order is placed, the system inventory level is unchanged after ordering (since the order could not be filled). However if the supplier is available, the system inventory level is increased by the amount ordered. These differences in the system inventory have implications for ordering decisions in future periods. In this thesis, we investigate a disruption to the transportation of product from supplier to customer.

1.1.2 Characteristics of Inventory Systems

This section provides a general discussion of several critical characteristics that are important in describing an inventory system and that largely determine the complexity of the corresponding inventory control model. See [34] and [23] for additional discussion about characteristics of inventory systems.

Almost all inventory related costs fall into one of three categories: ordering costs, holding costs, and penalty costs. Ordering costs generally include potentially two components, a fixed cost that is incurred each time an order is placed and a variable cost that is proportional to the amount ordered. The structure of the ordering costs can alter the form of an optimal policy. Holding costs are proportional to the amount of physical inventory held per period and include such costs as the cost of the physical space to store the inventory, breakage, spoilage, obsolescence, taxes, insurance, and the opportunity cost of alternative investments. Penalty costs are associated with a company's inability to meet demand when it occurs. One of the most common is a proportional cost to the number of items backordered in a period and often represents a loss of customer goodwill.

Demand processes used in inventory system modeling are categorized as constant versus variable, deterministic versus stochastic, and independent versus dependent. Different inventory management models may be used depending on the type of demand process. The ordering and supply process often has a variety of characteristics. A firm may require that at most a single order can be outstanding at any given time or that any number of orders may be outstanding. Order leadtimes may be zero-valued (i.e. instantaneous fulfillment), non-negative or positive, deterministic or stochastic, independent or dependent. Due to tractability issues, models that permit multiple outstanding orders at any given time generally must assume that the probability of order crossover during the leadtime is negligible or zero. That is, one assumes a delivery system such that orders will arrive in the order in which they were placed. Supplier availability dynamics must also be considered.

Inventory models may consider costs over a variety of planning horizon lengths. An inventory system may be in existence for a single time period, a finite time period, or expectedly forever (i.e. an infinite time horizon). Modeling methodologies vary depending on the length of the time horizon. For example, the newsvendor model is a famous single-period model while linear programming and dynamic programming are often used for finite or infinite horizon models. Finally, the status of the inventory system, that is, the amount of on-hand inventory, the number of backorders, the amounts and locations of all outstanding orders, and any other relevant information, must be reviewed in order to make ordering decisions. Review strategies are classified as continuous or periodic and if periodic, the inter-review times may be constant, deterministic or stochastic.

Once the critical characteristics of an inventory system have been identified and described, an appropriate model and solution methodology are constructed to answer two fundamental questions: (1) When to order? and (2) How much to order? The answers to these questions are determined with respect to an objective, for example to optimize some monetary measure. Constraints may be included in the model, such as minimum service levels or storage space capacities, but often by adding complexity.

1.2 Literature Review

1.2.1 General Inventory Management

Mathematical inventory control models date back to the early 20th century, where some of the earliest research concerned the development of the well-known economic order quantity (EOQ) model (see [17] and [38]). However, research on stochastic inventory models was largely not undertaken until needed by the US war efforts in the 1940's, producing published results in the 1950's (see [5] and [12]). Two seminal papers, [6] and [32], respectively prove the optimality of a basestock policy and an (s, S) policy for periodic-review inventory systems with stationary stochastic demand processes, constant deterministic order leadtimes, and a total expected discounted cost evaluation criterion. According to an (s, S) policy, when the inventory position decreases to or below s, an order is placed to increase the system inventory level up to S. Reviews of many well-known stochastic inventory models are presented in [16], [34], and [23].

The stochastic inventory control literature in which demand and leadtime uncertainties are represented by probability distributions is quite extensive. However, these distributions often lack attributes to represent rare or extreme events. Therefore, analytical models must be developed to explicitly model such events. Since we study a disruption to the supply system, in the next two sections we only discuss the literature pertaining to disruptions of supplier availability and disruptions in the transportation of product from supplier to customer.

We first comment on an inventory control model that is distinct from the majority of the inventory control literature due to its macro-economic perspective. Most of the inventory control literature investigates systems in which the decision maker can only control the order quantities and order timing, i.e. the rate of resupply. In contrast, the macro-economic model presented in [8] is a continuous-time Markov decision problem in which the decision variables are the rates of demand, rates of supply, and balancing inventory levels. Under the limiting assumption of instantaneous fulfillment, the objective is to maximize net benefits to the system. The system incurs costs and benefits which are both dependent on an exogenous continuous-time Markov process that represents the state of the world, and which are respectively dependent on the supply rate and demand rate.

1.2.2 Inventory Control Models with Supplier Disruptions

Supplier availability models generally assume that a supplier can either be available to supply product or not. The following models also restrictively assume that at most a single order can be outstanding at any given time and with the exception of [25], instantaneous order fulfillment when the supplier is available. The classical EOQ model in which a supply or demand disruption will occur at a known future time and last for a random duration is studied in [37], and optimal order quantities are developed. Similarly, optimal order quantities are developed for an EOQ model with supply uncertainty in [26]. In this model, periods of supplier availability are exponentially distributed and periods of supplier unavailability are constant or exponentially distributed. The existence of an optimal (s, S) policy is proved in [28] and [24] respectively for finite- and infinite-horizon periodic-review, discounted cost models in which the supplier's availability is modeled as a two-state discrete-time Markov chain (DTMC).

The following papers propose potentially sub-optimal policies and then optimize the policy parameters. An (s, S) policy is proposed for an economic production quantity (EPQ) model with stochastic supplier availability and constant demands in [21]. A (q, r) inventory policy is applied in [25] to a continuous review system in which the supplier availability is modeled by a continuous-time Markov chain (CTMC). Under a (q, r) policy, when the system inventory level reduces to r, an order of quantity q is placed. An (s, S) policy is applied in [4] to a similar system except that the supplier availability is modeled by a semi-Markov process, the unsatisfied demand is a mix of backorders and lost sales, and leadtimes are zero. In [27], the supplier's availability is modeled as a CTMC whose state is only revealed by the placement of an order at a fixed cost that arrives instantaneously or not at all. A (q, r, T) inventory policy is examined, where if the order of quantity q that was placed when the on-hand inventory reduces to r does not arrive, the decision maker waits for T time units before placing another order.

1.2.3 Inventory Control Models with Transportation Disruptions

There has been little work in the inventory literature on disruptions in the transportation of product from supplier to customer. In a departure from the models discussed above, we now focus on models that permit multiple outstanding orders at any given time and stochastic leadtimes. The earliest work to prove the optimality of an (s, S) inventory policy for a finite-horizon, periodic-review inventory system with multiple outstanding orders, stochastic leadtimes and a total expected discounted cost evaluation criterion is [19]. Order crossover is prohibited, meaning that orders must be received in the order in which they are placed. Under this key assumption, the inventory position (the sum of on-hand inventory and all outstanding order quantities) is a sufficient statistic rather than the amount of on-hand inventory, the number of backorders, and the amounts of all outstanding orders. The results of [19] are extended in [13] to the infinite horizon.

These models (as well as [24]) are generalized in [36] by allowing the leadtime distribution to depend on an exogenous system that is modeled by a DTMC. While they do not present it as such, this dependence allows for the explicit modeling of disruptions to transportation leadtimes. We utilize this feature to model border closures and the resulting border congestion in the model specializations in Chapters 3 and 4. When no fixed ordering cost is present, the optimality of a stationary, state-dependent basestock policy is proved in [36] for both the total expected discounted cost and long-run average cost models, where the basestock levels (also known as order-up-to levels) depend on the state of the exogenous system at the time of order placement. No expression or procedure is provided to determine the long-run average cost for an arbitrary stationary, state-dependent basestock policy, however, a specialized, yet complicated, algorithm developed in [9] for Markov-modulated demand models can be used to determine the optimal order-up-to levels as well as the optimal long-run average cost. The simple state-dependent leadtime model developed in [10] extends [36] to investigate the value of observability of the exogenous system.

1.3 Outline of Thesis

Chapter 2 extends the inventory control model in [36] and presents new results regarding the optimal inventory policy and the long-run-average cost. Chapter 3 specializes the inventory control model in [36] to an application of inventory control subject to border closures with negligible congestion and presents the results of a comprehensive numerical study. Chapter 4 describes an important extension to the model in Chapter 3 that incorporates both border closures and congestion and presents the results of another comprehensive numerical study.

Finally, Chapter 5 provides conclusions and a discussion of topics for future research.

We now list the key contributions of this thesis. The first five contributions correspond to a general periodic-review, infinite-horizon inventory control model with a stationary stochastic demand process, stochastic order leadtimes with Markov-modulated probability distributions, linear purchase costs, and bilinear holding and penalty costs. The last three contributions correspond to specific specializations of the general model.

- An expression for the long-run average cost of an arbitrary stationary, state-dependent basestock policy developed using Markov reward theory.

- A solution procedure to calculate an optimal state-invariant basestock policy and the associated long-run average cost.

- A sufficiency condition for the optimality of a state-invariant basestock policy.

- Monotonicity results for the optimal state-invariant order-up-to level with respect to a ratio of the holding and penalty costs, the individual holding and penalty costs, and stochastically larger demand.

- The optimal order-up-to level for supply states that exhibit a special property.

- An important and timely application of inventory control subject to border closures without congestion, a comprehensive numerical study, and managerial and policy insights.

- A proof of optimality of a state-invariant basestock policy for the border closure model without congestion and a monotonicity result for the optimal state-invariant order-up-to level with respect to the minimum leadtime.

- An extension to the previous application that incorporates both border closures and, importantly, the resulting congestion, as well as a comprehensive numerical study and managerial and policy insights.

CHAPTER II

BASESTOCK POLICIES IN AVERAGE COST INVENTORY MODELS WITH MULTIPLE SUPPLY STATES

2.1 Introduction

In most inventory control models in the literature, the system operator bases his/her ordering decision solely on the state of all outstanding orders, or when order crossover does not occur, on the state of the inventory position. In this chapter, we investigate an inventory system in which the probability distributions of order leadtimes are dependent on the state of an exogenous Markov process and the system operator has complete knowledge of both the state of all outstanding orders *and* the state of the exogenous system. We model a periodic-review, infinite-horizon inventory system, in which multiple orders may be outstanding at any given time and order crossover cannot occur. Ordering costs are linear in the amount ordered and stochastic demand that cannot be satisfied from on-hand inventory is fully backlogged. The objective of the system operator is to determine an inventory control policy that minimizes long-run average cost per period.

Since they are known to be optimal for this system (see [36]), we restrict our attention to stationary, state-dependent basestock policies, where the basestock (or order-up-to) levels depend on the state of the exogenous Markov process at the time of order placement. In a departure from the traditional dynammic programming approaches used in [36], we use Markov reward theory to develop a new expression for the long-run average cost of an arbitrary stationary, state-dependent basestock policy (Theorem 1). We then restrict our attention to stationary, *state-invariant* basestock policies, in which the order-up-to levels are equivalent for all exogenous system states. We show how to explicitly calculate the optimal state-invariant basestock level and the long-run average cost (Theorem 2) and provide a sufficient condition for the optimality of a state-invariant basestock policy with respect to all state-dependent basestock policies (Corollary 1). We provide three structural theorems about the monotonicity of the optimal state-invariant order-up-to level with respect to a cost ratio of holding and penalty costs, with respect to the individual holding and penalty costs, and with respect to stochastically larger demand (Theorems 3-5). We finally show in Corollary 2 that for states in which it is known with probability one that two consecutive orders will arrive in the same future period, the optimal order quantity is zero.

2.2 Problem Statement and Preliminaries

We consider an infinite-horizon, periodic-review inventory system in which a manufacturer periodically orders a single product from a supplier with unlimited supply (i.e. the supplier is always available). Order leadtimes are non-negative and stochastic with probability distributions that are dependent on the state of an exogenous supply system at the time of order placement. At the beginning of each period, the inventory and supply system states are observed and an order, if any, is placed. The ordering cost is immediately incurred. Next, some subset of the outstanding orders arrive and demand is realized. Demand is stochastic and is satisfied from on-hand inventory if possible; otherwise, it is fully backlogged. Finally the on-hand inventory holding cost or the backorder penalty cost is assessed. The objective is to minimize the long-run average cost over the set of all basestock policies. Under the long-run average cost criterion, there is no discounting of future costs.

The manufacturer orders in discrete quantities (for example, containers) at a cost of c per unit. Holding costs are $h > 0$ per unit per period for any inventory held. Penalty costs are $p > 0$ per unit per period for any backlogged demand. Let $\hat{x}_t$ be the on-hand inventory at the end of period t, and define the holding/penalty cost assessed at the end of period t to be

$$\hat{C}(\hat{x}_t) = \begin{cases} -p\hat{x}_t & \text{if } \hat{x}_t < 0, \\ h\hat{x}_t & \text{if } \hat{x}_t \geq 0. \end{cases} \tag{1}$$

Let D_t be a non-negative, integer random variable representing the demand in period t, where the demands in different periods are identically and independently distributed with probability mass function g and cumulative distribution function G. Demand is bounded such that $D_t \in S_D = \{d_1, d_2, ..., d_M\}$ where $M < \infty$ and $0 \leq d_1 < d_2 < ... < d_M < \infty$. Let $D^{(l)}$ be the cumulative demand over l periods with probability mass function g_l and cumulative distribution function G_l.

The exogenous supply system is modeled by a discrete-time Markov chain $\mathbf{I} = \{i_t, t \geq 0\}$, where i_t represents the state of the supply system at time t. This system is exogenous meaning that its evolution is independent of all other events. The Markov chain has state space $S_I = \{1, 2, ..., N < \infty\}$ and is time homogenous. For all $t \geq 0$, let $p_{ij} = P(i_{t+1} = j | i_t = i)$ be the one-step transition probability from supply state i to j and let $P_I = [p_{ij}]$ be the resulting stochastic matrix. Also for $l \geq 0$, define $[P_I^l]_{ij} = p_{ij}^{(l)} = P(i_{t+l} = j | i_t = i)$ to be the l-step transition probability. Note that we do not make assumptions about the periodicity of $\mathbf{I}$. Since the chain has finite state space, let π^I be the unique stationary distribution. We use the subscript $+$ to denote the next period, e.g. i_+ instead of i_{t+1}.

To track outstanding orders through the supply system, each order is given a *position* attribute. A general framework describing the concept of order positions and the transition dynamics from position to position (e.g. order movement functions) is given in [36]. For example, the order positions can represent geographical locations or the number of periods that the order has been outstanding. Let $\mathbf{z_t} = \{z_{kt}, 1 \leq k \leq K < \infty\}$ be the vector of outstanding orders where z_{kt} represents the cumulative order quantity in position k at time t. In addition to positions $1 \leq k \leq K$, we append position 0 to denote the current order and a dummy position γ to denote all orders that have arrived. We assume that for all t, z_{0t} is a non-negative integer. The decision variables compose the set $\{z_{0t}, t \geq 0\}$.

Let $M(k|i)$ be the order movement function. Given $i_t = i$, the order currently in position k moves to position $M(k|i)$ in period $t+1$ with probability one. If $M(k|i) = \gamma$, then the order in position k has arrived. Thus given $i_t = i$, the updated cumulative order quantity in position k at time $t+1$ is

$$z_{k,t+1} = \sum_{\{n : M(n|i) = k\}} z_{nt}. \tag{2}$$

Since z_{0t} is a non-negative integer, z_{kt} is a non-negative integer for all k and t. Given $i_t = i$, let $M^l(k|i)$ be the random variable representing the position to which the order in position

k will move at time $t+l$.

The on-hand inventory at time $t+1$ given $i_t = i$ is then the random variable

$$\hat{x}_{t+1} = \hat{x}_t + \sum_{\{k:M(k|i)=\gamma\}} z_{kt} - D_t. \tag{3}$$

The *inventory position* is defined to be the sum of all outstanding orders (prior to ordering) plus the on-hand inventory. At time t, the inventory position is

$$x_t = \sum_{1 \leq k \leq K} z_{kt} + \hat{x}_t. \tag{4}$$

We assume that leadtimes are stochastic and non-negative with a minimum value of $L \geq 0$. The random variable for the leadtime of the order placed at time t given $i_t = i$ is

$$L(i) = \min_{l \geq L} \left\{ M^{l+1}(0|i) = \gamma \right\}. \tag{5}$$

As is standard practice, we make the key assumption that order crossover is prohibited. Formally this requires that $P(L(i_+) \geq L(i) - 1) = 1$, which is ensured by appropriately constructing the order movement functions (e.g. $k \leq k' \Rightarrow M(k|i) \leq M(k'|i)$). We also assume that $L(i)$ is finite with probability one and that there are no states $i \in S_I$ such that $L(i_+) = L(i) - 1$ with probability one (we relax the second assumption for Lemma 6 and Corollary 2).

Let $S' = \{(i, \hat{x}, \mathbf{z}) \in S_I \times \mathbf{Z} \times (\mathbf{Z}^+)^K\}$ be the complete state space for each time period $t \geq 0$, where $\mathbf{Z}$ and $\mathbf{Z}^+$ are respectively the set of integers and the set of non-negative integers. A decision rule at time t is a function $\delta_t : S' \to \mathbf{Z}^+$ that maps each possible state $s \in S'$ at time t to a non-negative, integer-valued order quantity z_{0t}. Define a policy to be $\Delta = \{\delta_t, t \geq 0\}$. We will suppress subscripts and superscripts when appropriate, for example writing z_0 for z_{0t}.

In this research, we focus on inventory policies that minimize long-run average cost per period. Modern information technology systems increase the ease of monitoring and managing inventory throughout a supply chain, thus allowing inventory reviews and ordering decisions to be made often, for example, every day. The short inter-review times and ordering cycles make long-run average cost criterion Markov decision problems (MDPs) appropriate models. In many optimization models with economic criteria, future costs are often discounted to model the time value of money. For example, \$1 today is worth only \$$\lambda$ tomorrow, where λ is the discount rate such that $0 < \lambda < 1$. To capture the impacts of the time value of money, future costs are discounted in discounted MDPs and policies are evaluated by the expected total discounted cost criterion. In many inventory systems the per period discount rate can be very close to one, in which case long-run average cost MDPs are often utilized instead of discounted MDPs. For example, if a firm achieves an annual interest rate of 11.1% and the inventory reviews and order placements occur daily, then the daily discount rate is 99.97%. As the discount rate approaches one, limiting arguments can be made to relate the expected total discounted cost and the long-run average cost models (see Corollaries 8.2.4 and 8.2.5 in [29]). In these cases, the long-run average cost model can essentially be used to approximate the expected total discounted cost model.

Since the average cost model does not discount future costs, future costs associated with the current order can be assessed to the period in which the order is placed. Recall that $z_0 = \delta(i, \hat{x}, \mathbf{z})$. Following [36], the cost assessed to period t under policy Δ is then

$$r_\Delta(i, \hat{x}, \mathbf{z}) = c\delta(i, \hat{x}, \mathbf{z}) + C(i, x + \delta(i, \hat{x}, \mathbf{z})), \tag{6}$$

where

$$C(i, x + \delta(i, \hat{x}, \mathbf{z})) = \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+)) E\left[\hat{C}\left(x + \delta(i, \hat{x}, \mathbf{z}) - D^{(l+1)}\right)\right]. \tag{7}$$

Note that $C(i, x + \delta(i, \hat{x}, \mathbf{z}))$ is the expected cumulative holding and penalty costs incurred from the time the current order arrives until just before the order placed in the next period arrives.

In [36], under linear ordering costs the average cost optimality equation is stated to be

$$g + h(i, x) \quad = \quad \min_{y \geq x} \left\{ c(y - x) + C(i, y) + E[h(i_+, y - D)] \right\}, \tag{8}$$

where i is the state of the exogenous system, x is the inventory position, and the order quantity is $z_0 = y - x$. As standard practice for average cost Markov decision problems, g and h are respectively known as the gain and bias. The gain represents the average expected cost per period of the system in steady state. In the next section we will consider the gain more formally using Markov reward processes. Although we are not concerned with the bias in this thesis, the bias can be interpreted as the expected total difference between incurred costs and the gain. The existence of an optimal policy that is a stationary, state-dependent basestock policy is also proved for the average cost model (and for the total discounted expected cost, although we are not interested in this model) in [36]. Therefore an optimal policy $\mathbf{y}^* = \{\delta^*, t \geq 1\}$ has decision rules that only require i_t, x_t, and a set of parameters $\{y^*(i), i \in S_I\}$ (the basestock or order-up-to levels). The optimal ordering decision rule at time t is

$$\delta^*(i_t, x_t) = z_{0t} = \begin{cases} y^*(i_t) - x_t & \text{if } x_t < y^*(i_t), \\ 0 & \text{if } x_t \geq y^*(i_t). \end{cases} \tag{9}$$

Since the state of the exogenous system and the inventory position together represent a sufficient statistic, let $S^{\mathbf{y}} = S_I \times S_X^{\mathbf{y}}$ be the sufficient state space, where $S_X^{\mathbf{y}}$ is the state space of the inventory position. We use a superscript to show the dependence of the state spaces on the specific stationary, state-dependent basestock policy. If $\mathbf{y}^*$ is an optimal stationary, state-dependent basestock policy, then $g^{\mathbf{y}^*} \leq g^{\mathbf{y}}$ for all stationary, state-dependent basestock policies $\mathbf{y}$.

2.3 The Average Cost for a Class of Basestock Policies

In a departure from the traditional dynamic programming approach in [36], we now develop Theorem 1, which provides an expression for the long-run average cost of an arbitrary stationary, state-dependent basestock policy derived using Markov reward theory. We first present preliminary results useful to prove Theorem 1.

Let $\mathbf{W}$=$\{W_t : t \geq 0\}$ be a Markov chain with countable or finite state space S and transition probability matrix P. Let $r : S \to \Re$ be the cost function such that a cost of $r(s)$ is incurred at time t when $W_t = s$. The bivariate stochastic process $\{(W_t, r(W_t)) : t \geq 0\}$ is known as a Markov reward process (MRP). It is well known that for Markov decision processes, every stationary policy Δ produces a MRP (denoted $\mathbf{W}_\Delta$) with transition probability matrix P_Δ and cost function r_Δ (see [29]). This concept is central to our analysis.

We will again suppress subscripts and superscripts when appropriate, for example writing P for P_Δ.

For each initial state $W_0 = s \in S$, the total expected cost incurred from period 0 through period $T-1$ under policy Δ is

$$v_T^\Delta(s) = E_s\left\{\sum_{t=0}^{T-1} r_\Delta(W_t)\right\}, \tag{10}$$

where the expectation is conditioned on the initial state $W_0 = s$. The average expected cost of the system in steady state, more formally known as the *gain*, of policy Δ is

$$g^\Delta(s) = \lim_{T\to\infty}\frac{1}{T}v_T^\Delta(s) = \lim_{T\to\infty}\frac{1}{T}E_s\left\{\sum_{t=1}^{T-1} r_\Delta(W_t)\right\}. \tag{11}$$

Since there exists an optimal stationary, state-dependent basestock policy, we confine our interest to $\Delta = \mathbf{y}$. The resulting MRP, $\mathbf{W_y}$, has finite state space $S^\mathbf{y} = S_I \times S_X^\mathbf{y}$. Since demand is bounded and due to the nature of the policy, $S_X^\mathbf{y}$ is a finite set with smallest element $B_1 = \min_{i\in S_I}\{y(i)\} - d_M$ and largest element $B_2 = \max_{i\in S_I}\{y(i)\} - d_1$. The probability transition matrix is $P_\mathbf{y}$, and due to the independence of the demand process and exogenous system the one-step transition probability can be written as:

$$[P]_{(i,x)(j,x')} = p_{ij}P(D = max\{x, y(i)\} - x'). \tag{12}$$

The cost assessed to period t is

$$r_\mathbf{y}(i,x) = c(y(i)-x)^+ + C(i, x + (y(i)-x)^+), \tag{13}$$

where $(z)^+ = \max\{z, 0\}$.

Then for each state $s \in S$, the gain of policy $\mathbf{y}$ is

$$g^\mathbf{y}(s) = \lim_{T\to\infty}\frac{1}{T}v_T^\mathbf{y}(s) = \lim_{T\to\infty}\frac{1}{T}E_s\left\{\sum_{t=0}^{T-1} r_\mathbf{y}(W_t)\right\} = \lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1} P_\mathbf{y}^t r_\mathbf{y}(s) = [P_\mathbf{y}^* r_\mathbf{y}](s). \tag{14}$$

The limit exists since S is a finite set and P^* is defined to be the limiting matrix of $\mathbf{W}$, where the limiting matrix is defined by the Cesaro limit (see [29]) to be

$$P^* = \lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1} P^t. \tag{15}$$

Regardless of the periodicity characteristics of $\mathbf{W}$, the Cesaro limit exists for both countable- and finite-state Markov chains (and is equivalent to the regular limit if the chain is aperiodic). Furthermore if the Markov chain is irreducible and positive recurrent (which is true under our assumptions), then a unique stationary distribution π solves the system of equations $\pi = \pi P$ subject to $\sum_{s \in S} \pi_s = 1$ and $\pi_s \geq 0$ for all $s \in S$. A property of the limiting matrix is that $P^* P = P^*$. Therefore since the stationary distribution is unique, $P^* = \pi^T e^T$ where e is a column vector of ones. That is, the rows of P^* are identical and are each equivalent to the stationary distribution π. Finally, since $\mathbf{W}$ has finite state space and is irreducible, the gain is constant for all states $s \in S$ and is given by

$$g^{\mathbf{y}} = [P^*_{\mathbf{y}} r_{\mathbf{y}}] = \pi_{\mathbf{y}} r_{\mathbf{y}} = \sum_{(i,x) \in S_y} \pi^y_{(i,x)} \left[c(y(i) - x)^+ + C(i, x + (y(i) - x)^+) \right]. \qquad (16)$$

We now present a key lemma that will be used in the proof of Theorem 1.

LEMMA 1. $\sum_{(i,x) \in S} \pi_{(i,x)} \, (y(i) - x)^+ = E[D]$.

Proof. We note that for all $t \geq 0$, $x_{t+1} = x_t + (y(i_t) - x_t)^+ - D_t$. It follows that

$$\lim_{t \to \infty} E\left[(y(i_t) - x_t)^+ \right] = \lim_{t \to \infty} E[x_{t+1}] - \lim_{t \to \infty} E[x_t] + \lim_{t \to \infty} E[D_t], \qquad (17)$$

where E is the expectation operator conditioned on (i_0, x_0) and the limit is the Cesaro limit. For any bounded or non-negative function f,

$$\lim_{t \to \infty} E[f(i_t, x_t)] = \sum_{(i,x) \in S} f(i,x) \pi_{(i,x)}.$$

Therefore

$$\lim_{t \to \infty} E\left[(y(i_t) - x_t)^+ \right] = \sum_{(i,x) \in S} (y(i) - x)^+ \, \pi_{(i,x)},$$

$$\lim_{t \to \infty} E[x_t] = \sum_{(i,x) \in S} x \pi_{(i,x)},$$

$$\lim_{t \to \infty} E[D_t] = E[D],$$

where the last equality follows from the fact that the $\{D_t\}$ are independent and identically distributed. We note that

$$\begin{aligned}\lim_{t\to\infty} E[x_{t+1}] &= \lim_{t\to\infty} E\left[E[x_{t+1}|i_t, x_t]\right] \\ &= \sum_{(i,x)\in S} E[x'|i,x]\pi_{(i,x)},\end{aligned}$$

where $E[x'|i,x] = \sum_{(i',x')\in S} x'[P]_{(i,x)(i',x')}$. Thus,

$$\begin{aligned}\lim_{t\to\infty} E[x_{t+1}] &= \sum_{(i,x)\in S}\sum_{(i',x')\in S} x'[P]_{(i,x)(i',x')}\pi_{(i,x)} \\ &= \sum_{(i',x')\in S} x' \sum_{(i,x)\in S} \pi_{(i,x)}[P]_{(i,x)(i',x')} \\ &= \sum_{(i',x')\in S} x'\pi_{(i',x')},\end{aligned}$$

where the next to last equality follows from the fact that $\pi = \pi P$. Collecting terms into equation (17) produces the result. $\square$

We now present Theorem 1, which provides an expression used to calculate the long-run average cost of an arbitrary stationary, state-dependent basestock policy. This expression allows for direct cost comparisons between different stationary, state-dependent basestock policies. We use superscripts to emphasize the dependence of the state-space, $S^{\mathbf{y}}$, and the stationary distribution, $\pi^{\mathbf{y}}$, on the specific policy $\mathbf{y}$.

THEOREM 1. *Let* $\boldsymbol{y}$ *be any stationary, state-dependent basestock policy whose resulting MRP has state-space* S^y *and stationary distribution* π^y*. Then*

$$g^y = cE[D] + \sum_{(i,x)\in S^y} \pi^y_{(i,x)} C(i, x + (y(i) - x)^+).$$

Proof. From equation (16) and Lemma 1, we have

$$\begin{aligned}g^y &= \sum_{(i,x)\in S^y} \pi^y_{(i,x)} \left[c(y(i)-x)^+ + C(i, x + (y(i) - x)^+)\right] \\ &= cE[D] + \sum_{(i,x)\in S^y} \pi^y_{(i,x)} C(i, x + (y(i) - x)^+).\end{aligned}$$

$\square$

The long-run average cost is therefore composed of two parts, an expected purchase cost (which is independent of the policy $\mathbf{y}$) and an expected holding and penalty cost (which is dependent on the policy $\mathbf{y}$). For ease of notation, let the expected holding and penalty cost under policy $\mathbf{y}$ be denoted by $E[HPC_y]$, where

$$\begin{aligned} E[HPC_y] &= \sum_{(i,x)\in S^y} \pi^y_{(i,x)} C(i, x + (y(i) - x)^+) \\ &= g^y - cE[D]. \end{aligned}$$

2.4 Calculating the Optimal State-Invariant Basestock Levels and Average Cost

We restrict our attention to state-invariant basestock policies in this section. A state-invariant policy has order-up-to levels that are equivalent for all supply states. A firm may desire to implement such an ordering policy for its simplicity, despite the fact that it may be sub-optimal. Furthermore as we will show for the border closure model without congestion in Chapter 3, a state-invariant policy may in fact be optimal.

Theorem 2 shows how to calculate the optimal state-invariant order-up-to level and long-run average cost. Theorems 3 and 4 show that the optimal order-up-to level is monotonic in a cost ratio of the holding and penalty costs and in the individual holding and penalty costs. Corollary 1 provides a sufficient condition for the optimality of a state-invariant basestock policy, and finally Corollary 2 gives the optimal order-up-to level for supply states that exhibit a special leadtime property. Lemmas 2 and 3 given below are useful in the proof of Theorem 2.

LEMMA 2. *For all $i \in S_I$, $C(i, y)$ is convex in y and* $\lim_{|y|\to+\infty} C(i, y) = +\infty$.

Proof. The convexity of $C(i, y)$ follows from the convexity of $\hat{C}(x)$ and the definition of

$C(i, y)$. Since $\hat{C}(x) = \max\{-px, hx\}$,

$$
\begin{aligned}
C(i, y) &= \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+)) E\left[\hat{C}(y - D^{(l+1)})\right] \\
&= \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+)) E\left[\max\left\{-p(y - D^{(l+1)}), h(y - D^{(l+1)})\right\}\right] \\
&\geq \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+))) \max\left\{E[-p(y - D^{(l+1)})], E[h(y - D^{(l+1)})]\right\} \\
&= \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+)) \max\left\{-py + pE[D^{(l+1)}], hy - hE[D^{(l+1)}]\right\},
\end{aligned}
$$

which completes the proof. The third step is valid by Jensen's Inequality.

$\square$

LEMMA 3. *For all $i \in S_I$ and y,*

$$\Delta C(i, y) \equiv C(i, y+1) - C(i, y) = (p + h) \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+)) G_{l+1}(y) - p\delta_i,$$

where $\delta_i \equiv \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+)) > 0$.

Proof. Suppose $d > y$. From equation (1),

$$
\begin{aligned}
\Delta\hat{C}(y - d) &= \Delta\hat{C}(y + 1 - d) - \Delta\hat{C}(y - d) \\
&= -p(y + 1 - d) - (-p(y - d)) \\
&= -p - p(y - d - y + d) \\
&= -p.
\end{aligned}
$$

Now suppose $d \leq y$. Again from equation (1),

$$
\begin{aligned}
\Delta\hat{C}(y - d) &= \Delta\hat{C}(y + 1 - d) - \Delta\hat{C}(y - d) \\
&= h(y + 1 - d) - h(y - d)) \\
&= h + h(y - d - y + d) \\
&= h.
\end{aligned}
$$

Then,

$$\begin{aligned}
\Delta C(i,y) &= \sum_{l\geq 0} P(L(i)\leq l\leq L(i_+))E\left[\Delta\hat{C}\left(y-D^{(l+1)}\right)\right]\\
&= \sum_{l\geq 0} P(L(i)\leq l\leq L(i_+))\sum_{d=d_1}^{(l+1)d_M} g_{l+1}(d)\Delta\hat{C}\left(y-D^{(l+1)}\right)\\
&= \sum_{l\geq 0} P(L(i)\leq l\leq L(i_+))\left(\sum_{d=d_1}^{y} g_{l+1}(d)h+\sum_{d=y+1}^{(l+1)d_M} g_{l+1}(d)(-p)\right)\\
&= \sum_{l\geq 0} P(L(i)\leq l\leq L(i_+))\left(h\sum_{d=d_1}^{y} g_{l+1}(d)-p\left(1-\sum_{d=d_1}^{y} g_{l+1}(d)\right)\right)\\
&= \sum_{l\geq 0} P(L(i)\leq l\leq L(i_+))\left((p+h)\left(\sum_{d=d_1}^{y} g_{l+1}(d)\right)-p\right)\\
&= \sum_{l\geq 0} P(L(i)\leq l\leq L(i_+))\left((p+h)G_{l+1}(y)-p\right)\\
&= (p+h)\sum_{l\geq 0} P(L(i)\leq l\leq L(i_+))G_{l+1}(y)-p\delta_i,
\end{aligned}$$

where the expectation operator in the first equation is with respect to $D^{(l+1)}$. By assumption, there are no states $i \in S_I$ such that $L(i_+) = L(i) - 1$ with probability one. Therefore $P(L(i) \leq l \leq L(i_+)) \geq 0$ for all $i \in S_I$ and $l \geq 0$, and for each state $i \in S_I$, $P(L(i) \leq l \leq L(i_+)) > 0$ for at least one value of $l \geq 0$. Then by definition, $\delta_i > 0$. $\square$

Assume that we now restrict the set of feasible stationary, state-dependent basestock policies to those policies $\hat{\mathbf{y}}$ where $y(\hat{0}) = y(\hat{1}) = ... = y(\hat{N}) \equiv \hat{y}$. We refer to $\hat{\mathbf{y}}$ as a stationary state-invariant basestock policy and $\hat{y}$ as the state-invariant basestock or order-up-to level. Let $\hat{y}^*$ denote the smallest among all optimal state-invariant basestock levels.

Due to the independence of the exogenous Markov chain and the demand process, the one-step transition probability of $\mathbf{W}$ under a state-invariant basestock policy $\hat{\mathbf{y}}$ is

$$[P]_{(i,x)(j,x')} = p_{ij}P(D = \hat{y} - x'). \tag{18}$$

The following lemma presents two probabilistic relations that directly result from the independence of the exogenous Markov chain and the demand process. The lemma will be also useful in the proof of Theorem 2.

LEMMA 4.

(i) For $n \geq 1$ and for all $(i,x) \in S$ and $(j,x') \in S$, $[P^n]_{(i,x)(j,x')} = p_{ij}^{(n)} P(D = \hat{y} - x')$.

(ii) For all $(i,x) \in S$ and $(j,x') \in S$, $\pi_{(j,x')} = [P^*]_{(i,x)(j,x')} = \pi_j^I P(D = \hat{y} - x')$.

Proof. We prove part (i) by induction. For $n = 1$, select two arbitrary states $(i,x) \in S$ and $(j,x') \in S$. The result follows directly from equation (18). Assume the claim holds for arbitrary n. Select two arbitrary states $(i,x) \in S$ and $(j,x') \in S$. Then

$$\begin{aligned}
\left[P^{n+1}\right] &= \sum_{(k,x'') \in S} [P^n]_{(i,x)(k,x'')} [P]_{(k,x'')(j,x')} \\
&= \sum_{(k,x'') \in S} p_{ik}^{(n)} P(D = \hat{y}^* - x'') p_{kj} P(D = \hat{y} - x') \\
&= \sum_{(k \in S_I} p_{ik}^{(n)} p_{kj} \sum_{x'' \in S_X} P(D = \hat{y} - x'') P(D = \hat{y} - x') \\
&= p_{ij}^{(n+1)} P(D = \hat{y} - x') \sum_{x'' \in S_X} P(D = \hat{y} - x'') \\
&= p_{ij}^{(n+1)} P(D = \hat{y} - x')(1).
\end{aligned}$$

The second equation follows from the induction step for $n = 1$ and the induction assumption for arbitrary n. The fourth step is valid by the Chapman-Kolmogorov Equation and final step holds from the definition of the state space S_X and the law of total probability. This completes the proof of part (i).

For part (ii), again select two arbitrary states $(i,x) \in S$ and $(j,x') \in S$. Then

$$\begin{aligned}
\pi_{(j,x')} &= [P^*]_{(i,x)(j,x')} \\
&= \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \left[P^t\right]_{(i,x),(j,x')} \\
&= \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} p_{ij}^{(t)} P(D = \hat{y} - x') \\
&= P(D = \hat{y}^{-} x') \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} p_{ij}^{(t)} \\
&= P(D = \hat{y} - x') \pi_j^I.
\end{aligned}$$

The second equation follows from the definition of the limiting matrix for $\mathbf{W}$ and the third equation follows from part (i). The final step follows by taking the Cesaro limit of a finite or countable state-space Markov chain. Further, since $\mathbf{I}$ is irreducible with finite state-space, the Cesaro limit gives the stationary probabilities. This completes the proof of part (ii).

$\square$

The following theorem theorem provides a simple method for calculating the optimal state-invariant order-up-to level and long-run average cost. It is also clear from the theorem that the optimal that the optimal order-up-to level is dependent on the cost parameters through a single cost parameter which is the cost ratio $p/(p+h)$.

THEOREM 2.

(i) If

$$\tilde{y} = min\left\{ d_1 \leq y < \infty : \; y \in Z, \; \sum_{l \geq 0} G_{l+1}(y) \sum_{i \in S_I} \pi_i^I \frac{P(L(i) \leq l \leq L(i_+))}{\delta_i} \geq \frac{p}{p+h} \right\},$$

then $\tilde{y} = \hat{y}^*$.

(ii) $g^{\hat{\mathbf{y}}^*} = cE[D] + \sum_{i \in S_I} \pi_i^I C(i, \hat{y}^*)$.

Proof. By definition, $\hat{y}^* = \min\{\text{argmin}_{y:y \in Z}\{\sum_{i \in S_I} \pi_i^I C(i,y)\}\}$. By assumption, $\delta_i > 0$ for all $i \in S_I$. From Lemma 3, if $y < 0$, then $\Delta C(i,y) = -p\delta_i < 0$ for all $i \in S_I$ since $G_{l+1}(y) = 0$ for $y < d_1$ and $\delta_i > 0$. It follows from Lemma 2 that $\hat{y}^*$ is finite. We can rewrite the definition of $\hat{y}^*$ with the new bounds and two necessary conditions for optimality as $\hat{y}^* = \min\left\{ d_1 \leq y < \infty : y \in Z, \sum_{i \in S_I} \pi_i^I \Delta C(i,y) \geq 0 \right\}$. The result then follows from Lemma 3. Part (ii) follows from Theorem 1.

$$\begin{aligned}
g^{\hat{y}^*} &= cE[D] + \sum_{(i,x) \in S^{\hat{y}^*}} \pi_{(i,x)}^{\hat{y}^*} C(i, x + (\hat{y}^* - x)^+) \\
&= cE[D] + \sum_{i \in S_I} \sum_{x \in S_X^{\hat{y}^*}} \pi_{(i,x)}^{\hat{y}^*} C(i, \hat{y}^*) \\
&= cE[D] + \sum_{i \in S_I} C(i, \hat{y}^*) \sum_{x \in S_X^{\hat{y}^*}} \pi_{(i,x)}^{\hat{y}^*} \\
&= cE[D] + \sum_{i \in S_I} \pi_i^I C(i, \hat{y}^*)(1).
\end{aligned}$$

The final equation follows from Lemma 4 and the law of total probability.

□

THEOREM 3. *The optimal state-invariant order-up-to level ($\hat{y}^*$) is non-decreasing in the cost ratio ($p/(p+h)$).*

Proof. We prove the theorem by contradiction. Suppose α_1 and α_2 are real numbers between 0 and 1 such that $\alpha_1 \leq \alpha_2$. Let

$$\tilde{y_1} = \min\left\{d_1 \leq y < \infty :\ y \in Z,\ \sum_{l \geq 0} G_{l+1}(y) \sum_{i \in S_I} \pi_i^I \frac{P(L(i) \leq l \leq L(i_+))}{\delta_i} \geq \alpha_1\right\}.$$

$$\tilde{y_2} = \min\left\{d_1 \leq y < \infty :\ y \in Z,\ \sum_{l \geq 0} G_{l+1}(y) \sum_{i \in S_I} \pi_i^I \frac{P(L(i) \leq l \leq L(i_+))}{\delta_i} \geq \alpha_2\right\}.$$

Assume that $\tilde{y_2} < \tilde{y_1}$. Then

$$\alpha_1 \leq \alpha_2 \leq \sum_{l \geq 0} G_{l+1}(\tilde{y_2}) \sum_{i \in S_I} \pi_i^I \frac{P(L(i) \leq l \leq L(i_+))}{\delta_i} \leq \sum_{l \geq 0} G_{l+1}(\tilde{y_1} - 1) \sum_{i \in S_I} \pi_i^I \frac{P(L(i) \leq l \leq L(i_+))}{\delta_i} < \alpha_1,$$

which implies that $\alpha_1 < \alpha_1$. Therefore by contradiction, $\tilde{y_2} \geq \tilde{y_1}$, which implies the result by Corollary 1(*ii*). The third inequality follows since $G_{l+1}(y)$ is a cumulative distribution function and is non-decreasing in y for all l.

□

THEOREM 4. *The optimal state-invariant order-up-to level ($\hat{y}^*$) is non-decreasing in the penalty cost (p) and non-increasing in holding cost (h).*

Proof. Taking the derivative of the cost ratio with respect to the penalty cost, we have

$$\frac{d\left(\frac{p}{p+h}\right)}{dp} = \frac{(p+h)(1) - p(1)}{(p+h)^2} = \frac{h}{(p+h)^2} > 0$$

for positive h and p (which we have assumed). Therefore the cost ratio is increasing in p. The result then holds by Theorem 3. Similarly, taking the derivative of the cost ratio with respect to the holding cost, we have

$$\frac{d\left(\frac{p}{p+h}\right)}{dh} = \frac{(p+h)(0) - p(1)}{(p+h)^2} = \frac{-p}{(p+h)^2} < 0$$

for positive h and p. Therefore the cost ratio is decreasing in h. The result then holds by Theorem 3. □

Consider two random variables X and Y. If for all z,

$$P(X \leq z) \leq P(Y \leq z),$$

then we say that X is stochastically larger than Y and write $X \geq_{ST} Y$. The following lemma is useful in the proof of Theorem 5.

LEMMA 5 (Example 9.2(A) in [31]). *Let $X_1, ..., X_n$ be independent and $Y_1, ..., Y_n$ be independent. If $X_i \geq_{ST} Y_i$, then for any increasing function f*

$$f(X_1, ..., X_n) \geq_{ST} f(Y_1, ..., Y_n).$$

Consider two demand processes $\{D_{1t}, t \geq 0\}$ and $\{D_{2t}, t \geq 0\}$ in which the random variables in each process are separately non-negative and identically and independently distributed. Let $\hat{y_1}^*$ and $\hat{y_2}^*$ be the respective optimal state-invariant order-up-to levels under demand processes 1 and 2. The following theorem shows that stochastically larger demand leads to larger optimal state-invariant order-up-to levels.

THEOREM 5. *Assume $P(D_{kt} = d) > 0$ for all $k \in \{1, 2\}$, $t \geq 0$, and $d \geq 0$. If $D_{1t} \geq_{ST} D_{2t}$ for all $t \geq 0$, then $\hat{y_1}^* \geq \hat{y_2}^*$.*

Proof. The proof is by contradiction. Let $D_1^{(l)} = \sum_{t=0}^{l-1} D_{1t}$ and $D_2^{(l)} = \sum_{t=0}^{l-1} D_{2t}$ with respective cumulative distribution functions G_l^1 and G_l^2. Since $P(D_{kt} = d) > 0$ for $k \in \{1, 2\}$, $t \geq 0$, and $d \geq 0$, the summations that define $D_1^{(l)}$ and $D_2^{(l)}$ are increasing functions. By Lemma 5, $D_1^{(l)} \geq_{ST} D_2^{(l)}$ for all l and by the definition of stochastically larger, $G_l^1(y) \leq G_l^2(y)$ for all l and y. Let

$$\tilde{y_1} = \min\left\{ d_1 \leq y < \infty : \; y \in Z, \; \sum_{l \geq 0} G_{l+1}^1(y) \sum_{i \in S_I} \pi_i^I \frac{P(L(i) \leq l \leq L(i_+))}{\delta_i} \geq \frac{p}{p+h} \right\}$$

and

$$\tilde{y_2} = \min\left\{ d_1 \leq y < \infty : \; y \in Z, \; \sum_{l \geq 0} G_{l+1}^2(y) \sum_{i \in S_I} \pi_i^I \frac{P(L(i) \leq l \leq L(i_+))}{\delta_i} \geq \frac{p}{p+h} \right\}.$$

Assume $\tilde{y_1} < \tilde{y_2}$. Then

$$\begin{aligned}
\frac{p}{p+h} &\leq \sum_{l\geq 0} G^1_{l+1}(\tilde{y_1}) \sum_{i\in S_I} \pi^I_i \frac{P(L(i) \leq l \leq L(i_+))}{\delta_i} \\
&\leq \sum_{l\geq 0} G^1_{l+1}(\tilde{y_2}-1) \sum_{i\in S_I} \pi^I_i \frac{P(L(i) \leq l \leq L(i_+))}{\delta_i} \\
&\leq \sum_{l\geq 0} G^2_{l+1}(\tilde{y_2}-1) \sum_{i\in S_I} \pi^I_i \frac{P(L(i) \leq l \leq L(i_+))}{\delta_i} \\
&< \frac{p}{p+h},
\end{aligned}$$

which implies that $\frac{p}{p+h} < \frac{p}{p+h}$. Therefore by contradiction $\tilde{y_1} \geq \tilde{y_2}$ and by Corollary 1(ii), $\hat{y_1}^* \geq \hat{y_2}^*$. The second inequality follows since $G^k_{l+1}(y)$ is a cumulative distribution function and is non-decreasing in y for all k and l. □

In [36], the *myopic* cost function is defined as $H(i,y) = cE[D]+C(i,y)$. Let $y^+(i)$ denote the smallest among all minimizers of $H(i,y)$, known as myopic order-up-to level for state $i \in S_I$. Also considering all stationary, state-dependent basestock policies, let $y^*(i)$ denote the smallest among all unrestricted optimal order-up-to levels for exogenous system state i. The following corollary provides a method for calculating the myopic order-up-to levels and provides a sufficient condition for the optimality of a state-invariant basestock policy. We note from the corollary that the myopic order up-to levels exhibit similar properties to those in Theorems 3, 4 and 5.

COROLLARY 1.

(i) Let $\tilde{i} = min\{argmin_i\{y^+(i)\}\}$. *Then for all* $i \in S_I$, $y^+(\tilde{i}) \leq y^*(i) \leq y^+(i)$.

(ii) For each $i \in S_I$, *if*

$$\tilde{y} = min\left\{ d_1 \leq y < \infty : \; y \in Z, \; \sum_{l\geq 0} G_{l+1}(y) \frac{P(L(i) \leq l \leq L(i_+))}{\delta_i} \geq \frac{p}{p+h} \right\},$$

then $\tilde{y} = y^+(i)$.

(iii) *If* $y^+(0) = y^+(1) = ... = y^+(N) \equiv y^+$, *then* $y^+ = y^*(0) = y^*(1) = ... = y^*(N) = \hat{y}^*$ *and*

$$g^{\hat{y}^*} = cE[D] + \sum_{i \in S_I} \pi_i^I C(i, \hat{y}^*) = g^*,$$

where g^* *is the minimal gain over* all *stationary, state-dependent basestock policies.*

Proof. Part (i) restates Theorem 3(a) and 3(b) in [36]. The proof of part (ii) follows a similar proof as that of Theorem 2(i) in this chapter. In part (iii), the optimality of y^+ follows directly from part (i). The left equality in the expression for the gain in part (iii) holds by Theorem 2(ii) and the right equality follows from Theorem 1 under the unrestricted optimal policy $\hat{\mathbf{y}}^*$. □

In this chapter we have assumed that $P(L(i) \leq l \leq L(i_+)) \geq 0$ for all $i \in S_I$ and $l \geq 0$ and strictly positive for some value $l \geq 0$. Therefore by definition $\delta_i > 0$ for all $i \in S_I$. Suppose there exists a subset of states in S_I where this assumption does not hold. Note that for states $i \in S_I$ such that $L(i_+) = L(i) - 1$ with probability one, we cannot use the solution methodologies provided in Theorem 2 and Corollary 1 which require the assumption that $\delta_i > 0$. However for these states, Corollary 2 provides the optimal order-up-to level. The following lemma is useful in the proof of Corollary 2.

LEMMA 6. *Consider a state* $i \in S_I$ *such that* $L(i_+) = L(i) - 1$ *with probability one. Then*

(i) $P(L(i) \leq l \leq L(i_+)) = 0$ *for all* $l \geq 0$,

(ii) $\delta_i = 0$, *and*

(iii) $C(i, y) = 0$ *for all* y.

Proof. Given $i_t = i$, if $L(i_+) = L(i) - 1$ with probability one, then the orders placed at times t and $t + 1$ will arrive in the same future period with probability one, e.g. in period

$t + L(i) = t + 1 + L(i_+)$. Then for all $l \geq 0$, we have

$$\begin{aligned} P(L(i) \leq l \leq L(i_+)) &= P(L(i) \leq l \leq L(i) - 1) \\ &= P(L(i) \leq l, l < L(i)) \\ &= 0. \end{aligned}$$

By definition, $\delta_i = 0$ and from equation (7), $C(i, y) = 0$ for all y.

□

COROLLARY 2. *Consider a state $i \in S_I$ such that $L(i_+) = L(i) - 1$ with probability one. Then $y^+(i) = y^*(i) = -\infty$.*

Proof. For each state $i \in S_I$, recall that the myopic minimizer, $y^+(i)$, is the smallest among all minimizers of $H(i, y) = cE[D] + C(i, y)$. By Lemma 6, $C(i, y) = 0$ for all y. Then the minimum of $H(i, y)$ does not exist and

$$\lim_{y \to -\infty} H(i, y) = -\infty.$$

We therefore say that $H(i, y)$ is minimized at $y = -\infty$ and set $y^+(i) = -\infty$. Since this is clearly the smallest of all minimizers of $H(i, y)$ with respect to all states $i \in S_I$, $y^*(i) = y^+(i)$ by Corollary 1(i). □

2.5 Conclusions

In this chapter, we extend the inventory control literature by deriving an expression for the long-run average cost under an arbitrary stationary, state-dependent basestock policy for a periodic-review, infinite-horizon inventory control system in which order leadtimes are dependent on an exogenous system and ordering costs are linear in the amount ordered. We show how to calculate an optimal stationary, state-invariant basestock policy and the associated long-run average cost and provide a sufficient condition for the optimality of a stationary, state-invariant basestock policy. We provide structural theorems about the monotonicity of the optimal state-invariant order-up-to level with respect to a cost ratio of the holding and penalty costs, with respect to the individual holding and penalty costs, and with respect to stochastically larger demand. We finally show that for states in which it is known with probability one that two consecutive orders will arrive in the same future period, the optimal order quantity is zero.

The key feature of the inventory control model investigated in this chapter is the dependence of the leadtime probability distribution on the state of an exogenous Markov process. This dependence introduces an opportunity to concurrently model disruptions to the transportation of outstanding orders and the decision maker's ability to plan for and respond to leadtime disruptions.

CHAPTER III

AN INVENTORY CONTROL MODEL WITH POSSIBLE BORDER CLOSURES

3.1 Motivation

Modern global freight transportation and supply chain systems form an increasingly complex network of products, resources, companies, and nations. These systems are highly vulnerable to disruptions due to their design and to the volume and value of goods moved. According to a 2004 World Trade Organization report, the value of export merchandise transported globally in 2003 was an astonishing $7.3 trillion [1]. Disruptions to these supply chains not only result in increased operational and recovery costs, but may also adversely affect shareholder value. The results of an empirical study in [18] show that the mean decrease in a firm's market value is 10.28% over the two-day period following the public announcement of a supply chain disruption.

The terrorist attacks in the United States on September 11, 2001 provide a specific example of the dramatic impacts of a disruption. Border delays at the US-Canadian border quickly increased from a few minutes to an extreme 12 hours [7], and as a result Ford Motor Company was forced to intermittently idle production at five of its assembly plants due to the delays at US land borders [30]. Toyota came within hours of halting production at one plant since parts shipped by air from Germany were delayed due to the grounding of all US air traffic [33]. In the event of another security disruption, border closures are a feasible response that would severely impact international supply chains. A 2003 report from Booz Allen Hamilton presented the results of a port security wargame in which a terrorist attack using "dirty bombs" in intermodal containers was simulated [15]. The actions taken by the participating business and government leaders had significant consequences: every port in the United States was shut down for eight days, requiring 92 days to reduce the resulting backlog of container deliveries, and the forecasted total loss to the US economy was $58 billion, including the costs of spoilage, lost sales/contracts, and manufacturing slowdowns/production halts.

3.2 Problem Statement

Consider a supply chain consisting of a foreign supplier and a domestic manufacturer. Orders are shipped on a fixed transportation route from the supplier to a domestic port of entry for importation (e.g. a seaport or land border); the transit time is $L > 0$ periods. Assume that the inland transportation time between the port of entry and the manufacturer is negligible (see section 3.5 for an extension that allows positive inland transportation times). Upon arrival at the port of entry, if the border is open then the order arrives to the manufacturer without delay. Otherwise, the border is closed and the order is held at the port of entry until the border reopens (see section 3.6 for an extension that limits the maximum time an order can wait to cross the border). When the border reopens, all orders arriving to, or currently waiting at, the border cross and arrive at the manufacturer without further delay. Multiple orders may be outstanding at any given time, but order crossover does not occur. In this chapter, we assume that congestion at the border resulting from periods of border closure is negligible and has no effect on order leadtimes. The issue of border congestion is addressed in Chapter 4.

The manufacturer utilizes a periodic-review inventory policy and experiences periodic, stochastic, non-negative, integer-valued demand. Demand that cannot be satisfied from the on-hand inventory is fully backordered. Ordering costs are linear in the amount ordered and holding and penalty costs are respectively assessed for any on-hand inventory held or backordered demand. The manufacturer has complete knowledge of its on-hand inventory, backorders, outstanding orders, and the status of the border at the beginning of each period. The objective is to determine an ordering policy that minimizes that long-run average cost of the system and the minimum long-run average cost itself.

In this chapter we specialize the inventory control model presented in Chapter 2 to represent this border closure system without. The model is also more generally applicable to disruptions at a single choke-point in serial systems in which the choke-point exhibits an open or closed behavior. For example, it could model machine reliability in serial production systems in which a machine may be operating or not operating. Proposition 1 develops the probability mass function and the cumulative distribution function for the resultant leadtime random variable, $L(i)$ for all $i \in S_I$, as well as two other important quantities. Theorem 6 proves the optimality of a state-invariant basestock policy for the border closure model without congestion and Theorem 7 shows that the optimal order-up-to level is non-decreasing in the minimum leadtime. We present the results of a comprehensive numerical study that were determined using the procedures described in Chapter 2.

The results show that the optimal inventory policy and long-run average cost are much more sensitive to the expected duration of a disruption than to the occurrence likelihood of the disruption. This has important implications for the cooperation between business and government to reduce the duration of border closures through effective disruption management and contingency planning. The numerical results regarding the impacts on the optimal state-invariant order-up-to level with respect to the minimum leadtime, cost parameters, and demand distribution illustrate the monotonicity results proven in Chapter 2 and in this chapter. We observe that contingency planning for border closures is clearly important and provides greater benefits when the leadtime between the supplier and the international border is small due to the way in which the manufacturer manages demand and supply uncertainty over this leadtime. Finally we present three modeling extensions that model a positive inland transportation time, a maximum delay at the border, and multiple open border states representing increasing probabilities of closure.

3.2.1 Characteristics of the Border System

We now describe the border system with the following discrete-time Markov chain (DTMC) model. Let $\mathbf{I}$=$\{i_t, t \geq 0\}$ be a DTMC with state space $S_I = \{O, C\}$, where $i_t = O$ indicates that the border is open in period t and $i_t = C$ indicates that it is closed (see section 3.7 for an extension that includes multiple open states and a single closed state). The transition probability matrix is

$$P_I = \begin{pmatrix} 1 - p_{OC} & p_{OC} \\ p_{CO} & 1 - p_{CO} \end{pmatrix},$$

where we assume that $0 < p_{OC} < 1$ and $0 < p_{CO} < 1$, since the extreme values result in uninteresting systems. For $l \geq 0$, it is well known that the l-step transition probability matrix for a two-state Markov chain with state space $S_I = \{O, C\}$, $0 < p_{OC} < 1$, and $0 < p_{CO} < 1$, is

$$P_I^l = (p_{OC} + p_{CO})^{-1} \left\{ \begin{pmatrix} p_{CO} & p_{OC} \\ p_{CO} & p_{OC} \end{pmatrix} + (1 - p_{OC} - p_{CO})^l \begin{pmatrix} p_{OC} & -p_{OC} \\ -p_{CO} & p_{CO} \end{pmatrix} \right\}. \quad (19)$$

The stationary distribution of this chain is

$$\pi^I = \{\pi_O^I, \pi_C^I\} = \left\{ \frac{p_{CO}}{p_{OC} + p_{CO}}, \frac{p_{OC}}{p_{OC} + p_{CO}} \right\}.$$

Let $\mathbf{z_t} = \{z_{kt}, 0 \leq k \leq L-1\}$ be the vector of outstanding orders where z_{kt} represents the order quantity that has been outstanding for k time periods at period t for $k = \{0, 1, 2, ...L-1\}$. Since orders may accumulate at the border when it is closed, z_{Lt} represents the sum of all orders that have been outstanding for *at least* L periods. The order movement function describing this system is

$$M(k|O) = \begin{cases} k+1 & \text{if } 0 \leq k < L, \\ \gamma & \text{if } k = L, \end{cases}$$

and

$$M(k|C) = \begin{cases} k+1 & \text{if } 0 \leq k < L, \\ L & \text{if } k = L. \end{cases}$$

This order movement function prevents crossover and there are no border states $i \in S_I$ such that $L(i_+) = L(i) - 1$ with probability one.

Let $\mathbf{W}$=$\{W_t \equiv (i_t, x_t) : t \geq 0\}$ be the Markov chain on state space $S = S_I \times S_X$ that arises under the stationary, state-dependent basestock policy $\mathbf{y}$. Let the per period cost function be given by equation (6) and let the transition probability matrix be denoted by P. If optimal policy is a state-invariant basestock policy (which we show to be true in Theorem 6), the one-step transition probability of $\mathbf{W}$ is given in equation (18).

3.2.2 The Leadtime Probability Distribution

The following proposition provides the probability mass function, the cumulative distribution function, and two other important quantities for the leadtime random variable, $L(i)$ for all $i \in S_I$.

PROPOSITION 1.

(i) For all $i \in S_I$, the probability mass function of $L(i)$ is

$$P(L(i) = l) = \begin{cases} 0 & \text{if } l < L, \\ p_{iO}^{(L)} & \text{if } l = L, \\ p_{iC}^{(L)} p_{CC}^{l-L-1} p_{CO} & \text{if } l > L. \end{cases} \tag{20}$$

(ii) For all $i \in S_I$, the cumulative distribution function of $L(i)$ is

$$P(L(i) \leq l) = \begin{cases} 0 & \text{if } l < L, \\ p_{iO}^{(L)} & \text{if } l = L, \\ 1 - p_{iC}^{(L)} p_{CC}^{l-L} & \text{if } l > L. \end{cases} \tag{21}$$

(iii) For all $i \in S_I$,

$$P(L(i) \leq l \leq L(i_+)) = \begin{cases} 0 & \text{if } l < L, \\ p_{iO}^{(L)} & \text{if } l = L, \\ p_{iO}^{(L)} p_{OC} p_{CC}^{l-L-1} & \text{if } l > L. \end{cases} \tag{22}$$

(iv) For all $i \in S_I$,

$$\delta_i = 1 + \frac{p_{iC}^{(L)} - p_{iC}^{(L+1)}}{p_{CO}} > 0.$$

Proof. Throughout this proof, note that we have assumed $0 < p_{OC} < 1$ and $0 < p_{CO} < 1$ and therefore $0 < p_{OO} < 1$ and $0 < p_{CC} < 1$. To develop the probability mass function of $L(i)$, we consider how orders arrive to and cross the border. An order placed at time t when $i_t = i$ will arrive at time $t + L(i)$. From the order movement function, $P(L(i) = m) = 0$ for $0 \leq m \leq L-1$. The leadtime is exactly L if and only if $i_{t+L} = O$ and so $P(L(i) = L) = p_{iO}^{(L)}$.

Similarly, the leadtime is exactly $L+1$ if and only if $i_{t+L} = C$ and $i_{t+L+1} = O$. Therefore $P(L(i) = L+1) = p_{iC}^{(L)} p_{CO}$. Note that $P(L(i) = L+1) \neq p_{iO}^{(L+1)}$ since i_{t+L} cannot be O. Similarly for $m \geq 2$, $P(L(i) = L+m) = p_{iC}^{(L)} p_{CC}^{m-1} p_{CO}$. This completes the proof of part (i).

Let us consider the cumulative distribution function of $L(i)$ for $l \geq 0$. Since $P(L(i) \leq l) = \sum_{k=0}^{l} P(L(i) = k)$, it is clear from equation (20) that $P(L(i) \leq l) = 0$ for $l < L$. Similarly, if $l = L$, then $P(L(i) \leq L) = p_{iO}^{(L)}$. Finally, when $l > L$,

$$
\begin{aligned}
P(L(i) \leq l) &= P(L(i) = L) + \sum_{k=L+1}^{l} P(L(i) = k) \\
&= p_{iO}^{(L)} + \sum_{k=L+1}^{l} p_{iC}^{(L)} p_{CC}^{l-L-1} p_{CO} \\
&= p_{iO}^{(L)} + p_{iC}^{(L)} p_{CO} \sum_{k=0}^{l-L-1} p_{CC}^{l} \\
&= p_{iO}^{(L)} + p_{iC}^{(L)} p_{CO} \left(\frac{1 - p_{CC}^{l-L}}{1 - p_{CC}} \right) \\
&= p_{iO}^{(L)} + p_{iC}^{(L)} \left(1 - p_{CC}^{l-L} \right) \\
&= \left(p_{iO}^{(L)} + p_{iC}^{(L)} \right) - p_{iC}^{(L)} p_{CC}^{l-L} \\
&= 1 - p_{iC}^{(L)} p_{CC}^{l-L}.
\end{aligned}
$$

This completes the proof of part (ii). For part (iii), note that given $i_t = i$,

$$
\begin{aligned}
P(L(i) \leq l \leq L(i_+)) &= P(L(i) \leq l) - P(L(i) \leq l, L(i_+) < l) \\
&= P(L(i) \leq l) - P(L(i) \leq l, L(i_+) < l) \\
&= P(L(i) \leq l) - P(L(i_+) < l) \\
&= P(L(i) \leq l) - \sum_{j \in S_I} p_{ij} P(L(j) \leq l-1), \qquad (23)
\end{aligned}
$$

where the third equality holds since $P(L(i) \leq l, L(i_+) < l) = P(L(i_+) < l) - P(L(i) > l, L(i_+) < l)$ and $P(L(i) > l, L(i_+) < l) = 0$ since $P(L(i_+) \geq L(i) - 1) = 1$. From equations (21) and (23), it is clear that $P(L(i) \leq l \leq L(i_+)) = 0$ for $l < L$. Similarly, if $l = L$, then

$P(L(i) \le l \le L(i_+)) = p_{iO}^{(L)}$. When $l = L+1$,

$$\begin{aligned}
P(L(i) \le l \le L(i_+)) &= P(L(i) \le L+1) - \sum_{j \in S_I} p_{ij} P(L(j) \le L) \\
&= \left(1 - p_{iC}^{(L)} p_{CC}\right) - \sum_{j \in S_I} p_{ij} p_{jO}^{(L)} \\
&= \left(1 - p_{iC}^{(L)} p_{CC}\right) - \sum_{j \in S_I} p_{ij} \left(1 - p_{jC}^{(L)}\right) \\
&= 1 - p_{iC}^{(L)} p_{CC} - \sum_{j \in S_I} p_{ij} + \sum_{i \in S_I} p_{ij} p_{jC}^{(L)} \\
&= -p_{iC}^{(L)} p_{CC} + p_{iC}^{(L+1)} \\
&= -p_{iC}^{(L)} p_{CC} + \left(p_{iC}^{(L)} p_{CC} + p_{iO}^{(L)} p_{OC}\right) \\
&= p_{iO}^{(L)} p_{OC},
\end{aligned}$$

where the third to last equation holds by the Chapman-Kolmogorov Equation. Similarly when $l > L+1$,

$$\begin{aligned}
P(L(i) \le l \le L(i_+)) &= P(L(i) \le l) - \sum_{j \in S_I} p_{ij} P(L(j) \le l-1) \\
&= \left(1 - p_{iC}^{(L)} p_{CC}^{l-L}\right) - \sum_{j \in S_I} p_{ij} \left(1 - p_{iC}^{(L)} p_{CC}^{l-1-L}\right) \\
&= 1 - p_{iC}^{(L)} p_{CC}^{l-L} - \sum_{j \in S_I} p_{ij} + \sum_{j \in S_I} p_{ij} p_{jC}^{(L)} p_{CC}^{l-1-L} \\
&= -p_{iC}^{(L)} p_{CC}^{l-L} + \sum_{j \in S_I} p_{ij} p_{jC}^{(L)} p_{CC}^{l-1-L} \\
&= p_{CC}^{l-1-L} \left(-p_{iC}^{(L)} p_{CC} + p_{jC}^{(L+1)}\right) \\
&= p_{CC}^{l-1-L} \left(-p_{iC}^{(L)} p_{CC} + \left(p_{iC}^{(L)} p_{CC} + p_{iO}^{(L)} p_{OC}\right)\right) \\
&= p_{iO}^{(L)} p_{OC} p_{CC}^{l-1-L}.
\end{aligned}$$

This completes the proof of part (iii). Finally, from Lemma 3,

$$
\begin{aligned}
\delta_i &= \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+)) \\
&= p_{iO}^{(L)} + \sum_{l \geq L+1} p_{iO}^{(L)} p_{OC} p_{CC}^{l-L-1} \\
&= p_{iO}^{(L)} + p_{iO}^{(L)} p_{OC} \sum_{l \geq 0} p_{CC}^{l} \\
&= p_{iO}^{(L)} + \frac{p_{iO}^{(L)} p_{OC}}{1 - p_{CC}} \\
&= \left(1 - p_{iC}^{(L)}\right) + \frac{p_{iC}^{(L+1)} - p_{iC}^{(L)} p_{CC}}{1 - p_{CC}} \\
&= \frac{(1 - p_{iC}^{(L)})(1 - p_{CC}) + p_{iC}^{(L+1)} - p_{iC}^{(L)} p_{CC}}{1 - p_{CC}} \\
&= \frac{1 - p_{iC}^{(L)} - p_{CC} + p_{iC}^{(L)} p_{CC} + p_{iC}^{(L+1)} - p_{iC}^{(L)} p_{CC}}{1 - p_{CC}} \\
&= \frac{(1 - p_{CC}) + \left(p_{iC}^{(L+1)} - p_{iC}^{(L)}\right)}{1 - p_{CC}} \\
&= 1 - \frac{p_{iC}^{(L+1)} - p_{iC}^{(L)}}{p_{CO}} \\
&= 1 + \frac{p_{iC}^{(L)} - p_{iC}^{(L+1)}}{p_{CO}}.
\end{aligned}
$$

By assumption there are no states $i \in S_I$ for which $L(i_+) = L(i) - 1$ with probability one. Then for each $i \in S_I$, $P(L(i) \leq l \leq L(i_+)) > 0$ for at least one value of $l \geq 0$ and therefore $\delta_i > 0$. This completes the proof of part (iv). $\square$

3.3 Properties of the Optimal Policy

We now present Theorem 6 which shows that the optimal stationary, state-dependent basestock policy for the border closure model without congestion is actually a *state-invariant* basestock policy. Theorem 7 shows that the optimal state-invariant order-up-to level is non-decreasing in the minimum leadtime.

THEOREM 6. *The optimal basestock policy for the border closure model without congestion is state-invariant, that is $y^*(O) = y^*(C) = \hat{y}^*$.*

Proof. From Proposition 1(iv), $\delta_i > 0$ for all $i \in S_I$, and from the fourth step in the derivation

$$\delta_i = p_{iO}^{(L)} \left(1 + \frac{p_{OC}}{p_{CO}}\right). \tag{24}$$

Within the minimization in Corollary 1(ii), the second condition is

$$\sum_{l \geq 0} G_{l+1}(y) \frac{P(L(i) \leq l \leq L(i_+))}{\delta_i} \geq \frac{p}{p+h}.$$

From equations (22) and (24), the left-hand side of this condition can be written

$$\sum_{l \geq 0} G_{l+1}(y) \frac{P(L(i) \leq l \leq L(i_+))}{\delta_i} \quad = \quad \frac{G_{L+1}(y)}{1 + \frac{p_{OC}}{p_{CO}}} + \sum_{l > L} \left(\frac{p_{OC} p_{CC}^{l-L-1}}{1 + \frac{p_{OC}}{p_{CO}}} \right) G_{l+1}(y),$$

which is independent of i. Thus the same $\tilde{y}$ will solve the minimization in Corollary 1(ii) for both border states. □

The following lemmas will be useful in the proof of Theorem 7.

LEMMA 7. *Let $\{D_t, t \geq 0\}$ be identically and independently distributed non-negative random variables for $t \geq 0$ and let $D^{(l)} = \sum_{t=0}^{l-1} D_t$ be a random variable with cumulative distribution function G_l. If l and l' are integers such that $l \leq l'$, then $G_l(y) \geq G_{l'}(y)$ for all $y \geq 0$.*

Proof. Consider an arbitrary integer l and let $l' = l + k$ for some non-negative integer k. If $k = 0$, then $l' = l$ and the claim holds trivially. Suppose $k > 0$. Then $D^{(l')} \stackrel{d}{=} D^{(l)} + \hat{D}^{(l'-l)}$, where $\hat{D}^{(l'-l)} = \sum_{t=l}^{l'-1} D_t$ and $\hat{D}^{(l'-l)} \stackrel{d}{=} D^{(k)}$ since the $\{D_t\}$ are independently and identically distributed. For all $y \geq 0$,

$$
\begin{aligned}
G_l(y) - G_{l'}(y) &= P\left(D^{(l)} \leq y\right) - P\left(D^{(l')} \leq y\right) \\
&= P\left(D^{(l)} \leq y\right) - P\left(D^{(l)} + \hat{D}^{(l'-l)} \leq y\right) \\
&= P\left(D^{(l)} \leq y\right) - P\left(D^{(l)} \leq y - \hat{D}^{(l'-l)}\right) \\
&= \sum_{d=0}^{\infty} P\left(D^{(l)} \leq y | \hat{D}^{(l'-l)} = d\right) P\left(\hat{D}^{(l'-l)} = d\right) \\
&\qquad - \sum_{d=0}^{\infty} P\left(D^{(l)} \leq y - d | \hat{D}^{(l'-l)} = d\right) P\left(\hat{D}^{(l'-l)} = d\right) \\
&= \sum_{d=0}^{\infty} \left[P\left(D^{(l)} \leq y\right) - P\left(D^{(l)} \leq y - d\right)\right] P\left(\hat{D}^{(l'-l)} = d\right) \\
&= \sum_{d=0}^{\infty} \left[G_l(y) - G_l(y-d)\right] P\left(\hat{D}^{(l'-l)} = d\right) \\
&\geq 0.
\end{aligned}
$$

The fifth equality follows by the independence of $D^{(l)}$ and $\hat{D}^{(l'-l)}$. The final inequality follows since G_l is a cumulative distribution function and is a non-decreasing function.

□

LEMMA 8. *For positive integers L and L' such that $L \leq L'$, $T(L, y) \geq T(L', y)$ for all y, where $T(L, y) \equiv G_{L+1}(y) + p_{OC} \sum_{l \geq L+1} p_{CC}^{l-L-1} G_{l+1}(y)$.*

Proof. If $L = L'$, then the claim holds trivially. When $L < L'$, we will prove the claim by induction. Let $L' = L + k$ for some positive integers L and k. If $k = 1$, then

$$
\begin{aligned}
T(L, y) - T(L', y) &= G_{L+1}(y) + p_{OC} \sum_{l \geq L+1} p_{CC}^{l-L-1} G_{l+1}(y) \\
&\qquad - G_{L'+1}(y) - p_{OC} \sum_{l \geq L'+1} p_{CC}^{l-L'-1} G_{l+1}(y) \\
&= G_{L+1}(y) + p_{OC} \sum_{l \geq L+1} p_{CC}^{l-L-1} G_{l+1}(y) \\
&\qquad - G_{L+2}(y) - p_{OC} \sum_{l \geq L+2} p_{CC}^{l-L-2} G_{l+1}(y) \\
&= \left(G_{L+1}(y) - G_{L+2}(y)\right) + p_{OC} \sum_{l \geq L+1} p_{CC}^{l-L-1} \left(G_{l+1}(y) - G_{l+2}(y)\right) \\
&\geq 0.
\end{aligned}
$$

The final inequality follows from Lemma 7. Assume the claim holds for arbitrary $k > 1$, that is $T(L, y) \geq T(L + k, y)$. We will show the claim holds for $k + 1$.

$$\begin{aligned} T(L,y) - T(L',y) &= T(L,y) - T(L+k+1,y) \\ &= T(L,y) + T(L+k,y) - T(L+k,y) - T(L+k+1,y) \\ &= [T(L,y) - T(L+k,y)] + [T(L+k,y) - T(L+k+1,y)] \\ &\geq 0. \end{aligned}$$

The last inequality follows by the induction step for $k = 1$ and by the induction assumption for arbitrary k. This completes the proof by induction.

□

THEOREM 7. *For the border closure model without congestion, the optimal state-invariant order-up-to level ($\hat{y}^*$) is non-decreasing in the minimum leadtime (L).*

Proof. From Proposition 1, the inequality within the minimization in Corollary 1(ii) can be written as

$$G_{L+1}(y) + p_{OC} \sum_{l \geq L+1} p_{CC}^{l-L-1} G_{l+1}(y) \geq \left(1 + \frac{p_{OC}}{p_{CO}}\right)\left(\frac{p}{p+h}\right).$$

For ease of notation, let $\alpha = \left(1 + \frac{p_{OC}}{p_{CO}}\right)\left(\frac{p}{p+h}\right)$. Consider arbitrary positive integers L and L' such that $L \leq L'$. If $L = L'$, then the claim holds trivially. Consider $L < L'$ and let

$$\tilde{y} = \min\{d_1 \leq y < \infty : \; y \in Z, \; T(L, y) \geq \alpha\}$$

and

$$\tilde{y}' = \min\left\{d_1 \leq y < \infty : \; y \in Z, \; T(L', y) \geq \alpha\right\}.$$

Assume that $\tilde{y}' < \tilde{y}$. Then

$$\alpha \leq T(L', \tilde{y}') \leq T(L', \tilde{y} - 1) \leq T(L, \tilde{y} - 1) < \alpha,$$

which implies that $\alpha < \alpha$. The second inequality follows since $\tilde{y}' \leq \tilde{y} - 1$ by assumption and the cumulative distribution function $G_l(y)$ is non-decreasing in y for all $l \geq 0$. The third inequality follows by Lemma 8. Therefore by contradiction, $\tilde{y}' \geq \tilde{y}$.

□

3.4 Numerical Results and Discussion

3.4.1 Numerical Study Design

We now present the results of a comprehensive numerical study of the impacts on the optimal order-up-to level and long-run average cost by the system parameters. We will discuss the results of the numerical study in the context of the following supply chain. Consider an international supply chain subject to border closures in which a domestic manufacturer orders a single product from a single foreign supplier. Orders are measured in units of container loads and are placed each day. The containers are shipped by some mode of transportation where the leadtime from the supplier to the international border of the manufacturer's host nation is deterministically L days. The order remains at the border until the first day in which the border is open, at which point the order and all other orders arriving to, or waiting at, the border cross and immediately arrive at the manufacturer.

Table 1 displays the system parameters values that we study and Table 2 classifies the parameter combinations into 13 instances for reference purposes. The only parameter to remain constant throughout the numerical study is the per unit purchase cost, $c = \$150,000$. From Theorem 1, we note that the purchase cost does not affect the inventory policy. The purchase cost only contributes to the long-run average cost as a fixed cost, e.g. $cE[D]$. Varying its value only linearly changes the long-run average cost at rate $E[D]$ and therefore provides no important insights about the model.

Table 1: Numerical study design (border closure model without congestion).

Parameter	Values
Purchase Cost, c	\$150,000
Holding Cost, h	\$100, \$500
Penalty Cost, p	\$1,000, \$2,000
Minimum Leadtime, L	1, 7, 15
Transition Probability, p_{OC}	0.001, 0.003, 0.01, 0.02, 0.05, 0.1, 0.2,...,0.8, 0.9, 0.95
Transition Probability, p_{CO}	0.05, 0.1, 0.2,...,0.8, 0.9, 0.95
Demand Distribution	Poisson(Mean=0.5), Poisson(Mean=1)

Given a purchase cost of \$150,000, a holding cost of \$100 per day represents a 24.33% annual holding cost rate. We believe that this rate is reasonable for most industries (an

Table 2: Parameter instances (border closure model without congestion).

Instance	L	h	p	Demand Dist.
1	1	\$100	\$1,000	Poisson(0.5)
2	1	\$100	\$2,000	Poisson(0.5)
3	1	\$500	\$1,000	Poisson(0.5)
4	1	\$500	\$2,000	Poisson(0.5)
5	7	\$100	\$1,000	Poisson(0.5)
6	7	\$100	\$2,000	Poisson(0.5)
7	7	\$500	\$1,000	Poisson(0.5)
8	7	\$500	\$2,000	Poisson(0.5)
9	15	\$100	\$1,000	Poisson(0.5)
10	15	\$100	\$2,000	Poisson(0.5)
11	15	\$500	\$1,000	Poisson(0.5)
12	15	\$500	\$2,000	Poisson(0.5)
13	1	\$100	\$1,000	Poisson(1)

annual holding cost rate of 23% is actually cited as conservative for high-technology industries in [20]). While a holding cost of \$500 per day is excessive in reality, its extreme value highlights the effects that the holding cost can have on the policy and long-run average cost. The penalty costs of \$1,000 and \$2,000 per day respectively represent an annual penalty cost of 2.4 and 4.8 times the purchase cost. Although we do not consider customer service rates in this thesis, the greater the penalty cost, the higher the corresponding customer level.

We consider minimum leadtimes, that is, the leadtime from the supplier to the border, of 1, 7, and 15 days. A minimum leadtime of 1 day corresponds to supply chains which may utilize air cargo (such as for high-technology firms) or to supply chains in which the manufacturer and supplier are located in close proximity and the supply crosses a land border. A specific example of the latter is the automotive supply chain with suppliers in Canada and manufacturers in the Midwestern United States. The Canadian automotive supplier industry is predominantly based in the province of Ontario and the majority of the supply is shipped to the United States by truck. According to one automotive industry expert, the leadtime from Canadian suppliers to the US-Canadian border crossings (for example, at Niagara Falls, Windsor, and Sarnia) is typically less than one day. Even for supply that is shipped by rail, a reasonable estimate for the leadtime from supplier to the border is one to two days (for example, stamped parts shipped from General Motor's Oshawa complex in Canada to the United States). Since many of these manufacturing plants are located in the Midwestern United States, the leadtime from the US-Canadian border to the manufacturing facility can be considered negligible. A minimum leadtime of 7 days corresponds to a supply chain system with either longer physical travel time to the border or one that requires production time between order receipt and ship date. Finally, a minimum leadtime of 15 days corresponds to typical ocean carrier services from Asia to the Western United States. With growing Asian economies and increased outsourcing to Asia, this transportation mode and route are increasingly important to the world economy.

We study a wide range of border state transition probabilities in order gain a broad perspective of how the border system affects the optimal order-up-to level and long-run average cost. The transition probabilities correspond to an expected inter-closure time ranging from approximately three years to one day (respectively, $p_{OC} = 0.001$ and $p_{OC} = 0.95$) and an expected closure time ranging from 20 days to 1 day (respectively, $p_{CO} = 0.05$ and $p_{CO} = 0.95$). In reality, the expected inter-closure times are large and not on the order of days. In the comprehensive numerical study of border closure model with congestion in Chapter 4, we consider a restricted subset of realistic transition probabilities. For both models, we believe that the range of expected closure times represent realistic possibilities.

We consider per period demands that are approximately identically and independently distributed Poisson random variables with means of 0.5 and 1 units per day. We use a truncated Poisson distribution with a maximum realizable demand in any period of d_M units. A truncated Poisson distribution assigns Poisson probabilities to all demand realizations up through d_{M-1} and a probability of $1 - G(d_{M-1})$ to d_M where d_M is chosen such that $1 - G(d_{M-1}) < \epsilon$ for some $\epsilon > 0$. For example when the mean demand is 0.5 containers per day, the maximum realizable demand in any period is $d_M = 10$ and $P(D = 10) = 1 - G(9) = 1.63 \times 10^{-10}$.

We now present the results of the numerical study. Results are depicted in the figures and the corresponding numerical values are presented in tables in Appendix A.

3.4.2 Impact of the Transition Probabilities

The transition probabilities p_{OC} and p_{CO} are the key parameters describing the border system and offer different measures of border closure severity: expected closure duration and closure likelihood. Recall that if the border is in state i, then the expected number of periods until the border transitions to state j is $1/p_{ij}$. Therefore the expected duration of a border closure is given by $1/p_{CO}$ and the probability of a border closure is given by p_{OC}. We now present the optimal order-up-to level and the optimal long-run average cost versus the transition probabilities for the Instances 1-12 in Figures 1-25. Since the optimal policy is state-invariant, we only graph y^* in policy figures where $y^* = y^*(O) = y^*(C)$.

With the exception of Figure 13, in this thesis we do not include figures for the expected holding and penalty cost per day versus the system parameters since we are interested in changes in total long-run average cost per day. The form of the graph is identical to that for the long-run average cost per day but with scaled z-axes. It is important to understand why changes in the long-run average cost occur though. For example for Instance 1, when $p_{OC} = 0.01$ and p_{CO} decreases from 0.5 to 0.05, the optimal long-run average cost per day increases from \$75,223 to \$76,459, an increase of 1.64%. This cost increase is the result only of an increase in the expected holding and penalty cost per day from \$223 to \$1,236, a 555% increase. The tables in Appendix A can be used to determine the expected holding and penalty cost.

In each figure, we see clear trends in the optimal order-up-to level and long-run average cost. All else held constant, the optimal order-up-to level and long-run average cost increase as both p_{OC} increases and as p_{OC} decreases. In general the optimal order-up-to level and long-run average cost are much more sensitive to p_{CO} than to p_{OC}. We interpret this observation as the expected duration of a border closure ($1/p_{CO}$) much more negatively affects a firm's productivity as measured by cost and inventory than the probability of a border closure (p_{OC}). Note also that the greatest increases in the order-up-to level and long-run average cost occur when p_{CO} is small, corresponding to long expected closures. These results have important implications for the interaction between businesses and government. The primary focus of the US government has traditionally been on the prevention of security

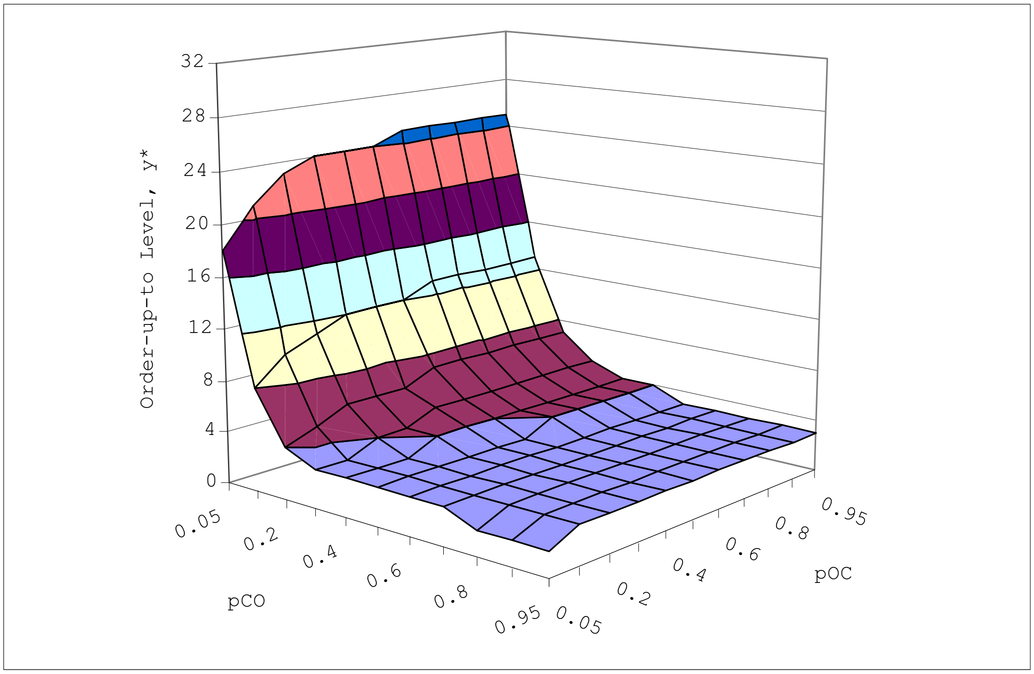

Figure 1: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 1: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

incidents that would lead to border closures. While prevention is critically important, it clear that businesses must engage and cooperate with government to design effective contingency plans that reduce the duration of a potential border closure and quickly return the system to a normal state of operation.

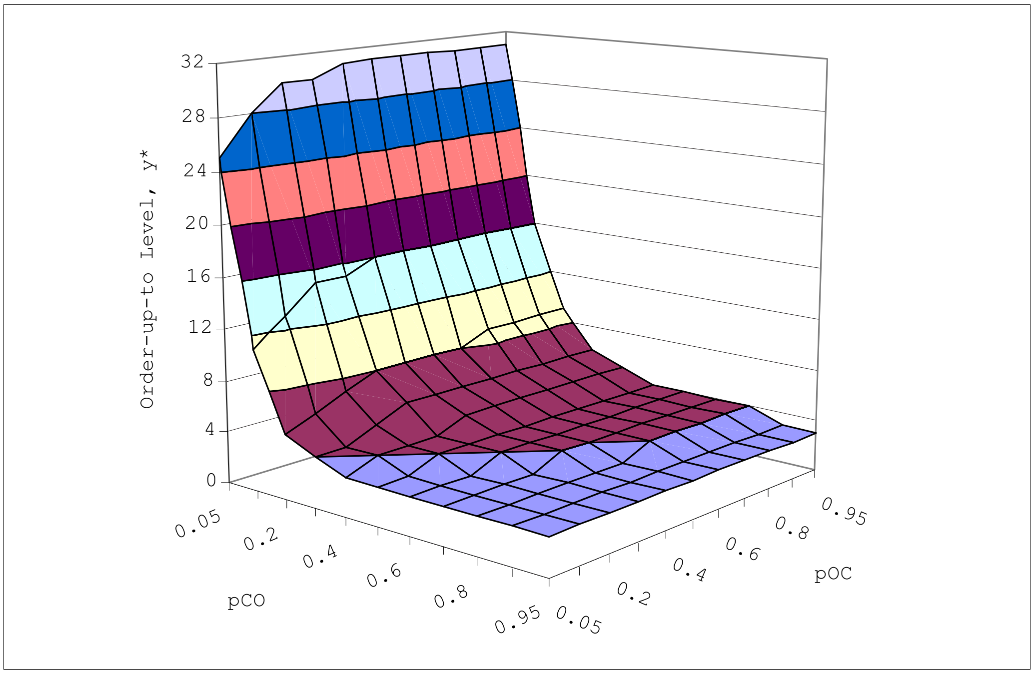

Figure 2: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 2: $L = 1$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

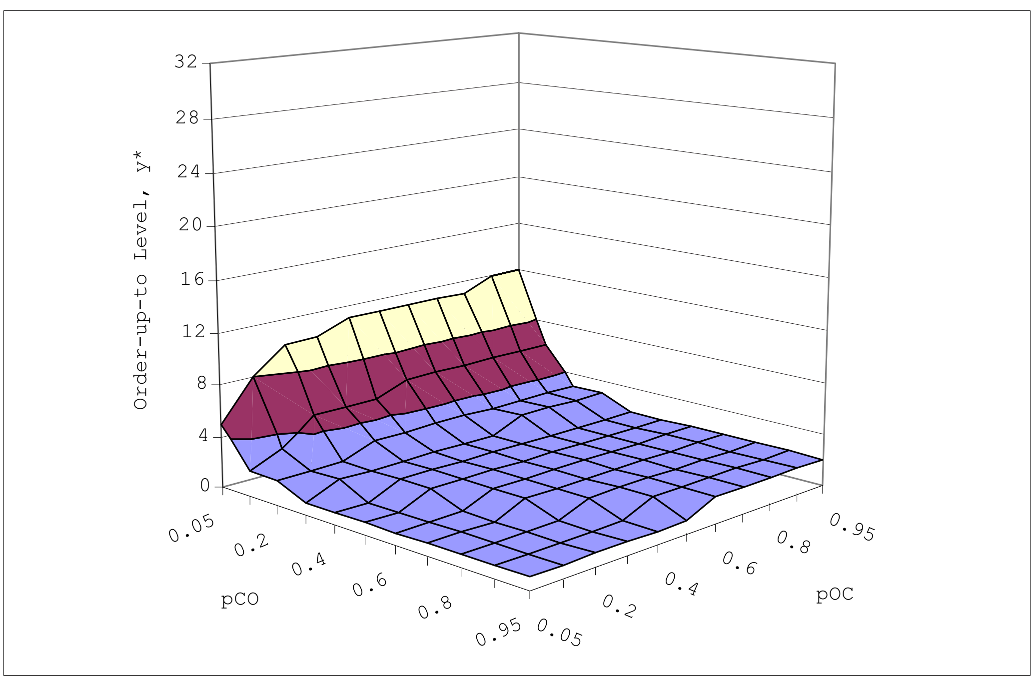

Figure 3: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 3: $L = 1$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

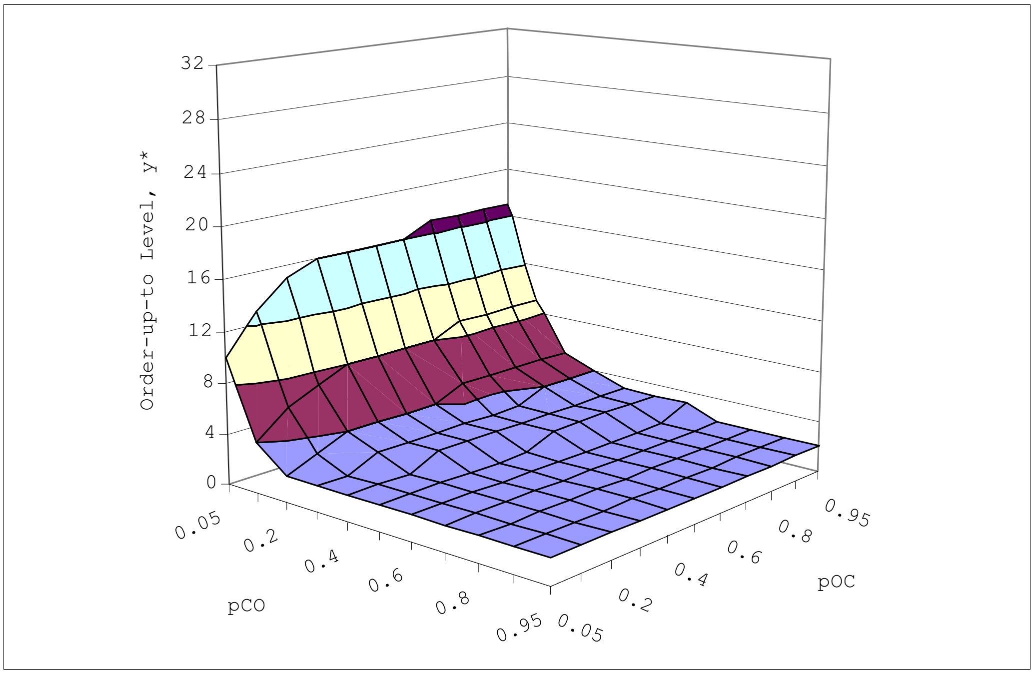

Figure 4: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC},p_{CO}) (Instance 4: $L = 1$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

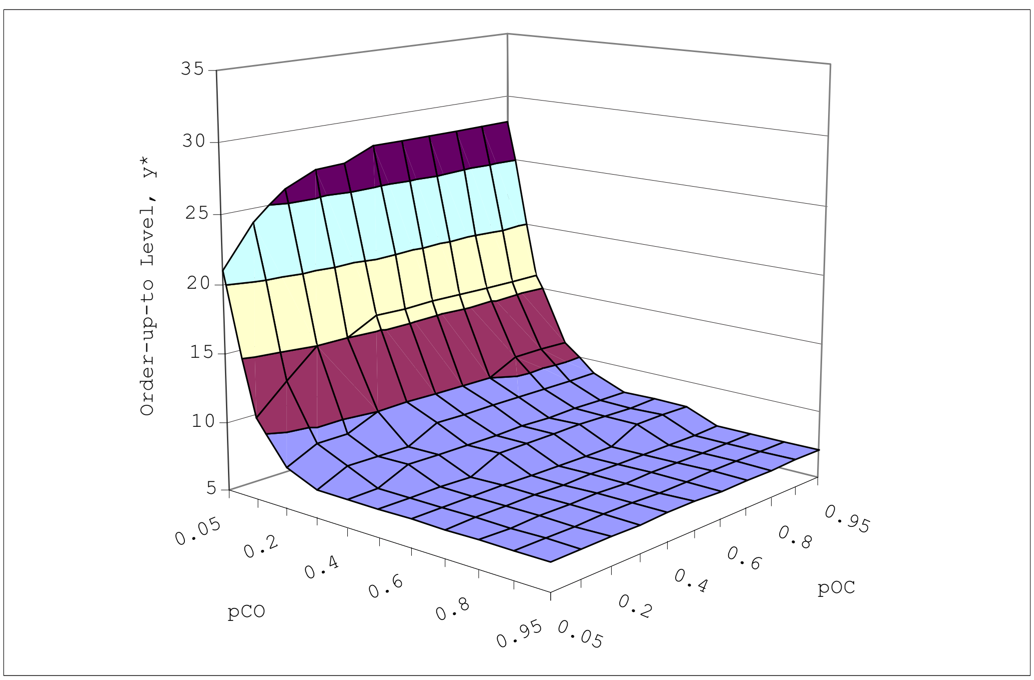

Figure 5: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC},p_{CO}) (Instance 5: $L = 7$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

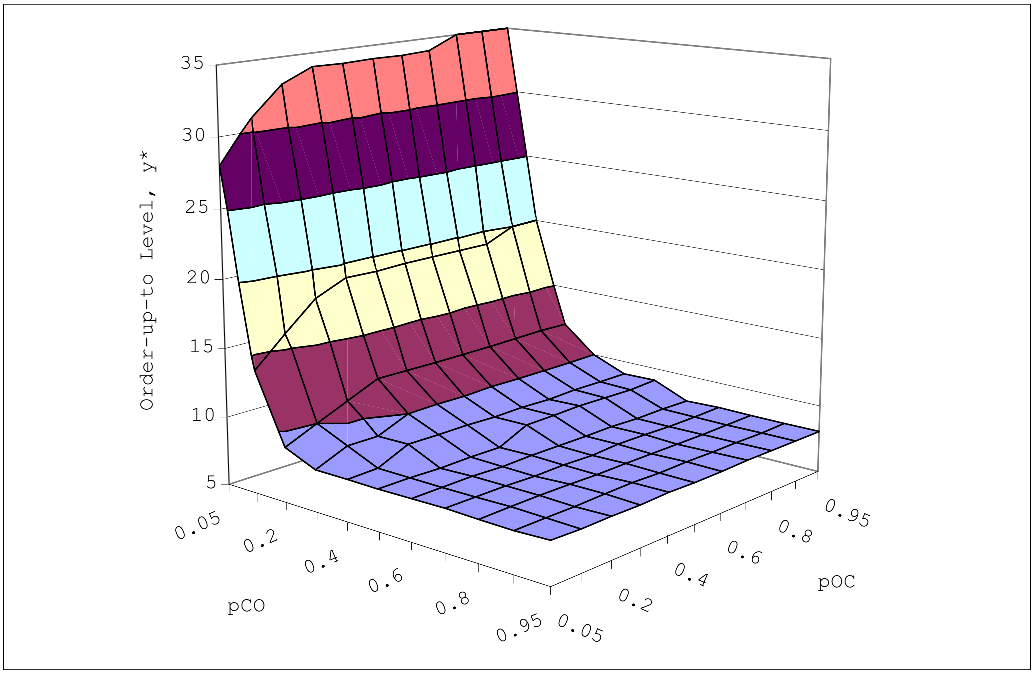

Figure 6: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC},p_{CO}) (Instance 6: $L = 7$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

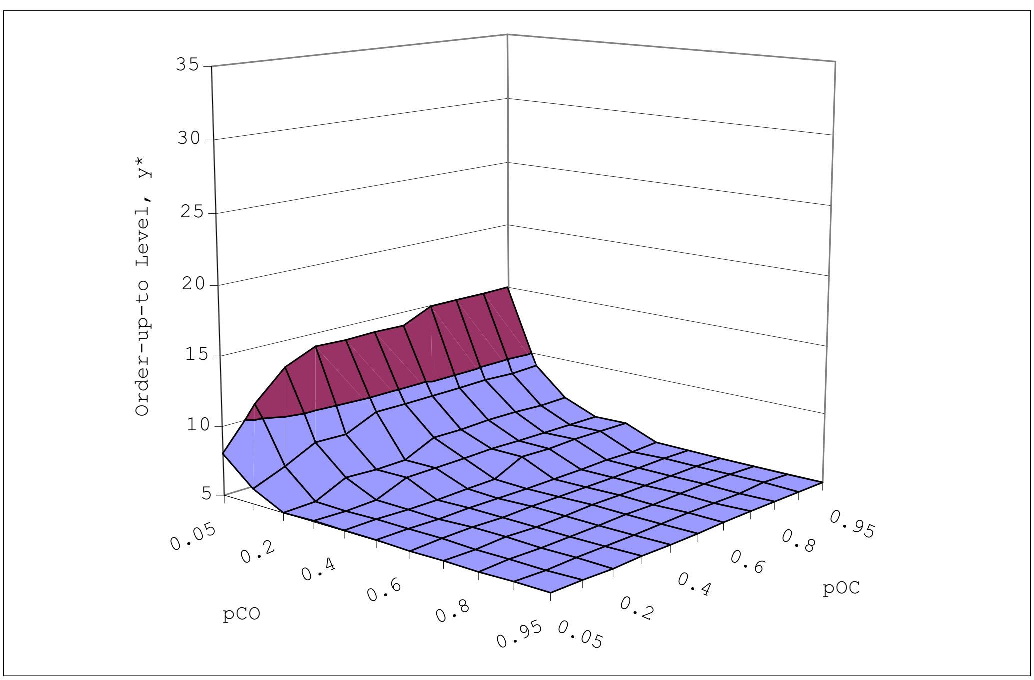

Figure 7: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC},p_{CO}) (Instance 7: $L = 7$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

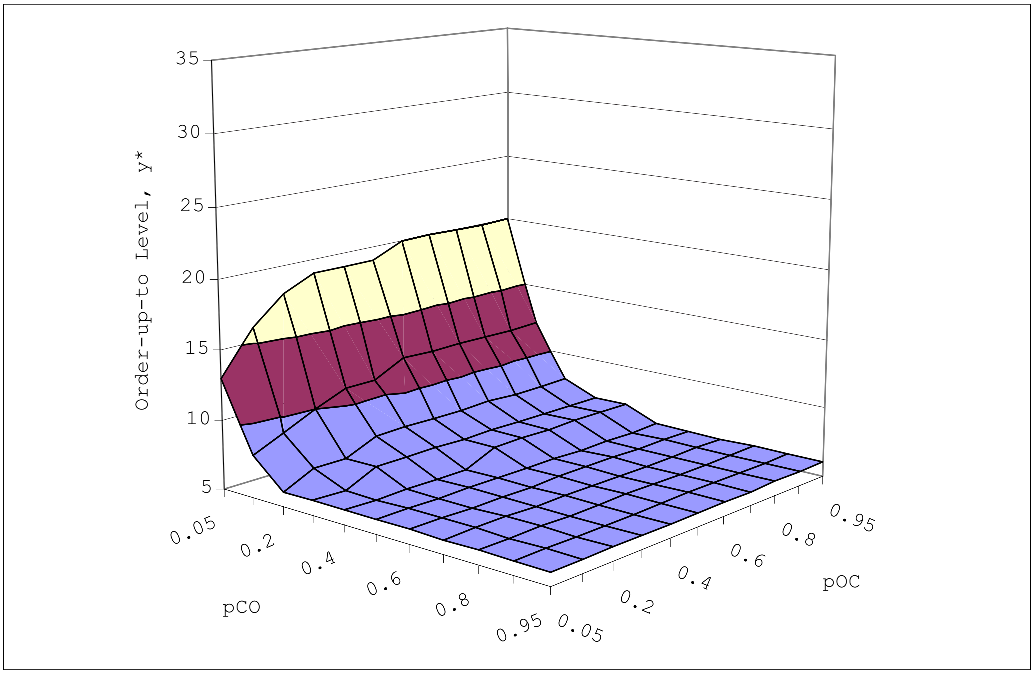

Figure 8: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 8: $L = 7$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

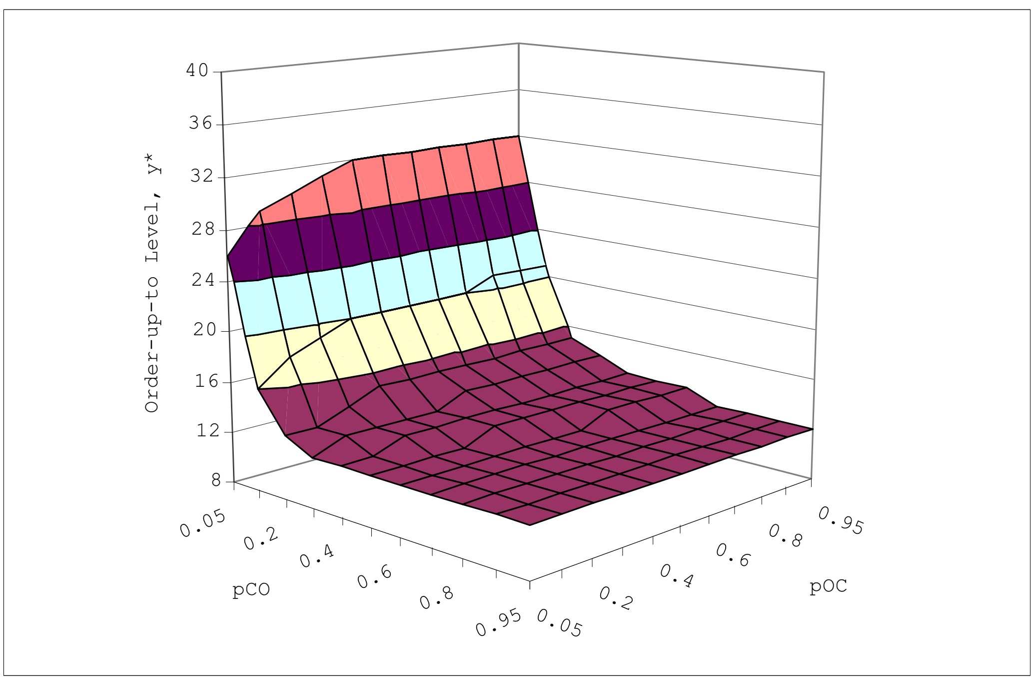

Figure 9: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 9: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

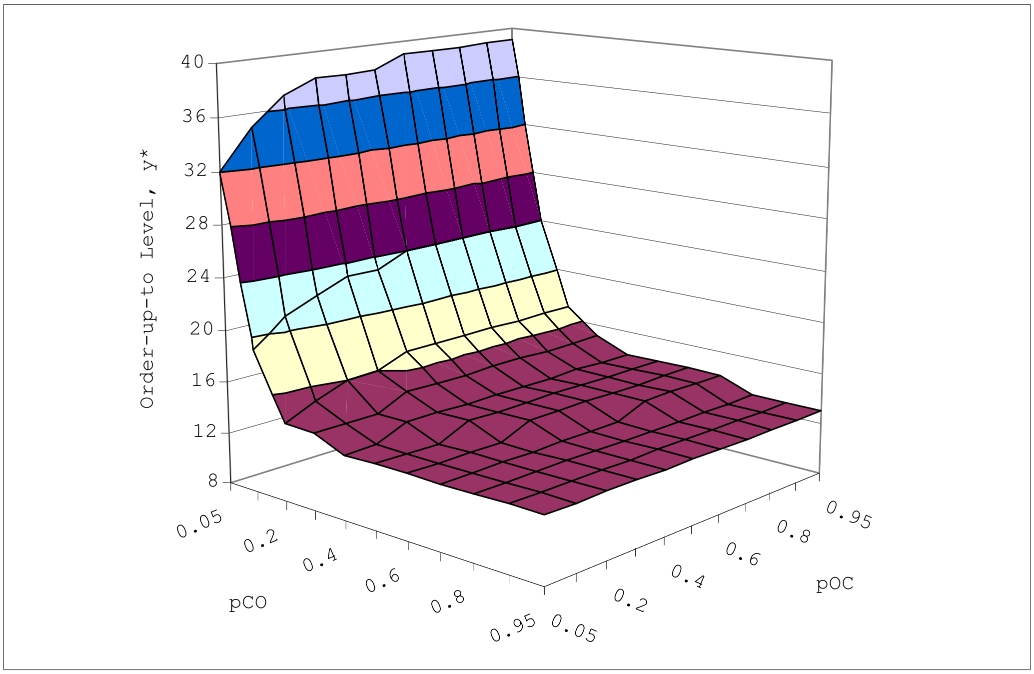

Figure 10: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 10: $L = 15$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

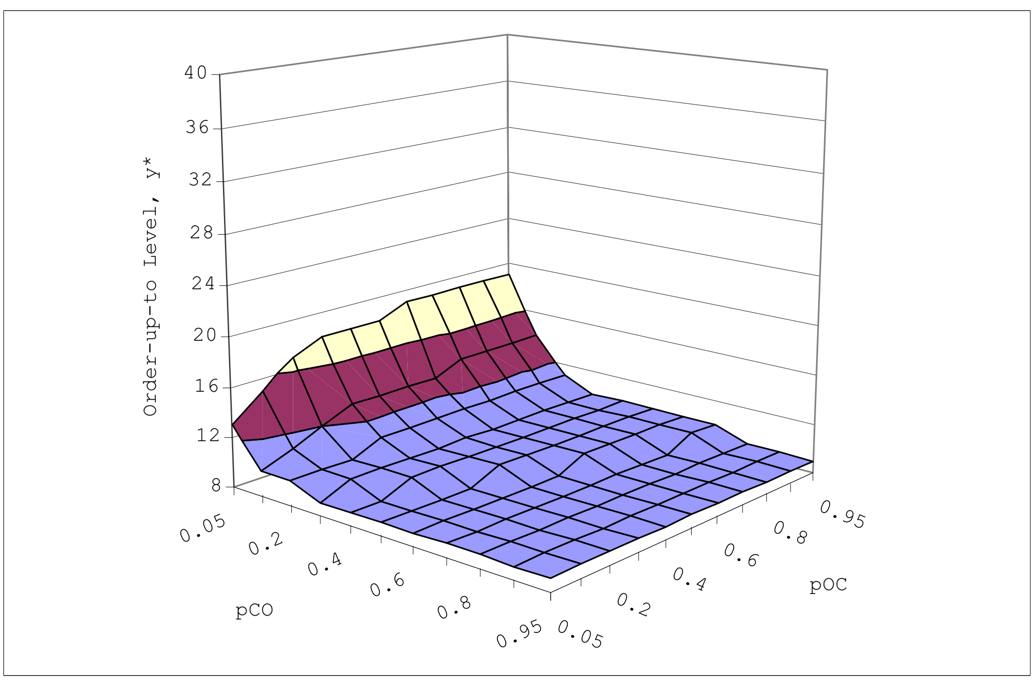

Figure 11: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 11: $L = 15$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

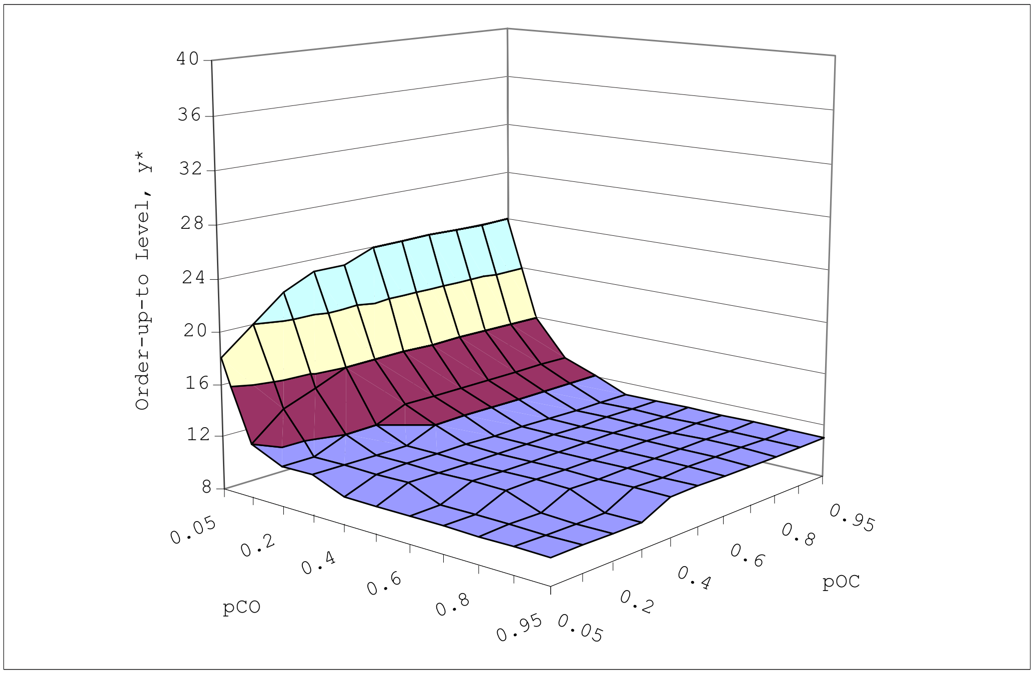

Figure 12: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 12: $L = 15$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

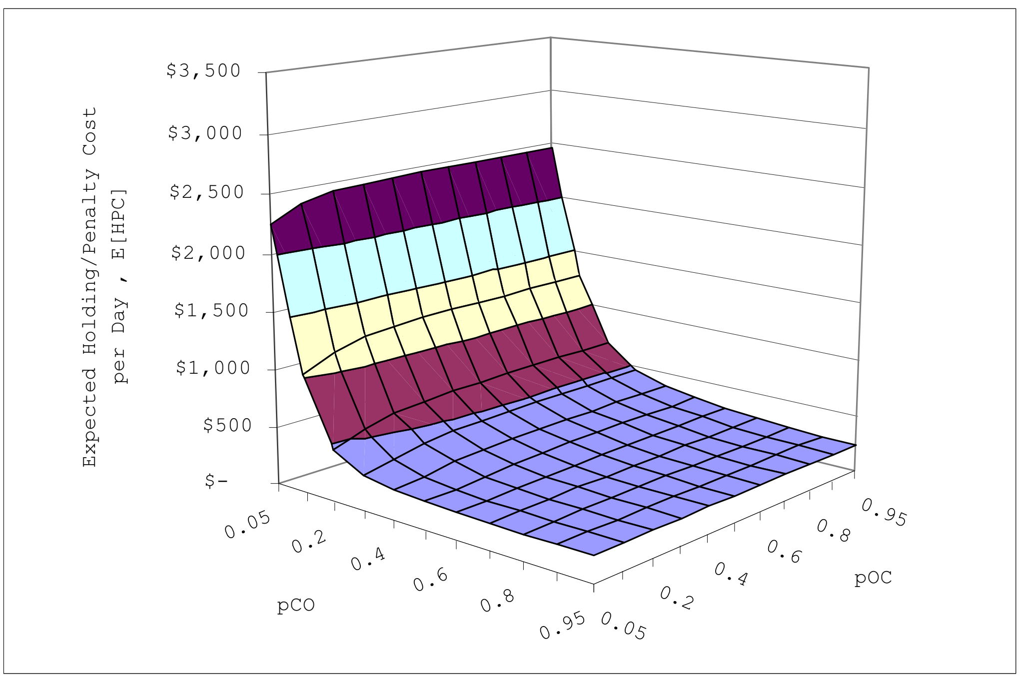

Figure 13: Optimal expected holding and penalty cost per day ($E[HPC]$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 1: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

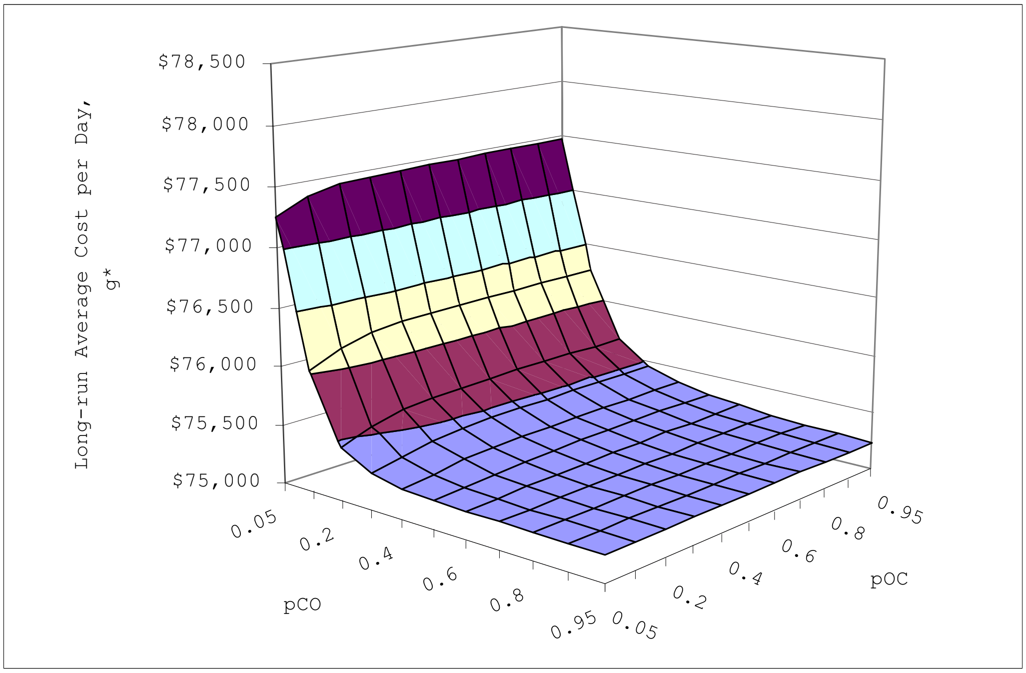

Figure 14: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 1: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

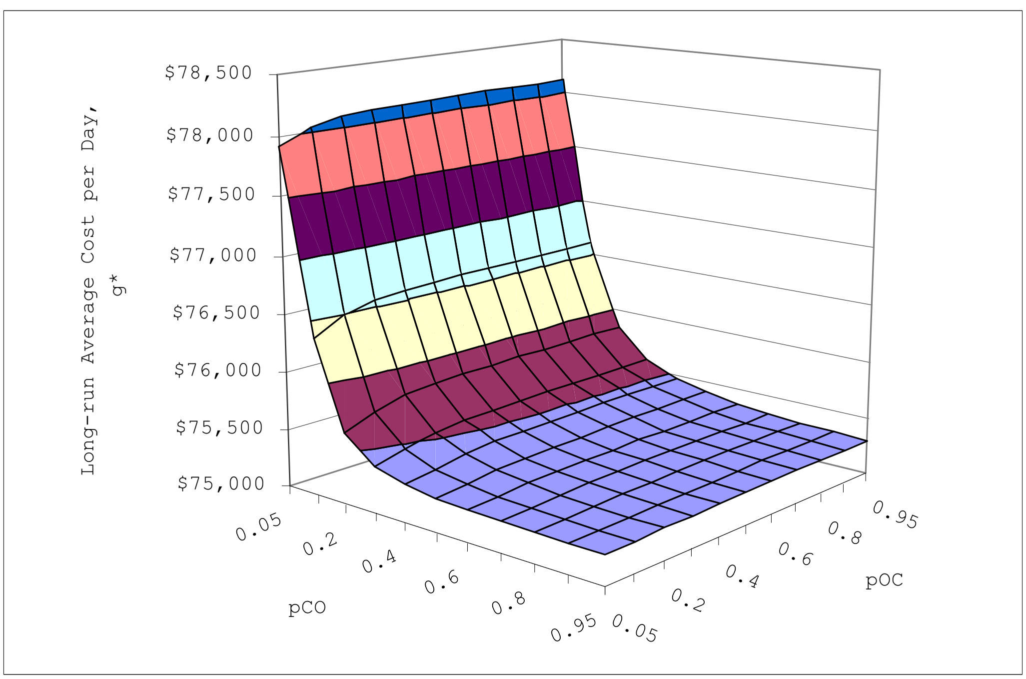

Figure 15: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 2: $L = 1$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

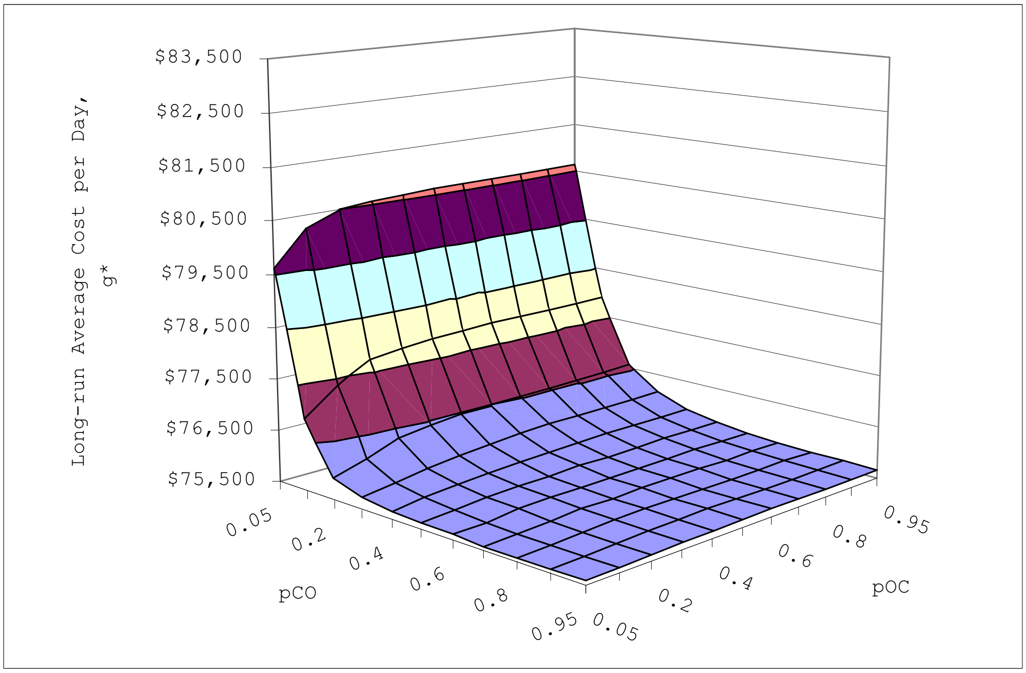

Figure 16: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 3: $L = 1$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

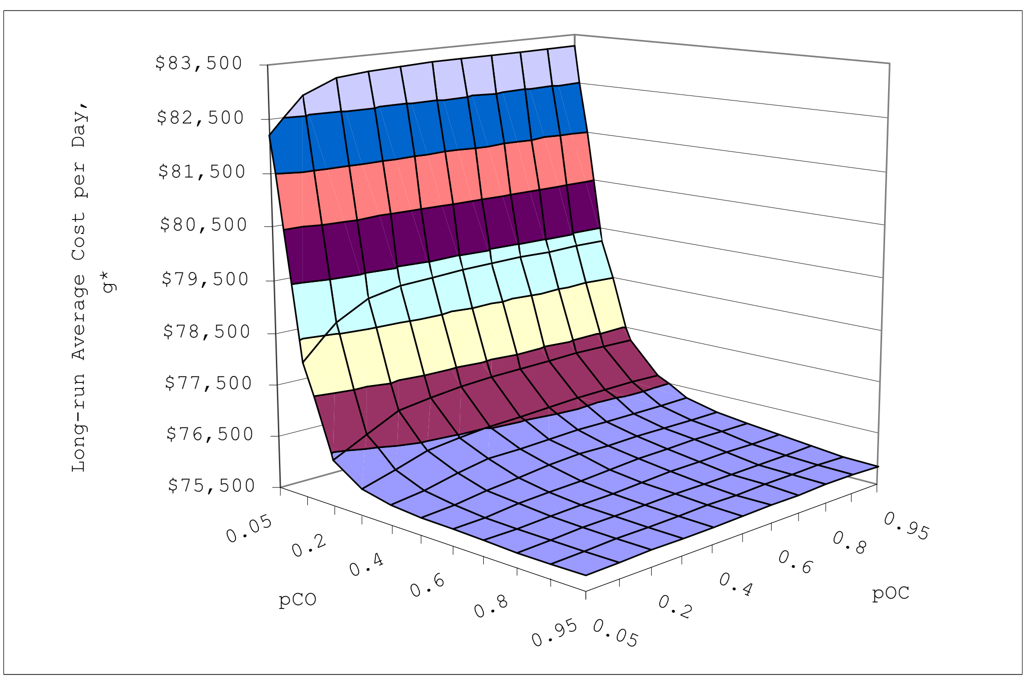

Figure 17: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 4: $L = 1$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

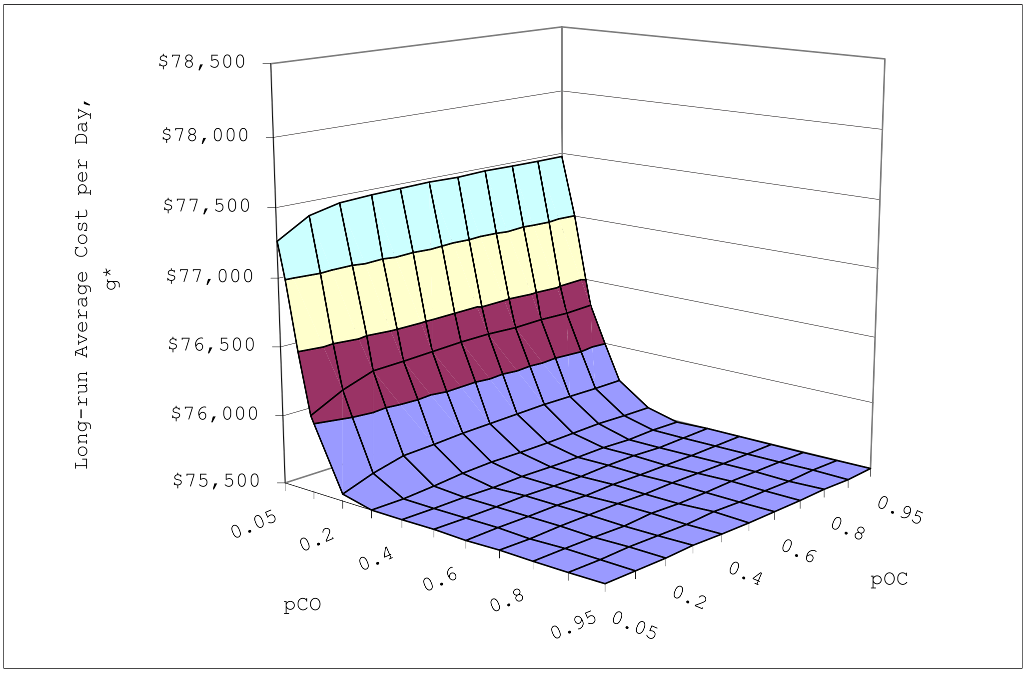

Figure 18: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 5: $L = 7$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

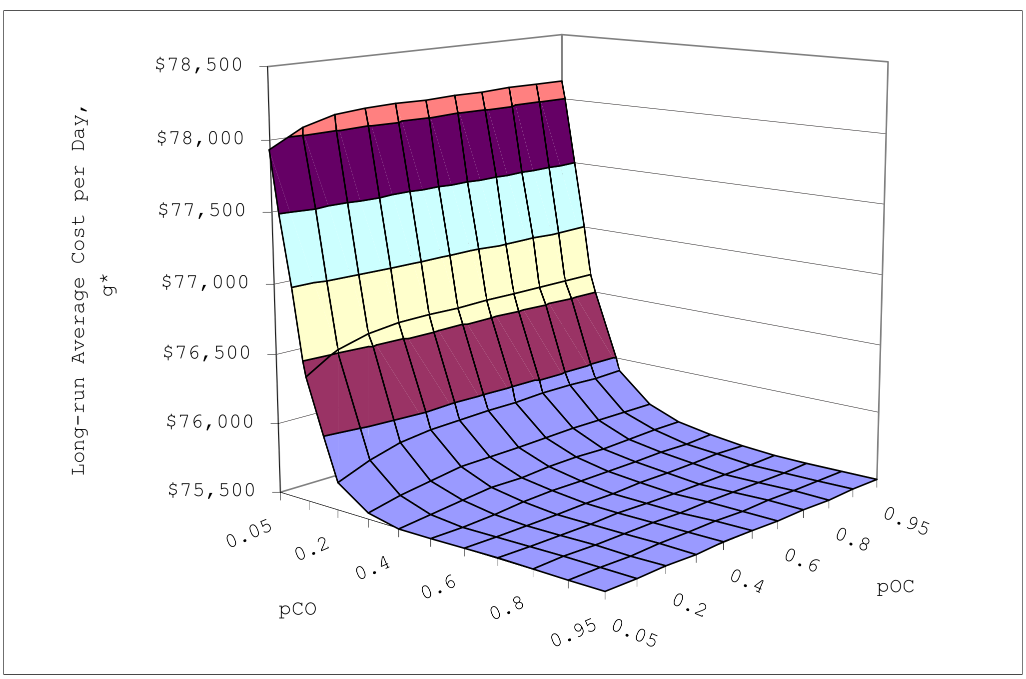

Figure 19: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 6: $L = 7$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

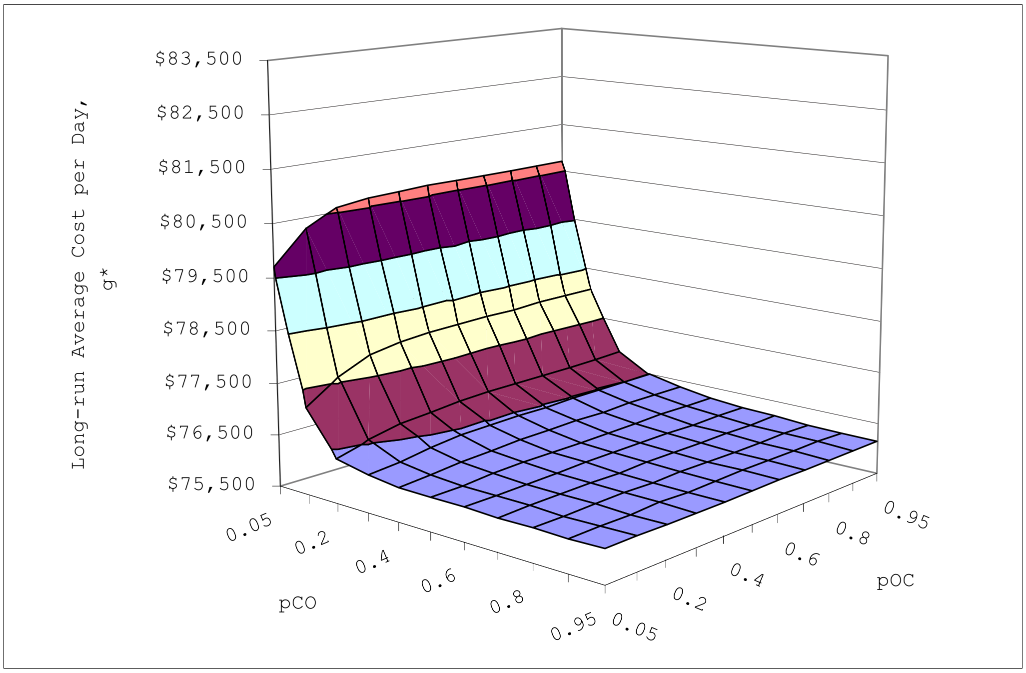

Figure 20: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 7: $L = 7$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

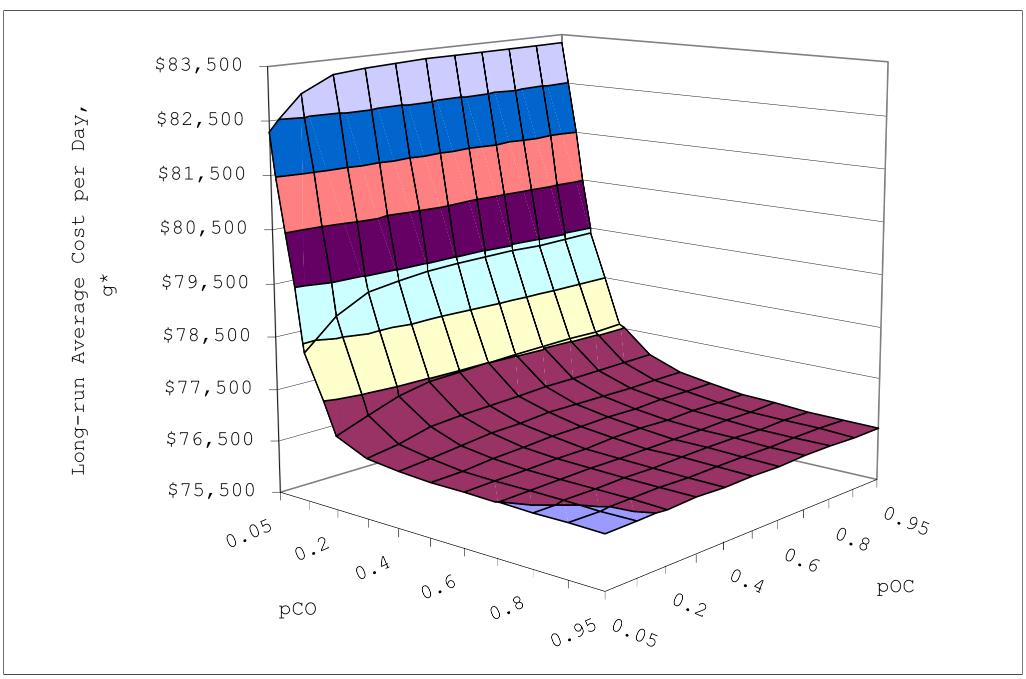

Figure 21: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 8: $L = 7$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

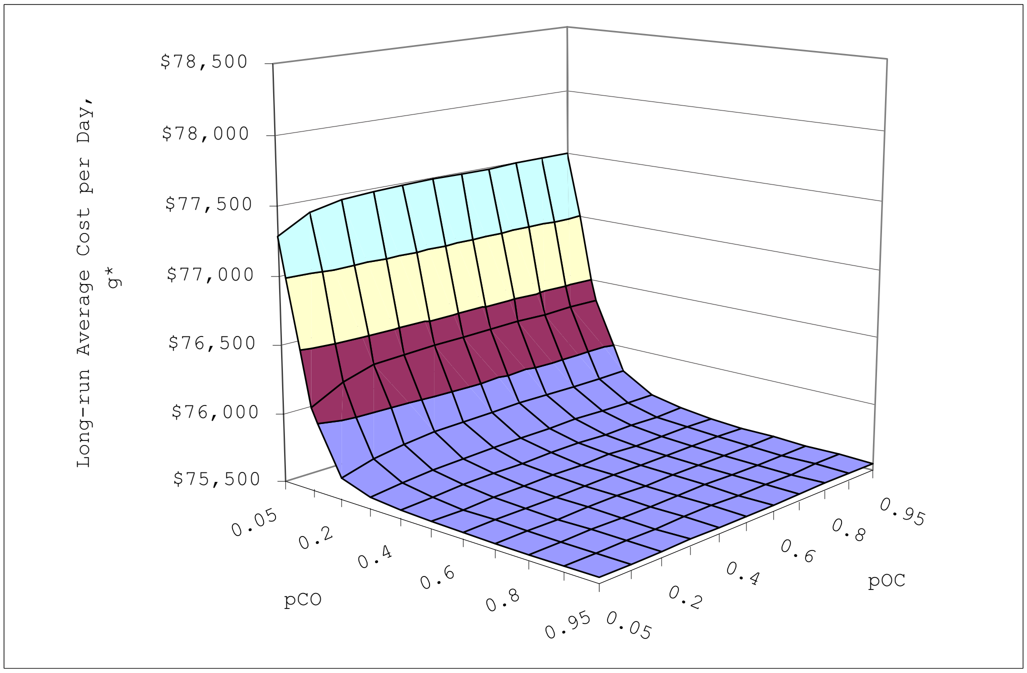

Figure 22: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 9: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

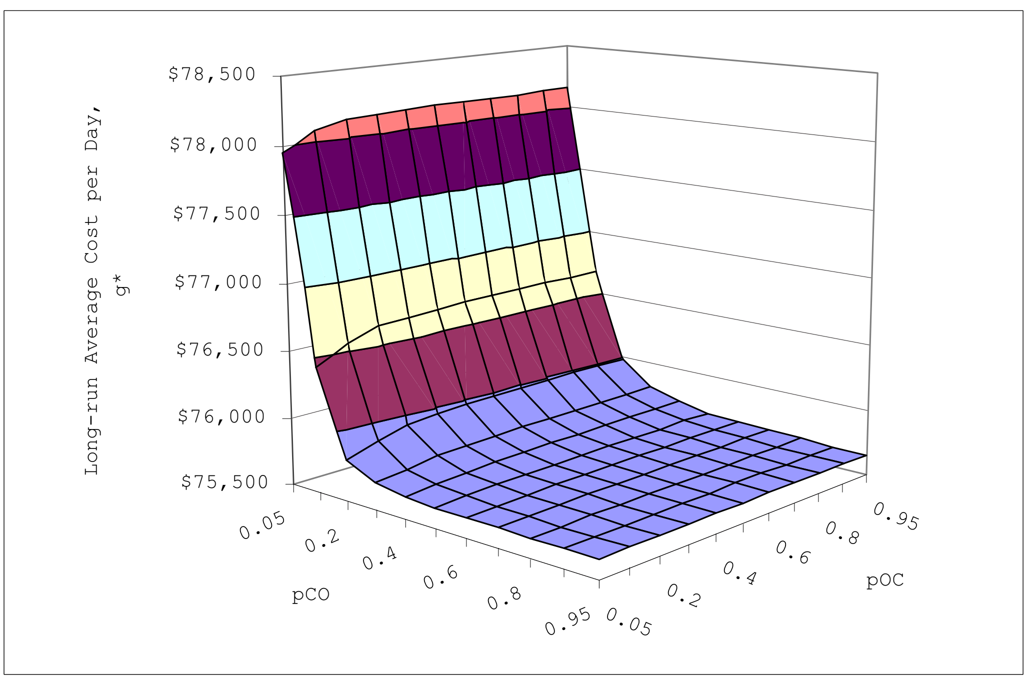

Figure 23: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 10: $L = 15$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

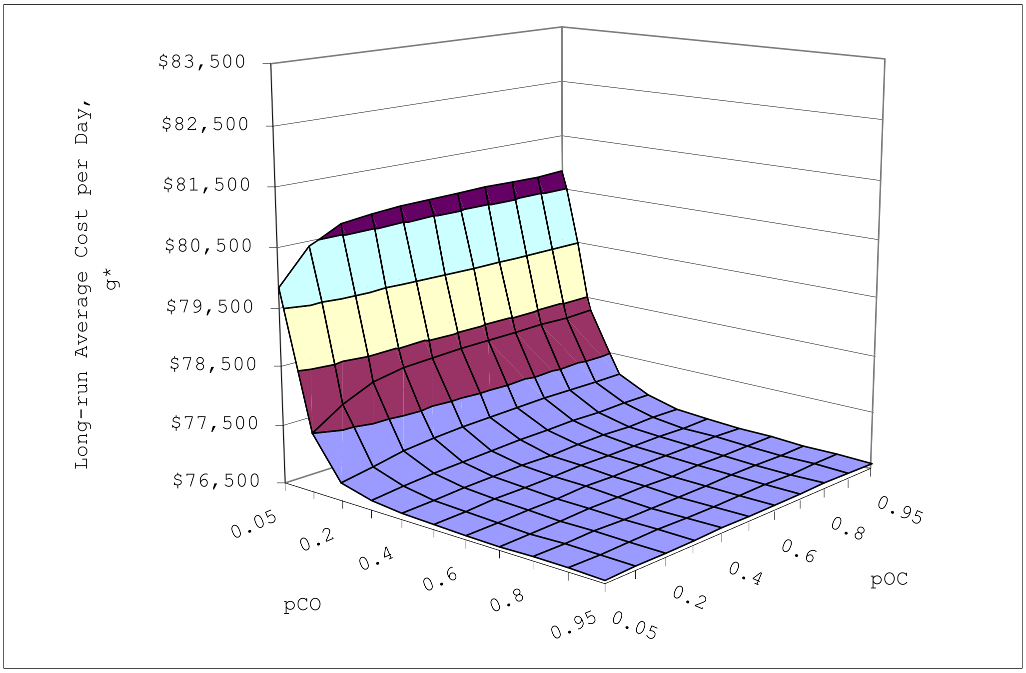

Figure 24: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 11: $L = 15$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

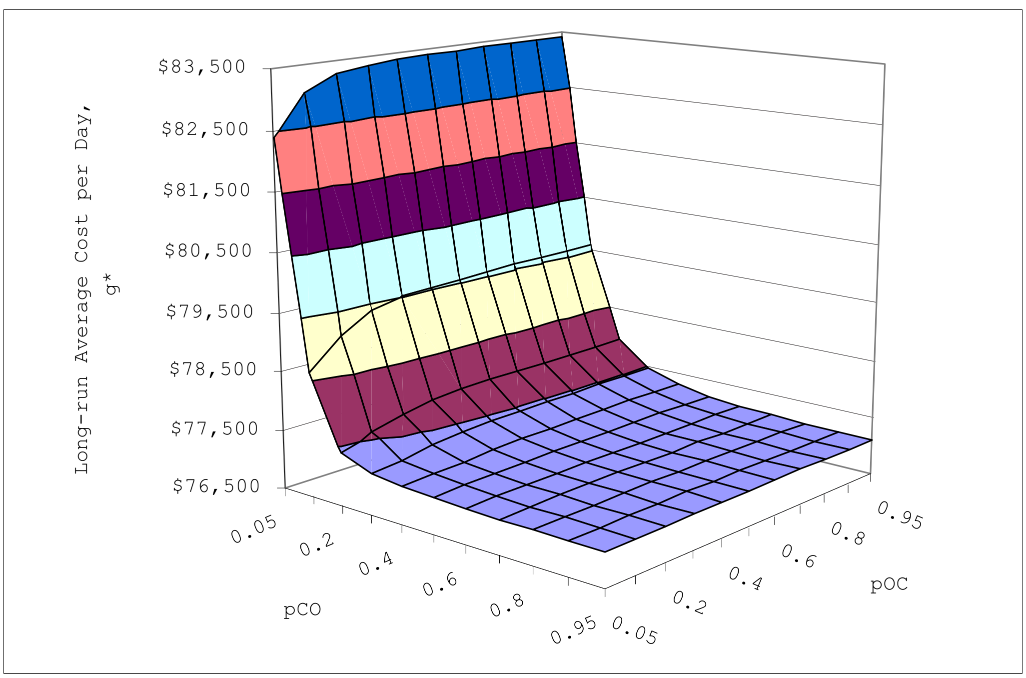

Figure 25: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 12: $L = 15$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

3.4.3 Impact of the Minimum Leadtime

Theorem 7 shows that the optimal order-up-to level is non-decreasing in the minimum leadtime. As the minimum leadtime increases, the speed with which the manufacturer can replenish its inventory obviously decreases. The manufacturer faces additional periods of demand prior to the arrival of the order, and therefore greater overall uncertainty about the total demand that will occur during order leadtimes as well as uncertainty about the future state of the border. All else held constant, we expect that the manufacturer will experience increased long-run average costs even as the manufacturer attempts to mitigate these increased risks by increasing the order-up-to levels. These expectations are confirmed when we compare Figures 1-25 based on the minimum leadtime and considering Figures 26 and 27, which display specific examples of how the optimal order-up-to level and long-run average cost change with the minimum leadtime. As expected, as the minimum leadtime increases, the optimal long-run average cost increases sub-linearly.

It is interesting to note that the optimal long-run average cost experiences the greatest increases when the minimum leadtime is small. As the minimum leadtime increases, knowledge about the future state of the border rapidly deteriorates. Recall that the minimum leadtime is a deterministic component of the total leadtime and the L-step transition probabilities for the exogenous system (which are key in equations (20) and (22)) geometrically approach the stationary probabilities. The probabilities from equation (22) that are used to calculate $C(i, y)$ provide less and less information information about the border as they geometrically approach

$$P(L(i) \leq l \leq L(i_+)) = \begin{cases} 0 & \text{if } l < L, \\ \pi_O^I & \text{if } l = L, \\ \pi_O^I p_{OC} p_{CC}^{l-L-1} & \text{if } l > L. \end{cases} \tag{25}$$

Note however that the probabilities for $l > L$ are not constant as L increases. These results confirm the conventional wisdom that shorter leadtimes are desirable.

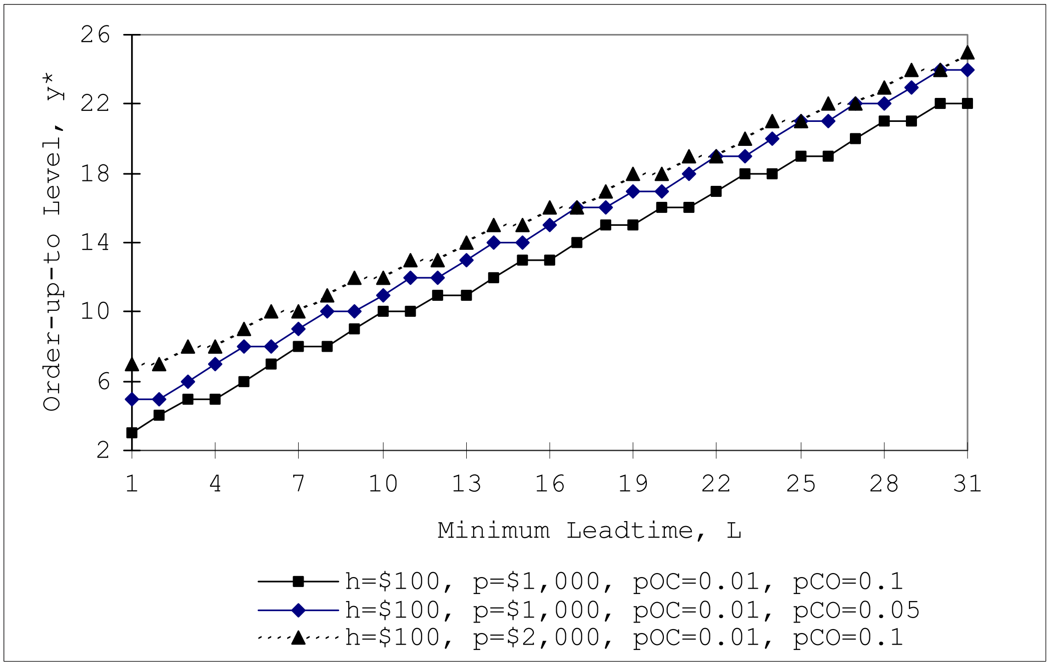

Figure 26: Optimal order-up-to level (y^*) vs. minimum leadtime (L).

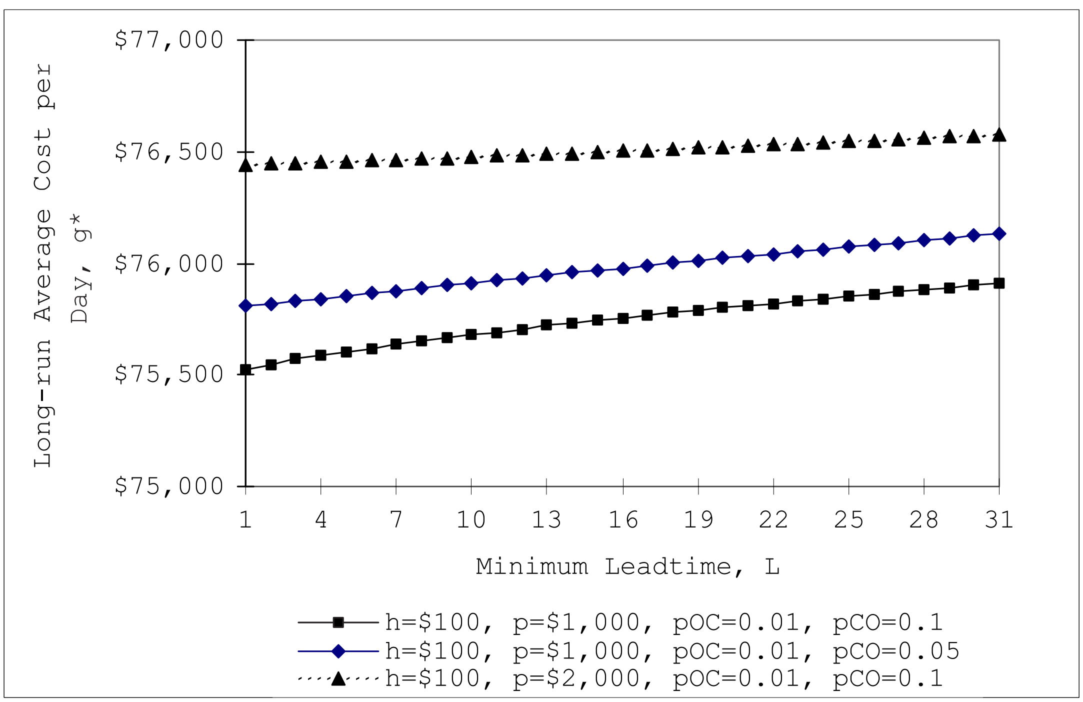

Figure 27: Optimal long-run average cost per day (g^*) vs. minimum leadtime (L).

3.4.4 Impact of the Holding and Penalty Costs

Theorem 2 shows that the optimal order-up-to level is only dependent on the cost parameters through the cost ratio $p/(p+h)$. In this section, we examine the impacts of this cost ratio and the individual holding and penalty costs on the optimal order-up-to level and long-run average cost. We first examine the impact of the cost ratio.

Theorem 6 proves that the optimal order-to-up to levels for the border closure model without congestion are state-invariant and so from Theorem 3, the optimal order-up-to level is non-decreasing in the cost ratio $(p/(p+h))$. The cost ratios for the four holding and penalty cost combinations we study in Instances 1-13 are presented in Table 3. Using Table 3, we can observe the impacts of the cost ratio in Figures 1-12. For example, fixing the transition probabilities and minimum leadtime, the order-up-to levels increase from smallest to largest for Instance 3,4,1, and 2, which respectively correspond to increasing cost ratios. Figure 28 also displays the optimal order-up-to level for three example parameter sets.

Table 3: Cost ratios for the studied holding (h) and penalty (p) cost combinations.

Instances	h	p	$CostRatio$
1,5,9	\$100	\$1,000	0.9090
2,6,10	\$100	\$2,000	0.9523
3,7,11	\$500	\$1,000	0.6666
4,8,12	\$500	\$2,000	0.8000

Cost ratios less than 0.5 imply that $h > p$ and ratio values greater than 0.5 imply $p > h$. Fixing the holding cost, a cost ratio ranging from 0 to 0.95 accounts for a wide range of possible penalty costs, approximately 0 to 20 times the holding cost. Fixing the penalty cost, a cost ratio ranging from 0.05 to 1 accounts for a wide range of possible holding costs, approximately 0 to 20 times the penalty cost.

We observe that the optimal order-up-to level is more sensitive to the cost ratio as it approaches 1. This has has two interpretations. The first interpretation considers a large penalty cost relative to the holding cost. Let α denote the cost ratio. Then for a given value of α, it is easily shown that $p = \left(\frac{\alpha}{1-\alpha}\right) h$. As α approaches 1 with a fixed holding cost, the

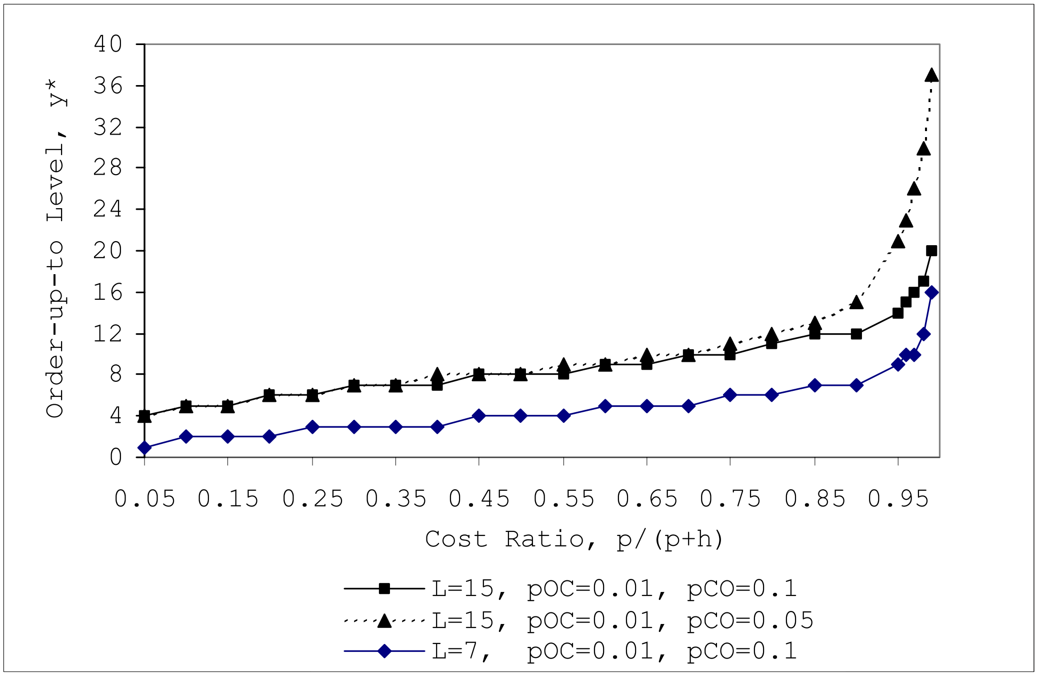

Figure 28: Optimal order-up-to level (y^*) vs. cost ratio ($p/(p+h)$).

penalty cost approaches infinity. This results in an increased order-up-to level that protects against costly backorders. The second interpretation considers a small holding cost relative to the penalty cost. Noting that $h = \left(\frac{1-\alpha}{\alpha}\right) p$, as α approaches 1 with a fixed penalty cost, the holding cost approaches 0. Since the holding cost is small, the order-up-to level can be increased to reduce the risk of backorders without incurring large holding costs.

Next, we examine the impact of the holding and penalty costs separately. Theorem 4 proves that the optimal order-to-up to level is non-decreasing in penalty cost and non-increasing in holding cost. Figures 29-32 present the optimal order-up-to levels and long-run average costs when we vary the holding cost from \$100 to \$2,200 while fixing $p = \$1,000$ and when we vary the penalty cost from \$100 to \$2,200 while while fixing $h = \$100$. The studied ranges for h and p correspond to approximately the same range of the cost ratio, respectively [0.91,0.33] and [0.5,0.96]. The results illustrate Theorem 4.

3.4.5 Impact of the Demand Distribution

Theorem 5 shows that stochastically larger demand increases the optimal state-invariant order-up-to level. It can be shown that a Poisson random variable is stochastically increasing

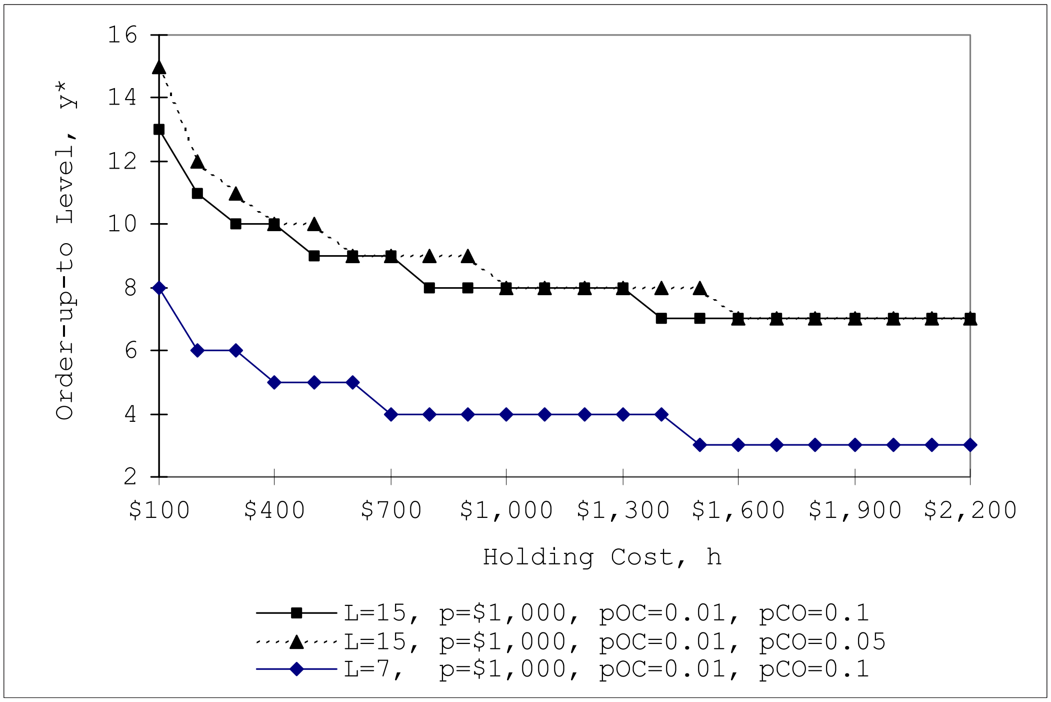

Figure 29: Optimal order-up-to level (y^*) vs. holding cost (h).

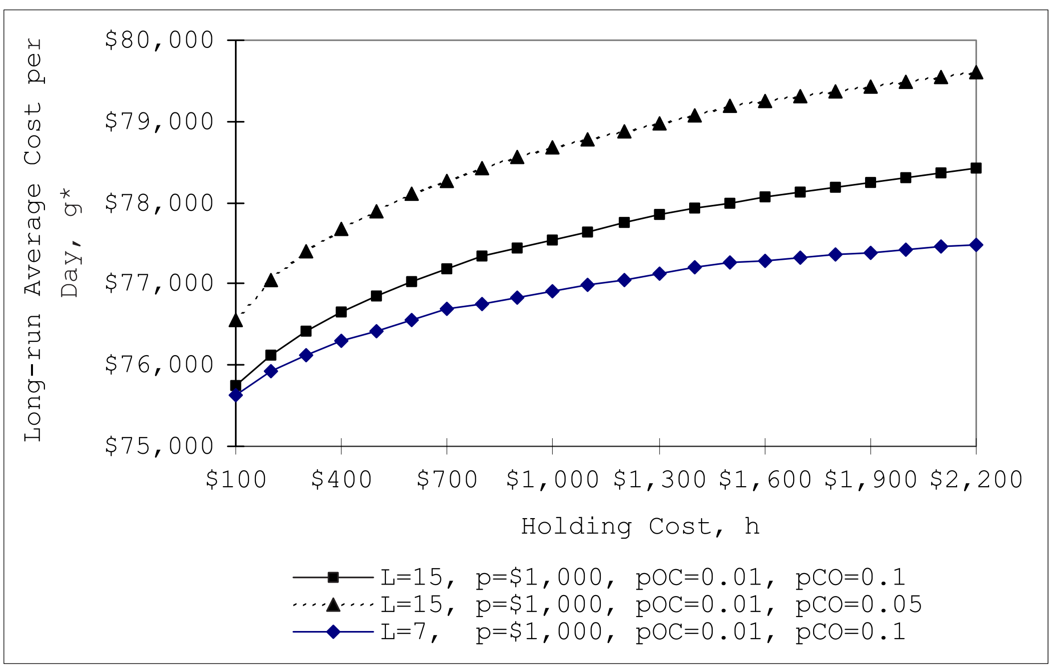

Figure 30: Optimal long-run average cost per day (g^*) vs. holding cost (h).

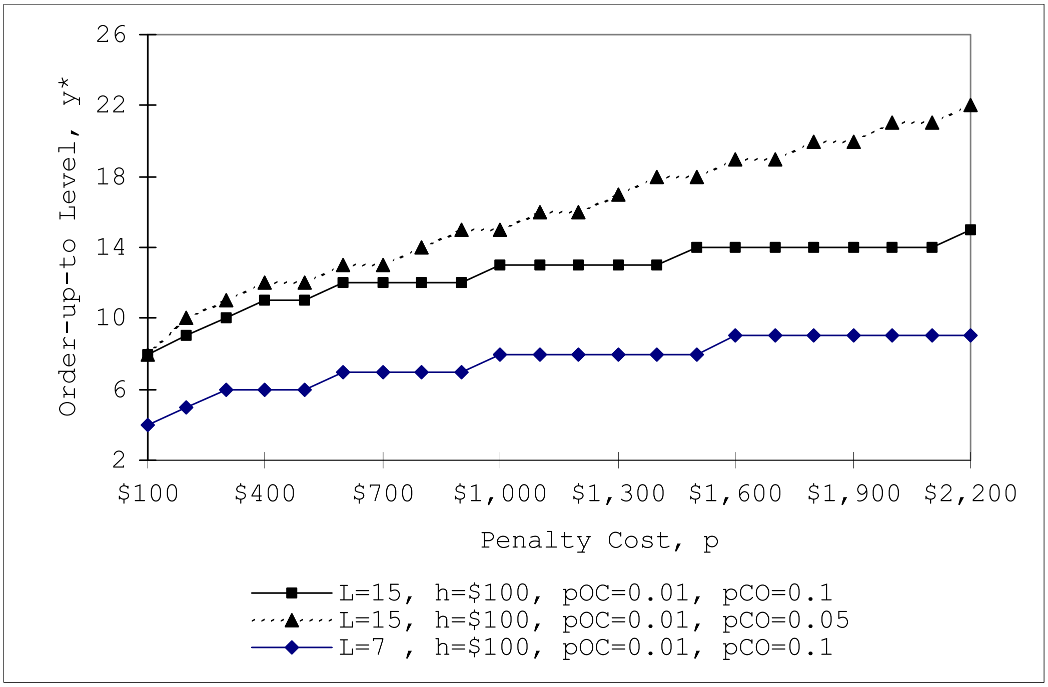

Figure 31: Optimal order-up-to level (y^*) vs. penalty cost (p).

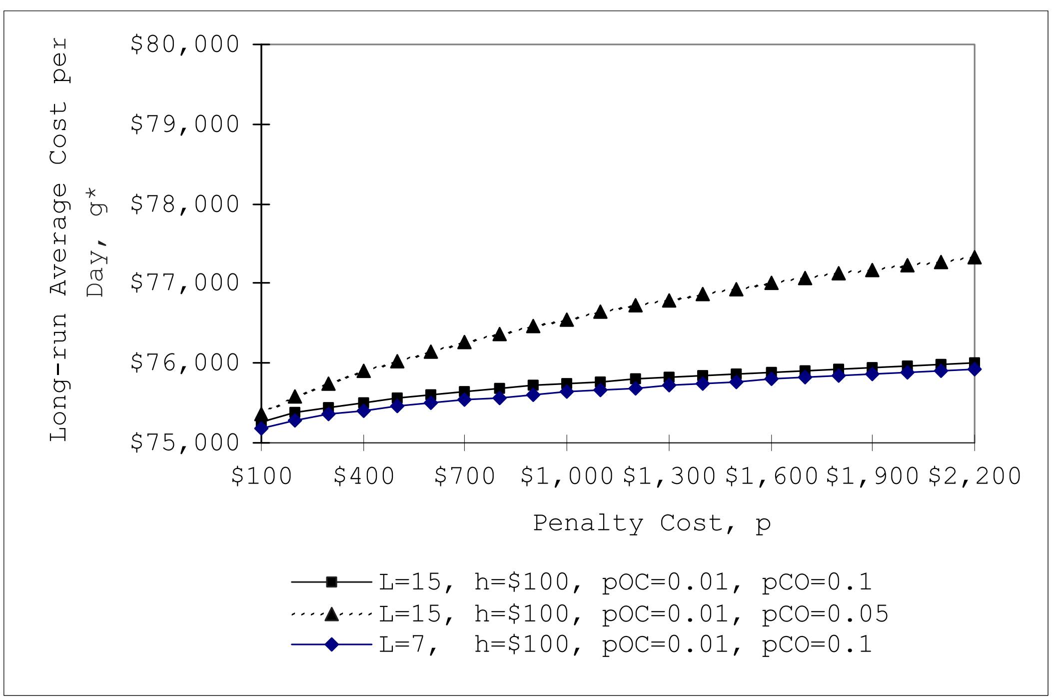

Figure 32: Optimal long-run average cost per day (g^*) vs. penalty cost (p).

in its mean (see Example 9.2(B) in [31]). If λ_1 and λ_2 are the respective means of two independent Poisson random variables, D_1 and D_2, such that $\lambda_1 \leq \lambda_2$, then $D_2 \geq_{ST} D_1$. In this section, we present results for Instance 13 in which the demand is Poisson distributed and the mean demand is doubled to one container per day (from 0.5 containers per day in Instance 1). Figures 33 and 34 display the optimal order-up-to level and long-run average cost per day versus the border state transition probabilities. These figures should be compared to Figures 1 and 14 respectively to see the impacts of stochastically larger demand.

For Poisson distributed demand, as the mean increases, so does the variance. Therefore the optimal order-up-to levels are greater in order to provide a buffer against this greater demand uncertainty. The long-run average cost is greater as well since at times the manufacturer may be holding more on-hand inventory (due to increased order-up-to levels and greater demand uncertainty) and at other times the manufacturer may be experiencing a greater number of backorders (due to greater demand uncertainty). Also due to greater demand uncertainty, the manufacturer becomes more sensitive to the risks of border closures and to potentially longer closures. As p_{OC} increases for fixed a value of p_{OC} and as p_{CO} decreases for fixed a value of p_{OC}, the optimal order-up-to level and long-run average cost increase faster in Instance 13 than in Instance 1. Also the maximum difference in order-up-to levels (over all transition probability pairs) doubles for the case of larger mean demand and variance. For example, consider Instances 1 and 13. The maximum and minimum order-up-to levels are respectively 25 and 2 for Instance 1, but they are respectively 4 and 49 for Instance 13. Therefore we observe that stochastically larger demand increases the optimal order-up-to level and greater demand variance contributes to the increase in the long-run average cost.

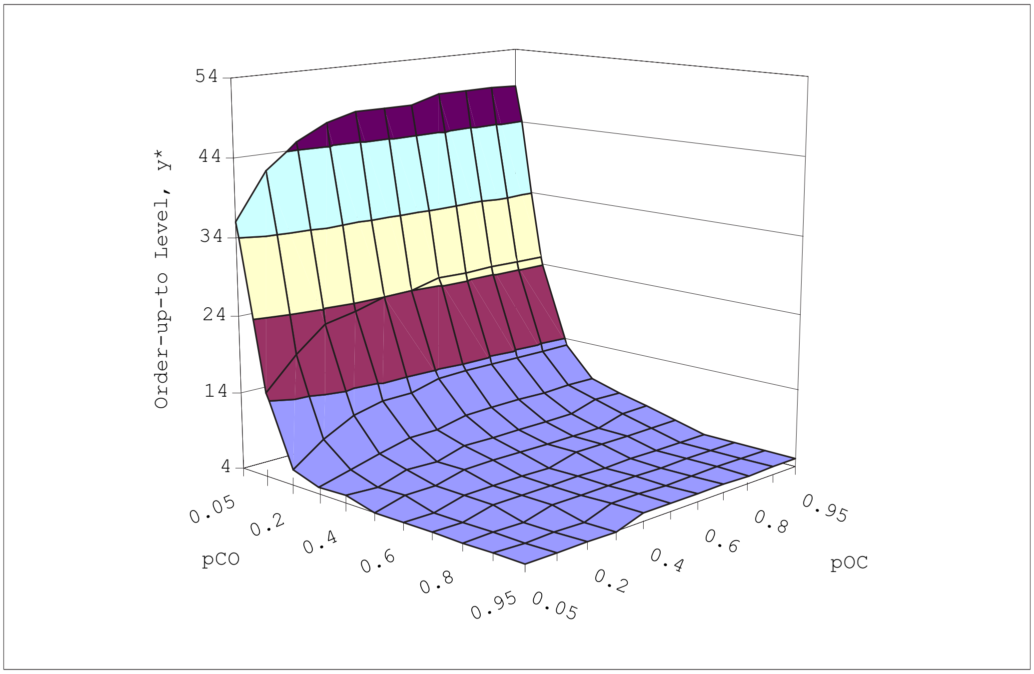

Figure 33: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 13: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(1)).

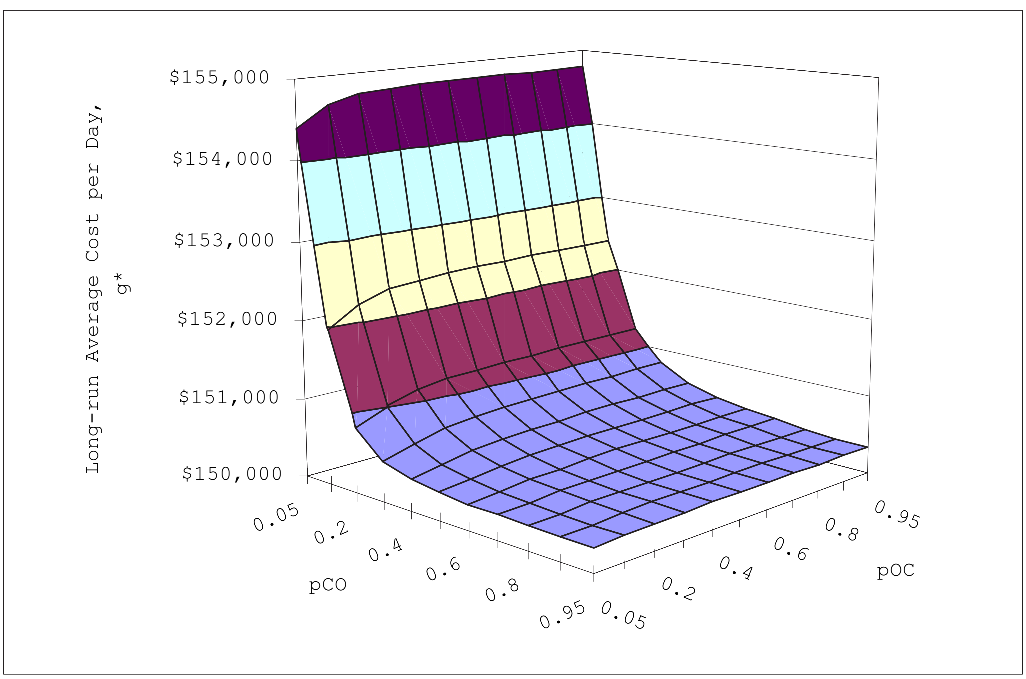

Figure 34: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 13: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(1)).

3.4.6 Impact of Contingency Planning

Border closures are typically considered to be rare-events and are therefore not included in regular operational planning models. Suppose that a firm optimizes its inventory policy without explicitly modeling border closures and implements them in a real-world environment in which the border may experience closures. We will refer to this policy as the *implemented policy*. Clearly the implemented policy may be sub-optimal for systems in which the border is actually subject to closure, and it is interesting to investigate how poor the implemented policy might be.

To address this question, we determine the optimal inventory policy using a model in which the probability of border closure is zero, e.g. $p_{OC} = 0$. We then use Theorem 2 to calculate the long-run average cost of the implemented policy in a system in which the actual probability of border closure is nonzero, e.g. $p_{OC} > 0$. We denote the long-run average cost under the implemented policy by g^y. When the implemented policy is sub-optimal, the long-run average cost under the true optimal policy will be less than that under the implemented policy. We interpret this cost reduction as the benefit of contingency planning for border closures. We use the term *contingency planning* to mean that the decision maker models border closures when determining optimal ordering policies (even if the resulting optimal policy is the same implemented policy).

There are clearly scenarios for which contingency planning for border closures is quite important. For example, consider the results in Table 4 when $L = 15$, $p_{OC} = 0.02$ and $p_{CO} = 0.05$. In this case $y = 12$ and $y^* = 20$. Since the implemented order-up-to level differs from optimal order-up-to level, the implemented policy is sub-optimal. The reduction in long-run cost per day resulting from contingency planning and the use of the true optimal policy is \$494 per day, or a reduction of 0.64%. This corresponds to a reduction in the long-run average cost per year of \$179,816 (assuming no discounting). The cost reductions due to contingency planning become even more dramatic when border congestion is modeled in Chapter 4.

Figures 35-37 display the reductions in long-run average cost per day that result from contingency planning for border closures versus the transition probabilities. While it is

Table 4: Implemented (y, g^y) vs. optimal (y^*, g^*) order-up-to level and long-run average cost per day ($h = \$100$, $p = \$1,000$).

		$p_{OC} = 0.02$, $p_{CO} = 0.05$			$p_{OC} = 0.05$, $p_{CO} = 0.05$		
L	y	y^*	$(y/y^*) * 100$	Cost Reduction	$y*$	$(y/y^*) * 100$	Cost Reduction
1	2	13	15%	\$865	18	11%	\$2,414
7	7	16	44%	\$561	21	33%	\$1,759
15	12	20	60%	\$494	26	46%	\$1,543

clear that contingency planning will result in greater cost reductions for higher holding and penalty costs, the behavior with respect to the minimum leadtime is unclear. Therefore the only system parameter that varies between the figures is the minimum leadtime, L.

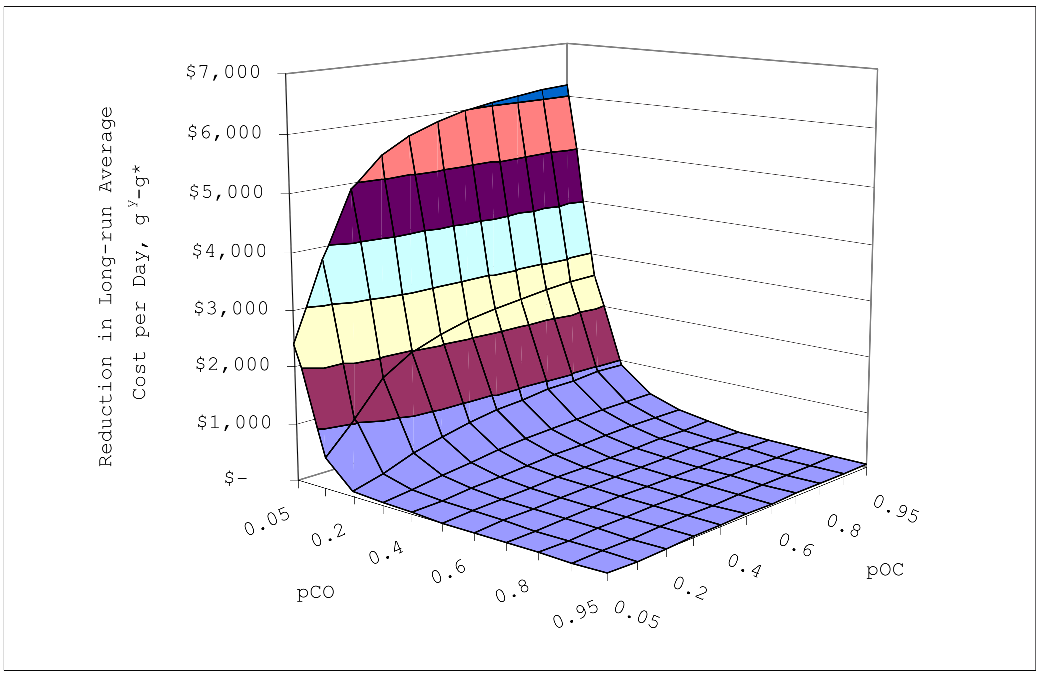

Figure 35: Reduction in long-run average cost per day from contingency planning ($g^y - g^*$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 1: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

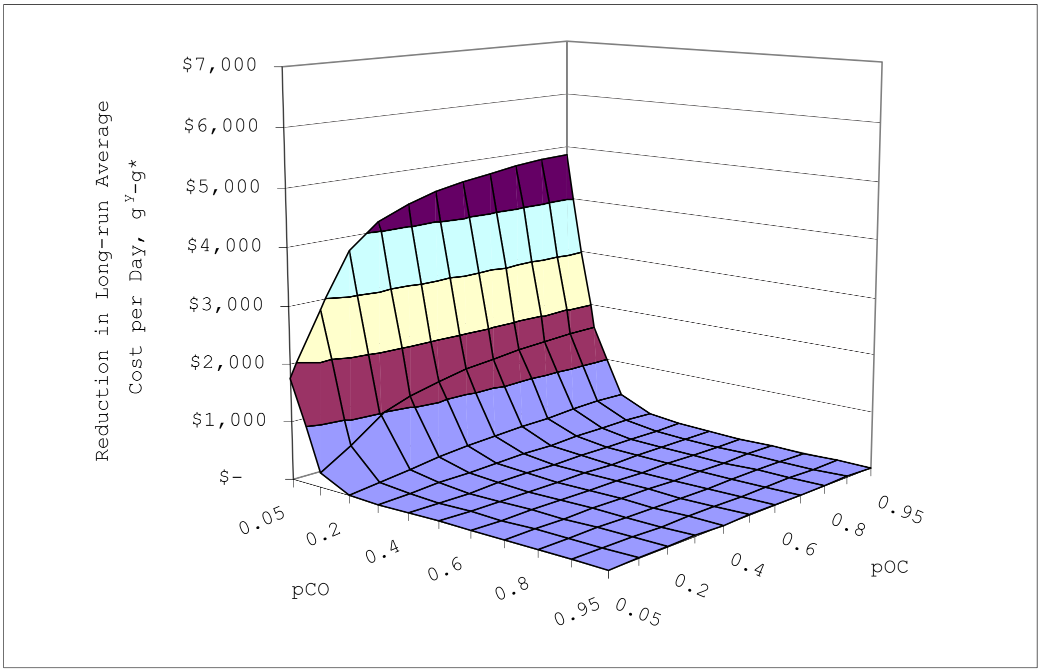

Figure 36: Reduction in long-run average cost per day from contingency planning ($g^y - g^*$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 5: $L = 7$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

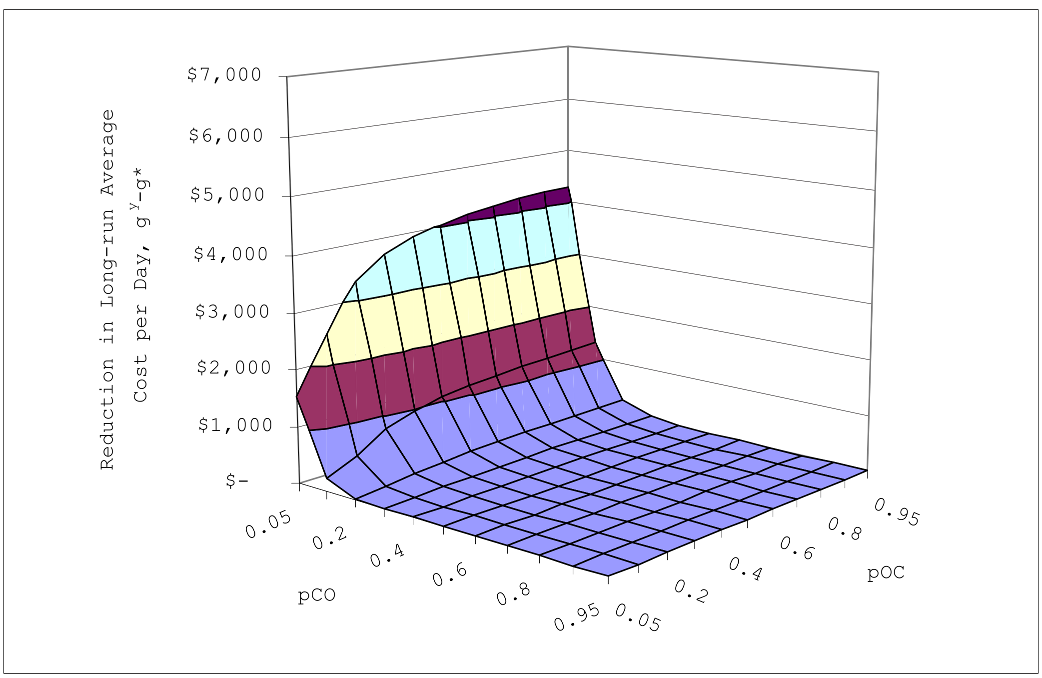

Figure 37: Reduction in long-run average cost per day from contingency planning ($g^y - g^*$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 9: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

Contingency planning for border closures results in greater reductions in the long-run average cost when the minimum leadtime is shorter. The initial intuition is that a shorter minimum leadtime should result in *smaller* reductions in the long-run average cost under the optimal policy since when a closure occurs, new orders will arrive sooner to reduce backordered demand. It is true that in systems with shorter minimum leadtimes, new orders will arrive sooner (or technically, no later) than new orders in systems with longer minimum leadtimes. To understand the intuition behind the greater cost reductions for smaller minimum leadtimes, we compare the optimal order-up-to levels when contingency planning occurs versus when it does not. We know from Theorem 7 that the optimal order-up-to level is non-decreasing in the minimum leadtime.

For some transition probability pairs, the order-up-to level obtained without contingency planning is still optimal for the system in which border closures may occur. In these cases, there is no benefit to contingency planning. We first observe that the implemented policy is sub-optimal over a greater range of transition probabilities when $L = 1$ than when $L = 7$ or 15. Comparing Tables 11, 15, 19 in Appendix A, we also observe that the optimal order-up-to level changes more frequently when $L = 1$ than when $L = 7$ or 15. Therefore, contingency planning not only results in costs reductions over a greater range of transition probabilities but also in greater cost reductions when $L = 1$ than when $L = 7$ or 15. We also observe that as the minimum leadtime increases, the implemented order-up-to level comprises a greater proportion of the optimal order-up-to level. Table 4 provides an example. Denote the implemented order-up-to level by y. The quantity $(y/y^*) * 100$ represents the proportion of the optimal order-to-level comprised by the implemented order-up-to level. As the minimum leadtime increases, the implemented order-up-to comprises a greater percentage of the optimal order-up-to level.

We interpret the effects discussed above as follows. When there is no possibility of border closures, a smaller minimum leadtime provides the manufacturer with greater responsiveness to changes in its inventory. The manufacturer takes advantage by implementing a small order-up-to level, knowing that it can quickly replenish its inventory when necessary in exactly L periods. As the minimum leadtime increases, the manufacturer manages the increasing demand uncertainty by increasing the order-up-to level. When the minimum leadtime increases in a system in which the border may actually close, the manufacturer manages both the increasing demand uncertainty and also the increasing supply uncertainty (e.g. about the future state of the border) by increasing the order-up-to level. Therefore when the implemented policy is utilized in a system subject to border closures, the additional inventory buffer against demand uncertainty due a longer minimum leadtime also serves to mitigate the effects of greater supply uncertainty. We therefore see the cost reductions from contingency planning decreasing as the minimum leadtime increases.

3.5 Extension: Border Closures with Positive Inland Transportation Times

An extension of the border closure model without congestion includes a positive inland transportation time. We present the model for this extension and a theorem that proves the optimal policy is state-invariant, but we do not study the model numerically. As will be seen from the structure of the leadtime probability distribution, a numerical study of this extension would provide the same insights as the model with negligible inland transportation time. Assume that the transportation time from the border to the manufacturer is deterministically $T > 0$ periods. The order movement function is

$$M(k|O) = \begin{cases} k+1 & \text{if } 0 \leq k < T-1; \\ \gamma & \text{if } k = T. \end{cases}$$

and

$$M(k|C) = \begin{cases} k+1 & \text{if } 0 \leq k < L \text{ and } L < k; \\ L & \text{if } k = L. \end{cases}$$

Following a similar derivation as in the proof of Proposition 1, we have the following results. The probability mass function of $L(i)$ is

$$P(L(i) = l) = \begin{cases} 0 & \text{if } l < L+T, \\ p_{iO}^{(L)} & \text{if } l = L+T, \\ p_{iC}^{(L)} p_{CC}^{l-L-T-1} p_{CO} & \text{if } L+T < l. \end{cases}$$

The cumulative distribution function is

$$P(L(i) \le l) = \begin{cases} 0 & \text{if } l < L+T, \\ p_{iO}^{(L)} & \text{if } l = L+T, \\ 1 - p_{iC}^{(L)} p_{CC}^{l-L-T} & \text{if } l > L+T. \end{cases}$$

Also,

$$P(L(i) \le l \le L(i_+)) = \begin{cases} 0 & \text{if } l < L+T, \\ p_{iO}^{(L)} & \text{if } l = L+T, \\ p_{iO}^{(L)} p_{OC} p_{CC}^{l-L-T-1} & \text{if } l > L+T, \end{cases}$$

and

$$\delta_i = 1 + \frac{p_{iC}^{(L)} - p_{iC}^{(L+1)}}{p_{CO}}.$$

THEOREM 8. *For the border closure model without congestion and with positive inland transportation time, $y^*(O) = y^*(C)$.*

Proof. The proof follows that of Theorem 6, letting the minimum leadtime be $L' = L + T$. □

3.6 Extension: Border Closures with a Maximum Delay

In the border closure model without congestion, while the border was closed, all order waiting at and arriving to the border were forced to wait at the border until it reopened. An alternative and less severe border closure model without congestion limits the maximum time an order can wait at the border to T periods. Therefore an order that arrives to a closed border at time $t + L$ will remain at the border until the period in which the border reopens, or until time $t + L + T$, whichever comes first. In the latter case, the order will cross the border even if the border remains closed. Once the border reopens, as before, all orders arriving to, or waiting at, the border cross and arrive at the manufacturer. In this section, we present the model for this extension but do not study it numerically.

The order movement function is

$$M(k|O) = \begin{cases} k+1 & \text{if } 0 \le k < L, \\ \gamma & \text{if } k \ge L \end{cases} \tag{26}$$

$$M(k|1) = \begin{cases} k+1 & \text{if } 0 \leq k < L+T, \\ \gamma & \text{if } k = L+T. \end{cases} \tag{27}$$

Following a similar derivation as in the proof of Proposition 1, we have the following results. The probability mass function of $L(i)$ is

$$P(L(i) = l) = \begin{cases} 0 & \text{if } l < L, \\ p_{iO}^{(L)} & \text{if } l = L, \\ p_{iC}^{(L)} p_{CC}^{l-L-1} p_{CO} & \text{if } L < l < L+T, \\ p_{iC}^{(L)} p_{CC}^{T-1} & \text{if } l = L+T, \\ 0 & \text{if } L+T < l. \end{cases}$$

The cumulative distribution function is

$$P(L(i) \leq l) = \begin{cases} 0 & \text{if } l < L, \\ 1 - p_{iC}^{(L)} p_{CC}^{l-L} & \text{if } L \leq l < L+T, \\ 1 & \text{if } L+T \leq l. \end{cases}$$

Also,

$$P(L(i) \leq l \leq L(i_+)) = \begin{cases} 0 & \text{if } l < L, \\ p_{iO}^{(L)} & \text{if } l = L, \\ p_{iO}^{(L)} p_{OC} p_{CC}^{l-1-L} & \text{if } L < l < L+T, \\ p_{iC}^{(L)} p_{CC}^{T} & \text{if } l = L+T, \\ 0 & \text{if } L+T < l, \end{cases}$$

and

$$\delta_i = p_{iO}^{(L)} + \frac{p_{iO}^{(L)} p_{OC} \left(1 - p_{CC}^{T-1}\right)}{1 - p_{CC}} + p_{iC}^{(L)} p_{CC}^{T}.$$

It is not clear whether the optimal policy will be state-invariant. This is a subject for future research.

3.7 *Extension: Border Closures with Multiple Open States and a Single Closed State*

Since September 11, 2001, the US Department of Homeland Security has implemented a color-coded Homeland Security Advisory System (HSAS) that was created to signal to government agencies, communities, and industry the current risk of a terrorist attack on the US homeland or to a specific government agency, community, or industry. It also provides protective measures to be implemented if appropriate. One of the protective measures for government agencies associated with the highest threat condition (highest level of risk) includes the following, "Monitoring, redirecting, or constraining transportation systems," [2]. We now present an extension of the border closure model without congestion that has multiple open states and a single closed state. We do not study this model numerically.

We model an exogenous system similar to the HSAS. Consider an exogenous system consisting of ν states that all represent an open border and a single state representing a closed border. Let $S_I = \{O_1, O_2, ..., O_\nu, C\}$ and define $S_O = \{O_1, O_2, ..., O_\nu\}$. The ν open states are such that $k \le k'$ implies $p_{O_k C} \le p_{O_{k'} C}$, meaning the open states are ordered according to increasing risk of border closure. The order movement function is defined as follows. For $k = 1, 2, ..., \nu$,

$$M(k|O_k) = \begin{cases} k+1 & \text{if } 0 \le k < L, \\ \gamma & \text{if } k = L, \end{cases}$$

and

$$M(k|C) = \begin{cases} k+1 & \text{if } 0 \le k < L, \\ L & \text{if } k = L. \end{cases}$$

Following a similar derivation as in the proof of Proposition 1, we have the following results. The probability mass function of $L(i)$ is

$$P(L(i) = l) = \begin{cases} 0 & \text{if } l < L, \\ 1 - p_{iC}^{(L)} & \text{if } l = L, \\ p_{iC}^{(L)} p_{CC}^{l-L-1} \sum_{j \in S_O} p_{Cj} & \text{if } L < l \le L+T, \\ 0 & \text{if } L+T < l. \end{cases}$$

The cumulative distribution function is

$$P(L(i) \le l) = \begin{cases} 0 & \text{if } l < L, \\ 1 - p_{iC}^{(L)} & \text{if } l = L, \\ 1 - p_{iC}^{(L)} p_{CC}^{l-L} & \text{if } l > L. \end{cases}$$

Also,

$$P(L(i) \le l \le L(i_+)) = \begin{cases} 0 & \text{if } l < L, \\ 1 - p_{iC}^{(L)} & \text{if } l = L, \\ \sum_{j \in S_O} p_{ij}^{(L)} p_{jC} p_{CC}^{l-L-1} & \text{if } l > L, \end{cases}$$

and

$$\delta_i = 1 - p_{iC}^{(L)} + \frac{1 - p_{iC}^{(L+1)}}{1 - p_{CC}}.$$

It is not clear whether the optimal policy will be state-invariant. In fact, since the open states represent increasing probabilities of border closure, the intuition is that the optimal policy would not be state-invariant. Rather, we would expect the order-up-to level to be larger for states with larger probabilities of closure. This type of policy would represent proactive planning for disruptions. The study of this model is a subject for future research.

3.8 Conclusions

In this chapter we use a specialization of the model in [36] to study an inventory control problem where an important and timely supply chain disruption, e.g. border closures, may occur. We derive the probability mass function and cumulative distribution function for the order leadtime random variables as well as two other important quantities in Proposition 1. The optimality of a state-invariant basestock policy is proved in Theorem 6. Theorem 7 shows that the optimal state-invariant order-up-to level is non-decreasing in the minimum leadtime. We present the results of a comprehensive numerical study that are determined using the procedures described in Chapter 2. The study investigates the impacts on the optimal order-up-to level and the long-run average cost of the border transition probabilities, the minimum leadtime, the economic parameters, and the demand distribution. We also examine the reduction in long-run average cost resulting from contingency planning for border closures.

The optimal inventory policy and long-run average cost are much more sensitive to the expected duration of a disruption than to the occurrence likelihood of a disruption. While prevention of a disruption is critically important, these results have important implications for business to engage and cooperate with government to reduce the duration of border closures through effective disruption management and contingency planning. The numerical results regarding the impacts on the optimal order-up-to level with respect to the minimum leadtime, holding and penalty cost parameters, and demand distribution illustrate the monotonicity results proven in Chapter 2 and in this chapter. Contingency planning for border closures is clearly important and provides greater benefits when the minimum leadtime is small due to the way in which the manufacturer manages demand and supply uncertainty over the minimum leadtime.

In the final sections, we present three interesting extensions of the border closure model without congestion including the addition of a positive inland transportation time, a maximum delay at the border, and multiple open states representing increasing risks of closure. We provide the probability mass function and cumulative distribution functions for the order leadtime and two other important quantities used in expected cost calculations. For the border closure model without congestion and with positive inland transportation time, we show that the optimal policy is state-invariant in Theorem 8.

CHAPTER IV

AN INVENTORY CONTROL MODEL WITH POSSIBLE BORDER CLOSURES AND CONGESTION

4.1 Introduction

The inventory control model presented in Chapter 3 included the simplifying assumption that border congestion was negligible, even after periods of border closure. In reality, congestion following border disruptions is an important consideration. A 2003 report from Booz Allen Hamilton presented the results of a port security wargame in which a terrorist attack using "dirty bombs" in intermodal containers was simulated [15]. The actions taken by the participating business and government leaders had significant consequences: every port in the United States was shut down for eight days, requiring 92 days to reduce the resulting backlog of container deliveries. Another example of border congestion following a closure occurred after the 10-day lock-out of dockworkers in the fall of 2002 at 29 Western US seaports. The resulting congestion and delays did not dissipate for months [3]. Congestion is clearly a major concern following border disruptions.

In this chapter we specialize the inventory control model presented in Chapter 2 to represent an inventory control model subject to border closure and the resulting congestion. We describe the border system in which the border may be open or closed and which includes a customer queue through which orders are processed. We develop the probability mass function for the order leadtime, which is more complex than in the border closure model without congestion. We prove by counter-example that the optimal policy for the border closure model with congestion is not state-invariant and discuss a special property exhibited by certain border states. We present the results of a comprehensive numerical study which are determined using the value iteration algorithm for Markov decision problems. Based on the results, we provide managerial and policy insights regarding business operations and the management of the infrastructure utilized by supply chains (e.g. ports of entry).

The optimal inventory policy is observed to be more reactive than proactive, meaning that the manufacturer tends to only change its order-up-to levels after a border closure has occurred and while congestion remains, rather than in anticipation of a border closure and congestion. The results show that the optimal order-up-to levels and long-run average cost are again much more sensitive to the expected duration of a disruption than to the occurrence likelihood of a disruption, and these quantities increase more than linearly with the utilization of the border queueing system. These results have important implications for business to engage and cooperate with government in contingency planning and disruption management and for business to encourage government investment to improve the processing capabilities of publicly owned and/or operated ports of entry in order to reduce the effects of post-disruption congestion. Contingency planning is again critically important for a manufacturer facing border closures and congestion, especially in supply chains with small leadtimes from the supplier to the international border. Additionally we observe that the optimal order-up-to levels and long-run average cost exhibit similar characteristics with respect to the leadtime from the supplier to the international border, the holding and penalty cost parameters, and the demand distribution.

4.2 Problem Statement

The problem statement is similar to that presented in section 3.2 and differences will be addressed in this section. Consider a supply chain consisting of a foreign supplier and a domestic manufacturer. Orders are shipped on a fixed transportation route from the supplier to a domestic port of entry for importation (e.g. a seaport or land border); the transit time is $L > 0$ periods. Assume that the inland transportation time between the port of entry and the manufacturer is negligible. Multiple orders may be outstanding at any given time, and order crossover does not occur. The inventory system is periodic-review and experiences periodic, stochastic non-negative, integer-valued demand. Demand that cannot be satisfied from the on-hand inventory is fully backordered. Ordering costs are linear in the amount ordered and holding and penalty costs are respectively assessed for any on-hand inventory held or backordered demand. The manufacturer has complete knowledge of its on-hand inventory, backorders, outstanding orders, and the state of the border system at any given time. The objective is to determine the ordering policy that minimizes that long-run average cost of the system and the minimum long-run average cost itself. We specialize the inventory control model presented in Chapter 2 to represent this inventory system.

4.2.1 Characteristics of the Border System

When congestion was negligible for the border closure model in Chapter 3, the only relevant information regarding the border was whether its status was open or closed. To incorporate non-negligible congestion, we augment the notion of the state of the border in the following manner using a simple deterministic queuing model. The state now consists of the border status (e.g. whether the border is open or closed) and the number of "customers" in a queue at the border waiting to be processed (e.g. waiting to cross). A customer represents some unit of work to be completed at the border. For example, if a seaport imports an average of $1,000$ containers per day, then we may define a customer to represent a collection of 100 containers or 500 containers. In these respective examples, on average respectively 10 or 2 customers are processed by the seaport each day.

We describe the border status system with the following discrete-time Markov chain model. Let the state space of the border status system be $S_I = \{O, C\}$ where $i_t = O$ indicates that the border is open in period t and $i_t = C$ indicates that it is closed. The transition probability matrix is

$$P_I = \begin{pmatrix} 1 - p_{OC} & p_{OC} \\ p_{CO} & 1 - p_{CO} \end{pmatrix},$$

where we again assume that $0 < p_{OC} < 1$ and $0 < p_{CO} < 1$, since the extreme values result in uninteresting systems. The stationary distribution of this chain is

$$\pi^I = \{\pi_O^I, \pi_C^I\} = \left\{ \frac{p_{CO}}{p_{OC} + p_{CO}}, \frac{p_{OC}}{p_{OC} + p_{CO}} \right\}.$$

Let $S_N = \{0, 1, 2...\}$. Define the complete border state space to be $S_B = S_I \times S_N$. For all $t \geq 0$, state $(i_t, n_t) \in S_B$, where i_t and n_t are respectively the border status and the number of customers in the border queue at time t. If S_X is the state space of the inventory position, then define the complete state space to be $S^y = S_B \times S_X^y$.

The customer queue at the border follows a first in, first out (FIFO) processing discipline. Let $a(i_t)$ and $b(i_t)$ respectively be the number of new customers arriving to the border and the maximum number of customers that can be processed in period t when $i_t = i$, where

$$a(i_t) = r_0, \tag{28}$$

and

$$b(i_t) = \begin{cases} r_1 & \text{if } i_t = O, \\ 0 & \text{if } i_t = C. \end{cases} \tag{29}$$

We assume r_0 and r_1 are finite, positive integer constants. Regardless of the border status, customers always arrive to the border; however, when the border is closed, no customers can be processed. The queue length at the border at time $t+1$ is

$$\begin{aligned} n_{t+1} &= (n_t + a(i_t) - b(i_t))^+ \\ &= (n_t + r_0 - b(i_t))^+ \\ &= \begin{cases} (n_t + r_0 - r_1)^+ & \text{if } i_t = O, \\ n_t + r_0 & \text{if } i_t = C, \end{cases} \end{aligned} \tag{30}$$

where $(x)^+ = max\{x, 0\}$.

We define the utilization of the border system to be

$$\begin{aligned} \rho &= \lim_{t\to\infty} \frac{E[a(i_t)]}{E[b(i_t)]} \\ &= \frac{r_0}{\pi_O^I r_1}, \end{aligned} \tag{31}$$

where E is the expectation operator conditioned on i_0, and the limit is the Cesaro limit. To ensure queue length stability, we assume $\rho < 1$. We are therefore only interested in systems such that

$$r_0 < \pi_O^I r_1. \tag{32}$$

Let $\mathbf{z_t} = \{z_{kt}, k \in S_l\}$ be the vector of outstanding orders where z_{kt} represents the cumulative order quantity in position k at time t and $S_l = \{-(L-1), ..., -1, 0\} \cup \{1, 2, 3, ...\}$

be the set of order positions. Positions $k \in \{-(L-1), ..., -1, 0\}$ correspond to orders that are in transit to the border from the supplier, e.g. z_{kt} corresponds to the total order quantity that is $-k$ periods away from arriving at the border. Position $k \in \{1, 2, 3, ...\}$ corresponds to customer k in the border queue (we describe the relation between customers and orders in the next paragraph). Since the queue follows a FIFO processing discipline, the customers are processed from highest to lowest position (this is done for ease of notation when we declare the order movement functions). In addition, position $-L$ represents the current order and the dummy position γ represents all orders that have arrived.

Orders and customers move through the border queueing system in the following manner. At time t, $a(i_t) = r_0$ new customers arrive at the border and join the border queue. The outstanding order in position 0 (e.g. the order arriving to the border in the current period) is *assigned* to the last of these arriving customers. Orders move through the queueing system with their assigned customers but do not affect the customers' movements. Then $\min\{n_t, b(i_t)\}$ customers in the queue are processed. Recall that $b(i_t)$ is maximum number of customers that can be processed in a period. Therefore, if the queue length is less than $b(i_t)$, only the n_t customers in the queue are processed. Finally, all outstanding order positions are updated.

Note that by assigning the order at the border to the last arriving customer, we are modeling a worst-case processing scenario. A simple modification assigns the order at the border to each arriving customer with discrete, uniform probability, e.g. since r_0 customers arrive in each period, the arriving order is assigned to each customer with probability $1/r_0$. We do not consider this modification in this thesis.

For all $(i, n) \in S_B$, the order movement function, $M(k|i, n)$, gives the position to which the order currently in position k will move in the next period. Given $(i_t, n_t) = (i, n)$ and $b(i) = b$, we define the order movement function as follows. If $b \geq n + r_0$, the entire queue is processed and

$$M(k|i,n) = \begin{cases} \gamma & \text{if } k \geq 0, \\ k+1 & \text{if } k < 0. \end{cases} \tag{33}$$

If $n < b < n + r_0$, the existing queue is processed but not all of new customers and

$$M(k|i,n) = \begin{cases} \gamma & \text{if } k > 0, \\ k+1 & \text{if } k \leq 0. \end{cases} \tag{34}$$

Finally if $b < n$, only part of the existing queue is processed and none of the new customers and

$$M(k|i,n) = \begin{cases} \gamma & \text{if } k > n-b, \\ r_0 + k & \text{if } 0 < k \leq n-b, \\ k+1 & \text{if } k \leq 0. \end{cases} \tag{35}$$

Note that this order movement function prevents order crossover. Given $(i_t, n_t) = (i, n)$, let $M^l(k|i,n)$ be the random variable representing the position to which the order in position k will move at time $t + l$.

4.2.2 The Leadtime Probability Distribution

The leadtime random variable of the order placed in period t given $(i_t, n_t) = (i, n)$ is

$$L(i,n) = \min_{l \geq L} \left\{ M^{l+1}(0|i,n) = \gamma \right\}. \tag{36}$$

Note that $L(i,n)$ is finite with probability one.

We now develop the probability mass function for the leadtime random variable, $L(i,n)$ for all states $(i,n) \in S_B$. The first task is to develop the probability mass function of $L(i,n)$. In the border closure model without congestion, only the border status was required to determine the leadtime probability mass function. If the border was closed when the order arrived to the border, the order waited at the border until it opened. If the border was open upon arrival, then *all* orders arriving to, or waiting at, the border crossed the border.

In the model with congestion, developing the probability mass function for the order leadtime is more difficult. If the border is open at time $t + l$ for some $l > 0$, all orders arriving to, or waiting at, the border are not necessarily processed in period $t + l$. Whether an order is processed depends on both the border status *and* on the specific position of the order in the customer queue.

We now present a proposition that provides necessary and sufficient conditions for an order placed at time t to cross the border in period $t + l$. The following random variable will be useful. Let $N_{ij}(t,l)$ be an integer random variable representing the number of visits to state j during the time interval $[t, t+l]$ given $i_t = i$. Recall that $P(L(i,n) = l) = 0$ for all $l < L$.

PROPOSITION 2. *Given* $(i_t, n_t) = (i, n)$*,* $L(i,n) = l$ *for* $l \geq L$ *if and only if the following two events occur:*

(i) $i_{t+l} = O$*.*

(ii) If $l = L$*, then*

$$n_{t+L} + r_0 \leq r_1. \tag{37}$$

If $l > L$*, then*

$$N_{i_{t+L}O}(t+L, l-L-1) = \beta, \tag{38}$$

where

$$\beta = \begin{cases} \lfloor \alpha \rfloor & \text{if } \alpha \notin Z, \\ \alpha - 1 & \text{otherwise,} \end{cases} \tag{39}$$

and

$$\alpha = \frac{n_{t+L} + r_0}{r_1}. \tag{40}$$

Proof. If $i_{t+l} = C$, then $b(C) = 0$ and the order cannot be processed in that period. Thus i_{t+l} must be O. The second condition accounts for the dynamics of the customer queue. Given $(i_t, n_t) = (i, n)$, an order placed at time t arrives to the border at time $t + L$ where the length of the queue is the random variable n_{t+L}. Then r_0 customers arrive and the order is assigned to the last arriving customer. We refer to this queue after the customer arrivals as the *full queue*. There are $n_{t+L} + r_0$ customers in the full queue and the order placed at time t is at the very end. If the order is to be processed in period $t + l$, then all customers that have arrived to the queue by the end of period $t + L$ must be completely processed by the end of period $t + l$, and moreover, the last of these customers must be processed *in* period $t + l$.

If the order is to be processed in period $t + L$, then the number of customers processed in period $t + L$ must be *at least* the number of customers in the full queue. That is,

$$n_{t+L} + r_0 \leq b(i_{t+L}) = b(O) = r_1. \tag{41}$$

The equalities hold since $i_{t+L} = O$ from part (i) and from equation (29).

If the order is to be processed in period $t + l$ for $l > L$, then the number of customers processed during the time interval $[t + L, t + l]$ must be at least the number of customers in the full queue. This condition is clearly necessary but not sufficient, since it allows the full queue to be completely processed in a period prior to period $t + l$. Therefore a second condition is required to ensure that by the end of period $t + l - 1$, there are still a positive number of customers remaining in the full queue. This means that the order has not yet been processed. These two conditions are represented by the following two inequalities:

$$N_{i_{t+L}O}(t + L, l - L)r_1 \geq n_{t+L} + r_0, \tag{42}$$

and

$$N_{i_{t+L}O}(t + L, l - L - 1)r_1 < n_{t+L} + r_0. \tag{43}$$

Note that by definition, $N_{ij}(t, l)$ equals $N_{ij}(t, l - 1)$ or $N_{ij}(t, l - 1) + 1$ with probability one. However given that the condition in part (i) holds, $N_{i_{t+L}O}(t + L, l - L) = N_{i_{t+L}O}(t + L, l - L - 1) + 1$ with probability one. Plugging into equation (42), we have

$$N_{i_{t+L}O}(t + L, l - L - 1)r_1 + r_1 \geq n_{t+L} + r_0$$

$$\Longleftrightarrow$$

$$N_{i_{t+L}O}(t + L, l - L - 1)r_1 \geq n_{t+L} + r_0 - r_1. \tag{44}$$

Combining equations (44) and (43), we have

$$n_{t+L} + r_0 - r_1 \leq N_{i_{t+L}O}(t + L, l - L - 1)r_1 < n_{t+L} + r_0,$$

and finally dividing by r_1, we have

$$\frac{n_{t+L} + r_0}{r_1} - 1 \leq N_{i_{t+L}O}(t + L, l - L - 1) < \frac{n_{t+L} + r_0}{r_1}$$

$$\Longleftrightarrow$$

$$\alpha - 1 \leq N_{i_{t+L}O}(t + L, l - L - 1) < \alpha. \tag{45}$$

Assume α is integer. Since $N_{i_{t+L}O}(t+L, l-L-1)$ is integer-valued, equation (45) holds if and only if $N_{i_{t+L}O}(t+L, l-L-1) = \alpha - 1$. Now assume that α is not integer. Then equation (45) holds if and only if $N_{i_{t+L}O}(t+L, l-L-1) = \lfloor \alpha \rfloor$. This completes the proof of part (ii). The conditions in parts (i) and (ii) are both necessary and sufficient. $\square$

We now need to determine the probability distributions of $N_{ij}(t,l)$ for all i and j in S_I, $t \geq 1$, and $l \geq 1$ as well as of $(i_{t+L}, n_{t+L} | i_t = i, n_t = n)$ for all $(i,n) \in S_B$. When $L > 1$, we believe that deriving the probability distribution of the random variables $(i_{t+L}, n_{t+L} | i_t = i, n_t = n)$, and therefore the probability distribution of $L(i,n)$, requires explicit enumeration of all border state sample paths of the length $L+1$ for each initial state $(i,n) \in S_B$. Investigating this probability distribution and approximations of the distribution is a subject for future research.

We first consider the probability distribution of the random variable $N_{ij}(t,l)$ for all i and j in S_I, $t \geq 1$, and $l \geq 1$. Due to the Markov property, $N_{ij}(t,l)$ is identically distributed for all t. The probability distribution of the number of visits to a specific state in a two-state Markov chain during l consecutive periods is derived in [14] and we present the derivation here for convenience. Consider a sequence of l periods following an initial period and a corresponding set of random variables $X_0, X_1, ..., X_l$ such that

$$X_t = \begin{cases} 1 & \text{if } i_t = O, \\ 0 & \text{if } i_t = C, \end{cases} \tag{46}$$

for $t = 0, 1, 2, ..., l$. From the transition probabilities of the border status Markov chain, for $t = 1, 2, ..., l$,

$$P(X_t = 0 | X_{t-1} = 1) = P(i_t = C | i_{t-1} = O) = p_{OC}, \tag{47}$$

and

$$P(X_t = 1 | X_{t-1} = 0) = P(i_t = O | i_{t-1} = C) = p_{CO}. \tag{48}$$

Let the number of visits to state O in l periods (not including the initial period) be

$$S = \sum_{t=1}^{l} X_t. \tag{49}$$

During a sequence of l state transitions, there will be a number of state changes from state O to state C and vice versa. Let Z be the total number of states changes (including from the initial state). If Z is even, define $\eta = \theta = 0.5Z$, and if Z is odd, define $\eta = \lceil 0.5Z - 1 \rceil$ and $\theta = \lceil 0.5Z \rceil$. The probability of s visits to state O in l periods following an initial visit to state O is

$$P(S = s|l \text{ periods}, i_0 = O) = \begin{cases} p_{OO}^{s} p_{CC}^{l-s} \sum_{z=1}^{Z_O} \binom{s}{\eta}\binom{l-s-1}{\theta-1}\left(\frac{p_{OC}}{p_{CC}}\right)^{\theta}\left(\frac{p_{CO}}{p_{OO}}\right)^{\eta} & \text{if } 0 \leq s < l, \\ p_{OO}^{l} & \text{if } s = l, \\ 0 & \text{otherwise,} \end{cases} \tag{50}$$

where $Z_O = l + 0.5 - |2s + 0.5 - l|$. Note that this expression for Z_O corrects the expression that is given in [14]. Similarly,

$$P(S = s|l \text{ periods}, i_0 = C) = \begin{cases} p_{CC}^{l} & \text{if } s = 0, \\ p_{OO}^{s} p_{CC}^{l-s} \sum_{z=1}^{Z_C} \binom{s-1}{\theta-1}\binom{l-s}{\eta}\left(\frac{p_{OC}}{p_{CC}}\right)^{\eta}\left(\frac{p_{CO}}{p_{OO}}\right)^{\theta} & \text{if } 0 < s \leq l, \\ 0 & \text{otherwise,} \end{cases} \tag{51}$$

where $Z_C = l + 0.5 - |2s - 0.5 - l|$.

From equations (50) and (51), we can write the probability distribution for $N_{ij}(t, l)$ for all $i \in S_I$, $j = O$, $t \geq 0$ and $l \geq 1$ is

$$P(N_{OO}(t, l) = 1 + s) = \begin{cases} p_{OO}^{s} p_{CC}^{l-s} \sum_{z=1}^{Z_O} \binom{s}{\eta}\binom{l-s-1}{\theta-1}\left(\frac{p_{OC}}{p_{CC}}\right)^{\theta}\left(\frac{p_{CO}}{p_{OO}}\right)^{\eta} & \text{if } 0 \leq s < l, \\ p_{OO}^{l} & \text{if } s = l, \\ 0 & \text{otherwise,} \end{cases} \tag{52}$$

$$P(N_{CO}(t, l) = s) = \begin{cases} p_{CC}^{l} & \text{if } s = 0, \\ p_{OO}^{s} p_{CC}^{l-s} \sum_{z=1}^{Z_C} \binom{s-1}{\theta-1}\binom{l-s}{\eta}\left(\frac{p_{OC}}{p_{CC}}\right)^{\eta}\left(\frac{p_{CO}}{p_{OO}}\right)^{\theta} & \text{if } 0 < s \leq l, \\ 0 & \text{otherwise.} \end{cases} \tag{53}$$

The following lemma will be useful in describing the probability mass function of $L(i, n)$.

LEMMA 9. *For a two-state DTMC, consider an initial period followed a consecutive sequence of periods during which there Z state changes (including from the initial period). If the initial period visits state i, then the last period visits state i if and only if Z is even.*

If the initial period visits state i, then the last period visits state $j \neq i$ if and only if Z is odd.

Proof. Consider the following two-state DTMC with transition probabilities $p_{ij} = p_{ji} = 1$ for $i \neq j$ and transition probability matrix P. Transitions in this DTMC represent the state changes during a consecutive sequence of periods (including from the initial period) in another independent two-state DTMC. From equation (19), the Z-step transition probability matrix is

$$P^Z = \begin{pmatrix} \frac{1}{2}\left(1+(-1)^Z\right) & \frac{1}{2}\left(1+(-1)^{Z+1}\right) \\ \frac{1}{2}\left(1+(-1)^{Z+1}\right) & \frac{1}{2}\left(1+(-1)^Z\right) \end{pmatrix}. \tag{54}$$

We first prove the forward direction of the if and only if claim. If the initial state is i and Z is even, then $[P^Z]_{(i,i)} = 1$ and $[P^Z]_{(i,j)} = 0$ for $i \neq j$. Similarly if the initial state if i and Z is odd, then $[P^Z]_{(i,i)} = 0$ and $[P^Z]_{(i,j)} = 1$ for $i \neq j$. Next we prove the reverse direction of the if and only if claim. If the initial and final states are i, then $\left[P^Z\right]_{(i,i)}$ must be one. From equation (54), $\left[P^Z\right]_{(i,i)} = \frac{1}{2}\left(1+(-1)^Z\right)$. This implies that $(-1)^Z = 1$ which only holds if Z is even. Similarly, if the initial state is i and the final states is $j \neq i$, then $\left[P^Z\right]_{(i,j)}$ must be one. From equation (54), $\left[P^Z\right]_{(i,j)} = \frac{1}{2}\left(1+(-1)^{Z+1}\right)$. This implies that $(-1)^Z = -1$ which only holds if Z is odd. This completes the proof. □

4.2.2.1 Special Case: Minimum Leadtime, $L = 1$

In order to calculate the probability mass function of $L(i,n)$ for all $(i,n) \in S_B$, we require the probability distribution on $(i_{t+L}, n_{t+L} | i_t = i, n_t = n)$. Consider the special case when the minimum leadtime is one period. In this case, the random variables i_{t+L} and n_{t+L} are independent. Recall from equation (30) that the queue length in the next period is independent from the border status in the next period. Given $(i_t, n_t) = (i,n)$, the queue length at time $t+L = t+1$ is known with probability one. In the following, we highlight the dependence of the random variable β on n_{t+L} by writing $\beta(n_{t+L})$. Note that when $L = 1$ and given $i_t = i$, the values of n_{t+1} and so $\beta(n_{t+1})$ are known with probability one.

To determine $P(L(i,n) = 1 | i_t = i, n_t = n)$ for all $l \geq 0$, we will consider four cases for the value of l. First recall that $P(L(i,n) = 0 | i_t = i, n_t = n) = 0$ since $0 < L$. Next consider the case when $l = L = 1$. From Proposition 2, we have

$$\begin{aligned}
P(L(i,n) = 1 | i_t = i, n_t = n) &= P(i_{t+1} = O, n_{t+1} + r_0 \le r_1 | i_t = i, n_t = n) \\
&= P(i_{t+1} = O, (n + r_0 - b(i))^+ + r_0 \le r_1 | i_t = i, n_t = n) \\
&= \begin{cases} p_{iO} & \text{if } (n + r_0 - b(i))^+ + r_0 \le r_1, \\ 0 & \text{otherwise.} \end{cases}
\end{aligned} \tag{55}$$

Next consider the case when when $l = L + 1$. From Proposition 2, we have
$P(L(i,n) = 2 | i_t = i, n_t = n)$

$$\begin{aligned}
&= P(i_{t+2} = O, N_{i_{t+1}O}(t+1, l-2) = \beta(n_{t+1}) | i_t = i, n_t = n) \\
&= \sum_{j \in S_I} P(i_{t+1} = j, i_{t+2} = O, N_{i_{t+1}O}(t+1, 0) = \beta(n_{t+1}) | i_t = i, n_t = n) \\
&= \sum_{j \in S_I} P(i_{t+2} = O, N_{jO}(t+1, 0) = \beta(n_{t+1}) | i_{t+1} = j, i_t = i, n_t = n) P(i_{t+1} = j | i_t = i, n_t = n) \\
&= \sum_{j \in S_I} P(i_{t+2} = O | N_{jO}(t+1, 0) = \beta(n_{t+1}), i_{t+1} = j, i_t = i, n_t = n) \\
&\qquad\qquad * P(N_{jO}(t+1, 0) = \beta(n_{t+1}) | i_{t+1} = j, i_t = i, n_t = n) p_{ij} \\
&= \sum_{j \in S_I} p_{jO} P(N_{jO}(t+1, 0) = \beta(n_{t+1}) | i_{t+1} = j, i_t = i, n_t = n) p_{ij} \\
&= \begin{cases} p_{iC} p_{CO} & \text{if } \beta(n_{t+1}) = 0, \\ p_{iO} p_{OO} & \text{if } \beta(n_{t+1}) = 1, \\ 0 & \text{otherwise.} \end{cases}
\end{aligned}$$

The final step follows since $N_{jO}(t+1, 0)$ represents the number of visits to state O only in period $t + 1$. Therefore, it can only be 0 (if $j = C$) or 1 (if $j = O$).

Finally consider the case when $l > L + 1$. From Proposition 2, we have

$P(L(i,n) = l | i_t = i, n_t = n)$

$$
\begin{aligned}
&= P(i_{t+l} = O, N_{i_{t+1}O}(t+1, l-2) = \beta(n_{t+1}) | i_t = i, n_t = n) \\
&= \sum_{j \in S_I} P(i_{t+1} = j, i_{t+l} = O, N_{i_{t+1}O}(t+1, l-2) = \beta(n_{t+1}) | i_t = i, n_t = n) \\
&= \sum_{j \in S_I} P(i_{t+l} = O, N_{i_{t+1}O}(t+1, l-2) = \beta(n_{t+1}) | i_{t+1} = j, i_t = i, n_t = n) P(i_{t+1} = j | i_t = i, n_t = n) \\
&= \sum_{j \in S_I} P(i_{t+l} = O | N_{jO}(t+1, l-2) = \beta(n_{t+1}), i_{t+1} = j, i_t = i, n_t = n) \\
&\qquad\qquad * P(N_{jO}(t+1, l-2) = \beta(n_{t+1}) | i_{t+1} = j, i_t = i, n_t = n) p_{ij} \\
&= \sum_{j \in S_I} \sum_{k \in S_I} P(i_{t+l-1} = k, i_{t+l} = O | N_{jO}(t+1, l-2) = \beta(n_{t+1}), i_{t+1} = j, i_t = i, n_t = n) \\
&\qquad\qquad * P(N_{jO}(t+1, l-2) = \beta(n_{t+1}) | i_{t+1} = j, i_t = i, n_t = n) p_{ij} \\
&= \sum_{j \in S_I} \sum_{k \in S_I} P(i_{t+l} = O | i_{t+l-1} = k, N_{jO}(t+1, l-2) = \beta(n_{t+1}), i_{t+1} = j, i_t = i, n_t = n) \\
&\qquad\qquad * P(N_{jO}(t+1, l-2) = \beta(n_{t+1}), i_{t+l-1} = k | i_{t+1} = j, i_t = i, n_t = n) p_{ij} \\
&= \sum_{j \in S_I} \sum_{k \in S_I} p_{kO} P(N_{jO}(t+1, l-2) = \beta(n_{t+1}), i_{t+l-1} = k | i_{t+1} = j, i_t = i, n_t = n) p_{ij} \\
&= \sum_{j \in S_I} \sum_{k \in S_I} P(N_{jO}(t+1, l-2) = \beta(n_{t+1}), i_{t+l-1} = k | i_{t+1} = j, i_t = i, n_t = n) p_{kO} p_{ij}.
\end{aligned}
$$

Recall that given $i_t = i$, $\beta(n_{t+1})$ is known with probability one. We calculate $P(N_{jO}(t+1, l-2) = \beta(n_{t+1}), i_{t+l-1} = k | i_{t+1} = j, i_t = i, n_t = n)$ using equations (52) and (53) and Lemma 9. Lemma 9 is important since it determines the set of values over which we sum in equations (52) and (53) (i.e. the lower and upper limits for z and whether only odd or even values of z should be added). For example if $j = k = O$, then the summation in equation (52) is over the set $\{2 \le z \le Z'_O,\ z \text{ even}\}$ where $Z'_O = \max\{2 \le z' \le Z_O :\ z' \text{ even}\}$.

Therefore for the special case when the minimum leadtime $L = 1$, we can calculate the probability distribution for $L(i,n)$ for all $(i,n) \in S_B$.

4.2.2.2 Remaining Cases: Minimum Leadtime, $L > 1$

Consider the remaining cases when the minimum leadtime is strictly greater than 1. When $L > 1$, deriving the probability distribution on $(i_{t+L}, n_{t+L} | i_t = i, n_t = n)$ requires explicit enumeration of all border state sample paths of the length $L+1$ for each initial state $(i,n) \in S_B$. This is due to the fact that border queue length is non-negative and that the number of customers processed in any period can range from 0 to r_1, depending on the length of the queue. From a practical standpoint, all sample paths can be enumerated and evaluated with a simple computer program in a reasonable amount of time for path lengths up to approximately 20. Beyond this range, some method of approximation seems to be necessary. Let the true joint probability mass function of $(i_{t+L}, n_{t+L}|i_t = i, n_t = n)$ be denoted by $f_{(i,n)}$.

To determine $P(L(i,n) = l|i_t = i, n_t = n)$ for all $l \geq 0$, we again consider four cases for the value of l. First recall that $P(L(i,n) = L|i_t = i, n_t = n) = 0$ for $l < L$. Next consider the case when $l = L$. From Proposition 2, we have

$$\begin{aligned} P(L(i,n) = L|i_t = i, n_t = n) &= P(i_{t+L} = O, n_{t+L} + r_0 \leq r_1|i_t = i, n_t = n) \\ &= P(i_{t+L} = O, n_{t+L} \leq r_1 - r_0|i_t = i, n_t = n) \qquad (56) \\ &= \sum_{0 \leq m \leq r_1 - r_0} P(i_{t+L} = O, n_{t+L} = m|i_t = i, n_t = n) \\ &= \sum_{0 \leq m \leq r_1 - r_0} f_{(i,n)}(O, m). \end{aligned}$$

Next consider the case when $l = L + 1$. From Proposition 2, we have
$P(L(i,n) = L + 1|i_t = i, n_t = n)$

$$\begin{aligned} &= P(i_{t+L+1} = O, N_{i_{t+L}O}(t + L, 0) = \beta(n_{t+L})|i_t = i, n_t = n) \\ &= \sum_{j \in S_I} \sum_{m \geq 0} P(i_{t+L} = j, n_{t+L} = m, i_{t+L+1} = O, N_{i_{t+L}O}(t + L, 0) = \beta(n_{t+L})|i_t = i, n_t = n) \\ &= \sum_{j \in S_I} \sum_{m \geq 0} P(i_{t+L+1} = O, N_{jO}(t + L, 0) = \beta(m)|i_{t+L} = j, n_{t+L} = m, i_t = i, n_t = n) \\ &\qquad\qquad * P(i_{t+L} = j, n_{t+L} = m|i_t = i, n_t = n) \\ &= \sum_{j \in S_I} \sum_{m \geq 0} P(i_{t+L+1} = O|N_{jO}(t + L, 0) = \beta(m), i_{t+L} = j, n_{t+L} = m, i_t = i, n_t = n) \\ &\qquad\qquad * P(N_{jO}(t + L, 0) = \beta(m)|i_{t+L} = j, n_{t+L} = m, i_t = i, n_t = n) f_{(i,n)}(j, m) \\ &= \sum_{j \in S_I} \sum_{m \geq 0} p_{jO} P(N_{jO}(t + L, 0) = \beta(m)|i_{t+L} = j, n_{t+L} = m, i_t = i, n_t = n) f_{(i,n)}(j, m) \\ &= \sum_{m \geq 0:\ \beta(m)=0} f_{(i,n)}(C, m) p_{CO} + \sum_{m \geq 0:\ \beta(m)=1} f_{(i,n)}(O, m) p_{OO} \\ &= \sum_{0 \leq m \leq r_1 - r_0} f_{(i,n)}(C, m) p_{CO} + \sum_{r_1 - r_0 < m \leq 2r_1 - r_0} f_{(i,n)}(O, m) p_{OO}. \end{aligned}$$

The second to last equation follows since $N_{jO}(t + L, 0)$ represents the number of visits to state O only in period $t + L$ and can therefore only take on values of 0 (if $j = C$) or 1 (if $j = O$). The final equation follows from equations (39) and (40). For example $\beta(m) = 0$ for values of $m \geq 0$ such that $\frac{m+r_0}{r_1} \leq 1$, which implies that $m \leq r_1 - r_0$. Since the customer queue length is non-negative, $m \geq 0$ as well.

Finally consider the case when $l > L+1$. From Proposition 2, we have $P(L(i,n) = l | i_t = i, n_t = n)$

$$\begin{aligned}
&= P(i_{t+l} = O, N_{i_{t+L}O}(t+L, l-L-1) = \beta(n_{t+L}) | i_t = i, n_t = n) \\
&= \sum_{j \in S_I} \sum_{m \geq 0} P(i_{t+L} = j, n_{t+L} = m, i_{t+l} = O, N_{i_{t+L}O}(t+L, l-L-1) = \beta(i_{t+L}) | i_t = i, n_t = n) \\
&= \sum_{j \in S_I} \sum_{m \geq 0} P(i_{t+l} = O, N_{jO}(t+L, l-L-1) = \beta(m) | i_{t+L} = j, n_{t+L} = m, i_t = i, n_t = n) \\
&\qquad\qquad * P(i_{t+L} = j, n_{t+L} = m | i_t = i, n_t = n) \\
&= \sum_{j \in S_I} \sum_{m \geq 0} P(i_{t+l} = O, N_{jO}(t+L, l-L-1) = \beta(m) | i_{t+L} = j, n_{t+L} = m, i_t = i, n_t = n) f_{(i,n)}(j,m) \\
&= \sum_{j \in S_I} \sum_{m \geq 0} P(i_{t+l} = O | N_{jO}(t+L, l-L-1) = \beta(m), i_{t+L} = j, n_{t+L} = m, i_t = i, n_t = n) \\
&\qquad\qquad * P(N_{jO}(t+L, l-L-1) = \beta(m) | i_{t+L} = j, n_{t+L} = m, i_t = i, n_t = n) f_{(i,n)}(j,m) \\
&= \sum_{j \in S_I} \sum_{m \geq 0} \sum_{k \in S_I} P(i_{t+l-1} = k, i_{t+l} = O | N_{jO}(t+L, l-L-1) = \beta(m), i_{t+L} = j, n_{t+L} = m, i_t = i, n_t = n) \\
&\qquad\qquad * P(N_{jO}(t+L, l-L-1) = \beta(m) | i_{t+L} = j, n_{t+L} = m, i_t = i, n_t = n) f_{(i,n)}(j,m) \\
&= \sum_{j \in S_I} \sum_{m \geq 0} \sum_{k \in S_I} P(i_{t+l} = O | i_{t+l-1} = k, N_{jO}(t+L, l-L-1) = \beta(m), i_{t+L} = j, n_{t+L} = m, i_t = i, n_t = n) \\
&\qquad\qquad * P(N_{jO}(t+L, l-L-1) = \beta(m), i_{t+l-1} = k | i_{t+L} = j, n_{t+L} = m, i_t = i, n_t = n) f_{(i,n)}(j,m) \\
&= \sum_{j \in S_I} \sum_{m \geq 0} \sum_{k \in S_I} P(N_{jO}(t+L, l-L-1) = \beta(m), i_{t+l-1} = k | i_{t+L} = j, n_{t+L} = m, i_t = i, n_t = n) \\
&\qquad\qquad * p_{kO} f_{(i,n)}(j,m).
\end{aligned}$$

We determine $P(N_{jO}(t+L, l-L-1) = \beta(m), i_{t+l-1} = k | i_{t+L} = j, n_{t+L} = m, i_t = i, n_t = n)$ using equations (52) and (53) and Lemma 9.

Therefore we can calculate the probability distribution for $L(i,n)$ for all $(i,n) \in S_B$ when $L > 1$ if we know the joint probability distribution of $(i_{t+L}, n_{t+L} | i_t = i, n_t = n)$, $f_{(i,n)}$, for all $(i,n) \in S_B$.

4.3 Numerical Results and Discussion

4.3.1 Numerical Study Design

We now present the results of a comprehensive numerical study for the border closure model with congestion. We investigate the impacts on the optimal order-up-to levels and long-run average cost by the system parameters. We will discuss the results of the numerical study in the context of the following supply chain. Consider an international supply chain subject to border closures and congestion in which a domestic manufacturer orders a single product from a single foreign supplier. Orders are measured in units of container loads and are placed each day. The containers are shipped by some mode of transportation where the leadtime from the supplier to the international border of the manufacturer's host nation is deterministically L days. The order arrives to the border and joins the end of a customer queue. If the border is open, some of the customers are processed. If the order has been assigned to one of these customers, it crosses the border and arrives immediately arrives at the manufacturer. Otherwise it remains in the customer queue until its assigned customer is processed.

Table 5 displays the system parameters values that we study, including the customer arrival and processing parameters, r_0 and r_1. The majority of the study parameters are identical to those studied for the border closure model without congestion in Chapter 3. Section 3.4.1 provides descriptions for the identical parameters, while new parameters are described below.

Table 5: Numerical study design (border closure model with congestion).

Parameter	Values
Purchase Cost, c	\$150,000
Holding Cost, h	\$100, \$500
Penalty Cost, p	\$1,000, \$2,000
Minimum Leadtime, L	1, 7, 15
Arrival Rate, r_0	10 customers per period
Maximum Service Rate, r_1	11 customers per period
Transition Probability, p_{OC}	0.001, 0.003, 0.01, 0.02
Transition Probability, p_{CO}	0.05, 0.1, 0.2, 0.3, 0.4, 0.5
Demand Distribution	Poisson(Mean=0.5), Poisson(Mean=1)

The values of r_0 and r_1 were selected to represent a realistic model of potential border congestion, specifically for seaports. Recall the port security wargame in [15] in which eight days of seaport closure resulted in 92 days of congestion and the 10-day closure of Western US seaports that resulted in months of congestion. Under these arrival and processing parameter values, a closure of 10 days results in a queue length of 100 customers. In the best case scenario, this queue will take 100 days to reduce to zero length. If the border remains open for 100 days, in each period $r_0 = 10$ customers arrive and $r_1 = 11$ are processed,

Table 6: Border utilization (ρ) vs. transition probabilities (p_{OC}, p_{CO}).

		p_{CO}					
		0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	0.911	0.911	0.912	0.914	0.918	0.927
	0.003	0.915	0.916	0.918	0.923	0.936	0.964
	0.01	0.927	0.932	0.939	0.955	1.000	1.091
	0.02	0.945	0.955	0.970	1.000	1.091	1.273

thereby reducing the queue length by one customer per day. We note that as r_1 becomes very large relative to r_0, the border closure model with congestion can be approximated by the border closure model without congestion. As r_1 increases relative to a fixed value of r_0, the probability that the border queue achieves a positive length decreases because the build up of a queue requires increasingly longer closures. For example, suppose $r_0 = 1$ and $r_1 = 1,000$. If $p_{CO} = 0.05$ and the border closes when the queue is initially empty, a positive queue can occur after 1,001 periods of consecutive closure which occurs with probability 5.03×10^{-23}. In section 4.3.5, we vary the value of r_1 to investigate the impacts of the border utilization rate.

For this numerical study, we consider a reduced set of transition probabilities compared to that in Chapter 3. The reduced set of transition probabilities represents a more realistic model of border closures and represents an expected inter-closure time ranging from approximately 3 years to 50 days (respectively $p_{OC} = 0.001$ and $p_{OC} = 0.02$) and an expected closure time ranging from 20 days to 2 days (respectively $p_{OC} = 0.05$ and $p_{OC} = 0.5$). We must also restrict our attention to transition probabilities that satisfy the border utilization constraint given in equation (32), e.g. the border utilization must be strictly less than one. As seen in Table 6, the excluded transition probability pairs are $(p_{OC}, p_{CO}) \in \{(0.01, 0.1), (0.01, 0.05), (0.02, 0.2), (0.02, 0.1), (0.02, 0.05)\}$. The remaining probability pairs result in border utilizations greater than 90%, which we believe is a good representation of many ports of entry. For example, may US seaports operate close to capacity throughout the year.

Table 7 lists the instance numbers and the parameter sets they represent. For labeling purposes for figures and tables, we append each instance number with the letter "C" to indicate the border closure model with congestion. Additionally for figures which do not correspond directly to a specific parameter instance, we write "Instance C" to differentiate the figures from similar ones for the border closure model without congestion. The probability mass function for $(i_{t+L}, n_{t+L}|i_t = i, n_t = n)$ is determined by explicit enumeration for all $(i, n) \in S_B$.

The numerical results in this section are determined using the value iteration algorithm for Markov decision problems. A detailed description of the algorithm can be found in [29]. For practical reasons we must constrain the allowable state space (S, where we remove the dependence on the policy y since it has yet to be determined) to be finite without substantially affecting the optimal solution. The state space of the border status is finite by assumption, but we must additionally constrain the state spaces of the border queue length (S_N) and the inventory position (S_X) to be finite. Given this restriction, the MDP for the border closure model with congestion is a communicating MDP, and it is known that the VIA can be used to solve unichain as well as communicating and weakly communicating MDPs. Also for practical reasons we must constraint the action space without substantially affecting the optimal solution. Specifically, we must constraint the set of order quantities (from which we can determine the order-up-to levels). As an example for Instance 9, we set $S_N = \{0, 1, ..., 200\}$ and $S_X = \{-100, -99, ...49, 50\}$ and consider order quantities in the set $\{0, 1, ..., min\{x, 151\}\}$ when the inventory position is $x \in S_X$.

The VIA terminates in a finite number of iterations with an ϵ-optimal policy, that is the long-run average cost at the termination of the VIA (denoted g_ϵ) satisfies the following inequality: $g_\epsilon - g^* < \epsilon$. Furthermore, we approximate the true optimal long-run average cost as in Theorem 8.5.6(b) in [29] (denoted g') and so $|g' - g^*| < \epsilon/2$. For any positive ϵ, no matter how small, the policy obtained at the termination of the VIA may be sub-optimal. However in this chapter, as is commonly done, we will refer to the policy obtained by the VIA algorithm and to the approximation of the optimal long-run average cost as the optimal policy and the optimal long-run average cost. We set $\epsilon = 0.01$ for this numerical study, which corresponds to a maximum difference between the approximate long-run average cost and the true optimal long-run average cost of less than one half of a cent (if the long-run average cost is measured in dollars).

For the instances considered in this research, reasonable computation times were obtained with the VIA. Continuing the example for Instance 9 from above when $p_{OC} = 0.003$ and $p_{CO} = 0.1$, the run-time of the VIA running on an Intel Pentium 4 Mobile CPU (1.60GHz) processor was 76.53 minutes. The run-times for the other instances for the different transition probability pairs were similar.

Table 7: Parameter instances (border closure model with congestion).

Instance	L	h	p	Demand Dist.
1C	1	\$100	\$1,000	Poisson(0.5)
2C	1	\$100	\$2,000	Poisson(0.5)
3C	1	\$500	\$1,000	Poisson(0.5)
4C	1	\$500	\$2,000	Poisson(0.5)
5C	7	\$100	\$1,000	Poisson(0.5)
6C	7	\$100	\$2,000	Poisson(0.5)
7C	7	\$500	\$1,000	Poisson(0.5)
8C	7	\$500	\$2,000	Poisson(0.5)
9C	15	\$100	\$1,000	Poisson(0.5)
10C	15	\$100	\$2,000	Poisson(0.5)
11C	15	\$500	\$1,000	Poisson(0.5)
12C	15	\$500	\$2,000	Poisson(0.5)
13C	1	\$100	\$1,000	Poisson(1)

4.3.2 Policy Structure

While it is known that a stationary, state-dependent basestock policy is optimal for this model, the following theorem proves by counter-example that the optimal policy is not state-invariant.

THEOREM 9. *For the border closure model with congestion, the optimal order-up-to levels ($y^*(i,n)$) are not state-invariant with respect to the border status (i) nor the customer queue length (n).*

Proof. The numerical results in this section provide counter-examples to the claim that the optimal order-up-to levels are state-invariant. See Table 8 for specific counter-examples. □

This result occurs for two reasons: the congestion resulting from closures and a characteristic of the way in which the customer queue is modeled. The latter reason is discussed in section 4.3.4. In the border closure model with congestion, once a period of closure ends, all orders do not simply cross the border and arrive at the manufacturer. They must be processed through the customer queue on a first come, first served basis, whose length increases for each period of closure. This has the potential effect of increasing the minimum leadtime (which is discussed in section 4.3.4). For example in the border closure model without congestion, if the border is closed for 10 periods, the longest delay experienced by any order is 10 periods and is experienced by the order that arrives to the border when it first closes. Now consider the same 10 period closure in the congestion model. If $r_0 = 10$ and $r_1 = 11$, the longest delay is now at least 100 periods and is experienced by the order that arrives in the period just before the border reopens. The manufacturer responds to the increased potential delays at the border by changing the order-up-to levels.

4.3.3 Impact of the Transition Probabilities

The transition probabilities p_{OC} and p_{CO} are two of the key parameters describing the border system and offer different measures of border closure severity, respectively, expected duration and occurrence likelihood. Recall that if the border is in state i, then the expected number of periods until the border transitions to state j is $1/p_{ij}$. Therefore the expected duration of a border closure is given by $1/p_{CO}$ and the probability of a border closure is given by p_{OC}. In the border closure model with congestion, we expect that the optimal policy will not be state-invariant due the new effects of the congestion caused by border closures. We also expect that the optimal order-up-to levels and the long-run average cost will be more sensitive to the transition probabilities than in the model without congestion.

We now present the optimal order-up-to levels and the optimal long-run average cost for the 12 parameter instances versus the transition probabilities. Recall that in the border closure model with congestion, order-up-to levels are denoted by $y^*(i, n)$, where i is the status of the border (i.e. Open or Closed) and n is the number of customers in the border queue. The results are displayed in Figures 38-74. Figure 62 displays the optimal expected holding and penalty cost per day to highlight the component of the long-run average cost that changes with the transition probabilities. As can be seen in the corresponding tables in Appendix B and in Table 8, the optimal order-up-to levels for open border states exhibit very little variation over the transition probabilities, unlike those for closed states. We therefore do not display the optimal order-order-up to levels for the open border state in the figures. In all figures and tables, we present the order-up-to levels for two queue lengths, 0 and 100 customers. A queue length of 100 customers represents the results of a 10-day border closure.

As we saw for the border closure model without congestion, the results in this section show that the expected duration of a border closure ($1/p_{CO}$) again much more negatively affects a firm's productivity as measured by cost and inventory than the probability of a border closure (p_{OC}). These results have important implications for the interaction between businesses and government to design effective contingency plans that reduce the duration of a potential border closure and reduce the resulting congestion, quickly returning the system to a normal state of operation. In the border closure model with congestion, the impacts of p_{OC} become more important than without congestion. The increases in the optimal order-up-to levels and long-run average cost are greater as p_{OC} changes in the border closure model with congestion than without congestion. This highlights the manufacturer's sensitivity to the potential congestion caused by border closures and the importance of including congestion in inventory planning models.

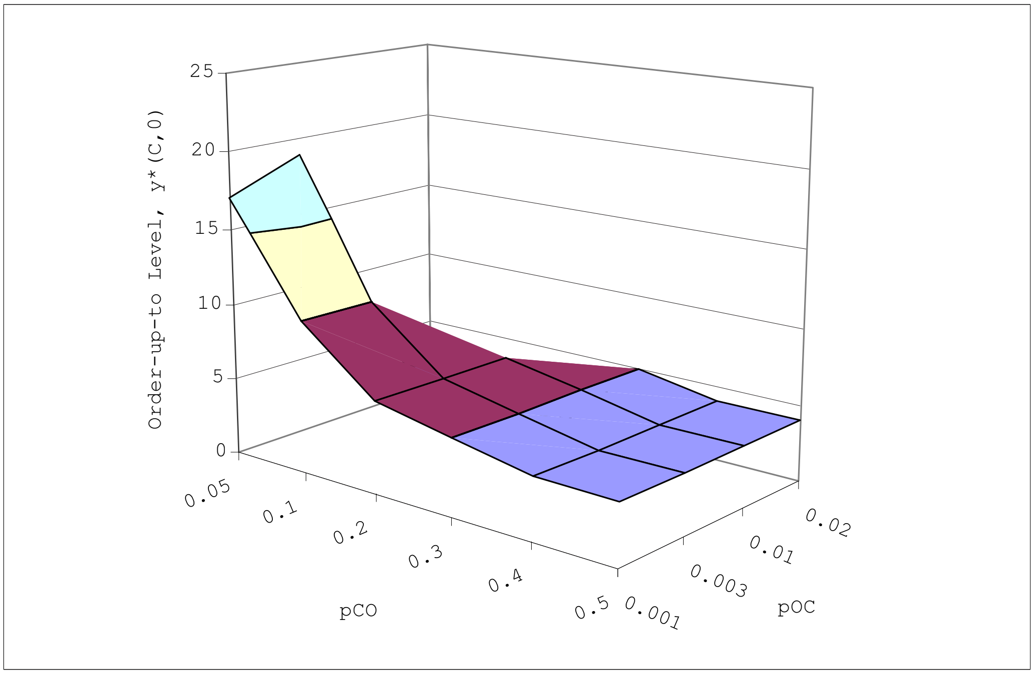

Figure 38: Optimal order-up-to level ($y^*(C,0)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 1C: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

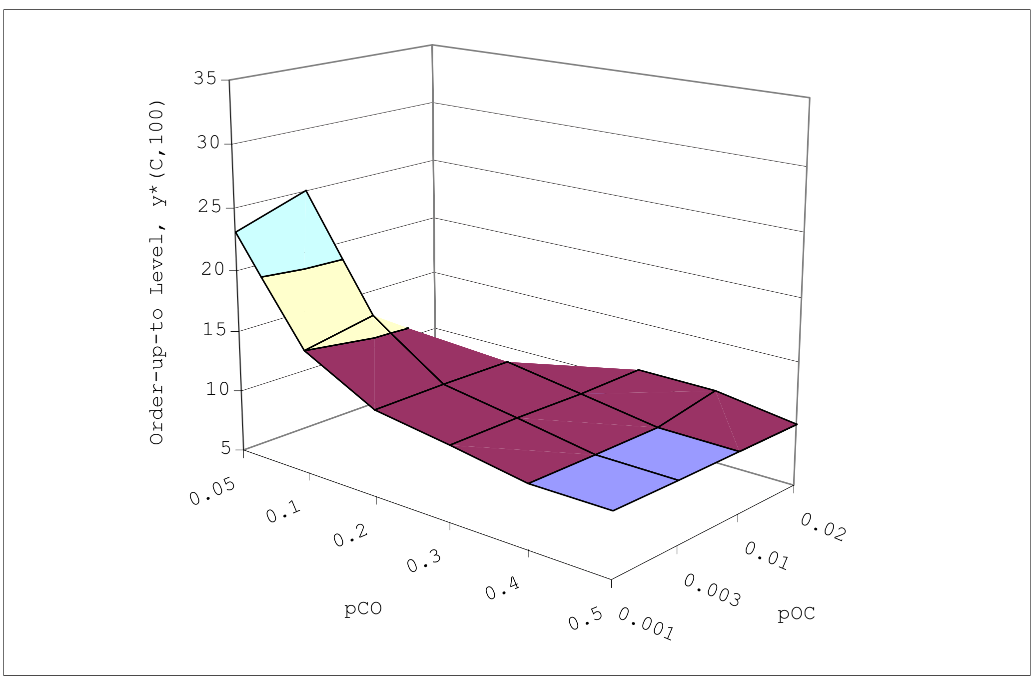

Figure 39: Optimal order-up-to level ($y^*(C,100)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 1C: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

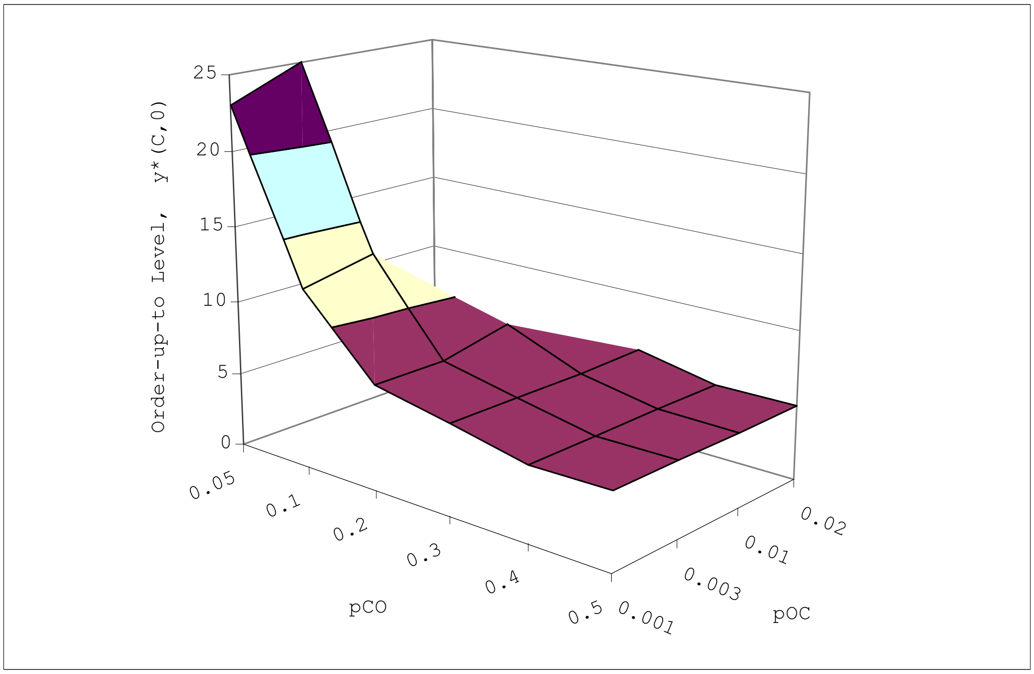

Figure 40: Optimal order-up-to level ($y^*(C, 0)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 2C: $L = 1$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

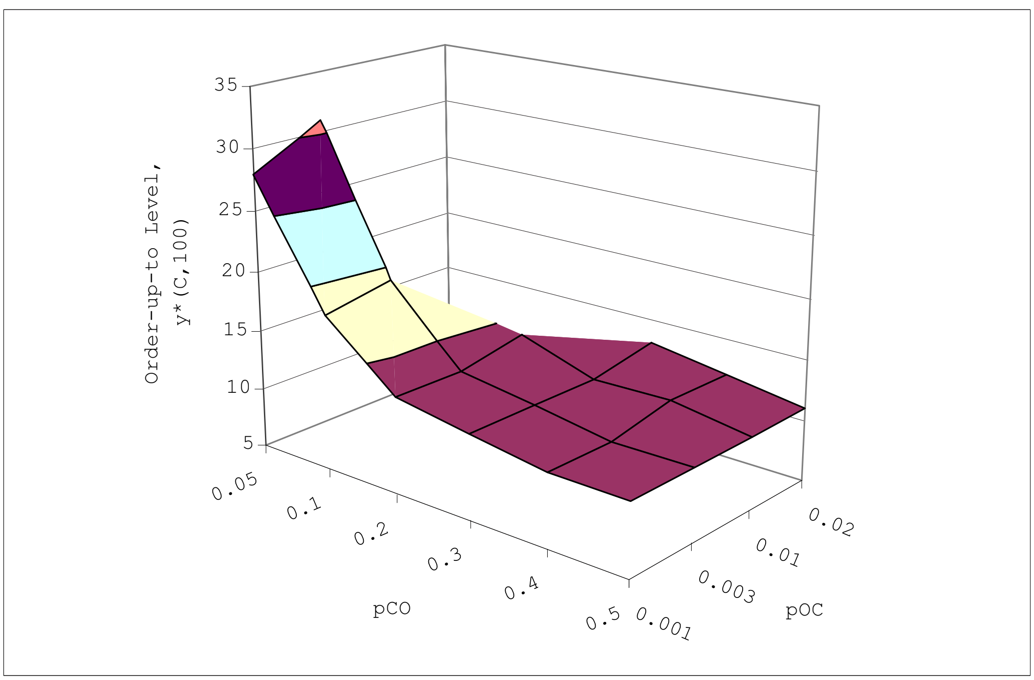

Figure 41: Optimal order-up-to level ($y^*(C, 100)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 2C: $L = 1$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

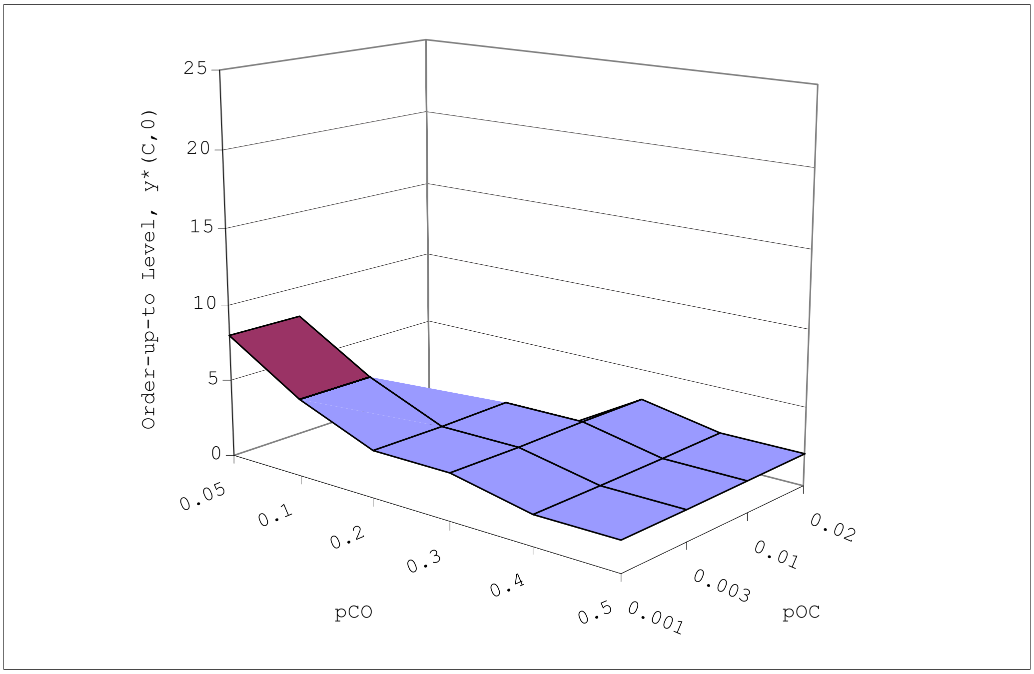

Figure 42: Optimal order-up-to level ($y^*(C,0)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 3C: $L = 1$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

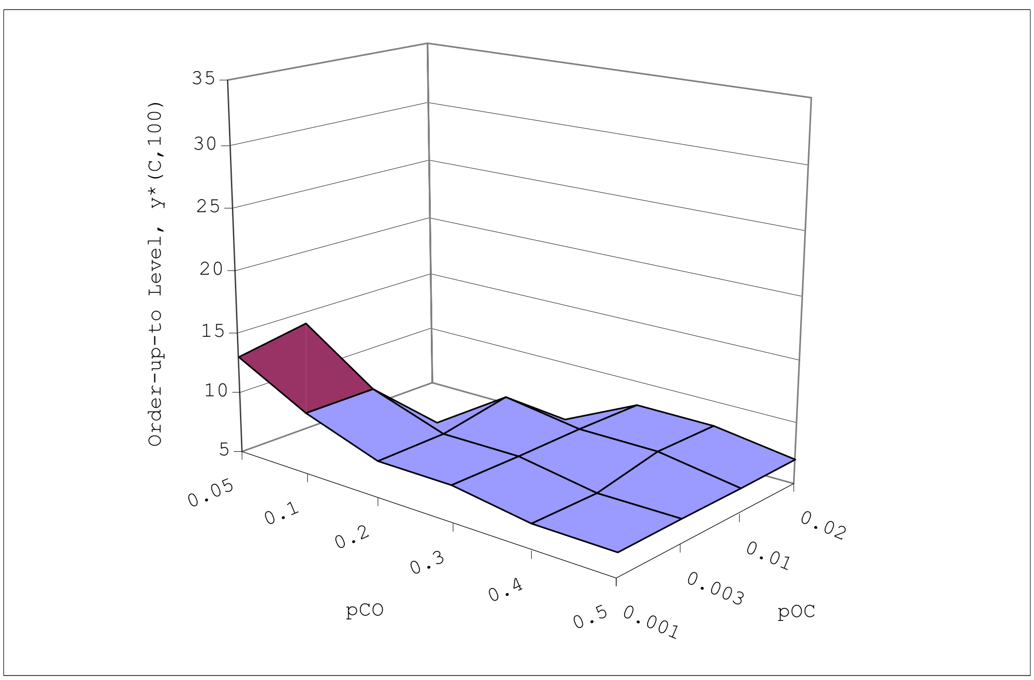

Figure 43: Optimal order-up-to level ($y^*(C,100)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 3C: $L = 1$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

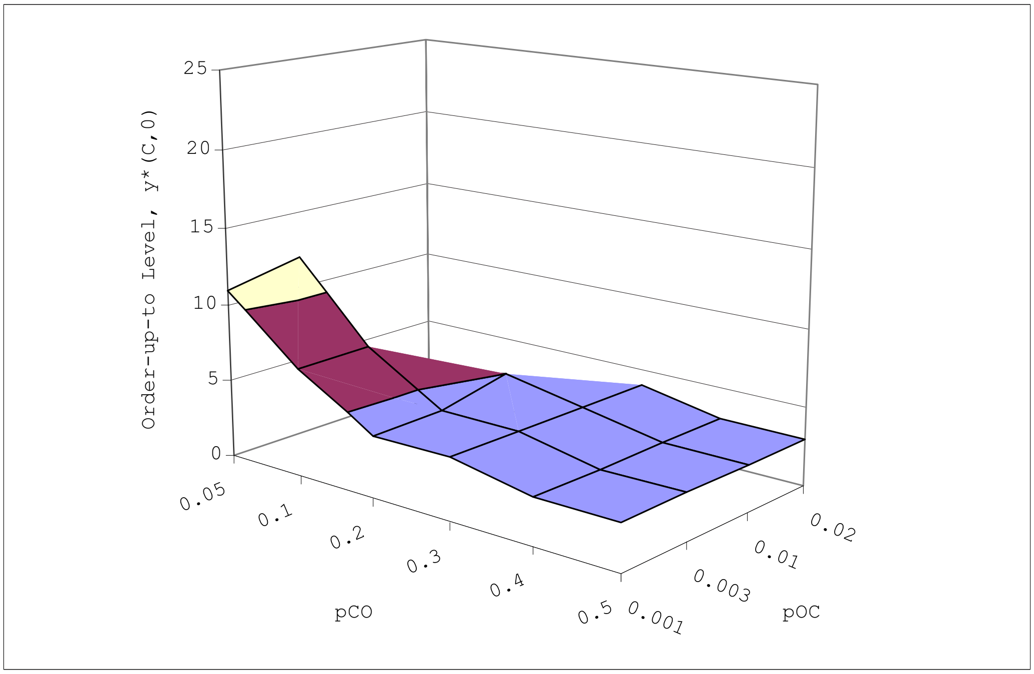

Figure 44: Optimal order-up-to level ($y^*(C,0)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 4C: $L = 1$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

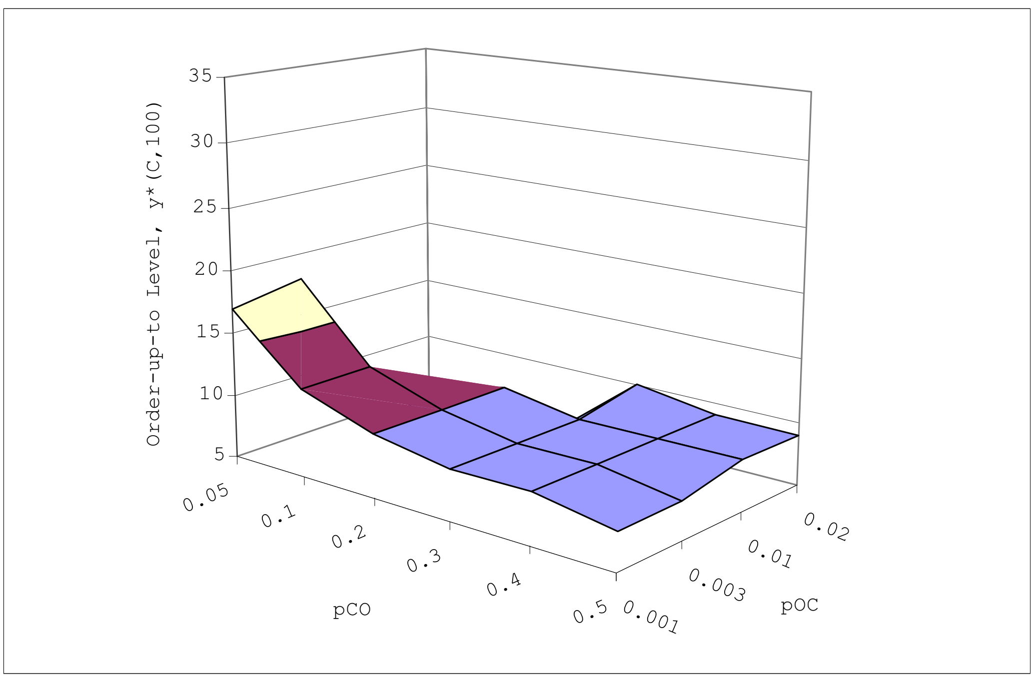

Figure 45: Optimal order-up-to level ($y^*(C,100)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 4C: $L = 1$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

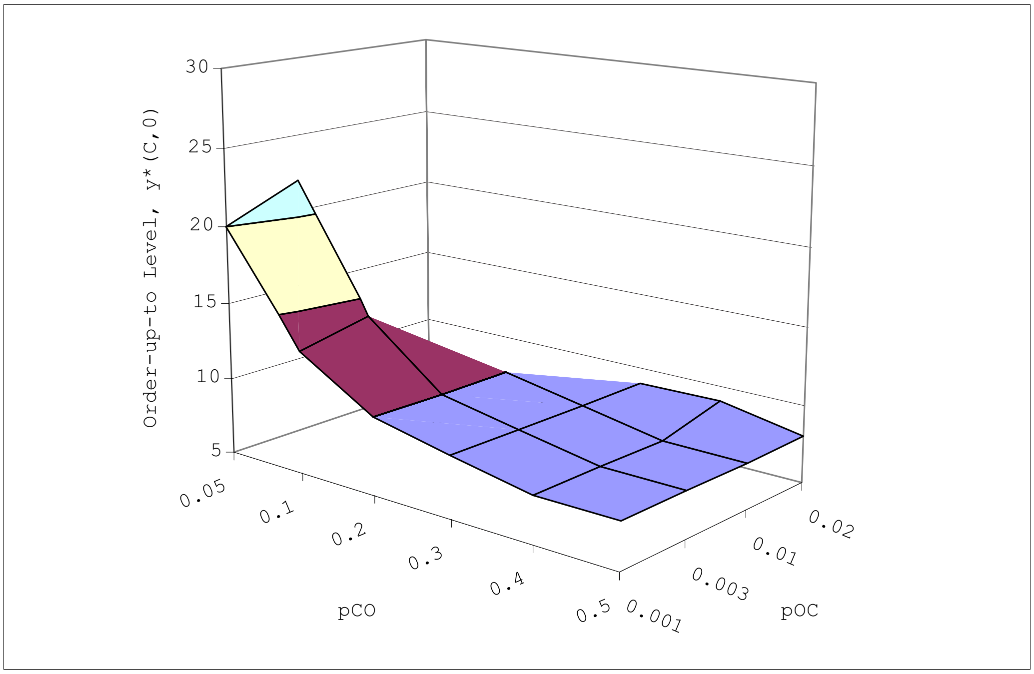

Figure 46: Optimal order-up-to level ($y^*(C, 0)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 5C: $L = 7$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

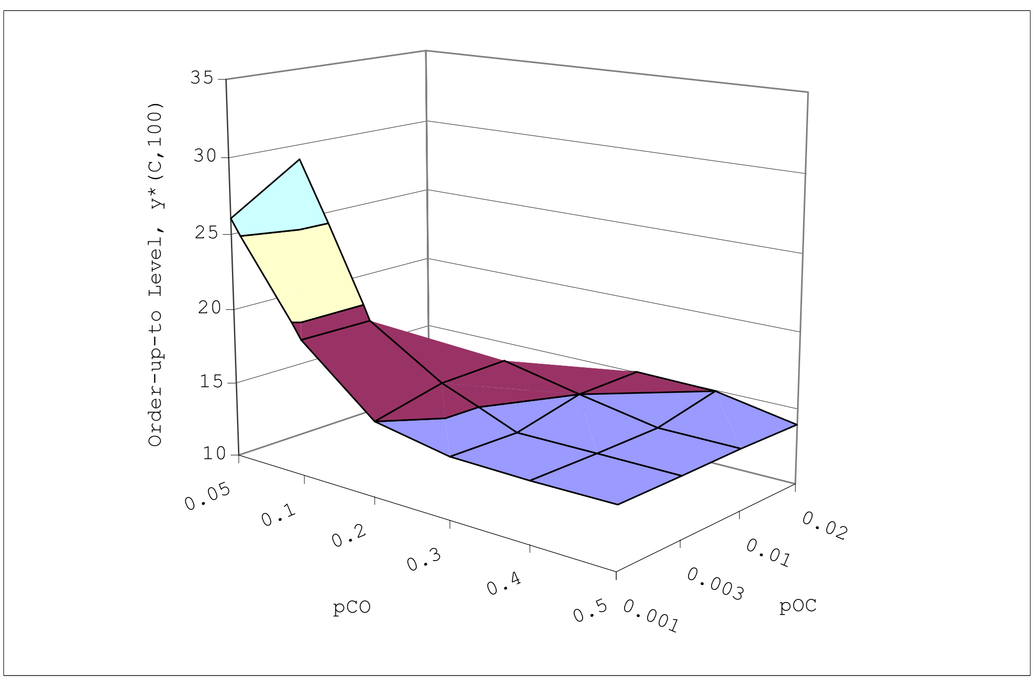

Figure 47: Optimal order-up-to level ($y^*(C, 100)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 5C: $L = 7$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

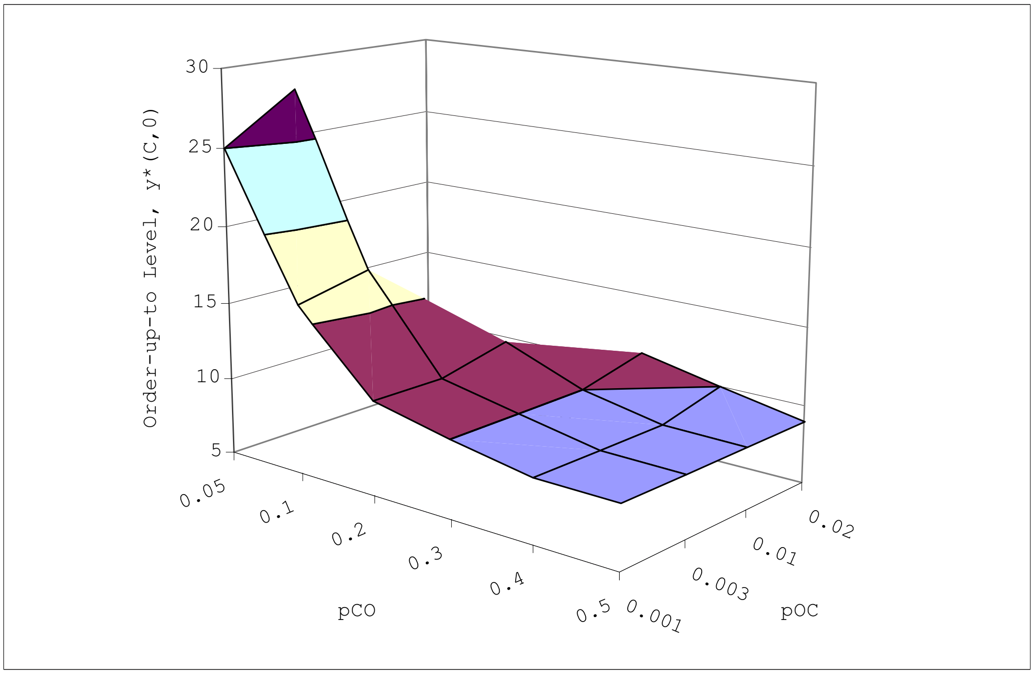

Figure 48: Optimal order-up-to level ($y^*(C, 0)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 6C: $L = 7$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

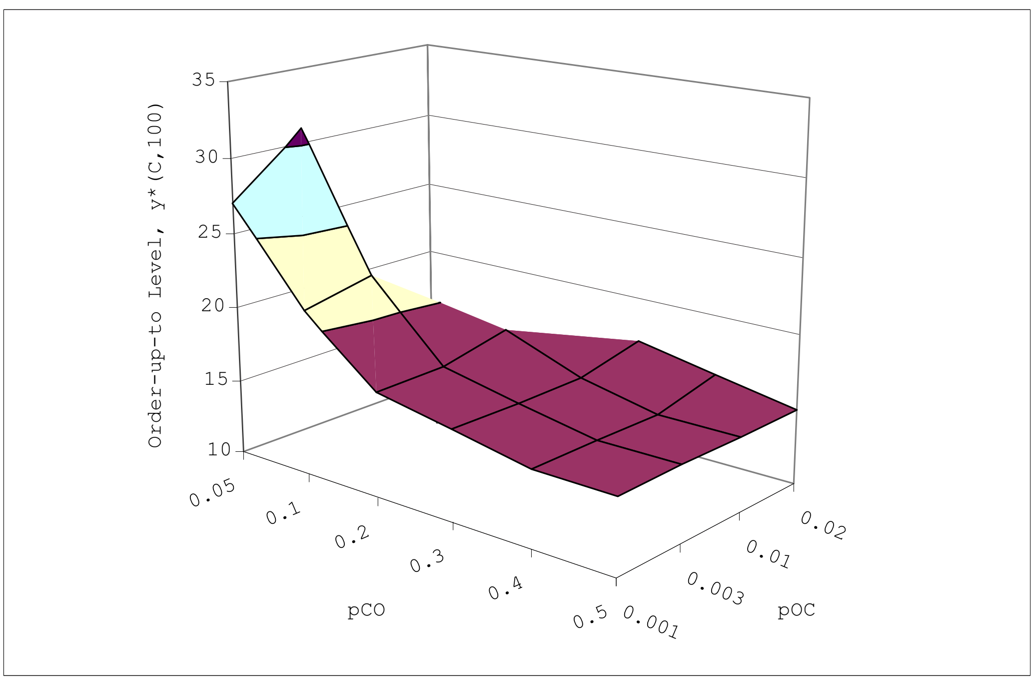

Figure 49: Optimal order-up-to level ($y^*(C, 100)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 6C: $L = 7$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

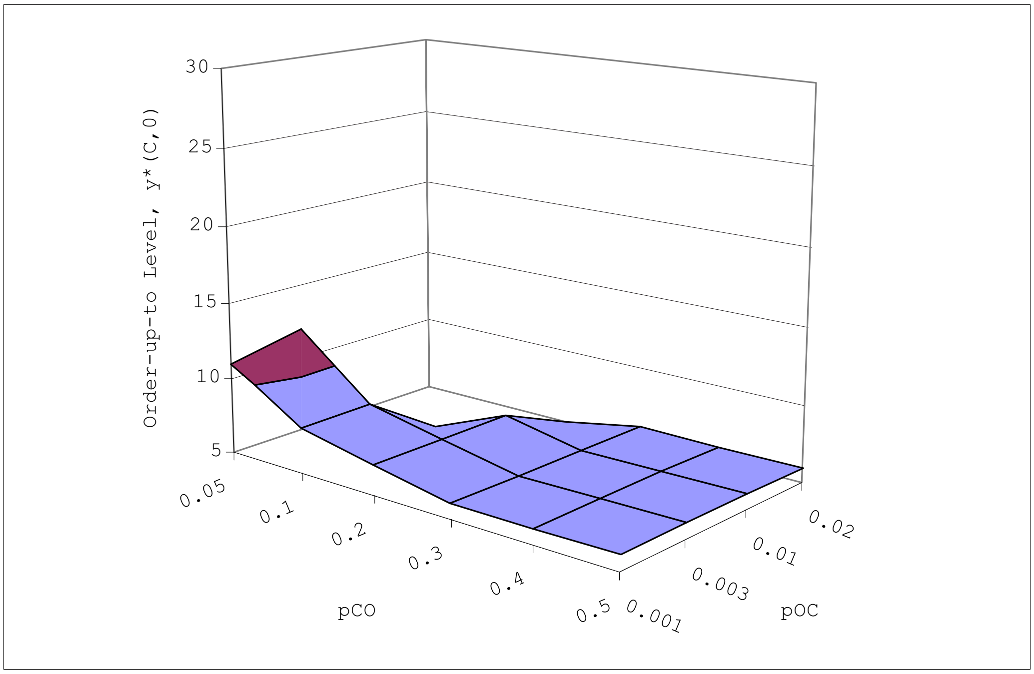

Figure 50: Optimal order-up-to level ($y^*(C, 0)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 7C: $L = 7$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

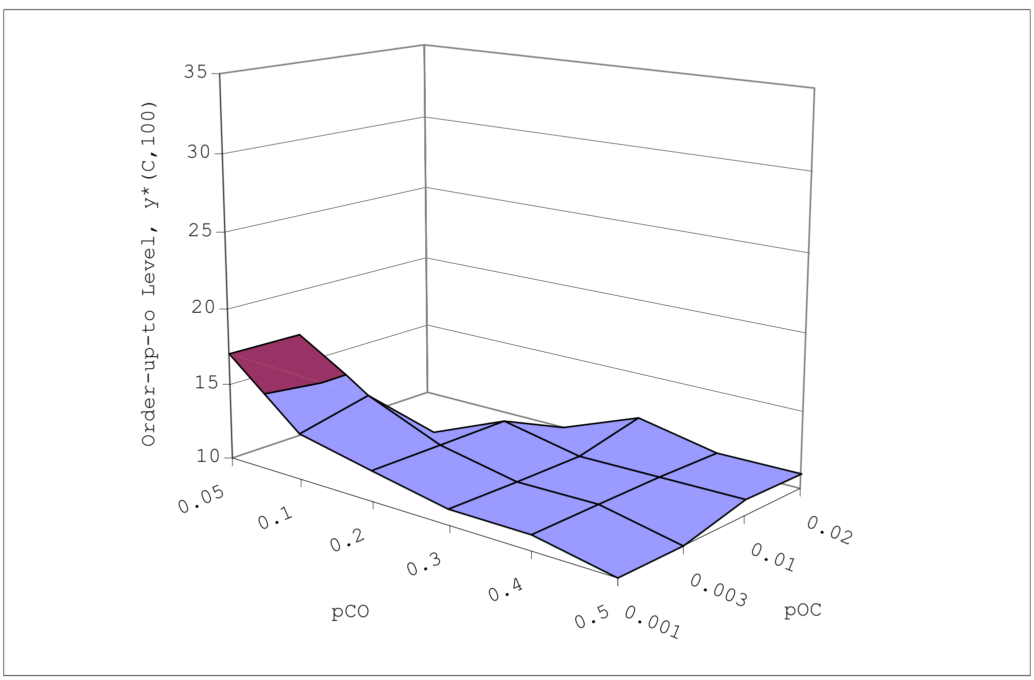

Figure 51: Optimal order-up-to level ($y^*(C, 100)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 7C: $L = 7$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

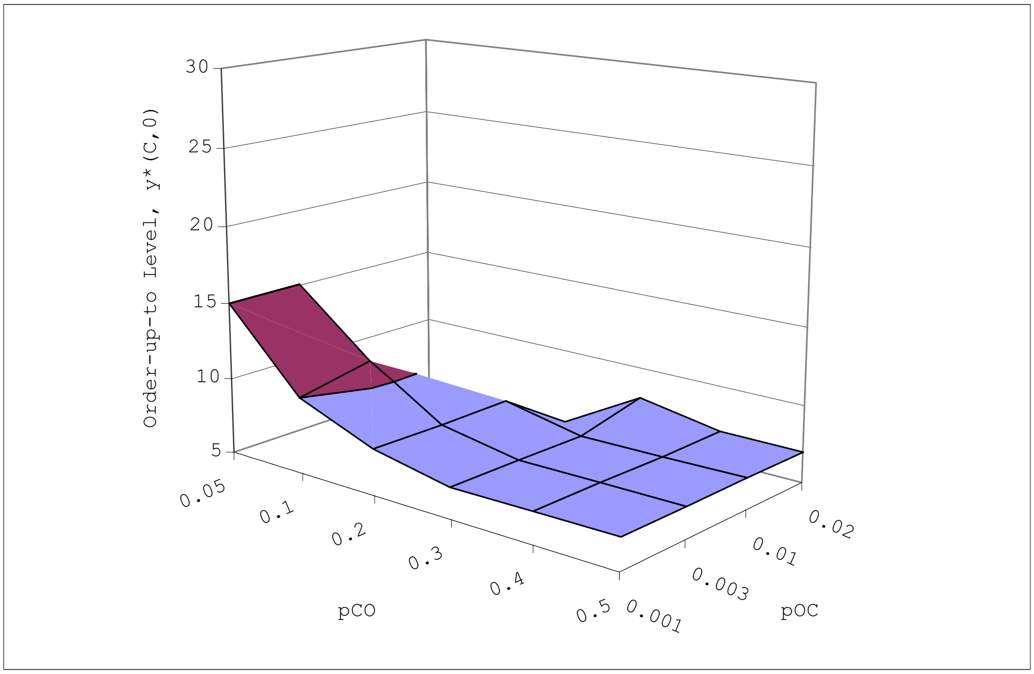

Figure 52: Optimal order-up-to level ($y^*(C,0)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 8C: $L = 7$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

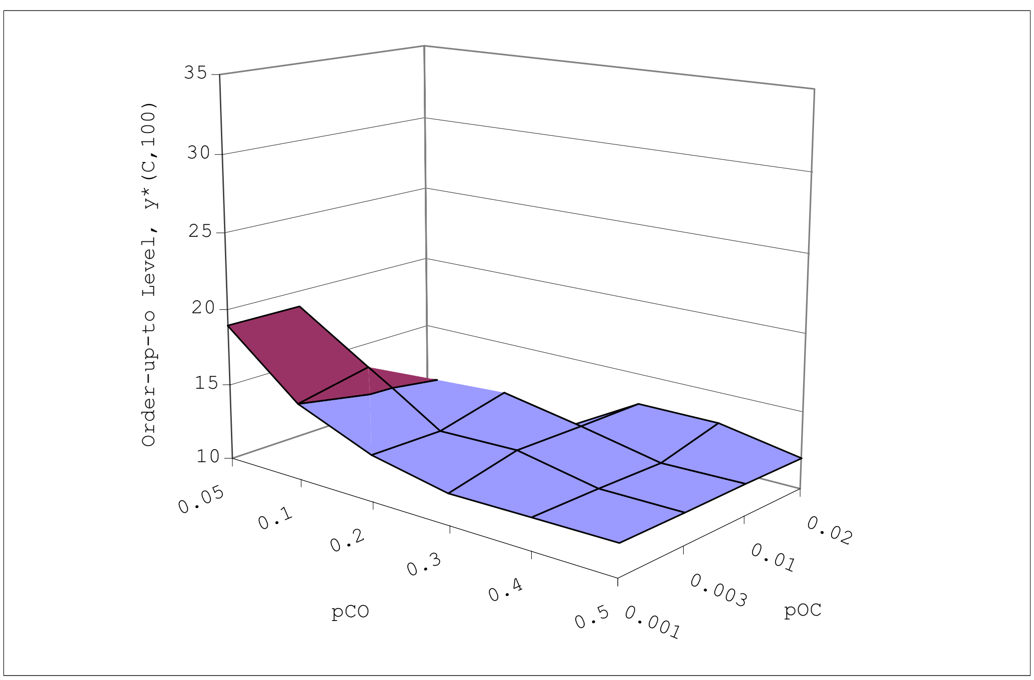

Figure 53: Optimal order-up-to level ($y^*(C,100)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 8C: $L = 7$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

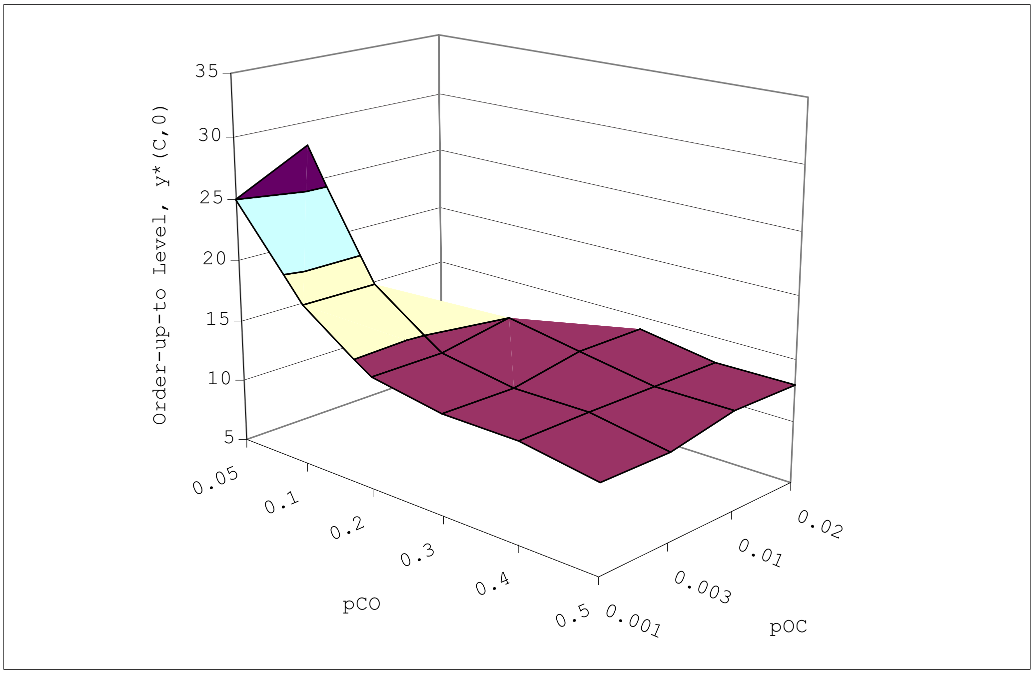

Figure 54: Optimal order-up-to level ($y^*(C, 0)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 9C: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

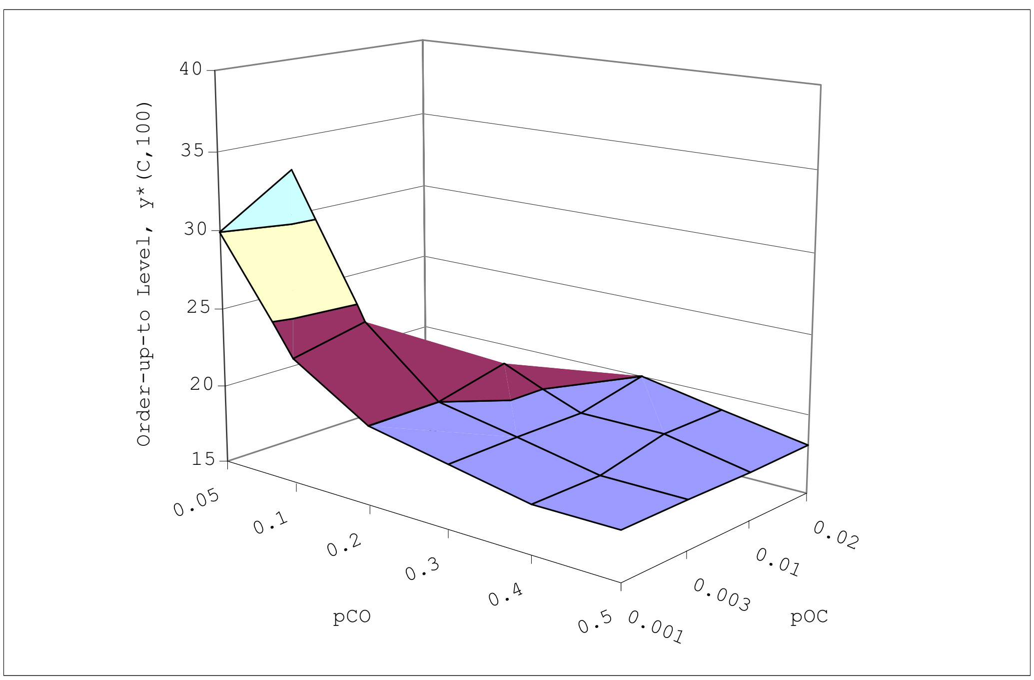

Figure 55: Optimal order-up-to level ($y^*(C, 100)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 9C: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

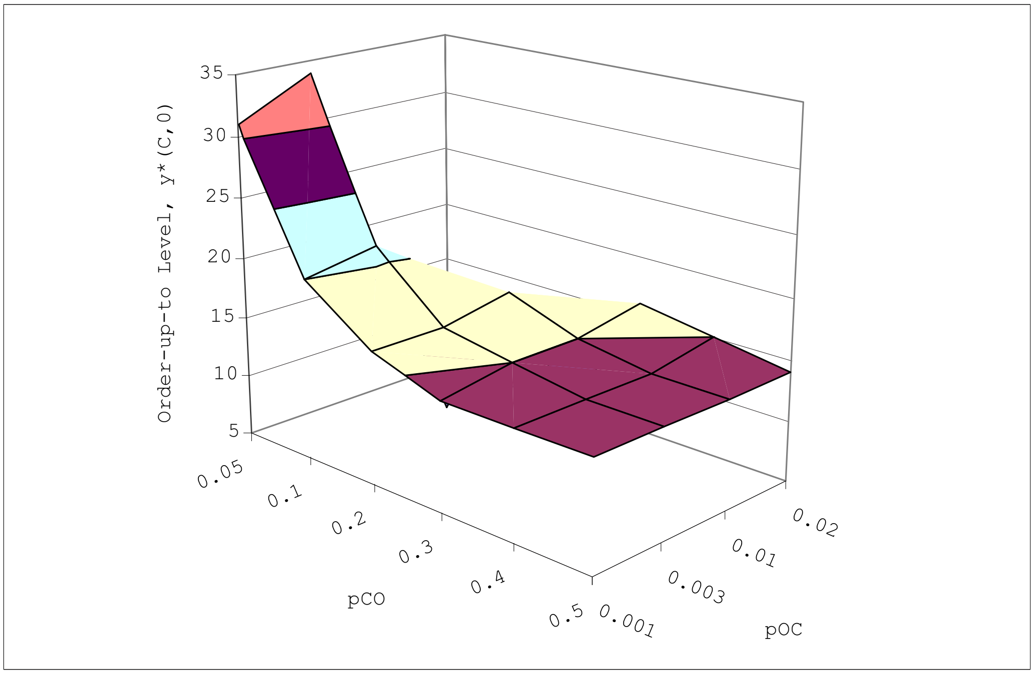

Figure 56: Optimal order-up-to level ($y^*(C,0)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 10C: $L = 15$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

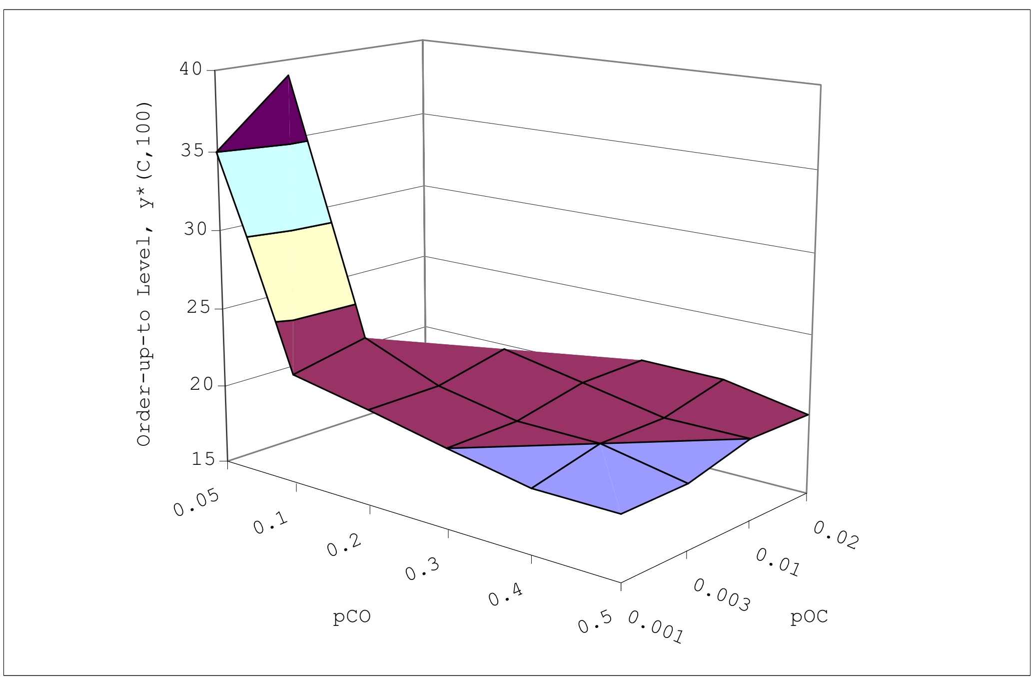

Figure 57: Optimal order-up-to level ($y^*(C,100)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 10C: $L = 15$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

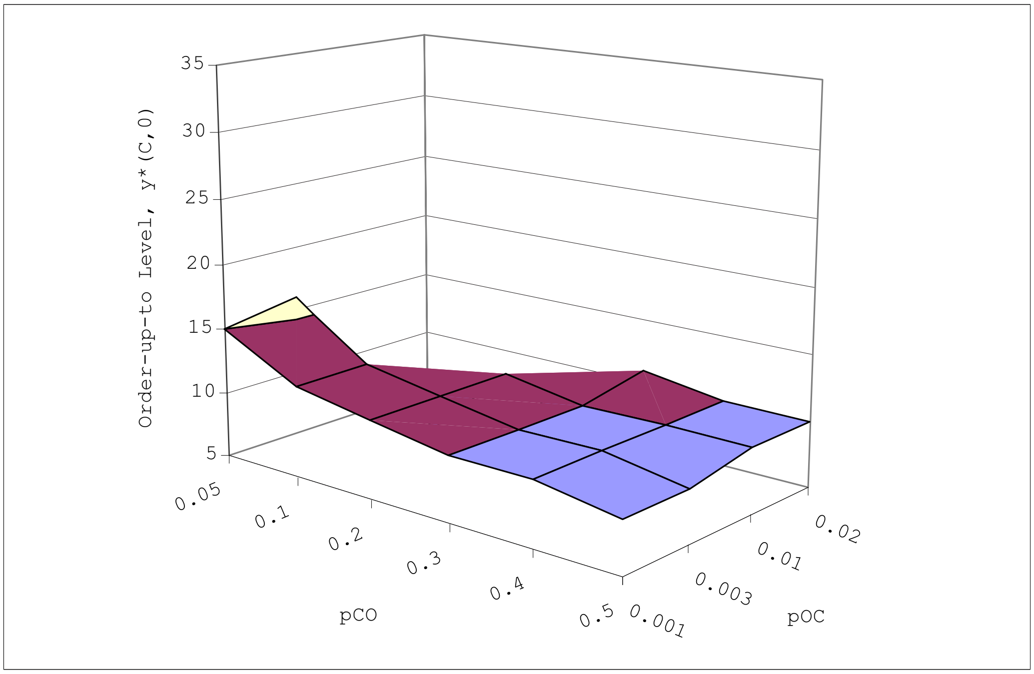

Figure 58: Optimal order-up-to level ($y^*(C, 0)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 11C: $L = 15$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

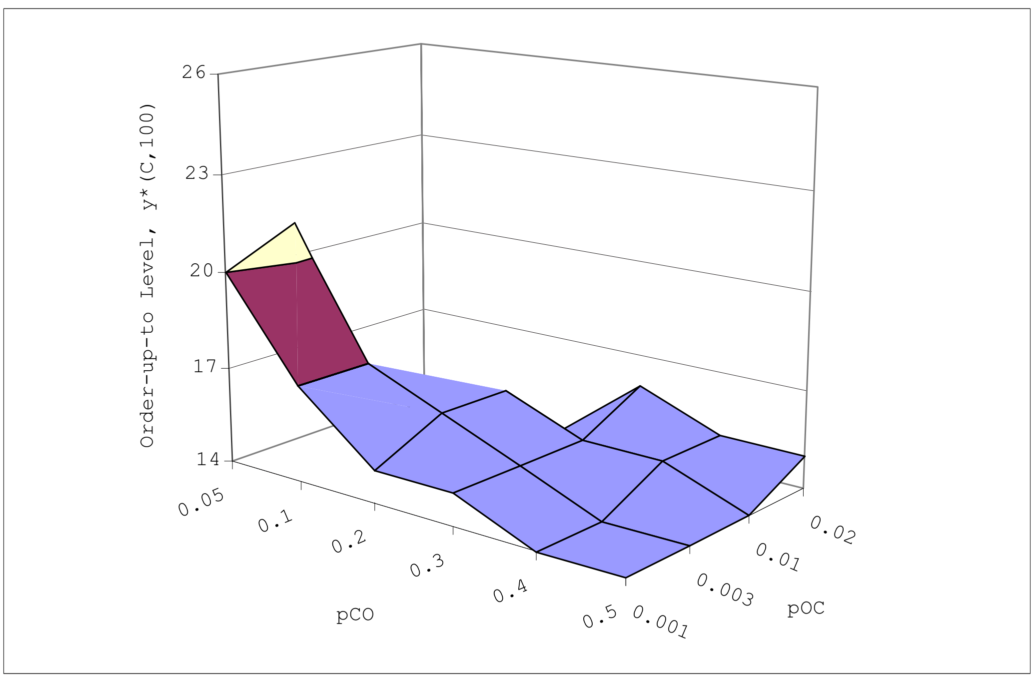

Figure 59: Optimal order-up-to level ($y^*(C, 100)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 11C: $L = 15$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

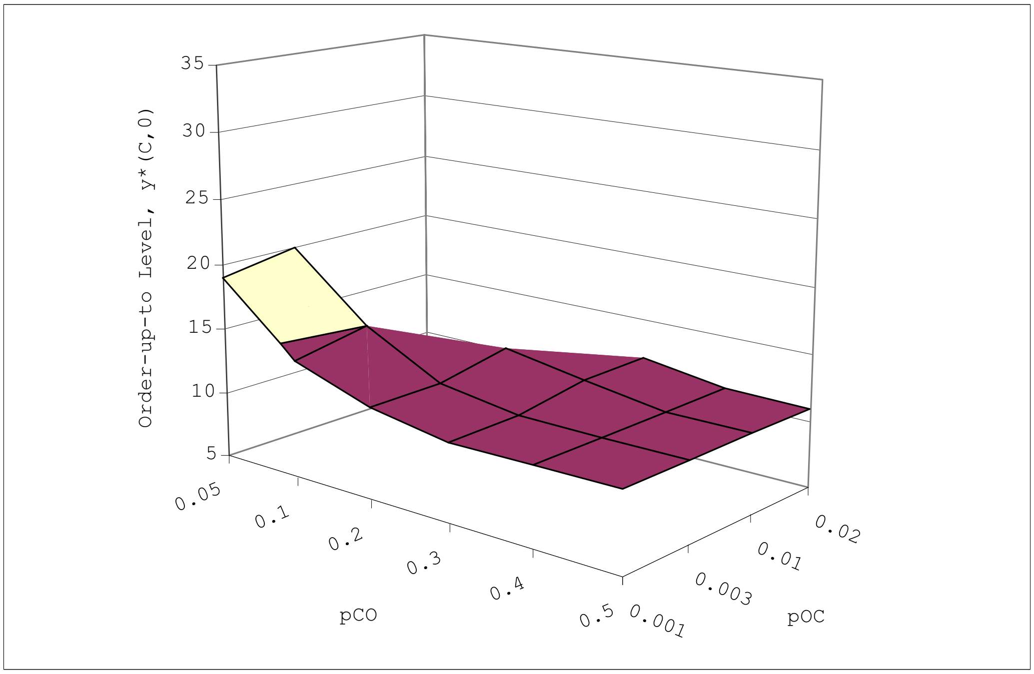

Figure 60: Optimal order-up-to level ($y^*(C, 0)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 12C: $L = 15$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

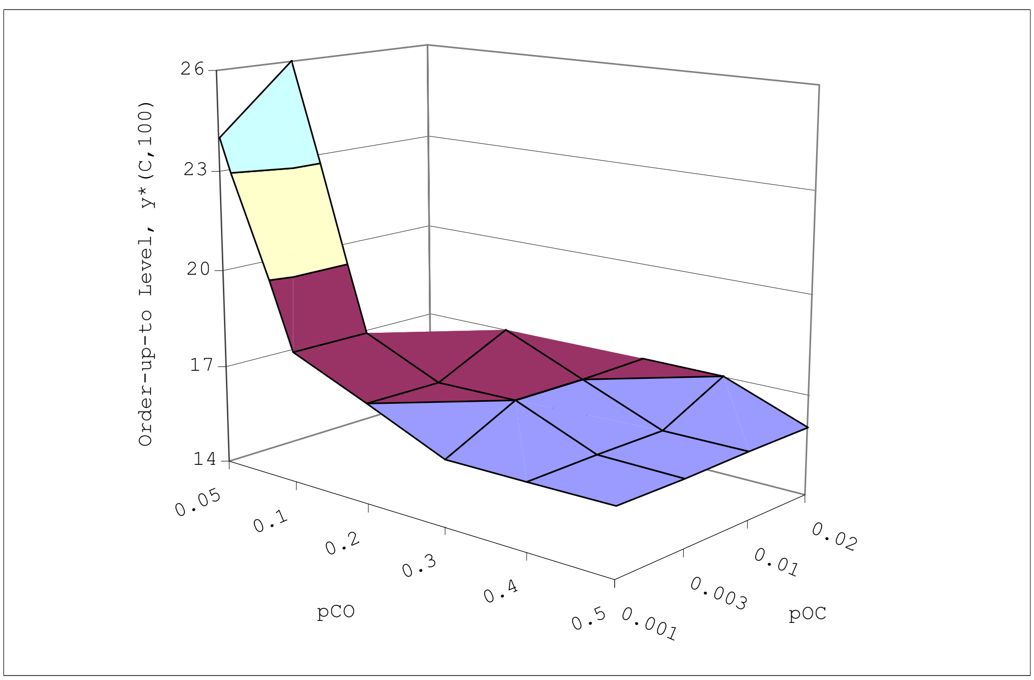

Figure 61: Optimal order-up-to level ($y^*(C, 100)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 12C: $L = 15$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

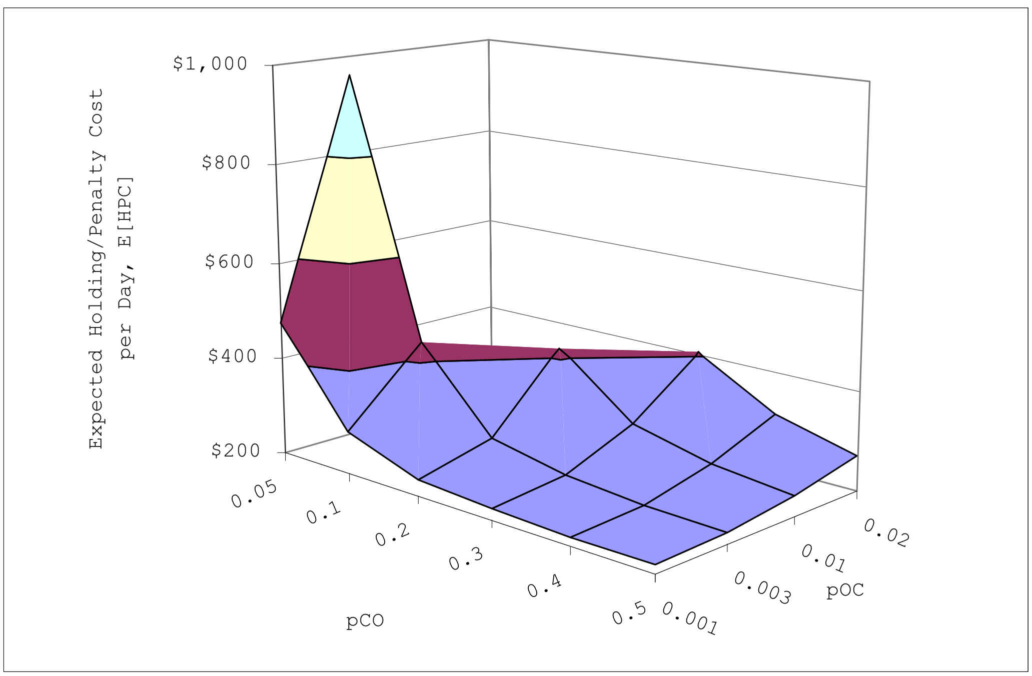

Figure 62: Optimal expected holding and penalty cost per day ($E[HPC]$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 1C: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

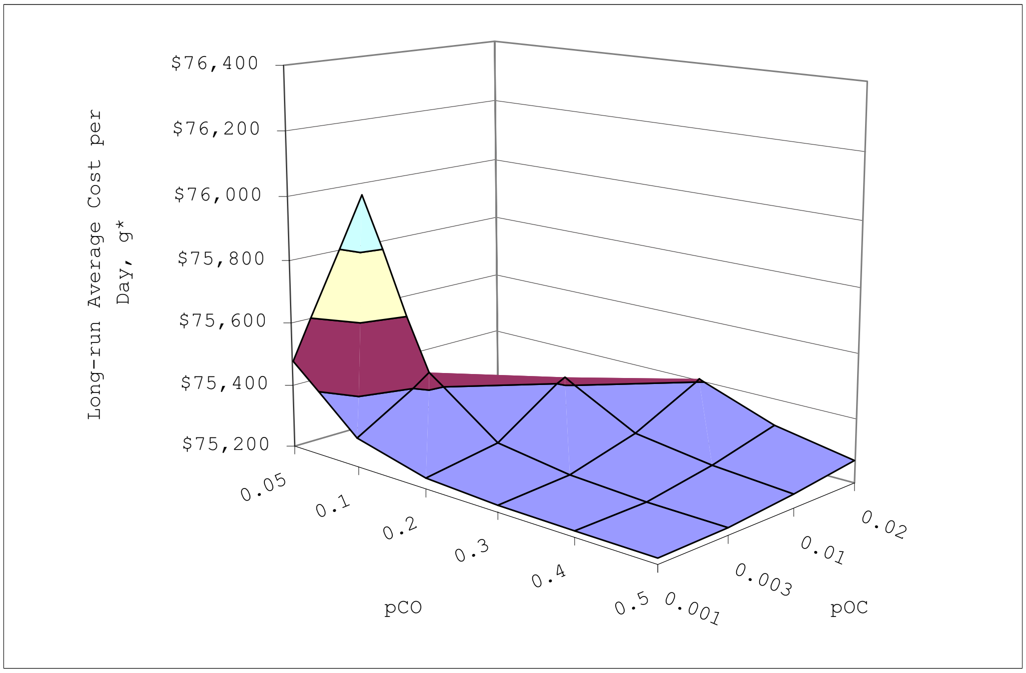

Figure 63: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 1C: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

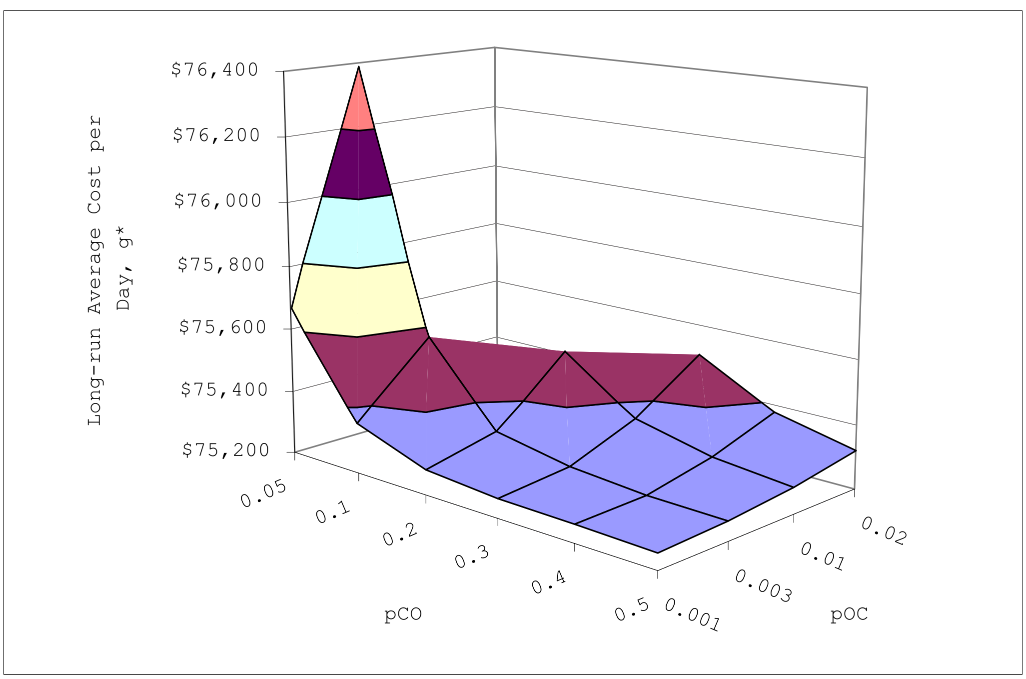

Figure 64: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 2C: $L = 1$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

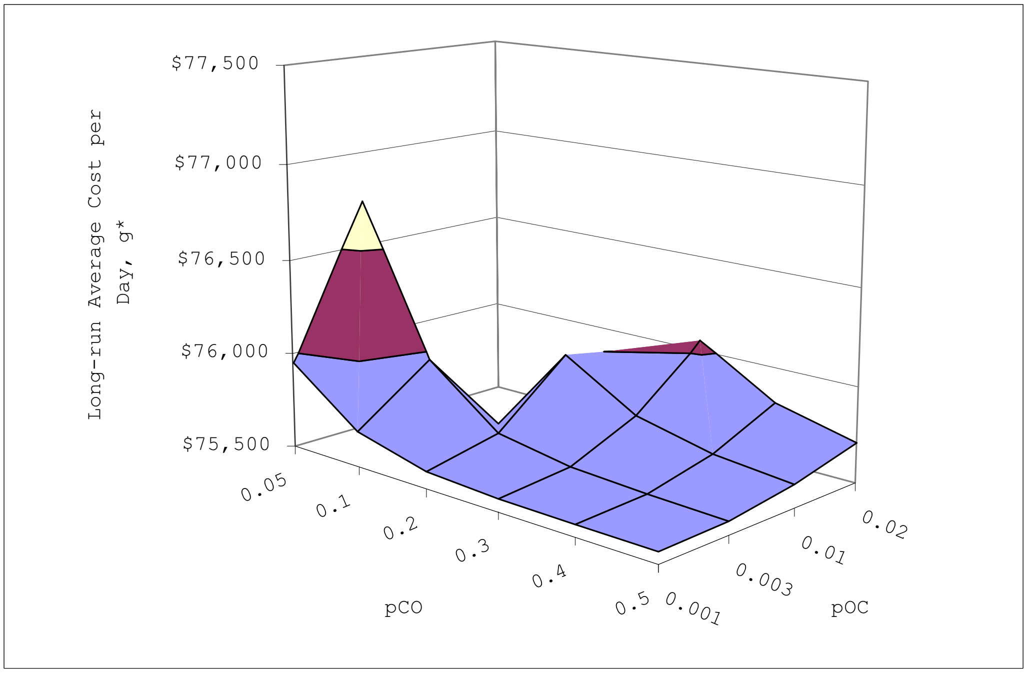

Figure 65: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 3C: $L = 1$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

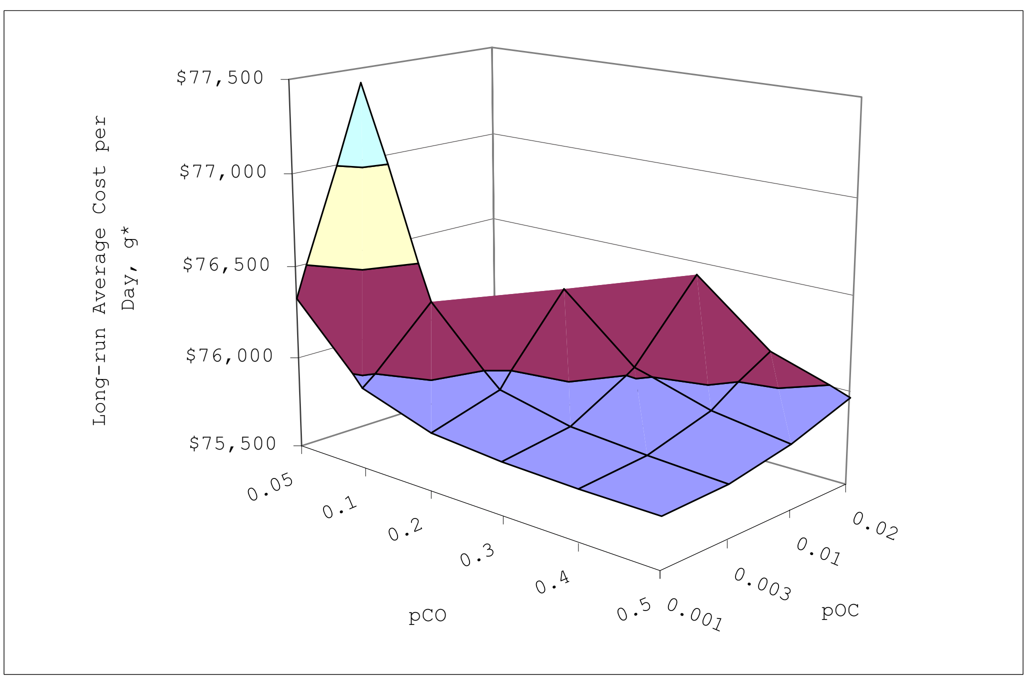

Figure 66: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 4C: $L = 1$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

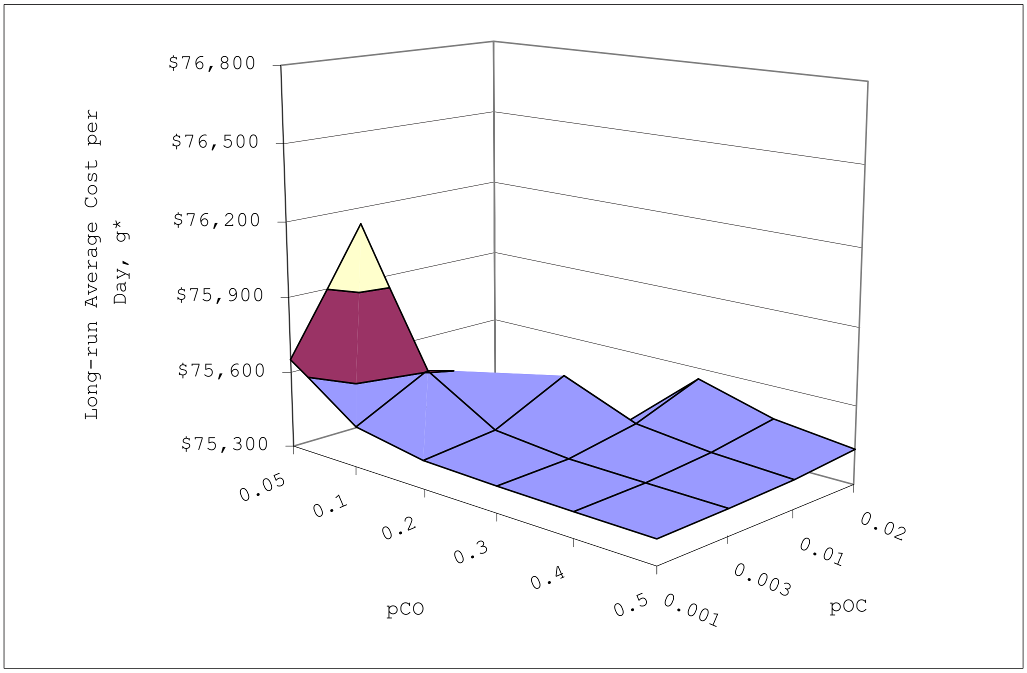

Figure 67: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 5C: $L = 7$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

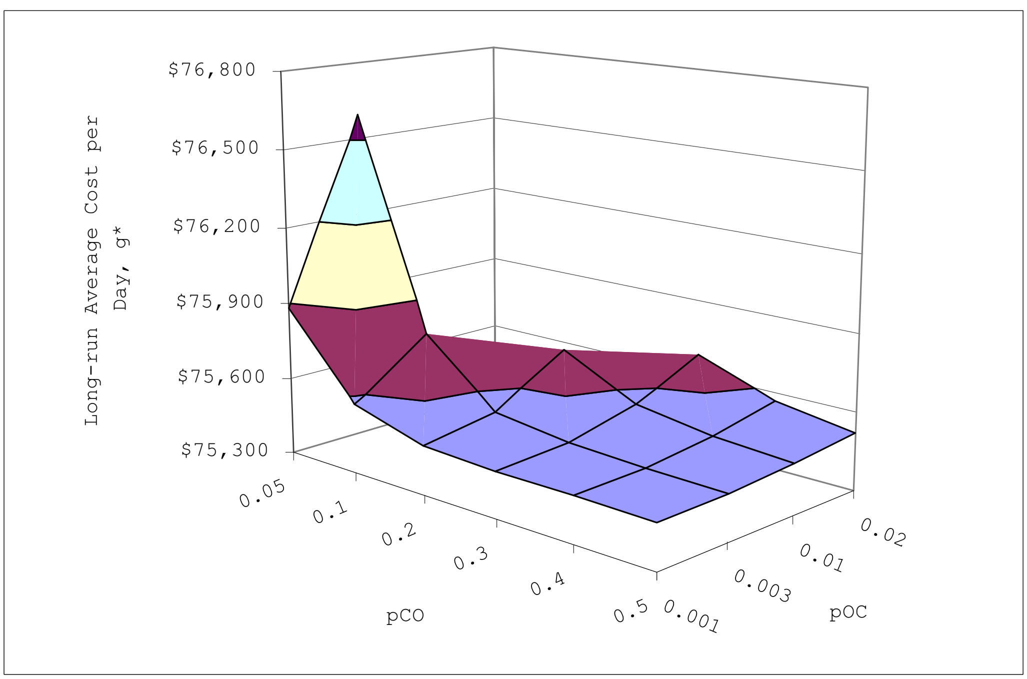

Figure 68: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 6C: $L = 7$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

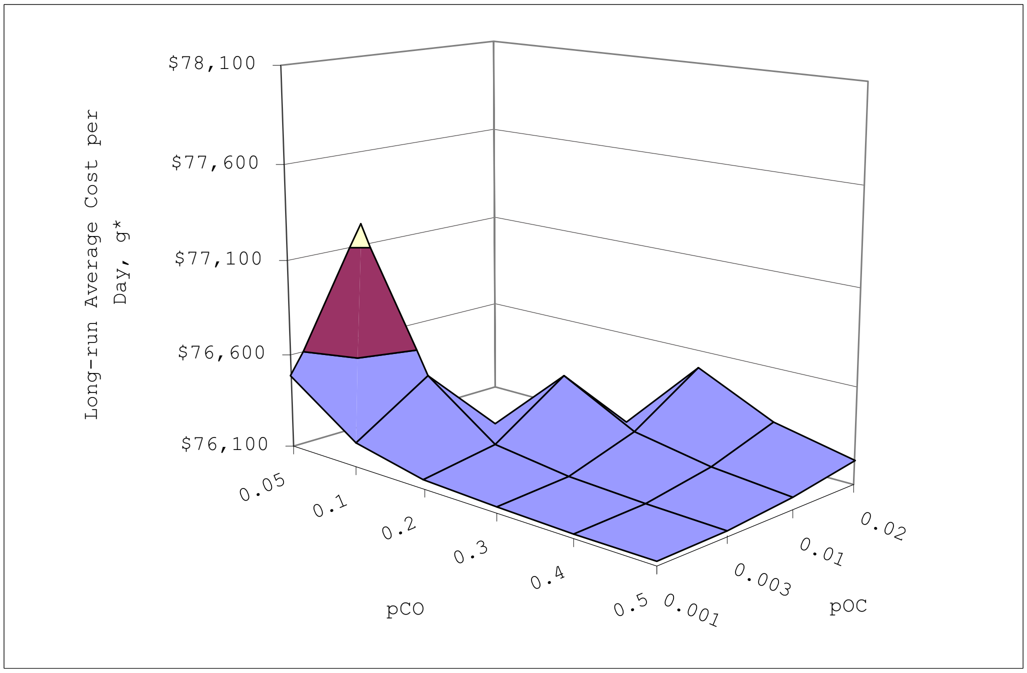

Figure 69: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 7C: $L = 7$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

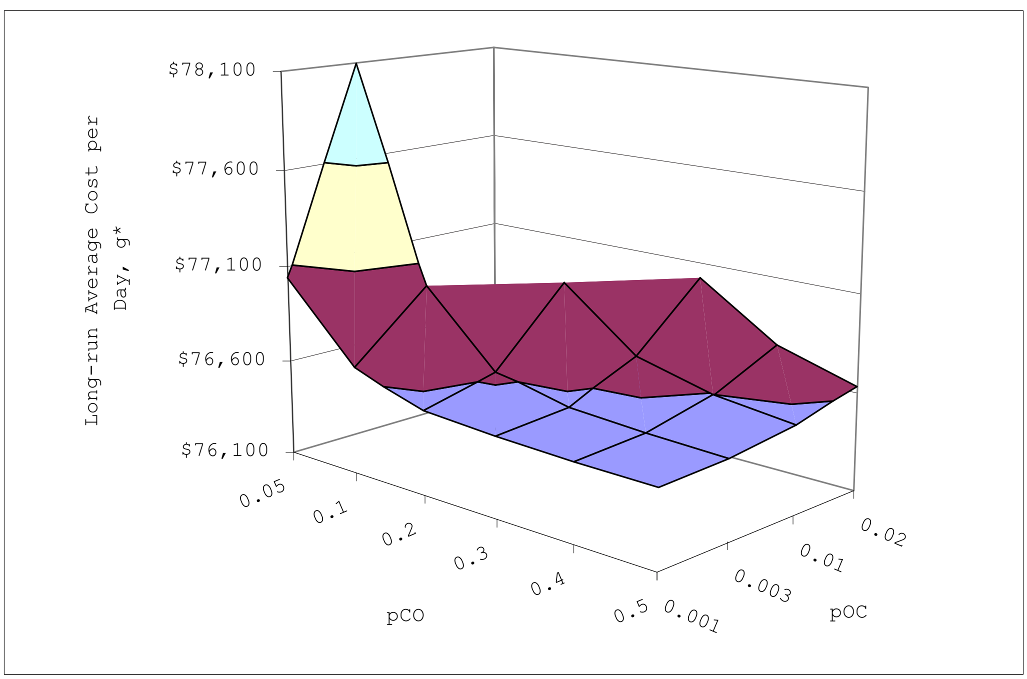

Figure 70: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 8C: $L = 7$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

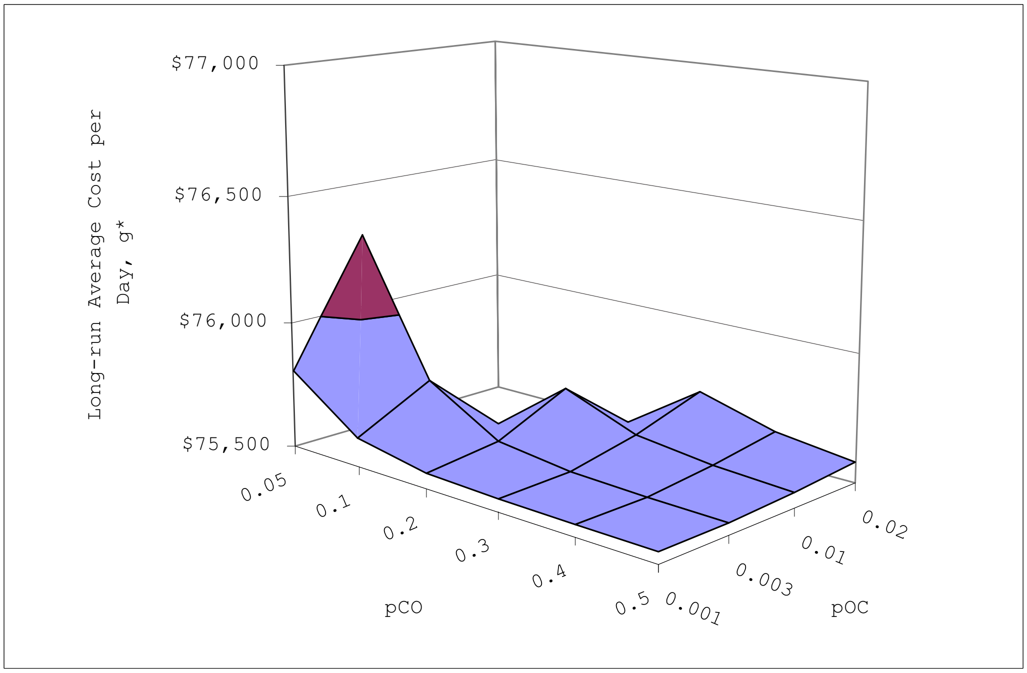

Figure 71: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 9C: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

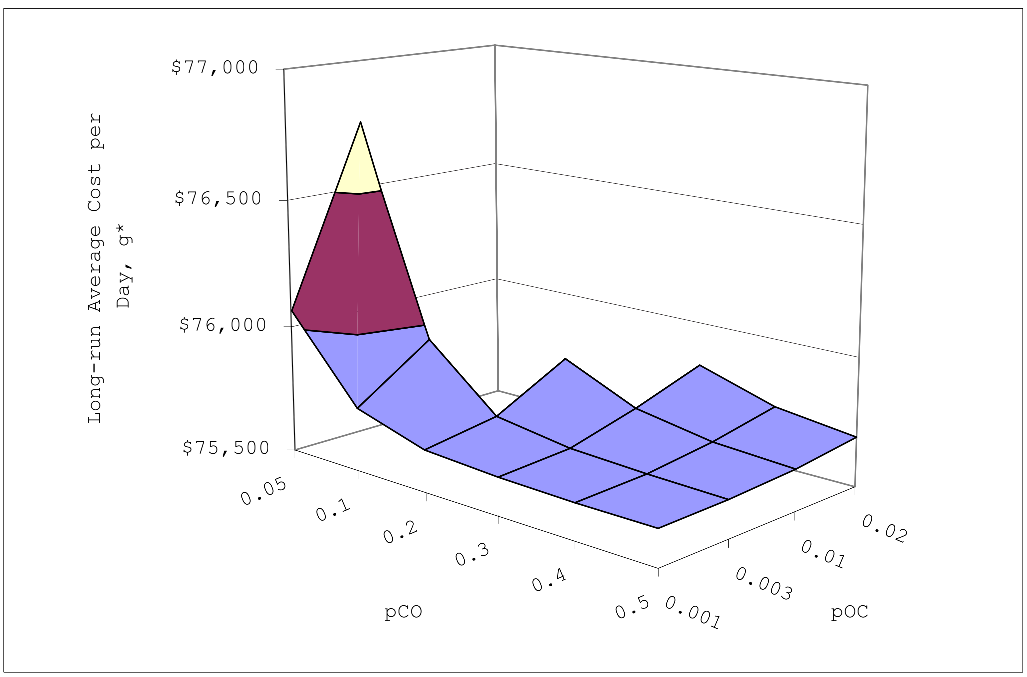

Figure 72: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 10C: $L = 15$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

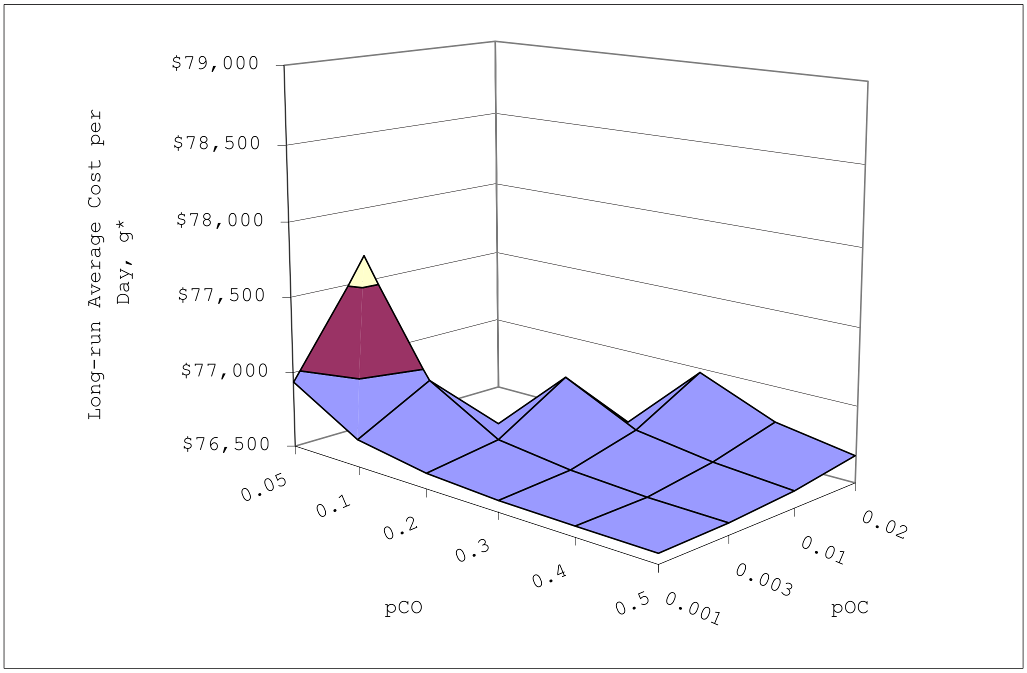

Figure 73: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 11C: $L = 15$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

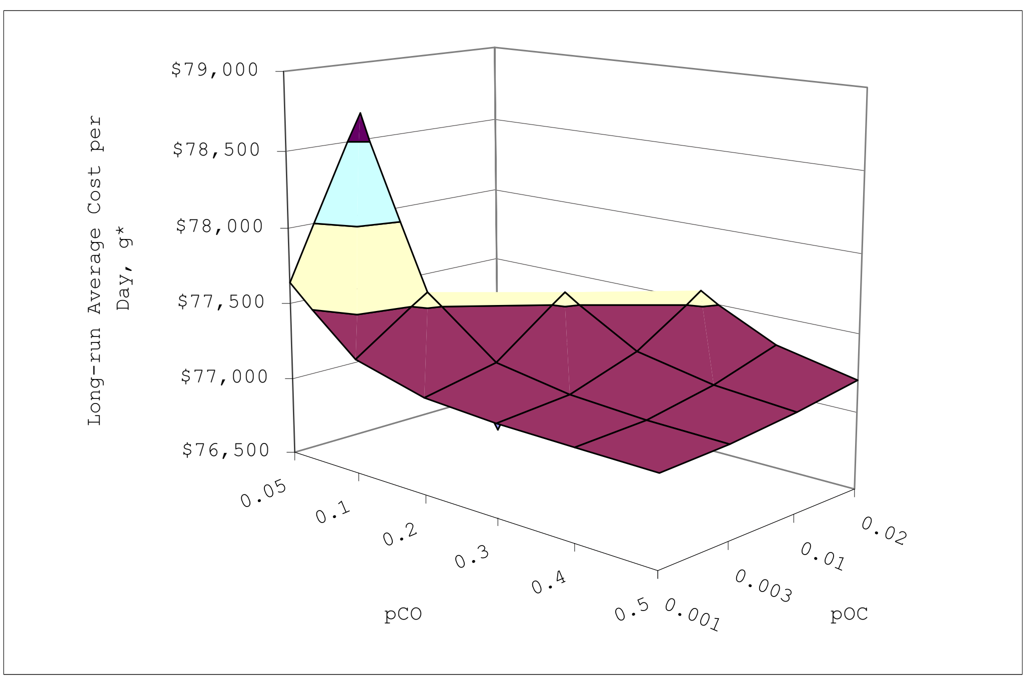

Figure 74: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 12C: $L = 15$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

We observe that the optimal ordering policy is more reactive than proactive. A reactive ordering policy is one that changes the order-up-to levels only in the event of a disruption and/or its aftermath. On the other hand, we define a proactive ordering policy to be one that changes the order-up-to levels in anticipation of a disruption and/or its aftermath. The optimal order-up-to levels for selected states for Instance 9C are given in Table 8. Recall from Chapter 3 that when there is no possibility of border closures, the optimal order-up-to level for Instance 9C is 12 containers. We will refer to the order-up-to level in this special case as the *no closure order-up-to level.* When the border is open and there are no customers in the queue, there is almost no variation from the no closure order-up-to level as the transition probabilities change. This indicates that there is little proactive planning by the manufacturer. When the border is open and there is 100 customers in the queue, the order-up-to levels are now all almost 50% greater than when the queue length was 0. A positive queue length can actually increase the minimum leadtime time (which is discussed further in the next section), and therefore we expect to see corresponding increases in the order-up-to levels. When the border is closed there is much greater variation from the no closure order-up-to level for all queue lengths (and from the optimal order-up-to level for the corresponding open state). When the border closes, the manufacturer immediately increases the order-up-to level to mitigate the risk of any resulting congestion. This indicates reactive planning by the manufacturer.

4.3.4 Impact of the Border Queue Length

In this section we consider the impact of the border queue length on the optimal order-up-to levels. Recall that the optimal long-run average cost is constant for all states and is therefore not considered in this section. Figures 75-77 display results for Instances 1, 5, and 9 when $p_{OC} = 0.003$ and $p_{CO} = 0.1$ and Figure 78 displays the results for Instance 9 when $p_{OC} = 0.003$ and $p_{CO} = 0.5$.

While we do not vary the arrival rate in this analysis, when comparing systems with different arrival rates based on queue length, the queue length should be scaled by the arrival rate. Consider a system in which $r_0 = 1$ and another in which $r_0 = 10$. For the first system,

Table 8: Optimal order-up-to levels for selected border states ($y^*(i, n)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 9C: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

			p_{CO}					
			0.5	0.4	0.3	0.2	0.1	0.05
g^*	p_{OC}	0.001	\$ 75,544	\$ 75,546	\$ 75,549	\$ 75,559	\$ 75,615	\$ 75,809
		0.003	\$ 75,548	\$ 75,552	\$ 75,562	\$ 75,592	\$ 75,761	\$ 76,294
		0.01	\$ 75,561	\$ 75,576	\$ 75,613	\$ 75,721		
		0.02	\$ 75,586	\$ 75,622	\$ 75,707			
$y^*(O, 0)$	p_{OC}	0.001	12	12	12	12	12	12
		0.003	12	12	12	12	12	13
		0.01	12	12	12	13		
		0.02	12	12	13			
$y^*(O, 100)$	p_{OC}	0.001	17	17	17	17	17	17
		0.003	17	17	17	17	18	19
		0.01	17	17	17	18		
		0.02	17	18	18			
$y^*(C, 0)$	p_{OC}	0.001	12	13	13	14	18	25
		0.003	12	13	13	14	18	28
		0.01	13	13	14	15		
		0.02	13	13	14			
$y^*(C, 100)$	p_{OC}	0.001	18	18	19	20	23	30
		0.003	18	18	19	20	24	33
		0.01	18	19	19	21		
		0.02	18	19	20			

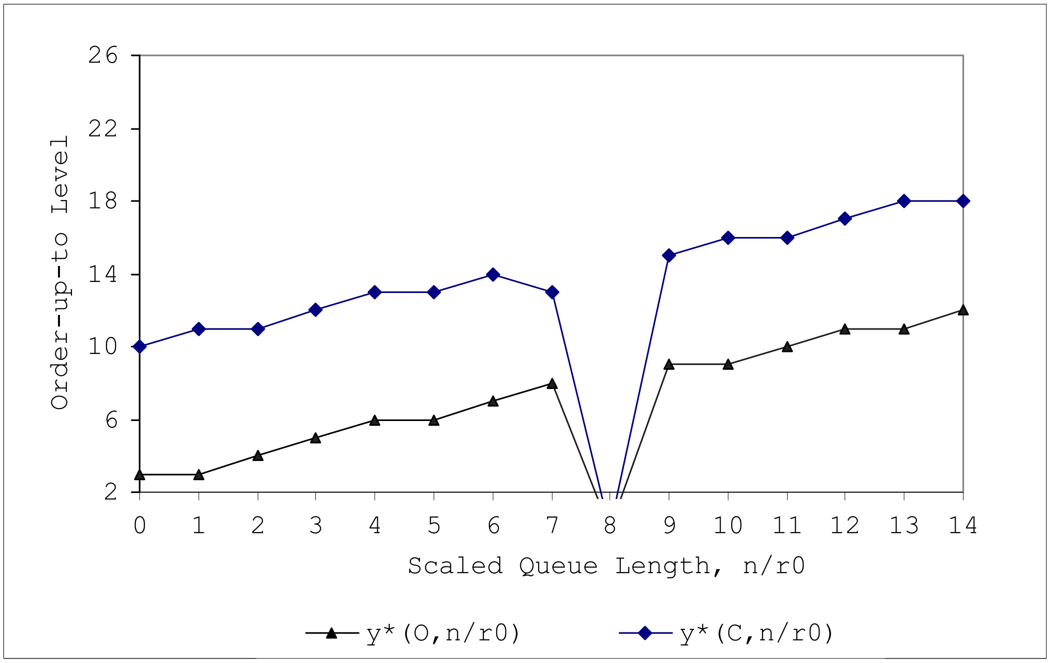

Figure 75: Optimal order-up-to levels $(y^*(O, n/r_0), y^*(C, n/r_0))$ vs. scaled queue length (n/r_0) (Instance 1C: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)), $p_{OC} = 0.003$, $p_{CO} = 0.1$).

a queue length of 100 represents 100 periods of closure. However for the second system, a queue length of 100 represents only 10 periods of closure. It is therefore not appropriate to directly compare these systems based on a queue length of 100. Since customers are an arbitrary units of work, one customer in the first system is equivalent to 10 customers in the second. By scaling the queue length by the arrival rate, we can directly compare different systems with respect to queue lengths.

As the queue length increases, the optimal order-up-to level increases. For any queue length, we can determine the minimum time for the queue to be reduced to zero. If the border is open in period t, then the minimum time for the queue to dissipate is $\lceil (n_t + r_0 - r_1)^+/r_1 \rceil$. If the border is closed in period t, then the minimum time for the queue to dissipate is $\lceil (n_t + r_0)/r_1 \rceil$. Therefore, the queue length can essentially increase the minimum leadtime to the minimum times just described. Even when the minimum time to reduce the queue is less than the minimum leadtime from supplier to the border, a positive queue still increases the chances of delays once the order reaches the border. Just as with

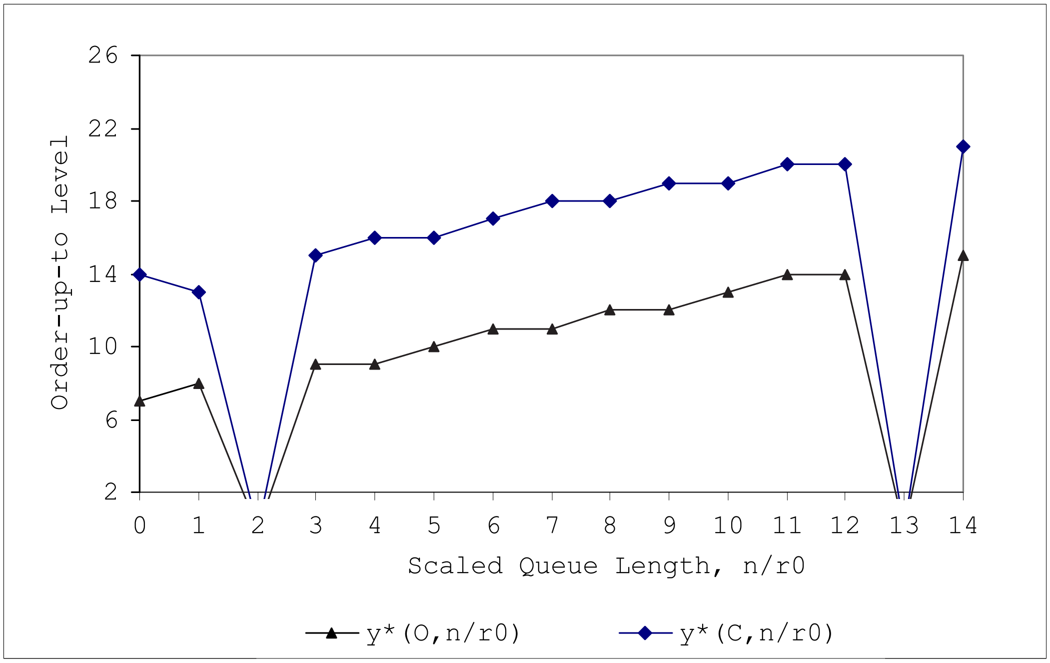

Figure 76: Optimal order-up-to levels $(y^*(O, n/r_0), y^*(C, n/r_0))$ vs. scaled queue length (n/r_0) (Instance 5C: $L = 7$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)), $p_{OC} = 0.003$, $p_{CO} = 0.1$).

the minimum leadtime, because the manufacturer may face additional periods of demand prior to the arrival of the order to the border, greater uncertainty about the demand over the leadtime, and about the state of the border at the end of the minimum leadtime, the manufacturer increases the order-up-to level to buffer against the uncertainty.

We note in Figure 78 that as p_{CO} increases, the border closures become less severe and the gap between the order-up-to levels for the open and closed states decreases. Since the manufacturer faces less severe border closures and less severe resulting congestion, its reaction in the event of a closure is also less severe.

The optimal order-up-to level exhibits an overall trend that increases with the queue length, however as Theorem 10 shows, the relation does not hold exactly.

THEOREM 10. *For the border closure model without congestion, the optimal order-up-to level $(y^*(i, n))$ is* not *monotonic in the customer queue length (n).*

Proof. The numerical results in this section provide counter-examples to the claim that the optimal order-up-to levels are monotonically non-decreasing in the queue length. □

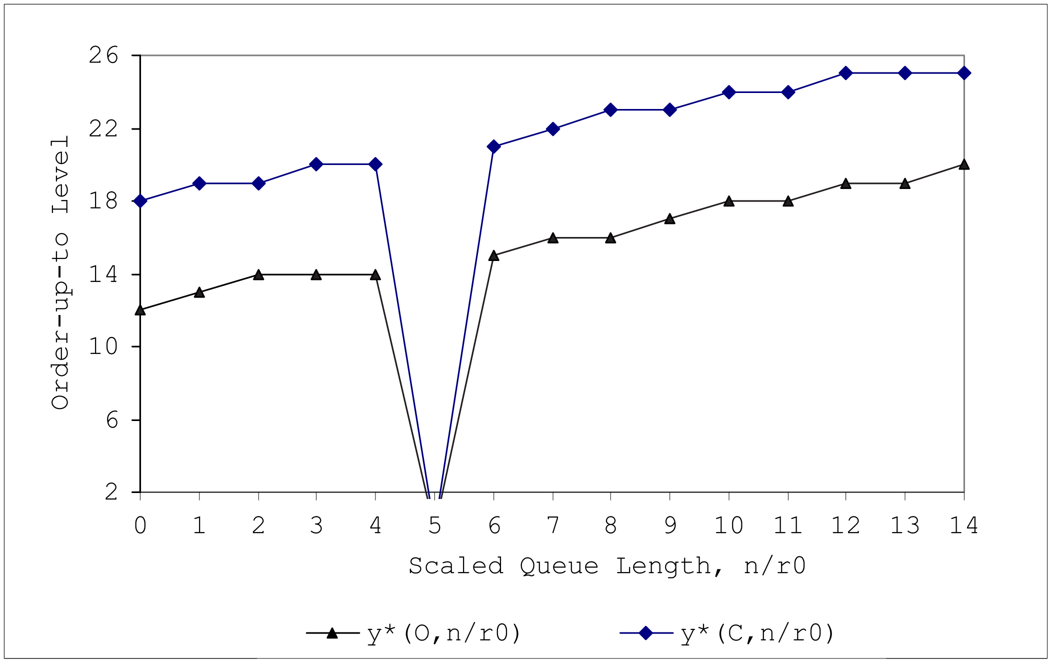

Figure 77: Optimal order-up-to levels $(y^*(O, n/r_0), y^*(C, n/r_0))$ vs. scaled queue length (n/r_0) (Instance 9C: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)), $p_{OC} = 0.003$, $p_{CO} = 0.1$).

We now discuss a property of certain queue lengths that provide the counter-examples for Theorem 10. Recall from Corollary 2 that for states $(i, n) \in S_B$ such that $L(i_+, n_+) = L(i, n) - 1$ with probability one, $y^*(i, n) = -\infty$. In the border closure model with congestion, there exist queue lengths such that for any border status, the border state has this property. Consider the case when $L = 1$, $r_0 = 10$ and $r_1 = 11$. Figure 79 shows one possible evolution of this border system over three periods, t, $t + 1$ and $t + 2$.

At the start of period t, the initial border state is $(i_t, n_t) = (O, 3)$ and the manufacturer places an order of quantity $z_{-1,t}$. This order is tracked throughout the figure by arrows. During period t, $r_0 = 10$ customers arrive to the end of the border queue. Since the border is open in period t, $r_1 = 11$ customers are processed according to a FIFO discipline leaving two customers. The system transitions to period $t + 1$. The state is updated to $(i_{t+1}, n_{t+1}) = (O, 2)$ and all order labels are updated according to the order movement function given in equations (33)-(35). An order for $z_{-1,t+1}$ units is placed. Since $L = 1$, the order placed in period t arrives to the border in period $t + 1$. The order is assigned

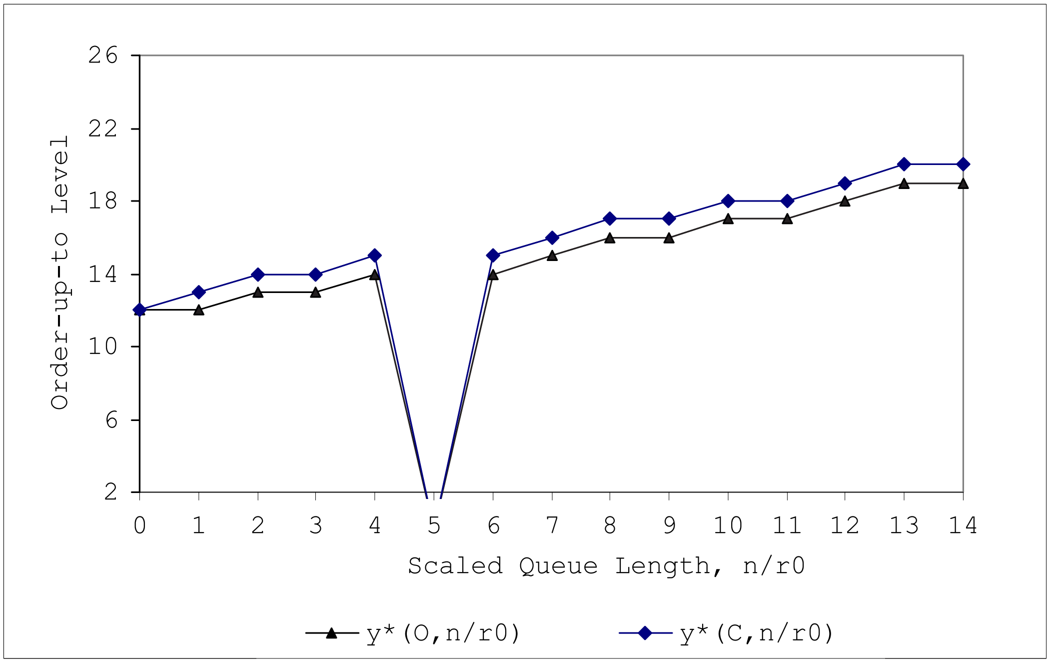

Figure 78: Optimal order-up-to levels $(y^*(O, n/r_0), y^*(C, n/r_0))$ vs. scaled queue length (n/r_0) (Instance 9C: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)), $p_{OC} = 0.003$, $p_{OC} = 0.5$).

to the last of the $r_0 = 10$ arriving customers. Again since the border is open, $r_1 = 11$ customers are processed leaving only the order placed at time t in the queue. Finally the system transitions to period $t + 2$, the state is updated to $(i_{t+2}, n_{t+2}) = (*, 1)$, and all order labels are updated. For illustrative purposes, the border status in period $t + 2$ is irrelevant and so we have indicated the status with an asterisk. Regardless of the border status, $r_0 = 10$ customers arrive to the border. The order placed at time $t + 1$, which arrives to the border in period $t + 2$, is assigned to the last of these arriving customers. There are now 11 customers in the queue, including both the orders placed at times t and $t+1$. Regardless of the sequence of future border statuses, the next time the border is open, all 11 of these customers will be processed and cross the border together. Therefore, the orders placed at time t and $t+1$ will arrive in the same period. The results are same when we consider all possible sample paths of border statuses for periods t and $t + 1$. Therefore $L(i_+, n_+) = L(i, 3) - 1$ holds with probability one for all $i \in S_I$.

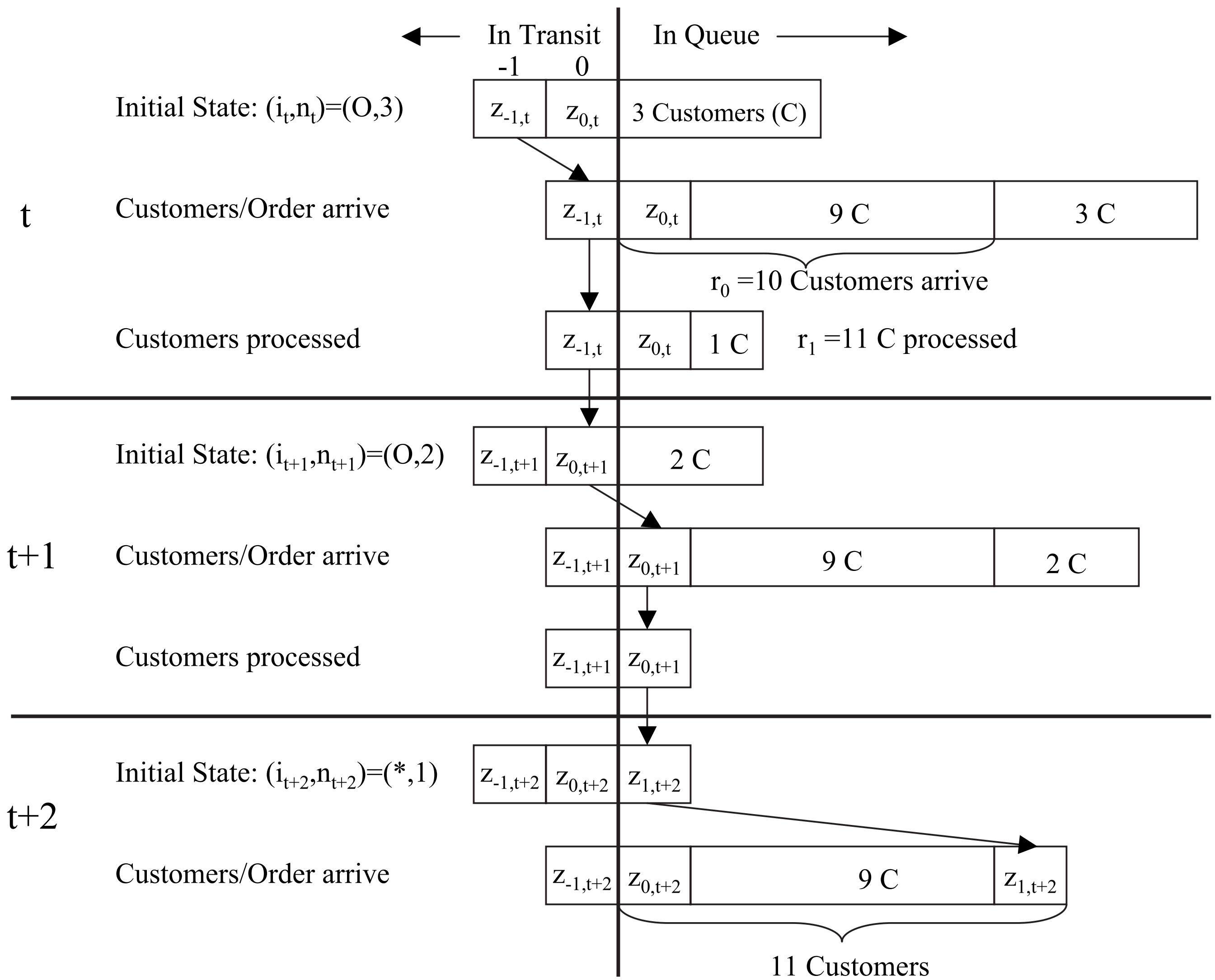

Figure 79: Three period sample path of border system states.

As the queue length approaches these special queue lengths from above, the optimal order-up-to levels often decrease slightly from the overall increasing trend. The order-up-to levels for queue lengths that are slightly larger than the special queue lengths are reduced in anticipation of reaching the special queue length in the near future. A specific example occurs in Figure 75 when the scaled queue length is 7, which in this case corresponds to an actual queue length of 70. A queue length of 69 is one of the special queue lengths. As a result, $y^*(i, 60) = 14$ and yet $y^*(i, 70) = 13$.

A proof of the following claim is a subject of future research, but is validated by the numerical study. Let n' be the smallest queue length for which $L(i_+, n'_+) = L(i, n') - 1$ holds with probability one for all $i \in S_I$ (when $r_0 = 10$ and $r_1 = 11$, $n' = L + 2$). Then $L(i_+, n''_+) = L(i, n'') - 1$ holds with probability one for all $i \in S_I$ and for all n'' such that $n'' = n' + kr_1$, where k is a positive integer.

4.3.5 Impact of the Border Utilization

Border utilization is a measure of the excess processing capacity at a port of entry, and utilization and excess processing capacity are inversely proportional. As the utilization increases, a port of entry's ability to reduce the congestion after a disruption diminishes. Therefore disruptions more negatively impact highly utilized ports of entry. In this section we investigate the impact of the border utilization on the optimal order-up-to levels and long-run average cost.

Recall that the border utilization is $\rho = \frac{r_0}{\pi_O^I r_1}$ and is therefore affected by the processing parameters r_0 and r_1 as well as the transition probabilities. We fix the arrival parameter ($r_0 = 10$) and the probability of transitioning from Open to Closed ($p_{OC} = 0.003$) and then vary the processing parameter (r_1) and the probability of transitioning from Closed to Open (p_{CO}). Table 9 displays the results to highlight the subtle differences in utilization values as the transition probabilities and arrival and processing parameters are varied. When $r_0 = 1$ and $r_1 = 24$, the optimal order-up-to levels and long-run average cost are equivalent to the border closure model without congestion. For this case, we state the order-up-to level for positive queue lengths as "NA" (for "not applicable") since queues are not possible in the border closure model without congestion. Figures 80-82 display the optimal long-run average cost and the order-up-to levels for the Open and Closed states with queue of 100 customers versus the border utilization. The optimal order-up-to levels for the Open state with small queues exhibit little variation as the utilization changes and the order-up-to levels for the Closed state with small queues exhibit similar trends as those in Figure 82 and are therefore not presented.

Table 9: Optimal order-up-to levels for selected border states ($y^*(i,n)$) and long-run average cost per day (g^*) vs. border utilization (ρ) (Instance 9C: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

p_{CO}	r_0	r_1	ρ	g^*	$y^*(O,100)$	$y^*(C,100)$
0.05	1	24	0.044	\$ 75,909	NA	NA
0.05	10	30	0.353	\$ 75,981	13	13
0.05	10	15	0.707	\$ 76,093	14	18
0.05	10	11	0.964	\$ 76,294	19	33
0.1	1	24	0.043	\$ 75,610	NA	NA
0.1	10	30	0.343	\$ 75,632	12	13
0.1	10	15	0.687	\$ 75,676	13	16
0.1	10	11	0.936	\$ 75,761	18	24
0.5	1	24	0.042	\$ 75,543	NA	NA
0.5	10	30	0.335	\$ 75,543	12	12
0.5	10	15	0.671	\$ 75,544	12	13
0.5	10	11	0.915	\$ 75,548	17	18

As the utilization increases, we see in the figures that optimal long-run average cost and the optimal order-up-to levels increase. For a fixed arrival rate, the utilization increases either because the processing parameter decreases, p_{OC} decreases, or p_{OC} increases. We have already observed the effects of the transition probabilities on the policy and long-run average cost. We now observe the impacts of the border queue parameters. As r_1 decreases relative to r_0, fewer customers can be processed in any open border period, which means that queues will require a greater number periods to process. We see that the long-run average cost and order-up-to levels increase more than linearly with the utilization, indicating the manufacturer's sensitivity to increasingly severe congestion effects from the border closures. This result has important implications for business to encourage government investment to improve the processing capabilities of publicly owned and/or operated ports of entry.

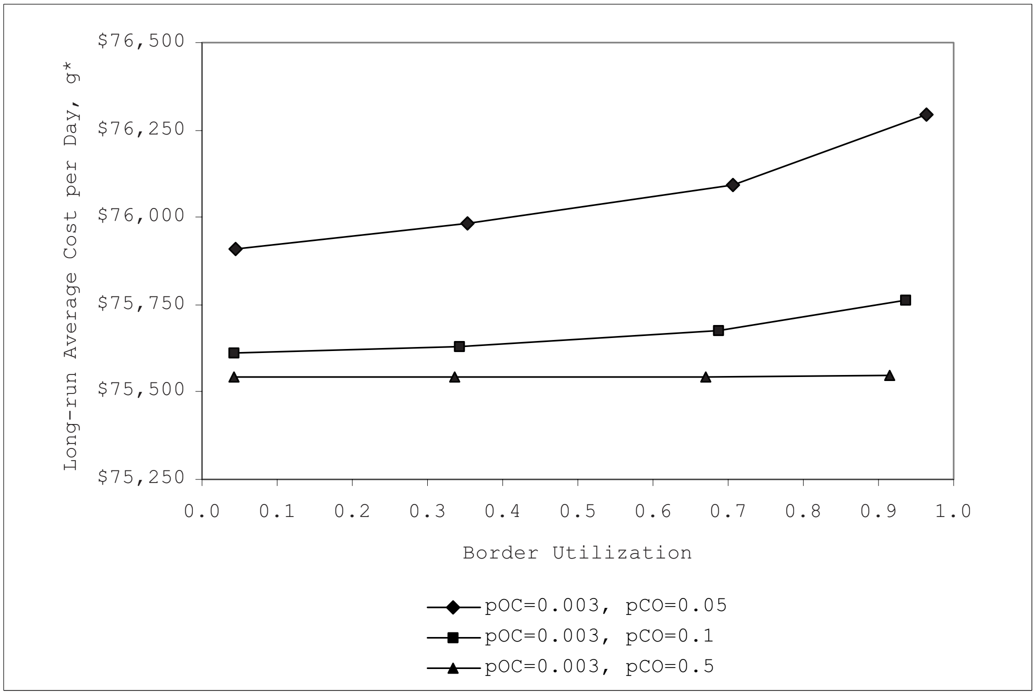

Figure 80: Optimal long-run average cost per day (g^*) vs. border utilization (ρ) (Instance 9C: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

It is interesting to see in Figure 81 that the order-up-to levels remain close across the three transition probabilities even as the utilization increases. This implies for the Open states that the transition probabilities are actually less important to decision making than the arrival and processing parameters. This reinforces the earlier insight that the optimal policy is more reactive than proactive.

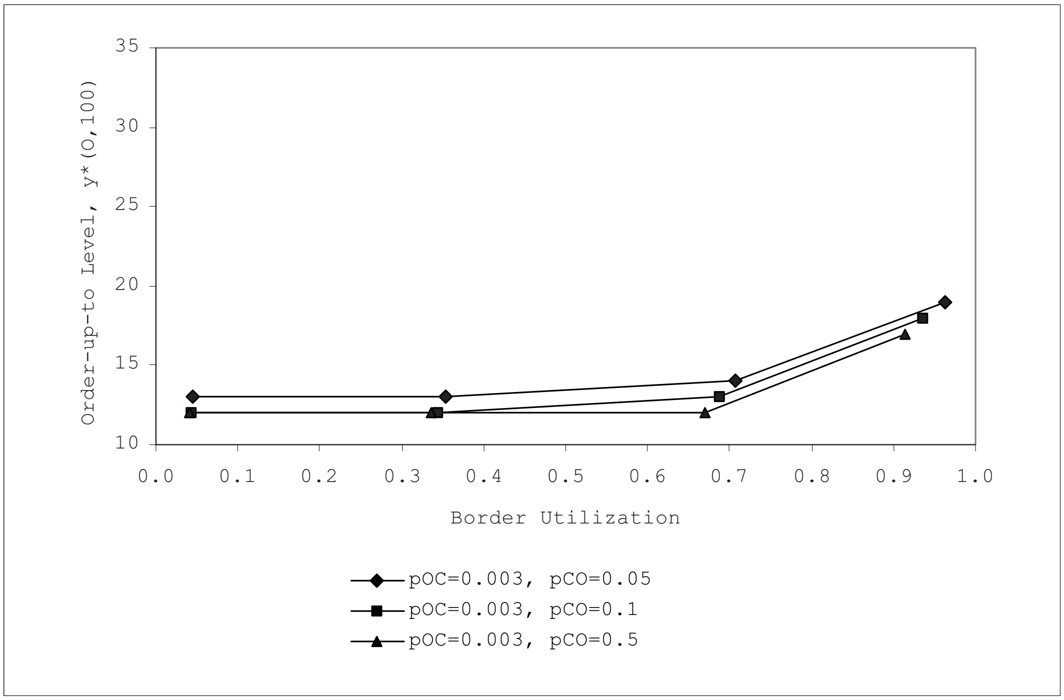

Figure 81: Optimal order-up-to level ($y^*(O, 100)$ vs. border utilization (ρ) (Instance 9C: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

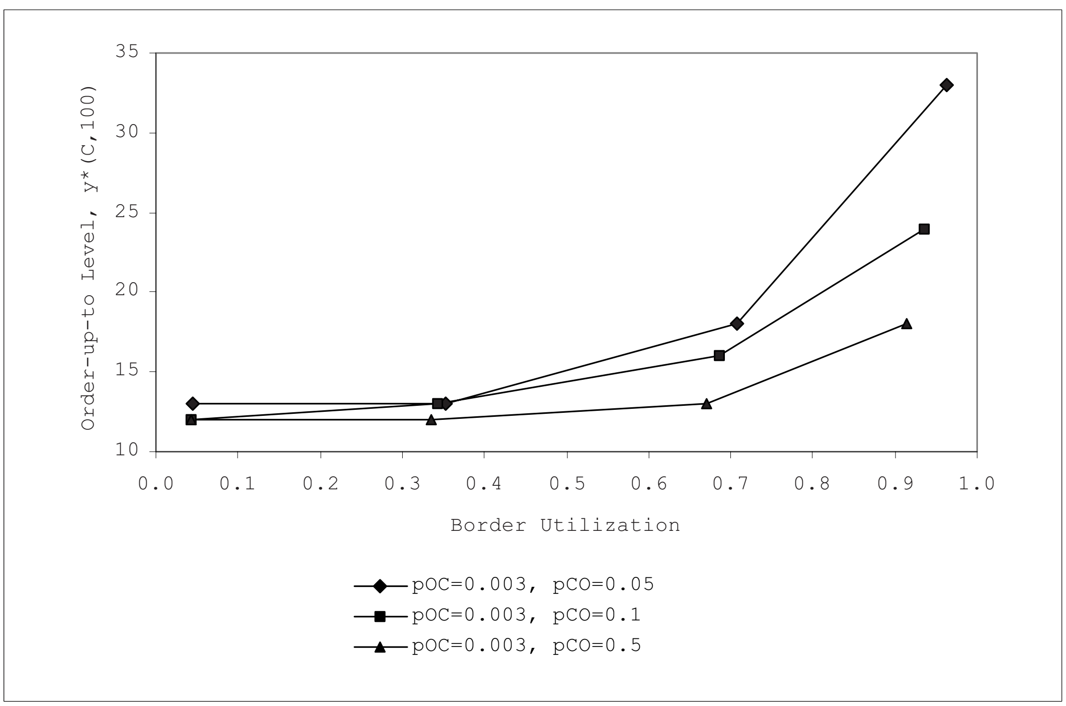

Figure 82: Optimal order-up-to level ($y^*(C, 100)$ vs. border utilization (ρ) (Instance 9C: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

4.3.6 Impact of the Minimum Leadtime

As the minimum leadtime increases, the manufacturer cannot replenish its inventory as quickly and faces additional periods of demand prior to arrival of an order, greater uncertainty about the leadtime demand, and greater uncertainty about the state of border system at the end of the minimum leadtime. The manufacturer therefore increases the order-up-to level to account for the potential additional demand and to buffer against the increased uncertainty and incurs greater long-run average costs per day. Figures 83-85 display the optimal order-up-to levels and long-run average cost versus the minimum leadtime.

We vary p_{CO} in each figure since it more negatively effects the manufacturer than p_{OC} and we see that the curve shapes are consistent across the studied values of p_{CO}. The greatest changes to the order-up-to levels and long-run average cost occur when the minimum leadtime is small. Therefore supply chains with shorter leadtimes between the supplier and the border have a greater incentive to reduce this leadtime than the supply chains with longer leadtimes from supplier to the border. We again see the reactive versus proactive policy characteristics when we compare Figures 83 and 84.

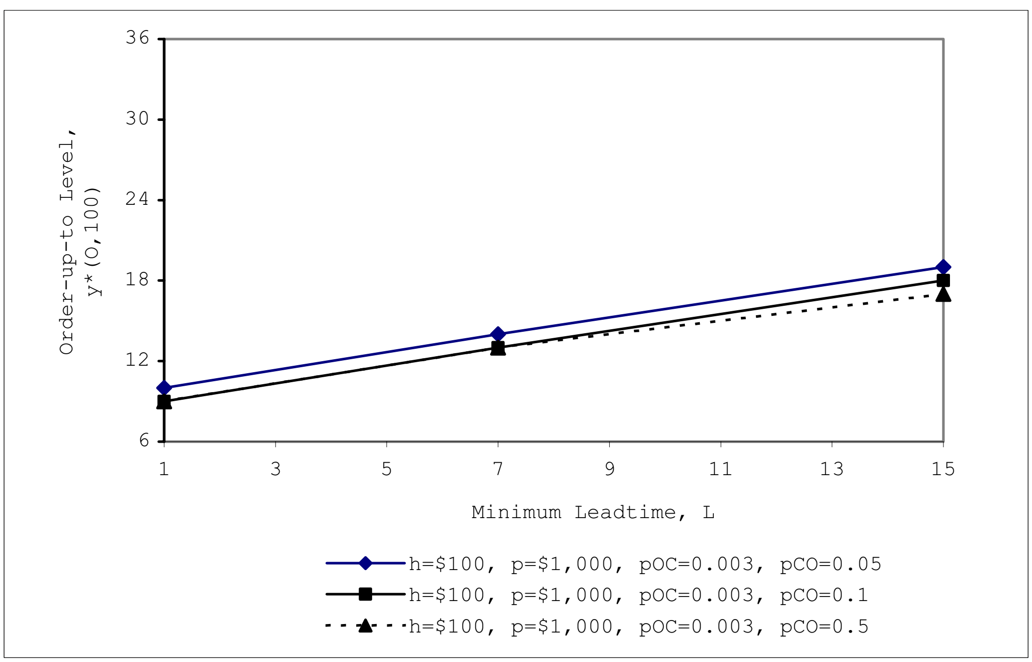

Figure 83: Optimal order-up-to level ($y^*(O, 100)$) vs. minimum leadtime (L) (Instance C).

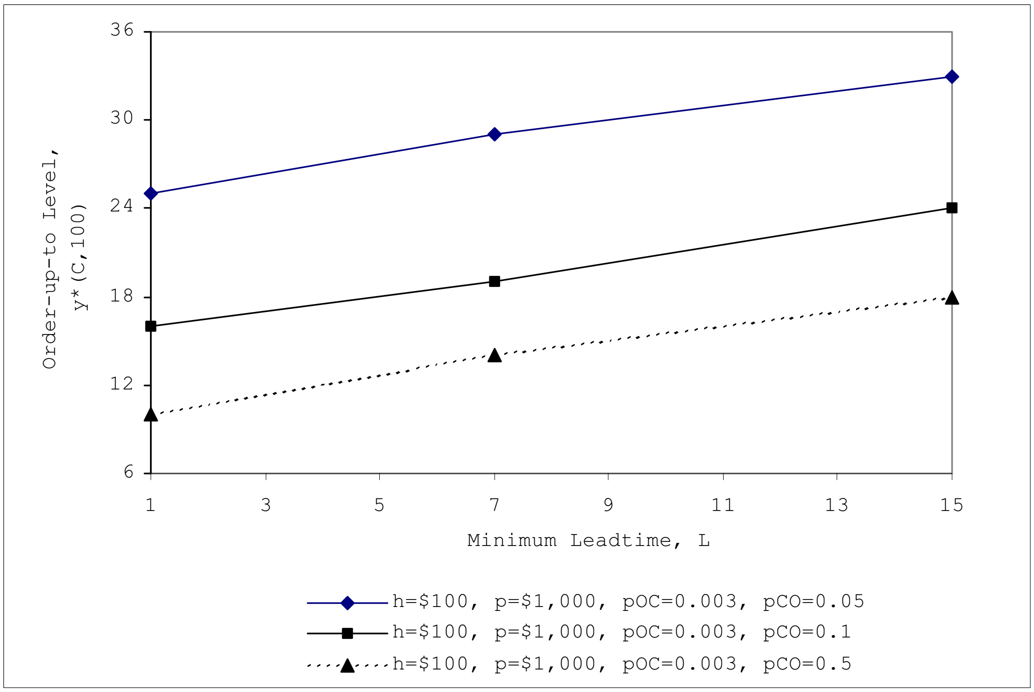

Figure 84: Optimal order-up-to level ($y^*(C, 100)$) vs. minimum leadtime (L) (Instance C).

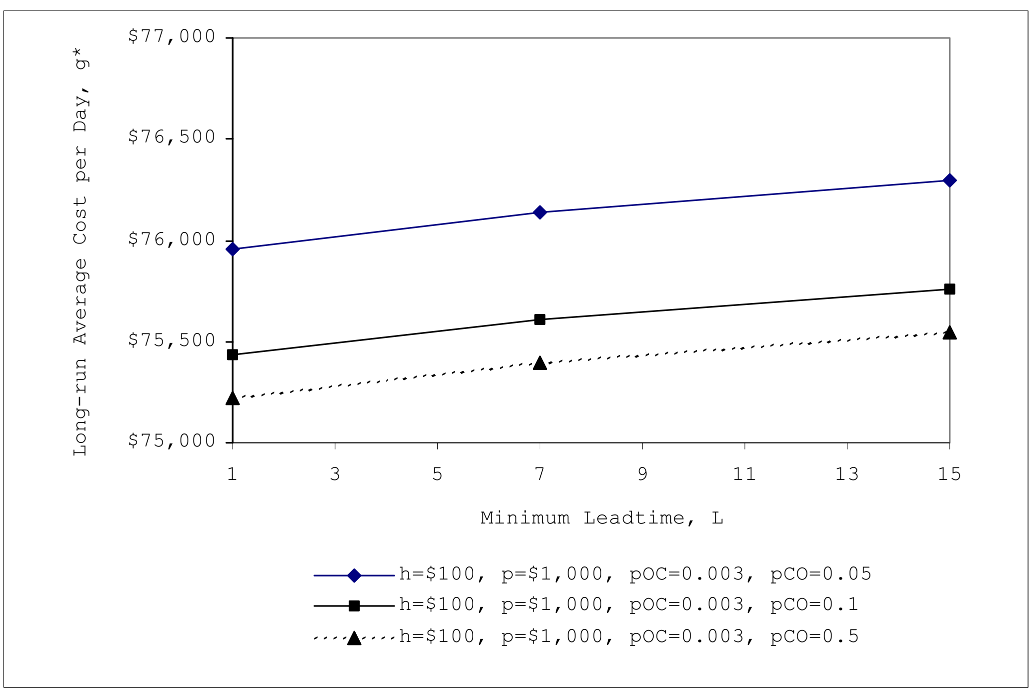

Figure 85: Optimal long-run average cost per day (g^*) vs. minimum leadtime (L) (Instance C).

4.3.7 Impact of the Holding and Penalty Costs

For this model, we do not explicitly consider the cost ratio $(p/(p+h))$ since the optimal policy is not state-invariant and we cannot use Theorem 2 to calculate the order-up-to levels. We therefore examine the impact of the holding and penalty costs separately. Figures 86-91 present the optimal order-up-to levels and long-run average costs when we vary the holding cost over values of \$100, \$300 and \$500 while fixing $p = \$1,000$ and when we vary the penalty cost over values \$500, \$1,000, and \$2,000 while while fixing $h = \$100$.

As the holding cost becomes small compared to the penalty cost, we observe a diverging of the optimal order-up-to levels for the three examples presented since it is less costly for the manufacturer to hold additional inventory as a buffer against backorders. As the holding cost increases, the long-run average costs for the three values of p_{CO} diverge, highlighting the varying levels the closure severity. The penalty cost invokes the opposite effects on the order-up-to level. As the penalty cost increases, backorders become more costly and so the manufacturer must increase the order-up-to level as a buffer. However when p_{CO} is small and there is greater uncertainty about the future state of the border, the manufacturer must further increase the order-up-to level as an additional buffer. The divergence of the long-run average costs as the penalty cost increases again highlights the varying levels closure severity.

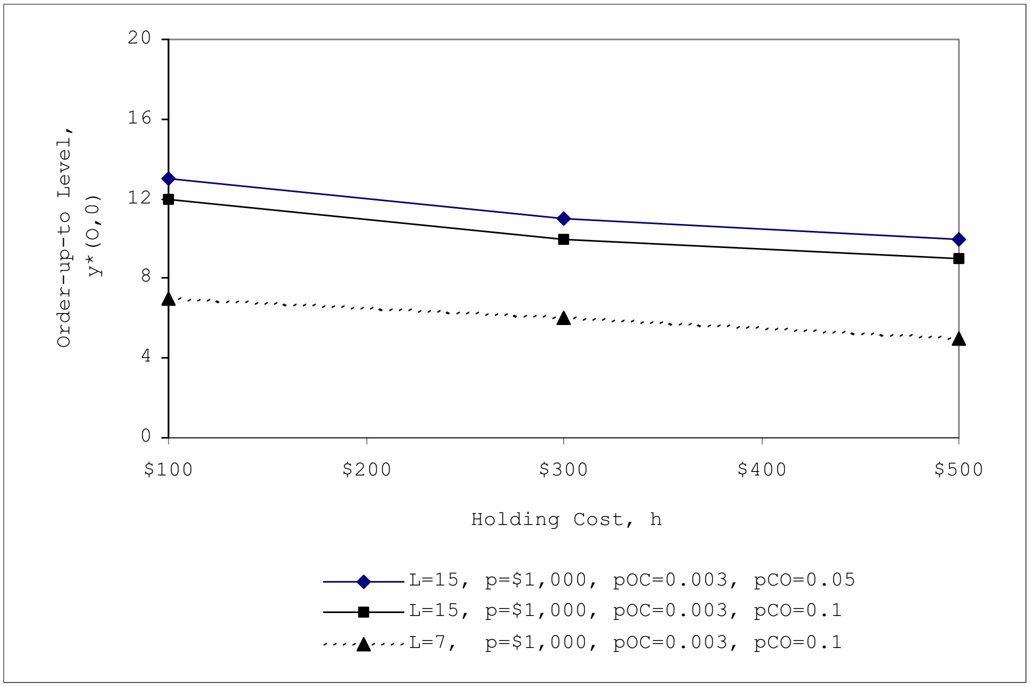

Figure 86: Optimal order-up-to level ($y^*(O,0)$) vs. holding cost (h) (Instance C).

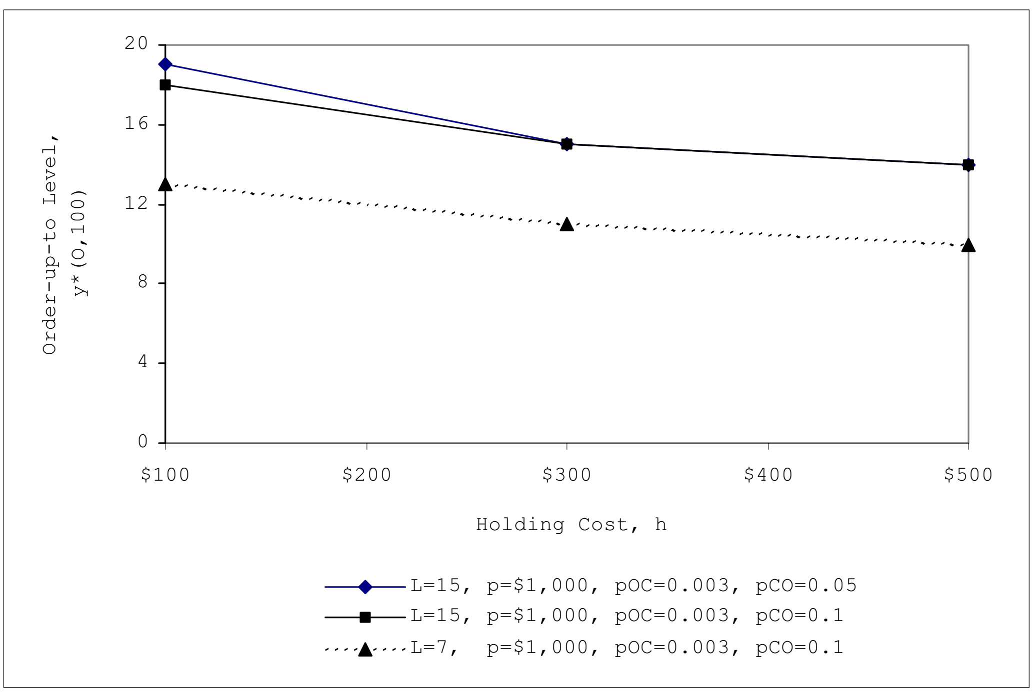

Figure 87: Optimal order-up-to level ($y^*(O,100)$) vs. holding cost (h) (Instance C).

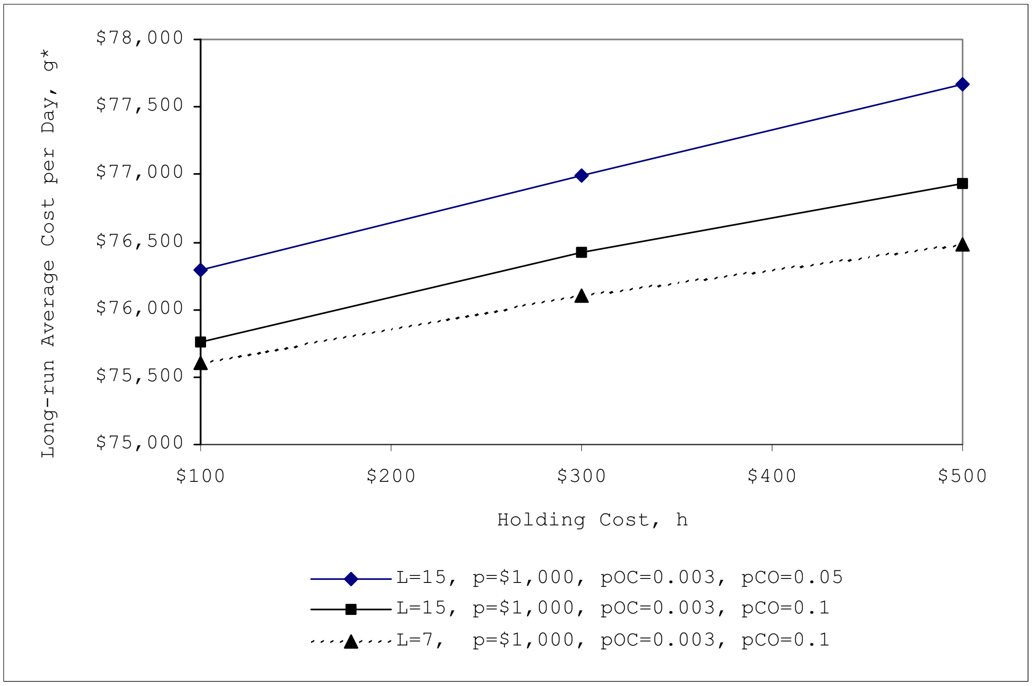

Figure 88: Optimal long-run average cost per day (g^*) vs. holding cost (h) (Instance C).

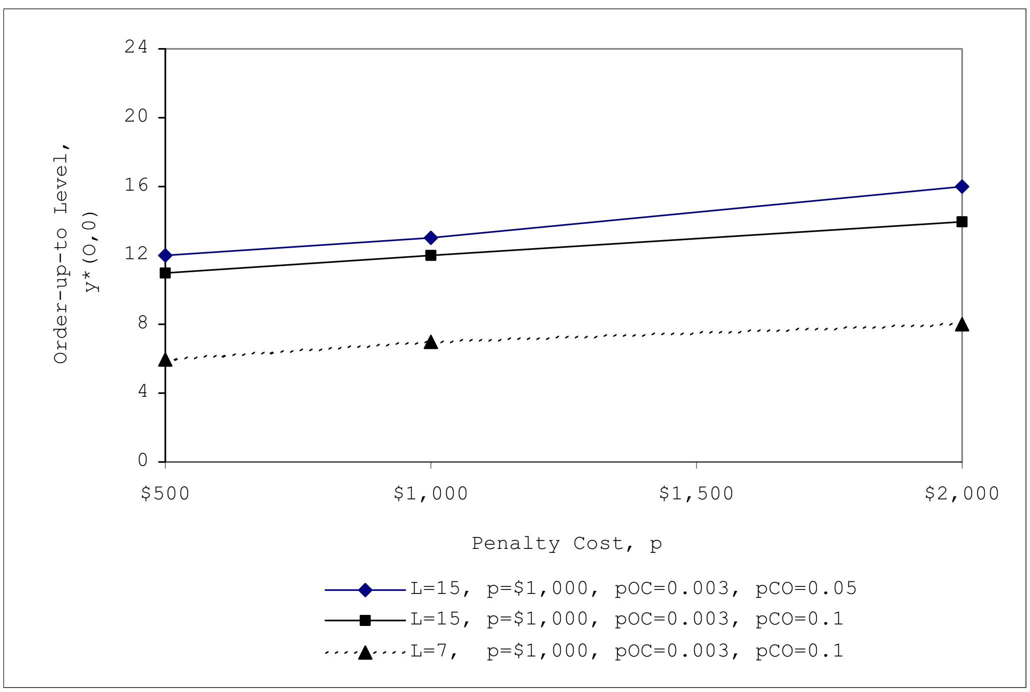

Figure 89: Optimal order-up-to level ($y^*(O, 0)$) vs. penalty cost (p) (Instance C).

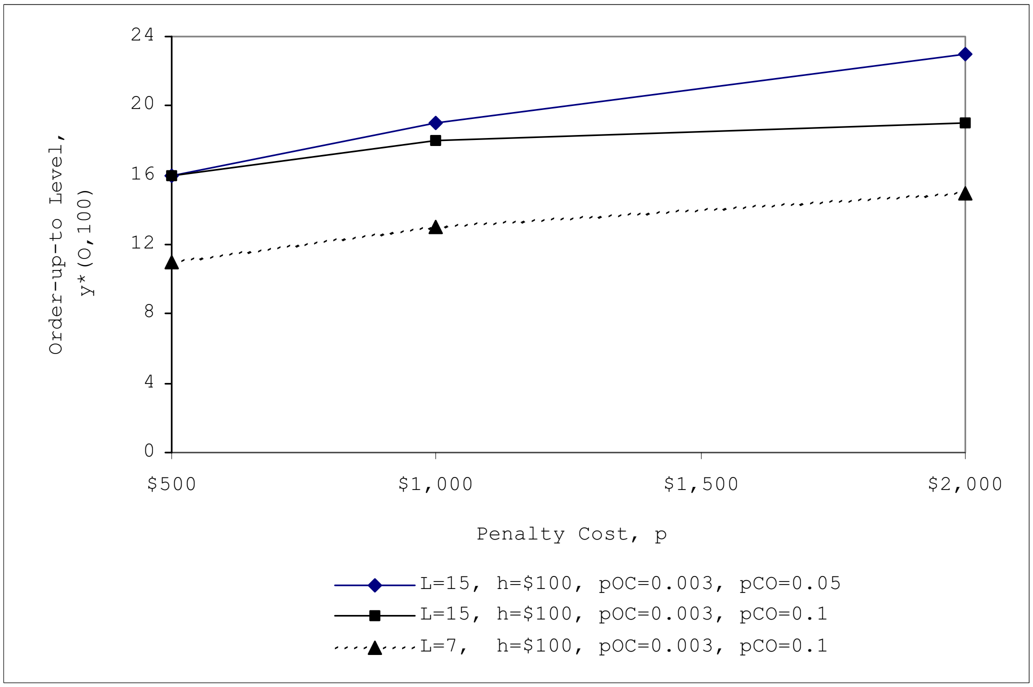

Figure 90: Optimal order-up-to level ($y^*(O, 100)$) vs. penalty cost (p) (Instance C).

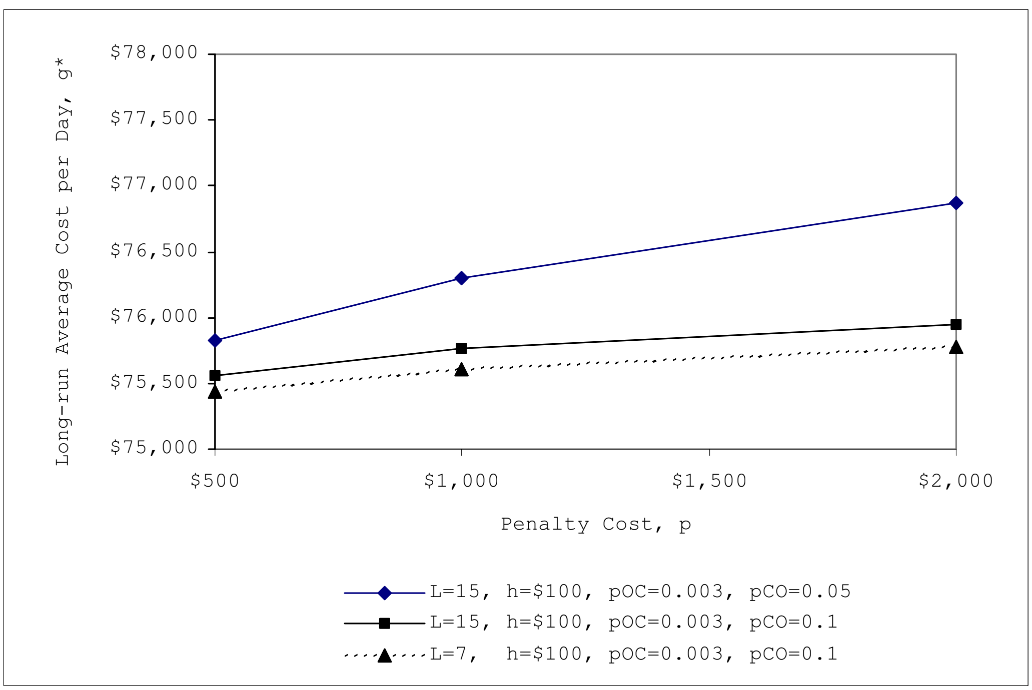

Figure 91: Optimal long-run average cost (g^*) vs. penalty cost (p) (Instance C).

4.3.8 Impact of the Demand Distribution

We finally present results for Instance 13C in which the demand is Poisson distributed and the mean demand one container per day. Figures 92-94 display the optimal order-up-to levels and the long-run average cost per day versus the border state transition probabilities.Comparing Instances 1C and 13C, the optimal order-up-to level and long-run average cost experience greater rates of changes with respect to transition probabilities when the demand mean and variance are larger. The expected demand, and perhaps more importantly the variance, are greater (recall that the mean and variance of a Poisson random variable are equivalent). Therefore the optimal order-up-to levels are greater in order to provide a buffer against this greater demand uncertainty. The long-run average cost is greater as well since at times the manufacturer may be holding more on-hand inventory (due to increased order-up-to levels and greater demand uncertainty) and at other time the manufacturer may be experiencing a greater number of backorders (due to greater demand uncertainty). Also due to greater demand uncertainty and the threat of backorders, the manufacturer becomes more sensitive to the risks of border closures and to potentially longer closures.

As p_{OC} increases for fixed a value of p_{OC} and as p_{CO} decreases for fixed a value of p_{OC}, the optimal order-up-to level and long-run average cost increase faster in Instance 13C than in Instance 1C. Also the maximum difference in order-up-to levels (over all transition probability pairs) for each state is larger for the case of larger mean demand and variance. For example, consider Instances 1C and 13C. The maximum and minimum order-up-to levels for state $(C, 0)$ are respectively 19 and 4 for Instance 1, but they are respectively 7 and 13 for Instance 13C. We therefore observe that stochastically larger demand increases the optimal order-up-to levels and greater demand variance contributes to the increase in the long-run average cost.

4.3.9 Impact of Contingency Planning

Border closures are typically not included in regular operational planning models. Suppose that a firm optimizes its inventory policy without explicitly modeling border closures and congestion and implements them in a real-world environment in which these disruptions

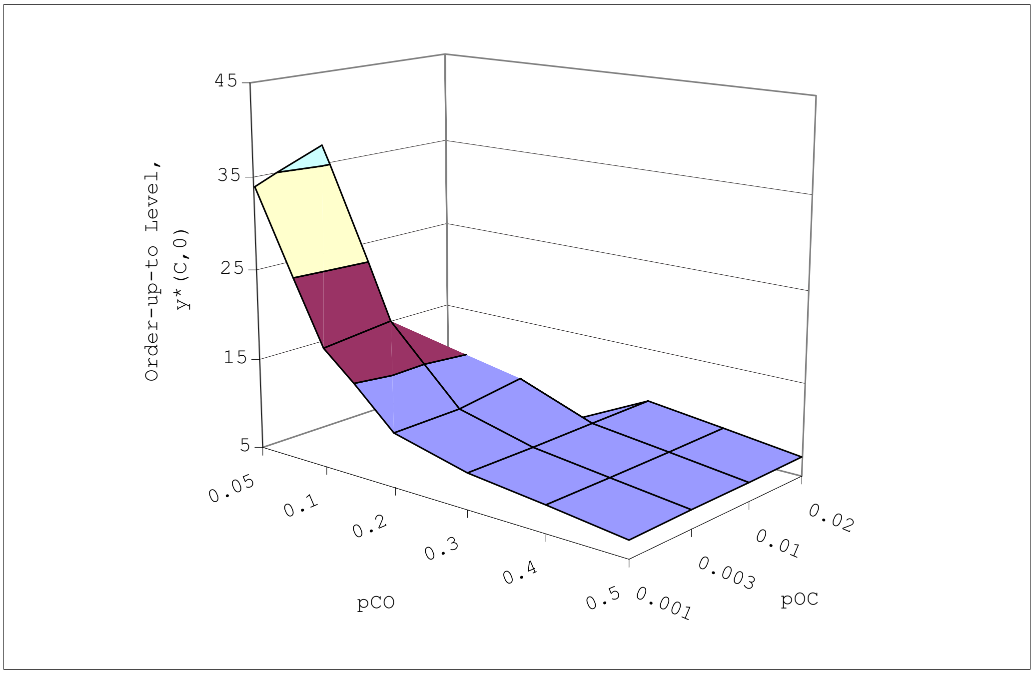

Figure 92: Optimal order-up-to level ($y^*(C,0)$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 13C: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(1)).

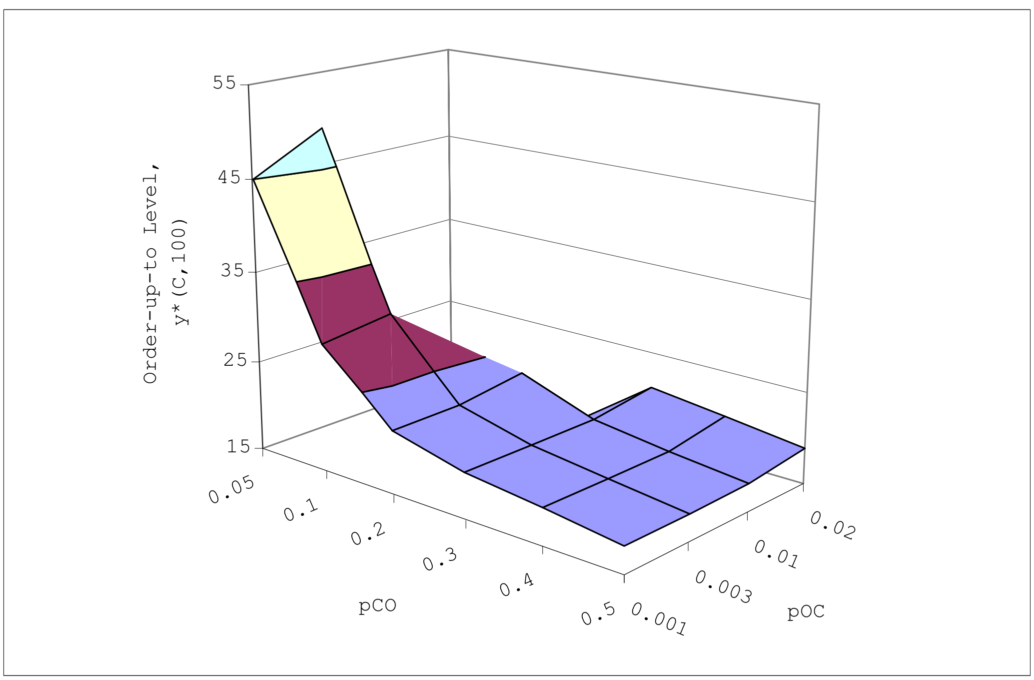

Figure 93: Optimal order-up-to level ($y^*(C,100)$) vs. transition probabilities (p_{OC}, p_{CO})(Instance 13C: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(1)).

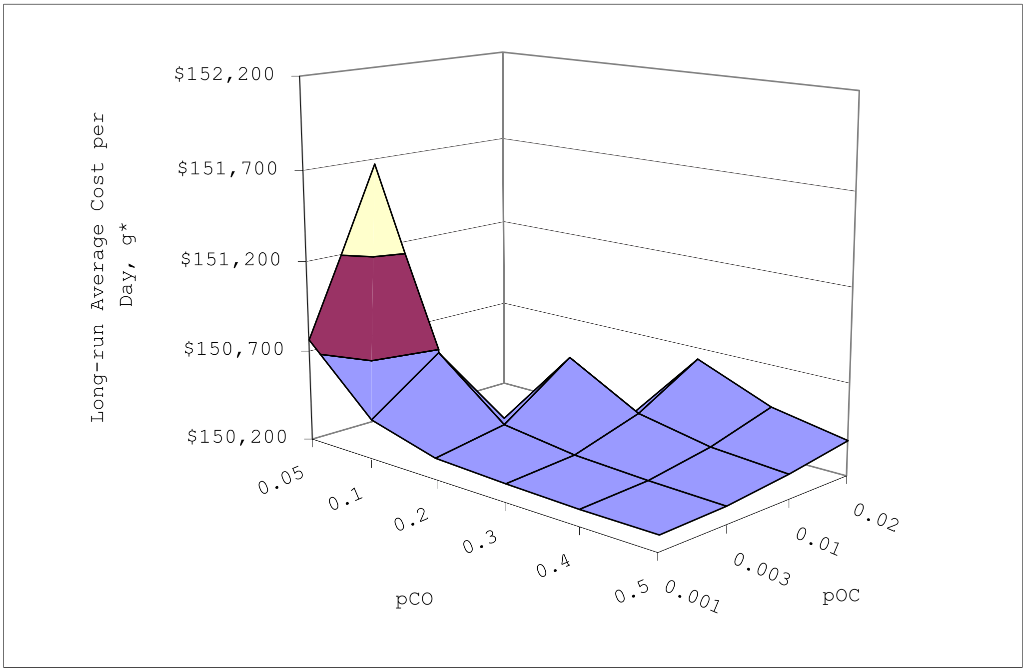

Figure 94: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO})(Instance 13C: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(1)).

may occur. We will refer to this policy as the *implemented policy*. Clearly the implemented policy may be sub-optimal for systems in which the border is actually subject to closure and congestion. In this section we investigate how poor the implemented policy might be.

We determine the optimal inventory policy using a model in which the probability of border closure is zero, e.g. $p_{OC} = 0$. We then calculate the long-run average cost of the implemented policy in a system in which the actual probability of border closure is nonzero, e.g. $p_{OC} > 0$. When there is no possibility of border closures, there is also no possibility congestion and positive-length queues. Therefore the implemented order-up-to level will be state-invariant with respect to both border status and to queue length.

When the implemented policy is sub-optimal, the long-run average cost under the true optimal policy will be less than that under the implemented policy. We interpret this cost reduction as the benefit of contingency planning for border closures and congestion. We use the term contingency planning to mean that the decision maker accounts for border closures and congestion when determining optimal ordering policies. Figures 95-97 display the cost reductions that result from contingency planning for border closures and congestion. While it is clear that contingency planning will result in greater cost reductions for higher holding and penalty costs, the behavior with respect to the minimum leadtime is unclear. Therefore the only system parameter that varies between the figures is the minimum leadtime, L.

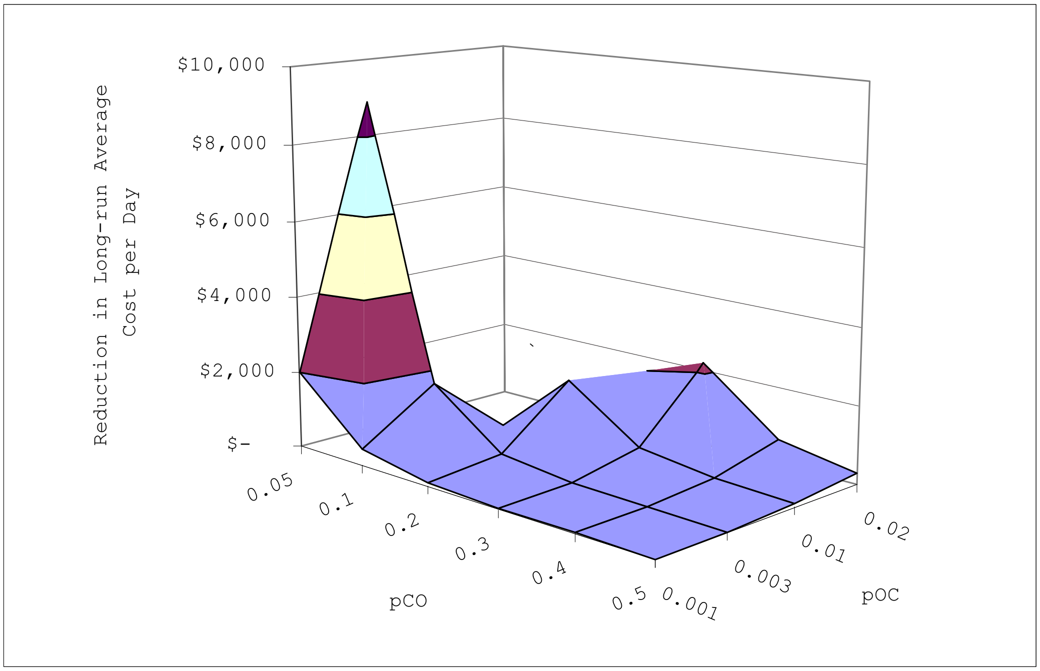

Figure 95: Reduction in long-run average cost per day from contingency planning ($g^y - g^*$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 1C: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

There are scenarios for which contingency planning for border closures and congestion is critically important. Table 10 displays the percent reduction in long-run average cost resulting from contingency planning as well as the annualized cost reductions for Instance 9C (assuming no discounting). Note that cost reductions of 1-2% correspond to annual reductions ranging from \$333,323 to \$526,092. The most impressive data point corresponds to the case when $p_{OC} = 0.003$ and $p_{CO} = 0.05$. The annual expected cost savings resulting from contingency planning in this case is a staggering \$2,606,718. These results, especially when compared to those for the border closure model without congestion, highlight the clear importance of contingency planning for both border closures and congestion.

Contingency planning for border closures and congestion results in greater reductions in the long-run average cost when the minimum leadtime is shorter. When there is no

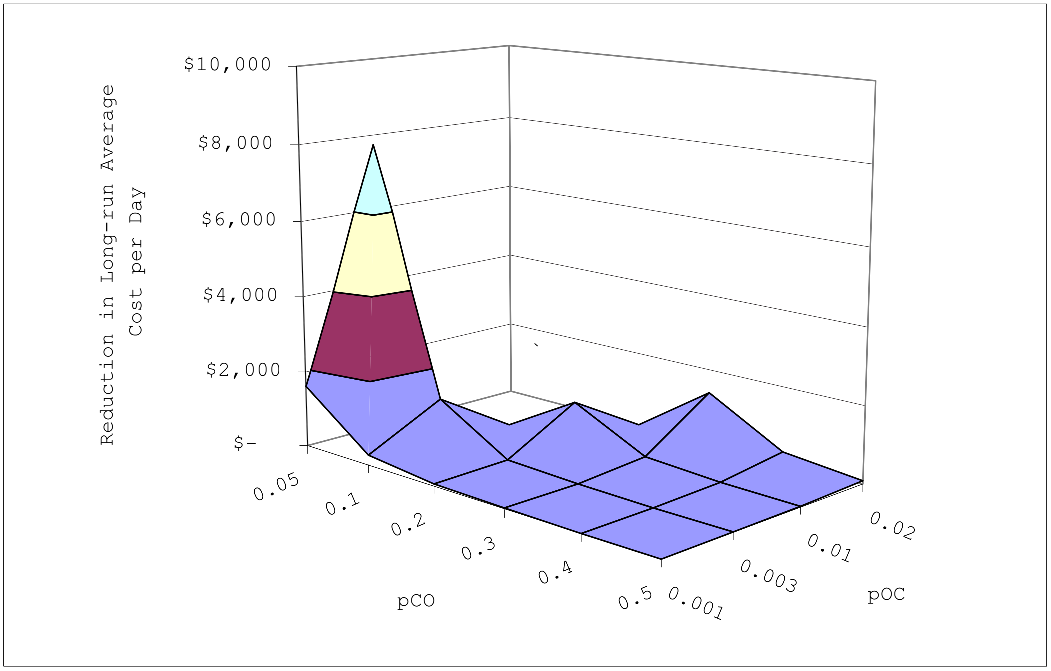

Figure 96: Reduction in long-run average cost per day from contingency planning ($g^y - g^*$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 5C: $L = 7$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

possibility of border closures, a smaller minimum leadtime provides the manufacturer with greater responsiveness to changes in its inventory level. The manufacturer takes advantage by implementing a small order-up-to level, knowing that it can quickly replenish its inventory when necessary in exactly L periods. As the minimum leadtime increases, the optimal implemented order-up-to level partially increases due to the fact the manufacturer will face additional periods of demand before the order arrives and due to greater overall uncertainty about the demand over the minimum leadtime. When the implemented policy is utilized in a system subject to border closures and congestion, this additional inventory buffer then also serves to mitigate the effects of greater uncertainty about the future state of the border. We therefore see the cost reductions from contingency planning decreasing as the minimum leadtime increases.

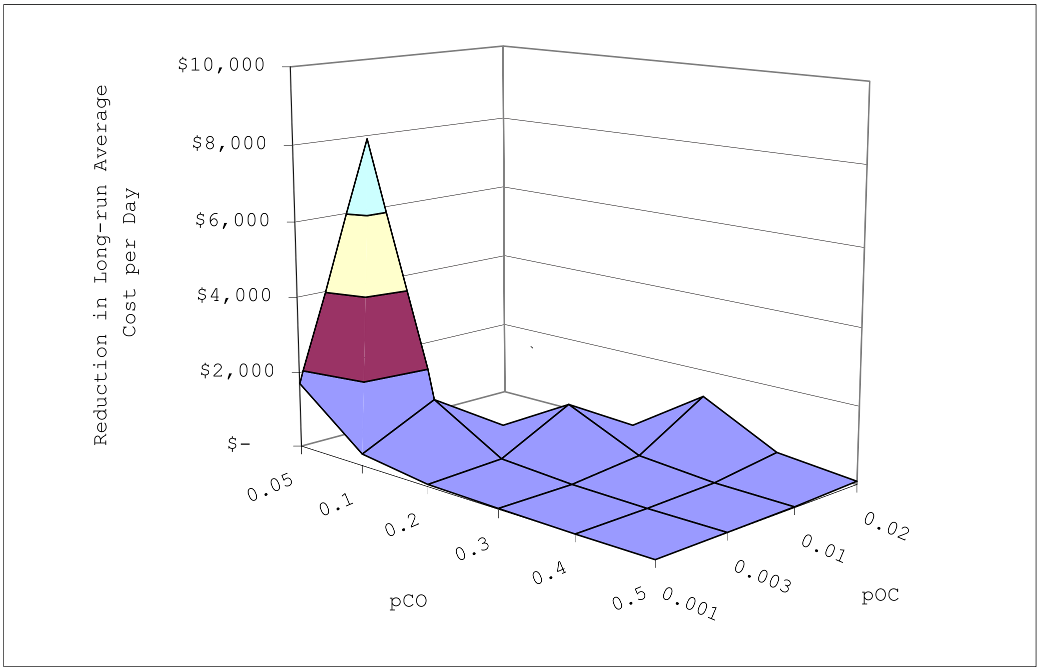

Figure 97: Reduction in long-run average cost per day from contingency planning ($g^y - g^*$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 9C: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

Table 10: Percent ($100 * (g^y - g^*)/g^*$) and annualized dollar reduction ($(g^y - g^*) * 365$) in long-run average cost from contingency planning vs. transition probabilities (p_{OC}, p_{CO}) (Instance 9C: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

		p_{CO}					
		0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	0.00 %	0.00 %	0.01 %	0.03 %	0.29 %	1.90 %
	0.003	0.00 %	0.01 %	0.02 %	0.12 %	1.36 %	9.36 %
	0.01	0.01 %	0.04 %	0.18 %	1.21 %		
	0.02	0.07 %	0.25 %	1.49 %			
p_{OC}	0.001	$ 158	$ 496	$ 1,756	$ 8,285	$ 80,803	$ 526,092
	0.003	$ 581	$ 1,824	$ 6,594	$ 32,878	$ 376,389	$ 2,606,718
	0.01	$ 3,683	$ 12,106	$ 49,342	$ 333,323		
	0.02	$ 18,114	$ 69,198	$ 412,558			

4.4 Conclusions

In this chapter we specialize the inventory control model presented in Chapter 2 to represent a border closure model with congestion. We describe the new border system. The border status may be open or closed and orders are processed through a new customer queue that models congestion. We develop the probability mass function for the order leadtime, which is more complex than in the border closure model without congestion.

The results of a comprehensive numerical study are determined using the value iteration algorithm for Markov decision problems. As is commonly done, we refer to the policy obtained at the termination of the value iteration algorithm as the optimal policy and we refer to a specific approximation to the long-run average cost as the optimal long-run average cost. The comprehensive numerical study investigates the impacts on the optimal order-up-to levels and the long-run average cost of the border transition probabilities, the customer queue length, the border utilization, the minimum leadtime, the economic parameters, and the demand distribution. We prove by counter-example that the optimal policy for the border closure model with congestion is not state-invariant and discuss a special property exhibited by certain border states for which the optimal order quantity is zero.

Some of the important observations and managerial and policy insights include that the optimal inventory policy is more reactive than proactive. That is, the manufacturer tends to only change its order-up-to levels after a border closure has occurred and while congestion remains, rather than in anticipation of a border closure and congestion. The optimal order-up-to levels and long-run average cost are much more sensitive to the expected duration of a disruption than to the occurrence likelihood of a disruption, and these quantities increase more than linearly with the border system utilization. These results have important implications for business to engage and cooperate with government in contingency planning and disruption management to decrease the length of a border closure and for business to encourage government investment to improve the processing capabilities of publicly owned and/or operated ports of entry in order to reduce the effects of post-disruption congestion.

We examine the reduction in long-run average cost resulting from contingency planning for border closures and congestion. The results show that contingency planning is critically important for a manufacturer facing border closures and congestion, especially in supply chains with small leadtimes from the supplier to the international border. Additionally we observe that the optimal order-up-to levels increase with the minimum leadtime, the penalty cost, and stochastically larger demand and decrease with the holding cost. The optimal long-run average cost increases with all of these system parameters.

CHAPTER V

CONCLUSIONS AND FUTURE RESEARCH

5.1 Conclusions

This thesis studies inventory control given the risk of major supply chain disruptions, specifically border closures and congestion. The first part contributes theoretical results for an inventory control model in which the order leadtime distributions are dependent on the state of an exogenous supply system at the time of order placement. The next two parts specialize this model to border closures with negligible congestion and to border closures with congestion respectively. We provide structural policy results, the results of comprehensive numerical studies, and important managerial and policy insights.

We first investigate an inventory system in which the probability distributions of order leadtimes are dependent on the state of an exogenous Markov process at the time of order placement. We utilize this feature to model border closures and congestion in the specialized models. Stationary, state-dependent basestock policies are known to be optimal for this inventory system under linear ordering costs, and we provide an expression for the long-run average cost of an arbitrary policy of this form. We then restrict our attention to state-invariant basestock policies and show how to calculate the optimal basestock (or order-up-to) level and long-run average cost. We provide a sufficient condition for the optimality of a state-invariant basestock policy and structural results about the monotonicity of the optimal state-invariant order-up-to level with respect to a cost ratio of the holding and penalty costs, with respect to the individual holding and penalty costs, and with respect to stochastically larger demand. We finally show that for states in which it is known with probability one that two consecutive orders will arrive in the same future period, the optimal order quantity is zero.

Motivated by the possibility of port of entry closures in the event of a security incident, we then specialize the inventory control model to a two-stage international supply chain.

We consider a simple scenario in which a domestic manufacturer orders a single product from a foreign supplier, and the orders must cross an international border that is subject to closure. We first assume that orders accumulated at the border during periods of closure and arrive at the manufacturer without further delay once the border reopens; that is, border congestion has negligible effects. The manufacturer's optimal inventory policy and long-run average cost are analyzed. We prove the optimality of a state-invariant basestock policy and show that the state-invariant order-up-to level is monotonic in the leadtime from the supplier to the international border. We present the results of a comprehensive numerical study that are determined using the procedures described in the first part of this thesis. The results show that the optimal inventory policy and long-run average cost are much more sensitive to the expected duration of a disruption than to the occurrence likelihood of a disruption. While the prevention of a disruption is critically important, this result has important implications for business to engage and cooperate with government in disruption management and contingency planning in order to reduce the duration of a closure. Contingency planning for border closures is shown to be clearly important and provides greater benefits when the leadtime from the supplier to the international border is small. The numerical results regarding the impacts on the optimal state-invariant order-up-to level with respect to the leadtime from the supplier to the international border, the holding and penalty cost parameters, and the demand distribution illustrate the theoretical monotonicity results. To conclude this part, we present three modeling extensions that model a positive inland transportation time, a maximum delay at the border, and multiple open border states representing increasing probabilities of closure.

Finally we extend the border closure model to include both border closures and the resulting congestion. We model the border processing system and congestion with a discrete-time, single-server queue with constant deterministic arrival rate and Markov-modulated (but otherwise deterministic) service rate. A key task is the development of the leadtime distribution, which is more complex than in the previous model. We prove by counter-example that the optimal policy for the border closure model with congestion is not state-invariant and observe that the order-up-to levels tend to increase when the border is closed and with the level of congestion. We present the results of a comprehensive numerical study which are determined using the value iteration algorithm for Markov decision problems. Based on these results, we provide managerial and policy insights regarding business operations and the management of the infrastructure utilized by supply chains (e.g. ports of entry). The optimal inventory policy is more reactive than proactive, meaning that the manufacturer tends to only change its order-up-to levels after a border closure has occurred and while congestion remains, rather than in anticipation of a border closure. The optimal order-up-to levels and long-run average cost are again more sensitive to the expected duration of a disruption than to the occurrence likelihood of a disruption, and these quantities increase more than linearly with the utilization of the border queueing system. These results have important implications for business to engage and cooperate with government in contingency planning and disruption management and for business to encourage government investment to improve the processing capabilities of publicly owned and/or operated ports of entry in order to reduce the effects of post-disruption congestion. Contingency planning is again critically important for a manufacturer facing border closures and congestion, especially in supply chains with small leadtimes from the supplier to the international border. Additionally we observe that the optimal order-up-to levels and long-run average cost exhibit similar characteristics with respect to the leadtime from the supplier to the international border, the holding and penalty cost parameters, and the demand distribution.

5.2 Future Research

There are several areas of future research.

- The numerical studies of the border closure models with and without congestion in Chapters 3 and 4 yield many observations about the optimal policy and long-run average cost, for example, regarding the monotonicity of the optimal order-up-to levels and the long-run average cost with respect to the transition probabilities. Formalizing many of these observations as proofs would be a useful avenue for future research.

- In sections 3.6-3.7, we present three modeling extensions applicable to both the border closure model with and without congestion. An investigation into the optimal policies and long-run average costs of these extensions is another topic of future research, with a special interest in the extension in section 3.7. Since this scenario considers a case with multiple open states representing increasing probabilities of closure, proactive decision making is more possible than in previous models.

- In the border closure model with congestion, we assume that when the border is open, a maximum of r_1 customers can be processed regardless of the length of the customer queue. In reality, we may expect to see an increase in a port of entry's processing capacity while the congestion exists. For example, work hours may be temporarily extended or additional personnel may be added during periods of high congestion. As the congestion increases, businesses may alternatively book orders on shipping lanes through other ports of entry to avoid the congestion. Therefore the arrival rate of r_0 customers per period may in reality decrease as congestion increases. Additionally, the costs and demand distribution may actually depend on the state of the supply system. Improving the sensitivity of the border processing system, costs, and demand distribution in this manner is a subject of future research.

- Even though the arrival and processing rates of the border queuing system depend on the state of the supply system, the current model uses a deterministic representation of the queuing system given the supply state. An extension for future research would be to model the arrival and processing rates as random variables whose distributions depend on the state of the supply system. It is believed that under realistic assumptions regarding a stochastic border queue, the border closure model with congestion would not include queue states in which it is known with probability one that two successively placed orders will arrive in the same future period, a characteristic of the model with a deterministic border queue given the supply state.

- We prove the optimality of a state-invariant basestock policy for the border closure model without congestion and showed by counter-example that a state-invariant basestock policy is not optimal for the border closure model with congestion. A topic of future research is to investigate necessary and/or sufficient conditions for the optimality of a state-invariant basestock policy for the border closure model with congestion.

- In Chapter 4, we use explicit enumeration to determine $f_{(i,n)}$, the probability distribution for $(i_{t+L}, n_{t+L} | i_t = i, n_t = n)$. This requires the explicit enumeration of all border state sample paths of the length $L+1$ for each initial state $(i, n) \in S_B$. As L grows large, enumeration clearly becomes inefficient. Investigating this probability distribution and approximations of the distribution is a subject for future research.

- In the border closure model with congestion, we assume that the manufacturer completely observes the state of the border. While it is feasible that the the manufacturer will know whether the border is open or closed, it may not know with certainty the length of the customer queue and where its orders are within the queue. Modeling the border closure model with congestion as a partially-observed Markov decision problem is a subject for future research. A comparison of the results of this model with those presented in this thesis would provide insight into the value of information regarding the level of congestion at a port of entry. This would have implications for the communication between business and ports of entry as well as for the effective assessment of congestion levels.

- We assume in this thesis that there only exists a single transportation route from supplier to manufacturer. An interesting and challenging subject for future research is to model the option to reroute in-transit orders through a secondary port of entry in the event that the primary port of entry closes.

- During the 2002 West Coast seaport closures in the United States, many businesses relied on emergency shipments via air carriers. Another potential subject for future research is the integration of emergency orders with regular orders in the border closure models and to study the structure of the optimal ordering policies for both order types.

APPENDIX A

NUMERICAL STUDY TABLES: BORDER CLOSURE MODEL WITHOUT CONGESTION

Table 11: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 1: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	2	2	2	2	2	2	2	2	2	2	3
	0.003	2	2	2	2	2	2	2	2	2	3	3
	0.01	2	2	2	2	2	2	2	3	3	3	7
	0.02	2	2	2	2	2	2	3	3	3	4	13
	0.05	2	2	2	3	3	3	3	3	4	8	18
	0.1	3	3	3	3	3	3	3	3	5	10	21
	0.2	3	3	3	3	3	3	3	4	6	11	23
	0.3	3	3	3	3	3	3	4	4	6	12	24
	0.4	3	3	3	3	3	3	4	5	6	12	24
	0.5	3	3	3	3	3	3	4	5	7	12	24
	0.6	3	3	3	3	3	4	4	5	7	13	25
	0.7	3	3	3	3	3	4	4	5	7	13	25
	0.8	3	3	3	3	3	4	4	5	7	13	25
	0.9	3	3	3	3	3	4	4	5	7	13	25
	0.95	3	3	3	3	3	4	4	5	7	13	25

Table 12: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 2: $L = 1$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	3	3	3	3	3	3	3	3	3	3	3
	0.003	3	3	3	3	3	3	3	3	3	3	4
	0.01	3	3	3	3	3	3	3	3	3	5	14
	0.02	3	3	3	3	3	3	3	3	4	7	19
	0.05	3	3	3	3	3	3	3	4	5	11	25
	0.1	3	3	3	3	3	3	4	4	6	13	28
	0.2	3	3	3	3	3	4	4	5	7	15	30
	0.3	3	3	3	3	4	4	4	6	8	15	30
	0.4	3	3	3	4	4	4	5	6	8	16	31
	0.5	3	3	3	4	4	4	5	6	8	16	31
	0.6	3	3	4	4	4	4	5	6	8	16	31
	0.7	3	3	4	4	4	4	5	6	9	16	31
	0.8	3	3	4	4	4	4	5	6	9	16	31
	0.9	3	3	4	4	4	4	5	6	9	16	31
	0.95	3	3	4	4	4	4	5	6	9	16	31

Table 13: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 3: $L = 1$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	1	1	1	1	1	1	1	1	1	1	1
	0.003	1	1	1	1	1	1	1	1	1	1	1
	0.01	1	1	1	1	1	1	1	1	1	1	2
	0.02	1	1	1	1	1	1	1	1	1	2	2
	0.05	1	1	1	1	1	1	1	1	2	2	5
	0.1	1	1	1	1	1	1	1	2	2	3	8
	0.2	1	1	1	1	1	2	2	2	2	5	10
	0.3	1	1	1	2	2	2	2	2	3	5	10
	0.4	1	1	2	2	2	2	2	2	3	5	11
	0.5	1	2	2	2	2	2	2	2	3	6	11
	0.6	2	2	2	2	2	2	2	2	3	6	11
	0.7	2	2	2	2	2	2	2	3	3	6	11
	0.8	2	2	2	2	2	2	2	3	3	6	11
	0.9	2	2	2	2	2	2	2	3	3	6	12
	0.95	2	2	2	2	2	2	2	3	3	6	12

Table 14: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 4: $L = 1$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	2	2	2	2	2	2	2	2	2	2	2
	0.003	2	2	2	2	2	2	2	2	2	2	2
	0.01	2	2	2	2	2	2	2	2	2	2	3
	0.02	2	2	2	2	2	2	2	2	2	2	5
	0.05	2	2	2	2	2	2	2	2	2	4	10
	0.1	2	2	2	2	2	2	2	2	3	6	13
	0.2	2	2	2	2	2	2	2	3	4	7	15
	0.3	2	2	2	2	2	2	3	3	4	8	16
	0.4	2	2	2	2	2	2	3	3	4	8	16
	0.5	2	2	2	2	2	2	3	3	4	8	16
	0.6	2	2	2	2	2	3	3	3	5	8	16
	0.7	2	2	2	2	2	3	3	3	5	9	17
	0.8	2	2	2	2	2	3	3	4	5	9	17
	0.9	2	2	2	2	2	3	3	4	5	9	17
	0.95	2	2	2	2	3	3	3	4	5	9	17

Table 15: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 5: $L = 7$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	7	7	7	7	7	7	7	7	7	7	7
	0.003	7	7	7	7	7	7	7	7	7	7	7
	0.01	7	7	7	7	7	7	7	7	7	8	10
	0.02	7	7	7	7	7	7	7	7	7	8	16
	0.05	7	7	7	7	7	7	7	7	8	11	21
	0.1	7	7	7	7	7	7	7	8	9	13	24
	0.2	7	7	7	7	7	7	8	8	9	15	26
	0.3	7	7	7	7	7	7	8	8	10	15	27
	0.4	7	7	7	7	7	8	8	9	10	16	27
	0.5	7	7	7	7	7	8	8	9	10	16	28
	0.6	7	7	7	7	7	8	8	9	10	16	28
	0.7	7	7	7	7	7	8	8	9	10	16	28
	0.8	7	7	7	7	8	8	8	9	11	16	28
	0.9	7	7	7	7	8	8	8	9	11	16	28
	0.95	7	7	7	7	8	8	8	9	11	16	28

Table 16: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 6: $L = 7$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	8	8	8	8	8	8	8	8	8	8	8
	0.003	8	8	8	8	8	8	8	8	8	8	9
	0.01	8	8	8	8	8	8	8	8	8	9	17
	0.02	8	8	8	8	8	8	8	8	8	11	22
	0.05	8	8	8	8	8	8	8	8	9	14	28
	0.1	8	8	8	8	8	8	8	9	10	16	31
	0.2	8	8	8	8	8	8	9	9	11	18	33
	0.3	8	8	8	8	8	8	9	10	12	19	34
	0.4	8	8	8	8	8	8	9	10	12	19	34
	0.5	8	8	8	8	8	9	9	10	12	19	34
	0.6	8	8	8	8	8	9	9	10	12	19	34
	0.7	8	8	8	8	8	9	9	10	12	19	34
	0.8	8	8	8	8	8	9	9	10	12	19	35
	0.9	8	8	8	8	8	9	9	10	12	20	35
	0.95	8	8	8	8	8	9	9	10	12	20	35

Table 17: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 7: $L = 7$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	5	5	5	5	5	5	5	5	5	5	5
	0.003	5	5	5	5	5	5	5	5	5	5	5
	0.01	5	5	5	5	5	5	5	5	5	5	5
	0.02	5	5	5	5	5	5	5	5	5	5	6
	0.05	5	5	5	5	5	5	5	5	5	6	8
	0.1	5	5	5	5	5	5	5	5	5	7	11
	0.2	5	5	5	5	5	5	5	5	6	8	13
	0.3	5	5	5	5	5	5	5	6	6	8	14
	0.4	5	5	5	5	5	5	5	6	6	9	14
	0.5	5	5	5	5	5	5	5	6	7	9	14
	0.6	5	5	5	5	5	5	6	6	7	9	14
	0.7	5	5	5	5	5	5	6	6	7	9	15
	0.8	5	5	5	5	5	5	6	6	7	9	15
	0.9	5	5	5	5	5	5	6	6	7	9	15
	0.95	5	5	5	5	5	5	6	6	7	9	15

Table 18: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 8: $L = 7$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	6	6	6	6	6	6	6	6	6	6	6
	0.003	6	6	6	6	6	6	6	6	6	6	6
	0.01	6	6	6	6	6	6	6	6	6	6	7
	0.02	6	6	6	6	6	6	6	6	6	6	8
	0.05	6	6	6	6	6	6	6	6	6	8	13
	0.1	6	6	6	6	6	6	6	6	7	9	16
	0.2	6	6	6	6	6	6	6	7	7	10	18
	0.3	6	6	6	6	6	6	6	7	8	11	19
	0.4	6	6	6	6	6	6	6	7	8	11	19
	0.5	6	6	6	6	6	6	7	7	8	12	19
	0.6	6	6	6	6	6	6	7	7	8	12	20
	0.7	6	6	6	6	6	6	7	7	8	12	20
	0.8	6	6	6	6	6	6	7	7	8	12	20
	0.9	6	6	6	6	6	6	7	7	8	12	20
	0.95	6	6	6	6	6	6	7	7	8	12	20

Table 19: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 9: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	12	12	12	12	12	12	12	12	12	12	12
	0.003	12	12	12	12	12	12	12	12	12	12	13
	0.01	12	12	12	12	12	12	12	12	12	13	15
	0.02	12	12	12	12	12	12	12	12	12	14	20
	0.05	12	12	12	12	12	12	12	12	13	16	26
	0.1	12	12	12	12	12	12	12	13	13	18	29
	0.2	12	12	12	12	12	12	13	13	14	19	30
	0.3	12	12	12	12	12	12	13	13	15	20	31
	0.4	12	12	12	12	12	13	13	13	15	20	32
	0.5	12	12	12	12	12	13	13	14	15	20	32
	0.6	12	12	12	12	12	13	13	14	15	20	32
	0.7	12	12	12	12	13	13	13	14	15	20	32
	0.8	12	12	12	12	13	13	13	14	15	21	32
	0.9	12	12	12	12	13	13	13	14	15	21	32
	0.95	12	12	12	12	13	13	13	14	15	21	32

Table 20: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 10: $L = 15$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	13	13	13	13	13	13	13	13	13	13	13
	0.003	13	13	13	13	13	13	13	13	13	13	14
	0.01	13	13	13	13	13	13	13	13	13	14	21
	0.02	13	13	13	13	13	13	13	13	14	16	27
	0.05	13	13	13	13	13	13	13	14	14	19	32
	0.1	13	13	13	13	13	13	13	14	15	21	35
	0.2	13	13	13	13	13	13	14	14	16	22	37
	0.3	13	13	13	13	13	14	14	15	16	23	38
	0.4	13	13	13	13	14	14	14	15	17	23	38
	0.5	13	13	13	13	14	14	14	15	17	24	38
	0.6	13	13	13	13	14	14	14	15	17	24	39
	0.7	13	13	13	14	14	14	14	15	17	24	39
	0.8	13	13	13	14	14	14	14	15	17	24	39
	0.9	13	13	13	14	14	14	14	15	17	24	39
	0.95	13	13	13	14	14	14	14	15	17	24	39

Table 21: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 11: $L = 15$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	9	9	9	9	9	9	9	9	9	9	9
	0.003	9	9	9	9	9	9	9	9	9	9	9
	0.01	9	9	9	9	9	9	9	9	9	9	10
	0.02	9	9	9	9	9	9	9	9	9	10	11
	0.05	9	9	9	9	9	9	9	9	10	10	13
	0.1	9	9	9	9	9	9	9	10	10	11	15
	0.2	9	9	9	9	9	9	10	10	10	12	17
	0.3	9	9	9	9	9	9	10	10	11	13	18
	0.4	9	9	9	9	9	10	10	10	11	13	18
	0.5	9	9	9	9	9	10	10	10	11	13	18
	0.6	9	9	9	9	10	10	10	10	11	13	19
	0.7	9	9	9	9	10	10	10	10	11	14	19
	0.8	9	9	9	9	10	10	10	10	11	14	19
	0.9	9	9	9	10	10	10	10	10	11	14	19
	0.95	9	9	9	10	10	10	10	10	11	14	19

Table 22: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 12: $L = 15$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	10	10	10	10	10	10	10	10	10	10	10
	0.003	10	10	10	10	10	10	10	10	10	10	11
	0.01	10	10	10	10	10	10	10	10	10	11	12
	0.02	10	10	10	10	10	10	10	10	11	11	13
	0.05	10	10	10	10	10	10	10	11	11	12	18
	0.1	10	10	10	10	10	11	11	11	11	14	20
	0.2	10	10	10	11	11	11	11	11	12	15	22
	0.3	10	10	11	11	11	11	11	11	12	16	23
	0.4	11	11	11	11	11	11	11	12	13	16	23
	0.5	11	11	11	11	11	11	11	12	13	16	24
	0.6	11	11	11	11	11	11	11	12	13	16	24
	0.7	11	11	11	11	11	11	11	12	13	16	24
	0.8	11	11	11	11	11	11	11	12	13	16	24
	0.9	11	11	11	11	11	11	11	12	13	16	24
	0.95	11	11	11	11	11	11	11	12	13	16	24

Table 23: Optimal expected holding and penalty cost per day ($E[HPC]$) vs. transition probabilities (p_{OC},p_{CO}) (Instance 1: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	$ 214	$ 214	$ 214	$ 214	$ 215	$ 215	$ 216	$ 217	$ 222	$ 254	$ 384
	0.003	$ 214	$ 215	$ 215	$ 215	$ 216	$ 217	$ 219	$ 223	$ 239	$ 322	$ 682
	0.01	$ 216	$ 216	$ 217	$ 218	$ 219	$ 223	$ 229	$ 243	$ 280	$ 526	$ 1,459
	0.02	$ 217	$ 218	$ 219	$ 221	$ 225	$ 231	$ 240	$ 260	$ 330	$ 748	$ 1,893
	0.05	$ 222	$ 223	$ 226	$ 231	$ 235	$ 242	$ 259	$ 304	$ 451	$ 1,024	$ 2,251
	0.1	$ 228	$ 229	$ 231	$ 235	$ 242	$ 256	$ 286	$ 362	$ 544	$ 1,152	$ 2,379
	0.2	$ 231	$ 232	$ 236	$ 242	$ 254	$ 278	$ 326	$ 411	$ 611	$ 1,220	$ 2,432
	0.3	$ 232	$ 234	$ 239	$ 248	$ 264	$ 294	$ 348	$ 437	$ 633	$ 1,239	$ 2,446
	0.4	$ 234	$ 236	$ 242	$ 253	$ 272	$ 306	$ 355	$ 451	$ 648	$ 1,248	$ 2,451
	0.5	$ 235	$ 238	$ 245	$ 257	$ 278	$ 316	$ 361	$ 456	$ 654	$ 1,254	$ 2,454
	0.6	$ 237	$ 239	$ 247	$ 260	$ 283	$ 322	$ 365	$ 460	$ 657	$ 1,256	$ 2,456
	0.7	$ 238	$ 241	$ 249	$ 263	$ 287	$ 323	$ 369	$ 463	$ 659	$ 1,258	$ 2,457
	0.8	$ 239	$ 242	$ 251	$ 266	$ 291	$ 324	$ 372	$ 465	$ 661	$ 1,259	$ 2,457
	0.9	$ 239	$ 243	$ 252	$ 268	$ 295	$ 325	$ 374	$ 467	$ 662	$ 1,259	$ 2,458
	0.95	$ 240	$ 243	$ 253	$ 269	$ 296	$ 325	$ 375	$ 468	$ 663	$ 1,260	$ 2,458

Table 24: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 1: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	\$ 75,214	\$ 75,214	\$ 75,214	\$ 75,214	\$ 75,215	\$ 75,215	\$ 75,216	\$ 75,217	\$ 75,222	\$ 75,254	\$ 75,384
	0.003	\$ 75,214	\$ 75,215	\$ 75,215	\$ 75,215	\$ 75,216	\$ 75,217	\$ 75,219	\$ 75,223	\$ 75,239	\$ 75,322	\$ 75,682
	0.01	\$ 75,216	\$ 75,216	\$ 75,217	\$ 75,218	\$ 75,219	\$ 75,223	\$ 75,229	\$ 75,243	\$ 75,280	\$ 75,526	\$ 76,459
	0.02	\$ 75,217	\$ 75,218	\$ 75,219	\$ 75,221	\$ 75,225	\$ 75,231	\$ 75,240	\$ 75,260	\$ 75,330	\$ 75,748	\$ 76,893
	0.05	\$ 75,222	\$ 75,223	\$ 75,226	\$ 75,231	\$ 75,235	\$ 75,242	\$ 75,259	\$ 75,304	\$ 75,451	\$ 76,024	\$ 77,251
	0.1	\$ 75,228	\$ 75,229	\$ 75,231	\$ 75,235	\$ 75,242	\$ 75,256	\$ 75,286	\$ 75,362	\$ 75,544	\$ 76,152	\$ 77,379
	0.2	\$ 75,231	\$ 75,232	\$ 75,236	\$ 75,242	\$ 75,254	\$ 75,278	\$ 75,326	\$ 75,411	\$ 75,611	\$ 76,220	\$ 77,432
	0.3	\$ 75,232	\$ 75,234	\$ 75,239	\$ 75,248	\$ 75,264	\$ 75,294	\$ 75,348	\$ 75,437	\$ 75,633	\$ 76,239	\$ 77,446
	0.4	\$ 75,234	\$ 75,236	\$ 75,242	\$ 75,253	\$ 75,272	\$ 75,306	\$ 75,355	\$ 75,451	\$ 75,648	\$ 76,248	\$ 77,451
	0.5	\$ 75,235	\$ 75,238	\$ 75,245	\$ 75,257	\$ 75,278	\$ 75,316	\$ 75,361	\$ 75,456	\$ 75,654	\$ 76,254	\$ 77,454
	0.6	\$ 75,237	\$ 75,239	\$ 75,247	\$ 75,260	\$ 75,283	\$ 75,322	\$ 75,365	\$ 75,460	\$ 75,657	\$ 76,256	\$ 77,456
	0.7	\$ 75,238	\$ 75,241	\$ 75,249	\$ 75,263	\$ 75,287	\$ 75,323	\$ 75,369	\$ 75,463	\$ 75,659	\$ 76,258	\$ 77,457
	0.8	\$ 75,239	\$ 75,242	\$ 75,251	\$ 75,266	\$ 75,291	\$ 75,324	\$ 75,372	\$ 75,465	\$ 75,661	\$ 76,259	\$ 77,457
	0.9	\$ 75,239	\$ 75,243	\$ 75,252	\$ 75,268	\$ 75,295	\$ 75,325	\$ 75,374	\$ 75,467	\$ 75,662	\$ 76,259	\$ 77,458
	0.95	\$ 75,240	\$ 75,243	\$ 75,253	\$ 75,269	\$ 75,296	\$ 75,325	\$ 75,375	\$ 75,468	\$ 75,663	\$ 76,260	\$ 77,458

Table 25: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 2: $L = 1$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	\$ 75,249	\$ 75,249	\$ 75,249	\$ 75,249	\$ 75,249	\$ 75,250	\$ 75,251	\$ 75,253	\$ 75,261	\$ 75,316	\$ 75,569
	0.003	\$ 75,249	\$ 75,249	\$ 75,250	\$ 75,250	\$ 75,250	\$ 75,252	\$ 75,254	\$ 75,261	\$ 75,285	\$ 75,446	\$ 76,143
	0.01	\$ 75,250	\$ 75,250	\$ 75,251	\$ 75,252	\$ 75,254	\$ 75,258	\$ 75,266	\$ 75,288	\$ 75,364	\$ 75,809	\$ 77,122
	0.02	\$ 75,251	\$ 75,252	\$ 75,253	\$ 75,255	\$ 75,259	\$ 75,266	\$ 75,282	\$ 75,324	\$ 75,457	\$ 76,084	\$ 77,554
	0.05	\$ 75,254	\$ 75,255	\$ 75,258	\$ 75,263	\$ 75,272	\$ 75,289	\$ 75,326	\$ 75,403	\$ 75,611	\$ 76,361	\$ 77,913
	0.1	\$ 75,259	\$ 75,261	\$ 75,266	\$ 75,275	\$ 75,291	\$ 75,322	\$ 75,370	\$ 75,474	\$ 75,717	\$ 76,490	\$ 78,041
	0.2	\$ 75,267	\$ 75,270	\$ 75,280	\$ 75,295	\$ 75,323	\$ 75,352	\$ 75,411	\$ 75,530	\$ 75,786	\$ 76,558	\$ 78,094
	0.3	\$ 75,273	\$ 75,278	\$ 75,291	\$ 75,312	\$ 75,334	\$ 75,366	\$ 75,440	\$ 75,559	\$ 75,808	\$ 76,578	\$ 78,108
	0.4	\$ 75,279	\$ 75,285	\$ 75,300	\$ 75,322	\$ 75,339	\$ 75,376	\$ 75,448	\$ 75,568	\$ 75,820	\$ 76,587	\$ 78,113
	0.5	\$ 75,284	\$ 75,290	\$ 75,308	\$ 75,323	\$ 75,343	\$ 75,384	\$ 75,453	\$ 75,574	\$ 75,829	\$ 76,591	\$ 78,116
	0.6	\$ 75,288	\$ 75,295	\$ 75,314	\$ 75,325	\$ 75,346	\$ 75,391	\$ 75,457	\$ 75,579	\$ 75,835	\$ 76,593	\$ 78,117
	0.7	\$ 75,292	\$ 75,300	\$ 75,315	\$ 75,326	\$ 75,349	\$ 75,397	\$ 75,461	\$ 75,583	\$ 75,837	\$ 76,595	\$ 78,119
	0.8	\$ 75,295	\$ 75,303	\$ 75,315	\$ 75,327	\$ 75,352	\$ 75,402	\$ 75,464	\$ 75,586	\$ 75,838	\$ 76,597	\$ 78,120
	0.9	\$ 75,298	\$ 75,307	\$ 75,315	\$ 75,328	\$ 75,354	\$ 75,406	\$ 75,466	\$ 75,589	\$ 75,839	\$ 76,598	\$ 78,121
	0.95	\$ 75,300	\$ 75,308	\$ 75,316	\$ 75,329	\$ 75,355	\$ 75,408	\$ 75,467	\$ 75,590	\$ 75,839	\$ 76,598	\$ 78,121

Table 26: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 3: $L = 1$, $h = \$500$, $p = \$1,000$, $D \sim \text{Poisson}(0.5)$).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	\$ 75,552	\$ 75,552	\$ 75,552	\$ 75,552	\$ 75,553	\$ 75,553	\$ 75,554	\$ 75,556	\$ 75,562	\$ 75,597	\$ 75,738
	0.003	\$ 75,553	\$ 75,553	\$ 75,553	\$ 75,554	\$ 75,555	\$ 75,556	\$ 75,559	\$ 75,565	\$ 75,583	\$ 75,684	\$ 76,089
	0.01	\$ 75,555	\$ 75,555	\$ 75,556	\$ 75,558	\$ 75,561	\$ 75,565	\$ 75,574	\$ 75,593	\$ 75,651	\$ 75,963	\$ 77,074
	0.02	\$ 75,558	\$ 75,559	\$ 75,561	\$ 75,564	\$ 75,569	\$ 75,578	\$ 75,595	\$ 75,632	\$ 75,741	\$ 76,263	\$ 78,087
	0.05	\$ 75,567	\$ 75,569	\$ 75,574	\$ 75,582	\$ 75,594	\$ 75,614	\$ 75,653	\$ 75,736	\$ 75,928	\$ 76,871	\$ 79,601
	0.1	\$ 75,581	\$ 75,584	\$ 75,594	\$ 75,608	\$ 75,630	\$ 75,667	\$ 75,734	\$ 75,827	\$ 76,109	\$ 77,376	\$ 80,240
	0.2	\$ 75,604	\$ 75,611	\$ 75,627	\$ 75,652	\$ 75,688	\$ 75,723	\$ 75,786	\$ 75,930	\$ 76,336	\$ 77,709	\$ 80,508
	0.3	\$ 75,624	\$ 75,633	\$ 75,655	\$ 75,683	\$ 75,705	\$ 75,745	\$ 75,823	\$ 75,999	\$ 76,425	\$ 77,797	\$ 80,580
	0.4	\$ 75,641	\$ 75,652	\$ 75,674	\$ 75,689	\$ 75,715	\$ 75,761	\$ 75,851	\$ 76,048	\$ 76,468	\$ 77,850	\$ 80,603
	0.5	\$ 75,656	\$ 75,667	\$ 75,677	\$ 75,694	\$ 75,723	\$ 75,774	\$ 75,873	\$ 76,084	\$ 76,499	\$ 77,869	\$ 80,616
	0.6	\$ 75,665	\$ 75,669	\$ 75,680	\$ 75,698	\$ 75,730	\$ 75,785	\$ 75,890	\$ 76,113	\$ 76,523	\$ 77,879	\$ 80,626
	0.7	\$ 75,665	\$ 75,670	\$ 75,682	\$ 75,702	\$ 75,735	\$ 75,794	\$ 75,905	\$ 76,133	\$ 76,541	\$ 77,887	\$ 80,633
	0.8	\$ 75,666	\$ 75,671	\$ 75,684	\$ 75,705	\$ 75,740	\$ 75,802	\$ 75,916	\$ 76,137	\$ 76,555	\$ 77,893	\$ 80,638
	0.9	\$ 75,667	\$ 75,672	\$ 75,686	\$ 75,708	\$ 75,744	\$ 75,808	\$ 75,926	\$ 76,140	\$ 76,567	\$ 77,898	\$ 80,642
	0.95	\$ 75,667	\$ 75,672	\$ 75,686	\$ 75,709	\$ 75,746	\$ 75,811	\$ 75,931	\$ 76,141	\$ 76,572	\$ 77,900	\$ 80,642

Table 27: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 4: $L = 1$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	\$ 75,759	\$ 75,759	\$ 75,759	\$ 75,760	\$ 75,760	\$ 75,761	\$ 75,762	\$ 75,765	\$ 75,775	\$ 75,836	\$ 76,103
	0.003	\$ 75,760	\$ 75,760	\$ 75,760	\$ 75,761	\$ 75,762	\$ 75,763	\$ 75,767	\$ 75,776	\$ 75,805	\$ 75,985	\$ 76,751
	0.01	\$ 75,761	\$ 75,762	\$ 75,763	\$ 75,765	\$ 75,768	\$ 75,773	\$ 75,785	\$ 75,814	\$ 75,907	\$ 76,463	\$ 78,660
	0.02	\$ 75,764	\$ 75,764	\$ 75,766	\$ 75,770	\$ 75,776	\$ 75,787	\$ 75,810	\$ 75,865	\$ 76,041	\$ 77,050	\$ 80,430
	0.05	\$ 75,770	\$ 75,772	\$ 75,777	\$ 75,785	\$ 75,799	\$ 75,825	\$ 75,878	\$ 76,002	\$ 76,379	\$ 78,070	\$ 82,217
	0.1	\$ 75,780	\$ 75,783	\$ 75,793	\$ 75,808	\$ 75,834	\$ 75,881	\$ 75,973	\$ 76,184	\$ 76,697	\$ 78,702	\$ 82,856
	0.2	\$ 75,796	\$ 75,803	\$ 75,820	\$ 75,846	\$ 75,890	\$ 75,967	\$ 76,115	\$ 76,374	\$ 77,000	\$ 79,034	\$ 83,122
	0.3	\$ 75,811	\$ 75,819	\$ 75,842	\$ 75,877	\$ 75,934	\$ 76,033	\$ 76,207	\$ 76,453	\$ 77,097	\$ 79,134	\$ 83,192
	0.4	\$ 75,823	\$ 75,833	\$ 75,860	\$ 75,902	\$ 75,969	\$ 76,083	\$ 76,231	\$ 76,509	\$ 77,163	\$ 79,176	\$ 83,217
	0.5	\$ 75,833	\$ 75,845	\$ 75,876	\$ 75,923	\$ 75,998	\$ 76,124	\$ 76,251	\$ 76,551	\$ 77,209	\$ 79,205	\$ 83,234
	0.6	\$ 75,842	\$ 75,855	\$ 75,889	\$ 75,941	\$ 76,022	\$ 76,134	\$ 76,266	\$ 76,584	\$ 77,225	\$ 79,225	\$ 83,245
	0.7	\$ 75,850	\$ 75,864	\$ 75,901	\$ 75,956	\$ 76,042	\$ 76,140	\$ 76,279	\$ 76,611	\$ 77,233	\$ 79,230	\$ 83,249
	0.8	\$ 75,857	\$ 75,872	\$ 75,911	\$ 75,969	\$ 76,059	\$ 76,144	\$ 76,289	\$ 76,619	\$ 77,240	\$ 79,234	\$ 83,251
	0.9	\$ 75,864	\$ 75,879	\$ 75,920	\$ 75,980	\$ 76,074	\$ 76,148	\$ 76,298	\$ 76,622	\$ 77,245	\$ 79,237	\$ 83,253
	0.95	\$ 75,867	\$ 75,882	\$ 75,924	\$ 75,985	\$ 76,079	\$ 76,150	\$ 76,302	\$ 76,623	\$ 77,247	\$ 79,238	\$ 83,253

Table 28: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 5: $L = 7$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	$ 75,393	$ 75,393	$ 75,393	$ 75,393	$ 75,393	$ 75,393	$ 75,394	$ 75,395	$ 75,397	$ 75,420	$ 75,536
	0.003	$ 75,393	$ 75,393	$ 75,393	$ 75,393	$ 75,394	$ 75,394	$ 75,395	$ 75,397	$ 75,406	$ 75,473	$ 75,805
	0.01	$ 75,394	$ 75,394	$ 75,394	$ 75,394	$ 75,395	$ 75,396	$ 75,398	$ 75,406	$ 75,434	$ 75,639	$ 76,478
	0.02	$ 75,394	$ 75,394	$ 75,394	$ 75,395	$ 75,396	$ 75,398	$ 75,403	$ 75,417	$ 75,471	$ 75,808	$ 76,907
	0.05	$ 75,395	$ 75,395	$ 75,396	$ 75,397	$ 75,400	$ 75,405	$ 75,417	$ 75,448	$ 75,548	$ 76,054	$ 77,267
	0.1	$ 75,396	$ 75,397	$ 75,398	$ 75,401	$ 75,405	$ 75,415	$ 75,435	$ 75,486	$ 75,622	$ 76,182	$ 77,394
	0.2	$ 75,398	$ 75,399	$ 75,402	$ 75,407	$ 75,415	$ 75,430	$ 75,463	$ 75,515	$ 75,676	$ 76,251	$ 77,447
	0.3	$ 75,400	$ 75,402	$ 75,405	$ 75,411	$ 75,422	$ 75,442	$ 75,470	$ 75,535	$ 75,697	$ 76,270	$ 77,460
	0.4	$ 75,402	$ 75,404	$ 75,408	$ 75,415	$ 75,427	$ 75,450	$ 75,476	$ 75,549	$ 75,708	$ 76,280	$ 77,466
	0.5	$ 75,403	$ 75,405	$ 75,410	$ 75,418	$ 75,432	$ 75,452	$ 75,480	$ 75,552	$ 75,715	$ 76,283	$ 77,470
	0.6	$ 75,405	$ 75,407	$ 75,412	$ 75,421	$ 75,436	$ 75,453	$ 75,483	$ 75,554	$ 75,721	$ 76,286	$ 77,471
	0.7	$ 75,406	$ 75,408	$ 75,414	$ 75,423	$ 75,439	$ 75,454	$ 75,486	$ 75,557	$ 75,725	$ 76,287	$ 77,472
	0.8	$ 75,407	$ 75,409	$ 75,415	$ 75,425	$ 75,442	$ 75,455	$ 75,489	$ 75,558	$ 75,727	$ 76,289	$ 77,472
	0.9	$ 75,408	$ 75,410	$ 75,417	$ 75,427	$ 75,442	$ 75,456	$ 75,491	$ 75,560	$ 75,728	$ 76,290	$ 77,473
	0.95	$ 75,408	$ 75,411	$ 75,417	$ 75,428	$ 75,442	$ 75,457	$ 75,492	$ 75,560	$ 75,728	$ 76,290	$ 77,473

Table 29: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 6: $L = 7$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	$ 75,471	$ 75,471	$ 75,471	$ 75,471	$ 75,471	$ 75,471	$ 75,471	$ 75,472	$ 75,477	$ 75,517	$ 75,734
	0.003	$ 75,471	$ 75,471	$ 75,471	$ 75,471	$ 75,471	$ 75,471	$ 75,473	$ 75,476	$ 75,490	$ 75,608	$ 76,212
	0.01	$ 75,471	$ 75,471	$ 75,471	$ 75,471	$ 75,472	$ 75,474	$ 75,477	$ 75,488	$ 75,532	$ 75,875	$ 77,136
	0.02	$ 75,471	$ 75,471	$ 75,472	$ 75,472	$ 75,474	$ 75,476	$ 75,483	$ 75,503	$ 75,588	$ 76,115	$ 77,570
	0.05	$ 75,472	$ 75,472	$ 75,473	$ 75,475	$ 75,478	$ 75,484	$ 75,500	$ 75,546	$ 75,695	$ 76,393	$ 77,928
	0.1	$ 75,473	$ 75,474	$ 75,475	$ 75,478	$ 75,484	$ 75,495	$ 75,523	$ 75,593	$ 75,783	$ 76,523	$ 78,056
	0.2	$ 75,475	$ 75,476	$ 75,479	$ 75,484	$ 75,493	$ 75,513	$ 75,558	$ 75,633	$ 75,847	$ 76,588	$ 78,109
	0.3	$ 75,477	$ 75,478	$ 75,482	$ 75,489	$ 75,501	$ 75,526	$ 75,567	$ 75,658	$ 75,875	$ 76,609	$ 78,123
	0.4	$ 75,478	$ 75,480	$ 75,484	$ 75,492	$ 75,507	$ 75,536	$ 75,573	$ 75,666	$ 75,883	$ 76,616	$ 78,128
	0.5	$ 75,479	$ 75,481	$ 75,486	$ 75,495	$ 75,512	$ 75,541	$ 75,579	$ 75,671	$ 75,889	$ 76,621	$ 78,131
	0.6	$ 75,480	$ 75,482	$ 75,488	$ 75,498	$ 75,516	$ 75,543	$ 75,583	$ 75,675	$ 75,894	$ 76,625	$ 78,133
	0.7	$ 75,481	$ 75,483	$ 75,490	$ 75,500	$ 75,520	$ 75,544	$ 75,586	$ 75,678	$ 75,897	$ 76,627	$ 78,134
	0.8	$ 75,482	$ 75,484	$ 75,491	$ 75,502	$ 75,523	$ 75,545	$ 75,589	$ 75,681	$ 75,900	$ 76,629	$ 78,135
	0.9	$ 75,483	$ 75,485	$ 75,492	$ 75,504	$ 75,525	$ 75,546	$ 75,592	$ 75,683	$ 75,902	$ 76,630	$ 78,136
	0.95	$ 75,483	$ 75,486	$ 75,493	$ 75,505	$ 75,526	$ 75,546	$ 75,593	$ 75,684	$ 75,903	$ 76,630	$ 78,136

Table 30: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 7: $L = 7$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	$ 76,116	$ 76,116	$ 76,116	$ 76,116	$ 76,116	$ 76,116	$ 76,116	$ 76,118	$ 76,122	$ 76,149	$ 76,276
	0.003	$ 76,116	$ 76,116	$ 76,116	$ 76,116	$ 76,116	$ 76,117	$ 76,118	$ 76,122	$ 76,134	$ 76,215	$ 76,579
	0.01	$ 76,116	$ 76,116	$ 76,117	$ 76,117	$ 76,118	$ 76,121	$ 76,125	$ 76,136	$ 76,175	$ 76,427	$ 77,480
	0.02	$ 76,117	$ 76,117	$ 76,118	$ 76,119	$ 76,121	$ 76,125	$ 76,134	$ 76,156	$ 76,230	$ 76,686	$ 78,368
	0.05	$ 76,119	$ 76,120	$ 76,121	$ 76,124	$ 76,129	$ 76,139	$ 76,159	$ 76,208	$ 76,367	$ 77,185	$ 79,700
	0.1	$ 76,122	$ 76,124	$ 76,127	$ 76,132	$ 76,141	$ 76,159	$ 76,194	$ 76,278	$ 76,535	$ 77,587	$ 80,317
	0.2	$ 76,128	$ 76,130	$ 76,136	$ 76,145	$ 76,161	$ 76,189	$ 76,246	$ 76,375	$ 76,661	$ 77,862	$ 80,583
	0.3	$ 76,133	$ 76,136	$ 76,143	$ 76,156	$ 76,176	$ 76,212	$ 76,283	$ 76,422	$ 76,734	$ 77,963	$ 80,653
	0.4	$ 76,137	$ 76,140	$ 76,150	$ 76,164	$ 76,188	$ 76,230	$ 76,311	$ 76,441	$ 76,783	$ 77,997	$ 80,678
	0.5	$ 76,140	$ 76,144	$ 76,155	$ 76,171	$ 76,198	$ 76,245	$ 76,333	$ 76,454	$ 76,807	$ 78,017	$ 80,693
	0.6	$ 76,143	$ 76,148	$ 76,159	$ 76,178	$ 76,207	$ 76,257	$ 76,344	$ 76,465	$ 76,816	$ 78,031	$ 80,704
	0.7	$ 76,146	$ 76,151	$ 76,163	$ 76,183	$ 76,214	$ 76,266	$ 76,347	$ 76,474	$ 76,823	$ 78,042	$ 80,711
	0.8	$ 76,148	$ 76,153	$ 76,167	$ 76,187	$ 76,220	$ 76,275	$ 76,349	$ 76,481	$ 76,828	$ 78,050	$ 80,713
	0.9	$ 76,151	$ 76,156	$ 76,170	$ 76,191	$ 76,225	$ 76,282	$ 76,352	$ 76,487	$ 76,833	$ 78,057	$ 80,714
	0.95	$ 76,152	$ 76,157	$ 76,171	$ 76,193	$ 76,227	$ 76,285	$ 76,352	$ 76,489	$ 76,835	$ 78,060	$ 80,715

Table 31: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 8: $L = 7$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	\$ 76,489	\$ 76,489	\$ 76,489	\$ 76,489	\$ 76,489	\$ 76,489	\$ 76,490	\$ 76,492	\$ 76,499	\$ 76,549	\$ 76,791
	0.003	\$ 76,489	\$ 76,489	\$ 76,489	\$ 76,489	\$ 76,490	\$ 76,491	\$ 76,493	\$ 76,498	\$ 76,519	\$ 76,667	\$ 77,362
	0.01	\$ 76,489	\$ 76,490	\$ 76,490	\$ 76,491	\$ 76,492	\$ 76,495	\$ 76,502	\$ 76,520	\$ 76,587	\$ 77,045	\$ 79,011
	0.02	\$ 76,490	\$ 76,491	\$ 76,492	\$ 76,493	\$ 76,496	\$ 76,502	\$ 76,515	\$ 76,550	\$ 76,676	\$ 77,509	\$ 80,559
	0.05	\$ 76,493	\$ 76,493	\$ 76,496	\$ 76,500	\$ 76,507	\$ 76,520	\$ 76,550	\$ 76,629	\$ 76,900	\$ 78,316	\$ 82,295
	0.1	\$ 76,496	\$ 76,498	\$ 76,502	\$ 76,509	\$ 76,522	\$ 76,547	\$ 76,600	\$ 76,735	\$ 77,129	\$ 78,862	\$ 82,933
	0.2	\$ 76,502	\$ 76,505	\$ 76,513	\$ 76,525	\$ 76,547	\$ 76,588	\$ 76,674	\$ 76,881	\$ 77,338	\$ 79,199	\$ 83,199
	0.3	\$ 76,507	\$ 76,511	\$ 76,521	\$ 76,538	\$ 76,566	\$ 76,619	\$ 76,727	\$ 76,923	\$ 77,432	\$ 79,286	\$ 83,266
	0.4	\$ 76,512	\$ 76,516	\$ 76,529	\$ 76,548	\$ 76,582	\$ 76,643	\$ 76,767	\$ 76,954	\$ 77,471	\$ 79,338	\$ 83,294
	0.5	\$ 76,516	\$ 76,521	\$ 76,535	\$ 76,557	\$ 76,595	\$ 76,663	\$ 76,789	\$ 76,976	\$ 77,498	\$ 79,358	\$ 83,312
	0.6	\$ 76,519	\$ 76,525	\$ 76,540	\$ 76,564	\$ 76,605	\$ 76,679	\$ 76,795	\$ 76,994	\$ 77,519	\$ 79,368	\$ 83,319
	0.7	\$ 76,522	\$ 76,528	\$ 76,545	\$ 76,571	\$ 76,614	\$ 76,692	\$ 76,800	\$ 77,008	\$ 77,535	\$ 79,376	\$ 83,322
	0.8	\$ 76,525	\$ 76,531	\$ 76,549	\$ 76,576	\$ 76,622	\$ 76,703	\$ 76,804	\$ 77,019	\$ 77,548	\$ 79,382	\$ 83,325
	0.9	\$ 76,527	\$ 76,534	\$ 76,552	\$ 76,581	\$ 76,629	\$ 76,713	\$ 76,807	\$ 77,029	\$ 77,559	\$ 79,387	\$ 83,327
	0.95	\$ 76,528	\$ 76,535	\$ 76,554	\$ 76,583	\$ 76,632	\$ 76,717	\$ 76,809	\$ 77,033	\$ 77,563	\$ 79,389	\$ 83,328

Table 32: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 9: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim \text{Poisson}(0.5)$).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	\$ 75,543	\$ 75,543	\$ 75,543	\$ 75,543	\$ 75,543	\$ 75,543	\$ 75,543	\$ 75,544	\$ 75,546	\$ 75,566	\$ 75,672
	0.003	\$ 75,543	\$ 75,543	\$ 75,543	\$ 75,543	\$ 75,543	\$ 75,543	\$ 75,544	\$ 75,546	\$ 75,553	\$ 75,610	\$ 75,909
	0.01	\$ 75,543	\$ 75,543	\$ 75,543	\$ 75,543	\$ 75,544	\$ 75,545	\$ 75,547	\$ 75,552	\$ 75,574	\$ 75,744	\$ 76,509
	0.02	\$ 75,543	\$ 75,543	\$ 75,544	\$ 75,544	\$ 75,545	\$ 75,547	\$ 75,550	\$ 75,561	\$ 75,603	\$ 75,885	\$ 76,927
	0.05	\$ 75,544	\$ 75,544	\$ 75,545	\$ 75,546	\$ 75,548	\$ 75,552	\$ 75,560	\$ 75,584	\$ 75,659	\$ 76,102	\$ 77,287
	0.1	\$ 75,545	\$ 75,546	\$ 75,547	\$ 75,549	\$ 75,552	\$ 75,559	\$ 75,574	\$ 75,610	\$ 75,716	\$ 76,226	\$ 77,415
	0.2	\$ 75,547	\$ 75,548	\$ 75,550	\$ 75,553	\$ 75,559	\$ 75,570	\$ 75,592	\$ 75,632	\$ 75,760	\$ 76,290	\$ 77,468
	0.3	\$ 75,549	\$ 75,550	\$ 75,552	\$ 75,557	\$ 75,564	\$ 75,579	\$ 75,598	\$ 75,646	\$ 75,783	\$ 76,311	\$ 77,481
	0.4	\$ 75,550	\$ 75,551	\$ 75,555	\$ 75,560	\$ 75,569	\$ 75,583	\$ 75,602	\$ 75,657	\$ 75,790	\$ 76,319	\$ 77,487
	0.5	\$ 75,551	\$ 75,553	\$ 75,556	\$ 75,562	\$ 75,572	\$ 75,585	\$ 75,605	\$ 75,662	\$ 75,795	\$ 76,324	\$ 77,489
	0.6	\$ 75,552	\$ 75,554	\$ 75,558	\$ 75,564	\$ 75,575	\$ 75,586	\$ 75,608	\$ 75,664	\$ 75,799	\$ 76,327	\$ 77,491
	0.7	\$ 75,553	\$ 75,555	\$ 75,559	\$ 75,566	\$ 75,577	\$ 75,587	\$ 75,610	\$ 75,665	\$ 75,802	\$ 76,330	\$ 77,492
	0.8	\$ 75,554	\$ 75,556	\$ 75,560	\$ 75,568	\$ 75,577	\$ 75,587	\$ 75,612	\$ 75,666	\$ 75,805	\$ 76,332	\$ 77,492
	0.9	\$ 75,555	\$ 75,557	\$ 75,561	\$ 75,569	\$ 75,577	\$ 75,588	\$ 75,613	\$ 75,667	\$ 75,807	\$ 76,332	\$ 77,493
	0.95	\$ 75,555	\$ 75,557	\$ 75,562	\$ 75,570	\$ 75,577	\$ 75,588	\$ 75,614	\$ 75,668	\$ 75,808	\$ 76,332	\$ 77,493

Table 33: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 10: $L = 15$, $h = \$100$, $p = \$2,000$, $D \sim \text{Poisson}(0.5)$).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	$ 75,639	$ 75,639	$ 75,639	$ 75,639	$ 75,639	$ 75,639	$ 75,639	$ 75,640	$ 75,644	$ 75,679	$ 75,879
	0.003	$ 75,639	$ 75,639	$ 75,639	$ 75,639	$ 75,639	$ 75,639	$ 75,640	$ 75,643	$ 75,654	$ 75,756	$ 76,294
	0.01	$ 75,639	$ 75,639	$ 75,639	$ 75,640	$ 75,640	$ 75,641	$ 75,644	$ 75,653	$ 75,689	$ 75,968	$ 77,156
	0.02	$ 75,639	$ 75,639	$ 75,640	$ 75,640	$ 75,642	$ 75,644	$ 75,649	$ 75,666	$ 75,731	$ 76,171	$ 77,590
	0.05	$ 75,640	$ 75,641	$ 75,642	$ 75,643	$ 75,646	$ 75,651	$ 75,664	$ 75,700	$ 75,808	$ 76,434	$ 77,948
	0.1	$ 75,642	$ 75,642	$ 75,644	$ 75,647	$ 75,652	$ 75,661	$ 75,684	$ 75,725	$ 75,877	$ 76,561	$ 78,076
	0.2	$ 75,645	$ 75,646	$ 75,648	$ 75,653	$ 75,661	$ 75,678	$ 75,698	$ 75,760	$ 75,933	$ 76,630	$ 78,129
	0.3	$ 75,647	$ 75,648	$ 75,652	$ 75,658	$ 75,669	$ 75,683	$ 75,708	$ 75,777	$ 75,957	$ 76,648	$ 78,143
	0.4	$ 75,649	$ 75,650	$ 75,655	$ 75,662	$ 75,674	$ 75,685	$ 75,714	$ 75,784	$ 75,969	$ 76,658	$ 78,148
	0.5	$ 75,650	$ 75,652	$ 75,657	$ 75,666	$ 75,675	$ 75,688	$ 75,720	$ 75,789	$ 75,973	$ 76,664	$ 78,152
	0.6	$ 75,652	$ 75,654	$ 75,660	$ 75,669	$ 75,676	$ 75,690	$ 75,724	$ 75,792	$ 75,977	$ 76,665	$ 78,154
	0.7	$ 75,653	$ 75,655	$ 75,661	$ 75,670	$ 75,677	$ 75,691	$ 75,727	$ 75,795	$ 75,979	$ 76,667	$ 78,154
	0.8	$ 75,654	$ 75,657	$ 75,663	$ 75,670	$ 75,677	$ 75,693	$ 75,730	$ 75,798	$ 75,981	$ 76,668	$ 78,155
	0.9	$ 75,655	$ 75,658	$ 75,664	$ 75,670	$ 75,678	$ 75,694	$ 75,733	$ 75,800	$ 75,983	$ 76,669	$ 78,155
	0.95	$ 75,656	$ 75,658	$ 75,665	$ 75,671	$ 75,678	$ 75,694	$ 75,734	$ 75,801	$ 75,984	$ 76,669	$ 78,155

Table 34: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 11: $L = 15$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	\$ 76,564	\$ 76,564	\$ 76,564	\$ 76,564	\$ 76,564	\$ 76,564	\$ 76,565	\$ 76,566	\$ 76,570	\$ 76,596	\$ 76,719
	0.003	\$ 76,564	\$ 76,564	\$ 76,564	\$ 76,564	\$ 76,565	\$ 76,565	\$ 76,567	\$ 76,570	\$ 76,581	\$ 76,658	\$ 77,011
	0.01	\$ 76,565	\$ 76,565	\$ 76,565	\$ 76,566	\$ 76,567	\$ 76,569	\$ 76,573	\$ 76,584	\$ 76,620	\$ 76,858	\$ 77,821
	0.02	\$ 76,566	\$ 76,566	\$ 76,567	\$ 76,568	\$ 76,570	\$ 76,574	\$ 76,582	\$ 76,602	\$ 76,670	\$ 77,073	\$ 78,642
	0.05	\$ 76,568	\$ 76,569	\$ 76,571	\$ 76,574	\$ 76,579	\$ 76,588	\$ 76,606	\$ 76,651	\$ 76,790	\$ 77,507	\$ 79,832
	0.1	\$ 76,572	\$ 76,574	\$ 76,577	\$ 76,582	\$ 76,591	\$ 76,607	\$ 76,640	\$ 76,716	\$ 76,890	\$ 77,841	\$ 80,424
	0.2	\$ 76,579	\$ 76,582	\$ 76,587	\$ 76,597	\$ 76,612	\$ 76,638	\$ 76,686	\$ 76,762	\$ 77,016	\$ 78,080	\$ 80,684
	0.3	\$ 76,585	\$ 76,588	\$ 76,596	\$ 76,608	\$ 76,628	\$ 76,662	\$ 76,700	\$ 76,793	\$ 77,073	\$ 78,158	\$ 80,752
	0.4	\$ 76,590	\$ 76,594	\$ 76,603	\$ 76,618	\$ 76,640	\$ 76,669	\$ 76,710	\$ 76,815	\$ 77,096	\$ 78,195	\$ 80,780
	0.5	\$ 76,594	\$ 76,599	\$ 76,609	\$ 76,625	\$ 76,651	\$ 76,673	\$ 76,718	\$ 76,831	\$ 77,113	\$ 78,220	\$ 80,797
	0.6	\$ 76,598	\$ 76,603	\$ 76,614	\$ 76,632	\$ 76,654	\$ 76,676	\$ 76,725	\$ 76,844	\$ 77,125	\$ 78,238	\$ 80,805
	0.7	\$ 76,602	\$ 76,606	\$ 76,619	\$ 76,638	\$ 76,655	\$ 76,679	\$ 76,730	\$ 76,854	\$ 77,135	\$ 78,245	\$ 80,808
	0.8	\$ 76,604	\$ 76,610	\$ 76,623	\$ 76,643	\$ 76,656	\$ 76,681	\$ 76,734	\$ 76,863	\$ 77,143	\$ 78,248	\$ 80,811
	0.9	\$ 76,607	\$ 76,612	\$ 76,626	\$ 76,644	\$ 76,657	\$ 76,683	\$ 76,738	\$ 76,870	\$ 77,149	\$ 78,251	\$ 80,813
	0.95	\$ 76,608	\$ 76,614	\$ 76,628	\$ 76,645	\$ 76,658	\$ 76,684	\$ 76,739	\$ 76,873	\$ 77,152	\$ 78,252	\$ 80,814

Table 35: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 12: $L = 15$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	\$ 77,065	\$ 77,065	\$ 77,065	\$ 77,065	\$ 77,065	\$ 77,066	\$ 77,066	\$ 77,068	\$ 77,075	\$ 77,124	\$ 77,362
	0.003	\$ 77,065	\$ 77,065	\$ 77,065	\$ 77,066	\$ 77,066	\$ 77,067	\$ 77,070	\$ 77,075	\$ 77,096	\$ 77,240	\$ 77,880
	0.01	\$ 77,066	\$ 77,067	\$ 77,067	\$ 77,068	\$ 77,070	\$ 77,073	\$ 77,081	\$ 77,099	\$ 77,164	\$ 77,552	\$ 79,363
	0.02	\$ 77,068	\$ 77,068	\$ 77,070	\$ 77,072	\$ 77,075	\$ 77,082	\$ 77,096	\$ 77,131	\$ 77,233	\$ 77,926	\$ 80,744
	0.05	\$ 77,072	\$ 77,074	\$ 77,077	\$ 77,082	\$ 77,090	\$ 77,105	\$ 77,137	\$ 77,190	\$ 77,388	\$ 78,618	\$ 82,399
	0.1	\$ 77,079	\$ 77,082	\$ 77,087	\$ 77,097	\$ 77,112	\$ 77,133	\$ 77,165	\$ 77,254	\$ 77,577	\$ 79,101	\$ 83,038
	0.2	\$ 77,092	\$ 77,096	\$ 77,106	\$ 77,118	\$ 77,129	\$ 77,153	\$ 77,205	\$ 77,344	\$ 77,730	\$ 79,396	\$ 83,302
	0.3	\$ 77,102	\$ 77,107	\$ 77,115	\$ 77,123	\$ 77,138	\$ 77,168	\$ 77,234	\$ 77,404	\$ 77,811	\$ 79,502	\$ 83,366
	0.4	\$ 77,109	\$ 77,111	\$ 77,117	\$ 77,127	\$ 77,144	\$ 77,179	\$ 77,255	\$ 77,438	\$ 77,864	\$ 79,534	\$ 83,399
	0.5	\$ 77,110	\$ 77,112	\$ 77,119	\$ 77,130	\$ 77,150	\$ 77,189	\$ 77,272	\$ 77,449	\$ 77,879	\$ 79,556	\$ 83,411
	0.6	\$ 77,111	\$ 77,113	\$ 77,120	\$ 77,133	\$ 77,154	\$ 77,196	\$ 77,286	\$ 77,457	\$ 77,889	\$ 79,571	\$ 83,417
	0.7	\$ 77,112	\$ 77,114	\$ 77,122	\$ 77,135	\$ 77,158	\$ 77,203	\$ 77,297	\$ 77,463	\$ 77,898	\$ 79,583	\$ 83,422
	0.8	\$ 77,112	\$ 77,115	\$ 77,123	\$ 77,137	\$ 77,161	\$ 77,208	\$ 77,306	\$ 77,469	\$ 77,904	\$ 79,592	\$ 83,425
	0.9	\$ 77,113	\$ 77,116	\$ 77,124	\$ 77,139	\$ 77,164	\$ 77,213	\$ 77,314	\$ 77,473	\$ 77,910	\$ 79,599	\$ 83,428
	0.95	\$ 77,113	\$ 77,116	\$ 77,125	\$ 77,139	\$ 77,166	\$ 77,215	\$ 77,317	\$ 77,475	\$ 77,912	\$ 79,602	\$ 83,429

Table 36: Reduction in long-run average cost per day from contingency planning ($g^y - g^*$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 1: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 5
	0.003	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 9	$ 36
	0.01	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 1	$ 13	$ 53	$ 239
	0.02	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 3	$ 13	$ 36	$ 134	$ 865
	0.05	$ 0	$ 0	$ 0	$ 1	$ 5	$ 12	$ 24	$ 46	$ 96	$ 526	$ 2,414
	0.1	$ 1	$ 2	$ 6	$ 11	$ 19	$ 32	$ 52	$ 88	$ 225	$ 1,066	$ 3,770
	0.2	$ 11	$ 13	$ 20	$ 29	$ 42	$ 62	$ 94	$ 182	$ 436	$ 1,666	$ 4,904
	0.3	$ 19	$ 23	$ 31	$ 43	$ 60	$ 86	$ 132	$ 251	$ 580	$ 1,981	$ 5,399
	0.4	$ 27	$ 31	$ 41	$ 55	$ 75	$ 104	$ 169	$ 304	$ 676	$ 2,172	$ 5,677
	0.5	$ 33	$ 38	$ 49	$ 65	$ 87	$ 118	$ 198	$ 350	$ 749	$ 2,299	$ 5,853
	0.6	$ 38	$ 44	$ 56	$ 73	$ 96	$ 133	$ 221	$ 386	$ 806	$ 2,393	$ 5,976
	0.7	$ 43	$ 49	$ 62	$ 80	$ 105	$ 148	$ 240	$ 415	$ 850	$ 2,463	$ 6,066
	0.8	$ 48	$ 53	$ 67	$ 86	$ 112	$ 161	$ 256	$ 438	$ 885	$ 2,518	$ 6,136
	0.9	$ 51	$ 57	$ 72	$ 92	$ 118	$ 173	$ 269	$ 458	$ 914	$ 2,562	$ 6,190
	0.95	$ 53	$ 59	$ 74	$ 94	$ 121	$ 178	$ 275	$ 466	$ 926	$ 2,580	$ 6,214

Table 37: Reduction in long-run average cost per day from contingency planning ($g^y - g^*$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 5: $L = 7$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0
	0.003	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0
	0.01	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 3	$ 126
	0.02	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 42	$ 561
	0.05	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 15	$ 252	$ 1,759
	0.1	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 4	$ 55	$ 581	$ 2,842
	0.2	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 1	$ 32	$ 143	$ 969	$ 3,757
	0.3	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 13	$ 51	$ 207	$ 1,179	$ 4,159
	0.4	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 23	$ 65	$ 253	$ 1,306	$ 4,384
	0.5	$ 0	$ 0	$ 0	$ 0	$ 0	$ 6	$ 30	$ 83	$ 286	$ 1,394	$ 4,527
	0.6	$ 0	$ 0	$ 0	$ 0	$ 0	$ 10	$ 36	$ 96	$ 311	$ 1,457	$ 4,628
	0.7	$ 0	$ 0	$ 0	$ 0	$ 0	$ 14	$ 41	$ 107	$ 330	$ 1,504	$ 4,701
	0.8	$ 0	$ 0	$ 0	$ 0	$ 1	$ 17	$ 45	$ 116	$ 347	$ 1,540	$ 4,758
	0.9	$ 0	$ 0	$ 0	$ 0	$ 3	$ 20	$ 48	$ 123	$ 362	$ 1,570	$ 4,802
	0.95	$ 0	$ 0	$ 0	$ 0	$ 4	$ 21	$ 50	$ 126	$ 368	$ 1,582	$ 4,821

Table 38: Reduction in long-run average cost per day from contingency planning ($g^y - g^*$) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 9: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0
	0.003	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 6
	0.01	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 9	\$ 129
	0.02	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 43	\$ 494
	0.05	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 15	\$ 210	\$ 1,543
	0.1	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 5	\$ 45	\$ 471	\$ 2,511
	0.2	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 2	\$ 26	\$ 111	\$ 791	\$ 3,334
	0.3	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 12	\$ 40	\$ 154	\$ 962	\$ 3,697
	0.4	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 2	\$ 19	\$ 50	\$ 191	\$ 1,070	\$ 3,899
	0.5	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 6	\$ 24	\$ 60	\$ 217	\$ 1,142	\$ 4,030
	0.6	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0	\$ 10	\$ 28	\$ 71	\$ 236	\$ 1,193	\$ 4,120
	0.7	\$ 0	\$ 0	\$ 0	\$ 0	\$ 1	\$ 12	\$ 32	\$ 79	\$ 251	\$ 1,232	\$ 4,187
	0.8	\$ 0	\$ 0	\$ 0	\$ 0	\$ 3	\$ 15	\$ 35	\$ 85	\$ 264	\$ 1,262	\$ 4,237
	0.9	\$ 0	\$ 0	\$ 0	\$ 0	\$ 4	\$ 17	\$ 37	\$ 91	\$ 274	\$ 1,287	\$ 4,278
	0.95	\$ 0	\$ 0	\$ 0	\$ 0	\$ 5	\$ 18	\$ 38	\$ 93	\$ 278	\$ 1,298	\$ 4,295

Table 39: Optimal order-up-to level (y^*) and long-run average cost per day (g^*) vs. minimum leadtime (L).

Parameters	$h = \$100$, $p = \$1,000$, $p_{OC} = 0.01$ $p_{CO} = 0.1$		$h = \$100$, $p = \$2,000$ $p_{OC} = 0.01$ $p_{CO} = 0.1$		$h = \$100$, $p = \$1,000$ $p_{OC} = 0.01$ $p_{CO} = 0.05$	
L	y^*	g^*	y^*	g^*	y^*	g^*
1	3	\$ 75,526	5	\$ 75,809	7	\$ 76,442
2	4	\$ 75,548	5	\$ 75,819	7	\$ 76,446
3	5	\$ 75,570	6	\$ 75,828	8	\$ 76,448
4	5	\$ 75,591	7	\$ 75,840	8	\$ 76,453
5	6	\$ 75,603	8	\$ 75,854	9	\$ 76,455
6	7	\$ 75,619	8	\$ 75,865	10	\$ 76,460
7	8	\$ 75,639	9	\$ 75,875	10	\$ 76,463
8	8	\$ 75,650	10	\$ 75,888	11	\$ 76,467
9	9	\$ 75,664	10	\$ 75,901	12	\$ 76,473
10	10	\$ 75,682	11	\$ 75,910	12	\$ 76,476
11	10	\$ 75,692	12	\$ 75,923	13	\$ 76,480
12	11	\$ 75,705	12	\$ 75,935	13	\$ 76,485
13	11	\$ 75,721	13	\$ 75,944	14	\$ 76,489
14	12	\$ 75,730	14	\$ 75,957	15	\$ 76,495
15	13	\$ 75,744	14	\$ 75,968	15	\$ 76,498
16	13	\$ 75,756	15	\$ 75,978	16	\$ 76,503
17	14	\$ 75,765	16	\$ 75,991	16	\$ 76,509
18	15	\$ 75,781	16	\$ 76,000	17	\$ 76,512
19	15	\$ 75,788	17	\$ 76,010	18	\$ 76,519
20	16	\$ 75,800	17	\$ 76,025	18	\$ 76,522
21	16	\$ 75,812	18	\$ 76,031	19	\$ 76,527
22	17	\$ 75,820	19	\$ 76,043	19	\$ 76,533
23	18	\$ 75,834	19	\$ 76,054	20	\$ 76,537
24	18	\$ 75,842	20	\$ 76,062	21	\$ 76,543
25	19	\$ 75,853	21	\$ 76,076	21	\$ 76,547
26	19	\$ 75,864	21	\$ 76,082	22	\$ 76,552
27	20	\$ 75,872	22	\$ 76,093	22	\$ 76,557
28	21	\$ 75,885	22	\$ 76,104	23	\$ 76,561
29	21	\$ 75,891	23	\$ 76,112	24	\$ 76,568
30	22	\$ 75,902	24	\$ 76,124	24	\$ 76,571

Table 40: Optimal order-up-to level (y^*) vs. cost ratio ($p/(p+h)$).

	$L = 15$ $p_{OC} = 0.01$, $p_{CO} = 0.1$	$L = 7$, $p_{OC} = 0.01$, $p_{CO} = 0.1$	$L = 15$, $p_{OC} = 0.01$, $p_{CO} = 0.05$
Cost Ratio	y*	y*	y*
0.05	4	1	4
0.10	5	2	5
0.15	5	2	5
0.20	6	2	6
0.25	6	3	6
0.30	7	3	7
0.35	7	3	7
0.40	7	3	8
0.45	8	4	8
0.50	8	4	8
0.55	8	4	9
0.60	9	5	9
0.65	9	5	10
0.70	10	5	10
0.75	10	6	11
0.80	11	6	12
0.85	12	7	13
0.90	12	7	15
0.95	14	9	21
0.96	15	10	23
0.97	16	10	26
0.98	17	12	30
0.99	20	16	37

Table 41: Optimal order-up-to level (y^*) and long-run average cost per day (g^*) vs. holding cost (h)

Parameters	$L = 15$, $p = \$1,000$, $p_{OC} = 0.01$, $p_{CO} = 0.1$		$L = 7$, $p = \$1,000$, $p_{OC} = 0.01$, $p_{CO} = 0.1$		$L = 15$, $p = \$1,000$, $p_{OC} = 0.01$, $p_{CO} = 0.05$	
h	$y*$	g^*	$y*$	g^*	$y*$	g^*
\$ 100	13	\$ 75,744	8	\$ 75,639	15	\$ 76,552
\$ 200	11	\$ 76,123	6	\$ 75,920	12	\$ 77,056
\$ 300	10	\$ 76,419	6	\$ 76,125	11	\$ 77,400
\$ 400	10	\$ 76,648	5	\$ 76,295	10	\$ 77,682
\$ 500	9	\$ 76,858	5	\$ 76,427	10	\$ 77,900
\$ 600	9	\$ 77,018	5	\$ 76,558	9	\$ 78,110
\$ 700	9	\$ 77,178	4	\$ 76,689	9	\$ 78,263
\$ 800	8	\$ 77,333	4	\$ 76,761	9	\$ 78,416
\$ 900	8	\$ 77,438	4	\$ 76,834	9	\$ 78,569
\$ 1,000	8	\$ 77,542	4	\$ 76,906	8	\$ 78,686
\$ 1,100	8	\$ 77,646	4	\$ 76,979	8	\$ 78,786
\$ 1,200	8	\$ 77,751	4	\$ 77,052	8	\$ 78,885
\$ 1,300	8	\$ 77,855	4	\$ 77,124	8	\$ 78,985
\$ 1,400	7	\$ 77,939	4	\$ 77,197	8	\$ 79,084
\$ 1,500	7	\$ 78,001	3	\$ 77,260	8	\$ 79,184
\$ 1,600	7	\$ 78,063	3	\$ 77,292	7	\$ 79,260
\$ 1,700	7	\$ 78,125	3	\$ 77,325	7	\$ 79,318
\$ 1,800	7	\$ 78,187	3	\$ 77,357	7	\$ 79,377
\$ 1,900	7	\$ 78,249	3	\$ 77,389	7	\$ 79,436
\$ 2,000	7	\$ 78,310	3	\$ 77,421	7	\$ 79,495
\$ 2,100	7	\$ 78,372	3	\$ 77,454	7	\$ 79,554
\$ 2,200	7	\$ 78,434	3	\$ 77,486	7	\$ 79,613

Table 42: Optimal order-up-to level (y^*) and long-run average cost per day (g^*) vs. penalty cost (p).

Parameters	$L = 15$, $h = \$100$, $p_{OC} = 0.01$, $p_{CO} = 0.1$		$L = 7$, $h = \$100$, $p_{OC} = 0.01$, $p_{CO} = 0.1$		$L = 15$, $h = \$100$, $p_{OC} = 0.01$, $p_{CO} = 0.05$	
p	$y*$	g^*	$y*$	g^*	$y*$	g^*
\$ 100	8	\$ 75254	4	\$ 75191	8	\$ 75367
\$ 200	9	\$ 75372	5	\$ 75285	10	\$ 75579
\$ 300	10	\$ 75449	6	\$ 75358	11	\$ 75748
\$ 400	11	\$ 75510	6	\$ 75409	12	\$ 75896
\$ 500	11	\$ 75562	6	\$ 75460	12	\$ 76027
\$ 600	12	\$ 75608	7	\$ 75501	13	\$ 76146
\$ 700	12	\$ 75644	7	\$ 75537	13	\$ 76260
\$ 800	12	\$ 75680	7	\$ 75572	14	\$ 76362
\$ 900	12	\$ 75716	7	\$ 75607	15	\$ 76461
\$ 1000	13	\$ 75744	8	\$ 75639	15	\$ 76552
\$ 1100	13	\$ 75770	8	\$ 75665	16	\$ 76639
\$ 1200	13	\$ 75796	8	\$ 75691	16	\$ 76721
\$ 1300	13	\$ 75822	8	\$ 75717	17	\$ 76796
\$ 1400	13	\$ 75849	8	\$ 75743	18	\$ 76869
\$ 1500	14	\$ 75870	8	\$ 75769	18	\$ 76937
\$ 1600	14	\$ 75889	9	\$ 75795	19	\$ 77001
\$ 1700	14	\$ 75909	9	\$ 75815	19	\$ 77062
\$ 1800	14	\$ 75929	9	\$ 75835	20	\$ 77119
\$ 1900	14	\$ 75949	9	\$ 75855	20	\$ 77174
\$ 2000	14	\$ 75968	9	\$ 75875	21	\$ 77225
\$ 2100	14	\$ 75988	9	\$ 75895	21	\$ 77275
\$ 2200	15	\$ 76005	9	\$ 75915	22	\$ 77322

Table 43: Optimal order-up-to level (y^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 13: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(1)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	7	7	7	7	7	7	7	7	7	7	7
	0.003	7	7	7	7	7	7	7	7	7	7	8
	0.01	7	7	7	7	7	7	7	7	7	9	26
	0.02	7	7	7	7	7	7	7	7	8	16	47
	0.05	7	7	7	7	7	7	8	9	12	30	69
	0.1	7	7	7	7	8	8	9	11	17	38	80
	0.2	7	8	8	8	9	9	11	14	21	43	87
	0.3	8	8	8	9	9	10	12	15	23	46	90
	0.4	8	8	8	9	10	11	13	16	24	47	92
	0.5	8	8	9	9	10	11	13	17	24	48	93
	0.6	8	8	9	9	10	11	13	17	25	48	93
	0.7	8	9	9	10	10	12	14	17	25	49	94
	0.8	8	9	9	10	11	12	14	18	25	49	94
	0.9	9	9	9	10	11	12	14	18	26	49	94
	0.95	9	9	9	10	11	12	14	18	26	49	94

Table 44: Optimal long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 13: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(1)).

		p_{CO}										
		0.95	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
p_{OC}	0.001	\$ 300,394	\$ 300,394	\$ 300,394	\$ 300,394	\$ 300,395	\$ 300,397	\$ 300,400	\$ 300,407	\$ 300,429	\$ 300,559	\$ 301,056
	0.003	\$ 300,394	\$ 300,395	\$ 300,396	\$ 300,397	\$ 300,399	\$ 300,403	\$ 300,412	\$ 300,433	\$ 300,498	\$ 300,882	\$ 302,300
	0.01	\$ 300,397	\$ 300,398	\$ 300,401	\$ 300,405	\$ 300,413	\$ 300,426	\$ 300,455	\$ 300,522	\$ 300,732	\$ 301,879	\$ 305,437
	0.02	\$ 300,401	\$ 300,403	\$ 300,408	\$ 300,417	\$ 300,431	\$ 300,458	\$ 300,513	\$ 300,643	\$ 301,015	\$ 302,849	\$ 307,089
	0.05	\$ 300,413	\$ 300,417	\$ 300,429	\$ 300,450	\$ 300,484	\$ 300,547	\$ 300,656	\$ 300,909	\$ 301,570	\$ 303,865	\$ 308,441
	0.1	\$ 300,430	\$ 300,438	\$ 300,461	\$ 300,499	\$ 300,548	\$ 300,640	\$ 300,812	\$ 301,156	\$ 301,919	\$ 304,322	\$ 308,911
	0.2	\$ 300,461	\$ 300,472	\$ 300,500	\$ 300,547	\$ 300,628	\$ 300,751	\$ 300,960	\$ 301,341	\$ 302,136	\$ 304,549	\$ 309,098
	0.3	\$ 300,472	\$ 300,485	\$ 300,522	\$ 300,584	\$ 300,666	\$ 300,802	\$ 301,022	\$ 301,412	\$ 302,209	\$ 304,612	\$ 309,141
	0.4	\$ 300,480	\$ 300,496	\$ 300,541	\$ 300,599	\$ 300,693	\$ 300,831	\$ 301,056	\$ 301,445	\$ 302,242	\$ 304,637	\$ 309,158
	0.5	\$ 300,487	\$ 300,505	\$ 300,552	\$ 300,611	\$ 300,705	\$ 300,847	\$ 301,073	\$ 301,464	\$ 302,259	\$ 304,650	\$ 309,166
	0.6	\$ 300,494	\$ 300,514	\$ 300,556	\$ 300,622	\$ 300,715	\$ 300,860	\$ 301,087	\$ 301,475	\$ 302,270	\$ 304,657	\$ 309,170
	0.7	\$ 300,499	\$ 300,520	\$ 300,560	\$ 300,631	\$ 300,723	\$ 300,867	\$ 301,093	\$ 301,484	\$ 302,276	\$ 304,662	\$ 309,173
	0.8	\$ 300,504	\$ 300,521	\$ 300,563	\$ 300,633	\$ 300,730	\$ 300,871	\$ 301,098	\$ 301,489	\$ 302,281	\$ 304,665	\$ 309,175
	0.9	\$ 300,507	\$ 300,521	\$ 300,566	\$ 300,634	\$ 300,732	\$ 300,874	\$ 301,101	\$ 301,491	\$ 302,284	\$ 304,667	\$ 309,176
	0.95	\$ 300,507	\$ 300,522	\$ 300,568	\$ 300,635	\$ 300,733	\$ 300,875	\$ 301,103	\$ 301,493	\$ 302,285	\$ 304,668	\$ 309,176

APPENDIX B

NUMERICAL STUDY TABLES: BORDER CLOSURE MODEL WITH CONGESTION

Table 45: Optimal order-up-to levels for selected border states ($y^*(i,n)$) and long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 1C: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

			p_{CO}					
			0.5	0.4	0.3	0.2	0.1	0.05
g^*	p_{OC}	0.001	\$ 75,217	\$ 75,218	\$ 75,222	\$ 75,233	\$ 75,289	\$ 75,476
		0.003	\$ 75,222	\$ 75,227	\$ 75,239	\$ 75,272	\$ 75,433	\$ 75,960
		0.01	\$ 75,242	\$ 75,260	\$ 75,299	\$ 75,418		
		0.02	\$ 75,274	\$ 75,315	\$ 75,406			
$E[HPC]$	p_{OC}	0.001	\$ 217	\$ 218	\$ 222	\$ 233	\$ 289	\$ 476
		0.003	\$ 222	\$ 227	\$ 239	\$ 272	\$ 433	\$ 960
		0.01	\$ 242	\$ 260	\$ 299	\$ 418		
		0.02	\$ 274	\$ 315	\$ 406			
$y^*(O,0,0)$	p_{OC}	0.001	2	2	2	2	2	3
		0.003	2	2	2	2	3	3
		0.01	2	2	3	3		
		0.02	3	3	3			
$y^*(O,100,0)$	p_{OC}	0.001	9	9	9	9	9	9
		0.003	9	9	9	9	9	10
		0.01	9	9	9	9		
		0.02	9	9	10			
$y^*(C,0,0)$	p_{OC}	0.001	4	4	5	6	10	17
		0.003	4	4	5	6	10	19
		0.01	4	4	5	6		
		0.02	4	4	5			
$y^*(C,100,0)$	p_{OC}	0.001	10	10	11	12	15	23
		0.003	10	10	11	12	16	25
		0.01	10	10	11	12		
		0.02	10	11	11			

Table 46: Optimal order-up-to levels for selected border states ($y^*(i, n)$) and long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 2C: $L = 1$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

			p_{CO}					
			0.5	0.4	0.3	0.2	0.1	0.05
g^*	p_{OC}	0.001	\$ 75,252	\$ 75,254	\$ 75,259	\$ 75,275	\$ 75,357	\$ 75,665
		0.003	\$ 75,259	\$ 75,265	\$ 75,281	\$ 75,327	\$ 75,568	\$ 76,392
		0.01	\$ 75,283	\$ 75,307	\$ 75,361	\$ 75,518		
		0.02	\$ 75,324	\$ 75,379	\$ 75,505			
$y^*(O, 0, 0)$	p_{OC}	0.001	3	3	3	3	3	3
		0.003	3	3	3	3	3	4
		0.01	3	3	3	3		
		0.02	3	3	3			
$y^*(O, 100, 0)$	p_{OC}	0.001	10	10	10	10	10	10
		0.003	10	10	10	10	10	13
		0.01	10	10	10	11		
		0.02	10	10	11			
$y^*(C, 0, 0)$	p_{OC}	0.001	5	5	6	7	12	23
		0.003	5	5	6	7	13	25
		0.01	5	5	6	8		
		0.02	5	5	6			
$y^*(C, 100, 0)$	p_{OC}	0.001	11	11	12	13	18	28
		0.003	11	11	12	13	19	31
		0.01	11	12	12	14		
		0.02	11	12	13			

Table 47: Optimal order-up-to levels for selected border states ($y^*(i,n)$) and long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 3C: $L = 1$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

			p_{CO}					
			0.5	0.4	0.3	0.2	0.1	0.05
g^*	p_{OC}	0.001	\$ 75,558	\$ 75,562	\$ 75,569	\$ 75,590	\$ 75,684	\$ 75,950
		0.003	\$ 75,572	\$ 75,583	\$ 75,606	\$ 75,670	\$ 75,964	\$ 76,737
		0.01	\$ 75,622	\$ 75,663	\$ 75,752	\$ 75,981		
		0.02	\$ 75,710	\$ 75,817	\$ 76,048			
$y^*(O,0,0)$	p_{OC}	0.001	1	1	1	1	1	1
		0.003	1	1	1	1	1	1
		0.01	1	1	1	1		
		0.02	1	1	1			
$y^*(O,100,0)$	p_{OC}	0.001	6	6	6	6	6	6
		0.003	6	6	6	6	7	7
		0.01	6	7	7	7		
		0.02	7	7	7			
$y^*(C,0,0)$	p_{OC}	0.001	2	2	3	3	5	8
		0.003	2	2	3	3	5	8
		0.01	2	2	3	3		
		0.02	2	2	3			
$y^*(C,100,0)$	p_{OC}	0.001	7	7	8	8	10	13
		0.003	7	7	8	8	10	14
		0.01	7	8	8	9		
		0.02	7	8	8			

Table 48: Optimal order-up-to levels for selected border states ($y^*(i,n)$) and long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 4C: $L = 1$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

			p_{CO}					
			0.5	0.4	0.3	0.2	0.1	0.05
g^*	p_{OC}	0.001	\$ 75,767	\$ 75,772	\$ 75,782	\$ 75,811	\$ 75,938	\$ 76,329
		0.003	\$ 75,784	\$ 75,798	\$ 75,831	\$ 75,919	\$ 76,302	\$ 77,423
		0.01	\$ 75,848	\$ 75,904	\$ 76,029	\$ 76,361		
		0.02	\$ 75,964	\$ 76,108	\$ 76,430			
$y^*(O,0,0)$	p_{OC}	0.001	2	2	2	2	2	2
		0.003	2	2	2	2	2	2
		0.01	2	2	2	2		
		0.02	2	2	2			
$y^*(O,100,0)$	p_{OC}	0.001	7	7	7	7	7	8
		0.003	7	7	7	8	8	8
		0.01	8	8	8	8		
		0.02	8	8	8			
$y^*(C,0,0)$	p_{OC}	0.001	3	3	4	4	7	11
		0.003	3	3	4	4	7	12
		0.01	3	3	4	5		
		0.02	3	3	4			
$y^*(C,100,0)$	p_{OC}	0.001	8	9	9	10	12	17
		0.003	8	9	9	10	12	18
		0.01	9	9	9	10		
		0.02	9	9	10			

Table 49: Optimal order-up-to levels for selected border states ($y^*(i,n)$) and long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 5C: $L = 7$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

			p_{CO}					
			0.5	0.4	0.3	0.2	0.1	0.05
g^*	p_{OC}	0.001	\$ 75,395	\$ 75,396	\$ 75,399	\$ 75,409	\$ 75,462	\$ 75,651
		0.003	\$ 75,399	\$ 75,403	\$ 75,412	\$ 75,442	\$ 75,605	\$ 76,137
		0.01	\$ 75,413	\$ 75,428	\$ 75,464	\$ 75,579		
		0.02	\$ 75,439	\$ 75,476	\$ 75,558			
$y^*(O,0,0)$	p_{OC}	0.001	7	7	7	7	7	7
		0.003	7	7	7	7	7	8
		0.01	7	7	7	7		
		0.02	7	7	7			
$y^*(O,100,0)$	p_{OC}	0.001	12	12	12	13	13	13
		0.003	13	13	13	13	13	14
		0.01	13	13	13	13		
		0.02	13	13	14			
$y^*(C,0,0)$	p_{OC}	0.001	8	8	9	10	13	20
		0.003	8	8	9	10	14	22
		0.01	8	8	9	10		
		0.02	8	9	9			
$y^*(C,100,0)$	p_{OC}	0.001	14	14	14	15	19	26
		0.003	14	14	14	16	19	29
		0.01	14	14	15	16		
		0.02	14	15	15			

Table 50: Optimal order-up-to levels for selected border states ($y^*(i,n)$) and long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 6C: $L = 7$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

			p_{CO}					
			0.5	0.4	0.3	0.2	0.1	0.05
g^*	p_{OC}	0.001	\$ 75,473	\$ 75,474	\$ 75,479	\$ 75,492	\$ 75,573	\$ 75,881
		0.003	\$ 75,477	\$ 75,482	\$ 75,495	\$ 75,538	\$ 75,775	\$ 76,595
		0.01	\$ 75,494	\$ 75,513	\$ 75,562	\$ 75,708		
		0.02	\$ 75,524	\$ 75,571	\$ 75,680			
$y^*(O,0,0)$	p_{OC}	0.001	8	8	8	8	8	8
		0.003	8	8	8	8	8	10
		0.01	8	8	8	8		
		0.02	8	8	8			
$y^*(O,100,0)$	p_{OC}	0.001	14	14	14	14	14	14
		0.003	14	14	14	14	15	17
		0.01	14	14	14	15		
		0.02	14	14	15			
$y^*(C,0,0)$	p_{OC}	0.001	9	9	10	11	16	25
		0.003	9	9	10	11	17	28
		0.01	9	9	10	12		
		0.02	9	10	11			
$y^*(C,100,0)$	p_{OC}	0.001	15	15	16	17	21	27
		0.003	15	15	16	17	22	31
		0.01	15	15	16	18		
		0.02	15	16	17			

Table 51: Optimal order-up-to levels for selected border states ($y^*(i,n)$) and long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 7C: $L = 7$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

			p_{CO}					
			0.5	0.4	0.3	0.2	0.1	0.05
g^*	p_{OC}	0.001	\$ 76,120	\$ 76,122	\$ 76,128	\$ 76,146	\$ 76,230	\$ 76,482
		0.003	\$ 76,128	\$ 76,136	\$ 76,155	\$ 76,211	\$ 76,476	\$ 77,218
		0.01	\$ 76,162	\$ 76,195	\$ 76,271	\$ 76,466		
		0.02	\$ 76,225	\$ 76,313	\$ 76,502			
$y^*(O,0,0)$	p_{OC}	0.001	5	5	5	5	5	5
		0.003	5	5	5	5	5	5
		0.01	5	5	5	5		
		0.02	5	5	5			
$y^*(O,100,0)$	p_{OC}	0.001	10	10	10	10	10	10
		0.003	10	10	10	10	10	10
		0.01	10	10	10	10		
		0.02	10	10	10			
$y^*(C,0,0)$	p_{OC}	0.001	6	6	6	7	8	11
		0.003	6	6	6	7	8	12
		0.01	6	6	6	7		
		0.02	6	6	6			
$y^*(C,100,0)$	p_{OC}	0.001	10	11	11	12	13	17
		0.003	10	11	11	12	14	17
		0.01	11	11	11	12		
		0.02	11	11	12			

Table 52: Optimal order-up-to levels for selected border states ($y^*(i,n)$) and long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 8C: $L = 7$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

			p_{CO}					
			0.5	0.4	0.3	0.2	0.1	0.05
g^*	p_{OC}	0.001	\$ 76,494	\$ 76,498	\$ 76,507	\$ 76,532	\$ 76,653	\$ 77,037
		0.003	\$ 76,505	\$ 76,517	\$ 76,545	\$ 76,624	\$ 76,989	\$ 78,098
		0.01	\$ 76,549	\$ 76,596	\$ 76,703	\$ 77,000		
		0.02	\$ 76,634	\$ 76,753	\$ 77,024			
$y^*(O,0,0)$	p_{OC}	0.001	6	6	6	6	6	6
		0.003	6	6	6	6	6	6
		0.01	6	6	6	6		
		0.02	6	6	6			
$y^*(O,100,0)$	p_{OC}	0.001	11	11	11	11	11	11
		0.003	11	11	11	11	11	12
		0.01	11	11	11	11		
		0.02	11	11	12			
$y^*(C,0,0)$	p_{OC}	0.001	7	7	7	8	10	15
		0.003	7	7	7	8	11	15
		0.01	7	7	7	8		
		0.02	7	7	8			
$y^*(C,100,0)$	p_{OC}	0.001	12	12	12	13	15	19
		0.003	12	12	13	13	16	19
		0.01	12	12	13	14		
		0.02	12	13	13			

Table 53: Optimal order-up-to levels for selected border states ($y^*(i,n)$) and long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 10C: $L = 15$, $h = \$100$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

			p_{CO}					
			0.5	0.4	0.3	0.2	0.1	0.05
g^*	p_{OC}	0.001	\$ 75,641	\$ 75,643	\$ 75,647	\$ 75,662	\$ 75,748	\$ 76,096
		0.003	\$ 75,646	\$ 75,651	\$ 75,665	\$ 75,711	\$ 75,944	\$ 76,874
		0.01	\$ 75,664	\$ 75,685	\$ 75,732	\$ 75,864		
		0.02	\$ 75,696	\$ 75,736	\$ 75,833			
$y^*(O,0,0)$	p_{OC}	0.001	13	13	13	13	13	14
		0.003	13	13	13	13	14	16
		0.01	13	13	14	14		
		0.02	13	14	14			
$y^*(O,100,0)$	p_{OC}	0.001	18	18	18	18	18	19
		0.003	18	18	18	19	19	23
		0.01	18	19	20	20		
		0.02	19	19	20			
$y^*(C,0,0)$	p_{OC}	0.001	14	14	14	16	20	31
		0.003	14	14	15	16	21	34
		0.01	14	14	15	17		
		0.02	14	15	16			
$y^*(C,100,0)$	p_{OC}	0.001	19	19	20	21	22	35
		0.003	19	20	20	21	23	39
		0.01	20	20	21	22		
		0.02	20	21	21			

Table 54: Optimal order-up-to levels for selected border states ($y^*(i,n)$) and long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 11C: $L = 15$, $h = \$500$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

			p_{CO}					
			0.5	0.4	0.3	0.2	0.1	0.05
g^*	p_{OC}	0.001	$ 76,568	$ 76,571	$ 76,578	$ 76,596	$ 76,683	$ 76,938
		0.003	$ 76,578	$ 76,587	$ 76,606	$ 76,664	$ 76,934	$ 77,673
		0.01	$ 76,613	$ 76,647	$ 76,724	$ 76,954		
		0.02	$ 76,677	$ 76,761	$ 76,968			
$y^*(O,0,0)$	p_{OC}	0.001	9	9	9	9	9	9
		0.003	9	9	9	9	9	10
		0.01	9	9	9	9		
		0.02	9	9	10			
$y^*(O,100,0)$	p_{OC}	0.001	13	13	13	13	13	14
		0.003	13	13	13	14	14	14
		0.01	14	14	14	14		
		0.02	14	14	14			
$y^*(C,0,0)$	p_{OC}	0.001	14	14	14	16	20	31
		0.003	14	14	15	16	21	34
		0.01	14	14	15	17		
		0.02	14	15	16			
$y^*(C,100,0)$	p_{OC}	0.001	19	19	20	21	22	35
		0.003	19	20	20	21	23	39
		0.01	20	20	21	22		
		0.02	20	21	21			

Table 55: Optimal order-up-to levels for selected border states ($y^*(i,n)$) and long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 12C: $L = 15$, $h = \$500$, $p = \$2,000$, $D \sim$ Poisson(0.5)).

			p_{CO}					
			0.5	0.4	0.3	0.2	0.1	0.05
g^*	p_{OC}	0.001	\$ 77,072	\$ 77,076	\$ 77,087	\$ 77,115	\$ 77,245	\$ 77,723
		0.003	\$ 77,087	\$ 77,101	\$ 77,132	\$ 77,219	\$ 77,575	\$ 78,958
		0.01	\$ 77,142	\$ 77,183	\$ 77,281	\$ 77,561		
		0.02	\$ 77,207	\$ 77,314	\$ 77,562			
$y^*(O,0,0)$	p_{OC}	0.001	10	10	10	10	10	11
		0.003	10	10	10	10	11	11
		0.01	10	11	11	11		
		0.02	11	11	11			
$y^*(O,100,0)$	p_{OC}	0.001	15	15	15	15	15	15
		0.003	15	15	15	15	15	16
		0.01	15	15	15	16		
		0.02	15	16	16			
$y^*(C,0,0)$	p_{OC}	0.001	11	11	11	12	14	19
		0.003	11	11	11	12	15	20
		0.01	11	11	12	13		
		0.02	11	11	12			
$y^*(C,100,0)$	p_{OC}	0.001	16	16	16	17	18	24
		0.003	16	16	17	17	18	26
		0.01	16	16	17	18		
		0.02	16	17	17			

Table 56: Optimal order-up-to levels $(y^*(O, n/r_0), y^*(C, n/r_0))$ vs. scaled queue length (n/r_0) ($h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

	$L = 1$, $p_{OC} = 0.003$, $p_{CO} = 0.1$		$L = 7$, $p_{OC} = 0.003$, $p_{CO} = 0.1$		$L = 15$, $p_{OC} = 0.003$, $p_{CO} = 0.1$		$L = 15$, $p_{OC} = 0.003$, $p_{CO} = 0.5$	
n/r_0	$y^*(O, n/r_0)$	$y^*(C, n/r_0)$	$y^*(O, n/r_0)$	$y^*(C, n/r_0)$	$y^*(O, n/r_0)$	$y^*(C, n/r_0)$	$y^*(O, n/r_0)$	$y^*(C, n/r_0)$
0	3	10	7	14	12	18	12	12
1	3	11	8	13	13	19	12	13
2	4	11	0	0	14	19	13	14
3	5	12	9	15	14	20	13	14
4	6	13	9	16	14	20	14	15
5	6	13	10	16	0	0	0	0
6	7	14	11	17	15	21	14	15
7	8	13	11	18	16	22	15	16
8	0	0	12	18	16	23	16	17
9	9	15	12	19	17	23	16	17
10	9	16	13	19	18	24	17	18
11	10	16	14	20	18	24	17	18
12	11	17	14	20	19	25	18	19
13	11	18	0	0	19	25	19	20
14	12	18	15	21	20	25	19	20

Table 57: Reduction in long-run average cost per day from contingency planning ($g^y - g^*$) vs. transition probabilities (p_{OC}, p_{CO}) (L=1,5, and 9, $h = \$100$, $p = \$1,000$, D ~Poisson(0.5)).

			p_{CO}					
			0.5	0.4	0.3	0.2	0.1	0.05
L=1	p_{OC}	0.001	\$ 7	\$ 13	\$ 27	\$ 76	\$ 412	\$ 1,977
		0.003	\$ 22	\$ 41	\$ 90	\$ 266	\$ 1,676	\$ 8,885
		0.01	\$ 90	\$ 182	\$ 454	\$ 1,743		
		0.02	\$ 271	\$ 632	\$ 2,184			
L=7	p_{OC}	0.001	\$ 1	\$ 3	\$ 8	\$ 34	\$ 275	\$ 1,614
		0.003	\$ 3	\$ 9	\$ 30	\$ 129	\$ 1,217	\$ 7,735
		0.01	\$ 19	\$ 55	\$ 197	\$ 1,120		
		0.02	\$ 80	\$ 270	\$ 1,396			
L=15	p_{OC}	0.001	\$ 0	\$ 1	\$ 5	\$ 23	\$ 221	\$ 1,441
		0.003	\$ 2	\$ 5	\$ 18	\$ 90	\$ 1,031	\$ 7,142
		0.01	\$ 10	\$ 33	\$ 135	\$ 913		
		0.02	\$ 50	\$ 190	\$ 1,130			

Table 58: Optimal order-up-to levels for selected border states ($y^*(i,n)$) and long-run average cost per day (g^*) vs. minimum leadtime (L) (Instance 9C: $L = 15$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(0.5)).

		L		
		1	7	15
$p_{CO} = 0.05$	g^*	\$ 75,960	\$ 76,137	\$ 76,294
	$y^*(O, 100)$	10	14	19
	$y^*(C, 100)$	25	29	33
$p_{CO} = 0.05$	g^*	\$ 75,433	\$ 75,605	\$ 75,761
	$y^*(O, 100)$	9	13	18
	$y^*(C, 100)$	16	19	24
$p_{CO} = 0.05$	g^*	\$ 75,222	\$ 75,399	\$ 75,548
	$y^*(O, 100)$	9	13	17
	$y^*(C, 100)$	10	14	18

Table 59: Optimal order-up-to levels for selected border states ($y^*(i,n)$) and long-run average cost per day (g^*) vs. holding cost (h) ($p = \$1,000$, D ~Poisson(0.5), $p_{OC} = 0.003$).

		h		
		\$100	\$300	\$500
$L = 15$	g^*	\$ 76,294	\$ 76,991	\$ 77,673
$p_{CO} = 0.05$	$y^*(O, 0)$	13	11	10
	$y^*(O, 100)$	19	15	14
$L = 15$	g^*	\$ 75,761	\$ 76,424	\$ 76,934
$p_{CO} = 0.1$	$y^*(O, 0)$	12	10	9
	$y^*(O, 100)$	18	15	14
$L = 7$	g^*	\$ 75,605	\$ 76,107	\$ 76,476
$p_{CO} = 0.05$	$y^*(O, 0)$	7	6	5
	$y^*(O, 100)$	13	11	10

Table 60: Optimal order-up-to levels for selected border states ($y^*(i,n)$) and long-run average cost per day (g^*) vs. penalty cost (p) ($h = \$100$, $D \sim$Poisson(0.5), $p_{OC} = 0.003$).

		p		
		\$500	\$1,000	\$2,000
$L = 15$	g^*	\$ 75,561	\$ 75,761	\$ 75,944
$p_{CO} = 0.05$	$y^*(O, 0)$	11	12	14
	$y^*(O, 100)$	16	18	19
$L = 15$	g^*	\$ 75,831	\$ 76,294	\$ 76,874
$p_{CO} = 0.1$	$y^*(O, 0)$	12	13	16
	$y^*(O, 100)$	16	19	23
$L = 7$	g^*	\$ 75,438	\$ 75,605	\$ 75,775
$p_{CO} = 0.05$	$y^*(O, 0)$	6	7	8
	$y^*(O, 100)$	11	13	15

Table 61: Optimal order-up-to levels for selected border states ($y^*(i,n)$) and long-run average cost per day (g^*) vs. transition probabilities (p_{OC}, p_{CO}) (Instance 13C: $L = 1$, $h = \$100$, $p = \$1,000$, $D \sim$ Poisson(1)).

			p_{CO}					
			0.5	0.4	0.3	0.2	0.1	0.05
g^*	p_{OC}	0.001	$ 150,287	$ 150,290	$ 150,297	$ 150,317	$ 150,417	$ 150,765
		0.003	$ 150,296	$ 150,305	$ 150,326	$ 150,386	$ 150,685	$ 151,669
		0.01	$ 150,330	$ 150,363	$ 150,438	$ 150,652		
		0.02	$ 150,387	$ 150,464	$ 150,641			
$y^*(O,0,0)$	p_{OC}	0.001	4	4	4	4	4	4
		0.003	4	4	4	4	4	4
		0.01	4	4	4	4		
		0.02	4	4	4			
$y^*(O,100,0)$	p_{OC}	0.001	16	16	16	16	16	16
		0.003	16	16	16	16	16	18
		0.01	16	16	16	17		
		0.02	16	17	17			
$y^*(C,0,0)$	p_{OC}	0.001	7	8	9	11	18	34
		0.003	7	8	9	11	19	37
		0.01	7	8	9	12		
		0.02	7	8	9			
$y^*(C,100,0)$	p_{OC}	0.001	18	19	20	22	29	45
		0.003	18	19	20	22	30	49
		0.01	18	19	20	23		
		0.02	19	20	21			

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