

# MECHANISMS OF INSTABILITY IN RAYLEIGH-BÉNARD CONVECTION

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# MECHANISMS OF INSTABILITY IN RAYLEIGH-BÉNARD CONVECTION

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*To my family.*

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# LIST OF SYMBOLS AND ABBREVIATIONS

Symbol or Abbreviation	Description
RBC	Rayleigh-Bénard convection
$Ra$	Rayleigh number
$Ra_c$	critical $Ra$ (onset of convection)
$Pr$	Prandtl number
$d$	depth of convection cell
$\Gamma$	cell aspect ratio
$\Delta T$	temperature difference across cell
$\Delta T_c$	critical $\Delta T$ (onset of convection)
$\epsilon$	reduced Rayleigh number, $\frac{Ra - Ra_c}{Ra_c}$
$q$	straight roll pattern wavenumber
$q_c$	critical $q$ (onset of instability)
SDC	spiral defect chaos
LETKF	Local Ensemble Transform Kalman Filter
$t_v, t_h$	vertical, horizontal diffusion time
CR	cross roll
ECK	Eckhaus
SV	skew-varicose
OSC	oscillatory
$\tau$	perturbation lifetime
LS	Lasershow Designer 2000 software

## SUMMARY

In many systems, instabilities can lead to time-dependent behavior, and instabilities can act as mechanisms for sustained chaos; an understanding of the dynamical modes governing instability is thus essential for prediction and/or control in such systems. In this thesis work, we have developed an approach toward characterizing instabilities quantitatively, from experiments on the prototypical Rayleigh-Bnard convection system.

We developed an experimental technique for preparing a given convection pattern using rapid optical actuation of pressurized  $\text{SF}_6$  gas. The technique and the convection apparatus are described in Chapter 2. Real-time analysis of convection patterns was developed as part of the implementation of closed-loop control of straight roll patterns; these methods are described in Chapter 3. Feedback control of the patterns via actuation was used to guide patterns to various system instabilities. Controlled, spatially localized perturbations were applied to the prepared states, which were observed to excite the dominant system modes. The lifetimes of excitations about a high-wavenumber state were measured as the pattern wavenumber was varied; a critical wavenumber was found from the observed dynamical slowing near the bifurcation, indicating where the localized skew-varicose instability occurs. These results are given in Chapter 4. In Chapter 5, we present the extracted spatial structure and growth rates of the modes dominating dynamics near various secondary instabilities of straight roll patterns. The modes were extracted from analysis of the evolutions of an experimentally prepared ensemble of states with nearby initial conditions. Chapter 6 describes the preliminary results of using a state estimation algorithm (LETKF) on experimentally prepared non-periodic patterns in a cylindrical convection cell.



# CHAPTER I

## INTRODUCTION

### *1.1 Pattern Formation and Instability*

Pattern-forming systems are ubiquitous. Astrophysical patterns can be seen on the surfaces of the Sun and planets (sun-spots, stripes on Jupiter, etc.). Patterns are observed in chemical systems (e.g., reaction-diffusion systems). Some of the most striking and most complex pattern-forming systems are biological, including the visible structures of plants and animals as well as internal systems such as organs and neuronal networks.

These are all examples of systems maintained out of equilibrium, systems in which dissipative mechanisms compete against a flow of energy into the system. Despite important physical differences, it turns out that many features observed across a variety of systems share a common mathematical description [5, 6]. It is hoped that the study of these shared features will help to build a general approach to characterizing and understanding non-equilibrium systems, rather than be left with treating systems on a case by case basis [7].

Generally speaking, a spatially extended, uniform state gives rise to a spatial pattern (with or without time dependence) as a result of instability. This so-called primary instability occurs when one or more system ‘control’ parameters exceed critical values. Control parameters can be thought of as the “knobs” accessible to the experimenter (or theorist); they can be adjusted systematically through some change in experimental conditions, for example. In many systems, the control parameter is related to the relative rates at which energy or some other relevant quantity (e.g., the

concentration of some chemical) is added to or dissipated from the system. At values different from critical, these control parameters can be used to define a distance above or below the onset of primary instability<sup>1</sup>. The particular pattern that emerges will depend on the governing equations (in addition to stochastic processes, such as thermal fluctuations) but, in many cases, a stationary pattern emerges with relatively simple spatial dependence (e.g., a small number of Fourier modes). However, as the distance above onset is increased, this primary pattern may itself undergo a transition to another state as a result of secondary instability. In principle, tertiary and higher instabilities could result in a succession of different time-independent patterns as the distance from onset is increased. In practice, instabilities often lead to complex (chaotic), time-dependent states where a quantitative understanding of pattern dynamics is limited.

## ***1.2 Rayleigh-Bénard Convection***

### **1.2.1 Introduction**

Rayleigh-Bénard convection (many times hereafter referred to as RBC) has long been an experimental and theoretical paradigm of pattern-forming systems (“the granddaddy of canonical examples” [8]). Its study is useful for understanding common aspects of pattern formation such as pattern competition, saturation, and the effects of physical boundaries. Additionally, RBC displays rich dynamics, ranging from stationary patterns to weakly chaotic evolution to highly turbulent states; the dynamical regime can be tuned by a small number of parameters. Moreover, convection is an important part of many physical systems and processes such as the weather, the Earth’s mantle, solar flares, and oceanic currents, to name just a few examples. For these

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<sup>1</sup>For clarity, consider as an example a system with a single control parameter: a pattern emerges when the control parameter exceeds (is ‘above’) a critical value; when the parameter is smaller than (‘below’) the critical value, there is no pattern. In systems with multiple control parameters, the quantitative meaning of ‘distance above’ and ‘distance below’ can be more ambiguous, but may still be used qualitatively to imply whether or not the system has undergone the instability of interest.

reasons, RBC is an excellent system on which to test new ideas and approaches for understanding nonlinear dynamics.

In its most basic description, RBC consists of a thin layer of fluid extending in the plane normal to the direction of gravitational acceleration (for many purposes, the lateral extent can be taken to be infinite); the fluid is heated from below and cooled from above. The temperature difference ( $\Delta T$ ) across the fluid gives rise to density differences, as regions of fluid near the bottom (top) boundary are warmed (cooled) and hence expand (contract). The density differences in turn result in a buoyancy force, which acts to reorganize the lighter fluid at the top boundary (and heavier fluid at the bottom). For small  $\Delta T$ , the fluid remains motionless; heat is transferred across the fluid through conduction, with a linear temperature profile across the fluid layer. When the temperature difference exceeds some critical value ( $\Delta T_c$ ), energy transfer through reorganization of hot/cold fluid becomes more efficient than thermal and viscous dissipation and fluid motion sets in. Bénard in 1900-1901 [9, 10] was the first to report careful experiments on the convective motion in a fluid heated from below, and Rayleigh provided a theoretical explanation of the buoyancy-driven motion in 1916 [11]. It turns out that surface tension was partly responsible for Bénard's observed fluid motion, a phenomenon now known as Bénard-Marangoni convection, but the work by Rayleigh was nonetheless very important in that it provided an accurate description of the mechanism of primary instability and the proper non-dimensional combination of fluid and other system properties forming the relevant control parameter. This parameter is now known as the Rayleigh number. Rayleigh treated the problem of a fluid with free boundaries; Jeffreys extended this approach to the more physical, but more challenging, case of rigid boundaries [12, 13].

The set of equations governing the evolution of the convecting fluid are known as the Boussinesq equations [14, 15]. The Boussinesq equations are derived from the full Navier-Stokes under the assumption that the temperature dependence of the fluid

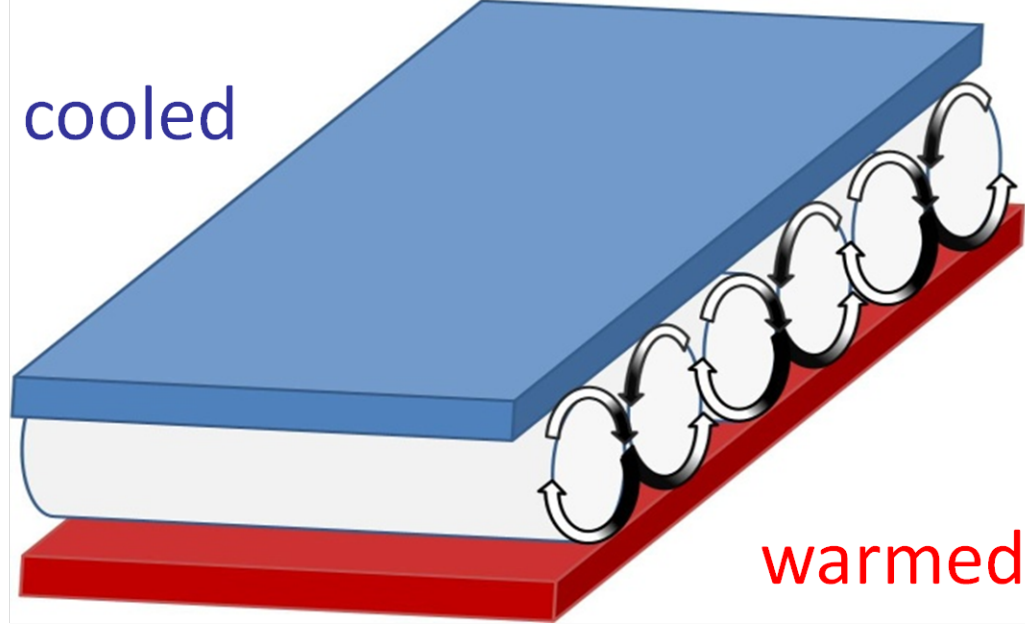


Figure 1.1: Rayleigh-Bénard convection showing straight convection rolls. Warm fluid rises to the top of the cell, where it cools, before falling back to the bottom. The arrows indicate the flow pattern, with bright and dark regions corresponding to warm and cool fluid, respectively.

properties can be neglected, except in the case of the density, which is assumed to have a linear temperature dependence. A derivation of the Boussinesq equations is given in Appendix A; the reader is referred to Ref. [16] for a more detailed treatment<sup>2</sup>.

The non-dimensional Boussinesq equations read:

$$\nabla \cdot \mathbf{V} = 0 \quad (1.1)$$

$$Pr^{-1} \left( \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} \right) = -\nabla P + \nabla^2 \mathbf{V} + \theta \hat{\mathbf{z}} \quad (1.2)$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{V} \cdot \nabla) \theta = \nabla^2 \theta + Ra V_z \quad (1.3)$$

Here,  $\mathbf{V}$  and  $\theta$  are the velocity and the deviation of the temperature from the linear conduction profile, respectively. In addition to these two fields that describe the fluid state (and the pressure  $P$ ), a pair of non-dimensional parameters ( $Ra$  and  $Pr$ ) are

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<sup>2</sup>For a discussion of non-Boussinesq effects on pattern formation and evolution, see Ref. [17] and the references therein or, more recently, Refs. [18, 19].

present in the Boussinesq equations. The Rayleigh number  $Ra$  is given by

$$Ra = \frac{\alpha g \Delta T d^3}{\nu \kappa} \quad (1.4)$$

where  $\alpha$  is the coefficient of fluid thermal expansion,  $g$  is the gravitational acceleration,  $\nu$  is the kinematic viscosity,  $\kappa$  is the thermal diffusivity, and  $\Delta T$  is the temperature difference of the two plates separated by a distance  $d$ . The Rayleigh number can be understood as a ratio of the strength of the buoyancy force to the magnitude of the dissipation processes: for a fixed  $d$ ,  $\alpha g \Delta T$  determines the size of the buoyancy force which tends to cause a locally warm (cool) and thus less (more) dense parcel of fluid to move to the top (bottom) of the cell;  $\nu$  sets the size of the viscous drag force opposing this motion; and  $\kappa$  determines the rate at which the temperature of the fluid parcel is equilibrated with its surroundings through thermal diffusion (conduction) alone. The Prandtl number  $Pr$  is given by

$$Pr = \frac{\nu}{\kappa} \quad (1.5)$$

and describes a ratio of the two dissipative mechanisms, viscous and thermal diffusion. In experiments, the fluid is bounded, so  $\Gamma$  is introduced as an additional parameter, defined as the ratio of lateral extent (e.g., cell radius) to cell depth. Note that, as illustrated by the Boussinesq equations and the accompanying parameters, it is customary to use non-dimensional units. For example, length units (such as  $\Gamma$ ) are non-dimensionalized by the fluid depth  $d$ ; other units will be defined as they are used. In the images showing patterns, the length scale will usually be described in terms of  $\Gamma$  or the non-dimensional pattern wavenumber  $q = \frac{2\pi}{\lambda/d}$ .

The onset of convective motion through an instability of the no-motion conducting state occurs at a critical value of the Rayleigh number, and hence a critical  $\Delta T$ , as mentioned above.  $Ra_c \approx 1708$  for rigid boundaries in the vertical direction, independent of the Prandtl number. The convective solution just above onset takes the form of “straight rolls” consisting of alternating upflowing hot fluid and downflowing

cold fluid (see Fig. 1.1) with a single spatial frequency in one of the lateral directions. The non-dimensional wavenumber of this critical Fourier mode is also independent of Prandtl number<sup>3</sup> and is given by  $q_c = 3.117$  [16]. In experiments, the convection pattern at onset will generally deviate somewhat from ideal; for example, strongly bounded systems will likely have a wavenumber different from  $q_c$ , and in cylindrical geometry, the convection pattern can take the form of a target pattern, as shown in Fig. 1.3 (and later in Fig. 2.2). At Rayleigh numbers above critical, it is convenient to use a parameter known as the reduced Rayleigh number,  $\epsilon \equiv \frac{Ra - R_c}{R_c}$  (which simplifies to  $\epsilon = \frac{\Delta T}{\Delta T_c} - 1$  under the Boussinesq approximation), to indicate the distance from onset. Pattern visualization is achieved via the shadowgraph method, which transforms the three-dimensional flow pattern into a two-dimensional intensity image showing regions of hot/cold fluid across the cell's lateral domain. The shadowgraph method is described in detail in Section 2.3.

### 1.2.2 Secondary Instability and the Busse Balloon

For  $\epsilon > 0$ , a range of possible pattern wavenumbers emerge, only some of which are observed experimentally. Starting in the 1960s, Busse and coworkers used the Boussinesq equations to construct a theoretical description of the different straight roll patterns arising near the primary convective instability [20, 21, 22]. This seminal work examined secondary instabilities, wherein the straight roll state loses stability. These instabilities cause the joining of existing hot upflow (cold downflow) regions or the introduction of new convection rolls, the overall result being a reorganization of the straight roll state. Effectively, these secondary instabilities limit the range of parameter values (the relevant parameters are the Rayleigh and Prandtl numbers as well as the pattern wavenumber) for which the straight roll pattern is predicted to be stable and can thus be expected to be observed in experiments; this volume

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<sup>3</sup>The Prandtl number appears as a coefficient on a nonlinear term in the Boussinesq equations and is thus neglected when investigating the linear stability of the conducting state.

of parameter space is bounded by what has been labeled eponymously the Busse (or stability) balloon. Which of the various instabilities form the balloon boundaries, and at what values those instabilities occur, is largely dependent on the Prandtl number. For this reason, the stability balloon is shown typically with fixed  $Pr$ , producing two-dimensional slices in  $(q, Ra)$  space. Figure 1.2 shows the Busse balloons at  $Pr = 7$  (water) and  $Pr = 0.71$  (air), from which one notices substantial differences both in the limiting instability types and where in the space they occur. See Refs. [21, 22, 23] for a detailed description of the various types of secondary instabilities.

### 1.2.3 Non-periodic and Time-dependent Patterns

Above the onset of convection, ( $\epsilon > 0$ ), the selection of the pattern that emerges and its spatiotemporal character are influenced heavily by the fluid's Prandtl number. This can be understood intuitively by noting the appearance of the Prandtl number in the evolution equation for the fluid velocity; small Prandtl numbers effectively enhance the strength of the non-linearity in this equation. The result for small- $Pr$  fluids ( $Pr \approx 1$  for typical gases) is the likely development of a flow pattern in the lateral directions (known as a mean flow) that couples positively with roll curvature [24, 25, 26]. This is in contrast to periodic straight rolls, which have no overall flow pattern in the horizontal directions and no roll curvature. Because convection rolls have a tendency to terminate at right angles to the convection cell boundaries (sidewalls)<sup>4</sup>, boundaries can become local sources of roll curvature, coupling back to the mean flow. The result is that ideal straight roll patterns are unlikely to be observed except very near onset.

Figure 1.3 shows shadowgraph snapshots during a progression of patterns at increasing values of  $\epsilon$ . The shadowgraph method is described in Section 2.3; roughly

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<sup>4</sup>The velocity must be zero at the cell boundaries (no-slip); aligning a given roll at a right angle to a boundary is a way of meeting that condition with little effect on the roll flow pattern (as opposed to aligning the convection roll parallel to the boundary).

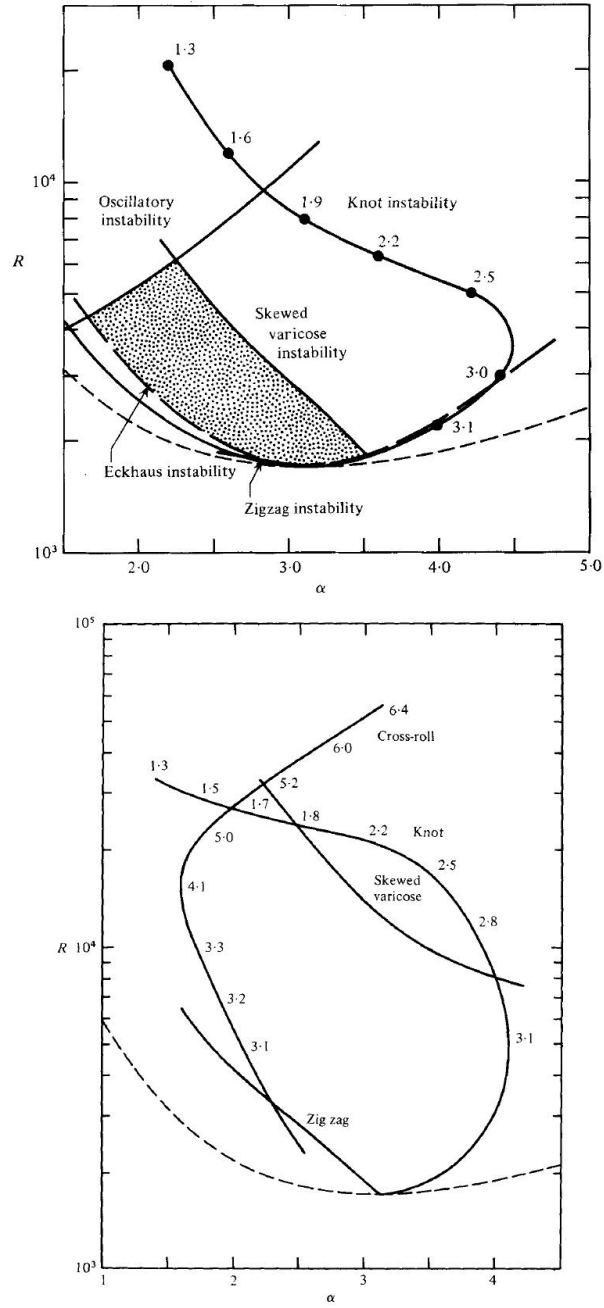


Figure 1.2: Stability balloons at Prandtl number 0.71 (air) and at 7.0 (water), reproduced from Ref. [1] with permission. The vertical axis is the Rayleigh number and the horizontal axis is the non-dimensional wavenumber. The boundaries of the different types of secondary instabilities are marked by the solid lines; the dashed lines indicate the onset of the primary (convective) instability for the given range of wavenumbers. The area enclosed by the intersecting boundaries forms the parameter region of stable straight roll patterns.



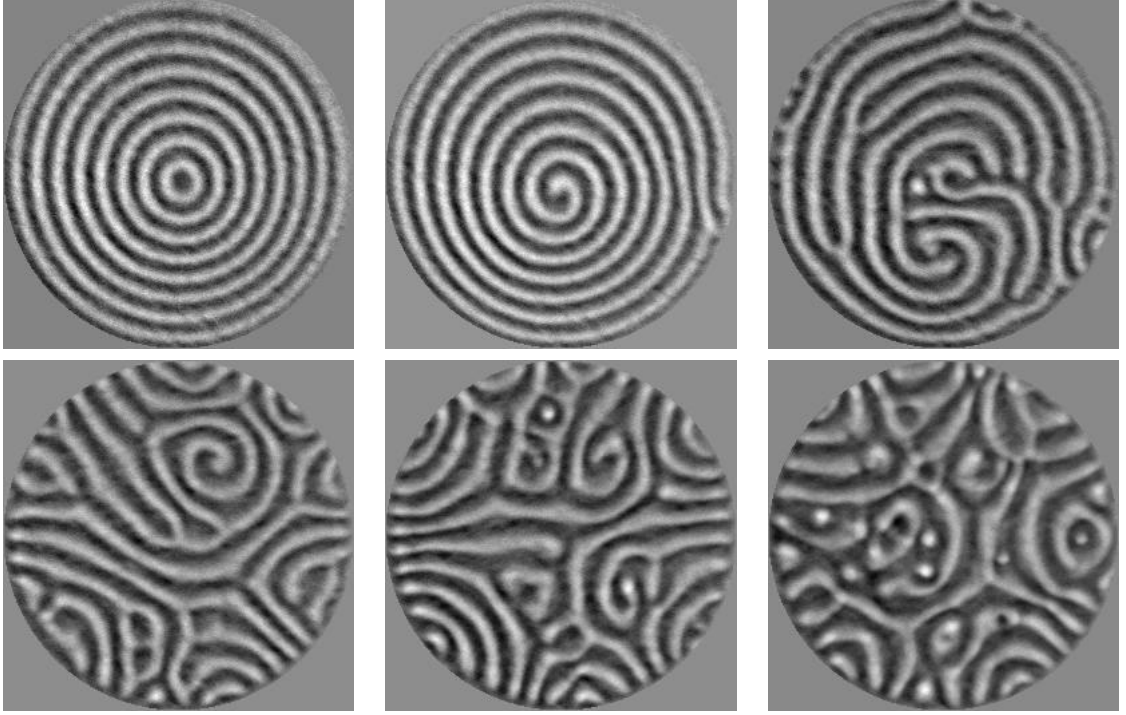


Figure 1.3: Snapshots of convection patterns as  $\epsilon$  is increased. Going clockwise starting with the image on the upper left, images are taken with  $\epsilon = 0.30$  (a “target” pattern),  $\epsilon = 0.56$ ,  $\epsilon = 0.71$ ,  $\epsilon = 0.95$ ,  $\epsilon = 1.3$ , and  $\epsilon = 1.8$ . The images are taken from a convection cell with  $d = 608 \mu\text{m}$  and  $\Gamma = 20$ .

speaking, bright (dark) areas on the image correspond to hot (cold) fluid. On the upper left is an example of a “target” pattern, the analog of straight rolls in a circular convection cell. This imprinting of the cell geometry onto the convection pattern is an example of what is known as sidewall forcing, which is most often due to a mismatch in the thermal conductivities between the boundary and the convective fluid. In this set of images,  $\epsilon$  has been increased rapidly, causing remnants of the initial circular symmetry to be present transiently, even at moderate  $\epsilon$ . Eventually, the pattern evolves to align the rolls at right angles to the sidewalls, and the spatial complexity of the pattern increases with increasing  $\epsilon$ . Point-like regions of fluid, like those present in the final image, are referred to as defects, and they play an important role in pattern evolution.

For sufficiently large aspect ratio convection cells, there exists a spatiotemporally

chaotic state known as the spiral defect chaos (SDC) state [2], a state characterized by rotating spirals and regions of locally parallel rolls, punctuated by defects; see Fig. 1.4. Intriguingly, it has been shown experimentally that the SDC state is bistable with the straight roll pattern [27] over a certain parameter range. That is, for certain parameter values, one can observe either a stationary, spatially periodic state, or a state exhibiting chaos in both space and time; because the physical sidewalls can be sources of roll curvature or defects, it is suspected that their presence tends to predispose the system toward SDC (i.e., the sidewalls put the system into the SDC basin of attraction) [28], meaning straight roll patterns are usually observed only under special experimental conditions (e.g., using sidewalls with non-uniform conductivity).

### ***1.3 Motivation and Research Objectives***

Chaotic convection states, like SDC, are particularly interesting because they allow one to study different ideas about chaotic evolution in an experimental system where conditions are well controlled, the evolution equations are known, and high-resolution measurements can be made. Several studies have attempted to characterize SDC through statistical measures, such as the number of defects or spirals [29] or through the Fourier power spectrum and correlation lengths/times [28]. What *dynamical* role these measures play, however, is unclear.

#### **1.3.1 Dynamical Systems**

The approach of dynamical systems theory is to picture a system state within a space spanned by the dynamical variables [7]; this space is termed the dynamical phase space. The phase space for RBC, in principle, consists of velocity vectors and temperature scalar values at every fluid particle position. System evolution is then visualized by a trajectory through points in the phase space. (Perhaps the best known example is the butterfly-like appearance of trajectories around the Lorenz attractor [30].) For simple patterns, or in certain representations, the dimensionality

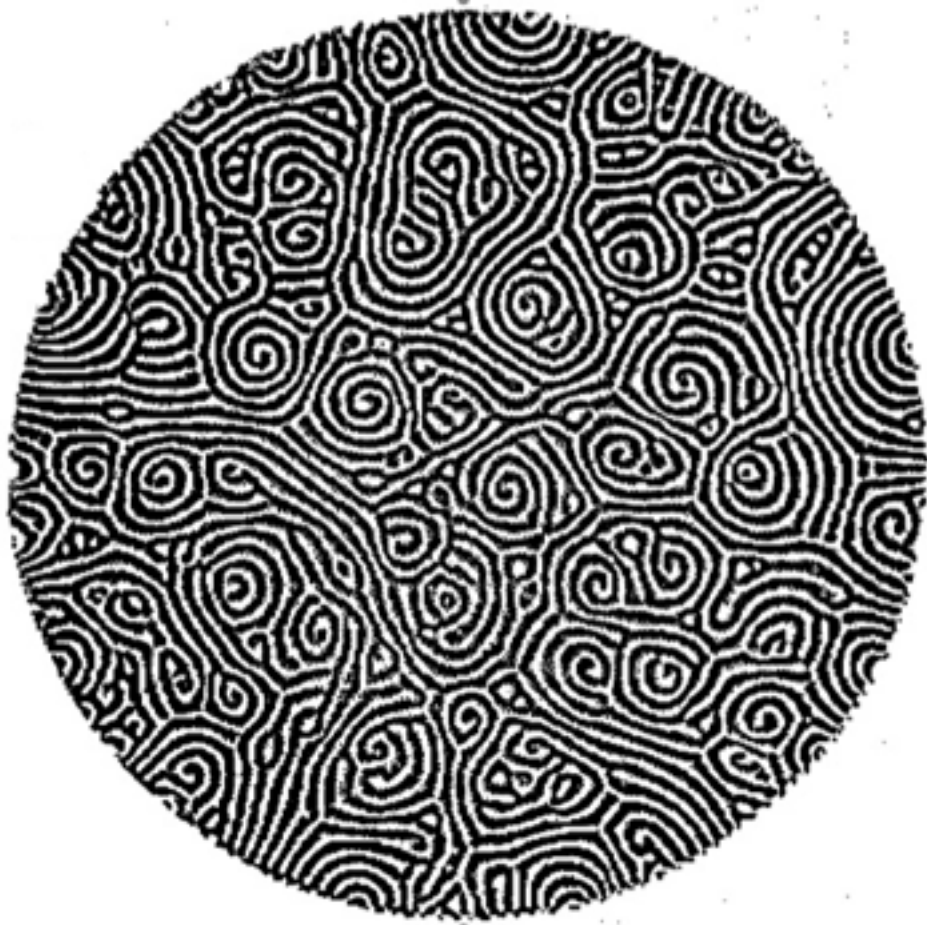


Figure 1.4: An image of the spatiotemporally chaotic spiral defect chaos state, taken from Ref. [2] (with permission). This experiment was with  $\text{CO}_2$  gas in a circular cell of  $\Gamma = 74.6$ , with  $\epsilon = 0.894$ .

of the space may reduce substantially, as is the case with the ideal straight roll state and the corresponding three-dimensional space embedding the Busse balloon. It is not known, however, for a general chaotic state, the degree to which the number of dimensions can be reduced while still capturing the full state evolution.

The different ways in which patterns can evolve from a given state are known as the dynamical modes (or local Lyapunov vectors); it is the number of these modes needed to capture an evolution that determines the dimension of a state, and it is the structure of these modes that describe the actual changes to a pattern. Together with each dynamical mode is a growth rate that indicates the rate at which small disturbances to the given state, in the form of the respective mode, can be expected to grow (positive growth rate) or decay (negative growth rate); it is therefore the modes with the largest growth rates that are likely to be responsible for pattern evolution.

Computational advances have recently allowed for the direct numerical computation of the dynamical modes of the SDC state [3]. Shown in Fig. 1.5 are the mid-plane temperature field and the corresponding temperature-component of the leading mode (Lyapunov vector) from a snapshot of a simulated spiral defect chaos state. Noticeable in the Lyapunov field is a high degree of spatial localization (i.e., most of the vector's magnitude is contained in a small area of space), indicating that the mechanism responsible for separating trajectories acts on a spatially localized scale through the creation/annihilation of defects occurring in straight roll regions of the pattern. In fact, the instabilities that were observed to contribute to chaotic evolution are instabilities of the straight roll pattern itself.

While the Busse balloon addresses the question of straight roll stability in the case of (infinite) ideal rolls, such patterns are rarely observed experimentally. More often is the case where a straight roll pattern is not strictly periodic: the presence of physical boundaries can be accommodated by an integer number of rolls with non-uniform spacing or can result in more than one pattern wavevector (so that rolls can

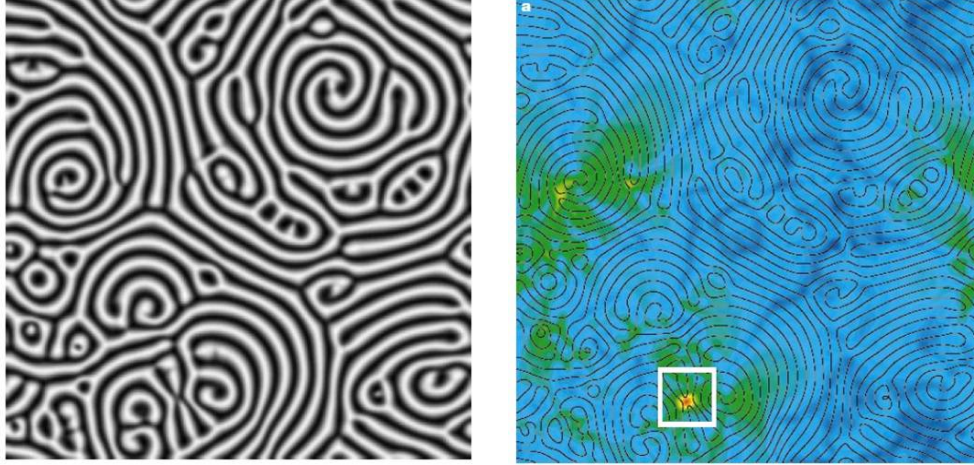


Figure 1.5: On the left, the mid-plane temperature of a simulated SDC state. On the right is the temperature-field component of the leading Lyapunov vector; red and blue indicate large and small magnitudes, respectively. From Ref. [3], reproduced with permission.

more closely meet all boundaries at right angles). The positive feedback between roll curvature and mean flow in low- $Pr$  fluids makes areas of these non-ideal patterns with locally higher or lower roll spacing most susceptible to pattern instabilities. The upshot is that instabilities in experimental straight roll patterns tend to occur in spatially localized, rather than global, regions, just as in SDC. While these straight roll instabilities can result simply in a change in pattern wavenumber or orientation, they can also introduce defects into the pattern that can cause further roll distortions and thus lead to a growing cascade of instabilities. The result of such a defect cascade can be a transition from stationary straight rolls to a time-evolving (chaotic) pattern such as SDC.

To date, there exists no general experimental approach to obtaining the dominant dynamical modes of nonlinear non-equilibrium systems; such an approach could be useful in systems where the governing equations are unknown or the system geometry does not lend itself easily to numerical modeling. The research presented in this thesis has been motivated by a desire to characterize experimentally the mechanisms of instability of the straight roll pattern; because these instabilities remain relevant

dynamically in more complex states (e.g., SDC), this work is a step toward characterizing complex (chaotic) pattern evolution directly from experimental measurements. That spatially localized instabilities continue to be important in the chaotic evolution of patterns suggests that it is through these types of instability events that patterns with nearby initial conditions diverge. It is natural, therefore, to expect that these events also play a role in limiting predictive capabilities. In this thesis, we present preliminary work aimed at applying a particular prediction algorithm to patterns which have been prepared with controlled initial conditions, as they evolve via roll instabilities. Many of the different techniques for extracting, analyzing, and characterizing the experimental data presented herein are developed generally, with the idea in mind that much of our approach can be carried over to other non-equilibrium systems, where understanding of the important system modes may prove useful for control and/or prediction of system dynamics.



## CHAPTER II

### EXPERIMENTAL APPARATUS AND METHODS

As discussed in the Introduction, the rich behavior observed in convection studies of low Prandtl number fluids ( $Pr \approx 1$  for a typical gas) makes gas convection of particular interest. However, high quality pattern imaging (see shadowgraph discussion below) of gases at atmospheric pressures is difficult due to their low density. For this reason, it is common to conduct convection experiments with pressurized gases [31, 32, 17]. This chapter provides an overview of the pressurized convection apparatus and other essential experimental components used in this thesis work. It has been written with the intent to focus on the novel features; accordingly, much of this chapter is devoted to describing in detail the components and techniques for actuation of the fluid flow using infrared laser light. The ability provided by this experimental tool, of manipulating convective patterns in a controlled fashion, is central to this thesis work. It is only appropriate to also give an explanation of shadowgraph visualization of the convection patterns, which account for the entirety of the experimental data. The experimental procedures are discussed in subsequent chapters, where appropriate. Further details regarding the design, maintenance, and operation of the experiment can be found in Appendix D.

#### *2.1 Convection Apparatus*

The design of our experimental apparatus is based on the description given by de Bruyn et al. [32] for shadowgraph visualization of compressed gas convection. A schematic is shown in Fig. 2.1. The convection cell is formed between two pressure windows which transmit visible light, allowing for flow visualization. The fluid, compressed sulfur hexafluoride ( $\text{SF}_6$ ) is bounded laterally by a set of sidewalls; together,



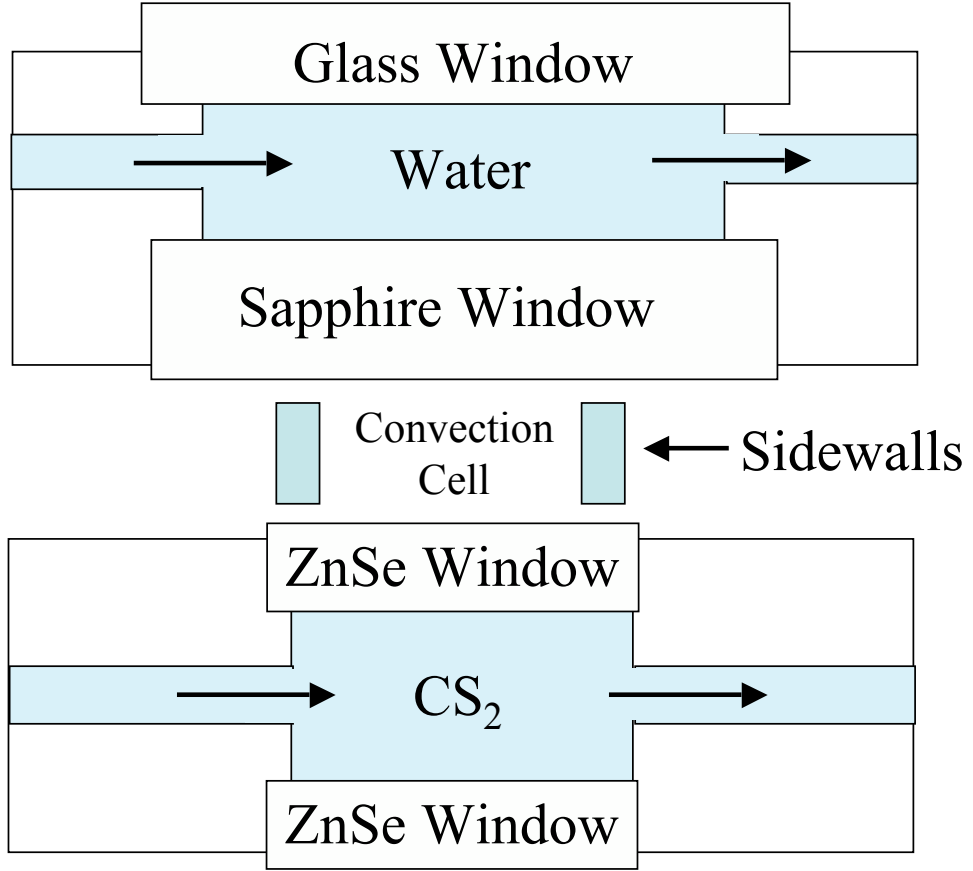


Figure 2.1: Schematic of the convection apparatus.

the sidewalls and pressure windows define the volume of the convection cell. A compressed O-ring sits in the plane of the convection cell around the sidewalls and forms the lateral pressure seal. The optical windows and convection cell are enclosed in the aluminum body of the convection apparatus. Detailed dimensions are given in Appendix A.

### 2.1.1 Optical Windows

The top boundary is formed by a 1.91 cm thick sapphire optical window of diameter 7.62 cm. Water circulating through a temperature bath flows over the sapphire window and maintains a temperature. While it is preferable to use a highly reflective bottom surface for good shadowgraph imaging, such as gold-plated aluminum,

our desire to direct infrared light from below (as described in Section 2.4) limits the choice of material. We use a 0.95 cm thick zinc selenide (ZnSe) optical window of diameter 5.08 cm and control the window temperature via circulating carbon disulfide (CS<sub>2</sub>). Both ZnSe and CS<sub>2</sub> have high transmission at the wavelength (10.6  $\mu\text{m}$ ) of the infrared laser light used in the experiments.

**Pressure Loading** Thick optical windows are needed to withstand the large gas pressures; a typical experiment is conducted near 200 psi. While zinc selenide is a material commonly used in CO<sub>2</sub> laser applications, thick material is difficult to find. The minimum thickness of the material for a given pressure load and desired radius is given by the following expression<sup>1</sup>:

$$T_{min} = \sqrt{\frac{1.1PR^2S}{M}} \quad (2.1)$$

where  $T$  is the material thickness,  $M$  is the material rupture modulus,  $P$  is the pressure differential across the window,  $R$  is the radius of the exposed face, and  $S$  is a safety factor (usually taken to be 4). Sapphire has a rupture modulus of over 50,000 psi, whereas  $M = 8,000$  for ZnSe. The limited availability and expense of thick ZnSe provides a practical constraint on the radial size, and thus, the available aspect ratio.

**BBAR Coating** Bulk ZnSe has a high internal transmission at the wavelength of laser light used in our experiments (10.6  $\mu\text{m}$ ); however, the high refractive index of ZnSe ( $\approx 2.4$ ) can result in a loss of  $\sim 30\%$  of incident light by reflection. A thin anti-reflection coating known as BBAR (BroadBand Anti-Reflection) increases the IR transmission<sup>2</sup> to  $> 99\%$ . However, the BBAR coating poses an additional

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<sup>1</sup>Many optical window vendors provide similar expressions; this form comes from the II-VI Infrared company.

<sup>2</sup>Transmission data is most readily available from optical window vendors. The following are a few companies with websites containing transmission spectra for materials used in our experiments: (a) ZnSe: II-VI Infrared, Janostech, Sciner Optics; (b) ZnS: International Crystal, Fairfield Crystal; (c) Sapphire: Valley Design, General Ruby; and (d) Germanium: Almaz Optics.

consideration on the pressure loading, as a large enough contact pressure can cause a cracking in the coating. A pressure of at least 230 psi can be used as long as the O-rings which support the windows (and which are in direct contact with the coating) are sufficiently thick.

### 2.1.2 Convection Cell

**Sidewalls** We use two different materials as lateral boundaries, or sidewalls. An important property of the sidewall material is the thermal conductivity; a large conductivity mismatch between the fluid and the sidewall can affect the dynamics of the interior pattern. Filter paper sidewalls have a conductivity ( $0.05 \text{ W/m}\cdot\text{K}$ ) on the same order as that of the convection gas ( $0.014 \text{ W/m}\cdot\text{K}$ ) and are used to minimize boundary forcing. Paper sidewalls are formed by stacking individual layers of paper out of which the desired cell geometry has been cut; a very small amount of vacuum grease acts as a glue between the layers. We use a rectangular cell with dimensions of  $25 \text{ mm} \times 15 \text{ mm}$  and depth of  $700 \text{ }\mu\text{m}$ . In experiments for state estimation, we have used plastic (polyethersulfone) sidewalls (radius of  $12 \text{ mm}$ , depth of  $608 \text{ }\mu\text{m}$ ); the conductivity of this material exceeds that of the gas by a factor of ten. These boundaries are used to meet approximately the infinitely-conducting boundary conditions used in the simulations of the Boussinesq equations. Plastic boundaries are formed from a single plastic sheet.

**Cell Alignment** The Rayleigh number dependence on the fluid depth is cubic, so it is highly important to minimize deviations in the cell depth over its horizontal extent. There are two main sources of depth variation. The first is that the pressurized gas can cause bowing of the optical windows forming the top and bottom boundaries, which can become important for windows of large radius. The second contribution to depth variation is possible misalignment of the two optical windows. Once the cell has been pressurized, the depth is adjusted by tightening one or more finely threaded

screws which connect the top and bottom halves of the apparatus. To measure the depth variation, an expanded beam of HeNe laser light is directed into the cell from above; reflections from the optical window surfaces forming the cell boundary combine to produce an interference pattern. Each fringe in the interference pattern indicates a height variation of one half-wavelength of the HeNe laser light [32]. The screws are carefully adjusted until only 25 or so fringes remain, which corresponds to a depth variation of less than  $10\text{ }\mu\text{m}$ .

**Determination of Cell Depth** The determination of the cell depth is crucial for subsequent determinations of the Rayleigh number. Rather than use the somewhat cumbersome, albeit direct, method of interferometry [32], we rely on a numerical program that estimates the depth based on the observed  $\Delta T_c$  at a particular mean temperature and pressure. This program, which contains a catalogue of values of fluid properties and interpolation formulae, was generously provided by the Santa Barbara group of Geunter Ahlers. Essentially, the values of the fluid properties and the critical temperature difference are substituted into the expression for the critical Rayleigh number, leaving the depth as the only unknown. An up-to-date version of this routine, containing more recent experimental measurements of fluid properties, was produced recently by a student in our group [18].

Shadowgraph images are taken as the temperature difference (or  $\epsilon$ ) across the cell is varied systematically. The onset of convection (and thus  $\Delta T_c$ ) is identified via an increase in either the rms intensity of the image or the power of the Fourier spectrum. Figure 2.2 shows both, as  $\epsilon$  is increased (from left to right). In this example with a circular convection cell, sidewall forcing causes a pattern of circularly concentric rolls to emerge at onset (rather than straight rolls). The Fourier spectrum shows a ring of wavenumbers present in the pattern above onset, reflecting the circular symmetry of the pattern. Images at far left show convection rolls just beginning to form near the

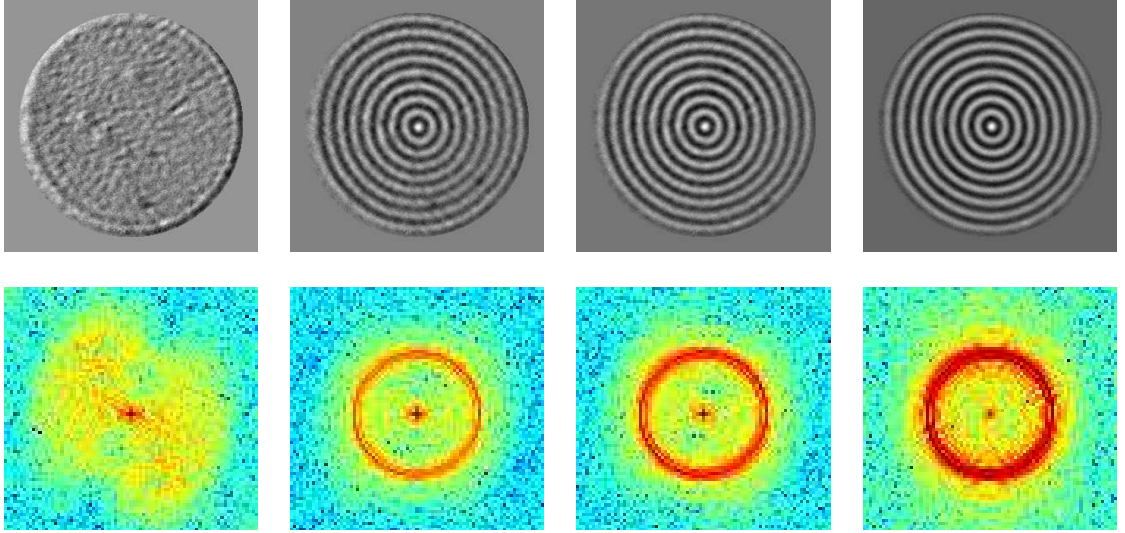


Figure 2.2: On the top, unfiltered shadowgraph images capture the onset of convection (in a cell of depth  $608\text{ }\mu\text{m}$ ,  $\Gamma = 20$ ). The Fourier power of each shadowgraph is shown in the bottom row of images, with red corresponding to large power and blue to small power. From left to right,  $\epsilon = -0.05, 0.02, 0.03$ , and  $0.04$

boundary; at slightly larger  $\epsilon$ , the rolls fill the cell and a clear structure emerges in the Fourier signal. The pattern here is time-independent at each fixed  $\epsilon$  but increases in amplitude as the driving is increased. We define  $\Delta T_c$  as the temperature difference at which a pattern is first seen over the cell domain; in this way,  $\Delta T_c$  is determined to within the uncertainty in  $\Delta T$  and the cell depth is then backed out, usually within  $10\text{ }\mu\text{m}$ . In the case of significant depth variation, a pattern will emerge in the thicker region of the cell first; a uniform pattern amplitude at onset is thus an independent check on the depth uniformity.

## 2.2 *Temperature and Pressure Measurement/Control*

Water flowing over the top window maintains that boundary at a temperature  $T_t$  while the circulating reagent-grade  $\text{CS}_2$  maintains a temperature  $T_b$  on the bottom. We therefore define the temperature difference across the cell as  $\Delta T = T_b - T_t$  and the mean temperature  $\bar{T} = \frac{1}{2}(T_b + T_t)$ . Typically,  $\Delta T$  is a few degrees Celsius, and  $\bar{T}$  is near room temperature. The cooling water above the sapphire window is

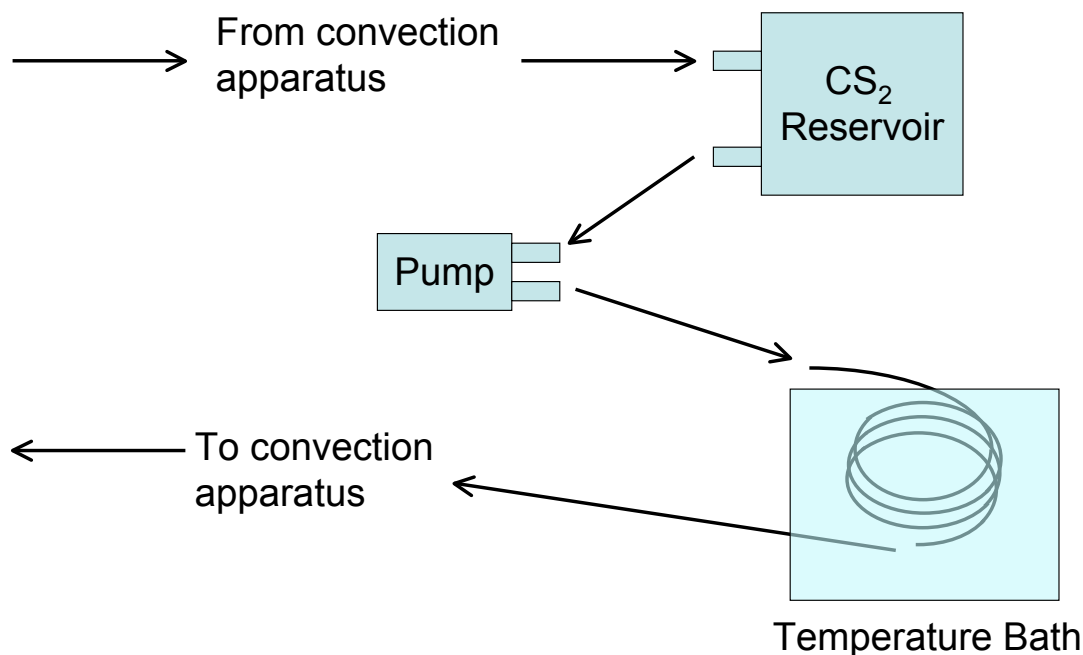


Figure 2.3: Schematic of closed-loop heating system for CS<sub>2</sub> through indirect heat exchange with temperature bath. Fluid from a reservoir is drawn into a pump, which forces the fluid through the copper coils sitting within a temperature bath, before circulating through the convection apparatus and returning to the reservoir.

pumped directly through a Neslab RTE-210 temperature bath (model 900685). The CS<sub>2</sub>, however, is a volatile fluid and is therefore pumped through a closed loop; it exchanges heat with a separate temperature bath during part of its circulation loop via a set of copper coils. A schematic of the CS<sub>2</sub> heating/cooling loop is shown in Fig. 2.3. We use a magnetic drive pump (Little Giant model 3-MD-MT-HC) to force the CS<sub>2</sub> through the loop. Both ZnSe and sapphire have thermal conductivities greater than 10 times that of the convective fluid, which is important in maintaining fixed temperatures at the vertical boundaries.

Two thermistors are used to measure the temperature of the top and bottom convection cell boundaries. The thermistors, which have temperature dependent resistances, are inserted into small cavities within the convection apparatus leading very near the pressure windows. The resistances are read via a digital multimeter

(Hewlett-Packard model 34401A) that communicates serially with Matlab and converted to temperature values. A 0 or 5 V signal sent from a data acquisition device (Measurement Computing model USB-1208FS) to a relay switches the multimeter connection between the two thermistors, so the top and bottom temperatures can be measured in rapid succession. The temperature dependences of the thermistors have been calibrated beforehand by measuring the resistances at a number of different temperatures (using an already-calibrated thermistor or thermometer to measure the temperatures); the data are fit with the Stein-Hart equation [33], which has a small number of free coefficients<sup>3</sup>. Resistance measurements are thus easily converted to temperature values using the Stein-Hart equation and the fit coefficients.

The top and bottom temperatures are computer-controlled via voltage signals output to the water baths. Because the heating of the CS<sub>2</sub> relies on indirect heat exchange, changes to the warm water bath temperature are slightly delayed in effect, relative to changes in the cold bath. Nonetheless, the control algorithms for the two temperatures are nearly the same. Changes to each bath setting are made by a proportional-integral-derivative (PID) control loop, using a combination of the current error between the desired and measured temperature (the proportional term), the running error over some time (the integral term), and the rate of change in the error (the derivative term). The difference between the top and bottom temperature control loops is the relative size of the coefficients on the different terms (in both cases, the integral term is much smaller than both the proportional and the differential terms). Note that the temperature measurement used for the error is from the thermistors near the convection cell, not from the water bath temperatures (which are likely somewhat different). An example illustrating the temperature control is shown in Fig. 2.4. The rms deviations of  $T_t$  and  $T_b$  are both less than 0.025° C, so we estimate

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<sup>3</sup>In the Stein-Hart equation, the inverse of the temperature is expanded in a polynomial in the logarithm of the resistance.

a variation in  $\Delta T$  of  $\leq 0.05^\circ \text{ C}$ .

Because the Rayleigh number has a strong implicit pressure dependence (through the fluid properties), a constant pressure within the convection cell is also important. A pressure transducer (Honeywell Sensotec model TJE) in-line with the gas feed produces voltage values between 0-5 V, corresponding proportionally to mechanical pressure values of 0-500 psi. These voltages are read via the data acquisition device. While we did explore connecting an external ballast volume of gas to help control pressure within the cell (using temperature control of the ballast volume to induce gas to flow into or out of the convection cell), it turns out that good control of the convection temperature is a sufficient method of pressure control. Even without temperature control (that is, using the baths without feedback), the pressure change is slight. Over the course of a day, the ambient room temperature does affect the pressure, but the effect is typically to change the pressure about the mean by  $\sim 1$  psi. With a pressure near 200 psi, this is less than a 2% deviation, which translates into about a 2% change in  $\Delta T_c$ .

## ***2.3 Flow Visualization***

There are two fields, velocity and temperature, that together describe the fluid state, neither of which can be measured easily. Instead we use as our primary measurement shadowgraph images that capture the convection patterns as two-dimensional intensity maps.

### **2.3.1 Shadowgraph**

The shadowgraph method is a relatively easily implemented experimental technique for visualization of transparent media with refractive index variation [34, 35, 36]. A schematic of our shadowgraph setup is given in Fig. 2.5. An incandescent bulb ( $\sim 100 \text{ W}$ ), connected to a DC power supply to avoid intensity fluctuations, illuminates an optical fiber sitting behind a small ( $750 \mu\text{m}$  diameter) pinhole. Emitted rays



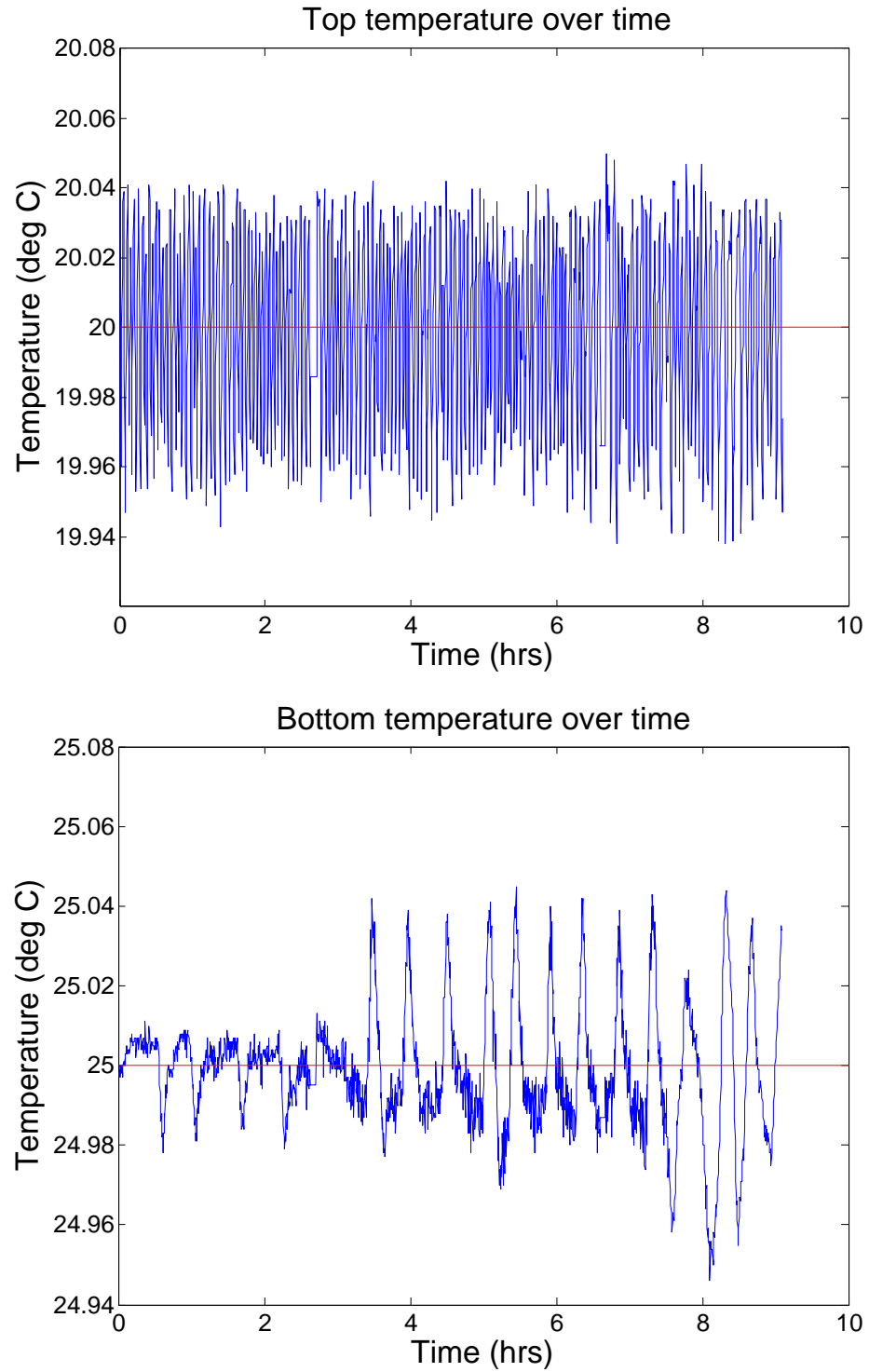


Figure 2.4: Temperature signals from above (top plot) and below (bottom plot) the convection cell. Both signals are under feedback control; the red lines indicate the set-point values of each:  $T_t = 20.0^{\circ}\text{C}$  and  $T_b = 25.0^{\circ}\text{C}$ .

of light are incident upon a beam splitter which directs them through a collimating lens (focal length of 50 cm). The resulting parallel rays of light are directed from above through the top of the convection cell and pass through the convecting fluid in the direction of gravity. As the light moves through the fluid, it is deflected away from regions of warmer fluid (lower index of refraction) and toward regions of cool fluid (higher index of refraction). The light is reflected at the surface of the bottom boundary and then passes upward through the fluid and back through the collimating lens before being collected in a CCD camera as a pattern of bright and dark regions. A geometric optics treatment relates the 2D shadowgraph intensity field to the variation of the fluid temperature/refractive index over the cell depth (note that the index of refraction is assumed to be wavelength-independent). Appendix C gives a derivation of the quantitative relationship between the temperature field and the observed intensity field and also comments on deviations from the geometric approximations; this relationship will be used in Chapter 6. Roughly, shadowgraphs can be understood as bright and dark intensities corresponding to regions of hot upflow and cold downflow, respectively (although this relationship can flip, depending on the optical distances, it will hold constant for this thesis); see Fig. 2.6.

### **2.3.2 Image Capturing**

A CCD camera, model 1312 from Digital Video Company, collects light after it passes through the shadowgraph system. The camera captures the shadowgraphs with 12-bit intensity resolution and spatial resolution of  $1030 \times 1300$  pixels. We use XCAP V2.2 image capturing software from EPIX to interact with the camera. Typical use is to set a fixed frame rate (the maximum at full resolution is about 12 fps), and manually initiate frame grabbing. More details about interacting with, and the use of XCAP are given in the following chapters and in Appendix D.

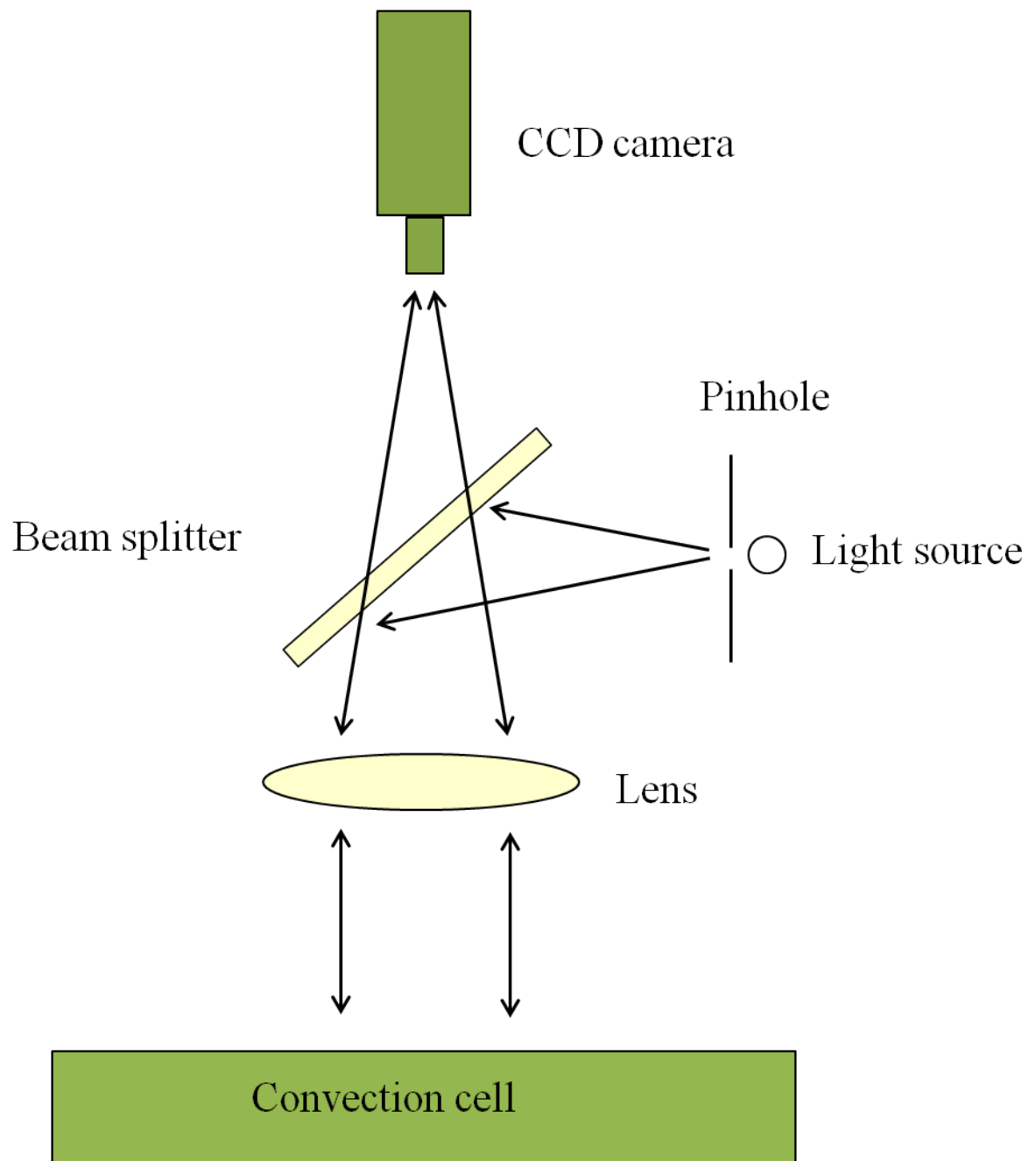


Figure 2.5: Shadowgraph schematic. Rays of light are collimated and directed into the convection cell. The light, after passing through the cell and reflecting off the bottom surface, is captured by a CCD camera.

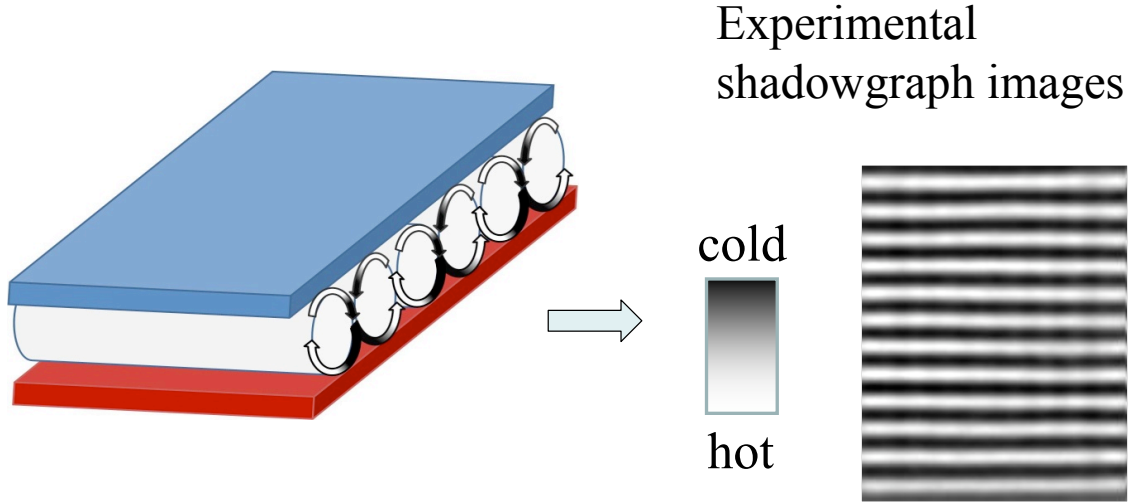


Figure 2.6: A shadowgraph intensity image consists of bright and dark regions that are related to hot upflowing and cold downflowing fluid, respectively. This example shows a straight roll pattern.

## 2.4 *Infrared Light Actuation*

Given in Fig. 2.7 is a schematic showing the different experimental components, including the convection apparatus and the shadowgraph system for flow visualization. In this section we describe the third major experimental component, that of infrared light actuation of the convective flow.

Control of RBC has been a goal in multiple studies, motivated by industrial applications (such as in crystal growth, where convection may be unwanted) or by interest in the evolution of particular convection patterns. Several investigations [37, 38, 39] have focused on suppressing the primary convective instability or increasing heat transport [40] through modulated heating of the bottom cell boundary. The work of Busse, Chen, and Whitehead [1, 41, 42, 43] instead studied the dynamics of imposed straight roll patterns by exposing the convection cell to perturbative light from above. The light was directed through a periodic mask held above the convection cell for several minutes as a steady temperature difference was established across the cell;

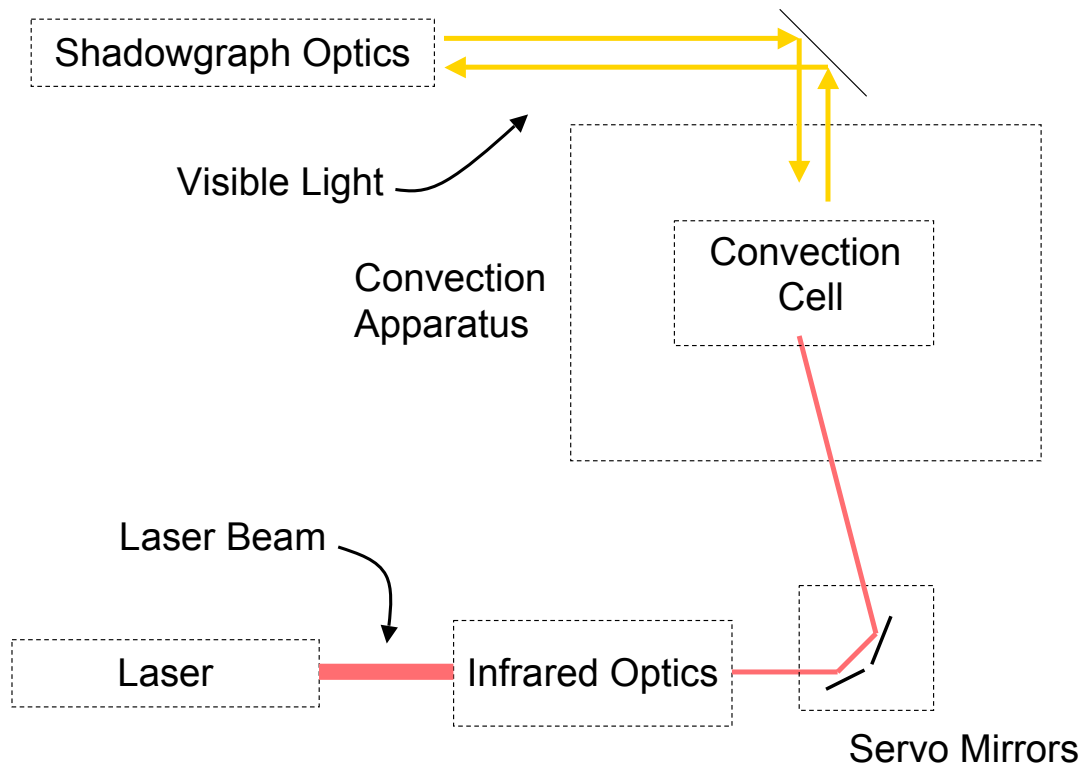


Figure 2.7: A schematic showing how the separate experimental components fit into the overall setup: the pressurized convection cell is enclosed within the convection apparatus; visible light, directed from above, is used with the shadowgraph optics to visualize the flow; actuation using infrared laser light is achieved with a CO<sub>2</sub> laser, optics to focus the laser beam, and computer-controlled mirrors to direct the laser light into the convection cell from below.

this acted to preferentially select certain periodic modes to grow in favor of other unstable modes. The evolution of the imposed patterns was then studied. An updated version of this protocol was used to investigate the evolution of hexagonal patterns in Bénard-Marangoni convection [44].

Our experimental approach follows and improves on this concept of manipulating fluid flow through selective optical actuation.  $\text{SF}_6$  is a greenhouse gas (it absorbs infrared light very strongly); in particular, it displays a large absorption peak at  $10.6 \mu\text{m}$  [45, 46], the same wavelength of light emitted by a  $\text{CO}_2$  laser. Laser light incident on  $\text{SF}_6$  is absorbed and heats the gas, thereby affecting the local temperature gradient, which in turn affects the convection pattern. The most extensive collection of data detailing the absorption of  $\text{CO}_2$  laser light by  $\text{SF}_6$  was compiled in the context of controlling boundary layer flow over airfoils [45, 46]. Laser light at  $10.6 \mu\text{m}$  was directed into a layer of  $\text{SF}_6$  gas; the variation in the absorption with pressure and temperature was fit as follows:

$$I(z) = I_0 e^{-\alpha z} \quad (2.2)$$

$$\alpha = p(5.64 \times 10^4 - 98.5T) \quad (2.3)$$

where  $I_0$  is the incident intensity,  $I(z)$  is the intensity after a distance  $z$ ,  $p$  is the pressure in atmospheres,  $T$  is the temperature in Kelvins, and  $\alpha$  is the absorption coefficient in  $\text{m}^{-1}$ . At room temperature (298 K) and a pressure of  $200 \text{ psi} = 13.6 \text{ atm}$ , this gives  $\alpha = 3.68 \times 10^5 \text{ m}^{-1}$ . One can then define an extinction length based on the depth at which the intensity falls to  $e^{-1}$  times the initial value,  $l = 1/\alpha$ , which gives  $l = 2.64 \times 10^{-6} \text{ m}$ . Alternatively, if defined by the distance at which the intensity falls by 90%,  $l = \ln(10)/\alpha = 6.26 \times 10^{-6} \text{ m}$ . While it is possible that saturation effects (due to feedback caused by the interaction of incident light with excited  $\text{SF}_6$  molecules) reduce the overall absorption, this is primarily a concern at low  $\text{SF}_6$  concentrations [45]. We therefore make a conservative estimate of the absorption

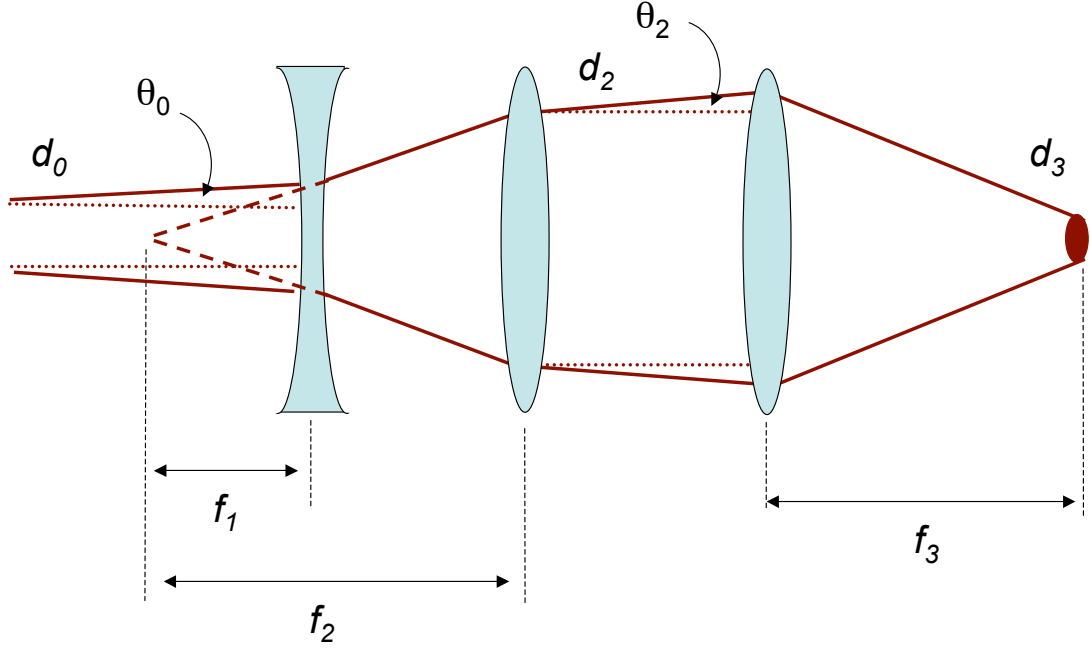


Figure 2.8: Optical system for focusing infrared laser light into convection cell. Three lenses are used to expand, collimate, and focus the light before it enters the cell.

depth of  $10\text{ }\mu\text{m}$ , which is less than 2% of the cell depths used in this thesis work.

#### 2.4.1 Optics

Laser light is directed into the convection cell from below. Because we want to manipulate the flow on a local scale, it is optimal to have the diameter of the laser light beam as small as possible when the light enters the convection cell. To that end, we designed a simple optical system that reduces the beam diameter. This optical system is shown in Fig. 2.8. The beam as emitted from the laser is too large to use directly; the beam size is reduced by three lenses which expand, collimate, and re-focus the beam.

We estimate the beam diameter  $d_3$  at the final location (the convection cell) as follows: let  $d_0$  denote the diameter of the infrared beam as emitted from the laser, and let  $\theta_0$  be the initial divergence of the beam. The beam is sent through a diverging lens of focal length  $f_1$  and then collimated by a converging lens. After exiting this

second lens, the diameter is  $d_2$ , with divergence  $\theta_2$ . These values are related to the initial diameter and divergence by

$$d_2 = \left(\frac{f_2}{f_1}\right)d_0 \quad (2.4)$$

$$\theta_2 = \left(\frac{f_1}{f_2}\right)\theta_0 \quad (2.5)$$

A third and final lens converges the beam, which reaches a minimum diameter  $d_3$

$$d_3 = 2\theta_2 f_3 = 2\left(\frac{f_1}{f_2}\right)f_3\theta_0 \quad (2.6)$$

We estimate the focal lengths of the lenses to be  $f_1 = 5$  cm,  $f_2 = f_3 = 12$  cm. Using the incident beam information provided by Synrad,  $d_0 = 3.5$  mm and  $\theta_0 = 2$  mrad, we find  $d_3 \approx 200\mu\text{m}$ .

Note that the analysis above is idealized in that it neglects the path of the ray after passing through the final lens. In practice, the beam enters the convection apparatus from below and thus must pass through a ZnSe window, carbon disulfide liquid, and another ZnSe window, in that order, before reaching the convection cell. In fact, this part of the path can have a converging effect on the beam, due to the high index of refraction of those materials (2.4 for ZnSe, 1.6 for CS<sub>2</sub>). We therefore verified the validity of this approximate beam diameter by measuring the beam size after passing it through the full path, but with paper in place of the convection cell; the size of the burn marks left by the light on the paper were measured using digital calipers.

A final note on the beam optics is that between the final converging lens and the convection apparatus, the beam must reflect off the rotating mirrors (that is, the mirrors are between final lens and the focal point). Figure 2.7 shows how the infrared optics and mirrors fit into the overall experimental setup. Practically speaking, this means that a longer focal length  $f_3$  is better, because it sets the distance between the final lens and the convection cell and therefore allows more space in which to fit the servo mirrors. If the mirrors are placed too close to the final lens, the mirrors may



not be able to capture the full beam ( $d_2 \sim 1$  cm). On the other hand, the final beam size is inversely proportional to  $f_3$ , so one must strike a compromise. Currently, the servo mirrors are necessarily too close to the convection cell to be able to deflect light to both above and below the convection cell, so the experimental setup is limited to unidirectional actuation. To add the capability to lase from above, a more complex optics setup, larger servo mirrors, or lenses of different focal lengths, are needed.

A natural measure of whether the beam is big or small is how it compares to the pattern wavelength. At onset, the critical wavenumber is  $q_c = 3.117 = 2\pi/(\lambda/d)$ , from which we get  $\lambda \approx 2d$ . The cell depths used in this thesis work are between 600 and 700  $\mu\text{m}$ , so the spatial extent of a “point-perturbation” is  $\approx 18\%$  of the pattern wavelength at onset. Away from onset, or during experiments when the wavelength is controlled, this fraction will be somewhat higher or lower.

## ***2.5 Servo Mirror and Laser Control***

**Lasershow Designer 2000** Immediately before entering the convection apparatus, the laser light is reflected from two gold-plated servo mirrors that are allowed to rotate about two orthogonal axes; these mirrors provide the ability to direct the light toward any point over the cell domain. Software developed in-house works with a commercial program (Lasershow Designer 2000, or LS for short) to synchronize mirror rotations and laser output in order to manipulate the convective flow on a time scale much faster than the typical dynamical time scale: the vertical thermal diffusion time is on the order of 2 seconds; the mirrors can scan through up to 30,000 pts/second.

In addition to a graphical interface, LS permits mirror control via a library of functions accessible from either Visual Basic or C++. We constructed source code built from these fundamental LS functions and compiled it into a Matlab executable, giving a way to seamlessly pass commands to the LS software (and thus, to the servo

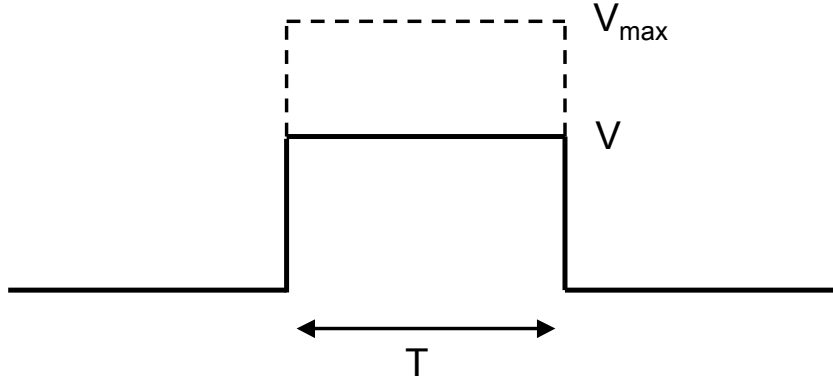


Figure 2.9: Voltage signal from QM2000 card.

mirrors and laser). See Appendix D for more details.

The LS program accepts as input a set of coordinates outlining the pattern to be lased as well as intensity values for all points. The program converts these inputs to a voltage signal which it emits from a PCI card called the QM2000; the signal is sent to an amplifier (built by Lighting Systems Design, Inc.), which in turn sends controlling signals to the two mirrors. Across two pins from the PCI card is a voltage signal related to the frequency of mirror rotations as well as the input light intensity. We copy this voltage signal and use it to control the laser output.

Figure 2.9 shows a representative voltage pulse between the two relevant pins of the QM2000. The height of the pulse  $V$  is directly proportional to the input intensity, on an 8-bit scale, up to  $V_{max} = 5$  Volts. The temporal width of the pulse is set by the number of points; if the mirrors rotate  $N$  times per second, then the time spent at each point is  $1/N \text{ s}^{-1}$ , and the width of a pulse corresponding to  $n$  continuous points is of length  $n/N \text{ s}^{-1}$ . The voltage signal repeats on an cycle set internally by the LS/servo system.

Figure 2.10 shows a schematic illustrating how the laser power is controlled by the QM2000 voltage. The voltage signal from the QM2000 is split and used both as a gate and as a controlling voltage to the UC1000 laser controller. A gate voltage

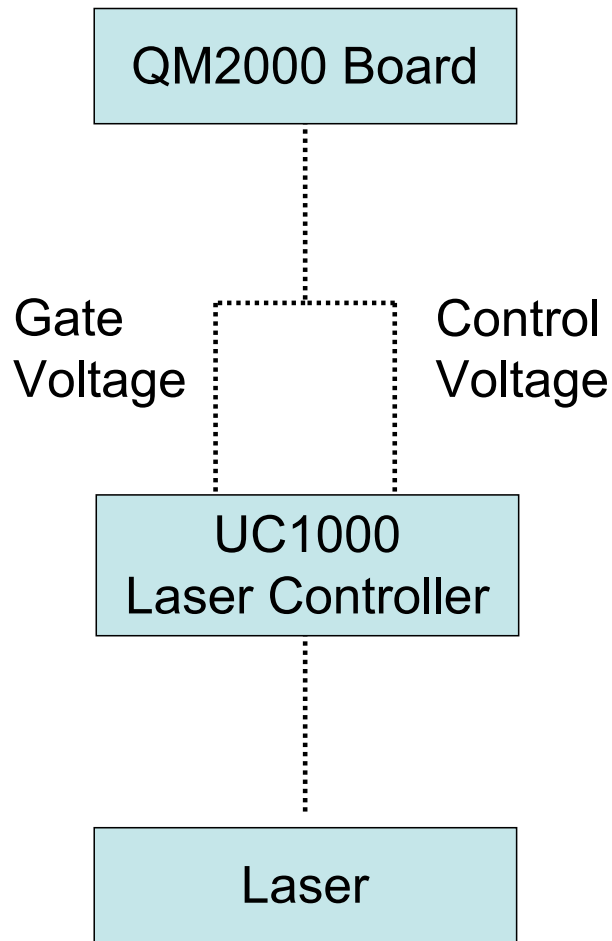


Figure 2.10: Schematic showing how the voltage from the QM2000 board is used, via the UC1000 controller, to control the laser output.

above 3.5 V allows lasing, and the lasing power is proportional to the input control voltage, on a 0-10 V scale.

**Laser** The CO<sub>2</sub> laser is a model J48-1W from Synrad Corporation which has a maximum power output of a few Watts. An interfacing control unit called a UC-1000 Laser Controller, made also by Synrad, was used to mediate the input voltage signal (from the QM2000) and the outgoing signal to the laser. The UC-1000 sends a Pulse-Width Modulated (PWM) voltage signal to the laser, based on a fixed 5-kHz cycle;

100% duty cycle is achieved by sending a constant logic-high signal, 0% duty cycle by sending a constant logic-low signal, and a 50% duty cycle by sending a periodic 5 kHz high/low signal<sup>4</sup>. The duty-cycle set by the UC-1000 is controlled via the input voltage signal from the QM2000 board, as shown in Fig. 2.10.

### Servo Mirrors

In order to properly reflect infrared light, the original (silver-backed) mirrors attached to the servo controllers were replaced. As part of the optical considerations, it was best to find large mirrors to capture the full expanded laser beam, subject to the constraint that the servo controllers have an expected inertial load; if the mirrors are too heavy, the servo system will constantly overshoot the desired rotation. The best solution is thin, gold-coated germanium wafer (any thin substrate would do). The mirrors are squares approximately  $1\text{ cm} \times 1\text{ cm}$ ,  $250\text{ }\mu\text{m}$  thick.

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<sup>4</sup>There is also a small tickle voltage the Controller maintains to keep the laser in a ready-to-lase state.

## CHAPTER III

### PREPARING STRAIGHT ROLL PATTERNS

As a step toward understanding the mechanisms of instability in spatiotemporally complex patterns, we first consider the instabilities of stationary straight roll patterns, which can lead to a loss of time-independence and a possible transition to chaotic behavior. For the range of control parameter  $\epsilon$  used in this thesis work, natural convection patterns are usually time-dependent and disordered. In order to study the evolution of straight roll patterns near a secondary instability, an ordered pattern must replace the initial pattern, and the state brought near a particular instability in a controlled fashion. This chapter presents the methods by which straight roll patterns are prepared for study. To facilitate this process, we prepare the straight rolls in a rectangular convection cell. (Convection rolls have a natural tendency to align at right angles to one of the sidewalls.) The experiments have been conducted with a convection cell of lateral dimensions of 25 mm  $\times$  15 mm and a depth of 700  $\mu$ m.

Pattern preparation consists of three parts. First is the mapping of the laser beam to the convection cell; i.e., how rotations of the servo mirrors (see Chapter 2) “map” laser light to a particular place in the convection cell. This mapping is crucial, as the convective gas ( $\text{SF}_6$ ) absorbs the laser light locally; thus, the ability to exhibit fine-tuned pattern manipulation is dependent on being able to direct the laser beam to a particular location within the cell, with a high degree of accuracy. Given an accurate mapping, an initial, idealized pattern is imposed over the existing, natural convection pattern. This step is something of a brute-force method, in that the actuation is nearly global over the convection cell and is implemented without regard

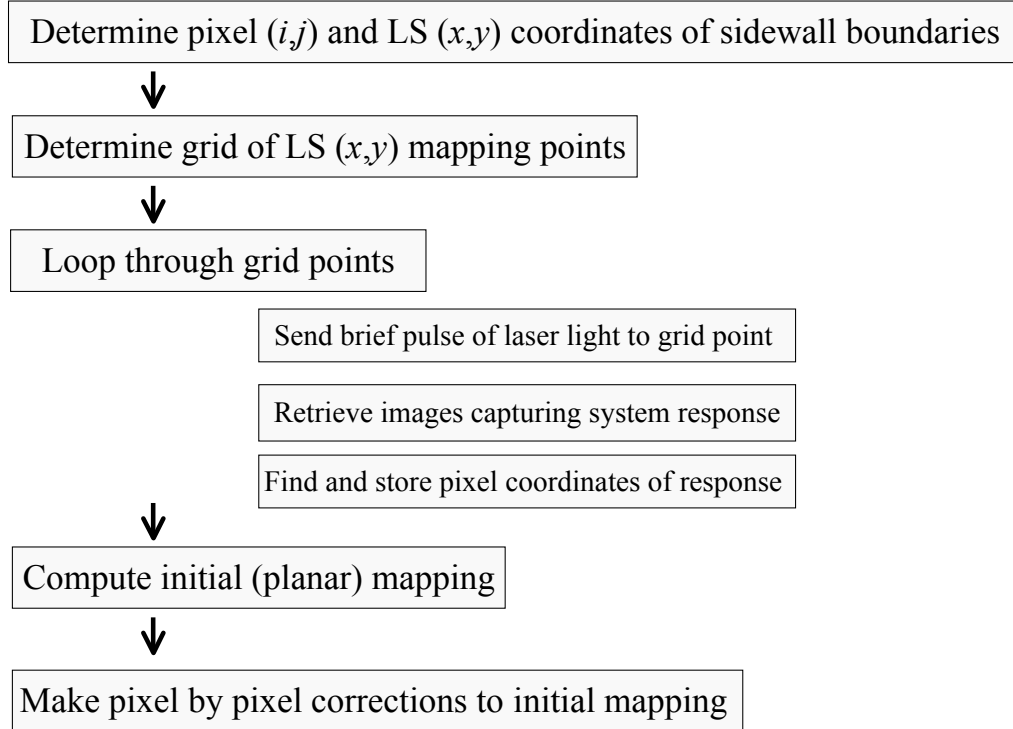


Figure 3.1: A schematic of the algorithm used to compute the mapping of the mirror coordinates to the convection cell.

to the pattern existing prior to imposition. Once the initial pattern has been imposed, a more sophisticated feedback loop is triggered; this loop guides the state to the final configuration using a minimal amount of actuation.

### 3.1 *Mapping of Laser Beam to Convection Cell*

There exists a two-dimensional space in the LS software (see Section 2.5) in which a pair of coordinates  $(x,y)$  refers to a particular pair of servo mirror rotations. A second pair of coordinates is used to describe a pixel location within a shadowgraph image. It is convenient to consider the digital shadowgraphs as matrices having coordinates  $(i,j)$  referring to the  $i$ th row and  $j$ th column and whose elements are the intensity values. The role of the mapping is to build an accurate relationship between the  $(x,y)$  and  $(i,j)$  coordinate spaces. Figure 3.1 shows an outline of the mapping routine.

The first step of the mapping is to define manually the borders of the convection

cell. (This is simpler than trying to automate this step.) For rectangular sidewalls, it simplifies things later to align the convection cell so that each sidewall segment lines up with either a constant row or constant column index of the shadowgraph images. It is then convenient to define the cell boundaries by the four corners of the cell (for circular cells, the alignment is not important, and the boundaries can be defined by the row/column limits).

Four pairs of  $(x,y)$  coordinates are found that correspond roughly to the corners of the convection cell. A brief laser pulse is directed into the convection cell and absorbed by the  $\text{SF}_6$ , causing a detectable localized response (see Fig. 3.2 and the discussion immediately below). The coordinates are varied in a “guess and check” manner until a response is observed near the cell boundary; these coordinates define the boundaries of the cell in  $(x,y)$  space (only roughly, because there will ultimately be a map over the entire convection cell).

Once the  $(x,y)$  borders have been defined, a grid of points over the domain of the cell is compiled to which brief laser pulses are directed, serially. As each coordinate pair is sent to LS to initiate the mirror rotation and lasing, the image-grabbing software is given a signal to begin taking images, capturing the perturbation response. The mapping algorithm then retrieves the small number of frames that hold the response. It should be noted that the mapping is usually done slightly below the onset of convection in order to isolate this response from any other convection pattern dynamics. Thus, the only frame to frame change should be due to the input disturbance. Difference images are formed for all successive pairs of images; the one with the largest rms intensity is assumed to contain the best signal of the perturbation response. An example is shown in Fig. 3.2, where a bright spot in the middle of the image on the left stands out above the raw shadowgraph image. The effect is much more visible when a background image (no perturbation) is subtracted; this image is shown on the right using a color map to further enhance the visibility of the structure. The

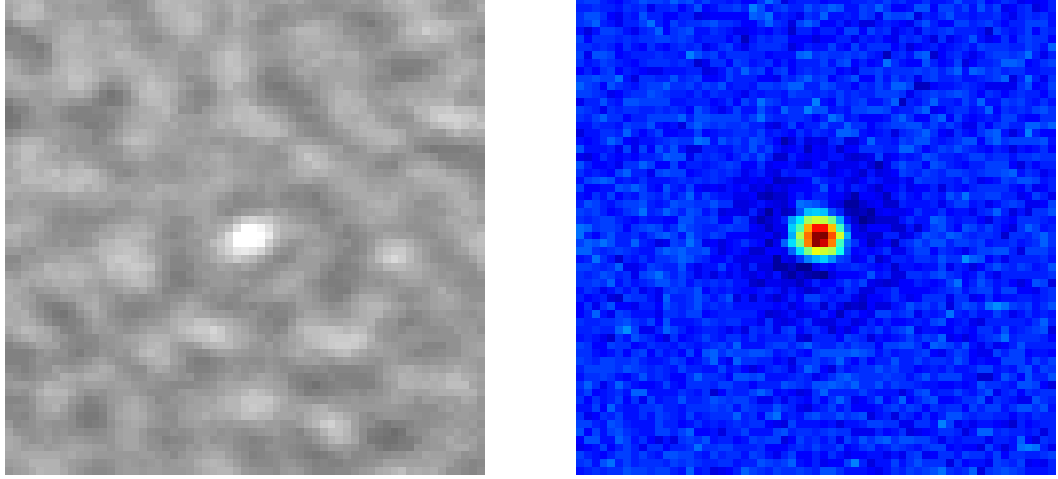


Figure 3.2: Shadowgraph snapshots during a mapping perturbation. The image on the left shows the raw image containing the intensity change; the image on the right shows the difference between the image on the left and a background image. The images show an area of  $\approx 5d \times 5d$ .

images have been cropped to show only a small portion of the convection cell, shortly after the perturbation has been applied; we estimate the initial disturbance to be of diameter  $\sim d/2$ .

The pixel location with the largest peak intensity is found from the difference images and the “center of mass” of the response is calculated, where the absolute intensity (deviation of intensity from the mean) plays the role of the mass. These “center of mass” pixel coordinates are stored, along with the original  $(x,y)$  coordinates. This process is repeated over the  $(x,y)$  grid, thus producing a discrete map of  $(x,y)$  coordinates to  $(i,j)$  shadowgraph locations over the convection cell.

A linear fit to the mapping data acts as a good first approximation of the relationship between the pixel and LS coordinates; one can imagine this step as fitting the data with  $(i,j,x)$  and  $(i,j,y)$  planes. The mapping thus takes the form of two matrices of size equal to the shadowgraph dimensions: one contains the  $x$  coordinate for any given  $(i,j)$  pair, the other contains the  $y$  coordinate for the same  $(i,j)$  pair. Now, it is possible that the procedure that attempted to find the perturbation responses



resulted in some errors due to, for example, some small particle or air bubble in the water flowing over the top boundary having passed in the view of the camera during automated image capture. To eliminate those spurious data, the program cycles through the data point by point, comparing the goodness of fit with and without the point, and throws out those points that seem anomalous.

After this preliminary mapping, a finer mesh of  $(x,y)$  perturbation points over the convection cell are used to make corrections to the linear approximation (the initial set of points can be used if the initial grid resolution is high enough). The pixel locations are found and stored in the same way as before for each point. Spurious points are again excluded, but now by throwing away those that disagree strongly with the initial mapping. For a given  $(i,j)$  pixel location, there is a corresponding  $(x,y)$  pair of values from the initial fit. Nearby measurements from the finer mesh may indicate that adjustments should be made to these values. Corrections are made by summing over the adjustments indicated from each individual perturbation point, with inverse distance weighting. All pixels are looped through and corrected, after which the mapping is smoothed. This loop then repeats until both the  $x$  and the  $y$  mappings converge.

Note that the final result is a mapping which allows the laser light to be directed to a location in the convection cell with sub-pixel resolution; in principle, the resolution could be as high as desired, by increasing the number of mapping points. In practice, it is suitable to use a few hundred points, which provides a resolution of  $\sim 0.2$  pixels (about 1% of a pattern wavelength).

### ***3.2 Straight Roll Imposition***

After the mapping is complete, a straight roll pattern is imposed, the wavenumber of which can then be controlled in order to guide the state to a given secondary instability; Fig. 3.3 shows a schematic summarizing the pattern imposition and control.

An idealized version of a straight roll convection pattern is first created digitally, which amounts to specifying the pixel locations of the roll pairs by choosing some pattern wavenumber and phase. This serves as the initial reference against which captured shadowgraph images are compared. To impose this ideal pattern, the roll positions are converted to output  $(x,y)$  coordinates via the mapping described above and are drawn for a short time. Because the actuation is in the form of heating from below, the pattern is drawn by sending a continuous line of laser light to each desired location of hot upflowing fluid. The light is maintained at the upflow location of each roll for about 1 s, after which the light is directed immediately to the next upflow location in the desired pattern. After a full iteration through the rolls, the lasing is turned off and a shadowgraph image is captured for comparison. If there are discrepancies, the process is repeated, with the lasing time increasing progressively after each iteration. If the process is successful, the algorithm calls for the next step in the experiment, which is usually to maintain the imposed pattern and apply perturbations, or to make adjustments, such as changing the pattern wavenumber. Figure 3.4 shows shadowgraph snapshots during pattern imposition. The original, natural pattern consists of evolving curved rolls and defects. Visible in the first few frames is the increase in intensity where the laser light is being directed. Through roll breakings and connections, the straight roll pattern emerges in about 30 - 60 s.

The physical walls can act as sources of defects and/or cause curving of nearby rolls (due to the tendency of rolls to end at right angles to the walls). This is a particular problem when controlling the pattern wavenumber because there is not generally an integer number of rolls fitting neatly into the convection cell and therefore it is difficult to enforce a constant wavenumber throughout the convection cell. To minimize these sidewall effects, the pattern is established over the central region of the cell, 2 wavelengths from either of the sidewalls in the wavevector region. The direction of the wavevector along the longer side of the cell has been chosen to allow

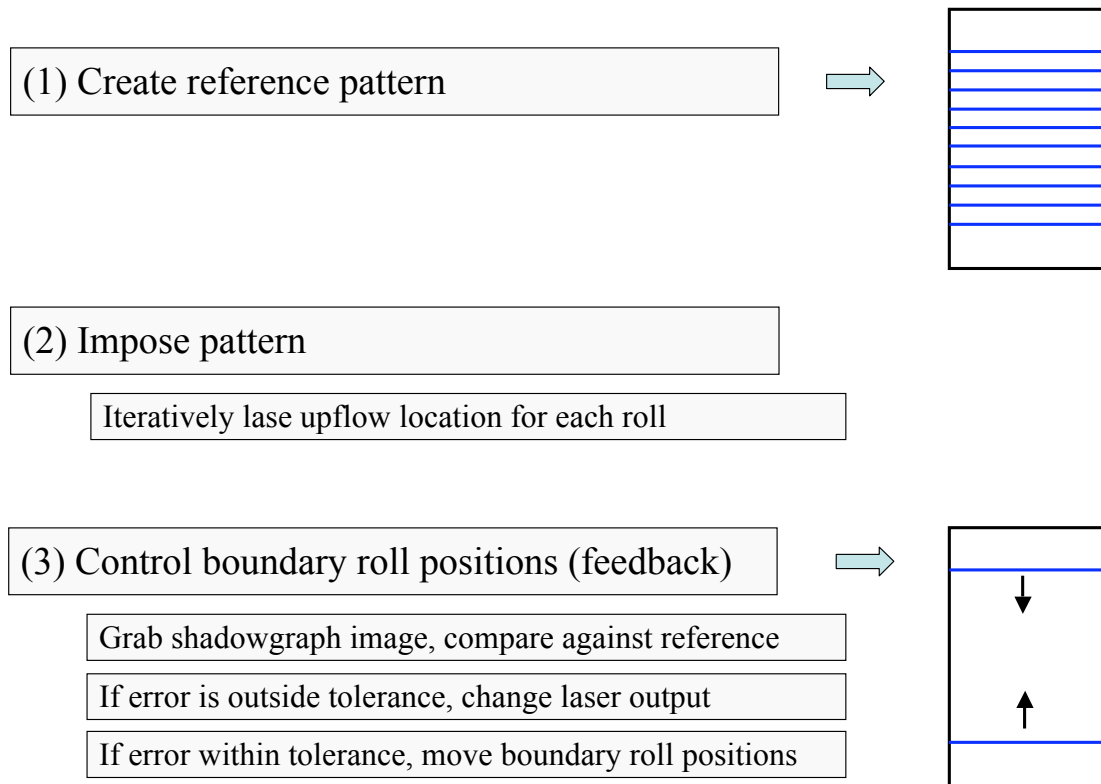


Figure 3.3: The pattern control involves three main components, as indicated by this schematic of the imposition/feedback algorithm.

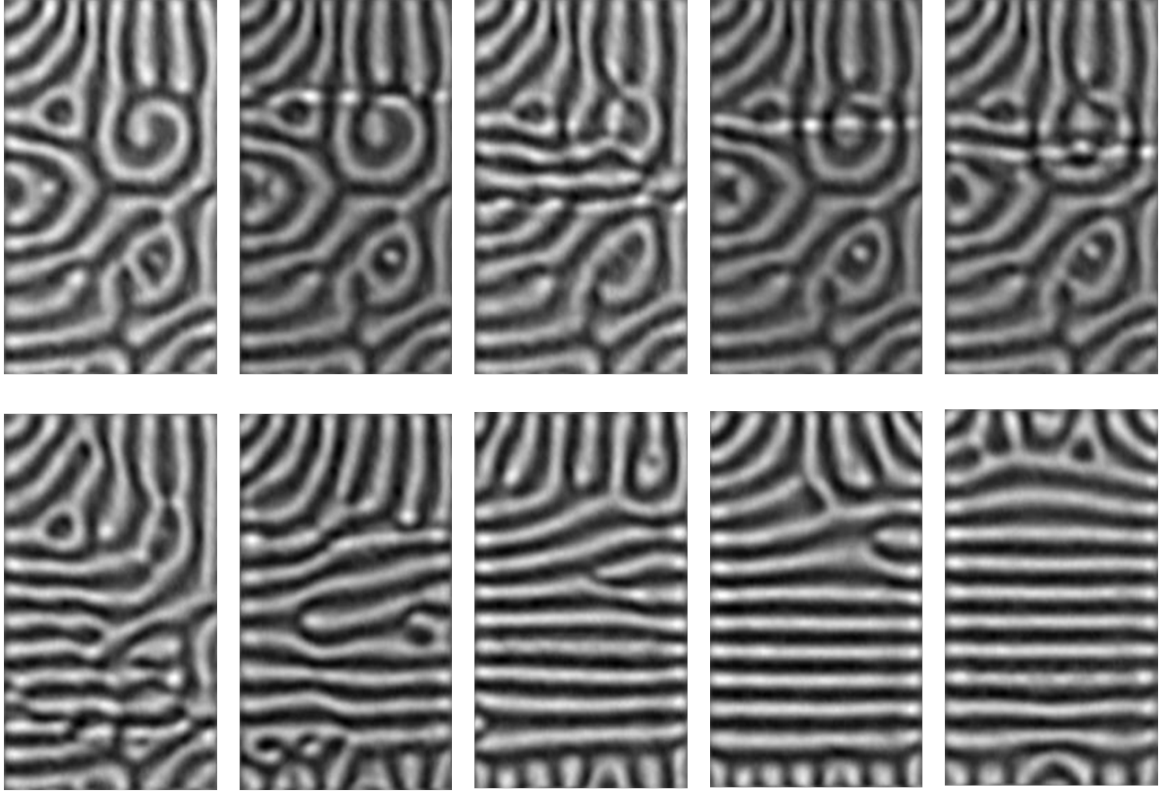


Figure 3.4: Snapshots during straight roll imposition, at approximately 3 s intervals. The images show the full 25 mm  $\times$  15 mm convection cell.

more interior roll pairs to fit in the cell, typically 7-9.

### 3.3 *Closed-loop Feedback Control*

For the study of the straight roll pattern dynamics, it is desirable that the effect of the optical actuation on the interior pattern dynamics is minimized. Thus, once the initial pattern has been imposed successfully, actuation is restricted to the two outer-most imposed convection rolls, which are thus deemed “boundary” rolls. Wavenumber adjustments are made by moving the positions of these two rolls: bringing them toward one another increases the wavenumber; moving them apart decreases the wavenumber. The interior roll spacing adjusts to the boundary roll positions. A feedback loop controls the relative positions of the boundary rolls and thus, in turn, the interior roll spacing. This method of changing the interior wavenumber without

direct forcing of all the rolls is advantageous because it reduces the risk of causing an instability in the bulk of the pattern.

In parallel with the feedback loop, the image-grabbing software takes shadowgraph images and deposits them in a specified directory shared by the feedback program (running in Matlab). When a new image is detected, a subroutine is called that performs an analysis to determine discrepancies between the reference and actual convection pattern. A second subroutine uses these errors to determine the outputs to be delivered to the laser controller and mirrors. The laser output affects the convection pattern and this process repeats until the desired pattern is achieved. One complete iteration of the feedback loop, including image analysis and calls to the laser and mirrors, is achieved in a few tenths of a second, which is much faster than the un-perturbed pattern evolution (as discussed below).

### **3.3.1 Shadowgraph Image Analysis**

The information captured by the images is reduced to a set of pixel coordinates at the center of each region of upflowing/downflowing fluid, and the image intensity along each roll. These two signals are used for comparison to the ideal reference image.

As mentioned above, the rectangular sidewalls are positioned so that each of the four sidewall segments aligns with a fixed row or column of the shadowgraph images. The major benefit of this is that each of the ideal straight rolls can be referenced with only one index (in most cases, a row index). Real rolls do not (generally) lie exactly along a fixed row, possibly due to slight non-parallelism of the sidewalls, but they do span the pixel columns of the convection cell. Figure 3.5 shows a straight roll pattern and the (smoothed) intensity profile across the center pixel column. The local maxima/minima of the intensity profile correspond to regions of hot and cold flow, respectively. The row index of each roll, for this column, is determined by finding locations of local maxima/minima of the intensity slices. The roll positions are

determined with sub-pixel resolution in one of two (similar) ways. When the intensity profile is sufficiently smooth, a numerical derivative of intensity (with respect to pixel position) can be computed and thus the maxima/minima found from places where the derivative crosses zero. Or, motivated by the procedures in Ref. [47], quadratic fits can be made locally to the intensity maxima/minima (once their positions have been determined roughly). An example is shown in the zoomed-in intensity profile in Fig. 3.6. The two methods give nearly identical results; we usually use the numerical derivative, for computational ease. Thus, for a given column, the row positions of all rolls can be calculated. Put together with intensity profiles over all columns, a determination is made of each roll position across the domain of the cell.

This procedure of calculating roll positions is done in each pass of the feedback loop and can be recorded for further use. When the program is set to change the pattern wavenumber, the primary interest is the roll spacing of the boundary rolls, but spacing of the interior rolls is also measured and will become important in Chapter 4.

### 3.3.2 Errors

The most significant source of error between the reference and observed convection pattern is improper roll spacing, which is governed by the positions of the boundary rolls. While there is a nominal desired pixel position for each boundary roll, it is more appropriate to enforce not the absolute position but rather the pattern wavelength (equivalently, the wavenumber). It must be noted that the sign of the error in roll spacing is important physically. When the pattern is brought near an instability boundary via a change in wavenumber, the natural tendency of the rolls is to relax to a wavenumber closer to the center of the Busse balloon. When the wavenumber is high, for example, and there is an error indicating the rolls are spaced closer together than desired, it is usually appropriate to decrease, rather than increase, the laser power. This holds for the converse situation at low wavenumbers. This allows the

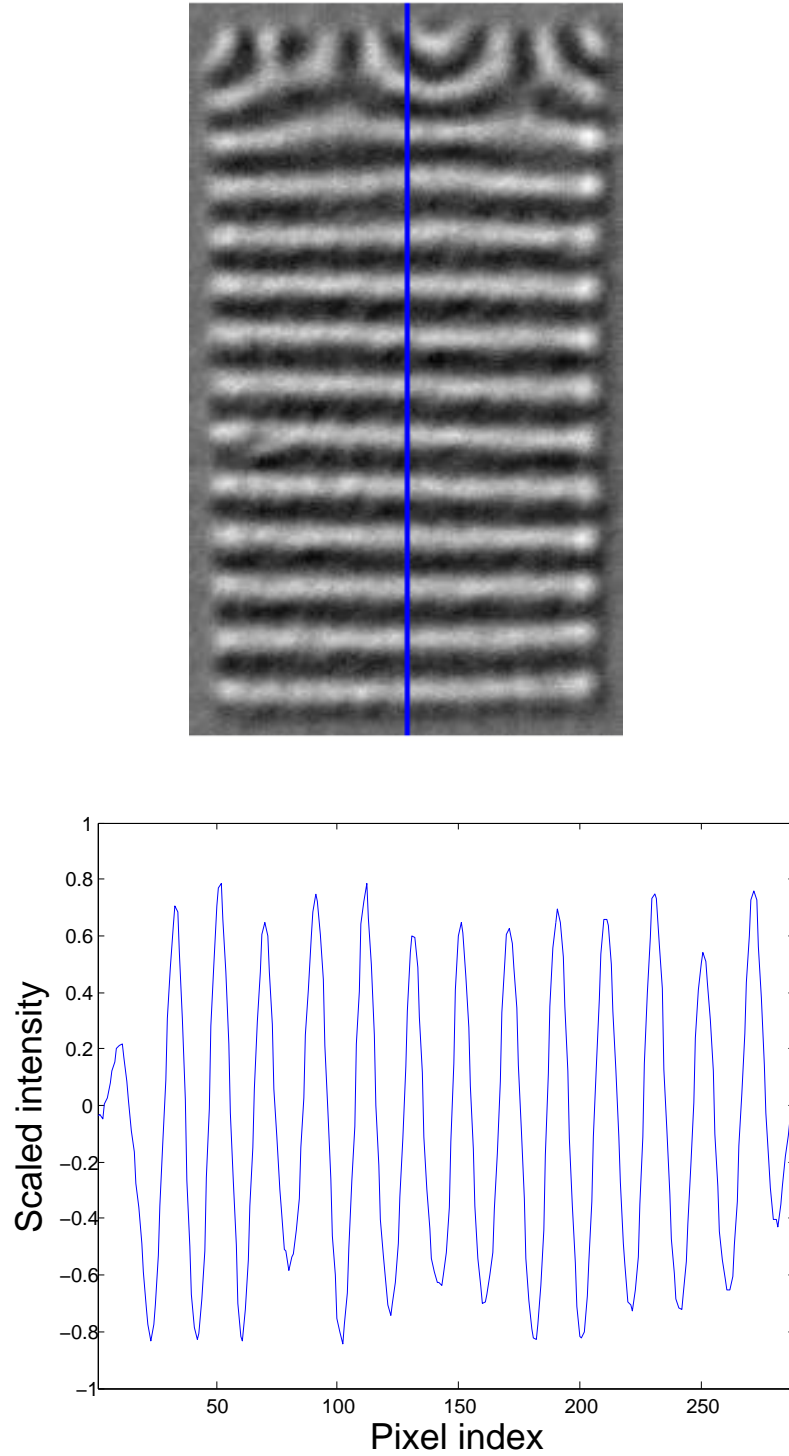


Figure 3.5: Shown above is the entire convection cell (and part of the cell boundaries, where the rolls terminate), showing the straight roll pattern under control (the curved rolls at the top are outside the region of control). Underneath the roll pattern is a profile of intensity values across the image, in the direction of the wavevector, as indicated by the line overlaid on the pattern.

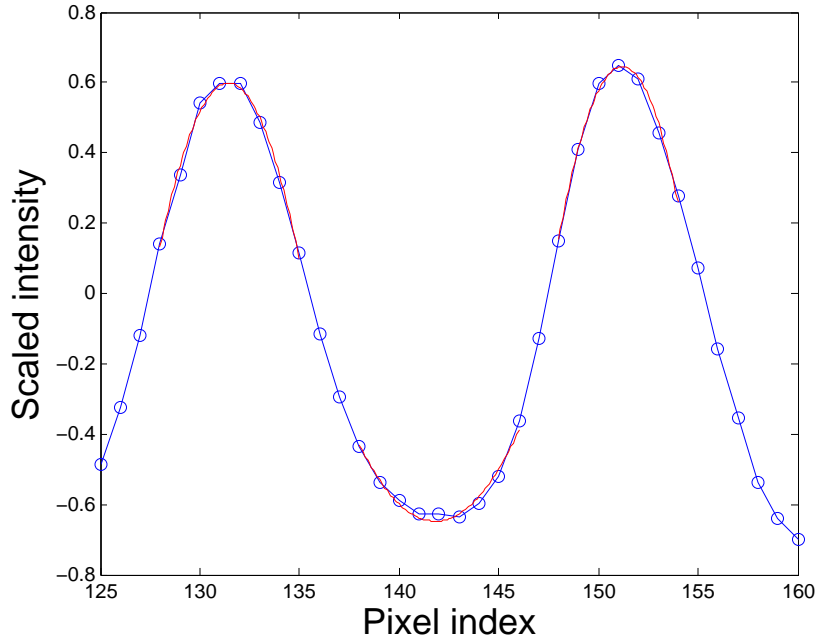


Figure 3.6: Local quadratic fits to the intensity maxima/minima. The intensity values are given by the circles in blue; the fits are shown in red.

natural system dynamics to adjust the spacing when possible, which in turn minimizes the amount of actuation required.

The shadowgraph intensity along boundary rolls is also monitored. At a high laser power, such as when imposing an initial pattern, the effect of the optical actuation is very apparent in the shadowgraph images through an increase in local intensity (see the first few frames in Fig. 3.4). Near instability, or when probing system dynamics, this is undesirable because it indicates a possibly large disturbance to the pattern. Therefore, an error is reported when the intensity of a boundary roll exceeds a reference value (a good reference is the average intensity of a nearby, unperturbed roll).

For each boundary roll, the position and intensity errors are normalized in order to be combined into a single error; i.e., the error in roll position is normalized by the pattern wavelength, and the intensity error is normalized by the intensity of a nearby, unperturbed roll. The two normalized errors are added with weighting coefficients



that reflect the relative error tolerances. Because the leading priority is to achieve the desired roll positions, we set a lower tolerance for errors in roll positions. The actual weighting values, found manually through trial and error, are those that consistently give a positive error when the control begins to break down<sup>1</sup>.

The laser power is changed in response to the combined error; the power change has the same sign as the error, but the magnitude depends on the current power output: when the power is low (high), the control algorithm is most sensitive to large positive (negative) errors. This design allows the algorithm to quickly adjust the laser power to the appropriate level. The utility of this feature can't be overstated, because if the feedback program is delinquent in fixing an error in roll position, the roll may begin to interact with grain boundaries emanating from the sidewalls (see the grain boundaries in the images of Figs. 3.4 and 3.5); on the other hand, over-forcing due to a large laser intensity can initiate instabilities. Either of these cases likely leads to dynamics of the interior rolls and a loss of the straight roll pattern.

In addition to changes in the laser intensity, adjustments may be made to the lased roll positions. Initially, laser light is directed to the nominal positions of the hot upflow of the boundary rolls. When there is an error between the observed and reference upflow position, the nominal position is moved slightly, opposite the direction of error. These small changes are made in each iteration of the loop and have the effect of moving the actual boundary roll position more quickly than an increase in power alone. When the spacing between the boundary rolls and the adjacent interior rolls assumes the desired value, the nominal boundary roll positions are updated with the actual positions. This allows the overall straight roll pattern to drift by some small amount, which is advantageous in that it may require less actuation to maintain the

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<sup>1</sup>The approach here is that it is not necessarily the magnitude of the combined error that is most important; rather, it is the *sign* of the error. With a very rapid feedback loop, many small adjustments can have the same effect as a small number of larger adjustments. In this way, a high analysis speed can, to a certain degree, make up for a lack of sophistication in the correction scheme.

boundary rolls at certain locations than others (due to interaction with the grain boundaries, for example).

**Error Tolerance** The tolerance for errors in the roll spacing is set to a constant 0.2 pixels, as dictated partly by the resolution in the laser mapping and in part by what has been shown to be achievable. This is only a small fraction of the pattern wavelength, less than 2% even at the largest wavenumbers (smallest wavelengths). When the spacing falls within this tolerance, the next step in the pattern control is triggered: either a further change in the pattern wavenumber or the application of selected perturbations, as discussed in Chapters 4 and 5.

### 3.3.3 Control near Boundaries

The physical sidewalls of the convection cell can be sources of forcing, due to the mismatch in conductivity, and can therefore introduce defects at the boundary through the joining of existing rolls or introduction of new rolls. Generally, these defects will glide across the cell domain and be carried out by the opposite sidewall, with the final result that the pattern wavenumber is changed. Therefore, when the wavenumber set-point is sufficiently high or low (the state is very near instability), the positions of the convection rolls near the sidewalls are monitored and controlled in a similar fashion to the boundary roll positions.

### 3.3.4 Approaching Instability

The initial pattern wavenumber is chosen to be within the stable band (for a given  $\epsilon$ ), and the state is brought near instability by either an increase or decrease in wavenumber. Because the wavenumber is controlled via the positions of the two boundary rolls, time must be allowed for the interior pattern to equilibrate after a change in the boundary roll spacing. The vertical diffusion time  $t_v = d^2/\nu$  is the typical time scale over which local changes in the pattern occur, due to thermal or

viscous diffusion (for  $Pr \approx 1$ ,  $t_v$  can be estimated using either  $\nu$  or  $\kappa$ ; we use the smaller of the two). Changes to the global pattern can be expected to occur on a timescale of  $t_h = \Gamma^2 t_v$ , where  $\Gamma$  is the cell aspect ratio and  $t_h$  is defined as the horizontal diffusion time. For a straight roll pattern of  $N$  rolls under control, the distance from the center of the pattern to one of the boundary rolls is  $\frac{N}{2}\lambda$ , where  $\lambda$  is the pattern wavelength. Typically,  $\lambda \sim 2d$ , so we can define an aspect ratio of the controlled pattern by  $\Gamma \sim \frac{N}{2}(2d) = N$ . Thus, the timescale over which we can expect the pattern to equilibrate is about  $N^2 t_v$ . In our straight roll experiments,  $t_v$  is around 2.7 s, and we have 7-9 roll pairs, meaning the pattern equilibrates on the order of a few minutes.

Changes in the wavelength are made in increments of the error tolerance (see above), a very small percentage of the pattern spacing. Rather than wait 2-3 minutes before every small change is made, however, the boundary roll positions are allowed to be incremented as soon as their positions fall within the error tolerance, as long as the pattern is far from instability. This procedure may cause transient uneven spacing, but patterns ultimately are allowed to equilibrate over several minutes. As the pattern nears instability, the time between spacing adjustments is progressively lengthened.

## CHAPTER IV

### PERTURBATION LIFETIMES

The Busse balloon predicts an area of  $(q, \epsilon)$  space in which ideal straight roll patterns are predicted to be linearly stable; i.e., infinitesimal disturbances are expected to decay. The secondary instability boundaries are defined by the  $(q, \epsilon)$  values at which patterns are predicted to be marginally stable. In between, we expect the time scale over which disturbances decay to be related to the distance from instability. For a small modulation about a stationary roll state in the vicinity of a secondary instability, the rate at which the modulation decays is expected to be linear in the phase-space distance from instability [48, 5]. (Basically, one can linearize the disturbance about the bifurcation point.) In RBC, distance from bifurcation is usually considered in terms of a temperature difference, just as  $\epsilon = \Delta T / \Delta T_c - 1$  measures the distance from onset of the primary instability. By using optical actuation, however, we have both  $\epsilon$  and the wavenumber  $q$  as control parameters that can be used to guide a stable pattern toward a given instability boundary.

In this Chapter, we present the decay rates of disturbances to an ensemble of prepared straight roll patterns at fixed  $\epsilon$ , over a range of wavenumbers. This serves both as an efficacy test of our experimental method of pattern manipulation near instability as well as providing an experimental measurement of distance from one of the more prevalent instabilities, the skew-varicose instability to roll regions of high wavenumber. This measurement sets the stage for Chapter 5, wherein the modes responsible for disturbance dynamics are determined experimentally.

## 4.1 *Experimental Procedure*

Using the feedback control algorithm described in Chapter 3, a straight roll pattern is prepared with a given wavenumber. Once the pattern adopts the wavenumber set-point, within some tolerance (see Chapter 3 for details), a subroutine is triggered from within the feedback program to construct a detailed wavenumber field of the pattern. This field is computed via the positions of all convection rolls in the pattern, determined from continual shadowgraph image analysis. While the local wavenumber is related to the local curvature, we find it more convenient, and equally informative for our uses, to consider the local wavenumber defined by the spacing of adjacent convection rolls. Distances between local intensity maxima/minima define wavelengths (rather, half-wavelengths) at discrete positions, and interpolation builds a wavenumber field over the full pattern domain. Figure 4.2 shows a straight roll pattern; shown in Fig. 4.3 is the corresponding wavenumber field, where the approximate positions of the hot upflow regions of the pattern have been marked by dashed lines. Note that the wavenumber values given are non-dimensionalized in terms of the cell depth,  $q = 2\pi/(\lambda/d)$ .

Visible from the wavenumber field are small differences in the local roll spacing, even over the length of two adjacent rolls. However, most of the variation is toward the top and bottom of the image, where the boundary rolls are located; in the interior, the spacing is relatively even. A probability distribution of all measured wavenumber values is shown in Fig. 4.4; the standard deviation is computed to be 0.10, only about the 4% of the mean,  $\bar{q} = 2.64$ .

The localized skew-varicose (SV) instability usually occurs in regions of large local wavenumbers [49] (considering the bulk, that is, away from the boundaries). It is thus reasonable to expect perturbations applied near maxima of the wavenumber field will tend to excite the least-stable mode of the system corresponding to the localized skew-varicose instability. We observe that perturbations directly between

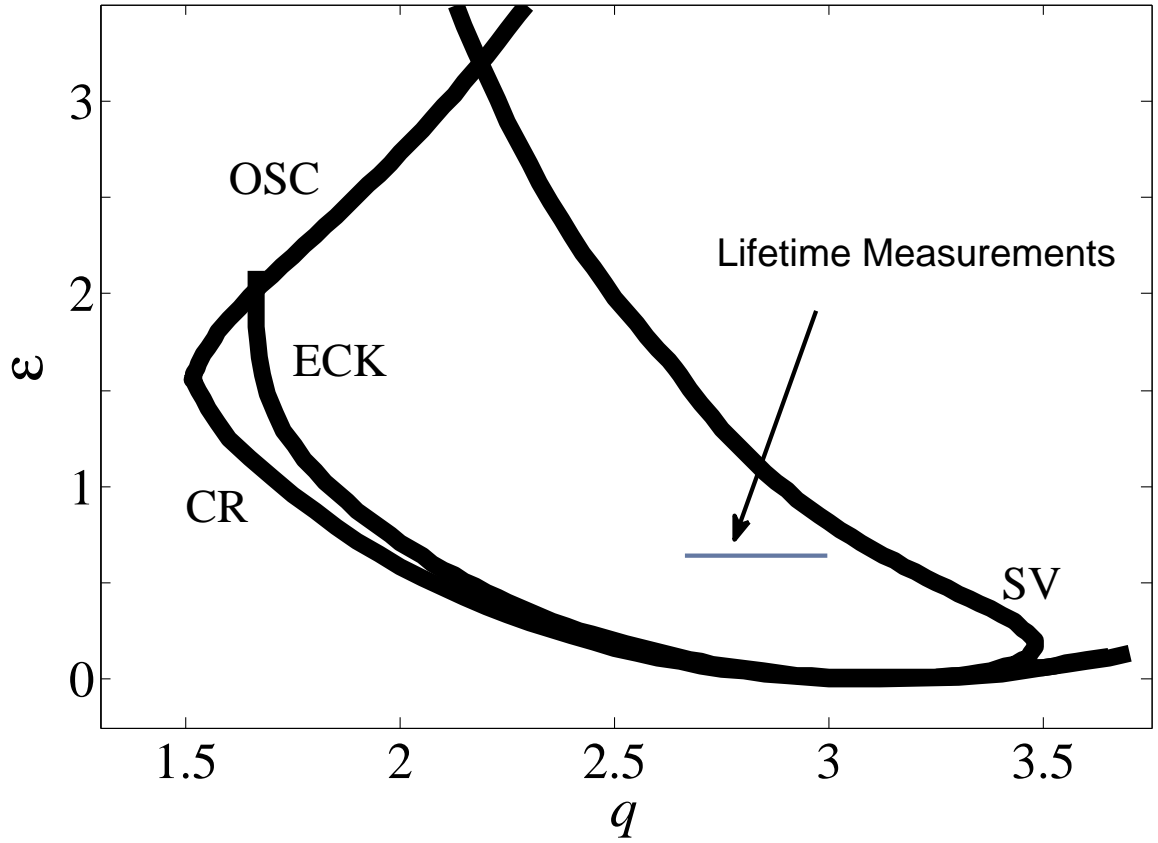


Figure 4.1: The Busse balloon showing various instabilities (CR, cross roll; ECK, Eckhaus; SV, skew-varicose; OSC, oscillatory) of the straight roll state, at a Prandtl number of 0.84, as in our experiments. The high-wavenumber region under study has been highlighted.

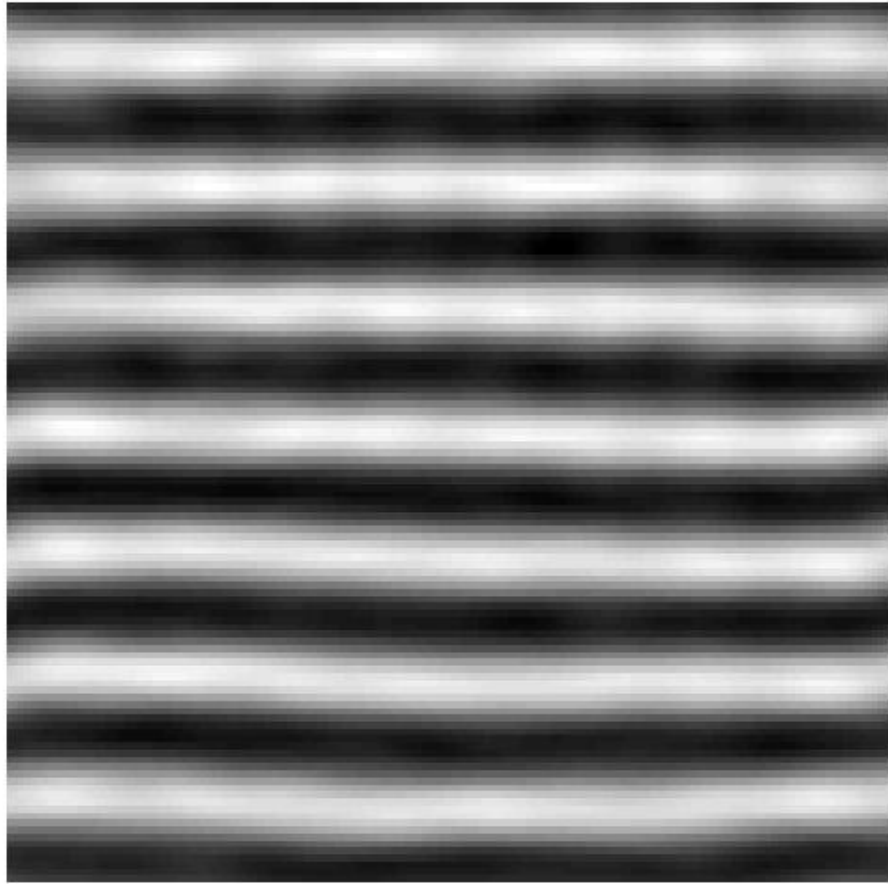


Figure 4.2: An image showing the straight roll pattern under control.

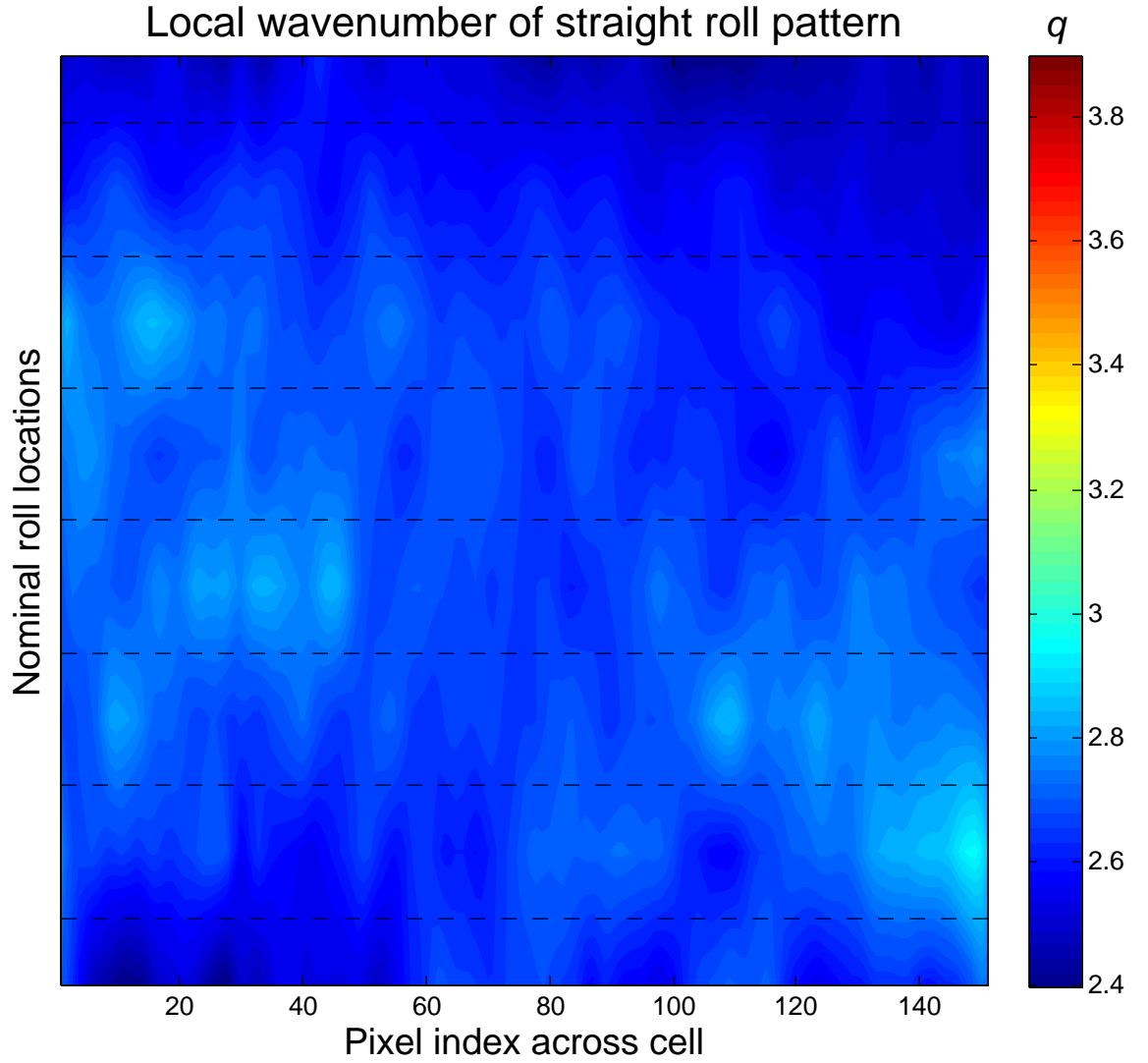


Figure 4.3: The local wavenumber field computed for the straight roll pattern in Fig. 4.2. The dashed lines indicate the approximate locations of upflowing fluid. The colormap scale has been chosen to match that of Fig. 4.6.



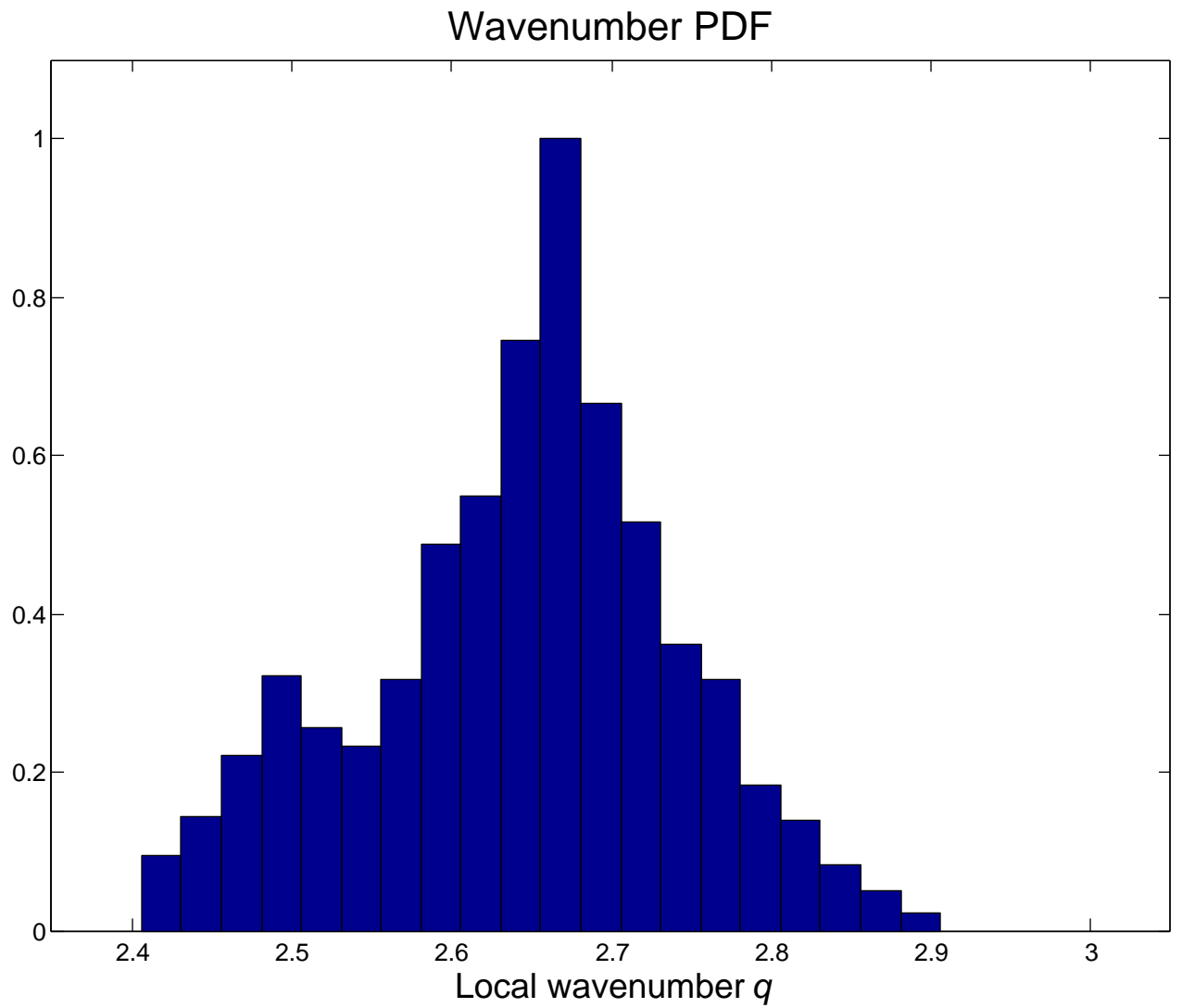


Figure 4.4: The probability distribution of local wavenumbers, as measured from the pattern in Fig. 4.2.

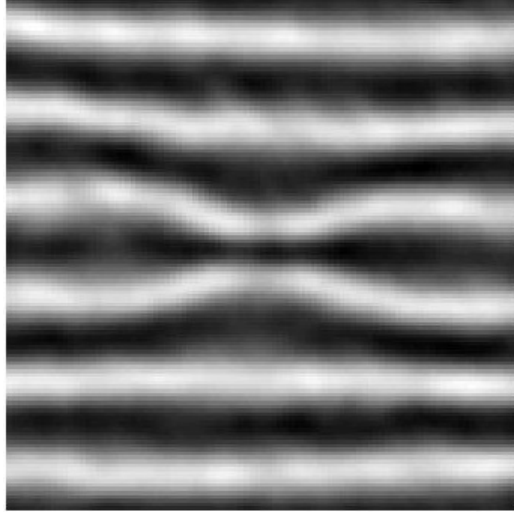


Figure 4.5: A pattern shortly after a point perturbation to the center cold roll (dark regions are cold down-flow; bright regions are warm up-flow).

adjacent regions of hot upflow are most likely to cause an instability; this makes intuitive sense, as one can imagine that this type of perturbation acts to add another region of upflow and thus increase the pattern wavenumber further. We therefore find the location in the central portion of the pattern (away from the boundaries) with the minimal separation between adjacent hot regions and apply a controlled perturbation to the region of cold flow directly between. The perturbation is in the form of heating of a single point in the pattern for about 100 ms, which results in an initially axisymmetric disturbance of diameter  $\sim d/2 = 350 \mu\text{m}$ . A stable pattern exhibits a visible response to the perturbation but relaxes to the pre-perturbation pattern after some period. The shadowgraph snapshot in Fig. 4.5 shows an example of a pattern response shortly after a perturbation; note the center two rolls of the pattern appear to bend toward one another, while the rest of the pattern deviates much less from an ideal straight roll pattern.

#### 4.1.1 Perturbation Strength

Necessarily, the applied perturbations are finite amplitude, as they need to be of large enough amplitude to overcome the modal damping and cause a visible pattern

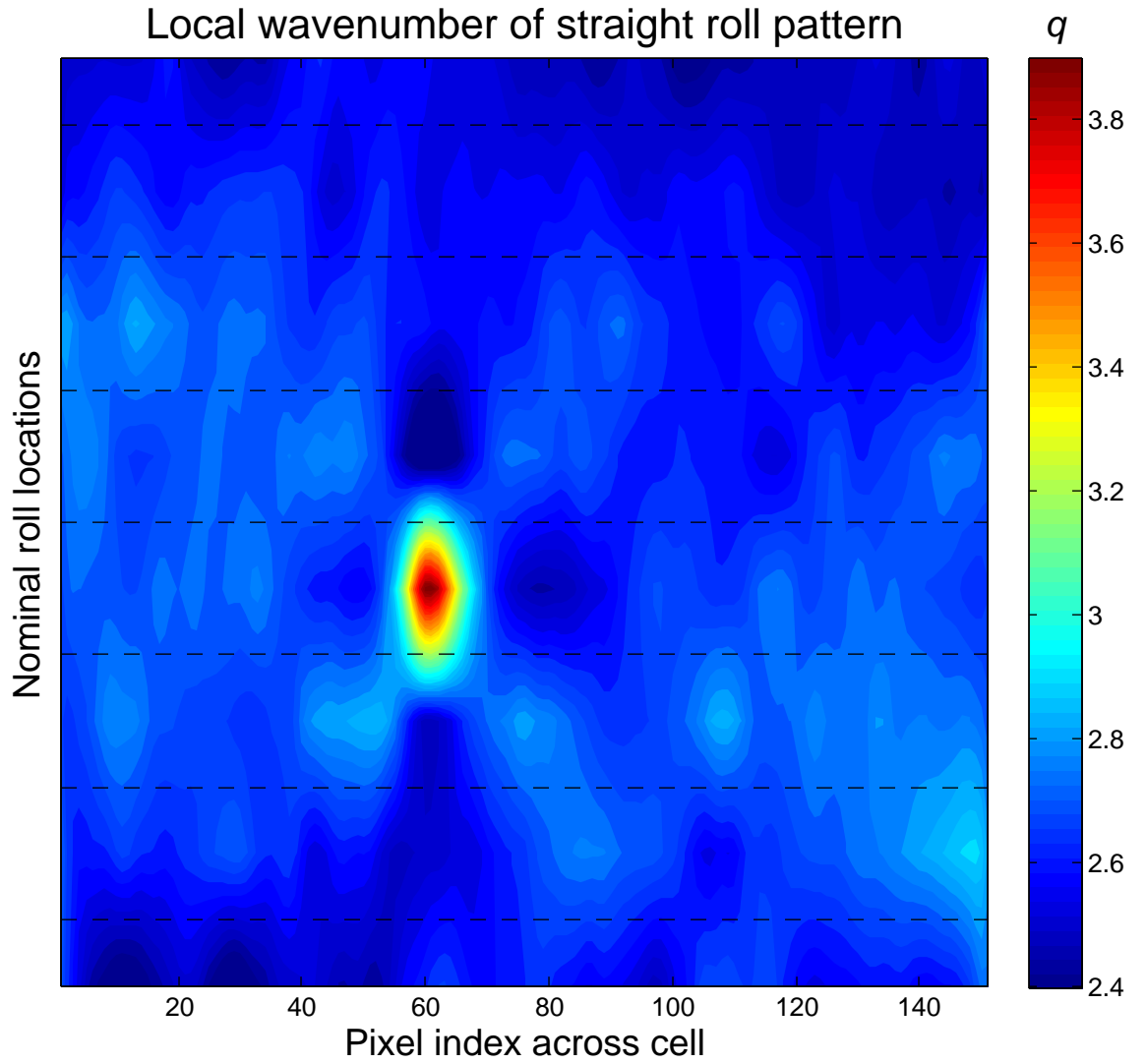


Figure 4.6: A local wavenumber field immediately following a perturbation. As in Fig. 4.3, dashed lines indicate the approximate locations of upflowing fluid in the base state.

response. However, a sufficiently strong perturbation may cause the pattern to undergo an instability; this is of particular concern when the pattern is already near an instability boundary and the modes are weakly damped. The amplitude of compromise is one that results in a brief period of nonlinear evolution, followed by linear decay. To account for any drift in laser intensity (from, say, a small change in the length of the laser's resonance cavity), we find it best to start with small perturbations, lasting  $\sim 100$  ms, and then slowly increase the duration of the pulse if the system response is weak.

## 4.2 *Lifetime Measurements*

Disturbance lifetimes are computed from snapshots of the pattern following a perturbation (see Fig. 4.5), as the pattern displays some dynamical response before relaxing back to the stationary state. Two related signals that can be used to define a lifetime are the rms intensity and the Fourier intensity, each of which can be computed directly from the images. We have chosen to use a third method, however, due to one significant drawback of either of these intensity measurements. Namely, there is an arbitrary choice of area from which either intensity signal is computed. A cleaner signal is obtained from considering a larger area, but because of the localized nature of the excited structures, pixels far from the central point of the perturbation are unlikely to contain much relevant information; conversely, an intensity signal computed from very few pixels is subject to a larger degree of fluctuation. (This is another way of saying that, a priori, the spatial extent of the excited structure is unknown.)

A different measure gives a clean and more reproducible signal. When the cold fluid between two adjacent hot upflow regions is perturbed, the upflow regions appear to bend toward one another and then relax back to their original positions at some rate that is dependent on distance from instability. Recall from Chapter 3 that the roll positions are computed as a step in the pattern control loop; the system response

is visible from this measurement in the form of a large dip in the local spacing. This response is most noticeable from the computed wavenumber field; Fig. 4.6 shows a wavenumber field following a point perturbation between regions of hot up-flowing fluid. Notice the spatial-localization of the disturbance.

A one-dimensional signal is formed by the separation of the upflow regions at their nearest point. This local spacing is converted to a wavenumber, which displays a large peak when or shortly after the perturbation is applied, followed by smooth decay to the equilibrium value over time. Note that because only the relative positions of the rolls is important in this measurement, any small amount of overall pattern translation (say, from a vibration of the apparatus) is ignored. The latter portion is well fit by one exponential term:  $q(t) = q_0 + Q \exp(-t/\tau)$ , where  $q_0$  is the relaxation value,  $Q$  is a perturbation amplitude, and we define  $\tau$  as the disturbance lifetime (equivalently,  $\tau^{-1}$  is the disturbance decay rate). An example is shown in Fig. 4.7. Deviation from the fit at the beginning of the decay, which becomes more noticeable near instability, is attributed to nonlinear evolution. The linear decay is typically on the order of a few to many seconds. In this example, the relaxation value is  $q_0 = 2.76$ , and the decay rate is  $\tau^{-1} = 0.31 \text{ s}^{-1}$ . In any cases where the applied perturbation causes the local wavenumber to shift (the pattern relaxes to a spacing much different than the initial one), those measurements are not used.

### 4.3 *Results*

The measured decay rates are shown in Fig. 4.8, as a function of the local wavenumber, with a linear fit. The scatter between individual points indicates well the uncertainty in any one lifetime value, which is estimated from multiple fits of any given response signal (using various segments of the full signal) to be  $\simeq 10\%$ . This repeatability of the measurements and the linear profile of the decay rates over the wavenumber range are good indicators that the straight roll patterns have been prepared without

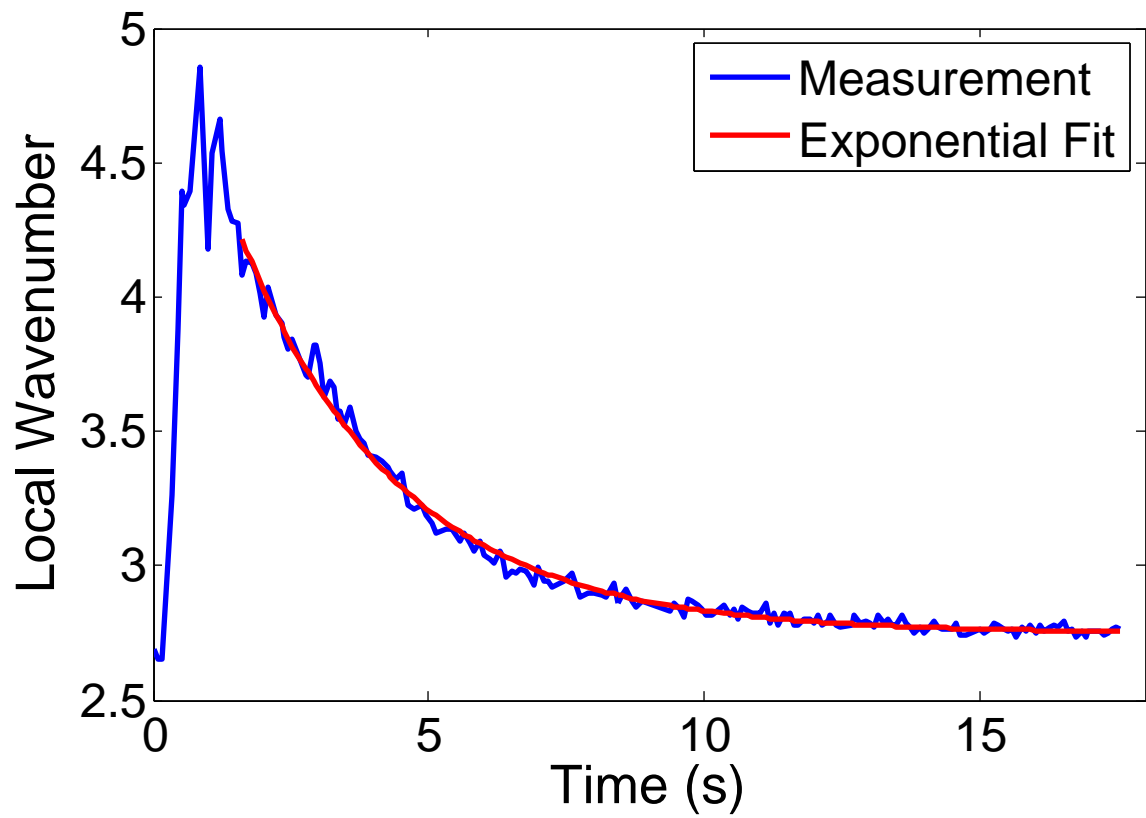


Figure 4.7: The local separation of adjacent upflow regions over time, after a perturbation to the cold downflow region in between. The red line shows an exponential fit.

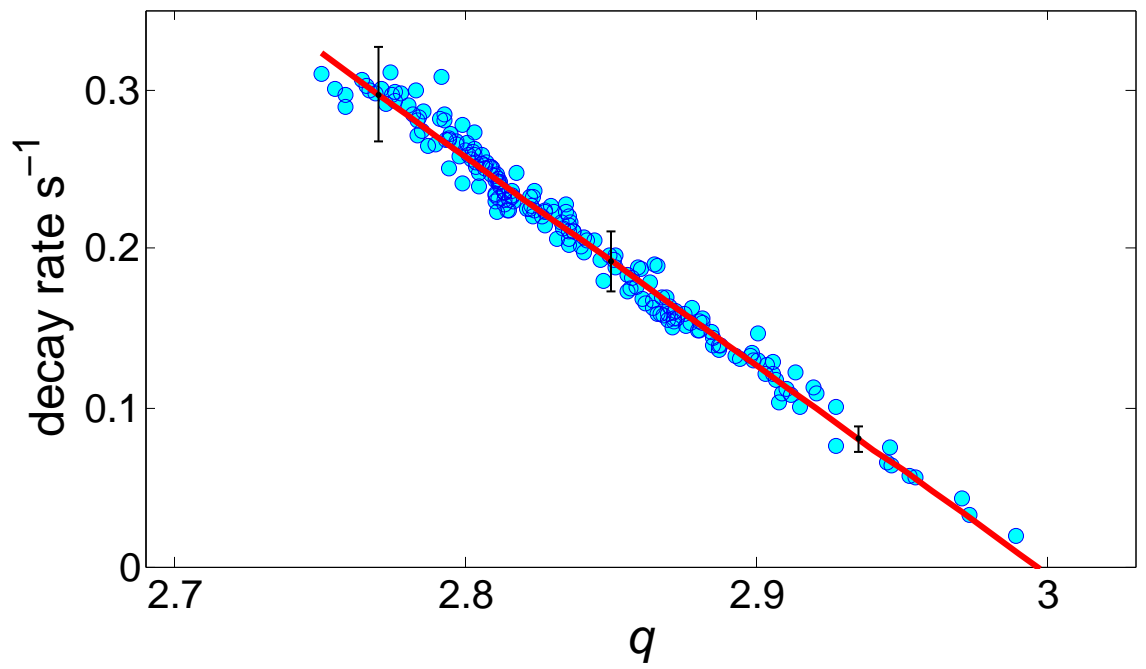


Figure 4.8: Decay rates (i.e., inverses of lifetimes) over a range of pattern wavenumbers. The error bars represent the 10% uncertainty in any one lifetime measurement.

introducing unwanted disturbances, such as thermal inertia effects from perturbative boundary heating.

The perturbation decay rate decreases by an order of magnitude over the range of wavenumbers studied here. This continuous slowing down of the dynamics near the expected instability implies that we are indeed capturing a signature of the least-stable mode, which is confirmed by the fact that localized skew-varicose instabilities occur when attempting to increase the wavenumber beyond this range. Perturbations to patterns with  $q$  slightly below the smallest values given here decayed too rapidly to retrieve a reproducible lifetime measurement; this is consistent with the phase space location being in the center of the Busse balloon, where patterns are far from any secondary instability, but it leaves open the possibility of a dominant mode in this lower range of  $q$  distinct from the one captured by these lifetime measurements.

The point at which the linear fit to the decay rates crosses zero indicates the critical value<sup>1</sup> of the local wavenumber, where perturbations are not expected to decay,  $q_c = 3.00$ ; this marks the boundary of the localized skew-varicose instability at  $\epsilon = 0.60$ . We can see from the Busse balloon (Fig. 4.1) that this value is slightly smaller than that predicted from global instability of an ideal, infinite pattern at this  $\epsilon$  ( $q_c = 3.15$ ). This discrepancy is unsurprising, insofar as the spatial-localization of the pattern response to perturbation indicates that the critical mode is not globally extended, as suggested by the Busse balloon analysis, and therefore one might expect the critical roll spacing to differ somewhat from that of the idealized pattern.

The exponential decay suggests that there is a single long-lived mode being excited from perturbations to downflow regions of these high- $q$  patterns, but it is unclear from these results if perturbations of this type will generally excite the dominant spatially localized modes. Moreover, it is desirable to characterize the spatial structures and

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<sup>1</sup>This critical wavenumber for the onset of the secondary skew-varicose instability should not be confused with the value of the critical wavenumber for the onset of convection.



time scales for not only the dominant modes but other important, sub-dominant modes. That is the subject of the next Chapter.

## CHAPTER V

### MODAL EXTRACTION

#### *5.1 Introduction*

In many systems, analysis of infinite, ideal patterns can provide a description of dynamics and pattern selection (e.g., the Busse balloon), but this approach can fail for systems with boundary forcing or in states far from the initial pattern onset. As discussed in the Introduction, and as illustrated in the previous chapter, one consequence of imperfect patterns is the spatial localization of the dominant system modes and thus spatially localized instabilities. In convection states exhibiting persistent chaos, such as the spiral defect chaos state (see the Introduction), we indeed do observe continual defect growth/annihilation on a local scale. The introduction of defects to an ordered state (straight rolls) can result in a cascade of instabilities that lead to global chaotic behavior, as in the case of a transition from a straight roll to the SDC state. It is therefore worthwhile to understand the mechanisms of spatially localized instabilities quantitatively.

In this chapter, we present the experimental and analysis procedures that we have developed for extracting the dynamical modes governing secondary instability of the straight roll state. Our approach uses the common idea of using an ensemble of states with nearby initial conditions to understand the important dynamical degrees of freedom, but is novel in that our ensemble is prepared experimentally. We repeatedly prepare a base straight roll pattern, to which selected perturbations are applied, giving an ensemble of patterns evolving from a set of nearby initial conditions. From the ensemble of different pattern evolutions, we compute a small set of basis patterns that, together, are sufficient to describe the observed pattern dynamics. By representing the

evolutions of all ensemble members in this low-dimensional basis computed directly from the different pattern evolutions, (rather than, say, a Fourier basis), we are able to construct a matrix representation of the operator that governs the evolution of disturbances to the base state. This eigenmodes and eigenvalues of this evolution operator, which is constructed from the experimental measurements, give us directly the dynamical modes of the base state as well as the growth rates of those modes.

We present the spatial modes and growth rates extracted from experiments at three different locations in  $(\epsilon, q)$  parameter space; these locations are shown in the context of the Busse balloon for our experimental conditions ( $Pr = 0.84$ ) in Fig. 5.1. Note that the measurements of perturbation lifetimes in Chapter 4 were taken at the same  $\epsilon$  and at wavenumbers surrounding  $D_I$ . To illustrate the analysis procedure, we show how the intermediate steps apply to the ensemble at  $D_I$ ; many of the conclusions will hold for all three ensembles, but any significant differences in the results of a given analysis step from one ensemble to another will be noted. Many images showing portions of the straight roll patterns have been cropped to show a square region of the pattern under control, usually 7-9 wavelengths; recall that all wavenumbers are given in non-dimensional units:  $q = 2\pi/(\lambda/d)$ .

## 5.2 *Creation of Perturbation Ensemble*

A given perturbation (as described in Section 4.1) excites one or more system modes. How many are excited, and which ones, may depend on the size, strength, and location of the perturbation. In order to excite all the dominant modes governing disturbances to a pattern, perturbations of differing strength are applied to evenly spaced points across a pattern half-wavelength (see Fig. 5.2). Note that all spatially localized modes are related to those excited from this ensemble through symmetries (see below) of the ideal straight roll pattern; an ensemble across a half-wavelength therefore comprises a “minimal” ensemble (however, as it turns out, it is useful to

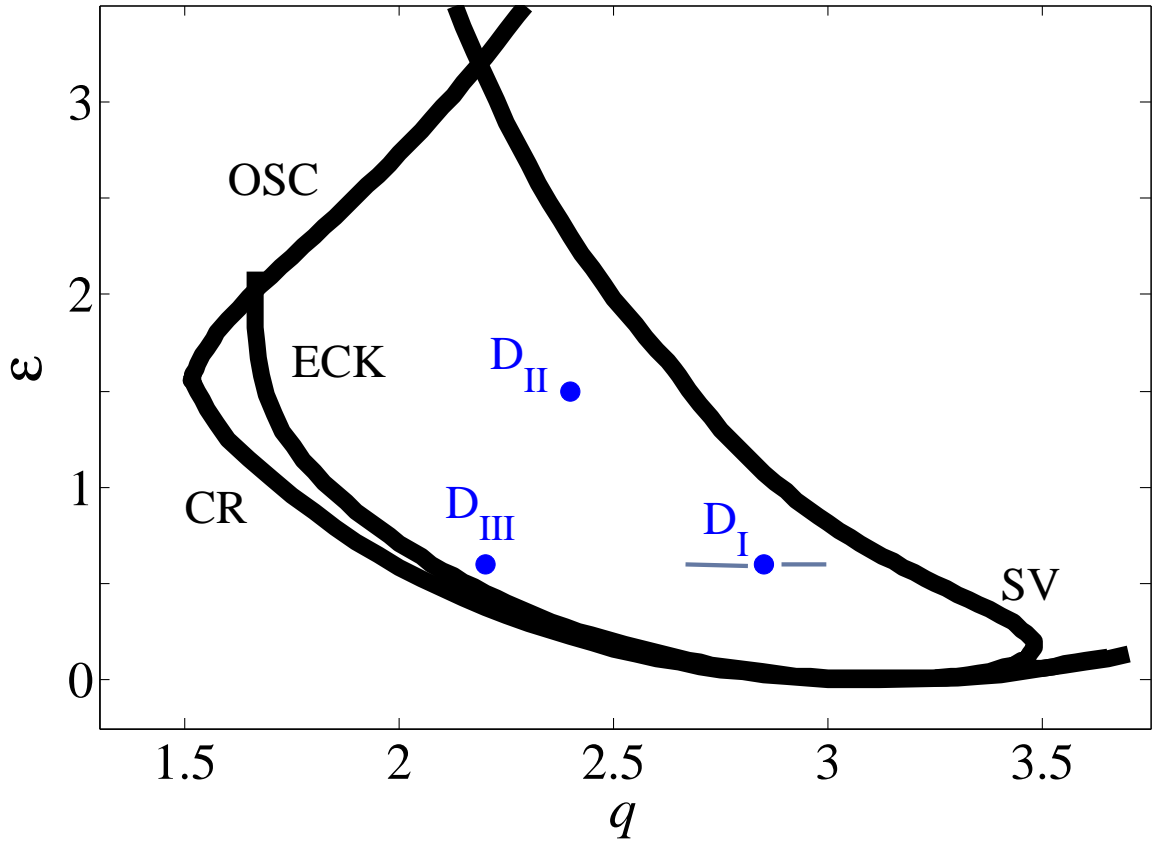


Figure 5.1: The Busse balloon for Prandtl number 0.84 as in our experiments. The parameter coordinates of the three separate experimental ensembles are indicated by  $D_I$ ,  $D_{II}$  and  $D_{III}$ .  $D_I = (0.60, 2.85)$ ,  $D_{II} = (1.50, 2.40)$ , and  $D_{III} = (0.60, 2.20)$ . Also marked is the wavenumber range around  $D_I$  over which perturbation lifetimes were measured (see Chapter 4).

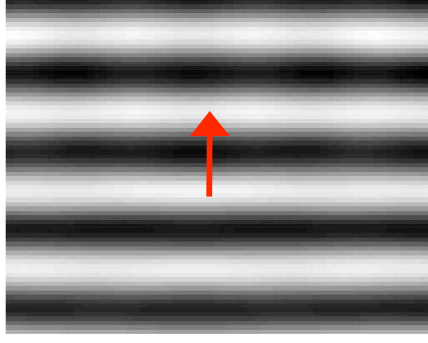


Figure 5.2: Perturbations are applied at different points along the direction of the pattern wavenumber (indicated by the arrow); perturbations across a half-wavelength constitute a minimal set.

consider a more extended ensemble). We define a grid of six perturbation locations over a pattern half-wavelength; because the spatial extent of the laser is 10% or more of the pattern wavelength (or 20% of the half-wavelength) in all cases reported here, there is no benefit from a finer partition. The perturbation locations are determined relative to the measured position of a central roll pair of the pattern, accounting for any slow drift of the pattern. While it is not known a priori how many distinct modes we may be able to extract, it is reasonable to expect the number to increase with the ensemble size. We therefore impose a large number ( $\sim 20$ ) of perturbations at each of the pre-determined locations.

### ***5.3 Shadowgraph Image Processing***

When a perturbation is applied, it excites modes of the system which modulate the base straight roll pattern. Shadowgraphs capture the pattern evolution after an imposed disturbance and thus contain snapshots of the composite structure of the excited modes over time. The following steps are taken to prepare the shadowgraphs for analysis.

### 5.3.1 Spatial Filtering

While the effect of the perturbations is visible on the raw shadowgraph images, the structures of the excited modes can be seen more clearly by subtracting the stationary straight roll pattern from all images following the perturbation. In Fig. 5.3 we show a pattern immediately following a perturbation and then a short time later. For comparison, the images are also shown after the straight roll pattern has been subtracted. A spatial window centered around the perturbation point is applied to all images to filter out intensity fluctuations far from the response. The images are then Fourier filtered. Rather than apply a wavenumber filter of the Fourier spectrum (which would eliminate structures at some length scales), we elect to keep the most dominant Fourier modes over the entire spectrum. An FFT is applied to each shadowgraph image capturing a given perturbation decay. The sum of Fourier amplitudes over the time series provides a Fourier spectrum representative of the most important modes present in the perturbation trajectory. Only those Fourier modes with amplitudes greater than 10% of the maximum are retained; thus, the number of Fourier modes is reduced from order  $10^4$  (the number of pixels in the original images) to the same few hundred over all snapshots of the perturbation trajectory (we have confirmed that including slightly more or fewer Fourier modes does not change the results). Figure 5.4 shows the decay of excited structures after perturbation to a pattern at  $D_I$  after the images have been spatially filtered.

### 5.3.2 Temporal Filtering

Due to memory limitations of our computer, the frame rate of the image-capturing software can fluctuate, at times, up to 30% of the nominal frame rate of about 10 frames per second. Although this amounts to a variation of less than 1/10 of a second between subsequent frames, small temporal errors will cause identical perturbations

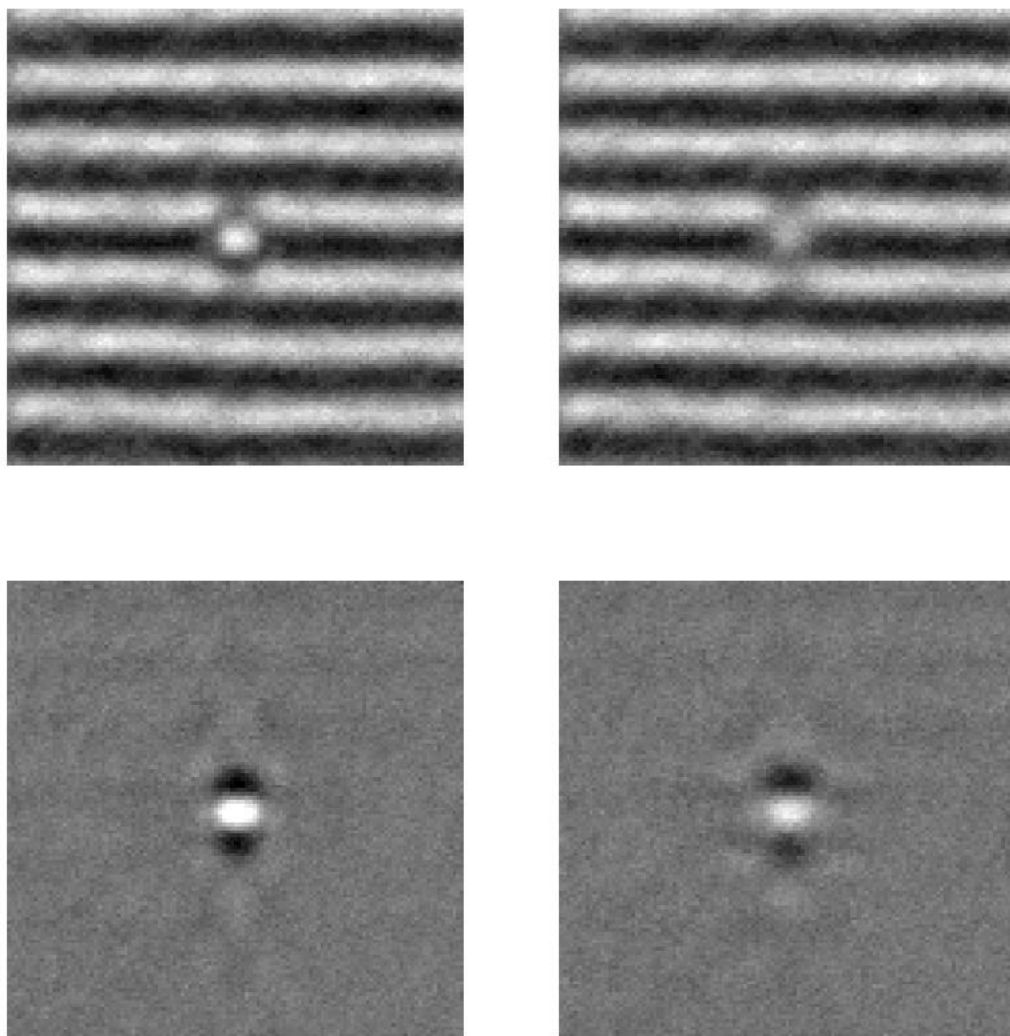


Figure 5.3: Images showing how the structures excited by perturbations can be visualized by subtracting off the stationary straight roll pattern. On the top are the raw images immediately following a perturbation and then at a short time (about 1 s) later. On the bottom are the same images after the straight rolls (with no perturbation) have been subtracted.

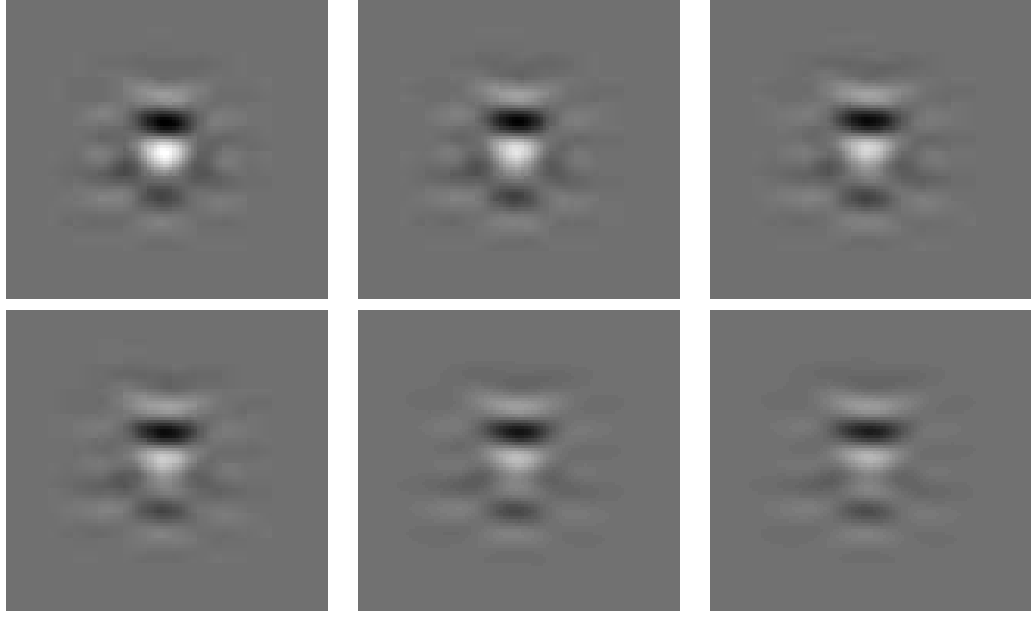


Figure 5.4: Snapshots (at  $D_I$ , with  $q = 2.85$ ) show the decay of the excited structures after an initial perturbation. This example is the result of a perturbation to a region of cold downflow and shows the structure at regularly spaced intervals over 2.5 seconds. The area shown covers over six wavelengths of the base straight roll pattern.

to appear to undergo different evolutions. Therefore, we interpolate between existing shadowgraphs to get images at any desired time after the initial disturbance is applied. The timestamp of each shadowgraph is scanned to find two images taken immediately prior and immediately after the time of interest; a linear interpolation of intensity values at all pixels provides a new shadowgraph image at the desired moment. Note that the evolution is much slower than the small times over which the interpolation is performed, so we expect the error introduced by constructing an interpolated shadowgraph image to be negligible.

For each perturbation trajectory, we define a reference time  $t_0$  as when, from the shadowgraphs, the perturbation amplitude appears largest. Relative to that time, each disturbance trajectory is reduced to a set of analysis shadowgraphs at regularly spaced time intervals of  $\Delta t$ . Thus, there exist snapshots of each ensemble member at times  $t = t_0, t_0 + \Delta t, t_0 + 2\Delta t$ , and so on.



## 5.4 *Construction of Karhunen-Loève Basis*

A Karhunen-Loève (KL) decomposition of the perturbation ensemble creates a set of basis modes in which to embed all perturbation trajectories. Note that the KL basis modes should not be confused with system eigenmodes; the KL modes are simply a choice of basis in which to represent all the perturbation evolutions. The essence of a KL decomposition is to build from a measurement a set of basis modes that capture the correlations present in the data. In our case, the measurement is a set of images showing the evolution of an ensemble of structures excited by perturbations. In this measurement set, there will be some pixel-to-pixel correlations across different images (for example, a particular group of pixels may all take on large intensity values in select images). A matrix of these pixel-to-pixel correlations is formed: this is the covariance matrix. An eigenvalue decomposition of this covariance matrix provides a set of KL modes, with each mode formed by a combination of pixels that are together correlated. The KL modes are ordered by the respective fraction accounted for of the total amount of measurement correlation; the modes themselves are minimally correlated.

We wish to reduce the correlations across excited structures present in the perturbation ensemble; the KL modes that capture the bulk of the correlation will then form the embedding basis. Note that, in contrast to the typical implementation of the KL decomposition [50, 51], this reduces covariance over different initial conditions rather than over time. This is an important difference, because time-averaged correlations (over a single trajectory) sample different states and can neglect structures that, although appearing infrequently (in time), have an important dynamical role [52]. Our KL basis captures the different structures about a single state (i.e., the different ways in which a pattern can change relative to a base pattern). Thus, our KL basis can be considered to be more representative of the dynamical, rather than statistical, modes.

Let  $\mathbf{x}_i$  denote the intensity vector formed from a shadowgraph image capturing

the structures excited from the  $i$ -th perturbation. We then construct the  $N \times M$  matrix  $X$ ,

$$X = \begin{bmatrix} \mathbf{x}_1 & | & \mathbf{x}_2 & | & \cdots & | & \mathbf{x}_M \end{bmatrix}$$

where  $M$  is the number of different initial conditions, and  $N$ , the number of rows, is equal to the original dimension of the data, i.e., the number of image pixels. The covariance matrix is then  $\mathbf{X}\mathbf{X}^T$ , where  $\mathbf{X}^T$  refers to the transpose of  $\mathbf{X}$ . The covariance matrix has dimensions  $N \times N$ . Finding the eigenvectors and eigenvalues of this large matrix ( $N \sim 10^4$ , the number of pixels) is a daunting task; instead, we perform an eigenvalue decomposition of the matrix  $\mathbf{X}^T\mathbf{X}$ , which has dimensions  $M \times M$ , with  $M \ll N$ . If  $\mathbf{\Lambda}$  is the diagonal eigenvalue matrix of  $\mathbf{X}^T\mathbf{X}$  and  $\mathbf{V}$  is a matrix with columns filled by the eigenvectors, we can write  $\mathbf{X}^T\mathbf{X}\mathbf{V} = \mathbf{\Lambda}\mathbf{V}$ . Multiplying by  $\mathbf{X}$  on the left gives  $\mathbf{X}\mathbf{X}^T\mathbf{X}\mathbf{V} = \mathbf{X}\mathbf{\Lambda}\mathbf{V} = \mathbf{\Lambda}\mathbf{X}\mathbf{V}$ , from which we conclude that  $\mathbf{X}\mathbf{V}$  are the eigenvectors of the original covariance matrix, sharing the same eigenvalue matrix  $\mathbf{\Lambda}$ . We therefore perform the computationally feasible calculation: we compute the eigenvectors  $\mathbf{V}$  of  $\mathbf{X}^T\mathbf{X}$  and transform those vectors by multiplying on the left by  $\mathbf{X}$ ;  $\mathbf{X}\mathbf{V}$  are the eigenvectors (KL modes) of the original covariance matrix  $\mathbf{X}\mathbf{X}^T$ .

Together with the KL eigenvectors are the corresponding eigenvalues; the eigenvalue of a given KL mode indicates the fraction of the total variance accounted for by that mode (more precisely, the eigenvalue should be normalized by the sum of all eigenvalues). There exist as many basis modes as there are original measurements (images). Modes with small eigenvalues are those that appear in few of the images, and therefore add little to the description of the modal trajectories. Moreover, including modes with very small contributions can contribute to spurious results, as it effectively introduces a dimension into the analysis that likely has little dynamical relevance. Again, it should be noted that although small-eigenvalue KL modes can carry significant dynamical information in a time-averaged decomposition, we do not expect this to be the case for our ensemble-average over nearby initial conditions,

where the slowest structural decay is likely due to the dominant modes of interest. Accordingly, we limit our embedding dimension to the smallest number of modes that capture 90% of the power of the KL eigenvalue spectrum.

While we are free to choose which of the ensemble images are used to construct the KL basis, it is appropriate to use the images near the end of the perturbation evolution, as this favors the longest-lived modal structures; other modes may be present in the early snapshots but are of less interest because they decay rapidly.

#### 5.4.1 Perturbation Averaging

At  $D_I$ , 23 individual perturbations (varying only in amplitude) were applied at each of the six perturbation locations of a pattern half-wavelength. As mentioned above, the choice of the number of different perturbations locations is guided by the spatial extent of the laser beam. A KL decomposition of the  $D_I$  ensemble therefore provides up to 138 basis modes. However, as Fig. 5.5 shows, the variance is unevenly distributed; the fraction of the total variance accounted for is plotted against the number of KL basis modes. Recall that, in our formulation, the variance is across the spatial structures present in the ensemble images. The first several modes account for over 50% of the variance; this suggests that the perturbation structures can be described by a lower-dimensional basis.

To isolate the possible dependence of the extracted modes and growth rates on the perturbation amplitudes, we decomposed the full ensemble of initial conditions into four sub-ensembles based on the maximum rms shadowgraph intensities. The KL modes across sub-ensembles were consistent, indicating that, at least over the perturbations considered here, there is no discernible dependence of the excited modal structures on the perturbation strength. Therefore, in the interest of minimizing any residual noise, all the perturbation trajectories at a given location are averaged; the largest number of KL modes is then equal to the number of perturbation locations.

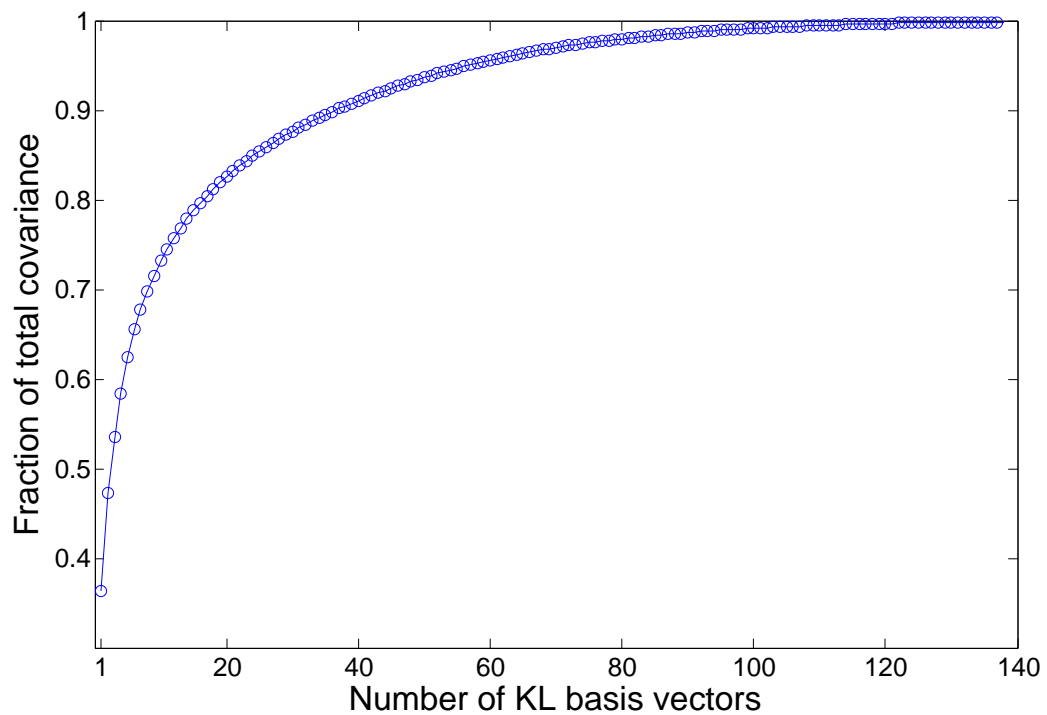


Figure 5.5: The fraction of the total variance accounted for by the KL basis modes at  $D_I$  as a function of the basis size.

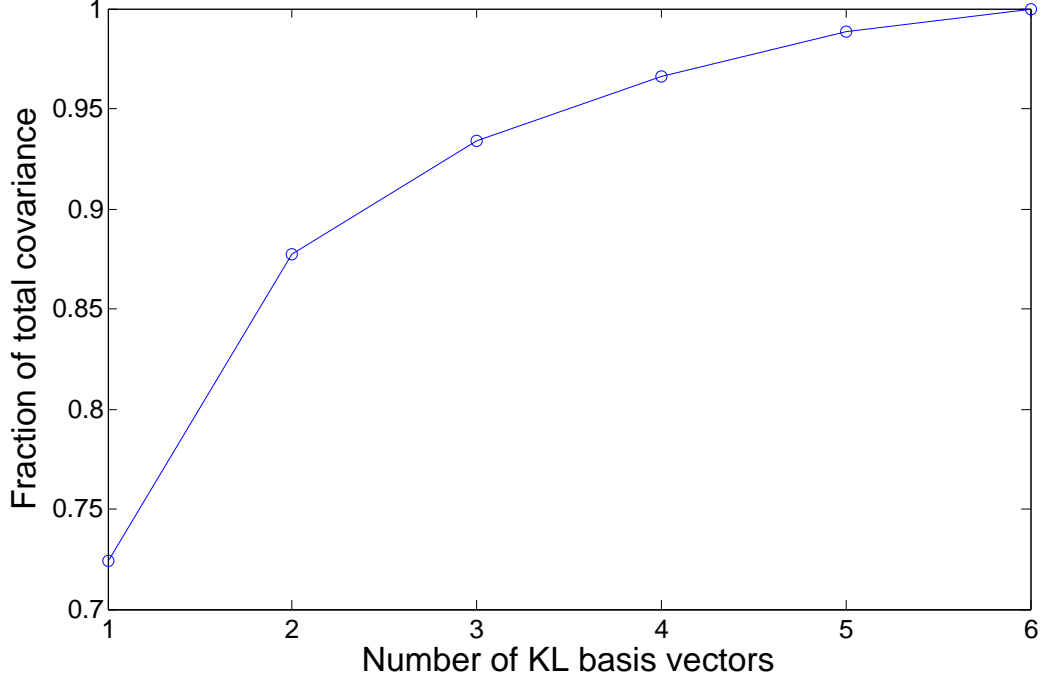


Figure 5.6: The fraction of the total variance accounted for by the KL basis modes at  $D_I$  as a function of the basis size, after averaging perturbations at each respective location.

The variance distribution for the resulting six KL basis modes at  $D_I$  is shown in Fig. 5.6. The corresponding basis images are shown in Fig. 5.7. The first few modes account for over 90% of the variance; this result holds for all three data sets.

### 5.5 Determination of Modal Structures and Growth Rates

To avoid confusion, we change notation slightly to distinguish between vectors in shadowgraph pixel space and those in the space spanned by the KL basis. Let  $\mathbf{b}^0$  denote the disturbance following an initial perturbation, expressed in a low-dimensional (KL) basis. After some time  $T$ , this state has evolved to  $\mathbf{b}^T$ . Then  $\mathbf{U}\mathbf{b}^0 = \mathbf{b}^T$ , where  $\mathbf{U}$  is the evolution operator. Using an ensemble of initial conditions we define

$$\mathbf{B}^0 = \left[ \mathbf{b}_1^0 \mid \mathbf{b}_2^0 \mid \cdots \mid \mathbf{b}_M^0 \right]$$

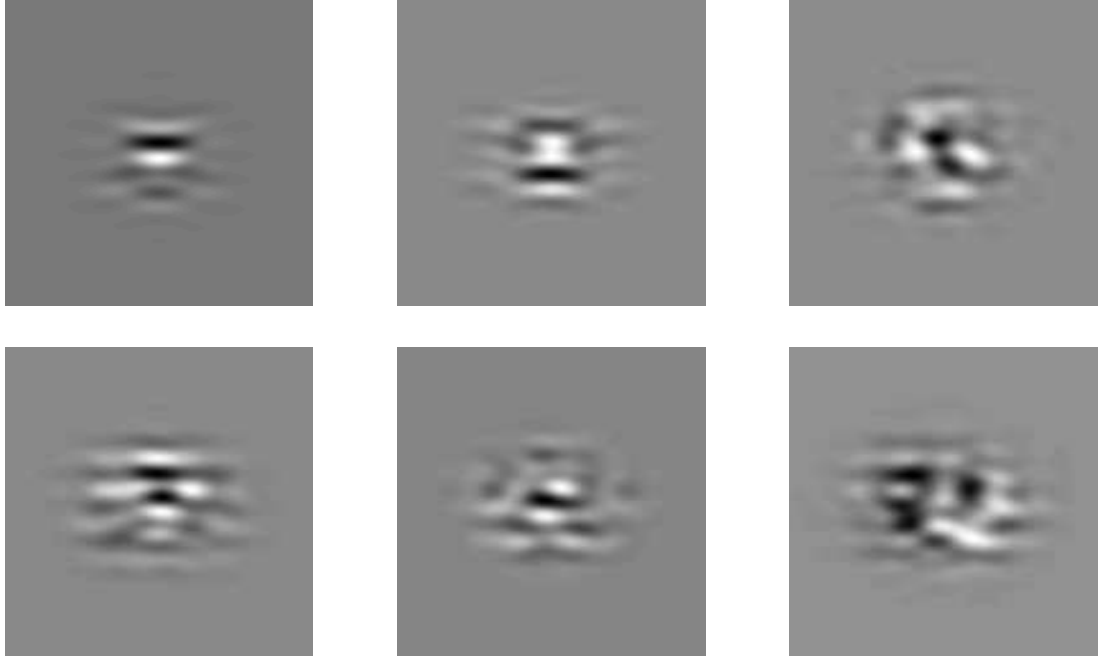


Figure 5.7: The KL basis images at  $D_I$ . As in Fig. 5.4, the area shown covers over six wavelengths of the base straight roll pattern.

and similarly,

$$\mathbf{B}^T = \left[ \mathbf{b}_1^T \mid \mathbf{b}_2^T \mid \cdots \mid \mathbf{b}_M^T \right]$$

This gives the over-determined (least-squares) problem for the evolution operator  $\mathbf{U}\mathbf{B}^0 = \mathbf{B}^T$ , which is solved by  $\mathbf{U} = \mathbf{B}^T(\mathbf{B}^0)^{-1}$ , where the reciprocal of  $\mathbf{B}^0$  is taken to refer to the generalized inverse of the non-square matrix.

During linear decay, each system eigenmode decays at a characteristic rate, so we can also write  $\mathbf{U}\mathbf{e}_i = \exp(\sigma T)\mathbf{e}_i$ , where  $\mathbf{e}_i$  are the eigenvectors and the eigenvalues  $\lambda_i$  are related to the growth rates by  $\lambda_i = \exp(\sigma_i T)$ .

Recall that we have, for each disturbance, snapshots at evenly spaced times over the disturbance decay; we select from the series of snapshots of each perturbation evolution an image representing the “initial” disturbance and, for convenience, re-label the time of that image as  $t = 0$ . The snapshot representing the “final” disturbance is then selected a time  $T$  later.

While these choices of “initial” and “final” times may appear arbitrary, they can have measurable effects on the convergence onto system eigenmodes. Using snapshots from the earliest stages of disturbance evolution may, for example, include nonlinear evolution, which will affect the extracted modal structures and/or growth rates. On the other hand, using snapshots long after the initial disturbance may result in the strength of the captured signal to appear too weak to stand out from the background noise.

With these possible effects in mind, the initial and final times are varied as a measure of the robustness of the extracted modes. We find that there is a clear range of 2-5 s over which the modal structures and the growth rates change little with respect to changes in the initial or final image times. This yields an estimate of the uncertainty of any one growth rate to be  $\sim 10\%$ .

### 5.5.1 Use of Pattern Symmetries

While the preliminary results suggest that the algorithm does indeed converge on system eigenmodes, we observe that the dominant mode is extracted as part of a complex conjugate pair. At  $D_I$ , for example, the growth rate with largest real component is  $\sigma = -0.13 \pm 0.03i \text{ s}^{-1}$ . While the proper interpretation of the imaginary part of  $\sigma$  is as a temporal frequency, this would imply a period of multiple minutes; the analysis takes place on the order of seconds, suggesting that we do not have the resolution to detect temporal oscillation on that time scale, and that this imaginary component may be spurious. To break the degeneracy of the real part of the leading growth rate, we consider the system symmetries.

Considerations of symmetry are useful for determining allowed patterns and in building a minimal description of the dynamics in both unbounded and bounded

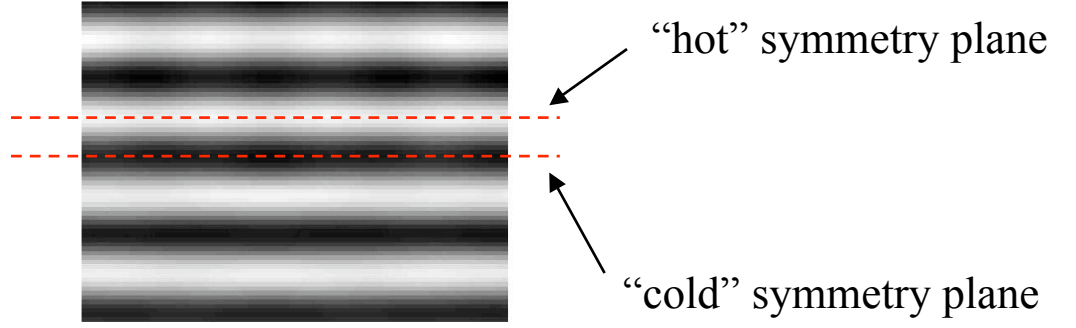


Figure 5.8: Two symmetry planes are defined over a half-wavelength of the pattern, one at the center of (hot) upflowing fluid, the other at the center of adjacent (cold) downflowing fluid. Four symmetric versions of each disturbance are formed by decomposing the disturbance into even/odd structures about either symmetry plane.

systems. The straight roll pattern contains the following symmetries: discrete translational symmetry (periodicity) in the direction of the wavevector, translational invariance in the direction perpendicular to the wavevector, and reflection symmetries across planes defined at the centers of hot/cold fluid. These symmetries are met approximately as long as the localized disturbances are far from the physical boundaries and far from locations where actuation is applied to rolls under control. Specifically, we examine the disturbance evolutions through the reflection symmetries by analyzing disturbances applied across a full pattern wavelength.

All disturbances can be decomposed in terms of symmetries related to a roll pair. We define two symmetry planes, one at the center of upflowing fluid, the other at the center of adjacent downflowing fluid (see Fig. 5.8). We can therefore define four symmetric versions of every initial disturbance, each even/odd about the hot/cold symmetry planes. Each of the corresponding four subspaces is invariant: disturbances retain their symmetry as they evolve.

The entire collection of initial and final conditions was decomposed using these symmetries, producing four independent ensembles. All eigenmodes extracted from the four ensembles are eigenmodes of the system, but in the cases when multiple eigenmodes share an eigenvalue (growth rate), we eliminate redundant representations



by computing the most spatially localized eigenmode structure.

**Fundamental Modal Structure** We wish to group all extracted modes by growth rates and define the fundamental mode as the most localized structure with a particular growth rate. For a given growth rate, there may exist up to four different extracted modal structures, one from each of the four symmetrized ensembles (although some symmetrized ensembles may not produce a mode with that growth rate). We group all of these modal structures together with copies of the same structures translated by an integer number of pattern wavelengths. The fundamental mode of a given group of modal structures is the linear combination of those structures which minimizes the  $p$ -norm, with  $0 < p < 2$ . The  $p$ -norm of vector  $\mathbf{v}$ , with  $i$ -th component  $v_i$ , is defined as  $(\sum_i \mathbf{v}_i^p)^{1/p}$ . Recall that eigenmode structures are defined up to an overall multiplicative constant: subject to an enforced normalization of the 2-norm (what would generally just be called the magnitude), minimizing the  $p$ -norm ( $p < 2$ ) produces a structure with the fewest number of non-zero components<sup>1</sup>. In our case, each component is a pixel element, so we are essentially minimizing the number of pixels that contribute to the fundamental modal structure. The linear combination of input modes is defined by a set of coefficients  $\{c_k\}$  describing the weight of each of the respective modal structures on the composite structure:  $\mathbf{x} = \sum_k^K c_k \mathbf{e}_k$ . A minimization routine was used to converge on the  $c_k$ .

### 5.5.2 Results

In all cases, one mode was extracted from each of the symmetrized ensembles; a mode even/odd about the hot/cold symmetry plane. There were two distinct growth rates extracted, each shared by a pair of the structures.

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<sup>1</sup>Consider a simple example in two dimensions to illustrate this point: a vector with one non-zero component,  $\mathbf{x} = (1, 0)$ , and another vector with two non-zero components,  $\mathbf{y} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ . Both have 2-norms (magnitudes) equal to 1, but  $\mathbf{x}$  has a smaller  $p$ -norm for any  $p < 2$ .

**High Wavenumber** The four symmetrized modes at  $D_I$  are shown in Fig. 5.9, with growth rates  $\sigma_1 = -0.13 \text{ s}^{-1}$  and  $\sigma_2 = -0.70 \text{ s}^{-1}$ . A second set of perturbations at high  $q$  was performed at  $D_{II}$ , with  $\epsilon$  increased relative to  $D_I$ . Again, two distinct growth rates are extracted;  $\sigma_1 = -0.15 \text{ s}^{-1}$  and  $\sigma_2 = -0.55 \text{ s}^{-1}$ . Mutual projections of the modes extracted at  $D_I$  and  $D_{II}$  indicate that the spatial structure of the two dominant modes changes little (after scaling by the wavelength) between these two locations. In Fig. 5.10, we compare a pair of symmetrized modes from the two ensembles, after the images have been scaled by the respective pattern wavelengths. There is also, in both cases, a large separation between the two growth rates. This suggests we can identify the dominant mode as the one representing the secondary instability at the high wavenumber boundary. The fundamental dominant and sub-dominant modes are given in Fig. 5.13. As the lifetime measurements suggested, the least-stable mode tended to be excited from perturbations to cold downflowing fluid. Note, however, that while the structures excited from these disturbances are even about the downflow symmetry plane, the most-localized representation of this mode does not obey that symmetry. The sub-dominant mode tended to be excited from perturbations to hot fluid. The structure of the dominant mode is consistent with a skew-varicose type instability. Figure 5.12 shows as an example the input modal structures for determining the fundamental representation of the dominant mode.

**Low Wavenumber** We also created an ensemble of perturbations to a low-wavenumber pattern, at  $D_{III}$ . As in the other cases, four symmetric modes were extracted; two dominant modes share a growth rates  $\sigma_1 = -0.20 \text{ s}^{-1}$  and the sub-dominant modes have  $\sigma_2 = -0.27 \text{ s}^{-1}$ . Shown in Fig. 5.11, the spatial structure of the dominant mode does not resemble any of the previously extracted modes, while the sub-dominant mode resembles the sub-dominant mode extracted at both  $D_I$  and  $D_{II}$ . The two fundamental modes are shown in Fig. 5.14. The closeness of the growth

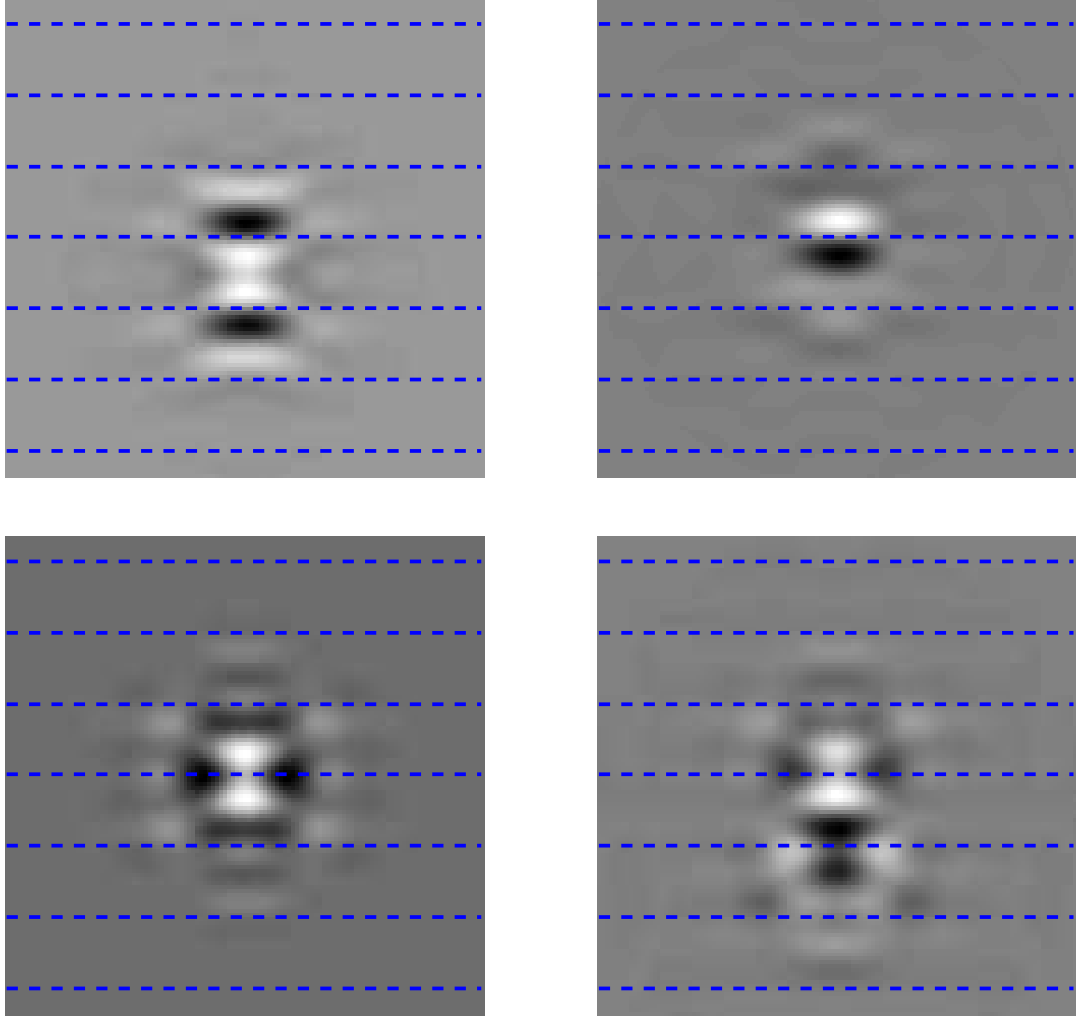


Figure 5.9: The four symmetric modes extracted at  $D_I$ . The top two images show the pair of structures sharing the largest growth rate; the two sub-dominant modes are shown below. Dashed lines mark the approximate locations of the hot upflow of the underlying base state; cold fluid lies between the dashed lines. (The distance between adjacent dashed lines is one pattern wavelength.)

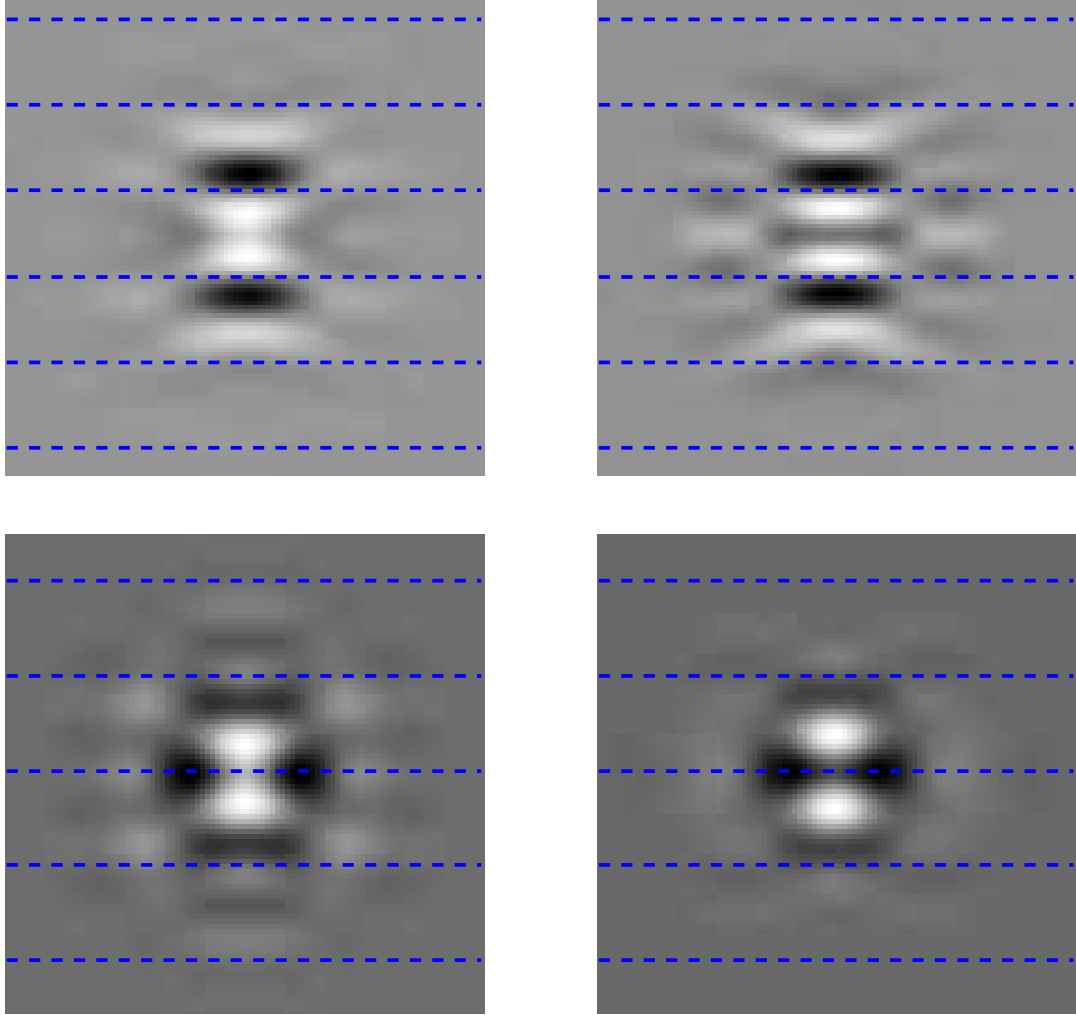


Figure 5.10: On the left, versions of the dominant (top) and sub-dominant (bottom) modes extracted at  $D_I$ . Next to these for comparison are modes extracted from the second high-wavenumber ensemble at  $D_{II}$ . The images have been scaled to show the patterns, over six wavelengths, using the same image size.

rates is consistent with the existence of two low-wavenumber instability types of the unbounded system that occur at very nearly the same parameter values, namely, the Eckhaus and cross-roll instabilities (see the Busse balloon in Fig. 5.1). The dominant mode is again excited from perturbations to a downflow; we find experimentally that sufficiently strong perturbations of this kind result in the growth of rolls perpendicular to the base pattern. We therefore identify the dominant low-wavenumber mode with the localized cross-roll instability and the sub-dominant mode with the localized Eckhaus instability.

It is worthwhile to note that the leading modes at  $D_I$ ,  $D_{II}$ , and  $D_{III}$  tended to be excited from perturbations applied to cold fluid while the sub-dominant mode tended to be excited via perturbations to hot fluid. As mentioned in the previous chapter, this is unsurprising, as heating of cold fluid tends to reduce the amplitude of the saturated roll state and can thus be seen as a destabilizing force.

Note that the symmetries imply that all extracted modes in each group can be represented as linear superpositions of the fundamental mode along with its translated and/or reflected copies. We verified that this is indeed the case as such representation was accurate, with mutual projection  $> 0.94$ . Three examples of the least-squares fits of more spatially extended modes are shown in Figs. 5.15.

## 5.6 *Conclusions*

The results presented indicate that two system eigenmodes describe the observed dynamics of any one location of phase space that was investigated. It is possible that closer to instability, more strongly damped modes may begin to play a dynamical role in the evolution of disturbances, but it is logical to expect modes to deform in a continuous way and thus for the dominant modes to change little in the neighborhood of any one  $(q, \epsilon)$  location. Nevertheless, for certain applications or in other systems, it may be of interest to build a spectrum including the fast-decaying eigenmodes.

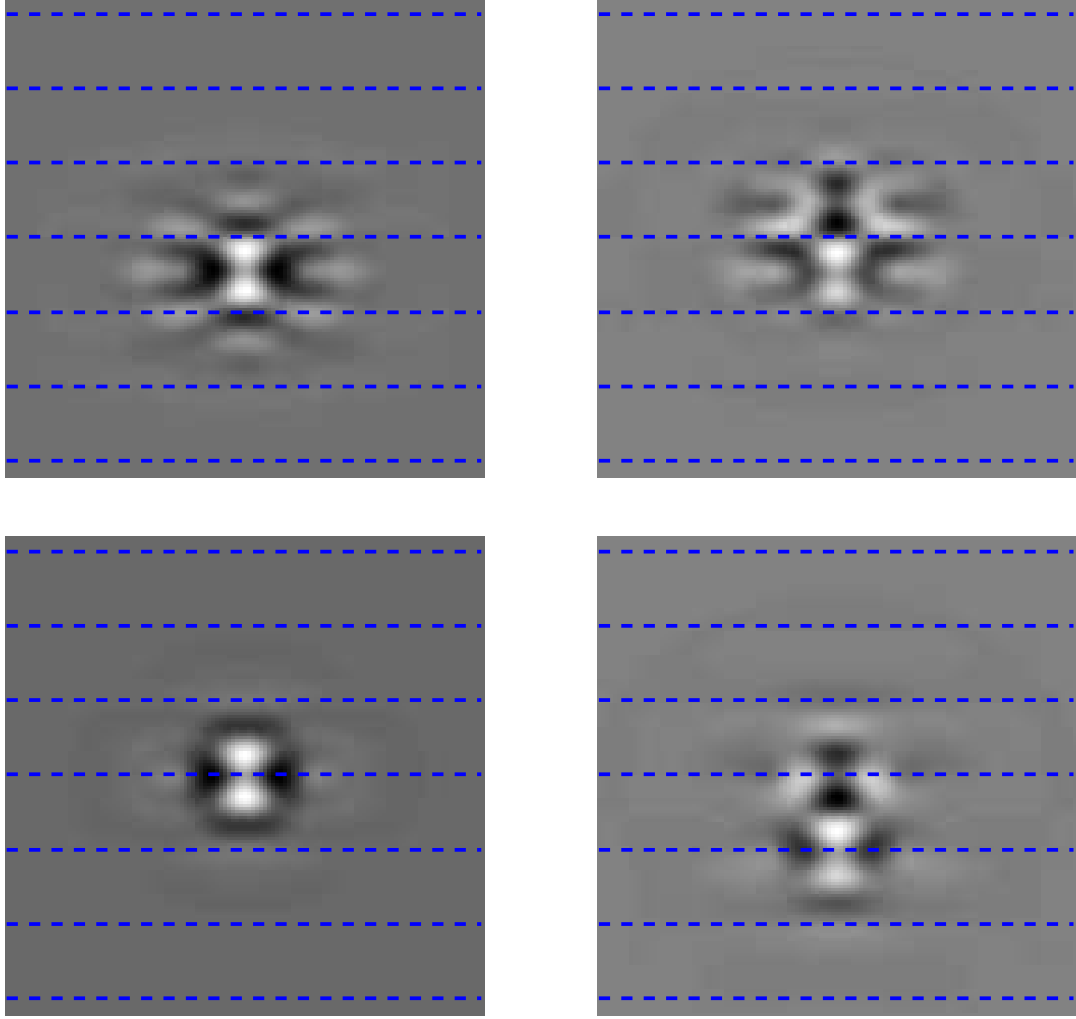


Figure 5.11: The four symmetric modes extracted at  $D_{III}$ . As in Fig. 5.9, the top two images show the modal structures sharing the largest growth rate, and the two subdominant modes are shown below. Again, the dashed lines show the approximate locations of hot upflow; the distance between adjacent dashed lines is one pattern wavelength.

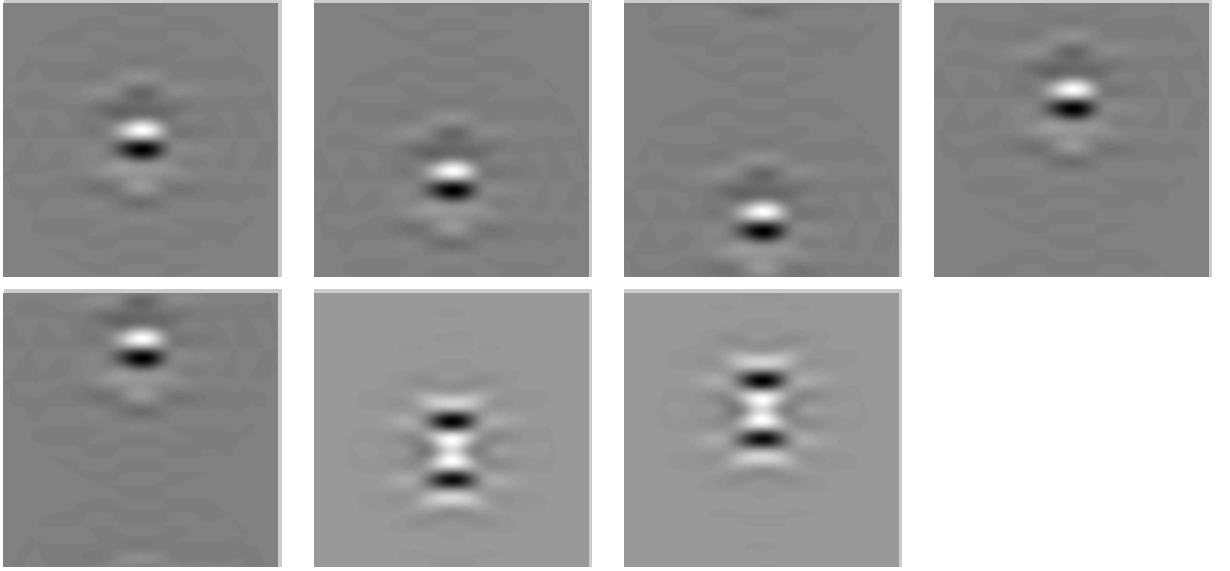


Figure 5.12: The set of possible representations of the dominant mode extracted at  $D_I$ . The most spatially localized (fundamental) mode is constructed from a linear combination of these modes.

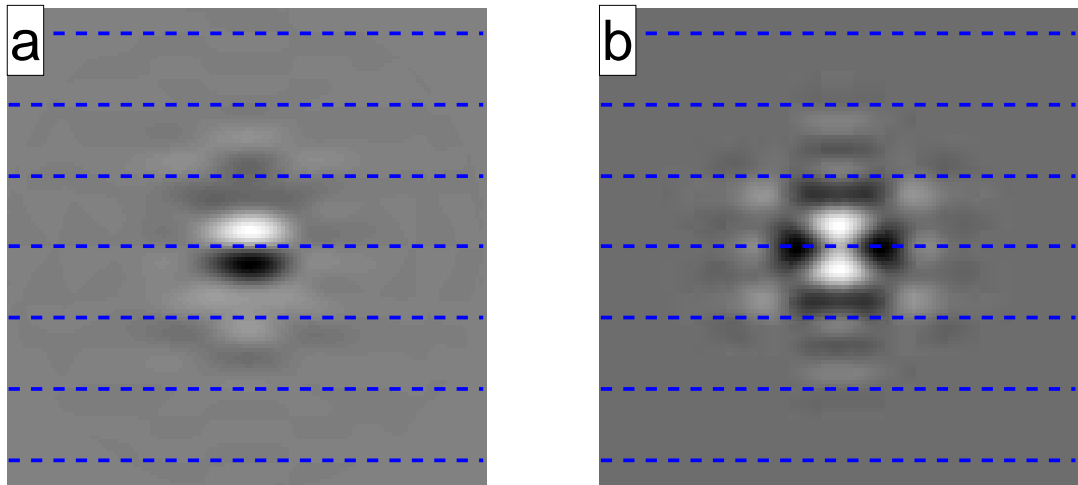


Figure 5.13: Fundamental representations of (a) the dominant and (b) sub-dominant mode high-wavenumber modes extracted at  $D_I$ .

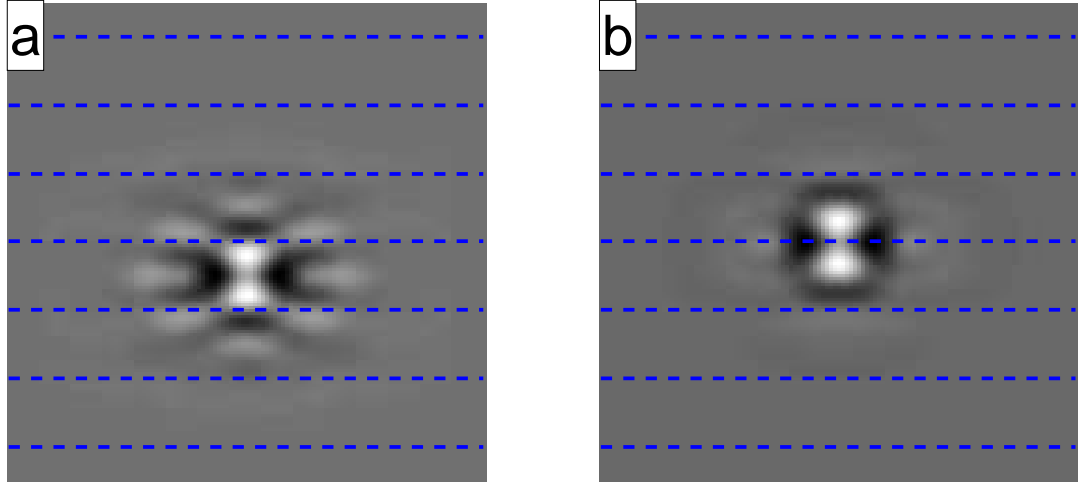


Figure 5.14: Fundamental representations of (a) the dominant and (b) sub-dominant mode high-wavenumber modes extracted at  $D_{III}$ .

To do this, it is likely necessary to use a very large ensemble of initial conditions in order to distinguish the heavily damped modes from background noise. An additional challenge to extracting modes with very small growth rates is the difficulty of separating the modes from possible Goldstone modes<sup>2</sup>, such as an overall translation or rotation of the pattern. It would be interesting to attempt an iterative version of this modal-extraction algorithm wherein the time range under consideration is incremented and/or modes are removed from the ensemble as they are converged.

While the observed spatial localization of defect creation/annihilation in chaotic convective states (such as spiral defect chaos) suggests a natural extension of the presented experimental approach to studying more complex convective patterns, the outlined procedure is general enough to be used in a variety of other dynamical systems, so long as an appropriate means of system actuation can be developed. It may be of particular use in systems where the governing equations are unknown or the system geometry make accurate modeling difficult (the voltage signals in cardiac

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<sup>2</sup>See, for example, the role of Goldstone modes in a nematic crystal system, as described in Ref. [53].



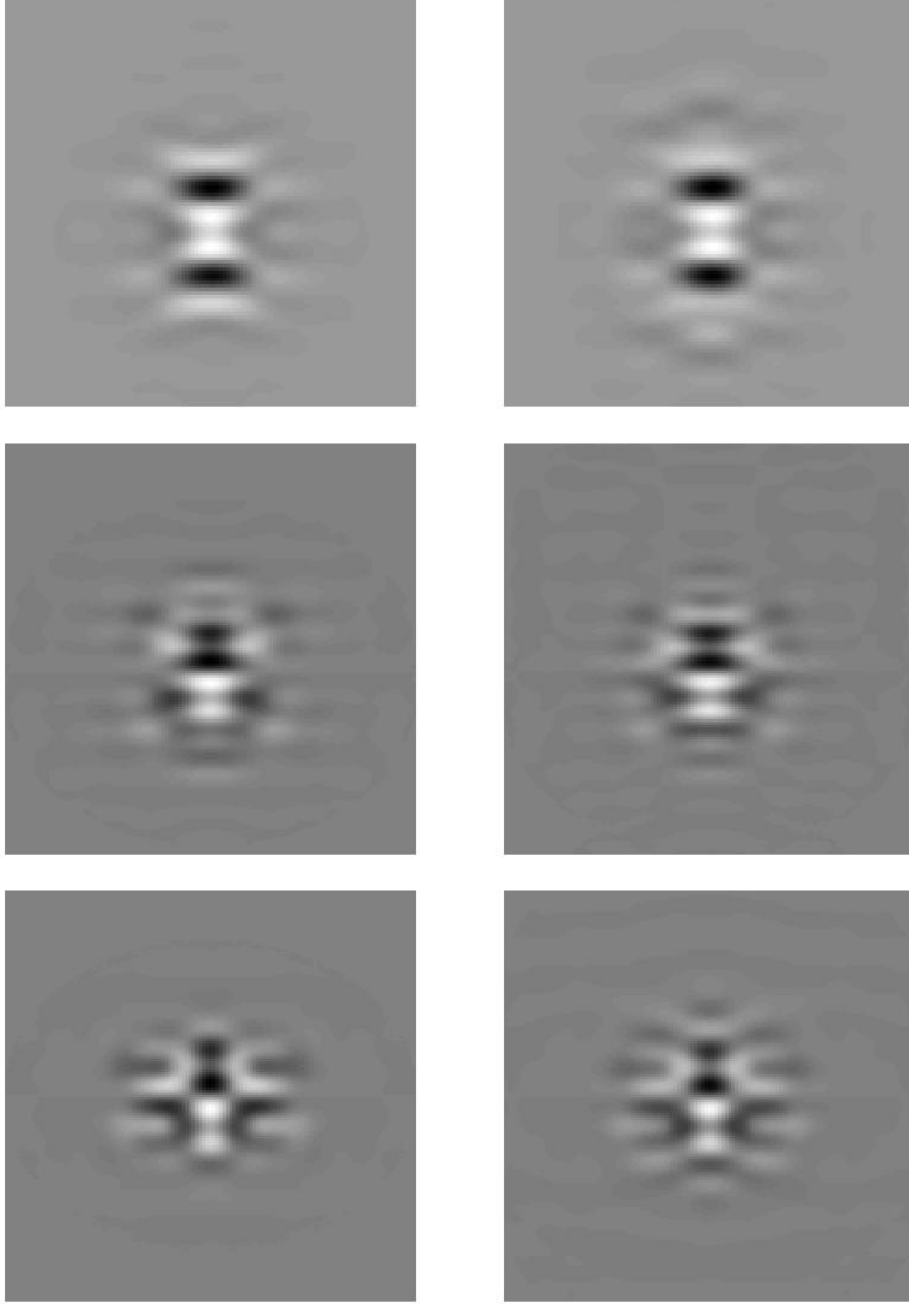


Figure 5.15: Examples of using the most spatially localized version of a mode to fit a more-extended version. On the left are the more-extended symmetric modes extracted, and on the right are the best fits using only translated copies of the corresponding spatially localized mode. From top to bottom, the modes are the dominant mode at  $D_I$  followed by the sub-dominant mode at  $D_I$  and then the dominant mode at  $D_{III}$ .

tissue is one such example). In addition to being of fundamental interest and of use in increasing predictive power, knowledge of the modes of instability could be particularly advantageous in system control, where small, controlled perturbations could be used to guide system dynamics [54].

## CHAPTER VI

### STATE ESTIMATION OF CHAOTIC PATTERNS

The Boussinesq model of Rayleigh-Bénard convection, derived from the well-known Navier-Stokes equations, has been shown to replicate much of the pattern behavior observed experimentally (see Refs. [55, 56] for a small sampling). Having high-resolution experimental access together with an accurate model of the governing equations makes RBC a good candidate for studying prediction in nonlinear systems. Moreover, the experimental technique developed as part of this thesis work provides the ability to impose patterns with nearby initial conditions, the evolutions of which can be used to investigate the limitations of certain parts of the prediction process. In this chapter, we present preliminary results of creating initial patterns with similar chaotic trajectories; these evolving patterns can be used to test the abilities of a recently developed state estimation algorithm.

State estimation refers to a step in the prediction process by which data is used to converge a model state to the true state of a system. While state estimation uses a system model (such as the Boussinesq equations) to evolve the model state forward in time as measurements are being taken, the way in which those measurements are used to converge the model state is considered as a separate process (known as data assimilation). The goal of state estimation is to converge on the true system state as accurately as possible, prior to forecasting the state forward in time according to the model<sup>1</sup>. The particular state estimation algorithm, known as the Local Ensemble

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<sup>1</sup>This is not strictly true, in the sense that the ultimate goal is to achieve accurate forecasts: the optimal state is that which, when evolved under the model equations, most accurately captures the behavior of the real system. When the model is accurate, this ceases to be a distinction.

Transform Kalman Filter (LETKF)<sup>2</sup>, takes advantage of the spatial-localization of dynamics on short times and has been tested on chaotic patterns in RBC [57]. This work is aimed at extending those tests by studying the ability of the LETKF to estimate accurately diverging states prepared with nearby initial conditions.

### ***6.1 Preparing Non-periodic Patterns***

Ordered patterns other than straight rolls are observed in our cylindrical convection cell ( $\text{SF}_6$  gas with plastic sidewalls of depth  $608\text{ }\mu\text{m}$  and an aspect ratio  $\Gamma = 20$ ). Two of the most common, which will be studied in this chapter, are the target and PanAm patterns. The target pattern emerges in our cell near onset from sidewall forcing, which imparts the circular symmetry to the convection rolls. An example is shown in Fig. 6.1. This axisymmetric pattern is stationary at small  $\epsilon$ , but as  $\epsilon$  is increased, eventually loses stability as the inner rolls move off center, causing a compression of the rolls in one direction [58]. Often, the final result is a large, slowly evolving spiral. A PanAm pattern resembles the logo of its namesake, the former airline; an example is shown in Fig. 6.2 in a small aspect ratio experiment [4]. The prototypical PanAm pattern is characterized by two foci formed at opposite sides of the cell, from which rolls appear to fan out, meeting the sidewalls at nearly  $90^\circ$  angles. We will generalize the use of the PanAm name to refer to patterns with a small number of wall foci and possibly other defects. In small aspect ratio cells, the PanAm pattern has been observed to be time independent or even time-periodic [4], with a repeating skew-varicose instability between the center rolls. In cells of larger aspect ratio (such as ours), it is common to observe time-evolving PanAm patterns, with multiple wall foci. An extensive accounting of the experimental conditions under which one observes target, spiral, and PanAm patterns is given in Ref. [59].

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<sup>2</sup>The algorithm and the actual computer code implementation have been developed by our collaborators at the University of Maryland.



Figure 6.1: An axisymmetric target pattern is observed in our convection cell near onset (cell aspect ratio  $\Gamma = 20$ , and  $\epsilon = 0.10$ ).

We wish to develop a reference convection pattern that can be imposed experimentally, from which there is sustained time-dependence. With small changes to that initial pattern, or with additional selected perturbations, the global evolution can be changed, thereby providing evolutions with which to test the state estimation sensitivity.

### 6.1.1 Imposing PanAm Pattern

We first consider using as an initial pattern the relatively simple PanAm pattern with 2 wall foci, which we do not observe occurring naturally and which can therefore be expected to undergo instability. We have found that, as  $\epsilon$  is increased, a pattern with rolls initially parallel to the sidewalls gives way to one with rolls perpendicular to the sidewalls. Near this transition ( $\epsilon \sim 0.5$ ) is where we see the emergence of a time-dependent PanAm pattern. An ideal PanAm pattern is constructed in terms of

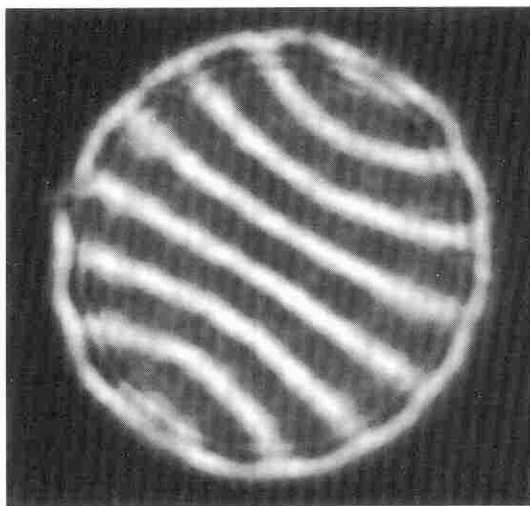


Figure 6.2: An example of a stationary PanAm pattern in a small aspect ratio ( $\Gamma = 7.66$ ) cylindrical cell of argon gas,  $\epsilon = 0.05$ . Reproduced from Ref. [4], with permission.

the locations of upflowing fluid; it is to these locations that laser light is directed. The imposition process is similar to that by which the straight roll pattern was achieved: a line of laser light is directed to the desired location of upflow of each convection roll at intervals of about 1 s, until the global pattern emerges. Snapshots from the time series over which the PanAm imposition takes place are given in Fig. 6.3.

Ultimately, at this aspect ratio (and this  $\epsilon$ ), a PanAm pattern with two foci can only be achieved and maintained with continual lasing. Even then, we still observe defects at the wall; in order for the roll curvature to meet the sidewalls at right angles, the pattern will tend to introduce defects at either the sidewalls or in the center of the pattern. Because it is undesirable to introduce dynamical effects through excess actuation (that is, *uncontrolled* dynamical effects), we instead consider imposing an initial target pattern.

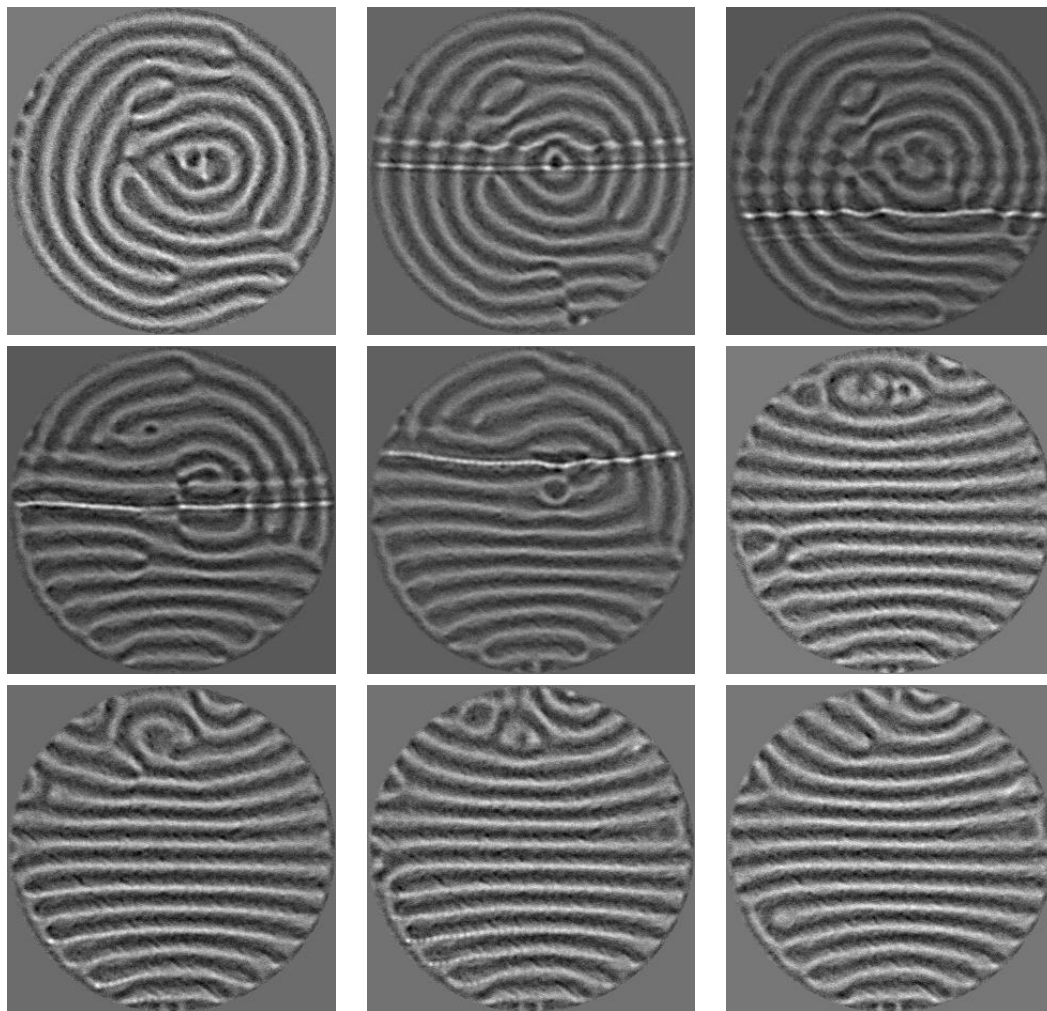


Figure 6.3: Snapshots showing the imposition of a panam pattern (in time, from left to right and top to bottom). Here, and in all other circular convection patterns in this chapter,  $\Gamma = 20$  (except where noted).

### 6.1.2 Imposing Target Pattern

The target patterns that emerges naturally at fixed  $\epsilon$  do not display time-dependence; our approach is to instead impose a target pattern over an existing, time-evolving pattern. Images in Fig. 6.4 show a series of shadowgraphs during imposition of the target pattern, with  $\epsilon$  again near 0.5.

To be able to prepare patterns with nearby initial conditions and similar trajectories, in a controlled way, it is first important to test the ability to prepare an initial pattern with a repeatable evolution. We have tested that an imposed axisymmetric target pattern will, as expected, undergo an instability wherein the inner rolls move off center, but the direction which they translate is apparently random, as it should be for the symmetric pattern. We introduce a bias to the initial pattern by drawing a slightly oblong pattern; that is, the pattern is slightly more compressed in one radial direction. This preferentially selects a particular direction for the rolls to further compress following the initial imposition. We show in Fig. 6.5 side-by-side snapshots during the evolutions of two different realizations of initially off-center target patterns. The images illustrate the similarity of the two pattern evolutions, with localized instabilities taking place at the same locations and times. The second pair of images in this time series shows the first instabilities, about two minutes after the initial pattern (the first pair of images); subsequent images are taken at approximately 4 s intervals.

However, pattern evolution is sensitive to changes in the initial conditions, which can be caused from either changes in the initial imposed pattern or substantial forcing from the incident laser light. A second pair of pattern evolutions (over three minutes) is shown in Fig. 6.6 and continued in Fig. 6.7. The initial pair of images is taken a full two minutes after the initial pattern imprinting; the next several images are taken every several seconds, showing the (relatively) rapid evolution of the patterns as they undergo instability. Both initial patterns were imposed following the same reference



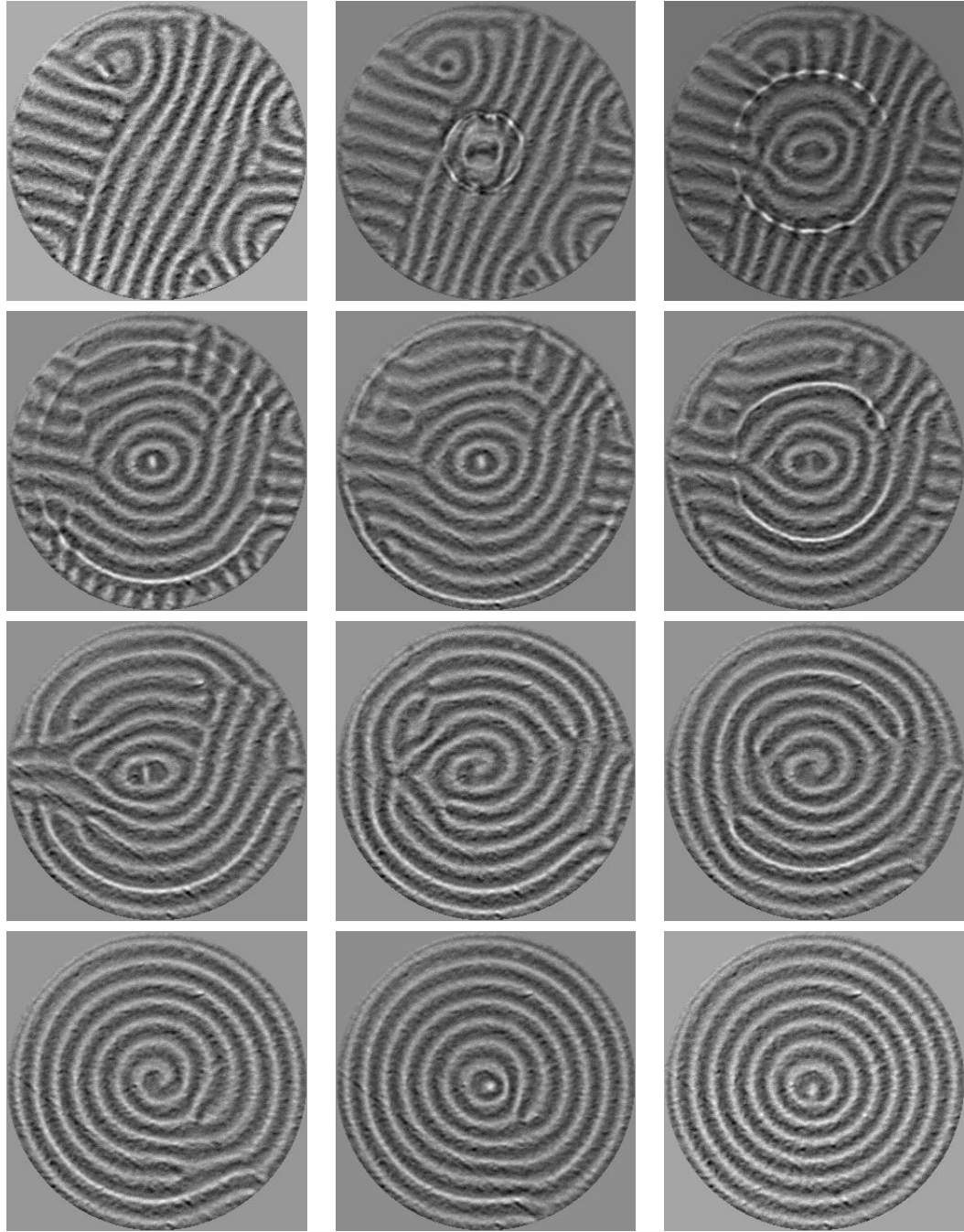


Figure 6.4: Snapshots showing the imposition of a target pattern.

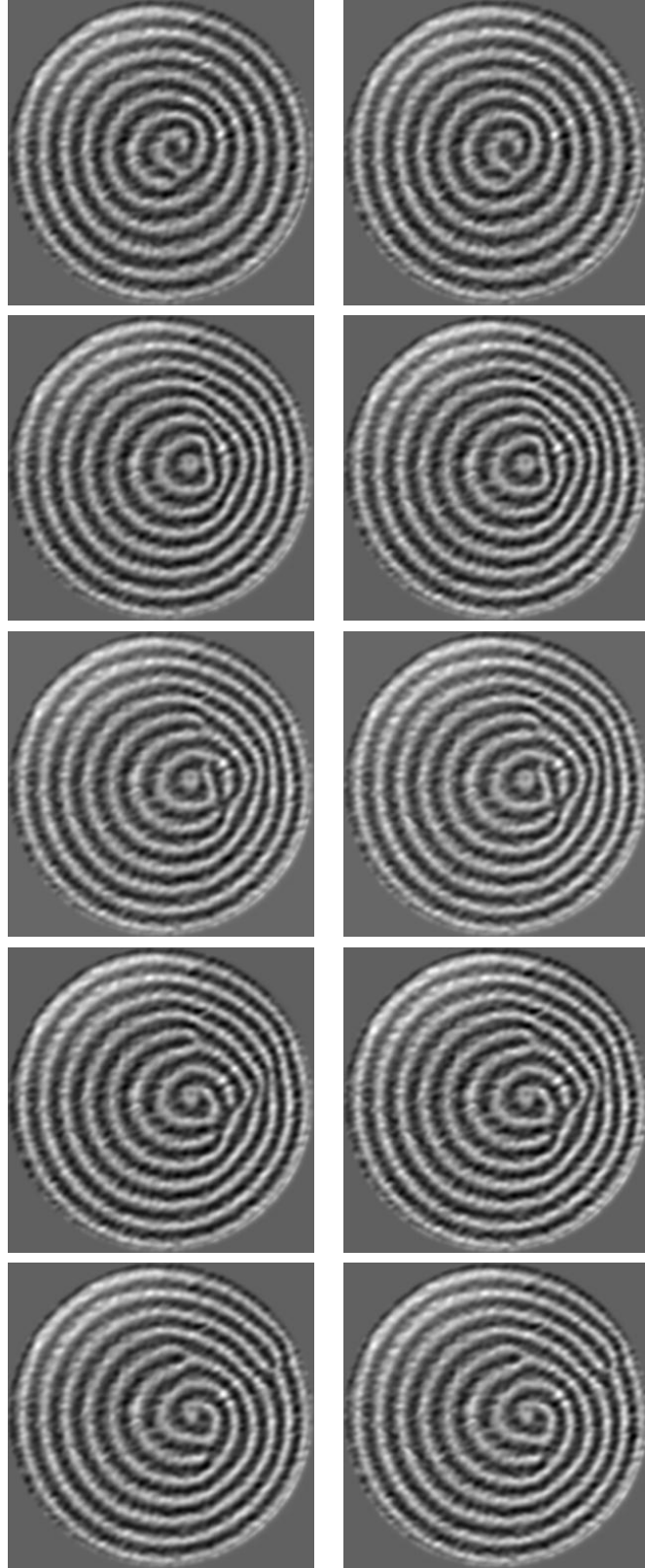


Figure 6.5: Shadowgraphs showing the evolution of two patterns from nearly identical imposed patterns. The images span over two minutes.

pattern as in Fig. 6.5, but the initial conditions were changed relative to that set by turning the lasing off after a slightly different amount of time<sup>3</sup>. Again, these two patterns show very similar trajectories; it is the slight initial differences, like those between these patterns and those in Fig. 6.5, that need to be resolved in order to consistently reproduce a given pattern evolution.

### 6.1.3 Long-term Pattern Evolution

Having been able to impose a target pattern with relative success, we examine the long-term behavior of an initial target pattern, to confirm that the pattern does not evolve toward a stationary pattern over long times. As mentioned in Chapter 3, the horizontal diffusion time is the time-scale over which diffusive processes occur over the lateral cell domain,  $t_h = \Gamma^2 t_v$ . Starting from a random initial pattern, this is the minimal time over which the pattern can be expected to evolve toward a steady-state pattern, if one exists. In our cell,  $\Gamma = 20$ , meaning  $t_h = 400t_v$ . Fig. 6.8 shows, at left, a pattern shortly after  $\epsilon$  has been increased from near zero to a value of 0.60. The asymmetric target pattern is a remnant of the target pattern that emerges close to onset. Shown in the center image, taken after  $4t_h$ , the pattern has lost any sign of circular symmetry, rolls instead forming right angles with the physical sidewalls in most places. The image on the right is taken  $30t_h$  after the initial image. These shadowgraphs give no indication that there is a time-independent state at  $\epsilon = 0.60$ .

---

<sup>3</sup>The process of imposing the target patterns here is open-loop, in that the laser is turned off manually once an initial pattern has been imposed. The difference in initial conditions is therefore largely due to the unintended difference in lasing time introduced by judging patterns by eye. Clearly, a more systematic (closed-loop) approach to pattern preparation will aid in increasing the level of reproducibility.

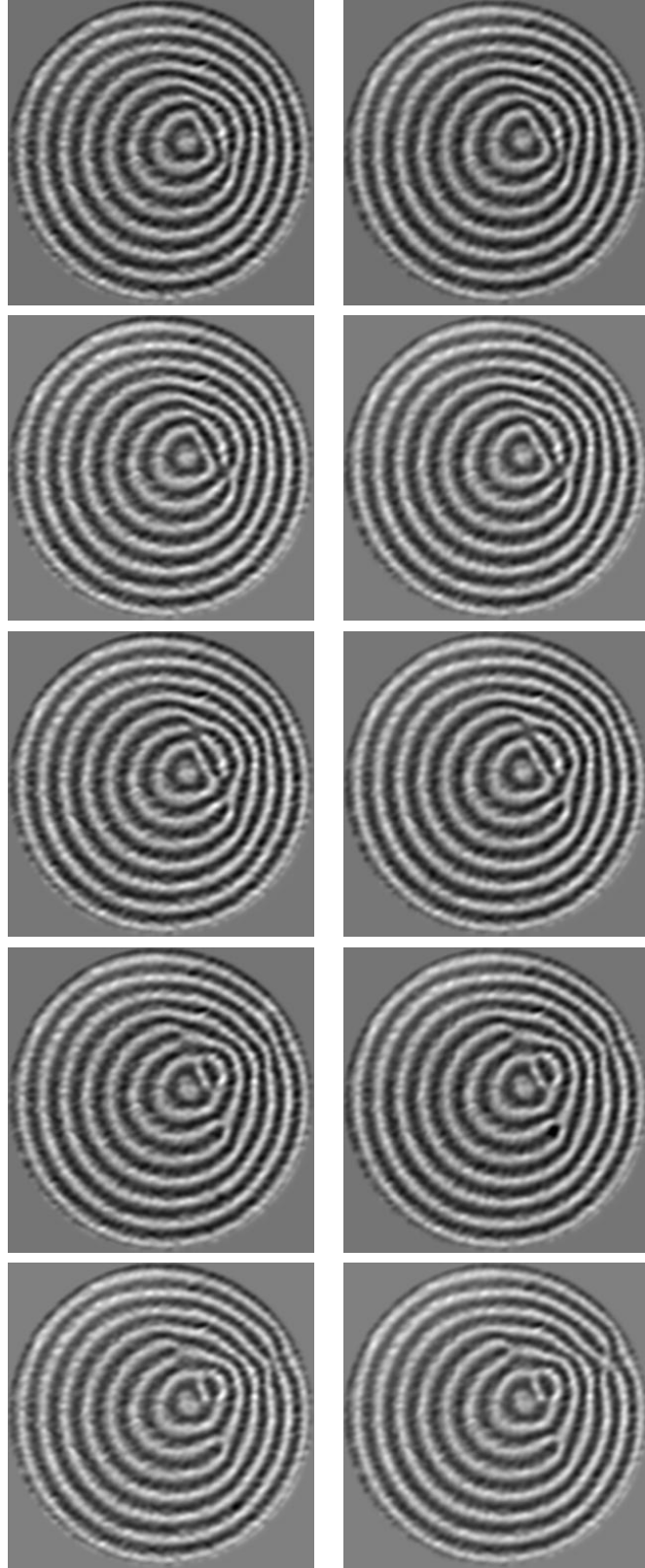


Figure 6.6: Shadowgraphs showing the evolution of two patterns from nearly identical imposed patterns, continued in Fig. 6.7.

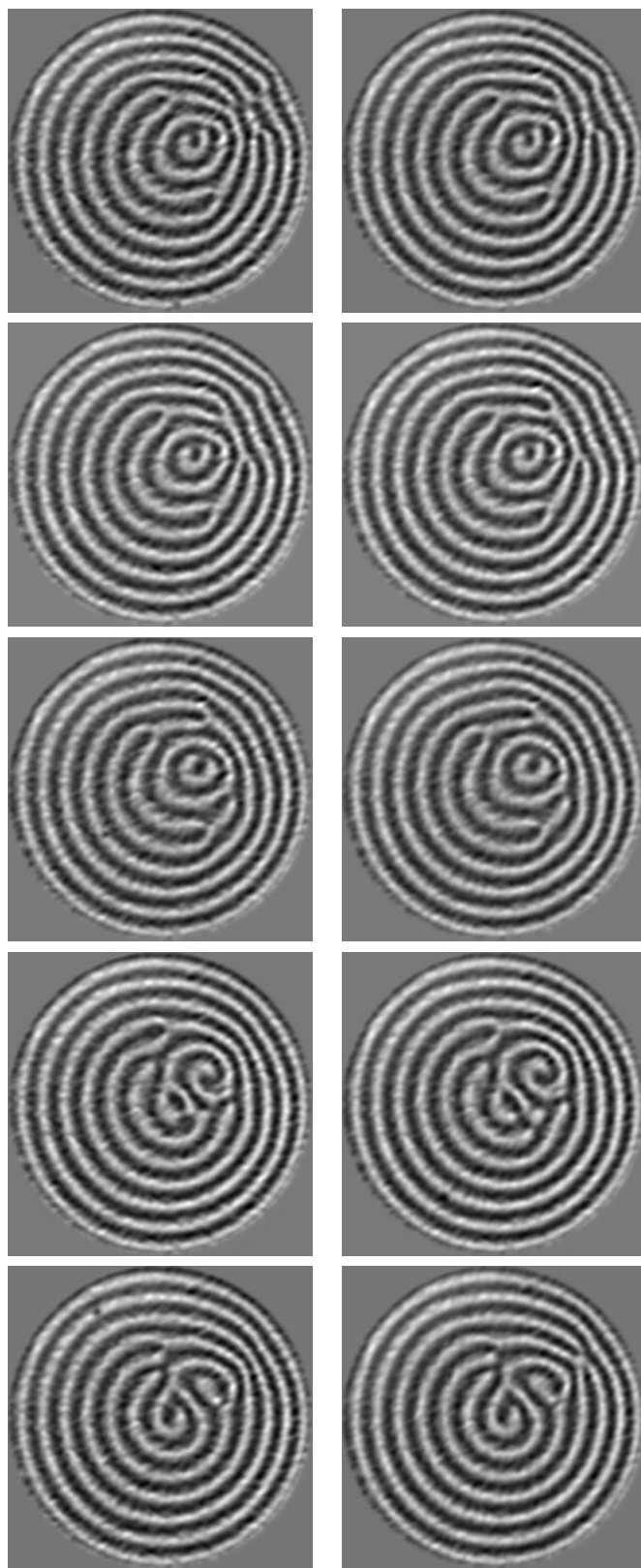


Figure 6.7: The continued evolution of the patterns in Fig. 6.6. The images were taken over the course of three minutes.

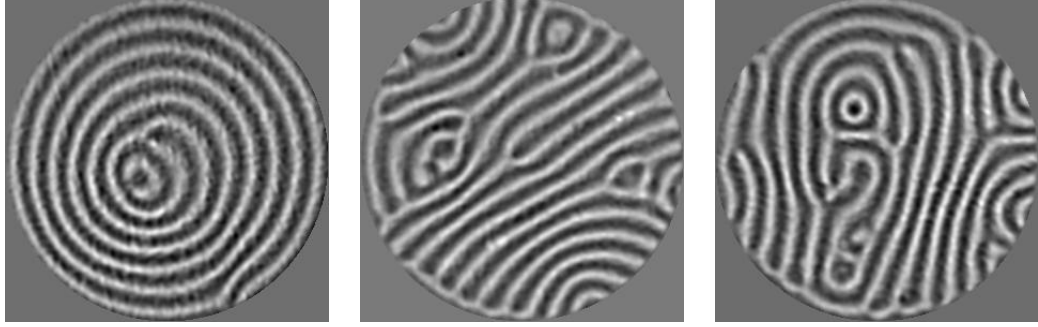


Figure 6.8: Snapshots at  $\epsilon = 0.60$ , showing sustained time-dependent patterns. Relative to the pattern on the left, the center pattern is taken after  $4t_h$ ; the pattern at the right is taken after  $30t_h$ .

## 6.2 Pattern Forecasting

Armed with a model of the governing equations for a system, one is able to make forecasts to future times, but first, one must use experimental measurements to determine accurately an initial system state. A whole class of algorithms devised to tackle this state estimation problem is based on variations of the Kalman filter. We provide below some background and motivation for the development of the LETKF; the reader is referred to Refs. [57, 60, 61, 62, 63] for historical and computational details.

### 6.2.1 Kalman Filtering

Consider a system state denoted by the vector  $\xi$ , together with some model of the governing equations  $\mathbf{M}$  such that  $\xi_{i+1} = \mathbf{M}(\xi_i)$ , where the index  $i$  is taken to refer to a time step. It may be the case that measurements do not access the state variables directly; rather, there is an observation mapping  $\mathbf{H}$  such that one makes measurements of  $\mathbf{H}(\xi)$ . Essentially, the task of the Kalman filter [64] is to find a trajectory through phase space evolving according to the model equations which, when projected into the observation space, best fits the time series of experimental measurements. The Kalman filter (built on Bayesian inference) does this by using measurements to evolve a Gaussian probability distribution around the best guess of the state; the

distribution is captured by a mean state and the covariance of deviations about that state. This process is recursive; as more measurements are assimilated, the model state eventually converges to the true state of the system, at which point forecasts can be made by integrating the model state forward in time.

**Extended Kalman Filter** When the model  $\mathbf{M}$  and the observation mapping  $\mathbf{H}$  are nonlinear, the probability distribution can no longer be expected to remain Gaussian over time. However, if the nonlinearities are weak or measurements are sufficiently frequent, it may work to linearize both the model and the observation transform about the current state, regaining the original Kalman filter equations. This additional step results in the so-called Extended Kalman Filter (EKF).

**Ensemble Kalman Filter** The computational limitation of the EKF is that, at some point in the minimization of the cost function, there is a required inversion of an  $N \times N$  matrix, where  $N$  is the number of observation locations (which can easily be of order  $10^5$  or more). The Ensemble Kalman Filter (EnKF) reduces the dimension of state representation by using an ensemble of state vectors from which a linear combination is formed to produce the best guess. The number of ensemble members  $k$  must be large enough to account for the dynamical degrees of freedom but, presumably, this will be smaller than  $N$ . In addition to reducing matrix sizes, this formulation has the feature that each of the ensemble members evolves under the full nonlinearity of  $\mathbf{M}$ ; effectively, the linearization exists in the approximation that the actual state can be represented by a linear combination of nearby states.

### 6.2.2 LETKF

As the number of degrees of freedom increases, the necessary EnKF ensemble size also grows and can reach a point where the computational bottleneck occurs in the simulation of all ensemble members. Recent evidence suggests, for example, that

the number of degrees of freedom of the spiral defect chaos convection state scales extensively with system size [65, 3, 66, 67]; it has been estimated that one would need to simulate  $\sim 1000$  ensemble members to perform an EnKF state estimation of this state. However, there also exists evidence from the SDC state that system dynamics can be spatially correlated over a small fraction of the total system size [28]. These observations are expected to be common properties of systems with spatiotemporal dynamics and thus act as motivations for the development of the LETKF<sup>4</sup>. The LETKF operates in much the same way as the EnKF, but instead of fitting the state with global ensemble members, the fitting in is done over many overlapping subsystem volumes, so that only measurements in a spatially localized region about any given grid point are considered when updating the model state. The main benefit of this approach is that the number of ensemble members needed is only enough to describe the (relatively) small number of degrees of freedom in any given small area of the system.

**Parameter Estimation** The model we use is the set of Boussinesq equations, which evolve temperature and velocity fields. In our experiments, measurements of the system take the form of shadowgraph intensity maps, related to the temperature field  $\theta$  by

$$I(x, y) = \frac{I_0(x, y)}{1 - 2dz_1 \left| \frac{dn}{dT} \right| \nabla_{\perp}^2 \bar{\theta}(x, y)} \quad (6.1)$$

where  $I(x, y)$  is the incident intensity,  $d$  is the cell depth,  $\nabla_{\perp}^2$  is the horizontal Laplacian,  $\bar{\theta}(x, y)$  is the integral of  $\theta(x, y, z)$  over the fluid depth, and  $z_1$  is the effective shadowgraph optical distance (see Appendix C for a derivation). We define the shadowgraph parameter  $a = 2dz_1 \left| \frac{dn}{dT} \right|$ . Note that the velocity field is updated only

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<sup>4</sup>Although we present the motivation in the context of convection, the LETKF is motivated by and expected to be applicable to a variety of chaotic systems, including the weather.



through its coupling to the temperature field inside the Boussinesq model. In addition to the temperature and velocity fields, the state estimation algorithm can also be used to predict global variables such as this shadowgraph parameter  $a$ , as well as the Prandtl and Rayleigh numbers. This is done by a simple extension of each state vector to include a variable component for each global parameter being fit.

### ***6.3 Preliminary Results***

A time series of shadowgraph images, with  $\epsilon = 0.60$  are assimilated into the LETKF at a rate of every  $t_v/4$  (2 Hz); the LETKF uses these measurements to update an ensemble of model state vectors. Here we use  $k = 8$  ensemble members. In between assimilation times, each ensemble state is integrated forward according to the Boussinesq model. The best guess of the real state is formed by an average over the ensemble.

To gauge convergence of the state, we attempt to fit the global Rayleigh number as well as the shadowgraph parameter  $a$  and look at the spread of those parameter values over the  $k$  ensemble states. In Fig. 6.9, we show the average Rayleigh number over the ensemble, along with the standard deviation about the mean. The same information is shown in Fig. 6.10 for  $a$ . Although the plots of deviations suggest that the state has converged in just a few  $t_v$  (consistent with earlier tests [57]), the behavior of the average values suggests otherwise. The estimated values for the Rayleigh number ( $\sim 2500$ ) and  $a$  ( $-0.05$ ) are, however, in the range of expected values. Note that the estimated Rayleigh number is smaller than the true Rayleigh number (for  $\epsilon = 0.60$ ,  $Ra = 2733$ ); this was also observed in Ref. [57] and was attributed to the algorithm implicitly accounting for possible model errors (non-Boussinesq effects) by using the Rayleigh number that allows the times series of measurements to be best fit.

Because the measurement is in the form of shadowgraph images, the ultimate comparison is of the actual shadowgraphs with those projected from the computed

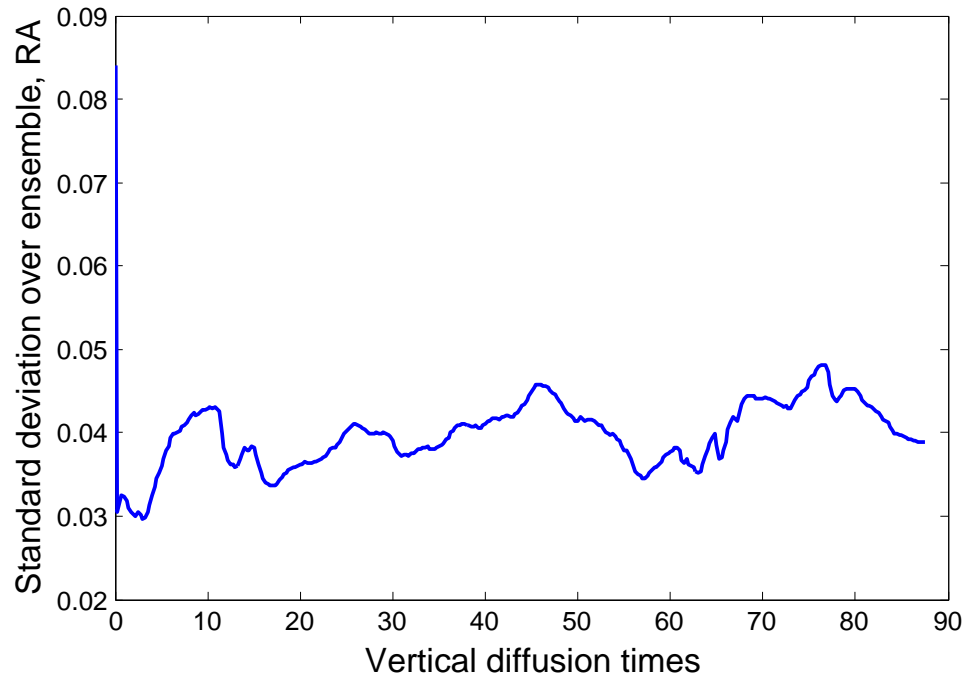
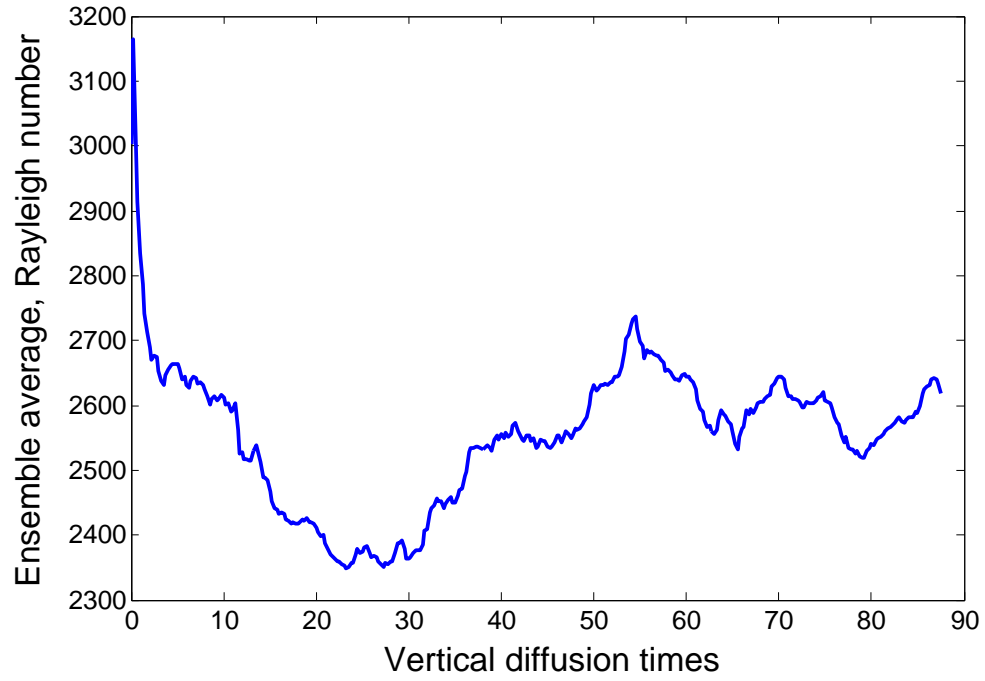


Figure 6.9: On top, the average Rayleigh number of the ensemble, as shadowgraphs are assimilated; below, the standard deviation in the ensemble Rayleigh numbers, relative to the average Rayleigh number at that time.

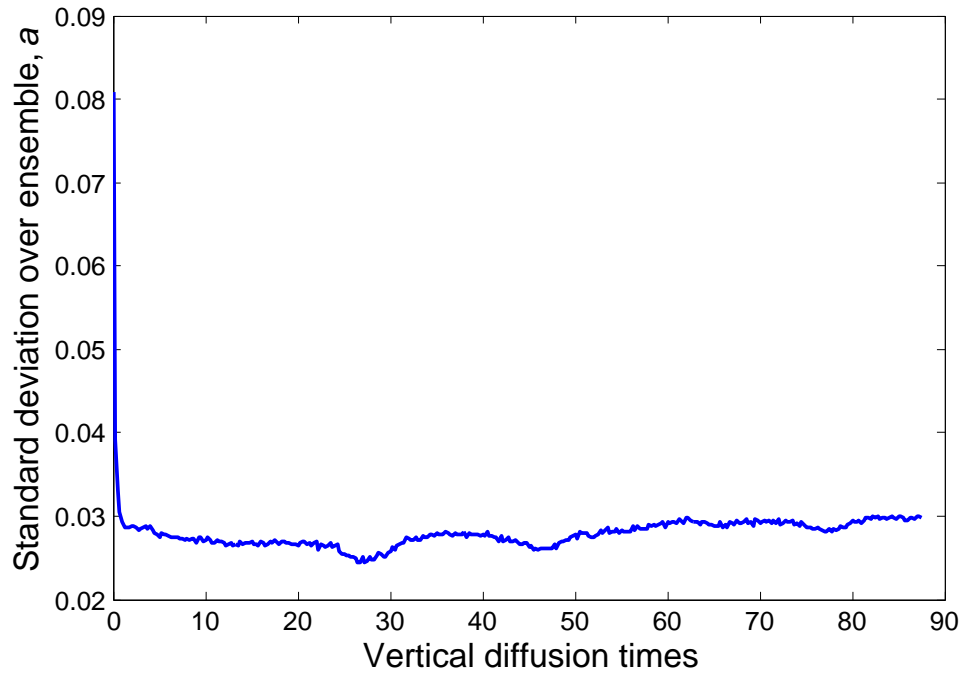
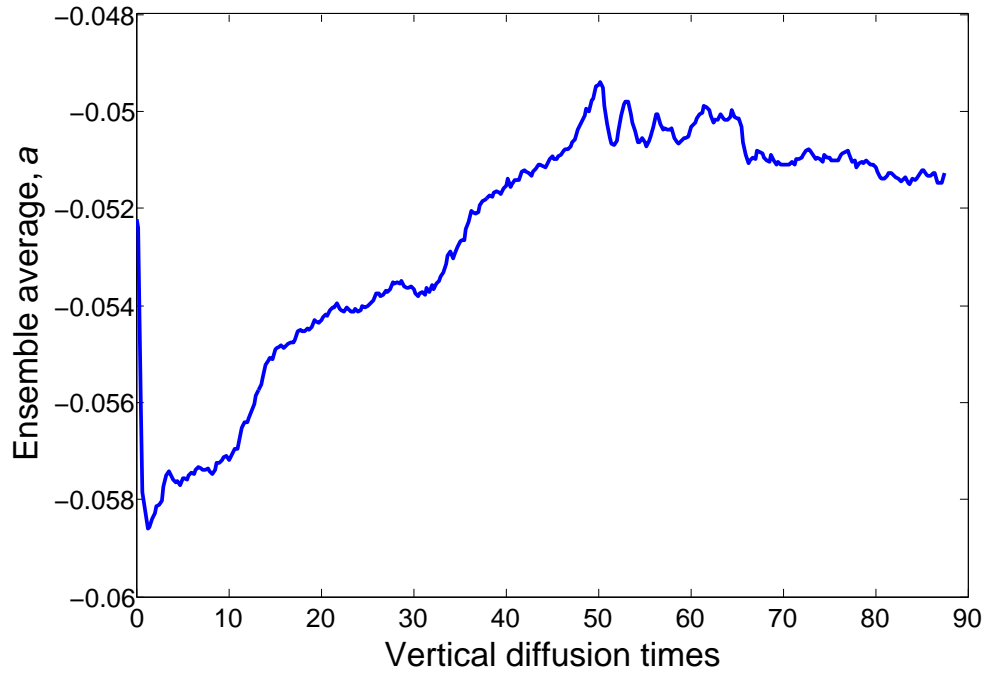


Figure 6.10: On top, the average value of the shadowgraph parameter  $a$  over the ensemble, as shadowgraphs are assimilated; below, the standard deviation in the ensemble  $a$  values, relative to the average value at that time.

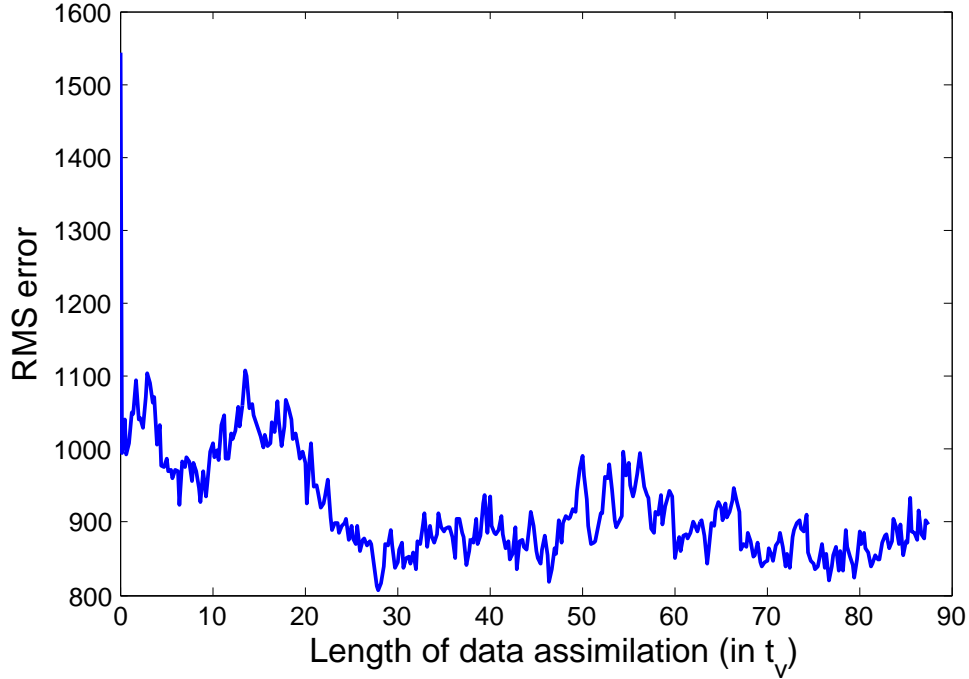


Figure 6.11: The root mean square error between the predicted and observed shadowgraph over assimilation (arbitrary units).

state. We define an rms (root mean square) error between the actual and predicted shadowgraphs at each time step; this is plotted in Fig. 6.11. After the first few  $t_v$ , the error shows little behavior, also suggesting convergence at early times. A few of the shadowgraphs are presented in Fig. 6.12. The best fit of the measurements by the LETKF effectively allows the measurement noise to be filtered out. The predicted shadowgraphs appear to differ from measurements most at times when the state is undergoing instability (center pair of images), but the discrepancies disappear with continued assimilation.

While these preliminary results are promising, there are several things undone. The next step, experimentally, is to increase the level of control over the initial conditions and develop an approach to creating a selected ensemble of patterns. While the fast convergence of the model state is important, the ultimate goal is to produce

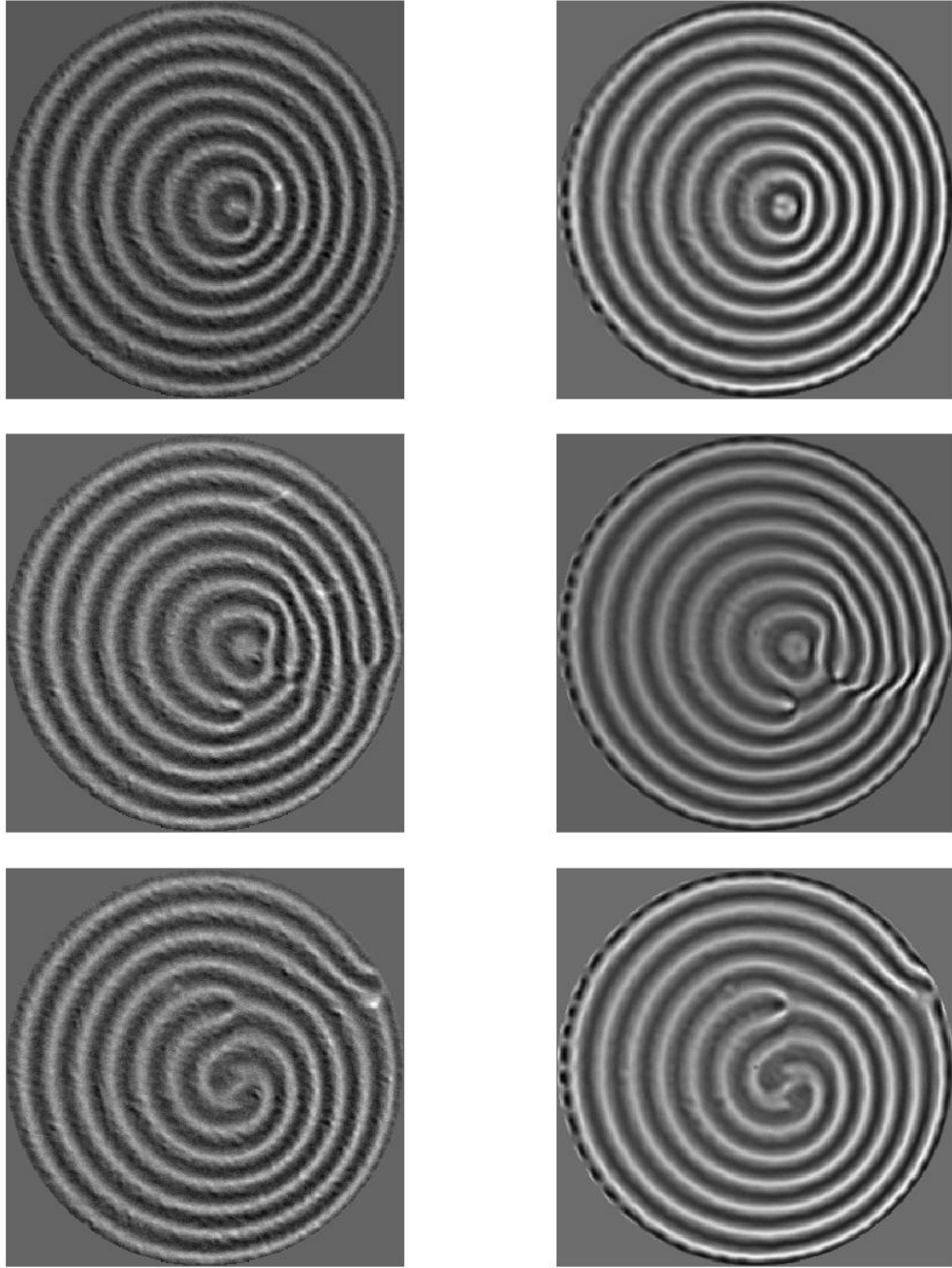


Figure 6.12: On the left are the actual shadowgraphs; on the right are those computed from the model state. These three images are taken at  $30t_v$ ,  $50t_v$ , and  $56t_v$  (from top to bottom).

better forecasts. With that idea in mind, and with an ensemble of pattern evolutions in hand, one can test the state estimation by comparing not only the convergence across the experimental ensemble, but also the accuracy of forecasts.

## CHAPTER VII

### CONCLUSIONS

Rayleigh-Bénard convection continues to be an extremely useful system for testing and developing ideas about pattern formation and dynamics. The technique of pattern actuation described herein takes advantage of the greenhouse effect of a commonly used convective fluid,  $\text{SF}_6$ , to provide the ability to repeatably create patterns experimentally. This adds one more tool to already well-established methods for studying the dynamics of convective patterns. Through pattern manipulation, we were able to repeatably prepare straight roll patterns near secondary instabilities, and excite the spatially localized dynamical modes by imposing selected perturbations to the patterns. The distance from onset of a particular localized instability (the skew-varicose) was quantified in terms by measured perturbation lifetimes. We extracted the dominant spatial modes and their growth rates directly from the ensemble of shadowgraph evolutions, at three locations in  $(q, \epsilon)$  phase space; the modes were concluded to be spatially localized versions of the global instabilities of infinite, ideal systems. We extended the pattern imposition to non-periodic spatial patterns with a circular boundary and showed preliminary results of estimating the fluid state (using the LETKF) from these prepared patterns.

Our approach to extracting dynamical information by studying the evolutions of many states with nearby initial conditions, although prevalent in theoretical and numerical investigations, has been limited experimentally because of the difficulty in preparing a state with a given set of initial conditions. We showed that, using an experimentally created ensemble of nearby states, the dominant system modes can be extracted directly from experimental measurements. The fact that the modes were

extracted without the use of a model set of governing equations suggests that the approach could extend to systems where the governing equations are unknown or the system geometry makes numerical modeling difficult. While pattern manipulation through optical means is specific to our experiment, we expect the approach to be applicable to a variety of systems, as long as an acceptable means of actuation can be devised. Striking examples of candidate systems exist in biological tissue, where control and prediction of the dynamics of voltage pulses in cardiac tissue or neurons are major goals [68].

It would be an improvement to the current experiment to add the ability to actuate the convective flow from above. Specifically, it would be very interesting to see if one would find any new dynamical response of straight roll patterns to perturbative heating from above. Early actuation tests indicated that this type of heating may suppress convection, as it effectively reduces the local temperature difference. Selective heating of both boundaries would likely lead to an increased level of pattern control, which would be helpful for studying more complex patterns.

Ultimately, a main goal is to extract dynamical modes systematically from a state exhibiting chaotic or even fully turbulent behavior. One can imagine a back-and-forth between using control to extract dynamical information and using that information to enhance system control. The rich variety of dynamics observed in Rayleigh-Bénard convection suggests a natural extension of our approach to extracting experimentally the dynamical modes of more complex convection states, such as spiral defect chaos, where spatially localized instabilities occur in regions of locally parallel rolls. While the global number of degrees of freedom may scale extensively with system size, it is likely that the number of distinct modes is some fraction of that total; distant spatially localized modes may be related in terms of translations, reflections, or rotations. Just as we saw during modal extraction of the straight roll state, we can expect considerations of symmetry to be useful in building a minimal description of the



important modes.

Chapter 6 lays the groundwork for further study into the prediction process. For example, how much measurement information is needed for an accurate convergence of a model state onto an experimental state near instability? Experimentally, the next major step is to refine the process of preparing initial patterns, pushing the limits of experimental repeatability of the initial conditions. The preliminary results are promising, in that we did observe a high degree of reproducibility in the evolutions of select imposed patterns. Because it is primarily through roll breakings and connections that patterns rearrange, we may expect the state estimation process (and thus prediction in general) to be sensitive to these instability events. Control over the pattern differences would allow one to test the predictability in a systematic way.

It is worthwhile to note that both the modal extraction and the state estimation procedure highlight an emerging way of thinking about the evolution of dynamical systems by embedding observed states in a space spanned by a well-chosen basis [69]. In the case of weakly stable modes of straight rolls near instability, a good choice is a basis formed from the slowly decaying structures; in state estimation, it is an evolving ensemble of states in the neighborhood surrounding the estimated state. Embedding system dynamics in a set of representative solutions is helpful by not only reducing the dimensionality of representation but also by emphasizing that understanding of the system evolution may simplify to understanding of special system solutions.

Together, accurate state estimation and knowledge of the system's important dynamical modes would provide enormous possibility for system control. As suggested in Ref. [54], the ability to control a nonlinear/chaotic system offers a significant advantage over a linear system. Namely, it is possible that a large variety of system behavior can be achieved, for the purpose at hand, through the careful choice and application of small changes to the system. Moreover, in systems where the important modes tend to be spatially localized, it follows that changes in the global state may

be induced by the proper choice of actuation on a sub-region of the system.

## APPENDIX A

### GOVERNING EQUATIONS

#### *A.1 Navier-Stokes*

The fluid is governed by the Navier-Stokes equations as well as the heat equation. This provides a set of three equations which describe, respectively, the movement of mass, momentum, and heat through the fluid volume:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (\text{A.1})$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \mu \nabla^2 \mathbf{u} \quad (\text{A.2})$$

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \kappa \nabla^2 T \quad (\text{A.3})$$

The first continuity equation is simply the conservation of mass in differential form which, for constant density, reduces to  $\nabla \cdot \mathbf{u} = 0$ . The second equation describes the conservation of momentum, and the third equation is a consequence of the conservation of energy.

#### *A.2 Uniform solution*

We will analyze the RB system in which two horizontally infinite plates are fixed at  $z = \pm \frac{1}{2}d$ . The boundary conditions appropriate for experimental conditions are that the temperatures at the top and bottom boundaries are fixed with  $\Delta T = T_{bot} - T_{top}$  and vanishing velocity at the rigid boundaries. The uniform state that meets these boundary conditions is a motionless state with a linear conducting profile

$$\mathbf{u} = 0 \quad (\text{A.4})$$

$$T_{cond} = T_{cond}(z) = \bar{T} - \frac{z}{d} \Delta T \quad (\text{A.5})$$

There will be small variation in the fluid density due to the temperature difference, so we expand the density

$$\rho(T) \approx \rho(T_0) + \frac{\partial \rho}{\partial T}(T - T_0) = \rho_0[1 - \alpha(T - T_0)] \quad (\text{A.6})$$

where  $\alpha = -\frac{1}{\rho} \frac{\partial \rho}{\partial T}$  is the coefficient of thermal expansion of the fluid. Note that while the density fluctuations will ultimately give rise to a non-uniform state, the incompressibility condition still holds reasonably well. That is, the velocity of the fluid will be much less than the speed of sound. The assumption that we can ignore the variation of other fluid properties (thermal diffusivity, viscosity, etc.) with temperature is known as the Oberbeck-Boussinesq approximation.

Substituting this state into the momentum equation, with the time-derivative set to zero, gives an equation for the pressure

$$\frac{\partial p}{\partial z} = -\rho_0 g(1 - \alpha \Delta T \frac{z}{d}) \quad (\text{A.7})$$

which can be integrated to give the steady pressure

$$p_{cond} = p_0 - \rho_0 g(z - \alpha \Delta T \frac{z^2}{2d}) \quad (\text{A.8})$$

Note that  $p_0$  is an arbitrary choice since pressure only shows up in the Navier-Stokes equations through a gradient.

### ***A.3 Disturbances to Conduction State***

We will introduce variables  $\mathbf{V}$ ,  $\theta$ , and  $P$  for the disturbance fields of velocity, temperature, and pressure, respectively.

$$v = V \quad (\text{A.9})$$

$$T = T_{cond} + \theta \quad (\text{A.10})$$

$$p = p_{cond} + P \quad (\text{A.11})$$

Substituting these expressions into the Boussinesq equations gives

$$\nabla \cdot \mathbf{V} = 0 \quad (\text{A.12})$$

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{V} + \alpha g \theta \hat{\mathbf{z}} \quad (\text{A.13})$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{V} \cdot \nabla) \theta = \kappa \nabla^2 \theta + \frac{\Delta T}{d} V_z \quad (\text{A.14})$$

#### A.4 Non-dimensionalization

In order to study the stability of the no-motion state, it is useful to first non-dimensionalize the governing equations. We introduce scales for all variables:

$$t = \tau t' \quad (\text{A.15})$$

$$\mathbf{r} = d \mathbf{r}' \quad (\text{A.16})$$

$$\mathbf{V} = \frac{d}{\tau} \mathbf{V}' \quad (\text{A.17})$$

$$\theta = T_0 \theta' \quad (\text{A.18})$$

$$P = P_0 P' \quad (\text{A.19})$$

We will substitute these equations terms but drop the primes for convenience.

From this point on, all variables will be dimensionless unless otherwise noted.

$$\left(\frac{d}{\tau^2}\right) \frac{\partial \mathbf{V}}{\partial t} + \left(\frac{d}{\tau^2}\right) (\mathbf{V} \cdot \nabla) \mathbf{V} = -(P_0) \frac{1}{\rho} \nabla P + \left(\frac{1}{d\tau}\right) \nu \nabla^2 \mathbf{V} + (T_0) \alpha g \theta \hat{\mathbf{z}} \quad (\text{A.20})$$

$$\left(\frac{T_0}{\tau}\right) \frac{\partial \theta}{\partial t} + \left(\frac{T_0}{\tau}\right) (\mathbf{V} \cdot \nabla) \theta = \left(\frac{T_0}{d^2}\right) \kappa \nabla^2 \theta + \left(\frac{1}{\tau}\right) \Delta T V_z \quad (\text{A.21})$$

or,

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\left(\frac{P_0 \tau^2}{d\rho}\right) \nabla P + \left(\frac{\nu \tau}{d^2}\right) \nabla^2 \mathbf{V} + \left(\frac{T_0 \tau^2 \alpha g}{d}\right) \theta \hat{\mathbf{z}} \quad (\text{A.22})$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{V} \cdot \nabla) \theta = \left(\frac{\kappa \tau}{d^2}\right) \nabla^2 \theta + \left(\frac{1}{T_0}\right) \Delta T V_z \quad (\text{A.23})$$

Let us choose  $\tau = \frac{d^2}{\kappa}$ ,  $P_0 = \frac{d\rho}{\tau^2}$ , and  $T_0 = \kappa \nu \alpha g d^3$  which reduces the equations to

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\nabla P + \left(\frac{\nu}{\kappa}\right) (\nabla^2 \mathbf{V} + \theta \hat{\mathbf{z}}) \quad (\text{A.24})$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{V} \cdot \nabla) \theta = \nabla^2 \theta + \frac{\alpha g \Delta T d^3}{\nu \kappa} V_z \quad (\text{A.25})$$

Here we can identify the two remaining non-dimensional parameters as the Prandtl number  $Pr = \frac{\nu}{\kappa}$  and the Rayleigh number  $Ra = \frac{\alpha g \Delta T d^3}{\nu \kappa}$ . The evolution equations are typically written in the following form

$$\nabla \cdot \mathbf{u} = 0 \tag{A.26}$$

$$Pr^{-1} \left( \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} \right) = -\nabla P + \nabla^2 \mathbf{V} + \theta \hat{\mathbf{z}} \tag{A.27}$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{V} \cdot \nabla) \theta = \nabla^2 \theta + Ra V_z \tag{A.28}$$

## APPENDIX B

### CAD DRAWINGS OF APPARATUS

Included in this chapter of the Appendix are the drawings for the parts comprising the convection apparatus. See Chapter 2 for a schematic of the apparatus. The design here is that the convection cell is sandwiched between two optical windows, each of which is held in a metal plate: these plates are shown in Figs. B.1, B.2, B.3, and B.4. Note that the top plate has four access ports, for connecting gas lines. Currently, we use only one of these ports; the others are sealed. An O-ring placed between the two plates forms the lateral pressure boundary (the optical windows form the vertical boundaries); screws connect the plates. The heating and cooling chambers (Fig. B.5) are identical, and screw into the top and bottom cell plates. An optical window on each chamber seals against the circulating fluid; the windows are held in place by two small metal plates (see Fig. B.6) which screw into the heating/cooling chambers.

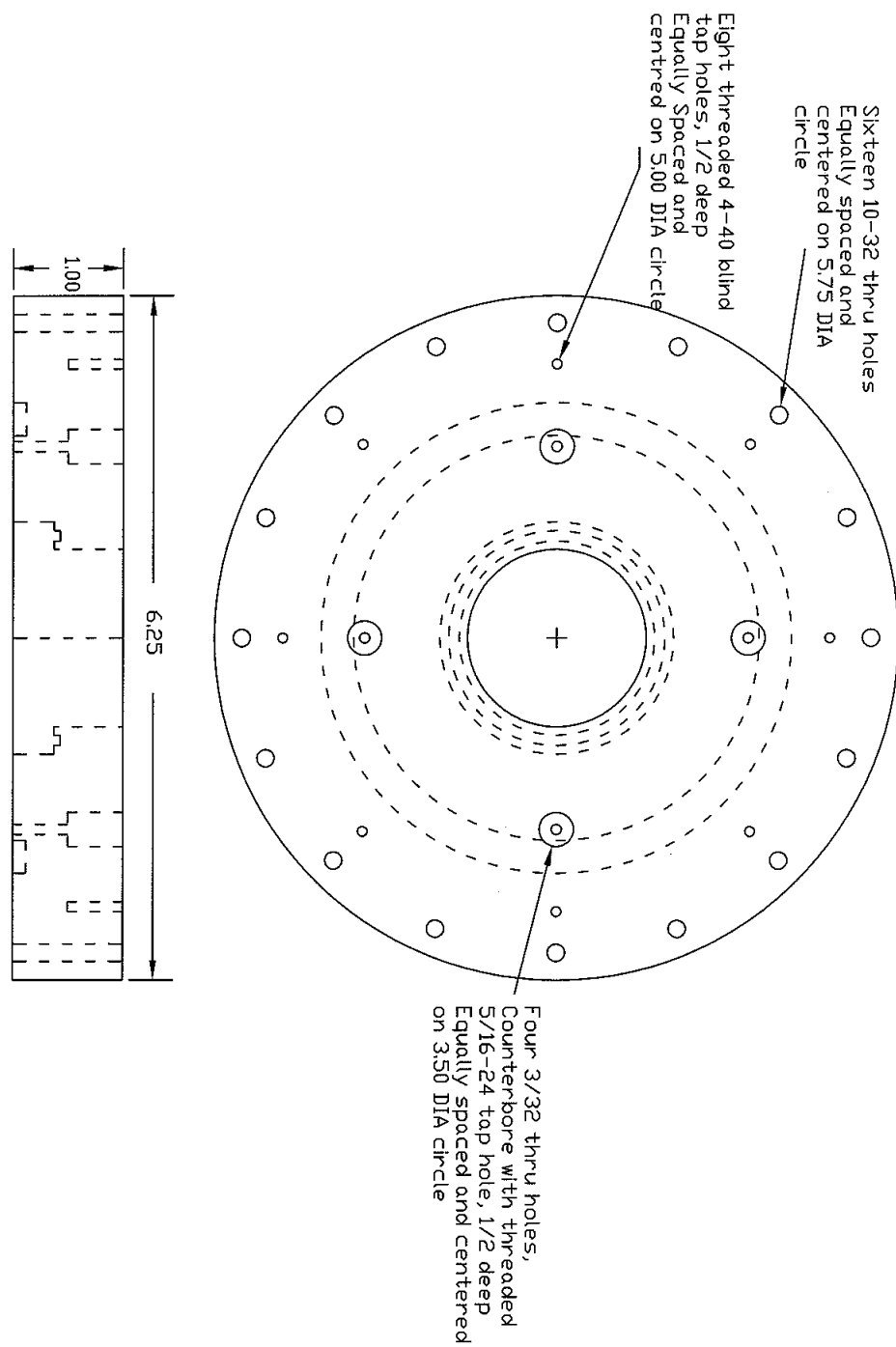


Figure B.1: The top cell plate, part I.



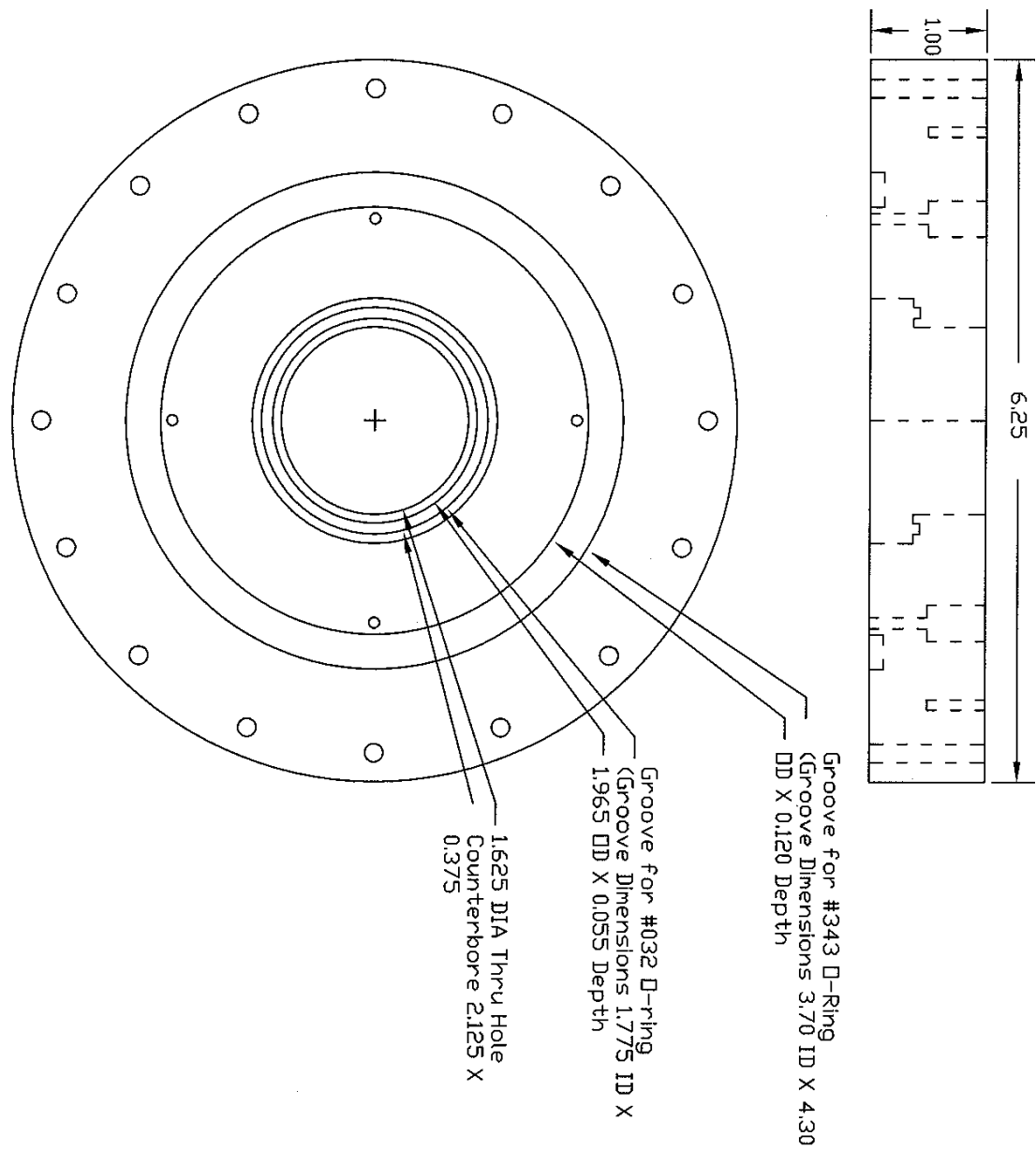


Figure B.2: The top cell plate, part II.

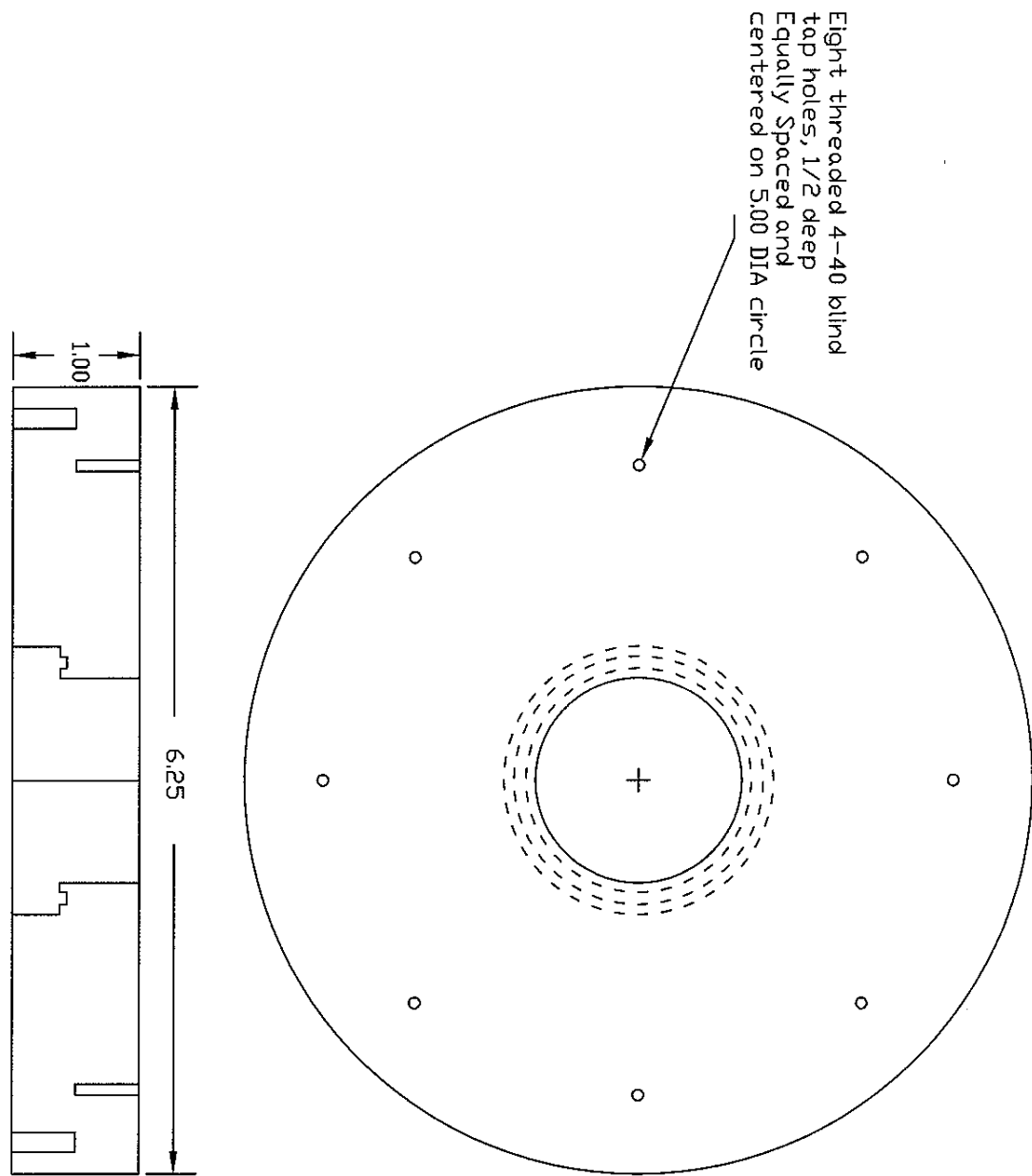


Figure B.3: The bottom cell plate, part I.

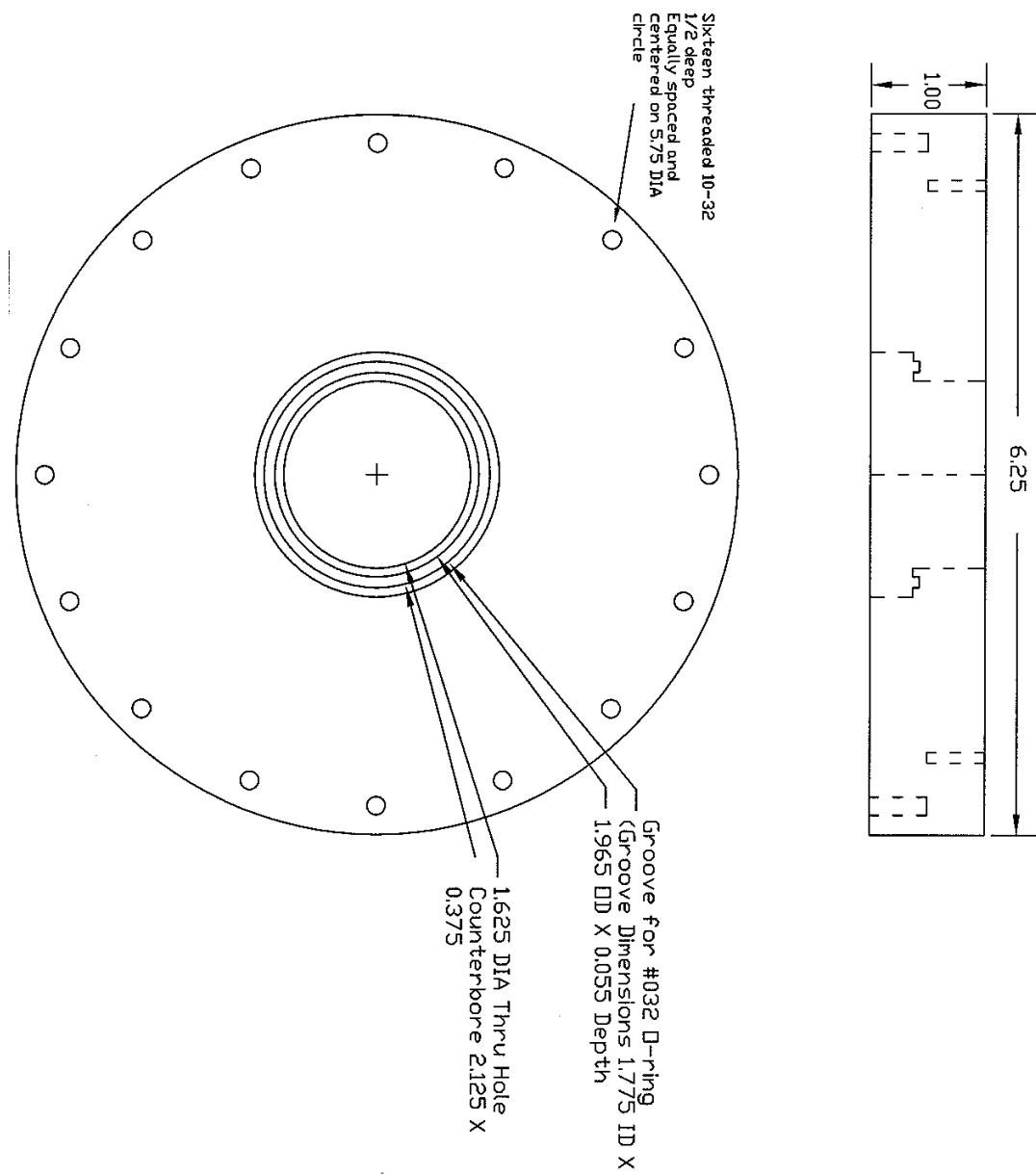


Figure B.4: The bottom cell plate, part II.

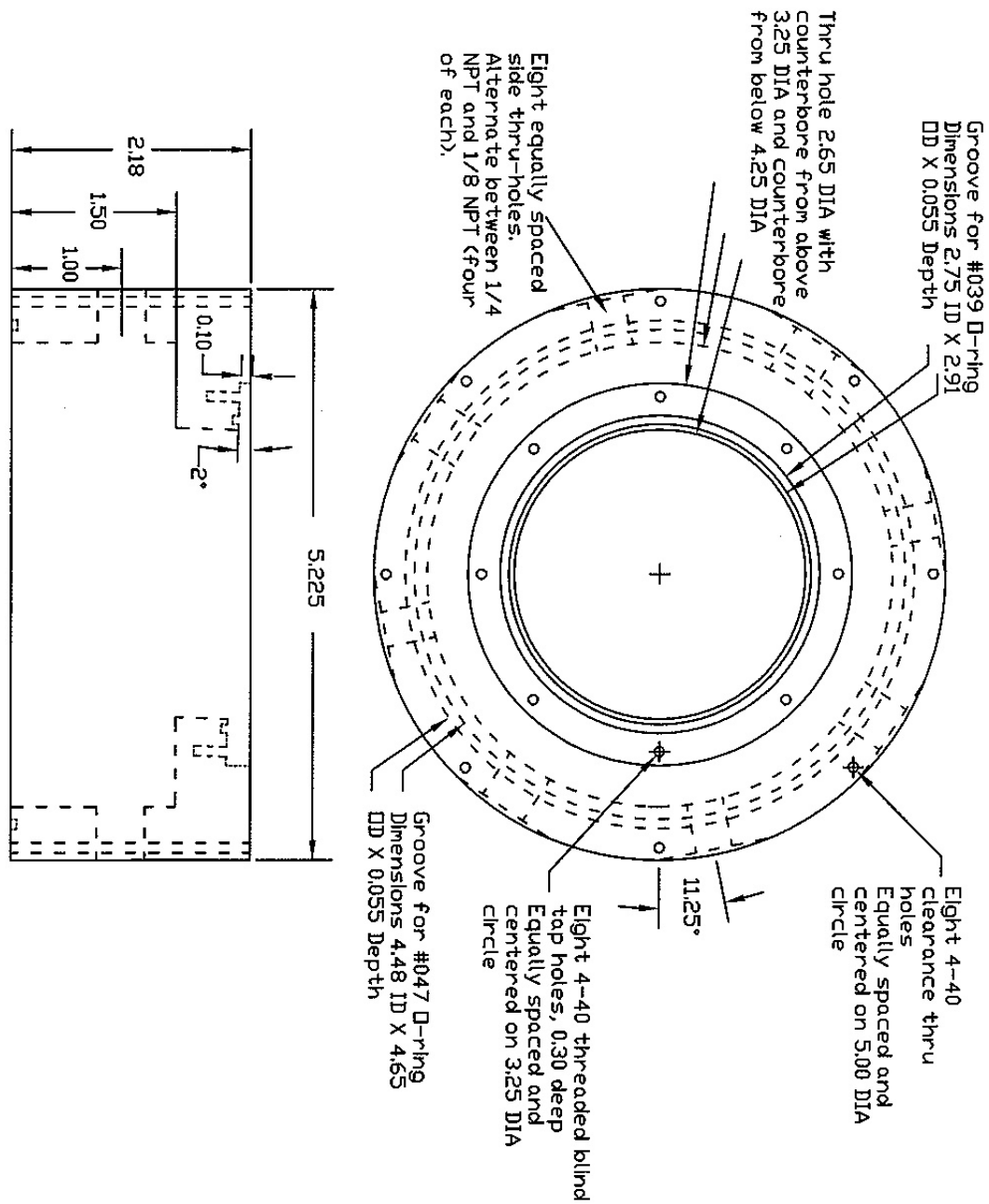


Figure B.5: The cooling (heating) chamber.

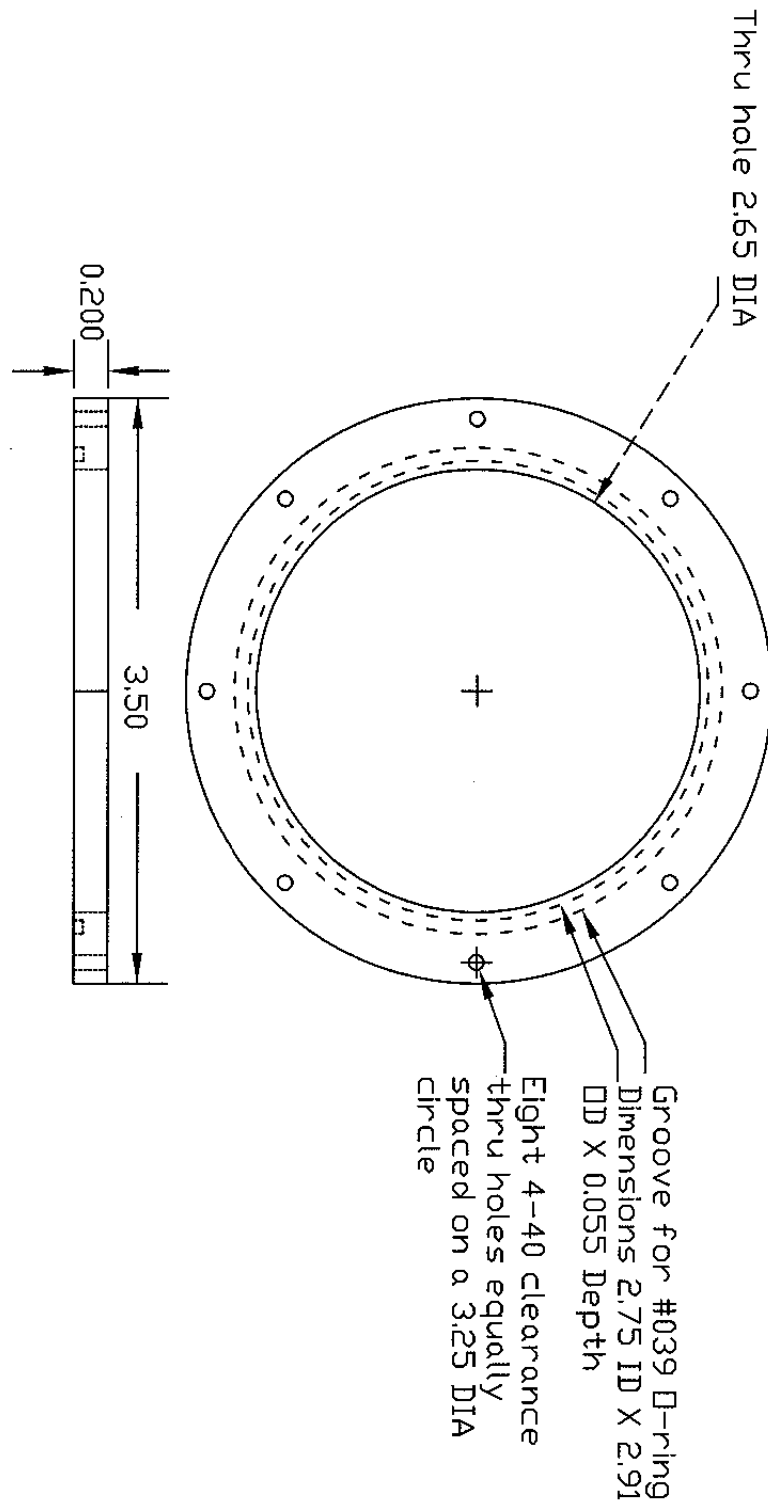


Figure B.6: The small plate used to hold the outer windows of the apparatus.

## APPENDIX C

### SHADOWGRAPH OPTICS

#### *C.1 Geometric Optics*

The following is a derivation of the relationship between the observed shadowgraph intensity field and the convective temperature field, assuming a valid geometric optics approximation. It follows Refs. [35, 36] and, most closely, Ref. [60].

The position of a ray of light  $\mathbf{r}$  is expressed as a function of the path length  $s$ . The ray path in a medium with inhomogenous refractive index  $n(x, y, z)$  is given by [70]

$$\frac{d}{ds}[n(\mathbf{r}(s))\frac{d\mathbf{r}(s)}{ds}] = \nabla n(\mathbf{r}) \quad (\text{C.1})$$

The left-hand side can be expanded to

$$\frac{dn}{ds} \frac{d\mathbf{r}}{ds} + n \frac{d^2\mathbf{r}}{ds^2} \quad (\text{C.2})$$

which we can re-write using

$$\frac{dn}{ds} = \frac{\partial n}{\partial x} \frac{dx}{ds} + \frac{\partial n}{\partial y} \frac{dy}{ds} + \frac{\partial n}{\partial z} \frac{dz}{ds} = \nabla n \cdot \frac{d\mathbf{r}}{ds} \quad (\text{C.3})$$

So we have

$$(\nabla n \cdot \frac{d\mathbf{r}}{ds}) \frac{d\mathbf{r}}{ds} + n \frac{d^2\mathbf{r}}{ds^2} = \nabla n \quad (\text{C.4})$$

Let us define  $\mathbf{v} \equiv \frac{d\mathbf{r}}{ds}$  and rearrange to get

$$\frac{d\mathbf{v}}{ds} = \frac{\nabla n}{n} - \left(\frac{\nabla n}{n} \cdot \mathbf{v}\right) \mathbf{v} \quad (\text{C.5})$$

And here we notice that the right-hand side produces the projection of  $\frac{\nabla n}{n}$  onto the plane perpendicular to  $\mathbf{v}$ .

We will assume that the refractive index is a constant value with a small perturbation field

$$n(x, y, z) = n_0 + \delta n(x, y, z) \quad (\text{C.6})$$

where  $n_0$  is the refractive index evaluated at  $\bar{T}$  of the fluid and from which follows

$$\frac{\nabla n(x, y, z)}{n(x, y, z)} \approx \nabla \delta n(x, y, z) \quad (\text{C.7})$$

Because the refractive index variation is small, we can further assume the displacement of the ray as it passes through the field is negligible but that it picks up a phase that corresponds to a slight angular deflection. We are interested in the horizontal components of the ray as it exits the fluid, so we choose  $\mathbf{v} = \hat{z}$

$$\frac{d\mathbf{v}}{ds} = \nabla_{\perp} \delta n \quad (\text{C.8})$$

where  $\nabla_{\perp}$  denotes the  $\nabla$  operator in the horizontal coordinates  $x, y$ . Again, ignoring deflections of the ray while in the fluid, we can integrate over the fluid depth to get the overall phase shift of the ray as it exits the fluid:

$$\mathbf{v}_{\perp} = 2d\nabla_{\perp} \overline{\delta n} \quad (\text{C.9})$$

The factor of 2 comes from traversing the fluid depth twice and  $d\overline{\delta n}$  comes from the integration and denotes the z-averaged value of the refractive index variation. This expression gives the direction of the ray as it exits the fluid.

The final position of the ray is then given by

$$\mathbf{r}_{\mathbf{f}} = \mathbf{r}_0 + z_1 \hat{\mathbf{z}} + 2dz_1 \nabla_{\perp} \overline{\delta n} \quad (\text{C.10})$$

a distance  $z_1 \gg d$  above the fluid.

Let us re-write this expression by defining

$$\mathbf{H}\mathbf{r}_0 = \mathbf{r}_0 + 2dz_1 \nabla_{\perp} \frac{\overline{\delta n}}{n_0 \mathbf{r}_0} \quad (\text{C.11})$$

Then the Jacobian of this operator is

$$JH = 2dz_1 \begin{pmatrix} 1 + \frac{d^2\bar{n}}{dx^2} & \frac{d^2\bar{n}}{dxdy} \\ \frac{d^2\bar{n}}{dxdy} & 1 + \frac{d^2\bar{n}}{dy^2} \end{pmatrix}$$

the determinant of which gives the change in the ray intensity at point  $\mathbf{r}_f$ . Expanding this out and keeping only the leading order term in refractive index variation gives

$$|\mathbf{JH}| = 1 + 2dz_1 \nabla_\perp^2 \bar{\delta n}(\mathbf{r}_0) \quad (\text{C.12})$$

and therefore the final light intensity is related to the incident intensity through

$$I(x, y) = \frac{I_0(x, y)}{1 + 2dz_1 \nabla_\perp^2 \bar{\delta n}(\mathbf{r}_0)} \quad (\text{C.13})$$

The refractive index is usually expressed conveniently in terms of the temperature deviation  $\theta$

$$\bar{\delta n} = \frac{dn}{dT} \bar{\theta} \quad (\text{C.14})$$

and because  $\frac{dn}{dT}$  is negative in most cases, it is convenient to put the intensity relationship in the following final form

$$I(x, y) = \frac{I_0(x, y)}{1 - 2dz_1 \left| \frac{dn}{dT} \right| \nabla_\perp^2 \bar{\theta}(x, y)} \quad (\text{C.15})$$

## ***C.2 Physical Optics Corrections***

In the geometric optics treatment, the deviations of light rays as they pass through the fluid are not considered. Rather, it is the phase change, or angular deviation of rays that give rise to a focusing effect of the rays in such a way as to reveal the convection pattern. This treatment predicts certain optical distances at which the strong focusing (the result of sharp dependence of refractive index on temperature, for example) causes the shadowgraph intensity to diverge, which is clearly unphysical. These locations are known as caustics. A more proper treatment of the shadowgraph



using physical optics is given in Ref. [36]. This section of the Appendix will review the relevant results to our experimental setup.

Let us take as an illustration the simplified case of an ideal straight roll pattern. The convection rolls act as cylindrical lenses and thus the authors define a geometric focal length  $f_{geo}$  at which the first unphysical caustic appears. We can understand this focal length from the expression above, in particular the term  $a = 2dz_1 \left| \frac{dn}{dT} \right| \nabla_{\perp}^2 \bar{\theta}(x, y)$  in the denominator. Geometric optics can be trusted quantitatively only in the limit that this term is  $\ll 1$ .

Following the authors of Ref. [36], we consider, for simplicity, a straight roll pattern defined by a single cosine term in the x-direction  $\cos(qx)$ , modulated by a single sine term in the z-direction (to meet boundary conditions). The temperature term  $\nabla_{\perp}^2 \bar{\theta}(x, y)$  is estimated as  $\sim 0.8q^2 \Delta T \sqrt{\epsilon}$ . We can then work backwards to find  $z_1 = a / (0.8q^2 \left| \frac{dn}{dT} \right| \Delta T \sqrt{\epsilon})$ .

Let us take experimental conditions similar to our experiments: the depth  $d = 0.06$  cm, mean temperature  $\bar{T} = 24.0$  deg C, pressure  $p = 215$  psi, and  $\epsilon = 1$ . We then find  $\Delta T_c = 2.96$  deg C, so  $\frac{dn}{dT} = -8.95 \times 10^{-5} / \text{deg C}$ . Assuming a critical wavenumber,  $q = 3.117/d = 52/\text{cm}$ , we find finally that  $f_{geo}$ , which occurs when  $a = 1$ , is equal to 18 cm. However, from the LETKF algorithm, we have an estimate of  $a \approx -0.06$ , which tells us that  $z_1 \approx 2$  cm. Thus  $z_1/f_{geo} < 0.10$  and we therefore conclude that the magnitude of physical-optics corrections to the geometric treatment is relatively small.

## APPENDIX D

### PROGRAM AND EXPERIMENT DETAILS

This chapter of the Appendix discusses some of the more technical and practical issues that arise in running the experiment and using the LETKF state estimation program. It is by no means an exhaustive description but is meant to address some of the less-obvious details. The names of some programs, software, and special commands have been set in italic text when appropriate, to distinguish them from idiomatic language. Other function names and files have been set in single quotation marks.

#### ***D.1 The Experiment***

##### **D.1.1 The Computers**

Two computers are used to run experiments. One computer (with Windows XP) runs a Matlab program which analyzes shadowgraph images and makes calls to the LaserShow program (call it computer #1) In parallel, computer #2 (with a Windows 2000 OS) runs the image-grabbing software (EPIX XCAP V2.2) and a Matlab program that monitors and regulates temperatures of the top and bottom convection cell boundaries. An ethernet cable connects the two computers so that images and other files in shared folders can be accessed from either computer. The two folders are stored in the ‘C:’ directory of computer #1 and can be accessed through ‘My Network Places’ on the older computer. To avoid confusion, keep in mind that use of the older computer (#2) is limited to setting the temperatures and initiating image taking (the imaging software, which does not run on newer systems, is the reason for maintaining this computer); most other mentions of function calls refer to using the newer computer armed with the LS software and QM2000 board. Almost all the Matlab files

that are used can be found inside ‘C:\Matlab\_Functions’ on computer #1, except for the temperature control files, which are in ‘E:\Matlab\_Functions’ on computer #2. I have attempted to organize the files intuitively and have added comments to the code that build on what is written here.

**Software Backup** The LS software exists on CD and has been shown to work on both Windows XP and Windows 2000. The QM2000 board is a PCI board that needs to be moved to the same computer hosting the software. The XCAP software can be downloaded from EPIX, but will not work without a parallel port dongle acting as a Product Key (XCAP checks for this every time it is opened). The imaging software will need to be updated for a newer OS. As of now, the XCAP software is loaded on the other convection experiment computer (on Windows 2000) but would need the Product Key to install the XCAP imaging board. In case of a computer #2 breakdown, the unused convection experiment computer could be used in its place.

**Cell Assembly/Disassembly** Changing the cell sidewalls requires planning. First, the water in the top cooling chamber should be emptied and removed (keeping the cell pressurized in the meantime, if possible). After de-pressurizing and opening the convection cell, the bottom ZnSe should be removed, cleaned, and replaced quickly (if at all; the ZnSe surfaces usually remain relatively clean), to minimize the amount of CS<sub>2</sub> that evaporates. Inevitably, there will be a pocket of air that gets trapped below the ZnSe window. To get it out, the entire apparatus should be tilted so that the air leaves through one of the outlets. This is when one should be aware of putting strain on the connecting tubes. As an aside—if one does not see a convection pattern even with a large temperature difference between the top and bottom baths, consider that there may be an air pocket keeping the CS<sub>2</sub> from contacting the ZnSe boundary.

When the apparatus is disassembled, the O-rings and other parts should be checked for signs of degradation, but the only parts that should need to be replaced

on a regular basis are the screws (it will usually be necessary to clean the interior of the top cooling chamber; the aluminum is corroded by the water which causes small flakes of material to build up). Note that the groove for the O-ring supporting the ZnSe cell-bounding is for a size #032 O-ring, but we use a 1xx-series O-ring that fits in the groove; this increases the contact area between the O-ring and window, which prevents cracking of the AR-coating, up to  $\sim 230$  psi.

There are 16 screws (set evenly in a circle) that hold the top and bottom of the convection apparatus together (see the CAD drawings). Tightening of these screws compresses the laterally sealing O-ring and forces the top/bottom windows together. Thus, it is important not to over-tighten, and to tighten evenly across the screws. After the sidewalls have been placed between the windows and all other sealing parts are in place, it makes sense to tighten the screws with under no pressure, to the point that the sidewalls are pressed between the windows (they should not shift if the entire apparatus is lifted/tilted). As the cell is pressurized, it is likely that the windows will be pushed apart slightly, and the screws will have to be tightened to re-trap the sidewalls between the windows (one will build an intuitive feel for how much torque is needed, but one can actually notice visually when the sidewalls are being compressed). When the sidewalls have been trapped between the windows at the approximate pressure at which the experiments will be done, the screws can be adjusted little by little until the two window surfaces forming the cell boundaries are parallel (checked using laser interferometry). It is helpful in doing the assembly to not attach the top cooling chamber until the alignment is done.

### **D.1.2 Setting Temperatures**

Control of a water bath is activated by pressing a button on the back of the water bath controllers labeled simply ‘ENABLE/DISABLE.’ A Matlab program, discussed below, sets the temperature by sending a DC voltage via the Measurement Computing

DAQ; each miliVolt is a tenth of a degree Celsius. Two wires are used – one carries the temperature-setting voltage, the other the ground voltage. The water bath manual can be consulted for information about which pins can be used to access/control settings.

Because the amount of RAM to run the imaging software and the Matlab temperature control is significant, the older computer will slow down after some time, so it may be necessary to restart the computer every few days of continual use (or to set the priority of XCAP “above normal”). At a minimum, programs should be shut down when not in use. Note, however, that control of the water baths should be disabled when the computer restarts, or the bath will see a zero voltage and therefore try to cool the water to 0° C.

The temperatures are measured using serial communication with the HP multimeter (the multimeter should be set to receive signals from the back side). The resistances of the thermistors are measured across pins in the back of the multimeter, and a relay switch (activated by a 5 V signal from one of the DAQs) flips between the top or the bottom thermistor. Incidentally, the pressure can be read from either one of the DAQs (look at the wire connections) or from the multimeter, but it is not possible under the current configuration to read both the resistances and the pressure at the same time.

A Graphical User Interface ‘Exp\_Params\_GUI’ runs in Matlab and is the gateway to measuring and changing temperatures. There are input fields for temperature set-points, and there are three main functional buttons. The primary way to use this interface is to set values for both top and bottom temperatures, and turn the program to ‘ON.’ The feedback program will change the temperatures of the water baths via changes in the voltages output . Pressing the ‘OFF’ button will turn off control (naturally). There are two auxillary buttons that can be used to ‘fix’ the respective temperatures. That is, changes are not allowed to the water bath temperature when

the respective temperature is ‘fixed.’ This is useful when control is wanted over only one of the water baths. For example, it is best to leave the bottom temperature near a set reference temperature. To do the mapping, the top temperature can be increased quickly to bring  $\Delta T$  below critical and then moved back afterward, all the while leaving the bottom temperature unchanged. The GUI can also be used to get temperature values without invoking control. This is initiated by pressing the ‘Just Info’ button. Note that each day that temperature control is used, the set/measured values are stored on computer #2 in a text file ‘C:\T\_Data\T\_data\_MM\_DD.txt’ where MM is the month and DD is the day.

### D.1.3 Carbon Disulfide Considerations

**Materials** Use of CS<sub>2</sub> as a heating fluid poses some challenges. An important consideration is that, because it is a strong solvent, only select materials are appropriate to use with CS<sub>2</sub>. Good materials include aluminum and Viton (and some similar materials); the best place to check is the chemical compatibility checklist from Cole Palmer, available online.

**Sealing** The relatively high vapor pressure means that it is difficult to entirely seal the fluid as it flows through the heating loop. ‘Torr Seal’ epoxy has been used around the joints connecting tubing to the pump, and Swagelok pieces are used at other junctions. The possibility of a larger-scale spill is also a concern. For example, it is necessary to pick up and/or tilt the convection apparatus from time to time, which puts strain on the Swagelok-tubing connections; eventually, the tubing can wear down and tear. One must remain aware of this whenever handling the apparatus. There are multiple valves which can be shut to stop the flow. In the event of a spill, these should be shut, the pump should be unplugged, an absorbent should be used to soak up any spilled CS<sub>2</sub> and keep it from evaporating into the lab. The plastic housing the pump and tubing should be closed and time given for the CS<sub>2</sub> to evaporate through

the vent.

**Re-filling** The continuous evaporation of  $\text{CS}_2$ , however small, means that more fluid will need to be added at some point (every 2-3 weeks, depending on how often the experiment has been running). The pump should (obviously) be turned off to do this. There is a small flap in the plastic surrounding the experiment, inside of which is a cap that can be unscrewed to gain access to the fluid. Slowly, pour  $\text{CS}_2$  until it starts approaching the level of the cap. Note that the evaporation may be minimized by unplugging the pump whenever the experiment is not running.

#### D.1.4 Imaging

XCAP is on computer #2, which has a PCI board installed for communicating with the digital camera (from the DVC company, model 1312). The graphical interface of this program, from the EPIX company, makes it relatively straightforward to set the frame rate, shutter rate, binning, etc.

During the mapping, XCAP should be set to record 5 '.tif' images in the mapping directory ('C:\Mapping\_Images'), at a frame rate of 4 fps ('Capture' —> 'Sequence Capture' —> 'Video to Image Files'). The images should be named 'mapping\_images' to match the filename searched for by Matlab. Initialize XCAP to expect a 'Script Remote Control' to be placed inside the mapping directory. XCAP scripts are basically pre-recorded steps that capture the user doing a series of tasks/manipulations and can be useful when the same series of steps are used many times. This script simply has recorded me pressing the 'Record' button on the image, and so, as the mapping perturbation points are looped through, Matlab will repeatedly copy the script into that folder, telling XCAP to begin taking images. Each time, XCAP will take the 5 images (as it has been directed by the steps above) and then stop.

After mapping, images for analysis are placed in a different folder ('C:\XCAP\_share'). The maximum number of images and the frame rate can be set to the desired values.

The frame rate can slow when the number of images in the folder becomes large, so it is sometimes appropriate to delete images as they are processed, if they will not be needed later (see the Matlab function ‘get\_img’).

As far as the shadowgraph settings go, it is easiest to arrange the camera wherever the best imaging can be realized. For some purposes, this will mean that the magnification will be small and the roll intensities will be more “saturated”; for the state estimation, it may be best to use a setting where the intensity maxima/minima are not so pronounced, which would indicate that one is operating near the “ $z_1 = 0$ ” limit (see the shadowgraph discussion above), the limit that geometric optics is most valid. The ultimate test is if the state estimation algorithm is able to converge onto a shadowgraph parameter (see the state estimation details).

### D.1.5 Mapping

Essentially, the mapping is a pair of matrices, each the size of the shadowgraph images: ‘X\_fit’ and ‘Y\_fit’. At each element is the  $x$  (or  $y$ ) value that corresponds to that pixel value. Usually, a few individual mappings are done and the results are averaged (and can be averaged with the already existing mapping). All the mapping files are contained in ‘C:\Matlab\_Function\mapping’.

Before calling the mapping functions, the lateral boundaries of the convection cell need to be defined manually by four pixel coordinates. These are stored in the ‘Compute\_Mapping.m’ source file in textitglobal variables  $I$  and  $J$ . The  $1 \times 2$  array  $I$  holds the lowest and highest rows spanned by the convection cell, as viewed on the shadowgraph images; likewise,  $J$  holds the lowest and highest columns. These can be found simply by taking a background image and finding where the intensity change indicates a cell boundary. Secondly, one must decide whether to recycle the stored mapping or start afresh. Unless the cell or camera has shifted, the old mapping is relevant. When the cell or camera has been shifted, however, it is important to



reset both  $I$ ,  $J$ , and to compute an entirely new mapping. There are a few lines of code within ‘Compute\_Mapping.m’ that should be commented out depending on if the previous mapping is being used or not (these lines are marked).

The mapping code exists under several different .m files. The hierarchy is as follows: The master file is ‘Do\_All\_Mapping( $n$ ),’ where  $n$  is the number of individual mappings that are averaged together ( $n = 1$  or  $2$  is sufficient if using the old mapping; if starting over, use  $3$  or  $4$ ). Inside this function, the  $x$  and  $y$  values that are to be perturbed serially are defined (these values may need to be changed if the sidewalls are replaced or the cell shifts substantially). This function passes arrays of these values to ‘Do\_Mapping\_Pts,’ which is the function that actually calls the LS software to draw the points, copies the ‘mapping\_images.scr’ file into the proper folder (causing XCAP to start imaging), and analyzes each shadowgraph image in order to find the system response to the perturbation. After cycling through all points, what is left is a set of  $(i,j)$  values, one pair for each  $(x,y)$  pair. These arrays are saved. After returning to ‘Do\_All\_Mapping,’ ‘Compute\_Mapping’ is called which opens these saved points and sends them to ‘Filter\_Mapping\_Pts,’ which filters out the spurious data and does an iterative smoothing (as described in the main text). The resulting maps ‘X\_fit’ and ‘Y\_fit’ are stored; ‘Compute\_Mapping’ reads these matrices and either averages them with the old mapping or stores it averages only over the current mapping iterations. Note that at this stage, the mapping exists as a set of *global* variables and is stored in a ‘.mat’ file in ‘C:\Mapping\_Images\latest\_mapping’; call the function ‘Reload\_Mapping’ to open this file and load the mapping variables (‘I’, ‘J’, ‘X\_fit’, and ‘Y\_fit’) back into the *global workspace*, if they have been cleared.

To convert between  $(i,j)$  and  $(x,y)$  once the mapping is complete, use ‘ $[x \ y] = \text{IJ\_to\_XY}(i, j)$ ’. For non-integer  $i$  or  $j$  values, the program simply interpolates using the surrounding elements. Note that this is different from IJ\_to\_XY2, which does the same thing, but uses only the initial planar fit (it is used for intermediate steps in

the mapping process).

### D.1.6 Lasing

The mirrors and tickle pulse should be turned on before lasing begins. A switch on the rear of the mirror driver enables the mirrors, and the tickle pulse is applied by flipping the switch on the UC1000 (which will turn an LED on the controller red). There is a power supply to the laser, which actually needs to be on to power not only the laser but the controller. There are no visible indicators on the laser to show the power is on<sup>1</sup>.

A function ‘Laser2\_source’ is called to pass coordinates and intensity values to the LS software: ‘Laser2\_source([ $N$  1 1],  $x$ ,  $y$ ,  $z$ ). Here, the arguments are (1) a  $1 \times 3$  array whose first value  $N$  should be equal to the number of points being passed and whose second two values should both usually be set to 1; (2) a  $1 \times N$  array holding the  $x$ -coordinates of the points; (3) a  $1 \times N$  array holding the  $y$ -coordinates of the points; (4) a  $1 \times N$  array holding the intensities of the points. The values should be between 0 and 255.

Patterns are usually lased by drawing the individual rolls sequentially:

- Determine the number of individual rolls to be lased and the  $i$  and  $j$  values at which the hot flow should be centered, for each. Store these values.
- Cycle through the number of rolls. Convert  $(i,j)$  pairs to  $(x,y)$  pairs. Call ‘Laser2\_source,’ passing the proper values.
- Turn off lasing and move to the next roll.

This cycle can be repeated until the convection pattern is achieved, and then the program can be set to stop or to prepare to perturb the pattern. It is best to

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<sup>1</sup>With the previous laser, a green light on the back of the laser would indicate power on, and a second light on the back of the laser would indicate it was receiving the tickle signal and it is ready to lase. When the laser is actually emitting light, the second light will take on a bright red color.

build Matlab ‘m-files’ that run through these steps automatically, with some degree of human intervention. See, for example, the ‘Target’ GUI, which has additional function calls (see ‘get\_img.m’ and ‘lase\_out.m’, for example) and comments that help to explain the process.

The effect of the laser can be enhanced by either increasing the intensity or by adding ‘pause( $t$ )’ statements after calling ‘Laser2\_source’ in order to increase the lasing time for any given roll, with  $t \sim$  hundredths of ms. For a convection roll across the cell domain, a few hundred points is usually appropriate. For a point perturbation, pass on the order of 10 points that all have the same coordinates (sending only 1 point will not cause a noticeable effect, presumably because of the rise/fall time of the laser).

**Turning Off Laser** It is best not to let the laser remain on unnecessarily, as it can heat the glue holding the mirrors. In other words, the laser should be on only when actively perturbing or imposing a pattern. To turn off the laser from within Matlab (that is, rather than physically flipping switches), call ‘Laser2\_source’ and pass some  $x$  and  $y$  pair, such as (0,0), with zero brightness. In this case, also set the second element of the first variable to 0; this acts as a backup to turn drawing off: ‘Laser2\_source([1,0,1], 0, 0, 0)’. Because the current laser does not have any visible indicator if the laser is on or off, it is important to be sure to send this call (or something equivalent) whenever done lasing.

**‘Laser2\_source’** The mirror rotations are controlled by the LaserShow Designer 2000 software package. There is a graphical interface (use the Desktop or the Start Menu of Computer # 1), but, almost exclusively, we use one single function within Matlab to communicate with the software. Note that the pattern coordinates ( $x,y$ ) should be set between the limits of  $\pm 8000$ .

The function ‘Laser2\_source’ is a *MEX-file*. Changes must be made to the C++ source file and the *project* must be re-compiled. This can be done in Visual Studio,

for example. The function must then be re-built in Matlab with the *mex* command. For example, the code is set up, currently, to keep the laser on in between input points; if one wanted to turn the laser off in between, it would have to be done via changes to the C++ source code. There is a README file in the same directory as the source code ('C:\Matlab\_Functions\Laser2') that explains in much more detail.

**Laser Detection?** There are slight timing differences between the mirror rotations and switching of on/off the laser. I looked into a few options for laser detection on a fast time scale but did not pursue beyond initial inquiries. Two of the seemingly appropriate detection devices are part "P3257-30MCT" (photoconductive detector) and B749 (photon drag detector) from Hamamatsu Photonics. Much earlier, we had looked into using an acousto-optic modulator (aom) that would (presumably) allow somewhat more synchronized timing. The appropriate aom models are 37027-3 or 37027-5 (the latter has a larger aperture, both cost 2400\$ at last inquiry) which would also need a driver (39027-35DMA05-A, OEM model for 1200\$ or 39027-35DSA05-A, the complete driver/power supply/rack box, for 1900\$).

**Mirrors** As discussed in the main text, we use for mirrors two pieces of gold-coated germanium wafer. There is in the lab additional gold-coated wafer, in case the current pair of mirrors are degraded or break. They should be attached to the rotation arms of the servo system with a small amount of glue. If the mirrors/glue combination is too big, or the mirrors are poorly attached, one may notice a constant high-pitched noise when the mirrors are powered on; this signals that the servo system is having trouble calibrating the mirror positions and is consequently constantly overshooting. The positions of the mirrors is based on optimizing the amount of laser light being captured (as transmitted from the final focusing lens) along with having the focal point of the beam be at the location of the convection cell. If some optical component or the apparatus is changed, it may be necessary to re-position the servo mirrors.

### D.1.7 Other Notes

- We purchased our ZnSe windows from Design Research Optics, and my main contact there was the president, Bruce Sunderland. He can be reached by either `bruce@designresearchoptics.com` or `sales@designresearchoptics.com`.
- For questions about the LS software, contact William Benner at Pangolin (`wrb@pangolin.com`). He is the one who sent the Software Development Kit that contained the Visual Basic and C++ source code for interacting with LS. However, we have not been in contact for a few years.
- If the image frame rate seems to change during a time series of images, look at the function ‘`write_file_times.m`’. This can be used to look at the time-stamps of a given series of images; one can determine the “true” frame rate over the time series, or one can compute shadowgraphs at the desired frame rate (by interpolating between actual shadowgraphs).
- The critical temperature difference at a particular mean temperature and pressure can be found by using the `RBC.c` program provided by G. Ahlers group. This program exists on the computer used with the other convection experiment. Open ‘Cygwin’ and change the directory to ‘`G:\RBC_Program`.’ The executable is named simply ‘`RBC`’ (invoke by ‘`./RBC`’). It is currently set to take a mean temperature and pressure as arguments and to compute the  $\Delta T_c$ , given a particular cell depth, which can be changed by modifying ‘`RBC.c`.’ After editing, run *make* to re-compile the code.

## D.2 The LETKF Computer Code

The Boussinesq simulation is pieced together from code written during more than one generation of Fortran (77 and 90). Two open-source Fortran compilers, *gfortran* and *g95*, were used in attempts to compile the code, but neither was successful, the

reason being presumably the inability to process the older subroutines together with the newer code into a single executable. Instead, we use a commercial compiler, Lahey f95 (the same compiler as used by the Maryland group).

The simulation uses a pseudo-spectral method [71]. A backward Euler step is used for linear terms and a second order Adams-Bashforth step for non-linear terms. The fields are expressed in terms of Fourier components in  $\phi$  and Chebyshev polynomials in  $z$ .

### D.2.1 Compiling

The simulation code consists of several separate files; a *Makefile* script containing certain *flags* calls the compiler which then produces corresponding ‘.o’ *object* (binary) files for all input Fortran files (it also makes an executable, but that can be ignored). The *Makefile* can be invoked by the *make* command. The *Makefile* only needs to be called whenever there has been a change to one of the simulation files.

Meanwhile, the state estimation code is contained in a set of C++ files. A script called a link file calls the built-in gcc C++ compiler via a *mpicc* command to produce the corresponding *object* files. Additionally, it makes accessible to the code (hence the term *link*) a group of needed *libraries* (discussed below). With the simulation *object* files already existing, the link file builds a single executable, currently named ‘kalman.’

There are a small number of necessary external libraries. One is known as the FFTW (Fastest Fourier Transform in the West). A second set of libraries adds the capability of parallel processing, using the protocol called *MPICH2*; these libraries come from Argonne National Lab. The directory locations of these libraries of these (and others, which are probably system-dependent) are present in the *link* file.

### D.2.2 Header Files

The main C++ file is `mpifekf.c`. Under normal circumstances, this is the only file containing source code which needs to be changed. Additionally, specific parameters may need to be set within a small number of *header* files.

The input shadowgraph images need to be cropped and circular, leaving no background region apart from that in the corners of the resulting square. The images are 8-bit bitmap images; a Matlab program (`'tif_to_bmp'`) is used to convert the images from the original 12-bit intensities, crop the images, and write the final bitmap files. Within the `'def.h'` header file, the size of the input shadowgraphs needs to be set. This number should be the number of pixels along one side of the square images.

Other parameters should be set inside the `'parameters.h'` file, brief descriptions of which are given in the following list; terms appearing in the file but not listed below should not be changed.

- **Radius** The size of the cell radius in terms of the depth. This is the same as the aspect ratio.
- **Prandtl number** Self-explanatory
- **tdelh** The time step of the simulation, in terms of the vertical diffusion time. Note that the actual time step is twice this value.
- **xdir** if 1.0, Dirichlet boundary conditions, if 0, Neuman boundary conditions.
- **Rayleigh number** Self-explanatory

The number of grid points (**PARAM\_NR** in the radial direction, **PARAM\_NT** in the  $\theta$  direction, and **PARAM\_NZ** in the z-direction) can be set inside `'dimensions.h'`. Note that two points are added implicitly to **PARAM\_NZ** for the two vertical boundaries. Some combinations may not work with the Fortran simulation

code, but it is uncommon to change these numbers. The current values work for an aspect ratio of 20, but If the aspect ratio were to double, **PARAM\_NR** and **PARAM\_NT** should roughly double; **PARAM\_NZ** can remain constant.

### D.2.3 Main Program

The main file is ‘mpifekf.c. In addition to the main LETKF algorithm, there are many smaller subroutines that can be used to make specific calculations (such as to create an ensemble, open a field and simulate it forward in time, compute the vorticity, etc.). By default these subroutines are simply commented out. At the beginning of the function the following parameters are defined.

**DATA\_FILE** The first of two output files containing information about the parameter estimates, e-dimension, and other useful data/parameters.

**RMS\_FILE** One of the ‘.dat files written. The rms error between predicted and actual shadowgraphs is written to the file and can be monitored; convergence of this error can be used to monitor convergence to a state.

**MEASUREMENT\_BASE** This is the directory containing the shadowgraph images. Be sure to include the correct number of zeros, such that the indexing goes like 001, 002... as opposed to 1, 2,

**OUTPUT\_DIR** The path to the folder where ensemble members and estimated shadowgraphs are put.

**INIT\_ENSEMBLE** The path to a set of random states used to initialize the ensemble.

**OPEN\_INITIAL** Set to 1 to open the initial ensemble. This is the usual case, unless creating a new ensemble with a new aspect ratio, for example, in which case set to 0.



**ANA\_ENSEMBLE\_SAVE** Rate at which to save analysis ensemble members (as ‘.vtk’ files). Looking at the ensemble is useful for debugging.

**BG\_ENSEMBLE\_SAVE** Rate at which to save the background ensemble members (as ‘.vtk’ files).

**RA** Either the actual value, or the mean of an initial guess of the Rayleigh number.

**PR** The Prandtl number.

**MEANH1** The initial guess of H1, the parameter in the shadowgraph observation operator.

**SEED** The seed for the random number generator.

**MAXSIM** The maximum number of frames to assimilate.

**DFRAME** The frequency of frames to assimilate. For example, when set to 1, every frame is used, when set to 2, every other frame is used, and so on.

**INIT\_FRAME** The initial frame number.

**T** The amount of time (in terms of the vertical diffusion time) between assimilated frames. This should usually be set to some multiple of DFRAME. For example, if the time between shadowgraphs is  $t_v/8$ , then set this to  $(1/8)(1/2tdelh)$ , where *tdelh* is set in ‘parameters.h’.

**NEUMAN** This is set to 0 and means that the simulation is using conducting boundary conditions.

**K** The number of ensemble members.

**S** The number of observations within a disk. To use all pixels, set to SGN\*SGN; to use half, set to SGN\*SGN/2, etc.

**R** The noise level. If, for example, the magnitude of noise on an 8-bit scale (0-255) is 10, then  $R = 10/256$ .

**ARTIFICIAL\_NOISE** Nonzero to add noise to the shadowgraphs.

**FILTER\_TYPE** Set to either DI (Direct Insertion), GLOBAL, or, LOCAL.

**LOCAL\_REGION\_RADIUS** The distance away from an observation point inside of which data are used to form a local observation vector. This is, roughly, the correlation length.

**FALLOFF\_RADIUS** The standard deviation (width) of a Gaussian weighting of observation points around a given point.

**GLOBILIZATION\_METHOD** Set to CENTER.

**GLOBILIZATION\_RADIUS** Set to 0.

**EKF\_SPACE** Set to BOTH.

**INFLATION\_TYPE** The artificial inflation of error variance. Set to STANDARD\_VARIANCE to use multiplicative inflation by epsilon. It is best to use a large inflation at the beginning and decrease later, eventually reaching some constant value (see EPS, DEPS, FINAL immediately below).

**EPS** The initially constant, high value of epsilon used for variance inflation.

**DEPS** The slope of epsilon as a function of frame, going between EPS and FINAL.

**FINALEPS** The final, constant, smaller value of epsilon.

**AUGTRA** Set to 1 means to attempt to fit the Rayleigh number, otherwise 0.

**AUGTPR** Same as above, but for Prandtl number. Leave set to 0 because of computational expense of fitting.

**AUGTH1** Same as above, but for H1, the parameter in the shadowgraph observation operator.

**RA\_SPREAD** The initial spread .

**PR\_SPREAD** Same as above, but for Prandtl number. Leave set to 0 because of computational expense of fitting.

**SVDH1** Same as two terms above, but for H1.

**PARAM\_SPREAD\_START** Set to 0.

#### D.2.4 Other Comments and Considerations

- An assimilation rate of roughly four Hz works well.
- The program outputs ‘.vtk’ files from time to time. These files contain system states in the form of grid locations with temperature and velocity values. They can be visualized in Paraview, an open-source program.
- In addition to the rms error and the estimated state parameters, convergence of the e-dimension is a way to check convergence onto a state. It falls between one and (K-1) and represents the dimension of the ensemble members (see Ref. [72]).
- The maximum aspect ratio that has been used is 20. The time for the simulation scales as  $\Gamma^2 \ln(\Gamma)$ .

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