Single Stage Heuristics for Perishable Inventory Control in Two-Echelon Supply Chains

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We study the problem of determining stocking levels for fixed-life perishable products in a two-echelon supply chain. We consider both serial chains and distribution networks consisting of a warehouse and *n* non-identical retail locations. Inventory retains constant utility throughout its lifetime, lead-times are deterministic, there are no fixed ordering costs, and unmet demand is backlogged. Although an extensive literature exists for the nonperishable product case, the consideration of perishability significantly complicates the problem. For instance, a major complication is the need to track the age of inventory as well as its position in the supply chain, adding a dimension to the already burdensome state space of dynamic programming formulations. We provide accurate single-stage heuristics for determining the stocking levels for two-echelon supply chains. We use these heuristics to develop insight and intuition into the proper management of perishable inventory. Our heuristics are robust, easy-to-use, and simple enough to be implemented using spreadsheet applications.

1. Introduction

The control of perishable products is increasingly important in supply chain management. In the past few decades, the need to properly manage such products has increased in many industries. For instance, in the technology and fashion industries, continually decreasing product lifecycles have increased the need for agile manufacturing practices. Shrinking margins place ever increasing stress on managing the $200 billion perishable product sales in the US grocery industry, which loses up to 15% of product due to spoilage. Numerous instances of perishable products exist in practice, such as photographic films, pharmaceuticals, blood, biotechnology products, foodstuffs, radioactive materials, electronic wafer fabricators, and many chemicals. Fashion and technology goods may also be viewed as deteriorating or perishable products over sufficiently long time horizons. Recognizing the economic and social importance of perishable products, researchers have conducted substantial work in perishable inventory theory. However, with the exception of a handful of situationally specific papers, the multiechelon perishable inventory problem has yet to be addressed. The following example motivates our study.

Example 1: A firm consisting of a two-echelon serial supply chain (a distribution and retail stage) sells a perishable product from the retail stage. The firm purchases a product from an upstream supplier for $20 per unit, the lead-time between the stages is a single period, and unsold product perishes 3 periods after arriving at the distribution stage. The firm faces uncertain demand (per period) that follows the negative binomial distribution with a mean of 10 units per period and a coefficient of variation of 0.632. If demand exceeds the on-hand inventory at the retail stage, the firm incurs a backordering cost of $20 per unit. Holding costs at the distribution and retail stages are $0.5 and $1 per period, respectively. The firm seeks inventory stocking levels that minimize their expected long-term operating costs per period.

The current state of the literature gives scant guidance under this scenario. One approach is to treat the inventory as nonperishable. Using the technique for determining optimal base-stock levels in a serial supply chain described in Federgruen and Zipkin (1984), the firm will stock 12 units at the warehouse and 23 units at the retailer. Alternately, the firm may choose to use Nahmias' (1979) single location heuristic for each stage. For the retail stage, this procedure is straightforward, while for the warehouse, one can utilize backorder costs analogously to the method described in Shang and Song (2003). Using this heuristic, the firm stocks 10 units at the warehouse but only 12 units at the retailer. Either of these ad-hoc policies creates significant errors. For this scenario, the base-stock levels that result in the minimal total costs are 4 units at the warehouse and 24 at the retailer. The increase in costs associated with the two ad-hoc policies, for product lifetimes of 2, 3, and 5 periods, are presented in Figure 1.

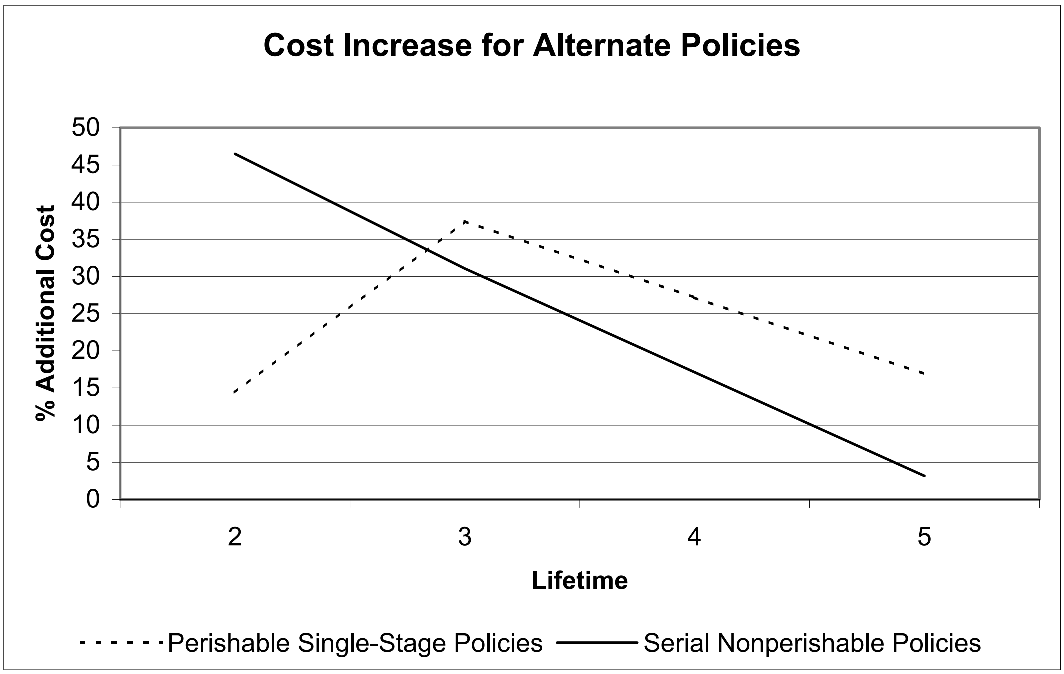

Figure 1: Additional Costs Incurred by Existing Policies

Motivated by the large cost penalties of the currently available policies, we provide managers with a simple and robust heuristic for solving problems such as the one described above. Specifically, our heuristic derives base-stock levels for two-level serial and distribution systems when the product is perishable with fixed lifetimes.

To accomplish this, we link the calculation of stocking policies for supply chains of nonperishable products with the perishable inventory theory. We construct computationally simple and independent single-stage stocking level heuristics for each supply chain installation, and test the performance of our heuristics via simulation. Our approach yields costs that exceed the best found policy (via the simulation study presented in Section 5) by 2.18% for serial systems and 2.99% for distribution systems. The simple structure of our heuristic allows us to develop insights into the proper management of perishable inventory in a two-level supply chain. For instance, our experiments suggest that for serial supply chains, downstream installations tend to behave as if they held nonperishable inventory; however, in distribution systems, the stocking level of inventory at retail installations must account for additional costs arising from the allocation decision. Thus, the inventory management of perishable products for a serial system is quite different than for a distribution system.

We argue that the main driver of the differences in managing serial versus distribution systems is, in the latter case, a significant opportunity cost is assessed for inventory expiring at one retailer rather than being used to satisfy demand at a second retailer. For expensive products with short lifetimes, this opportunity cost is sufficiently large to prevent exploitation of the common risk-pooling effect at the warehouse stage. Thus, extending the product's lifetime is most valuable when there are fewer downstream customers.

The remainder of the paper is organized as follows. In Section 2 we review the related literature, and in Section 3 we describe the setting and foundation for our model. Section 4 presents our heuristics, and Section 5 describes the simulation methodology for testing them. In Section 6 we present our numerical results and observations based on a wide range of test cases over various parameter values and demand distributions. We conclude with managerial insights in Section 7.

2. Literature Review

Our approach in finding well performing critical number policies for the perishable two-echelon distribution system draws from both perishable and nonperishable inventory theory. From the perishable inventory perspective, we utilize the concept of echelon inventory, as introduced by Clark and Scarf (1960), and characterize our stocking policies as echelon base-stock levels. To set the warehouse echelon base-stock levels, we apply a regression-based heuristic similar to the one in Ehrhardt (1979) for nonperishable, single stage (s,S) policies. Our allocation policy and stocking heuristics are similar to those in Lystad and Ferguson (2006), which are based on newsvendor calculations for calculating base-stock levels for distribution network of nonperishable products. Their approach is to construct a set of serial supply chains whose costs bound the cost of a nonperishable distribution network from above and below, and average the resulting serial chain base-stock policy levels for each echelon. We use a similar approach for determining the warehouse stocking levels of perishable inventory, but impose a further modification when determining the retailer base-stock levels.

In comparison to the amount of work on managing nonperishable inventory, the work on how to manage perishable inventory is generally limited to a small set of specific applications. The seminal works in this area are Van Zyl (1964), Nahmias (1975a) and Fries (1975). Van Zyl (1964) derives the optimal stocking policy for products with lifetimes of two periods. Nahmias (1975a) and Fries (1975) independently consider the expected single-period cost of a single installation controlling a single product with a lifetime of r periods. In both of these studies, it is assumed that all items arrive fresh from the supplier. Thus, in addition to the traditional per unit holding and shortage costs, the firm incurs an outdating cost for units of inventory held at the end of the $(r+1)^{\text{st}}$ period after they were ordered. This "fresh from the supplier" assumption may hold when the retailer is replenished directly from a manufacturer but is often violated in the more common case where a retailer is replenished from a warehouse or distribution center.

An important finding of Nahmias (1975a) is the existence of a bounded ordering region. Specifically, he shows that once the system enters a particular region, it never leaves. The generation of an optimal ordering policy however, is complicated by the need to retain a multi-dimensional state variable (the quantity of inventory held of ages $(1,\ldots,r)$. Nahmias (1982) notes that, due to this complication, the computation of optimal policies for large r is prohibitively complex.

Luckily, in light of well performing myopic heuristics, determining an optimal policy for the single location fixed-life perishable inventory problem is seldom necessary (Nahmias 1976, Lian and Liu 1999). Several approximate methods to solve the single-stage perishable inventory problem have been introduced. Nahmias (1975b) considers three policies that only utilize information on the total quantity of system inventory as opposed to the quantities of inventory at each possible remaining lifetime. Of these, a critical number policy is found to be both accurate and simple to implement. Nahmias (1976) approximates the problem with a myopic critical number policy through the use of an upper bound on the expected quantity of product that will perish before being sold or consumed. The upper bound is determined through a modified newsvendor policy that we also utilize in our model.

Cohen (1976) provides an optimal critical number policy for two period lifetimes by constructing the stationary distribution of inventory. Nandakumar and Morton (1993) create myopic upper and lower bounds and take a ratio of the tightest ones to select the order quantity. Tekin, Gurler, and Berk (2001) show that, in a continuous review system, incorporating the age of inventory into the ordering policy improves performance. Cooper (2001) provides additional bounds on the outdated quantity, and provides numerical evidence that the critical number policies are nearly as good as the optimal policies. In this work, we also utilize fixed critical number policies.

Our work differs from the aforementioned literature primarily by expanding the analysis to two-echelon systems. The consideration of extended supply chains raises a number of additional complexities. The multidimensional state vector now requires a second dimension to account for the position of each unit of inventory in the supply chain. Additionally, the existing literature assumes that all incoming inventory is fresh. Unfortunately, in multi-echelon problems where safety stock is held at the upper levels, downstream levels receive products with various inventory ages. Due in part to these difficulties, papers considering multi-echelon inventory theory for perishable products are sparse. Ferguson and Ketzenberg (2006) and Ketzenberg and Ferguson (2006) explicitly consider the effects of uncertain remaining lifetime of inventory upon receipt at the downstream stage of a two-echelon supply chain, and the value that information sharing imparts to the system. Goh, Greensberg, and Matsuo (1993) consider a two-stage system where both supply and demand are stochastic, and inventory may fill two separate types of age segregated demand. Fujiwara et al. (1997) also analyze a two-echelon serial system where the upstream stage holds a product that is decomposed into multiple subproducts. Their model allows for emergency expedition of orders in the event of a stockout and for the lifetime of the product to vary by the installation at which it is held.

However, their work is restricted by the assumptions that the demands for all products are identically distributed, and the subproducts are produced from a unit of the master product at a constant ratio.

Contributions considering traditional distribution systems are even less frequent. In addition to the two-dimensional state space vector, arborescent systems also require the specification of an allocation policy. Prastacos (1981) extends the work of Yen (1965) by considering the myopic allocation policies of perishable inventory in distribution networks. Prastacos shows that both stockouts and outages are minimized when inventory is allocated to equalize the probability of demand exceeding inventory for each age at each location. Leiberman (1958) and Pierskalla and Roach (1972) show that with constant product utility, issuing the oldest inventory first (FIFO) is optimal. Prastacos' (1981) allocation, as well as ours, assumes constant product utility; thus we also both employ FIFO policies. Unlike our work, Prastacos does not develop stocking policies, and assumes random supply. To the best of our knowledge, this work is the first to develop stocking policies for traditional two-echelon distribution systems with perishable inventory.

3. Model

We consider a two-echelon supply chain with *n* retail sites, labeled with index $i \in \{1, 2, \ldots, n\}$, and a single warehouse denoted by *W*. Inventory is fresh when it arrives to the warehouse, but at the end of *r* periods after arrival, it must be disposed of at a cost of *p* per unit. Before the age of the inventory exceeds *r* periods, it maintains a constant utility to the customer over its lifetime, i.e., the customer values a two-day old unit the same as a three-day old unit. Let D_i^t denote the stochastic demand over *t* periods at retailer *i* (we omit the superscript when *t* = 1), with respective probability and cumulative distribution functions $f_i^{(t)}$ and $F_i^{(t)}$. We assume that the demand process for each retailer is stationary over time, with the demand processes being independent across retailers. In each period, the following sequence of events occurs: previously shipped replenishments arrive at each level, demand occurs at each retailer, excess demand is fully backordered, inventory is aged (and disposed of if necessary), replenishment orders are placed, costs are assessed, and replenishment orders are shipped. The inventory positions are reviewed every period and a centralized decision maker places replenishment orders based on knowledge of the entire supply chain's inventory positions.

We assume linear per unit local inventory holding costs (h_i) and backordering costs (b_i), and zero ordering costs throughout the system. We also assume that inventory in transit from the warehouse to any of the retailers incurs a holding cost of h_W per period. Therefore we utilize echelon base-stock policies at each installation with reorder points s_i. Before costs are assessed in each period, the following variables are measured:

B_i = number of units backordered at installation i

O = inventory disposed of in a over all installations

J_i = on-hand inventory at installation i

T_i = inventory in transit to installation i

I_i = echelon inventory at installation i, $I_i = J_i$ for $i = 1, \dots, n$

$I_W = J_W + \sum_{i=1}^{n} \left(T_i + I_i \right)$

IP_i = echelon inventory-transit position at installation $i = I_i - B_i + T_i$

IO_i = inventory orders outstanding for installation $i = \max(s_i - IP_i, 0)$

The objective is to minimize the long-run expected total cost per period,

$$\min \mathrm{E}\left[h_W I_w + \sum_{i=1}^{n} \left(b_i B_i + h_i I_i \right) + pO \right] \tag{1}$$

Replenishments for each level in the supply chain arrive Li periods after being shipped. Product is shipped from the warehouse in a first in, first out (FIFO) order, minimizing the outstanding orders (IO_i) in successive allotments of increasing remaining life. By minimizing the outstanding orders, the allocation policy allocates scarce inventory to installations on the basis of their relative need. In other words, the FIFO policy ships inventory that is more likely to expire to retailers who are more likely to use the inventory to satisfy demand before that unit's lifetime is exceeded. We do not claim that this allocation policy is optimal with regard to the total cost, although we note that such an allocation minimizes expected backorders and stockouts (Prastacos, 1981). The policy's nonperishable analog has also been used previously by McGavin, Schwarz, and Ward (1993) for identical retailers and Lystad and Ferguson (2006) for non-identical retailers.

4. Echelon Base-Stock Heuristic for Perishable Distribution Networks

In this section, we present a heuristic for determining echelon base-stock levels for a two-echelon distribution network. Based on the approach introduced by Lystad and Ferguson (2006) for nonperishable inventory systems, we begin by constructing two serial supply chain systems whose mean costs bound the mean cost of the distribution system from above and below. We then use a "power approximation" regression model, constructed and tested using simulation, to identify robust echelon base-stock levels for these serial chains. Unfortunately, the serial chain stocking policies are not close approximations for the stocking levels in a distribution networks. Thus, we introduce a second adjustment for determining the retailer base-stock levels. The heuristic resulting from the power regressions and the retailer stocking level adjustment provides near-optimal echelon base-stock levels for the distribution system.

4.1 Bounding Serial Systems

To determine the upper bound, we remove risk pooling opportunities from the warehouse by constraining the centralized decision maker such that he must specify which retailer each unit of inventory will eventually be shipped to as that unit of inventory is ordered from the supplier. This approach was introduced by Graves (1996), who noted that since it may be desirable to un-commit stock, this assignment rule will not perform as well as a dynamic allocation policy. The restriction decomposes the distribution network into a set of n independent serial systems, one system for each retailer. We introduce the labels W_i to denote a warehouse installation that exclusively serves retailer i. We refer to these serial chains as "decomposed".

The construction of a lower bound on the mean system cost is based on an approach used by Federgruen and Zipkin (1984), who assume instantaneous and costless transshipment opportunities within an echelon. Under this assumption, the distinctions between installations within an echelon are artificial and the retailers may be treated collectively as a single virtual installation that fills all system demands, as shown on the left-hand side of Figure 2. We refer to this system as "collapsed". We use a combination of the stocking levels of these two serial systems as an approximation for the stocking levels of the distribution system.

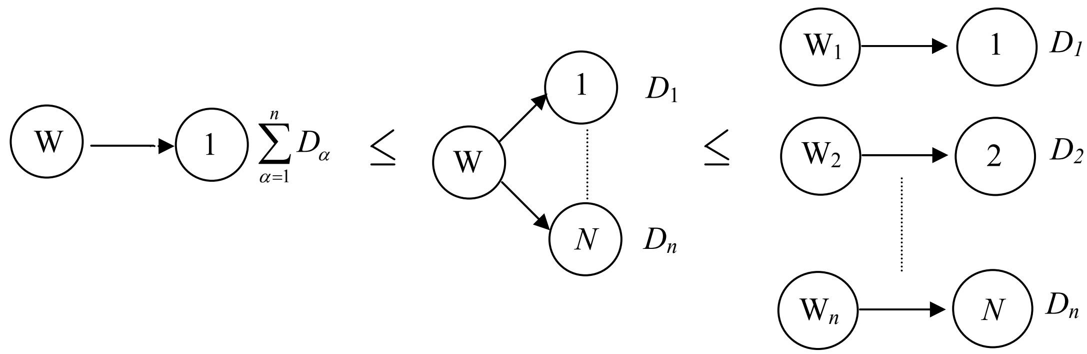

Figure 2: Collapsed and Decomposed Systems

It still remains, however, to determine stocking levels for the serial supply chain relaxations when inventory is perishable. In our numerical studies, we found that a systematic bias was induced by traditional serial chain stocking policies developed for nonperishable products as an approximation for the stocking levels of a serial chain carrying perishable products. To adjust for this bias, we use a modification of the power approximation method developed by Ehrhardt (1979). Our method is described below.

4.2 Power Approximation for Perishable Inventory in Serial Systems

We formulate a regression model to approximate the echelon base-stock levels for the serial chains. We initially create two regressors to utilize in our model. First, we calculate a distribution stage echelon base stock level by treating the supply chain as a single installation that receives fresh inventory from its upstream supplier. Nahmias (1976) shows that a good heuristic for stocking levels in this problem is the solution to the equation

$$\frac{dG_i(s)}{ds} + pF_i^{(r)}(s) - pF_i^{(r+1)}(s) = 0 \qquad (2)$$

where

$$G_i(s) = \sum_{x=0}^{s} h(s-x) f(x) + \sum_{x=s+1}^{\infty} b(x-s) f(x)$$

We next calculate retailer base stock levels by using (2) assuming that the inventory ages by L_i periods (i.e. the distribution center stocks no inventory). This approach is motivated by noting that for nonperishable products, keeping no safety stock at the upstream stage of a two-echelon system is frequently a good and simple heuristic (Graves, 1996). This observation suggests that the inventory arriving to the downstream stage of a two-echelon network has a remaining lifetime of approximately $r - L_i$. Hence we calculate Nahmias' heuristic twice, first for the warehouse base-stock level $\left(s_W^N\right)$ and then for the retailer base-stock level $\left(s_R^N\right)$ with the reduced remaining lifetime.

Next we treat the inventory in the serial chains as nonperishable. In this case, optimal solutions are given by Clark and Scarf (1960) over a finite horizon and Federgruen and Zipkin (1984) over an infinite horizon. Approximately optimal solutions may be calculated via the simple newsvendor heuristics of Shang and Song (2003). These heuristics have the attractive property that they can be expressed in closed-form. Thus, we use the Shang and Song approximation for our second regressor by setting

$$s_R^S = F_R^{-1}\left(\frac{b + h_W}{b + h_W + h_R}\right) \tag{3}$$

and

$$s_W^S = \frac{1}{2}\left(F_W^{-1}\left(\frac{b}{b + h_W + h_R}\right) + F_W^{-1}\left(\frac{b}{b + h_W}\right)\right) \tag{4}$$

for the retail and warehouse installations, respectively.

An initial pilot study also suggested that demand variance $\left(\sigma^2\right)$, product lifetime (r), backordering cost (b), outdating cost (p), and coefficient of variation $\left(\sigma/\mu\right)$ may also be significant. These regressors were entered into the two regression models below

$$s_R^E = v_1^1 \left(s_R^N\right)^{v_1^2} \left(s_R^S\right)^{v_1^3} \left(\sigma^2\right)^{v_1^4} (r)^{v_1^5} (b)^{v_1^6} (p)^{v_1^7} \left(\sigma/\mu\right)^{v_1^8} \tag{5}$$

$$s_W^E = V_2^1 \left(s_W^N\right)^{V_2^2} \left(s_W^S\right)^{V_2^3} \left(\sigma^2\right)^{V_2^4} (r)^{V_2^5} (b)^{V_2^6} (p)^{V_2^7} \left(\sigma/\mu\right)^{V_2^8} \tag{6}$$

where v_i^x are variables to be fitted. We make (5) and (6) linear by taking their logarithms, and use least-squares regression to fit the models over a grid of 144 values for s_R^E and s_W^E. These values were determined by the simulation model described in Section 5, and were taken over the following parameter values: demand following the Poisson and negative binomial distributions with mean $\mu \in \{10, 20\}$ and variance $\sigma^2 \in \{\mu, 2\mu, 4\mu\}$, $r \in \{2,3\}$, $b \in \{5, 10, 20\}$, and $p \in \{0.5, 5, 10, 20\}$. The Poisson and negative binomial distributions were selected because of their versatility in representing a spectrum of low to high coefficients of variation. Our cost parameters are similar to those used in Nahmias (1976), and Nandakumar and Morton (1993) for perishable systems, and Cachon (2001), Axsater et. al (2002), Shang and Song (2003) and Lystad and Ferguson (2006) for nonperishable systems. The models (5) and (6) were refined by discarding insignificant factors (p-value < 0.01) and were refitted, yielding the approximations

$$s_R^R = 1.402\left(s_R^S\right)^{.8306}\left(\sigma^2\right)^{.0551}\left(b\right)^{.0153} \tag{7}$$

$$s_W^R = 1.707\left(s_W^N\right)^{1.0399}\left(s_W^S\right)^{-.2702}\left(\sigma^2\right)^{.0964}\left(b\right)^{.0399}\left(r\right)^{-.0935} \tag{8}$$

The proportions of explained variation for these regressions are 0.988 and 0.997, respectively (although we note that because the regressors are not independent, these values solely represent the fit of the resulting equations). The model in (7) and (8) was tested against a second grid of 162 values for s_R^R and s_W^R, over the following parameter values: demand following the Poisson and negative binomial distributions with mean $\mu \in \{5, 15, 30\}$ and variance $\sigma^2 \in \{\mu, 2\mu, 3\mu\}$, $r \in \{2,3\}$, $b \in \{2, 8, 32\}$, and $p \in \{2, 8, 32\}$. The average error in the resulting serial system costs is 3.43% compared to the best found results via the simulation study in Section 5 (full data are presented in Tables A.1 and A.2 in the Appendix).

Investigation of Equation (7) gives rise to the following observation:

Observation 1: *The retailer base-stock levels of a two-installation serial chain may be calculated independently of any consideration of perishability.*

Our intuitive explanation is as follows. When the central decision maker sets the stocking level at the warehouse, s/he in effect determines the total system stock and thus the likelihood that demand over the lifetime of the product falls below this level. Thus the full responsibility of outdating is realized at the warehouse. With the total system stock decision made, the retailer, unable to affect the outdating process, operates as if it held nonperishable inventory.

4.3 Retailer Base-Stock Approximations for a Distribution System

Observation 1 does not apply to distribution systems. In the absence of a rebalancing relaxation (where inventory is collected from the retailers and redistributed each period), carrying an additional unit of inventory at one retailer may cause that unit to expire rather than being used to satisfy demand at an alternate retailer. This opportunity cost is not present in serial systems. Thus, we include an adjustment to our serial system policy for setting the retailer base-stock levels in a distribution system. To develop this adjustment, we relax our problem by making the following simplifying assumptions:

1) All inventory units at each retailer are as fresh as is possible (e.g., with $r-1$ periods of life remaining).
2) The warehouse carries sufficient stock to ensure that all orders are filled in the following period.

These assumptions allow an approximation for the cost of allocating a unit of inventory to a retail stage. This cost is comprised of three elements: additional expected holding costs, a reduction in potential backordering costs, and an increase in expected outdating costs. The first element is simply the probability that the demand in one period is less than the amount of inventory held at the retailer times the echelon holding cost at that retailer, $h_i F_i(s_i)$. By moving a unit of inventory from the warehouse to the retailer, savings in expected backorder costs are achieved. To capture this, our second element is equal to the probability a stockout occurs times the backorder cost rate: $b_i(1-F_i(s_i))$. The previous two terms are analogous to the costs in the traditional newsvendor problem. To capture the impact of perishable products, we add the increase in the cost associated with perishability. The probability that the unit expires at the retailer is $F_i^{(r-L_i)}(s_i)$.

However, the unit perishes regardless of whether it was at the retailer or at the warehouse if the total cumulative demand at each other location is less than the inventory held at that location. In this case, we do not penalize the allocation decision. Thus the final cost element in our adjustment is the outdate cost times the joint probability that a unit expires at the retailer and at least one other retailer has sufficient demand so the unit could have avoided perishing:

$$pF_i^{(r-L_i)}(s_i)\left(1-\prod_{j\neq i}F_j^{(r-L_j)}(s_j)\right)$$

The combined cost of allocating a unit of inventory to a retail stage is

$$h_iF_i(s_i)-b_i(1-F_i(s_i))+pF_i^{(r-L_i)}(s_i)\left(1-\prod_{j\neq i}F_j^{(r-L_j)}(s_j)\right) \tag{9}$$

We select as our stocking level adjustment, the inventory quantity that minimizes the cost in (9):

$$s_i^A=\arg\min\left[h_iF_i(s_i)-b_i(1-F_i(s_i))+pF_i^{(r-L_i)}(s_i)\left(1-\prod_{j\neq i}F_j^{(r-L_j)}(s_j)\right)\right] \tag{10}$$

4.4 Policy

Our heuristic calculates the echelon base-stock levels as simple averages of the preceding calculations. For the warehouse, we follow the technique in Lystad and Ferguson (2006) and average the warehouse echelon base-stock level under the collapsed serial chain $\left(s_W^R\right)$ with the sum of the warehouse echelon base-stock levels over the decomposed serial chains $\left(s_{W_i}^R\right)$. That is,

$$s_W^H=\frac{1}{2}\left(s_W^R+\sum_{i=1}^n s_{W_i}^R\right). \tag{11}$$

For the retail stages, we take a weighted sum of the retailer base stock level under the decomposed serial chains ($s_{W_i}^R$) and the stocking level found by our approximation (s_i^A). The weights were calculated by regression, fitting the model

$$s_i^H=V^9s_i^R+V^{10}s_i^A \tag{12}$$

with data from experiments taken over the following parameter values: demand following the Poisson and negative binomial distributions with mean $\mu\in\{10, 20\}$ and variance $\sigma^2 \in \{\mu, 2\mu, 4\mu\}$, $r \in \{2,3\}$, $b \in \{5, 10, 20\}$, and $p \in \{0.5, 5, 10, 20\}$. The resulting model,

$$s_i^H = 0.281 s_i^R + 0.782 s_i^A \tag{13}$$

was tested against a second grid of values for s_i^H over the following parameter set: demand following the Poisson and negative binomial distributions with mean $\mu \in \{5, 15\}$ and variance $\sigma^2 \in \{\mu, 2\mu, 3\mu\}$, $r \in \{2,3\}$, $b \in \{2, 8, 32\}$, and $p \in \{2, 8, 32\}$. The average error in the resulting distribution system costs is 3.51% (the entire set of results from this experiment is presented in Tables A.3 and A.4 in the Appendix).

5. Simulation Methodology

Our simulation methodology is based on an unequal variance, two-stage screening-subset selection procedure presented in Nelson et al. (2001). We first create a set of base-stock level candidates covering a range of the expected minimizing base-stock levels (as predicted by our heuristic), ± at least 5 inventory units for each installation. For the parameter settings in these examples, this range covers at least 50% of the cumulative distribution of the lead-time demand at each installation.

For each stocking level, we initially make a long simulation run of our model over 20,020 periods. We use the method of batch means (Law and Kelton, 2000) by batching periods into groups of 20 to obtain sample averages (costs) that are approximately i.i.d. and normally distributed. We remove the first batch (20 periods) to mitigate initialization effects. The remaining data points are used in the initial screening phase which identifies a temporary "winning" system with the lowest estimated cost and eliminates poorly-performing systems with high costs.

Potential sets of stocking levels that survive the initial screening are subjected to a second round of simulation experiments, where we retain our batch sizes and generate a sufficient number of data points to eliminate all but one of the systems. After this experiment, the set of stocking levels that has the lowest per period cost is selected. With a 1.5 GHz processor, the joint processes completes in an average of 27.3 seconds for each experiment. This procedure ensures that the cost of the selected system is within δ of the cost of the (unknown) best system with probability $\geq 1-\alpha$. The *indifference* *parameter* δ was set at 0.2% of the estimated cost associated with the temporary winner from the initial screening stage. Since δ is a realization of a random variable, this method is an approximate implementation of the method of Nelson et al. (2001).

6. Problem Design and Experimental Results

6.1 Symmetric Two-Echelon Networks

Our first experimental design considers two network topologies, with either two or four symmetric retailers. We test the heuristics using a full factorial design over a range of backorder costs, outdate costs, lifetimes, and demand variances. We assume the total system demand is distributed according to the Poisson or the negative binomial distribution with a mean of 20 units per period across topologies, and that the demand processes at the retailers are i.i.d. Our remaining parameters are $b_i \in \{5,10,20\}$, $p \in \{5,10,20\}$, $r \in \{2,3\}$, and $\sigma_i^2 \in \{2\mu, 4\mu\}$ for negative binomial demand. The demand, backorder, and holding cost parameter values are similar to those used in nonperishable works by Jackson (1988), Cachon (2001), Axsater et al. (2002), Shang and Song (2003), and Lystad and Ferguson (2006). The demand, holding cost, outdate cost, and backorder cost parameter values are similar to those used by Nahmias (1976) and Nandakumar and Morton (1993) for single-stage perishable systems. In the latter works, the authors note that a perishable system quickly resembles a nonperishable system as lifetimes exceed two periods. We also observe this convergence as noted in the following observation.

Observation 2: *In both the serial supply chain and distribution network cases, increasing lifetimes cause a system to quickly converge to its nonperishable analog.*

This trend is apparent when comparing the inventory stocking levels in Tables A.1 and A.2 in the Appendix. In Table A.1, the best found stocking levels decrease quickly with outdate cost (while keeping other parameters constant). In the analogous results in Table A.2, the stocking levels decrease at a significantly slower rate, suggesting that the costs imposed by expired inventory are somewhat small. To illustrate the effects of increasing product lifetimes on total system costs, we plot the average total cost per period as lifetimes increase for systems with $b = 20$, $p = 20$, $h_W = h_i = 0.5$ and $r \in \{2, 3, 5\}$. Serial systems with $\mu = 10$, $\sigma^2 \in \{10, 20, 40\}$ are presented in Figure 3. Two-retailer distribution systems with $\mu_i = 10$, $\sigma_i^2 \in \{10, 20, 40\}$ are presented in Figure 4.

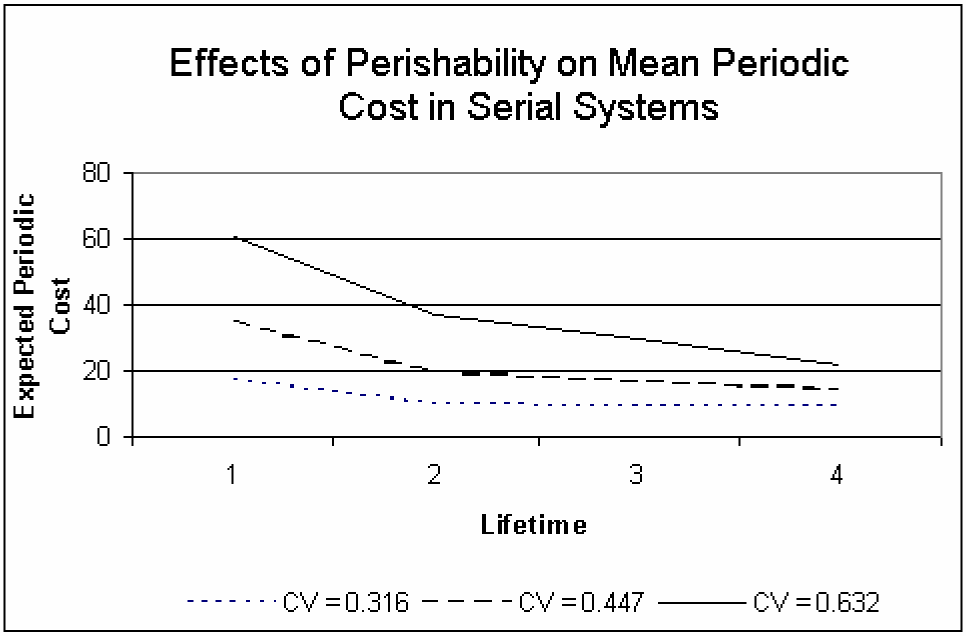

Figure 3: Effects of Perishability for Serial Systems

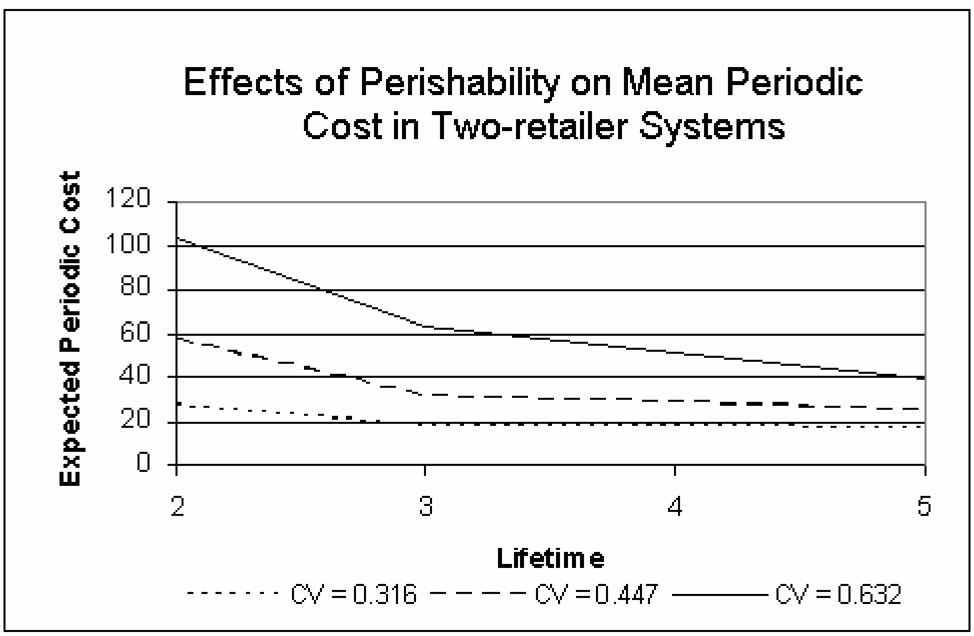

Figure 4: Effects of Perishability for Two-retailer Systems

Clearly, the decrease in costs associated with increasing lifetimes occurs due to a decreasing frequency of outdating. The expected periodic cost converges to that of a nonperishable system as the lifetimes increase, but significant additional costs associated with perishability exist when either lifetimes are short or demand variance is high. Because the majority of perishable associated costs are eliminated by the third period of lifetime, we set our lifetimes to either 2 or 3 periods. These assignments are equivalent to the ones in Nahmias (1976) and Nandakumar and Morton (1993), adjusted to account for the lifetime lost in transit to the retail stages. The results of these tests are sumarized in Tables 1 and 2, for the two and four retailer systems, respectively, with the entire set of results displayed in Tables A.5 and A.6 in the Appendix.

% Error for Two-retailer Systems		
	Lifetime	
p	1	2
0.5	0.94	1.09
5	0.54	0.92
10	0.93	1.59
20	1.94	1.07

Table 1: Two-retailer Summary

% Error for Four-retailer Systems		
	Lifetime	
p	1	2
0.5	5.34	9.02
5	3.66	4.11
10	4.42	3.44
20	3.28	1.16

Table 2: Four-retailer Summary

The results above lead to the following observation:

Observation 3: *The heuristic performs well, with small error rates. These errors are increasing as the number of retailers increases. The most significant errors correspond to low outdating costs*.

While the heuristic's error can exceed an average of 5%, it does so only for low outdating costs. Since the local holding cost rate at the retailers in these tests is set to one unit while the outdating cost is only 0.5 units, the firm prefers to dispose of expiring inventory rather than carry it an additional period. In practice such scenarios are rare, although we note the backorder to holding cost ratios in our study are greater than 5. Neglecting the cases where the holding cost exceeds the outdating cost, the heuristic's error is approximately 2.26%. The increase in the error when the number of retailers increases is partly due to discretization effects, where rounding errors imposed to ensure integer valued base-stock levels become more problematic because of the smaller inventory quantities. However, these errors are only slightly greater than those arising from leading heuristics for nonperishable distribution systems (see for comparison Lystad and Ferguson, 2006).

Observation 4: *All else held constant, increasing the outdate cost, backorder cost, or demand variance also increases the total system cost. Further, decreasing the lifetime tends to increase costs, except when the outdating cost is less than the local holding cost.*

This observation is in line with intuition and previous work in both nonperishable multiechelon and perishable single location systems. Increasing the variance of demand increases both the expected number of backorders and expected number of outdates. Increasing the costs of either of these increases mean system costs directly.

Observation 5: *As product lifetime increases, the mean system cost drops faster for two-retailer systems than for four-retailer systems.*

Previous work in nonperishable multi-echelon systems suggests that increasing the number of retailers inhibits the exploitation of risk-pooling benefits, leading to increased costs. This effect is apparent when lifetimes are long and our stocking levels approach those of a nonperishable system. When lifetimes are short, however, less inventory is held at the retailers because of the opportunity cost effect. The greater drop in system costs associated with the two-retailer systems results from the ability to exploit risk-pooling opportunities at the retailers.

Observation 6: *As product lifetime decreases, the system-wide inventory savings associated with risk-pooling for systems with fewer, large-volume retailers decrease.*

The increase in opportunity cost effects begins to dominate the risk pooling advantages the two-retailer network enjoys compared to the four-retailer network. The downstream risk-pooling savings can no longer be captured in light of the increased opportunity costs, thus the incentives for developing transshipment opportunities increase as lifetimes decrease. However, it should be mentioned that in practice transshipment opportunities may consume valuable lead-time. As noted above, unless lifetimes are very short, the system may be treated as if the inventory were nonperishable. Thus, transshipments that route inventory back through a distribution point, such as the traditional rebalancing relaxation in nonperishable work (e.g., see Clark and Scarf, 1960; Federgruen and Zipkin, 1984; and Axsater et al., 2002), are particularly inappropriate in a perishable context. Rather, it is the ability to directly *satisfy customer demand* from multiple sources that becomes increasingly attractive rather than the ability to *rebalance inventories*.

6.2 Asymmetric Backorder Rates

In this section, we consider a warehouse that serves the perishable product to two retailers with differing backorder rates. We continue to utilize Poisson and negative binomial demands with $\mu_i = 10$. We set $b_1 = 5$, $b_2 \in \{10, 20\}$, $p \in \{0.5, 5, 10, 20\}$, $r \in \{2, 3\}$, and $\sigma_i^2 \in \{2\mu, 4\mu\}$ for negative binomial demand distributions. These assignments generate a total of 48 cases. The results of the tests are summarized in

Tables 3 and 4 for problems with lifetimes of 2 or 3 periods, respectively. The complete set of results is given in Tables A.7 and A.8 in the Appendix.

% Errors for Two-period Lifetimes		
	b_2	
σ^2	10	20
10	0.77	1.15
20	0.95	0.64
40	1.31	2.35

Table 3: Asymmetric Backorder Cost Summary for Two-period Lifetimes

% Errors for Three-period Lifetimes		
	b_2	
σ^2	10	20
10	1.34	0.56
20	0.35	0.73
40	1.14	1.04

Table 4: Asymmetric Backorder Cost Summary for Three-period Lifetimes

Observation 7: *The proposed heuristic approach is robust to asymmetry in backordering costs.*

The errors of the heuristic under asymmetric backorder cost profiles are similar to those of under symmetric backorder cost profiles. Thus the heuristic is robust to asymmetry in backordering costs of even up to 400%, atypical for a vast majority of practical examples.

Observation 8: *Although system-wide echelon inventories for asymmetric backorder cases are approximately the same as for symmetric backorder cases, more inventory is held at the warehouse in the asymmetric cases. Also, backorder asymmetry decreases total system costs.*

Lystad and Ferguson (2006) show for the management of nonperishable products, backorder cost asymmetry decreases echelon stocking levels and inventory costs. We find similar results for perishable products. The system controller may exploit virtual pooling effects as inventories increase at the retailer with high backorder cost. However, these virtual pooling savings are inhibited by the danger of inventory expiring at the high backorder retailer; hence the decision maker holds a portion of inventory at the warehouse rather than allocating it to the retail stages. This limits the cost savings that may be achieved through the risk-pooling effects.

6.3 Asymmetric Demand Rates

We next consider a warehouse that serves the perishable product to two retailers with varying demand rates. As before, we assume Poisson or negative binomial demands, with means $\mu_1 = 5$ and $\mu_2 = 15$. The remaining parameters are $b_i \in \{5,10,20\}$, $p \in \{0.5,5,10,20\}$, $r \in \{2,3\}$, and $\sigma_i^2 = 2\mu$ for negative binomial demands. The assignments result in a total of 48 problems. The results of these tests are summarized in Tables 5 and 6 for problems with lifetimes of 2 or 3, respectively. The complete set of results is contained in Tables A.9 and A.10 in the Appendix.

% Error for Two-period Lifetimes			
	b		
σ^2	5	10	20
15	0.76	0.96	1.61
30	1.00	0.56	0.18

Table 5: Asymmetric Demand Summary for Two-period Lifetimes

% Error for Three-period Lifetimes			
	b		
σ^2	5	10	20
15	2.31	2.61	4.53
30	3.31	1.79	1.94

Table 6: Asymmetric Demand Summary for Three-period Lifetimes

Observation 9: *The proposed heuristic methodology is robust to asymmetry in demand.*

The errors in the heuristic under asymmetry in demand are approximately the same as under symmetric demand, and continue to be on par with those found for heuristics for nonperishable inventory control.

Observation 10: *We find no significant differences in total system costs between the symmetric and asymmetric demand cases. Slightly greater inventory is held in the asymmetric demand problems.*

Lystad and Ferguson (2006) show that in nonperishable cases, demand rate asymmetry decreases echelon stocking levels and inventory costs. In this case, we find a slight increase in the warehouse echelon base-stock level with demand asymmetry. The presence of the opportunity cost effect prevents the exploitation of virtual risk-pooling as one retailer captures most of the system demand.

6.4 Importance of Perishable Inventory Policies

In the previous sections, we presented heuristics to set inventory base-stock levels in supply chains when inventory is perishable. We also argued that, beyond very short lifetimes, the systems behave essentially as if they held nonperishable products. A natural question is: when does a firm need to consider the perishable aspect of its inventory and utilize the more complex inventory control policies?

To illustrate the importance of inventory control policies that account for perishability, we compare our heuristic to the nonperishable newsvendor heuristics of Shang and Song (2003) and Lystad and Ferguson (2006). We consider scenarios with $b = 20$, $p = 20$, $h_W = h_i = 0.5$, and $r \in \{2, 3, 5\}$. The results for serial systems (having single retailers) with $\mu = 10$ and $\sigma^2 \in \{10, 20, 40\}$ are presented in Figure 5. The results for two-retailer distribution systems with $\mu_i = 10$ and $\sigma_i^2 \in \{10, 20, 40\}$ are presented in Figure 6.

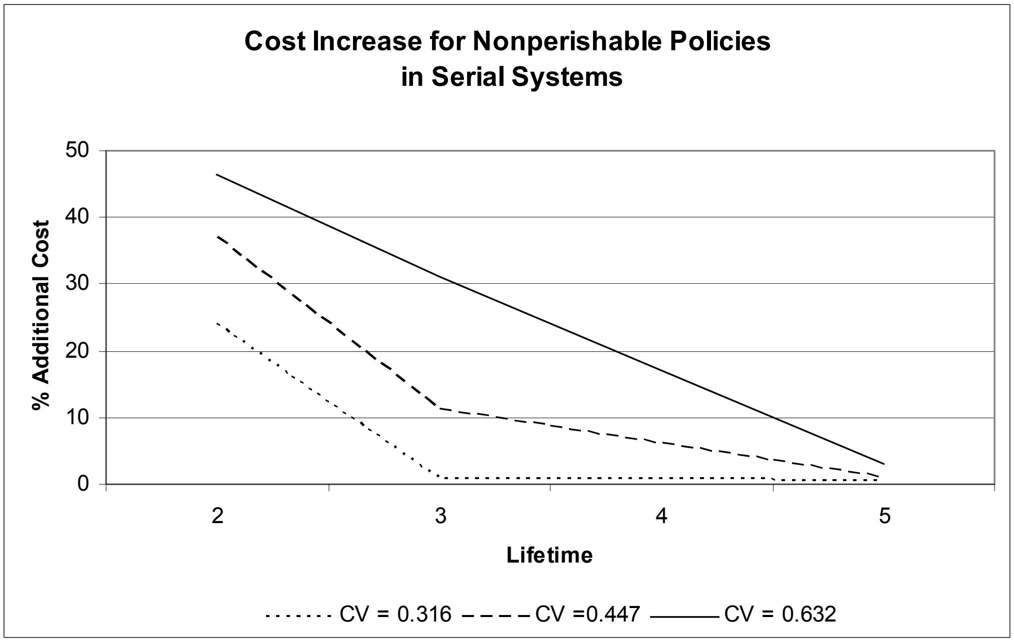

Figure 5: Nonperishable Heuristics in Perishable Serial Systems

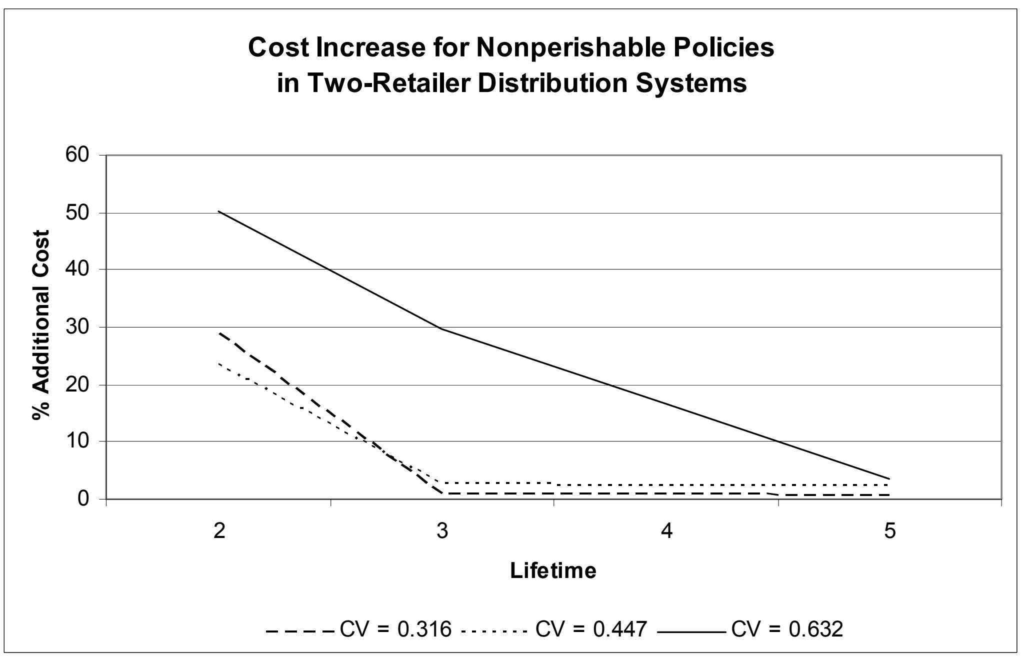

Figure 6: Nonperishable Heuristics in Perishable Two-retailer Distribution Systems

Figures 5 and 6 reinforce our claim that supply chains resemble nonperishable systems as the likelihood of outdating decreases. They additionally show that when the opportunity of outdating is significant, the use of nonperishable inventory control policies yields significantly higher costs. Thus, failure to account for the perishable aspect of inventory may lead to costly mistakes. We found that the use of nonperishable policies increases costs by about 23% for products with the shortest lifetimes and lowest demand variances; this impact decreases with lifetime but increases with the demand variance.

7. Concluding Remarks

Managers faced with perishable inventory may make costly mistakes when relying on nonperishable policies and intuition. In this paper, we present a simple heuristic for two-echelon supply chains of fixed-life perishable inventory. We consider both serial chains and distribution networks with n non-identical retailers, and show the qualitative behaviors of the two types of topologies are distinct. Our heuristic treats installations within the supply chain independently, allowing for simple, closed-form solutions that may be applied using spreadsheet applications.

Relative to the best found (via simulation) base-stock levels, our heuristic yields a 2.18% and 2.99% average errors for the mean total cost per period in serial and distribution systems, respectively. Under serial systems, the retail stage of a two-echelon network behaves as if it holds nonperishable inventory because the warehouse determines the expected number of outdates per period through the setting of its echelon base-stock level. For the parameter settings under consideration, the heuristic echelon base-stock levels were close to their nonperishable inventory analogs. As the lifetime increases, the likelihood of outdating decreases quickly since the probability of stock exceeding demand over the lifetime of the inventory unit approaches zero.

For distribution systems, however, the allocation of inventory to the retailers may increase the probability of outdating; a unit of inventory held in stock at one retailer may expire while a "younger" unit is used to satisfy demand at another retailer. We refer to this as an opportunity cost of carrying inventory at the retail stages, and find that this opportunity cost has important implications for the control of distribution networks. In particular, it inhibits the exploitation of risk-pooling benefits at the warehouse. As the product lifetime increases, this opportunity cost effect diminishes, creating greater savings for systems with fewer retailers. Compared to the nonperishable product case, the risk-pooling effect (increasing the number of retailers while keeping the total system demand constant) has a smaller benefit on the stocking levels and the total amount of inventory in the system. This suggests that investments made to extend product lifetimes (make the product less perishable) are more valuable when significant risk-pooling opportunities exist. Alternately, strategies allowing customer demands to be filled from multiple retail sites remove this opportunity cost effect entirely, in addition to enabling the traditional cost savings.

This work opens a promising stream of future research. For instance, we have assumed throughout this paper that inventory lifetimes are deterministic, while random lifetimes are common in practice for some products. We have likewise assumed that inventory expiration costs are constant, regardless of the location of the units when they perish. In a production process, the increase in inventory value suggests that outdates late in the process are more costly than outdates early in the process. Extensions beyond the previous model variations are also numerous. For instance, by comparing the costs of various network topologies, the value of expedition strategies, delayed differentiation, demand variance reduction, and forecasting accuracy in supply chains managing perishable goods may be explored.

Appendix

Table A.1: Serial System to Create Regressions

r	μ	σ^2	b	p	s_W^*	s_1^*	C^*
2	10	10	5	0.5	10	15	6.31
2	10	10	5	5	10	14	7.41
2	10	10	5	10	8	15	8.30
2	10	10	5	20	8	14	9.60
2	10	10	10	0.5	12	16	7.44
2	10	10	10	5	10	16	9.36
2	10	10	10	10	10	15	10.87
2	10	10	10	20	8	16	13.05
2	10	10	20	0.5	12	17	8.39
2	10	10	20	5	10	17	11.42
2	10	10	20	10	9	17	13.60
2	10	10	20	20	8	17	17.08
2	10	20	5	0.5	13	16	8.85
2	10	20	5	5	8	17	12.23
2	10	20	5	10	7	16	14.49
2	10	20	5	20	3	19	17.62
2	10	20	10	0.5	14	18	10.51
2	10	20	10	5	9	18	15.98
2	10	20	10	10	8	18	19.81
2	10	20	10	20	4	20	25.20
2	10	20	20	0.5	15	20	12.07
2	10	20	20	5	10	20	19.84
2	10	20	20	10	8	20	25.73
2	10	20	20	20	6	20	34.68
2	10	40	5	0.5	12	18	11.54
2	10	40	5	5	6	17	18.80
2	10	40	5	10	0	20	23.30
2	10	40	5	20	1	17	28.78
2	10	40	10	0.5	15	21	13.83
2	10	40	10	5	6	21	25.21
2	10	40	10	10	4	20	32.85
2	10	40	10	20	4	17	43.00
2	10	40	20	0.5	18	23	16.02
2	10	40	20	5	7	24	32.19
2	10	40	20	10	1	27	43.95
2	10	40	20	20	1	23	60.69
2	20	20	5	0.5	19	24	9.02
2	20	20	5	5	18	24	9.68
2	20	20	5	10	17	24	10.27

Table A.1, Continued							
2	20	20	5	20	17	24	11.14
2	20	20	10	0.5	20	26	10.67
2	20	20	10	5	19	26	11.96
2	20	20	10	10	18	26	13.00
2	20	20	10	20	17	26	14.64
2	20	20	20	0.5	21	27	12.26
2	20	20	20	5	19	28	14.33
2	20	20	20	10	19	27	16.03
2	20	20	20	20	17	27	18.74
2	20	40	5	0.5	21	28	12.76
2	20	40	5	5	17	29	15.31
2	20	40	5	10	16	28	17.20
2	20	40	5	20	15	27	19.81
2	20	40	10	0.5	23	31	15.19
2	20	40	10	5	19	30	19.57
2	20	40	10	10	17	30	22.81
2	20	40	10	20	14	31	27.50
2	20	40	20	0.5	25	33	17.39
2	20	40	20	5	20	33	23.99
2	20	40	20	10	16	34	29.17
2	20	40	20	20	16	32	36.97
2	20	80	5	0.5	21	31	16.94
2	20	80	5	5	13	31	23.88
2	20	80	5	10	10	31	28.37
2	20	80	5	20	3	35	34.12
2	20	80	10	0.5	24	34	20.25
2	20	80	10	5	16	34	31.40
2	20	80	10	10	5	41	39.18
2	20	80	10	20	8	35	49.57
2	20	80	20	0.5	27	38	23.35
2	20	80	20	5	18	37	39.66
2	20	80	20	10	13	38	51.21
2	20	80	20	20	11	35	69.09
3	10	10	5	0.5	10	15	6.55
3	10	10	5	5	10	15	6.60
3	10	10	5	10	10	15	6.64
3	10	10	5	20	10	15	6.70
3	10	10	10	0.5	12	15	7.92
3	10	10	10	5	12	15	7.99
3	10	10	10	10	11	15	8.09
3	10	10	10	20	10	16	8.20
3	10	10	20	0.5	11	17	9.12
3	10	10	20	5	11	17	9.36
3	10	10	20	10	11	17	9.47

Table A.1, Continued							
3	10	10	20	20	12	16	9.80
3	10	20	5	0.5	10	17	9.62
3	10	20	5	5	11	16	10.11
3	10	20	5	10	8	17	10.52
3	10	20	5	20	9	16	11.09
3	10	20	10	0.5	11	19	11.67
3	10	20	10	5	9	20	12.64
3	10	20	10	10	9	19	13.42
3	10	20	10	20	9	18	14.76
3	10	20	20	0.5	13	21	13.65
3	10	20	20	5	11	20	15.23
3	10	20	20	10	10	20	16.70
3	10	20	20	20	8	21	18.96
3	10	40	5	0.5	10	18	12.80
3	10	40	5	5	6	19	15.18
3	10	40	5	10	4	19	16.89
3	10	40	5	20	3	18	19.25
3	10	40	10	0.5	12	21	15.64
3	10	40	10	5	7	21	19.80
3	10	40	10	10	5	22	22.95
3	10	40	10	20	2	22	27.32
3	10	40	20	0.5	13	23	18.25
3	10	40	20	5	7	25	24.66
3	10	40	20	10	8	22	29.55
3	10	40	20	20	3	25	36.92
3	20	20	5	0.5	18	25	9.18
3	20	20	5	5	19	24	9.17
3	20	20	5	10	19	24	9.15
3	20	20	5	20	19	24	9.18
3	20	20	10	0.5	19	26	11.03
3	20	20	10	5	20	26	11.01
3	20	20	10	10	20	26	11.01
3	20	20	10	20	20	26	11.00
3	20	20	20	0.5	20	28	12.79
3	20	20	20	5	21	27	12.68
3	20	20	20	10	21	27	12.87
3	20	20	20	20	20	27	12.80
3	20	40	5	0.5	20	28	13.40
3	20	40	5	5	19	29	13.48
3	20	40	5	10	18	29	13.60
3	20	40	5	20	18	29	13.71
3	20	40	10	0.5	21	31	16.16
3	20	40	10	5	21	30	16.51
3	20	40	10	10	19	32	16.65

Table A.1, Continued							
3	20	40	10	20	21	30	17.09
3	20	40	20	0.5	22	33	18.92
3	20	40	20	5	22	33	19.41
3	20	40	20	10	22	32	19.90
3	20	40	20	20	20	33	20.61
3	20	80	5	0.5	18	31	18.47
3	20	80	5	5	17	30	19.53
3	20	80	5	10	17	30	20.36
3	20	80	5	20	14	30	21.81
3	20	80	10	0.5	22	35	22.56
3	20	80	10	5	18	34	24.65
3	20	80	10	10	16	35	26.46
3	20	80	10	20	13	35	29.28
3	20	80	20	0.5	24	37	26.52
3	20	80	20	5	20	38	30.07
3	20	80	20	10	18	38	33.29
3	20	80	20	20	16	36	38.15

Table A.2: Serial System Experimental Data Used to Test Regression Models

r	μ	σ^2	b	p	s_W^*	s_1^*	C^*	s_W^H	s_1^H	C^H
2	5	5	2	2	4	7	3.55	6	7	3.81
2	5	5	2	8	3	7	4.36	5	7	4.47
2	5	5	2	32	1	8	5.88	2	7	5.90
2	5	5	8	2	5	9	5.82	6	9	5.89
2	5	5	8	8	4	9	8.35	5	9	8.37
2	5	5	8	32	2	9	13.43	2	9	13.43
2	5	5	32	2	7	10	8.01	6	11	8.01
2	5	5	32	8	5	10	13.28	5	11	13.59
2	5	5	32	32	2	11	25.81	3	11	26.41
2	5	10	2	2	4	8	5.22	4	9	5.37
2	5	10	2	8	1	8	6.87	2	9	7.23
2	5	10	2	32	0	7	9.31	0	7	9.31
2	5	10	8	2	6	10	9.05	6	12	9.16
2	5	10	8	8	1	12	14.24	2	12	14.32
2	5	10	8	32	0	10	23.56	0	11	23.50
2	5	10	32	2	8	13	13.17	7	15	13.45
2	5	10	32	8	3	14	24.08	4	15	24.30
2	5	10	32	32	0	14	48.54	0	14	49.35
2	5	15	2	2	3	6	5.90	2	8	5.92
2	5	15	2	8	0	7	7.97	0	8	8.30
2	5	15	2	32	0	4	10.63	0	4	10.63
2	5	15	8	2	5	10	10.79	5	11	10.86
2	5	15	8	8	1	10	17.68	1	11	17.89
2	5	15	8	32	0	7	29.21	0	7	29.21
2	5	15	32	2	7	13	16.34	7	15	16.42
2	5	15	32	8	3	13	31.30	3	15	32.25
2	5	15	32	32	0	12	63.90	0	12	63.90
2	15	15	2	2	14	19	5.84	12	19	6.49
2	15	15	2	8	13	19	6.10	12	19	6.53
2	15	15	2	32	12	19	6.78	11	19	7.34
2	15	15	8	2	16	21	9.26	16	21	9.42
2	15	15	8	8	15	21	10.39	14	21	10.81
2	15	15	8	32	12	22	13.27	12	21	13.89
2	15	15	32	2	17	24	12.44	17	24	12.52
2	15	15	32	8	16	24	15.47	15	24	15.58
2	15	15	32	32	14	23	23.17	13	24	23.31
2	15	30	2	2	13	20	8.52	13	20	8.52
2	15	30	2	8	11	20	9.68	12	20	9.74
2	15	30	2	32	9	20	11.86	9	20	11.90
2	15	30	8	2	17	24	14.07	15	24	14.24
2	15	30	8	8	11	26	18.28	12	24	18.55
2	15	30	8	32	9	24	26.65	9	24	26.65

Table A.2, Continued

2	15	30	32	2	18	29	19.66	18	29	19.74
2	15	30	32	8	15	28	29.29	13	29	29.96
2	15	30	32	32	9	29	51.38	9	29	52.71
2	15	45	2	2	11	21	10.36	11	21	10.36
2	15	45	2	8	5	23	12.54	8	21	12.60
2	15	45	2	32	1	23	16.25	4	21	16.26
2	15	45	8	2	15	25	17.66	14	26	17.65
2	15	45	8	8	12	24	25.05	10	26	24.97
2	15	45	8	32	7	23	38.99	4	26	39.11
2	15	45	32	2	18	31	25.51	17	31	25.72
2	15	45	32	8	12	31	41.25	12	31	41.50
2	15	45	32	32	8	29	78.35	6	31	79.12
2	30	30	2	2	27	33	8.18	25	32	9.15
2	30	30	2	8	27	33	8.23	25	32	9.15
2	30	30	2	32	26	33	8.43	25	32	9.75
2	30	30	8	2	30	37	12.72	28	36	13.95
2	30	30	8	8	29	37	13.12	28	36	14.08
2	30	30	8	32	28	37	14.37	27	36	15.52
2	30	30	32	2	31	41	16.92	32	39	17.32
2	30	30	32	8	31	40	18.36	31	39	18.69
2	30	30	32	32	29	40	22.17	30	39	22.57
2	30	60	2	2	28	37	11.77	26	35	13.02
2	30	60	2	8	26	38	12.22	25	35	13.69
2	30	60	2	32	25	37	13.40	23	35	15.22
2	30	60	8	2	32	43	18.84	30	41	20.37
2	30	60	8	8	28	44	21.16	27	41	24.01
2	30	60	8	32	26	43	26.40	24	41	30.62
2	30	60	32	2	34	49	25.78	32	47	28.76
2	30	60	32	8	31	48	31.97	29	47	35.61
2	30	60	32	32	27	48	46.76	25	47	52.97
2	30	90	2	2	26	38	14.67	25	38	14.84
2	30	90	2	8	23	38	16.12	23	38	16.21
2	30	90	2	32	19	38	18.83	19	38	18.94
2	30	90	8	2	29	47	24.11	30	44	24.48
2	30	90	8	8	26	44	29.59	26	44	29.88
2	30	90	8	32	21	44	41.17	21	44	41.48
2	30	90	32	2	35	52	33.44	33	52	34.32
2	30	90	32	8	28	52	47.12	28	52	47.53
2	30	90	32	32	23	51	78.62	22	52	78.83
3	5	5	2	2	5	7	3.43	6	7	3.63
3	5	5	2	8	4	7	3.54	5	7	3.60
3	5	5	2	32	4	7	3.84	4	7	3.84
3	5	5	8	2	5	9	5.53	6	9	5.56
3	5	5	8	8	5	9	6.03	5	9	6.03

Table A.2, Continued										
3	5	5	8	32	4	9	7.33	4	9	7.33
3	5	5	32	2	7	10	7.47	7	11	7.72
3	5	5	32	8	6	10	8.83	6	11	9.18
3	5	5	32	32	5	10	12.38	5	11	13.50
3	5	10	2	2	5	9	5.39	5	9	5.39
3	5	10	2	8	3	8	5.46	4	9	6.09
3	5	10	2	32	0	9	6.52	1	9	6.57
3	5	10	8	2	5	11	8.48	5	12	8.65
3	5	10	8	8	4	12	10.88	4	12	10.88
3	5	10	8	32	1	11	14.53	0	12	14.54
3	5	10	32	2	7	15	12.63	7	15	12.63
3	5	10	32	8	5	13	16.66	4	15	16.95
3	5	10	32	32	4	12	27.97	1	15	27.89
3	5	15	2	2	2	7	5.63	3	8	5.79
3	5	15	2	8	0	8	6.50	2	8	7.11
3	5	15	2	32	0	6	8.17	0	6	8.17
3	5	15	8	2	5	10	10.10	5	11	10.25
3	5	15	8	8	3	9	13.59	2	11	13.70
3	5	15	8	32	0	9	20.22	0	10	20.51
3	5	15	32	2	7	14	15.12	6	15	15.17
3	5	15	32	8	4	13	22.98	4	15	23.99
3	5	15	32	32	0	13	41.39	0	14	41.84
3	15	15	2	2	14	19	5.82	12	19	6.10
3	15	15	2	8	14	19	5.80	12	19	6.07
3	15	15	2	32	14	19	5.80	12	19	6.07
3	15	15	8	2	17	21	9.10	15	21	9.38
3	15	15	8	8	16	22	9.15	15	21	9.41
3	15	15	8	32	17	21	9.16	15	21	9.40
3	15	15	32	2	17	24	12.12	17	24	12.12
3	15	15	32	8	17	24	12.17	17	24	12.17
3	15	15	32	32	17	24	12.42	17	24	12.42
3	15	30	2	2	14	20	8.38	14	20	8.38
3	15	30	2	8	13	20	8.41	14	20	8.41
3	15	30	2	32	12	21	8.56	13	20	8.55
3	15	30	8	2	17	24	13.52	17	24	13.52
3	15	30	8	8	15	24	13.97	15	24	13.97
3	15	30	8	32	14	25	15.02	14	24	15.06
3	15	30	32	2	19	28	18.55	18	29	18.54
3	15	30	32	8	16	29	20.01	16	29	20.01
3	15	30	32	32	14	29	23.97	14	29	23.97
3	15	45	2	2	12	20	10.08	12	21	10.13
3	15	45	2	8	12	19	10.35	11	21	10.36
3	15	45	2	32	9	21	10.96	10	21	11.08
3	15	45	8	2	15	25	16.73	16	26	16.81

Table A.2, Continued

3	15	45	8	8	14	24	18.29	14	26	18.33
3	15	45	8	32	10	26	21.79	10	26	21.79
3	15	45	32	2	18	31	23.75	18	31	23.75
3	15	45	32	8	14	31	27.96	14	31	27.96
3	15	45	32	32	10	32	38.58	10	31	38.62
3	30	30	2	2	27	33	8.17	23	32	9.77
3	30	30	2	8	26	34	8.18	23	32	9.74
3	30	30	2	32	27	33	8.17	23	32	9.75
3	30	30	8	2	30	37	12.72	27	36	14.02
3	30	30	8	8	29	37	12.73	27	36	13.96
3	30	30	8	32	30	37	12.68	27	36	13.96
3	30	30	32	2	32	41	16.81	31	39	17.42
3	30	30	32	8	32	40	16.86	31	39	17.54
3	30	30	32	32	31	41	16.80	31	39	17.43
3	30	60	2	2	28	38	11.74	26	35	12.62
3	30	60	2	8	29	37	11.73	26	35	12.61
3	30	60	2	32	28	37	11.76	25	35	13.06
3	30	60	8	2	31	44	18.59	28	41	21.07
3	30	60	8	8	33	43	18.59	28	41	21.13
3	30	60	8	32	31	44	18.71	28	41	21.18
3	30	60	32	2	35	49	25.14	32	47	26.84
3	30	60	32	8	36	48	25.18	32	47	26.77
3	30	60	32	32	34	49	25.70	31	47	28.00
3	30	90	2	2	26	39	14.48	26	38	14.47
3	30	90	2	8	26	39	14.52	26	38	14.50
3	30	90	2	32	25	38	14.65	25	38	14.65
3	30	90	8	2	33	44	23.36	31	44	23.46
3	30	90	8	8	29	46	23.63	30	44	23.81
3	30	90	8	32	28	46	24.41	29	44	24.58
3	30	90	32	2	36	51	31.88	34	52	31.88
3	30	90	32	8	34	52	33.15	33	52	33.19
3	30	90	32	32	31	53	36.79	30	52	36.71

Table A.3: Distribution System Experimental Data Used to Test Regression Model

r	μ	σ^2	b	p	s_W^*	s_1^*	C^*	s_W^H	s_1^H	Ca	ErrA
2	5	5	2	2	9	7	6.30	11	7	6.75	7.09%
2	5	5	2	8	9	6	7.48	11	6	8.21	9.77%
2	5	5	2	32	8	5	9.82	7	5	10.32	5.10%
2	5	5	8	2	11	8	10.50	12	8	10.48	-0.21%
2	5	5	8	8	9	8	14.25	10	8	14.48	1.59%
2	5	5	8	32	8	7	22.33	7	7	22.90	2.58%
2	5	5	32	2	12	10	14.60	12	10	14.60	0.00%
2	5	5	32	8	9	10	23.00	12	9	23.53	2.32%
2	5	5	32	32	8	9	42.75	8	9	42.75	0.00%
2	5	10	2	2	9	7	9.32	11	7	9.59	2.94%
2	5	10	2	8	7	6	11.95	9	6	12.18	1.89%
2	5	10	2	32	6	5	16.13	5	4	17.15	6.36%
2	5	10	8	2	12	10	16.51	16	9	17.55	6.33%
2	5	10	8	8	8	9	24.80	8	9	24.80	0.00%
2	5	10	8	32	7	7	40.59	7	7	40.59	0.00%
2	5	10	32	2	14	13	24.26	17	12	24.84	2.38%
2	5	10	32	8	11	11	42.50	14	11	44.34	4.33%
2	5	10	32	32	7	10	81.91	8	9	84.58	3.26%
2	5	15	2	2	7	6	10.62	3	8	10.80	1.67%
2	5	15	2	8	6	4	14.18	3	6	14.48	2.13%
2	5	15	2	32	4	3	18.83	0	4	19.84	5.39%
2	5	15	8	2	10	9	19.69	10	10	19.93	1.22%
2	5	15	8	8	7	7	31.49	5	9	32.03	1.71%
2	5	15	8	32	5	5	50.92	1	6	53.71	5.49%
2	5	15	32	2	13	13	30.17	15	13	30.24	0.22%
2	5	15	32	8	9	11	56.02	10	12	58.33	4.13%
2	5	15	32	32	7	8	110.09	3	10	113.43	3.03%
2	15	15	2	2	27	19	10.40	22	19	11.46	10.23%
2	15	15	2	8	28	18	10.60	24	18	11.43	7.84%
2	15	15	2	32	27	18	11.25	23	18	12.33	9.65%
2	15	15	8	2	30	21	16.50	25	22	17.74	7.48%
2	15	15	8	8	29	21	17.68	25	21	19.59	10.80%
2	15	15	8	32	28	20	20.94	23	21	23.32	11.35%
2	15	15	32	2	32	24	22.41	29	24	23.10	3.09%
2	15	15	32	8	29	24	26.05	26	24	27.53	5.67%
2	15	15	32	32	27	23	35.29	26	23	37.01	4.87%
2	15	30	2	2	26	20	15.12	26	19	15.16	0.28%
2	15	30	2	8	25	19	16.53	26	18	16.62	0.54%
2	15	30	2	32	25	17	19.69	24	17	19.77	0.44%

Table A.3, Continued

2	15	30	8	2	30	24	25.09	29	23	25.71	2.47%
2	15	30	8	8	27	23	30.99	24	23	31.89	2.90%
2	15	30	8	32	26	21	43.18	24	21	43.07	-0.25%
2	15	30	32	2	34	28	35.51	33	27	35.70	0.52%
2	15	30	32	8	27	27	48.01	28	26	51.10	6.44%
2	15	30	32	32	28	24	80.06	26	24	84.63	5.70%
2	15	45	2	2	24	19	18.41	26	19	18.49	0.41%
2	15	45	2	8	23	17	21.48	24	18	21.71	1.03%
2	15	45	2	32	21	15	27.19	22	16	28.00	2.97%
2	15	45	8	2	27	25	31.59	27	25	31.59	0.00%
2	15	45	8	8	25	23	42.78	25	23	42.78	0.00%
2	15	45	8	32	23	19	64.57	21	21	65.52	1.48%
2	15	45	32	2	33	29	46.17	33	29	46.17	0.00%
2	15	45	32	8	27	27	70.07	29	27	71.12	1.49%
2	15	45	32	32	22	25	128.32	24	25	133.22	3.82%
3	5	5	2	2	9	7	6.11	10	7	6.24	2.24%
3	5	5	2	8	8	7	6.20	8	7	6.20	0.00%
3	5	5	2	32	8	7	6.57	6	7	6.93	5.41%
3	5	5	8	2	10	9	9.89	9	9	9.90	0.08%
3	5	5	8	8	9	9	10.54	7	9	11.14	5.68%
3	5	5	8	32	10	8	12.01	8	8	12.60	4.91%
3	5	5	32	2	12	10	13.52	13	10	13.66	1.02%
3	5	5	32	8	11	10	15.15	11	10	15.15	0.00%
3	5	5	32	32	9	10	19.64	11	9	19.95	1.57%
3	5	10	2	2	9	7	8.92	11	7	9.13	2.34%
3	5	10	2	8	8	7	9.45	12	6	10.02	5.94%
3	5	10	2	32	8	6	10.88	6	6	11.30	3.81%
3	5	10	8	2	11	10	15.37	11	10	15.37	0.00%
3	5	10	8	8	10	9	17.87	11	9	17.96	0.50%
3	5	10	8	32	9	8	24.07	4	9	26.72	11.02%
3	5	10	32	2	12	13	22.10	16	12	22.76	3.01%
3	5	10	32	8	11	12	28.90	11	12	28.90	0.00%
3	5	10	32	32	9	11	46.04	7	11	47.10	2.29%
3	5	15	2	2	8	6	10.11	4	8	10.34	2.24%
3	5	15	2	8	8	5	11.73	4	7	11.94	1.82%
3	5	15	2	32	6	3	14.62	1	5	15.24	4.29%
3	5	15	8	2	10	9	18.36	9	10	18.40	0.22%
3	5	15	8	8	4	10	24.18	4	10	24.18	0.00%
3	5	15	8	32	6	6	35.11	1	6	40.91	16.51%
3	5	15	32	2	13	13	27.49	13	13	27.49	0.00%
3	5	15	32	8	9	13	42.05	9	13	42.05	0.00%

Table A.3, Continued											
3	5	15	32	32	9	9	72.53	3	11	72.65	0.16%
3	15	15	2	2	27	19	10.35	21	19	12.02	16.15%
3	15	15	2	8	27	19	10.35	21	19	11.94	15.35%
3	15	15	2	32	27	19	10.39	21	19	12.07	16.25%
3	15	15	8	2	31	21	16.41	24	22	18.41	12.23%
3	15	15	8	8	31	21	16.43	24	22	18.14	10.42%
3	15	15	8	32	31	21	16.31	24	22	18.14	11.24%
3	15	15	32	2	32	24	21.93	28	24	23.37	6.59%
3	15	15	32	8	32	24	21.78	28	24	23.00	5.59%
3	15	15	32	32	31	24	22.09	28	24	23.44	6.15%
3	15	30	2	2	26	20	14.89	27	19	14.93	0.24%
3	15	30	2	8	26	20	14.94	27	19	14.99	0.29%
3	15	30	2	32	26	20	15.04	25	19	15.29	1.63%
3	15	30	8	2	30	24	24.40	28	24	24.64	0.96%
3	15	30	8	8	29	24	24.54	26	24	25.24	2.87%
3	15	30	8	32	28	24	25.05	27	23	25.86	3.20%
3	15	30	32	2	33	28	33.35	33	27	33.73	1.12%
3	15	30	32	8	32	28	34.46	31	27	35.06	1.74%
3	15	30	32	32	31	27	38.36	28	27	39.56	3.14%
3	15	45	2	2	25	19	17.90	25	20	18.05	0.84%
3	15	45	2	8	24	19	18.08	26	19	18.20	0.67%
3	15	45	2	32	22	19	18.86	25	19	19.07	1.14%
3	15	45	8	2	28	25	29.85	30	25	30.02	0.56%
3	15	45	8	8	28	24	31.36	27	25	31.48	0.38%
3	15	45	8	32	26	23	35.91	25	24	36.20	0.79%
3	15	45	32	2	32	30	42.44	32	30	42.44	0.00%
3	15	45	32	8	28	30	47.31	28	30	47.31	0.00%
3	15	45	32	32	26	28	62.84	26	28	62.84	0.00%

Table A.4: Two Symmetric Retailer Experimental Data

r	μ	σ^2	b	p	s_W^*	s_1^*	C^*	s_W^H	s_1^H
2	10	5	0.5	18	15	11.56	18	15	11.56
2	10	5	5	17	15	12.89	16	15	12.92
2	10	5	10	18	14	14.00	16	14	14.59
2	10	5	20	17	14	15.49	14	14	16.92
2	10	10	0.5	19	16	13.92	19	16	13.92
2	10	10	5	18	16	16.19	17	16	16.25
2	10	10	10	17	16	18.22	18	15	18.17
2	10	10	20	17	15	21.11	17	15	21.11
2	10	20	0.5	20	17	15.94	20	17	15.94
2	10	20	5	18	17	19.67	18	17	19.67
2	10	20	10	18	16	22.96	18	16	22.96
2	10	20	20	18	16	27.43	17	16	27.60
2	20	5	0.5	19	17	16.50	19	17	16.50
2	20	5	5	19	15	21.07	15	16	21.35
2	20	5	10	18	14	24.59	15	15	24.69
2	20	5	20	16	14	29.60	12	15	29.89
2	20	10	0.5	20	19	19.87	22	18	20.56
2	20	10	5	19	17	27.68	18	17	27.89
2	20	10	10	19	16	33.48	18	16	33.41
2	20	10	20	13	17	42.35	14	16	42.34
2	20	20	0.5	19	22	23.26	23	20	23.75
2	20	20	5	20	19	34.95	19	19	35.11
2	20	20	10	18	18	44.20	17	18	44.84
2	20	20	20	17	17	57.58	14	18	58.11
3	10	5	0.5	20	14	11.76	17	15	11.92
3	10	5	5	20	14	11.82	17	15	11.95
3	10	5	10	18	15	11.91	17	15	12.03
3	10	5	20	19	14	11.87	17	15	11.93
3	10	10	0.5	19	16	14.35	18	16	14.52
3	10	10	5	19	16	14.35	18	16	14.53
3	10	10	10	19	16	14.38	17	16	14.91
3	10	10	20	19	16	14.49	17	16	14.98
3	10	20	0.5	20	17	16.64	18	17	17.25
3	10	20	5	20	17	16.67	18	17	17.23
3	10	20	10	20	17	16.73	18	17	17.21
3	10	20	20	20	17	16.92	18	17	17.33
3	20	5	0.5	20	16	17.45	17	17	17.55
3	20	5	5	19	16	17.73	19	16	17.73
3	20	5	10	19	16	18.15	17	16	18.27
3	20	5	20	19	15	18.98	17	16	19.02
3	20	10	0.5	21	18	21.30	20	18	21.35
3	20	10	5	22	17	22.30	19	18	22.27
3	20	10	10	21	17	22.98	20	17	22.99
3	20	10	20	20	17	24.67	19	17	24.65
3	20	20	0.5	21	20	25.21	22	20	25.11
3	20	20	5	21	19	27.00	21	19	27.00
3	20	20	10	21	19	28.52	19	19	28.87
3	20	20	20	18	19	31.35	18	19	31.35

Table A.5: Four Symmetric Retailer Experimental Data

r	μ	σ^2	b	p	s_W^*	s_1^*	C^*	s_W^H	s_1^H
2	5	5	0.5	15	8	11.14	16	8	11.13
2	5	5	5	17	7	12.83	14	7	13.26
2	5	5	10	19	6	14.15	14	7	14.23
2	5	5	20	17	6	15.98	12	7	16.29
2	5	10	0.5	20	8	13.47	21	9	14.72
2	5	10	5	19	7	16.16	14	8	16.33
2	5	10	10	18	7	18.27	14	8	18.43
2	5	10	20	15	7	21.40	11	8	21.72
2	5	20	0.5	19	9	16.09	20	10	17.10
2	5	20	5	19	8	20.52	16	9	20.78
2	5	20	10	17	8	24.06	13	9	24.45
2	5	20	20	18	7	28.89	14	8	29.39
2	10	5	0.5	18	8	15.54	19	9	16.09
2	10	5	5	19	6	19.87	16	8	20.97
2	10	5	10	16	6	22.59	8	8	23.49
2	10	5	20	13	6	26.40	9	7	27.08
2	10	10	0.5	20	9	18.92	20	11	20.45
2	10	10	5	19	7	25.79	16	9	27.15
2	10	10	10	20	6	30.31	13	9	33.10
2	10	10	20	12	7	36.70	8	8	37.24
2	10	20	0.5	22	10	22.95	22	12	24.08
2	10	20	5	21	8	33.97	14	11	35.82
2	10	20	10	20	7	41.52	15	10	45.77
2	10	20	20	16	7	51.31	9	10	56.67
3	5	5	0.5	18	7	11.45	19	8	12.29
3	5	5	5	18	7	11.54	16	8	11.62
3	5	5	10	18	7	11.65	13	8	11.67
3	5	5	20	17	7	11.81	13	8	11.85
3	5	10	0.5	18	8	13.92	18	9	14.79
3	5	10	5	17	8	14.13	15	9	14.28
3	5	10	10	17	8	14.33	12	9	14.47
3	5	10	20	16	8	14.69	12	9	14.88
3	5	20	0.5	18	9	16.94	17	10	17.79
3	5	20	5	17	9	17.35	15	10	17.87
3	5	20	10	20	8	17.71	16	9	17.67
3	5	20	20	20	8	18.38	16	9	18.45
3	10	5	0.5	16	8	16.35	17	10	18.72
3	10	5	5	18	7	17.07	13	9	17.58
3	10	5	10	17	7	17.68	10	9	18.42
3	10	5	20	16	7	18.82	11	8	19.02
3	10	10	0.5	18	9	20.20	19	11	23.01
3	10	10	5	19	8	21.49	18	10	23.68
3	10	10	10	17	8	22.74	15	10	25.15
3	10	10	20	15	8	24.77	11	9	25.16
3	10	20	0.5	19	10	24.73	19	12	26.48
3	10	20	5	20	9	27.34	13	12	29.21
3	10	20	10	18	9	29.54	14	11	30.99
3	10	20	20	14	9	33.51	9	11	34.30

Table A.6: Asymmetric Backorder Cost Experiment Data

r	μ	σ^2	p	s_W^*	s_1^*	s_2^*	C^*	s_W^H	s_1^H	s_2^H	C^H
2	10	10	0.5	21	14	16	12.54	21	14	15	12.55
2	10	10	5	20	14	15	14.36	19	14	15	14.37
2	10	10	10	19	14	15	15.92	17	14	15	16.14
2	10	10	20	19	13	14	18.24	16	14	15	18.54
2	10	20	0.5	21	14	17	13.50	21	14	17	13.50
2	10	20	5	19	14	17	16.17	19	15	16	16.51
2	10	20	10	19	14	16	18.30	18	14	16	18.41
2	10	20	20	19	13	15	21.66	16	14	16	22.06
2	20	10	0.5	23	16	18	17.98	24	15	17	18.05
2	20	10	5	19	15	17	24.34	19	15	16	24.43
2	20	10	10	18	14	16	29.04	19	15	15	29.57
2	20	10	20	15	14	16	35.62	15	15	15	36.07
2	20	20	0.5	24	16	20	19.56	24	16	19	19.61
2	20	20	5	19	15	19	28.08	19	16	18	28.30
2	20	20	10	18	14	18	34.50	19	15	17	34.74
2	20	20	20	17	13	17	43.69	16	15	17	44.06
2	40	10	0.5	24	17	21	23.83	29	15	18	24.28
2	40	10	5	16	15	18	38.90	19	15	17	39.13
2	40	10	10	13	14	17	48.98	16	15	15	49.88
2	40	10	20	12	12	15	61.98	12	14	15	62.56
2	40	20	0.5	24	17	24	26.03	28	16	21	26.31
2	40	20	5	17	15	21	45.25	16	18	20	46.41
2	40	20	10	14	14	20	58.78	12	17	19	60.77
2	40	20	20	13	12	17	77.33	11	15	18	79.15
3	10	10	0.5	20	14	16	12.94	19	14	16	12.96
3	10	10	5	20	14	16	12.99	18	14	16	13.20
3	10	10	10	20	14	16	13.02	19	14	15	13.22
3	10	10	20	20	14	16	13.06	19	14	15	13.34
3	10	20	0.5	21	14	17	14.15	19	14	17	14.22
3	10	20	5	20	14	17	14.17	19	14	17	14.29
3	10	20	10	20	14	17	14.26	19	14	17	14.31
3	10	20	20	20	14	17	14.36	19	14	17	14.45
3	20	10	0.5	20	17	18	19.25	23	15	17	19.33
3	20	10	5	20	16	18	19.83	22	15	17	19.94
3	20	10	10	20	16	17	20.58	20	16	17	20.58
3	20	10	20	18	16	17	21.58	20	15	16	21.67
3	20	20	0.5	20	16	21	21.18	24	15	19	21.27
3	20	20	5	21	16	19	22.20	21	16	19	22.20
3	20	20	10	20	16	19	23.21	21	16	18	23.55
3	20	20	20	18	16	19	25.02	19	16	18	25.28
3	40	10	0.5	21	17	20	26.28	27	15	18	26.91
3	40	10	5	17	16	19	30.78	21	15	17	31.06
3	40	10	10	16	15	18	34.43	20	15	16	34.82
3	40	10	20	15	14	17	39.73	17	15	16	39.77
3	40	20	0.5	22	16	23	28.89	26	16	22	29.06
3	40	20	5	17	16	22	35.24	18	18	21	35.43
3	40	20	10	16	15	21	40.50	17	17	20	40.82
3	40	20	20	16	14	19	48.02	16	16	18	49.10

Table A.7: Asymmetric Demand Experiment Data

r	σ_1^2	σ_2^2	b	p	s_W^*	s_1^*	s_2^*	C^*	s_W^H	s_1^H	s_2^H	C^H
2	5	15	5	0.5	20	8	21	11.28	21	8	20	11.26
2	5	15	5	5	20	7	20	12.77	18	7	22	12.95
2	5	15	5	10	18	7	21	13.97	17	7	22	14.09
2	5	15	5	20	19	6	20	15.70	17	6	21	15.84
2	5	15	10	0.5	21	9	22	13.49	22	9	22	13.57
2	5	15	10	5	20	8	22	16.07	18	8	24	16.44
2	5	15	10	10	18	8	23	18.25	17	8	24	18.42
2	5	15	10	20	15	8	25	21.19	15	8	25	21.19
2	5	15	20	0.5	23	10	23	15.50	23	10	23	15.50
2	5	15	20	5	20	9	24	19.65	19	9	26	20.28
2	5	15	20	10	20	8	23	23.01	17	9	26	23.21
2	5	15	20	20	19	8	23	27.94	15	9	26	28.60
2	10	30	5	0.5	22	9	23	16.03	25	8	21	16.40
2	10	30	5	5	17	8	23	20.79	20	7	22	20.92
2	10	30	5	10	17	7	22	24.08	15	7	24	24.37
2	10	30	5	20	16	6	21	28.65	17	6	21	28.61
2	10	30	10	0.5	24	11	25	19.31	25	10	24	19.33
2	10	30	10	5	19	9	25	27.06	19	9	25	27.06
2	10	30	10	10	18	8	24	33.00	16	9	26	33.36
2	10	30	10	20	17	7	23	41.17	12	9	27	41.61
2	10	30	20	0.5	26	12	27	22.37	27	11	26	22.46
2	10	30	20	5	19	11	28	34.21	19	11	28	34.21
2	10	30	20	10	15	11	29	43.28	15	11	29	43.28
2	10	30	20	20	15	9	27	56.64	14	10	28	56.81
3	5	15	5	0.5	19	8	21	11.59	20	8	19	11.90
3	5	15	5	5	19	8	21	11.70	19	8	20	11.88
3	5	15	5	10	19	8	21	11.81	17	8	21	12.23
3	5	15	5	20	21	7	20	11.94	18	7	21	12.11
3	5	15	10	0.5	21	9	22	14.05	20	10	21	14.83
3	5	15	10	5	21	8	22	14.25	19	9	22	14.54
3	5	15	10	10	21	8	22	14.38	17	9	24	14.70
3	5	15	10	20	21	8	22	14.68	18	8	24	14.77
3	5	15	20	0.5	21	10	23	16.36	21	10	23	16.45
3	5	15	20	5	22	9	23	16.70	18	10	25	16.96
3	5	15	20	10	22	9	23	17.03	14	10	29	18.00
3	5	15	20	20	20	9	24	17.63	15	9	29	19.46
3	10	30	5	0.5	20	9	23	16.98	25	9	20	17.75
3	10	30	5	5	20	8	23	17.66	22	7	23	18.13
3	10	30	5	10	19	8	23	18.21	20	7	24	18.63
3	10	30	5	20	19	7	23	19.15	20	6	24	19.87
3	10	30	10	0.5	21	10	25	20.82	26	10	23	21.27
3	10	30	10	5	20	10	25	22.23	20	9	27	22.37
3	10	30	10	10	20	9	25	23.32	18	9	28	23.78
3	10	30	10	20	20	8	25	25.16	17	8	28	25.76
3	10	30	20	0.5	22	12	28	24.50	26	11	26	24.60
3	10	30	20	5	21	11	28	26.90	19	11	30	27.26
3	10	30	20	10	21	10	27	28.97	15	11	33	29.91
3	10	30	20	20	19	10	28	32.13	16	10	32	33.03

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