GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION
SPONSORED PROJECT INITIATION

Date: June 9, 1979

Project Title: Applications of Nonstandard Analysi to Mathematical Physics

Project No: G-37-616 *Green card*

Project Director: Dr. Alan D. Sloan

Sponsor: National Science Foundation

Agreement Period: From 7/1/79 Until 12/31/81 (Grant Period)

Type Agreement: Grant No. MCS-7902730 dated 5/21/79

Amount: $17,642 NSF
5,482 GIT(G-37-325)
$23,124 TOTAL

Reports Required:
Annual Progress Report(s); Final Project Report

Sponsor Contact Person (s):

Technical Matters

NSF Program Official

Dr. William G. Rosen
Program Director
Modern Analysis and Probability Program
Mathematical Sciences Section
Division of Mathematical and Computer Sciences
Directorate for Mathematical and Physical
 Sciences, and Engineering
National Science Foundation
Washington, DC 20550
202/632-7377

Contractual Matters
 (thru OCA)
NSF Grants Official

Aileen Buie
MPE/STIA Branch
Division of Grants and Contracts
Directorate for Administration
National Science Foundation
Washington, D.C. 20550
202/632-4344

Defense Priority Rating: N/A

Assigned to: Mathematics (School/Laboratory)

COPIES TO:

Project Director
Division Chief (EES)
School/Laboratory Director
Dean/Director—EES
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EES Information Office
EES Reports & Procedures
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Project Code (GTRI)
Other

CA—3 (3/76)

GEORGIA INSTITUTE OF TECHNOLOGY

OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION/CLOSEOUT SHEET

Date _____ 4/22/85 _____

Project No. _____ G-37-616 _____   School/~~Lab~~ _____ Math _____

Includes Subproject No.(s) _____ N/A _____

Project Director(s) _____ Dr. Alan D. Sloan _____ GTRC /~~GIT~~

Sponsor _____ National Science Foundation _____

Title _____ Application of Nonstandard Analysis to Mathematical Physics _____

Effective Completion Date: _____ 12/31/83 _____ (Performance) _____ 12/31/83 _____ (Reports)

Grant/Contract Closeout Actions Remaining:

- [ ] None
- [ ] Final Invoice or Final Fiscal Report
- [ ] Closing Documents
- [X] Final Report of Inventions   (For our records)
- [ ] Govt. Property Inventory & Related Certificate
- [ ] Classified Material Certificate
- [ ] Other _____

Continues Project No. _____   Continued by Project No. _____

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_____ Jones _____

FORM OCA 69.285

GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA. GEORGIA 30332

MATHEMATICS

September 18, 1980

Ms. Aileen Buie
MPE/STIA Branch
Division of Grants and Contracts
Directorate for Administration
National Science Foundation
Washington, D.C. 20550

Dear Ms. Buie:

This is the Annual Progress Report for the period 7/1/79-6/30/80 for NSF grant number MCS-7902730, dated 5/21/79. Dr. Alan D. Sloan is the Project Director.

1. The paper titled "Strong Convergence of Schrodiger Propagators" was accepted for publication in the Transactions of the American Mathematical Society.

2. Dr. Sloan continued joint work with Dr. Berger on partial differential equations. The connection between this work and the grant objectives was described in:

an invited talk by Dr. Sloan to the Second Victoria Symposium on Nonstandard Analysis at the University of Victoria, June 23-28, 1980. The title of this talk was "Explicit Solutions of Partial Differential Equations."

3. This work was further described in: The abstract of Dr. Berger's presentation of "Explicit Solutions for Cauchy Problems," at the 86th Annual Meeting of the AMS, San Antonio, Texas, January 3-6, 1980. The abstract, by Berger and Sloan, appears in Abstracts of the AMS 1(1980), 81.

4. Dr. Berger and Dr. Sloan prepared a manuscript titled "A Method of Generalized Characteristics" for submission to the Transactions of the AMS."

5. Dr. Berger and Dr. Sloan prepared lecture notes titled "Radical Differential Calculus, Volume 1," Chapter 1 and conducted a two quarter seminar at Georgia Tech on this subject. Two copies are enclosed.

6. Dr. Berger and Dr. Sloan are completing Chapter 4 to Volume 1 of "Radical Differential Calculus".

Sincerely,

Alan D. Sloan

Principal Investigator: Alan D. Sloan, Georgia Institute of Technology

NSF Contract No: MCS-7902730

Period Covered: July 82-June 83

Title of Proposal: Stochastic Analysis & Characteristic Methods for
                    Evolution Equations

List of manuscripts submitted or published under sponsorship during period:
 A Method of Generalized Characteristics, appeared in Memoirs of the AMS,
 July, 1982, p. 1-37.
 Characteristic Methods for Multidimensional Evolution Equations, submitted
 to Journal of Differential Equations, April 1983.

Brief summary of findings:

    In a series of papers, the authors provide a generalization of both
the classical calculus and the second order calculus of Itô to calculi of
higher order, for the purpose of developing characteristic methods suitable
for the analysis of initial value evolution equations such as

$$\frac{\partial u}{\partial t} = Au, \; u(x,0) = f(x).$$

Here A is a linear partial differential operator in the space variables, x,
which determines the propagation of the given initial data, f. The authors'
technique has been successfully applied to evolution equations with coefficients
which depend on time continuously and on space polynomially with initial data
in various classical function spaces. Continued development of the authors'
generalization will provide a comprehensive theory of more general equations
which permits explicit computation of the solution in many cases.

APPENDIX VI

<table>
<tr><td>NATIONAL SCIENCE FOUNDATION
Washington, D.C. 20550</td><td colspan="2">**FINAL PROJECT REPORT**
NSF FORM 98A</td></tr>
</table>

PLEASE READ INSTRUCTIONS ON REVERSE BEFORE COMPLETING

PART I–PROJECT IDENTIFICATION INFORMATION

1. Institution and Address	2. NSF Program Applied Mathematics	3. NSF Award Number MCS 790 2730
Georgia Institute of Technology 225 North Avenue Atlanta, Georgia 30332	4. Award Period From 7/1/79 To 12/31/83	5. Cumulative Award Amount 62,750

6. Project Title

Stochastic Analysis and Characteristic Methods for Evolution Equations

PART II–SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

The Method of Brownian Characteristics was extended from second order diffusion equations to evolution equations of arbitrary order.

New Product Integrals were developed and corresponding product formulas were found. Applications of these occurred in three areas:(a) classification of constant coefficient elliptic semigroup generators in a geometric manner;(b)description of support properties of solutions to evolution equations having degenerate elliptic generators; and (c)new central limit theorems for products of random matrices.

This material was presented at Hebrew University,Technion and Weizmann Institute and at technical meetings at Cornell,University of Alabama and University of Colorado.

Six publications connerning this project have appeared and three more have been accepted for publication.

PART III–TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM	
				Check (✓)	Approx. Date
a. Abstracts of Theses			–		
b. Publication Citations					
c. Data on Scientific Collaborators					
d. Information on Inventions					
e. Technical Description of Project and Results	//////				
f. Other *(specify)*	//////				

2. Principal Investigator/Project Director Name *(Typed)* Alan D. Sloan Marc Berger	3. Principal Investigator/Project Director Signature	4. Date 10/25/84

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