"In presenting the dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institution shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.

"

NUMERICAL SOLUTIONS FOR THE GRILLAGE BEAM PROBLEM

A THESIS

Presented to

the Faculty of the Graduate Division

by

Norman Thomas Williams

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Civil Engineering

Georgia Institute of Technology

November, 1959

3 A
12 R

NUMERICAL SOLUTIONS FOR THE GRILLAGE BEAM PROBLEM

Approved:

Austin B. Caseman

James R. Fincher

R. Kenneth Jacobs

Date Approved by Chairman: December 4, 1959

ACKNOWLEDGMENT

The author wishes to express his deep appreciation to Dr. F. W. Schutz for his encouragement and many suggestions while serving as thesis advisor prior to his departure on a year's leave of absence. Dr. Schutz was a continued inspiration throughout this study. The many valuable comments and criticisms offered by the reading committee are gratefully acknowledged. Dr. A. B. Caseman, Professor J. R. Fincher, and Dr. R. K. Jacobs generously gave of their time to serve in this capacity.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

SUMMARY

The grillage beam problem is one which is encountered many times in the design of modern structures. Its solution is generally accomplished by approximate means. A rigorous treatment of the problem involves a highly theoretical mathematical treatment which is beyond the scope of most practical design offices.

This study undertakes an investigation of the grillage beam problem from a numerical approach. Two basic numerical solutions are developed which involve numerical integration. The first solution is based on the use of apparent spring constants. This involves treating each beam of the grillage system separately and considering cross beams as being elastic supports. The computation proceeds through the grillage system in a systematic manner and each spring constant value is corrected in an iterative manner. At the end of each cycle the system is made to satisfy the statics of the grid. Most problems will converge to the correct solution. The limitation of this approach is that some problems will not converge to the solutions but require a trial and error approach. This phenomenon is discussed and explained.

The other method developed is the correction configuration solution. This is a general approach and does not have special limitations. In this procedure the overall grid deformation configuration is assumed by treating only the deflections at each node point in the first cycle assumptions. Then the loads causing this configuration are calculated for each beam. By inspection it is seen that loads at cross node points of two beams do not agree. An averaging correction is made and these new loads applied to the system. An iterative solution evolves which systematically corrects the loads and then the deflections in each cycle. Problems are generally observed to diverge by this process and answers must be extrapolated after the completion of three cycles. A check cycle is next conducted using these extrapolated values in order to ensure desired accuracy. Accuracy which is sufficient for most engineering problems has been found after three cycles on all problems attempted by this method. However, the correctness of the first assumption partially controls this. Some special grillage systems will converge to an answer but these are the exceptions.

CHAPTER I

INTRODUCTION

The grillage beam problem is one that is encountered countless times in design and analysis engineering. Although the grillage beam system is often present in the floors of buildings and the decks of bridges, it is not discussed in the standard text books, and standard engineering literature contains very limited information of a practical nature on the grillage system. Approximate methods of analysis are most often used. Other approaches are of such a highly theoretical mathematical nature that they are limited in practice.

It was with this background that the investigation and development of numerical solutions to the grillage beam problem were attempted. Solutions tend to be long and time-consuming by the very nature of both the grillage problem and the numerical approach. It appeared that the best method of approach would be that of attempting to develop a solution from the numerical beam solutions presented by Dr. Newmark (1)*. This approach was undertaken and the results are presented herein. The general background of the methods used are discussed and examples cited. It is believed that this work is self-contained and the methods developed are useable without recourse to other matter for reference.

---

*Numbers in parentheses indicate references listed in the "Literature Cited" section in the bibliography.

CHAPTER II

REVIEW OF DR. NEWMARK'S NUMERICAL PROCEDURE FOR COMPUTING DEFLECTIONS, MOMENTS AND BUCKLING LOADS

The numerical procedure presented by Dr. Newmark (2) is actually a bookkeeping arrangement that allows fast accurate numerical integration to be systematically performed. It is a step-by-step calculation system of performing integration by the classic concept of obtaining an area under a curve. The numerical value of this area is taken as the value of the desired integral. The following well-recognized beam principles are utilized in this process:

1. Shear is equal to the integral of loading times distance.

$$V = \int L\,dx$$

2. Moment is equal to the integral of shear times distance.

$$M = \int V\,dx$$

3. Slope is equal to the integral of moment times distance divided by the modulus of elasticity times the beam moment of inertia.

$$\Theta = \int M\,dx/\,EI$$

4. Deflection is equal to the integral of slope times distance.

$$\delta = \int \Theta \, dx$$

Thus it can be seen that an orderly progression can be made from the known loading to the resulting deflection of any given beam. By the nature of the Newmark bookkeeping system, the process requires the use of only concentrated loads. Distributed loads are handled by the selection of equivalent concentrated loads which produce the same shears and moments at certain specified sections. Formulas for these equivalent concentrated loads are given in Fig. 1. Due to the nature of problems in this study the straight line formulas will be used almost exclusively. The sign convention is as shown in Fig. 2. This convention is used throughout this thesis.

The general steps involved in the use of the Newmark process are as follows:

1. At the top of the calculation sheet sketch the beam with its applied loading. Divide the beam into an arbitrary number of divisions. It is easiest if the divisions are taken to be of equal length and if a division falls on each concentrated load. Uniform loads are replaced by equivalent concentrated loads as discussed above.

2. Label horizontal lines downward as follows:

Loads

Shear Trial

Moment Trial

Moment Correction

Moment

Shear

Angle Change Ordinate (M/EI)

Equivalent Concentrated Angle Change (E.C. M/EI)

Slope

Deflection Trial

Deflection Correction

Deflection

3. Start the calculations by recording the loading values in the first line. The value at each division point may be composed of two numerical figures. Each division line may have a loading composed first of a concentrated load and secondly of an equivalent concentrated load caused by uniform loading in either or both of the adjacent divisions.

4. The shear is generally unknown and as such a value is assumed for any one division. Note that this is not true for a cantilever structure where the shear is known. The remaining values are calculated by algebraically adding or subtracting

successive loads from the preceding line. Always remember to add values if proceeding from left to right but subtract (change signs) when proceeding from right to left.

5. Complete the moment trial line next. The moment over a support for a simple beam is known to be zero. This is used as the starting point. The shear trial values are added across the line in succession. If the moment at the far end is zero then the values are correct for a simple beam. This will only be the case when the shear is correctly assumed in step four above.

6. If the far end moment value is not zero, make a correction. This is done in the moment correction line. A linear correction is applied.

7. Add the moment correction to the moment trial to produce the final moments at each division point. The algebraic difference in the final moments produces the true average shear value.

8. Complete the angle change ordinate line. The numerical value of the angle change ordinate (M/EI) is merely the moment value with the sign reversed, providing the EI term is placed under the common factor column and the beam is of constant cross section.

9. Find the equivalent concentrated angle change values by use of the formulas of Fig. 1. If the moment diagram consists of straight lines (only concentrated loads applied) then use the straight line formulas, but if the applied loading is uniform use the curve formulas.

10. Assume the slope for one division and calculate the remaining values. This is analogous to the method of handling the shear line.

11. Record the first deflection value, noting that the deflection is known to be zero over a support. The slope values are then successively added to obtain the remaining deflection values. If the deflection is zero at the far support, the values are correct. This is true only when the correct value of slope is assumed in step ten above.

12. Otherwise apply a linear correction and then add the two deflection lines to obtain the true deflection.

The calculations are simplified by removing all common factors from the arithmetical section. The common factors are placed to the right of the bookkeeping framework. An example of the Newmark process is contained in Fig. 3. This is an example of a simple beam loaded only with concentrated loads. The beam is of uniform cross section. The calculations follow the general steps already outlined. Values must be multiplied by their appropriate common factor to obtain answers in useable units.

This has necessarily been a short review of the Newmark method. It is also applicable to indeterminate beams, beams of variable cross section, and columns. More detailed information is available in Dr. Newmark's paper (3).

CHAPTER III

REVIEW OF ITERATION PROCEDURE FOR BEAMS ON ELASTIC FOUNDATIONS

The iteration procedure for beams on elastic foundations (4) is a numerical procedure which involves no new principles. This approach follows directly from the Newmark method discussed in Chapter II. It basically consists of assuming the deflected shape and from this assumption calculating the forces exerted by the elastic foundation. Then the deflections caused by the forces are calculated. If the calculated deflections agree with those assumed, the problem is solved. Otherwise the calculated deflections are used as the assumed deflections in the next trial. The solution to most problems will converge to an answer. The divergent problem will be discussed later.

One of the fundamental concepts of this method is that of the spring constant, which is defined as the force exerted by the elastic foundation per unit amount of deformation. Thus, it has units of a force per length (Kips per inch). It is important that this value be known. It is rather difficult in some cases to decide on an accurate value for a spring constant. In other cases such as a single beam supported on one or more cross beams, the spring constant may be accurately calculated.

The general steps required for solution of a single beam on an elastic foundation by the iteration procedure are as follows:

1. At the top of the calculation sheet sketch the beam. Indicate the presence of the elastic foundation by sketching springs at the appropriate division points. Note that the elastic foundation can act only at a division point due to the nature of the Newmark calculation procedure. If it acts at other points than division points then either change the location of division lines or approximate the elastic action as closely as possible at appropriate division points.

2. Label horizontal lines downward as follows:

Assumed Deflection

Spring Loading

Actual Loading

Total Loading

Shear

Moment Trial

Correction Moment

Moment

Angle Change Ordinate (M/EI)

Equivalent Concentrated Angle Change (E.C. M/EI)

Slope

Deflection

Correction Deflection

Deflection

3. Start by assuming a deflection in inches. Multiply this assumed deflection by the spring constant to give the spring loading. This is the resisting force offered by the elastic foundation.

4. Add the actual beam loading to obtain the total loading acting on the beam. Complete the usual Newmark process as previously outlined.

5. Compare the calculated deflection with the assumed deflection. If they are the same the problem is solved. If not, another trial is indicated.

6. Use the calculated deflections found at the end of the preceding trial as the assumed deflections in the next trial.

In most cases the procedure outlined will converge on the correct answer by this approach. Generally three to four cycles are sufficient to obtain an answer to the desired accuracy. However, using this method, the solution to some problems will diverge. The criterion for solution behavior may be stated as follows:

1. The problem will converge when the elastic foundation is weaker (not as stiff) than the main beam.

2. The problem will slowly converge, oscillate or slowly diverge when the elastic foundation is approximately as stiff as the main beam.

3. The problem will diverge when the elastic foundation is stiffer than the main beam.

A detailed study of this divergent or convergent question has been made (5). In general, the question of whether a given solution will diverge, oscillate, or converge is dependent on the numerical value of the spring constant as compared with the stiffness of the beam. The numerical value of the first critical spring constant is the critical value and is analogous to the natural frequency of vibrations for the laterally vibrating main beam. The above-mentioned reference fully covers this secondary problem and exact methods are presented to explain and analyze this phenomenon.

Diverging problems may still be solved by the iterative method but a trial and error approach must be used. An average of eight or nine cycles is usually required. Divergent problems may best be solved by the step-by-step solution which will be reviewed in the next chapter.

CHAPTER IV

REVIEW OF STEP-BY-STEP PROCEDURE FOR BEAMS ON ELASTIC FOUNDATIONS

The iteration procedure described in the preceding chapter does not efficiently solve those problems which are of a divergent nature. The solution of such problems by this method basically evolves into a trial and error approach. This process is frequently quite laborious. The step-by-step procedure (6) is the most effective numerical method for the solution of those elastic foundation problems which are divergent. It also holds true for convergent problems but is usually not as efficient as the iteration approach.

The step-by-step procedure is an adaptation of the numerical calculation system given by Dr. Newmark (7). The computations are conducted vertically instead of horizontally and possible errors are introduced into the system by assuming numerical values for all unknown quantities. Obviously a correct assumption does not result in an introduced error. Then corrections are applied to the system as a whole for any introduced errors. Finally, the original and corrective calculations are so combined as to result in the correct solution.

The general step-by-step process is as follows:

1. At the top of the calculation page sketch the beam and divide it into the desired divisions. A large number of divisions results in a more accurate solution. The elastic foundation must act as independent single springs at each division point. Where such is the case, obviously no error is introduced. However, when the elastic foundation is distributed, slight errors are introduced by this assumption. In such a case the assumed concentrated spring constant will have a value equal to the product of the spring constant of the distributed elastic foundation and the effective length over which it acts. For interior division points this length is the division length while for exterior points it is half the division length. When the elastic foundation has a spring constant which is variable along the beam, the assumed concentrated spring constant must be found by use of the equivalent concentrated load formulas given in Fig. 1.

2. Compute the equivalent concentrated applied loads at each division point. (This is only necessary when loads are applied at locations other than the division points.) Otherwise, record the applied concentrated loads.

3. Label the framework for the calculations. Draw vertical lines through each division point and label horizontal lines down as follows:

Loads

Deflections

Spring Loads

Summation of Loads

Shear

Moment

Angle Change Ordinate

Equivalent Concentrated Angle Change

Slope

Deflection

4. Proceed with the computation by listing all known quantities in the first two divisions. Note that there are two unknown values. Assume values for these unknown quantities. Note that an incorrectly assumed value will introduce errors into the calculations.

5. Complete the calculations across the framework by proceeding vertically from one division to the next. No other quantities need be assumed. On completion note that the two known end conditions of the beam are violated. This error is due to the incorrect values introduced by the preceding step. The original calculations are now completed.

6. Compute a corrective calculation for a unit change of each of the assumed quantities. This involves two separate calculations

on beams containing no loads and results in obtaining the effects of a unit amount of the introduced error.

7. Two equations may be formed from the results of the above step. Solve these two simultaneous equations so as to combine the original and corrective calculations in such a way that the known end conditions are satisfied. Add the correct proportions of each correction to the original calculations to obtain the true answer. The numerical details involved in setting up these simultaneous equations and combining the corrections with the original calculations are illustrated and explained by the example which follows.

The above general procedure is simpler than it might appear and can best be followed by an example. In Fig. 4, the numbers in parentheses indicate the order of computation. The step-by-step process is conducted as follows:

1. At the top of Fig. 4 sketch the beam and record the pertinent data regarding the moment of inertia and spring constant for the elastic foundation. Then the horizontal lines are labeled as shown and the known loads applied to the beam.

2. In division one record the four quantities known to have zero value. The moment and deflection are obviously zero.

3. All pertinent quantities in division one are known. To proceed to division two, the values of shear and slope are

required. Since both of these quantities are unknown, it is necessary to assume some value. Thus, assume the shear to be ten. Add the shear to the moment in division one to obtain the moment in division two. Change signs to obtain the angle change ordinate.

4. Assume the slope to be twenty and add to the deflection in division one to obtain the deflection in division two. Multiply this deflection by the common factor to obtain the deflection in inches and record this value at the top of division two.

5. Multiply the deflection by the spring constant to obtain the spring load in division two. Add the spring load to the applied load to obtain the total load at division two.

6. To the shear assumed add the total load in division two. This gives the shear to be recorded between divisions two and three. Proceed down to the angle change ordinate as previously done. The equivalent concentrated angle change in division two may now be computed. The straight line formula is used due to the applied concentrated loading.

7. Next find the slope and deflection. Repeat this process until the end of the beam is reached. There are two discrepancies at the end. Neither the deflection nor the moment is zero. This violates the known end conditions of a simple beam. The error is due to assuming incorrect values for the

shear and slope at the left end. Corrections must now be made for each of these values.

8. Compute correction 'A' for the error in shear. This is done exactly as the original calculations except that the beam has no load. A unit shear and a zero slope are assumed. The calculations are shown in Fig. 5. Note discrepancies in deflection and moment at the right end.

9. Compute correction 'B' for the error in slope. This is done the same way. It is shown in Fig. 6. The opposite assumption of a unit slope and zero shear is made. Note the discrepancies in deflection and moment at the right end.

10. By the use of simultaneous equations find the correct amounts of each correction calculation to add to the original calculation in order to produce zero deflection and moment at the right end. This is done in Fig. 6.

11. Combine the three deflection values in the amounts found above to obtain the correct deflections. Fig. 6 gives the correct deflections.

To show the accuracy of the step-by-step method, the problem is checked in Fig. 7 by the Newmark numerical method, using the answers found above. The true deflections are found to agree with the assumed deflections. The solution is correct.

CHAPTER V

AVAILABLE NUMERICAL SOLUTIONS FOR THE GRILLAGE BEAM PROBLEM

The grillage beam problem is not covered in the standard engineering textbooks in English and has not received the attention in this country that it has in Germany and France. As such, the available solutions are limited in scope. Only within the past year has a specialized book appeared which covers the grillage problem (8).

The basic methods of solution can be placed in categories as follows:

1. Solutions which are based on a type of relaxation principle such as moment distribution.

2. An analysis which utilizes plate theory to explain grid action.

3. Methods which over-simplify the problem as to grid construction or type of deflection.

4. A harmonic analysis approach to the solution of differential euqations for the applied beam loading.

5. A system of equating deflections at cross grillage node points and thereby setting up a series of simultaneous equations.

The relaxation method applies to all problems of the grillage variety. The arithmetical work is laborious and too lengthy for all but the simplest problems. No general solution is possible and this prevents the use of a computer.

The plate theory approach is complicated and mainly applicable to grillage systems composed of the same size beams. The difficulty of extending this idea to a system composed of several different sized beams precludes its use.

Over-simplification of the problem can easily lead to completely erroneous results and experience is needed to be able to ascertain which simplifications are permissible. This over-simplification solution is not recommended unless one has considerable experience in grillage type problems.

The harmonic analysis method has only recently been developed (9), and the mathematical complexity of it renders an explanation beyond the scope of this discussion.

The simplest grillage beam solution (10) known to the writer involves the solution of a set of simultaneous equations which have as the unknowns the interaction loads between beams at the node points. This method is an efficient one for small grids. However, as the grid size is increased the number of simultaneous equations rapidly rises until soon the use of a computer is needed. The preparatory work required in setting up the equations is usually laborious. Upon solution of the equations only the interaction loads are known and further effort is required to obtain moments, shears, and deflections. For small grids this solution is still the best available, but its limitations must be recognized for the most efficient use.

The procedure is based upon the application of the method of consistent deformations. The intersecting beams must have the same deflection and interaction load at the common point of intersection. This forms the basis of the simultaneous equations. Each beam is considered independent of the grid and then forced to conform to the grid pattern. The general approach is as follows:

1. Consider each beam independently of the grid. Find the deflection at each node point due to the applied loading on the individual beam. Note that any applied loading on other beams is not received directly but in the form of interaction loads at the node points. Next find the deflection at each node point due to unit interaction loads. This is done successively for loads at each node point. This procedure can be conducted easiest by the moment-area method or the Newmark numerical approach. These values are known as deformation coefficients and are labelled as follows:

$$\delta_{A1}^{P1}$$

The subscript indicates the beam under consideration and the superscript indicates the load on that beam causing the deflection at the node point for which the equation is written. Note that the load may be either a unit interaction load or the applied problem load.

2. Write an equation for each node point. The deflection of the node point on one beam is equated to that of the other common intersecting beam. The only unknowns are the interaction loads and there are the same number of equations as unknowns.

3. Solve these equations for the interaction loads. Each beam is then loaded with the known loads and interaction loads and solved for moment and deflection. The grillage beam problem is now solved.

The required individual equations are found by writing the equation for the deflection at each node point on each beam. A general example of this follows:

$$\delta_{A1} = \delta_{A1}^{load} + \delta_{A1}^{P1} P_{A1} + \delta_{A1}^{P2} P_{A2} + \cdots$$

This equation merely states that the total deflection of point one on beam A-A is equal to the deflection caused by the applied loading plus the sum of the individual deflections caused by the interaction loads acting at the node points along beam AA. Such an equation is written for point A on beam 1-1 and the two equated to give the final equation for node point AA-1 and 11-A.

The main difficulty in applying this method lies in the sign convention to be used for the simultaneous equations. For convenience, downward deflection should be taken as positive in most problems, since the usual deflection of standard grids is downward in most practical problems and thus positive signs may be maintained. The direction of all interaction loads is assumed. A downward interaction load produces downward deflection and the appropriate term is thus positive. It is an incorrect procedure to assign positive signs to all terms and expect them to end correctly. An absolute check is to load the beams with the loads and solve for the deflection at the nodes. A comparison of the node deflections is positive proof of the correctness of the solution.

A grillage beam example has been worked out using this method and is presented to enable complete understanding of the process. Refer to Fig. 8. The computation follows these steps:

1. The grillage system is sketched in Fig. 8. The directions of the interaction loads are assumed as shown. Downward deflection is taken to be positive.

2. The deflection coefficients are calculated next in Fig. 9.

Due to the symmetry of the grillage system only one computation is needed. (Usually several separate calculations are involved here.) The effect of the applied ten kip load is available from this figure and is merely ten times the deflection caused by a one kip load.

3. In Fig. 10 the simultaneous equations are set up and solved. One equation is written for each node point. The deflection of one beam at the node is equated to the deflection on the cross beam at the same node.

4. These equations are solved for the interaction loads. The problem is now solved except for the final deflections which are obtained by the principle of superposition and shown at the bottom of Fig. 10.

The problem just solved is the simplest possible for this method. Likewise, this method is the best for this particular problem.

CHAPTER VI

APPARENT SPRING CONSTANT SOLUTION FOR THE GRILLAGE BEAM PROBLEM

The apparent spring constant solution for the grillage beam problem is a direct extension of the iteration procedure for beams on elastic foundations and the step-by-step procedure for beams on elastic foundations. It would appear that if a single beam supported partially by an elastic foundation can be solved, then a series of these solutions can be so combined as to produce a solution to the grid. The apparent spring constant solution does just this in an iterative process for most grillage systems. There are exceptions to this statement and such problems will be discussed in this chapter.

Heretofore the term spring constant has meant the force per unit of deflection exerted by the elastic foundation against the main beam. This force per unit of deflection has truly been a constant since it has been implied that plastic conditions or rupture never occurred. From the original non-deflected position of the main beam, this force was constantly exerted by the assumed spring as the beam deflected under load. In other words, the base of this imaginary spring remained stationary as the beam deflected. For purposes of this discussion, it has been assumed that a nonlinear elastic foundation does not apply.

Now consider the grillage system. Each beam may be considered as an elastic support for the other beams. But the difference is that as the grillage system deflects the effect is that the elastic support is lowered with the system. This causes the spring constant to be variable. Since the spring constant is defined as a force per unit of deflection, it actually becomes smaller as the system deflects. The concept of an apparent spring constant is used in order to avoid having a spring constant that varies with deflection in some unknown manner. An apparent spring constant is good for only one condition of loading and changes with the loading. It really is an average value which takes into account the deflection of the system as a whole. Thus, an apparent spring constant may be defined as the interaction load divided by the true deflection of the respective node point.

An iteration procedure is conducted by proceeding systematically around the grillage system and correcting the apparent spring constants each time. Care must be taken to proceed in such a way that the statics of the system are not violated at the end. Different loading conditions on the same grillage may cause a different systematic procedure to be necessary. This will be shown in this chapter.

The general steps necessary to solve a grillage beam problem by the apparent spring constant method are:

1. Decide on the systematic manner to proceed around the grillage system. Upon completion of one cycle, check to see that the compatability of deflections is satisfied. The end must connect with the beginning point. (If not, there must be some provision for a correction to be made.) It is important here to ensure that the procedure to be followed is reasonable. A helpful guide to this will be presented later in this chapter.

2. Either assume or calculate spring constants for each node. The closer the assumed spring constant is to the true apparent spring constant the faster the solution. Calculate the spring constant as if the beam was not part of the remaining grid in order to obtain a value of approximately the same order as the apparent spring constant. This will give a value which is too high in most cases. Another approach is to assume all spring constants for the first cycle. In general, closer values are obtained by a rough calculation than by a straight assumption.

3. Take the beams as free bodies acted upon by loads and partially supported by elastic foundations. Solve the system by one of the methods previously presented. Proceed through the grillage system and correct previous values of the apparent spring constant. This is done on solution of each beam. At the end of the first cycle, correct the initial value of the applicable apparent spring constant. Continue this process

for three cycles. Plot the values of each of the apparent spring constants. If the values appear to be converging to an answer, continue the process. It is possible that divergency or impossible solutions may result. This will be taken up later.

4. The correct solution will be reached when the initial cycle value of the apparent spring constant is equal the value calculated. The deflections of the node points on each cross beam will also agree.

The manner of proceeding around the grillage system has been stated to be of paramount importance. Evidence for this statement will now be given. In the following examples note that not only the geometry of the grillage system but also the applied loading determines the procedure. Consider the grillage system of Fig. 11 (a). The correct procedure for this system is:

1. Consider beam B-B with the applied load and elastically supported by beams 1-1 and 2-2. Solve this beam as previously shown.

2. Consider beam 2-2 with the applied load taken to be the spring load as found in step one above. Beam 2-2 is elastically supported by beam A-A. Solve this system. Correct the apparent spring constant at point two on beam B-B. The new apparent

spring constant is the interaction load divided by the deflection found in this step.

3. Consider beam 1-1 with the applied load taken to be the spring load as found in step one above. Beam 1-1 is elastically supported by beam A-A. Solve this system and correct the apparent spring constant at point one on beam B-B.

4. Consider beam A-A with the applied loads taken to be the spring loads found in steps two and three above. Solve for deflection.

5. The entire system must now be tied together. Do this by obtaining new apparent spring constants for points one and two on beam A-A. The new spring constants are equal to the spring loads found in steps two and three above divided by the deflection found in step four.

6. The first cycle is now completed. Repeat using the new values of apparent spring constants. The system is a complete one and the statics of the grillage system have not been violated.

The grillage system of Fig. 11 (b) should be solved in a similar manner. Beam C-C with elastic supports at points one and two is first solved. Then beam 2-2 is loaded at point C and solved considering elastic supports at points A and B. Correct the apparent spring constant at point two on beam C-C.

Next solve beam 1-1 in a similar fashion and correct the apparent spring constant at point one on beam C-C. Beams A-A and B-B are now loaded with the spring forces found above. The deflections are found and the remaining apparent spring constants corrected. The systematic approach to this grid is now complete.

The grillage system of Fig. 11 (c) involves a somewhat different approach for a systematic solution. The easiest method in this case is to solve all of the main beams with the applied loading. The secondary beams are to be considered as elastic supports. Then apply the spring forces found above to the secondary beams. Use the deflections thus found to obtain the new apparent spring constants. This completes the systematic procedure.

The apparent spring constant method is an iteration procedure, and as such, is self-eliminating as far as errors are concerned. The method will now be demonstrated by an example. Problem One shown in Fig. 12 is the grillage system already discussed. The beam sizes and dimensions are shown. The initial spring constant values were determined by the use of the moment-area principle. Beam B-B is then solved. The problem is of a converging nature and the solution is reached in three cycles. Beam 2-2 is next solved. The problem is known to be divergent since the spring is much stiffer than the actual beam. Nine trials were required to solve the problem by the trial and error method. These are shown in Fig. 16 through 24. Note that the method is not systematic and an attempt to bracket and close in on the answer must be made. In contrast to the trial and error approach, the step-by-step solution is presented in Figs. 25, 26, and 27. The step-by-step solution is the most efficient approach to this problem.

Beam 1-1 is solved in three trials in Fig. 28 through 30. New apparent spring constants are calculated as the iterative procedure progresses. Beam 1-1 is also solved by the step-by-step idea in Fig. 31 through 33. Beam A-A is now loaded and the remaining spring constants are evaluated again. This completes cycle one. Figure 35 shows the new spring constants.

Cycle two and three are completed in Fig. 36 through 55. The process is seen to be convergent and as such is continued. Cycle four is completed and in Fig. 63 the error is checked. The error is the difference in deflection of the cross beam and the main beam at the node points. It is less than 0.01 inches in all cases. Fig. 63 is a plot of the values of the spring constants against the number of cycles. The percentage of error is shown. The accuracy is considered sufficient for most engineering purposes.

Unfortunately, the apparent spring constant method is not applicable to all grillage beam problems. The nature of the difficulty lies in the definition of the apparent spring constant. As long as the elastic spring acts so as to resist the deflection of the main beam, the method is applicable. The method will not iterate correctly if the elastic spring exerts a downward force on a downwardly deflecting beam. The evaluation technique used for the spring constant is not correct in this situation.

An example of this type problem is presented as problem two in Fig. 64. The iteration procedure has already been discussed. The numerical calculations are conducted as outlined in Fig. 65 through 88. Only the first four cycles are included. Succeeding cycles were calculated in the same manner but are omitted to conserve space and prevent duplication of calculation. Fig. 89 is a graph of spring constant values versus cycles, and it presents the results of thirteen cycles of computation. The graph appears to be converging on the correct value but this assumption is in error. Fig. 90 shows the results of dividing one beam's node deflection by the deflection of the cross beam node point. From this it is evident that the interior deflection values are not converging but are remaining almost constant. The explanation lies in the value of the spring constant for the interior node. Due to the configuration of the grid and the applied loading, beams 1-1 and 2-2 are below beam B-B if it is imagined that the beams are not connected at the interior nodes. To connect the grillage system these beams must be pulled up by beam B-B. This results in a downward force on beam B-B and invalidates the process.

The problem can still be solved by the apparent spring constant procedure but must be so done by a trial and error approach. This method was used and the correct solution found in eleven cycles. The last cycle is presented in Fig. 91 through 95. The correct solution can be found in a problem of this type, but it is difficult.

The apparent spring constant method can be used for all grillage beam problems but is not an automatic convergent iterative solution. It is arithmetically long and may evolve into a trial and error approach. Still it is a valid procedure.

CHAPTER VII

CORRECTION CONFIGURATION SOLUTION FOR THE GRILLAGE BEAM PROBLEM

The correction configuration solution for the grillage beam problem is an iterative type approach which eliminates human errors. Unlike the apparent spring constant method, it is not a special approach but holds true for all type grid problems. It is a general method. This does not imply that all problems converge to a solution for such is not the case. On the contrary, the large majority of grid systems will diverge by this method. However, this fact does not hinder the usefulness of the method or detract from its accuracy, since a true answer may be successfully extrapolated. A solution is generally possible within four cycles and certain short cuts are available which establish this method as being a feasible one for certain grillage problems.

One of the first concepts to be developed is that of reversing the Newmark procedure. The basic Newmark approach is to proceed from known loads to deflections in an orderly systematic manner. There is one and only one configuration associated with a given set of loads. Similarly, for a known condition of loading and a known configuration, there is associated one and only one set of loads. This is the case in the grillage problem. The known condition of loading is that loads can only occur at points of known application and at node points. Then if a configuration is assumed the magnitude of these loads may be calculated by reversing the Newmark procedure.

The general steps involved in reversing the Newmark procedure are as follows:

1. At the top of the calculation sheet sketch the beam and divide it into an arbitrary number of divisions. It should be divided in such a way that a division falls at each possible loading location. This is not essential to the solution but simplifies the work.

2. Label horizontal lines down in the following manner and order:

Loads

Shear

Moment

Angle Change Ordinate

Equivalent Concentrated Angle Change

Slope

Deflection

Deflection

3. Calculations proceed from the bottom line to the top in a line by line manner. This approach enables the computer to

maintain the basic Newmark framework. In the bottom deflection line record all the assumed deformation configurations in inch units. There must be a deflection for each vertical division line. At the ends the deflection will be zero.

4. Divide each of these deflections by the $h^3/6EI$ common factor and record the value so obtained in the upper deflection line.

5. Record the slope next between division lines. Its value is the algebraic difference in the deflection values.

6. The difference in slope figures gives the equivalent concentrated angle change values. Record these on the division lines. Note that there is no equivalent concentrated angle change value for either end of the beam. Only the interior values can be obtained.

7. The main difficulty in this process is to proceed upward to the angle change line. First record the end values since they are known to be zero for a simple beam. Next observe that due to the known concentrated loading the moment diagram will consist of straight lines. The use of the formulas in Fig. 1 is warranted. This provides the key to solution and enables a series of simultaneous equations to be written. The solution of these equations yields the values to be recorded in the angle change line.

8. Find the moment line values by changing the sign and recording the same values as found above in step seven.

9. Record the shear between divisions. Each shear value is the difference between adjacent moment values.

10. Record the loads as the difference between adjacent shear values. The problem is now solved.

These steps closely follow the Newmark method in a reverse procedure. The only disadvantage is that of the necessity of solving simultaneous equations. There will be the same number of simultaneous equations as there are interior division lines provided the problem deals with simple beams. This disadvantage is eliminated later. This process is used on the first illustrative problem for the sake of basic understanding.

The corrective configuration method is an iterative idea based on assuming a deformation configuration and then calculating the loads associated with that configuration. These loads are then observed to disagree with statics. For example, loads at the node points of cross beams may not agree and there may be interior loads at locations where there are known to be no loads. The loads are corrected so as to agree with statics. This is done by an averaging method. The beams are then solved for deflections under this loading. The deflections are found to disagree with the principle of consistent deformations since node deflections of cross beams may not be the same. The cycle is completed when the deflections are averaged to agree with compatability requirements. A new configuration is obtained. The cycle proceeds from deformation configuration to loads to deformation configuration. In one cycle, both loads and deflections are corrected.

The general steps in this process are as follows:

1. The direction and magnitude of interaction loads at all node points are assumed. This is necessary in order to obtain an initial deformation configuration of the same approximate magnitude as the true one. It has been found easier to obtain an answer of the approximate order of magnitude as the true one by assuming interaction loads than by assuming deflections of the grillage system. This is a personal choice and does not affect the correctness of the method or solution. The basic idea being to start off as closely as possible to the answer.

2. The deflection ratio values associated with each possible loading application are now calculated. A known load is applied to each beam and a separate calculation conducted for each position of loading. This is a required step and will be explained in step four below.

3. Each beam is solved for the deflections resulting from the assumed interaction loads and the known loading. Note that in

general cross beam node deflections do not agree. Average these values to obtain the new node deflections.

4. For each beam there is now a deflection for each node point. However, this leaves unknown deflections at every division point which is not a node. These unknown deflections must be obtained. Several methods present themselves here. One way is to draw a smooth curve through the known deflection points and graphically obtain the values of the unknown deflections. This method has the advantage of satisfying the fact that the deflected beam will have a smooth curve shape. However, the inaccuracy of this and other similar selective methods renders the calculations useless. It is essential to start with a configuration which is closely associated with a beam loaded only at the same locations as the grillage beam under consideration. In order to achieve this, the calculations of step two are required. By the use of simultaneous equations the desired deflection values are determined. There will be one equation for each node point. The first configuration has now been found. This configuration could have been assumed, but the author's experience indicates the calculation method described to be the preferable approach.

5. The loads associated with the grillage configuration found above are next calculated by the reverse Newmark procedure previously described.

6. These loads are averaged and forced to agree with statics.

7. The new loads are applied to the system and the deflections computed as in step four above. This completes one cycle.

8. Three cycles are completed and the results plotted. Corrected values are then interpolated from the plot and a check cycle calculated to observe the accuracy of the interpolated values.

9. If the accuracy is not sufficient for the desired purpose repeat steps one through eight.

It is obvious that this process is quite long. The required solution of several sets of simultaneous equations renders the calculations quite tedious. Several steps are possible which simplify the process and will be presented after a problem is solved by the original approach.

Problem three starts with Fig. 96. This is the same problem that was solved by the apparent spring constant method and presented as problem one. The interaction loads are assumed as shown at the bottom of Fig.96. Next the deflection ratio is determined in Fig. 97. Values are shown for both main and support beams. The beams are then loaded with the assumed interaction loads and solved for the deflections. Fig. 103 contains the averaging calcuations for the deflections. The deflections are averaged according to the relative deflection values caused by the same loads. In this particular case it is the same as averaging in proportion to the beam moments of inertia. This is shown in Fig. 99. Simultaneous equations must now be set up to convert the calculated deflection into the averaged deflection. At the same time the interior deflection values must agree with a no interior load condition. This is done by the application of two corrections to the original deflection line. The final result is the new grillage configuration and is shown as the last line of Fig. 98 through 102.

These deflected shapes are then solved for the resulting loading. Note that small loads appear in the interior divisions. Loads here should be zero. Slight errors in the calculations result in these loads. The loads found in Fig. 104 through 107 are averaged as indicated in Fig. 108. These loads are then applied to the beams and the deflections averaged as previously done. This completes the cycle.

This process is continued for three cycles. The results are plotted in Fig. 134 and 135. Fig. 134 shows the plot of interaction loads and Fig. 135 shows the plot of deflection values. The first value is the assumed one made as a start. Both plots are clearly diverging. An interpolation is made in Fig. 137 by averaging adjacent values and then double weighting the first average. The final average is taken to be close to the true deflection value. One more cycle is run as a check.

In running the next cycle, certain changes are introduced which tend to considerably shorten the work involved. The calculations are conducted as shown in Fig. 138 instead of conducting a reverse Newmark process to obtain the loads associated with a configuration. The loads are found directly by the use of simultaneous equations formed with the deflection ratio data of Fig. 97. This approach considerably shortens the numerical work. The cycle is continued to completion. The new deflections are found in Fig. 145 and the difference is seen to be less than 0.01 inches from the values obtained by the apparent spring constant method.

Problem four is the same grillage system composed of four identical beams. The purpose in this problem is to show that all problems do not diverge but may oscillate for all practical purposes even though this requires a special system of equally sized beams symmetrically arranged. Figs. 147 through 159 contain the calculations for the first three cycles. The shorter method discussed above is used. Fig. 160 shows a graph of deflections against cycles. The same method of averaging adjacent values and then averaging these figures is used. By symmetry BB-1 and AA-2 must have the same deflection. As a result, the usual values were averaged to obtain similar figures. Another cycle is started using these deflection results. Fig. 163 indicates the loads found. These are seen to satisfy the symmetry of the problem and their close agreement before averaging indicates correct values.

Problem five starting with Fig. 164 is the same as problem two. By the apparent spring constant method this problem presented difficulties and had to be solved by a trial and error approach. The solution is self-explanatory and follows the same general pattern. The correction configuration method easily handles this problem.

CHAPTER VIII

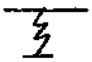
CONCLUSIONS

This study has presented two different numerical solutions to the grillage beam problem. The first is the apparent spring constant method and is not a general method. It basically is a special approach but in theory is applicable to many problems. Its usefulness as an efficient analysis procedure is limited. The second method developed is the correction configuration method. This solution is of a general nature and is applicable to all type grillage problems. It is considered a valuable analysis method and efficient for many problems. As in all statically indeterminate problems, particular solutions are better adapted to certain problems due to their inherent characteristics. Experience is needed for efficient choice.

APPENDIX A

GLOSSARY OF ABBREVIATIONS

a, b, c	Heights of diagram in equivalent concentrated load formulas
C.F.	Common factor
Corr	Correction
dx	Increment of beam length
E	Modulus of elasticity
E.C.	Equivalent concentrated
h	Division length in Newmark process
I	Moment of inertia
K	Spring constant in kips per inch
L	Loading
M	Moment at a given section in a beam
M/EI	Angle change ordinate
Q	Spring load
R	Reaction in equivalent concentration formulas
V	Shear at a given section in a beam
WF	Wide flange beam
Y	Deflection

δ	Deflection
Σ	Summation of
Θ	Slope at a given section in a beam (or average slope in a given division)
$\int$	Integral sign
$\downarrow$	Load applied to a beam
	Simple pin support for a beam
	Spring support for a beam

APPENDIX B

ILLUSTRATIONS

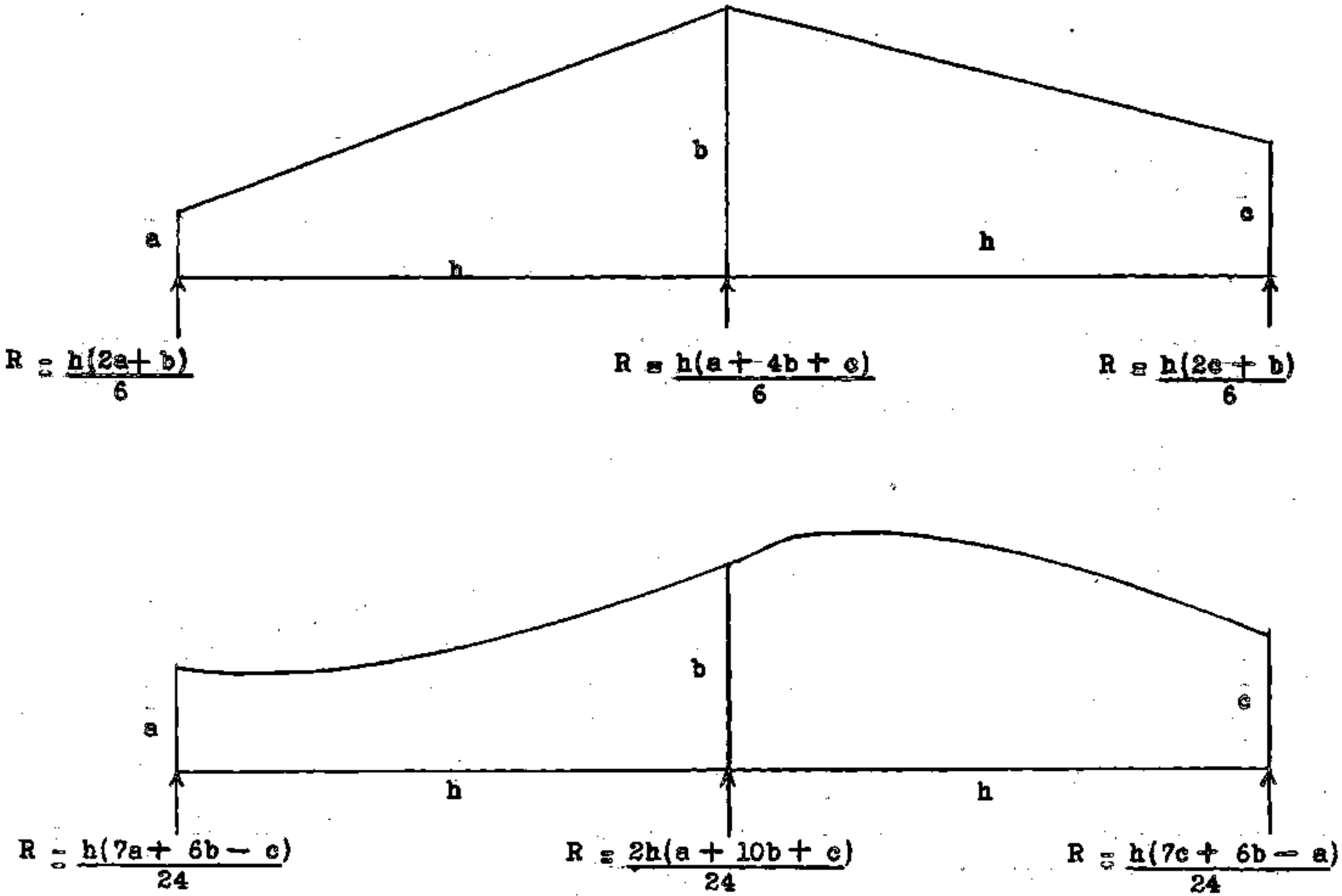


Figure 1. Equivalent Concentrated Load Formulas

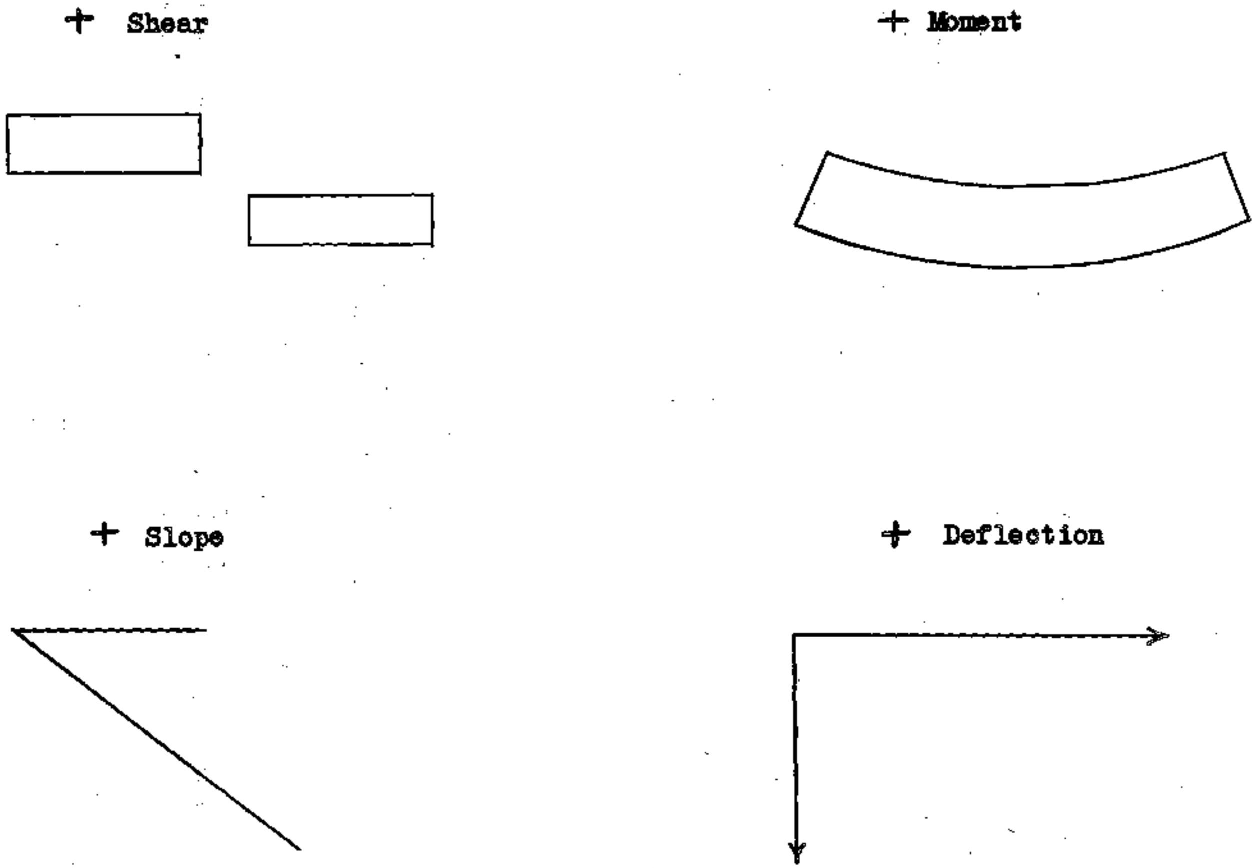
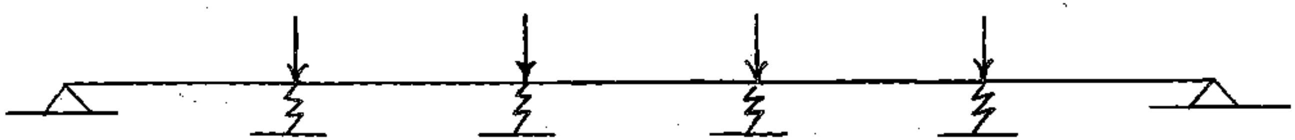


Figure 2. Sign Convention

C.F.

Loads	0		-1		-2		0		-1		0	
V Trial		0		-1		-3		-3		-4		
M Trial	0		0		-1		-4		-7		-11	h
M Corr	0		2.2		4.4		6.6		8.8		11	h
M	0		2.2		3.4		2.6		1.8		0	h
V		2.2		1.2		-.8		-.8		-1.8		
M/EI	0		-2.2		-3.4		-2.6		-1.8		0	h/EI
E.C. M/EI	-2.2		-12.2		-18.4		-15.6		-9.8		-1.8	$h^2/6EI$
Slope		12.2		0		-18.4		-34		-43.8		$h^2/6EI$
Y Trial	0		12.2		12.2		-6.2		-40.2		-84	$h^3/6EI$
Y Corr	0		16.8		33.6		50.4		67.2		84	$h^3/6EI$
Y	0		29		45.8		44.2		27		0	$h^3/6EI$

Figure 3. Newmark Process Example

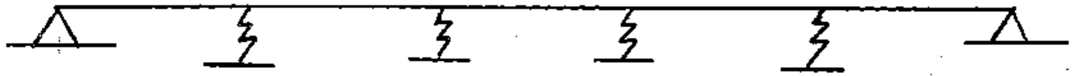

C.F.

	1		2		3		4		5		6	
Division	1		2		3		4		5		6	
Loads			-2		-2		-2		-3			
Y	(1) 0		(6)0.2993		-.2783		-2.4876		-6.805		$\boxed{-13.0025}$	
Sp. Load			(7)0.5986		-.5566		-4.9752		-13.61			
Σ Load			(8) -1.4014		-2.5566		-6.9752		-16.61			
V		Assume (2) 10		(9) 8.5986		6.042		-.9332		-17.5432		
M	(1) 0		(3) 10		(10)18.5986		24.6406		23.7074		$\boxed{6.1642}$	h
M/EI	(1) 0		(4) -10		(11) -18.5986		-24.6406		-23.7074		-6.1642	h/EI
E.C. M/EI			(12) -58.5986		-109.035		-140.8684		-125.6344			$h^2/6EI$
Slope		Assume 20		(13) -38.5986		-147.6336		-288.502		-414.1364		$h^2/6EI$
Y	(1) 0		(5) 20		(14)-18.5986		-166.2322		-454.7342		-868.87	$h^3/6EI$

Main Beam 14WF61 with I of 641.5 K (spring constant) equals 2 kips per inch

Common factor $h^3/6EI$ equals .014964925

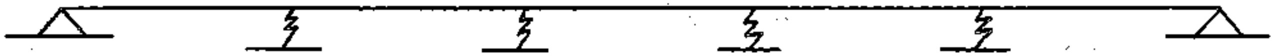
Figure 4. Basic Step-By-Step Process Calculation



Y	0		0		-0.0898		-0.3564		-0.8657		$\boxed{-1.6119}$	
Spring Load			0		-0.1796		-0.7128		-1.7314			
V		Assume 1		1		0.8204		0.1076		-1.6238		
M	0		1		2		2.8204		2.928		$\boxed{1.3042}$	h
M/EI	0		-1		-2		-2.8204		-2.928		-1.3042	h/EI
E.C. M/EI	-1		-6		-11.8204		-16.2096		-15.8366			$h^2/6EI$
Slope		Assume 0		-6		-17.8204		-34.03		-49.8666		$h^2/6EI$
Y	0		0		-6		-23.8204		-57.85		-107.7166	$h^3/6EI$

Obvious errors exist at the right end for both deflection and moment which are known to be zero. This can be combined with Correction 'B' so as to eliminate these errors in the original calculation.

Figure 5. Correction 'A' To Step-By-Step Calculation



												C.F.
Y	0		0.0149		0.0295		0.0404		0.0395		$\boxed{.0116}$	
Sp. Load			0.0298		0.0590		0.0808		0.0790			
V		Assume 0		0.0298		0.0888		0.1696		0.2486		
M	0		0		0.0298		0.1186		0.2882		$\boxed{0.5368}$	h
M/EI	0		0		-.0298		-.1186		-.2882		-.5368	h/EI
E.C. M/EI			-.0298		-.2378		-.7924		-1.8082			$h^2/6EI$
Slope		Assume 1		0.9702		0.7324		-.06		-1.8682		$h^2/6EI$
Y	0		1		1.9702		2.7026		2.6426		0.7744	$h^3/6EI$

Simultaneous Correction Equations

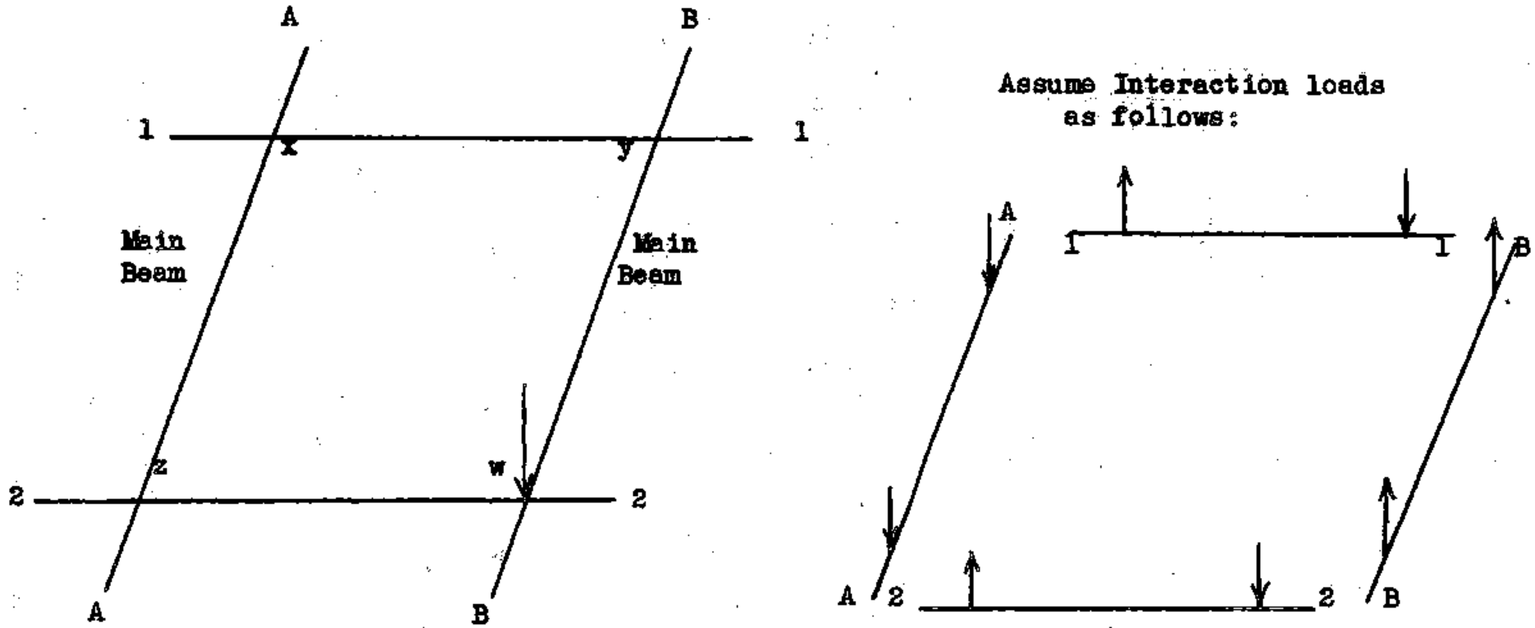
$$-1.6119A + .0116B = 13.0025 \qquad A = -8.0091$$
$$1.3042A + .5368B = -6.1642 \qquad B = 7.9755$$

Y Original	0	0.2993	-.2783	-2.4876	-6.805	-13.0025
Y Corr A	0	0	0.7192	2.8544	6.9334	12.9099
Y Corr B	0	0.1188	0.2352	0.3222	0.315	0.0925
True Y Value	0	0.4181	0.6761	0.689	0.4434	0

Figure 6. Correction 'B' To Step-By-Step Calculation And True Deflection Determination

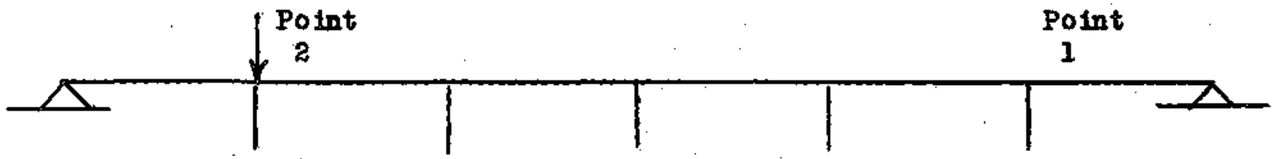
Loads			-2		-2		-2		-3			
Y	0		0.4181		0.6761		0.689		0.4434		0	
Sp. Load			0.8362		1.3522		1.378		0.8868			
Σ Loads	0		-1.1638		-.6478		-.622		-2.1132			
V		1.8116		0.6478		0		-.622		-2.7352		
M Trial	0		1.8116		2.4594		2.4594		1.8374		-.8978	h
Corr M	0		0.1795		0.3591		0.5386		0.7182		0.8978	h
M	0		1.9911		2.8185		2.998		2.5556		0	h
M/EI	0		-1.9911		-2.8185		-2.998		-2.5556		0	h/EI
E.C. M/EI	-1.9911		-10.7829		-16.2631		-17.3661		-13.2204		-2.5556	$H^2/6EI$
Slope		27.046		16.2631		0		-17.3661		-30.5865		$h^2/6EI$
Y	0		27.046		43.3091		43.3091		25.943		-4.6435	$h^3/6EI$
Corr Y	0		0.9287		1.8574		2.7861		3.7148		4.6435	$h^3/6EI$
Y	0		27.9747		45.1665		46.0952		29.6578		0	$h^3/6EI$
Y	0		0.418		0.676		0.689		0.443		0	inches

Figure 7. Check Solution Of Step-By-Step Answers



All beams 10WF45 with I of 248.6

Figure 8. Grillage System To Be Solved By Equation



														C.F.
Load			-1		0		0		0		0			
V		0		-1		-1		-1		-1		-1		
M Trial	0		0		-1		-2		-3		-4		-5	h
Corr M	0		0.833		1.666		2.499		3.333		4.166		5	h
M	0		0.833		0.666		0.499		0.333		0.166		0	h
M/EI	0		-.833		-.666		-.499		-.333		-.166		0	h/EI
E.C. M/EI	-.833		-3.998		-3.996		-2.995		-1.997		-.997		-.166	$h^2/6EI$
Slope		7.994		3.996		0		-2.995		-4.992		-5.989		$h^2/6EI$
Y	0		7.994		11.99		11.99		8.995		4.003		-1.986	$h^3/6EI$
Corr Y	0		0.331		0.662		0.993		1.324		1.655		1.986	$h^3/6EI$
Y	0		8.325		12.652		12.983		10.319		5.658		0	$h^3/6EI$
Y	0		0.32148		0.48857		0.50135		0.39848		0.21849		0	inches

Figure 9. Deflection Coefficient Calculation

$\delta^{W}_{BB-2} = 3.2148$ $\delta^{W}_{BB-1} = 2.1849$

Node BB-2 and 22-B equation:

$3.2148 - 0.32148W - 0.21849Y = 0.32148W - 0.21849\ Z$

Node BB-1 and 11-B equation:

$2.1849 - 0.21849W - 0.32148Y = 0.32148Y - 0.21849\ X$

Node AA-1 and 11-A equation:

$0.21849\ Y - 0.32148X = 0.32148X + 0.21849Z$

Node AA-2 and 22-A equation:

$0.32148Z + 0.21849X = 0.21849W - 0.32148Z$

The simultaneous solution of these equations results in:

$X = 0$ $Y = 1.698$ $W = 5$ $Z = 1.698$

Thus the following node deflections are correct:

BB-1	0.5465	inches	BB-2	1.2365	inches
11-B	0.5459	inches	22-B	1.2365	inches
AA-1	0.3709	inches	AA-2	0.5459	inches
11-A	0.3709	inches	22-A	0.5465	inches

Accuracy is sufficient for most engineering purposes.

Figure 10. Simultaneous Equation Solution

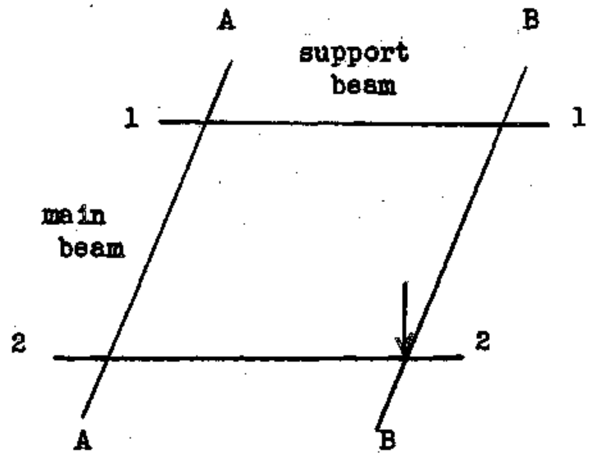


Figure 11a.
Top Grillage
Figure

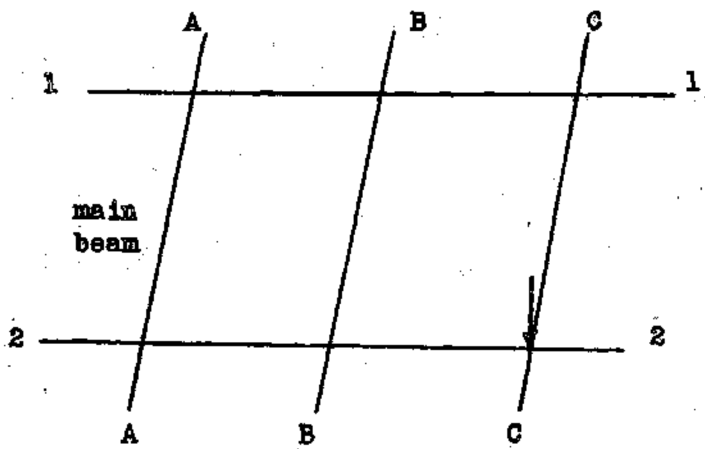


Figure 11b.
Middle Grillage
Figure

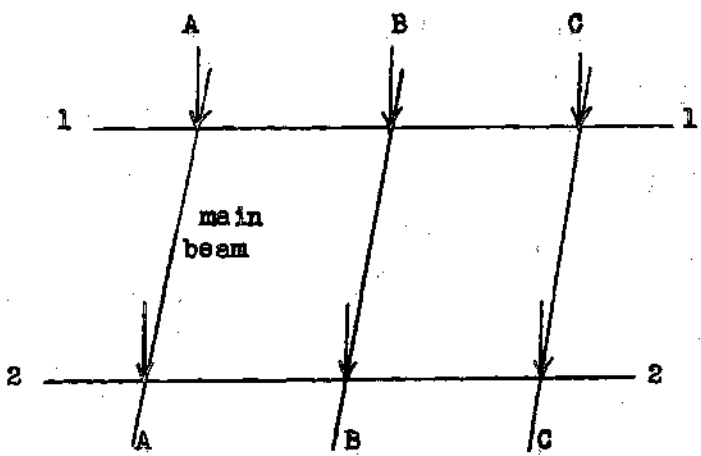
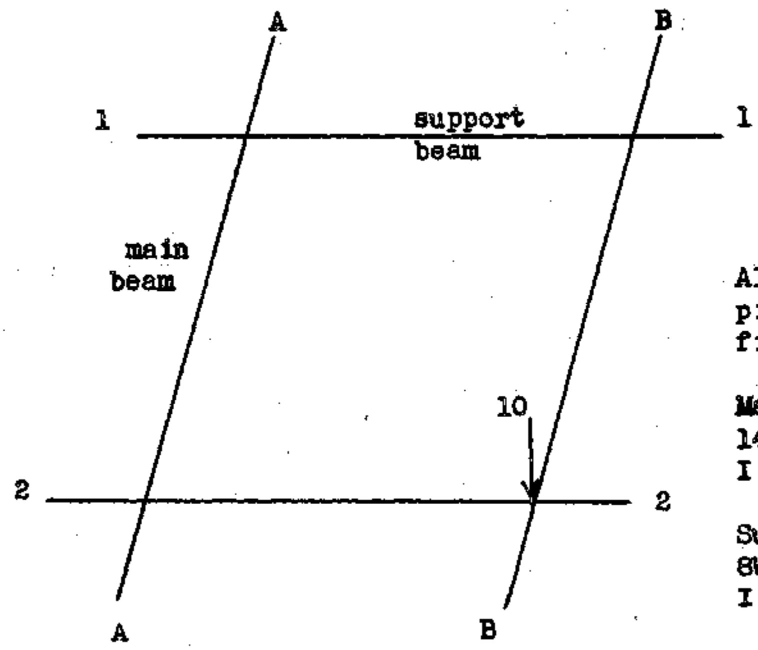


Figure 11c.
Bottom Grillage
Figure

Figure 11. Possible Grillage Systems



All beams pinned to fixed supports.

Main beam
14WF61
I of 641.5

Support beam
8WF17
I of 56.4

Dimensions

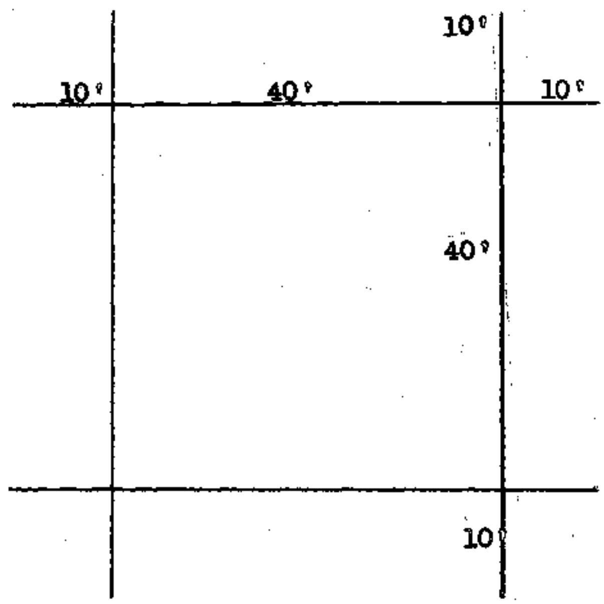
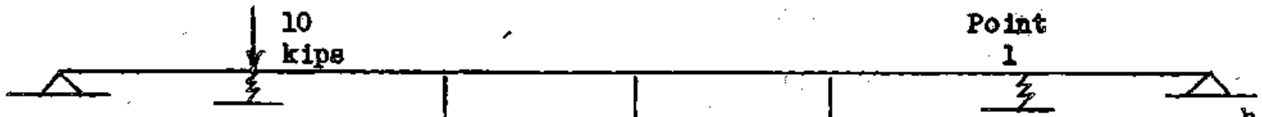


Figure 12. Problem One Grillage System



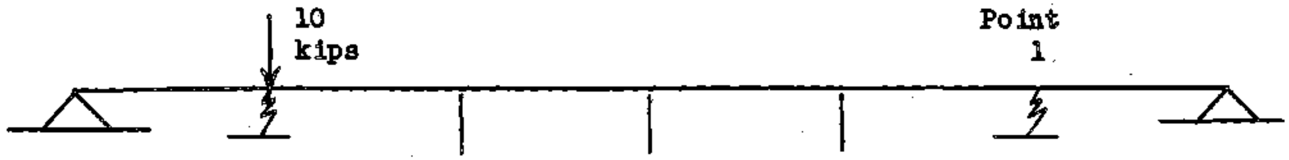
h = 120"
K = .705
kips/inch

Loads			-10		0		0		0		0			
Assumed Y (inches)			0.2								0.1			
Spring Load			0.141								0.0705			
Total Loads	0		-9.859		0		0		0		0.0705		0	
V Trial		0		-9.859		-9.859		-9.859		-9.859		-9.7885		
M Trial	0		0		-9.859		-19.718		-29.577		-39.436		-49.2245	h
Corr M	0		8.2041		16.4082		24.6123		32.8163		41.0204		49.2245	h
M	0		8.2041		6.5492		4.8943		3.2393		1.5844		0	h
V		8.2041		-1.6549		-1.6549		-1.655		-1.6549		-1.5844		
M/EI	0		-8.2041		-6.5492		-4.8943		-3.2393		-1.5844		0	h/EI
E.C. M/EI	-8.2041		-39.3656		-39.2952		-29.3657		-19.4359		-9.5769		-1.5844	$h^2/6EI$
Slope		39.3656		0		-39.2952		-68.6609		-88.0968		-97.6737		$h^2/6EI$
Y	0		39.3656		39.3656		.0704		-68.5905		-156.6873		-254.361	
Y Corr	0		42.393		84.786		127.179		169.572		211.965		254.361	
Y	0		81.7586		124.1516		127.2494		100.9815		55.2777		0	$h^3/6EI$

Y = 1.224 inches Y = .828 inches

Use these values in next trial.

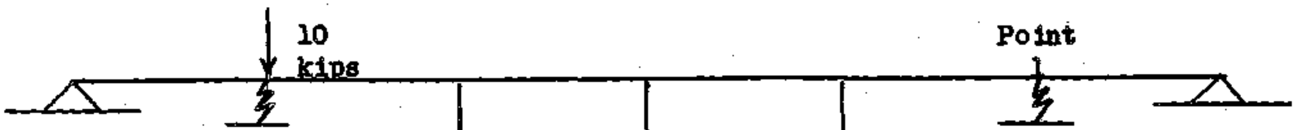
Figure 13. Problem One - Beam B-B Cycle 1 Trial 1



Loads	0		-10		0		0		0		0		0	
Y Assumed			1.2								0.8			
Spring Loads			0.846								0.564			K = .705 kips/inch
Total Loads	0		-9.154		0		0		0		0.564		0	
V Trial		0		-9.154		-9.154		-9.154		-9.154		-8.59		
M Trial	0		0		-9.154		-18.308		-24.462		-36.616		-45.206	h
Corr M	0		7.534		15.068		22.602		30.136		37.67		45.206	h
M	0		7.534		5.914		4.294		2.674		1.054		0	h
M/EI	0		-7.534		-5.914		-4.294		-2.674		-1.054		0	h/EI
E.C. M/EI	-7.534		-36.05		-35.484		-25.764		-16.044		-6.89		-1.054	$h^2/6EI$
Slope		36.05		0		-35.484		-61.248		-77.292		-84.182		$h^2/6EI$
Y	0		36.05		36.05		0.566		-60.682		-137.974		-222.156	
Y Corr	0		37.026		74.052		111.078		148.104		185.13		222.156	
Y			73.076								47.156			$h^3/6EI$

Y = 1.094 inches　　　　Y = .707 inches

Figure 14. Problem One - Beam B-B Cycle 1 Trial 2

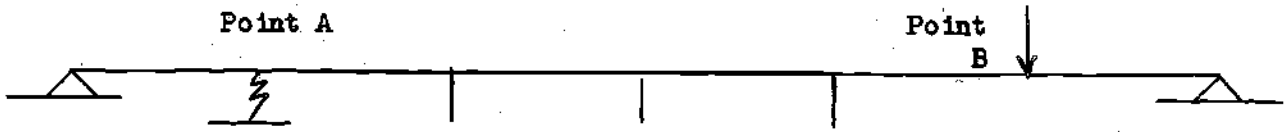


Loads			-10		0		0		0		0		K = .705	
Assumed Y			1.1								0.72		kips/inch	
Spring Load			0.776								0.507			
Total Loads	0		-9.224		0		0		0		0.507			
V Trial		0		-9.224		-9.224		-9.224		-9.224		-8.717		
M Trial	0		0		-9.224		-18.448		-27.672		-36.896		-45.613	h
Corr M	0		7.602		15.204		22.806		30.408		38.01		45.613	h
M	0		7.602		5.98		4.358		2.736		1.114		0	h
M/EI	0		-7.602		-5.98		-4.358		-2.736		-1.114		0	h/EI
E.C. M/EI	-7.602		-36.388		-35.88		-26.148		-16.416		-7.192		-1.114	$h^2/6EI$
Slope		36.388		0		-35.88		-62.028		-78.444		-85.636		$h^2/6EI$
Y	0		36.388		36.388		.508		-61.52		-139.964		-225.6	
Corr Y	0		37.6		75.2		112.8		150.4		188		225.6	
Y	0		73.988								48.036			

Y = 1.106 inches Y = .72 inches

Solution accurate enough.

Figure 15. Problem One - Beam B-B Cycle 1 Trial 3



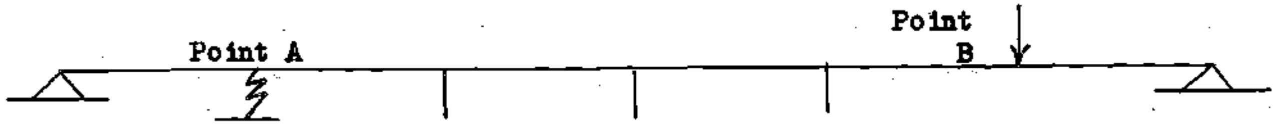
K = 8.02 kips/inch

Assumed Y			0.04											
Loads	0		0.32		0		0		0		-.776			
V Trial		0		0.32		0.32		0.32		0.32		-.456		
M Trial	0		0		0.32		0.64		0.96		1.28		0.824	h
Corr M	0		-.137		-.274		-.411		-.548		-.685		-.824	h
M	0		-.137		0.046		0.229		0.412		0.595		0	h
M/EI	0		0.137		-.046		-.229		-.412		-.595		0	h/EI
E.C. M/EI	0.137		0.502		-.276		-1.374		-2.472		-2.792		-.595	$h^2/6EI$
Slope		0.778		0.276		0		-1.374		-3.846		-6.638		$h^2/6EI$
Y	0		0.778		1.054		1.054		-.32		-4.166		-10.804	
Corr Y	0		1.8		3.6		5.4		7.2		9		10.804	
Y	0		2.578		4.654		6.454		6.88		4.834		o	$h^3/6EI$

Y = .438 inches

Note that this problem is of a diverging nature. Must use a trial and error approach.

Figure 16. Problem One - Beam 2-2 Cycle 1 Trial 1



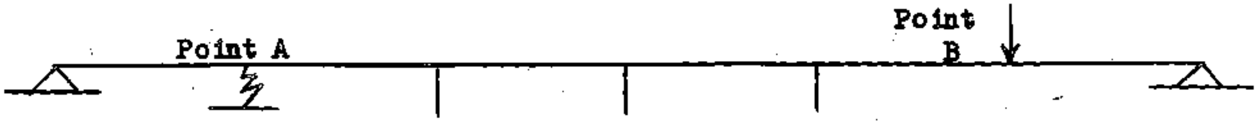
K = 8.02 kips/inch

Assumed Y			0.02											
Loads			0.16		0		0		0		-.776			
V Trial		0		0.16		0.16		0.16		0.16		-.616		
M Trial	0		0		0.16		0.32		0.48		0.64		0.024	h
Corr M	0		-.004		-.008		-.012		-.016		-.02		-.024	h
M	0		-.004		0.152		0.308		0.464		0.62		0	h
M/EI	0		0.004		-.152		-.308		-.464		-.62		0	h/EI
E.C. M/EI	0.004		-.136		-.912		-1.848		-2.784		-2.944		-.62	$h^2/6EI$
Slope		1.048		.912		0		-1.848		-4.632		-7.576		$h^2/6EI$
Y	0		1.048		1.96		1.96		.112		-4.52		-12.096	
Corr Y	0		2.016		4.032		6.048		8.064		10.08		12.096	
Y	0		3.064								5.56		0	$h^3/6EI$

Y = .522 inches

Next trial use Y (assumed) of 0.05 inches.

Figure 17. Problem One - Beam 2-2 Cycle 1 Trial 2



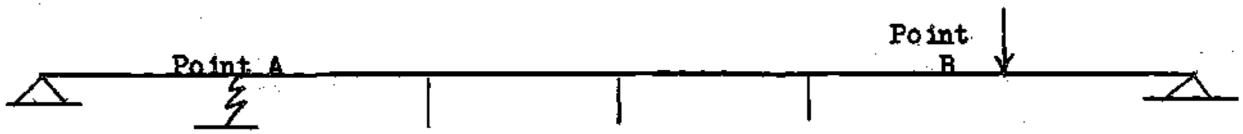
K = 8.02 kips/inch

Assumed Y			0.05											
Loads	0		0.401		0		0		0		-.776		0	
V Trial		0		0.401		0.401		0.401		0.401		-.375		
M Trial	0		0		0.401		0.802		1.203		1.603		1.228	h
Corr M	0		-.204		-.408		-.612		-.816		-1.02		-1.228	h
M	0		-.204		-.007		0.19		0.387		0.583		0	h
M/EI	0		0.204		0.007		-.19		-.387		-.583		0	h/EI
E.C. M/EI	0.204		0.823		0.042		-1.14		-2.321		-2.719		-.583	$h^2/6EI$
Slope		0.275		1.098		1.14		0		-2.321		-5.04		$h^2/6EI$
Y	0		0.275		1.373		2.513		2.513		0.192		-4.848	
Corr Y	0		0.808		1.616		2.424		3.232		4.04		4.848	
Y	0		1.083								4.232		0	$h^3/6EI$

Y = .1845 inches

Next trial assume Y of .048 inches.

Figure 18. Problem One - Beam 2-2 Cycle 1 Trial 3

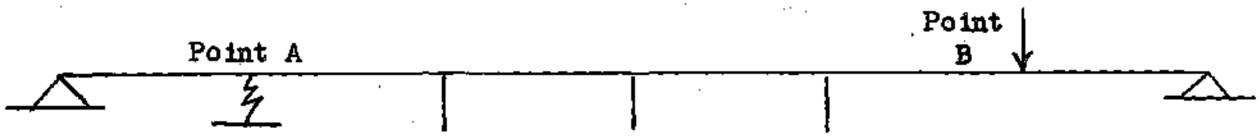


Assumed Y			0.048								$K \cong 8.02$ kips/inch			
Loads	0		0.385		0		0		0		-.776		0	
V Trial		0		0.385		0.385		0.385		0.385		-.391		
M Trial	0		0		0.385		0.77		1.155		1.54		1.149	h
Corr M	0		-.191		-.382		-.573		-.764		-.955		-1.149	h
M	0		-.191		0.003		0.197		0.391		0.585		0	h
M/EI	0		0.191		-.003		-.197		-.391		-.585		0	h/EI
E.C. M/EI	0.191		0.761		-.018		-1.182		-2.346		-2.731		-.585	$h^2/6EI$
Slope		-.743		0.018		0		-1.182		-3.528		-6.259		$h^2/6EI$
Y	0		-.743		-.725		-.725		-1.907		-5.435		-11.694	
Corr Y	0		1.949		3.898		5.847		7.796		9.745		11.694	
Y	0		1.206								4.31		0	$h^3/6EI$

Y = .205 inches

Next trial assume Y of 0.052 inches.

Figure 19. Problem One - Beam 2-2 Cycle 1 Trial 4

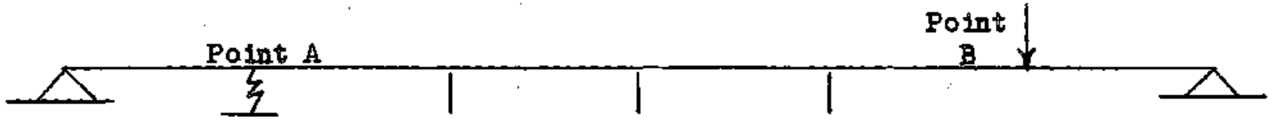


Assumed Y			0.052								K = 8.02 kips/inch			
Loads	0		0.417		0		0		0		-0.776		0	
V Trial		0		0.417		0.417		0.417		0.417		-0.359		
M Trial	0		0		0.417		0.834		1.251		1.668		1.309	h
Corr M	0		-0.218		-0.436		-0.654		-0.872		-1.09		-1.309	h
M	0		-0.218		-0.019		0.18		0.379		0.578		0	h
M/EI	0		0.218		0.019		-0.18		-0.379		-0.578		0	h/EI
E.C. M/EI	0.218		0.891		0.114		-1.08		-2.274		-2.691		-0.578	$h^2/6EI$
Slope		-1.005		-0.114		0		-1.08		-3.354		-6.045		$h^2/6EI$
Y	0		-1.005		-1.119		-1.119		-2.199		-5.553		-11.598	
Y Corr	0		1.933		3.866		5.799		7.732		9.665		11.598	
Y	0		0.928								4.112		0	$h^3/6EI$

Y = 0.158 inches

Next trial assume Y of 0.054 inches.

Figure 20. Problem One - Beam 2-2 Cycle 1 Trial 5

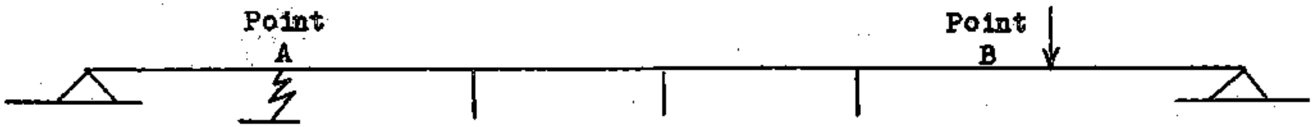


Assumed Y			0.054								K = 8.02 kips/inch			
Loads	0		0.433		0		0		0		-0.776		0	
V Trial		0		0.433		0.433		0.433		0.433		-0.343		
M Trial	0		0		0.433		0.866		1.299		1.732		1.389	h
Corr M	0		-0.231		-0.462		-0.693		-0.924		-1.155		-1.389	h
M	0		-0.231		-0.029		0.173		0.375		0.577		0	h
M/EI	0		0.231		0.029		-0.173		-0.375		-0.577		0	h/EI
E.C. M/EI	0.231		0.953		0.174		-1.038		-2.25		-2.683		-0.577	$h^2/6EI$
Slope		-1.127		-0.174		0		-1.038		-3.288		-5.971		$h^2/6EI$
Y	0		-1.127		-1.301		-1.301		-2.339		-5.627		-11.598	
Corr Y	0		1.933		3.866		5.799		7.732		9.665		11.598	
Y	0		0.806										0	$h^3/6EI$

Y = 0.137 inches

Next trial assume Y of 0.056 inches.

Figure 21. Problem One - Beam 2-2 Cycle 1 Trial 6

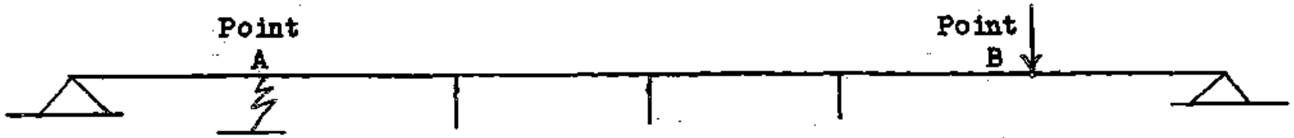


Assumed Y			0.056								K = 8.02 kips/ inch			
Loads	0		0.449		0		0		0		-0.776		0	
V Trial		0		0.449		0.449		0.449		0.449		-0.327		
M Trial	0		0		0.449		0.898		1.347		1.796		1.469	h
Corr M	0		-0.245		-0.49		-0.735		-0.98		-1.225		-1.469	h
M	0		-0.245		-0.041		0.163		0.367		0.571		0	h
M/EI	0		0.245		0.041		-0.163		-0.367		-0.571		0	h/EI
E.C. M/EI	0.245		1.021		0.246		-0.978		-2.202		-2.651		-0.571	$h^2/6EI$
Slope		-1.267		-0.246		0		-0.978		-3.18		-5.831		$h^2/6EI$
Y	0		-1.267		-1.513		-1.513		-2.491		-5.671		-11.502	$h^3/6EI$
Corr Y	0		1.917		3.834		5.751		7.668		9.585		11.502	$h^3/6EI$
Y	0		0.65										0	$h^3/6EI$

Y = 0.1105 inches

Next trial assume Y of 0.06 inches.

Figure 22. Problem One - Beam 2-2 Cycle 1 Trial 7

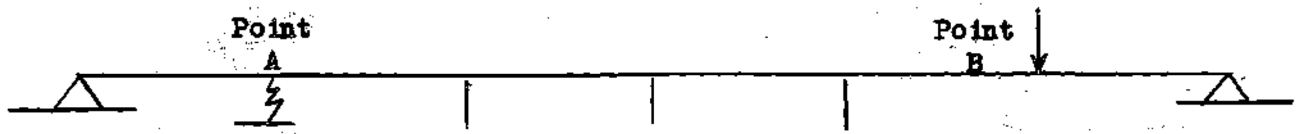


Assumed Y			0.06								K = 8.02 kips/inch			
Loads	0		0.4812		0		0		0		-0.776		0	
V Trial		0		0.4812		0.4812		0.4812		0.4812		-0.2948		
M Trial	0		0		0.4812		0.9624		1.4436		1.9248		1.63	h
Corr M	0		-0.27		-0.54		-0.81		-1.08		-1.35		-1.63	h
M	0		-0.27		-0.0588		0.1524		0.3636		0.5748		0	h
M/EI	0		0.27		0.0588		-0.1524		-0.3636		-0.5748		0	h/EI
E.C. M/EI	0.27		1.1388		0.3528		-0.9144		-2.1816		-2.6628		-0.5748	$h^2/6EI$
Slope		-1.4916		-0.3528		0		-0.9144		-3.096		-5.7588		$h^2/6EI$
Y	0		-1.4916		-1.8444		-1.8444		-2.7588		-5.8548		-11.6136	
Corr Y	0		1.9356		3.8712		5.8068		7.7424		9.678		11.6136	
Y	0		0.444										0	$h^3/6EI$

Y = 0.0756 inches

Next trial assume Y of 0.061 inches.

Figure 23. Problem One - Beam 2-2 Cycle 1 Trial 8



K = 8.02 kips/inch

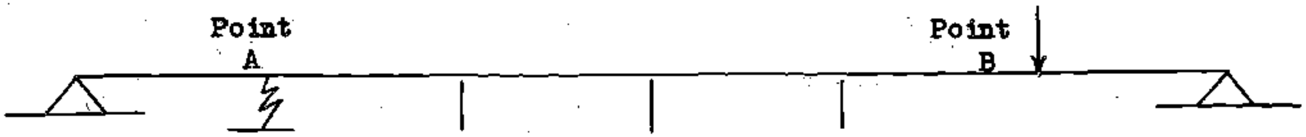
Assumed Y			0.061											
Loads	0		0.488		0		0		0		-0.776		0	
V Trial		0		0.488		0.488		0.488		0.488		-0.288		
M Trial	0		0		0.488		0.976		1.464		1.952		1.664	h
Corr M	0		-0.277		-0.554		-0.831		-1.108		-1.385		-1.664	h
M	0		-0.277		-0.066		0.145		0.356		0.567		0	h
M/EI	0		0.277		0.066		-0.145		-0.356		-0.567		0	h/EI
E.C. M/EI	0.277		1.174		0.396		-0.87		-2.136		-2.624		-0.567	$h^2/6EI$
Slope		-1.57		-0.396		0		-0.87		-3.006		-5.63		$h^2/6EI$
Y	0		-1.57		-1.966		-1.966		-2.836		-5.842		-11.472	
Corr Y	0		1.912		3.824		5.736		7.648		9.56		11.472	
Y	0		0.342								3.718		0	$h^3/6EI$

Y = 0.0583 inches Y = 0.635 inches

Accurate enough for an early cycle.

New support beam spring constant = 0.776 / 0.635 or 1.22 kips per inch.

Figure 24. Problem One - Beam 2-2 Cycle 1 Trial 9

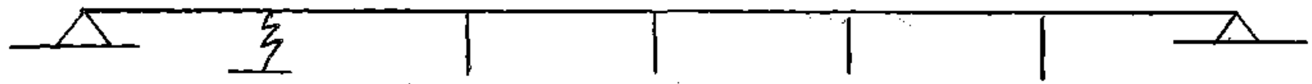


K = 8.02 kips/inch

Load	0		0		0		0		0		-0.776		0	
Y	0		2.5531								-414.529		$\boxed{-777.6212}$	
Spring Load			20.475		0		0		0		0			
Total Load	0		20.475		0		0		0		-0.776		0	
Shear		Assume 10		30.475		30.475		30.475		30.475		29.699		
Moment	0		10		40.475		70.95		101.425		131.9		$\boxed{161.599}$	h
M/EI	0		-10		-40.475		-70.95		-101.425		-131.9		-161.599	h/EI
E.C. M/EI	-10		-80.475		-242.85		-425.7		-608.55		-790.624		-455.098	$h^2/6EI$
Slope		Assume 15		-65.475		-308.325		-734.025		-1342.57		-2133.199		$h^2/6EI$
Y	0		15		-50.475		-358.8		-1092.825		-2435.4		-4568.599	

Obvious errors exist for deflection and moment at the right end. This indicates that the assumed shear and slope made in division one are incorrect. Two corrections must now be made.

Figure 25. Problem One - Step-By-Step Solution Beam 2-2 Cycle 1



Y	0		0								-20.425		$\boxed{-35.744}$	
Spring Loads			0		0		0		0		0			
Shear		Assume 1		1		1		1		1		1		
Moment	0		1		2		3		4		5		$\boxed{6}$	h
M/EI	0		-1		-2		-3		-4		-5		-6	h/EI
E.C. M/EI	-1		-6		-12		-18		-24		-30		-17	$h^2/6EI$
Slope		Assume 0		-6		-18		-36		-60		-90		$h^2/6EI$
Y	0		0		-6		-24		-60		-120		-210	$h^3/6EI$

Simultaneous equations obtained from Corrections 'A' and 'B'.

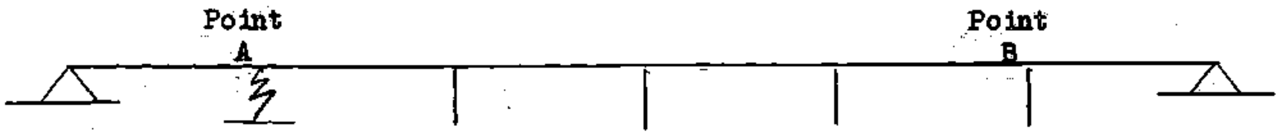
$$-35.744A - 28.0208B = 777.7212$$

$$6A + 6.825B = -161.599$$

$$A = -10.275$$

$$B = -14.6445$$

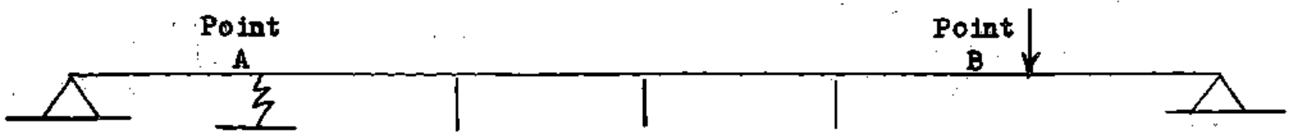
Figure 26. Problem One - Correction 'A' Step-By-Step Solution Beam 2-2 Cycle 1



Y	0		0.1702								-14.0184		$\boxed{-28.0208}$	
Loads			1.365		0		0		0		0			
V		Assume 0		1.365		1.365		1.365		1.365		1.365		
M	0		0		1.365		2.73		4.095		5.46		$\boxed{6.825}$	h
M/EI	0		0		-1.365		-2.73		-4.095		-5.46		-6.825	h/EI
E.C. M/EI	0		-1.365		-8.19		-16.38		-24.57		-32.76		-19.11	$h^2/6EI$
Slope		Assume 1		-0.365		-8.555		-24.935		-49.505		-82.265		$h^2/6EI$
Y	0		1		0.635		-7.92		-32.855		-82.36		-164.625	
Y Original			2.5531								-414.529		-777.6212	
Y Corr 'A'			0								209.8691		367.2699	
Y Corr 'B'			-2.4925								205.2924		410.3516	
True Y Values			0.0606								0.6325		0	

New support spring constant = 0.776 / 0.6325 or 1.22 kips/inch which checks with Fig. 24 value.

Figure 27. Problem One - Correction 'B' Step-By-Step Solution Beam 2-2 Cycle 1



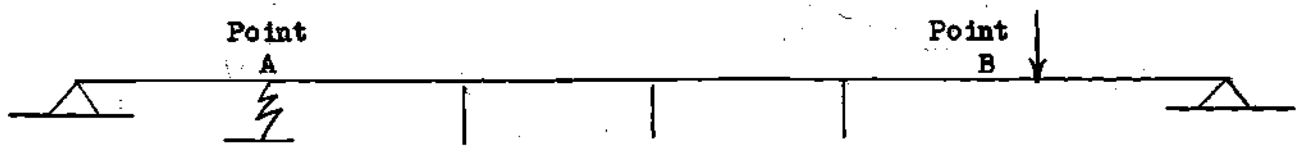
K = 8.02 kips/inch

Assumed Y			0.031											
Loads	0		0.249		0		0		0		-0.507		0	
V Trial		0		0.249		0.249		0.249		0.249		-0.258		
M Trial	0		0		0.249		0.498		0.747		0.996		0.738	h
Corr M	0		-0.123		-0.246		-0.369		-0.492		-0.615		-0.738	h
M	0		-0.123		0.003		0.129		0.255		0.381		0	h
M/EI	0		0.123		-0.003		-0.129		-0.255		-0.381		0	h/EI
E.C. M/EI	0.123		0.489		-0.018		-0.774		-1.53		-1.779		-0.381	$h^2/6EI$
Slope		-0.489		0		-0.018		-0.792		-2.322		-4.101		$h^2/6EI$
Y	0		-0.489		-0.489		-0.507		-1.299		-3.621		-7.722	$h^3/6EI$
Corr Y	0		1.287		2.574		3.861		5.148		6.435		7.722	$h^3/6EI$
Y	0		0.798										0	

Y = 0.136 inches

Next trial assume Y of 0.034 inches.

Figure 28. Problem One - Beam 1-1 Cycle 1 Trial 1



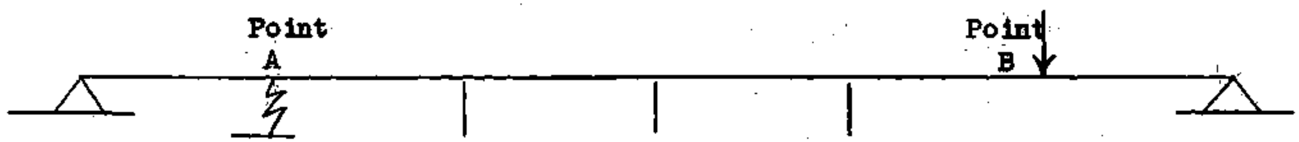
K = 8.02 kips/inch

Assumed Y			0.034											
Loads	0		0.273		0		0		0		-0.507		0	
V Trial		0		0.273		0.273		0.273		0.273		-0.234		
M Trial	0		0		0.273		0.546		0.819		1.092		0.858	h
Corr M	0		-0.143		-0.286		-0.429		-0.572		-0.715		-0.858	h
M	0		-0.143		-0.013		0.117		0.247		0.377		0	h
M/EI	0		0.143		0.013		- 0.117		-0.247		-0.377		0	h/EI
E.C. M/EI	0.143		0.585		0.078		-0.702		-1.482		-1.755		-0.377	$h^2/6EI$
Slope		-0.585		0		0.078		-0.624		-2.106		-3.861		$h^2/6EI$
Y	0		-0.585		-0.585		-0.507		-1.131		-3.237		-7.098	$h^3/6EI$
Corr Y	0		1.183		2.366		3.549		4.732		5.915		7.098	$h^3/6EI$
Y	0		0.598										0	$h^3/6EI$

Y = 0.102 inches

Next trial assume Y of 0.04 inches.

Figure 29. Problem One - Beam 1-1 Cycle 1 Trial 2



Assumed Y			0.04											
Loads	0		0.32		0		0		0		-0.507		0	
V Trial		0		0.32		0.32		0.32		0.32		-0.187		
M Trial	0		0		0.32		0.64		0.96		1.28		1.093	h
Corr M	0		-0.182		-0.364		-0.546		-0.728		-0.91		-1.093	h
M	0		-0.182		-0.044		0.094		0.232		0.37		0	h
M/EI	0		0.182		0.044		-0.094		-0.232		-0.37		0	h/EI
E.C. M/EI	0.182		0.772		0.264		-0.564		-1.392		-1.712		-0.37	$h^2/6EI$
Slope		-1.036		-0.264		0		-0.564		-1.956		-3.668		$h^2/6EI$
Y	0		-1.036		-1.3		-1.3		-1.864		-3.82		-7.488	
Corr Y	0		1.248		2.496		3.744		4.992		6.24		7.488	
Y	0		0.212								2.42		0	$h^3/6EI$

K = 8.02 kips/inch

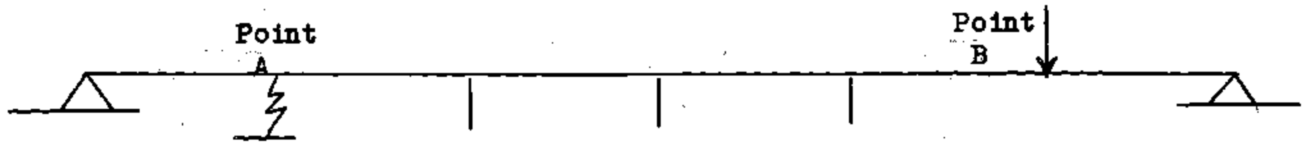
Y = 0.036 inches

Y = 0.411 inches

Accurate enough for trial one.

New spring constant equals 0.507 / 0.411 or 1.23 kips/inch which checks.

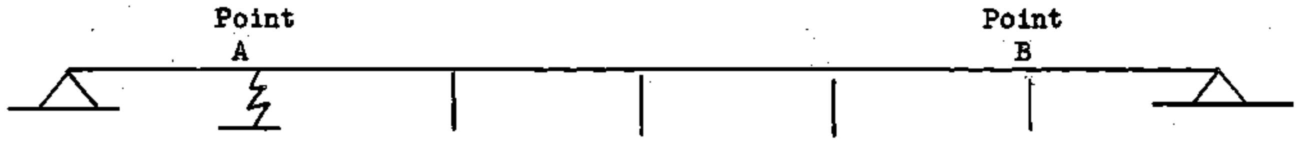
Figure 30. Problem One - Beam 1-1 Cycle 1 Trial 3



Loads	0		0		0		0		0		-0.507		0	
Y			0.3404								-48.4621		$\boxed{-91.6994}$	
Spring Load			2.73											
Total Loads	0		2.73		0		0		0		-0.507		0	
V		Assume 1		3.73		3.73		3.73		3.73		3.223		
M	0		1		4.73		8.46		12.19		15.92		$\boxed{19.143}$	h
M/EI	0		-1		-4.73		-8.46		-12.19		-15.92		-19.143	h/EI
E.C. M/EI	-1		-8.73		-28.38		-50.76		-73.14		-95.013		-54.206	$h^2/6EI$
Slope		Assume 2		-6.73		-35.11		-85.87		-159.01		-254.023		$h^2/6EI$
Y	0		2		-4.73		-39.84		-125.71		-284.72		-538.743	

Errors exist in deflection and moment for the right end and two corrections must be made as before.

Figure 31. Problem One - Step-By-Step Solution Beam 1-1 Cycle 1



Y	0		0								-40.8504		$\boxed{-71.4882}$	
Spring Load			0											
V		Assume 2		2		2		2		2		2		
M	0		2		4		6		8		10		$\boxed{12}$	h
M/EI	0		-2		-4		-6		-8		-10		-12	h/EI
E.C. M/EI	-2		-12		-24		-36		-48		-60		-34	$h^2/6EI$
Slope		Assume 0		-12		-36		-72		-120		-180		$h^2/6EI$
Y	0		0		-12		-48		-120		-240		-420	$h^3/6EI$

Simultaneous equations obtained from Corrections 'A' and 'B'.

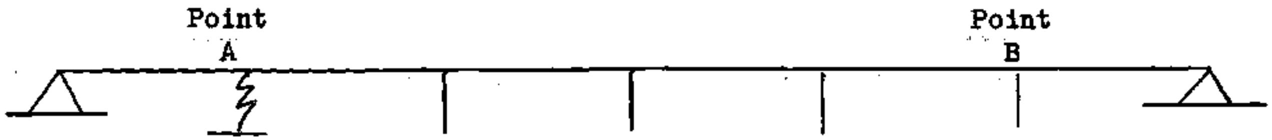
$$-71.4882A - 28.0208B = 91.6994$$

$$12A + 6.825B = -19.143$$

$A = -0.5898$

$B = -1.7678$

Figure 32. Problem One - Correction 'A' To Step-By-Step Solution Beam 1-1 Cycle 1



Y			0.17021								-14.0184		$\boxed{-28.0208}$	
Spring Load			1.365											
V		Assume 0		1.365		1.365		1.365		1.365		1.365		
M	0		0		1.365		2.73		4.095		5.46		$\boxed{6.825}$	h
M/EI	0		0		-1.365		-2.73		-4.095		-5.46		-6.825	h/EI
E.C. M/EI	0		-1.365		-8.19		-16.38		-24.57		-32.76		-19.11	$h^2/6EI$
Slope		Assume 1		-0.365		-8.555		-24.935		-49.505		-82.265		$h^2/6EI$
Y	0		1		0.635		-7.92		-32.855		-82.36		-164.625	

Original Y	0	0.34042	-48.46219	-91.6994
Corr 'A' Y	0	0	24.0935	42.1637
Corr 'B' Y	0	-0.30089	24.7817	49.5351
True Y Value	0	0.03953	0.4131	0

New spring constant equals 0.507 / 0.4131 or 1.227 kips per inch. Checks.

Figure 33. Problem One - Correction 'B' To Step-By-Step Solution Beam 1-1 Cycle 1

Loads	0		-0.488		0		0		0		-0.32		0	
V Trial		0		-0.488		-0.488		-0.488		-0.488		-0.808		
M Trial	0		0		-0.488		-0.976		-1.464		-1.952		-2.76	h
Corr M	0		0.46		0.92		1.38		1.84		2.3		2.76	h
M	0		0.46		0.432		0.404		0.376		0.348		0	h
M/EI	0		-0.46		-0.432		-0.404		-0.376		-0.348		0	h/EI
E.C. M/EI	-0.46		-2.272		-2.592		-2.424		-2.256		-1.768		-0.348	$h^2/6EI$
Slope		0		-2.272		-4.864		-7.288		-9.544		-11.312		$h^2/6EI$
Y	0		0		-2.272		-7.136		-14.424		-23.968		-35.28	
Corr Y	0		5.88		11.76		17.64		23.52		29.4		35.28	
Y	0		5.88								5.432		0	$h^3/6EI$

Y = 0.088 inches   Y = 0.0813 inches

New spring constant equals 0.488 / 0.088 or 5.55 kips per inch for point 2 on beam A-A.

New spring constant equals 0.32 / 0.0813 or 3.94 kips per inch for point 1 on beam A-A.

Figure 34. Problem One Beam A-A Cycle 1 Trial 1

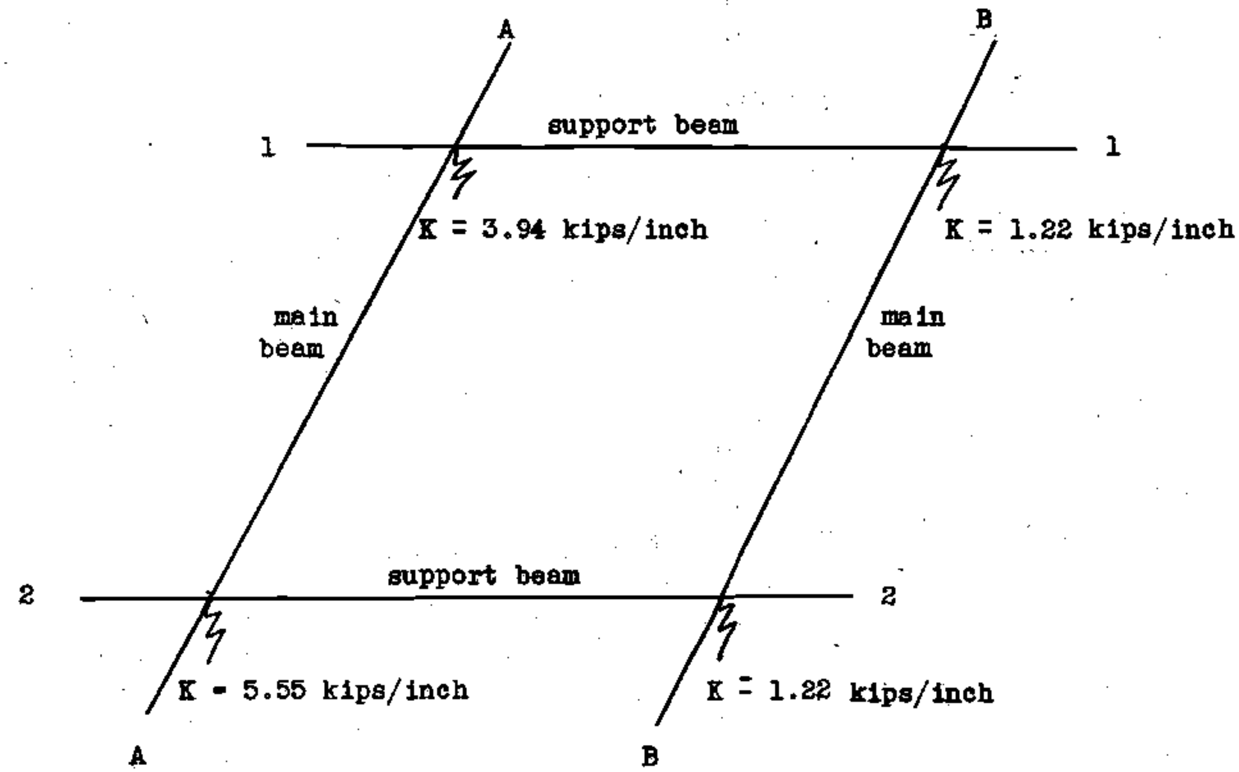
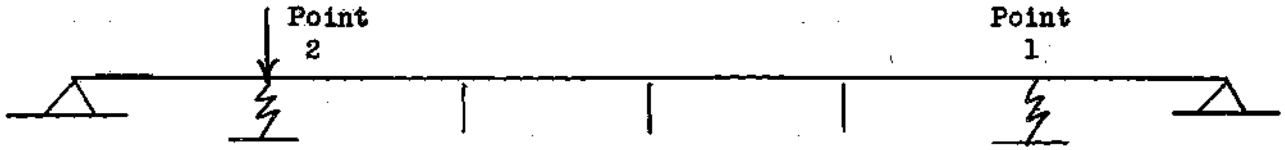


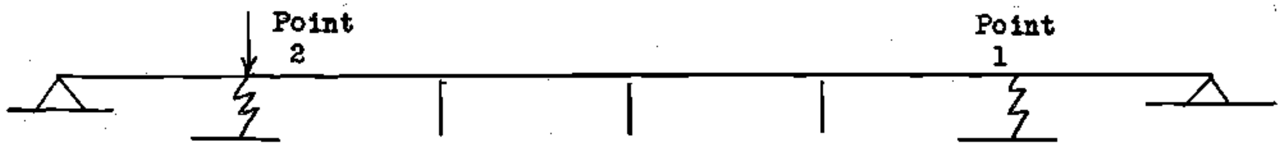
Figure 35. Problem One - Spring Constant Values End Of Cycle 1



Assumed Y		1.1				0.64		
Spring Load		1.34				0.78		
Loads	0	-10.0	0	0	0	0	0	
Total Loads	0	-8.66	0	0	0	0.78	0	
V Trial	0	-8.66	-8.66	-8.66	-8.66	-7.88		
M Trial	0	0	-8.66	-17.32	-25.98	-34.64	-42.52	h
Corr M	0	7.086	14.172	21.258	28.344	35.43	42.52	h
M	0	7.086	5.512	3.938	2.364	0.79	0	h
M/EI	0	-7.086	-5.512	-3.938	-2.364	-0.79	0	h/EI
E.C. M/EI	-7.086	-33.856	-33.072	-23.628	-14.184	-5.524	-0.79	$h^2/6EI$
Slope	66.928	33.072	0	-23.628	-37.812	-43.336		$h^2/6EI$
Y	0	66.928	100	100	76.372	38.56	-4.776	$h^3/6EI$
Corr Y	0	0.796	1.592	2.388	3.184	3.98	4.776	$h^3/6EI$
Y	0	67.724	101.592	102.388	79.556	42.54	0	$h^3/6EI$

Y = 1.01 inches Y = 0.63 inches

Figure 36. Problem One - Beam B-B Cycle 2 Trial 1

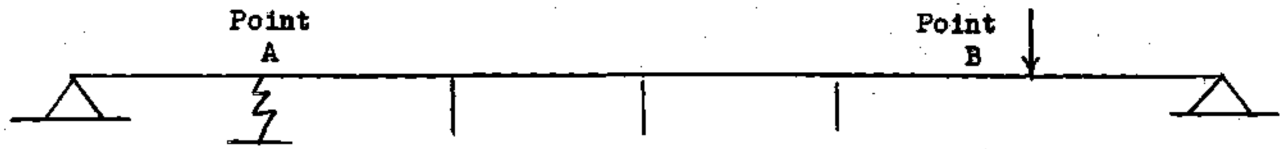


Assumed Y			1.01								0.63			
Spring Load			1.23								0.77			
Loads	0		-10		0		0		0		0		0	
Total Loads	0		-8.77		0		0		0		0.77		0	
V Trial		0		-8.77		-8.77		-8.77		-8.77		-8		
M Trial	0		0		-8.77		-17.54		-26.31		-35.08		-43.08	h
Corr M	0		7.18		14.36		21.54		28.72		35.9		43.08	h
M	0		7.18		5.59		4		2.41		0.82		0	h
M/EI	0		-7.18		-5.59		-4		-2.41		-0.82		0	h/EI
E.C. M/EI	-7.18		-34.31		-33.54		-24		-14.46		-5.69		-0.82	$h^2/6EI$
Slope		67.85		33.54		0		-24		-38.46		-44.15		$h^2/6EI$
Y	0		67.85		101.39		101.39		77.39		38.93		-5.22	$h^3/6EI$
Corr Y	0		0.87		1.74		2.61		3.48		4.35		5.22	$h^3/6EI$
Y	0		68.72								43.28		0	$h^3/6EI$

Y = 1.028 inches Y = 0.648 inches

Sufficient accuracy.

Figure 37. Problem One - Beam B-B Cycle 2 Trial 1

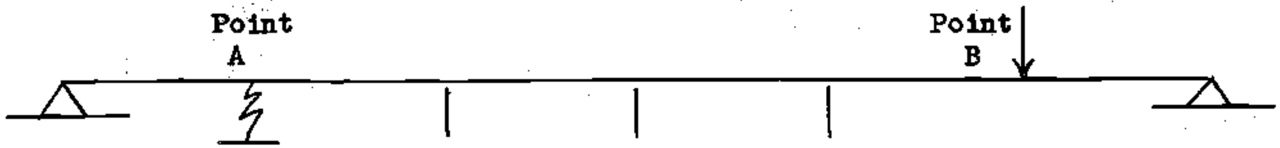


Assumed Y			0.12											
Total Loads	0		0.665		0		0		0		-1.23		0	
V Trial		0		0.665		0.665		0.665		0.665		-0.565		
M Trial	0		0		0.665		1.33		1.995		2.66		2.095	h
Corr M	0		-0.349		-0.698		-1.047		-1.396		-1.745		-2.095	h
M	0		-0.349		-0.033		0.283		0.599		0.915		0	h
M/EI	0		0.349		0.033		-0.283		-0.599		-0.915		0	h/EI
E.C. M/EI	0.349		1.426		0.198		-1.698		-3.594		-4.259		-0.915	$h^2/6EI$
Slope		-1.624		-0.198		0		-1.698		-5.292		-9.551		$h^2/6EI$
Y	0		-1.624		-1.822		-1.822		-3.52		-8.812		-18.363	$h^3/6EI$
Corr Y	0		3.06		6.12		9.18		12.24		15.3		18.363	$h^3/6EI$
Y	0		1.436										0	$h^3/6EI$

Y = 0.244 inches

Next trial assume Y of 0.13 inches.

Figure 38. Problem One - Beam 2-2 Cycle 2 Trial 1

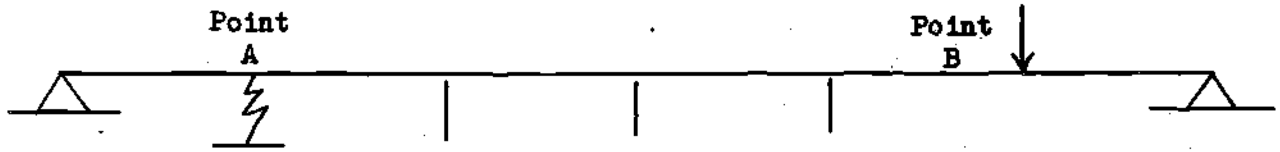


Assumed Y			0.13								K = 5.55 kips/inch			
Total Loads	0		0.721		0		0		0		-1.23		0	
V Trial		0		0.721		0.721		0.721		0.721		-0.509		
M Trial	0		0		0.721		1.442		2.163		2.884		2.375	h
Corr M	0		-0.396		-0.792		-1.188		-1.584		-1.98		-2.375	h
M	0		-0.396		-0.071		0.254		0.579		0.904		0	h
M/EI	0		0.396		0.071		-0.254		-0.579		-0.904		0	h/EI
E.C. M/EI	0.396		1.655		0.426		-1.524		-3.474		-4.195		-0.904	$h^2/6EI$
Slope		-2.081		-0.426		0		-1.524		-4.998		-9.913		$h^2/6EI$
Y	0		-2.081		-2.507		-2.507		-4.031		-9.029		-18.222	$h^3/6EI$
Corr Y	0		3.037										18.222	$h^3/6EI$
Y	0		0.956										0	$h^3/6EI$

Y = 0.163 inches

Next trial assume Y of 0.134 inches.

Figure 39. Problem One - Beam 2-2 Cycle 2 Trial 2



K ≅ 5.55 kips/inch

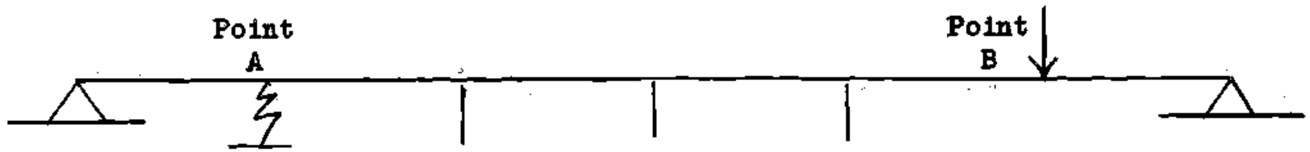
Assumed Y		0.134						
Total Loads	0	0.744	0	0	0	-1.23	0	
V Trial	0	0.744	0.744	0.744	0.744	-0.486		
M Trial	0	0	0.744	1.488	2.232	2.976	2.49	h
Corr M	0	-0.415	-0.83	-1.245	-1.66	-2.075	-2.49	h
M	0	-0.415	-0.086	0.243	0.572	0.901	0	h
M/EI	0	0.415	0.086	-0.243	-0.572	-0.901	0	h/EI
E.C. M/EI	0.415	1.746	0.516	-1.458	-3.432	-4.176	-0.901	$h^2/6EI$
Slope	-2.262	-0.516	0	-1.458	-4.89	-9.066		$h^2/6EI$
Y	0	-2.262	-2.778	-2.778	-4.236	-9.126	-18.192	$h^3/6EI$
Corr Y	0	3.032				15.16	18.192	$h^3/6EI$
Y	0	0.77				6.034	0	$h^3/6EI$

Y = 0.1312 inches   Y = 1.03 inches

Sufficient accuracy.

New support beam spring constant equals 1.23 / 1.03 or 1.194 kips/inch for point 2.

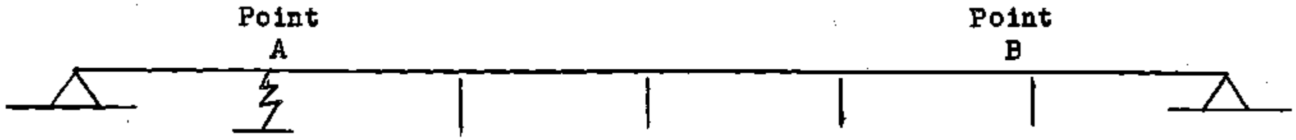
Figure 40. Problem One - Beam 2-2 Cycle 2 Trial 3



Load											-1.23			
Y	0		0.34042								-39.304		-73.6894	
Spring Load			1.8893											
Total Load	0		1.8893		0		0		0		-1.23		0	
V		Assume 1		2.8893		2.8893		2.8893		2.8893		1.6593		
M	0		1		3.8893		6.7786		9.6679		12.5572		14.2165	h
M/EI	0		-1		-3.8893		-6.7786		-9.6679		-12.5572		-14.2165	h
E.C. M/EI	-1		-7.8893		-23.3358		-40.6716		-58.0074		-74.1132		-40.9902	$h^2/6EI$
Slope		Assume 2		-5.8893		-29.2251		-69.8967		-127.904		-202.0173		$h^2/6EI$
Y	0		2		-3.8893		-33.1144		-103.0111		-230.9152		-432.9325	$h^3/6EI$

Two corrections required due to errors in deflection and moment at right end.

Figure 41. Problem One - Step-By-Step Solution Beam 2-2 Cycle 2



Y	0		0.1702								-9.4388		-19.0762	
Loads	0		0.9446		0		0		o		0		0	
V		Assume 0		0.9446		0.9446		0.9446		0.9446		0.9446		
M	0		0		0.9446		1.8892		2.8338		3.7784		4.723	h
M/EI	0		0		-0.9446		-1.8892		-2.8338		-3.7784		-4.723	h/EI
E.C. M/EI	0		-0.9446		-5.6676		-11.3352		-17.0028		-22.6704		-13.2244	$h^2/6EI$
Slope		Assume 1		0.0554		-5.6122		-16.947		-33.9502		-56.6206		$h^2/6EI$
Y	0		1		1.0554		-4.5568		-21.5042		-55.4544		-112.075	$h^3/6EI$

Simultaneous equations obtained from corrections 'A' and 'B' are:

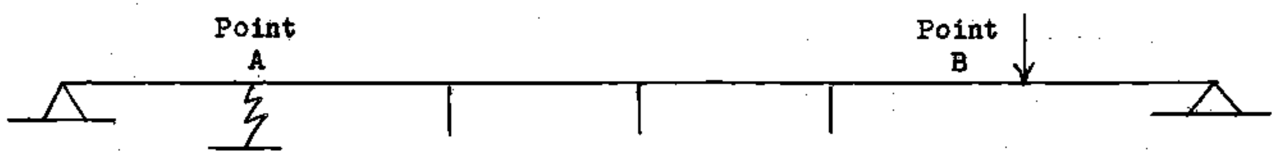
$$-35.744A - 19.0762B \approx 73.6894$$

$$6A \quad 4.723B \approx -14.2165$$

Correction 'A' same as Fig. 26. $A = -1.4134$ $B = -1.2146$

Original Y	0	0.34042	-39.304	-73.6894
Corr 'A' Y	0	0	28.8686	50.5205
Corr 'B' Y	0	-0.2067	11.4643	23.1699
True Y Values		0.13372	1.0289	0

Figure 42. Problem One - Correction 'A' and 'B' To Step-By-Step Solution Beam 2-2 Cycle 2



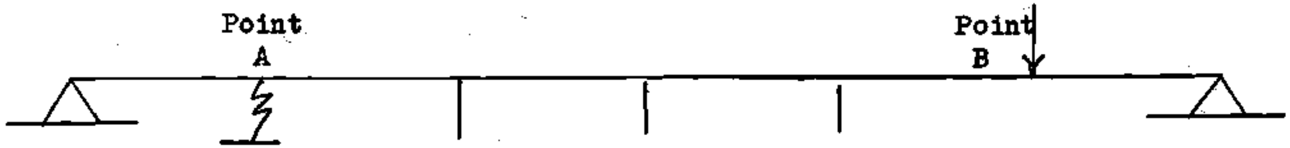
K = 3.94 kips/inch

Assumed Y			0.1											
Total Loads	0		0.394		0		0		0		-0.77		0	
V Trial		0		0.394		0.394		0.394		0.394		-0.376		
M Trial	0		0		0.394		0.788		1.182		1.576		1.2	h
Corr M	0		-0.2		-0.4		-0.6		-0.8		-1		-1.2	h
M	0		-0.2		-0.006		0.188		0.382		0.576		0	h
M/EI	0		0.2		0.006		-0.188		-0.382		-0.576		0	h/EI
E.C. M/EI	0.2		0.806		0.036		-1.128		-2.292		-2.686		-0.576	$h^2/6EI$
Slope		-0.842		-0.036		0		-1.128		-3.42		-6.106		$h^2/6EI$
Y	0		-0.842		-0.878		-0.878		-2.006		-5.426		-11.532	$h^3/6EI$
Corr Y	0		1.922								9.61		11.532	$h^3/6EI$
Y	0		1.08										0	$h^3/6EI$

Y = 0.184 inches

Next trial assume Y of 0.12 inches.

Figure 43. Problem One - Beam 1-1 Cycle 2 Trial 1



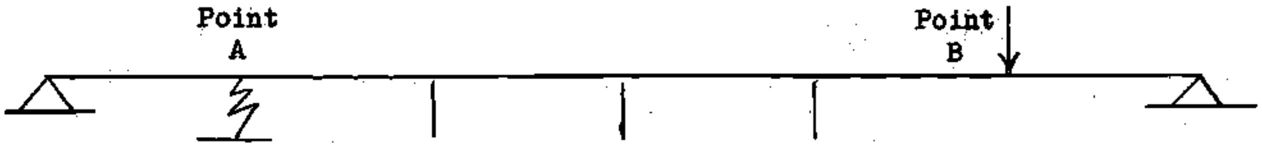
K = 3.94 kips/inch

Assumed Y			0.12											
Total Loads	0		0.472		0		0		0		-0.77		0	
V Trial		0		0.472		0.472		0.472		0.472		-0.298		
M Trial	0		0		0.472		0.944		1.416		1.888		1.59	h
Corr M	0		-0.265		-0.53		-0.795		-1.06		-1.325		-1.59	h
M	0		-0.265		-0.058		0.149		0.356		0.563		0	h
M/EI	0		0.265		0.058		-0.149		-0.356		-0.563		0	h/EI
E.C. M/EI	0.265		1.118		0.348		-0.894		-2.136		-2.608		-0.563	$h^2/6EI$
Slope		-1.466		-0.348		0		-0.894		-3.03		-5.638		$h^2/6EI$
Y	0		-1.466		-1.814		-1.814		-2.708		-5.738		-11.376	$h^3/6EI$
Corr Y	0		1.896										11.376	$h^3/6EI$
Y	0		0.43										0	$h^3/6EI$

Y = 0.0732 inches

Next trial assume Y of 0.11 inches.

Figure 44. Problem One - Beam 1-1 Cycle 2 Trial 2



Assumed Y			0.11											
Total Loads	0		0.433		0		0		0		-0.77		0	
V Trial		0		0.433		0.433		0.433		0.433		-0.337		
M Trial	0		0		0.433		0.866		1.299		1.732		1.395	h
Corr M	0		-0.232		-0.464		-0.696		-0.928		-1.16		-1.395	h
M	0		-0.232		-0.031		0.17		0.371		0.572		0	h
M/EI	0		0.232		0.031		-0.17		-0.371		-0.572		0	h/EI
E.C. M/EI	0.232		0.959		0.186		-1.02		-2.226		-2.659		-0.572	$h^2/6EI$
Slope		-1.145		-0.186		0		-1.02		-3.246		-5.905		$h^2/6EI$
Y	0		-1.145		-1.331		-1.331		-2.351		-5.597		-11.502	$h^3/6EI$
Corr Y	0		1.917		3.834		5.751		7.668		9.585		11.502	$h^3/6EI$
Y	0		0.772								3.988		0	$h^3/6EI$

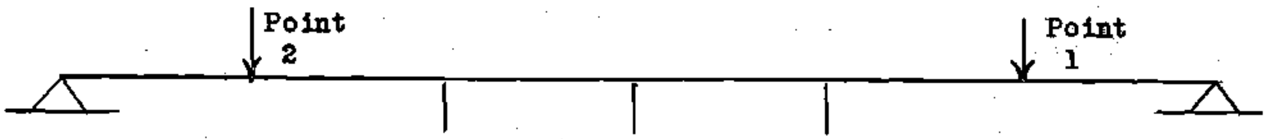
Y = 0.131 inches

Y = 0.68 inches

Sufficient accuracy.

New support beam spring constant for point one equals 0.77 / 0.68 or 1.13 kips/inch.

Figure 45. Problem One - Beam 1-1 Cycle 2 Trial 3



Loads	0	-0.744	0	0	0	-0.433	0	
V Trial	0	-0.744	-0.744	-0.744	-0.744	-1.177		
M Trial	0	0	-0.744	-1.488	-2.232	-2.976	-4.153	h
Corr M	0	0.692	1.384	2.076	2.768	3.46	4.153	h
M	0	0.692	0.64	0.588	0.536	0.484	0	h
M/EI	0	-0.692	-0.64	-0.588	-0.536	-0.484	0	h/EI
E.C. M/EI	-0.692	-3.408	-3.84	-3.528	-3.216	-2.472	-0.484	$h^2/6EI$
Slope	7.248	3.84	0	-3.528	-6.744	-9.216		$h^2/6EI$
Y	0	7.248	11.088	11.088	7.56	0.816	-8.4	$h^3/6EI$
Corr Y	0	1.4	2.8	4.2	5.6	7	8.4	$h^3/6EI$
Y	0	8.648				7.816	0	$h^3/6EI$

Y = 0.129 inches Y = 0.117 inches

New spring constant point two equals 0.744 / 0.129 or 5.76 kips/inch.

New spring constant point one equals 0.433 / 0.117 or 3.7 kips/inch.

Figure 46. Problem One - Beam A-A Cycle 2 Trial 1

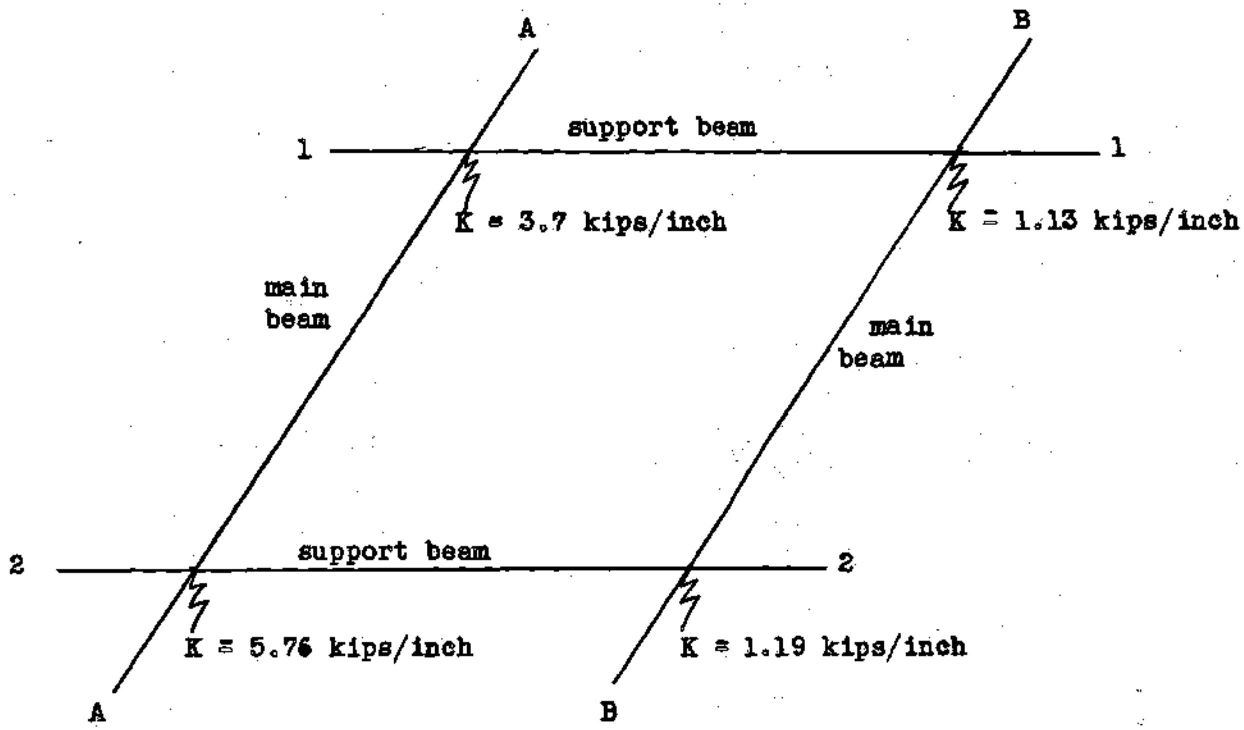
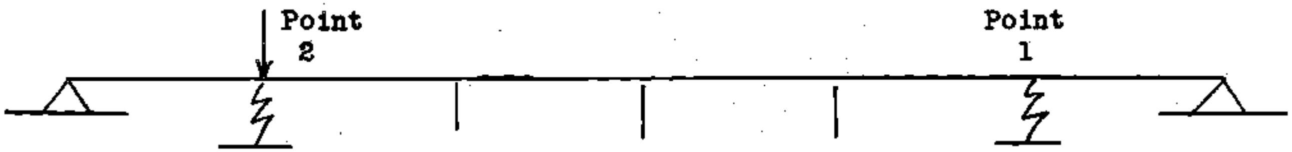


Figure 47. Problem One - Spring Constant Values At End Of Cycle 2

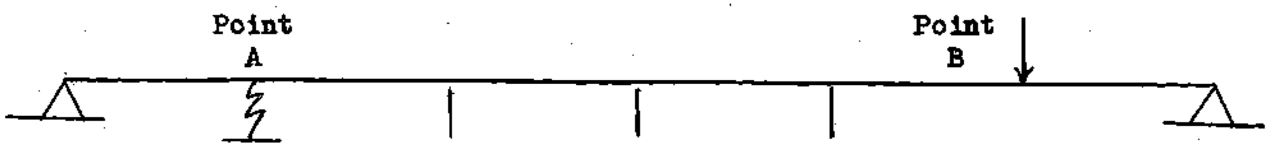


Assumed Y			1.03								0.65			
Loads			-10											
Spring Load			1.225								0.735			
Total Loads	0		-8.775		0		0		0		0.735		0	
V Trial		0		-8.775		-8.775		-8.775		-8.775		-8.04		
M Trial	0		0		-8.775		-17.55		-26.325		-35.1		-43.14	h
Corr M	0		7.19		14.38		21.57		28.76		35.95		43.14	h
M	0		7.19		5.605		4.02		2.435		0.85		0	h
M/EI	0		-7.19		-5.605		-4.02		-2.435		-0.85		0	h/EI
E.C. M/EI	-7.19		-34.365		-33.63		-24.12		-14.61		-5.835		-0.85	$h^2/6EI$
Slope		67.995		33.63		0		-24.12		-38.73		-44.565		$h^2/6EI$
Y	0		67.995		101.625		101.625		77.505		38.775		-5.79	$h^3/6EI$
Corr Y	0		0.965		1.93		2.895		3.86		4.825		5.79	$h^3/6EI$
Y	0		68.96								43.6		0	$h^3/6EI$

Y = 1.03 inches   Y = 0.653 inches

Sufficient accuracy.

Figure 48. Problem One - Beam B-B Cycle 3 Trial 1

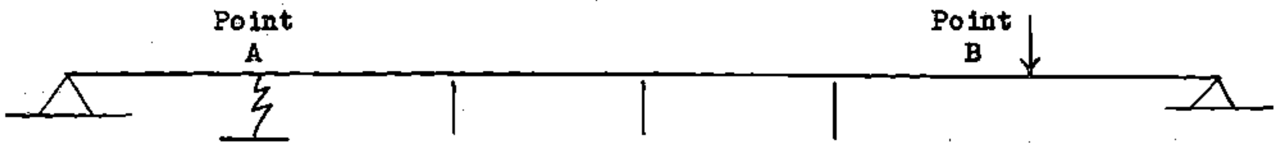


Assumed Y			0.129											
Total Loads	0		0.743		0		0		0		-1.225		0	
V Trial		0		0.743		0.743		0.743		0.743		-0.482		
M Trial	0		0		0.743		1.486		2.229		2.972		2.49	h
Corr M	0		-0.415		-0.83		-1.245		-1.66		-2.075		-2.49	h
M	0		-0.415		-0.087		0.241		0.569		0.897		0	h
M/EI	0		0.415		0.087		-0.241		-0.569		-0.897		0	h/EI
E.C. M/EI	0.415		1.747		0.522		-1.446		-3.414		-4.157		-0.897	$h^2/6EI$
Slope		-2.269		-0.522		0		-1.446		-4.86		-9.017		$h^2/6EI$
Y	0		-2.269		-2.791		-2.791		-4.237		-9.097		-18.114	$h^3/6EI$
Corr Y	0		3.019		6.038		9.057		12.076		15.095		18.114	$h^3/6EI$
Y	0		0.75								5.998		0	$h^3/6EI$

Y = 0.128 inches Y = 1.02 inches

New spring constant for point two support beam equals 1.225 / 1.02 or 1.2 kips/inch.

Figure 49. Problem One - Beam 2-2 Cycle 3 Trial 1



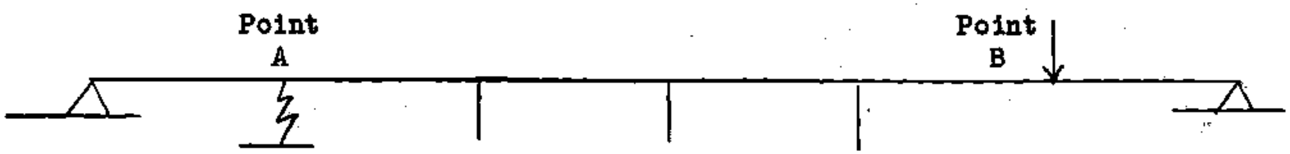
K = 3.7 kips/inch

Assumed Y			0.118											
Total Loads	0		0.437		0		0		0		-0.735		0	
V Trial		0		0.437		0.437		0.437		0.437		-0.298		
M Trial	0		0		0.437		0.874		1.311		1.748		1.45	h
Corr M	0		-0.24		-0.48		-0.72		-0.96		-1.2		-1.45	h
M	0		-0.24		-0.043		0.154		0.351		0.548		0	h
M/EI	0		0.24		0.043		-0.154		-0.351		-0.548		0	h/EI
E.C. M/EI	0.24		1.003		0.258		-0.924		-2.106		-2.543		-0.548	$h^2/6EI$
Slope		-1.261		-0.258		0		-0.924		-3.03		-5.573		$h^2/6EI$
Y	0		-1.261		-1.519		-1.519		-2.443		-5.473		-11.046	$h^3/6EI$
Corr Y	0		1.841		3.682		5.523		7.364		9.205		11.046	$h^3/6EI$
Y	0		0.58										0	$h^3/6EI$

Y = 0.099 inches

Next trial assume Y of 0.11 inches.

Figure 50. Problem One - Beam 1-1 Cycle 3 Trial 1



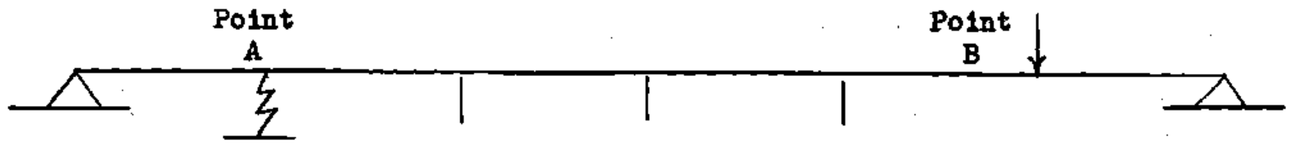
K = 3.7 kips/inch

Assumed Y			0.11											
Total Loads	0		0.407		0		0		0		-0.735		0	
V Trial		0		0.407		0.407		0.407		0.407		-0.328		
M Trial	0		0		0.407		0.814		1.221		1.628		1.3	h
Corr M	0		-0.216		-0.432		-0.648		-0.864		-1.08		-1.3	h
M	0		-0.216		-0.025		0.166		0.357		0.548		0	h
M/EI	0		0.216		0.025		-0.166		-0.357		-0.548		0	h/EI
E.C. M/EI	0.216		0.889		0.15		-0.996		-2.142		-2.549		-0.548	$h^2/6EI$
Slope		-1.039		-0.15		0		-0.996		-3.138		-5.687		$h^2/6EI$
Y	0		-1.039		-1.189		-1.189		-2.185		-5.323		-11.01	$h^3/6EI$
Corr Y	0		1.835		3.67		5.505		7.34		9.175		11.01	$h^3/6EI$
Y	0		0.796										0	$h^3/6EI$

Y = 0.1357 inches

Next trial assume Y of 0.112 inches.

Figure 51. Problem One - Beam 1-1 Cycle 3 Trial 2

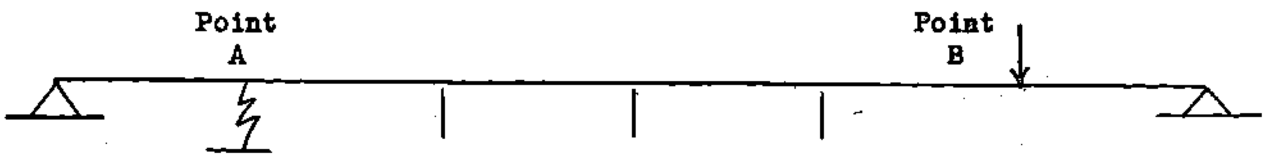


Assumed Y			0.112											
Total loads	0		0.415		0		0		0		-0.735		0	
V Trial		0		0.415		0.415		0.415		0.415		-0.32		
M Trial	0		0		0.415		0.83		1.245		1.66		1.34	h
Corr M	0		-0.223		-0.446		-0.669		-0.892		-1.115		-1.34	h
M	0		-0.223		-0.031		0.161		0.353		0.545		0	h
M/EI	0		0.223		0.031		-0.161		-0.353		-0.545		0	h/EI
E.C. M/EI	0.223		0.923		0.186		-0.996		-2.12		-2.533		-0.545	$h^2/6EI$
Slope		-1.109		-0.186		0		-0.996		-3.086		-5.619		$h^2/6EI$
Y	0		-1.109		-1.295		-1.295		-2.261		-5.347		-10.966	$h^3/6EI$
Corr Y	0		1.828		3.656		5.484		7.312		9.14		10.966	$h^3/6EI$
Y	0		0.719										0	$h^3/6EI$

Y = 0.1224 inches

Next trial assume Y of 0.113 inches.

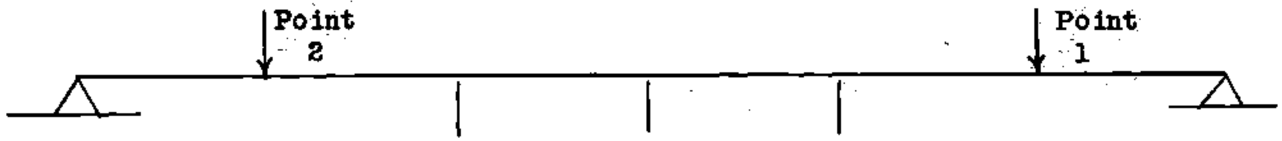
Figure 52. Problem One Beam 1-1 Cycle 3 Trial 3



Assumed Y			0.113									K = 3.7 kips/inch		
Total Loads	0		0.418		0		0		0		-0.735		0	
V Trial		0		0.418		0.418		0.418		0.418		-0.317		
M Trial	0		0		0.418		0.836		1.254		1.672		1.355	h
Corr M	0		-0.226		-0.452		-0.678		-0.904		-1.13		-1.355	h
M	0		-0.226		-0.034		0.158		0.35		0.542		0	h
M/EI	0		0.226		0.034		-0.158		-0.35		-0.542		0	h/EI
E.C. M/EI	0.226		0.938		0.204		-0.948		-2.1		-2.518		-0.542	$h^2/6EI$
Slope		-1.142		-0.204		0		-0.948		-3.048		-5.566		$h^2/6EI$
Y	0		-1.142		-1.346		-1.346		-2.294		-5.342		-10.908	
Corr Y	0		1.818		3.636		5.454		7.272		9.09		10.908	
Y	0		0.676								3.748		0	$h^3/6EI$
			Y = 0.115 inches								Y = 0.64 inches			

New spring constant for support beam point one equals 0.735 / 0.64 or 1.149 kips/inch.

Figure 53. Problem One - Beam 1-1 Cycle 3 Trial 4



Total Loads			-0.743								-0.418			
V Trial		0		-0.743		-0.743		-0.743		-0.743		-1.161		
M Trial	0		0		-0.743		-1.486		-2.229		-2.972		-4.133	h
Corr M	0		0.689		1.378		2.067		2.756		3.445		4.133	h
M	0		0.689		0.635		0.581		0.527		0.473		0	h
M/EI	0		-0.689		-0.635		-0.581		-0.527		-0.473		0	h/EI
E.C. M/EI	-0.689		-3.391		-3.81		-3.486		-3.162		-2.419		-0.473	
Slope		7.201		3.81		0		-3.486		-6.648		-9.067		$h^2/6EI$
Y	0		7.201		11.011		11.011		7.525		0.877		-8.19	
Corr Y	0		1.365		2.73		4.095		5.46		6.835		8.19	
Y	0		8.566		13.741		15.106		12.985		7.712		0	
			Y = 0.128 inches								Y = 0.1153 inches			

New spring constant main beam point one equals 0.418 / 0.1153 or 3.53 kips/inch.

New spring constant main beam point two equals 0.743 / 0.128 or 5.8 kips/inch.

Figure 54. Problem One - Beam A-A Cycle 3 Trial 1

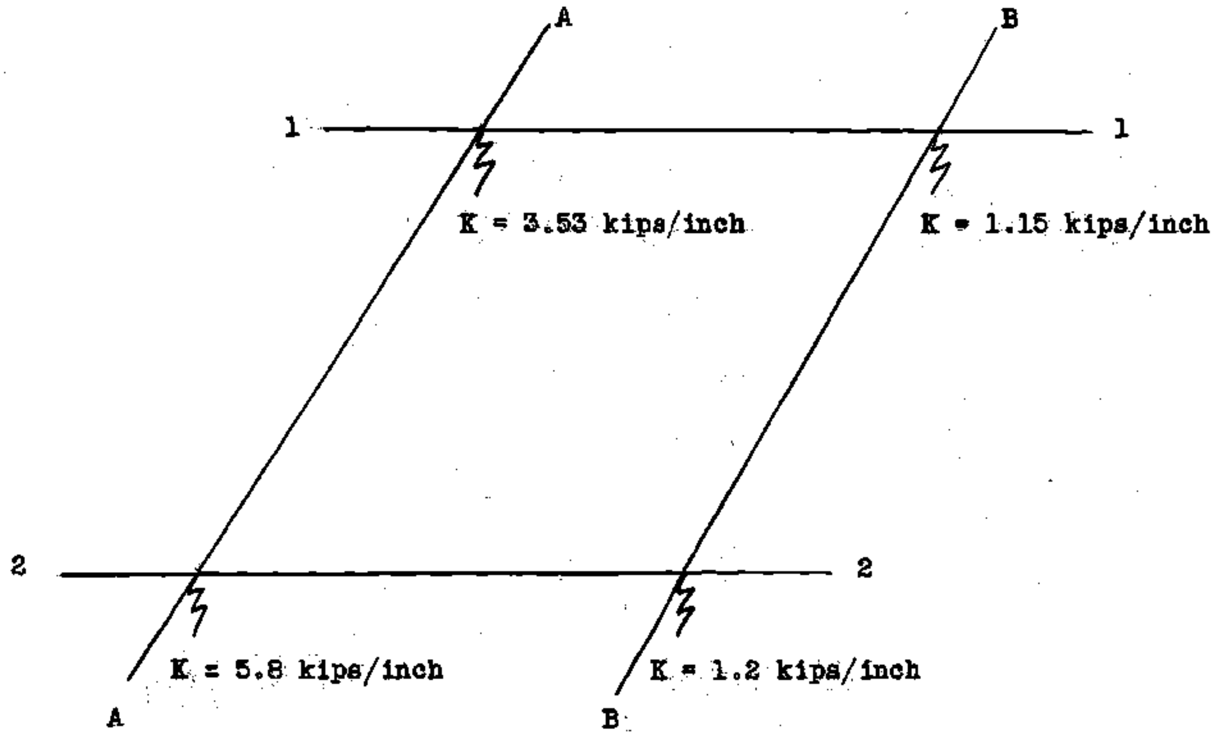
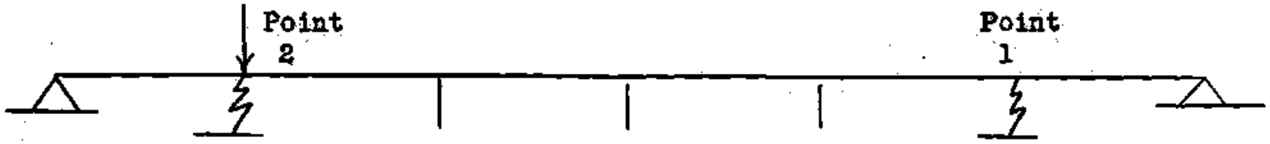


Figure 55. Problem One - Spring Constant Values End Of Cycle 3



K = 1.2 kips/inch

K = 1.15 kips/inch

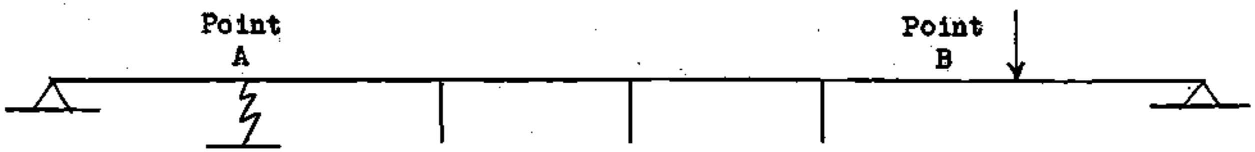
Assumed Y			1.028								0.649			
Spring Loads			1.23								0.746			
Total Loads	0		-8.77		0		0		0		0.746		0	
V Trial		0		-8.77		-8.77		-8.77		-8.77		-8.024		
M Trial	0		0		-8.77		-17.54		-26.31		-35.08		-43.104	h
Corr M	0		7.184		14.368		21.552		28.736		35.92		43.104	h
M	0		7.184		5.598		4.012		2.426		0.84		0	h
M/EI	0		-7.184		-5.598		-4.012		-2.426		-0.84		0	h/EI
E.C. M/EI	-7.184		-34.334		-33.588		-24.072		-14.556		-5.786		-0.84	
Slope		67.922		33.588		0		-24.072		-38.628		-44.414		$h^2/6EI$
Y	0		67.922		101.51		101.51		77.438		38.81		-5.604	
Corr Y	0		0.934		1.868		2.802		3.736		4.67		5.604	
Y	0		68.856								43.48		0	

Y = 1.03 inches

Y = 0.649 inches

Sufficient accuracy.

Figure 56. Problem One - Beam B-B Cycle 4 Trial 1

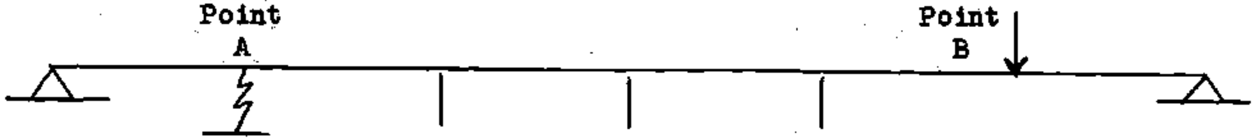


Assumed Y			0.129								K = 5.8 kips/inch			
Total Loads	0		0.748		0		0		0		-1.23		0	
V Trial		0		0.748		0.748		0.748		0.748		-0.482		
M Trial	0		0		0.748		1.496		2.244		2.992		2.51	h
Corr M	0		-0.42		-0.84		-1.26		-1.68		-2.1		-2.51	h
M	0		-0.42		-0.092		0.236		0.564		0.892		0	h
M/EI	0		0.42		0.092		-0.236		-0.564		-0.892		0	h/EI
E.C. M/EI	0.42		1.772		0.552		-1.416		-3.384		-4.132		-0.892	h^2/6EI
Slope		-2.324		-0.552		0		-1.416		-4.8		-8.932		h^2/6EI
Y	0		-2.324		-2.876		-2.876		-4.292		-9.092		-18.024	
Corr Y	0		3.004		6.008		9.012		12.016		15.02		18.024	
Y	0		0.68										0	h^3/6EI

Y = 0.116 inches

Next trial assume Y of 0.128 inches.

Figure 57. Problem One - Beam 2-2 Cycle 4 Trial 1



Assumed Y			0.128											
Total Loads	0		0.742		0		0		0		-1.23		0	
V Trial		0		0.742		0.742		0.742		0.742		-0.488		
M Trial	0		0		0.742		1.484		2.226		2.968		2.48	h
Corr M	0		-0.413		-0.826		-1.239		-1.652		-2.065		-2.48	h
M	0		-0.413		-0.084		0.245		0.574		0.903		0	h
M/EI	0		0.413		0.084		-0.245		-0.574		-0.903		0	h/EI
E.C. M/EI	0.413		1.736		0.504		-1.47		-3.444		-4.186		-0.903	$h^2/6EI$
Slope		-2.24		-0.504		0		-1.47		-4.914		-9.1		$h^2/6EI$
Y	0		-2.24		-2.744		-2.744		-4.214		-9.128		-18.228	$h^3/6EI$
Corr Y	0		3.038		6.076		9.114		12.152		15.19		18.228	$h^3/6EI$
Y	0		0.798								6.062		0	$h^3/6EI$

Y = 0.136 inches

Next trial assume Y of 0.1284 inches.

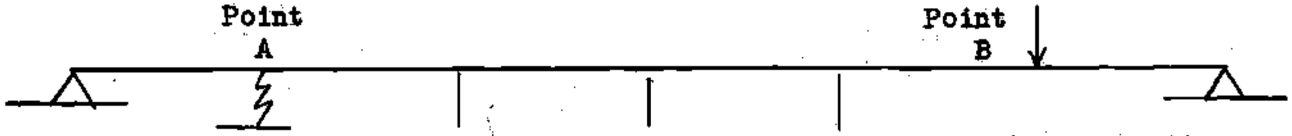
Figure 58. Problem One - Beam 2-2 Cycle 4 Trial 2

Assumed Y			0.1284											
Total Loads	0		0.745		0		0		0		-1.23		0	
V Trial		0		0.745		0.745		0.745		0.745		-0.485		
M Trial	0		0		0.745		1.49		2.235		2.98		2.495	h
Corr M	0		-0.416		-0.832		-1.248		-1.664		-2.08		-2.495	h
M	0		-0.416		-0.087		0.242		0.571		0.9		0	h
M/EI	0		0.416		0.087		-0.242		-0.571		-0.9		0	h/EI
E.C. M/EI	0.416		1.751		0.522		-1.452		-3.426		-4.171		-0.9	$h^2/6EI$
Slope		-2.273		-0.522		0		-1.452		-4.878		-9.049		$h^2/6EI$
Y	0		-2.273		-2.795		-2.795		-4.247		-9.125		-18.174	
Corr Y	0		3.029		6.058		9.087		12.116		15.145		18.174	
Y	0		0.756								6.02		0	

Y = 0.129 inches Y = 1.024 inches

New spring constant for point 2 on beam B-B equals 1.23 / 1.024 or 1.2 kips/inch.

Figure 59. Problem One - Beam 2-2 Cycle 4 Trial 3



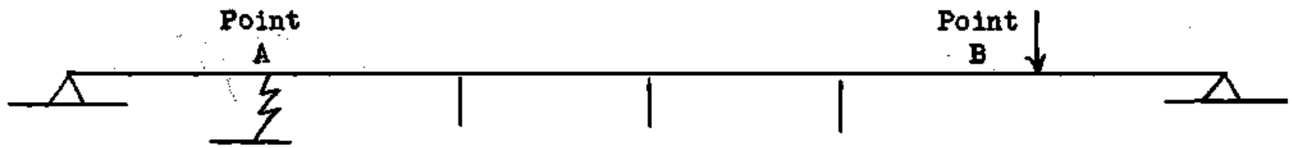
K = 3.53 kips/inch

Assumed Y			0.115											
Total Loads	0		0.406		0		0		0		-0.746		0	
V Trial		0		0.406		0.406		0.406		0.406		-0.34		
M Trial	0		0		0.406		0.812		1.218		1.624		1.284	h
Corr M	0		-0.214		-0.428		-0.642		-0.856		-1.07		-1.284	h
M	0		-0.214		-0.022		0.17		0.362		0.554		0	h
M/EI	0		0.214		0.022		-0.17		-0.362		-0.554		0	h/EI
E.C. M/EI	0.214		0.708		0.132		-1.02		-2.172		-2.578		-0.554	
Slope		-0.84		-0.132		0		-1.02		-3.192		-5.77		$h^2/6EI$
Y	0		-0.84		-0.972		-0.972		-1.992		-5.184		-10.954	
Corr Y	0		1.826		3.652		5.478		7.304		9.13		10.954	
Y	0		0.986										0	$h^3/6EI$

Y = 0.168 inches

Next trial assume Y of 0.12 inches.

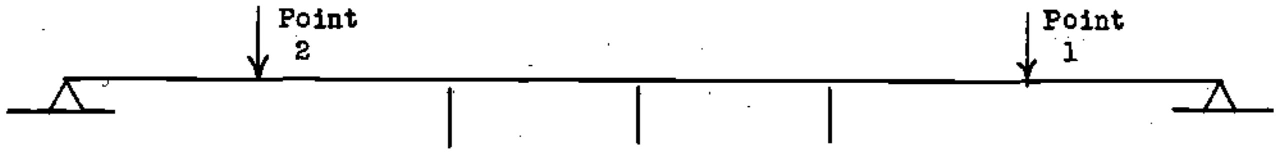
Figure 60. Problem One - Beam 1-1 Cycle 4 Trial 1



Assumed Y			0.12											
Total Loads	0		0.424		0		0		0		-0.746		0	
V Trial		0		0.424		0.424		0.424		0.424		-0.322		
M Trial	0		0		0.424		0.848		1.272		1.696		1.374	h
Corr M	0		-0.229		-0.458		-0.687		-0.916		-1.145		-1.374	h
M	0		-0.229		-0.034		0.161		0.356		0.551		0	h
M/EI	0		0.229		0.034		-0.161		-0.356		-0.551		0	h/EI
E.C. M/EI	0.229		0.95		0.204		-0.986		-2.136		-2.56		-0.551	$h^2/6EI$
Slope		-1.154		-0.204		0		-0.986		-3.122		-5.682		$h^2/6EI$
Y	0		-1.154		-1.358		-1.358		-2.344		-5.466		-11.148	
Corr Y	0		1.858		3.716		5.574		7.432		9.29		11.148	
Y	0		0.704								3.824		0	$h^3/6EI$
			Y = 0.12 inches								Y = 0.652 inches			

New point one support beam spring constant equals 0.746 / 0.652 or 1.144 kips/inch.

Figure 61. Problem One - Beam 1-1 Cycle 4 Trial 2



Total Loads			-0.745								-0.424			
V Trial		0		-0.745		-0.745		-0.745		-0.745		-1.169		
M Trial	0		0		-0.745		-1.49		-2.235		-2.98		-4.149	h
Corr M	0		0.6915		1.383		2.0745		2.766		3.4575		4.149	h
M	0		0.6915		0.638		0.5845		0.531		0.4775		0	h
M/EI	0		-0.6915		-0.638		-0.5845		-0.531		-0.4775		0	h/EI
E.C. M/EI	-0.6915		-3.404		-3.828		-3.507		-3.186		-2.441		-0.4775	$h^2/6EI$
Slope		7.232		3.828		0		-3.507		-6.693		-9.134		$h^2/6EI$
Y	0		7.232		11.06		11.06		7.553		0.86		-8.274	$h^3/6EI$
Corr Y	0		1.379		2.758		4.137		5.516		6.895		8.274	$h^3/6EI$
Y	0		8.611								7.755		0	$h^3/6EI$

Y = 0.1289 inches   Y = 0.116 inches

New spring constant for point two on beam A-A equals 0.745 / 0.1289 or 5.78 kips/inch.

New spring constant for point one on beam A-A equals 0.424 / 0.116 or 3.66 kips/inch/

Figure 62. Problem One - Beam A-A Cycle 4 Trial 1

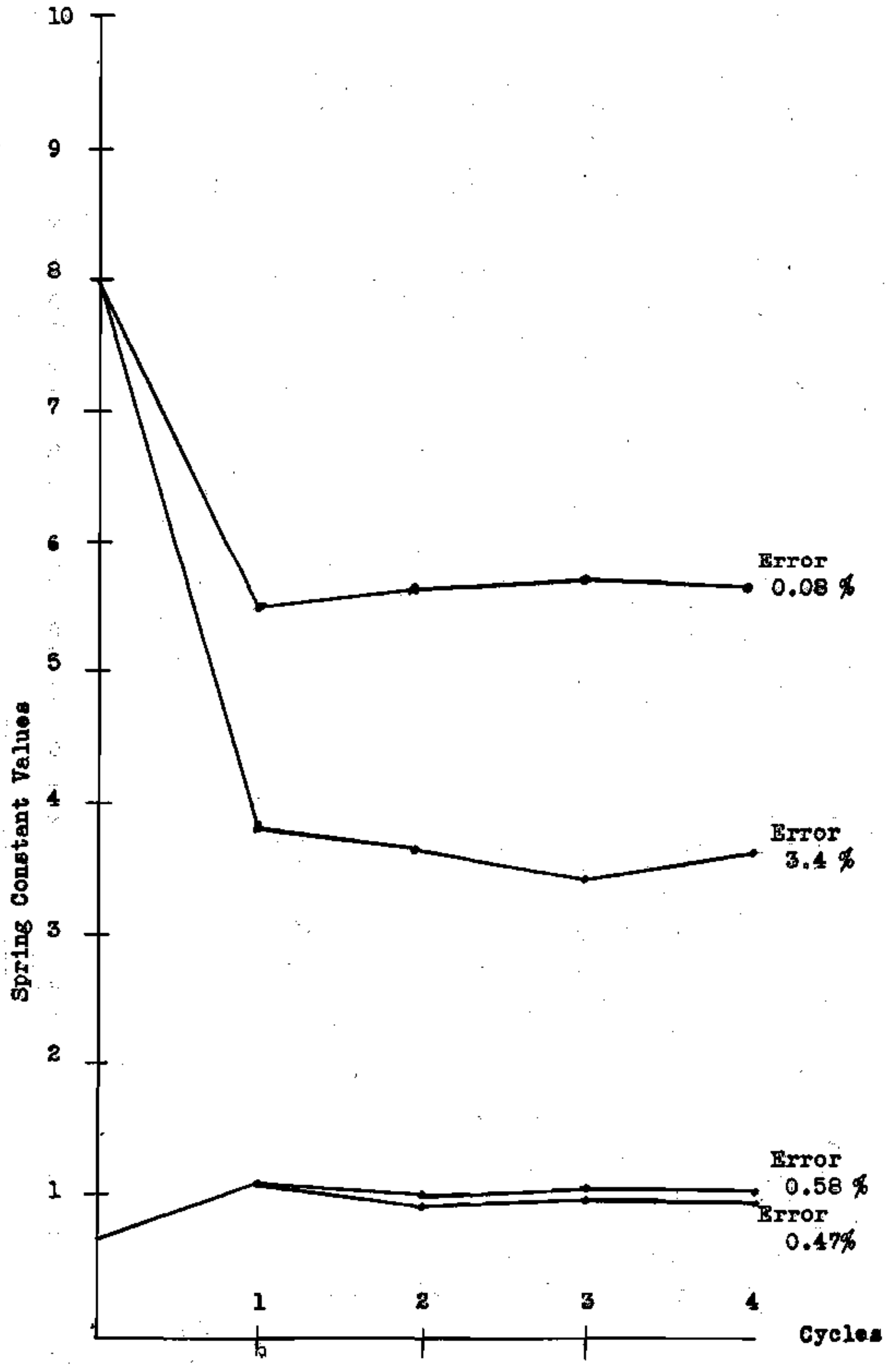
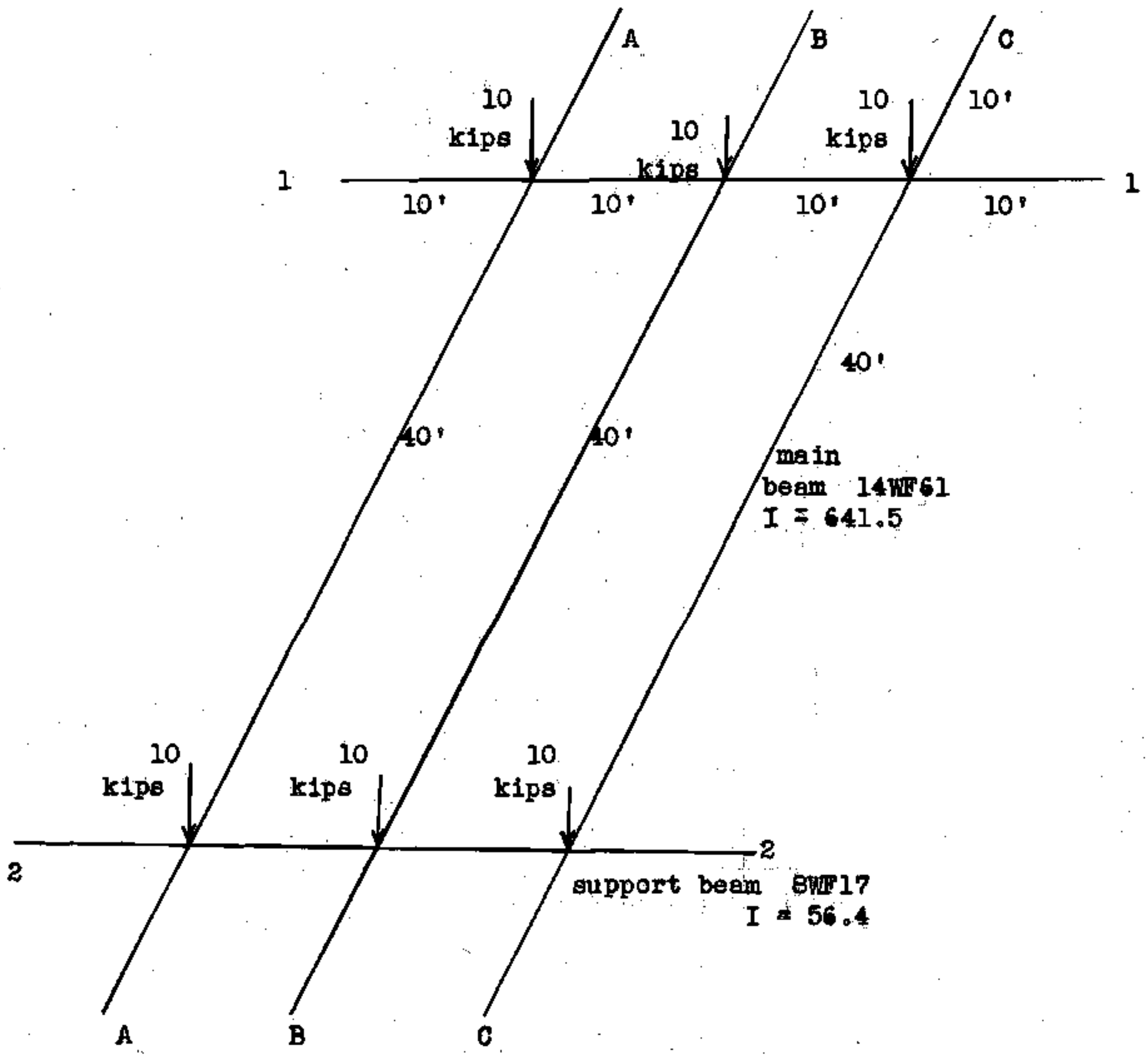


Figure 63. Problem One - Graph Of Spring Constants Versus Cycles



All beams pinned connected to fixed supports.

Figure 64. Problem Two - Grillage System

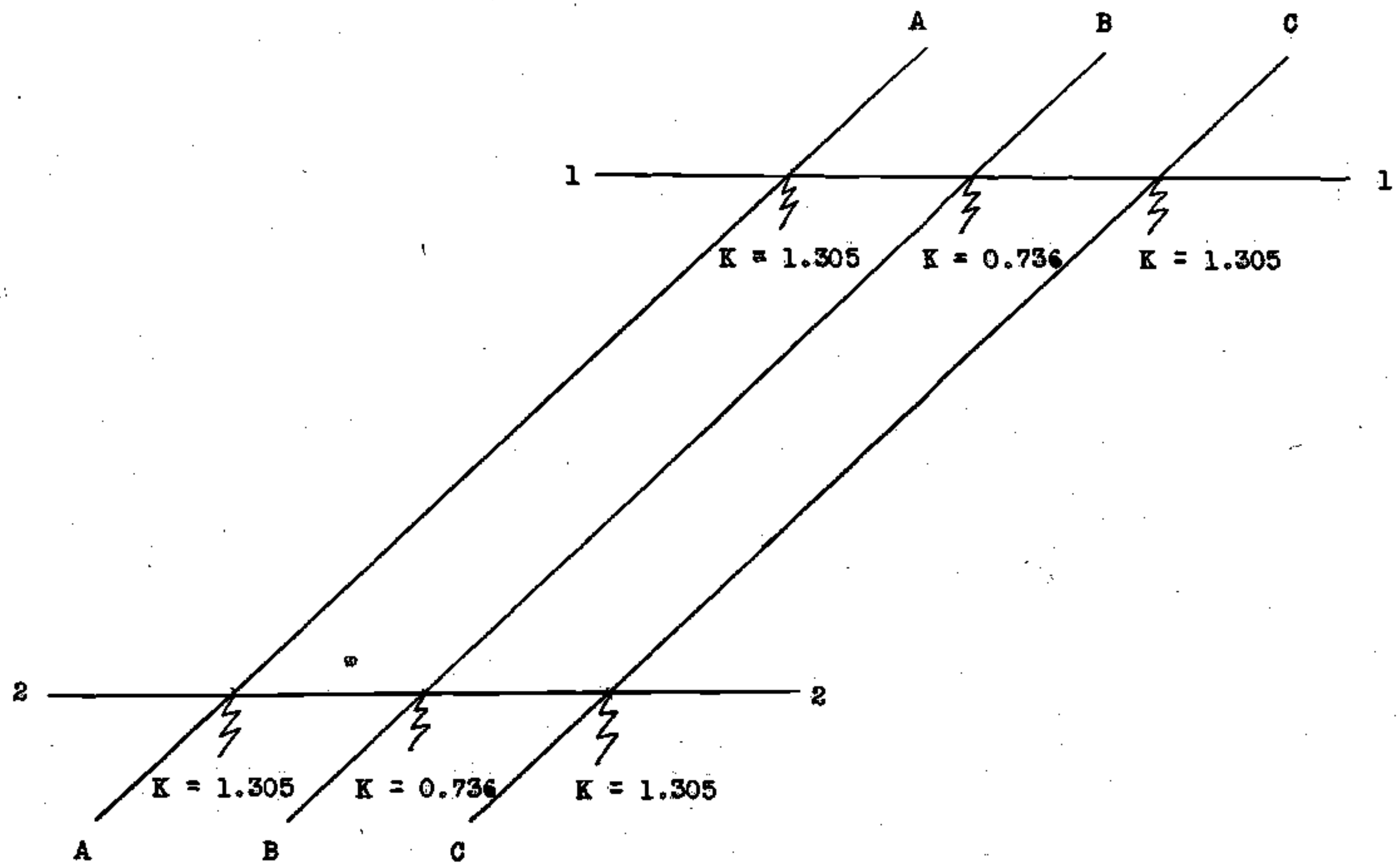
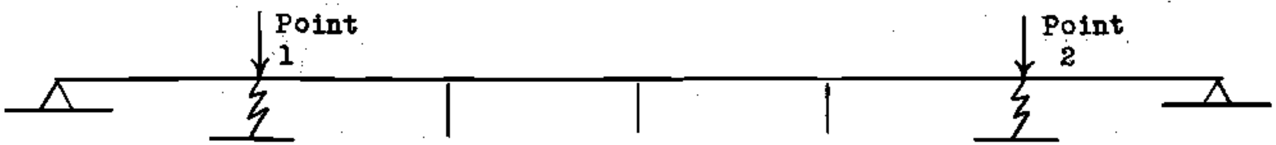


Figure 65. Problem Two - Initial Spring Constants From Moment-Area Calculation

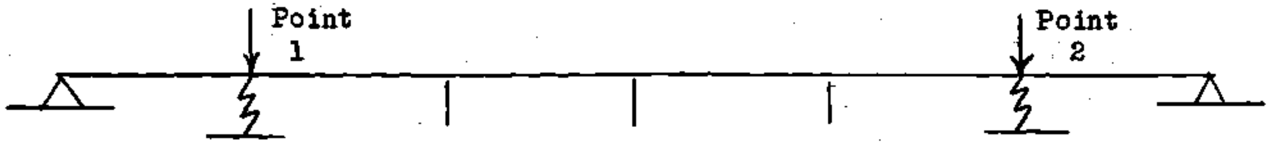


Assumed Y			1.30								1.30			
Loads	0		-10		0		0		0		-10		0	
Spring Loads			1.7								1.7			
Total Loads	0		-8.3		0		0		0		-8.3		0	
V Trial		0		-8.3		-8.3		-8.3		-8.3		-16.6		
M Trial	0		0		-8.3		-16.6		-24.9		-33.2		-49.8	h
Corr M	0		8.3		16.6		24.9		33.2		41.5		49.8	h
M	0		8.3		8.3		8.3		8.3		8.3		0	h
M/EI	0		-8.3		-8.3		-8.3		-8.3		-8.3		0	h/EI
E.C. M/EI	-8.3		-41.5		-49.8		-49.8		-49.8		-41.5		-8.3	$h^2/6EI$
Slope		116.2		74.7		24.9		-24.9		-74.7		-116.2		$h^2/6EI$
Y	0		116.2		190.9		215.8		190.9		116.2		0	$h^3/6EI$

Y = 1.74 inches

Next trial assume Y of 1.7 inches.

Figure 66. Problem Two - Beam A-A & C-C Cycle 1 Trial 1

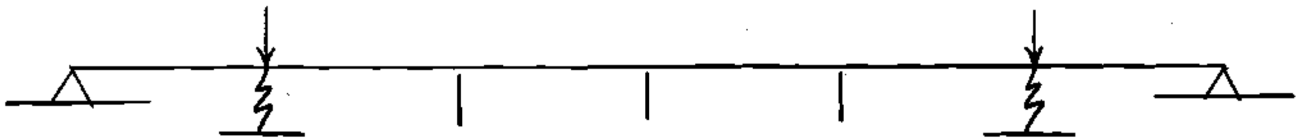


Assumed Y			1.7								1.7			
Loads	0		-10		0		0		0		-10		0	
Spring Load			2.22								2.22			
Total Loads	0		-7.78		0		0		0		-7.78		0	
V Trial		0		-7.78		-7.78		-7.78		-7.78		-15.56		
M Trial	0		0		-7.78		-15.56		-23.34		-31.12		-46.68	h
Corr M	0		7.78		15.56		23.34		31.12		38.9		46.68	h
M	0		7.78		7.78		7.78		7.78		7.78		0	h
M/EI	0		-7.78		-7.78		-7.78		-7.78		-7.78		0	h/EI
E.C. M/EI	-7.78		-38.9		-46.68		-46.68		-46.68		-38.9		-7.78	$h^2/6EI$
Slope		108.92		70.02		23.34		-23.34		-70.02		-108.92		$h^2/6EI$
Y	0		108.92		178.94		202.28		178.94		108.92		0	$h^3/6EI$

Y = 1.628 inches

Next trial assume Y of 1.63 inches.

Figure 67. Problem Two - Beams A-A & C-C Cycle 1 Trial 2

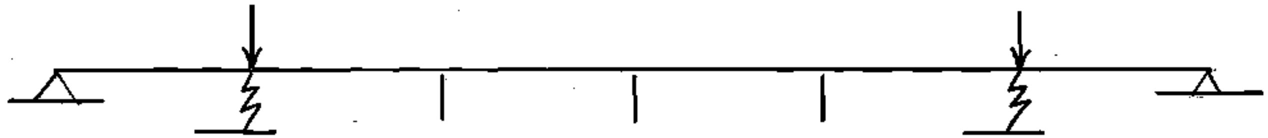


Assumed Y			1.63									1.63			
Loads	0		-10		0		0		0		-10		0		
Spring Load			2.13									2.13			
Total Loads	0		-7.87		0		0		0		-7.87		0		
V Trial		0		-7.87		-7.87		-7.87		-7.87		-15.74			
M Trial	0		0		-7.87		-15.74		-23.61		-31.48		-47.22		h
Corr M	0		7.87		15.74		23.61		31.48		39.35		47.22		h
M	0		7.87		7.87		7.87		7.87		7.87		0		h
M/EI	0		-7.87		-7.87		-7.87		-7.87		-7.87		0		h/EI
E.C. M/EI	-7.87		-39.35		-47.22		-47.22		-47.22		-39.35		-7.87		$h^2/6EI$
Slope		110.18		70.83		23.61		-23.61		-70.83		-110.18			$h^2/6EI$
Y	0		110.18		181.01		204163		181.01		110.18		0		$h^3/6EI$

Y = 1.65 inches

Next trial assume Y of 1.65 inches.

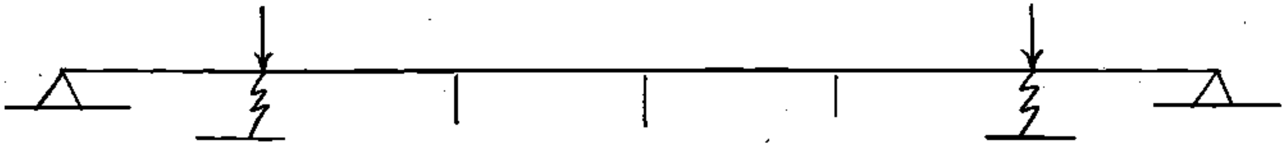
Figure 68. Problem Two - Beams A-A & C-C Cycle 1 Trial 3



Assumed Y			1.65								1.65			
Loads	0		-10		0		0		0		-10		0	
Spring Load			2.15								2.15			
Total Load	0		-7.85		0		0		0		-7.85		0	
V Trial		0		-7.85		-7.85		-7.85		-7.85		-15.7		
M Trial	0		0		-7.85		-15.7		-23.55		-31.4		-47.1	h
Corr M	0		7.85		15.7		23.55		31.4		39.25		47.1	h
M	0		7.85		7.85		7.85		7.85		7.85		0	h
M/EI	0		-7.85		-7.85		-7.85		-7.85		-7.85		0	h/EI
E.C. M/EI	-7.85		-39.25		-47.1		-47.1		-47.1		-39.25		-7.85	$h^2/6EI$
Slope		109.9		70.65		23.55		-23.55		-70.65		-109.9		$h^2/6EI$
Y	0		109.9		180.55		204.1		180.55		109.9		0	$h^3/6EI$

Y = 1.642 inches   Sufficient accuracy.

Figure 69. Problem Two - Beams A-A & C-C Cycle 1 Trial 4

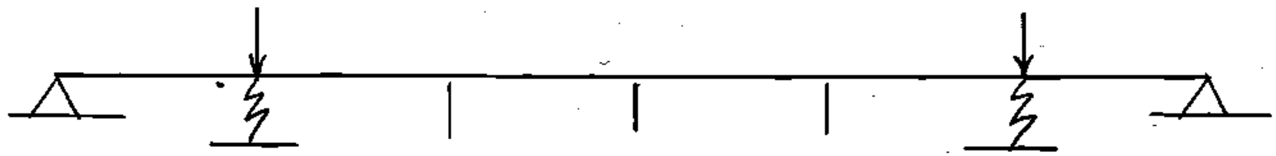


Assumed Y			1.9								1.9			
Loads	0		-10		0		0		0		-10		0	
Spring Load			1.4								1.4			
Total Load	0		-8.6		0		0		0		-8.6		0	
V Trial		0		-8.6		-8.6		-8.6		-8.6		-17.2		
M Trial	0		0		-8.6		-17.2		-25.8		-34.4		-51.6	h
Corr M	0		8.6		17.2		25.8		34.4		43		51.6	h
M	0		8.6		8.6		8.6		8.6		8.6		0	h
M/EI	0		-8.6		-8.6		-8.6		-8.6		-8.6		0	h/EI
E.C. M/EI	-8.6		-43		-51.6		-51.6		-51.6		-43		-8.6	$h^2/6EI$
Slope		120.4		77.4		25.8		-25.8		-77.4		-120.4		$h^2/6EI$
Y	0		120.4		197.8		223.6		197.8		120.4		0	$h^3/6EI$

Y = 1.8 inches

Next trial assume Y of 1.8 inches.

Figure 70. Problem Two - Beam B-B Cycle 1 Trial 1

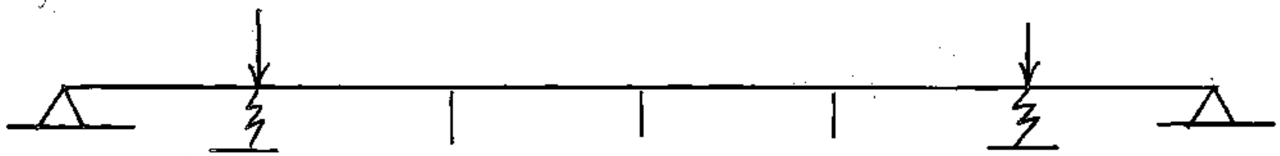


Assumed Y			1.8								1.8			
Loads	0		-10		0		0		0		-10		0	
Spring Loads			1.325								1.325			
Total Loads	0		-8.675		0		0		0		-8.675		0	
V Trial		0		-8.675		-8.675		-8.675		-8.675		-17.35		
M Trial	0		0		-8.675		-17.35		-26.025		-34.7		-52.05	h
Corr M	0		8.675		17.35		26.025		34.7		43.375		52.05	h
M	0		8.675		8.675		8.675		8.675		8.675		0	h
M/EI	0		-8.675		-8.675		-8.675		-8.675		-8.675		0	h/EI
E.C. M/EI	-8.675		-43.375		-52.05		-52.05		-52.05		-43.375		-8.675	
Slope		121.45		78.075		26.025		-26.025		-78.075		-121.45		$h^2/6EI$
Y	0		121.45		199.525		225.55		199.525		121.45		0	$h^3/6EI$

Y = 1.815 inches

Next trial assume Y of 1.81 inches.

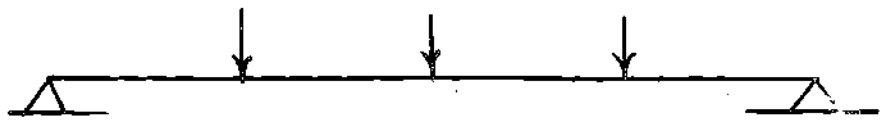
Figure 71. Problem Two - Beam B-B Cycle 1 Trial 2



Assumed Y			1.81								1.81			
Loads	0		-10		0		0		0		-10		0	
Spring Loads			1.33								1.33			
Total Loads	0		-8.67		0		0		0		-8.67		0	
V Trial		0		-8.67		-8.67		-8.67		-8.67		-17.34		
M Trial	0		0		-8.67		-17.34		-26.01		-34.68		-52.02	h
Corr M	0		8.67		17.34		26.01		34.68		43.35		52.02	h
M	0		8.67		8.67		8.67		8.67		8.67		0	h
M/EI	0		-8.67		-8.67		-8.67		-8.67		-8.67		0	h/EI
E.C. M/EI	-8.67		-43.35		-52.02		-52.02		-52.02		-43.35		-8.67	$h^2/6EI$
Slope		121.38		78.03		26.01		-26.01		-78.03		-121.38		$h^2/6EI$
Y	0		121.38		100.41		225.42		199.41		121.38		0	$h^3/6EI$

Y = 1.814 inches   Sufficient accuracy.

Figure 72. Problem Two - Beam B-B Cycle 1 Trial 3



Loads	0		-2.15		-1.33		-2.15		0	
V Trial		0		-2.15		-3.48		-5.63		
M Trial	0		0		-2.15		-5.63		-11.26	h
Corr M	0		2.815		5.63		8.445		11.26	h
M	0		2.815		3.48		2.815		0	h
M/EI	0		-2.815		-3.48		-2.815		0	h/EI
E.C. M/EI	-2.815		-14.74		-19.55		-14.74		-2.815	$h^2/6EI$
Slope		24.515		9.775		-9.775		-24.515		$h^2/6EI$
Y	0		24.515		34.29		24.515		0	$h^3/6EI$

$Y = 4.17$ $\quad Y = 5.84$ $\quad Y = 4.17$

New exterior spring constant equals 2.15 / 4.17 or 0.516 kips/inch.

New interior spring constant equals 1.33 / 5.84 or 0.228 kips/inch.

Figure 73. Problem Two - Beams 1-1 & 2-2 Cycle 1

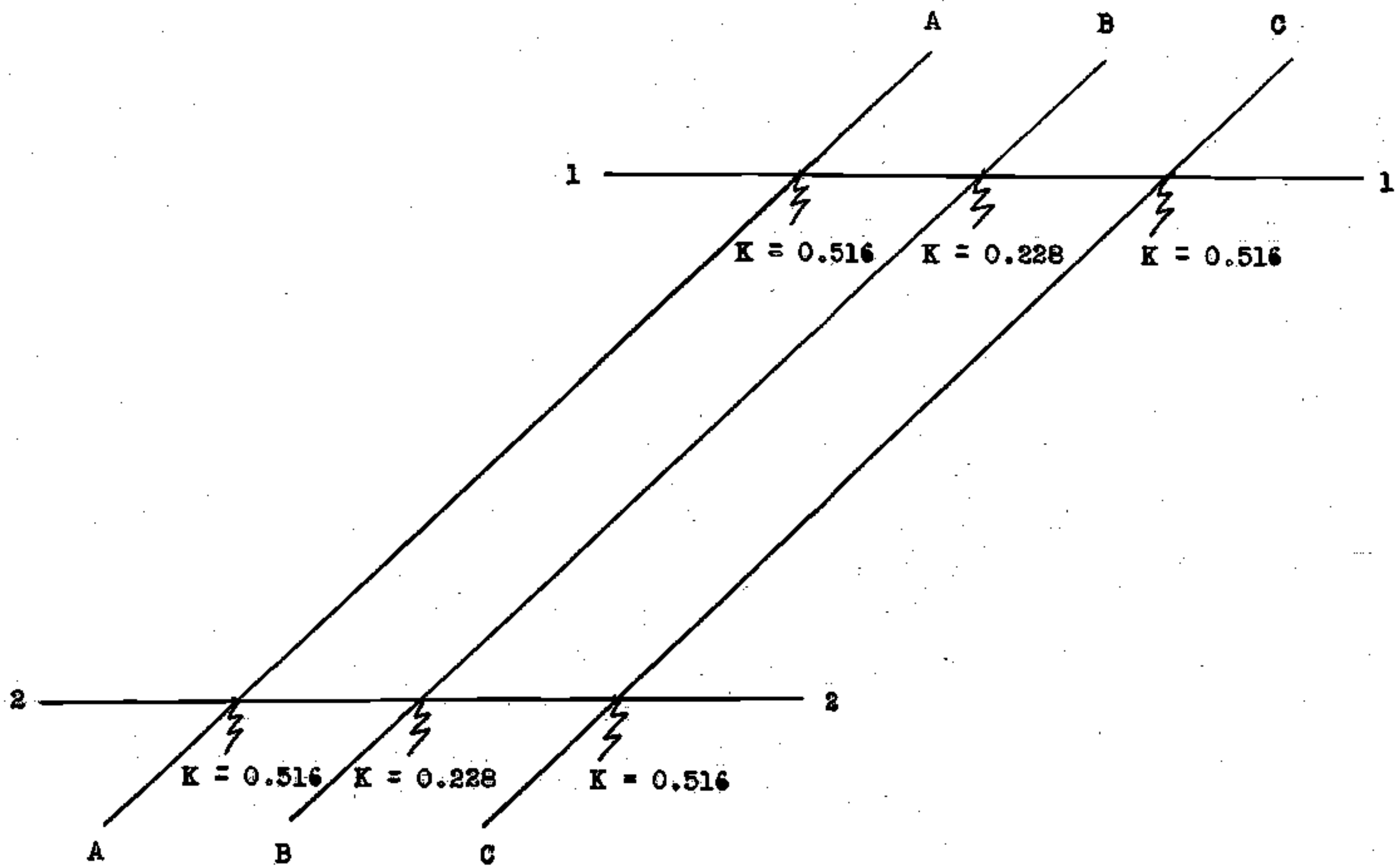
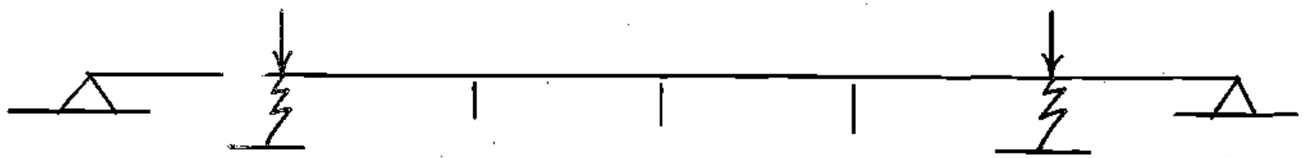


Figure 74. Problem Two - Spring Constant Values End Of Cycle 1

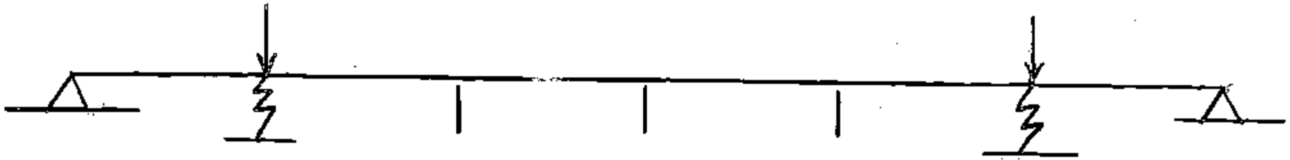


Assumed Y			1.0								1.0		K = 0.516	
Loads	0		-10		0		0		0		-10		0	
Spring Loads			0.516								0.516			
Total Loads	0		-9.484		0		0		0		-9.484		0	
V Trial		0		-9.484		-9.484		-9.484		-9.484		-18.968		
M Trial	0		0		-9.484		-18.968		-28.452		-37.936		-56.904	h
Corr M	0		9.484		18.968		28.452		37.936		47.42		56.904	h
M	0		9.484		9.484		9.484		9.484		9.484		0	h
M/EI	0		-9.484		-9.484		-9.484		-9.484		-9.484		0	h/EI
E.C. M/EI	-9.484		-47.42		-56.904		-56.904		-56.904		-47.42		-9.484	$h^2/6EI$
Slope		132.776		85.356		28.452		-28.452		-85.356		-132.776		$h^2/6EI$
Y	0		132.776		218.132		246.584		218.132		132.776		0	$h^3/6EI$

Y = 1.98 inches

Next trial assume Y of 1.9 inches.

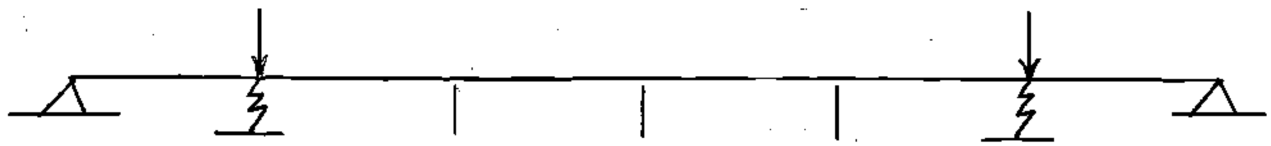
Figure 75. Problem Two - Beams A-A & C-C Cycle Two Trial 1



Assumed Y			1.9								1.9			
Loads	0		-10		0		0		0		-10		0	
Spring Loads			0.98								0.98			
Total Loads	0		-9.02		0		0		0		-9.02		0	
V Trial		0		-9.02		-9.02		-9.02		-9.02		-18.04		
M Trial	0		0		-9.02		-18.04		-27.06		-36.08		-54.12	h
Corr M	0		9.02		18.04		27.06		36.08		45.1		54.12	h
M	0		9.02		9.02		9.02		9.02		9.02		0	h
M/EI	0		-9.02		-9.02		-9.02		-9.02		-9.02		0	h/EI
E.C. M/EI	-9.02		-45.1		-54.12		-54.12		-54.12		-45.1		-9.02	$h^2/6EI$
Slope		126.28		81.18		27.06		-27.06		-81.18		-126.28		$h^2/6EI$
Y	0		126.28		207.46		234.52		207.46		126.28		0	$h^3/6EI$

Y = 1.89 inches   Sufficient accuracy.

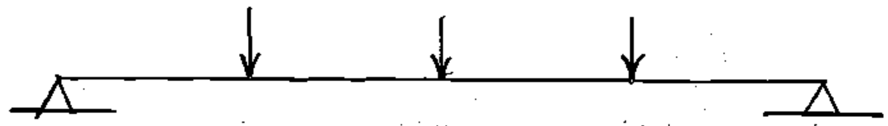
Figure 76. Problem Two - Beams A-A & C-C Cycle 2 Trial 2



Assumed Y			2								2		K = 0.228	
Loads	0		-10		0		0		0		-10		0	
Spring Load			0.456								0.456			
Total Loads	0		-9.544		0		0		0		-9.544		0	
V Trial		0		-9.544		-9.544		-9.544		-9.544		-19.088		
M Trial	0		0		-9.544		-19.088		-28.632		-38.176		-57.264	h
Corr M	0		9.544		19.088		28.632		38.176		47.72		57.264	h
M	0		9.544		9.544		9.544		9.544		9.544		0	h
M/EI	0		-9.544		-9.544		-9.544		-9.544		-9.544		0	h/EI
E.C. M/EI	-9.544		-47.72		-57.264		-57.264		-57.264		-47.72		-9.544	$h^2/6EI$
Slope		133.616		85.896		28.632		-28.632		-85.896		-133.616		$h^2/6EI$
Y	0		133.616		219.512		248.144		219.512		133.616		0	$h^3/6EI$

Y = 1.999 inches   Sufficient accuracy.

Figure 77. Problem Two - Beam B-B Cycle 2 Trial 1



Loads	0		-0.98		-0.456		-0.98		0	
V Trial		0		-0.98		-1.436		-2.416		
M Trial	0		0		-0.98		-2.416		-4.832	h
Corr M	0		1.208		2.416		3.624		4.832	h
M	0		1.208		1.436		1.208		0	h
M/EI	0		-1.208		-1.436		-1.208		0	h/EI
E.C. M/EI	-1.208		-6.268		-8.16		-6.268		-1.208	$h^2/6EI$
Slope		10.348		4.08		-4.08		-10.348		$h^2/6EI$
Y	0		10.348		14.428		10.348		0	$h^3/6EI$
			Y = 1.763		Y = 2.46		Y = 1.763			

New spring constant exterior point equals 0.98 / 1.763 or 0.556 kips/inch.

New spring constant interior point equals 0.456 / 2.46 or 0.185 kips/inch.

Figure 78. Problem Two - Beams 1-1 & 2-2 Cycle 2 Trial 1

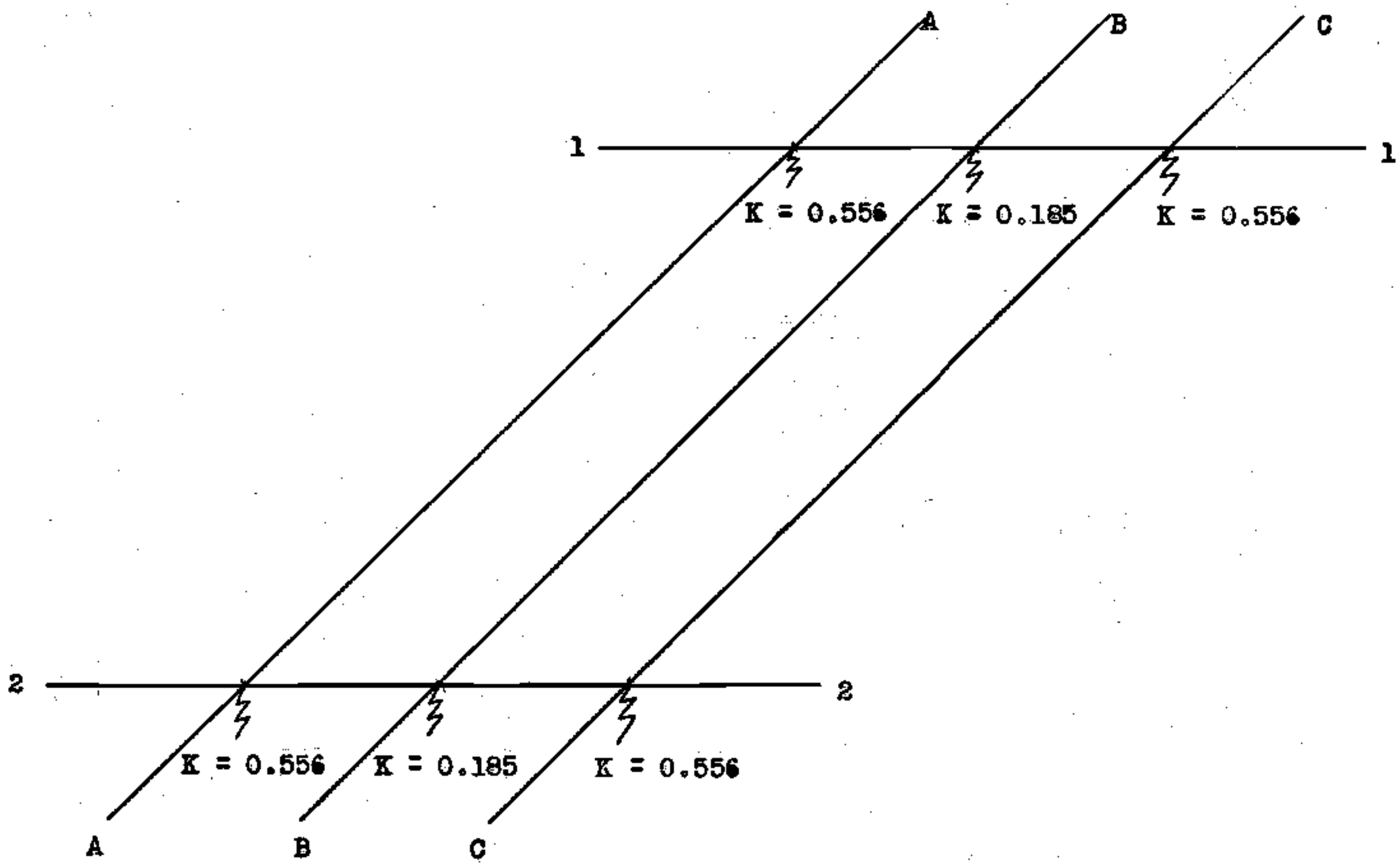
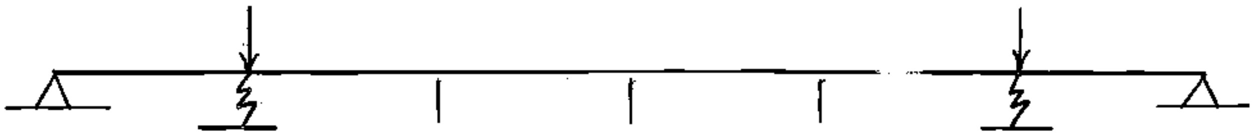


Figure 79. Problem Two - Spring Constant Values End Of Cycle 2

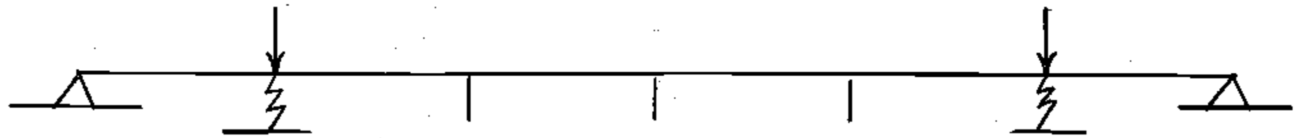


Assumed Y		1.85				1.85	K = 0.556	
Loads	0	-10	0	0	0	-10	0	
Spring Loads		1.03				1.03		
Total Loads	0	-8.97	0	0	0	-8.97	0	
V Trial	0	-8.97	-8.97	-8.97	-8.97	-17.94		
M Trial	0	0	-8.97	-17.94	-26.91	-35.88	-53.82	h
Corr M	0	8.97	17.94	26.91	35.88	44.85	53.82	h
M	0	8.97	8.97	8.97	8.97	8.97	0	h
M/EI	0	-8.97	-8.97	-8.97	-8.97	-8.97	0	h/EI
E.C. M/EI	-8.97	-44.85	-53.82	-53.82	-53.82	-44.85	-8.97	$h^2/6EI$
Slope	125.58	80.73	26.91	-26.91	-80.73	-125.58		$h^2/6EI$
Y	0	125.58	206.31	233.22	206.31	125.58	0	$h^3/6EI$

Y = 1.88 inches

Next trial assume Y of 1.88 inches.

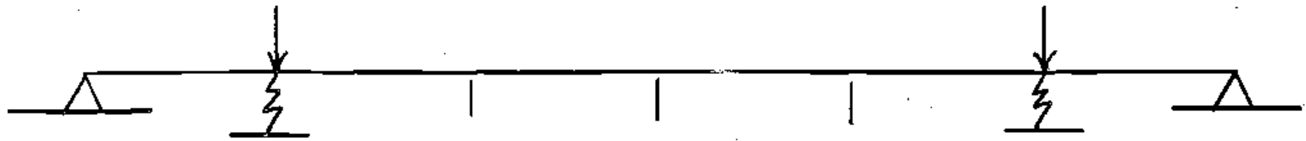
Figure 80. Problem Two - Beams A-A & C-C Cycle 3 Trial 1



Assumed Y			1.88								1.88			
Loads	0		-10		0		0		0		-10		0	
Spring Loads			1.045								1.045			
Total Loads	0		-8.955		0		0		0		-8.955		0	
V Trial		0		-8.955		-8.955		-8.955		-8.955		-17.91		
M Trial	0		0		-8.955		-17.91		-26.865		-35.82		-53.73	h
Corr M	0		8.955		17.91		26.865		35.82		44.775		53.73	h
M	0		8.955		8.955		8.955		8.955		8.955		0	h
M/EI	0		-8.955		-8.955		-8.955		-8.955		-8.955		0	h/EI
E.C. M/EI	-8.955		-44.775		-53.73		-53.73		-53.73		-44.775		-8.955	$h^2/6EI$
Slope		125.37		80.595		26.865		-26.865		-80.595		-125.37		$h^2/6EI$
Y	0		125.37		205.965		232.83		205.965		125.37		0	$h^3/6EI$

Y = 1.875 inches Sufficient accuracy.

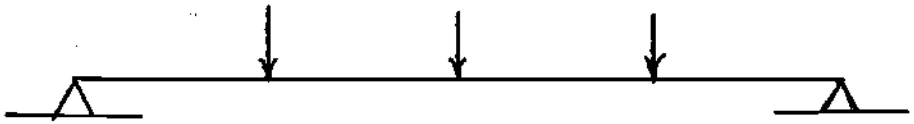
Figure 81. Problem Two - Beams A-A & C-C Cycle 3 Trial 2



Assumed Y			2								2		K = 0.185	
Loads	0		-10		0		0		0		-10		0	
Spring Load			0.37								0.37			
Total Loads	0		-9.63		0		0		0		-9.63		0	
V Trial		0		-9.63		-9.63		-9.63		-9.63		-19.26		
M Trial	0		0		-9.63		-19.26		-28.89		-38.52		-57.78	h
Corr M	0		9.63		19.26		28.89		38.52		48.15		57.78	h
M	0		9.63		9.63		9.63		9.63		9.63		0	h
M/EI	0		-9.63		-9.63		-9.63		-9.63		-9.63		0	h/EI
E.C. M/EI	-9.63		-48.15		-57.78		-57.78		-57.78		-48.15		-9.63	$h^2/6EI$
Slope		134.82		86.67		28.89		-28.89		-86.67		-134.82		$h^2/6EI$
Y	0		134.82		221.49		250.38		221.49		134.82		0	$h^3/6EI$

Y = 2.017 inches   Sufficient accuracy.

Figure 82. Problem Two - Beam B-B Cycle 3 Trial 1

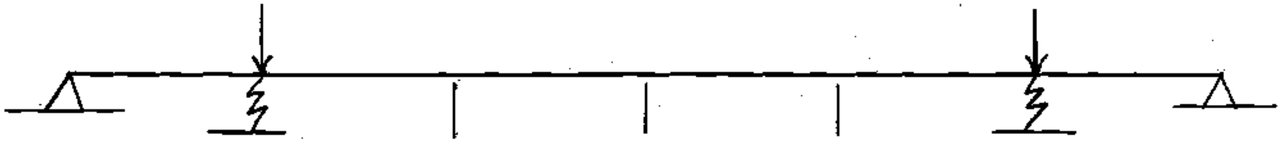


Loads	0		-1.045		-0.37		-1.045		0	
V Trial		0		-1.045		-1.415		-2.46		
M Trial	0		0		-1.045		-2.46		-4.92	h
Corr M	0		1.23		2.46		3.69		4.92	h
M	0		1.23		1.415		1.23		0	h
M/EI	0		-1.23		-1.415		-1.23		0	h/EI
E.C. M/EI	-1.23		-6.335		-8.12		-6.335		-1.23	$h^2/6EI$
Slope		10.395		4.06		-4.06		-10.395		$h^2/6EI$
Y	0		10.395		14.455		10.395		0	$h^3/6EI$
			Y = 1.77		Y = 2.462		Y = 1.77			

Exterior point spring constant equals 1.045 / 1.77 or 0.59 kips/inch.

Interior point spring constant equals 0.37 / 2.462 or 0.15 kips/inch.

Figure 63. Problem Two - Beams 1-1 & 2-2 Cycle 3

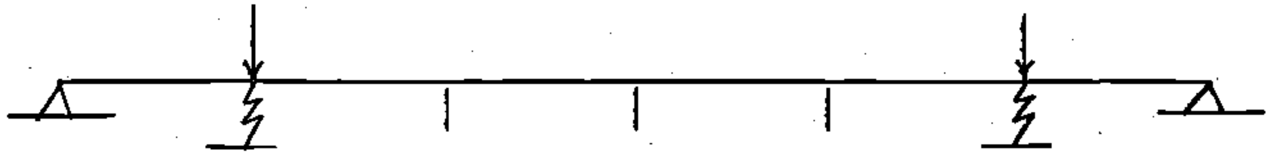


Assumed Y			1.82								1.82		K = 0.59	
Loads	0		-10		0		0		0		-10		0	
Spring Loads			1.075								1.075			
Total Loads	0		-8.925		0		0		0		-8.925		0	
V Trial		0		-8.925		-8.925		-8.925		-8.925		-17.85		
M Trial	0		0		-8.925		-17.85		-26.775		-35.7		-53.55	h
Corr M	0		8.925		17.85		26.775		35.7		44.625		53.55	h
M	0		8.925		8.925		8.925		8.925		8.925		0	h
M/EI	0		-8.925		-8.925		-8.925		-8.925		-8.925		0	h/EI
E.C. M/EI	-8.925		-44.625		-53.55		-53.55		-53.55		-44.625		-8.925	$h^2/6EI$
Slope		124.95		80.325		26.775		-26.775		-80.325		-124.95		$h^2/6EI$
Y	0		124.95		205.275		232.05		205.275		124.95		0	$h^3/6EI$

Y = 1.87 inches

Next trial assume Y of 1.87 inches.

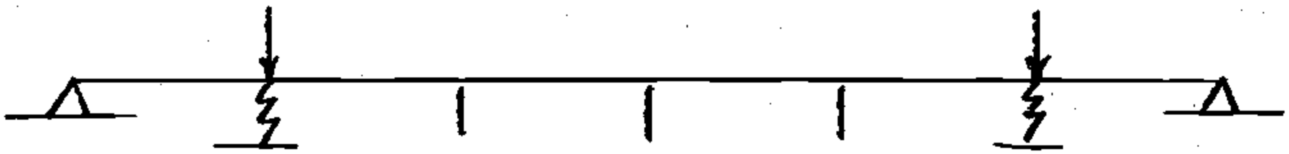
Figure 84. Problem Two - Beam A-A & C-C Cycle 4 Trial 1



Assumed Y			1.87								1.87			
Loads	0		-10		0		0		0		-10		0	
Spring Load			1.103								1.103			
Total Loads	0		-8.897		0		0		0		-8.897		0	
V Trial		0		-8.897		-8.897		-8.897		-8.897		-17.794		
M Trial	0		0		-8.897		-17.794		-26.691		-35.588		-53.382	h
Corr M	0		8.897		17.794		26.691		35.588		44.485		53.382	h
M	0		8.897		8.897		8.897		8.897		8.897		0	h
M/EI	0		-8.897		-8.897		-8.897		-8.897		-8.897		0	h/EI
E.C. M/EI	-8.897		-44.485		-53.382		-53.382		-53.382		-44.485		-8.897	$h^2/6EI$
Slope		124.558		80.073		26.691		-26.691		-80.073		-124.558		$h^2/6EI$
Y	0		124.558		204.631		231.322		204.631		124.558		0	$h^3/6EI$

Y = 1.863 inches Sufficient accuracy.

Figure 85. Problem Two - Beams A-A & C-C Cycle 4 Trial 2

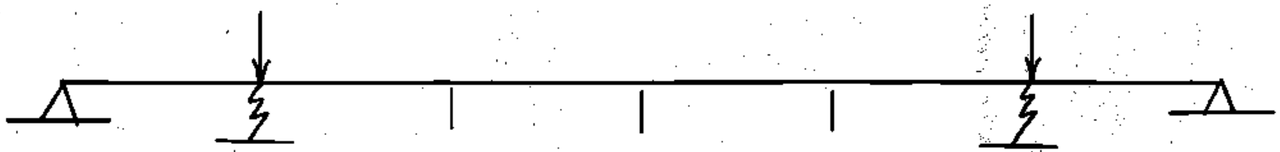


Assumed Y			2								2		K = 0.15	
Loads	0		-10		0		0		0		-10		0	
Spring Load			0.3								0.3			
Total Loads	0		-9.7		0		0		0		-9.7		0	
V Trial		0		-9.7		-9.7		-9.7		-9.7		-19.4		
M Trial	0		0		-9.7		-19.4		-29.1		-38.8		-58.2	h
Corr M	0		9.7		19.4		29.1		38.8		48.5		58.2	h
M	0		9.7		9.7		9.7		9.7		9.7		0	h
M/EI	0		-9.7		-9.7		-9.7		-9.7		-9.7		0	h/EI
E.C. M/EI	-9.7		-48.5		-58.2		-58.2		-58.2		-48.5		-9.7	$h^2/6EI$
Slope		135.8		87.3		29.1		-29.1		-87.3		-135.8		$h^2/6EI$
Y	0		135.8		223.1		252.2		223.1		135.8		0	$h^3/6EI$

Y = 2.03 inches

Next trial assume Y of 2.03 inches.

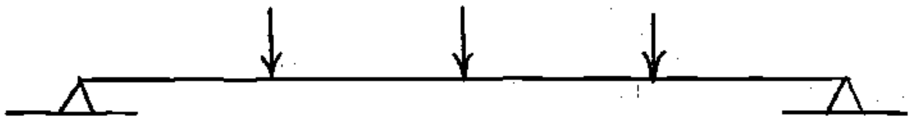
Figure 86. Problem Two - Beam B-B Cycle 4 Trial 1



Assumed Y			2.03								2.03			
Loads	0		-10		0		0		0		-10		0	
Spring Loads			0.305								0.305			
Total Loads	0		-9.695		0		0		0		-9.695		0	
V Trial		0		-9.695		-9.695		-9.695		-9.695		-19.39		
M Trial	0		0		-9.695		-19.39		-29.085		-38.78		-58.17	h
Corr M	0		9.695		19.39		29.085		38.78		48.475		58.17	h
M	0		9.695		9.695		9.695		9.695		9.695		0	h
M/EI	0		-9.695		-9.695		-9.695		-9.695		-9.695		0	h/EI
E.C. M/EI	-9.695		-48.475		-58.17		-58.17		-58.17		-48.475		-9.695	
Slope		135.73		87.255		29.085		-29.085		-87.255		-135.73		$h^2/6EI$
Y	0		135.73		222.985		252.07		222.985		135.73		0	$h^3/6EI$

Y = 2.028 inches Sufficient accuracy.

Figure 87. Problem Two - Beam B-B Cycle 4 Trial 2



Loads	0		-1.103		-0.305		-1.103		0	
V Trial		0		-1.103		-1.408		-2.511		
M Trial	0		0		-1.103		-2.511		-5.022	h
Corr M	0		1.2555		2.511		3.7665		5.022	h
M	0		1.2555		1.408		1.2555		0	h
M/EI	0		-1.2555		-1.408		-1.2555		0	h/EI
E.C. M/EI	-1.2555		-6.43		-8.143		-6.43		-1.2555	$h^2/6EI$
Slope		10.5015		4.0715		-4.0715		-10.5015		$h^2/6EI$
Y	0		10.5015		14.573		10.5015		0	$h^3/6EI$

Y = 1.79 Y = 2.48 Y = 1.79 inches

New exterior point spring constant equals 1.103 / 1.79 or 0.616 kips/inch.

New interior point spring constant equals 0.305 / 2.48 or 0.123 kips/inch.

This completes Cycle 4.

Figure 88. Problem Two - Beam 1-1 & 2-2 Cycle 4

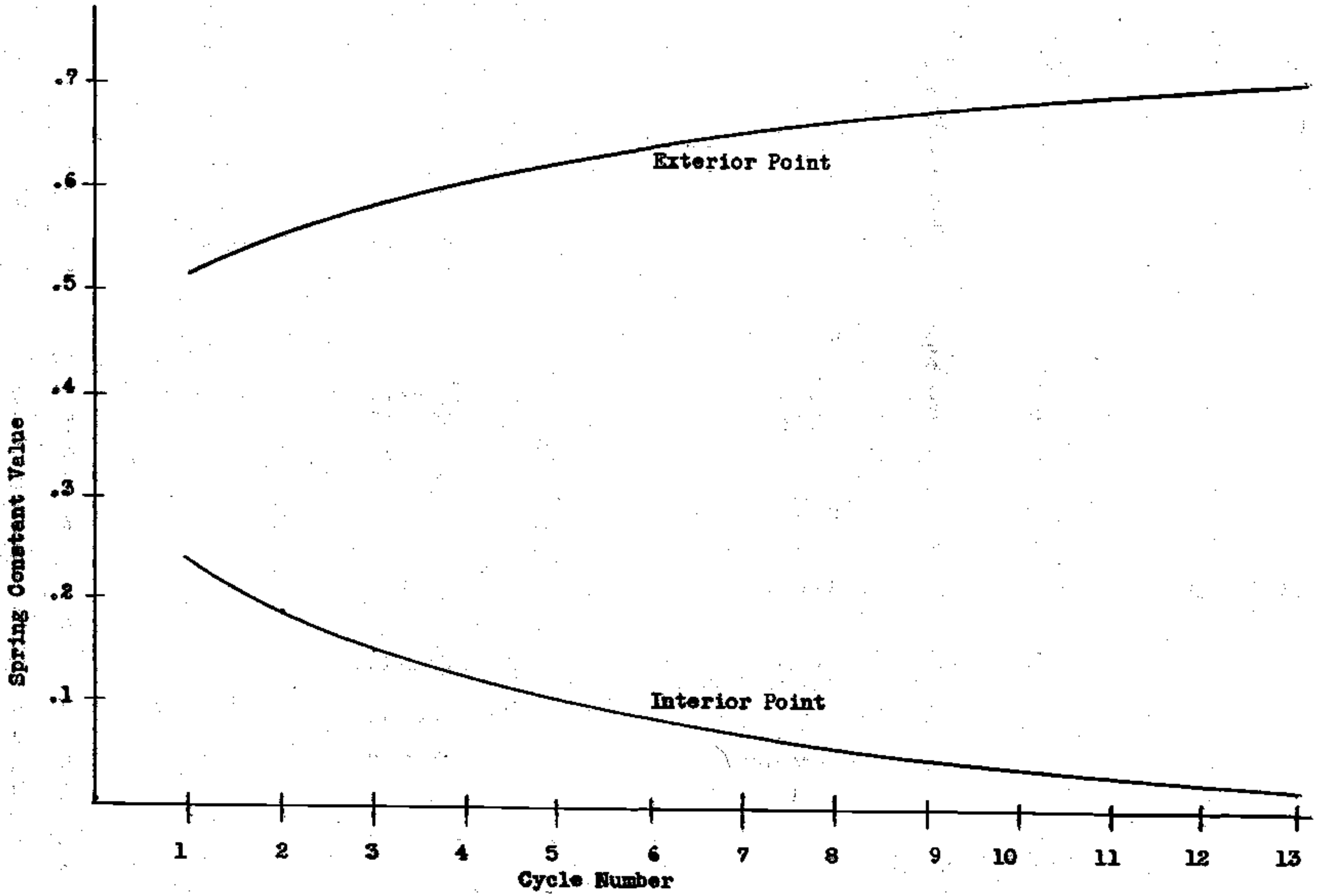
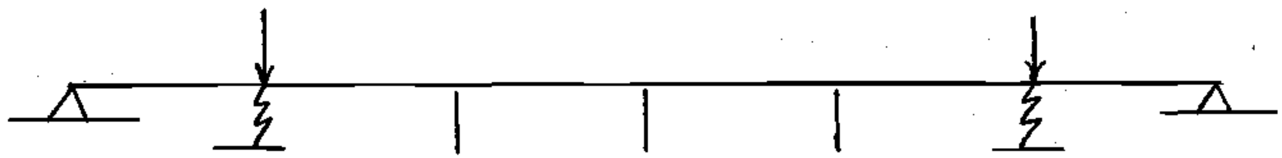


Figure 89. Problem Two - Graph of Spring Constants Versus Cycles

Cycle Number	Exterior Point Y of main beam / Y of support beam	Interior Point Y of main beam / Y of support beam
1	39.37649 %	31.0616 %
2	107.2036	81.2601
3	105.9322	81.9252
4	104.0782	81.774
5	103.00333	81.1646
6	102.7222	82.4617
7	101.939	82.3859
8	101.9444	83.0586
9	100.8264	82.80
10	100.8264	83.004
11	101.0341	83.3991
12	100.6341	83.2853
13	100.5852	83.4288

The error is clearly shown by this comparison. The correct solution would indicate one hundred per cent at each point. Instead the interior point has a seventeen per cent error after thirteen cycles and is not improving . This indicates a negative spring constant at the interior point and a trial and error approach is needed. The support beam actually pulls down on the main beam in this problem.

Figure 90. Comparison of Deflection Values

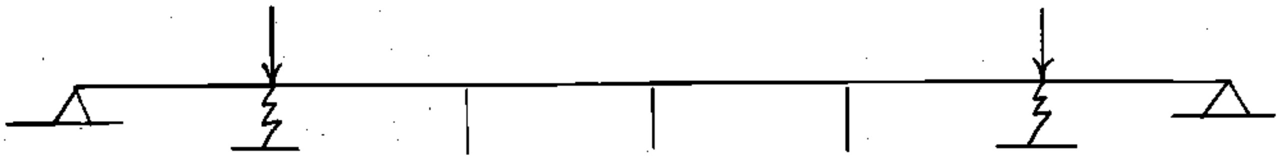


Spring Constant			1.1 kips/inch								1.1 kips/inch			
Loads	0		-10		0		0		0		-10		0	
Assumed Y			1.71								1.71			
Spring Loads			1.88								1.88			
Total Loads	0		-8.12		0		0		0		-8.12		0	
E.C. M/EI	-8.12		-40.6		-48.72		-48.72		-48.72		-40.6		-8.12	$h^2/6EI$
Slope		113.68		73.08		24.36		-24.36		-73.08		-113.68		$h^2/6EI$
Y	0		113.68		186.76		211.12		186.76		113.68		0	$h^3/6EI$

Y = 1.70 inches

Next trial assume Y of 1.7 inches.

Figure 91. Problem Two - Beams A-A & C-C Cycle 25 Trial 1

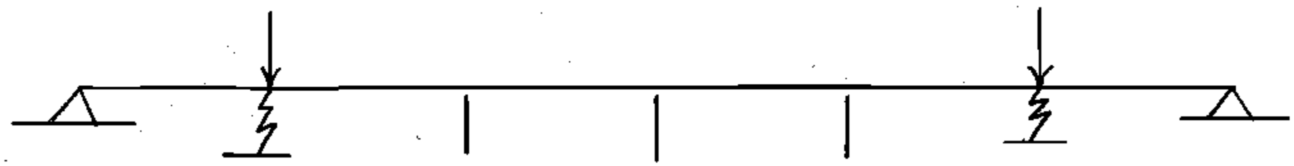


Spring Constant			1.1 kips/inch								1.1 kips/inch			
Assumed Y			1.70								1.70			
Loads	0		-10		0		0		0		-10		0	
Spring Load			1.87								1.87			
Total Load	0		-8.13		0		0		0		-8.13		0	
E.C. M/EI	-8.13		-40.65		-48.78		-48.78		-48.78		-40.65		-8.13	$h^2/6EI$
Slope		113.82		73.17		24.39		-24.39		-73.17		-113.82		$h^2/6EI$
Y	0		113.82		186.99		211.38		186.99		113.82		0	$h^3/6EI$

Y = 1.702 inches

Sufficient accuracy.

Figure 92. Problem Two - Beams A-A & C-C Cycle 25 Trial 2

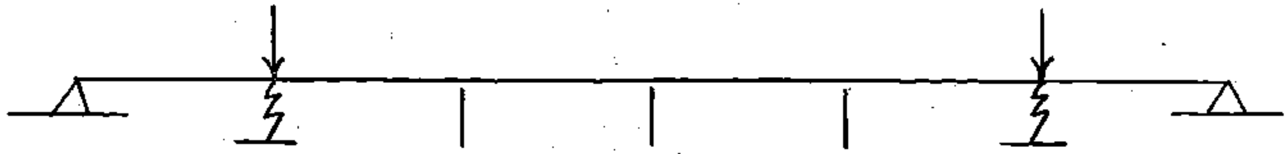


Spring Constant			-0.4 kips/inch								-0.4 kips/inch			
Assumed Y			2.2								2.2			
Loads	0		-10		0		0		0		-10		0	
Spring Loads			-0.88								-0.88			
Total Loads	0		-10.88		0		0		0		-10.88		0	
M/EI	0		-10.88		-10.88		-10.88		-10.88		-10.88		0	h/EI
E.C. M/EI	-10.88		-54.4		-65.28		-65.28		-65.28		-54.4		-10.88	
Slope		152.32		97.92		32.64		-32.64		-97.92		-152.32		$h^2/6EI$
Y	0		152.32		250.24		282.88		250.24		152.32		0	$h^3/6EI$

Y = 2.27 inches

Next trial assume Y of 2.27 inches.

Figure 93. Problem Two - Beam B-B Cycle 25 Trial 1



Spring Constant		-0.4 kips/inch				-0.4 kips/inch		
Loads	0	-10	0	0	0	-10	0	
Assumed Y		2.27				2.27		
Spring Loads		-0.908				-0.908		
Total Loads	0	-10.908	0	0	0	-10.908	0	
M/EI	0	-10.908	-10.908	-10.908	-10.908	-10.908	0	
E.C. M/EI	-10.908	-54.54	-65.448	-65.448	-65.448	-54.54	-10.908	
Slope		152.712	98.172	32.724	-32.724	-98.172	-152.712	$h^2/6EI$
Y	0	152.712	250.884	283.608	250.884	152.712	0	$h^3/6EI$

Y = 2.28 inches

Sufficient accuracy.

Figure 94. Problem Two - Beam B-B Cycle 25 Trial 2

Loads	0		-1.87		0.908		-1.87		0	
V Trial		0		-1.87		-0.962		-2.832		
M Trial	0		0		-1.87		-2.832		-5.664	h
Corr M	0		1.416		2.832		4.248		5.664	h
M	0		1.416		0.962		1.416		0	h
E.C. M/EI	-1.416		-6.626		-6.68		-6.626		-1.416	$h^2/6EI$
Slope		9.966		3.34		-3.34		-9.966		$h^2/6EI$
Y	0		9.966		13.306		9.966		0	$h^3/6EI$
			Y = 1.698		Y = 2.267		Y = 1.698			

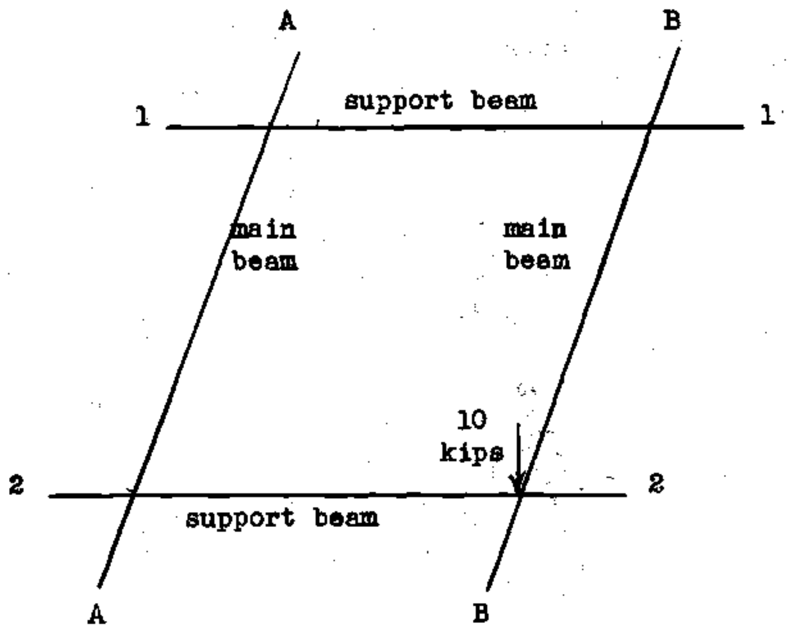
Exterior point deflection error check equals 1.702 / 1.698 or 0.267 per cent error.

Interior point deflection error check equals 2.284 / 2.267 or 0.758 per cent error.

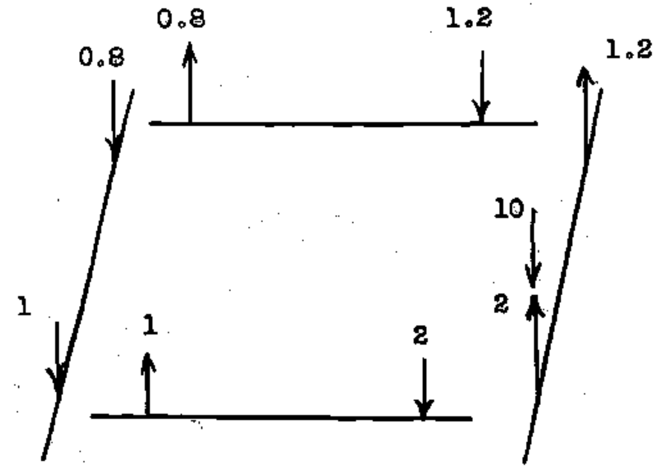
The error in both cases is less than one per cent and the problem is solved.

Figure 95. Problem Two - Beams 1-1 & 2-2 Cycle 25

Problem Three Grillage System.



Assume Initial Interaction Loads As Follows:



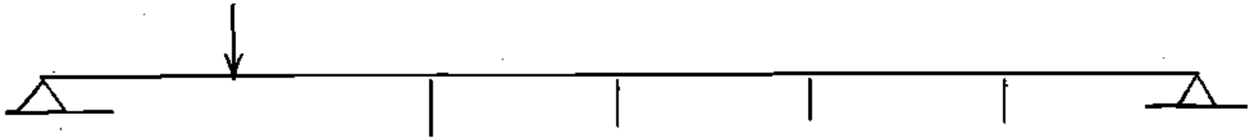
Each beam pinned to fixed support at both ends.

Main beams are 14WF61 with I of 641.5

Support beams are 8WF17 with I of 56.4

Grillage system is identical to that of Problem One. All dimensions are the same.

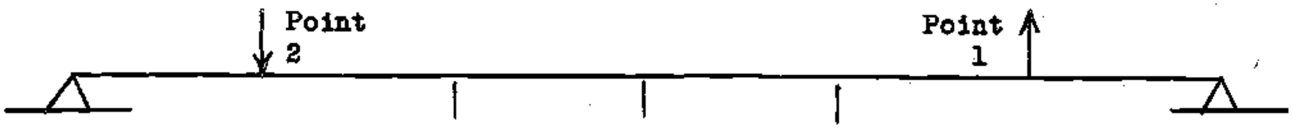
Figure 96. Problem Three Grillage System



Loads	0		-2		0		0		0		0		0	
V Trial		0		-2		-2		-2		-2		-2		
M Trial	0		0		-2		-4		-6		-8		-10	h
Corr M	0		1.66		3.32		4.98		6.64		8.3		10	h
M	0		1.66		1.32		0.98		0.64		0.3		0	h
M/EI	0		-1.66		-1.32		-0.98		-0.64		-0.3		0	h/EI
E.C. M/EI	-1.66		-7.96		-7.92		-5.88		-3.84		-1.84		-0.3	$h^2/6EI$
Slope		7.96		0		-7.92		-13.8		-17.64		-19.48		$h^2/6EI$
Y	0		7.96		7.96		0.04		-13.76		-31.4		-50.88	
Corr Y	0		8.48		16.96		25.44		33.92		42.4		50.88	
Y	0		16.44		24.92		25.48		20.16		11		0	$h^3/6EI$
Main Beam	0		0.246		0.373		0.381		0.301		0.1645		0	inches
Y	0		1		1.516		1.55		1.225		0.668		0	inches
Support	0		2.8		4.24		4.34		3.43		1.87		0	inches
Beam Y	0		1		1.516		1.55		1.225		0.668		0	inches

Second deflection line gives deflection values in inches for a one inch deflection at division two. This deflection is caused by a load at division two.

Figure 97. Problem Three - Deflection Ratio Calculation



Loads	0		-8		0		0		0		1.2		0	
V Trial		0		-8		-8		-8		-8		-6.8		
M Trial	0		0		-8		-16		-24		-32		-38.8	h
Corr M	0		6.46		12.92		19.38		25.84		32.3		38.8	h
M	0		6.46		4.92		3.38		1.84		0.3		0	h
M/EI	0		-6.46		-4.92		-3.38		-1.84		-0.3		0	h/EI
E.C. M/EI	-6.46		-30.76		-29.52		-20.28		-11.04		-3.04		-0.3	h^2/6EI
Slope		60.28		29.52		0		-20.28		-31.32		-34.36		h^2/6EI
Y Trial	0		60.28		89.8		89.8		69.52		38.2		3.84	h^3/6EI
Corr Y	0		-0.64		-1.28		-1.92		-2.56		-3.2		-3.84	h^3/6EI
Y	0		59.64		88.52		87.88		66.96		35		0	h^3/6EI
Y	0		0.893		1.323		1.312		1.0		0.524		0	inches
Corr 'A'	0		0.098		0.1486		0.152		0.12		0.0655		0	inches
Corr 'B'	0		-0.019		-0.0348		-0.044		-0.043		-0.0284		0	inches
Total	0		0.972		1.4368		1.42		1.077		0.5611		0	inches

Figure 98. Problem Three - Beam B-B Cycle 1

In the correction configuration method the deflections of cross beams at the node points must be averaged by some method. The best method is to average according to the relative deflection that an equal load causes on each beam or in this particular case according to the relative I value (this is the same as averaging according to relative deflections).

641.5 / 56.4 equals 11.4 Thus the main beam is 11.4 times as stiff as the support beam if the grillage connection is ignored.

641.5 / (641.5 + 56.4) = 91.9 %

56.4 / 697.9 = 8.1%

Therefore proportion as follows:

91.9 per cent of the main beam value

8.1 per cent of the support beam value

Figure 98 correction values were obtained as follows:

0.972 - 0.893 = 0.079 0.561 - 0.524 = 0.037

0.079 = A + 0.668B

0.037 = B + 0.668A

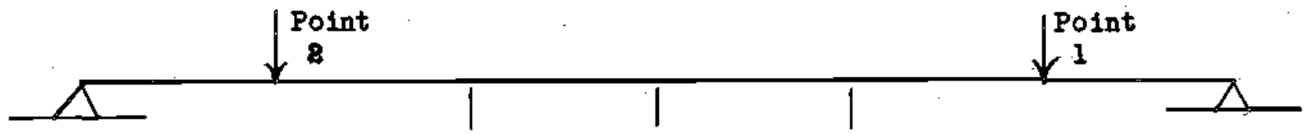
Solving simultaneously gives: A = 0.098 B = -0.0284

These values were multiplied by the deflection ratio figures of Figure 97 to obtain the values recorded in the correction lines.

Figure 99. Problem Three - Deflection Proportion & Corr

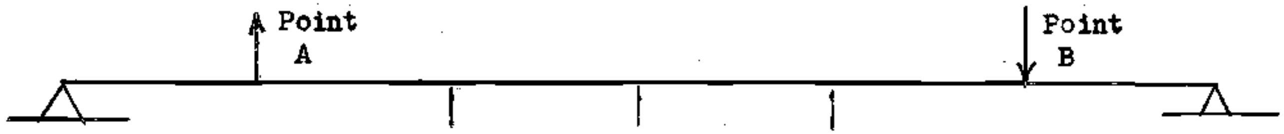
Loads	0		1		0		0		0		-2		0	
V Trial		0		1		1		1		1		-1		
M Trial	0		0		1		2		3		4		3	h
Corr M	0		-0.5		-1		-1.5		-2		-2.5		-3	h
M	0		-0.5		0		0.5		1		1.5		0	h
M/EI	0		0.5		0		-0.5		-1		-1.5		0	h/EI
E.C. M/EI	0.5		2		0		-3		-6		-7		-1.5	h^2/6EI
Slope		-2		0		0		-3		-9		-16		h^2/6EI
Y Trial	0		-2		-2		-2		-5		-14		-30	h^3/6EI
Corr Y	0		5		10		15		20		25		30	h^3/6EI
Y	0		3		8		13		15		11		0	h^3/6EI
Y	0		0.511		1.363		2.218		2.56		1.873		0	inches
Corr 'A'	0		0.553		0.839		0.857		0.678		0.369		0	inches
Corr 'B'	0		-0.849		-1.555		-1.97		-1.926		-1.27		0	inches
Total	0		0.215		0.647		1.105		1.312		0.972		0	inches

Figure 100. Problem Three - Beam 2-2 Cycle 1



Loads	0		-1		0		0		0		-0.8		0	
V Trial		0		-1		-1		-1		-1		-1.8		
M Trial	0		0		-1		-2		-3		-4		-5.8	h
Corr M	0		0.96		1.92		2.88		3.84		4.8		5.8	h
M	0		0.96		0.92		0.88		0.84		0.8		0	h
M/EI	0		-0.96		-0.92		-0.88		-0.84		-0.8		0	h/EI
E.C. M/EI	-0.96		-4.76		-5.52		-5.28		-5.04		-4.04		-0.8	$h^2/6EI$
Slope		10.28		5.52		0		-5.28		-10.32		-14.36		$h^2/6EI$
Y	0		10.28		15.8		15.8		10.52		0.2		-14.16	
Corr Y	0		2.36		4.72		7.08		9.44		11.8		14.16	
Y	0		12.64		20.52		22.88		19.96		12		0	$h^3/6EI$
Y	0		0.189		0.307		0.342		0.2984		0.1795		0	inches
Y Corr 'A'	0		0.05846		0.0885		0.0906		0.0716		0.039		0	
Y Corr 'B'	0		-0.0325		-0.0595		-0.0754		-0.0737		-0.0486		0	
Total Y	0		0.215		0.336		0.3572		0.2963		0.17		0	inches

Figure 101. Problem Three - Beam A-A Cycle 1



Loads	0		0.8		0		0		0		-1.2		0	
V Trial		0		0.8		0.8		0.8		0.8		-0.4		
M Trial	0		0		0.8		1.6		2.4		3.2		2.8	h
Corr M	0		-0.46		-0.92		-1.38		-1.84		-2.3		-2.8	h
M	0		-0.46		-0.12		0.22		0.56		0.9		0	h
M/EI	0		0.46		0.12		-0.22		-0.56		-0.9		0	h/EI
E.C. M/EI	0.46		1.96		0.72		-1.32		-3.36		-4.16		-0.9	$h^2/6EI$
Slope		-2.68		-0.72		0		-1.32		-4.68		-8.84		$h^2/6EI$
Y	0		-2.68		-3.4		-3.4		-4.72		-9.4		-18.24	$h^3/6EI$
Corr Y	0		3.04		6.08		9.12		12.16		15.2		18.24	$h^3/6EI$
Y	0		0.36		2.68		5.72		7.44		5.8		0	$h^3/6EI$
Y	0		0.0613		0.456		0.975		1.268		0.988		0	inches
Y Corr 'A'	0		0.7099		1.074		1.1		0.869		0.473		0	
Y Corr 'B'	0		-0.602		-1.103		-1.394		-1.363		-0.9		0	
Total Y	0		0.17		0.428		0.681		0.774		0.561		0	inches

Figure 102. Problem Three - Beam 1-1 Cycle 1

Simultaneous correction equations for Fig. 100:

$-0.296 = A + 0.668B$

$-0.901 = B + 0.668A$ $\quad A = 0.553 \quad B = -1.27$

Simultaneous correction equations Fig. 101:

$0.026 = A + 0.668B$

$-0.0095 = B + 0.668A$ $\quad A = 0.05846 \quad B = -0.0486$

Simultaneous correction equations for Fig. 102:

$0.1087 = 1A + 0.668B$

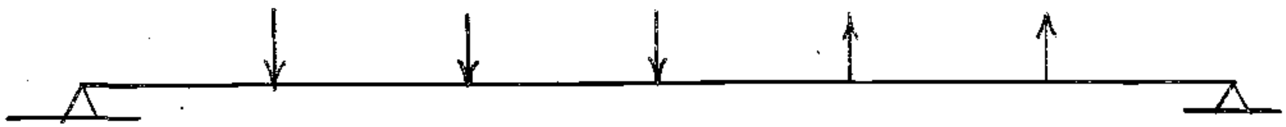
$-0.427 = 1B + 0.668A$ $\quad A = 0.7099 \quad B = -0.9$

Deflection averaged according to relative values of Fig. 99.

Node	Value	Average	Node	Value	Average
BB-1	0.524	0.481	AA-1	0.1795	0.165
11-B	0.988	0.08	11-A	0.0613	0.005
New value		0.561 inches	New value		0.17 inches
BB-2	0.893	0.82	AA-2	0.189	0.174
22-B	1.873	0.152	22-A	0.511	0.041
New value		0.972 inches	New value		0.215 inches

Use these values of deflections to start Cycle 2 and obtain loads.

Figure 103. Problem Three - Deflection Average Cycle 1

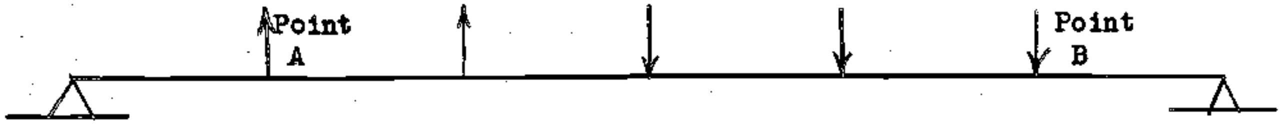


Loads	0		-8.862		-0.002		-0.026		0.148		1.338		0	
V		7.127		-1.735		-1.737		-1.763		-1.615		-0.277		
M	0		7.127		5.392		3.655		1.892		0.277		0	h
M/EI	0		-7.127		-5.392		-3.655		-1.892		-0.277		0	h/EI
			A		B		C		D		E			
E.C. M/EI	-7.127		-33.9		-32.2		-21.9		-11.5		-3		-0.277	$h^2/6EI$
Slope		65		31.1		-1.1		-23		-34.5		-37.5		$h^2/6EI$
Y	0		65		96.1		95		72		37.5		0	$h^3/6EI$
Y	0		0.972		1.4368		1.42		1.077		0.5611		0	inches

Simultaneous equations needed to obtain values for angle change (M/EI) line.

$4A + B = -33.9$	Solution
$A + 4B + C = -32.2$	$A = -7.127$ $B = -5.392$
$B + 4C + D = -21.9$	$C = -3.655$ $D = -1.892$
$C + 4D + E = -11.5$	$E = -0.277$
$D + 4E = -3$	

Figure 104. Problem Three - Beam B-B Cycle 2



Loads	0		0.59		0.012		-0.024		-0.018		-1.048		0	
V		-0.311		0.279		0.297		0.273		0.255		-0.793		
M	0		-0.311		-0.032		0.265		0.538		0.793		0	h
M/EI	0		0.311		0.032		-0.265		-0.538		-0.793		0	h/EI
			A		B		C		D		E			
E.C. M/EI			1.276		0.162		-1.49		-3.21		-3.71			$h^2/6EI$
Slope		1.262		2.538		2.7		1.21		-2		-5.71		$h^2/6EI$
Y	0		1.262		3.8		6.5		7.71		5.71		0	$h^3/6EI$
Y	0		0.215		0.647		1.105		1.312		0.972		0	inches

Simultaneous equations needed to obtain values for the angle change (M/EI) line:

$4A + B = 1.276$

$A + 4B + C = 0.162$

$B + 4C + D = -1.49$

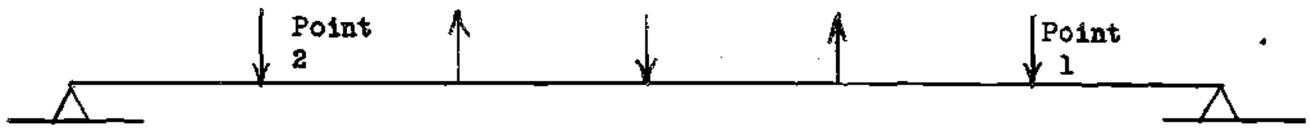
$C + 4D + E = -3.21$

$D + 4E = -3.71$

Solutions

$A = 0.311$
$B = 0.032$
$C = -0.265$
$D = -0.538$
$E = -0.793$

Figure 105. Problem Three - Beam 2-2 Cycle 2



			A		B		C		D		E			
Loads	0		-1.508		0.047		-0.074		0.082		-0.408		0	
V		1.303		-0.205		-0.158		-0.232		-0.15		-0.558		
M	0		1.303		1.098		0.94		0.708		0.558		0	h
M/EI	0		-1.303		-1.098		-0.94		-0.708		-0.558		0	h/EI
E.C. M/EI			-6.31		-6.62		-5.55		-4.33		-2.94			$h^2/6EI$
Slope		14.38		8.07		1.45		-4.1		-8.43		-11.37		$h^2/6EI$
Y	0		14.38		22.45		23.9		19.8		11.37		0	$h^3/6EI$
Y	0		0.215		0.336		0.3572		0.2963		0.17		0	inches

Simultaneous equations needed to obtain values for the angle change (M/EI) line:

$4A + B = -6.31$

$A + 4B + C = -6.62$

$B + 4C + D = -5.55$

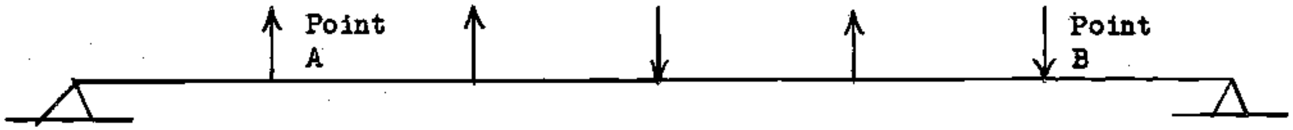
$C + 4D + E = -4.33$

$D + 4E = -2.94$

Solutions:

$A = -1.303$
$B = -1.098$
$C = -0.94$
$D = -0.708$
$E = -0.558$

Figure 106. Problem Three - Beam A-A Cycle 2



Loads	0		0.242		0.064		-0.054		0.014		-0.576		0	
V		-0.126		0.116		0.18		0.126		0.14		-0.436		
M	0		-0.126		-0.01		0.17		0.296		0.436		0	h
M/EI	0		0.126		0.01		-0.17		-0.296		-0.436		0	h/EI
			A		B		C		D		E			
E.C. M/EI			0.514		-0.022		-0.95		-1.79		-2.04			$h^2/6EI$
Slope		0.998		1.512		1.49		.54		-1.25		-3.29		$h^2/6EI$
Y	0		0.998		2.51		4		4.54		3.29		0	$h^3/6EI$
Y	0		0.17		0.428		0.681		0.774		0.561		0	inches

Simulteneous equations needed to obtain values for the angle change (M/EI) line:

$4A + B = 0.514$

$A + 4B + C = -0.022$

$B + 4C + D = -0.95$

$C + 4D + E = -1.79$

$D + 4E = -2.04$

Solutions:

$A = 0.126$
$B = 0.01$
$C = -0.17$
$D = -0.296$
$E = -0.436$

Figure 107. Problem Three - Beam 1-1 Cycle 2

Straight average of interaction loads:

Node	Load
BB-1	1.338 up
11-B	0.576 down
2	1.914
	0.957
AA-1	0.408 down
11-A	0.242 up
2	0.65
	0.325

Node	Load
BB-2	1.138 up
22-B	1.048 down
2	2.186
	1.093
AA-2	1.508 down
22-A	0.59 up
2	2.098
	1.049

Next apply these interaction loads to each beam and then average the resulting deflections to complete Cycle 2.

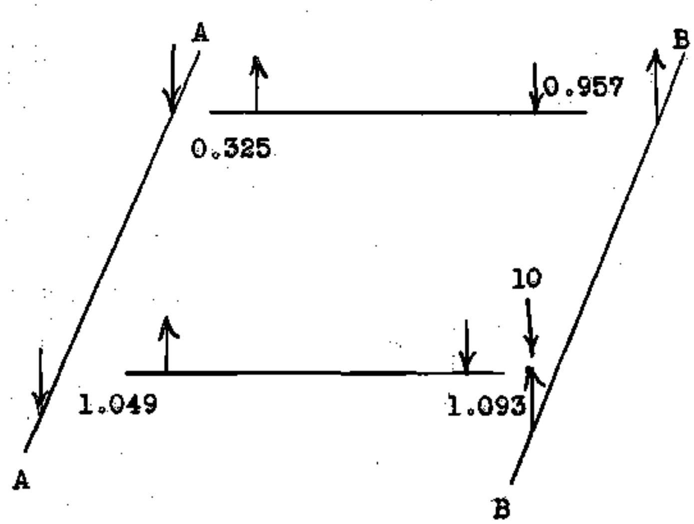
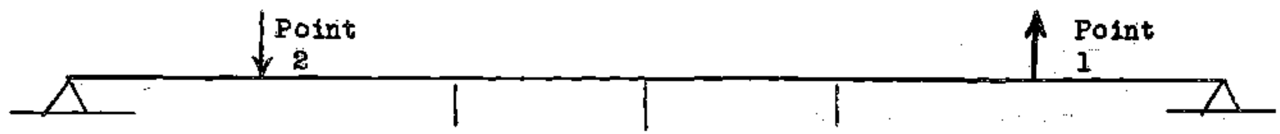
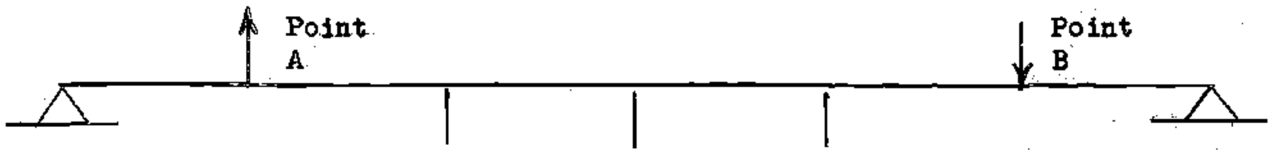


Figure 108. Problem Three - Cycle 2 Interaction Loads



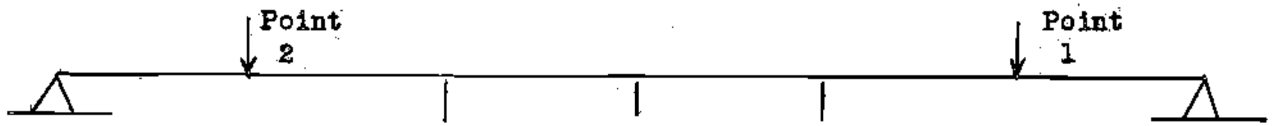
Loads	0		-8.907		0		0		0		0.957		0	
V Trial		0		-8.907		-8.907		-8.907		-8.907		-7.95		
M Trial	0		0		-8.907		-17.814		-26.721		-35.628		-43.578	h
Corr M	0		7.263		14.526		21.789		29.052		36.315		43.578	h
M	0		7.263		5.619		3.975		2.331		0.687		0	h
M/EI	0		-7.263		-5.619		-3.975		-2.331		-0.687		0	h/EI
E.C. M/EI	-7.263		-34.671		-33.714		-23.85		-13.986		-5.079		-0.687	$h^2/6EI$
Slope		68.385		33.714		0		-23.85		-37.836		-42.915		$h^2/6EI$
Trial Y	0		68.385		102.099		102.099		78.249		40.413		-2.502	$h^3/6EI$
Corr Y	0		0.417		0.834		1.251		1.668		2.085		2.502	$h^3/6EI$
Y	0		68.802		102.933		103.35		79.917		42.498		0	$h^3/6EI$
Y	0		1.03		1.54		1.546		1.194		0.635		0	inches
Y Corr 'A'	0		-0.11445		-0.1735		-0.1774		-0.14		-0.0764		0	
Y Corr 'B'	0		0.0744		0.1365		0.1727		0.1689		0.11145		0	
Total Y	0		0.99		1.503		1.541		1.223		0.67		0	inches

Figure 109. Problem Three - Beam B-B



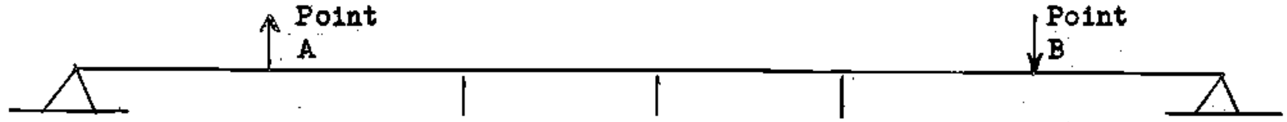
Loads	0	1.049	0	0	0	-1.093	0	
V Trial		0	1.049	1.049	1.049	1.049	-0.044	
M Trial	0	0	1.049	2.098	3.147	4.196	4.152	h
Corr M	0	-0.692	-1.384	-2.076	-2.768	-3.46	-4.152	h
M	0	-0.692	-0.335	0.022	0.379	0.736	0	h
M/EI	0	0.692	0.335	-0.022	-0.379	-0.736	0	h/EI
E.C. M/EI	0.692	3.103	2.01	-0.132	-2.274	-3.323	-0.736	$h^2/6EI$
Slope		-5.113	-2.01	0	-0.132	-2.406	-5.729	$h^2/6EI$
Y Trial	0	-5.113	-7.123	-7.123	-7.255	-9.661	-15.39	$h^3/6EI$
Corr Y	0	2.565	5.13	7.695	10.26	12.825	15.39	$h^3/6EI$
Y	0	-2.548	-1.993	0.572	3.005	3.164	0	$h^3/6EI$
Y	0	-0.4337	-0.3392	0.0974	0.5114	0.5385	0	inches
Y Corr 'A'	0	0.4372	0.6628	0.6776	0.5355	0.292	0	
Y Corr 'B'	0	0.1065	0.1953	0.2471	0.2417	0.15946	0	
Total Y	0	0.11	0.5189	1.022	1.288	0.99	0	inches

Figure 110. Problem Three - Beam 2-2



Loads	0	-1.049	0	0	0	-0.325	0	
V	0	-1.049	-1.049	-1.049	-1.049	-1.374		
M Trial	0	0	-1.049	-2.098	-3.147	-4.196	-5.57	h
Corr M	0	0.928	1.856	2.784	3.713	4.641	5.57	h
M	0	0.928	0.807	0.686	0.566	0.445	0	h
M/EI	0	-0.928	-0.807	-0.686	-0.566	-0.445	0	h/EI
E.C. M/EI	-0.928	-4.519	-4.842	-4.117	-3.395	-2.346	-0.445	$h^2/6EI$
Slope	9.361	4.842	0	-4.117	-7.512	-9.858		$h^2/6EI$
Y Trial	0	9.361	14.203	14.203	10.086	2.574	-7.284	$h^3/6EI$
Corr Y	0	1.214	2.428	3.642	4.856	6.07	7.284	$h^3/6EI$
Y	0	10.575	16.631	17.845	14.942	8.644	0	$h^3/6EI$
Y	0	0.1582	0.2488	0.267	0.2236	0.1293	0	inches
Y Corr 'A'	0	-0.11923	-0.1807	-0.1848	-0.1460	-0.0796	0	
Y Corr 'B'	0	0.071	0.1302	0.1648	0.1612	0.10634	0	
Total Y	0	0.11	0.198	0.247	0.239	0.156	0	inches

Figure 111. Problem Three - Beam A-A



Loads	0	0.325	0	0	0	-0.967	0	
V Trial	0	0.325	0.325	0.325	0.325	-0.632		
M Trial	0	0	0.325	0.65	0.975	1.3	0.668	h
Corr M	0	-0.111	-0.222	-0.333	-0.445	-0.556	-0.668	h
M	0	-0.111	0.103	0.317	0.53	0.744	0	h
M/EI	0	0.111	-0.103	-0.317	-0.53	-0.744	0	h/EI
E.C. M/EI	0.111	0.341	-0.618	-1.901	-3.181	-3.506	-0.744	$h^2/6EI$
Slope	0.277	0.618	0	-1.901	-5.082	-8.588		$h^2/6EI$
Y Trial	0	0.277	0.895	0.895	-1.006	-6.088	-14.676	$h^3/6EI$
Corr Y	0	2.446	4.892	7.338	9.784	12.23	14.676	$h^3/6EI$
Y	0	2.723	5.787	8.233	8.778	6.142	0	$h^3/6EI$
Y	0	0.4634	0.985	1.401	1.494	1.045	0	inches
Y Corr 'A'	0	-0.1027	-0.1556	-0.1591	-0.1258	-0.0686	0	
Y Corr 'B'	0	-0.2047	-0.3754	-0.4749	-0.4645	-0.30644	0	
Total Y	0	0.156	0.454	0.767	0.9037	0.67	0	inches

Figure 112. Problem Three - Beam 1-1

Simultaneous correction equations for Fig. 109:

$-0.04 = A + 0.668B$

$0.035 = B + 0.668A$ $\qquad A = -0.11445 \qquad B = 0.11145$

Simultaneous correction equations for Fig. 110:

$0.5437 = A + 0.668B$

$0.4515 = B + 0.668A$ $\qquad A = 0.4372 \qquad B = 0.15946$

Simultaneous correction equations for Fig 111:

$-0.0482 = A + 0.668B$

$0.0267 = B + 0.668A$ $\qquad A = -0.11923 \qquad B = 0.10634$

Simultaneous correction equations for Fig. 112:

$-0.3074 = A + 0.668B$

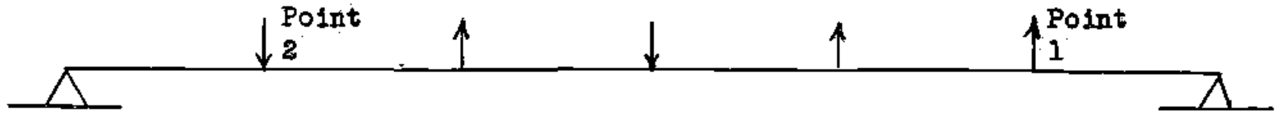
$-0.375 = B + 0.668A$ $\qquad A = -0.1027 \qquad B = -0.30644$

Average of deflection values as follows:

Node	Deflection	Average	Node	Deflection	Average
BB-1	0.635	0.583	BB-2	1.03	0.947
11-B	1.045	0.084	22-B	0.5385	0.044
New value		0.667			0.991
AA-1	0.1293	0.1188	AA-2	0.1582	0.145
11-A	0.4634	0.0375	22-A	-0.4337	-0.035
New value		0.1563			0.11

Cycle 2 is now completed.

Figure 113. Problem Three - Deflection Average Cycle 2



Loads	0		-8.0182		0.0656		-0.0656		0.0662		0.0002		0	
V		6.6488		-1.3694		-1.3038		-1.3694		-1.3032		-1.303		
M	0		6.6488		5.2794		3.9756		2.6062		1.303		0	h
M/EI	0		-6.6488		-5.2794		-3.9756		-2.6062		-1.303		0	h/EI
			A		B		C		D		E			
E.C. M/EI			-31.8746		-31.7408		-23.789		-15.7034		-7.8182			$h^2/6EI$
Slope		66.1547		34.2801		2.5393		-21.2497		-36.953		-44.7713		$h^2/6EI$
Y	0		66.1547		100.4348		102.9741		81.7244		44.7713		0	$h^3/6EI$
Y	0		0.99		1.503		1.541		1.223		0.67		0	inches

Simultaneous equations needed to obtain angle change (M/EI) line values:

$4A + B = -31.8746$

$A + 4B + C = -31.7408$

$B + 4C + D = -23.789$

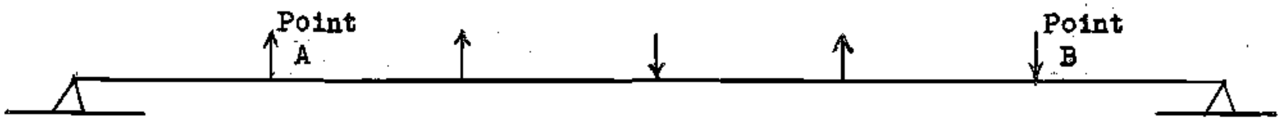
$C + 4D + E = -15.7034$

$D + 4E = -7.8182$

Solution:

$A = -6.6488$
$B = -5.2794$
$C = -3.9756$
$D = -2.6062$
$E = -1.303$

Figure 114. Problem Three - Beam B-B Cycle 3



Loads	0		0.7387		0.0038		-0.0102		0.012		-1.2073		0	
V		-0.4158		0.3229		0.3267		0.3165		0.3285		-0.8788		
M	0		-0.4158		-0.0929		0.2338		0.5503		0.8788		0	h
M/EI	0		0.4158		0.0929		-0.2338		-0.5503		-0.8788		0	h/EI
			A		B		C		D		E			
E.C. M/EI			1.7561		0.5534		-1.3929		-3.3136		-4.0654			$h^2/6EI$
Slope		0.6462		2.4023		2.9557		1.5628		-1.7508		-5.8162		$h^2/6EI$
Y	0		0.6462		3.0485		6.0042		7.567		5.8162		0	$h^3/6EI$
Y	0		0.11		0.5189		1.022		1.288		0.99		0	inches

Simultaneous equations needed for angle change (M/EI) line values:

$4A + B = 1.7561$

$A + 4B + C = 0.5534$

$B + 4C + D = -1.3929$

$C + 4D + E = -3.3136$

$D + 4E = -4.0654$

Solution:

$A = 0.4158$
$B = 0.0929$
$C = -0.2338$
$D = -0.5503$
$E = -0.87879$

Figure 115. Problem Three - Beam 2-2 Cycle 3

Loads	0	-0.0935	0.0399	0.0003	-0.0403	-1.176	0	
V	0.2606	0.1671	0.207	0.2073	0.167	-1.009		
M	0	0.2606	0.4277	0.6347	0.842	1.009	0	h
M/EI	0	-0.2606	-0.4277	-0.6347	-0.842	-1.009	0	h/EI
		A	B	C	D	E		
E.C. M/EI		-1.4701	-2.6061	-3.8089	-5.0117	-4.878		$h^2/6EI$
Slope	7.3505	5.8804	3.2743	-0.5346	-5.5463	-10.4243		$h^2/6EI$
Y	0	7.3505	13.2309	16.5052	15.9706	10.4243	0	$h^3/6EI$
Y	0	0.11	0.198	0.247	0.239	0.156	0	inches

Simultaneous equations needed to obtain values for the angle change (M/EI) line:

$4A + B = -1.4701$

$A + 4B + C = -2.6061$

$B + 4C + D = -3.8089$

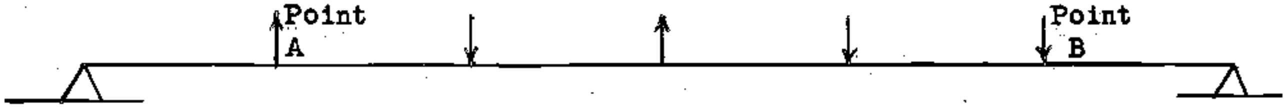
$C + 4D + E = -5.0117$

$D + 4E = -4.878$

Solutions:

A = -0.2606
B = -0.4277
C = -0.6347
D = -0.842
E = -1.009

Figure 116. Problem Three - Beam A-A Cycle 3



Loads	0		0.3966		-0.0065		0.0073		-0.0062		-0.7359		0	
V		-0.2051		0.1915		0.185		0.1923		0.1861		-0.5498		
M	0		-0.2051		-0.0136		0.1714		0.3637		0.5498		0	h
M/EI	0		0.2051		0.0136		-0.1714		-0.3637		-0.5498		0	h/EI
			A		B		C		D		E			
E.C. M/EI			0.8342		0.0882		-1.0358		-2.1761		-2.5632			$h^2/6EI$
Slope		0.9165		1.7507		1.8389		0.8031		-1.373		-3.9362		$h^2/6EI$
Y	0		0.9165		2.6672		4.5061		5.3092		3.9362		0	$h^3/6EI$
Y	0		0.156		0.454		0.767		0.9037		0.67		0	inches

Simultaneous equations needed to obtain values for the angle change (M/EI) line:

$4A + B = 0.8342$

$A + 4B + C = 0.0882$

$B + 4C + D = -1.0358$

$C + 4D + E = -2.1761$

$D + 4E = -2.5632$

Solutions:

A = 0.20515
B = 0.0136
C = -0.1714
D = -0.3637
E = -0.54987

Figure 117. Problem Three - Beam 1-1 Cycle 3

Straight average of interaction loads:

Node	Value
BB-1	0.0002 up
11-B	0.7359 down
2	0.7361
	0.368

Node	Value
BB-2	1.9818 up
22-B	1.2073 down
2	3.1891
	1.594

Node	Value
AA-1	1.176 down
11-A	0.3966 up
2	1.5726
	0.786

Node	Value
AA-2	0.0935 down
22-A	0.7387 up
2	0.8322
	0.416

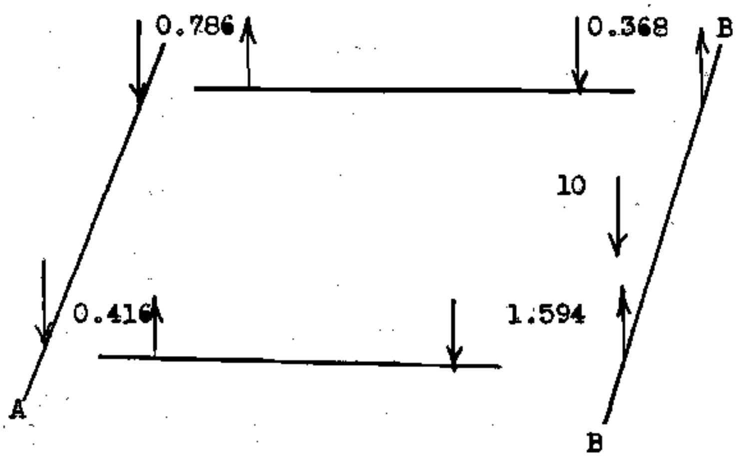
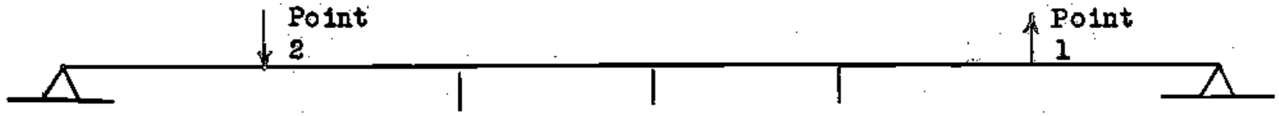


Figure 118. Problem Three - Interaction Load Average

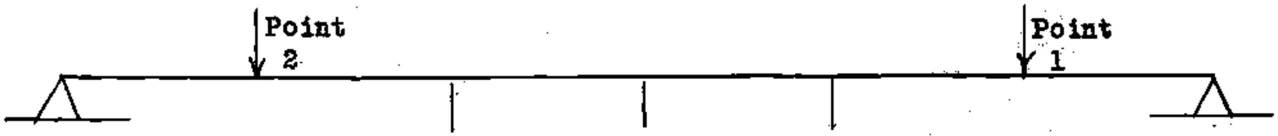


Loads	0	-8.406	0	0	0	0.368	0	
V Trial	0	-8.406	-8.406	-8.406	-8.406	-8.038		
M Trial	0	0	-8.406	-16.812	-25.218	-33.624	-41.662	h
Corr M	0	6.9436	13.8873	20.8309	27.7746	34.7183	41.662	h
M	0	6.9436	5.4813	4.0189	2.5566	1.0943	0	h
M/EI	0	-6.9436	-5.4813	-4.0189	-2.5566	-1.0943	0	h/EI
E.C. M/EI	-6.9436	-33.2557	-32.8877	-24.1135	-15.3396	-6.9338	-1.0943	$h^2/6EI$
Slope	66.1434	32.8877	0	-24.1135	-39.4531	-46.3869		$h^2/6EI$
Y Trial	0	66.1434	99.0314	99.0314	74.9179	35.4648	-10.9221	
Corr Y	0	1.8203	3.6407	5.461	7.2814	9.1017	10.9221	
Y	0	67.964	102.6721	104.4924	82.1993	44.5665	0	$h^3/6EI$
Y	0	1.017	1.5365	1.5637	1.2301	0.6669	0	inches
Y Corr 'A'	0	0.2107	0.3194	0.3266	0.2581	0.1407	0	
Y Corr 'B'	0	-0.1427	-0.2616	-0.3311	-0.3238	-0.2136	0	
Total Y	0	1.085	1.5943	1.5592	1.1644	0.594	0	inches

Figure 119. Problem Three - Beam B-B

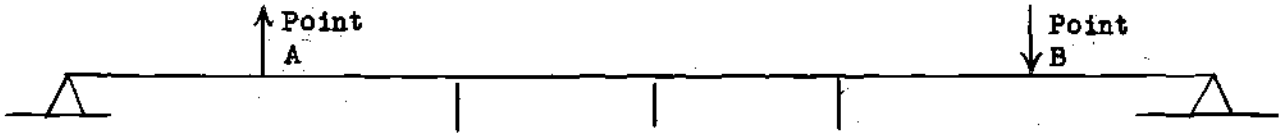
Loads	0	0.416	0	0	0	-1.594	0	
V	0	0.416	0.416	0.416	0.416	-1.178		
M Trial	0	0	0.416	0.832	1.248	1.664	0.486	h
Corr M	0	-0.081	-0.162	-0.243	-0.324	-0.405	-0.486	h
M	0	-0.081	0.254	0.589	0.924	1.259	0	h
M/EI	0	0.081	-0.254	-0.589	-0.924	-1.259	0	h/EI
E.C. M/EI	0.081	0.07	-1.524	-3.534	-5.544	-5.96	-1.259	h^2/6EI
Slope	4.988	5.058	3.534	0	-5.544	-11.504		h^2/6EI
Y Trial	0	4.988	10.046	13.58	13.58	8.036	-3.468	h^3/6EI
Corr Y	0	0.578	1.156	1.734	2.312	2.89	3.468	h^3/6EI
Y	0	5.566	11.202	15.314	15.892	10.926	0	h^3/6EI
Y	0	0.9474	1.9067	2.6066	2.705	1.8597	0	inches
Y Corr 'A'	0	-0.4412	-0.6688	-0.6838	-0.5404	-0.2947	0	
Y Corr 'B'	0	-0.3206	-0.588	-0.744	-0.7277	-0.48	0	
Total Y	0	0.1856	0.6499	1.1788	1.4369	1.085	0	inches

Figure 120. Problem Three - Beam 2-2



Loads	0		-0.416		0		0		0		-0.786		0	
V		0		-0.416		-0.416		-0.416		-0.416		-1.202		
M Trial	0		0		-0.416		-0.832		-1.248		-1.664		-2.866	h
Corr M	0		0.4776		0.9553		1.4329		1.9106		2.388		2.866	h
M	0		0.4776		0.5393		0.6009		0.6626		0.724		0	h
M/EI	0		-0.4776		-0.5393		-0.6009		-0.6626		-0.724		0	h/EI
E.C. M/EI	-0.4776		-2.4497		-3.2357		-3.6055		-3.9753		-3.5586		-0.724	$h^2/6EI$
Slope		9.2909		6.8412		3.6055		0		-3.9753		-7.5339		$h^2/6EI$
Y Trial	0		9.2909		16.1321		19.7376		19.7376		15.7623		8.2284	$h^3/6EI$
Y Corr	0		-1.3714		-2.7428		-4.1142		-5.4856		-6.857		-8.2284	$h^3/6EI$
Y	0		7.9195		13.3893		15.6234		14.252		8.9053		0	$h^3/6EI$
Y	0		0.1185		0.2003		0.2338		0.2133		0.1332		0	inches
Y Corr 'A'	0		0.2082		0.3156		0.3227		0.255		0.1391		0	
Y Corr 'B'	0		-0.1411		-0.2588		-0.3275		-0.3203		-0.2113		0	
Total Y	0		0.1856		0.2571		0.229		0.148		0.061		0	inches

Figure 121. Problem Three - Beam A-A



Loads	0		0.786		0		0		0		-0.368		0	
V		0		0.786		0.786		0.786		0.786		0.418		
M Trial	0		0		0.786		1.572		2.358		3.144		3.562	h
Corr M	0		-0.5936		-1.1873		-1.7809		-2.3746		-2.9683		-3.562	h
M	0		-0.5936		-0.4013		-0.2089		-0.0166		0.1757		0	h
M/EI	0		0.5936		0.4013		0.2089		0.0166		-0.1757		0	h/EI
E.C. M/EI	0.5936		2.7757		2.4077		1.2535		0.0996		-0.6862		-0.1757	$h^2/6EI$
Slope		-5.1834		-2.4077		0		1.2535		1.3531		0.6669		$h^2/6EI$
Y Trial	0		-5.1834		-7.5911		-7.5911		-6.3376		-4.9845		-4.3176	$h^3/6EI$
Corr Y	0		0.7196		1.4392		2.1588		2.8784		3.598		4.3176	$h^3/6EI$
Y	0		-4.4638		-6.1519		-5.4323		-3.4592		-1.3865		0	$h^3/6EI$
Y	0		-0.7598		-1.0471		-0.9246		-0.5888		-0.2359		0	inches
Y Corr 'A'	0		0.4811		0.7293		0.7457		0.5893		0.3213		0	
Y Corr 'B'	0		0.3397		0.6229		0.7882		0.7709		0.5085		0	
Total Y	0		0.061		0.3051		0.6093		0.7714		0.594		0	inches

Figure 122. Problem Three - Beam 1-1

Simultaneous correction equations for Fig. 119:

0.068 = A   0.668B

-0.0729 = B   0.668A   A = 0.2107   B = -0.21366

Simultaneous correction equations for Fig. 120:

-0.7618 = A   0.668B

-0.7747 = B   0.668A   A = -0.4412   B = -0.48

Simultaneous correction equations for Fig 121:

0.0671 = A   0.668B

-0.0722 = B   0.668A   A = 0.2082   B = -0.2113

Simultaneous correction equations for Fig. 122:

0.8208 = A   0.668B

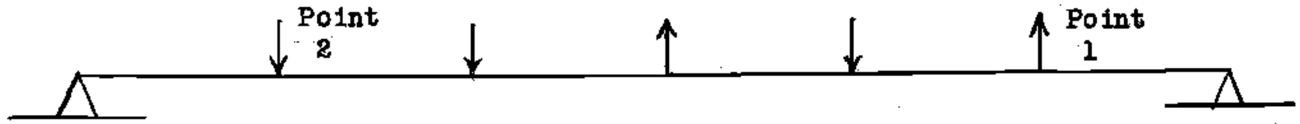
0.8299 = B   0.668A   A = 0.4811   B = 0.5085

Deflection average according to Fig. 99 values:

Node	Value	Average	Node	Value	Average
BB-1	0.6669	0.6128	BB-2	1.017	0.9346
11-B	-0.2359	-0.0191	22-B	1.8597	0.1506
		0.5937			1.0852
AA-1	0.1332	0.1224	AA-2	0.1185	0.1089
11-A	-0.7598	-0.0615	22-A	0.9474	0.0767
		0.0609			0.1856

This completes Cycle 3.

Figure 123. Problem Three - Deflection Average Cycle 3



Loads	0		-10.1321		-0.0342		0.0196		-0.0051		2.1463		0	
V		8.1004		-2.0317		-2.0659		-2.0463		-2.0514		0.0949		
M	0		8.1004		6.0687		4.0028		1.9565		-0.0949		0	h
M/EI	0		-8.1004		-6.0687		-4.0028		-1.9565		0.0949		0	h/EI
			A		B		C		D		E			
E.C. M/EI			-38.4703		-36.3781		-24.0363		-11.7341		-1.577			$h^2/6EI$
Slope		72.503		34.0327		-2.3454		-26.3817		-38.1158		-39.6928		$h^2/6EI$
Y	0		72.503		106.5357		104.1903		77.8086		39.6928		0	$h^3/6EI$
Y	0		1.085		1.5943		1.5592		1.1644		0.594		0	inches

Simultaneous equations needed to obtain values for the angle change (M/EI) line:

$4A + B = -38.4703$

$A + 4B + C = -36.3781$

$B + 4C + D = -24.0363$

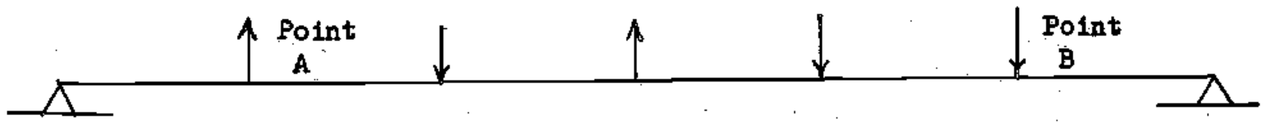
$C + 4D + E = -11.7341$

$D + 4E = -1.577$

Solutions:

$A = -8.1004$
$B = -6.0687$
$C = -4.0028$
$D = -1.9565$
$E = 0.09489$

Figure 124. Problem Three - Beam B-B Cycle 4



Loads	0		0.7249		-0.0049		0.0081		-0.0054		-1.2562		0	
V		-0.3937		0.3312		0.3263		0.3344		0.329		-0.9272		
M	0		-0.3937		-0.0625		0.2638		0.5982		0.9272		0	h
M/EI	0		0.3937		0.0625		-0.2638		-0.5982		-0.9272		0	h/EI
			A		B		C		D		E			
E.C. M/EI			1.6373		0.3796		-1.5909		-3.5838		-4.307			$h^2/6EI$
Slope		1.0904		2.7277		3.1073		1.5164		-2.0674		-6.3744		$h^2/6EI$
Y	0		1.0904		3.8181		6.9254		8.4418		6.3744		0	$h^3/6EI$
Y	0		0.1856		0.6499		1.1788		1.4369		1.085		0	inches

Simultaneous correction equations for values of angle change (M/EI) line:

$4A + B = 1.6373$

$A + 4B + C = 0.3796$

$B + 4C + D = -1.5909$

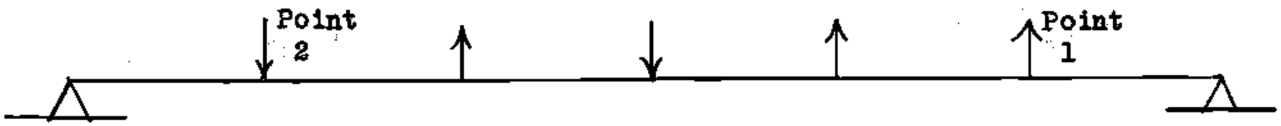
$C + 4D + E = -3.5838$

$D + 4E = -4.307$

Solution:

$A = 0.3937$
$B = 0.0625$
$C = -0.2638$
$D = -0.5982$
$E = -0.9272$

Figure 125. Problem Three - Beam 2-2 Cycle 4



Loads	0		-2.1483		0.0003		-0.0055		0.0086		0.9668		0	
V		1.6288		-0.5195		-0.5192		-0.5247		-0.5161		0.4507		
M	0		1.6288		1.1093		0.5901		0.0654		-0.4507		0	h
M/EI	0		-1.6288		-1.1093		-0.5901		-0.0654		0.4507		0	h/EI
			A		B		C		D		E			
E.C. M/EI			-7.6245		-6.6555		-3.5349		-0.401		1.7374			$h^2/6EI$
Slope		12.4023		4.7778		-1.8777		-5.4126		-5.8136		-4.0762		$h^2/6EI$
Y	0		12.4023		17.1801		15.3024		9.8898		4.0762		0	$h^3/6EI$
Y	0		0.1856		0.2571		0.229		0.148		0.061		0	inches

Simultaneous equations needed to obtain values for the angle change (M/EI) line:

$4A + B = -7.6245$

$A + 4B + C = -6.6555$

$B + 4C + D = -3.5349$

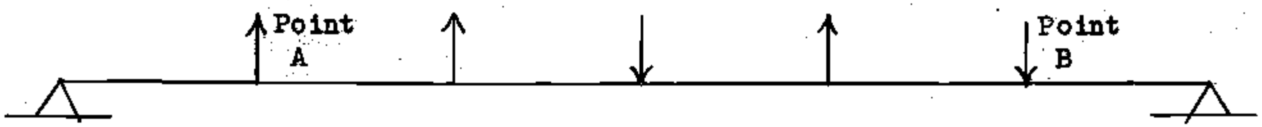
$C + 4D + E = -0.401$

$D + 4E = 1.7374$

Solution:

$A = -1.6288$
$B = -1.1093$
$C = -0.5901$
$D = -0.0654$
$E = 0.4507$

Figure 126. Problem Three - Beam A-A Cycle 4



Loads	0		0.4477		0.0072		-0.0108		0.0078		-0.7271		0	
V		-0.2539		0.1938		0.201		0.1902		0.198		-0.5291		
M	0		-0.2539		-0.0601		0.1409		0.3311		0.5291		0	h
M/EI	0		0.2539		0.0601		-0.1409		-0.3311		-0.5291		0	h/EI
			A		B		C		D		E			
E.C. M/EI			1.0756		0.3532		-0.8349		-1.9945		-2.4475			$h^2/6EI$
Slope		0.3584		1.434		1.7872		0.9523		-1.0422		-3.4897		$h^2/6EI$
Y	0		0.3584		1.7924		3.5796		4.5319		3.4897		0	$h^3/6EI$
Y	0		0.061		0.3051		0.6093		0.7714		0.594		0	inches

Simultaneous needed to obtain values for the angle change (M/EI) line:

$4A + B = 1.0756$

$A + 4B + C = 0.3532$

$B + 4C + D = -0.8349$

$C + 4D + E = -1.9945$

$D + 4E = -2.4475$

Solution:

$A = 0.25389$
$B = 0.0601$
$C = -0.1409$
$D = -0.3311$
$E = -0.529098$

Figure 127. Problem Three - Beam 1-1 Cycle 4

Average of interaction loads:

Node	Load
AA-1	0.9668 up
11-A	0.4477 up
	0.2595 up on AA-1 down on 11-A

Node	Load
AA-2	2.1483 down
22-A	0.7249 up
	1.4366

Node	Load
BB-1	2.1463 up
11-B	0.7271 down
	1.4367

Node	Load
BB-2	0.1321 down
22-B	1.2562 down
	0.562 up on BB-2 down on 22-B

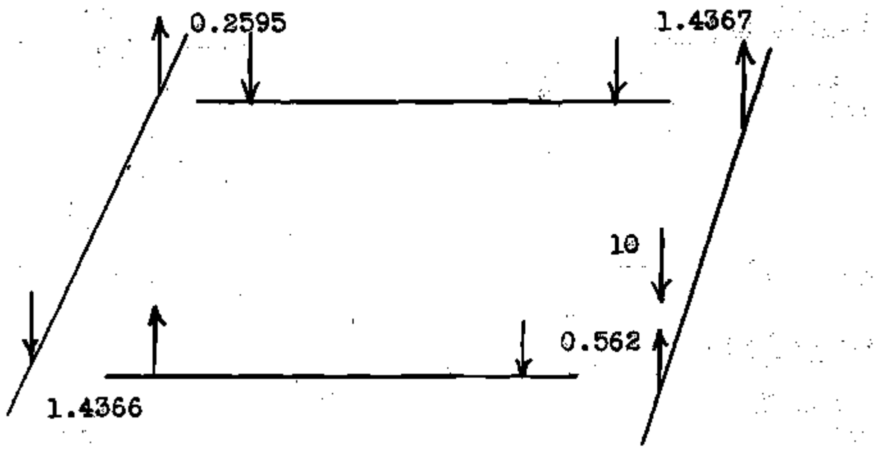
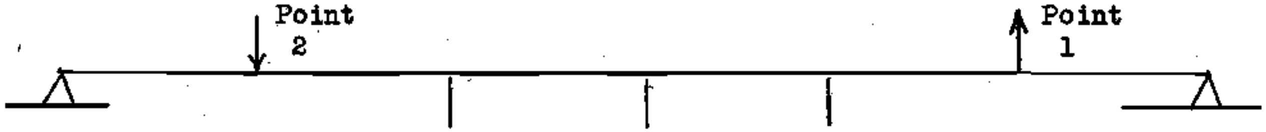
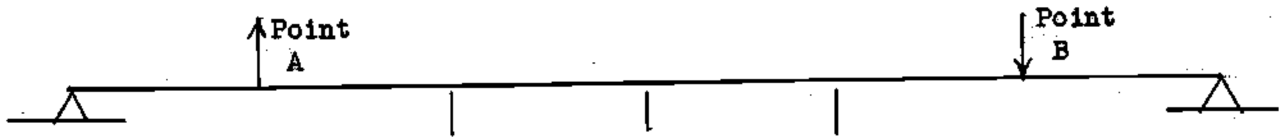


Figure 128. Problem Three - Interaction Load Average Cycle 4



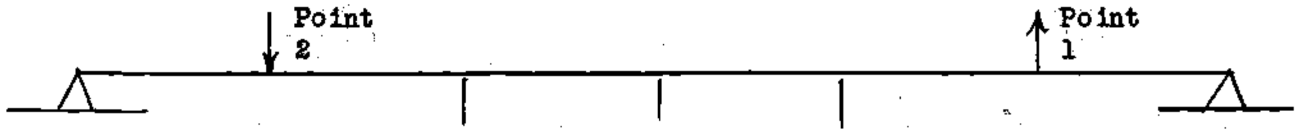
Loads	0		-9.438		0		0		0		1.44		0	
Shear		0		-9.438		-9.438		-9.438		-9.438		-7.998		
M Trial	0		0		-9.438		-18.876		-28.314		-37.752		-45.75	h
Corr M	0		7.625		15.25		22.875		30.5		38.125		45.75	h
M	0		7.625		5.812		3.999		2.186		0.373		0	h
M/EI	0		-7.625		-5.812		-3.999		-2.186		-0.373		0	h/EI
E.C. M/EI	-7.625		-36.312		-34.872		-23.994		-13.116		-3.678		-0.373	$h^2/6EI$
Slope		71.184		34.872		0		-23.994		--37.11		-40.768		$h^2/6EI$
Y Trial	0		71.184		106.056		106.056		82.062		44.952		4.164	$h^3/6EI$
Corr Y	0		-0.694		-1.388		-2.082		-2.776		-3.47		-4.164	$h^3/6EI$
Y	0		70.49		104.668		103.974		79.286		41.482		0	$h^3/6EI$
Y	0		1.0548		1.5663		1.5559		1.1865		0.6207		0	inches

Figure 129. Problem Three - Beam B-B



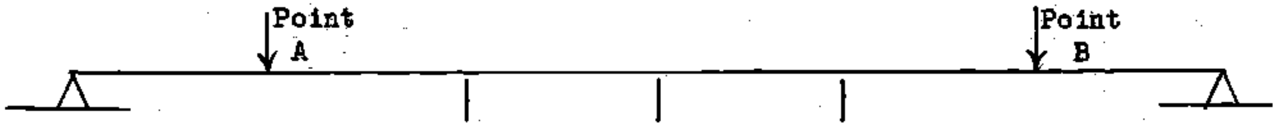
Loads	0		1.44		0		0		0		-0.562		0	
Shear		0		1.44		1.44		1.44		1.44		0.878		
M Trial	0		0		1.44		2.88		4.32		5.76		6.638	h
Corr M	0		-1.106		-2.212		-3.318		-4.425		-5.531		-6.638	h
M	0		-1.106		-0.772		-0.438		-0.105		0.229		0	h
M/EI	0		1.106		0.772		0.438		0.105		-0.229		0	h/EI
E.C. M/EI	1.106		5.196		4.632		2.629		0.629		-0.811		-0.229	$h^2/6EI$
Slope		-9.828		-4.632		0		2.629		3.258		2.447		$h^2/6EI$
Y Trial	0		-9.828		-14.46		-14.46		-11.831		-8.573		-6.126	$h^3/6EI$
Corr Y	0		1.021		2.042		3.063		4.084		5.105		6.126	$h^3/6EI$
Y	0		-8.807		-12.418		-11.397		-7.747		-3.468		0	$h^3/6EI$
Y	0		-1.499		-2.1137		-1.9399		-1.3186		-0.5903		0	inches

Figure 130. Problem Three - Beam 2-2



Loads	0		-1.44		0		0		0		0.26		0	
Shear		0		-1.44		-1.44		-1.44		-1.44		-1.18		
M Trial	0		0		-1.44		-2.88		-4.32		-5.76		-6.94	h
Corr M	0		1.156		2.313		3.469		4.626		5.783		6.94	h
M	0		1.156		0.873		0.589		0.306		0.023		0	h
M/EI	0		-1.156		-0.873		-0.589		-0.306		-0.023		0	h/EI
E.C. M/EI	-1.156		-5.497		-5.237		-3.535		-1.836		-0.398		-0.023	h^2/6EI
Slope		10.734		5.237		0		-3.535		-5.371		-5.769		h^2/6EI
Y	0		10.734		15.971		15.971		12.436		7.065		1.296	h^3/6EI
Corr Y	0		-0.216		-0.432		-0.648		-0.864		-1.08		-1.296	h^3/6EI
Y	0		10.518								5.985		0	h^3/6EI
Y	0		0.1574								0.0896		0	inches

Figure 131. Problem Three - Beam A-A



Loads	0		-0.26		0		0		0		-1.44		0	
Shear		0		-0.26		-0.26		-0.26		-0.26		-1.7		
M Trial	0		0		-0.26		-0.52		-0.78		-1.04		-2.74	h
Corr M	0		0.456		0.913		1.369		1.826		2.283		2.74	h
M	0		0.456		0.653		0.849		1.046		1.243		0	h
M/EI	0		-0.456		-0.653		-0.849		-1.046		-1.243		0	h/EI
E.C. M/EI	-0.456		-2.477		-3.917		-5.095		-6.276		-6.018		-1.243	$h^2/6EI$
Slope		11.489		9.012		5.095		0		-6.276		-12.294		$h^2/6EI$
Y Trial	0		11.489		20.501		25.596		25.596		19.32		7.026	$h^3/6EI$
Corr Y	0		-1.171		-2.342		-3.513		-4.684		-5.855		-7.026	$h^3/6EI$
Y	0		10.318								13.465		0	$h^3/6EI$
Y	0		1.7562								2.2919		0	inches

Figure 132. Problem Three - Beam 1-1

Deflection value average according to Fig. 99:

Node	Value	Average	Node	Value	Average
BB-1	0.6207	0.57	BB-2	1.0548	0.969
11-B	2.2919	0.185	22-B	-0.5903	-0.047
		0.755			0.922
AA-1	0.0896	0.0823	AA-2	0.1574	0.1446
11-A	1.7562	0.1422	22-A	-1.499	-0.1214
		0.2245			0.0232

This completes four cycles of operation. Plots of both deflection and interaction load values follow.

Figure 133. Deflection Average Cycle 4 of Problem Three

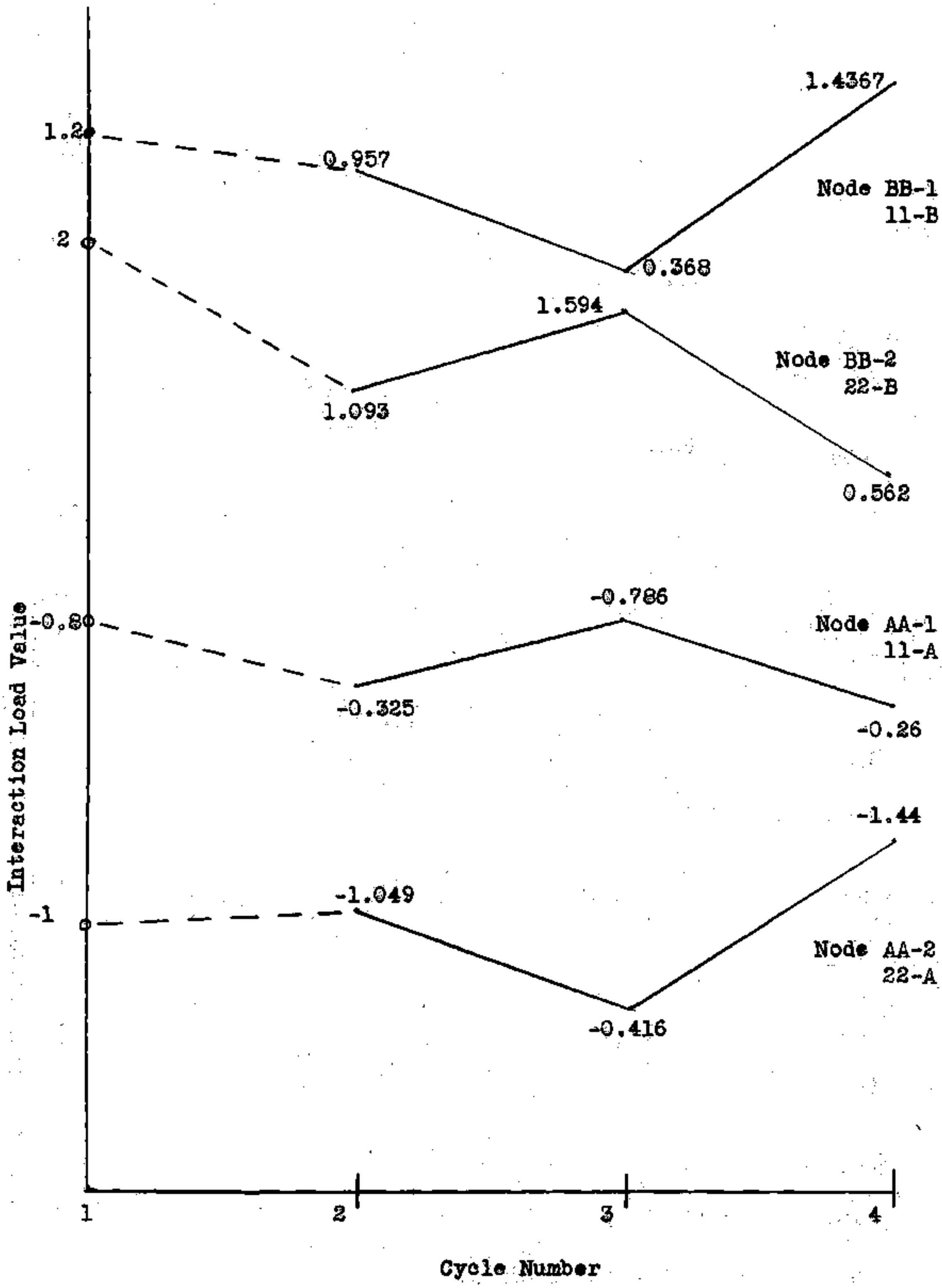


Figure 134. Problem Three - Interaction Load Graph

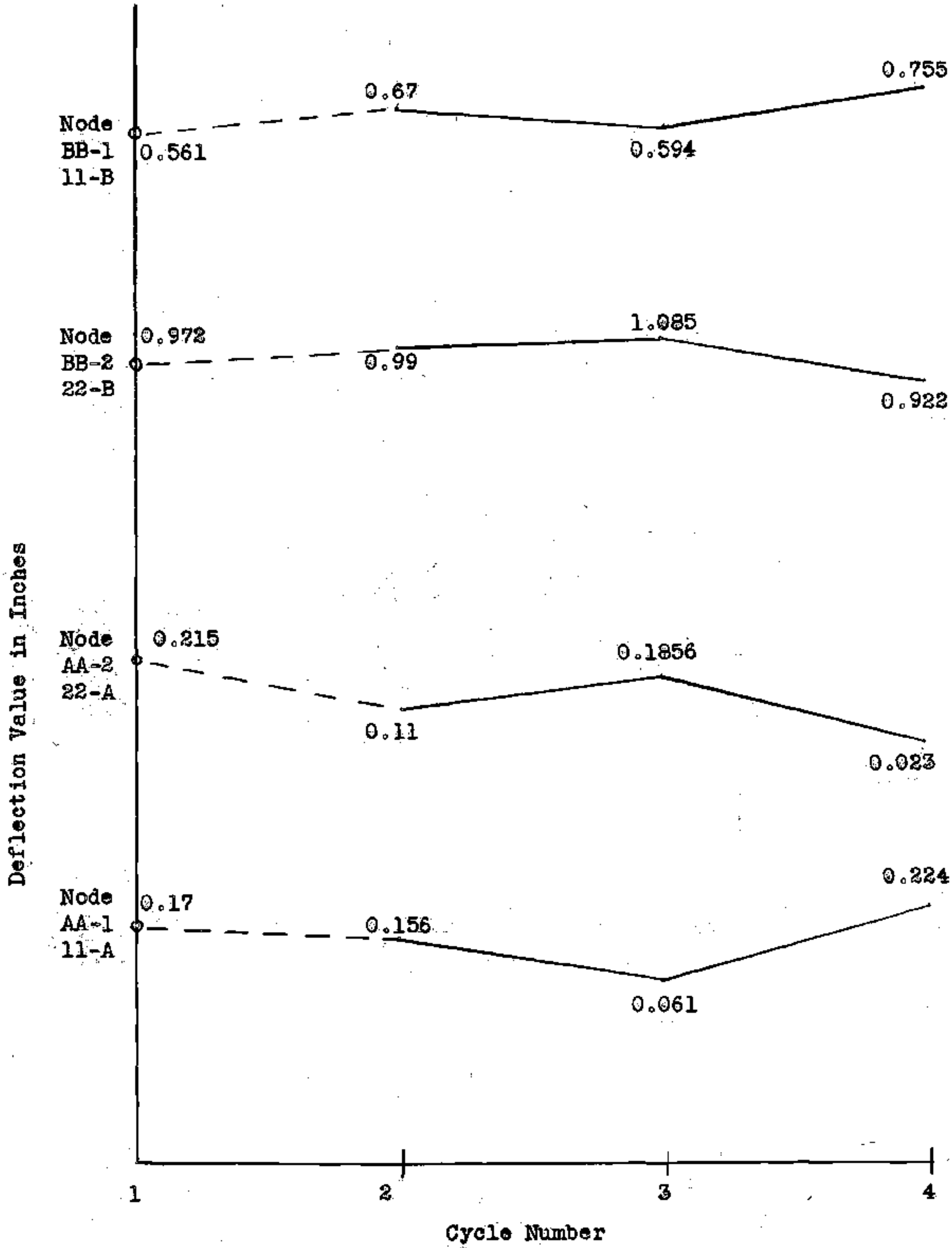


Figure 135. Problem Three - Deflection Graph

Interpolation of interaction load values :

BB-1	0.957	0.368	
11-B	0.368	1.436	2(0.662) = 1.324
	2 | 1.325	2 | 1.804	0.902
	0.662	0.902	3 | 2.226
			0.742 = New value

BB-2	1.093	1.594	
22-B	1.594	0.562	2(1.343) = 2.686
			1.078
	2 | 2.687	2 | 2.156	3 | 3.764
	1.343	1.078	
			1.254 = New value

AA-1	0.325	0.786	
11-A	0.786	0.26	2(0.555) = 1.11
	2 | 1.111	2 | 1.046	0.523
	0.555	0.523	3 | 1.633
			0.544 = New value

AA-2	1.049	0.416	
22-A	0.416	1.44	2(0.732) = 1.464
	2 | 1.465	2 | 1.856	0.928
	0.732	0.928	3 | 2.392
			0.797 = New value

Figure 136. Problem Three - Interaction Load Interpolation

Interpolation of deflection values:

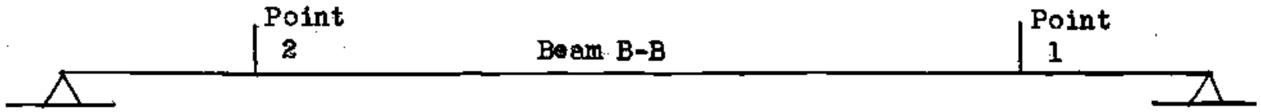
BB-1	0.67	0.594	
11-B	0.594	0.755	2(0.632) = 1.264
	2 | 1.264	2 | 1.349	0.674
	0.632	0.674	3 | 1.938
			0.646 = New value

BB-2	0.99	1.085	
22-B	1.085	0.922	2(1.037) = 2.074
	2 | 2.075	2 | 2.007	1.003
	1.037	1.003	3 | 3.077
			1.026 = New value

AA-2	0.11	0.1856	
22-A	0.1856	0.023	2(0.1478) = 0.2956
	2 | 0.2956	2 | 0.2086	0.1043
	0.1478	0.1043	3 | 0.3999
			0.133 = New value

AA-1	0.156	0.061	
11-A	0.061	0.224	2(0.108) = 0.216
	2 | 0.217	2 | 0.285	0.142
	0.108	0.142	3 | 0.358
			0.119 = New value

Figure 137. Problem Three - Interpolation of Deflection Values

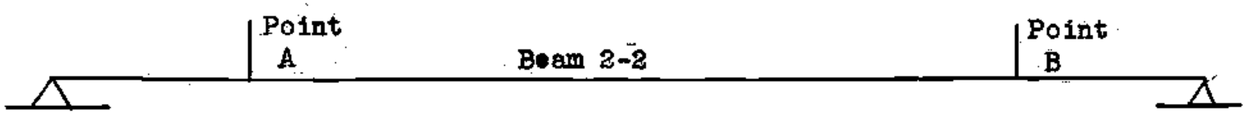


Y	1.026	0.646
Load	X	Y

$$0.123X + 0.08225Y = 1.026$$

$$0.08225X + 0.123Y = 0.646$$

X = 8.7348 down Y = 0.5882 up



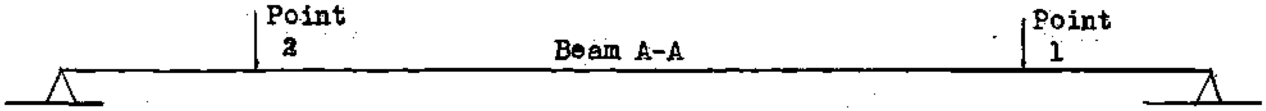
Y	0.133	1.026
Load	X	Y

$$1.4X + 0.935Y = 0.133$$

$$0.935X + 1.4Y = 1.026$$

X = 0.7119 up Y = 1.2083 down

Figure 138. Problem Three - Beam B-B & Beam 2-2 Cycle 5

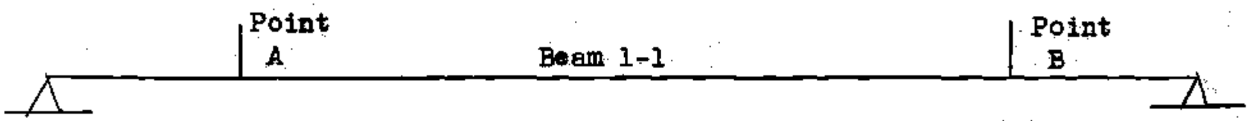


Y	0.133	0.119
Load	X	Y

$$0.123X + 0.08225Y = 0.133$$

$$0.08225X + 0.123Y = 0.119$$

X = 0.7856 down   Y = 0.4422 down



Y	0.119	0.646
Load	X	Y

$$1.4X + 0.935Y = 0.119$$

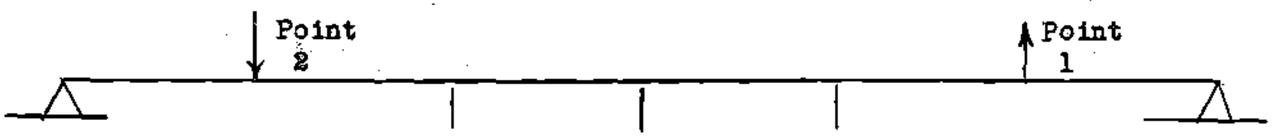
$$0.935X + 1.4Y = 0.646$$

X = 0.4028 up   Y = 0.7305 down

Figure 139. Problem Three - Beam A-A & Beam 1-1 Cycle 5

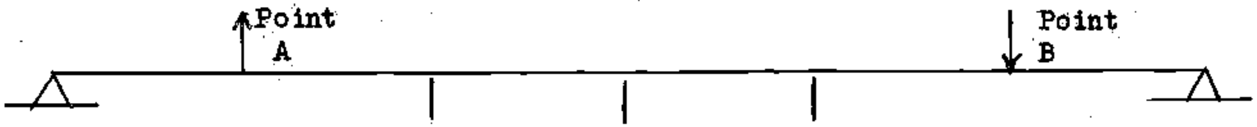
Node		Load	Node		Load
BB-1		0.5882 up	BB-2		1.2652 up
11-B		0.7305 down	22-B		1.2083 down
	2	1.3187		2	2.4735
		0.659 = New value			1.236 = New value
AA-1		0.4422 down	AA-2		0.7856 down
11-A		0.4028 up	22-A		0.7119 up
	2	0.8450		2	1.4975
		0.422 = New value			0.748 = New value

Figure 140. Problem Three - Interaction Load Average Cycle 5



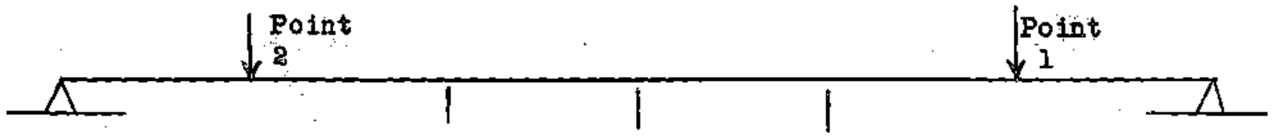
Loads	0		-8.764		0		0		0		0.659		0	
V Trial		0		-8.764		-8.764		-8.764		-8.764		-8.105		
M Trial	0		0		-8.764		-17.528		-26.292		-35.056		-43.161	h
Corr M	0		7.193		14.387		21.58		28.774		35.967		43.161	h
M	0		7.193		5.623		4.052		2.482		0.911		0	h
M/EI	0		-7.193		-5.623		-4.052		-2.482		-0.911		0	h/EI
E.C. M/EI	-7.193		-34.395		-33.737		-24.313		-14.891		-6.126		-0.911	$h^2/6EI$
Slope		68.132		33.737		0		-24.313		-39.204		-45.33		$h^2/6EI$
Y	0		68.132		101.869		101.869		77.556		38.352		-6.978	$h^3/6EI$
Corr Y	0		1.163		2.326		3.489		4.652		5.815		6.978	$h^3/6EI$
Y	0		69.295								44.167		0	$h^3/6EI$
Y	0		1.0369								0.6609		0	inches

Figure 141. Problem Three - Beam B-B Cycle 5



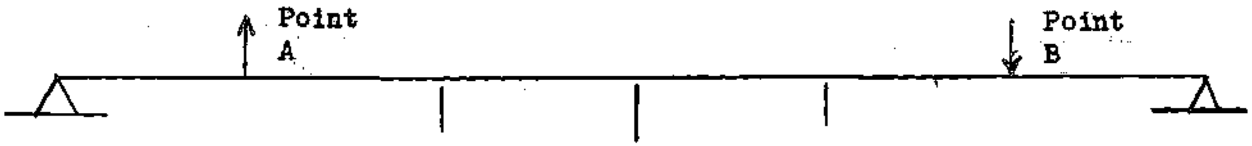
Loads	0		0.748		0		0		0		-1.236		0	
V Trial		0		0.748		0.748		0.748		0.748		-0.488		
M Trial	0		0		0.748		1.496		2.244		2.992		2.504	h
Corr M	0		-0.417		-0.834		-1.251		-1.669		-2.086		-2.504	h
M	0		-0.417		-0.086		0.245		0.575		0.906		0	h
M/EI	0		0.417		0.086		-0.245		-0.575		-0.906		0	h/EI
E.C. M/EI	0.417		1.754		0.516		-1.469		-3.451		-4.199		-0.906	h^2/6EI
Slope		-0.801		0.953		1.469		0		-3.451		-7.65		h^2/6EI
Y Trial	0		-0.801		0.152		1.621		1.621		-1.83		-9.48	h^3/6EI
Y Corr	0		1.58		3.16		4.74		6.32		7.9		9.48	h^3/6EI
Y	0		0.779								6.07		0	h^3/6EI
Y	0		0.1325								1.0331		0	inches

Figure 142. Problem Three - Beam 2-2 Cycle 5



Loads	0		-0.748		0		0		0		-0.422		0	
V Trial		0		-0.748		-0.748		-0.748		-0.748		-1.17		
M Trial	0		0		-0.748		-1.496		-2.244		-2.992		-4.162	h
Corr M	0		0.693		1.387		2. 08		2.774		3.468		4.162	h
M	0		0.693		0.639		0.584		0.53		0.476		0	h
M/EI	0		-0.693		-0.639		-0.584		-0.53		-0.476		0	h/EI
E.C. M/EI	-0.693		-3.411		-3.833		-3.505		-3.18		-2.434		-0.476	$h^2/6EI$
Slope		7.244		3.833		0		-3.505		-6.685		-9.119		$h^2/6EI$
Y Trial	0		7.244		11.077		11.077		7.572		0.887		-8.232	$h^3/6EI$
Corr Y	0		1.372		2.744		4.116		5.488		6.86		8.232	$h^3/6EI$
Y	0		8.616								7.747		0	$h^3/6EI$
Y	0		0.1289								0.1159		0	inches

Figure 143. Problem Three - Beam A-A Cycle 5



Loads	0		0.422		0		0		0		-0.659		0	
V Trial		0		0.422		0.422		0.422		0.422		-0.237		
M Trial	0		0		0.422		0.844		1.266		1.688		1.451	h
Corr M	0		-0.241		-0.483		-0.725		-0.967		-1.209		-1.451	h
M	0		-0.241		-0.061		0.119		0.299		0.479		0	h
M/EI	0		0.241		0.061		-0.119		-0.299		-0.479		0	h/EI
E.C. M/EI	0.241		1.025		0.366		-0.714		-1.794		-2.215		-0.479	$h^2/6EI$
Slope		-0.677		0.348		0.714		0		-1.794		-4.009		$h^2/6EI$
Y Trial	0		-0.677		-0.329		0.385		0.385		-1.409		-5.418	$h^3/6EI$
Corr Y	0		0.903		1.806		2.709		3.612		4.515		5.418	$h^3/6EI$
Y	0		0.226								3.106		0	$h^3/6EI$
Y	0		0.0385								0.5286		0	inches

Figure 144. Problem Three - Beam 1-1 Cycle 5

Cycle 5 average of deflection values:

BB-1	0.6609	0.6073	BB-2	1.0369	0.9529
11-B	0.5286	0.0428	22-B	1.0331	0.0836
New value		0.6501			1.0365
AA-1	0.1159	0.1065	AA-2	0.1289	0.1184
11-A	0.0385	0.0031	22-A	0.1325	0.0107
New value		0.1096			0.1291

Comparison of the deflection values of Cycle 4 and Cycle 5 follows:

Node BB-1 and 11-B differs by 0.004 inches

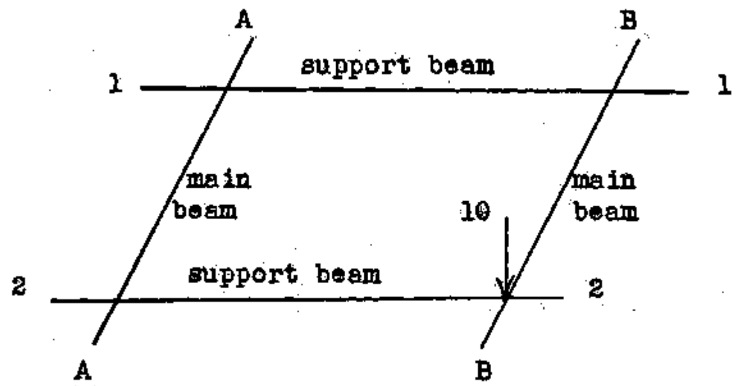
Node BB-2 and 22-B differs by 0.01 inches

Node AA-1 and 11-A differs by 0.0094 inches

Node AA-2 and 22-A differs by 0.004 inches

In all cases the error is less then 0.01 inches and is considered sufficiently small for most engineering purposes. The problem is solved.

Figure 145. Problem Three - Deflection Average



All beams are pinned to fixed supports.

Main beam = 10WF45 with I of 248.6

Support beam = 10WF45 with I of 248.6

Dimensions are the same as Problem One.

Assume initial interaction loads as follows:

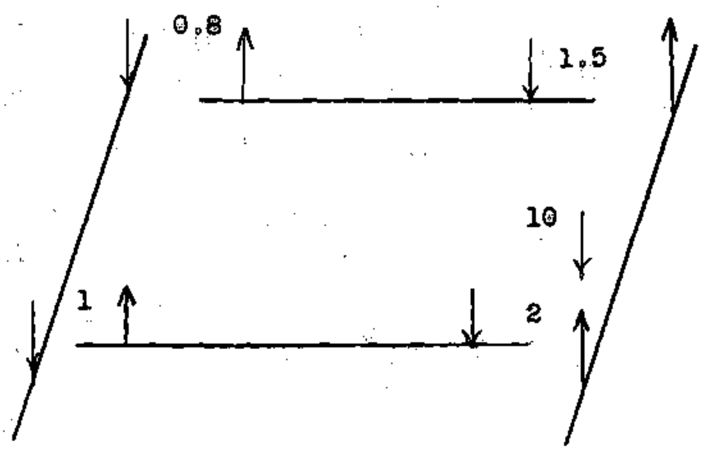
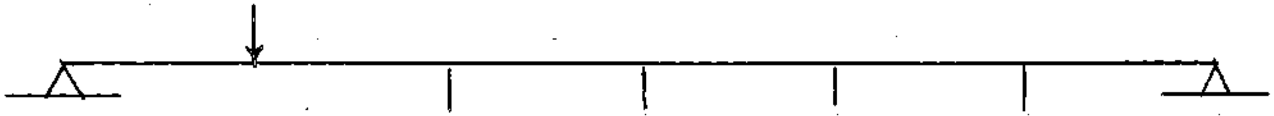
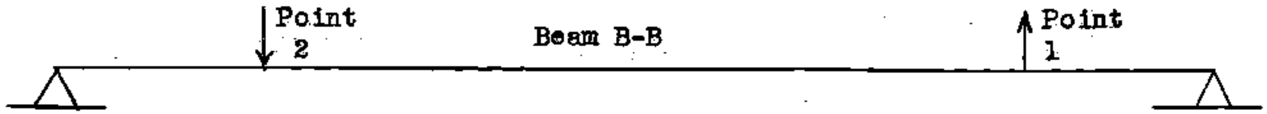


Figure 146. Problem Four - Grillage System

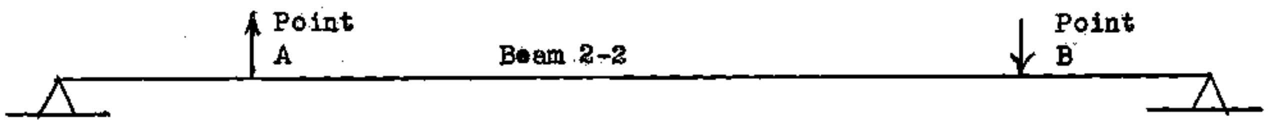


Load	0		-1		0		0		0		0		0	
V		0		-1		-1		-1		-1		-1		
M Trial	0		0		-1		-2		-3		-4		-5	h
Corr M	0		0.833		1.666		2.499		3.333		4.166		5	h
M	0		0.833		0.666		0.499		0.333		0.166		0	h
M/EI	0		-0.833		-0.666		-0.499		-0.333		-0.166		0	h/EI
E.C. M/EI	-0.833		-3.998		-3.996		-2.995		-1.997		-0.997		-0.166	$h^2/6EI$
Slope		7.994		3.996		0		-2.995		-4.992		-5.989		$h^2/6EI$
Y Trial	0		7.994		11.99		11.99		8.995		4.003		-1.986	$h^3/6EI$
Corr Y	0		0.331		0.662		0.993		1.324		1.655		1.986	$h^3/6EI$
Y	0		8.325		12.652		12.983		10.319		5.658		0	$h^3/6EI$
Y	0		0.32148		0.48857		0.50135		0.39848		0.21849		0	inches

Figure 147. Problem Four - Deflection Ratio Calculation

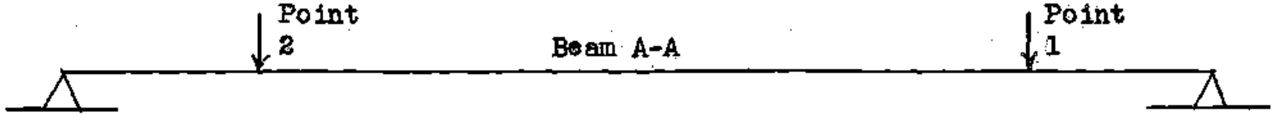


Loads	-8	1.5
Y Pt 2	2.5718	1.7479
Y Pt 1	-0.3277	-0.4822
Final Y	2.2441	1.2657

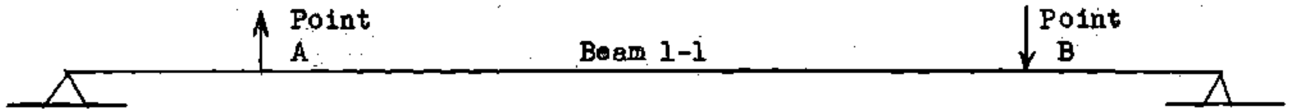


Loads	1	-2
Y Pt A	-0.3214	-0.2184
Y Pt B	0.4368	0.6428
Final Y	0.1154	0.4244

Figure 148. Problem Four - Beam B-B & Beam 2-2 Cycle 1



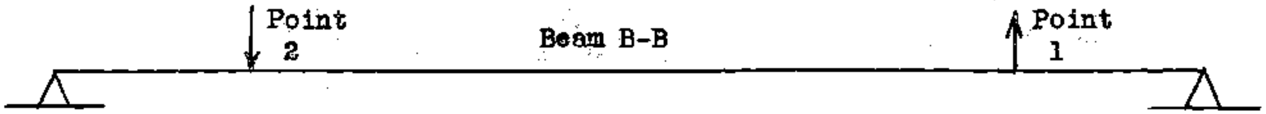
Loads	-1	-0.8
Y Pt 2	0.3214	0.2184
Y Pt 1	0.1748	0.2571
Final Y	0.4962	0.4755



Loads	0.8	-1.5
Y Pt A	-0.2571	-0.1748
Y Pt B	0.3277	0.4822
Final Y	0.0706	0.3074

BB-1	1.2657	BB-2	2.2441	AA-1	0.4755	AA-2	0.4962
11-B	0.3074	22-B	0.4244	11-A	0.0706	22-A	0.1154
Average = 0.7865		Average = 1.3342		Average = 0.273		Average = 0.3058	

Figure 149. Problem Four - Beam A-A & Beam 1-1 Cycle 1

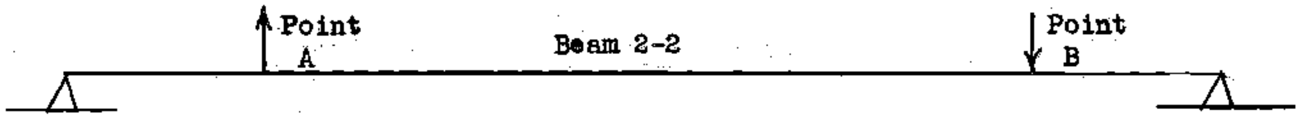


Y	1.3342	0.7865
Load	X	Y

$$0.32148X + 0.21849Y = 1.3342$$

$$0.21849X + 0.32148Y = 0.7865$$

New Loads	X = 4.6223 down	Y = 0.6948 up



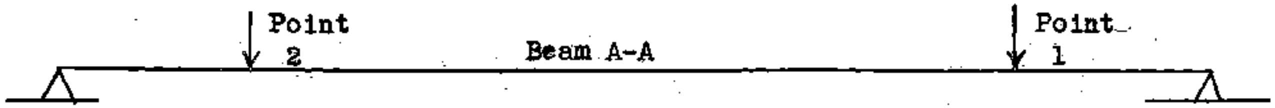
Y	0.3058	1.3342
Load	X	Y

$$0.32148X + 0.21849Y = 0.3058$$

$$0.21849X + 0.32148Y = 1.3342$$

New Loads	X = 3.4739 up	Y = 6.5113 down

Figure 150. Problem Four - Beam B-B & Beam 2-2 Cycle 2



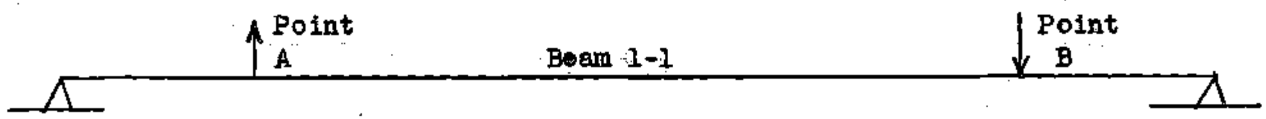
Y 0.3058 0.273

Load X Y

$$0.32148X + 0.21849Y = 0.3058$$

$$0.21849X + 0.32148Y = 0.273$$

New Loads X = 0.6952 down Y = 0.3769 down



Y 0.273 0.7865

Load X Y

$$0.32148X + 0.21849Y = 0.273$$

$$0.21849X + 0.32148Y = 0.7865$$

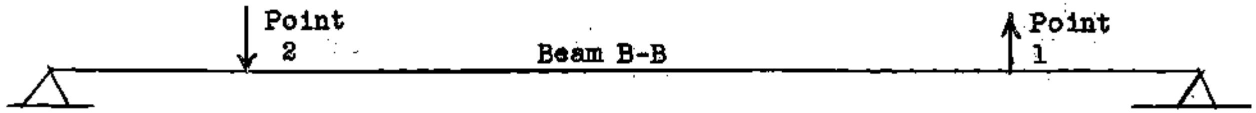
New Loads X = 1.5117 up Y = 3.4742 down

Figure 151. Problem Four - Beam A-A & Beam 1-1 Cycle 2

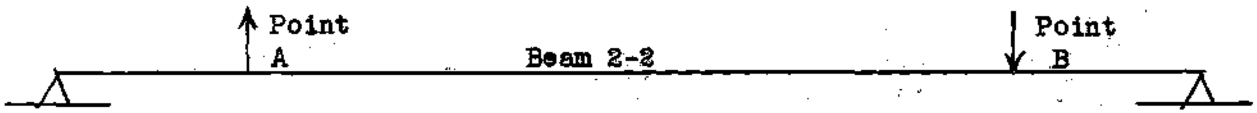
Node Point	Load Value	Direction	Node Point	Load Value	Direction
BB-1	0.6948	up	BB-2	5.3777	up
11-B	3.4742	down	22-B	6.5113	down
2	4.1690		2	11.889	
New value	2.084			5.944	
AA-1	0.3769	down	AA-2	0.6952	down
11-A	1.5117	up	22-A	3.4739	up
2	1.8886		2	4.1691	
New value	0.944			2.084	

These loads are used to obtain the final cycle two deflection values.

Figure 152. Problem Four - Interaction Load Average Cycle 2

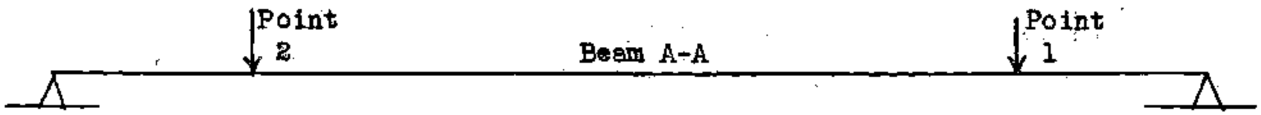


Loads	-4.056	2.084
Y Pt 2	1.3039	0.8861
Y Pt 1	-0.4553	-0.6699
Final Y	0.8486	0.2162

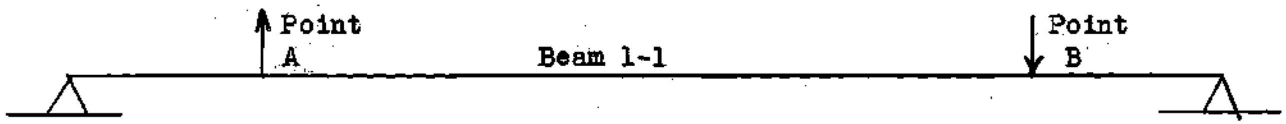


Loads	2.084	-5.944
Y Pt A	-0.6699	-0.4553
Y Pt B	1.2987	1.9108
Final Y	0.6288	1.4555

Figure 153. Problem Four - Beam B-B & Beam 2-2 Cycle 2



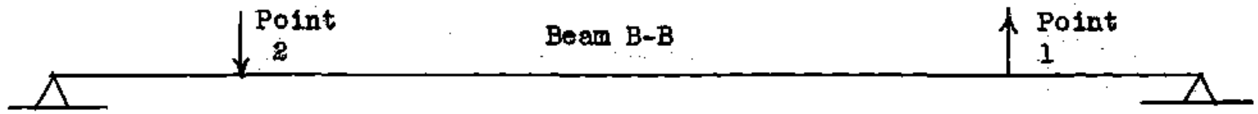
Load	-2.084	-0.944
Y Pt 2	0.6699	0.4553
Y Pt 1	0.2062	0.3034
Final Y	0.8761	0.7587



Load	0.944	-2.084
Y Pt A	-0.3034	-0.2062
Y Pt B	0.4553	0.6699
Final Y	0.1519	0.4637

BB-1	0.2162	BB-2	0.8486	AA-1	0.7587	AA-2	0.8761
11-B	0.4637	22-B	1.4555	11-A	0.1519	22-A	0.6288
Average	0.3399	Average	1.152	Average	0.4553	Average	0.7524

Figure 154. Problem Four - Beam A-A & Beam 1-1 Cycle 2

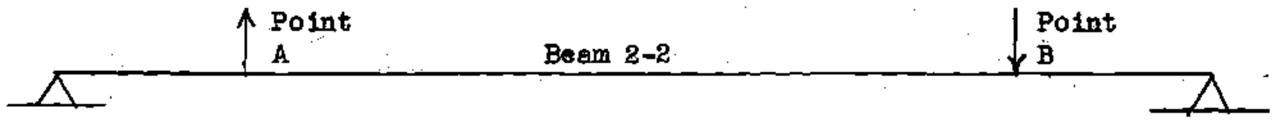


Y	1.152	0.3399
Loads	X	Y

$$0.32148X + 0.21849Y = 1.152$$

$$0.21849X + 0.32148Y = 0.3399$$

X = 5.3231 down　　　　Y = 2.5602 up



Y	0.7524	1.152
Loads	X	Y

$$0.32148X + 0.21849Y = 0.7524$$

$$0.21849X + 0.32148Y = 1.152$$

X = 0.1763 up　　　　Y = 3.7036 down

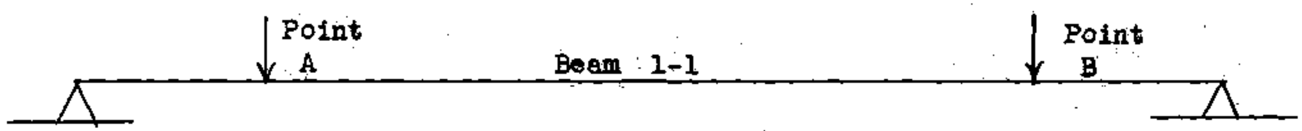
Figure 155. Problem Four - Beam B-B & Beam 2-2 Cycle 3

Y	0.7524	0.4553
Load	X	Y

$$0.32148X + 0.21849Y = 0.7524$$

$$0.21849X + 0.32148Y = 0.4553$$

X = 2.5603 down Y = 0.3237 up



Y	0.4553	0.3399
Load	X	Y

$$0.32148X + 0.21849Y = 0.4553$$

$$0.21849X + 0.32148Y = 0.3399$$

X = 1.2965 down Y = 0.1763 down

Figure 156. Problem Four - Beam A-A & Beam 1-1 Cycle 3

Node	Load Value	Direction
BB-1	2.5602	up
11-B	0.1763	down
2	2.7365	
	1.3682 = New value	

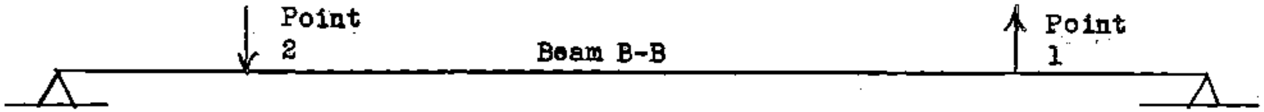
Node	Load Value	Direction
AA-1	0.3237	up
11-A	1.2965	down
2	1.6202	
	0.8101 = New value	

Node	Load Value	Direction
BB-2	4.6769	up
22-B	3.7036	down
2	8.3805	
	4.1902 = New value	

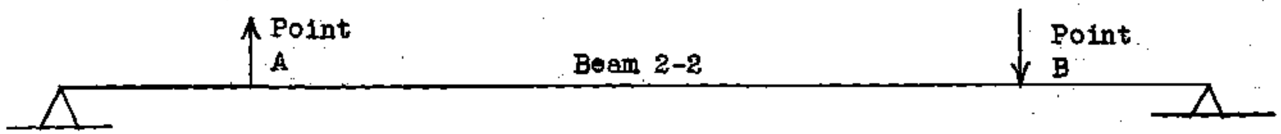
Node	Load Value	Direction
AA-2	2.5603	down
22-A	0.1763	up
2	2.7366	
	1.3683 = New value	

These loads are used to obtain the final cycle three deflections

Figure 157. Problem Four - Interaction Load Average Cycle 3



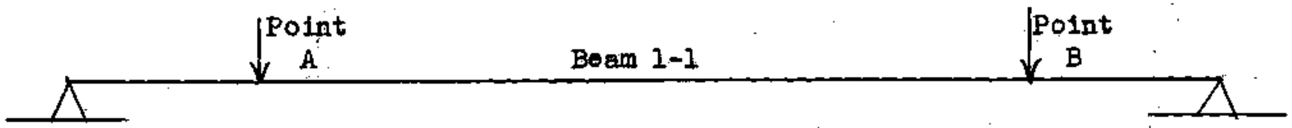
Loads	-5.8098	1.3682
Y Pt '2'	1.8677	1.2693
Y Pt '1'	-0.2989	-0.4398
Final Y	1.5688	0.8295



Loads	1.3682	-4.1902
Y Pt 'A'	-0.4398	-0.2989
Y Pt 'B'	0.9155	1.347
Final Y	0.4757	1.0481

Figure 158. Problem Four - Beam B-B & Beam 2-2 Cycle 3

Loads	-1.3682	0.8101
Y Pt '2'	0.4398	0.2989
Y Pt '1'	-0.1769	-0.2604
Final Y	0.2629	0.0385



Loads	-0.8101	-1.3682
Y Pt 'A'	0.2604	0.1769
Y Pt 'B'	0.2989	0.4398
Final Y	0.5593	0.6167

BB-1	0.8295	BB-2	1.5688	AA-1	0.0385	AA-2	0.2629
11-B	0.6167	22-B	1.0481	11-A	0.5593	22-A	0.4757
Average	0.7231		1.3084		0.2989		0.3693

Use the above computed average values as the final cycle three values.

Figure 159. Problem Four - Beam A-A & Beam 1-1 Cycle 3

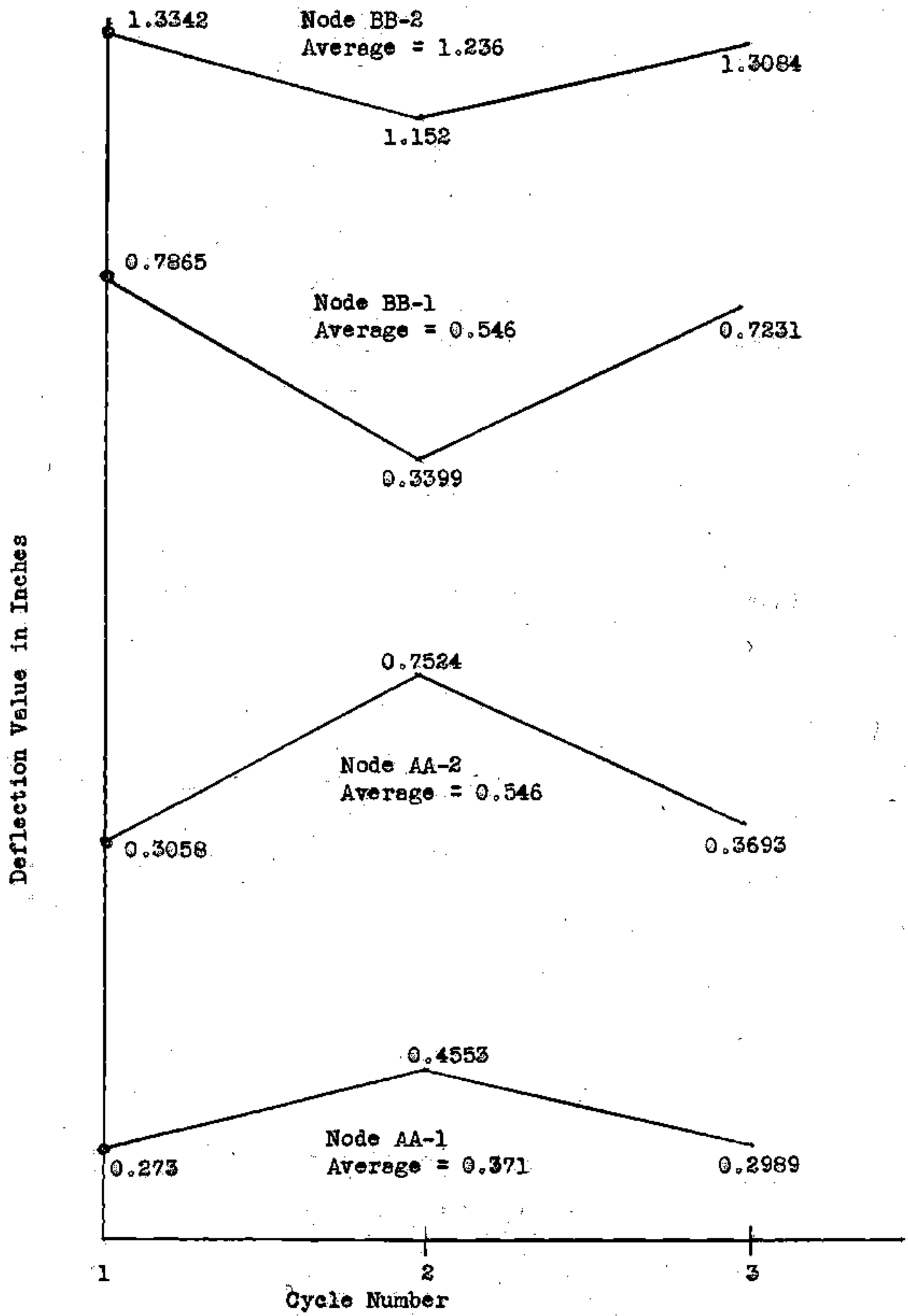
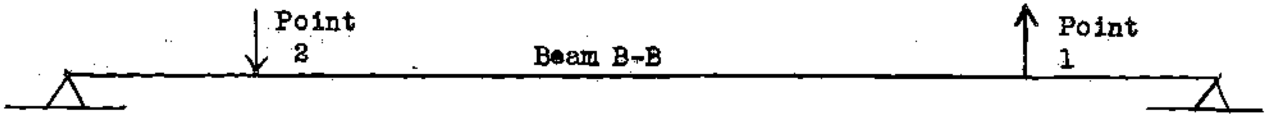


Figure 160. Problem Four - Deflection Versus Cycle

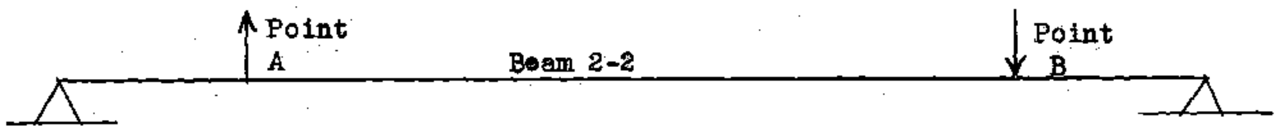


Y	1.236	0.546
Load	X	Y

$$0.32148X + 0.21849Y = 1.236$$

$$0.21849X + 0.32148Y = 0.546$$

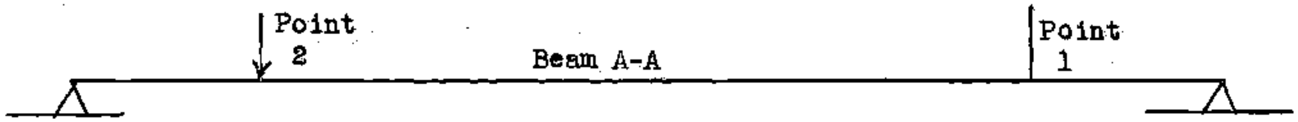
X = 4.9994 down Y = 1.6989 up



Y	0.546	1.236
Loads	1.6989 up	4.9994 down

This case is exactly the reverse of Beam B-B.

Figure 161. Problem Four - Beam B-B & Beam 2-2 Cycle 4

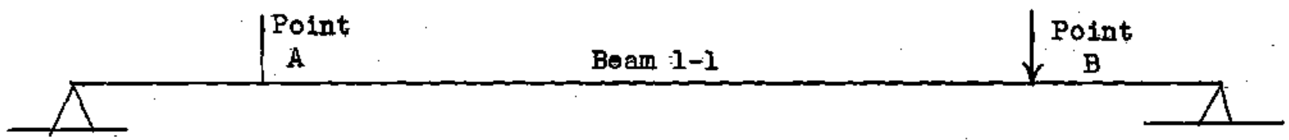


Y	0.546	0.371
Load	X	Y

0.32148X 0.21849Y = 0.546

0.21849X 0.32148Y = 0.371

X = 1.6983 down Y = 0



Y	0.371	0.546
Load	0	1.6983 down

Note that this is the reverse of Beam A-A.

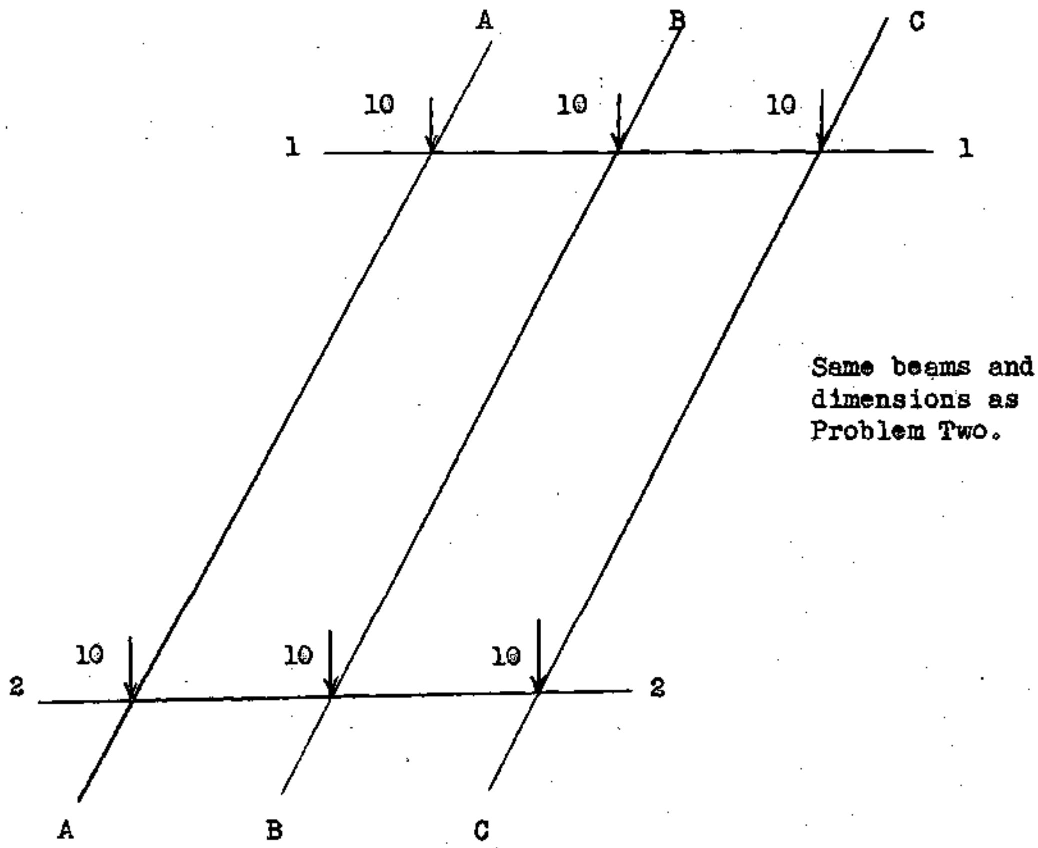
Figure 162. Problem Four - Beam A-A & Beam 1-1 Cycle 4

Node	Load Value	Direction
BB-1	1.6989	up
11-B	1.6983	down
	2 | 3.3972	
	1.6986 = New value	
AA-1	0	
11-A	0	
	0	

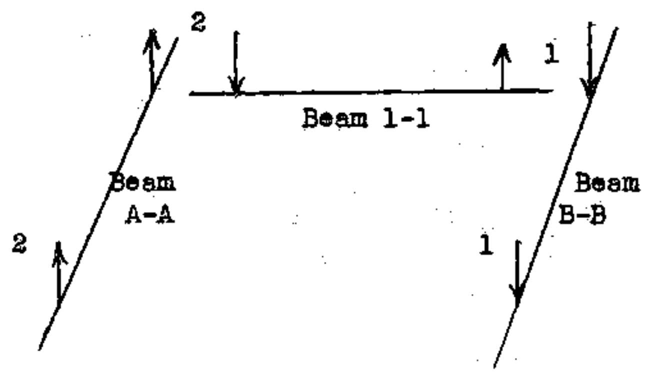
Node	Load Value	Direction
BB-2	5.0006	up
22-B	4.9994	down
	2 | 10	
	5 = New value	
AA-2	1.6983	down
22-A	1.6989	up
	2 | 3.3972	
	1.6986 = New value	

These values are obviously correct due to the symmetry of the problem. Inspection shows that the deflections at cross node points will agree. The problem is solved.

Figure 163. Problem Four - Deflection Average Cycle 4

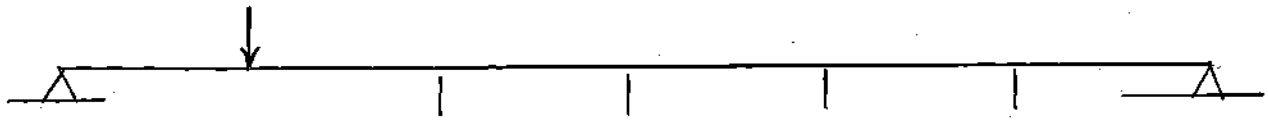


Assume initial interaction loads as follows:



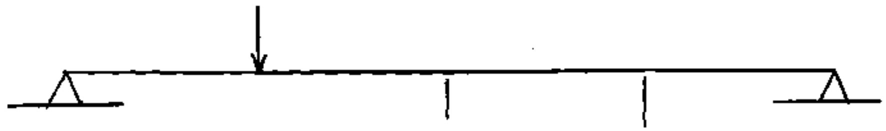
Due symmetry Beams A-A and C-C and also Beams 1-1 and 2-2 have the same interaction loads. Advantage of this symmetry is taken in the solution.

Figure 164. Problem Five - Grillage System



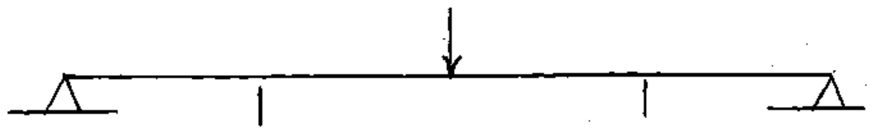
Load	0		-1		0		0		0		0		0	
V Trial		0		-1		-1		-1		-1		-1		
M Trial	0		0		-1		-2		-3		-4		-5	h
Corr M	0		0.833		1.666		2.499		3.333		4.166		5	h
M	0		0.833		0.666		0.499		0.333		0.166		0	h
M/EI	0		-0.833		-0.666		-0.499		-0.333		-0.166		0	h/EI
E.C. M/EI	-0.833		-3.998		-3.996		-2.995		-1.997		-0.977		-0.166	$h^2/6EI$
Slope		7.994		3.996		0		-2.995		-4.992		-5.989		$h^2/6EI$
Y	0		7.994		11.99		11.99		8.995		4.003		-1.986	$h^3/6EI$
Corr Y	0		0.331		0.662		0.993		1.324		1.655		1.986	$h^3/6EI$
Y	0		8.325		12.652		12.983		10.319		5.658		0	$h^3/6EI$
Y	0		0.12458		0.1893		0.19428		0.1544		0.08467		0	inches

Figure 165. Problem Five - Deflection Ratio Calculation For Beams A-A, B-B, C-C



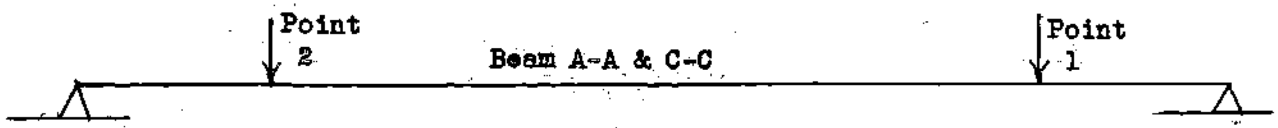
Load	0		-1		0		0		0	
V Trial		0		-1		-1		-1		
M Trial	0		0		-1		-2		-3	h
Corr M	0		0.75		1.5		2.25		3	h
M	0		0.75		0.5		0.25		0	h
M/EI	0		-0.75		-0.5		-0.25		0	h/EI
E.C. M/EI	-0.75		-3.5		-3		-1.5		-0.25	$h^2/6EI$
Slope		3.5		0		-3		-4.5		$h^2/6EI$
Y	0		3.5		3.5		0.5		-4	$h^3/6EI$
Corr Y	0		1		2		3		4	$h^3/6EI$
Y	0		4.5		5.5		3.5		0	$h^3/6EI$
Y	0		0.76595		0.93617		0.59574		0	inches

Figure 166. Problem Five - Deflection Ratio Calculation For Beams 1-1 And 2-2

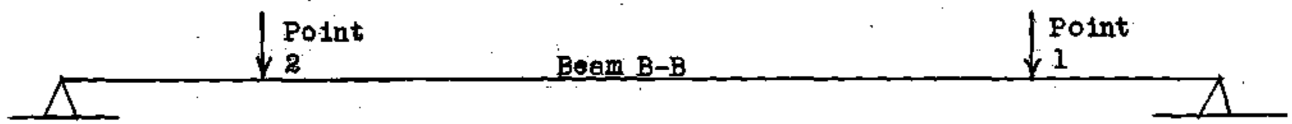


Load	0		0		-1		0		0	
V Trial		0		0		-1		-1		
M Trial	0		0		0		-1		-2	h
Corr M	0		0.5		1		1.5		2	h
M	0		0.5		1		0.5		0	h
M/EI	0		-0.5		-1		-0.5		0	h/EI
E.C. M/EI	-0.5		-3		-6		-3		-0.5	$h^2/6EI$
Slope		3		0		-6		-9		$h^2/6EI$
Y	0		3		3		-3		-12	$h^3/6EI$
Corr Y	0		3		6		9		12	$h^3/6EI$
Y	0		6		9		6		0	$h^3/6EI$
Y	0		1.02127		1.53190		1.02127		0	inches

Figure 167. Problem Five - Deflection Ratio Calculation For Beams 1-1, 2-2

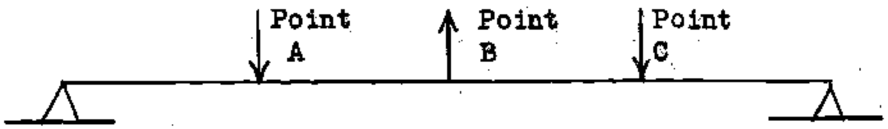


Loads	-8	-8
Y Pt '2'	0.9966	0.6773
Y Pt '1'	0.6773	0.9966
Final Y	1.6739	1.6739



Loads	-11	-11
Y Pt '2'	1.3703	0.9313
Y Pt '1'	0.9313	1.3703
Final Y	2.3016	2.3016

Figure 168. Problem Five - Beam A-A , B-B, C-C Cycle 1



Beam 1-1 & 2-2

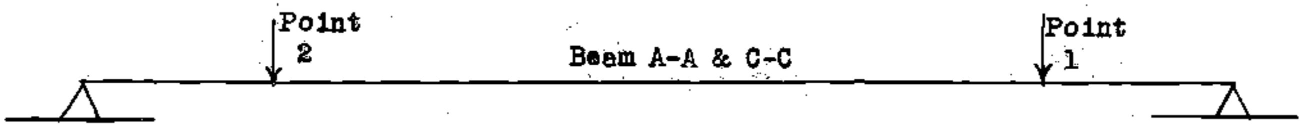
Loads	-2	1	-2
Y Pt 'A'	1.5319	1.8723	1.1914
Y Pt 'B'	-1.0212	-1.5319	-1.0212
Y Pt 'C'	1.1914	1.8723	1.5319
Final Y	1.7021	2.2127	1.7021

Proportion deflections according to the ratio of stiffness at the node point as determined in Fig. 165 through 167. These ratios are as follows:

AA-1	86%	BB-1	92.48%
11-A	14%	11-B	7.52%

Node Point	Value	Proportion	Node Point	Value	Proportion
AA-1	1.6739	1.4441	BB-1	2.3016	2.1285
11-A	1.7021	0.2383	11-B	2.2127	0.1663
		1.6824 = New Value		New Value =	2.2948

Figure 169. Problem Five- Completion of Cycle 1

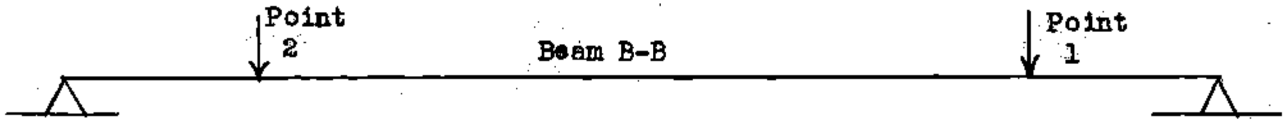


Y	1.6824	1.6824
Load	X	Y

$$0.12458X + 0.08467Y = 1.6824$$

$$0.08467X + 0.12458Y = 1.6824$$

X = 8.04 down   Y = 8.04 down



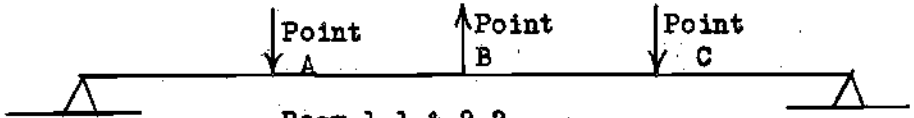
Y	2.2948	2.2948
Load	X	Y

$$0.12458X + 0.08467Y = 2.2948$$

$$0.08467X + 0.12458Y = 2.2948$$

X = 10.967 down   Y = 10.967 down

Figure 170. Problem Five - Beams A-A, B-B, C-C Cycle 2



Beam 1-1 & 2-2

Y	1.6824	2.2948	1.6824
Load	X	Y	Z

$$0.76595X + 1.02127Y + 0.59574Z = 1.6824$$

$$0.93617X + 1.5319Y + 0.93617Z = 2.2948$$

$$0.59574X + 1.02127Y + 0.76595Z = 1.6824$$

X = 1.3443 down Z = 1.3443 down

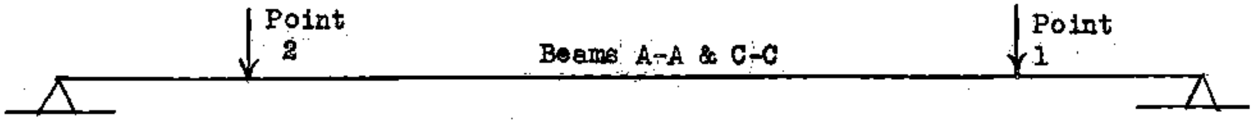
Y = 0.14508 up

Average of loads is calculated as follows:

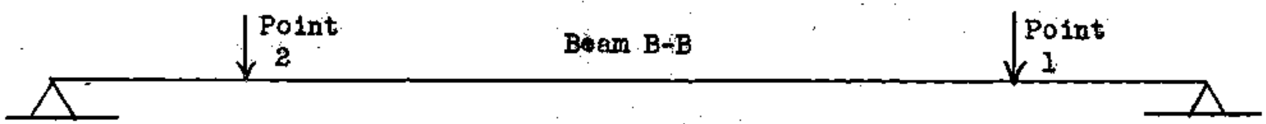
Outside node	AA-1	1.96 up	Inside node	BB-1	0.967 down
	11-A	1.3443 down		11-B	0.14508 up
	2	3.3043		2	1.11208
		1.652 = New Value			0.556 = New value

Use these loads for the final Cycle 2 deflections.

Figure 171. Problem Five - Beams 1-1 & 2-2 Cycle 2

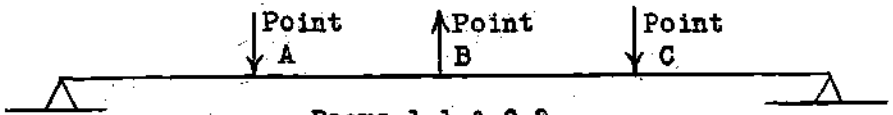


	Point 2	Point 1
Loads	-8.348	-8.348
Y Pt '2'	1.0399	0.7068
Y Pt '1'	0.7068	1.0399
Final Y	1.7467	1.7467



	Point 2	Point 1
Loads	-10.556	-10.556
Y Pt '2'	1.315	0.8937
Y Pt '1'	0.8937	1.315
Final Y	2.2087	2.2087

Figure 172. Problem Five - Beam B-B , A-A , C-C Cycle 2

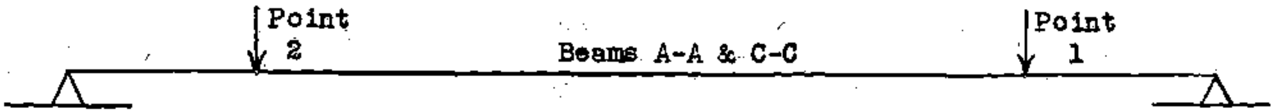


Beams 1-1 & 2-2

Loads	0	-1.652	0.556	-1.652	0
Y Pt 'A'		1.2653	1.5465	0.9841	
Y Pt 'B'		-0.5678	-0.8517	-0.5678	
Y Pt 'C'		0.9841	1.5465	1.2653	
Final Y		1.6816	2.2413	1.6816	

		Value	Average			Value	Average
Outside Node	AA-1	1.7467	1.5021	Inside Node	BB-1	2.2087	2.0426
	11-A	1.6816	0.2354		11-B	2.2413	0.1685
			1.7375				2.2111

Figure 173. Problem Five - Beams 1-1 & 2-2, Deflection Average Cycle 2

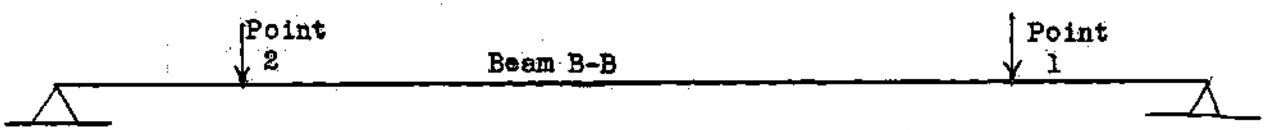


Y	1.7375	1.7375
Load	X	Y

$$0.12458X + 0.08467Y = 1.7375$$

$$0.08467X + 0.12458Y = 1.7375$$

X = 8.303 down   Y = 8.303 down



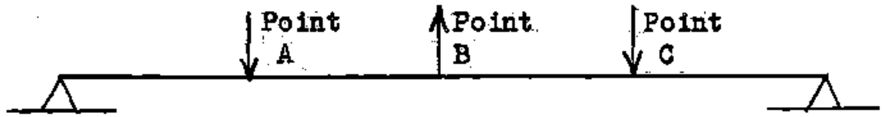
Y	2.2111	2.2111
Load	X	Y

$$0.12458X + 0.08467Y = 2.2111$$

$$0.08467X + 0.12458Y = 2.2111$$

X = 10.567 down   Y = 10.567 down

Figure 174. Problem Five - Beams A-A, B-B, C-C Cycle 3



Beams 1-1 & 2-2

Y	0	1.7375	2.2111	1.7375	0
Load		X	Y	Z	

$$0.76595X + 1.02127Y + 0.59574Z = 1.7375$$

$$0.93617X + 1.53190Y + 0.93617Z = 2.2111$$

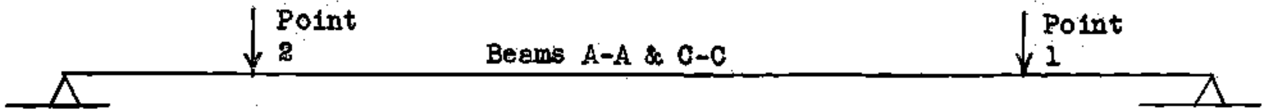
$$0.59574X + 1.02127Y + 0.76595Z = 1.7375$$

X = 2.3208 down   Z = 2.3208 down

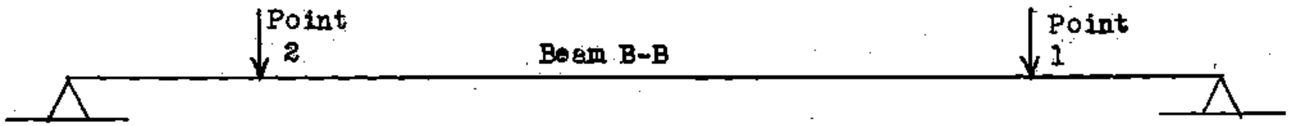
Y = 1.3931 up

Outside Node	AA-1	1.697 up	Inside Node	BB-1	0.567 down
	11-A	2.3208 down		11-B	1.3931 up
	2	4.0178		2	1.9601
		2.0089			0.98

Figure 175. Problem Five - Beams 1-1 & 2-2, Load Average Cycle 3



Loads	-7.9911	-7.9911
Y Pt '2'	0.9955	0.6766
Y Pt '1'	0.6766	0.9955
Final Y	1.6721	1.6721



Loads	-10.98	-10.98
Y Pt '2'	1.3678	0.9297
Y Pt '1'	0.9297	1.3678
Final Y	2.2975	2.2975

Figure 176. Problem Five - Beams A-A, B-B, C-C Cycle 3

Beams 1-1 & 2-2

Loads	0	-2.0089	0.98	-2.0089	0
Y Pt 'A'		1.5387	1.8806	1.1967	
Y Pt 'B'		-1.0008	-1.5012	-1.0008	
Y Pt 'C'		1.1967	1.8806	1.5387	
Final Y		1.7346	2.26	1.7346	

Outside Node		Value	Average		Value	Average
	AA-1	1.6721	1.438	BB-1	2.2975	2.1247
	11-A	1.7346	0.2428	11-B	2.26	0.1699
			1.6808			2.2946

Figure 177. Problem Five - Beams 1-1 & 2-2, Deflection Average Cycle 3

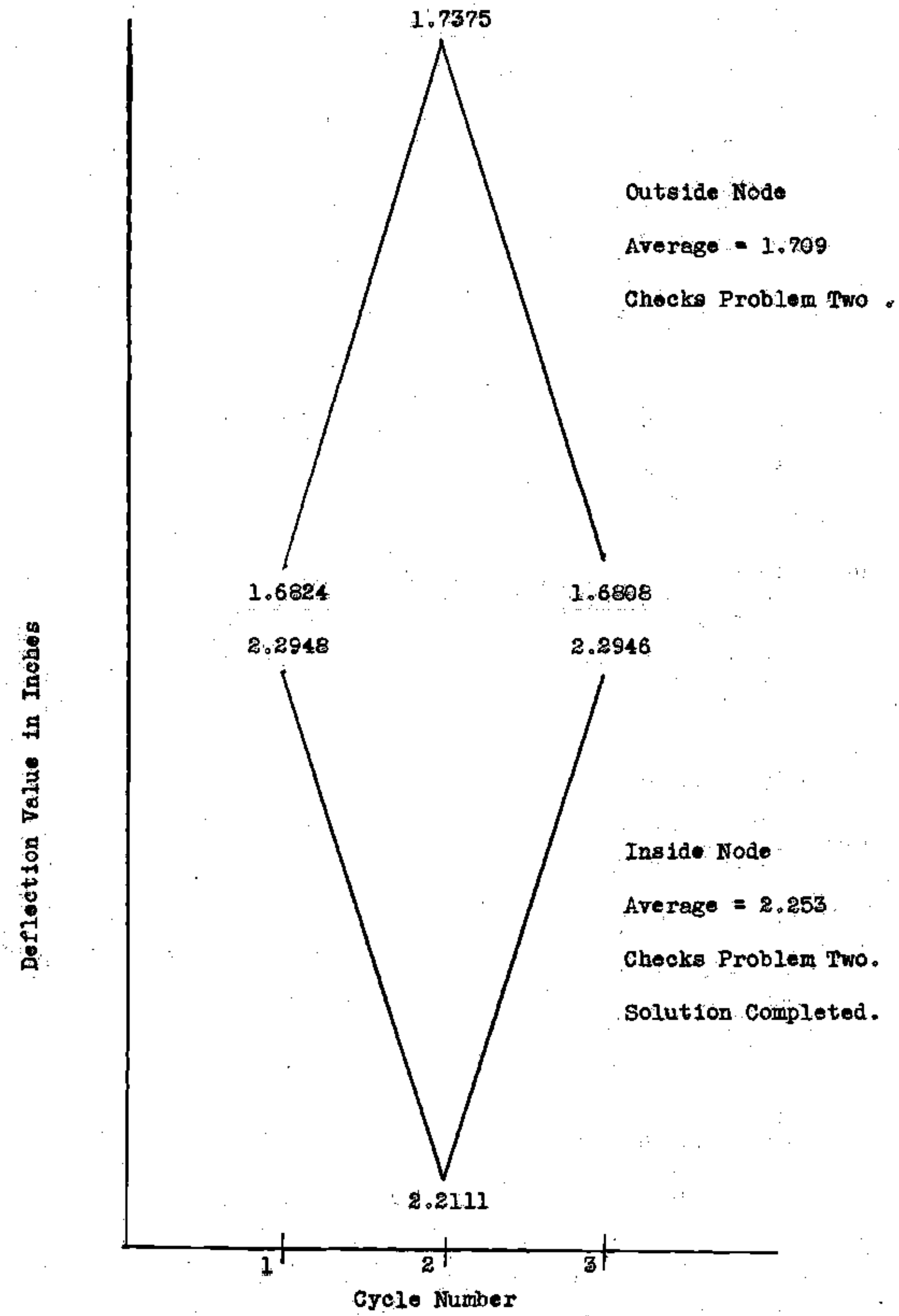


Figure 178. Problem Five - Deflection Versus Cycle

APPENDIX C

LITERATURE CITED

1. Newmark, N. M., "Numerical Procedure for Computing Deflections, Moments, and Buckling Loads," Transactions, American Society of Civil Engineers, 108, 1943, pp. 1161.

2. Ibid., p. 1162.

3. Ibid., pp. 1166-1171.

4. Schutz, F. W., Jr., An Iteration Procedure for Bars on Elastic Foundations. University of Illinois, Urbana, Illinois, Unpublished M.S. Thesis, 1950.

5. Ibid.

6. Ibid.

7. Newmark, op. cit.

8. Hendry, A. W., and Jaeger, L. G., The Analysis of Grid Frameworks and Related Structures. London: Chatto and Windus, Ltd., 1958.

9. Ibid.

10. Fowler, R. J., and Pandya, A. H., "The All-Welded Diagonal Grid Applied to Plane and Spatial Structures," Arc Welding in Design, Manufacture and Construction, James F. Lincoln Arc Welding Foundation, p. 411.

OTHER REFERENCES

Maltor, H., "Numerical Solutions for Beams on Elastic Foundations," Journal of the Structural Division, American Society of Civil Engineers, March 1958, pp. 1562.