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Principal Investigator Dr. A. S. Debs

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Final Report April 1975

METHODOLOGIES FOR TESTING, TUNING, AND EVALUATION OF ON-LINE STATE ESTIMATION FOR THE BPA DITTMER CONTROL CENTER

by

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Final Report Prepared for the
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1. INTRODUCTION

This Final Report discusses the developments and findings during the period September 15 - December 2, 1974, for the BPA-sponsored project entitled "Investigation of Methods for Optimizing the Performance of State Estimation for Real-Time Applications at BPA's Dittmer Control Center." These developments were totally related to problems of implementation of the BPA on-line state estimator whereby actual data from a portion of the main-grid system was obtained and analyzed. The tests made relied heavily on developments under the National Science Foundation (NSF) sponsored project[6] during the summer of 1973 and 1974 with BPA's endorsement and provision of facilities. Hence, the attempt is made here to provide an integrated picture of all these developments for the purpose of clarity and proper understanding of these projects.

The main problem of on-line implementation relates to the theoretical modeling assumptions used in all the formulations of state estimation techniques. In one approach, the state estimator is tested with actual system data to see if it performs according to theoretical predictions. If it does, there is no modeling problem and the issue is resolved. This approach was not followed since apriori engineering judgement as well as previous studies[9] indicated that modeling problems exist to a certain extent. Hence, the approach that was followed consisted first, of studying the effect of modeling inaccuracy on state estimation performance, and second, of developing the computational tools to validate and tune the models in order to achieve acceptable performance.

As the problem of performance acceptability is resolved, the issue centers on the other aspects of on-line implementation. Two primary aspects are treated. The first is that of the overall role of state estimation in the control center. And the second is that of computational requirements.

The modeling problem with its implications is treated in Section 2. In Section 3, the techniques of parameter estimation used are developed mathematically and commented upon. Test results of a part of BPA's main-grid are presented and discussed in Section 4. And in Section 5, an integrated set of procedures for on-line implementation is developed and elaborated on. Conclusions and recommendations follow in Section 6.

2. THE MODELING PROBLEM

2.1 Sources and Extent of Modeling Inaccuracies

Two sets of model parameters are used in state estimation. The first set is that of admittances in the equivalent pi-section representation of transmission lines and transformers. The second set corresponds to the statistical parameters describing measurement and sensor errors.

Reasons for network parameter errors are several. Baumann(9) reports in a German study that standard formulas used to compute transmission line impedences contain errors to the extent of $\underset{\sim}{\sim}$5%. These can be due to truncation errors in the Taylor series expansion formulas. The exclusive use of positive-sequence impedences to represent untransposed lines and the neglect of mutual coupling between parallel lines can cause modeling problems. As for transformers, knowledge of the leakage impedences and its dependence on tap-settings is another problem. Certainly, aging, weather, and temperature effects are neglected. The end result is a network model with parameter errors of perhaps 10% in some bad cases.

In the case of the statistical parameters of measurement errors one is interested in reliable information on sensor and meter calibration curves. Since these are not always available, a thorough analysis of the sources of sensor errors is required. Internal studies at BPA have indicated that sensor errors are due primarily to two components. One is transducer bias and the other is potential and current transformer bias. The errors themselves are primarily bias errors in the sense that they do not change appreciably from one time instant to the next, or even over long periods of time.

An important problem here is to distinguish between the two types of modeling errors. They both reflect themselves as bias-type errors. In our efforts and formulations below the attempt has been and will be made to resolve this question.

2.2 Treatment of Modeling Problems

The approach taken in our work relies on a three-stage approach to the treatment of modeling problems.

Stage 1: Model Validity Assessment

The Chi-Square test provides the quickest means to see if the overall model is sufficiently accurate. Here one looks at the function

$$J = \frac{1}{M} \sum_{i=1}^{m} \frac{(z_i - h_i(\hat{x},p))^2}{\sigma_i^2} \tag{1}$$

where

$z_i \triangleq i^{th}$ measurement, $i=1,...,m$

$\hat{x} \triangleq$ state-vector estimate, $\dim[\hat{x}]=n$

$p \triangleq$ parameter vector

$\sigma_i^2 \triangleq$ error variance of i^{th} measurement

and where

$$z_i = h_i(x,p)+v_i \tag{2}$$

with

$$v_i \triangleq \text{measurement error with zero mean and variance } \sigma_i^2$$

and

$$h_i(x,p) \triangleq \text{nonlinear function relating the measured quantity } z_i \text{ to the state vector } x \text{ and parameter vector } p.$$

The function J is chi-square distributed with m-n degrees of freedom. This implies that

$$E[J] = \frac{m-n}{m} < 1. \tag{3}$$

The test itself consists of computing the state estimate $\hat{x}$ and then evaluating J. If J is of the order of $\frac{m-n}{m}$ then the model is valid and state estimation is adequate. However, if $J \gg \frac{m-n}{m}$ then a modeling problem is present.

If a modeling problem is detected the next step is to attempt to pinpoint the source of the trouble. The simplest procedure is to examine the vector of residuals r whose i^{th} component is

$$r_i \triangleq \frac{(z_i - h_i(\hat{x},p))}{\sigma_i} \tag{4}$$

On the average, $|r_i| \leq 1$. Hence, if $|r_i| \gg 1$ then the parameters associated with $h_i(x,p)$ are in question. These could be parameters which appear in the function $h_i(x,p)$, as well as, parameters of measured quantities that are strongly coupled with z_i. From this one draws a candidate list of parameters that may be erroneous to a significant extent.

An alternative approach to identifying erroneous parameters is the one proposed in Ref.'s (13, 16). However this approach is of the "bad-data suppression" type which is primarily geared to detect a single highly erroneous parameter.*

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*Actually two or more erroneous parameters can be detected provided they are highly uncoupled.

Stage 2: Checking of Individual Components

In this stage careful analysis is undertaken to see if improvements in parameter values can be obtained. This is a stage of directly "measuring" individual parameters. The easiest parameters to check here are those of measurement errors. Suspected meters can be field-tested and calibrated in a rather routine manner. Next, one considers transformer parameters with special emphasis on leakage impedences, tap settings at both sides of the transformer and the effect of tap settings on leakage impedences. This can be best achieved through a careful evaluation of manufacturers design and test data. Direct tests on installed transformers cannot be ruled out especially when serious anomalies are still present in the model. As for transmission line data, the cases to watch for are those of parallel lines with mutual coupling. Otherwise, little can be done other than to check the accuracy of calculations provided in the data book.

Stage 3: Parameter Estimation

Parameter estimation provides the means to improve on the accuracy of the system models using masses of actual system data over representative operating conditions. It represents an important tuning process which ascertains the statistical validity of the models used over a wide range of operating conditions. In our assessment, this has the following advantages:

a. For on-line state estimation, the estimated and measured quantities will correspond closely to each other within the statistical accuracy of measuring instruments. This is essential to develop operator or dispatcher confidence in state estimation calculations and predictions.

b. Bad-data rejection and identification will become much easier to perform. As will be shown later, modeling errors can seriously degrade state estimator performance. This can cause the bad-data detection algorithm to be almost useless.

c. Improvements on some network models can be achieved. These can now be used in the applications or planning programs.

d. Once the software is developed, parameter estimation can become a routine function requiring only little man-hour effort and some computer time.

2.3 Sensitivity Analysis

Through sensitivity analysis one can determine those parameters which can cause serious errors in state estimation calculations. This can provide a further refinement over the results obtained in model validity assessments, thus limiting the set of parameters to be estimated.

Mathematical derivations of sensitivity analysis formulas showing sensitivity of state estimates to various sets of parameters are available in several references and will not be attempted here.[11] Instead, by means of a simple illustrative example, the contribution of parameter errors to the overall error in a given measurement is computed.

Denoting by Δp the expected error in the parameter vector one can write

$$\begin{aligned} z &= h(x,\ p+\Delta p)+v \\ &= h(x,p)+\left.\frac{\partial h}{\partial x}\right|_{x,p} \Delta p+v. \end{aligned} \tag{5}$$

The vector $\left.\frac{\partial h}{\partial x}\right|_{x,p}\Delta p$ represents a bias-type error which is added to metering error v. In the example chosen we consider a pi-section representation of a line or a transformer as shown in figure 1.

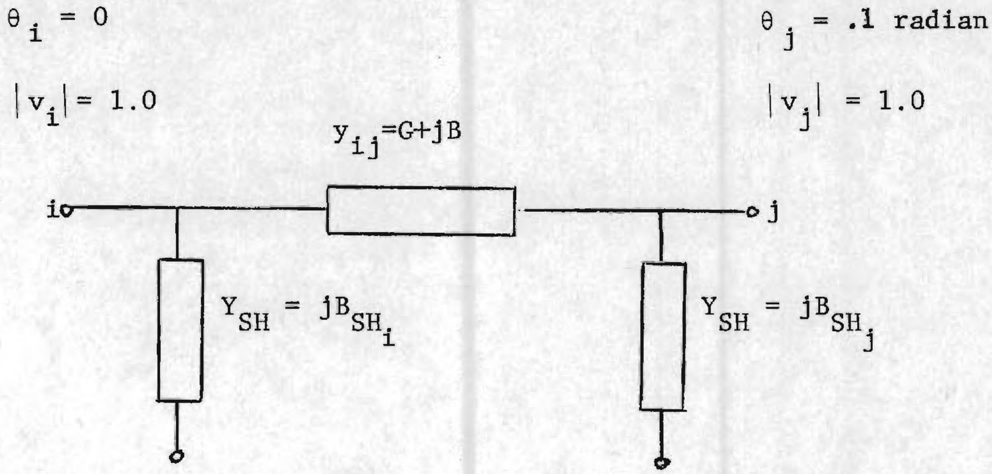

Fig. 1: Pi-Section Representation of a Typical Network Element

In the case of a transmission line, the following typical values are selected:

$$B = 100 \text{ pu}$$

$$G = 10 \text{ pu}$$

$$B_{SH_i} = B_{SH_j} = .01 \text{ pu.}$$

For the case where $\theta_j = .1r$ and parameter errors are 5% of nominal values one obtains:

$$\left|\frac{\partial T_{ij}}{\partial B} \Delta B\right| \cong .5 \text{ p.u.}$$

$$\left|\frac{\partial T_{ij}}{\partial G} \Delta G\right| \cong .25 \times 10^{-4} \text{ p.u.}$$

$$\left|\frac{\partial U_{ij}}{\partial B} \Delta B\right| \cong .25 \times 10^{-3} \text{ p.u.}$$

$$\left|\frac{\partial U_{ij}}{\partial G} \Delta G\right| \cong .5 \times 10^{-2} \text{ p.u.}$$

$$\left|\frac{\partial U_{ij}}{\partial B_{SH_i}} \Delta B_{SH_i}\right| \cong .5 \times 10^{-3}$$

where T_{ij} and U_{ij} are the real and reactive flows from bus i to bus j respectively.

In the case of a transformer the following values are selected:

$B = 100$ p.u.

$G = 0.0$

$E =$ Tap Ratio $= 1.0$.

Assuming $\theta_j = .1r$ and 5% parameter errors one obtains:

$$\left|\frac{\partial T_{ij}}{\partial B} \Delta B\right| \cong .5 \text{ p.u.}$$

$$\left|\frac{\partial U_{ij}}{\partial B} \Delta B\right| \cong .25 \times 10^{-3} \text{p.u.}$$

$$\left|\frac{\partial U_{ij}}{\partial E} \Delta E\right| \cong 5\text{p.u.}$$

The conclusions of the above results are

a. For transmission lines the sensitivity of real-flow to line susceptance errors is at least two orders of magnitude larger than the sensitivity to errors in all other parameters.

b. If the measurement error standard deviation has the reasonable value of .1 p.u., then the error due to line susceptance is five standard deviations which is quite unacceptable.

c. In the case of transformers, sensitivity to an error in tap positions is at least an order of magnitude larger than that with respect to leakage susceptance.

Therefore, the order of priority in estimating network parameters should be:

1. Tap settings of transformers
2. Line susceptance for transmission lines or leakage susceptance of transformers
3. (Possibly) line conductance of transmission lines.

All other parameter errors can be reasonably neglected.

The above results are consistent with findings in Ref.(11) and also with our own simulations as will be discussed later in the report. (For a detailed derivation of all sensitivity relations refer to Appendix A)

3. PARAMETER ESTIMATION

3.1 Problem Formulation

Denoting by $z(k)$ the vector of measurements at time sample k, $k=1,\ldots N$, one can write

$$z(k) = h(x(k),p) + v(k);\ k=1,\ldots,N \tag{6}$$

where

$x(k) \triangleq$ state vector with dimension n

$p \triangleq$ parameter vector with dimension e

$v(k) \triangleq$ error in $z(k)$ with zero mean and diagonal covariance matrix $R(k)$.

Given value of the parameter vector is denoted by p^o. This can be related to the true (but unknown) parameter vector p by

$$p^o = p + w \tag{7}$$

where w represents the error in the knowledge of p and is modeled as a zero-mean random vector with diagonal covariance matrix M_o.

Denoting by $\hat{x}(k)$ and $\hat{p}$ the weighted least squares estimates of $x(k)$ and p, $k=1,\ldots,N$, then by definition $\hat{x}(k)$ and $\hat{p}$ should minimize

$$J = \sum_{k=1}^{N} [(z(k)-h(\hat{x}(k),\hat{p})^T R^{-1}(k)(z(k)-h(\hat{x}(k),\hat{p})] + (p^o-\hat{p})M^{-1}(p^o-\hat{p}). \tag{8}$$

3.2 Solution Methods

(a) Suboptimal Kalman Filter Approach

In this state and parameter estimates are updated with every new snap-shot measurement vector $z(k)$. Let $\hat{x}^n(n)$ and $\hat{p}^n$ be the estimates of $\hat{x}(n)$ and $\hat{p}$ at the minimum of

$$J_n = (p^o-\hat{p})M^{-1}(p^o-\hat{p}) + \sum_{i=1}^{n} (z(k)-h(\hat{x}(i),\hat{p})^T R^{-1}(i)(z(i)-h(\hat{x}(i),\hat{p})) \tag{9}$$

Also let M_n be the covariance of $\hat{p}^n$ then, it can be shown that $\hat{x}^{n+1}(n+1)$ and $\hat{p}^{n+1}$ will minimize.*

$$\begin{aligned} L_{n+1} &= (\hat{p}^n-\hat{p}^{n+1})M_n^{-1}(\hat{p}-\hat{p}^{n+1}) \\ &\quad + (z(n+1)-h(\hat{x}^{n+1}(n+1),\ \hat{p}^{n+1})R^{-1}(n+1)(z(n+1)- \\ &\quad h(\hat{x}^{n+1}(n+1),\ \hat{p}^{n+1})). \end{aligned} \tag{10}$$

Minimization of L_{n+1} w.r.t. $\hat{p}^{n+1}$ and $\hat{x}^{n+1}(n+1)$ can be accomplished via a Netwon-Raphson type algorithm. For simplicity let $\hat{p}_i$ and $\hat{x}_i$ represent the i^{th} iteration in computing $\hat{p}^{n+1}$ and $\hat{x}^{n+1}(n+1)$, then this algorithm is given by,

$$\begin{bmatrix} \hat{x}_{i+1} \\ \hat{p}_{i+1} \end{bmatrix} = \begin{bmatrix} \hat{x}_i \\ \hat{p}_i \end{bmatrix} + \Sigma_i \begin{bmatrix} H_i^T R^{-1}(z(n+1)-h(\hat{x}_i,\ \hat{p}_i)) \\ G_i^T R^{-1}(z(n+1)-h(\hat{x}_i,\ \hat{p}_i))+M^{-1}(\hat{p}^n-\hat{p}_i) \end{bmatrix} \tag{11}$$

where

$$\Sigma_i = \begin{bmatrix} (H_k^i)^T R^{-1} H_k^i & (H_k^i)^T R^{-1}(k) G_k^i \\ (G_k^i)^T R^{-1}(k) H_k^i & (G_k^i)^T R^{-1}(k) G_k^i + M_n^{-1} \end{bmatrix}^{-1}$$

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*This is true using linearized equations only.

$$\triangleq \begin{bmatrix} \Sigma^i_{xx} & \Sigma^i_{xp} \\ \Sigma^i_{px} & \Sigma^i_{pp} \end{bmatrix}$$

$$M_{n+1} \triangleq \Sigma^i_{pp}$$

$$\hat{x}_o \triangleq \hat{x}^n$$

$$\hat{p}_o \triangleq \hat{p}^n$$

$$i = 0,1,2,\ldots$$

(b) Decoupled State Parameter Approach

Minimization of J as expressed in Eq. (8) requires that at the solution $\hat{x}(k)$ and $\hat{p}$ be zero i.e.

$$0 = \frac{\partial J}{\partial \hat{x}(k)} = -2H_k^T R^{-1}(k)(z(k)-h(\hat{x}(k),\hat{p})); \; k=1,\ldots,N, \tag{12}$$

$$0 = \frac{\partial J}{\partial \hat{p}} = -2\sum_{k=1}^{N} [G_k^T R^{-1}(k)(z(k)-h(\hat{x}(k),\hat{p}))]-2M^{-1}(p^o-\hat{p}) \tag{13}$$

where

$$\frac{\partial h}{\partial x}\Big|_{\hat{x}(k),\hat{p}} \quad \text{and} \quad G_k = \frac{\partial h}{\partial p}\Big|_{\hat{x}(k),\hat{p}}.$$

In the decoupled approach one preceeds according to the following steps:

<u>Step 1</u>: By holding $\hat{p}$ to be constant solve for $\hat{x}(k)$ using Eq. (9). This can be accomplished by means of the state estimation interative algorithm

$$\hat{x}^{i+1}(k) = \hat{x}^i(k) + [(H_k^i)^T R^{-1}(x) H_k^i]^{-1}(H_k^i)^T R^{-1}(z(k)-h(\hat{x}^i(k),\hat{p})) \tag{14}$$

where i=1,2,..., and

$$H_k^i = \frac{\partial h(x,p)}{\partial x}\Big|_{\hat{x}^i(k),\hat{p}}$$

<u>Step 2</u>: Hold $\hat{x}(k)$ at the values obtained in Step 1 and solve for

$\hat{p}$ iteratively according to Eq. (10) by means of the algorithm

$$\hat{p}^{i+1} = \hat{p}^{i} + \left(\sum_{k=1}^{N} [(G_k^i)^T R^{-1}(k) G_k^i] + M^{-1}\right)^{-1} (M^{-1}(p^o - \hat{p}^i)$$

$$+ \sum_{k=1}^{N} (G_k^i)^T R^{-1}(k)(z(k) - h(\hat{x}(k), \hat{p}^i)) \tag{15}$$

where

$$G_k^i = \frac{\partial h(x,p)}{\partial p}\bigg|_{\hat{x}(k),\hat{p}^i} .$$

The summation terms in Eq.(12) are obtained sequentially during step 1. This implies that only one iteration can be used in Eq. (15).

Step 3: If $|\hat{p}^{i+1} - \hat{p}^{i}| < \epsilon$, where ϵ is a given positive constant, then the process is stopped. Otherwise, go back to Step 1 with the new parameter values.

If convergence occurs the necessary minimization conditions (12) and (13) are obviously satisfied. Due to the nonlinearity of the equations, there is no guarantee that the global minimum is attained. However, by calculating the chi-square performance index J given in the previous section, one can determine if a statistically acceptable solution has been obtained, In addition, engineering judgement as to the reasonableness of solutions can be exercised in order to make sure that adequate answers are obtained.

3.3 Parameter Estimation Programs

Two parameter estimation programs were developed during the summer of 1974 corresponding to the two approaches discussed above. With a few minor modifications the programs can be considered of the production type. Both programs are written for BPA's CDC-6400 computer using Fortran IV. In both cases sparsity techniques are employed to minimize the amount of computer core storage and to increase computational speeds.

The use of sparsity techniques poses no difficulties in the decoupled approach. However, for the recursive approach the matrix M_n and its inverse become full following the first snapshot. As a result an approximation is used whereby M_n is diagonalized at the end of every snapshot computation. This is the main reason the Kalman Filter approach is called suboptimal.

4. INITIAL TEST RESULTS

4.1 Introduction

During the summer of 1973 a limited number of computer simulations were conducted to study the issue of inaccurate modeling and the feasibility of parameter estimation. Results of this effort are provided in a technical paper.[4] In that paper two rather significant conclusions were obtained. First, inaccuracies in the network parameter models of the order of 5-10% can cause significant statistical degradation in state estimator performance. And second, these inaccuracies can be corrected for by means of parameter estimation leading to state estimator performance which is almost indistinguishable from that where a perfect model is used.

The above conclusions, however, were based on computer simulations and not on an actual system. Furthermore, parameter errors were introduced in a limited number of parameters. The limitation was mainly due to the fact that sparsity techniques were not implemented at that time. As soon as sparsity techniques were implemented in the summer of 1974, simulation tests were conducted whereby all network parameters contained a certain amount of error. Following that in November of 1974 data from eight remote stations of BPA's SCADA (Supervisory Control and Data Acquisition) system became available. This made it possible, for the first time, to test state estimation at BPA using adequate and reliable data. It is recalled here, that an earlier test in 1970 was made. However, at that time, SCADA was not in operation and many inadequacies in the data were present. Furthermore, parameter estimation programs were not available to improve on the network model.

In the following discussion, results of simulations as well as tests using actual system data are presented.

4.2 Simulation Results of Parameter Estimation Programs

The use of sparsity techniques in the parameter estimation programs permitted more realistic simulation tests whereby all network parameters were inaccurate to a certain degree. Here, a Gaussian random number generator was used to introduce errors in each parameter p_i, $i=1,\ldots\ell$. The error had a mean of zero and a standard deviation of $\alpha_i|p_i|$; where α_i ranged from 0.02 to 0.1 for a particular test case. For the results shown below $\alpha_i=0.05$ for all transmission line admittance terms and $\alpha_i=0.02$ for transformer tap ratios. The initial parameter covariance matrix M_o was diagonal with the i^{th} diagonal term given by:

$$(M_o)_{ii} = \alpha_i^2 |p_i|^2.$$

The network chosen for simulations is the same one used in Ref. (4) It is representative of a portion of BPA's main grid network and associated data acquisition system.

In Figure (2), performance of the Kalman Filter algorithm is shown (dashed line). In the test simulated here, all transmission line parameters were randomly perturbed using a Gaussian random number generator. The errors had a mean of zero and a standard deviation of 5% of nominal value. As for transformers, a 2% random error was introduced in the tap ratios. For the first four time samples, no parameter estimation was attempted. These samples were used primarily to determine the set of parameters that should be estimated. The decision on which parameter to estimate was based on the analysis of residuals of line-flow measurements. Basically, if the residual of the real or reactive measurement became greater than 3 then the line susceptance and shunt capacitance are included in the candidate set. In the case of transformers, tap ratios were included automatically. The use of several time samples (in this case four of them) to decide on the candidate parameter set tended to make this set slightly larger than the case of basing the candidate list on one sample. The idea here is to exclude any parameters to which the performance index is insensitive and include as many parameters otherwise. This process normally did eliminate most of the radial portions of the network from consideration. And this is an expected result.

Starting with the fifth time sample, the parameter estimator was turned on. It is clear from Figure (2) that system performance immediately improves from values of the order 14-13 to values of the order of .6-.7. The improvement is attained at the first time sample parameter estimation is performed. However, this does not mean that the parameters become all accurate after one time sample. Parameter estimation at different operating conditions will continuously tend to improve the accuracy of parameter estimates.

Performance of the Decoupled State-Parameter algorithm is shown in Figure (3). The performance at the last time sample (10 samples were used) is plotted as a function of major loop iterations. Here all the line susceptances and shunt capacitances, as well as, transformer ratios were estimated. After four iterations, the performance changed from a value of 14.0 to 0.52. This performance is slightly better than that of the Kalman Filter approach.

4.3 Experimental Tests Using Actual Data

4.3.1 Experimental Setup

Data used for the reported results was collected on November 8, 1974.

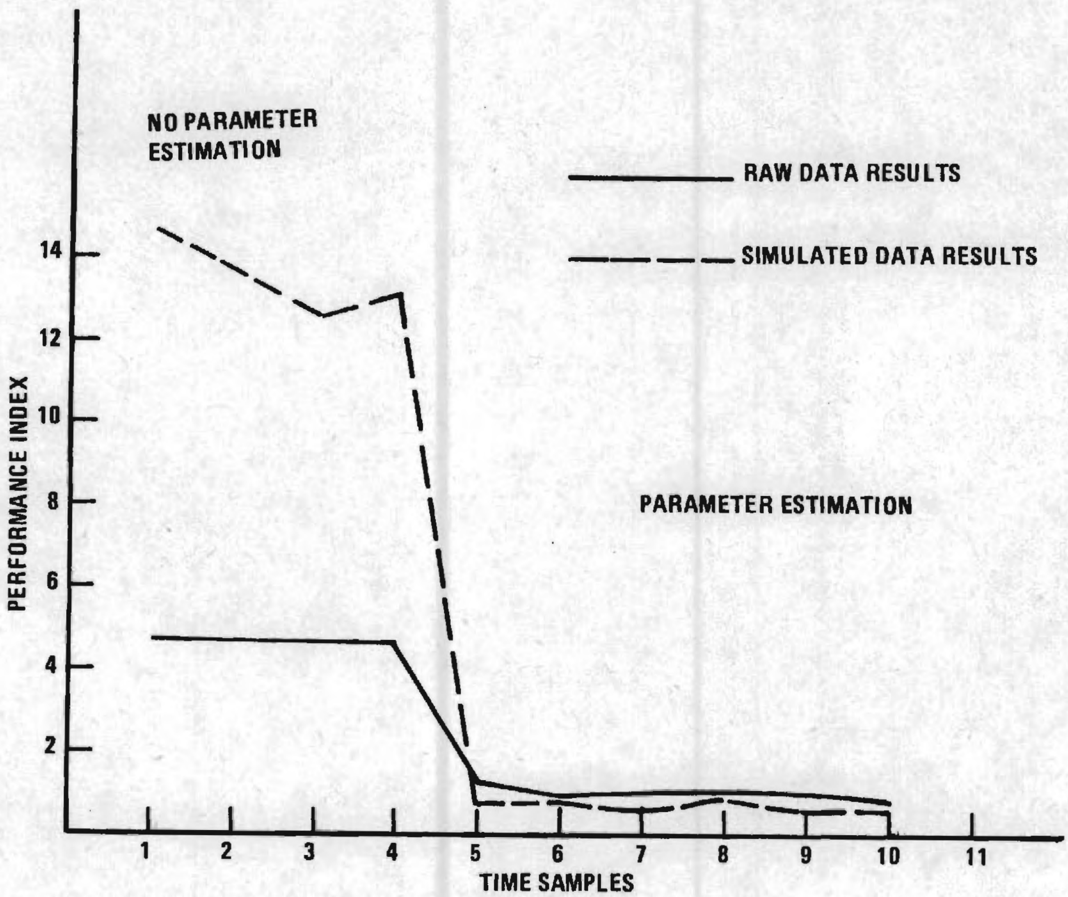


Fig. 2. Performance of Suboptimal Kalman Filter Algorithm
Using Simulated as well as Actual Measurement Data

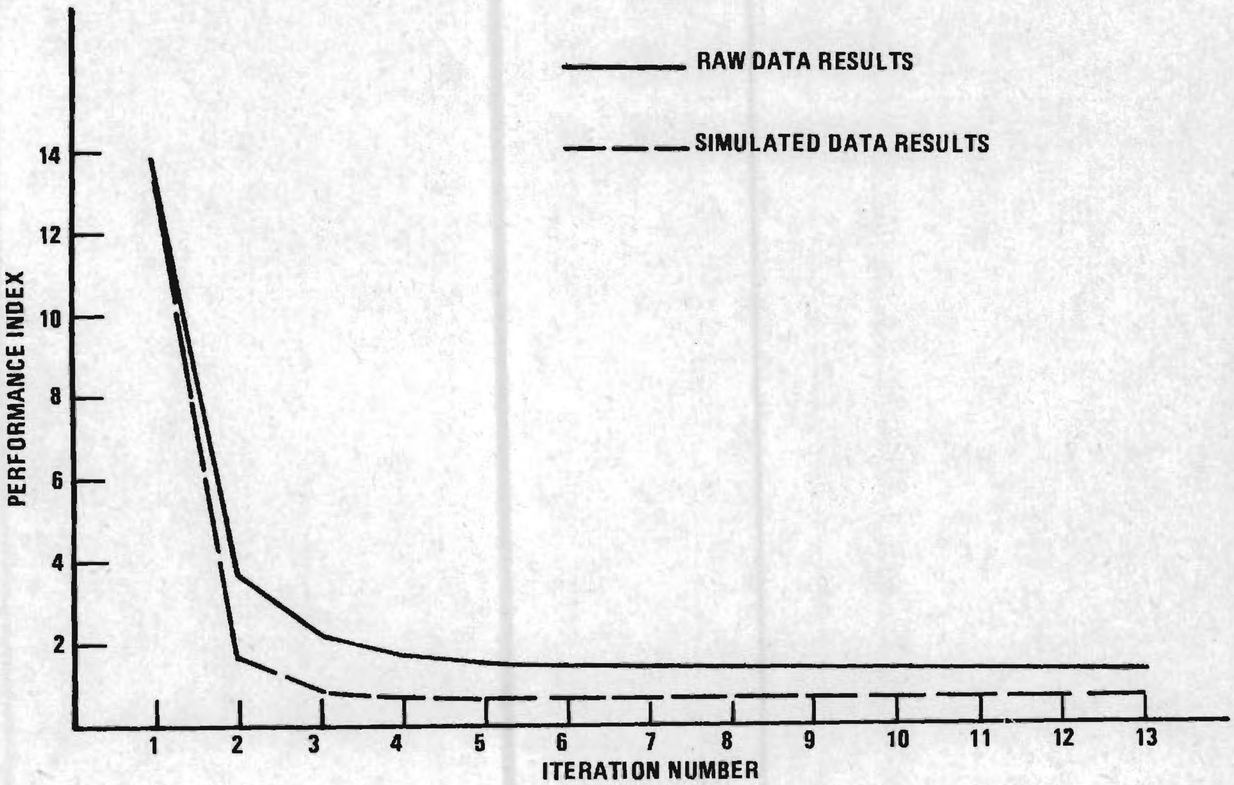


Fig. 3: Performance of the Decoupled State-Parameter Algorithm Using Simulated as well as Actual Measurement Data

It consisted of three 10 minute scans. Each scan consisted of 40 snapshots taken at 15 second intervals. One scan was conducted at noon time, the second at 7:00 p.m., and the third around midnight in order to observe the system at widely different operating conditions. It was readily noted that during each of the 10 minute periods the system operating conditions hardly changed. Hence, only five snapshots were retained for each of the three time periods.

At the time of conducting experimental tests, data from 8 remote stations became available. The network monitored by these stations is shown in Figure 4. It consists of 17 busses and 34 branches. 66 measurement quantities were transmitted every 15 seconds during a 10 minute interval upon request from the operator. Network status (or configuration) was determined directly from station diagrams displayed to the operator. In the actual on-line system, network configuration will be determined directly from status readings by SCADA I.

The 66 measurements mentioned consist of a mix of 12 voltage (KV), 21 real and 17 reactive line flow (MW and MVAR) measurements and 9 real and 7 reactive injection measurements. Transformer tap settings were also monitored. There are approximately twice as many measurements as there are state variables. This two-to-one redundancy is fairly evenly distributed over the entire system providing considerable back-up in cases of lost measurements due to various types of failures.

In testing the various components of SCADA I hardware very careful attention was given to the calibration of measurement instrumentation. From our point of view, it was crucial to know fairly accurately the expected errors in the various measurements. The BPA staff on their part, conducted independent tests to see if tranducers satisfy the required specifications under a variety of conditions. And after looking at calibration curves and internal memo's and talking to various individuals we became convinced that the formula

$$\sigma_z^2 = (.006z)^2 + (.005x(\text{full scale}))^2 \tag{16}$$

where z is a MW or MVAR measurement and σ_z^2 is the corresponding error variance, is quite adequate. For KV measurements the following formula was used:

$$\sigma_v = .005x(\text{full scale}). \tag{17}$$

From a sensitivity analysis point of view, the state estimates, where the model is exactly known, are quite insensitive to errors in the measurement covariance matrix. This is not true, however, when parameter estimation is attempted. A measurement covariance matrix with diagonal entries considerably smaller than corresponding true values will weigh the measurements too heavily causing significant model changes. Alternatively, variances which are greater than true values will tend to improve the model slightly causing no significant improvement in the state estimation process.

Preliminary checks on measurement accuracies were conducted prior to testing using the state and parameter estimation programs. One such check is shown in Table I whereby the injection measurement is subtracted from the sum of line flow measurements, around a given bus. The resulting error is compared with the cumulative σ defined by

$$\sigma = [\sigma_{INJ}^2 + \Sigma\,\sigma_{FLOW}^2]^{\frac{1}{2}}. \tag{18}$$

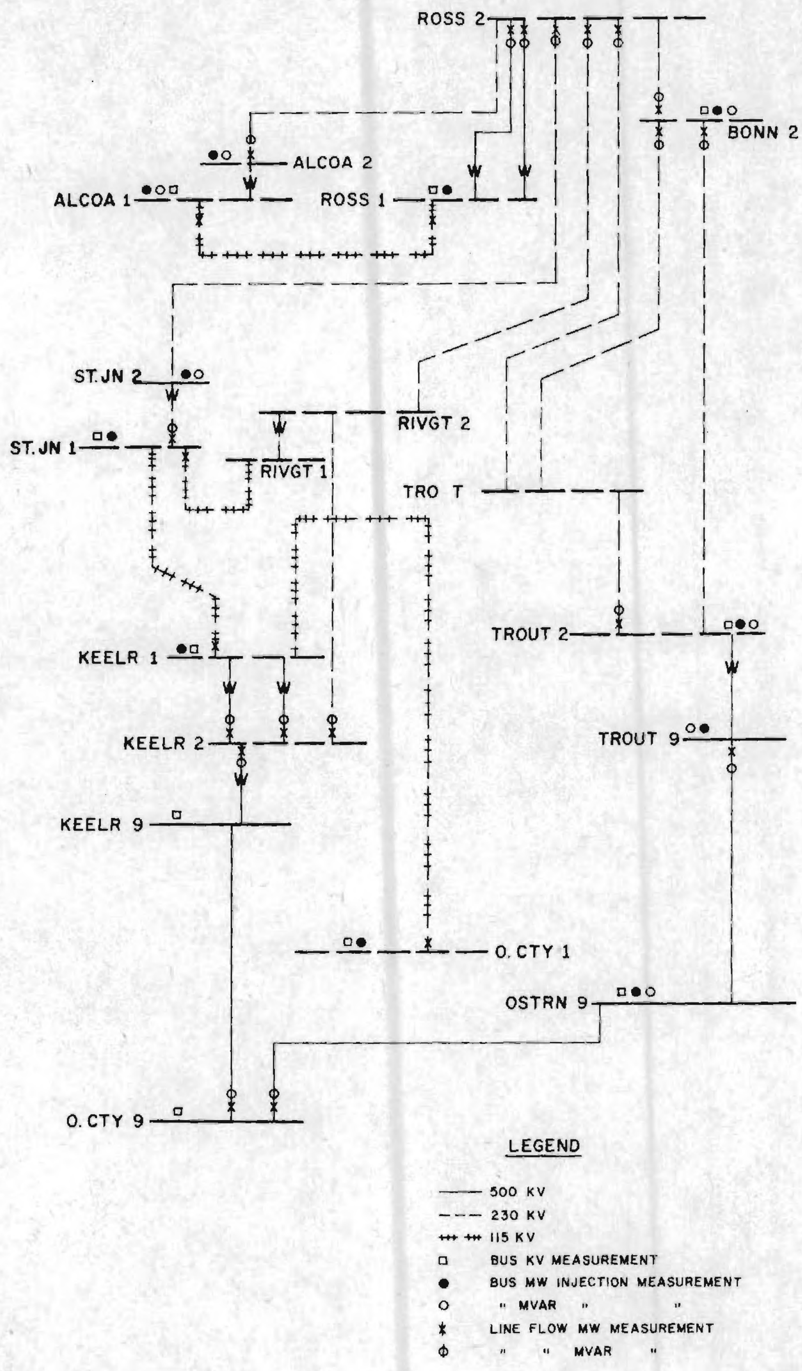


Fig. 4: Network Used in Experimental Tests

TABLE I: Preliminary Evaluation of Expected Errors in MW and MVAR Measurements

BUS	QUANTITY	ΣFLOW-INJ.	σ	ERROR
KEELR2	MW	4.6 MW	10.56	.434
BONN2	MW	11.3 MW	7.3	1.57
KEELR2	MVAR	3.32 MVAR	8.1	.40
OCTY1	MW	0.9	1.7	.53

It is clear from this table that our choice of σ values is reasonable. This method of checking did point out some inconsistencies. For example, we were able to determine that the reactive injection measurement at BONN2 needed recalibration. This measurement was not included for estimation purposes. It also pointed out errors in scale-factor conversion coefficients used to translate digital octal readings to MW, MVAR or KV quantities. It is, however, limited to those cases where all flows and injections are measured at a bus.

4.3.2 Comparison with Simulations

Figures (2) and (3) contain results of both parameter estimation approaches using actual as well as simulated data. In both cases parameter estimation significantly improves performance.

As will be discussed later, the experiments indicated that transformer data contained errors primarily in the series leakage susceptance as well as the tap setting at the fixed end. (The variable end was directly monitored.) Due to time limitations, only the decoupled algorithm was upgraded to perform the estimation of these parameters. The earlier version of the algorithm attempted the estimation of tap ratios only at transformer branches.

It is clear from Figure (3), that simulated and actual results correspond closely to one another. This confirms our earlier predictions

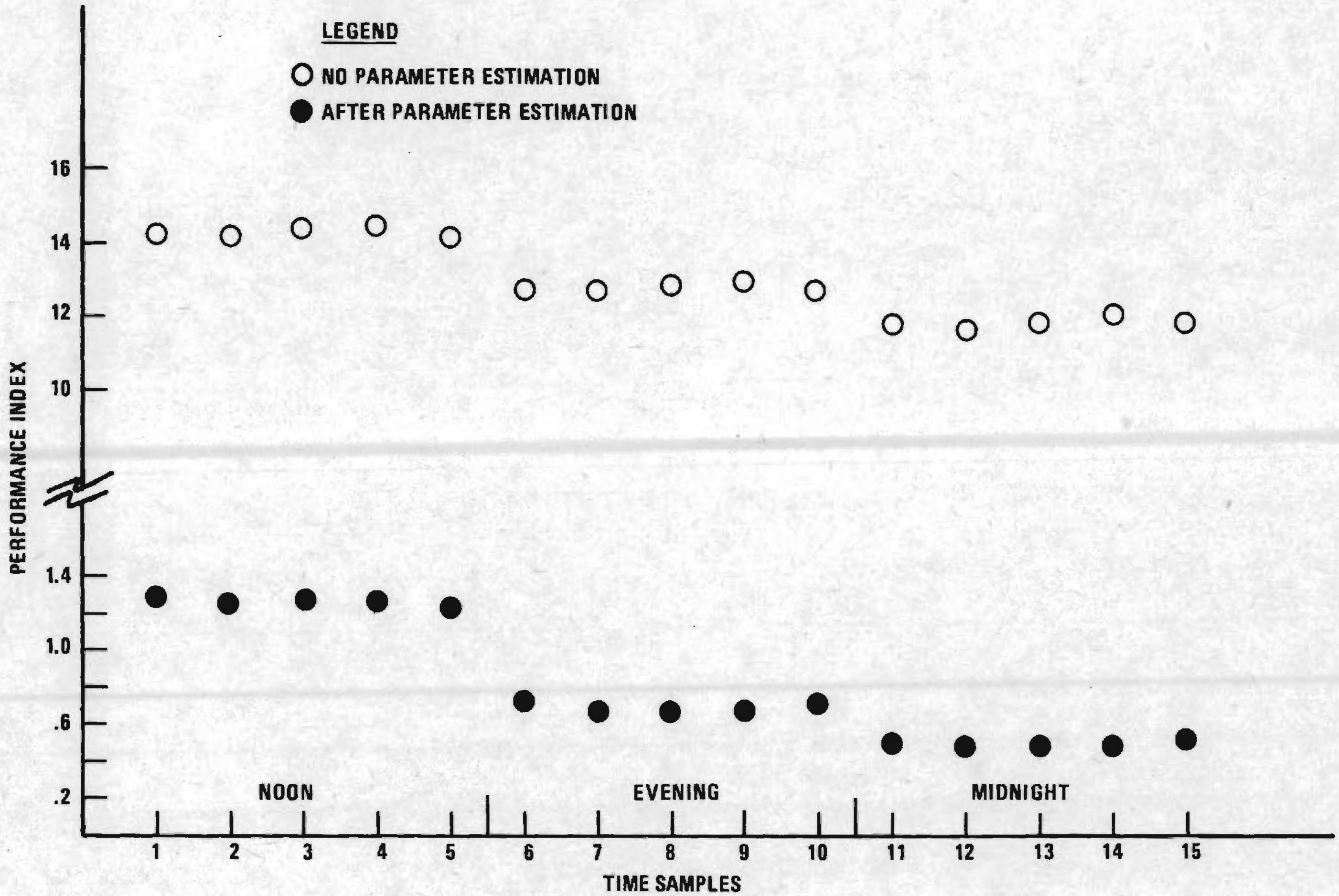


Fig. 5: Comparison of State Estimator Performance before and after Parameter Estimation

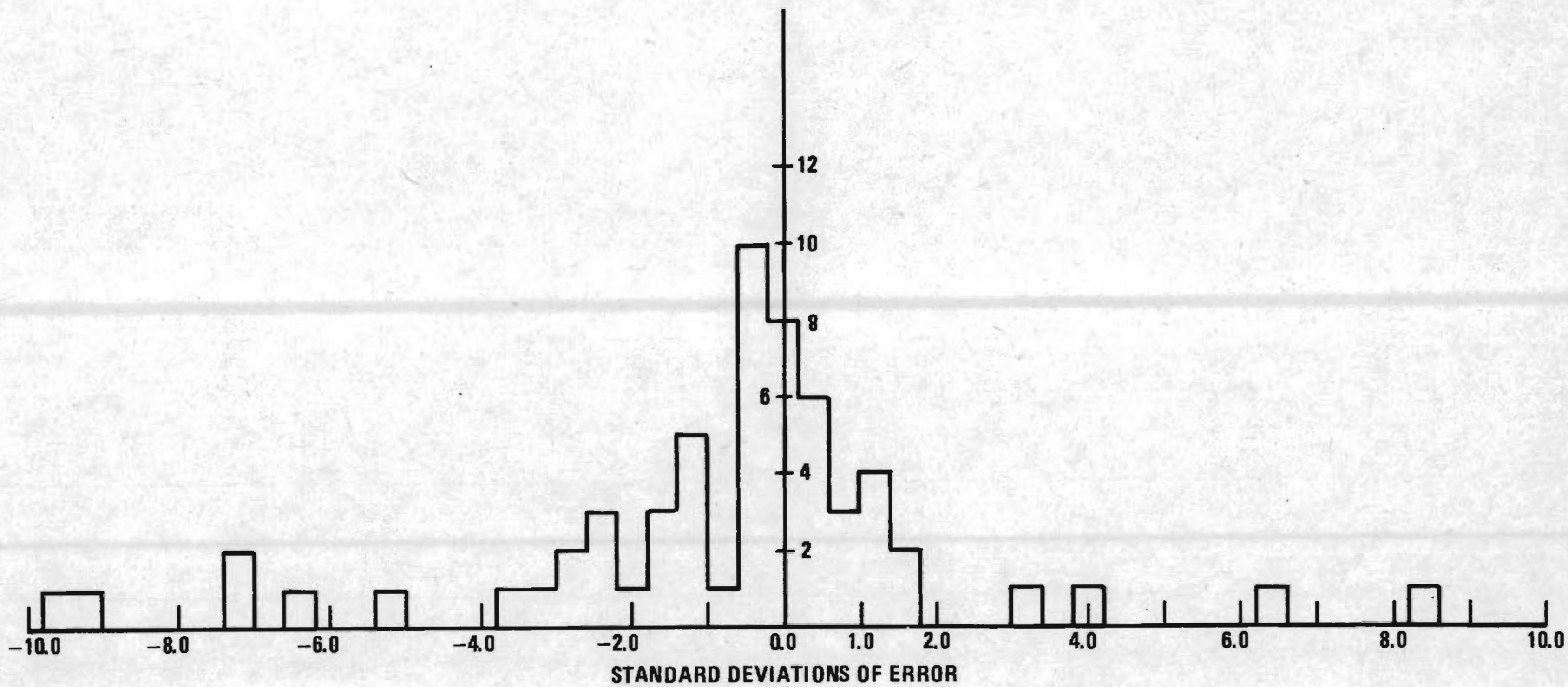


Fig. 6(a): Histogram of State Estimator Normalized Residuals Prior to Parameter Estimation (Time sample No. 15)

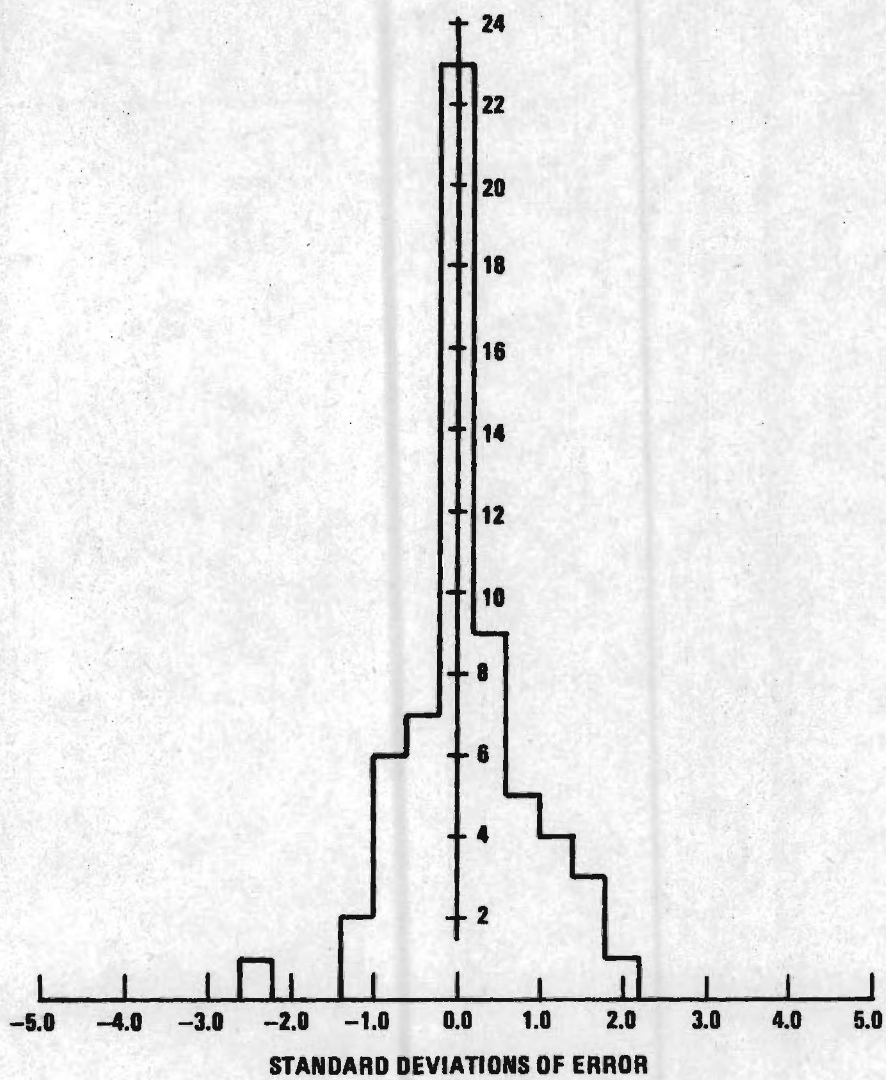


Fig. 6(b): Histogram of State Estimator Normalized Residuals After Parameter Estimation (Time Sample No. 15) about a general level of network parameter inaccuracy of 5-10%.

In Figure (2), the actual results were obtained after deleting all reactive measurements at the transformer banks. This caused the sensitivity of the performance index to transformer parameter errors to be small. Unfortunately, time did not permit to upgrade this algorithm to estimate transformer leakage susceptance and the tap at the fixed end. The algorithm, however, did estimate transmission line series susceptances and shunt capacitances. The performance improved from a value of 4.5 to .8-1.4.

For the remainder of the report, all results pertain to the decoupled State-Parameter Algorithm.

4.3.3 Overall Performance

The performance index defined in Eq. (1) is plotted as a function of time samples before and after performing parameter estimation as shown in Figure (5). Two aspects can be noted here. First, an improvement in performance of at least one order of magnitude is observed due to parameter estimation. The second is that a slight reduction in measurement errors is observed as loading conditions decrease in magnitude from noon to evening and then to midnight. This demonstrates the assumption that measurement errors generally increase with increasing load conditions. It is noted that the formulas for measurement variances given in Eqs. (12) and (13) were used only for the first time sample and then the variances were held constant. A variable σ for each time sample would, most probably, yield a fairly constant performance index.

Figure (6) shows the histograms of normalized residuals r_i defined as:

$$r_i = \frac{z_i - h_i(\hat{x},\hat{p})}{\sigma_i} \; ; \; i=1,\ldots,m.$$

It is clear from this figure that all the large errors which are statistically unacceptable have been eliminated by the parameter estimation process. Furthermore the distribution of errors following parameter estimation is approximately Gaussian. It is felt that this is an interesting result which justifies to a good extent, The Gaussian assumption about measurement noise.

4.3.4. Transmission Line Parameters

The parameter estimation program attempted the estimation of line susceptances only. From a sensitivity analysis point of view it can be shown that small errors in line conductances and shunt capacitances are of little consequence. This was confirmed by simulation tests whereby very minor performance degradation was observed due to these parameters.

Table II shows the initial and final values of transmission line susceptances. Changes greater than 10% in these values occured in 4 of the 14 lines shown. A change of 38.1% was observed on the last line in the list. Comparison of measured with estimated flows, both before and after parameter estimation, is provided in Figure (7). As can be expected, parameter estimation does the job of reducing the values of the residuals.

4.3.5 Transformer Parameters

In the case of transformers several modifications to our prior assessments had to be made. In Ref. (4) it was advocated that transformer tap settings should be estimated. No attempt was made to estimate leakage susceptances. After considerable evaluation of results it was concluded that it is more meaningful to estimate a) transformer leakage susceptance and b) the fixed tap settings which are constant. The variable tap settings are directly measured.

TABLE II: Comparison of Given and Estimated Susceptances (B) of all Transmission Lines

LINE	GIVEN B	ESTIMATED B	DIFFERENCE	%DIFFERENCE
BONN2-ROSS2	17.7648	18.1002	.3354	1.91%
BONN2-TRO T	29.9539	30.1022	.1482	.443%
O CTY9-KEELR9	199.8990	228.1128	28.2137	12.65%
BONN2-TROUT2	22.5949	22.1232	-.4717	-2.14%
RIVGT2-ROSS2	107.8550	127.4719	19.6169	15.30%
ROSS2-TRO T	42.8996	40.1044	-2.7953	-7.22%
TROUT2 TRO T	92.1934	90.7880	-1.4053	-1.53%
ROSS2-ALCOA2	160.6944	159.9392	-.7553	-.47%
OSTRN9-O CTY9	178.7614	212.7244	33.9631	15.9%
ST JN2-ROSS2	89.0228	88.7825	-.2404	-.27%
TROUT9-OSTRN9	195.1089	195.0905	-.0184	-.01%
KEELR2-RIVGT2	66.6669	61.6149	-5.052	-8.2%
KEELR1-O CTY1	7.4684	6.8251	-.6432	-9.45%
KEELR1-ST JN1	14.7558	23.8421	9.0683	38.1%

TABLE III: Comparison of Given and Estimated Transformer Series Admittances (B) of all Transformers

TRANSFORMER	GIVEN B	ESTIMATED B	DIFFERENCE	%DIFFERENCE
TROUT9-TROUT2	83.6854	83.6776	.0078	.009%
ROSS1-ROSS2(1)	33.3146	37.4421	4.13	11.02%
ROSS1-ROSS2(2)	33.7209	36.7527	3.032	8.25%
ALCOA1-ALCOA2	34.5639	34.4084	-.1555	-.452%
KEELR9-KEELR2	87.7133	92.1847	4.47	4.85%
ST JN2-ST JN1	38.0219	37.93	4.47	4.58%
KEELR2-KEELR1(1)	37.3351	42.8753	-.092	-.242%
KEELR2-KEELR1(2)	38.2457	37.5748	-.671	-1.78%

TABLE IV: Comparison of Given and Estimated Transformer Tap Ratios for all Transformers

TRANSFORMER	GIVEN TAP	ESTIMATED TAP	DIFFERENCE	%DIFFERENCE
TROUT9-TROUT2	.9762	.9762	0.0	0.0
ROSS1-ROSS2(1)	1.0122	1.0066	-.0056	-.55%
ROSS1-ROSS2(2)	1.000	1.0061	.0061	-.60%
ALCOA1-ALCOA2	.9756	.9758	.0002	.02%
KEELR9-KEELR2	.9762	.9741	-.0021	-.22%
ST JN2-ST JN1	.9685	.9687	.0002	.02%
KEELR2-KEELR1(1)	1.050	1.0037	-.0436	-4.3%
KEELR2-KEELR1(2)	1.025	1.0038	-.0212	-2.1%

Tables III and IV compare initial and final values of transformer susceptances and tap ratios respectively. Figure (8) provides comparisons of measured vs estimated flows at various transformers. Some fairly serious discrepencies can be observed at the ROSS1-ROSS2 and KEELR1-KEELR2 transformer banks.

4.3.6 Discussion of Results

Based on the above results and also the mass of information acquired during the testing period, it can be safely concluded that our prior suspicions regarding the network modeling problem were justified. This could not be more true then in the case of transformers. By deleting the reactive flows at the transformers banks from the estimation process, the estimates of these flows have errors of approximately several hundred MVAR's. In Fig. (8) the error is still over 100 MVAR's although these measurements were included in the estimation process.

In the case of transmission lines the case of 38.1% error seems to be amomalous. However, the other errors seem to be within theoretical predictions.

The main result of the above tests, we feel, is that we have a good tool to work with to simultaneously improve the accuracy of network parameters and identify sources of discrepancy in the model.

5. PROCEDURES FOR ON-LINE IMPLEMENTATION

5.1 Implementation Phases and Requirements

Successful implementation of BPA's on-line state estimator will depend on several software developments and the coordination of several activities. The primary activities involved are:

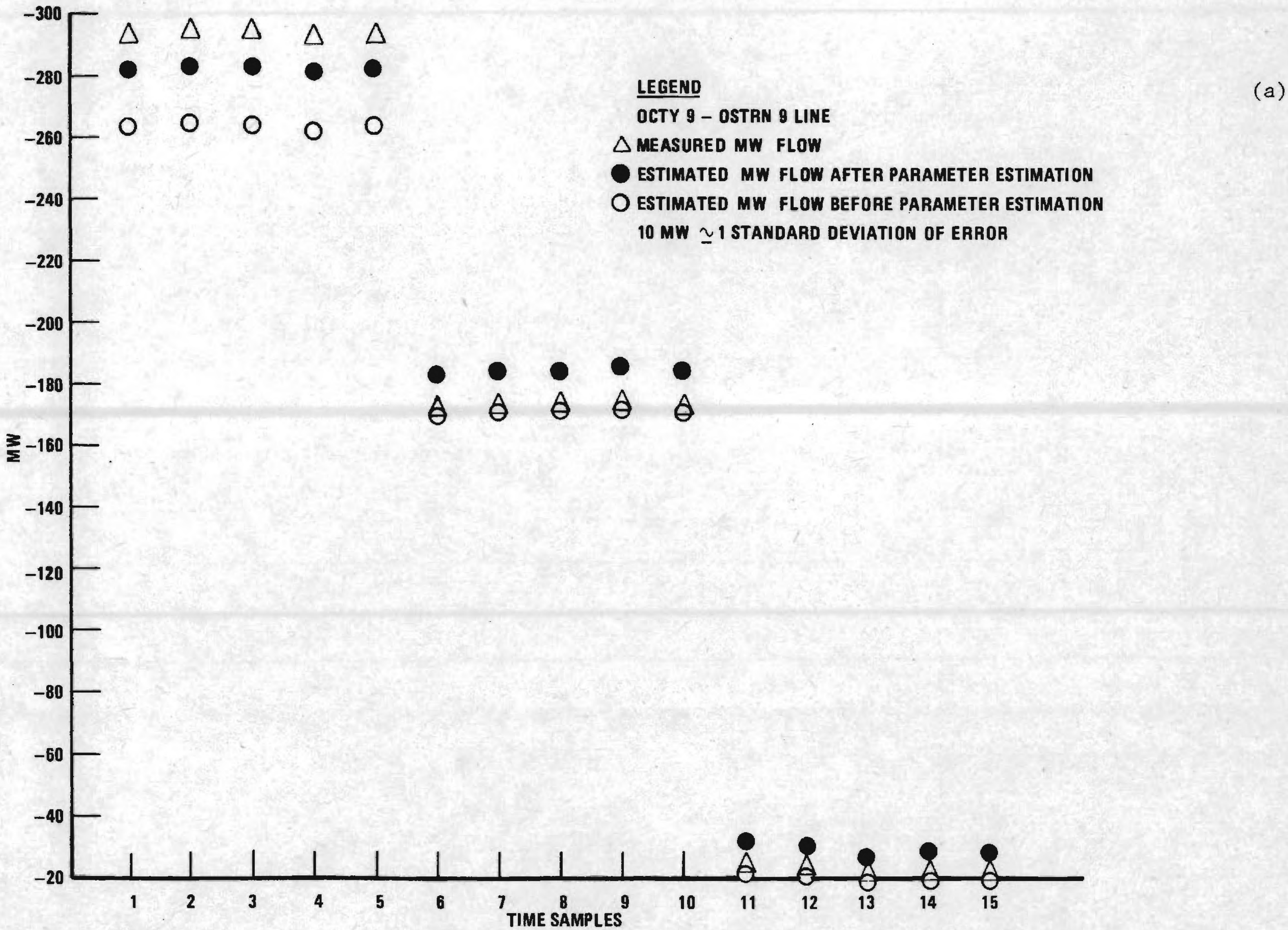
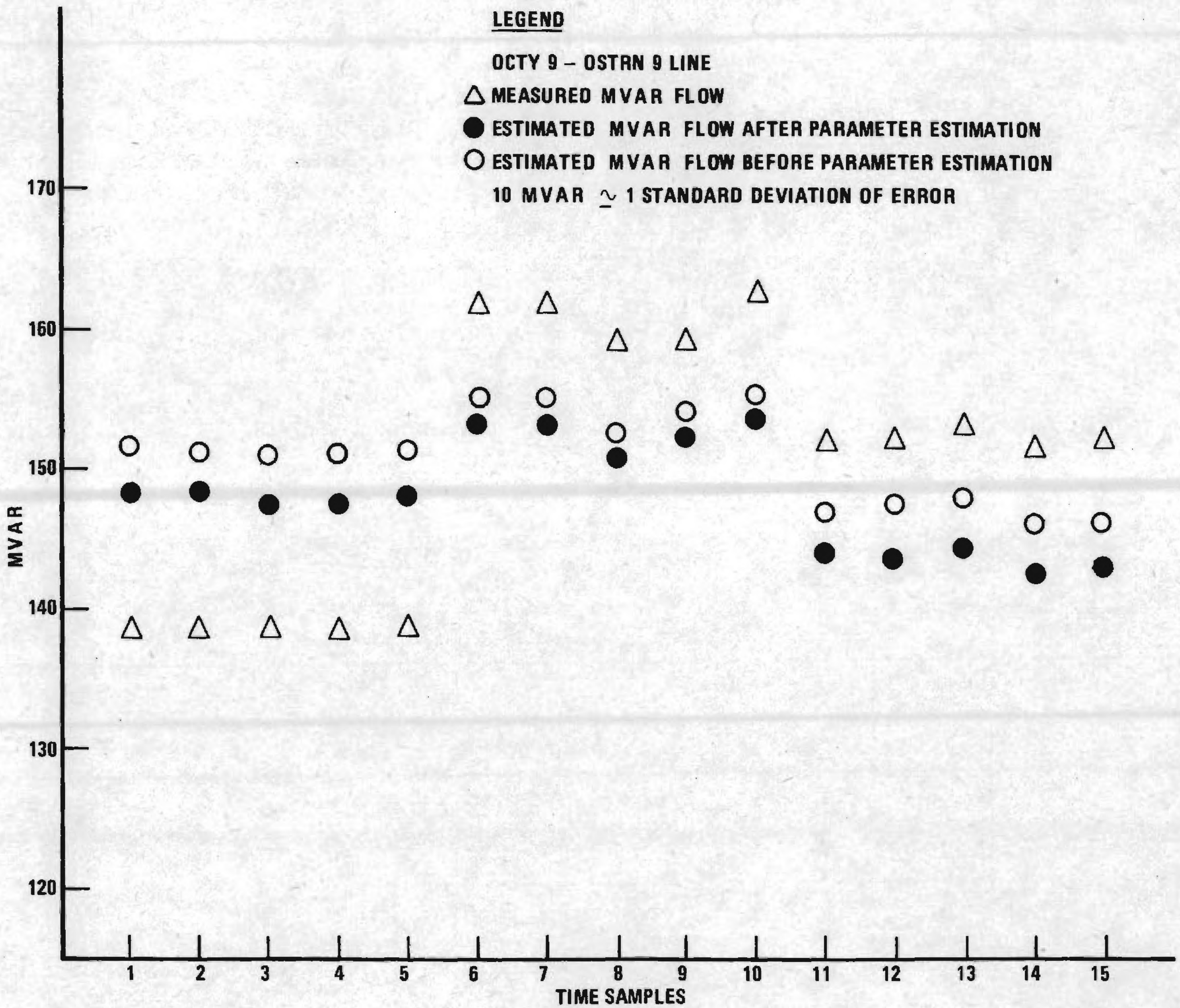
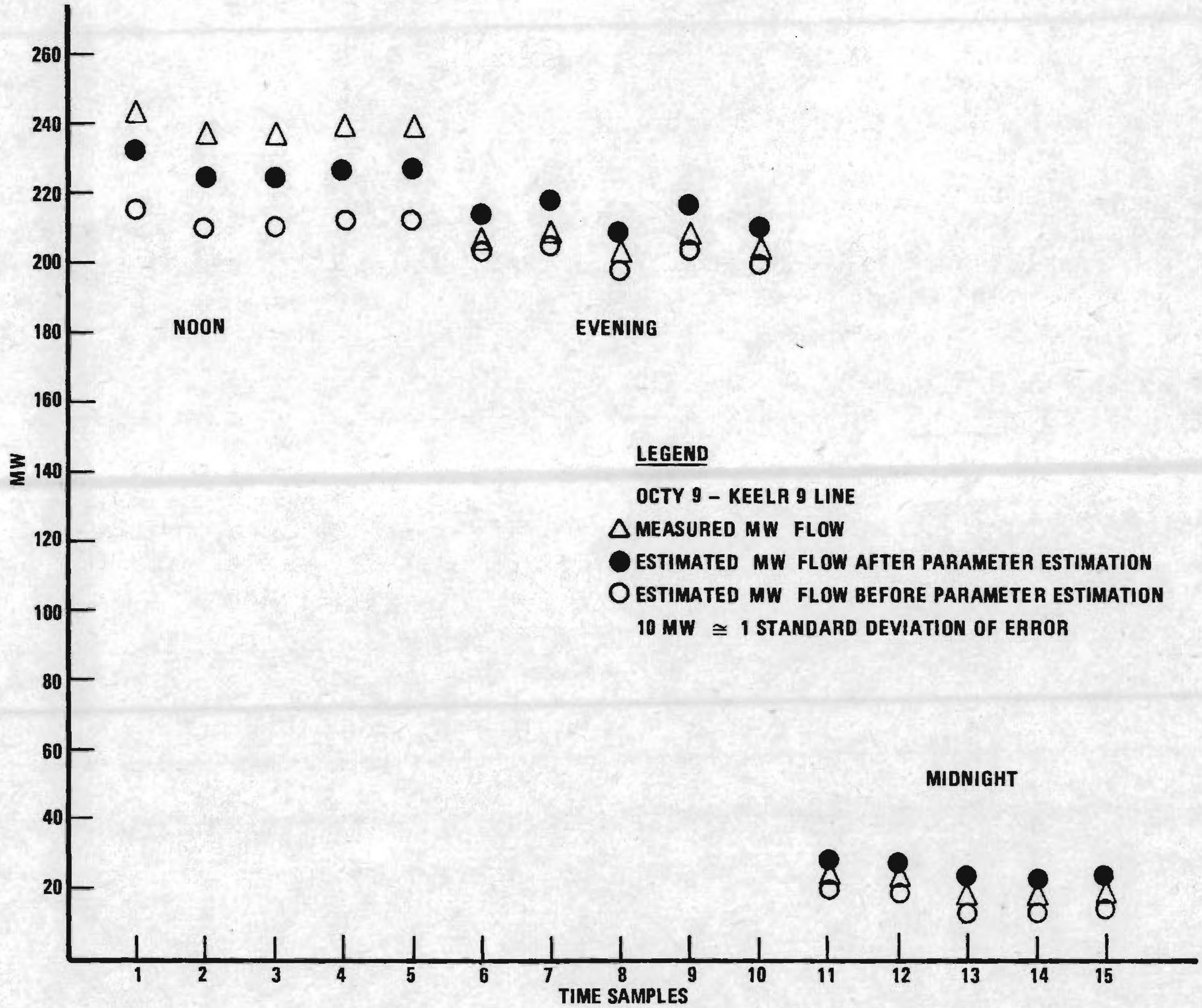


Fig. 7(a-e): Comparison of Measured with Estimated Flows on Transmission Line before and after Parameter Estimation.

(b)

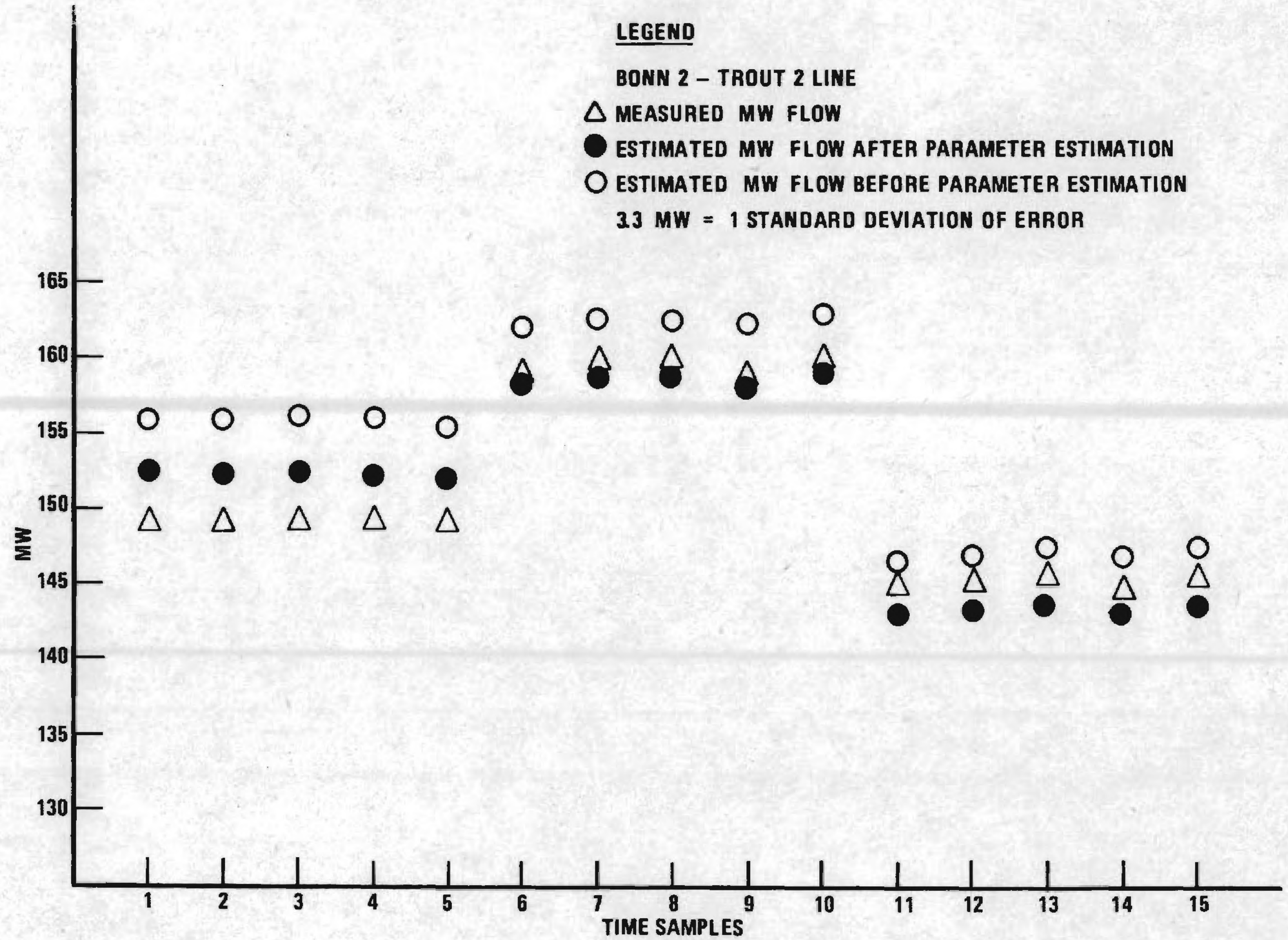

LEGEND
OCTY 9 – OSTRN 9 LINE
MEASURED MVAR FLOW
ESTIMATED MVAR FLOW AFTER PARAMETER ESTIMATION
ESTIMATED MVAR FLOW BEFORE PARAMETER ESTIMATION
10 MVAR ~ 1 STANDARD DEVIATION OF ERROR
MVAR
170
160
150
140
130
120
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
TIME SAMPLES

(c)

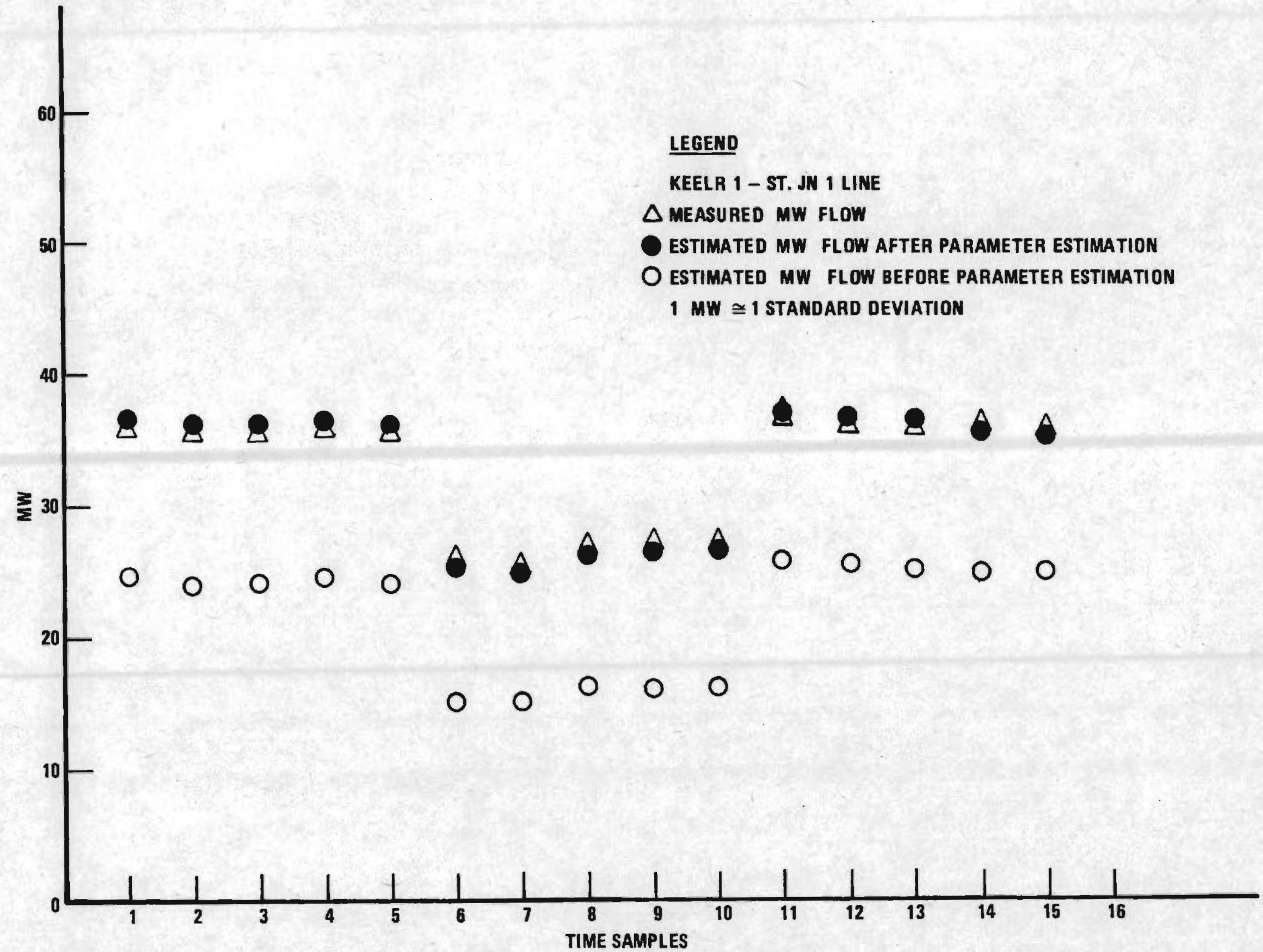


MW
260
240
220
200
180
160
140
120
100
80
60
40
20
NOON
EVENING
MIDNIGHT
LEGEND
OCTY 9 – KEELR 9 LINE
△ MEASURED MW FLOW
● ESTIMATED MW FLOW AFTER PARAMETER ESTIMATION
○ ESTIMATED MW FLOW BEFORE PARAMETER ESTIMATION
10 MW ≅ 1 STANDARD DEVIATION OF ERROR
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
TIME SAMPLES

(d)


LEGEND
BONN 2 – TROUT 2 LINE
MEASURED MW FLOW
ESTIMATED MW FLOW AFTER PARAMETER ESTIMATION
ESTIMATED MW FLOW BEFORE PARAMETER ESTIMATION
3.3 MW = 1 STANDARD DEVIATION OF ERROR
MW
165
160
155
150
145
140
135
130
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
TIME SAMPLES

(e)


LEGEND
KEELR 1 – ST. JN 1 LINE
MEASURED MW FLOW
ESTIMATED MW FLOW AFTER PARAMETER ESTIMATION
ESTIMATED MW FLOW BEFORE PARAMETER ESTIMATION
1 MW ≅ 1 STANDARD DEVIATION
MW
0
10
20
30
40
50
60
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
TIME SAMPLES

(a)

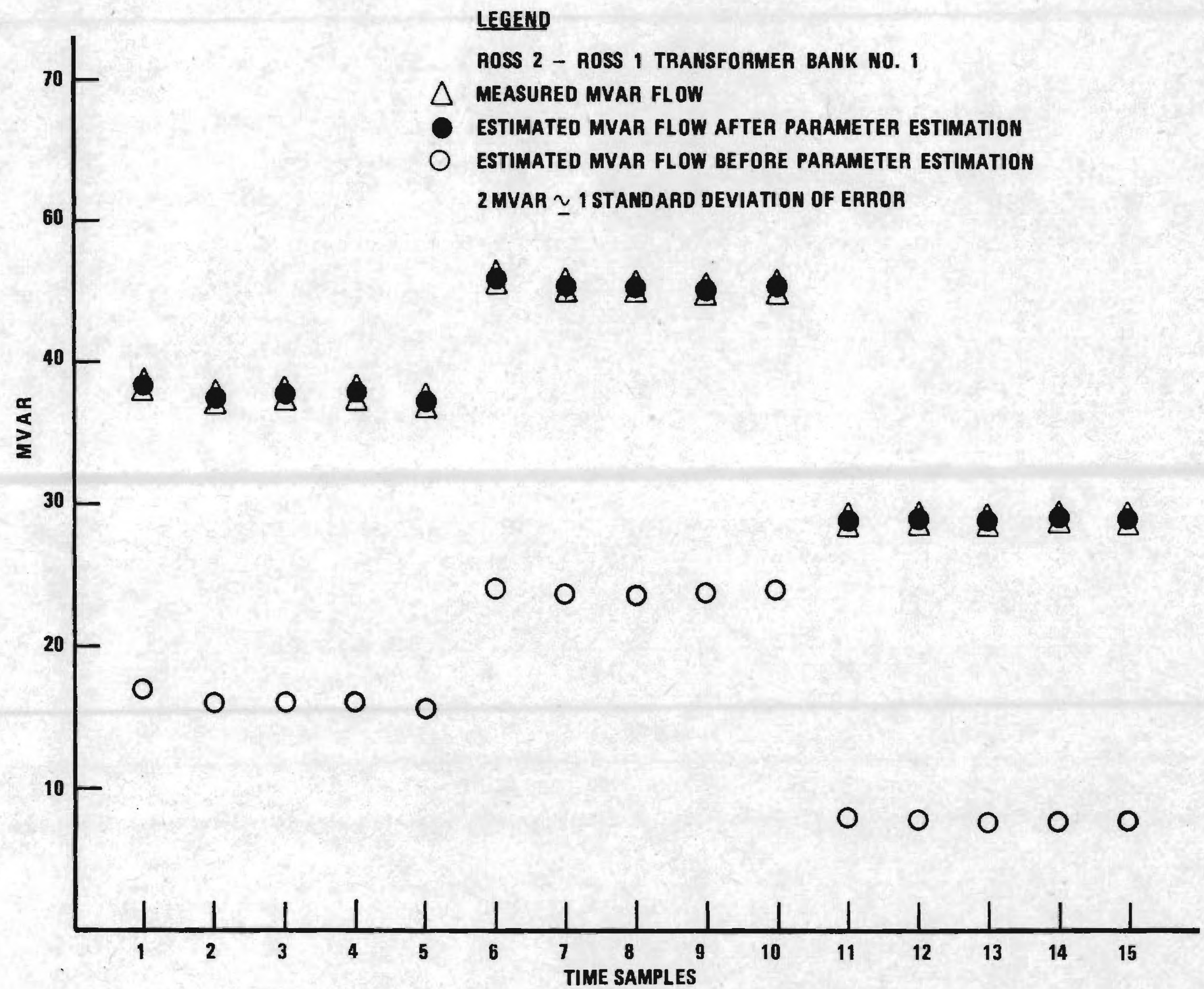
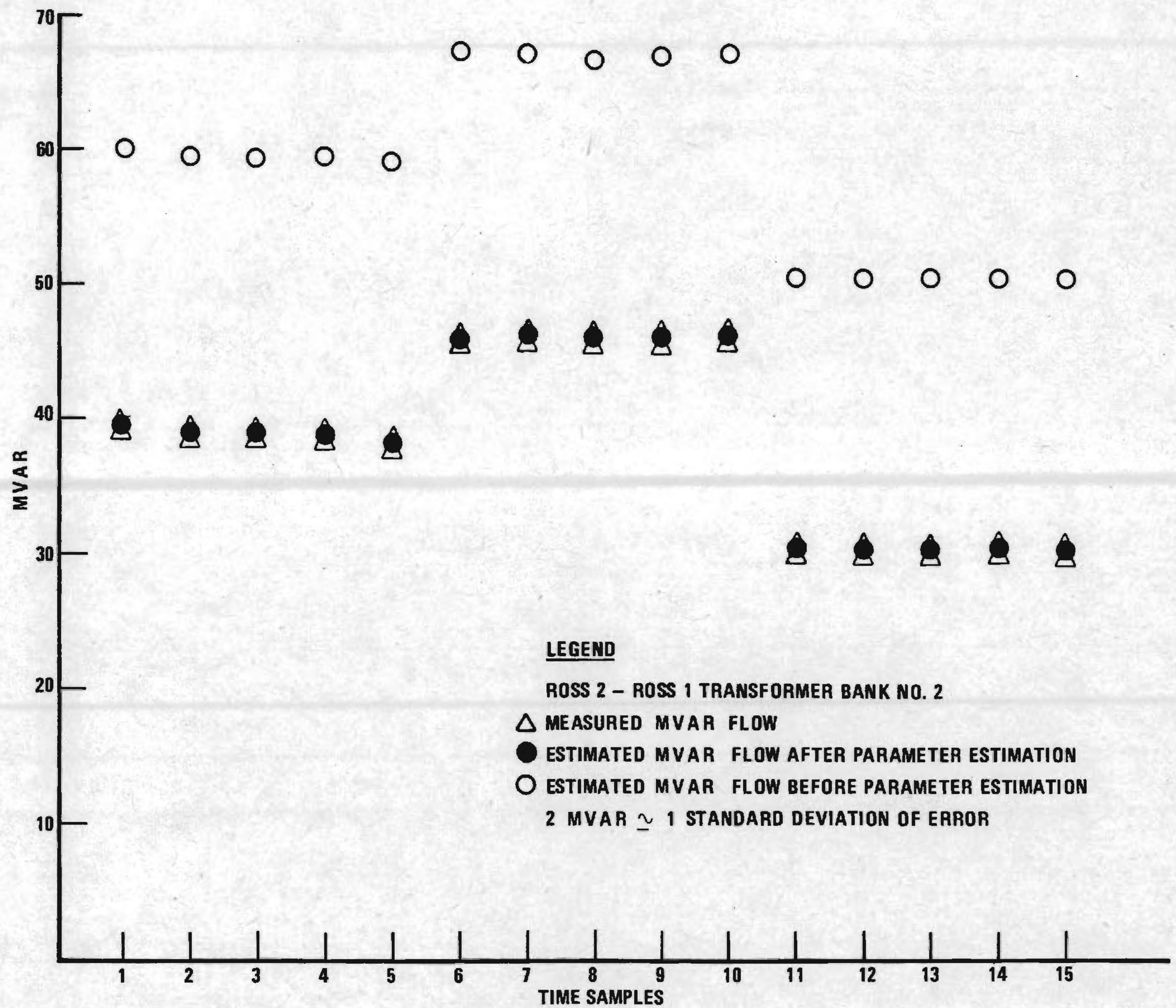
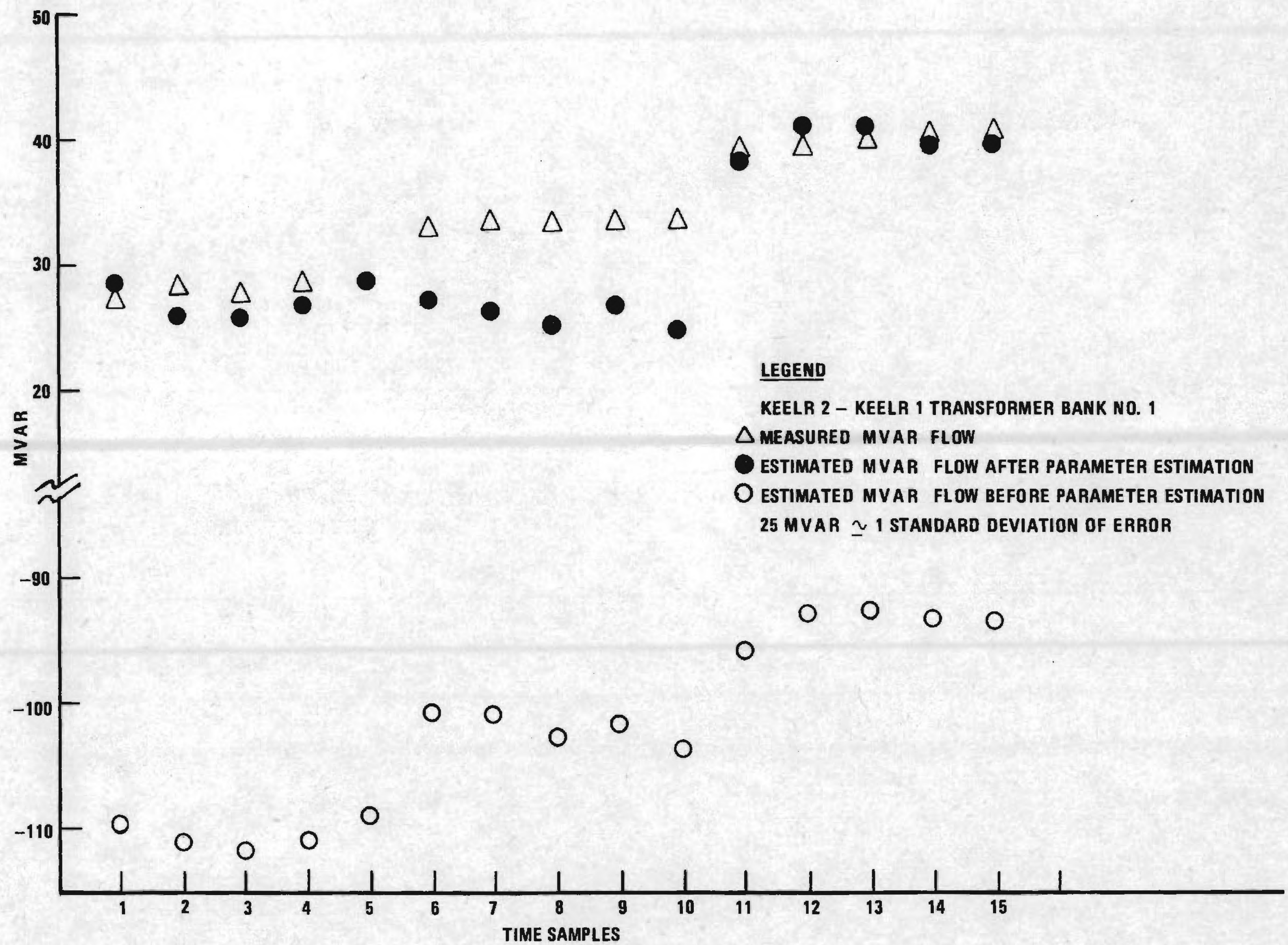


Fig. 8(a-d): Comparison of Measured with Estimated Flows on Some Transformer Banks

(b)

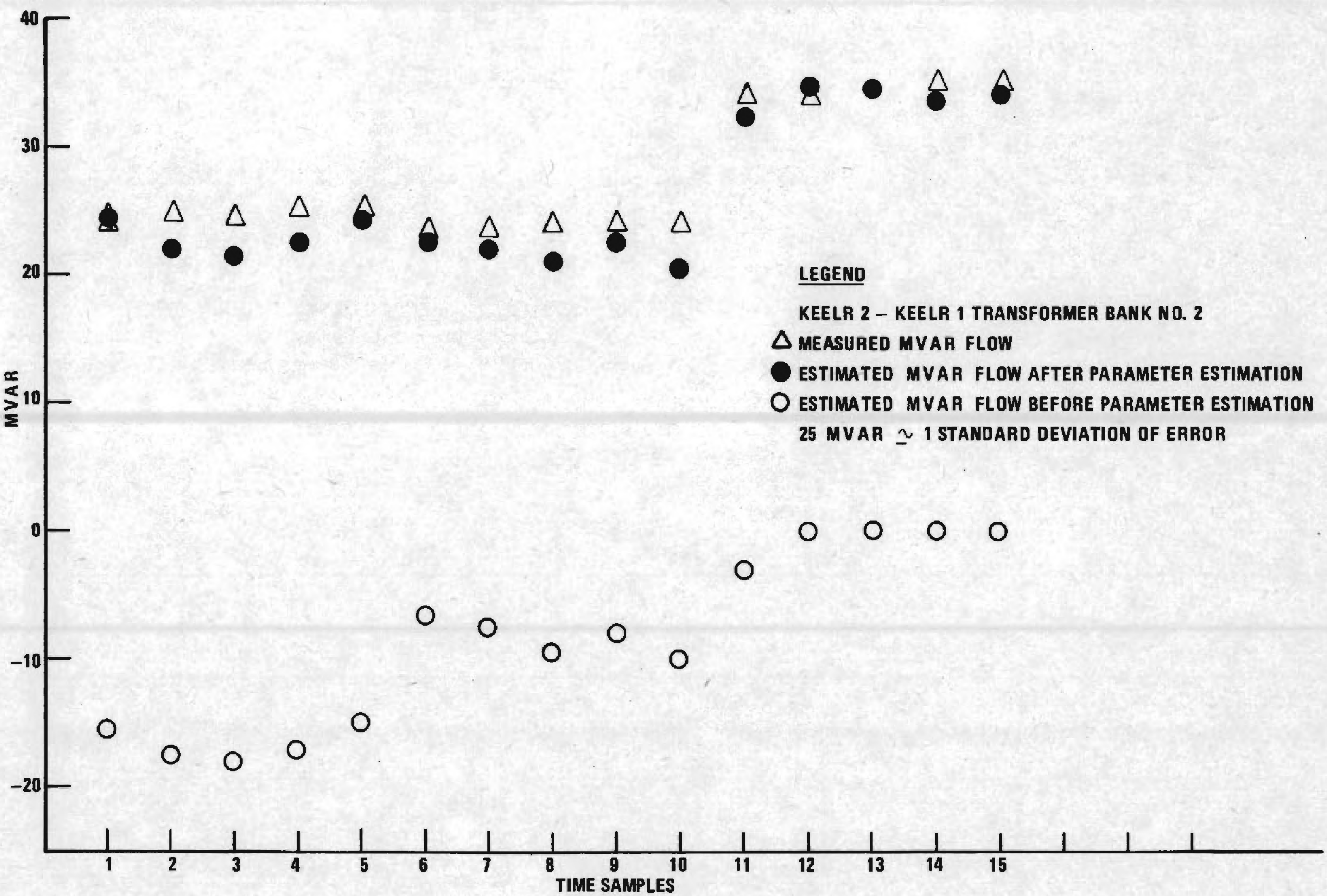

MVAR
70
60
50
40
30
20
10
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
TIME SAMPLES
LEGEND
ROSS 2 – ROSS 1 TRANSFORMER BANK NO. 2
MEASURED MVAR FLOW
ESTIMATED MVAR FLOW AFTER PARAMETER ESTIMATION
ESTIMATED MVAR FLOW BEFORE PARAMETER ESTIMATION
2 MVAR ≃ 1 STANDARD DEVIATION OF ERROR

(c)



50
40
30
20
MVAR
-90
-100
-110
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
TIME SAMPLES
LEGEND
KEELR 2 – KEELR 1 TRANSFORMER BANK NO. 1
△ MEASURED MVAR FLOW
● ESTIMATED MVAR FLOW AFTER PARAMETER ESTIMATION
○ ESTIMATED MVAR FLOW BEFORE PARAMETER ESTIMATION
25 MVAR ≃ 1 STANDARD DEVIATION OF ERROR

(d)



40
30
20
10
0
-10
-20
MVAR
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
TIME SAMPLES
LEGEND
KEELR 2 – KEELR 1 TRANSFORMER BANK NO. 2
MEASURED MVAR FLOW
ESTIMATED MVAR FLOW AFTER PARAMETER ESTIMATION
ESTIMATED MVAR FLOW BEFORE PARAMETER ESTIMATION
25 MVAR ≃ 1 STANDARD DEVIATION OF ERROR

a. Implementation of all software associated with the Sequential State Estimator,

b. Implementation of parameter estimation software,

c. Evaluation of operator requirements and confidence in state estimation,

d. Evaluation of scheduling requirements of performing state estimation based on collected system-wide data, and

e. Tuning of all kinds of relevant parameters prior to commissioning of the on-line estimator.

In the following sections we shall elaborate on the above items with emphasis on what has been achieved so far and what needs to be developed.

5.2 The Sequential State Estimator (SSE)

This program will be used for on-line state estimation purposes. The major components of the program dealing primarily with state estimation per se are fully developed and tested. During the course of the present project improved procedures for tuning the estimator to provide better performance weredeveloped and tested. The general approach to this tuning process is described in Appendix B.

The components of SSE that are in good working order are:

a. Pre-processing subroutines for initialization, divergence checks, limit checks and others

b. Ordering subroutine which retains and orders the busses of the observable portion of the network

c. Sequential estimation subroutine

d. Output subroutines

e. Subroutine which interfaces with bus-load forecasting program.

Remaining portions of the overall package contain subroutines for the detection and diagnosis of bad data. Full implementation and testing of these has not been performed yet.

Finally, programs which generate network configuration from SCADA outputs are yet to be fully developed and completed. This is a fairly difficult and obviously crucial program. It is essential that it becomes available as soon as possible.

5.3 Parameter Estimation

The two parameter estimation programs which have been developed are compatible with the state estimation package. In essense they use the same inputs and produce the same outputs except for outputs on network admittance parameters. Further refinements in these programs are necessary. These are:

a. Program cleanup which requires deletion of some unnecessary subroutines

b. Increase in capability to handle large networks. This may involve some extended core storage programming

c. Implementation of a better bus/parameter ordering scheme

d. Detailed program documentation

A by-product of the parameter estimation program is the weighted least squares estimation program using sparsity techniques. Any improvements on the parameter estimation package will automatically apply to this program.

5.4 Operator Requirements

During the full-scale testing period, serious effort should be associated with overall operator requirements from the points of view of

a. Display requirements

b. Understanding of the estimation program, and

c. Developing his confidence in the results.

All of these items are interrelated since a good deal of interaction, his confidence in the estimation process will, hopefully, grow.

5.5 Frequency of Updates

Studies which use actual system-wide data should be performed to determine the required frequency of updating the state estimates. These studies should be based on the following requirements:

a. Operator display

b. Bus-load forecasting

c. Security monitor, and

d. Voltage scheduling

The objective should be to satisfy all these requirements while minimizing computer time requirements. It is hoped that this will considerably reduce computational time requirements of state estimation. The data obtained during the study period indicated the changes in system operating conditions are very small even over a range of ten minutes. However, this may be characteristic only of the particular area monitored. Not enough information was available for system-wide assessment over a day or so.

The approach which we proposed contemplated using information about the time behavior of bus loads and generations. And by means of a simple prediction scheme (simplified bus-load forecaster) the decision is made when the next state estimate is necessary. This would make the process of when to compute state estimates adaptive. The predictor will determine also when to attempt security assessment next. Hence, there are overall savings in computer time for both state estimation and the rest of the applications programs.

Alternatively, however, display and bus-load forecasting requirements are better satisfied with periodic state estimator updates. Hence, an optimum solution should be reached which takes into account all of the above factors.

5.6 Testing and Tuning

Prior to final commissioning of the on-line estimator many of the tests conducted so far and others should be performed on a system-wide and thorough basis. These will consist of:

a. Very careful calibration of instrumentation and testing of any discrepancies arising in the metering process.

b. Evaluation of any inadequacies in the network configuration which is based on status information.

c. Careful checking of network models with special emphasis on transformers, status of capacitor banks, and all the required model changes due to status changes (e.g., 3-winding transformers).

d. Testing of statistical model validity without attempting any parameter estimation.

e. Tuning of network models by means of parameter estimation.

f. Examination, through field tests, analysis of data, discussions with operators, etc. of any serious discrepancies in the network models which might arise following parameter estimation.

g. Tuning of the sequential state estimator using all the validated models.

6. CONCLUSIONS AND RECOMMENDATIONS

The basic conclusions of the present study are:

a. Modeling problems do exist as far as network parameters are concerned,

b. These problems cause the state estimator to provide fairly unreliable results whereby considerable discrepencies between some measured and estimated quantities exist.

c. Parameter estimation can clean-up the network parameter models leading to statistically acceptable results.

d. There is still a significant role for field tests and engineering judgement in the modeling area. This should be exercised. Parameter estimation can easily pinpoint the areas of possible trouble and discrepencies.

e. Improved tuning procedures for the sequential state estimator were developed and implemented.

In my recommendations for future developments the following items are stressed:

a. The final stages of software development as described in the previous section should be undertaken with speed to insure early state estimator implementation.

b. Closer interaction among analysts, system operators, and programmers will be required to insure a proper understanding of the whole process and to develop the required confidence in it.

c. The immediate next step will be the model validation of all parameters associated with security monitoring. The present NSF supported work (with BPA's endorsement) is quite significant in developing the basic concepts. These will have to be tested for feasibility from the practical point of view. It is hoped that as a result of concerted effort a valid security monitor will result.

REFERENCES

1. A. S. Debs and W. H. Litzenberger, "The BPA State Estimator Project: Tuning of the Network Model," paper submitted for presentation at Summer Power Meeting, to be held in July, 1975.

2. A. S. Debs and W. H. Litzenberger, "Implementation of the BPA On-Line State Estimator," paper submitted for presentation at Power System Computation Conference, to be held in Cambridge, England in September 1975.

3. A. S. Debs, R. E. Larson, and L. P. Hajdu, "On-Line Sequential State Estimation for Power Systems," Proceedings of 1972 Power System Computation Conference held in Grenoble, France.

4. A. S. Debs, "Estimation of Steady-State Power System Model Parameters," IEEE Transactions on Power Apparatus and Systems, vol. PAS-93, No. 5, Sept./Oct. (1974).

5. K. A. Clements and R. J. Ringlee, "Treatment of Parameter Uncertainty in Power System State Estimation," Paper C-74-311-7 presented at Summer Power Meeting held in Anaheim, California (July, 1974).

6. A. S. Debs, "Estimation of Model Parameters for Power System Monitoring and Control," proposal submitted to National Science Foundation (Engineering Division), and resulted in Grant GK-37474 December (1972).

7. F. C. Schweppe--Discussion of Paper in Ref. 1 above.

8. N. Sato and W. F. Tinney, "Techniques for Exploiting the Sparsity of the Network Admittance Matrix, IEEE Trans. on Power Apparatus and Systems, Vol. 82, December 1963.

9. Richard Bauman, et. al., "The Main Factors of Influence in the Mathematical Modeling of Actual Conditions in Power Systems," ETZ-A Vol. 87, No. 32, Nov. 1966, pp. 818-823, Translated into English by approval of Bonneville Power Administration, c/o H. W. Dommel.

10. A. S. Debs, "Power System Model Inaccuracies and Sequential Tracking Estimation," Technical Memorandum PAU 949-3-1 prepared for Bonneville Power Administration by Systems Control, Inc., Contract 14-03-79968, Exhibit F, October 1971.

11. T. Stuart and C. J. Herget, "Sensitivity Analysis of Weighted Least Squares State Estimation for Power Systems," IEEE Trans. on Power Apparatus and Systems, Vol. PAS-92, No. 5, Sept./Oct. (1973).

12. K. A. Clements, et. al., "The Effects of Measurment Non-Simulatansity, Bias, and Parameter Uncertainty on Power System State Estimation," 1973 PICA Conference Proceedings, June 1973.

13. F. C. Schweppe, "Role of System Identification in Electric Systems," paper presented at Power Systems Computation Conference held in Grenoble, France (Sept. 1972).

14. A. S. Debs, "Estimation of External Network Equivalents from Internal System Data," paper T74 339-8 to appear in IEEE Trans. on Power Apparatus and Systems and presented at Summer Power Meeting, Anaheim, California, July (1973).

15. A. S. Debs, "Data Base for Real-Time Power System Control," invited paper to Allerton Conference on Circuits and Systems held in Montecello, Illinois (October 1973).

16. H. D. Merrill and F. C. Schweppe, "On-line system model error correction," IEEE Winter Power Meeting 1973, C73-106-2.

17. E. Hanschin, Editor, "Real-Time Control of Electric Power Systems," Elsivice Publishing Co., New York, 1973.

18. N. M. Peterson, W. F. Tinney, and D. W. Bree, "Iterative Linear AC Power Flow Solutions for Fast Approximate Outage Studies," IEEE Trans. PAS-91, No. 5, Sept./Oct. 1972.

19. R. A. Pyne, "Bus Load Forecasting and Its Intended Use in System Security Analysis," 1974 Proceedings of IEEE Conference on Decision and Control held in Phoenix, Arizona, November, 1974.

APPENDIX A

SENSITIVITY RELATIONS

Definitions:

$T_{ij} \triangleq$ Real power flow from node i to node j

$U_{ij} \triangleq$ Reactive power flow from node i to node j

$G_{ij} + j\,B_{ij} \triangleq$ Series admittance of branch i-j

$BSH_i \triangleq$ Shunt admittance of branch i-j referred to node i

$a_i \triangleq$ Tap setting in p.u. of the fixed-end tap of a transformer

$E_i + j\,F_j \triangleq$ Complex voltage at node i.

$$\frac{\partial T_{ij}}{\partial B_{ij}} = E_i F_j - E_j F_i$$

$$\frac{\partial T_{ij}}{\partial G_{ij}} = E_i^2 + F_i^2 - E_i E_j - F_i F_j$$

$$\frac{\partial U_{ij}}{\partial B_{ij}} = E_i^2 - F_i^2 + E_i E_j + F_i F_j$$

$$\frac{\partial U_{ij}}{\partial G_{ij}} = E_i F_j - E_j F_i$$

$$\frac{\partial U_{ij}}{\partial BSH_i} = -E_i^2 - F_i^2$$

$$\frac{\partial T_{ij}}{\partial a_i} = +\frac{1}{a_i}\left(\left(2(E_i^2 + F_i^2) - (E_i E_j + F_i F_j)\right)G_{ij} + (E_i F_j - E_j F_i)B_{ij}\right)$$

$$\frac{\partial U_{ij}}{\partial a_i} = \frac{1}{a_i}\left(B_{ij}(2(E_i^2 + F_i^2) - E_i E_j - F_i F_j) + 2BSH_i(E_i^2 + F_i^2) + G_{ij}(F_i E_j - F_j E_i)\right)$$

$$\frac{\partial T_{ij}}{\partial a_j} = \frac{1}{a_j}\left(G_{ij}(E_i E_j + F_i F_j) + B_{ij}(F_i E_j - F_j E_i)\right)$$

$$\frac{\partial U_{ij}}{\partial a_j} = \frac{1}{a_j} (G_{ij}(F_i E_j - F_j E_i) - B_{ij}(E_i E_j + F_i F_j))$$

APPENDIX B

TUNING PROCEDURES FOR THE SEQUENTIAL STATE ESTIMATOR

In Ref. (1) a fairly detailed description of the basic algorithm of sequential state estimation is provided. It is argued there that the diagonalization of the "covariance" matrix can lead to improper performance. Hence, a set of tuning procedures were developed in order to ensure the statistical validity of the results. Following is a description of such procedures.

B. Description of the Algorithm

Denote by z_i the next measurement to be sequentially processed, and by $\hat{x}^{j,i}$ to be the estimate of x at the j^{th} iteration following the processing of z_i. The algorithm now simply is:

$$\hat{x}^{j,i} = \hat{x}^{j,i-1} + w^{j,i}(z_i - h_i(\hat{x}^{j,i-1})), \tag{B1}$$

$$w^{j,i} = \frac{P^{j,i-1}H_i}{\mu(z_i)\cdot\sigma_i^2 + H_i^T P^{j,i-1} H_i}, \tag{B2}$$

$$P^{j,i} = \mathrm{Diag}[(I - w^{j,i}H_i)P^{j,i-1}], \tag{B3}$$

where

$$z_i = h_i(x) + v_i$$

v_i = zero mean Gaussian random variable whose variance is σ_i^2

$$H_i = \frac{\partial h_i}{\partial x}\Big|_{\hat{x}^{j,i}}$$

$\mu(z_i)$ = tuning parameter associated with measurement z_i.

As will be shown below the measurement z_i may be processed more than once within an interation. This will become clear following the description of the ordering procedure.

C. Ordering of Measurements within a Single Iteration

The ordering process is based on the following rules:

a) The order of priority is on measurement equations with the least number of state variables

b) A MW quantity is followed by the corresponding MVAR quantity

c) The next measurement to be processed should update, at most, the variables of a single bus which previously, have not been updated.

The simplest application of these rules is when line flow measurements are sufficient to determine all state variables. The ordering of these measurements is illustrated in figure B1. In Figure B1(a) the node at measurement no. 1 is the slack bus. Processing of every subsequent measurement will, at most,

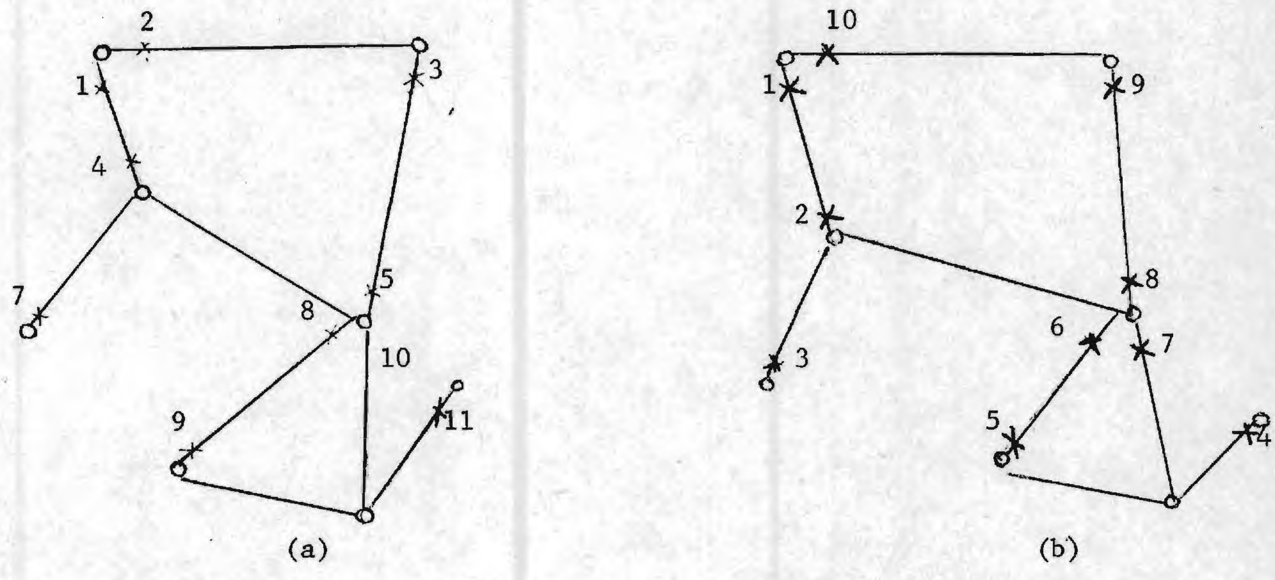


Fig. B1: Ordering of Line Flow measurements (x indicates line flow is measured at corresponding bus) (a) Adequate ordering (b) Inadequate ordering--measurements 4 and 5 will update state variables at two "new" busses.

update the variables at one more bus that have not been updated before. Thus, if a subset of line flow measurements is sufficient for a load-flow solution application of the above rules is straighforward. The ordering is as follows:

1. Process all KV measurements in any arbitrary order, then

2. Process all line flow (real, then reactive) measurements according to rule (3), and finally,

3. Process all injection measurements in any arbitrary order.

In the actual program, all line flow measurements are processed twice. This is achieved by processing first the near-end and then the far end line flow measurements. Normally, all line flows at a bus are processed at the same as well as the opposite ends. Then one moves to a neighboring bus whose state variables have already been updated and so on.

Special attention is paid to those cases where line flows do not constitute a sufficient set. In figure B2, it is necessary to process the injections 3 and 4 before continuing to process the remaining line flow measurements.

The main implication of this ordering process is that voltage and line flow measurements constitute the backbone of the overall measurement system. With these measurements the bulk of the network should be observable.

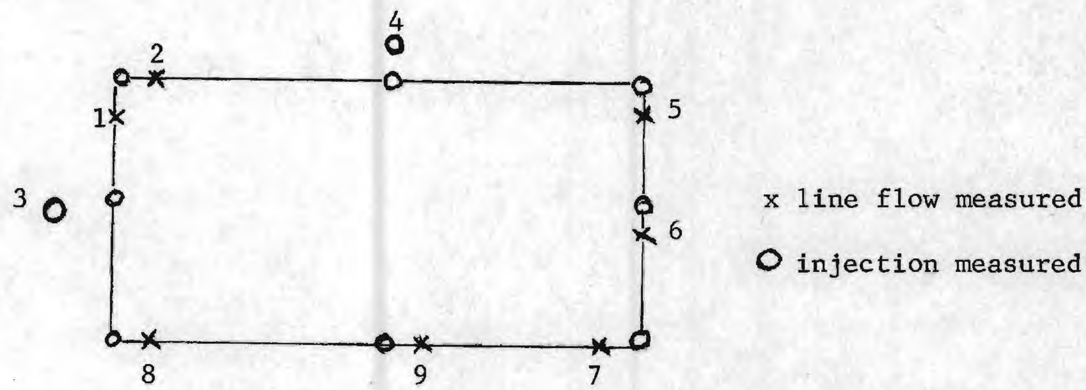


Fig. B2: Ordering of Injections in Cases where Line Flow Measurements Do Not Constitute a Sufficient Set

D. Tuning Parameters

One scalar parameter $\mu(z_i)$ is associated with each of the five measurement categories: voltage, real and reactive flows, and real and reactive injections. Normally for real flows, μ is set at the value of 1.0. Typical values for the other μ's range from 1 to .1. They are adjusted in a few simulations to insure adequate performance. It should be noted however, that this constitutes a fine tuning process. Experience has shown that estimator performance is quite insensitive to changes in the μ values.

E. Processing of Iterations

An iteration is defined by the set of all sequential operations according to the above ordering scheme. At the end of an iteration all line flow measurements would have been processed twice and the remaining measurements once. Here the diagonal P matrix is updated according to the equation

$$P^{j+1} = P^{j} + Q \tag{B4}$$

where

$$Q = \left[\begin{array}{c|c} \alpha_E I & 0 \\ \hline 0 & \alpha_F I \end{array}\right]$$

where I is the identity matrix and α_E and α_F are two constants associated with the real and imaginary parts of complex bus voltages. Typically, $\alpha_E \simeq 10^{-2}$-10^{-3} and $\alpha_F \simeq 10\alpha_E$. Larger values of α_E and α_F will accelerate the convergence process. Initially P^o is set equal to Q.

Normally, convergence takes place in two iterations starting from a flat voltage start. Following that, at least one iteration is performed without updating P_j according to Eq. (B4). This has shown to improve performance somewhat.