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Initial Evaluation: Dynamical Modelling of the Lockheed-Martin Phase Locker

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This report summarizes my findings on the current status of dynamical modelling of laser arrays of the type relevant to the Lockheed-Martin system.

Relevant Literature

My goal in undertaking the literature search, my primary focus was to identify mathematical descriptions for the time evolution of laser arrays. I identified several useful papers; the main features of the most useful of these are cited in this section. A larger listing is provided at the end of this document. The mathematical descriptions fall into three categories: differential equations, delay differential equations, and discrete iterative maps.

Braiman *et al.* theoretically investigate the phase locking properties of a set of lasers coupled via evanescent overlap of the electric fields additionally subject to a common injected field. They report that the natural dephasing effects of evanescent coupling can be overcome by injecting the additional (common) field. The model used in this paper has several forerunners, most particularly those by Basov *et al.*, Perel and Rogova, and Spencer and Lamb.

Braiman *et al.* introduce a dynamical model consisting of a set of coupled ordinary differential equations, treating the complex electric field and gain for each laser as relevant dynamical variables, having adiabatically eliminated the polarization. For the purpose of analytic investigation they furthermore reduce the dynamical equations by considering a limit valid when the amplitude of the injected field is sufficiently small. The result is a pure phase model, i.e. a set of differential equations which directly and exclusively follow the phases of the electric fields; the remaining degrees of freedom (the electric field amplitudes and the laser gain variables) completely decouple. The resulting phase equations are analytically tractable, and comparisons with the full dynamical model are consistent in the appropriate limiting cases.

The dynamical model developed by Wang and Winful to study coupled lasers also has the form of a set of ordinary differential equations. It follows the complex electric fields and the (real) gain fields continuously in time:

$$
\begin{aligned}
\dot{E}_i &= \frac{1}{2}\left(\Gamma G_i - \frac{1}{\tau_p}\right) E_i - \frac{\kappa c}{n}\left[E_{i+1}\sin\left(\phi_{i+1}-\phi_i\right) + E_{i-1}\sin\left(\phi_{i-1}-\phi_i\right)\right] \\
\dot{\phi}_i &= \omega_{th} - \omega_i + \frac{\kappa c}{n}\left[\frac{E_{i+1}}{E_i}\cos\left(\phi_{i+1}-\phi_i\right) + \frac{E_{i-1}}{E_i}\cos\left(\phi_{i-1}-\phi_i\right)\right] \\
\dot{N}_i &= \frac{J}{ed} - \frac{N_i}{\tau_s} - \Gamma G_i \left|E_i\right|^2
\end{aligned}
$$

where E_i is the (real) amplitude of the complex electric field in the i^{th} laser, ϕ_i is the corresponding (real) phase, and N_i is the scaled linear excess gain above threshold of the i^{th} laser; τ_p and τ_s are the photon and carrier lifetimes, and κ is the coupling constant. In their studies, Wang and Winful had in mind the problem of coupled semiconductor lasers; however, their mathematical formulation is also appropriate (with minor changes) for solid state lasers and fiber lasers. They found that for two lasers which were coupled evanescently – the only case they consider – the antiphase state was stable over a fairly wide range of physically accessible parameters, while the inphase state was typically unstable.

Williams *et al.* develop a theoretical model to compare against their experiments on an erbium-doped fiber ring laser. Their model is a stochastic delay differential equation. Numerical simulations of the model appear to reproduce at a qualitative level a wide range of nonlinear dynamical behavior observed in the experiments. Even though many longitudinal modes are active inside the cavity, they find that a model limited to only two supermodes corresponding to two orthogonal polarization states gives a good description. Starting from the Maxwell-Bloch equations for a single fiber, they derive a set of coupled delay-differential equations by translating the spatial dependence on the propagation direction of the Maxwell-Bloch partial differential equations into a purely temporal dependence on time-delayed quantities. The resulting model equations are cumbersome (and for that reason I don't reproduce them here), and sufficiently complicated that only numerical simulations were reported.

The above fiber ring laser model describes a single laser only. The work of Lewis *et al.* considers the problem of coupling together two such lasers. The type of synchronization they sought is somewhat different than the type envisioned for power combining. Specifically, Lewis *et al.* were pursuing the possibility of synchronized chaos for the purposes of secure communications. Therefore, the kind of coupling they studied was a one-way, feed-forward coupling applicable to a transmitter-receiver pair. They find that synchronization is theoretically achieved for even very small levels of injected signal. Their model for coupled doped fiber ring lasers takes the form of coupled delay differential equations. As with the previously discussed model of Williams *et al.*, the explicit equations are cumbersome; they can be found in Sections II and III of Lewis *et al.*. The general form of the equations can be represented in a reasonably compact form, by making use of the propagation map $\mathbf{M}$ which depends on the independent variables of the system:

$$\begin{aligned}
\mathcal{E}_T(\tau+\tau_R) &= \mathbf{M}\left(w_T(\tau), \mathcal{E}_T(\tau)\right) \\
\frac{dw_T(\tau)}{dt} &= Q - \gamma\left[w_T(\tau) + 1 + |\mathcal{E}_T(\tau)|^2\left(e^{Gw_T(\tau)} - 1\right)\right]
\end{aligned}$$

$$\begin{aligned}\mathcal{E}_R(\tau+\tau_R) &= \mathbf{M}\left(w_R(\tau), c\mathcal{E}_T(\tau)+(1-c)\mathcal{E}_R(\tau)\right)\\ \frac{dw_R(\tau)}{dt} &= Q-\gamma\left[w_R(\tau)+1+|c\mathcal{E}_T(\tau)+(1-c)\mathcal{E}_R(\tau)|^2\left(e^{Gw_R(\tau)}-1\right)\right]\end{aligned}$$

Here, $\mathcal{E}$ is the complex electric field and w is the gain; the subscripts T and R refer to the transmitter and receiver lasers, respectively.

The descriptions given above represent time-continuous dynamical models. That is to say, time is always continuous but these models attempt to represent the evolution as it unfolds at each instant of time. A different kind of dynamical description is sometimes employed to great effect in the study of nonlinear systems. In the particular context of laser arrays, such a model has been developed (and only recently published) for coupled laser arrays by Rogers *et al.*. The model follows the evolution of dynamical variables in finite time steps equal to the roundtrip time of the propagating electromagnetic wave. The model is derived under conditions which allow that the system may be operating in the high gain regime, so that the laser output may be time dependent and even erratically pulsing. This in turn implies that the field variables can varying significantly on a rather short time scale, perhaps even comparable to the roundtrip time itself. Under these circumstances, a discrete-time iterative map approach is indicated, and in any event passage to a time-continuous set of differential equations by way of the slowly varying wave amplitude (SVWA) approximation is not valid.

The resulting equations are

$$E_n(t+T) = \sum_{m=1}^{N} A_{nm}E_m(t) \tag{1}$$

$$G_n(t+T) = G_n(t)+\epsilon\left[G_n^p - G_n(t)\right] - \frac{2\epsilon}{I_{sat}}\left(1-e^{-G_n(t)}\right)I_n(t) \tag{2}$$

where A is the propagation matrix which includes the effects of coupling, ϵ is the ratio of the round trip time to the fluorescence time, I_{sat} is the saturation intensity and I_n is the electric field intensity in the n^{th} fiber. The derivation of this model consists of two parts: one traces the propagating electromagnetic waves through one complete round trip from some conveniently chosen reference plane and the other is a Rigrod analysis for the (relatively slow) gain dynamics.

The propagation matrix elements A_{nm} depend on the details of the system architecture. In the 2005 JQE paper, the coupling is presumed to be due to evanescent overlap of the electric fields occurring between neighboring ports within the physical coupling regime. This assumption has to be reconsidered in the case of injection coupling. That said, the mathematical structure of the displayed equations is general enough to include injection coupling of the type embodied by the Lockheed-Martin Phase Locker; however, the details are sufficiently different in a number of respects, and a simple adoption of the model isn't possible. On the other hand, the Lockheed-Martin system dynamics can be derived in close correspondence, using the JQE derivation as a blueprint.

Physical and Mathematical Considerations in Formulating the Problem

This section lays out the basic physics problem and the relevant considerations required to develop a quantitative dynamical description of the Lockheed-Martin Phase Locker system.

A convenient characteristic of the Lockheed-Martin system is that its architecture admits a modular description. That is, the Phase Locker can be viewed as one stand-alone component and similarly the set of lasers in the region outside the Phase Locker. This physical property of the composite system will be reflected in the mathematical structure of the dynamical model. It follows that, for example, alterations in the Phase Locker properties (such as different degrees of rotation and/or percentage of incident transmission) can be separately developed and refined, and the resulting mathematical description can be "plugged in" to the integrated system model.

The basic structure of the physical system is a set of lasers which evolve independently over some spatial region and are coupled over another, spatially distinct, region of space. Depending on the type of lasers employed, the number of independent variables (per laser) needed to describe the system may be as large as five. Under most circumstances, it is likely that variations within the cavity itself can be ignored over the time scale of a single round trip. This is strictly true only for the case of single-mode operation, but the bulk of the published literature uses this level of approximation even when a large number of modes are active, as happens for example in typical fiber laser systems. Mathematically, this allows a reduction in complexity which obviates the need for partial differential equations and instead allows either ordinary differential equations – either with or without time-delay – or iterative maps.

For either semiconductor lasers or (most) fiber lasers, it is possible to adiabatically eliminate the polarization degrees of freedom, and to capture the relevant dynamics using three degrees of freedom, namely the complex electric field and gain variables. These variables typically evolve on widely different time scales. The ratio of the corresponding time scales (round trip time and fluorescence time) then arises as a natural small parameter. In physical terms, this means that the gain evolves on a much slower time scale than the electric field. In practical terms, from a modelling standpoint, it becomes important to develop at least some analytic understanding of the problem; otherwise, numerical simulations have to be carried out with very small time steps corresponding to the fastest evolving variable. In contrast, making direct use of the natural small parameter can increase simulation times by three or four orders of magnitude.

At the present time, there remains a significant gap between physically comprehensive models of laser arrays and models which have been carefully vetted against laboratory experiments. I expect this gap to remain reasonably wide in the near term. On the other hand, some piecemeal progress that would yield useful levels of physical insight is possible. Two areas that are particularly important when it comes to guiding laboratory realizations are (i) the role of noise (both quenched disorder and dynamical noise) and (ii) the consequences of multiple active modes.

Phase Locker

The essential role of the Phase Locker is to take an input beam (or a set of input beams) and re-inject a modified profile back into the beam line. Consider first the case of a single laser beam incident, and call the incident electric field amplitude E_{in}. As the wave propagates around the Phase Locker, it is modified and eventually re-injected as F_{out}. The distinction between the variable names keeps straight the fact that they represent waves advancing in counter-propagating directions. A complete characterization of the Phase Locker may be represented by an expression of the form

$$F_{out}(t) = f\left\{E_{in}\left(t - T_{PL}\right)\right\}$$

where T_{PL} is the time it takes for light to propagate through the Phase Locker.

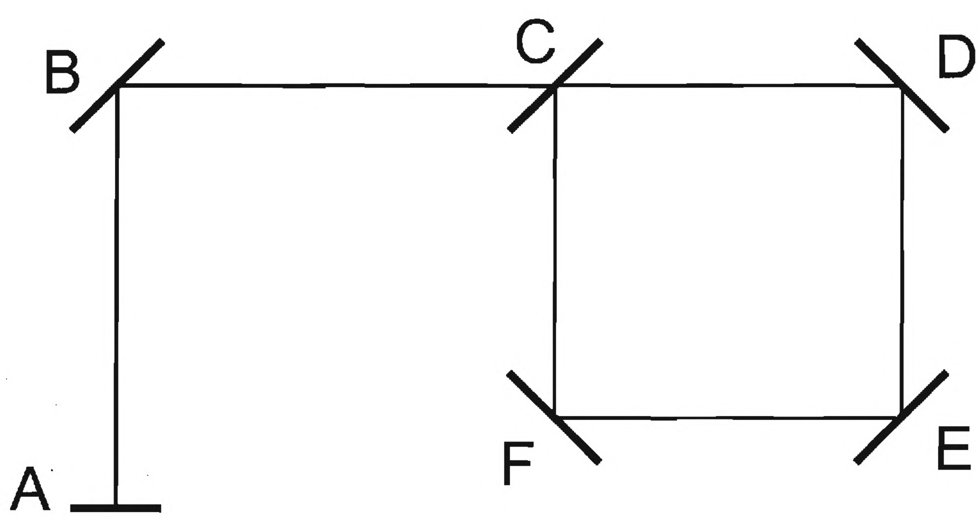
For an array of input beams, a similar characterization can be made. As a particular example, consider the following characterization of the Phase Locker as described in the Lockheed-Martin white paper dated 30 March 2007. The incident ring of four beams impinge on a 50/50 splitter; one part propagates clockwise and the other counterclockwise around the Phase Locker in a time T_{PL}, with each beam cyclically shifted to the position of its nearest neighbor. If I denote the complex electric field amplitude of the n^{th} incident beam as E_n, and the corresponding re-injected field as F_n, then after a single pass

$$\begin{pmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \\ F_4(t) \end{pmatrix} = \frac{1}{2} e^{i\phi} \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} E_1(t-T_{PL}) \\ E_2(t-T_{PL}) \\ E_4(t-T_{PL}) \\ E_4(t-T_{PL}) \end{pmatrix}$$

where ϕ is the free-propagation phase shift acquired in going around the Phase Locker: $\phi = \omega T_{PL}$ if ω is the laser frequency. What happens next depends on the particular architecture of the full system: for example, some fraction of the emerging beams may suffer additional passes through the Phase Locker; in any event at least some of the power is re-injected into the laser cavity in the opposite direction as the incident beams, and these propagate through the amplifier, to be reflected (for example) at one end of the cavity.

Example

I now present a specific theoretical realization of an explicit mathematical model which embodies the above considerations. It is (of course!) unknown the degree to which these equations might accurately capture the behavior of the Lockheed-Martin system. Comparisons with laboratory measurements presumably will lead to at least some modifications of



Consider the arrangement depicted in the figure. A set of otherwise independent beams created in the cavity AB is directed through the Phase Locker configuration CDEF. Four of the mirrors are perfectly reflecting; mirror B is partially reflecting (and the transmitted portion represents the systems output); mirror C is a 50-50 splitter. To deduce a corresponding set of dynamical equations, one follows the beams through one complete round trip. The reference plane can be chosen for convenience, and in the following I choose a point just inside mirror A.

The sequence of events is the (1) propagation with gain through arm AB; (2) partial reflection into arm BC and partial transmission out of the system; (3) splitting into counter-propagating beams at point C; (4) free propagation around the Phase Locker with corresponding cyclic rotation of the beams; (5) recombination of the counter-propagating beams at C; (6) reflection of the backward-propagating beams at mirror B; (7) re-injection into the gain arm BA; (8) reflection at A.

Denote by $E_n(x,t)$ the complex electric field amplitude of the n^{th} fiber at position x and time t, for the forward (n the diagram, the path ABC) propagating beam; similarly, denote by $F_n(x,t)$ the amplitude for the reverse beam (path CBA). The gain of the n^{th} fiber is G_n and depending on the situation may be modelled as a parameter or as an independent dynamical variable.

Tracking through the event sequence, we have in the first step amplitude amplification and a phase shift

$$E_n(B^-,t_1) = e^{G_n}e^{I\phi_1}E_n(A,t_0)$$

If R is the field reflection coefficient of mirror B,

$$E_n(B^+,t_1) = RE_n(B^-,t_1)$$

Each (still independent) beam acquires an additional phase shift in arm BC:

$$E_n(C^-,t_2) = e^{I\phi_2}E_n(B^+t_1)$$

One half of each beam propagates around the Phase Locker and is shifted into the beam line of its neighbor; counter-propagating halves are taken to have opposite shifts; all beams acquire the same free-propagation phase shift:

$$F_n(C^-,t_3) = \frac{1}{2}e^{I\phi_3}E_{n-1}(C^-,t_2) + \frac{1}{2}e^{I\phi_3}E_{n+1}(C^-,t_2)$$

where the subscript labelling is taken modulo N. There is another phase shift in traversing path CB:

$$F_n(B^+, t_4) = e^{I\phi_2} F_n(C^-, t_3)$$

Finally, the (independent) beams pass through the gain region a second time:

$$F_n(A, t_5) = e^{G_n} e^{I\phi_1} F_n(B^+, t_4)$$

The complete round trip ends with a final reflection off of the mirror (or grating) at point A, which generates a final phase advance of π radians. Combining the full sequence of expressions over one round trip yields an equation of the form Eq.(1), with an explicit realization for the propagation matrix elements A_{nm}.

This description is sufficient to quantitatively determine the temporal evolution of the laser array provided the corresponding gain factors G_n are treated as parameters rather than dynamical variables. This simple view is quite common in the existing literature. On the other hand, the dynamical character of the gain can be crucial to understanding (and correctly predicting) the output behavior, a fact pointed out by many researchers. The recent theory paper by Bochove provides a recent discussion of this issue: he presents examples where neglecting the gain dynamics leads to incorrect predictions of which supermode is selected in simple array configurations.

The gain dynamics can be derived using standard methods such as the Rigrod analysis. A typical result is given by Eq.(2) (see the discussion of the paper by Rogers *et al.*). To derive this result, one starts with the partial differential equations (plus boundary conditions) for the population inversion and intensities of the two counter-propagating electromagnetic fields can be solved subject to the condition that the latter evolve on a much more rapid time scale than the former. The result is an ordinary differential equation for the gain, and Eq.(2) has the typical form. The details of the equation, however, will depend on the physics of the laser used, for example three-level vs. four-level schemes.

Integration with Laboratory Measurements

Any claims of reliability for a theoretical model rest on direct comparisons with laboratory measurements. In this section, I identify specific quantities that would allow for such direct comparisons. Ultimately, these could serve to validate the dynamical model at which point the model itself would be useful for testing and even guiding modifications of the hardware system. Before reaching that stage, however, I expect that side-by-side comparisons between theory and experiment will lead to refinements of the model that will increase its accuracy and reliability.

The most valuable diagnostic is to tune across a transition. This requires a control parameter on the one hand and an output quantity which suffers a radical (ideally, discontinuous) change at one or more critical values of the control parameter. In dynamical systems crossing such a critical point is called a bifurcation.

One obvious and accessible control parameter is the pump strength (or, equivalently in the case of a pure amplifier, the gain factor). A less obvious but potentially important control parameter is the percentage of incident light which is directed into the Phase Locker. It is an open and interesting question as to how large the feedback needs to be in order to achieve the desired locking behavior. Creating this control parameter would also serve to demonstrate at least one of the stated advantages of the Lockheed-Martin system as detailed in the White Paper dated 30 March 07, namely that a Phase Locker which "operates with relatively low circulating power".

With either of these control parameters, the most natural and accessible output quantity is the total output power, or a normalized version of this to yield the output power efficiency. Also useful would be a measurement of fringe contrast (as a primary indicator that directly demonstrates that the output beam is truly phase coherent). The same output can be measured as a function of pump power, for a fixed coupling percentage, and also as a function of coupling percentage for fixed pump strength.

More exotic measurements are possible, for example blocking one or more of the beams either prior to entering – or once inside – the Phase Locker.

Possible Next Steps

Proof of Concept. It is well-known that lasers can be injection-locked by even a weak master signal, i.e. a laser which is not itself subject to feedback from the target laser. It has also been established that arrays of lasers can display coherent behavior when they mutually interact without the benefit of a master signal. The Lockheed-Martin Phase Locker is based on an interesting idea that might be called mutual injection locking. That said, it is not obvious that the Lockheed-Martin Phase Locker will generate the desired coherent behavior, even in principle. Instead, the lasers might dynamically pull and push one another's phases in an endless meandering without ever settling down to a single globally coherent stable phase profile. A dynamical model of the array could be used to establish proof of concept for the Phase Locker scheme, while eliminating all the various disordering sources that might mask the effect in the laboratory.

Sensitivity Analysis. Once the proof of concept is established, a sensitivity analysis could be carried out to determine which disordering influences are most likely to quench the effect and which are relatively benign. Similarly, it would be very useful to map out transition points at which the system behavior changes suddenly, for example the transition between cw and pulsing output.

Experimental Tests. It would be most useful to have a model which accurately predicts the behavior of the actual laboratory system. The model should be as simple as possible, capturing the essential physics but also ignoring inessential details. The best way to arrive at such a model is through direct comparisons testing the theoretical predictions against experimental measurements. It is likely that this would lead to modifications of whatever model one uses at the start, but presumably the process of refinement and testing would eventually converge to a good (and useful) model.

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