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A RATIO DELAY STUDY OF

LONG CYCLE OPERATIONS PERFORMED

BY A VARIABLE LABOR FORCE

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By

William Judd Halladay, Jr.

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ABSTRACT

Ratio Delay is a statistical technique developed by L. H. C. Tippett in 1934 to determine allowances and avoidable delays for time standards application and work methods improvement. Random samples of worker or machine operations are obtained by instantaneous observations, and the nature of the operation is recorded according to predetermined work classifications. The percentage of any activity is the ratio of the number of occurrences of that activity divided by the total number of occurrences of all activities observed. The chance variability of the sample percentage may be predicted from the Binomial distribution which is the appropriate mathematical model.

Since 1934, Ratio Delay has proven to be practical in studies involving a constant labor force performing repetitive work elements. The purpose of this experiment was to test the hypothesis that Ratio Delay will give valid, unbiased estimates of activity percentages when used to measure operations involving a variable labor force performing long cycle operations, and to determine the effective sample size with which the reliability of the Ratio Delay percentage estimates can be assessed.

This hypothesis, that Ratio Delay will give unbiased estimates, was tested by taking random samples from a known

universe and statistically analyzing the difference between the sampling results and the known universe values. Fifty Ratio Delay studies were made by taking 366 samples per study from a universe composed of a condensed production all-day time study of operations having the desired characteristics. The validity of the sample percentage estimates was tested against the known universe percentages with the normal Z statistic. The effective sample size was determined by comparison of the root-mean-square standard deviation of the fifty percentage estimates with the theoretical Binomial distribution standard error of a percentage.

The results of the experiment indicate that Ratio Delay will give valid, unbiased estimates of activity percentages when used to measure a variable labor force performing long cycle operations. Further, the effective sample size, say N, can be determined and the variability of the percentage estimate can be expressed by an adaptation of the standard error of a percentage formula for the Binomial distribution. The value of N is the product of the total number of observations times the average number of crews being observed in the study. A crew is defined as one or more workers performing a task in which the activity of any crew member is dependent upon the activity of others in the same crew, and independent of others in different crews.

The author recommends that additional experiments be made on other types of operations as each type has its own peculiarities. With additional experiments, the crew definition can be more rigorously tested than was possible in this experiment.

CHAPTER I

INTRODUCTION

What It Is.--Ratio Delay is a statistical sampling tool used to investigate machine operations and human activity to determine allowances and the nature of avoidable delays. The work in question is sampled by random observations and the type of activity is recorded. The frequency of occurrence of each activity is divided by the total number of observations to obtain an estimate of the percentage of time spent on the activity in question.

An English statistician, L. H. C. Tippett (1), first used Ratio Delay to determine the down-time of weaving and spinning machines in eleven English cotton mills in 1934. The fundamental basis of Ratio Delay has been stated by Tippett (2) as follows:

> If a machine may at any time be either working or stationary, and large numbers of 'snap-readings', or records of its state, are taken at instants separated by random intervals, the percentage number of readings that record the machine as working will tend to equal the percentage time it is in that state.
>
> If the snap-readings are randomly distributed over a sufficiently long time, the relationship holds whether the machine stoppages or operations of the operatives are short or long, many or few, regular or irregular. This is the basis of the snap-reading method.

Precision.--The precision on the percentages obtained is, of course, directly proportional to the square root of the number of observations taken. The error is composed of a random sampling error and systematic error components. Random sampling introduces error by not recording or accounting for all of the delays or situations, and the systematic errors are due to changes in the extent of the delay from time to time. Concerning the systematic errors, Tippett (3) states the following:

> These may be due to variations in actual conditions from time to time or to the operatives working abnormally when under observation but they may usually be reduced to relative unimportance by taking readings at representative periods extended over a sufficient length of time, and by gaining the confidence of the operatives.

Concerning the random errors he states (4) that:

> ...aside from the systematic errors, even if all the conditions that may affect the results remain constant, replicate determinations of the percentage snap-readings of a given kind vary; these variations are called random errors of observations.

These random errors of observation can be reduced to any desired level by increasing the number of readings, n. Then, however, the systematic errors become increasingly important, so that it is not practically possible to reduce the total error indefinitely.

> Unless exceptional precautions are taken, the total error is not likely to be reduced much below two per cent except when the percentage measured is very high or very low (i.e., between zero and five per cent, and between ninety-five and one hundred per cent) or comparisons are being made under conditions in which systematic errors are constant (5).

R. L. Morrow introduced the "snap-reading" technique in the United States and in 1940 renamed it Ratio Delay (6). Concerning Ratio Delay inaccuracy, Morrow listed nine rules to guide the user.

1. Only homogeneous groups should be combined such as delays or similar operations performed on similar types of machines, or delays of operators on work of a similar nature (7).

2. A large number of observations is recommended, and studies are best adapted to large groups of machines or operators. When the number of observations on the job was about five hundred, a fairly reliable result was obtained. Over three thousand observations gave very accurate results.

3. Results from a few hundred observations may be used, if the frequency distribution conforms to the binomial law.

4. The accuracy of the results may be determined in any case.

5. As the percentage of delay time increases, more observations are necessary for a given accuracy.

6. Data are more reliable if the observations are taken over a long period of time.

7. Observations must be taken at random intervals and distributed over all hours of the day and week.

8. Intervals between samples must be sufficient to give independent readings (8).

9. A truly random time of observations must be used--otherwise a periodic stop synchronized with the clock, such as a rest period, might be recorded every day, if the observer made his rounds at the time of the rest period (9).

A rather unique thought concerning sufficiency of samples is expressed by Levinson (10) as follows:

"Reaching conclusions about an entire population from the statistics of a sample is like enlarging a photograph. Small defects are magnified into serious blemishes."

Nadler and Denholm (11) suggest that Ratio Delay may produce more accurate results than continuous observations or a production time study since of necessity the Ratio Delay study is applied over a longer period of time. J. S. Petro (12) also believes that Ratio Delay is more accurate than a continuous time study because the worker is not under pressure due to a stop watch. Waddell (13) suggests that Ratio Delay "...enables you to get the facts without watching everything and everybody all of the time."

<u>How To Use Ratio Delay</u>.--The first step in planning a Ratio Delay study is to become familiar with the work in question, determine which delays are expected, and code the delay as to its avoidability (14). The delays should be recorded in detail and later grouped into more easily handled classifications.

The second step is to calculate the number of observations that are consistent with the reliability desired. Thus some discussion on the mathematical aspects of Ratio Delay is in order.

Statistically speaking, Ratio Delay sampling is similar to success or failure type sampling characterized by measurements with go-no-go gages, and thus the mathematical model for this type of data is the Binomial distribution. Regardless of the number of possible work classifications, each occurrence of a given work classification can be considered a success and its non-occurrence as a failure. Then for each work classification there is a number of occurrences or successes, while the number of failures is the occurrence of any of the other possible work classifications. From the Binomial distribution, the formulae for the accuracy of Ratio Delay can be derived as shown below:

p = fraction of successes

$100p$ = percentage of successes

q = fraction of failures

$100q$ = percentage of failures

n = observations taken

np = number of successes

σnp = the standard deviation of the number of successes and is indicative of the dispersion of the number of successes

$$\sigma np = \sqrt{npq} = \sqrt{np(1-p)}$$

By multiplying each side of the equation by $\frac{1}{n}$ we obtain the standard deviation of a percentage such as the percentage of delay in the Ratio Delay studies.

$$p = \sqrt{\frac{p(1-p)}{n}}$$

With a large number of samples, the Binomial distribution closely approximates the Normal distribution and we can use the following Normal distribution areas, depending on the number of standard deviations employed (15):

$\pm 1\,\sigma_p$ includes 68 per cent of the cases or a confidence limit of 68 per cent

$\pm 2\,\sigma_p$ includes 95 per cent of the cases or a confidence limit of 95 per cent

$\pm 3\,\sigma_p$ includes 99.7 per cent of the cases or a confidence limit of 99.7 per cent

The usual confidence limit is set at the 95 per cent level ($\pm 2\,\sigma_p$). However, other confidence limits may be employed if so desired.

Transposing the above formula and solving for n we obtain:

$$n = \frac{p(1-p)}{(\sigma_p)^2}$$

This formula can be used to find n if an estimate of p is used and if σ_p is specified. If we choose to work at a confidence level of 95 per cent ($\pm 2\,\sigma_p$), and if it is desired that the error in the estimate of the percentage occurrence of the activity in question be no greater than

T fraction of the mean, then $Tp = 2\sigma_p$ and $\sigma_p^{\,2} = (Tp)^2/4$. Therefore:

$$n = \frac{p(1-p)}{\sigma_p^{\,2}} = \frac{p(1-p)}{\left(\frac{Tp}{2}\right)^2} = \frac{4(1-p)}{T^2 p}$$

The tolerance T is a statement of the maximum desired error in p, while the confidence limit ($\pm 2\sigma_p$ or $\pm 3\sigma_p$) is a statement of the desired probability that the actual error will not exceed the tolerance, T. For example, if in computing n we set our confidence limit at the 95 per cent level and the tolerance T at 7 per cent, then we will be 95 per cent confident that the resulting error in the estimate of p will be less than 7 per cent. In this example, if we estimate the percentage delay to be 30, then the necessary sample size is calculated below to be 1905.

$$n = \frac{4(1-0.3)}{(0.07^2)(0.03)} = 1905$$

Using the above formulae after the Ratio Delay study is completed, we can determine the reliability of the results using the observed percentage occurrences. An example of this procedure is given below.

Table 1. Sample Ratio Delay Summary

Item	Number of Observations	Per Cent of Total	Per Cent of Productive
Productive Work	700	70	----
Allowable Delay	200	20	28.6
Avoidable Delay	100	10	14.3
Total	1000	100	----

The standard deviation of the percentage is calculated as shown:

$$\sigma_{\text{productive work}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.3)(0.7)}{1000}} = 0.0145$$

Using a confidence level of 95 per cent, we calculate the reliability of our Ratio Delay estimate of the productive time in terms of the percentage of the total time as follows:

$$\bar{p} \pm 2\sigma_{\bar{p}} = 0.70 \pm 2(0.0145) = 0.70 \pm 0.029$$

Thus, for a 480 minute day, we can expect the following time interval to include the correct value with a confidence of 95 per cent:

productive work = 336 minutes $\pm$ 13.9 minutes

One important point that should be stated here is that the personal percentage allowance is determined as a percentage of the productive observations and not of the total observations. This is the allowance value that is added to productive elements in subsequent time studies.

After preliminary calculations have been completed, the third step in Ratio Delay planning is determining the random times at which the observations are to be made. Morrow (16) states that the time interval between observations should be long enough to give independent readings of delays. That is, the time interval should be longer than the longest time interval of the suspected delays. However, this author believes that the times should be randomly selected without attempting to provide for independent observations. If possible, the time interval should be long enough to allow an observer to complete one observation tour. On short cycle operations, the time interval can be a matter of a few minutes. To obtain accurate results, the observations should be planned to cover all phases of work during the day and week (17).

Since the observations must be made at random intervals, the observer must have some accurate and definite method of obtaining random times for study. One recommended method is to select random numbers from tables such as Tippett's Tracts for Computers and convert these numbers into minutes or hours.

This author recommends dividing the production time into a number of time intervals that correspond to the average number of observation tours required during the production time. The time intervals would be numbered consecutively and one random number for each time interval would be selected from the random numbers table. If two time selections were made that were too close to allow for two complete observation tours, the second time selected would apply to the next higher time interval. As an illustration, consider an observer making ten minute tours sixteen times during an eight-hour day beginning at seven a.m. If the random time for the first time interval was 7:28 and the second interval time was 7:36, the observer would be unable to make the second tour as he would not finish the first one until 7:38. In this case, the 0.06 minute random number used for the second time interval would be moved up to the third time interval and become 8:06, and another number would be selected for the second interval. The use of this procedure is not recommended in cases where the work cycle is very long or non-repetitive in nature because of the chance of introducing a bias in the estimated percentages.

The final step in Ratio Delay planning is to make a form on which the observer can record his observations. The form should show the time of observation and the nature of the expected delays.

Use of P Charts With Ratio Delay.--P charts, or fraction defective charts, have been used extensively in Statistical Quality Control applications in industry. A good example is testing samples with a go-no-go gauge and plotting the percentage of pieces that fail to meet specifications.

Since P charts essentially measure success or failure, the formulae and distribution stem from the Binomial distribution in the same manner as the Ratio Delay formulae (18). The similarity is evident from the formulae for the upper and lower control limits of the P chart (19):

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

The value $\bar{p}$ is the average of all of the percentage successes, p; and n is the sample size in each p value plotted.

There are several advantages in using P charts with the Ratio Delay study:

1. The control limits will confine all observations to the same chance distribution, and mistakes in recording and coding the delay may be found (20).

2. Changes in conditions from day to day, or changes during longer periods of operations are indicated by plotting the samples against time (21).
3. After the original Ratio Delay study has been completed, additional observations can be compared statistically with the original results to determine if a significant change has come about. If there is not a significant change, a complete new study is not needed.

Allowances.--The purpose of the Ratio Delay study is usually to determine accurate allowances and the extent of avoidable delays. The allowances found are added then to the normal or leveled times, as determined by time study, to produce the standard time allowed for work completion. The avoidable delays are brought to light that they may be studied and eliminated. Generally, a Ratio Delay study will not yield the normal or leveled time, but only the allowances and the nature of the avoidable delays. For example, the Ratio Delay study will record a drill press as working, but will not record the feed or spindle speed employed, which determines how fast the machine is working. There have been occasions however, where a Ratio Delay observer has recorded the pace or effort of the operator being studied, but this author believes that this may not be in keeping with an instantaneous observation required in a Ratio Delay study.

Lowry (22) defines an allowance found by a Ratio Delay study as "...an extra time which is added to the leveled time to care for various items which require the operator's time and which are not a regular part of any one job." Leveled time is found by multiplying the time study average element time by a factor to correct the "graded" effort to an average or "normal" effort.

In general, allowances may be divided into four classes: personal, fatigue, unavoidable delays, and special (23). However, since Ratio Delay study is involved with all types of allowances and delays, it might be well to consider a more detailed list such as found in Ralph Presgrave's _The Dynamics of Time Study_ (24).

1. Operator personal time
2. Fatigue
3. Unavoidable delays
    A. Delays in which the operator is idle such as power shut down or machine breakage
    B. Delays in which the operator is busy such as repairs
    C. Operator idle time waiting for machine to complete its cycle
    D. Production loss due to machine interference
    E. Preparatory work such as set-up
    F. Incidental work such as oiling or cleaning

G. Waiting for help

H. Operator slow down due to insufficient material

4. Special

A. Loss of production caused by faulty pieces produced

B. Loss of production due to faulty materials, machines, or conditions

C. Incentive factors which are added to the normal time to permit an operator to earn incentive wages

Not all of the above items will be found in every situation, but the list can be used to design a Ratio Delay study for maximum effectiveness. The list should prevent the omission of a critical allowance. For example, the observer might record incorrectly as productive work such operations as set-up, cleaning, oiling, etc. It would seem best to record the items in detail and later group them into more easily handled classifications. The actual application of the percentage for each classification will be a matter of management policy.

It should be emphasized that a Ratio Delay study will not determine either the fatigue allowance or the special allowances. It would be impossible for a Ratio Delay observer to record whether a worker is fatigued or not, or if he were producing a faulty part.

The fatigue allowance can best be determined from a detailed time study. Lowry, Maynard, and Stegemerten (25) suggest the following procedure in a detailed time study to obtain a fatigue allowance:

1. Perform an all-day time study
2. Level the operator's work during the beginning of the time study
3. Compute the delay allowance from the formula below:

$$\text{per cent delay} = 100\left(\frac{OL}{MS} - 1\right)$$

O = net overall time

L = leveling factor from the beginning of the study

M = number of pieces produced

S = leveled time per piece

The formula is simply a ratio of the total productivity a worker would achieve if he worked all day at his beginning pace to the standard productivity achieved. The formula is based on the assumption that "...the time lost due to fatigue will be an ever-increasing amount as the day progresses..." (26) which is not the case in every situation. The authors do state that rest periods will affect fatigue, but their formula does not indicate this change. Also, it would be very difficult to substantiate the end of the "beginning" of the day.

The fatigue allowance has been discussed in some detail, for while it cannot be found in a Ratio Delay study, the amount of fatigue produced is directly connected with the amount of idle time and personal time needed. As the idle time increases, fatigue and personal time required decreases. There may be so much idle time necessary that a fatigue allowance is not needed and rest periods might cover all the personal time required. This situation would apply especially in incentive situations where a worker desired to spend as much time as possible at productive labor.

In many plants, where all work ceases during rest periods, it would be well to avoid taking Ratio Delay observations during the period because the status of the operation is already known (27). However, in situations where the machines might continue to run during an operator's absence during a rest period, the observation should be made.

The application of Ratio Delay to the remaining items in the allowance list are self explanatory with the exception of the idle productive time caused by machine interference. Jones (28) has defined machine interference idleness "...as the time a machine is nonproductive while awaiting servicing by its operator who is tending some other machine in his assignment." Jones has computed three tables based on the Binomial distribution that indicate the per cent of the time the machine is running, and machine and operator idle time for a given number of machines. To determine the machine and operator effectiveness, the tables are entered with the number of machines assigned to the operator, the work load of the operator based on the servicing time required per machine, and the cycle time of the machine. These tables can be supplemented and checked as to their validity in a particular situation by performing a Ratio Delay study of the down-time of the various machines assigned to the operator. These studies would be especially valuable if the machines had different cycle and service times.

As the above tables suggest, there are many ways of determining the allowances. Unfortunately, many of the methods are nothing but rather rough guesses. This author has come in contact with one rather amusing method employed by a large consulting firm. The time study observer would first average all of the element times, then discard all of the element times below the average and calculate a new average from the remaining times. The fatigue, personal, and unavoidable allowance percentage was the ratio of the first average over the second average.

On a higher plane of accuracy are tables for various machines to cover personal, fatigue and unavoidable delay allowances. Two examples are Morrow's (29) allowance of 25 per cent on short cycle, power-press, hand feed operations, and Nordhoff's (30) 5 per cent for automatic screw machines. These percentages, however, are only estimates and should be used with extreme care, especially in incentive situations.

The necessity of obtaining these allowances is, of course, evident from the fact that the time standards cannot be set on the basis of the assumption that the worker is always working and never idle for any reason. However, a study of the nonproductive times and avoidable delays is not quite so easy to see at first glance. As Abruzzi (31) aptly puts it, "Many plant managers are not aware of the fact that the study of nonproductive time and its causes is probably the most effective means of increasing production output per man-hour and thus achieving sizable economies in production costs." One way to see the necessity of the study of avoidable delays is to calculate the monetary value of such delays from the following formula (32):

$$\frac{\text{Delay cost}}{\text{Year}} = \left(\frac{\text{Variable operator cost}}{\text{Hour}}\right)\left(\text{Per cent delay}\right)\left(\frac{\text{Normal operator hours}}{\text{Year}}\right)$$

The formula reveals that, based on a 40 hour week and a one dollar per hour variable operator cost, each minute lost per day on one operator is worth $4.33 per year. In a large plant of 10,000 employees, one minute lost per day by each operator paid one dollar per hour would cost management $43,000 per year. Since most Ratio Delay studies can be completed in a month by one observer, the savings can be quite substantial.

Application of Ratio Delay.--Since the introduction of Ratio Delay in this country in 1940 by Mr. Morrow, there have been numerous applications in industry. Most of the applications have been concerned with the study of machine down-time. However, Ratio Delay will also give reliable data in the field of human activity. This accent on machines is due primarily to Tippett's original studies of textile machinery and Morrow's testing of Ratio Delay in 1940. Several of Morrow's (33) early applications are listed below:

1. A study of machining operations in the Eagle Pencil Company
2. A study of machinery in the J. E. Ogden Company
3. A toolroom study to determine the number of machines operated in a day
4. A study of a power press lathe in a metal products factory
5. A study of the use of trucks in material handling
6. An analysis of ironing shirts in a laundry
7. A study of sewing machines in an underwear factory
8. A study of the use of jigs in an Airplane Wing Assembly

The application of Ratio Delay to machine down-time is still predominate in industry. Some recent applications have been a study of crane efficiency (34), distribution of material handling machinery (35), machine productivity at the Wolverine Tube Company (36), and a study of elevator traffic at the Eastman Kodak Company (37). However, there have been a few applications in the field of Human Activity. The Carrier Corporation used Ratio Delay to study the activity of their clerical employees. Ratio Delay was used in the Harper Hospital in Detroit to study nurse activity (38). The Wolverine Tube Company has used Ratio Delay to study the non-engineering duties of their engineering staff (39). The Eastman Kodak Company has used Ratio Delay to set incentive time standards on indirect labor such as the stock room, packaging department, and adjustment department (40). However, Charles Bogenrief (41), writing in Factory Management, states that Ratio Delay time standards are better than "guesstimates" but not accurate enough for incentive purposes. One unique example is the application of Ratio Delay to determine the percentage of unsafe practices to reduce accidents at the Monsanto Chemical Company (42).

All of the above applications of Ratio Delay have proven its reliability, and point to an ever increasing field of study. Waddell (43), an editor of Factory Management and Maintenance, states that the use of Ratio Delay to study human activity "...is barely in its infancy."

CHAPTER II

NATURE OF THE PROBLEM

Introduction.--Most of the examples of Ratio Delay applications cited in the previous section are concerned with studies of one machine or worker, or groups of machines or workers that meet the following conditions:

1. A stable worker force where the number of workers under observation remained constant.
2. A repetitive short cycle operation where the elements of the operation are frequently repeated and the time required for the element is of a short duration.

The purpose of this paper is to determine the feasibility of using Ratio Delay where the operating characteristics are the opposite of the above conditions; namely, where the situation involves a variable labor force performing long cycle operations.

If Ratio Delay is to be employed, then it must be both valid and reliable. That is to say that the estimates of the delay percentages must be unbiased (accurate), and we must be able to predict the variability or reliability of these estimates.

The question of delay percentage accuracy can be determined by comparison of Ratio Delay percentages with known values of the percentages. If the estimates differ significantly from the known values, then one is forced to say that the Ratio Delay estimate is biased. A significance test of the difference between the two values must take into consideration the number of samples on which the Ratio Delay percentage estimate is based since an increase in sample size decreases the variability of the sample percentage. Thus, it is clear that one must be certain of the sample size employed to measure Ratio Delay validity and reliability. The sample size determination would be more certain if the labor force involved were constant; but where the labor force varies, then the sample size becomes a problem. To illustrate, consider the table of the beginning of a theoretical Ratio Delay study shown below:

Table 2. Theoretical Ratio Delay Study

	Observation Number	Number of Workers Classified as to Delay			Total Workers
		Code			
		D	E	F	
	1	5	3	4	12
	2	5	5	0	10
Total	2	10	8	4	22

In this example, there are two observations, giving a total of 22 observed workers. Is the effective sample size, N, equal to 22, two, or some other figure? In order to algebraically define this sample size problem, we will use the following terminology:

n = the number of observations

N = the effective sample size

K = the number of effective samples per observation

so that

$$N = nK$$

Since a variable labor force introduces difficulty in determining the correct sample size, the definition and value of K becomes the focal point in this thesis. Only by using some value of K can we adapt the Binomial distribution standard deviation to compensate for the variable sample size per observation. The problem, then, is whether or not K can be defined as above and can be determined practically in Ratio Delay application.

The variable labor force problem is also directly allied with non-repetitive, long cycle operations. If the operations are of a repetitive nature, then the labor force is more inclined to remain at a constant level. It may well be that long cycle operations could increase the systematic error to the extent that the random samples would have to be extended to cover such a time duration that Ratio Delay would become impractical.

Literature Survey.--At this point we should consider how other engineers have treated the sample size when studying groups of workers. Most authors have avoided any direct reference to the difference between the number of samples and the number of observations, and have stated that only homogeneous groups should be combined.[1] However, C. L. Brisely (46), writing in Factory Management and Maintenance, obtained 136 samples with only 34 observations on four draw benches. A New York food bottling plant obtained 40 samples per observation in an assembly line operation (47). In this assembly line were 13 labeling machines, 10 filling machines, six blowing machines, four cap tightening machines, and seven cappers for a total of 40 machines. Morrow (48) reports that in a Ratio Delay study of an underwear factory, the observations on individual operators were totaled to obtain the sample value N.

Adam Abruzzi (49), reporting on a study in the ladies' garment industry, stated that the worker force varied considerably.

> In this case the preliminary study showed that: (1) some workers shifted from one type of operation to another; (2) other workers did not report until

---

[1]See bibliography references 11, 27, 35, 44 and 45.

> afternoon; (3) still other workers were sent home early because materials were not ready; (4) an average of 40 per cent of the workers were absent on the two days covered.

Despite this labor force variation, Abruzzi did not mention that there may be a difference between the observations and samples as defined in this thesis. He did say, however, that when it is not possible or economically desirable to study only homogeneous operations that "...unbiased results can still be obtained if each type of operation is represented in the basic sample in proportion to the number of work stations involved." (50). In an example, Abruzzi weighted his sample size according to sub-populations of types of operations and different operating floors. In his discourse, it is not clear whether his weighting process was based upon weighted observations or weighted samples per observation. In the absence of any distinction between samples and observations, it must be assumed that they are the same value.

Since sub-populations and the degree of homogeneity seem to be so important, we might quote a definition by Barnes and Correll (51):

> Machine classifications should be listed and then subdivided into groups of similar operations. By this it is meant, the classifying of the operations into homogeneous groups for which a single delay percentage may be found.... The determining factor ...is the amount of variation in delay percentage which may be expected within the group itself.

Thus, in summary, it may be said that there does not seem to be an adequate distinction between observations and samples per observation in the Ratio Delay literature.

CHAPTER III

EXPERIMENT DESIGN

Purpose.--The purpose of this experiment is to test the hypothesis that Ratio Delay will give valid, unbiased estimates of activity percentages when used to measure operations involving a variable labor force performing long cycle operations, and to determine the effective sample size with which the reliability of the Ratio Delay percentage estimates can be assessed. This hypothesis can be tested by comparison of Ratio Delay percentages, based on random samples, with actual percentages from a known universe.

Universe Selection.--To obtain a universe with the desired characteristics, the author examined several months of production (continuous) time studies made on the flight line at Lockheed Aircraft Corporation in Marietta, Georgia. In these production time studies, all worker activity was classified and each classification was identified by a letter symbol. Worker activity described by these classifications was recorded for individual workers in increments of two minute intervals. At the end of the studies, departmental activity was represented by various percentages of each classification.

After examining these production time studies, the author selected 24 consecutive shifts as a homogeneous population with the desired universe characteristics. During these shifts, which cover 13 working days, various equipment was installed in one production type aircraft. For this operation the labor force varied from a maximum force of 21 workers to one worker, with an average of 6.6 workers. The operation was non-repetitive in that individual pieces of equipment varied considerably, and the operation would not be repeated until installation in the next aircraft. The installation could be described as a long cycle operation, since much of the equipment required over two hours to install.

To obtain a practical universe for sampling, the production time studies were condensed into data indicating total worker activity by shifts. This was accomplished by combining individual time studies, and eliminating references to individual workers. In addition, the number of worker activity classifications was reduced from the original 28 to ten classifications; nine descriptive and one miscellaneous classification. The universe was further condensed by recording only the end points of time intervals within which the worker activity was constant. An example of one shift in the universe is shown in the Appendix in Table 8.

Universe Description.--The ten work classification symbols and universe percentages are shown below. The description of each classification is omitted because of the nature of the work on a military type aircraft.

Symbol	Per Cent	Symbol	Per Cent
A	43.55	C-2	4.13
C-4	14.41	YY	2.25
W	7.46	M	2.43
H	3.72	X	1.19
G	3.51	O	17.37

Graphs of the percentages of each classification by shifts are shown in Figures 12, 13, and 14 in the Appendix. These graphs indicate that the percentage times for all of the classifications do not form a stable (random) distribution and this would seem to be characteristic of long cycle operations. The total elapsed production time for the 24 shifts was 182 hours and 36 minutes. Pertinent data on each shift are shown in Table 10 in the Appendix.

Sampling Procedure.--Random samples from this universe were selected on a shift basis with an average of one sample per half-hour of elapsed production time. Thus, a total of 16 random samples was drawn from each eight-hour shift. Since the universe was divided into two minute time intervals, this method of sampling amounted to an average of one sample per fifteen recordings, or 6.67 per cent of the total universe.

Randomness was insured by reading the "observation" time from time-marked balls selected from a continuously revolving bowl. Small balls, ten millimeters in diameter, were marked with the production time in two minute intervals, for the entire elapsed time for the shift. The electric motor-driven bowl, shown in Figure 1, mixed the balls continuously so that each ball, or observation time, had an equal opportunity of being selected. As each ball was selected, the observation time was noted and the ball was returned to the revolving bowl. This time was then used to enter the particular shift in the universe being sampled. For this sample time, the number of workers in each classification was noted and recorded on the form shown in Table 9. Then another ball was selected and the procedure repeated until a total of 16 samples was drawn from each shift. The balls were replaced after each sample as required for random sampling.

As will be discussed later, fifty complete Ratio Delay studies were made in the manner described above. Since the total universe elapsed time was 10,956 minutes, and one observation time was required for each 30 minutes on the average, there were 366 observation times per complete Ratio Delay study. Therefore, the total number of observations for all 50 studies was 18,300.

Figure 1. Electric Motor Driven Bowl

Statistical Testing.--To study the random error in Ratio Delay percentage activity estimates, 50 complete Ratio Delay studies were taken from the universe. At the completion of each study, the per cent of each work classification was determined as 100 times the quotient of the total number of workers observed in that particular work classification divided by the total number of workers observed in all ten work classifications. These 50 studies thus gave 50 estimates of the percentage values for each work classification. These estimates were averaged to obtain grand average estimates for each activity. Graphs of the 50 estimates for each of the ten work classifications are shown in Figures 2 through 11, and in some cases indicate possible skewness. Although the shape was not tested because of the small sample size, the shape appears to be reasonably normal.

The validity, or lack of bias, of the Ratio Delay estimates can be determined by a significance test of the difference between the grand average of each of the work classification percentages and the corresponding universe percentages. The calculation of the Z value given in the following formula was used to test the null hypothesis that the mean of the distribution of Ratio Delay percentage estimates is equal to the universe mean.

$$Z = \frac{\bar{p}-p'}{\sqrt{\frac{p'(1-p')}{nK}}}$$

where:

Z = a statistic assumed to be normally distributed.

$\bar{p}$ = the grand mean of the 50 Ratio Delay estimates of the work classification percentage in question.

p' = the universe percentage for the work classification in question.

n = the number of observations in one Ratio Delay study.

K = the average number of effective samples per observation.

A table of normal curve areas can then be used to assess the significance of the Z values.

The reliability of the Ratio Delay estimates can be determined by comparing the theoretical Binomial distribution standard deviation with the actual root-mean-square standard deviation of the 50 Ratio Delay study percentages. The two standard deviation formulae are shown below:

$$\sigma t = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\sigma s = \sqrt{\frac{\Sigma p^2 - \frac{(\Sigma p)^2}{N'}}{N' - 1}}$$

where:

σ_t = the theoretical Binomial distribution standard deviation based on the number of observations per Ratio Delay study.

σ_s = the root-mean-square standard deviation of 50 Ratio Delay study percentages.

N' = the number of Ratio Delay studies made.

n and $\bar{p}$ are as previously defined.

As described above, the theoretical standard deviation, σ_t, does not take into consideration the number of effective samples or men per observation. The other standard deviation, σ_s, inherently reflects this value of effective samples per observation. Therefore, the σ_s value should be smaller than the σ_t value. In order to equate these two standard deviations, the value K is introduced into the σ_t formula as shown below:

$$\sigma_T = (\sigma_t)\frac{1}{\sqrt{K}} = \sqrt{\frac{\bar{p}(1-\bar{p})}{nK}} = \sqrt{\frac{\bar{p}(1-\bar{p})}{N}}$$

where:

σ_T = the Binomial distribution standard deviation based on the effective sample size for one Ratio Delay study.

K = the average number of effective samples per observation.

N = the average effective sample size of one Ratio Delay study.

With the K value we can equate the two standard deviation formulae or express their relationship.

$$\sigma_s = \sigma_T = \frac{\sigma_t}{\sqrt{K}}$$

or

$$K = \frac{\sigma_t^2}{\sigma_s^2}$$

If it is not possible to determine a practical K value where the number of samples per observation varies, then it will not be possible to estimate the variability of any Ratio Delay percentage. Stated another way, if K cannot be accurately determined in operations involving a variable labor force, the reliability of the Ratio Delay estimate cannot be assessed accurately.

Thus, with an analysis of the K value and the normal distribution Z value, the reliability and validity of Ratio Delay percentage estimates can be determined.

CHAPTER IV

RESULTS

Reliability Results.--The reliability results will be discussed prior to the validity results because the latter require the K values that are determined in the reliability calculations.

Table 3 contains the K values and other mathematical determinations. The data are arranged into three divisions of work classifications that contain similar ranges of K values. The first division has an average $\bar{K}$ value of 3.17; the second division has a $\bar{K}$ value of 1.75; and the third division has a $\bar{K}$ value of 0.98.

If regulations permitted the further defining of the work classifications presented in this paper, the reader would see that the divisions bring together those work classifications having a common opportunity of occurrence. The first division of work classification contains only those activity classifications in which all of the workers are divided into a number of crews where each crew performs a certain operation that is independent of the activities of the other crews. The second division of work classifications, O, contains the miscellaneous

Table 3. Results of Experiment Calculations

Code	p'	$\bar{p}$	$\bar{p}-p'$	Z	σ_s	95 Per Cent Confidence Interval for σ_s	σ_t	K	95 Per Cent Confidence Interval for K
A	43.55	43.42	-0.13	0.64	1.48	1.23 - 1.83	2.59	3.07	2.00 - 4.42
C-4	14.41	14.37	-0.04	0.24	0.90	0.75 - 1.12	1.83	4.18	2.71 - 5.98
W	7.46	7.61	+0.15	1.34	0.91	0.76 - 1.13	1.39	2.33	1.51 - 3.34
H	3.72	3.59	-0.13	1.64	0.62	0.51 - 0.77	0.97	2.50	1.62 - 3.58
G	3.51	3.49	-0.02	1.91	0.60	0.50 - 0.74	0.96	2.59	1.67 - 3.70
C-2	4.13	4.13	+0.002	0.02	0.56	0.46 - 0.69	1.00	3.52	2.28 - 5.04
X	1.19	1.16	-0.03	0.64	0.28	0.23 - 0.35	0.56	4.01	2.59 - 5.74
O	17.37	17.27	-0.10	0.44	1.50	1.25 - 1.86	1.98	1.75	1.13 - 2.50
YY	2.25	2.56	+0.31	2.89	0.94	0.79 - 1.17	0.83	0.77	0.50 - 1.10
M	2.43	2.34	-0.09	0.71	0.73	0.61 - 0.90	0.79	1.18	0.77 - 1.69

classifications. In this case, the workers are not necessarily divided into independent crews, but all may act as one crew on certain occasions. For example, the rest period is in this classification and all of the workers have their rest period at the same time. In the third classification, the activity was such that the workers generally acted as one crew. For example, the symbol YY refers to the clean-up operation which was generally performed by all of the workers at the end of the shift. These divisions indicate that the nature of the work classification determines the value of K. Before the experiment, K was defined as the number of effective samples per observation. As a result of the experiment, K can be defined further as the number of crews of workers in an operation. Here a crew refers to one or more workers performing a task in which the activity of any crew-member is dependent upon the activity of others in the same crew and independent of others in different crews; or stated statistically, there is perfect correlation of the activity of workers within a crew and zero correlation of the activities between crews. In Ratio Delay terminology, K becomes the average number of crews per observation of the particular work classification being studied. Several examples may illustrate this definition of K. Since crews may act as a unit in more than one classification during a study, the work classifications will be divided into divisions that have a common K value. If all of the workers take a rest period, then there is only one crew at this time, and the K value becomes one. If ten men work independently of each other on ten tasks, the K value becomes ten. If the men work in pairs as crews, as on many maintenance operations, then the number of crews would be the K value.

How does this concept of the number of crews agree with the experimental K values shown in Table 3? This table shows that the representative K value of each division is well within the confidence intervals of the K values for each work classification in that division. The first division of work classification has an average $\bar{K}$ value of 3.17. During the operations concerned with the first division K values, the workers generally are in crews of two men each. Thus, by dividing the average number of workers per observation, 6.59, by a crew of two men each, the average number of crews becomes 3.29. This number of crews agrees very closely with the experimental value of 3.17.

The third division $\bar{K}$ value is 0.96. During the work activity described by this division, such as cleaning up at the end of a shift, all of the workers act as one crew. Thus again, the number of crews agrees very closely with the experimental $\bar{K}$ value.

Table 4. Comparison of Experimental and Theoretical K Values

Work Classification Division	Types of Activities	Experimental K	Approximate Experimental 95 Per Cent Confidence Interval on K	Theoretical K
1	Workers divided into several crews	3.17	2.48 - 3.86	3.29
2	Workers may be in one or several crews	1.75	1.13 - 2.50	$1.00<K<3.29$
3	Workers generally act as one crew	0.98	0.71 - 1.25	1.00

The second division, or miscellaneous activities, contains the rest period, where all workers act as one crew, as well as other activities where the workers function as independent crews. It seems logical then to expect the number of crews to fall somewhere between 1.0 and 3.29. The experimental K value of 1.75, which is almost midway between 1.0 and 3.29, is quite reasonable for the collection of activities grouped in this miscellaneous category.

In order to see the overall picture of the expermental K values versus the theoretical K values, Table 4 on page 40 was prepared.

Validity Results.--Accepting the definition of K, we can proceed with the testing of the validity of our Ratio Delay percentage estimates. As in the reliability testing, the work classifications were divided into three divisions with homogeneous work characteristics. For each division, a representative K value was selected for use in the Z statistic used to test the difference between the experimental and universe percentages.

Table 5. Work Classification Division K Values For Validity Testing

Division	K Value
1	3.17
2	1.75
3	1.00

The null hypothesis being tested is that the mean of the distribution of Ratio Delay percentage estimates is equal to the universe mean. If this hypothesis is true, the distribution of the ten sample Z values listed in Table 3 should conform to the expected frequencies dictated by the Normal distribution. This comparison is shown in Table 6 which follows.

Table 6. Expected Percentage Occurrence Z Values Versus Actual Percentage Occurrence

Range of Z Values	Expected Percentage Occurrence	Actual Percentage Occurrence
1	68	60
2	95	90
3	99.7	100

The agreement shown in Table 6 offers no reason for rejecting the null hypothesis and therefore it is reasonable to conclude that the Ratio Delay percentage estimates obtained in this experiment are valid (unbiased).

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The purpose of this experiment was to test the null hypothesis that Ratio Delay will give valid, unbiased estimates of activity percentages when used to measure operations involving a variable labor force performing long cycle operations, and to determine the effective sample size with which the reliability of the Ratio Delay percentage estimates can be assessed. The results of the experiment indicate that the null hypothesis cannot be rejected and the conclusion is therefore reached that the percentage estimates are unbiased. Thus, the effect of long cycle operations does not seem to bias the Ratio Delay estimates if the observations are made at random intervals. Moreover, the crew definition of K adapts the Binomial distribution formula to a variable labor force condition so that the reliability of the percentage estimates can be assessed.

The heart of the problem is proving the validity of the crew definition of K. In this paper, the value of K, corresponding to the number of independent crews, is the value that corresponded with the theoretical K values. However, if additional Ratio Delay experiments are made, the definition, no doubt, will be improved and refined.

Accepting this definition, two additional problems arise. First, how does an observer determine an accurate crew value in a Ratio Delay study; and secondly, where is the dividing line between using and not using a K value?

In answer to the first question, the observer should become familiar with the work involved so that he could develop work classification definitions that could be combined into divisions that are homogeneous with regard to the number of crews. Then, during a study, the observer would record the number of men in each classification and the number of crews found. The following table illustrates a form which the observer might use.

Table 7. Sample Ratio Delay Form for Study of Variable Labor Force Operations

Work Symbol	Random Observation Time and Number of Crews		
	7:02	7:36	8:04
D			4
E		4	
F		2	7
Crews		3	4
R	19		
S			
Crews	1		

In Table 7, the work classifications are divided into two divisions that are homogeneous with regard to the average number of crews performing the work classifications. This division implies that at a given time, the activity of all crew members will fall into either division one or division two, but not into both divisions at the same time. For example, it may be that if one crew member is performing classification D, his activities may cause others in the same crew to have either an E or F classification, but not an R or S classification. The example shows that there are several crews performing operations in the first division and only one crew performing operations in the second division. One explanation might be that the first division includes all productive work classifications and the second division is composed of rest periods or clean-up elements where all the workers act as one crew. Thus, for each work classification division there is an average number of crews that can be calculated as the mean value of the number of crews per observation of the work classification division. This average number of crews is the K value used for each work classification in the division to assess the reliability of the percentage estimate in question.

The problem of when to use the K value should be the object of further research. At present, with only the results of this one experiment to rely upon, this author believes that K values should be employed where management normally assigns a variable labor force to an operation, or where it is suspected that the workers perform operations as crews. It may be that such factors as absenteeism and material shortages will cause a stable labor force to behave as a variable force. Also, it should be stated that while crew member activity is directly correlated, it is not implied that all crew member activities must be placed in the same work classifications. The activities of the crew members are correlated due to work environment and the activity of one member may result necessarily in the idleness of another member.

As this is the first experiment concerning Ratio Delay studies of a variable labor force, the author recommends that additional experiments be made. Such experiments should be designed to obtain actual crew values for each observation and thus determine an average crew value for each work classification division. With this observed average crew value, it would be possible to test the K definition much more rigorously than was possible with the universe in this experiment. Additional experiments should be made on several types of operations as each type has its own peculiarities.

With only one experiment on one type of operation, it would be assuming too much to state conclusively that the crew definition applies to all variable labor force operations. However, the author believes that future experiments will tend to substantiate further the findings of this thesis; namely, that Ratio Delay can be used to obtain accurate work percentages in studies of variable labor forces on non-repetitive operations, and that the reliability of the percentage estimates can be assessed accurately.

APPENDIX

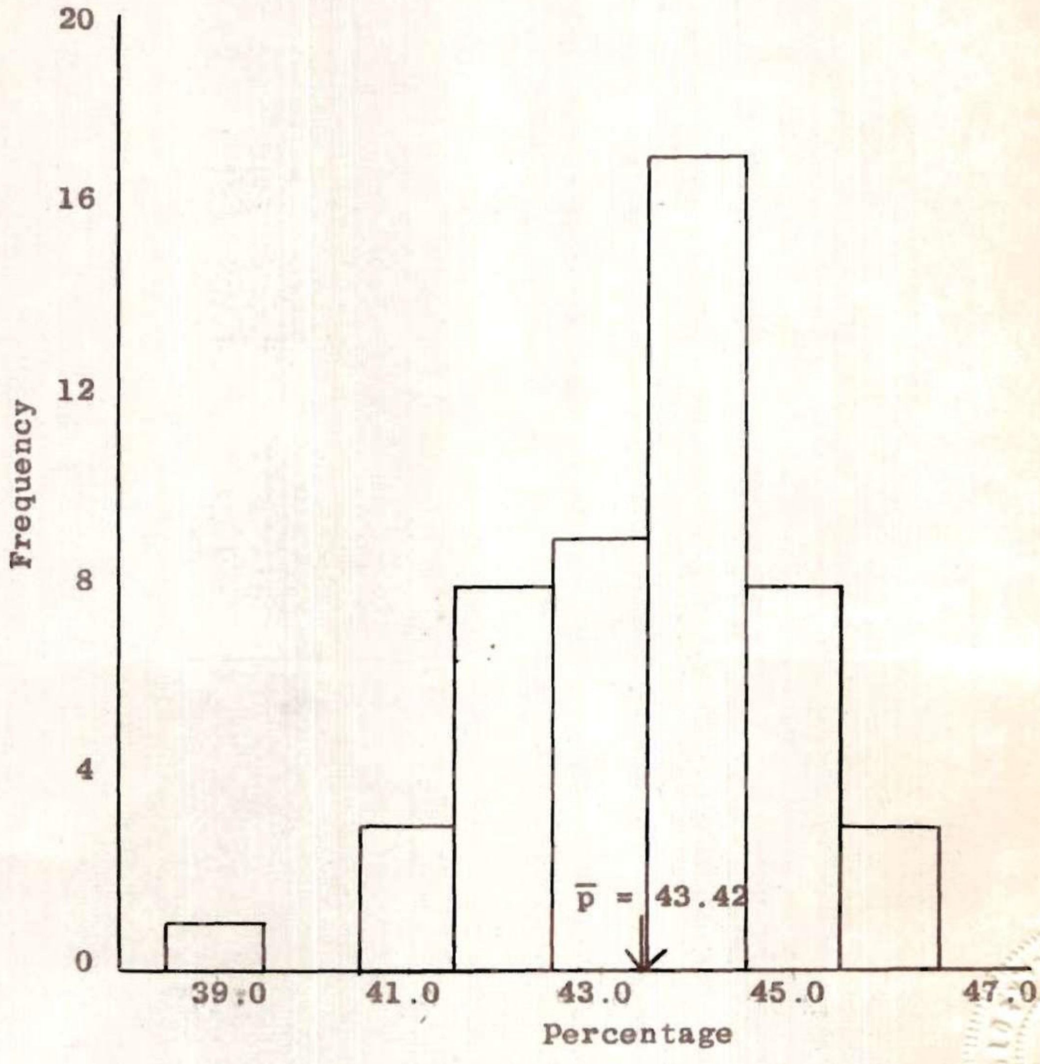

Figure 2. Graph of the A Work Classification Percentage Estimates

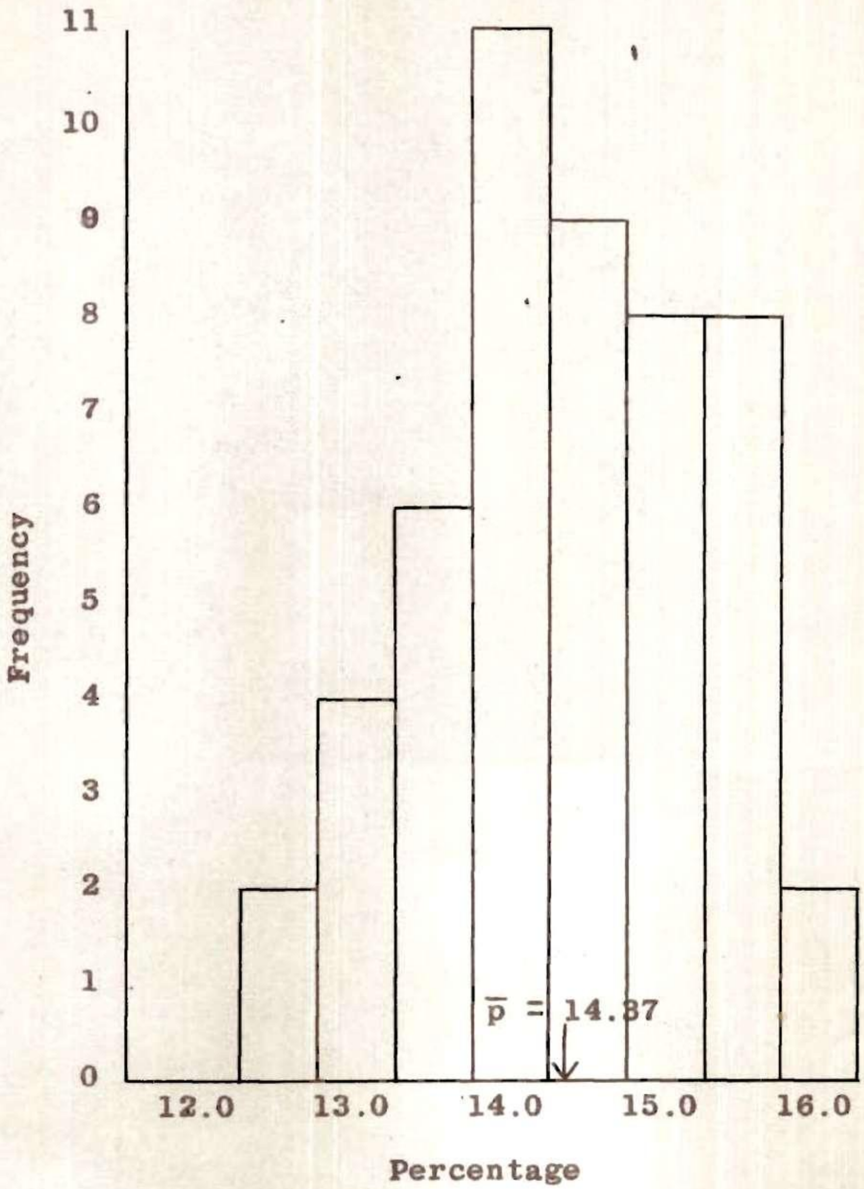

Figure 3. Graph of the C-4 Work Classification Percentage Estimates

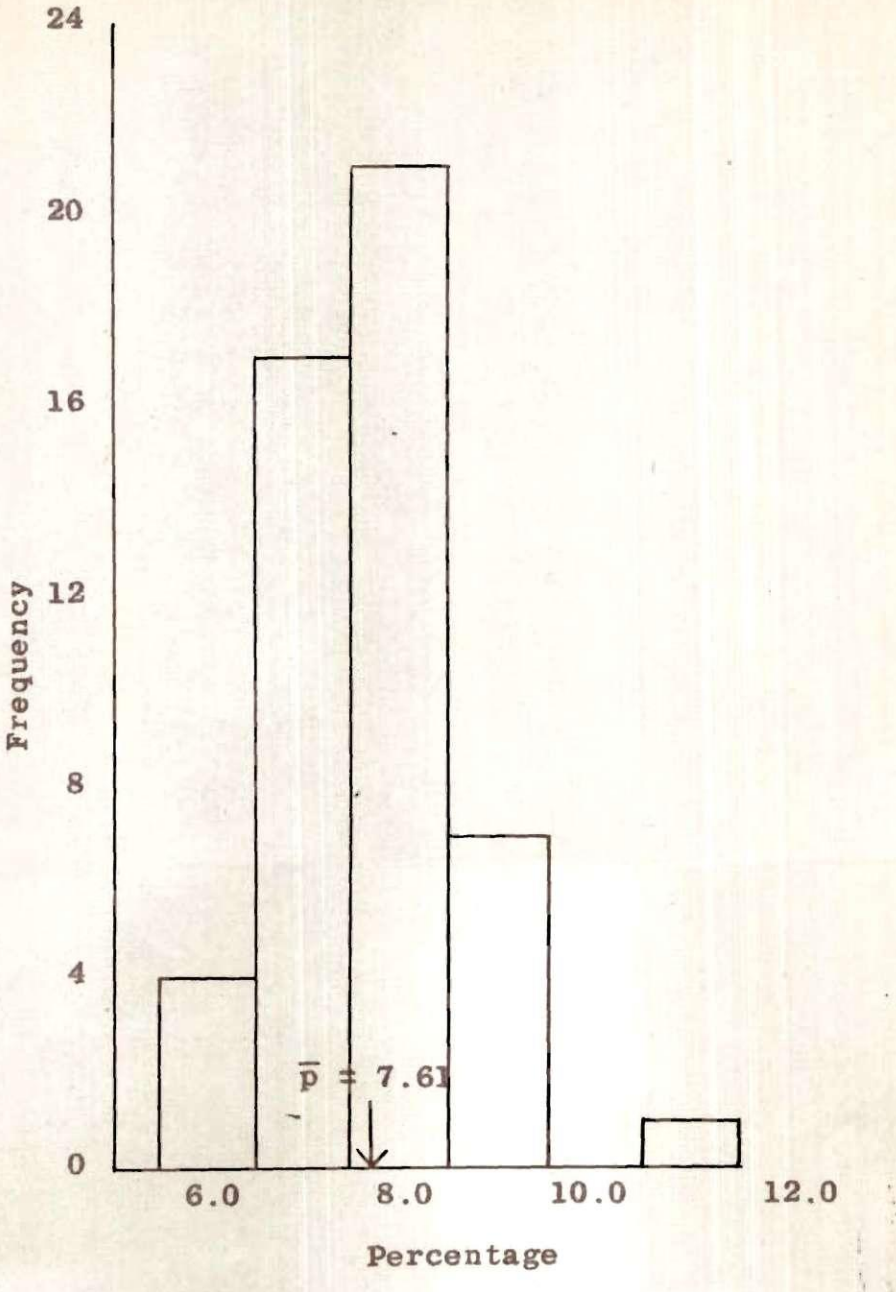

Figure 4. Graph of the W Work Classification Percentage Estimates

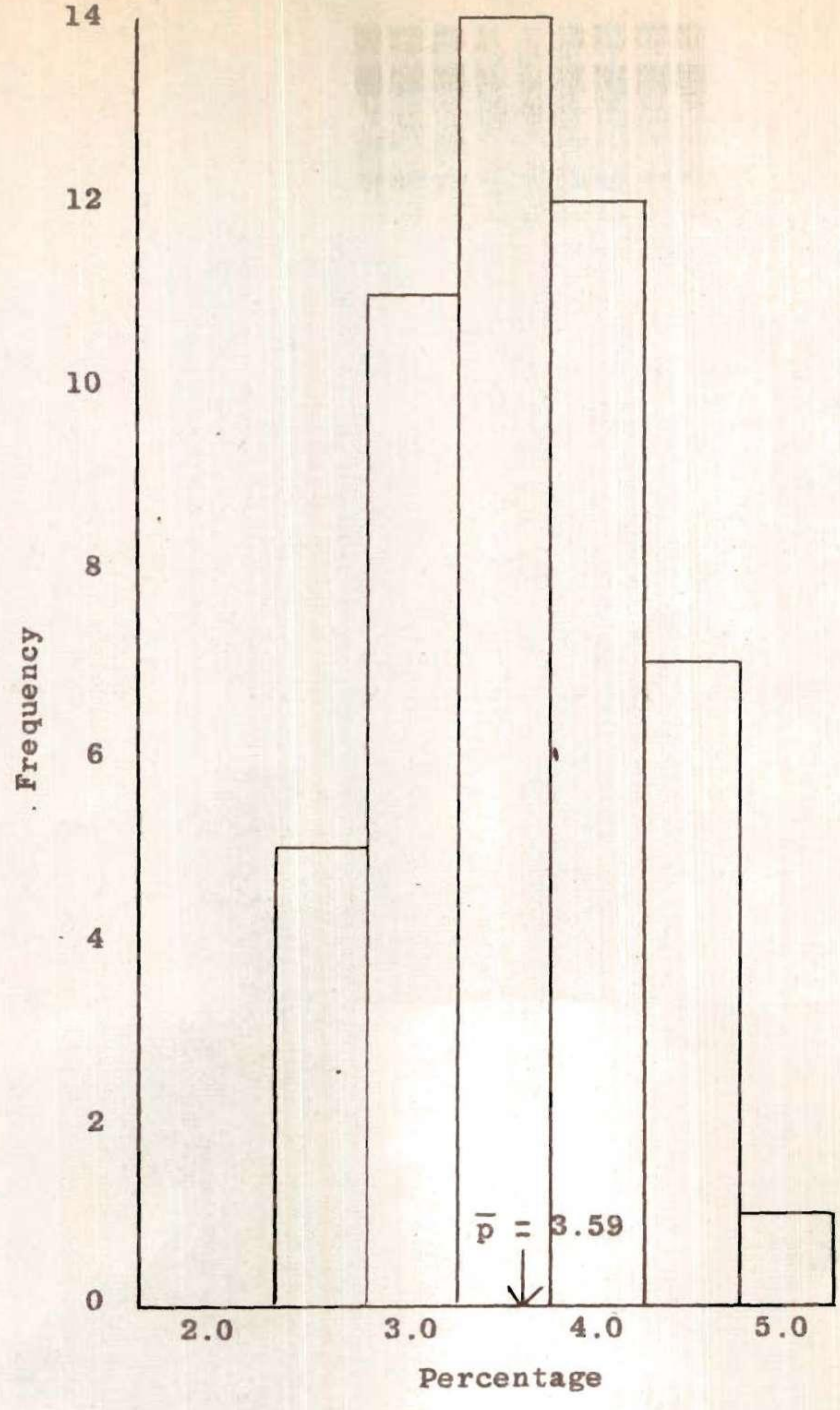

Figure 5. Graph of the H Work Classification Percentage Estimates

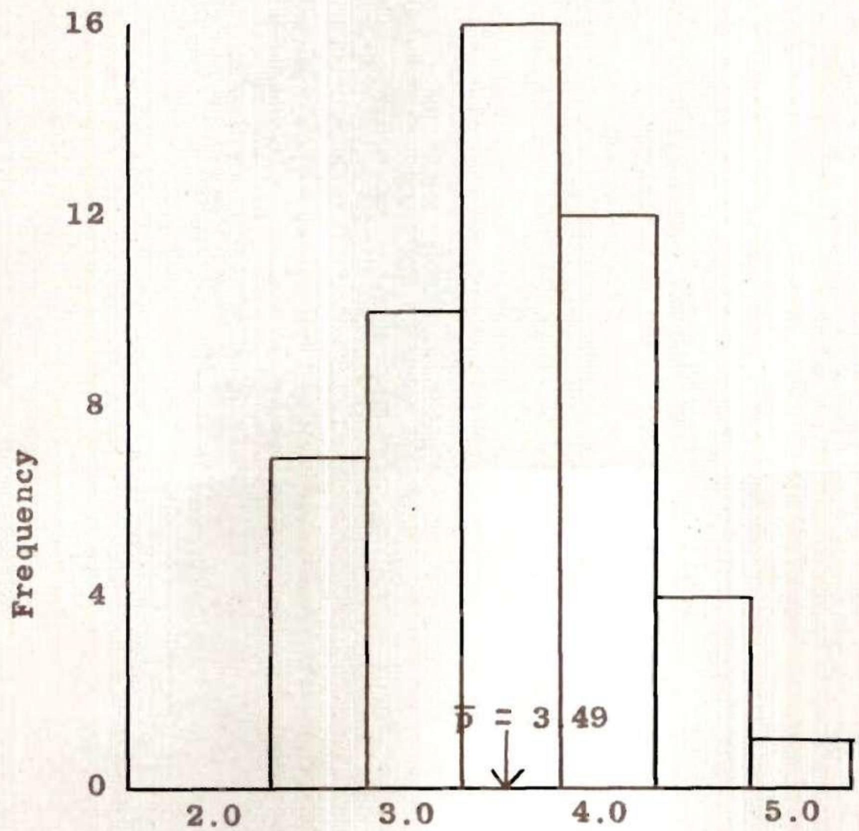

Figure 6. Graph of the G Work Classification Percentage Estimates

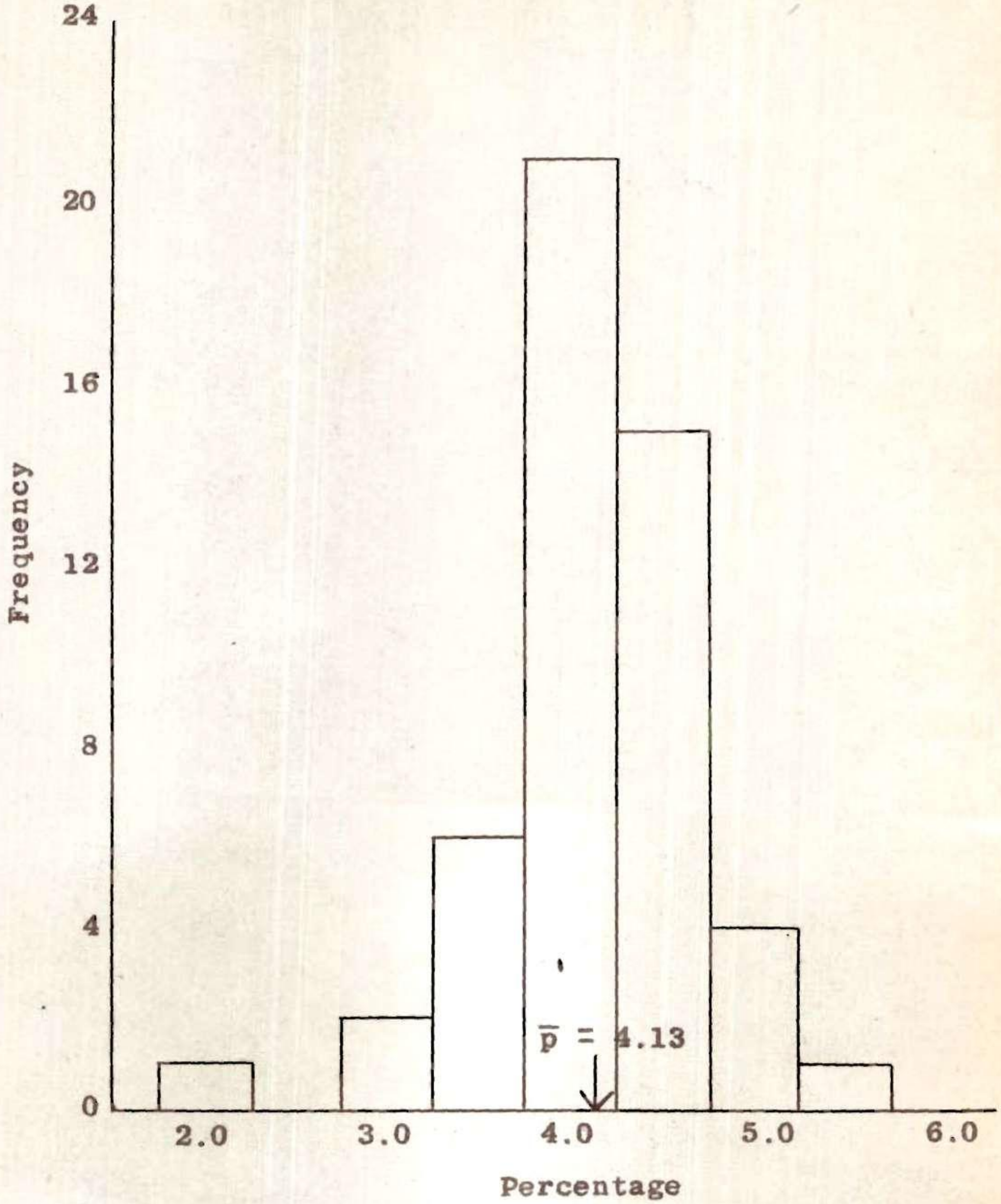

Figure 7. Graph of the C-2 Work Classification Percentage Estimates

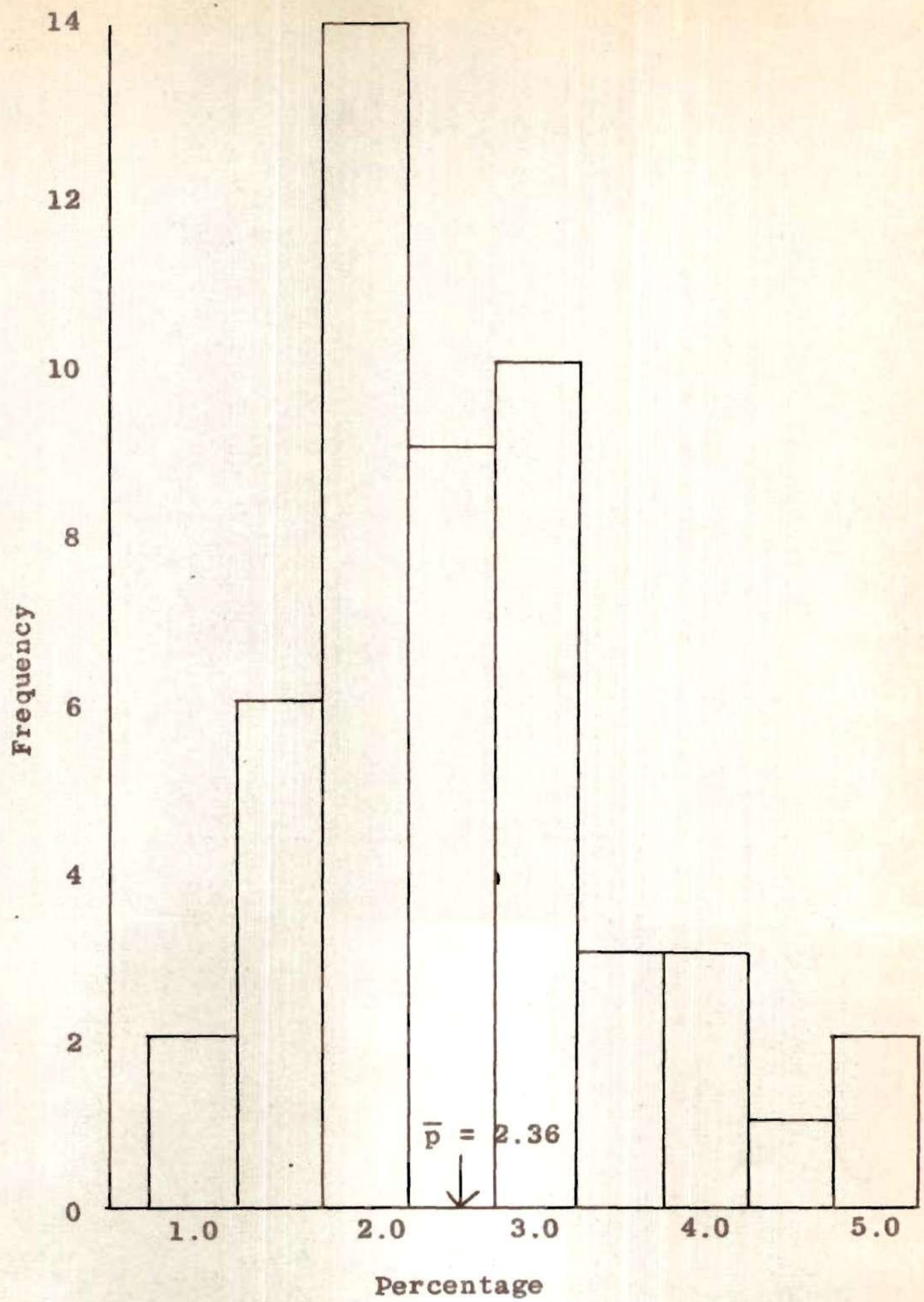

Figure 8. Graph of the YY Work Classification Percentage Estimates

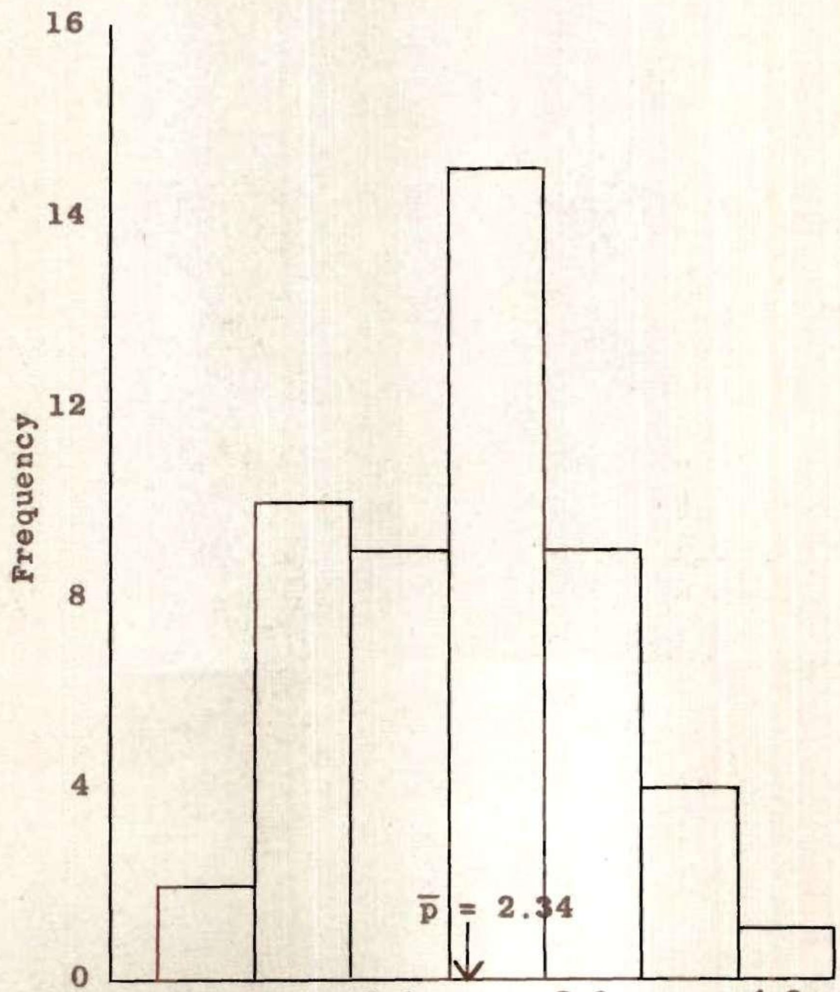

Figure 9. Graph of the M Work Classification Percentage Estimates

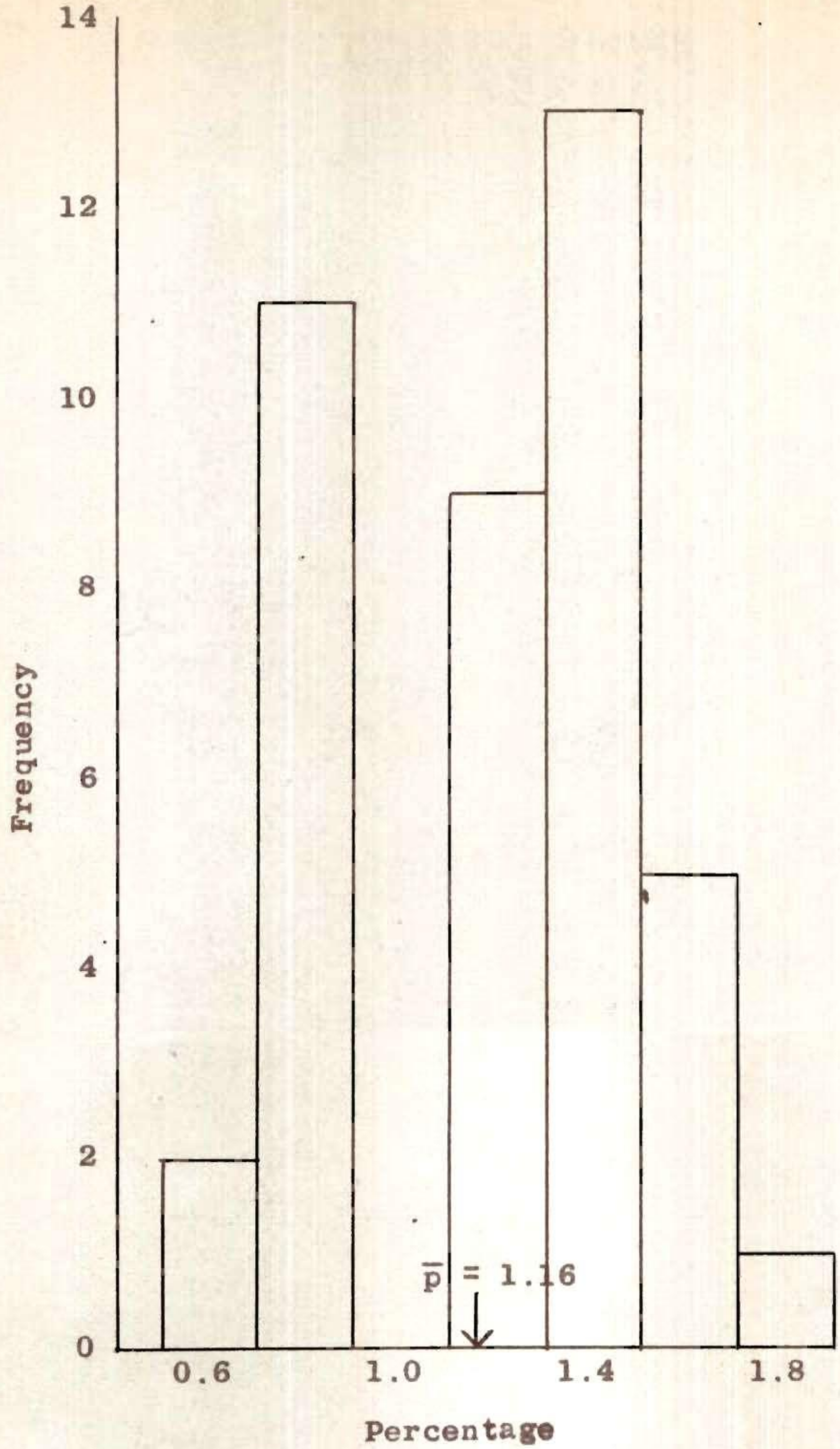

Figure 10. Graph of the X Work Classification Percentage Estimates

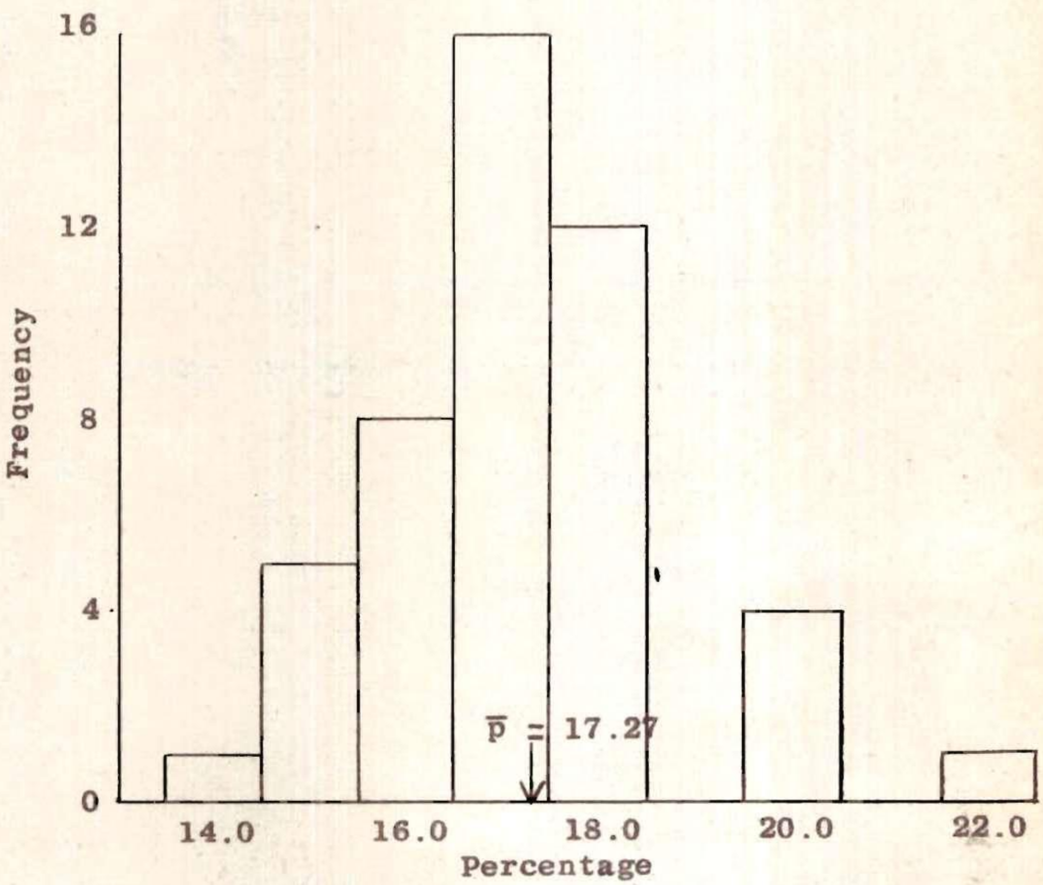

Figure 11. Graph of the O Work Classification Percentage Estimates

fication

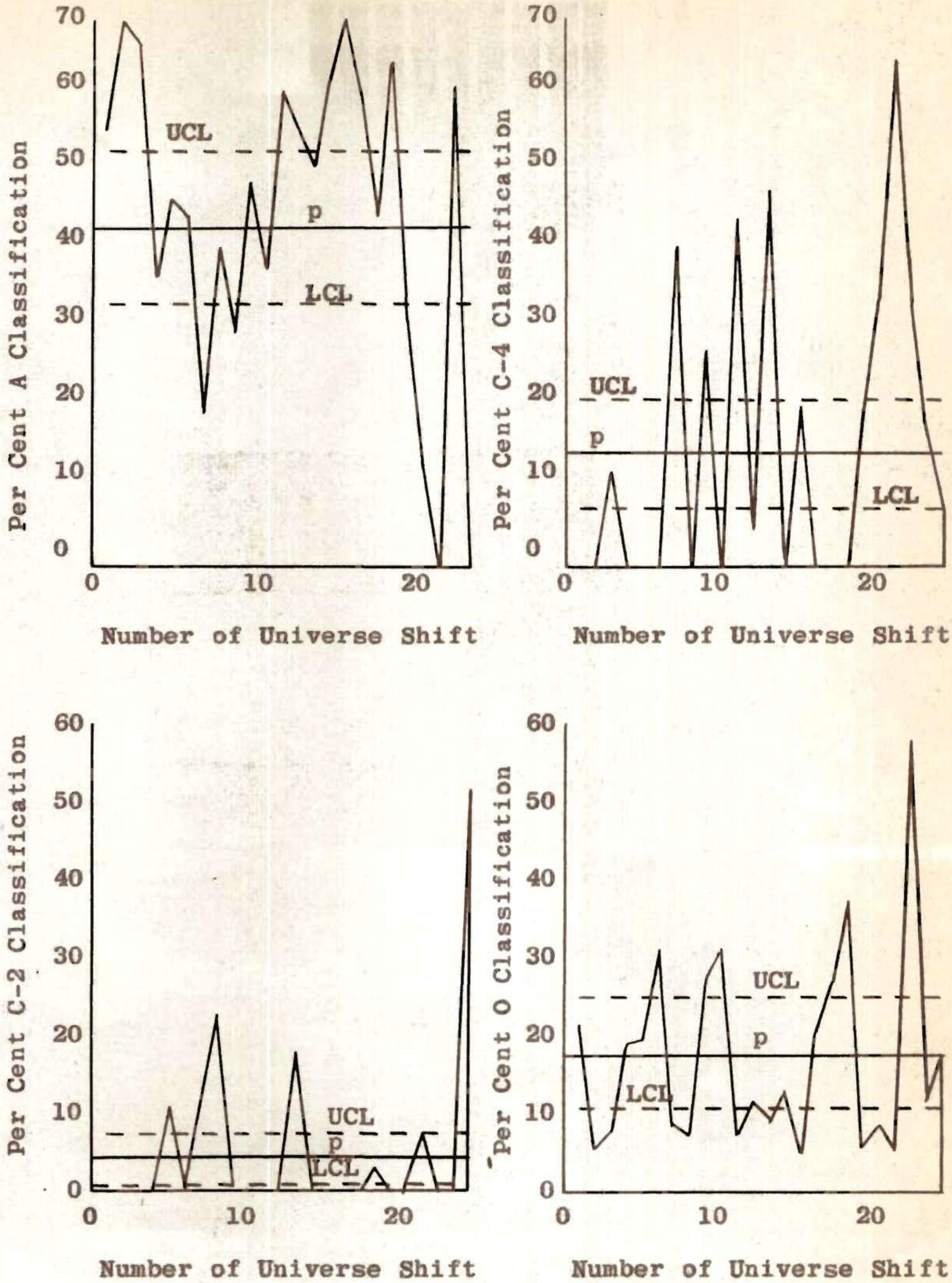

Figure 12. Graphs Showing Universe Percentages Per Shift of A, C-4, C-2, and O Work Classification

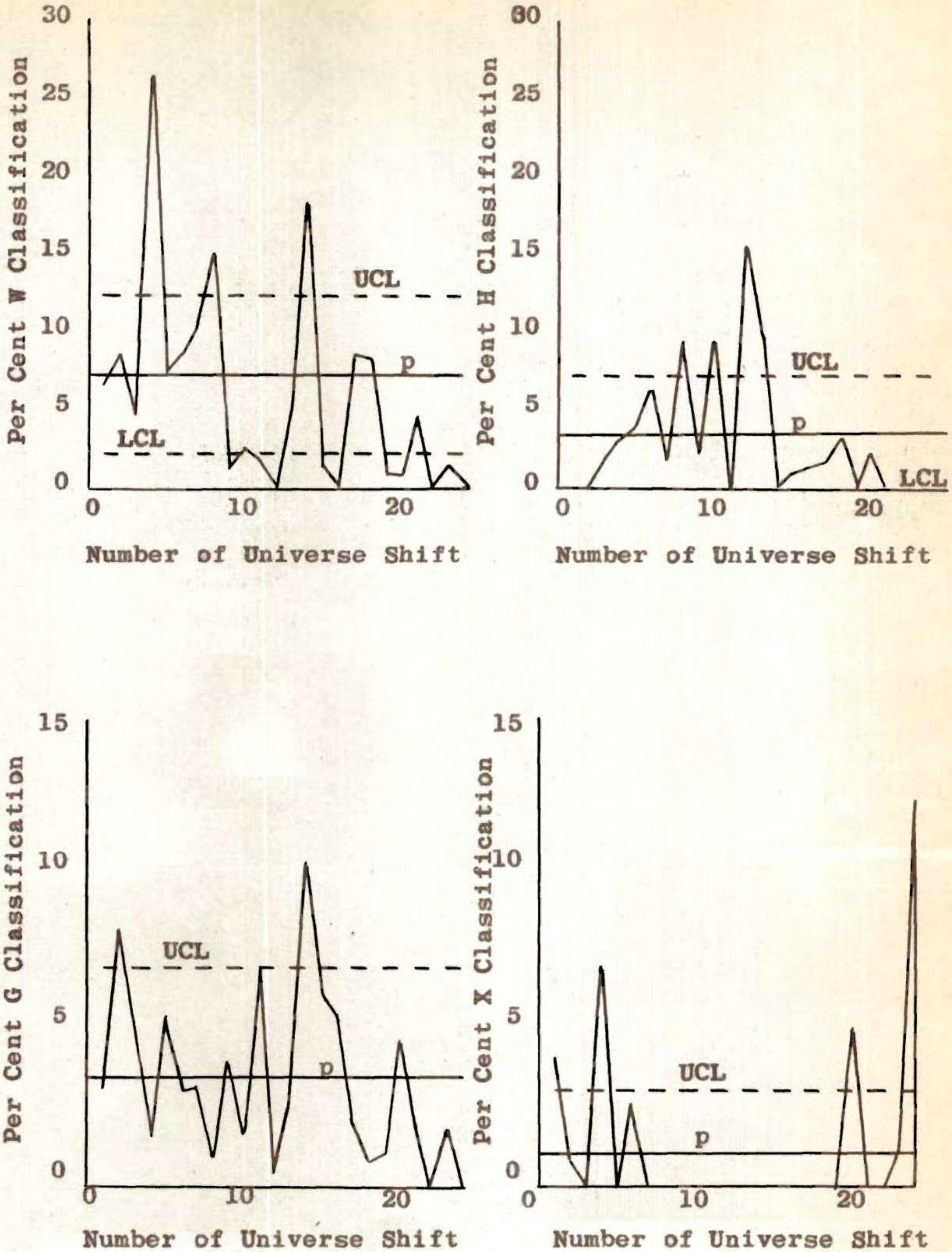

Figure 13. Graphs Showing Universe Percentages Per Shift of W, H, G, and X Work Classifications

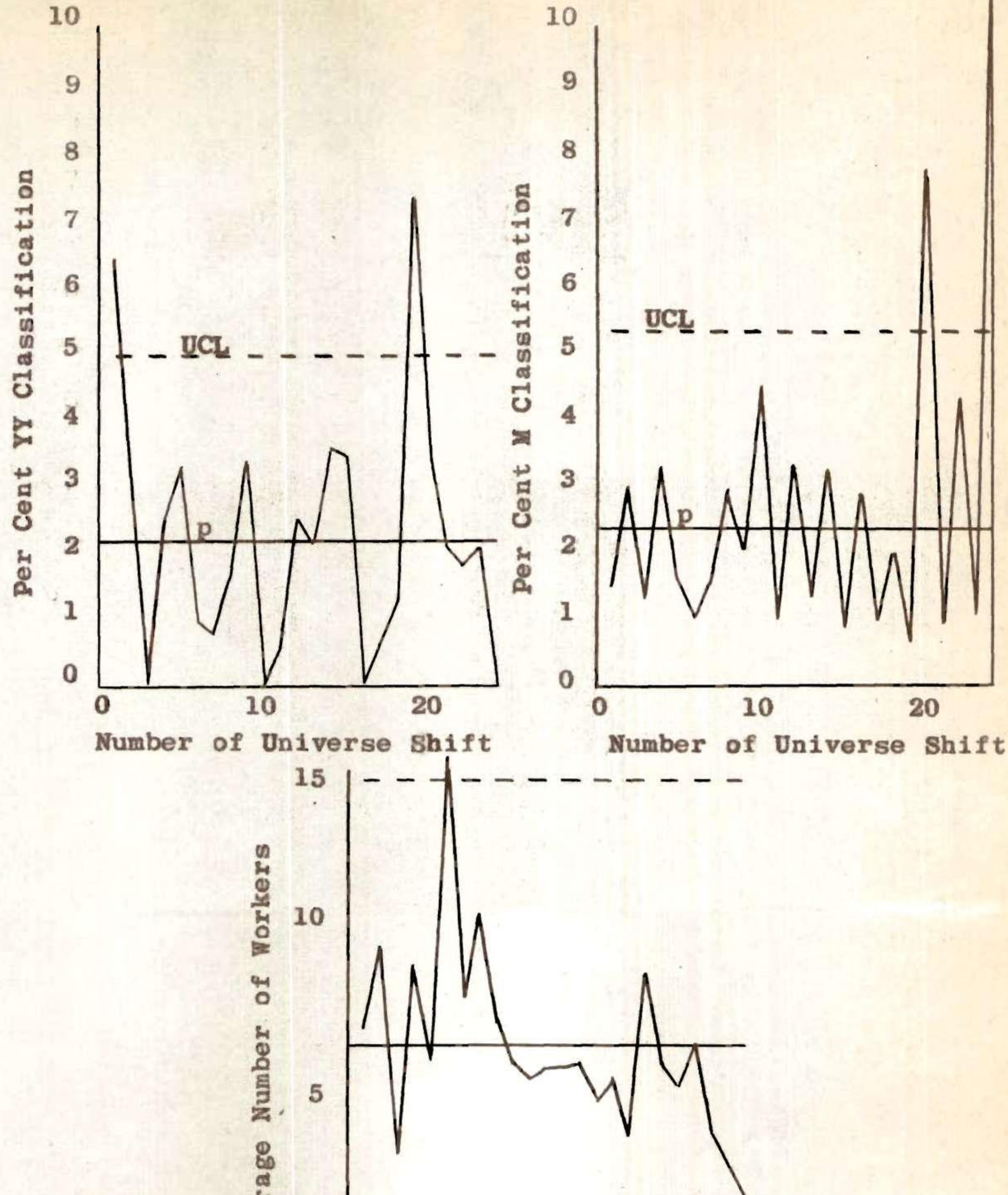

Figure 14. Graphs Showing Universe Percentages Per Shift of YY and M Work Classifications and Average Number of Workers Per Shift

Table 8. Record of Work Activity From One Shift in the Universe

From	To	A	C-4	W	H	G	C-2	YY	M	X	O
345	352								7		
352	356				3				4		
356	400				3	4					
400	428	3				4					
428	436	3	4								
436	448	3	2								2
448	458	2	2								3
458	500	3	2								2
500	524	1	4	1	1						
524	550	3	4								
550	600										7
600	612	4	2								2
612	624	3	4				1				
624	636	3	2				1				2
636	730	3	4				1				
730	815						Lunch				
815	830	3	6				1				
830	842	3	3	3			1				
842	900	3	6				1				
900	906	2	2	5	1						
906	924	2	2	3	1						2
924	936	2	4	2			2				
936	948		4	2			2				
948	1000		4				2				2
1000	1024		2	4			2				
1024	1030	6					2				
1030	1040										8
1040	1048		3	3			2				
1048	1100		2	4			2				
1100	1124		4	2			2				
1124	1220		6				2				
1220	1228		6					2			
1228	1230							8			

Table 9. Example of Sixteen Samples Taken From the Universe Shift in Table 8

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	T
A	3	1	6	3	3	3				3	3			1			26
C4	4	4	4	4	6	2	2	2	6	6	4	6	2	4			56
W		1				4	4						4	1			14
H		1												1			2
G	4																4
C2	1	2	2	2	2	2	2	1	1	2	2						19
YY																	
M																	
X																	
O	2	2															4

Table 10. Pertinent Data of Each Shift in the Universe

Shift Number	Date of Production Time Study	Time of Production Time Study	Number of Production Time Study Observers
1	7-20-1953	12:00 M to 3:45 PM	1
2	7-21-1953	7:00 AM to 3:45 PM	2
3	7-21-1953	3:45 PM to 12:30 PM	2
4	7-22-1953	6:00 AM to 3:45 PM	1
5	7-22-1953	3:45 PM to 12:30 PM	3
6	7-23-1953	7:00 AM to 3:45 PM	2
7	7-23-1953	3:45 PM to 12:30 PM	2
8	7-24-1953	7:00 AM to 3:45 PM	2
9	7-24-1953	3:45 PM to 12:30 PM	2
10	7-27-1953	7:00 AM to 2:36 PM	2
11	7-27-1953	3:45 PM to 12:30 PM	2
12	7-28-1953	7:00 AM to 3:45 PM	1
13	7-28-1953	3:45 PM to 12:30 PM	2
14	7-29-1953	7:00 AM to 3:45 PM	2
15	7-29-1953	3:45 PM to 12:30 PM	2
16	7-30-1953	7:00 AM to 3:45 PM	2
17	7-30-1953	3:45 PM to 12:30 PM	2
18	7-31-1953	7:00 AM to 3:45 PM	1
19	7-31-1953	3:45 PM to 12:30 PM	2
20	8- 3-1953	7:00 AM to 3:45 PM	2
21	8- 3-1953	3:45 PM to 12:30 PM	2
22	8- 4-1953	7:00 AM to 3:45 PM	2
23	8- 4-1953	3:45 PM to 12:30 PM	1
24	8- 5-1953	7:00 AM to 11:00 AM	1

SAMPLE CALCULATIONS FOR WORK CLASSIFICATION A

$$p = \frac{100(\text{total men observed doing A in one study})}{\text{total men observed in one study}} = \frac{985}{2375} = 41.47$$

$$\overline{p} = \frac{\Sigma p}{N'} = \frac{2170.87}{50} = 43.4174$$

$$p' = \frac{100(\text{total man minutes of A classification})}{\text{total man minutes in universe}} = \frac{31{,}423}{72{,}156} = 43.549$$

$$\sigma_t = \sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = \sqrt{\frac{43.4174(56.5826)}{366}} = \sqrt{6.71221141} = 2.5907$$

$$\sigma_s = \sqrt{\frac{\Sigma \overline{p}^2 - \frac{(\Sigma \overline{p})^2}{N'}}{N'-1}} = \sqrt{\frac{94{,}360.5755 - \frac{(2170.87)^2}{50}}{50-1}} = \sqrt{2.18457959} = 1.4746$$

$$K = \frac{\sigma_t^2}{\sigma_s^2} = \frac{6.71221141}{2.18457959} = 3.0725$$

$$UCL_{\sigma_s} = \frac{\sigma_s}{1-1.96\sqrt{2N'}} = \frac{1.4746}{1-1.96\sqrt{2(50)}} = 1.83407$$

$$LCL_{\sigma_s} = \frac{\sigma_s}{1+1.96\sqrt{2N'}} = \frac{1.4746}{1+1.96\sqrt{2(50)}} = 1.23294$$

$$UCL_K = \frac{\sigma_t^2}{(LCL\ \sigma_s)^2} = \frac{6.71221141}{(1.23294)^2} = 4.41551$$

$$LCL_K = \frac{\sigma_t^2}{(UCL\ \sigma_s)^2} = \frac{6.71221141}{(1.83407)^2} = 1.99541$$

$$\overline{K} = \frac{\text{total of K values in work classification division}}{\text{number of work classifications in division}} =$$

$$\frac{22.19777}{7} = 3.17111$$

$$s = \sqrt{\frac{p'(1-p')}{N'(n)\overline{K}}} = \sqrt{\frac{43.549(56.451)}{50(366)3.17111}} = .20579$$

$$Z = \frac{\overline{p}-p'}{S} = \frac{43.4174 - 43.549}{.20579} = .6414$$

GLOSSARY OF SYMBOLS

A = universe work classification code.

C-2 = universe work classification code.

C-4 = universe work classification code.

D = theoretical work classification code.

E = theoretical work classification code.

F = theoretical work classification code.

G = universe work classification code.

H = universe work classification code.

K = number of effective samples per observation.

K = number of crews of workers per observation of the particular work classification being studied. A crew is one or more workers performing a task in which the activity of any crew member is dependent upon the activity of others in the same crew and independent of others in different crews.

$\bar{K}$ = average K value for work classification division.

L = leveling factor for the beginning of a fatigue study.

LCL = lower control limit.

M = number of pieces produced.

M = actual work classification code symbol.

N = the effective sample size.

N' = the number of Ratio Delay studies made.

n = the number of observations in one Ratio Delay study.

O = net overall time.

O = actual work classification code symbol.

$\overline{p}$ = mean per cent of fifty work classification percentages estimates.

p = fraction of success; per cent delay or work classification.

$\overline{p}$ = mean p value.

q = fraction of failures.

R = the range between the maximum and minimum x in a sample.

R = theoretical work classification code symbol.

= standard deviation.

S = leveled time per piece.

S = theoretical work classification code symbol.

σ_s = the root-mean-square standard deviation of 50 Ratio Delay percentages.

S = standard deviation of the work classification percentages based on the effective sample size of all studies made

Σ = an operator signifying "sum of."

σ_T = the Binomial distribution standard deviation based on the effective sample size for one Ratio Delay study.

σ_t = the theoretical Binomial distribution standard deviation of a work classification percentage based on the number of observations per Ratio Delay study.

T = tolerance; maximum desired error in p.

UCL = upper control limit.

W = actual work classification code symbol.

$\overline{x}$ = the mean value of a variable x.

X = actual work classification code symbol.

YY = actual work classification code symbol.

Z = the normal curve deviate.

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