ACCURACY OF A NEW METHOD FOR MEASUREMENT OF AN ACOUSTIC IMPEDANCE

A THESIS

Presented to

The Faculty of the Graduate Division

by

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In Partial Fulfillment

of the Requirements for the Degree

Master of Science

In the School of Aerospace Engineering

Georgia Institute of Technology

August, 1970

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AN ACOUSTIC IMPEDANCE

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ACKNOWLEDGMENTS

The author would like to express here his appreciation to his thesis advisor, Dr. Warren C. Strahle, for his guidance and understanding, and for his encouragement in the difficult periods of the research.

Also appreciated is the work of Mr. Brady R. Daniel who was instrumental in the construction and operation of the experimental apparatus. He generously donated many free hours of time toward the completion of this project. The contribution of Mr. Charlie R. Lord of the electronics laboratory is also acknowledged.

Dr. Ben T. Zinn and Dr. James E. Hubbartt are due a special note of thanks for their sincere interest and constructive comments as readers.

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SUMMARY

Two possible experimental procedures are described for the measurement of the acoustic impedance of a vaporizing liquid. Theoretical investigation of the first procedure shows that the use of a vaporizing spray in an impedance tube would require experimental precision far beyond present capabilities. Consequently this approach is rejected. The second procedure involves an impedance tube terminated by a plane vaporizing surface. The design of this experiment is described and initial experiments to determine the impedance of the end plate assembly without mean air flow and without a vaporizing fluid are performed. It is found that the experimental precision of the impedance measurement is rather poor. Reasons for the imprecise results are deduced and recommendations for experiment improvement are given.

NOTATION

A	complex constant
B	complex constant
c	mean speed of sound
D	diffusion coefficient
D_1	diameter of a droplet
f	frequency
h	height of the plexiglass wall
i	imaginary number $\sqrt{-1}$
K	proportional to the wave number, defined by K_r and K_i
K_r	dimensionless circular frequency, $K_r = \frac{\omega L}{c}$
K_i	decay factor
L	length of the tube
L_v	heat of vaporization
$\tilde{m}$	instantaneous mass flow rate
$\bar{m}$	mean mass flow rate
m	perturbation of the mass flow rate, $m = \tilde{m} - \bar{m}$
M	mach number
$\dot{M}$	mass flow rate vaporizing per droplet
$\bar{P}$	ambient pressure
$\tilde{p}$	instantaneous pressure at a point
p	perturbation of the pressure at a point, or acoustic pressure, $p = \tilde{p} - \bar{P}$

P	dimensionless acoustic pressure, $P = p/\bar{P}$
P'	number defined by $P' = \|P\| e^{i\psi}$
Re	Reynolds number
Sc	Schmidt number
t	time
T	temperature
$\tilde{u}$	instantaneous speed of the gas
u	perturbation of the speed of the gas
U	dimensionless perturbation of the speed of the gas, $U = u/c$
x	abscissa
X	dimensionless abscissa, $X = x/L$
Y	number defined by $Y = \frac{P_1' + P_3'}{2P_2'} = \frac{p_1 + p_3}{2p_2}$
z	acoustic impedance, $z = p/u$
Z	dimensionless acoustic impedance, $Z = \frac{P}{u} = \frac{zc}{\bar{P}}$
zz	quantity defined by equation (9)
γ	ratio of specific heats
μ	viscosity
ψ	phase of P, taking the phase of P_1 (at X_1) as zero
Ψ	complex constant
$\tilde{\rho}$	instantaneous density
$\bar{\rho}$	mean density
ρ	perturbation of the density
ω	circular frequency
Ω	dimensionless circular frequency, $\Omega = \frac{\omega L}{c}$

Subscripts:

v	vaporization
j	index related to the three transducers used in the experiment

CHAPTER I

INTRODUCTION

Several theoretical treatments have appeared in the literature[1-4] concerning the change of vaporization rate of a liquid due to acoustic waves in the ambient fluid. However, no experiments have been successfully designed to measure this effect. This thesis explores two methods.

Both methods use an impedance tube, which is an enclosure where a system of one dimensional standing waves is produced; then measurements of the pressures of various locations enable the acoustic impedance to be computed at any point of this tube. In particular, if a liquid vaporizes at a given point of the tube in a mean one dimensional air flow, it is possible to compute the acoustic impedance at this point for any given vaporization rate.

The vaporization process of interest is one which occurs in flight vehicle engines in the form of a vaporizing spray. An initial attempt is made to model this situation experimentally. It is found theoretically in the second chapter that the influence of the spray on the acoustic wave pattern is too small to be evaluated. This method is therefore to be rejected.

A second approach explored is to use an impedance tube terminated by a flat porous plate through which a vaporizing fluid is allowed to pass. It can be shown that the presence of a vaporization process induces a small but noticeable change of the acoustic impedance of the flat porous plate. It is therefore important to be able to determine this impedance with precision, even without vaporization.

The third chapter of this thesis is concerned with the design of the experiment, including the vaporization devices. The fourth and fifth chapters present measurements of the end plate impedance without mean air flow in the tube, and without the flow of a vaporizing fluid. The measurements are taken using a new method similar to the one investigated in Ref. 5. Several fixed microphones are employed, instead of the classical single traversing microphone. The fifth chapter presents results of this acoustic experiment, and remarks on their accuracy.

CHAPTER II

EVALUATION OF THE FIRST EXPERIMENT

The first method investigated is depicted in Figure 1. A low speed air flow (≈40in/sec) passes horizontally between two plexiglass walls. Perpendicular to the flow a spray of droplets is introduced by a vibrating hypodermic needle[6]. The droplets vaporize, as they fall due to gravity, and the flow passage is divided, therefore, into region I containing air and region II containing air and vapor. Acoustic waves are produced by an acoustic driver located at the left hand end.

The feasibility of determining the effect of the vaporizing spray may be determined theoretically. Since $\tilde{m}_2=\tilde{\rho}_2\tilde{u}_2$ at point 2 downstream of the droplets, if small amplitude disturbances are allowed

$$m_2 = \rho_2\bar{u}_2 + \bar{\rho}_2 u_2 \qquad (1)$$

This equation may be transformed, using three relations. Assume isentropic oscillations $P_2/\bar{P} = \gamma(\rho_2/\bar{\rho}_2)$. Note that the acoustic pressure is assumed the same on both sides of the droplets, since 1). The momentum equation applied to a small control surface containing the droplets and their vaporization process[8] shows that the change in pressure across

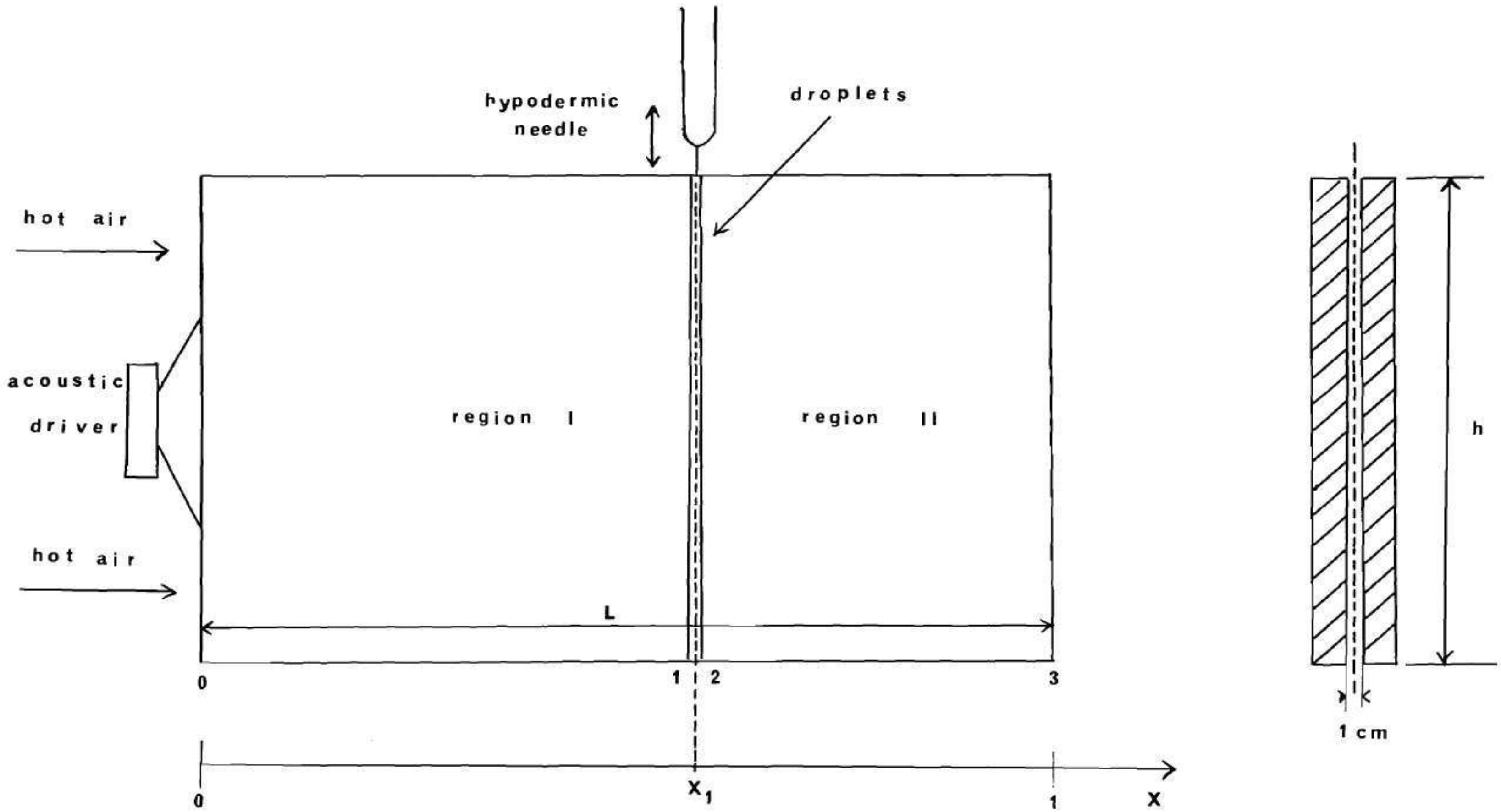

Figure 1. First Experiment

the droplets depends on the change in Mach number. This change being very low, as shown later, a strong expansion wave does not exist; 2). The wavelength of the traveling wave is very large compared to the control surface considered so that there are no wave propagation phenomena in the control volume. Furthermore, wave scattering from the droplets is negligible. Thus, $p_{II} \simeq p_I$. Finally, use the definition of the dimensionless acoustic impedance, $Z = \frac{P/\bar{P}}{u/c}$. Equation (1) then becomes

$$\frac{m_v/\bar{m}_v}{P/\bar{P}} = \frac{1}{\gamma} + \frac{\bar{m}_2}{\bar{m}_v}\frac{1}{M_2 Z_2} - \frac{\bar{m}_1}{\bar{m}_v}\frac{1}{M_1 Z_1} \qquad (2)$$

The quantity on the left hand side of Equation (2) is the quantity of interest. From theory[1] it is dependent upon frequency and is a quantity of the order of unity. Measurable quantities on the right hand side of Equation (2) are Z_1 and Z_2.

Assuming a pressure wave in region I of the form

$$P_I = A_I \sin(\Omega_I X - \Psi_I)e^{i\omega t}$$

where P_I, A_I, Ψ_I are complex numbers, and a corresponding waveform in region II, consider the determination of Z_1 and Z_2. The acoustic impedance of the end of the tube determines the value of Ψ_{II}. For convenience in computation, assume that the end of the tube behaves as an ideal open end with a zero impedance. Then, at $X = 1$, $P_{II} = 0$ or $\sin(\Omega_{II}-\Psi_{II}) = 0$. Consequently, $\Psi_{II}=\Omega_{II}$ or $\Psi_{II}=\Omega_{II}+\pi$. At $X = X_1$, the relation $P_{II}=P_I$ proved earlier gives A_{II}. Finally,

$$P_{II} = \frac{A_I \sin[\Omega_I X_1 - \Psi_I]}{\sin[\Omega_{II}(X_1-1)]} \sin[\Omega_{II}(X-1)]e^{i\omega t}$$

Using the acoustic equation[7] $\partial P/\partial X = -\bar{\rho}\ \partial u/\partial t$

$$\frac{u}{c} = -\frac{\bar{P}A}{i\bar{\rho}c^2}\cos[\Omega X-\Psi]e^{i\omega t}$$

so that, from the definition of the dimensionless acoustic impedance, $Z = -i\gamma \tan(\Omega X-\Psi)$. Consequently, Equation (2) becomes

$$\frac{m_v/\bar{m}_v}{P/\bar{P}} =$$

$$\frac{1}{\gamma}\{1+ \frac{i\bar{m}_2}{\bar{m}_v M_2}[\frac{1}{\tan[\Omega_{II}(X_1-1)]} - \frac{c_2}{c_1}\frac{1}{\tan[\Omega_I X_1-\Psi_I]}]\} \qquad (3)$$

It now remains to be seen whether normal experimental errors would yield an acceptable computation of the ratio $\frac{m_v/\bar{m}_v}{P/\bar{P}}$. The measurable acoustic quantities in Equation (3) are c_1, c_2, Ω_I, Ω_{II}, and Ψ_I. To evaluate the ratio $\bar{m}_v/\bar{m}_2$, consider the following development.

Consider first droplets of 750 microns in diameter since Dabora[6] has determined that droplets of this size remain equally spaced during their downward travel. That is, there is no risk of droplet coalescence. For computational simplicity assume the liquid is water at 212°F and the air is at 900°R and 1 atm. The formula for the vaporization rate in dry air is[8]

$$\dot{M} = 2\eta D_{\ell}\mu \ln[1+ \frac{c_p(T-T_{\ell})}{L_v}](1+0.276\ Re^{1/2}Sc^{1/2}) \qquad (4)$$

The following numerical values are chosen from Reference 9.

μ (water vapor)	= 4.42×10^{-2} lbm/ft/hr
μ (air)	= 6.6×10^{-2} lbm/ft/hr
L_v	= 1205.5 BTU/lbm
c_p	= 6007 BTU/lbm/°F
ρ_{air}	= 0.113×10^{-4} lbm/ft^3
$D_{vapor-air}$	= 1.337 ft^2/hr

Consequently, $Sc = 1.117$, $Re = 19.5$ and, using Equation (4), $\dot{M} = 7 \times 10^{-8}$ lbm/sec. This is the initial vaporization rate. Assuming the droplet velocity in the Reynolds number in Equation (4) to remain unchanged and noting $\dot{M} = -\rho_L \frac{d}{dt}(\pi D_1^3/6)$ an integration for the lifetime of the droplet yields 14.1 sec. A simpler formula for the lifetime results if the Reynolds number is assumed constant in Equation (4). In this case the lifetime is 9.68 sec which is of the same order of magnitude as the more precise estimate. According to Dabora[6] the liquid velocity at the needle exit should equal the Stoke's velocity (drag = weight). This velocity of 36.7 ft/sec can be obtained pressurizing the fluid ahead of the orifice. Integrating Stokes formula to account for the changing droplet size it is found that for a fall of 2 inches the time required is 0.00446 sec and the droplet size at the end of the fall is D_1 = 749.8 microns. For a fall of 40 in (1 m) the time is 0.0896 sec and the diameter has only devreased to 746.5 microns. In these computations the air velocity has been assumed small in comparison with the droplet velocity. To produce these droplets by a vibrating needle requires a vibration frequency of 1100 cycles per second[6], which is also the droplet number production rate. For a 40 in/sec (1 m/sec) air flow it is therefore determined that $\bar{m}_v/\bar{m}_1 = 4.8 \times 10^{-4}$ if the distance between the plexiglass walls is about 0.4 in (1 cm), a reasonable minimum distance to prevent splashing of the spray on the walls. A higher speed airflow would result in downstream displacement of the droplets and destruction of the one dimensional situation, although it would increase the vapor production. Using multiple needles would be possible but there are experimental difficulties in making them all perform properly[6]. It therefore appears that the experiment is constrained to operate with small mass flow ratios, say $\bar{m}_v/\bar{m}_1 \simeq \bar{m}_v/\bar{m}_2 < 10^{-3}$.

Repeating the calculation for 100 μ droplets yields $\bar{m}_v/\bar{m}_2 = 8.3 \times 10^{-4}$ where in this case vaporization is complete after a fall of only 1.07 in. It is concluded that there is no experimental arrangement of this type to produce $\bar{m}_v/\bar{m}_2$

greater than roughly 10^{-3}.

Returning to Equation (3) and noting that $M_2 \simeq 0.003$, it is seen that the term in braces

$$\left[\frac{1}{\tan[\Omega_{II}(X_1 - 1)]} - \frac{c_2}{c_1} \frac{1}{\tan[\Omega_I X_1 - \Psi_1]} \right]$$

must be small, of the order of 10^{-5} in order that the vaporization ratio $\frac{m_v/\bar{m}_v}{P/\bar{P}}$ be of order unity[1]. The temperature is not important in this evaluation: Since $\frac{\bar{m}_2}{\bar{m}_v M_2} = \frac{\bar{\rho}_2 c_2}{\bar{m}_v}$ which is inversely proportional to the square root of the temperature T_2, assuming a perfect adiabatic gas, and inversely proportional to $\bar{m}_v$ which is a logarithmic function of T_2.

The small term in braces is the difference of two terms of order unity which contain numerical errors due to the experimental error. It is concluded that this method is not satisfactory for the measurement of the change in vaporization rate.

CHAPTER III

DESIGN OF THE SECOND EXPERIMENT

The first experiment was shown to be impractical, due to the too low ratio of vaporized mass flow rate to air flow rate. This ratio can be raised to the order of 10^{-1} if the configuration of Figure 2 is used. Here the vaporizing surface which covers the end plate of an impedance tube. Air flows downward toward the plate which is constructed of porous metal through which the vaporizing fluid is forced. Escape of the air and vapor is provided by a cylindrical passage between the tube wall and the end plate. In this second experiment, it can be shown that the presence of a vaporization process induces a small but noticeable change of the acoustic impedance of the flat porous plate. This chapter is concerned with the design of the experiment, including the vaporization devices and the acoustic apparatus. The following chapters will discuss the acoustic problem and measurements.

In order to match the Reynolds number of droplets in a rocket engine, a constraint on the design was selected as $10 \leq Re \leq 100$, where the reference length is the end plate radius, selected as 3 in. For ambient air the velocity in the tube approaching the end plate varies from 0.16 to 1.6

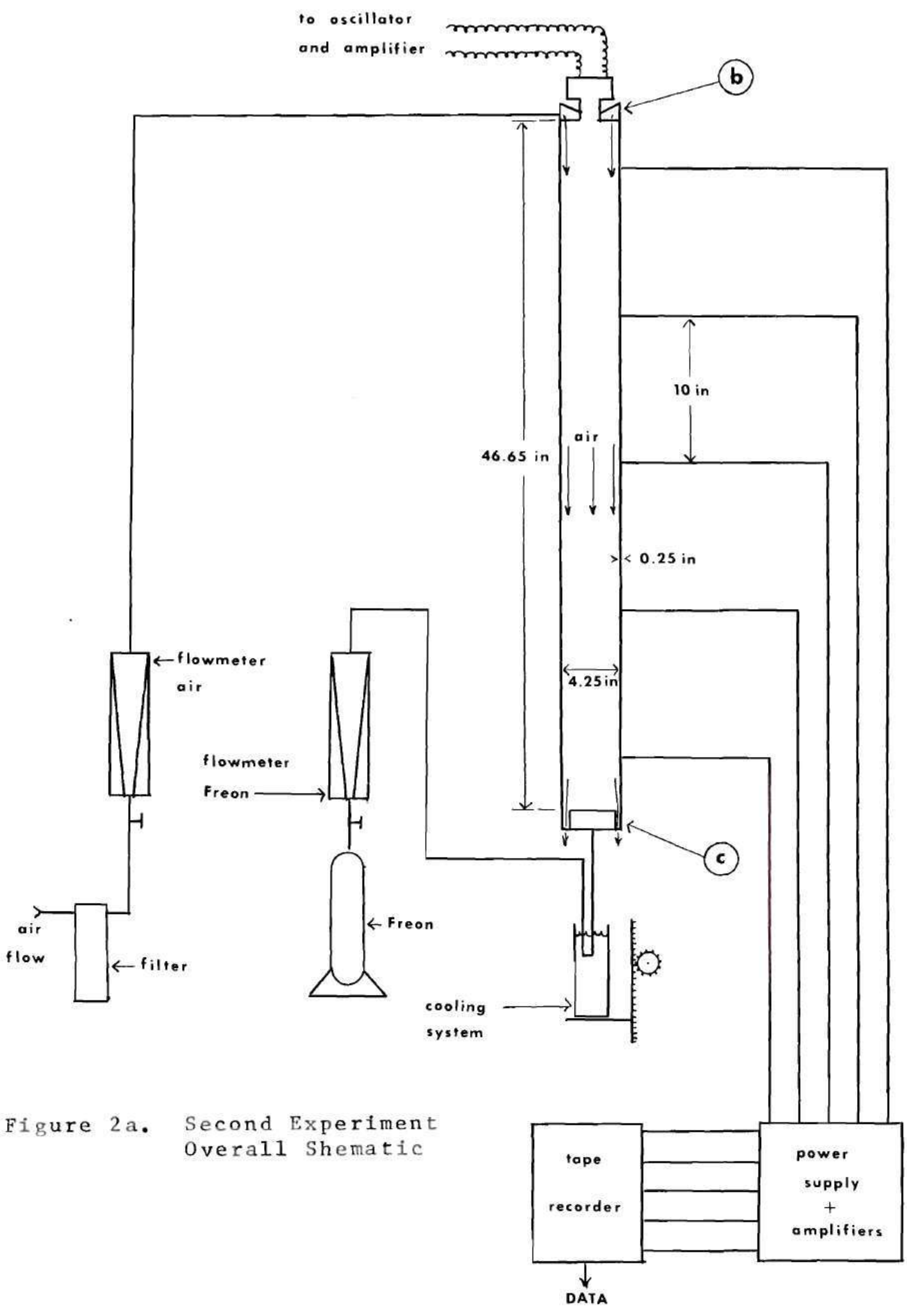

Figure 2a. Second Experiment
Overall Shematic

in/sec as the Reynolds number goes from 10 to 100. Obviously, the velocities occurring in a rocket chamber are not matched. The maximum air flow rate is 0.77 ft^3/min.

Freon 114 was selected as the most promising non-toxic, non flammable fluid as the vaporizing substance. Freon 114 has the lowest storage pressure at ambient temperature of the Freon family. It has a boiling point at one atmosphere of 38.4°F and a vapor pressure at 72°F of 2 atm. Further data on Freon 114 are given in Reference 10. The storage pressure in standard Freon bottles is slightly above the vapor pressure at the ambient temperature and this vapor pressure is used to pump the Freon. Before the expansion of the Freon from the storage bottle to the atmosphere, it is necessary that the Freon be cooled below its boiling point, as shown in Figure 3. The path consists of cooling at constant pressure, Then expansion of the liquid to 1 atm, by heat transfer from the experimental apparatus. A shematic of the cooling system (at constant pressure) is shown in Figure 4.

The flow rate required can be determined using the method of Reference 11, in which it is assumed that the end plate surface behaves as a stagnation point. Using this method, the estimated flow rate is 0.106 cm^3/min at an air flow Reynolds number of 10, and 0.335 cm^3/min at an air flow Reynolds number of 100 (Recall that $10 \leq Re \leq 100$ to match the Reynolds number of droplets in a rocket engine). The flow-

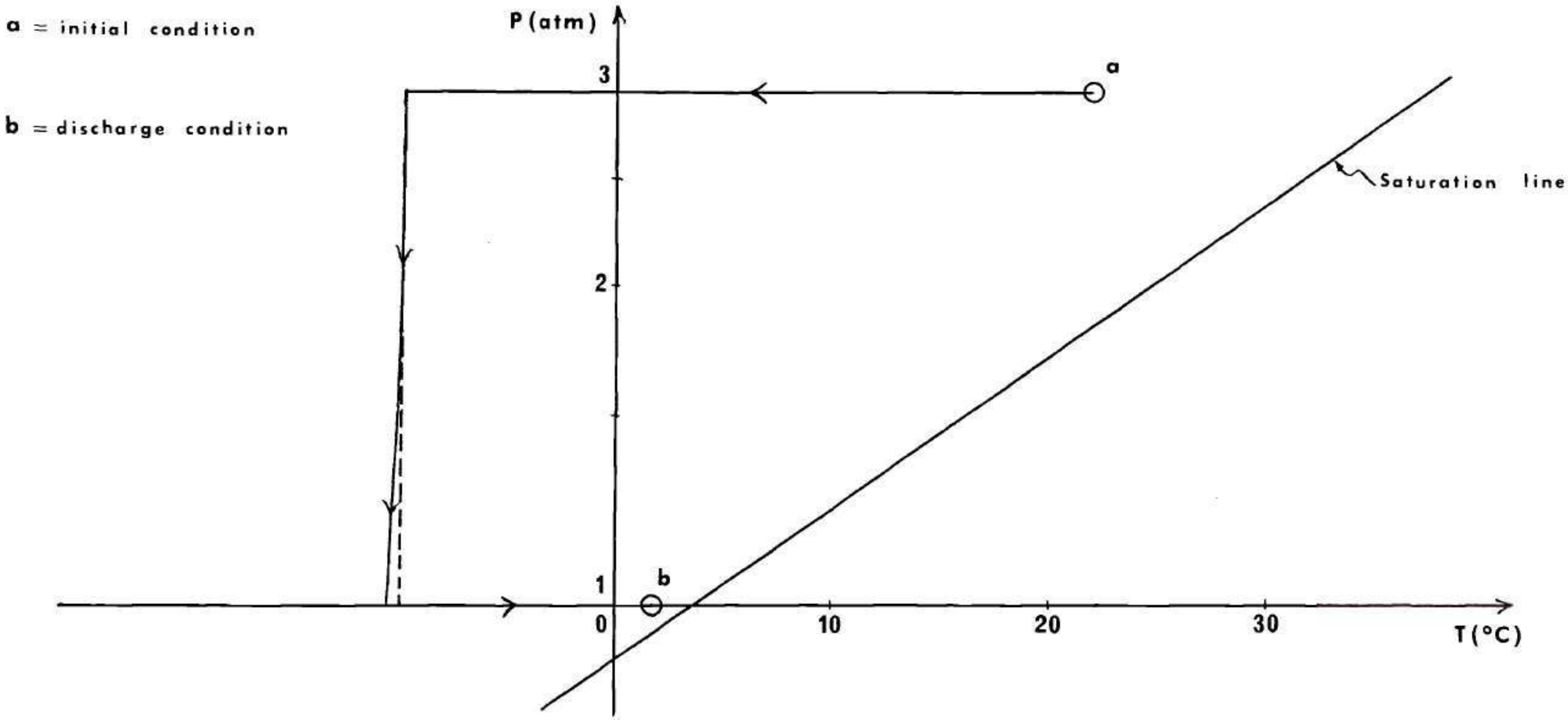

Figure 3. Temperature vs. Pressure for Freon 114

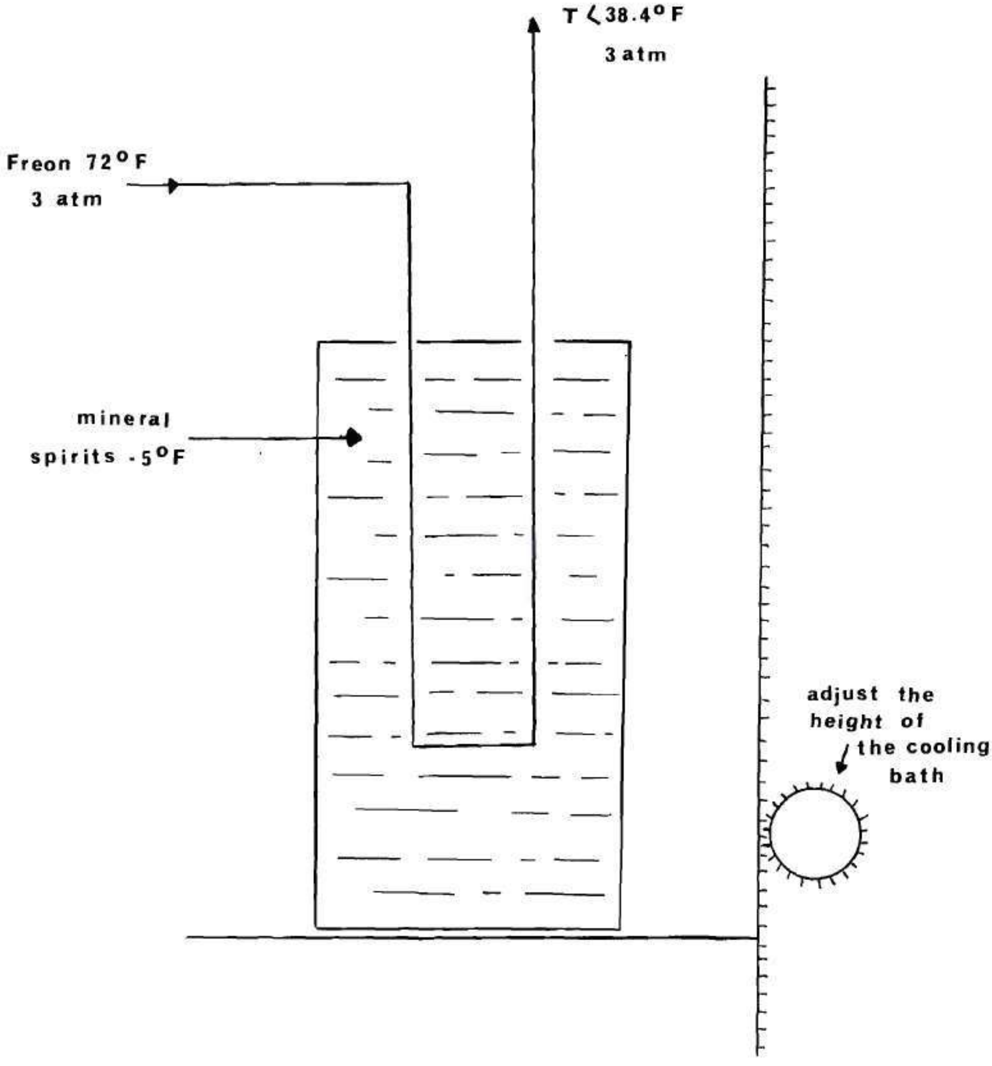

Figure 4. Design of the Cooling System

meter required for the Freon can be roughly sized using these figures.

Actual construction of the air and Freon flow systems were not included in the scope of this thesis. However, the acoustic system was completed. Sound generation is performed by a University 50 watt, 16 ohm driver which yields a satisfactory signal above 100 cps. A Dynasciences Corporation, Model PC 125, oscillator is used to generate a sine wave of a selected frequency and the signal is fed to a Krohn-Hite amplifier, Model DCA, 50R. To adapt the amplifier impedance to the drive impedance a Krohn-Hite matching transformer, Model MT-56, is used. As shown later, the acoustic pressure is measured at five points as located in Figure 2. Bruel and Kjaer condenser microphones, Type 4135, are used. Calibration of the transducers is accomplished with a Bruel and Kjaer Microphone Amplifier, Type 2604. The 5 power supplies and amplifiers for the pressure transducers are grouped into a compact single unit. Each channel has seven attenuation settings ranging from gains of 1 to 1000, in increments of a factor of $\sqrt{10}$. To record the signals an Ampex, Model FR 1300, 14 channel tape recorder is used. A two channel Tektronix, Model 535A, oscilloscope is used to determine appropriate amplifier gain settings, determine resonance frequencies, check signal distortion and check phase differences between two signals.

From the recorded signals it is necessary to determine the amplitudes and phases of the five channels used. The output voltage is read by a digital voltmeter, Data Technology Corporation, Model 360B. The phase difference between two channels is determined using an Aerometrics, Model PM 730, phasemeter. The error in phase angle is roughly ±0.7°. The primary error in the amplitude arises from a ±2% error in the tape recorder.

Calibration of the acoustic system is accomplished by introducing a known acoustic source to each microphone. The signals used ranged from 110 to 160 db re 2×10^{-4} μ bar. 180 db is the maximum signal rating for the transducers and below 110 db the signal is not strong enough for the tape recorder. Each microphone is provided a correction factor (K) to add to the readings on the Bruel and Kjaer microphone amplifier, which is used as a standard. It was checked with another acoustic standard, a Dynasciences Corporation, Model PC 125, acoustic calibrator and some differences were noted. The K factors were modified in accordance with the results in Appendix I.

To calibrate the channel amplifiers a microphone is placed near the acoustic driver and a db level is determined on the B&K microphone amplifier. The microphone signal is then put through a channel on the five channel box. The output voltage is recorded for all attenuation settings, several db settings, and several frequencies. The frequencies of interest were determined as the resonant frequencies of the acoustic tube 186, 312, 443, 585, 882, and 1025 cps. The results of this calibration are given in Appendix II. The assumption is then made that the output voltage is linearly transformed to the tape recorder.

A photograph of the experimental setup is shown in Figure 5.

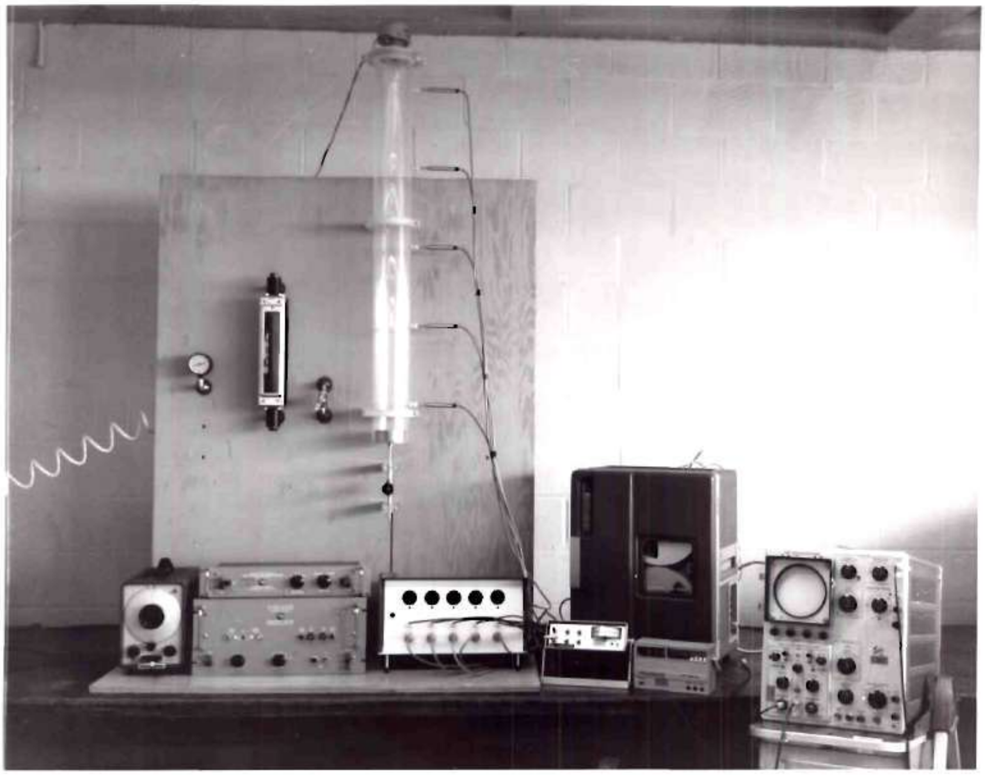

Figure 5. Experimental Setup

CHAPTER IV

THEORY OF THE IMPEDANCE MEASUREMENT

In the second experiment, the presence of a vaporization process induces a small but noticeable change of the acoustic impedance of the flat porous plate. It is therefore important to be able to determine this impedance with precision, even without vaporization. This chapter presents the theory of this determination.

The classical method of measuring the acoustic impedance at the end of a tube uses a traversing microphone[7]. This approach is undesirable in the present program because of the presence of a mean flow and a vaporization in its future version, and the consequent mechanical complexity of the device. An attempt is therefore made to measure to impedance by five fixed transducers.

In the future version, the presence of a mean flow will affect the acoustics of the system[7], and in particular the fundamental mode in which we are interested. However, the effect is proportional to the Mach number. Since $M \simeq 0.003$, the effect is negligible in this experiment. The standing wave in the tube will consist of two traveling waves, the wave produced by the driver and the wave reflected from the tube end[7]. These waves are damped due to wall effects. Using complex notation the general expression for the standing waveform is given by

$$P = (Ae^{iKX} + Be^{-iKX})e^{i\omega t}$$

with

$$K = K_r + iK_i$$

A and B are also complex numbers. The objective is the measurement of the three unknowns A, B, and K. Note that K_r is unknown because c is unknown in the absence of a temperature measurement.

Let us write the acoustic pressure as $P = |P|e^{i(\omega t+\psi)}$. The time origin may be chosen so that, at $X = X_1$, the location of the top transducer, $\psi_1 = 0$. Thus, measurement of the pressure amplitude $|P|$ and phase relative to transducer number 1 yields a complex number P'. Three such measurements at three different points yield three equations

$$P'_j = Ae^{iKX_j} + Be^{-iKX_j} \qquad j = 1,2,3 \qquad (5)$$

for A, B, and K. Solving for A and B using j = 1 and 2,

$$A = \frac{P'_1 e^{-iKX_2} - P'_2 e^{-iKX_1}}{2i \sin K(X_1 - X_2)}$$

$$B = \frac{P'_2 e^{iKX_1} - P'_1 e^{iKx_2}}{2i \sin K(X_1 - X_2)} \qquad (6)$$

Placing Equations (6) into j = 3 of Equations (5)

$$P_1' \sin K(X_2-X_3) + P_2' \sin K(X_3-X_1) + P_3' \sin K(X_1-X_2) = 0 \quad (7)$$

In general Equation (7) cannot be solved analytically. However, for equal transducer spacings with $\Delta X = X_3-X_2 = X_2=X_1$, there results as a nontrivial solution

$$Y = \cos K\,\Delta X = \frac{P_1' + P_3'}{2P_2'} = \frac{p_1 + p_3}{2p_2} \quad (8)$$

Equation (8) is the fundamental equation which determines K from the measured P_1', P_2' and P_3'. Using five transducers there are four possible determinations of K at any given frequency, using the first three, the second three, the last three or the first, third, and last transducers. After some algebra the inversion of Equation (8) for K_r and K_i gives

$$K_i = \frac{\ell n(zz + \sqrt{zz^2-1})}{\Delta X}$$

$$K_r = \varepsilon\,\frac{\cos^{-1}(Y_r/zz)}{\Delta X} + n\,\frac{\pi}{\Delta X} \qquad n = \text{integer}$$

$$\varepsilon = \pm 1$$

$$zz = \frac{1 + Y_r^2 + Y_i^2 + \sqrt{(1+Y_r^2-Y_i^2)^2 - 4Y_r^2}}{2} \quad (9)$$

Y_r and Y_i are the real and imaginary parts of Equation (8), determined experimentally. The choice of ε and n can be most easily accomplished by comparison with the theoretical value of K_r ($K_r = \frac{\omega L}{c}$) which can be closely approximated if the speed of sound can be computed. Then if there is an integer n_K such that

$$(2n_K-2)\pi < K_{r_{\text{theoretical}}} \Delta X \leq (2n_K-1)\pi$$

it follows that $\varepsilon=1$, $n=2(n_K-1)$. If there is an integer n_K such that

$$(2n_K-1)\pi < K_{r_{\text{theoretical}}} \Delta X \leq 2n_K\pi$$

it follows that $\varepsilon=-1$, $n=2n_K$. This problem in the choice of ε and n arises because with fixed transducers there is difficulty in knowing how many wavelengths exist between transducers. A and B may now be computed from Equation (6).

The computation of the impedance follows from its definition and the relation between the pressure and velocity[7] through $\partial p/\partial x = - \rho\ \partial u/\partial t$. The result is

$$Z = \gamma \frac{K_r}{K} \left[\frac{Ae^{iKX} + Be^{-iKX}}{-Ae^{iKX} + Be^{-iKX}} \right] \tag{10}$$

A computer program has been developed to handle the solution of Equation (6) - (10). For a listing of the program and an explanation of its use see Appendix III.

Some comments concerning accuracy of the results are in order here. If Y_i is near zero, $zz \simeq 1$ from Equations (9). Consequently K_i is known with bad precision, and it is not possible to check that K_i is small. For this reason the calculation for zz is carried out in double precision on the computer. Checking the calculation with known pressure profiles K can be determined with an accuracy of 5×10^{-6}. Another problem occurs in the computation of A and B. Equations (6) use Equations (5) with j = 1 and 2. But clearly j = 1, 3 or j = 2, 3 could also be used. Because of experimental errors the results will be different. The computer program calculates three values of A and B from the three possibilities and takes an arithmetic average. This average is the expression of the least mean square method in the A and B planes, respectively.

Another problem occurs if Z is large (rigid, closed end tube). Rewriting Equation (10)

$$Z = \gamma \frac{K_r}{K} \left[\frac{A/B \; e^{2iKX} + 1}{A/B \; e^{2iKX} - 1} \right]$$

it follows that Z is large if $A/B \simeq e^{-2iKX}$. As an example consider

$$\gamma = 1.405 \qquad K = 3 + 0.01\, i$$

$$X = 0.9 \qquad B = 1$$

Z is then large if $A \simeq 0.646 + 0.786\, i$. Computations for Z, varying the real and imaginary parts of A, are shown in Figure 6. It is clear that near A = 0.6462 + 0.7868i a 10^{-4} error in A can make a drastic error in the computed impedance. Pressure profiles have been introduced into the main computer program of Appendix III which simulate Re(Z) = 10, 100, 1000 and 10000. The accuracy with which this may be done is shown in the first two columns of the following table:

Re (Z)	Precision of program		10^{-7} Var. on P_1 gives	
	on A,B	on Z	on A,B	on Z
10	$5.\times10^{-7}$	$5.\times10^{-6}$	10^{-6}	$5.\times10^{-5}$
100	$5.\times10^{-7}$	10^{-4}	$2.\ 10^{-6}$	10^{-2}
1000	$5.\times10^{-7}$	$5.\times10^{-2}$	$2.\ 10^{-6}$	10^{-1}
10000	$5.\times10^{-7}$	1.	$2.\ 10^{-6}$	1 to 100

Then a 10^{-7} error in $|P_1|$ is introduced at point X_1 and the resulting variation Z is noted in the last two columns of the table. The error is primarily in $I_m(Z)$ since the pressure profiles were selected so that $I_m(Z) \simeq 0$. Nevertheless, if $|Z| > 100$, say, there can be expected a rather poor precision in its determination, particularly when P cannot be experimentally determined within 1%.

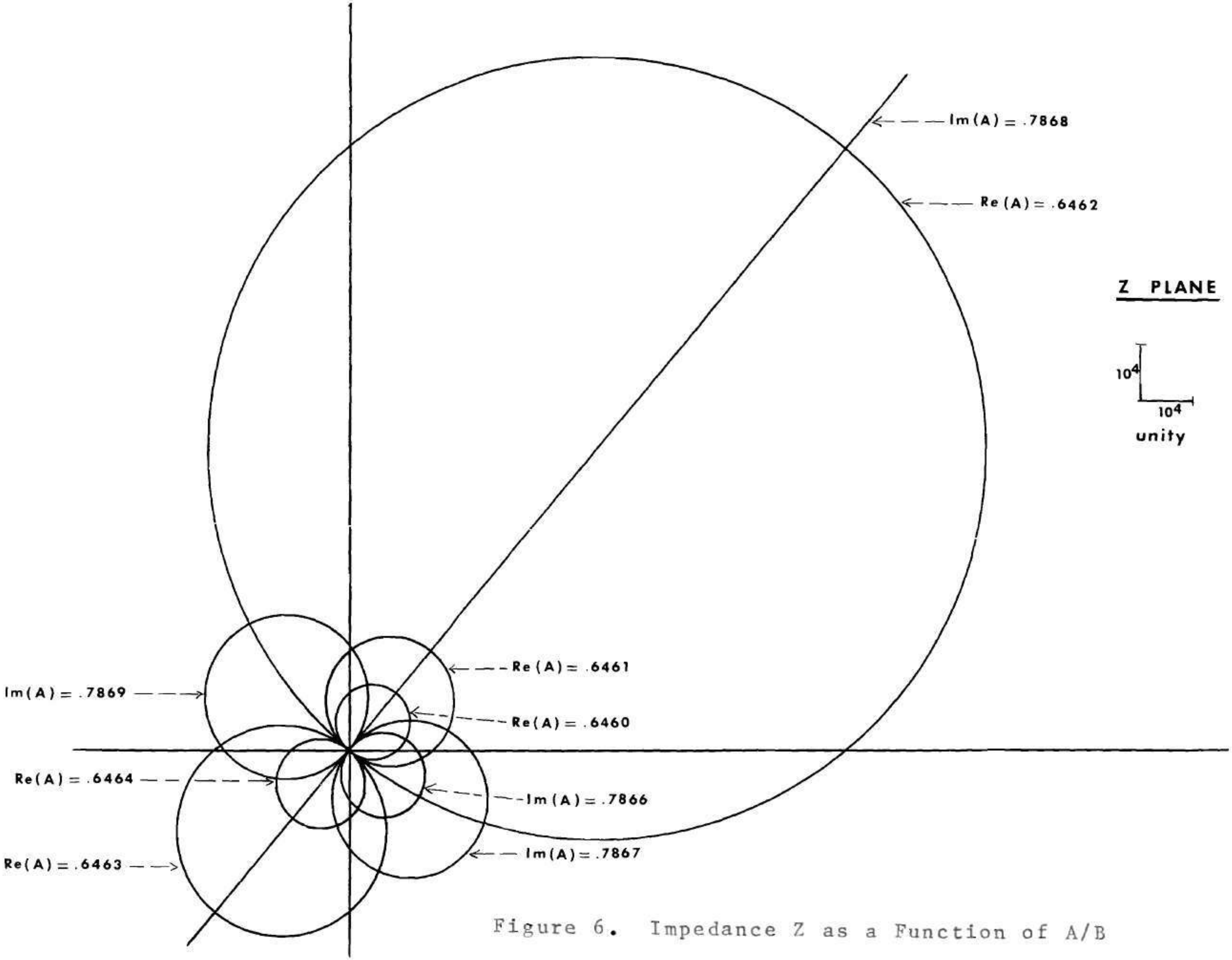

Figure 6. Impedance Z as a Function of A/B

CHAPTER V

EXPERIMENTAL RESULTS

The calibrations of Appendix II reduce to straight lines on a plot of voltage output vs absolute pressure as illustrated for channel 3 in Figure 7. The best straight line fits with pressure linear in voltage output for all channels are shown on the middle line of each box on Table 1. There is a slight variation with frequency. Generally the calibration can be performed within an error of 0.1 db. This corresponds to an error in absolute pressure of approximately 1%.

The pressures obtained from all five channels as a function of frequency are shown in Table 1. They are obtained by putting the voltage, data of the experiment (on the top line of each box, on Table 1) into the calibration formula (on the middle line of each box). These pressures are on the lower line of each box. Table 2 shows the results of the phase measurement along with a repeat of the pressure data of Table 1. The data of Table 2 are used in the computer program of Appendix III, and the computational results are shown in the output of Table 3.

As expected K_i is small compared with K_r, showing small acoustic losses. The three values of A and B are shown in Table 3 for each run along with the average value. The

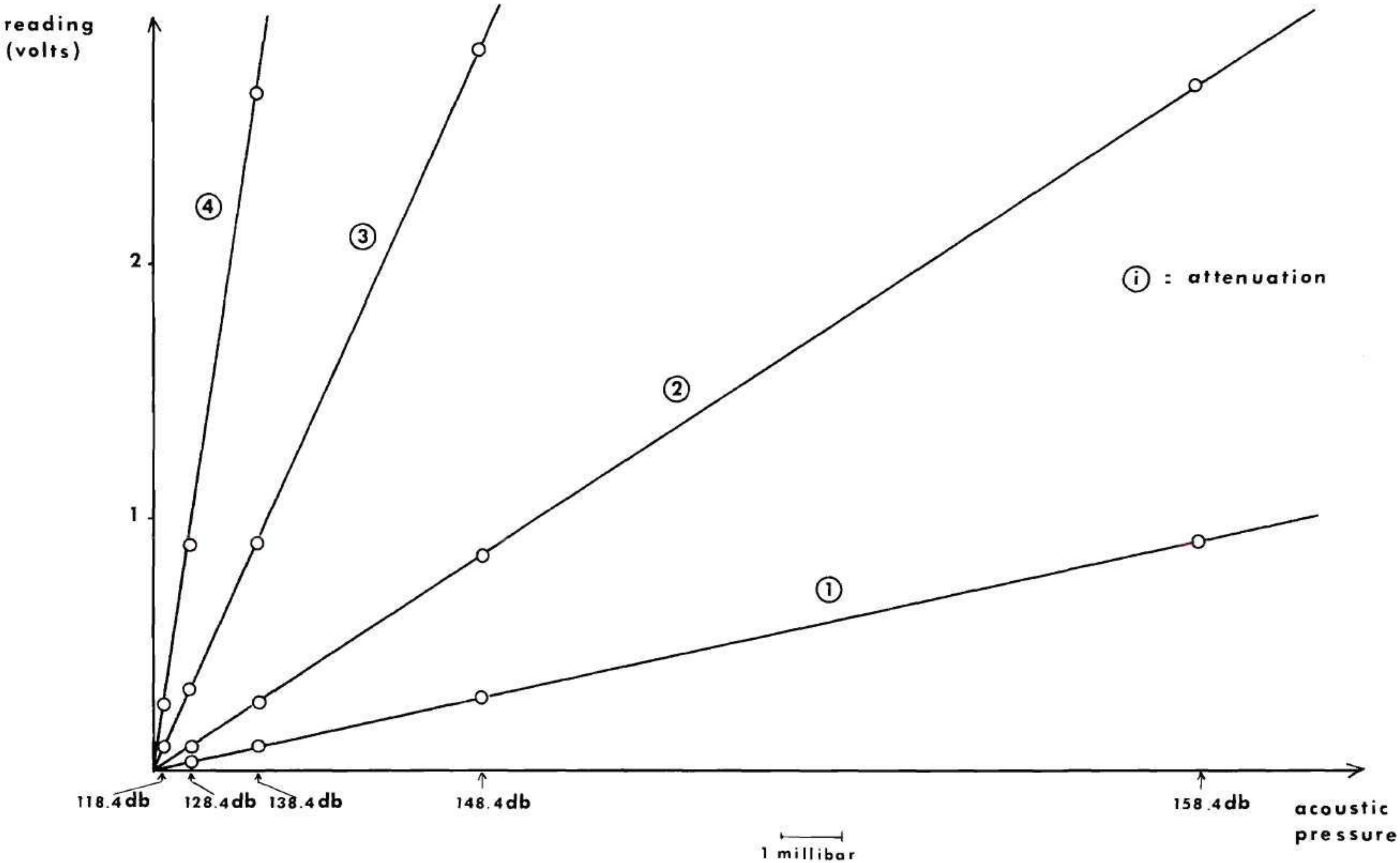

Figure 7. Calibration Curves of the Third Channel at 186 cps.

Table 1. Transformation from the Amplitude Reading (volts) to the Pressure Level (μ bar)

Reading (volts) and attenuation
Transformation formula
Pressure (μ bar)

frequency (cps)	channel				
	1	2	3	4	5
186	1.154 3 P=1625V+10 1886	.723 3 P=1313V+9 958	1.280 4 P=623.0V 797	1.082 3 P=1647V+15 1796	.893 3 P=1948V+13 1752
312	1.225 3 P=1627V+4 1996	1.026 3 P=1319V+3 1356	1.039 3 P=1850V+6 2427	1.140 4 P=555V+1.23 633.9	1.310 3 P=1949V+9 2561
443	.992 3 P=1623V+2 1611	.535 2 P=4356V+14 2345	1.135 4 P=620V+2 705.7	1.064 3 P=1657V+4 1766	1.193 3 P=1951V+2 2329
585	1.070 3 P=1617V+4 1733	.704 2 P=4394V 3093	.651 2 P=6110V+13 3991	.723 2 P=5575V 4030	1.688 3 P=1952V+2 3297
882	.607 4 P=545V+4 335	1.261 3 P=1315V+1 1659	1.046 3 P=1836V+8 1928	1.400 4 P=552V+4 776	1.286 4 P=667V-4 853
1025	.643 3 P=1618V 1040	1.274 3 P=1320V 1681	1.179 4 P=627V 740	1.076 3 P=1645V+6 1775	.584 4 P=673V 393

Table 2. Results of the Experiment

phase (degrees) / amplitude (μ bar

frequency (cps)	channel and abscissa				
	X_1=.0750167	X_2=.2893503	X_3=.5036838	X_4=.7180174	X_5=.9323509
186	0 / 1886	-16.6 / 958	-151.7 / 797	-169.5 / 1796	175.3 / 1752
312	0 / 1996	-156.5 / 1356	-175.8 / 2427	40.8 / 633.9	4.7 / 2561
443	0 / 1611	-166.5 / 2345	16.3 / 705.7	5. / 1766	-173.7 / 2329
585	0 / 1733	-164.2 / 3093	9 / 3991	-162 / 4030	9.6 / 3297
882	0 / 335	-160. / 1659	-12. / 1928	162.5 / 776	162.3 / 853
1025	0 / 1040	-172. / 1681	170.8 / 740	7. / 1775	-5.5 / 393

Table 3. Results of the Program

```
FREQUENCY =    186,000

TEST NO 1

   X =   .0750167  MAG(P) = 1886.0   PHASE(P) =      .0
   X =   .2893503  MAG(P) =  958.0   PHASE(P) =   -16.6
   X =   .5036838  MAG(P) =  797.0   PHASE(P) =  -151.7
   Y =       .64866903     -.01240699
   AA=   .14209254405331402 4+001
   UU=   .26570464754357490 5-003
   ZZ=   .10001328435000740 3+001
   KR THEOR.=      4.02108127       KR EXPER.=     4.03611571
                                    KI EXPER.=      .07604840
   A =      810.9536    -39.6650    B = 1108.4565   135.0231
   A =      810.9533    -39.6649    B = 1108.4562   135.0230
   A =      810.9532    -39.6652    B = 1108.4561   135.0233

   A =      810.9534    -39.6650    B = 1108.4563   135.0231
      MAG(Z) =       1.3452          PHASE(Z) =    27.1349

TEST NO 2

   X = .2893503    MAG(P) =     958.0   PHASE(P) =     -16.6
   X = .5036838    MAG(P) =     797.0   PHASE(P) =    -151.7
   X = .7180174    MAG(P) =    1796.0   PHASE(P) =    -169.5
   Y =      .64707354      .07979716
   AA= .14250717483043243 2+001
   UU= .10869932353357247 9-001
   ZZ= .10054202764781289 4+001
   KR THEOR.=     4.02108127            KR EXPER.=   4.06666738
                                        KI EXPER.=    .48555640
   A =    948.1633     38.9556          B =   831.5995    136.3410
   A =    817.3788    -83.2029          B =   916.5183    163.3556
   A =    933.3364   -170.8136          B =   931.3468     54.6304

   A =    899.6262    -71.6870          B =   893.1549    118.1090
      MAG(Z) =      1.3333               PHASE(Z) =      55.3307
```

Table 3. Results of the Program (Continued)

```
TEST NO 3

   X =   .5036838    MAG(P) = 797.0    PHASE(P) =  -151.7
   X =   .7180174    MAG(P) =1796.0    PHASE(P) =  -169.5
   X =   .9323509    MAG(P) =1752.0    PHASE(P) =   175.3
   Y =        .68194750     -.06005530
   AA=   .1468659016105509 76+001
   UU=   .6703238428637774 35-002
   ZZ=   .1003346021285098 72+001
   KR THEOR.=      4.02108127           KR EXPER.=      3.84204420
                                        KI EXPER.=       .38156459
   A =      916.1474        119.8140    B = 908.6818     46.3458
   A =      916.1474        119.8137    B = 908.6817     46.3458
   A =      916.1476        119.8138    B = 908.6817     46.3460

   A =      916.1475        119.8139    B = 908.6817     46.3459
      MAG(Z) =            1.4999         PHASE(Z) =    45.1198


TEST NO 4

   X =   .0750167    MAG(P) =  1886.0    PHASE(P) =        .0
   X =   .5036838    MAG(P) =   797.0    PHASE(P) =   -151.7
   X =   .9323509    MAG(P) =  1752.0    PHASE(P) =    175.3
   Y =      -.11996873        -.03769016
   AA=   .1015813043675599 63+001
   UU=   .1441261863036975 10-002
   ZZ=   .1000720371463995 42+001
   KR THEOR.=      4.02108127           KR EXPER.=    3.94471076
                                        KI EXPER.=     .08854153
   A =      828.3503        -1.4541     B = 1112.8911    91.5156
   A =      828.3503        -1.4541     B = 1112.8911    91.5156
   A =      828.3503        -1.4541     B = 1112.8911    91.5156

   A =      828.3503        -1.4541     B = 1112.8911    91.5156
      MAG(Z) =          1.4669           PHASE(Z) =    27.5963
```

Table 3. Results of the Program (Continued)

```
FREQUENCY =    312.000

TEST NO 1

   X =   .0750167    MAG(P) =   1996.0    PHASE(P) =        .0
   X =   .2893503    MAG(P) =   1356.0    PHASE(P) =    -156.5
   X =   .5036838    MAG(P) =   2427.0    PHASE(P) =    -175.8
   Y =        .16967312       -.00230605
   AA=   .102879428580346488+001
   UU=   .547549659706244998-005
   ZZ=   .100000273774455091+001
   KR THEOR.=      6.74503952           KR EXPER.=    6.53326792
                                        KI EXPER.=     .01091745
   A =    1019.5775      366.7428       B =   1496.8974   -110.5920
   A =    1019.5780      366.7418       B =   1496.8967   -110.5912
   A =    1019.5779      366.7428       B =   1496.8976   -110.5918

   A =    1019.5778      366.7425       B =   1496.8972   -110.5917
      MAG(Z) =          2.6909           PHASE(Z) =    24.1418


TEST NO 2

   X =   .2893503    MAG(P) =   1356.0    PHASE(P) =    -156.5
   X =   .5036838    MAG(P) =   2427.0    PHASE(P) =    -175.8
   X =   .7180174    MAG(P) =    633.9    PHASE(P) =      40.8
   Y =       .15881496       .01446867
   AA=   .102543153320053560+001
   UU=   .214757772285104208-003
   ZZ=   .100010737312164892+001
   KR THEOR.=      6.74503952           KR EXPER.=    6.58470702
                                        KI EXPER.=     .06837051
   A =     1119.5097     357.3352       B = 1405.6723     -95.3780
   A =     1085.3777     589.0588       B = 1368.2175     113.6123
   A =     1046.2904     364.9835       B = 1472.3223     -78.6463

   A =     1083.7259     437.1258       B = 1415.4040     -20.1373
      MAG(Z) =          2.4726           PHASE(Z) =    22.2606
```

Table 3. Results of the Program (Continued)

TEST NO 3

```
X =   .5036838    MAG(P) =   2427.0    PHASE(P) =   -175.8
X =   .7180174    MAG(P) =   633.9     PHASE(P) =     40.8
X =   .9323509    MAG(P) =   2561.0    PHASE(P) =      4.7
Y =       .09530137        -.04882113
AA=   .101146585304982844+001
UU=   .240529604124648573-002
ZZ=   .100120192570791956+001
KR THEOR.=        6.74503952           KR EXPER.=     6.88396508
                                       KI EXPER.=      .22872838
A =        1283.2256      100.0176     B = 1230.7254    159.2346
A =        1283.2267      100.0173     B = 1230.7246    159.2342
A =        1283.2258      100.0178     B = 1230.7255    159.2347

A =        1283.2260      100.0176     B = 1230.7252    159.2345
     MAG(Z) =            2.0809         PHASE(Z) =   27.3007
```

TEST NO 4

```
X =   .0750167    MAG(P) =   1996.0    PHASE(P) =       .0
X =   .5036838    MAG(P) =   2427.0    PHASE(P) =   -175.8
X =   .9323509    MAG(P) =   2561.0    PHASE(P) =      4.7
Y =      -.93768893         .02551255
AA=   .187991140168023346+001
UU=   .519532769091319852-002
ZZ=   .100259429865270688+001
KR THEOR.=        6.74503952           KR EXPER.=     6.48474425
                                       KI EXPER.=      .16800066
A =      1319.5823        365.8434     B = 1183.0588    -135.6409
A =      1129.1228        268.8670     B = 1339.3036    -136.1564
A =       949.9395        315.1233     B = 1474.9331     -17.1297

A =      1132.8815        316.6113     B = 1332.4318     -96.3090
    MAG(Z) =              3.1089        PHASE(Z) =   36.7003
```

Table 3. Results of the Program (Continued)

```
FREQUENCY =   443.000

TEST NO 1

   X =  .0750167   MAG(P) =  1611.0   PHASE(P) =       .0
   X =  .2893503   MAG(P) =  2345.0   PHASE(P) =   -166.5
   X =  .5036838   MAG(P) =   705.7   PHASE(P) =     16.3
   Y =       -.48429536     .07283767
   AA=  .123984731379972203+001
   UU=  .691636131374878890-002
   ZZ=  .100345222173940538+001
   KR THEOR.=      9.57709134          KR EXPER.=   9.67860639
                                       KI EXPER.=    .38756917
   A =     1329.5989    543.1717       B =  986.2412   112.2863
   A =     1314.8620     61.5104       B = 1080.9186  -199.7945
   A =      976.6632    289.9548       B = 1332.7239    91.5341

   A =     1207.0414    298.2123       B = 1133.2946     1.3419
      MAG(Z) =          2.7638          PHASE(Z) =    49.5566


TEST NO 2

   X =  .2893503   MAG(P) =  2345.0   PHASE(P) =   -166.5
   X =  .5036838   MAG(P) =   705.7   PHASE(P) =     16.3
   X =  .7180174   MAG(P) =  1766.0   PHASE(P) =      5.0
   Y =     -.43250316       -.16401427
   AA=  .121395966330526323+001
   UU=  .328501568327785612-001
   ZZ=  .101629235795256207+001
   KR THEOR.=      9.57709134          KR EXPER.=   9.37971687
                                       KI EXPER.=    .84106395
   A =     1501.3415    928.2124       B = 768.6944   -28.6615
   A =     1501.3412    928.2108       B = 768.6940   -28.6611
   A =     1501.3405    928.2117       B = 768.6940   -28.6618

   A =     1501.3410    928.2116       B = 768.6941   -28.6615
       MAG(Z) =         2.9535          PHASE(Z) =  68.5078
```

Table 3. Results of the Program (Continued)

TEST NO 3

```
   X =   .5036838    MAG(P) =    705.7    PHASE(P) =     16.3
   X =   .7180174    MAG(P) =   1766.0    PHASE(P) =      5.0
   X =   .9323509    MAG(P) =   2329.0    PHASE(P) =   -173.7
   Y =      -.46330145        .02418989
   AA=   .12152333777232797 7+001
   UU=   .74492937611157577 3-003
   ZZ=   .10003723953489078 4+001
   KR THEOR.=       9.57709134          KR EXPER.=    9.57534742
                                        KI EXPER.=     .12732491
   A =      1280.9306      358.0667     B = 1127.8069     -87.9741
   A =      1181.0310      297.2900     B = 1210.9342    -127.9072
   A =      1187.8026      407.8063     B = 1195.4251     -31.7550

   A =      1216.5880      354.3876     B = 1178.0554     -82.5454
      MAG(Z) =              3.9967        PHASE(Z) =     17.6848
```

TEST NO 4

```
   X =   .0750167    MAG(P) =   1611.0    PHASE(P) =        .0
   X =   .5036838    MAG(P) =    705.7    PHASE(P) =     16.3
   X =   .9323509    MAG(P) =   2329.0    PHASE(P) =   -173.7
   Y =      -.52952471       -.03381687
   AA=   .12815399853510883 2+001
   UU=   .15882001881037348 1-002
   ZZ=   .10007937850467016 8+001
   KR THEOR.=       9.57709134          KR EXPER.=    9.69246888
                                        KI EXPER.=     .09294319
   A =      1158.1202     240.1141      B = 1217.1843     -48.1890
   A =      1114.3890     202.4153      B = 1265.6331     -51.3202
   A =      1163.4198     181.4747      B = 1239.4977     -96.9411

   A =      1145.3097     208.0014      B = 1240.7717     -65.4834
      MAG(Z) =             3.3272         PHASE(Z) =     20.6201
```

Table 3. Results of the Program (Continued)

```
FREQUENCY =   585.000

TEST NO 1

   X =  .0750167   MAG(P) =  1733.0   PHASE(P) =       .0
   X =  .2893503   MAG(P) =  3093.0   PHASE(P) =   -164.2
   X =  .5036838   MAG(P) =  3991.0   PHASE(P) =      9.0
   Y =     -.91019212     .15266986
   AA=  .185175776116342661+001
   UU=  .955905010171439152-001
   ZZ=  .104670459109394562+001
   KR THEOR.=     12.64694917          KR EXPER.=    12.24793983
                                       KI EXPER.=     1.42045841
   A =    2947.8187     1522.4212      B = 1180.8026   -111.2670
   A =    1577.9561      360.7932      B = 1562.6576     85.1742
   A =     653.5620      676.9771      B = 1612.1762    873.0749

   A =    1726.4456      853.3972      B = 1451.8788    282.3274
      MAG(Z) =           1.6084         PHASE(Z) =      99.7786


TEST NO 2

   X =  .2893503   MAG(P) = 3093.0   PHASE(P) =   -164.2
   X =  .5036838   MAG(P) = 3991.0   PHASE(P) =      9.0
   X =  .7180174   MAG(P) = 4030.0   PHASE(P) =   -162.0
   Y =     -.88344094       -.12486334
   AA=  .179605873171887659+001
   UU=  .592399952979458288-001
   ZZ=  .102919385700554287+001
   KR THEOR.=     12.64694917        KR EXPER.=  12.14417064
                                     KI EXPER.=   1.12465604
   A =    2722.5905     1524.7995    B = 1275.6751  -198.7907
   A =    2722.5896     1524.8016    B = 1275.6747  -198.7909
   A =    2722.5907     1524.8028    B = 1275.6749  -198.7916

   A =    2722.5903     1524.8013    B = 1275.6749  -198.7911
      MAG(Z) =          2.3331        PHASE(Z) =    100.8448
```

Table 3. Results of the Program (Continued)

```
TEST NO 3

   X =   .5036838    MAG(P) =   3991.0    PHASE(P) =       9.0
   X =   .7180174    MAG(P) =   4030.0    PHASE(P) =    -162.0
   X =   .9323509    MAG(P) =   3277.0    PHASE(P) =       9.6
   Y =       -.89127902        .13685470
   AA=   .18131074814106762 4+001
   UU=   .72268611234397739 9-001
   ZZ=   .10355040372854167 4+001
   KR THEOR.=     12.64694917          KR EXPER.=  12.16551471
                                       KI EXPER.=   1.23961537
   A =      3699.5894     6179.8989    B =  397.8446   -192.7381
   A =      4463.8852     2892.9008    B =  664.4281   -394.7351
   A =      2987.1077     1568.5667    B = 1184.0461   -162.8101

   A =      3716.8607     3547.1221    B =  748.7729   -250.0944
      MAG(Z) =            4.2816        PHASE(Z) =   72.3989


TEST NO 4

   X =   .0750167    MAG(P) =   1773.0    PHASE(P) =        .0
   X =   .5036838    MAG(P) =   3991.0    PHASE(P) =       9.0
   X =   .9323509    MAG(P) =   3277.0    PHASE(P) =       9.6
   Y =        .62991627       -.03044879
   AA=   .13977216293624699 6+001
   UU=   .15354544501142270 8-002
   ZZ=   .10007674327485453 4+001
   KR THEOR.=     12.64694917          KR EXPER.=  12.58135116
                                       KI EXPER.=    .09138759
   A =      2067.4614      731.8848    B = 1924.0204    -68.3382
   A =      1916.4865      508.4149    B = 2098.6096   -214.0945
   A =      1743.3446      687.8730    B = 2213.6054      3.3387

   A =      1909.0975      642.7242    B = 2078.7451    -93.0313
      MAG(Z) =            6.1354        PHASE(Z) =   29.5652
```

Table 3. Results of the Program (Continued)

```
FREQUENCY =   882.000

TEST NO 1

   X =  .0750167   MAG(P) =    335.0   PHASE(P) =       .0
   X =  .2893503   MAG(P) =   1629.0   PHASE(P) =   -160.0
   X =  .5036838   MAG(P) =   1928.0   PHASE(P) =    -12.0
   Y =     -.58765313      .34245395
   AA=  .146261089712096060+001
   UU=  .166588658008988783+000
   ZZ=  .108008733813936953+001
   KR THEOR.=    19.06770778          KR EXPER.=  19.30213618
                                      KI EXPER.=   1.85502538
   A =     -858.7740     -509.9441    B = -688.7990    72.9119
   A =     -858.7790     -509.9390    B = -688.7979    72.9117
   A =     -858.7802     -509.9420    B = -688.7994    72.9097

   A =     -858.7777     -509.9417    B = -688.7988    72.9111
      MAG(Z) =          1.4010         PHASE(Z) =   91.4480


TEST NO 2

   X =  .2893503   MAG(P) =   1659.0   PHASE(P) =   -160.0
   X =  .5036838   MAG(P) =   1928.0   PHASE(P) =    -12.0
   X =  .718074    MAG(P) =    776.0   PHASE(P) =    162.5
   Y =     -.56518120     -.20870333
   AA=  .136298686578819983+001
   UU=  .622867031394315490-001
   ZZ=  .103067293703649343+001
   KR THEOR.=    19.06770778          KR EXPER.=  19.27829361
                                      KI EXPER.=   1.15265334
   A =    -1980.1441      767.8135    B = -424.7850     51.8790
   A =    -1493.1708     -918.0146    B = -526.2512   -266.8620
   A =     -670.5241     -236.5195    B = -939.0921     93.9006

   A =    -1381.2797     -130.2402    B = -630.0428    -40.3608
      MAG(Z) =          1.8381         PHASE(Z) =   73.7715
```

Table 3. Results of the Program (Continued)

```
TEST NO 3

   X =  .5036838    MAG(P) =  1928.0    PHASE(P) =    -12.0
   X =  .7180174    MAG(P) =   776.0    PHASE(P) =    162.5
   X =  .9323509    MAG(P) =   853.0    PHASE(P) =    162.3
   Y =      -.68693875      -.12098547
   AA=  .148652232605982082+001
   UU=  .270785583507004946-001
   ZZ=  .101344884347987713+001
   KR THEOR.=     19.06770778          KR EXPER.=  18.51128483
                                       KI EXPER.=    .76433134
   A =     -1859.8222      70.9163     B = -311.1569   430.5788
   A =     -1953.4611    -281.1778     B = -336.9973   346.8733
   A =     -1695.0984    -327.8542     B = -449.8636   392.0349

   A =     -1836.1272    -179.3719     B = -366.0059   389.8290
      MAG(Z) =            7.5012        PHASE(Z) =   53.6996


TEST NO 4

   X =  .0750167    MAG(P) =   335.0    PHASE(P) =       .0
   X =  .5036838    MAG(P) =  1928.0    PHASE(P) =    -12.0
   X =  .9323509    MAG(P) =   853.0    PHASE(P) =    162.3
   Y =     -.13514079       .04003391
   AA=  .101986574626703169+001
   UU=  .163247911426662764-002
   ZZ=  .100081590670525747+001
   KR THEOR.=     19.06770778          KR EXPER.=  18.63783026
                                       KI EXPER.=    .09422921
   A =     -1131.9288    -119.2405     B = -748.3518   480.5179
   A =      -796.3926     381.7147     B = -554.3341   947.6118
   A =      -970.9573    -146.3284     B = -892.5096   515.9677

   A =      -966.4262      38.7153     B = -731.7318   648.0324
      MAG(Z) =            8.5351        PHASE(Z) =   38.0754
```

Table 3. Results of the Program (Continued)

```
FREQUENCY =    1025.000

TEST NO 1

   X =   .0750167    MAG(P) =   1040.0    PHASE(P) =      .0
   X =   .2893503    MAG(P) =   1681.0    PHASE(P) =  -172.0
   X =   .5036838    MAG(P) =    740.0    PHASE(P) =   170.8
   Y =      -.09606574      -.02203575
   AA=   .10097142002403633O+001
   UU=   .49009481464166417 2-003
   ZZ=   .10002450173905599 9+001
   KR THEOR.=      22.15918422         KR EXPER.=  21.53744078
                                       KI EXPER.=    .10327955
   A =      -895.9608      -630.5575   B = -724.7538   360.4879
   A =      -701.6606      -977.4825   B = -648.1250    10.4376
   A =      -820.6571      -616.1635   B = -796.7046   354.2685

   A =      -806.0928      -741.7012   B = -723.1945   241.7313
      MAG(Z) =          12.6428          PHASE(Z) =   -44.3621


TEST NO 2

   X =   .2893503    MAG(P) =   1681.0    PHASE(P) =  -172.0
   X =   .5036838    MAG(P) =    740.0    PHASE(P) =   170.8
   X =   .7180174    MAG(P) =   1775.0    PHASE(P) =     7.0
   Y =       -.06668833         .00126829
   AA=   .1004448941228522 91+001
   UU=   .1615752827752633 74-005
   ZZ=   .1000000807876087 54+001
   KR THEOR.=      22.15918422         KR EXPER.=  21.67486548
                                       KI EXPER.=    .00593058
   A =      -880.1067      -549.9387   B = -774.8504   313.3073
   A =      -880.1078      -549.9365   B = -774.8517   313.3049
   A =      -880.1071      -549.9387   B = -774.8508   313.3073

   A =      -880.1072      -549.9379   B = -774.8510   313.3065
      MAG(Z) =          7.5286           PHASE(Z) =   -34.0054
```

Table 3. Results of the Program (Continued)

```
TEST NO 3

   X =   .5036838    MAG(P) =    740.0    PHASE(P) =    170.8
   X =   .7180174    MAG(P) =   1775.0    PHASE(P) =      7.0
   X =   .9323509    MAG(P) =    393.0    PHASE(P) =     -5.5
   Y =       -.09209376      .03419524
   AA=   .1009650575768713O5+001
   UU=   .117930491348411795-002
   ZZ=   .100058947871416483+001
   KR THEOR.=     22.15918422          KR EXPER.=  21.55619884
                                       KI EXPER.=    .16019076
   A =      -906.1895     -661.2070    B = -694.4929    321.3182
   A =      -906.1711     -661.1882    B = -694.4736    321.3212
   A =      -906.1912     -661.2023    B = -694.4933    321.3143

   A =      -906.1839     -661.1991    B = -694.4866    321.3179
      MAG(Z) =            13.7690       PHASE(Z) =    -17.4620


TEST NO 4

   X =   .0750167    MAG(P) = 1040.0    PHASE(P) =         .0
   X =   .5036838    MAG(P) =   740.0    PHASE(P) =     170.8
   X =   .9323509    MAG(P) =   393.0    PHASE(P) =      -5.5
   Y =       -.95865029       -.12948553
   AA=   .193577687414791910+001
   UU=   .101296304528206702+000
   ZZ=   .104942665514470648+001
   KR THEOR.=     22.15918422        KR EXPER.=  22.96366668
                                     KI EXPER.=    .73047043
   A =      -415.4815     -869.1096  B =   91.5219      115.4497
   A =     - 36.9221     -885.4733   B =   56.8184      206.0798
   A =       104.9610     -740.9022  B =  139.4041      367.7427

   A =      -115.8142     -831.8284  B =   95.9148      229.7574
      MAG(Z) =             2.6951     PHASE(Z) =   164.7745
```

dispersion generally increases with frequency. Another problem concerning accuracy was uncovered during the data reduction. In Equation (8), if P_1' and P_3' are nearly 180° out of phase and of comparable magnitude, the experimental precision on Y is poor as shown in Figure 8.

The phase and magnitude of Z are shown on Figures 9 and 10. Clearly, the precision of the measurement decreases with a frequency increase, as more wavelengths occur between transducers. It is not immediately obvious why this should be so. However some findings are the following:

1. As frequency increases more pairs of transducers 1 and 3 are 180° out of phase yielding the accuracy problem mentioned above. However, rejecting those measurements does not appear to reduce the data scatter.
2. Nonlinearities at acoustic resonance may alter the waveform and it was specifically assumed that only the fundamental component of the wave was present. However, an independent spectral analysis of the tape yielded only very small higher harmonic content.
3. The first and fifth transducers were rather near the acoustic source and end plate, respectively. Since these may not be located in a region of a sufficiently one dimensional waveform, errors may occur. A check on this assumption could not be made

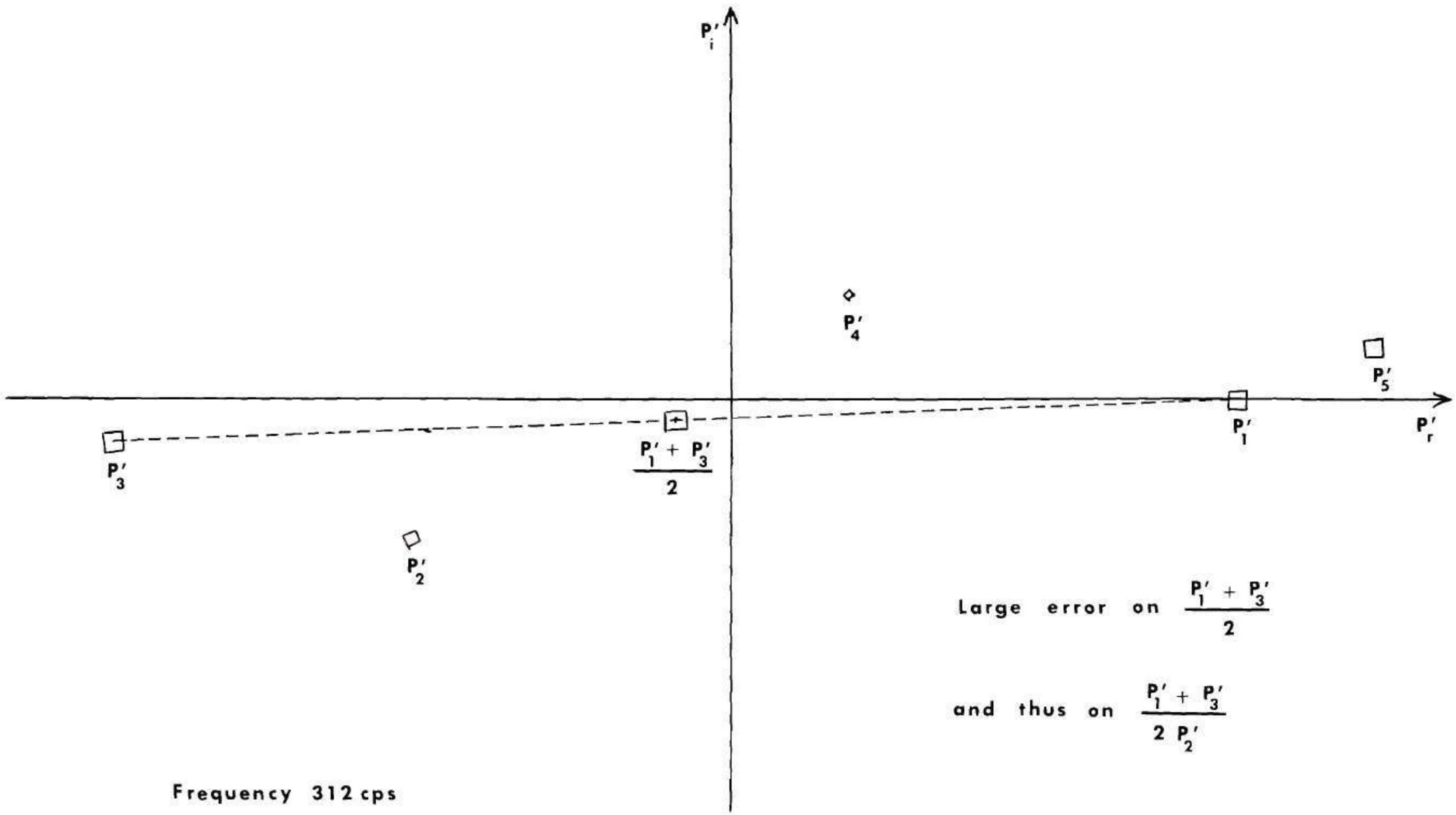

Figure 8. Pressure on Complex Plane

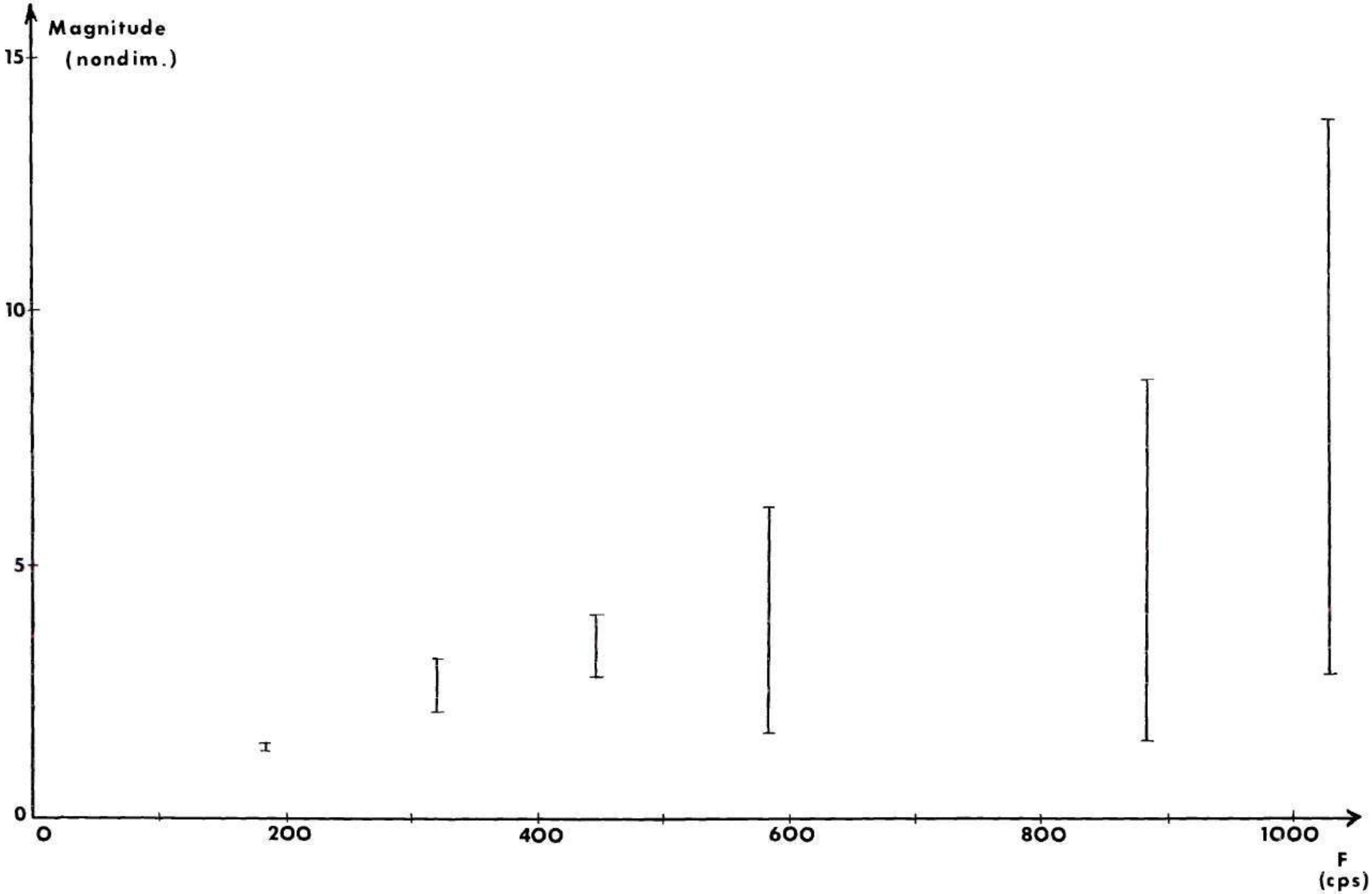

Figure 9. Magnitude of the Impedance vs. the Frequency

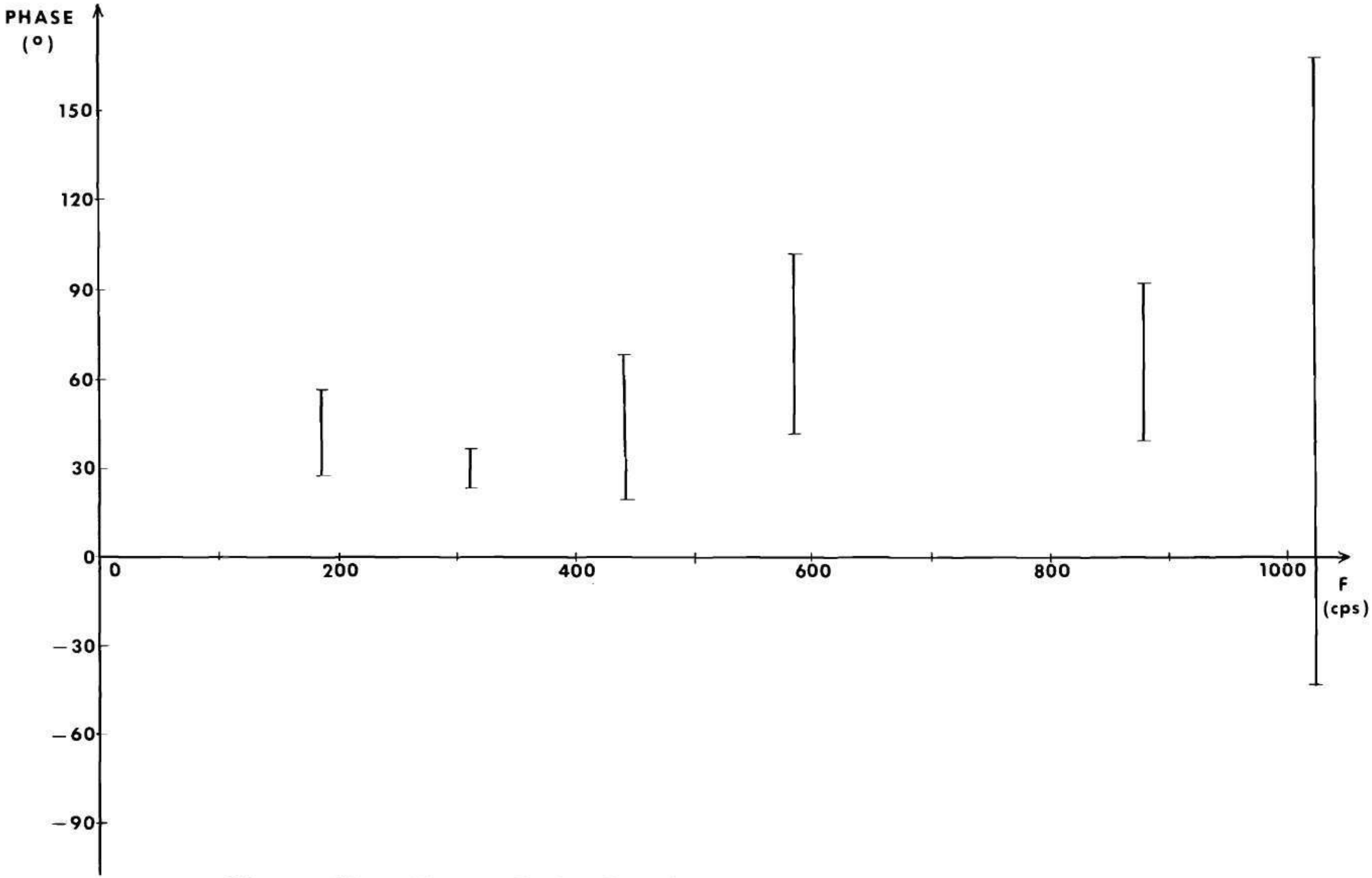

Figure 10. Phase of the Impedance vs. the Frequency

because rejection of data from these two transducers yield only one measurement with the middle three transducers.

It should finally be noted that the precision obtained in the impedance measurement is insufficient to extract the vaporization response with sufficient accuracy. Therefore, changes in the experiment should be made to improve the accuracy.

CHAPTER VI

SUMMARY AND CONCLUSIONS

1. Measurement of the impedance of a vaporizing spray was shown to be not feasible due to an insufficient feedback to the acoustic wave in an impedance tube using a practical experimental setup for spray injection.
2. An experiment was designed to measure the impedance of a large vaporizing surface upon which a mean flow was impinging.
3. Measurement of the impedance of the vaporization surface assembly without air or liquid flow was accomplished using an impedance tube with fixed pressure transducers.
4. The precision of the impedance measurement was poor, especially at the higher frequencies. It is recommended that
   a) The transducers be moved closer together so that no more than one wavelength is under surveillance by the transducers.
   b) The two outside transducers be moved farther from the driver and endplate.
   c) The tape recordings be filtered to remove components of the wave at frequencies higher than the driving frequency.

These changes should improve the precision of the measurements.

APPENDIX I

CALIBRATION OF THE MICROPHONES

	1	2	3	4	5
Serial No.	176067	176152	176154	176330	276991
Given correction factor K	33.0 db	37.7 db	37.5 db	33.7 db	36.5 db
120 db (calibrator)	121.6 db	124.6 db	121.7 db	122.4 db	121.0 db
130 db (calibrator)	131.6	134.5	131.6	132.5	130.8
140 db (calibrator)	141.6	144.6	141.6	142.4	141.0
150 db (calibrator)	151.6	154.6	151.7	152.5	150.9
Thus, mean correction to add, k	-1.6	-4.6	-1.6	-2.5	-0.9

Correction factor to	31.4 db	33.1 db	35.9db	31.2 db	35.6 db

APPENDIX II

CALIBRATION TABLES OF THE FIVE CHANNELS

CALIBRATION OF CHANNEL No. 1

Voltage output (Volt) for a given pressure input (dB).

p	p	Attenuation							Frequency
(μbar)	(dB)	1	2	3	4	5	6	7	(cps)
16634.	158.4	1.024	3.07						
5260.	148.4	.321	.961	3.23					
1663.4	138.4	.104	.303	1.013	3.02				186
526.0	128.4	.034	.096	.317	.946	3.16			
166.34	118.4	.012	.031	.102	.301	1.006	3.01		
52.60	108.4	.005	.010	.032	.095	.317	.948	3.17	
16634.	158.4	1.003	3.01						
5260.	148.4	.321	.958	3.23					
1663.4	138.4	.102	.302	1.012	3.02				312
526.0	128.4	.035	.097	.321	.956	3.20			
166.34	188.4	.013	.031	.101	.302	1.010	3.02		
52.60	108.4	.004	.010	.032	.096	.320	.956	3.21	

CALIBRATION OF CHANNEL No. 1 (Continued)

p	p	Attenuation							Frequency
(μbar)	(dB)	1	2	3	4	5	6	7	(cps)
16634.	158.4	1.019	3.06						
5260.	148.4	.322	.962	3.24					
1663.4	138.4	.104	.305	1.019	3.04				443
526.0	128.4	.034	.097	.323	.959	3.22			
166.34	118.4	.012	.031	.102	.302	1.009	3.02		
52.60	108.4	.004	.010	.032	.096	.320	.956	3.20	
16634.	158.4	1.025	3.07						
5260.	148.4	.323	.965	3.25					
1663.4	138.4	.105	.306	1.024	3.05				585
526.0	128.4	.032	.096	.323	.960	3.23			
166.34	118.4	.012	.031	.102	.303	1.017	3.04		
52.60	108.4	.007	.010	.032	.096	.321	.960	3.21	

CALIBRATION OF CHANNEL No. 1 (Continued)

p	p	Attenuation							Frequency
(μbar)	(dB)	1	2	3	4	5	6	7	(cps)
16634.	158.4	1.017	3.06						
5260.	148.4	.324	.972	3.27					
1663.4	138.4	.103	.307	1.028	3.04				882
526.0	128.4	.033	.096	.322	.950	3.22			
166.34	118.4	.011	.030	.101	.297	1.005	3.01		
52.60	108.4	.003	.009	.031	.094	.318	.952	3.19	
16634.	158.4	1.014	3.05						
5260.	148.4	.323	.968	3.25					
1663.4	138.4	.102	305	1.025	3.02				1025
526.0	128.4	.034	.097	.325	.953	3.24			
166.34	118.4	.010	.030	.101	.299	1.015	3.04		
52.60	108.4	.005	.011	.032	.095	.322	.964	3.23	

CALIBRATION OF CHANNEL No. 2

Voltage output (Volt) for a given pressure input (dB).

p (μbar)	p (dB)	Attenuation 1	2	3	4	5	6	7	Frequency (cps)
11776.	155.4	.893	2.68						
3724.	145.4	.282	.846	2.83					
1177.6	135.4	.088	2.63	.881	2.63				186
372.4	125.4	.027	.082	.277	.830	2.77			
117.76	115.4	.008	.025	.087	.265	.885	2.65		
37.24	105.4	.003	.007	.027	.082	.280	.83	2.75	
11776.	155.4	.884	2.65						
3724.	145.4	.281	.842	2.82					
1177.6	135.4	.089	.265	.887	2.65				312
372.4	125.4	.029	.084	.280	.838	2.79			
117.76	115.4	.009	.026	.088	.265	.887	2.66		
37.24	105.4	.003	.008	.027	.083	.279	.84	2.78	

CALIBRATION OF CHANNEL No. 2 (Continued)

p	p	Attenuation							Frequency
(μbar)	(dB)	1	2	3	4	5	6	7	(cps)
11776.	155.4	.900	2.70						
3724.	145.4	.281	.842	2.82					
1177.6	135.4	.089	.267	.897	2.67				443
372.4	125.4	.028	.084	.281	.839	2.80			
117.76	115.4	.009	.026	.088	.263	.880	2.64		
37.24	105.4	.003	.009	.028	.083	.279	.838	2.78	
11776.	155.4	.894	2.68						
3724.	145.4	.284	.849	2.84					
1177.6	135.4	.090	.268	.896	2.66				585
372.4	125.4	.027	.084	.283	.839	2.81			
117.76	115.4	.010	.026	.089	.264	.880	2.64		
37.24	105.4	.006	.008	.027	.082	.276	.830	2.75	

CALIBRATION OF CHANNEL No. 2 (Continued)

p	p	Attenuation							Frequency
(μbar)	(dB)	1	2	3	4	5	6	7	(cps)
11776.	155.4	.893	2.68						
3724.	145.4	.282	.845	2.83					
1177.6	135.4	.089	.267	.894	2.64				882
372.4	125.4	.028	.084	.282	.834	2.81			
117.76	115.4	.011	.027	.090	.264	.894	2.68		
37.24	105.4	.007	.010	.028	.082	.278	.833	2.76	
11776.	155.4	.900	2.70						
3724.	145.4	.283	.851	2.85					
1177.6	135.4	.088	.264	.884	2.60				1025
372.4	125.4	.029	.085	.283	.834	2.82			
117.76	115.4	.011	.027	.090	.264	.894	2.68		
37.24	105.4	.006	.010	.028	.082	.277	.832	2.76	

CALIBRATION OF CHANNEL No. 3

Voltage output (Volt) for a given Pressure input (dB).

p	p	Attenuation							Frequency
(μbar)	(dB)	1	2	3	4	5	6	7	(cps)
16634.	158.4	.902	2.71						
5260.	148.4	.283	.852	2.84					
1663.4	138.4	.088	.266	.890	2.67				186
526.0	128.4	.030	.084	.279	.835	2.80			
166.34	118.4	.010	.027	.089	.267	.890	2.66		
52.60	108.4	.003	.008	.027	.084	.277	.840	2.83	
16634.	158.4	.888	2.68						
5260.	148.4	.283	.849	2.84					
1663.4	138.4	.091	.268	.893	2.68				312
526.0	128.4	.028	.084	.281	.841	2.82			
166.34	118.4	.009	.026	.089	.265	.890	2.67		
52.60	108.4	.002	.008	.027	.083	.280	.846	2.81	

CALIBRATION OF CHANNEL No. 3 (Continued)

p (μbar)	p (dB)	Attenuation 1	2	3	4	5	6	7	Frequency (cps)
16634.	158.4	.904	2.72						
5260.	148.4	.286	.855	2.86					
1663.4	138.4	.089	.267	.894	2.68				
526.0	128.4	.030	.085	.283	.845	2.84			443
166.34	118.4	.009	.026	.088	.265	.885	2.65		
52.60	108.4	.002	.008	.027	.083	.280	.846	2.82	
16634.	158.4	.899	2.72						
5260.	148.4	.284	.851	2.86					
1663.4	138.4	.091	.270	.899	2.70				
526.0	128.4	.030	.086	.283	.844	2.84			585
166.34	118.4	.009	.026	.089	.265	.893	2.67		
52.60	108.4	.005	.009	.028	.083	.283	.845	2.83	

CALIBRATION OF CHANNEL No. 3 (Continued)

p (μbar)	p (dB)	Attenuation 1	2	3	4	5	6	7	Frequency (cps)
16634.	158.4	.892	2.68						
5260.	148.4	.285	.857	2.86					
1663.4	138.4	.089	.269	.899	2.66				
526.0	128.4	.027	.084	.282	.836	2.83			882
166.34	118.4	.008	.026	.089	.264	.896	2.68		
52.60	108.4	.003	.008	.027	.082	.280	.843	2.83	
16634.	158.4	.900	2.72						
5260.	148.4	.288	.863	2.88					
1663.4	138.4	.091	.269	.897	2.65				
526.0	128.4	.030	.086	.283	.835	2.85			1025
166.34	118.4	.011	.027	.090	.264	.897	2.69		
52.60	108.4	.004	.009	.028	.083	.280	.845	2.82	

CALIBRATION OF CHANNEL No. 4

Voltage output (Volt) for a given pressure input (dB).

p	p	Attenuation							Frequency
(μbar)	(dB)	1	2	3	4	5	6	7	(cps)
14998.	157.5	.903	2.70						
4743.	147.5	.284	.852	2.87					
1499.8	137.5	.088	.265	.894	2.68				
474.3	127.5	.027	.082	.279	.840	2.81			186
149.98	117.5	.008	.026	.088	.267	.890	2.67		
47.43	107.5	.005	.009	.028	.084	.280	.841	2.81	
14998.	157.5	.893	2.68						
4743.	147.5	.280	.847	2.86					
1499.8	137.5	.091	.268	.901	2.70				
474.3	127.5	.029	.085	.284	.851	2.85			312
149.98	117.5	.011	.027	.090	.268	.899	2.69		
47.43	107.5	.002	.008	.028	.085	.286	.848	2.84	

CALIBRATION OF CHANNEL No. 4 (Continued)

p (μbar)	p (dB)	Attenuation 1	2	3	4	5	6	7	Frequency (cps)
14998.	157.5	.913	2.74						
4743.	147.5	.284	.850	2.86					
1499.8	137.5	.089	.267	.904	2.71				443
474.3	127.5	.028	.084	.284	.850	2.84			
149.98	117.5	.009	.026	.089	.267	.900	2.69		
47.43	107.5	.002	.008	.028	.084	.287	.853	2.86	
14998.	157.5	.895	2.69						
4743.	147.5	.286	.855	2.90					
1499.8	137.5	.089	.269	.905	2.71				585
474.3	127.5	.028	.084	.286	.853	2.87			
149.98	117.5	.008	.026	.089	.267	.900	2.69		
47.43	107.5	.005	.009	.028	.084	.283	.849	2.84	

CALIBRATION OF CHANNEL No. 4 (Continued)

p	p	Attenuation							Frequency
(μbar)	(dB)	1	2	3	4	5	6	7	(cps)
14998.	157.5	.899	2.70						
4743.	147.5	.284	.850	2.86					
1499.8	137.5	.089	.269	.904	2.71				882
474.3	127.5	.028	.084	.285	.844	2.86			
149.98	117.5	.011	.027	.090	.265	.900	2.69		
47.43	107.5	.002	.008	.028	.083	.283	.848	2.83	
14998.	157.5	.900	2.70						
4743.	147.5	.286	.858	2.88					
1499.8	137.5	.091	.270	.904	2.67				1025
474.3	127.5	.028	.084	.285	.840	2.85			
149.98	117.5	.010	.027	.090	.264	.899	2.69		
47.43	107.5	.003	.008	.028	.083	.284	.849	2.83	

CALIBRATION OF CHANNEL No. 5

Voltage output (Volt) for a given pressure input (dB).

p	p	Attenuation							Frequency
(µbar)	(dB)	1	2	3	4	5	6	7	(cps)
18032.	159.1	.918	2.75						
5702.	149.1	.289	.865	2.92					
1803.2	139.1	.089	.269	.910	2.72				186
570.2	129.1	.030	.085	.286	.856	2.85			
180.32	119.1	.008	.026	.090	.272	.907	2.72		
57.02	109.1	.003	.008	.028	.085	.283	.850	2.84	
18032.	159.1	.899	2.70						
5702.	149.1	.290	.866	2.92					
1803.2	139.1	.092	.272	.915	2.73				312
570.2	129.1	.030	.086	.288	.860	2.86			
180.32	119.1	.011	.028	.092	.274	.913	2.73		
57.02	109.1	.005	.009	.028	.085	.285	.855	2.85	

CALIBRATION OF CHANNEL No. 5 (Continued)

p	p	Attenuation							Frequency
(μbar)	(dB)	1	2	3	4	5	6	7	(cps)
18032.	159.1	.924	2.77						
5702.	149.1	.289	.868	2.92					
1803.2	139.1	.091	.273	.919	2.74				
570.2	129.1	.028	.086	.291	.868	2.90			443
180.32	119.1	.011	.028	.091	.271	.905	2.71		
57.02	109.1	.005	.009	.029	.086	.286	.860	2.86	
18032.	159.1	.916	2.75						
5702.	149.1	.289	.870	2.92					
1803.2	139.1	.091	.274	.925	2.75				
570.2	129.1	.028	.086	.291	.868	2.90			585
180.32	119.1	.011	.027	.091	.271	.908	2.71		
57.02	109.1	.005	.009	.028	.085	.285	.855	2.85	

CALIBRATION OF CHANNEL No. 5 (Continued)

p	p	Attenuation							Frequency
(μbar)	(dB)	1	2	3	4	5	6	7	(cps)
18032.	159.1	.903	2.72						
5702.	149.1	.291	.875	2.95					
1803.2	139.1	.090	.273	.920	2.71				
570.2	129.1	.029	.087	.294	.867	2.92			882
180.32	119.1	.009	.027	.093	.277	.931	2.79		
57.02	109.1	.002	.008	.028	.085	.290	.870	2.90	
18032.	159.1	.900	2.70						
5702.	149.1	.288	.864	2.91					
1803.2	139.1	.090	.272	.915	2.69				
570.2	129.1	.028	.084	.285	.839	2.83			1025
180.32	119.1	.009	.027	.092	.271	.922	2.75		
57.02	109.1	.002	.008	.028	.085	.291	.872	2.90	

APPENDIX III

LISTING AND USE OF THE COMPUTER PROGRAM

The data required consist of 2+2n cards where n is the number of frequencies for which experiments were run.

Card #1	F15.8	Length of the impedance tube (ft)
	I2	Number of frequencies, n
	F10.7	Dimensionless value of X at which the impedance measurement is computed (usually X = 1.0)
Card #2	5(F10.7)	Dimensionless X locations of the five transducers
Card #3	F10.3	Frequency (cps)
Card #4	5(F10.3,F6.1)	Five pairs of values of the pressure amplitude (any units) followed by the phase (degrees) for the five transducers

Repeat cards #3 and #4 n times

The program gives four results for each frequency. Test #1 is the result for channels 1, 2 and 3, Test #2 is for 2, 3 and 4, Test #3 is for 3, 4 and 5, and Test #4 is for channels 1, 3 and 5. The value KR THEOR = $\frac{\omega L}{c}$ where c = 1130 ft/sec. KR EXPER and KI EXPER are the experimentally determined values of K. The first three pairs of values for A and B are determined from p_j measurements with $j = 2,3$, $j = 3,1$, and $j = 1,2$, respectively. The fourth pair is the average which is used in these impedance calculation.

The following is a program listing:

```
      COMPLEX P(3)
      DIMENSION X(5),AP(2,3),AX(5),AAP(2,5)
      COMPLEX CI,A,B,K,Y,DEN,Z,A1,A2,A3,B1,B2,B3
      REAL KR,KI
      DOUBLE PRECISION AA,UU,ZZ
      PI=3.141592
      CI=CMPLX(0,1)
      READ(5,100)RL,N,XX
  100 FORMAT(F15.8,I2,F10.7)
      READ(5,110)(AX(I),I=1,5)
  110 FORMAT(5(F10.7))
      C=1130.
      GAM=1.405
      DO 10 II=1,N
      READ(5,101)F
  101 FORMAT(F10.3)
      WRITE(6,1110)F
 1110 FORMAT(1H1,11HFREQUENCY =,F10.3/////////)
      READ(5,200)((AAP(I,J),I=1,2),J=1,5)
  200 FORMAT(5(F10.3,F6.1))
      RKTH=2.*PI*F*RL/C
      DO 9 JJ=1,4
      DO 1 J=1,3
      DO 16 I=1,2
      IF(JJ-2)13,14,15
   13 AP(I,J)=AAP(I,J)
      X(J)=AX(J)
      GO TO 16
   14 AP(I,J)=AAP(I,J+1)
      X(J)=AX(J+1)
      GO TO 16
   15 IF(JJ-4)17,18,18
   17 AP(I,J)=AAP(I,J+2)
      X(J)=AX(J+2)
      GO TO 16
   18 AP(I,J)=AAP(I,2*J-1)
      X(J)=AX(2*J-1)
   16 CONTINUE
    1 P(J)=AP(1,J)*CEXP(CI*AP(2,J)*PI/180.)
      Y=(P(1)+P(3))/2./P(2)
      AA=1.+CABS(Y)*CABS(Y)
      YR=REAL(Y)
```

```
      YI=AIMAG(Y)
      UU=YI*YI*(.5+(1.+YR*YR+YI*YI*.5)/(1.-YR*YR+SQRT(AA*AA-4
                                    *YR*YR)))
      ZZ=SQRT(1.+UU)
      KR=ACOS(REAL(Y)/ZZ)/(X(2)-X(1))
      KI=ALOG(SQRT(1.+UU)+SQRT(UU))/(X(2)-X(1))
      RKT=RKTH*(X(2)-X(1))/PI
      DO 2 NK=1,100
      RNK1=RKT-2.*FLOAT(NK)+1.
      RNK2=RKT-2.*FLOAT(NK)
      IF(RNK1)3,3,4
    3 KR-KR+2.*FLOAT(NK-1)*PI/(X(2)-X(1))
      GO TO 6
    4 IF(RNK2)5,5,2
    5 KR=-KR+2.*FLOAT(NK)*PI/(X(2)-X(1))
      GO TO 6
    2 CONTINUE
    6 CONTINUE
      WRITE(6,1000)JJ
 1000 FORMAT(1H ,7HTEST NO,I2/)
      WRITE(6,1500)(X(J),(AP(I,J),I=1,2),J=1,3)
 1500 FORMAT(1H ,10X,3HX =,F10.7,4X,8HMAG(P) =,F8.1,3X,10H
                                    PHASE(P) =,F8.11)
      TEST=ABS(X(1)+X(3)-2.*X(2))-.0001
      IF(TEST)11,12,12
   12 WRITE(6,1200)
 1200 FORMAT(1H ,37HX(3)-X(2) = X(2)-X(1) IS NOT VERIFIED)
   11 CONTINUE
      WRITE(6,1111)Y,AA,UU,ZZ
 1111 FORMAT(1H ,10X,3HY =,2(F15.8)/1H ,10X,3HAA=,D25.18/1H
                                    ,10X,3HUU=,D125.18/1H ,10X,3HZZ
                                    =,D25.18)
      WRITE(6,2000)RKTH,KR
 2000 FORMAT(1H ,10X,10HKR THEOR.=,F15.8,10X,10HKR EXPER.=
                                    ,F15.8)
      WRITE(6,3000)KI
 3000 FORMAT(1H ,45X,10HKI EXPER.=,F15.8)
      K=CMPLX(KR,KI)
      DEN=2.*CI*CSIN(K*(X(2)-X(3)))
      A1-(P(2)*CEXP(-CI*K*X(3))-P(3)*CEXP(-CI*K*X(2)))/DEN
      B1=(P(3)*CEXP(CI*K*X(2))-P(2)*CEXP(CI*K*X(3)))/DEN
      DEN=2.*CI*CSIN(K*(X(3)-X(1)))
      A2=(P(3)*CEXP(-CI*K*X(1))-P(1)*CEXP(-CI*K*X(3)))/DEN
      B2=(P(1)*CEXP(CI*K*X(3))-P(3)*CEXP(CI*K*X(1)))/DEN
      DEN=2.*CI*CSIN(K*(X(1)=X(2)))
      A3=(P(1)*CEXP(-CI*K*X(2))-P(2)*CEXP(-CI*K*X(1)))/DEN
      B3=(P(2)*CEXP(CI*K*X(1))-P(1)*CEXP(CI*K*X(2)))/DEN
      A=(A1+A2+A3)/3.
      B=(B1+B2+B3)/3.
      Z=-GAM*KR/K*(A*CEXP(2.*CI*K*XX)+B)/A*CEXP(2.*CI*K*XX)-B)
      ARGZ=CABS(Z)
      ZR=REAL(Z)
```

```
      ZI=AIMAG(Z)
      PHZ=ATAN 2(ZR,ZI)*180./PI
      WRITE(6,3500)A1,B1,A2,B2,A3,B3
 3500 FORMAT(3(1H ,10X,3HA =,2(F13.4),6X,3HB =,2(F13.4)/))
      WRITE(6,3600)A,B
 3600 FORMAT(1H ,10X,3HA =,2(F13.4),6X,3HB =,2(F13.4))
      WRITE(6,4000)ARGZ,PHZ
 4000 FORMAT(1H ,13X,8HMAG(Z) =,F15.4,10X,10HPHASE(Z) =
                                ,F15.4///////)
    9 CONTINUE
   10 CONTINUE
      STOP
      END

END OF COMPILATION:          NO DIAGNOSTICS.
```

APPENDIX IV

ADDITIONAL FIGURES

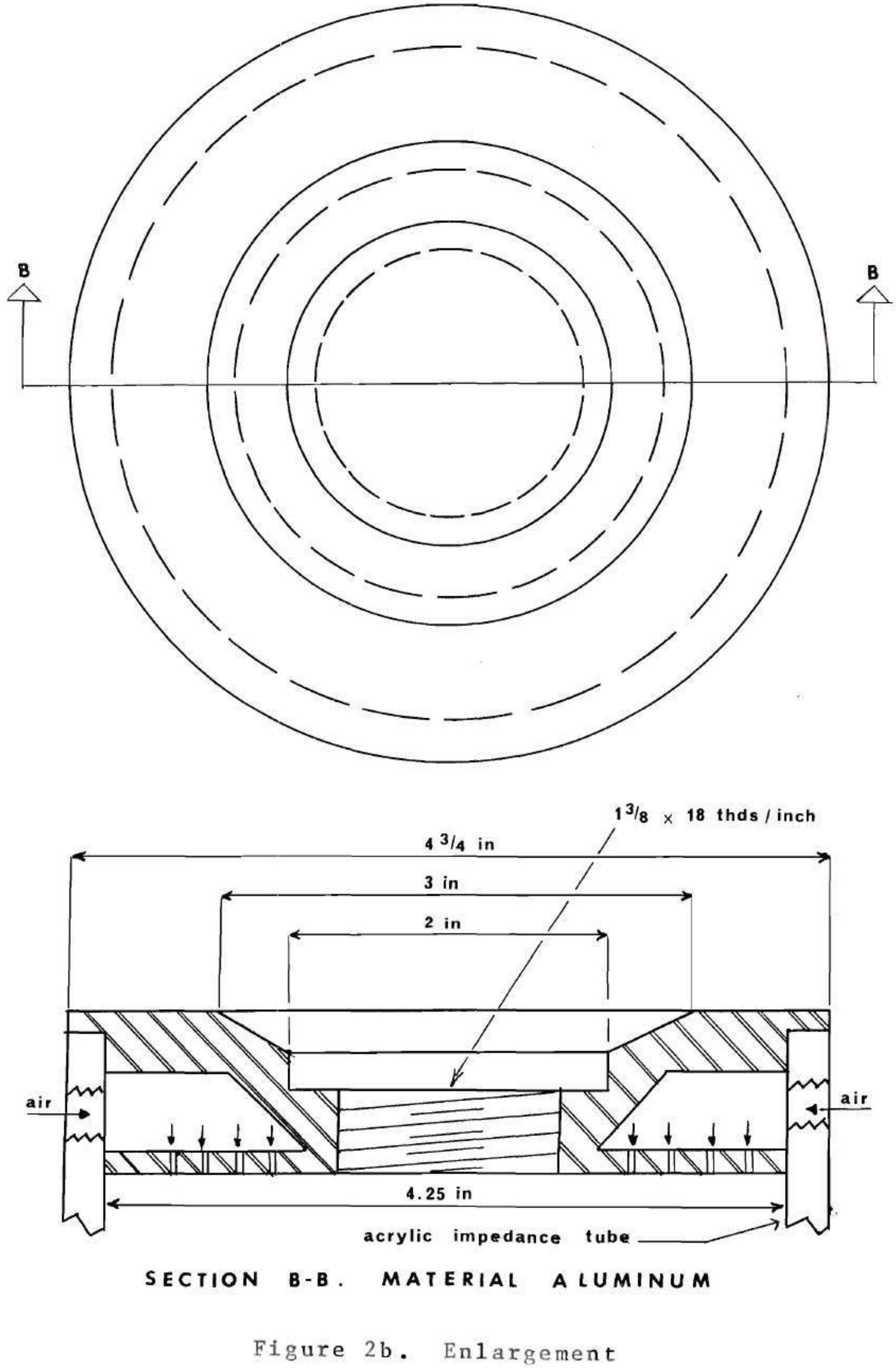

Figure 2b. Enlargement

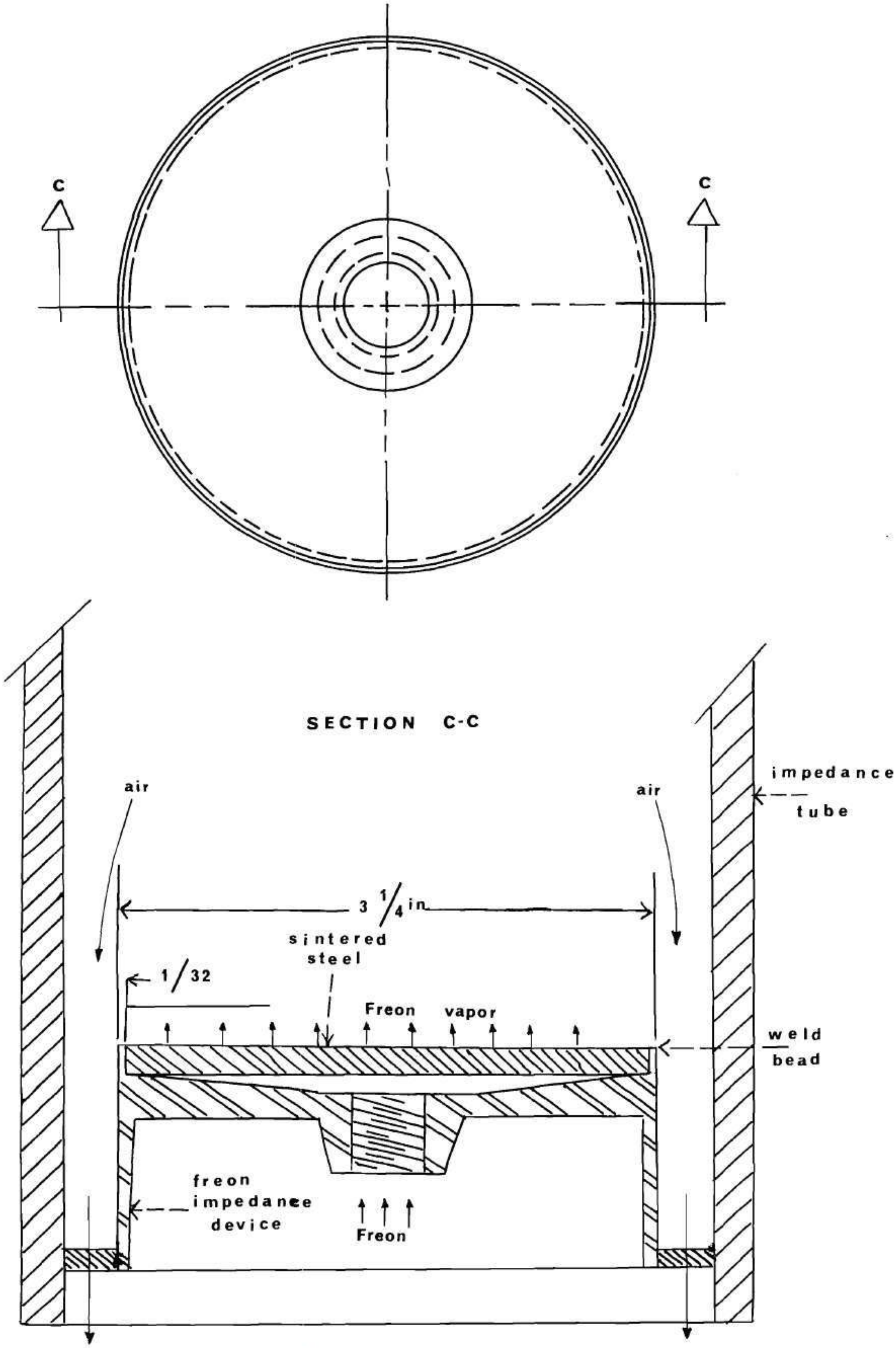

Figure 2c. Enlargement

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