The Excitement of Scattering Amplitudes

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Physics Colloquium
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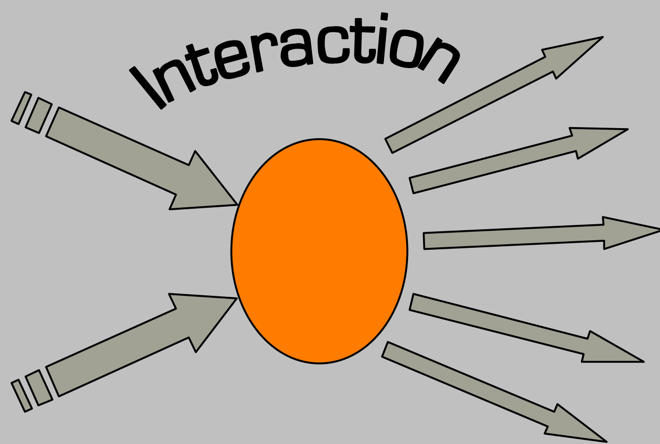
Scattering Amplitudes

Scattering amplitudes
encode the processes of
elementary particle interactions

for example
electrons, photons, gluons,...



$$e^- + \gamma \;\rightarrow\; e^- + \gamma$$

gluon + gluon $\rightarrow$ quark + anti-quark

Experimentally...

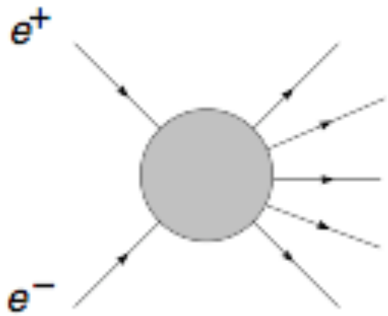
Accelerate particles to high energies...

collide them....

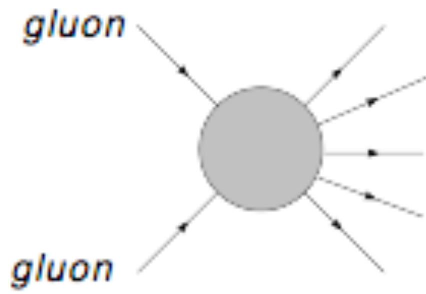
see what comes out!

smack!!

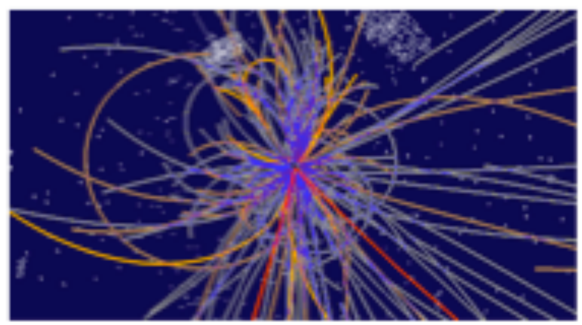
Particle Physics Experiments

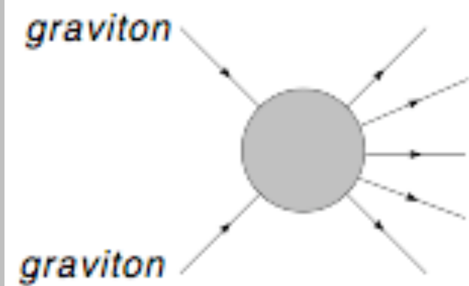


LEP at CERN



Fermilab, LHC



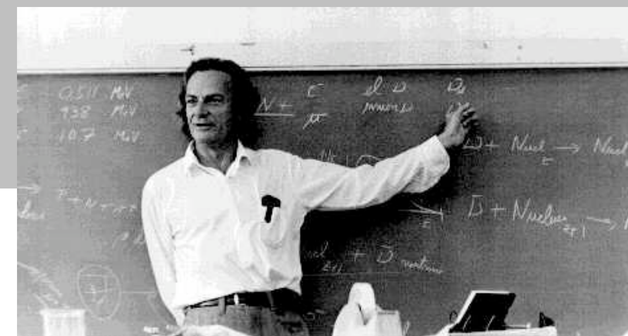


(gedanken)

How to calculate scattering amplitudes?

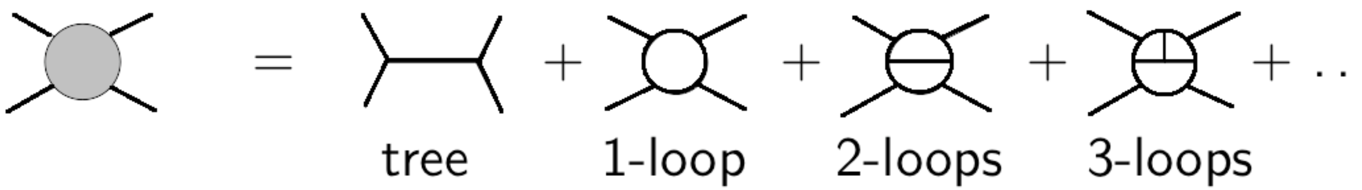
Interaction is perturbation on "free theory"

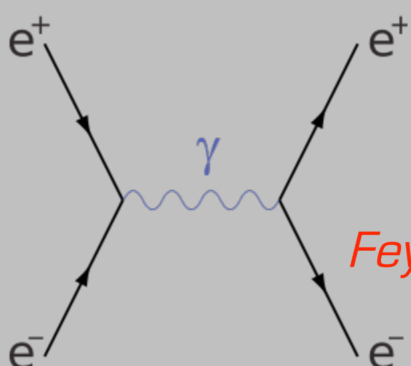
Governed by expansion in small coupling



Perturbation expansion in quantum field theory:

Amplitude = sum of Feynman diagrams





Feynman rules

mathematical expression

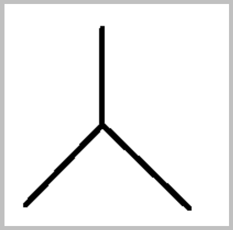
Calculate Feynman diagrams !

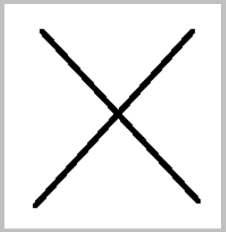
Example

gluon amplitudes at leading order (tree level).

The Gluon Feynman Rules

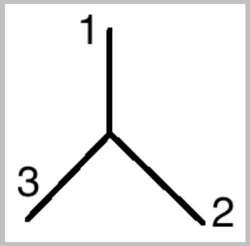
interaction vertices
=
building blocks



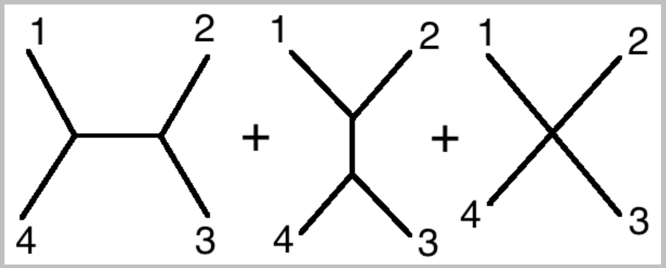


Add all tree diagrams with n external lines
Without crossing any lines (color-ordered ampl)

$n=3$



$n=4$



How many diagrams for given n ???

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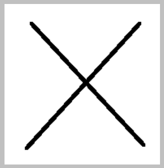
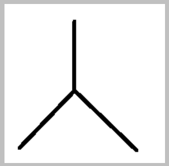
Lower bound: let's ignore  and just use 

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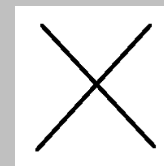
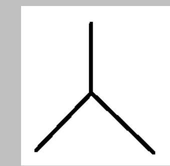
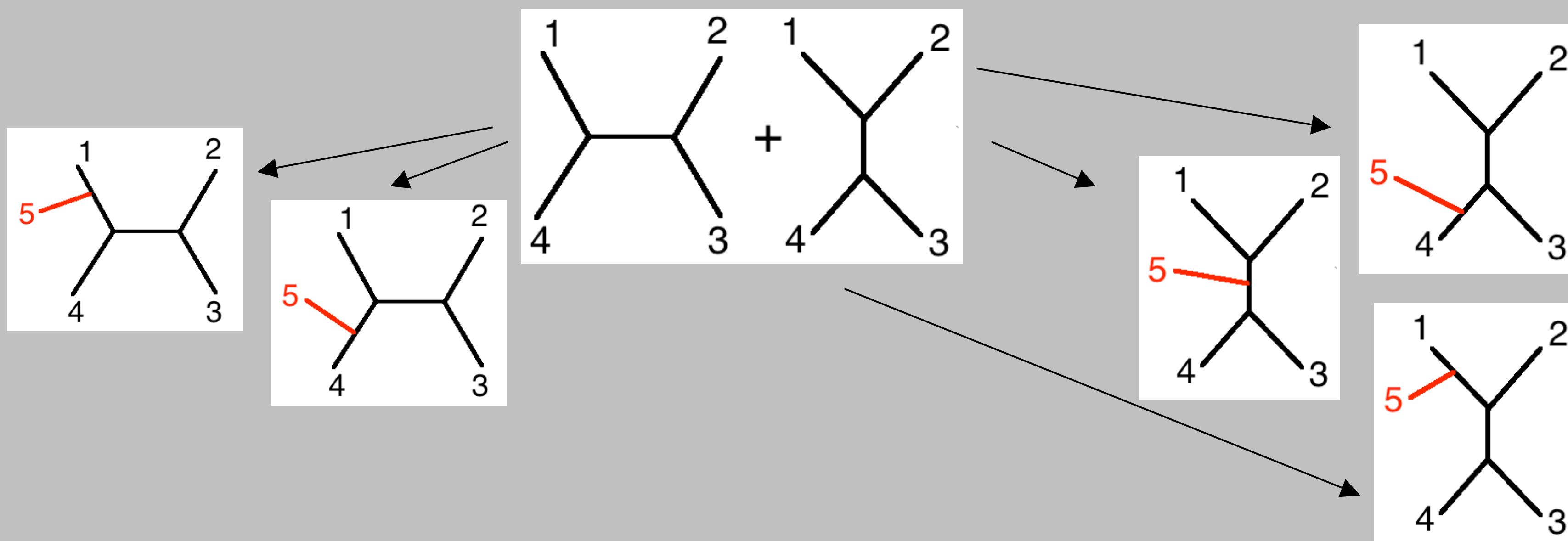
Lower bound: let's ignore  and just use 

From $n=4$ to $\mathbf{n=5}$



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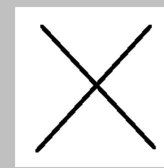
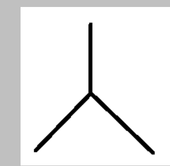
Lower bound: let's ignore  and just use 

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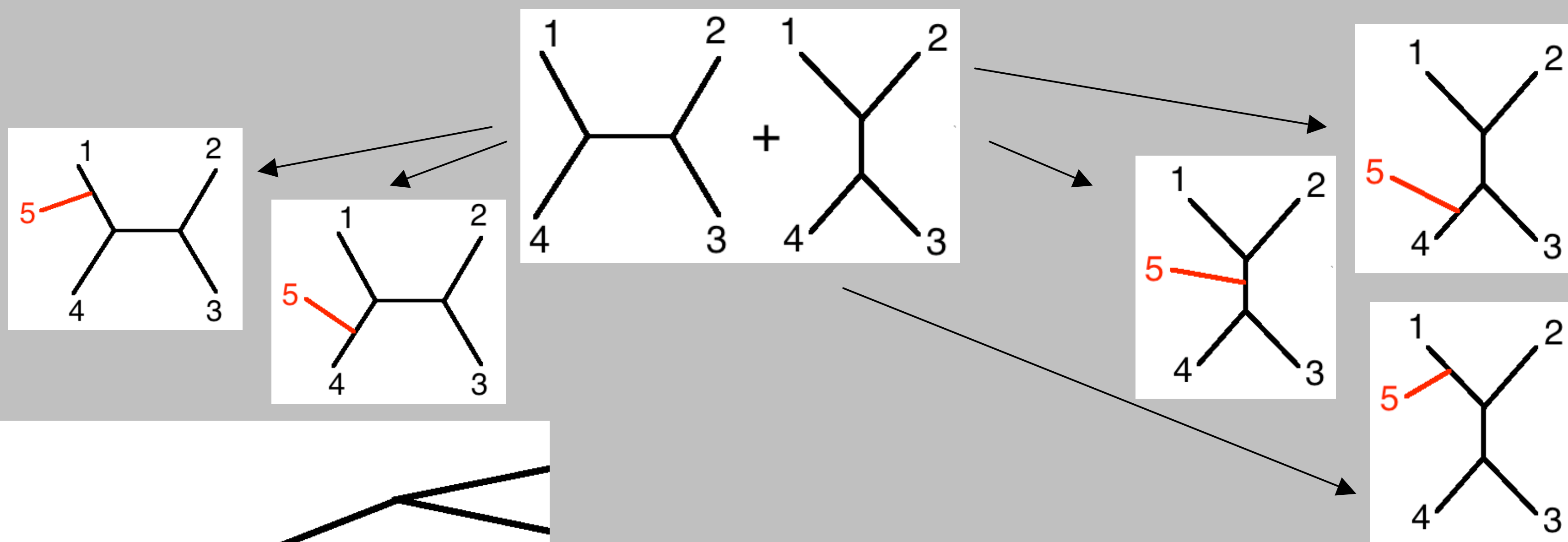
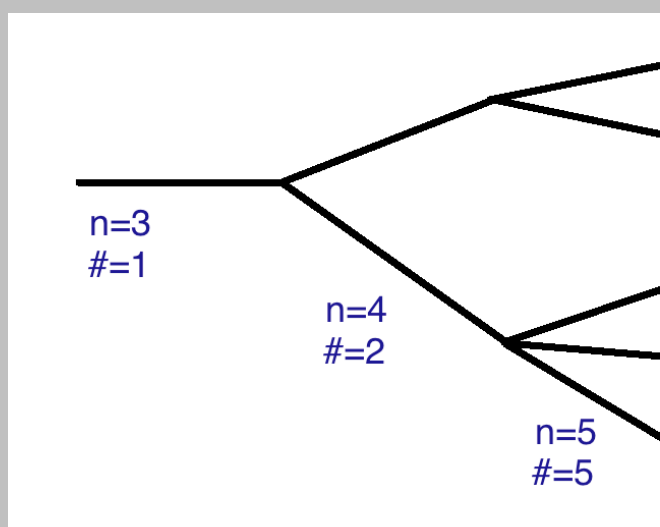
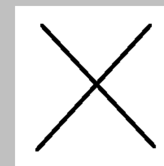
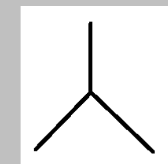




1, 2, 5, …?

HOW MANY DIAGRAMS FOR GIVEN n ???

LOWER BOUND: LET'S IGNORE  AND JUST USE 

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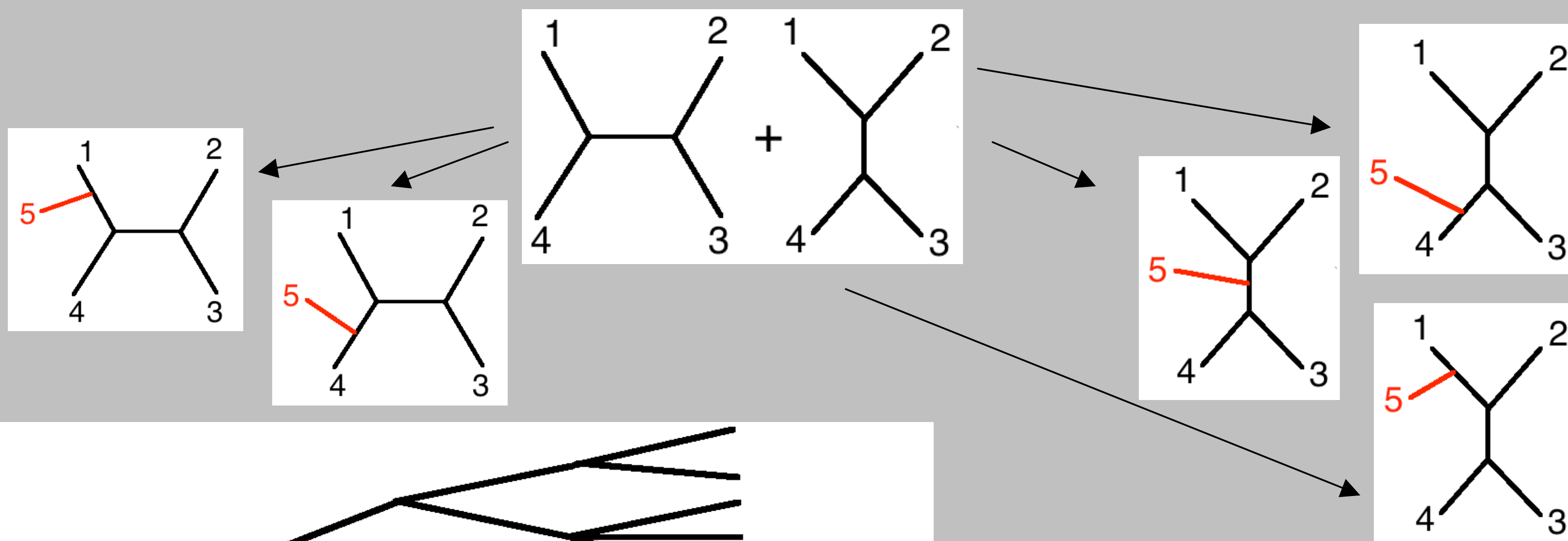
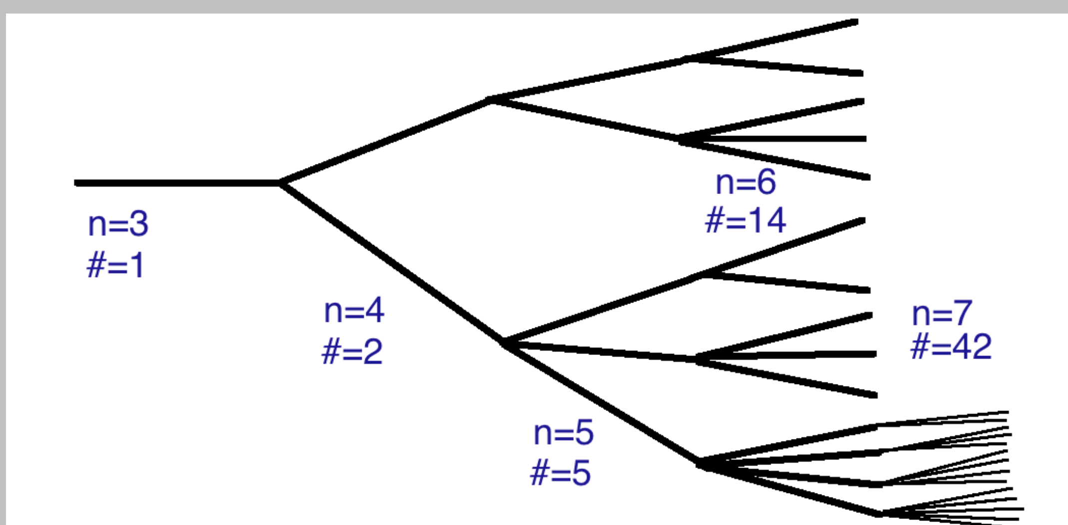
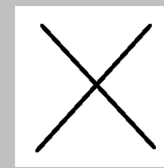
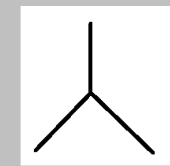




1, 2, 5, 14, 42,132,…

How many diagrams for given n ???

Lower bound: let's ignore  and just use 

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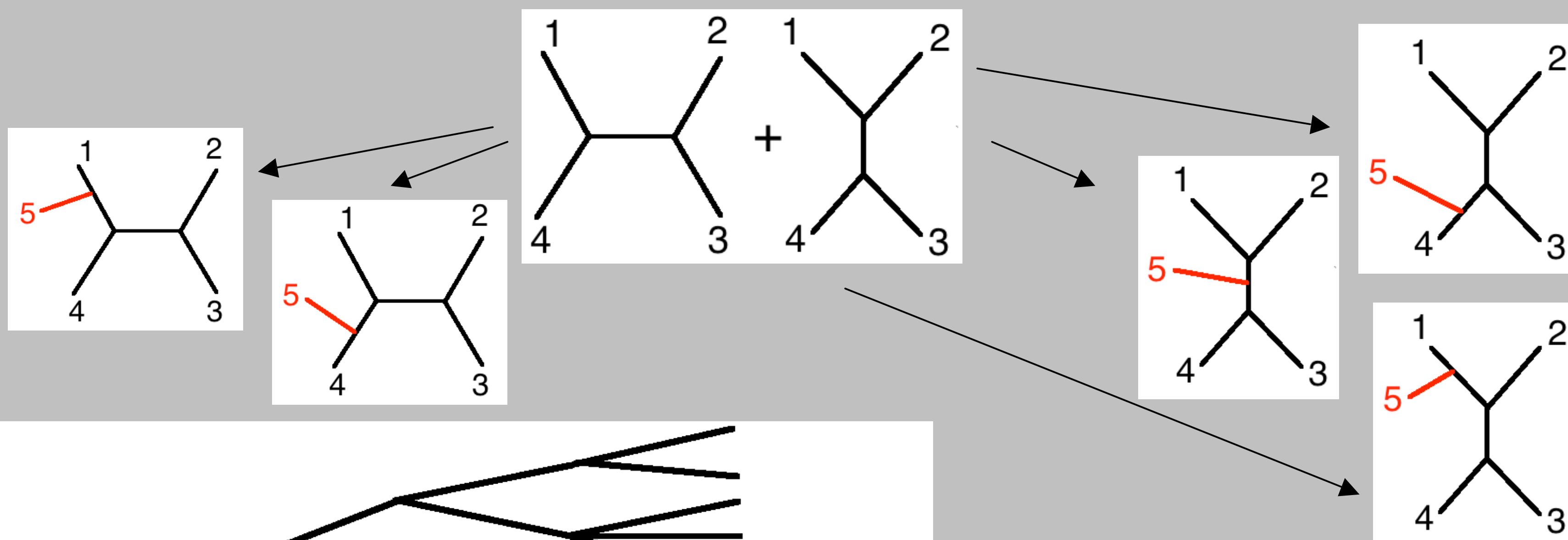

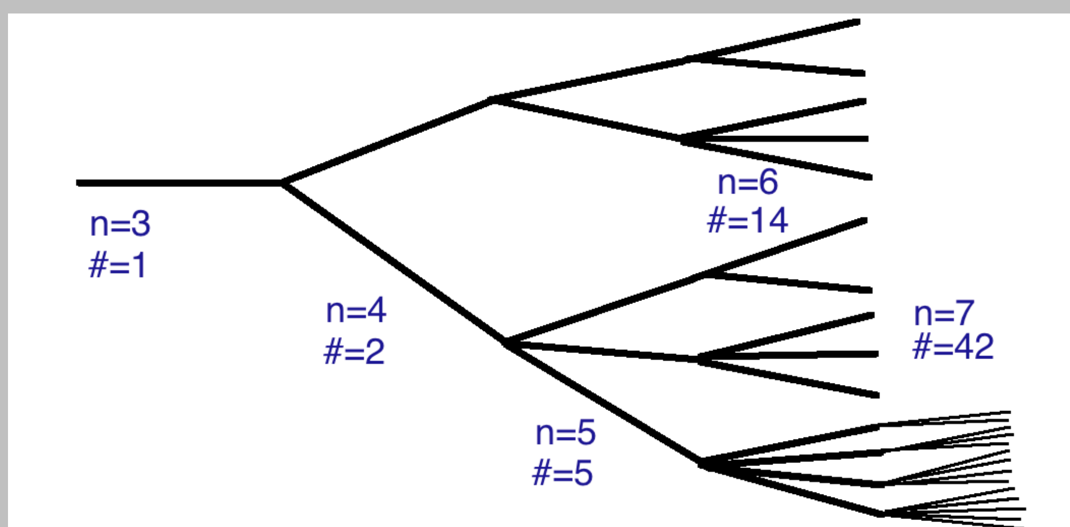

1, 2, 5, 14, 42,132,…

Catalan numbers!

How many diagrams for given n ???

Catalan numbers

(1,) 1, 2, 5, 14, 42, 132,....

Lower bound on # diagrams

$$C_n = \frac{1}{n+1}\binom{2n}{n} \sim \frac{4^n}{n^{3/2}\,\pi^{1/2}}$$

Counting problem solved by two Univ of Michigan undergrads as part of their Winter'11 research project.

Nico Wagner & William Murdoch

As n grows

- number of diagrams grow rapidly
- each diagram corresponds to an increasingly complicated expression

Gluon amplitudes

Scattering of gluon+gluon $\longrightarrow$ k gluons

k	=	1	2	3	4	5	6	7
# trivalent	=	1	2	5	14	42	132	429
# diagrams	=	1	3	10	38	149	...	

at tree level (leading order in perturbation theory)

Now write down mathematical expression for each diagram and simplify the sum....

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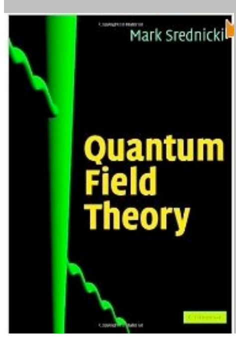
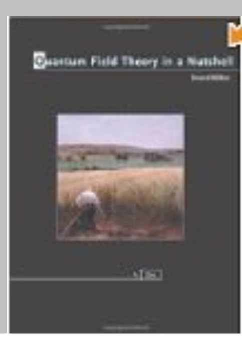
Result: $A_N(p_1^-, p_2^-, p_3^+,\ldots p_N^+) = \dfrac{\langle 12\rangle^4}{\langle 12\rangle \langle 23\rangle \ldots \langle N1\rangle}$ one single simple term!!!

$N=k+2$





$\langle 12\rangle \sim (2p_1.p_2)^{1/2}$

complex version

Why so simple?

Better way to calculate?

The surprising simplicity and enticing mathematical structure – and of course the practical relevance for particle physics – is what motivates current studies of scattering amplitudes.

Three Main Research Branches

1) Try to push as far as possible in a very controlled simple theory: "(planar) N=4 Super Yang Mills Theory" (SYM) (gluons, gluinos, scalars - all massless)

Goals: `Solve' theory at all loop order.
Compact expressions? Understand why!

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Goals: Point-particle quantum gravity perturbatively sensible?
Gravity as (gauge theory)^2
Relations to string theory?

New on-shell methods for calculating scattering amplitudes

Tree level amplitudes
satisfy on-shell *recursion relations*

$$A_n = \sum_k A_{n-k} A_k$$

Has been an immensely powerful approach

- even revealed new symmetries of amplitudes

dual conformal symmetry in planar N=4 SYM

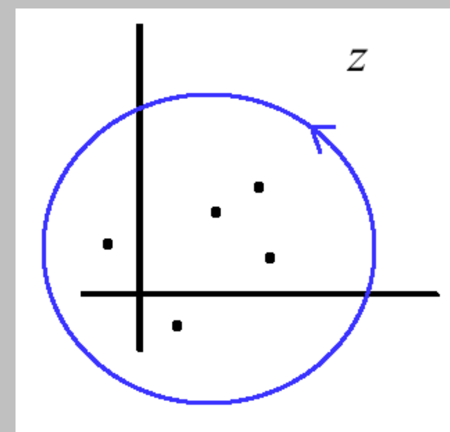
Outline of method

Think of amplitude $A_n(p_i)$ as function of momenta.

Shift (some of) the momenta: $p_i \longrightarrow \hat{p}_i = p_i + z\, q_i$

such that $\hat{p}_i^2 = 0$ and $\sum \hat{p}_i = 0$

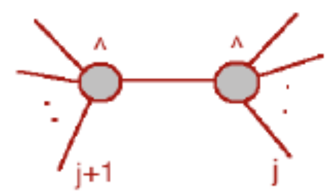
(on-shell) (momentum conservation)



Tree-level: $A_n(z)$ *only simple poles*

If $\hat{A}_n(z) \rightarrow 0$ as $z \rightarrow \infty$ then

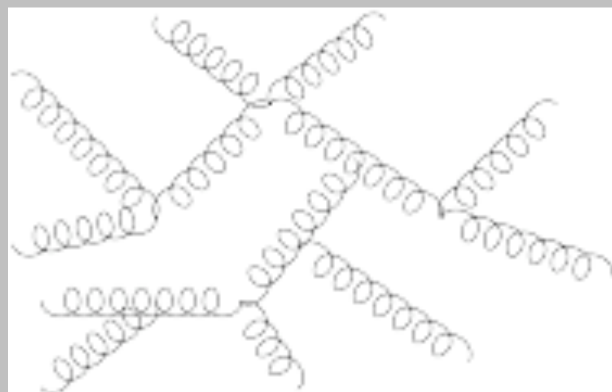
$$0 = \oint_C \frac{\hat{A}_n(z)}{z} \quad \Longrightarrow \quad A_n = \hat{A}_n(0) = \sum_I \hat{A}_L(z_I) \frac{1}{P_I^2} \hat{A}_R(z_I) = \sum_j$$



Britto, Cachazo, Feng, Witten (2004)

Example of recursion

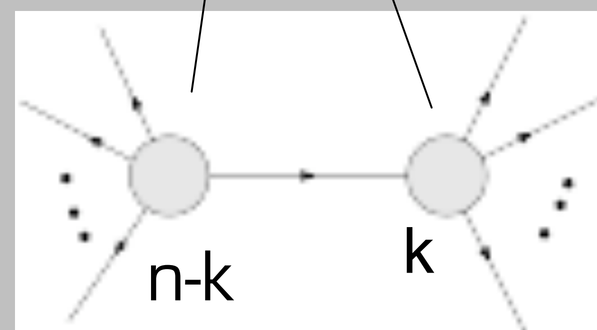
Replace sum of



$$A_n = \sum_k A_{n-k} A_k$$

On-shell amplitudes

with sum of



This method is called "BCFW recursion relation".

Get result

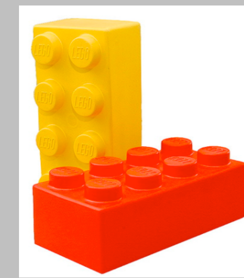
$$A_n(p_1^-, p_2^-, p_3^+, \ldots p_n^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle \langle 23\rangle \ldots \langle n1\rangle}$$

from **one** diagram!!!

Britto, Cachazo, Feng, Witten (2004)

ANOTHER EXAMPLE

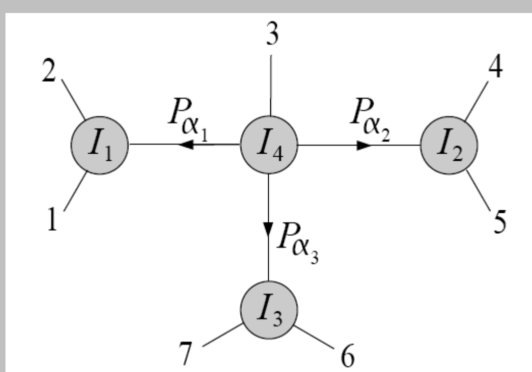
THE *LEGO* APPROACH

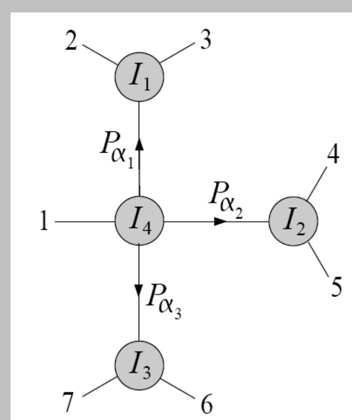


Built higher-pt amplitudes from the *simplest* lower-pt input => all tree **gluon** amplitudes can be expressed using building blocks

$$A_N(p_1^-, p_2^-, p_3^+, \ldots p_N^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle \langle 23\rangle \ldots \langle N1\rangle}$$

This method is called "CSW expansion". Cachazo, Svrcek, Witten (2004)



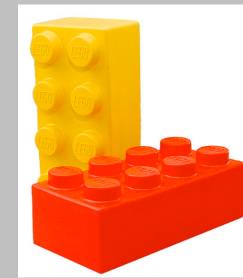


$A_N(p_1^-, p_2^-, \ldots, p_j^-, p_{j+1}^+, \ldots p_N^+)$

But **why** does it work?

And **when** (i.e. in which theories) does it work?

The *Lego* Approach



When does it work?

"on-shell constructibility"

- It works for **gluon** amplitudes. [CSW 2004, Risager 2005]

- It works in **N=4 SYM**. [HE, Freedman, Kiermaier 2008]

- Derived **very simple sufficient criterion** for validity in *general* 4d theories w/ or w/o SUSY, w/or w/o masses, + (non-)renormalizable couplings.

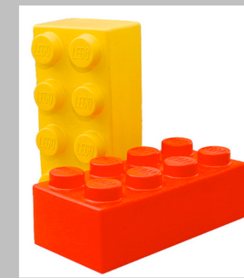
$$4 - n - c + \Sigma\, h_i < 0$$

[Cohen, HE, Kiermaier 2010]

h_i = helicity

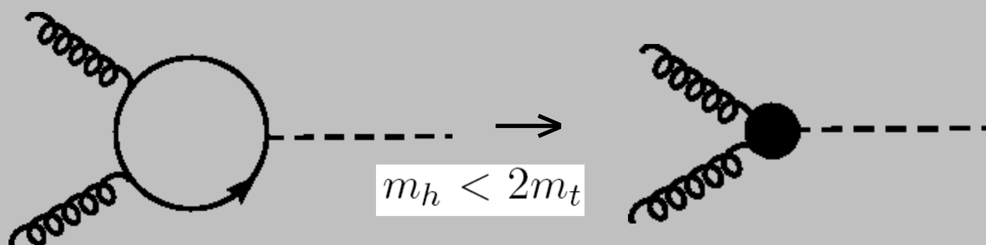
c = mass-dimension of product of couplings

Example



Gluon-Higgs fusion

Gluon-Higgs fusion: g g $\rightarrow$ higgs



$$h\,\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu}$$

dim-5 operator

CSW-expansion used in literature for Gluon-Higgs fusion.

[Dixon, Glover, Khoze'04; Berger, Del Duca, Dixon'06]

$A_N(p_1^-, p_2^-, p_3^+, \ldots p_N^+, h)$ = very simple building block

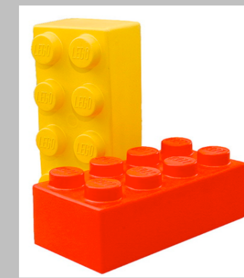
$A_N(p_1^-, p_2^-, p_3^-, p_4^+, \ldots p_N^+, h)$

Proof of validity by simple criterion with $n=N+1$, $c=-1$, $\Sigma\, h_i = N-6$

$$4-n-c+\Sigma\, h_i = 4-(N+1)-(-1)+N-6 = -2 < 0$$

[Cohen, HE, Kiermaier 2010]

Further Examples



- *Top quark amplitudes with anomalous magnetic moment*

[Larkoski & Peskin 2010]

BCFW w/ aux field *or* CSW-type expansion

- *Amplitudes on the Coulomb branch of N=4 SYM.*

[Craig, HE, Kiermaier, Slatyer 2011]

Massive external states, rich structure.

Can reconstruct massive amplitudes from massless ones!

[Kiermaier, 2011]

So far tree level

Indicated idea and applications of tree -level on-shell (recursive) methods.

tree level = leading order

Beyond leading order: Need Loop Amplitudes

also simplicity at loop-level?

Example of what QFT & Feynman loop diagrams are good for

Magnetic moment electron $\vec{\mu} = g_S \, \mu_B \, \vec{S}/\hbar$

where $g_S = 2$.

Quantum corrections:

$$g_S = 2\,(1 + \alpha/2\pi + \ldots),$$

where $\alpha = 1/137$ is the fine structure constant.

So

$$g_S - 2 = 0.00115965218 11(74) \quad \text{(exp)}$$

Can calculate g_S with Feynman diagrams.
Fantastic agreement with experiment!

- # Three-loop correction to electron g-2

Predrag Cvitanovic - acceptance speech, 1993 NKT Research Prize in Physics, Dansk Fysisk Selskab Årsmøde

“One day terror struck; I was invited to Caltech to give a talk. I could go to any other place and say that Kinoshita and I had calculated thousands of diagrams and that the answer was, well, the answer is:

$$+(0.92 \pm 0.02)\,(\alpha/\pi)^3$$

(Cvitanovic & Kinoshita '74)

But in front of Feynman? He is going to ask me why + and not - ? Why do 100 diagrams give a result of the order of unity, and not 10 or 100 or any other number? It might be the most precise agreement between fundamental theory and experiment in all of physics, but what does it mean ? ”

(Travaglini slides’2010)

Predrag Cvitanovic again:

"So in fear of God I went into deep trance and after a month I came up with this: if gauge invariance of QED guarantees that all UV and IR divergences cancel, why not also the finite parts?

And indeed; when the diagrams we had computed were grouped into gauge-invariant subsets, a rather surprising thing happens: while the finite part of each Feynman diagram is of order 10 to 100, every subset adds up to approximatively

$$\pm 1/2\,(\alpha/\pi)^n$$

...For me, the above is the most intriguing hint that something deeper than what we know underlies quantum field theory... "

(Travaglini slides'2010)

Trees → Loops

It can be very useful to build **gauge invariant** objects out of gauge invariant quantities!

Already saw this with recursive methods at tree level.

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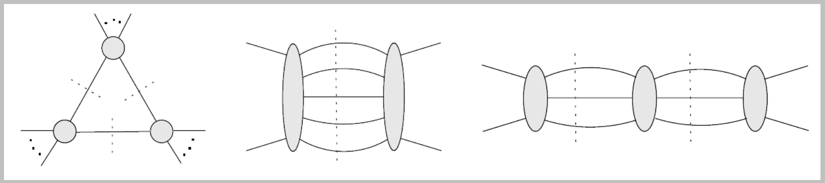
At loop level:



- Generalized unitarity methods Loops from trees
  [Bern, Dixon, Kosower, Dunbar,..]
- Recursion relations at loop level in planar N=4 SYM.
  [Arkani-Hamed, Bourjaily, Cachazo, Trnka 2010] [Boels 2010]
- MHV vertex method at loop-level
  [Bullimore 2010] [H.E., Freedman, Kiermaier, in progress]

Exciting and useful progress!

Recall

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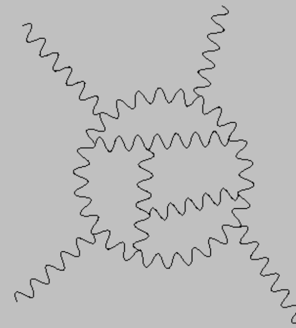
Gravity

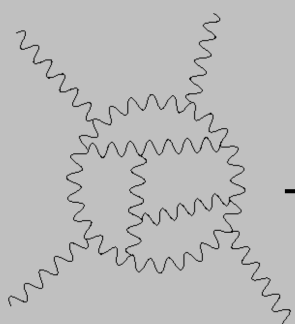


= ∞

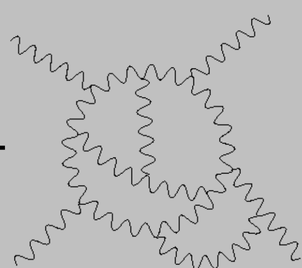
Perturbative gravity

Standard lore: graviton Feynman diagrams divergent.
Point-particle theory of gravity *not* OK as quantum theory
(a motivation for string theory)

What if all divergences cancel when all Feynman diagrams are summed – then amplitude finite...?!?



+



+ .... = finite ?????

Theory would be perturbatively well-defined

Gravity

Is maximally supersymmetric gravity

"***N=8 supergravity***"

perturbatively finite in 4d?

4-graviton scattering amplitudes in this theory known to be *finite* at loop orders L=1,2,3,4.
(pretty wild! - but why finite?)

[Bern, Carrasco, Dixon, Johansson, Roiban]

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What does the symmetries of the theory have to say about this?

Supergravity

Instead: **Study "candidate counterterms"**

L-loop divergence $\leftrightarrow$ counterterm of mass dimension $(2L+2)$

for example: R^4 at 3-loop order

R^4 = contraction of 4 Riemann tensors $R_{\mu\nu\rho\sigma}$

N=8 supergravity

Counterterm operators MESSY

but we can study the **amplitudes** they would produce.

(HE, Freedman, Kiermaier 2010)

Candidate counterterms are

- local operators
- $\mathcal{N} = 8$ SUSY
- $SU(8)_R$-invariant ← global symmetry
- $E_{7(7)}$-compatible ← global spontaneously broken symmetry

Exceptional group

These conditions become constraints on the matrix elements of the counterterms.

N=8 supergravity

SUSY + SU(8) eliminate several operators

Explicit 4-pt calc. shows finite

L	$n=4$	5	6			
3	R^4	None $\longrightarrow$				
4	$\cancel{D^2 R^4}$	$\cancel{R^5}$	None $\longrightarrow$			
5	$D^4 R^4$	$\cancel{D^2 R^5}$	$\cancel{R^6}$	None $\longrightarrow$		
6	$D^6 R^4$	$\cancel{D^4 R^5}$	$\cancel{D^2 R^6}$	$\cancel{R^7}$	None $\longrightarrow$	
7	$D^8 R^4$	$\cancel{D^6 R^5}$	$D^4 R^6$	$\cancel{D^2 R^7}$	R^8	
8	$D^{10} R^4$	$D^8 R^5$	$D^6 R^6$	$D^4 R^7$	$D^2 R^8$	R^9

[HE, Freedman, Kiermaier 2010]

[Howe, Heslop, Drummond 2010]

N=8 supergravity

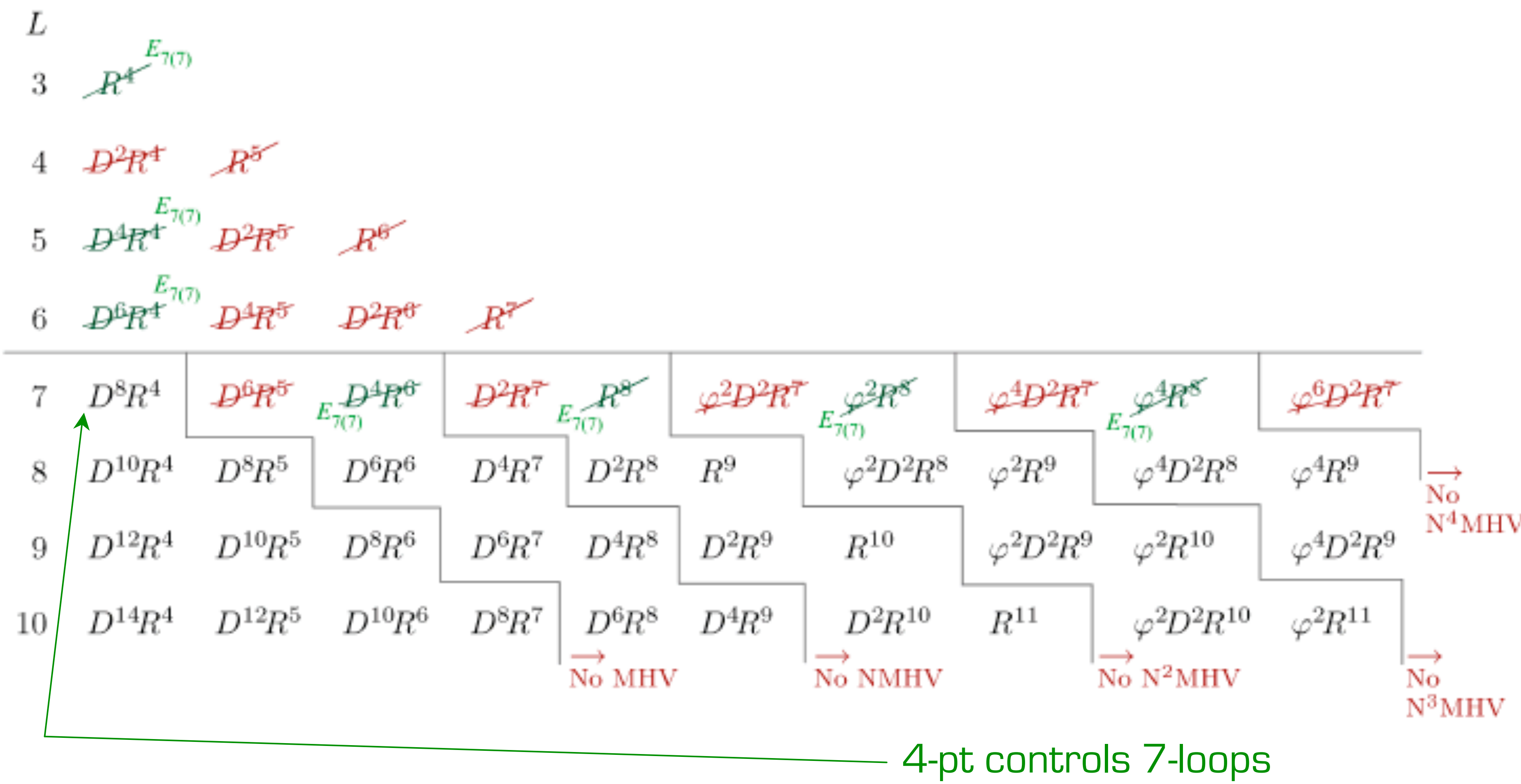
$E_{7(7)}$-symmetry manifest itself in low-energy theorems
(developed in pion-physics in the 1960's)

N=8 supergravity has 70 scalars ("pions").
In the limit of small momentum of a single scalar,
amplitude must vanish.

$\Rightarrow$ test of candidate counterterms!

$$\lim_{p_1 \to 0} \langle \varphi \, \overline{\varphi} \; 3^- \, 4^- \, 5^+ \, 6^+ \rangle_{\mathcal{O}} = ?$$

$E_{7(7)}$-symmetry constraints then eliminate the remaining operators below 7-loop order



[HE, Kiermaier, 1007.4813]

[Beisert, HE, Freedman, Kiermaier, Morales, Stieberger, 1009.1643]

Complimentary stringy approach [Green, Miller, Vanhove]

If operator is $f(\phi)D^{2k}R^4$ for k=0,2,3 (L=3,5,6 in 4d)

then $f(\phi)$ satisfies Laplacian equation

$$\Delta f = - \lambda f$$

We reproduce the specific value of λ from our soft-scalar limits. Consistency check.

N=8 supergravity

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We learn that based on SUSY and global symmetries, SU(8) and $E_{7(7)}$, perturbative N=8 supergravity in 4d *cannot have UV divergences until at first 7-loop order!*

This conclusion reached without calculating a single loop amplitude!

So.... What do you think??

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calculate?

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Amplitude excitement

- Field interesting in its own right: *new insights into QFT*?
- Solve *all order* amplitudes in planar N=4 SYM?
- Relevant for *particle physics processes at the LHC*.
- *Fundamental physics* applications:

  Perturbative gravity?

  Origin of simple structure in string theory?
- New methods carry over to *other areas of physics*?

And it is FUN too!