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NEAR OPTIMUM CONTROL OF FLEXIBLE ROBOT ARMS ON FIXED PATHS

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ABSTRACT

This paper presents the analysis and modification of near optimum trajectories for robotic manipulators moving along pre-defined paths. Modifications of trajectories are done such that the vibrations due to flexibility of arms and other components of the manipulator are minimized. Ultimately, the productivity of robotic manipulators depends on the speed of the task execution. Higher productivity requires higher speed of operation and in turn better control and trajectory generation algorithms. Today trajectory generation algorithms do not consider the dynamic characteristics of the manipulators. In order to utilize the available capability in the optimum manner the trajectory generation algorithms need to consider the dynamics of the manipulator, actuator constraints, nature of the task, and flexibility of arms and compliance of the joint connections.

In the search for an optimal trajectory that will meet all of the above requirements while optimizing some criterion, some simplifying assumptions have to be made and/or some of the requirements have to be kept out of the formulation so that the defined problem can be solved or some feasible solutions obtained. Once the simplified problem is solved, one may consider modifying the original solution in such a way that the excluded requirements are also satisfied to some extent.

In this paper the minimum time control solution of a two link flexible arm with actuator constraints is presented. We solved the minimum time problem with no constraints on the flexible modes and show the time improvement due to the use of light-weight arms. The objective is to modify the trajectory, such that flexible vibrations are bounded while changing the solution from the previous one as little as possible. Practical ways of trajectory modifications for flexible arms are discussed.

I. INTRODUCTION

Today, most trajectory planning algorithms do not consider the dynamics of the manipulators, rather constant and/or piece wise constant accelerations for the overall task are used and an overall maximum allowable speed is set [5,6,7]. However, robotic manipulators are highly nonlinear dynamic systems, so it is expected that affordable accelerations and maximum speeds will vary as a function of states. For the traditional schemes to work, the trajectory must be planned for the worst possible case. The capabilities of the system will be used only a small part of the time. Bobrow et.al. [1] first reported that for every point on any path, there is an associated maximum allowable speed and maximum affordable acceleration and deceleration for every speed in the affordable range, and these values can drastically vary from one state to another. Incorporating the manipulator dynamics into the trajectory planning level, they found the minimum time trajectories for different manipulator models [1,2] with limited actuator capabilities moving along pre-defined paths. Shin and McKay [3] solved the same problem independently.

Light-weight manipulators with the same actuator capabilities will be faster. The main problem associated with the light-weight structures is the flexible vibrations. Fig. 1 conceptually shows the performance improvement in terms of increased speed and faster task executions.

In this paper we show the performance improvements due to:

1. use of light-weight arms

2. incorporating the manipulator dynamics into the trajectory planning level

3. discuss flexible vibrations during a near minimum time trajectory execution and considerations of path modifications such that flexible vibrations will be bounded. This problem is similar in nature to the one raised by Hollerbach [8] and Kiriazov et. al [9].

II. FLEXIBLE MANIPULATOR DYNAMIC MODEL IN JOINT AND PATH VARIABLES

A general dynamic modelling technique for flexible robotic manipulators was developed by Book using a recursive Lagrangian-assumed modes method. Homogeneous transformation matrices are used for kinematic relations of the system [4]. A two link flexible robotic manipulator is modelled using that technique (Fig. 2). In the model no actuator dynamics is considered, rather the net torque input to the links is considered as the input variable. No friction at joints nor in the structural vibrations are explicitly considered. Flexibility of each link is approximated with one assumed mode for each link. The dynamic model of the manipulator may be expressed in general terms as :

$$[J]_{4x4} \quad \underline{\ddot{q}}=\underline{f}(\underline{q},\underline{\dot{q}}) + \underline{Q} \qquad (2\text{-}1)$$

where

$\underline{q}^T$: $[\theta_1, \theta_2, \delta_1, \delta_2]$ Joint angles and flexible mode time variables

$\underline{Q}^T$: $[T_1, T_2, 0, 0]$ Net input torques

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$[J]_{4x4}$: Generalized Inertia Matrix symmetric, pos. definite.

$\underline{f}^T$: $[f_1, f_2, f_3, f_4]$ Nonlinear dynamic terms including centrifugal, gravitational, effective spring and Coriolis forces.

The problem is to find the minimum time trajectories for a given manipulator with limited actuator capabilities moving along a fixed path, with state constraints (bounded flexible vibration constraints). Once the path to be moved along is specified as a combination of Cartesian variables (x and y for the 2 d.o.f. case), distance along the path S can be specified as

$$S=S(x,y) \quad . \tag{2-2}$$

From the inverse kinematic formulation, the corresponding joint angles for a rigid arm of the same dimensions can be found as

$$\underline{\theta}=\underline{\theta}\ (s) \quad , \quad \underline{\theta}^T=[\theta_1, \theta_2] \tag{2-3}$$

Similarly, once the speed $\dot{S}(S)$ along the path is known

$$\dot{\underline{\theta}}=\dot{\underline{\theta}}\ (s,\dot{s}) \tag{2-4}$$

and

$$\ddot{\underline{\theta}}=\ddot{\underline{\theta}}\ (s,\dot{s},\ddot{s}) \tag{2-5}$$

Knowing the relations (2-3)-(2-5) in analytical or numerical form, the manipulator dynamics in part can be expressed in path variables under the assumption that somehow the joint relationships specified in (2-3)-(2-5) will be maintained. These joint variables specify the torques and flexible states as follows

$$\begin{bmatrix} C_{11}(s,\underline{\delta}) \\ C_{12}(s,\underline{\delta}) \end{bmatrix}_{2x1} \ddot{S} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} - \begin{bmatrix} C_{21}(s,\dot{s},\underline{\delta},\dot{\underline{\delta}},\vec{e}_t,\vec{e}_n,\rho) \\ C_{22}(s,\dot{s},\underline{\delta},\dot{\underline{\delta}},\vec{e}_t,\vec{e}_n,\rho) \end{bmatrix} \tag{2-6a}$$

$$\begin{bmatrix} \ddot{\delta}_1 \\ \ddot{\delta}_2 \end{bmatrix} = \begin{bmatrix} J_{33} & J_{34} \\ J_{43} & J_{44} \end{bmatrix}^{-1} \begin{bmatrix} f_3-g_3+h_1(s,\ddot{s},\vec{e}_t) \\ f_4-g_4+h_2(s,\ddot{s},\vec{e}_t) \end{bmatrix} \tag{2-6b}$$

where

$$f_i=f_i(s,\dot{s},\underline{\delta},\dot{\underline{\delta}}) \tag{2-7}$$

$$J_{ij}=J_{ij}(s,\underline{\delta}) \tag{2-8}$$

$$g_i=g_i(s,\dot{s},\vec{e}_t,\vec{e}_n,\rho) \tag{2-9}$$

$\vec{e}_t, \vec{e}_n$: Unit tangent and normal vectors along the path.

ρ : Curvature of the path at a point.

Note that once the path to be followed has been defined, the degrees of freedom of the rigid manipulator reduces to one, no matter how many joints it has. Then the manipulator dynamics can be expressed as a second order non-linear ordinary differential equation. If the flexiblitiy of links are included in the model but not in the definition of the path, as is the case here, there will be additional flexible dynamics coupled with each other and the rigid dynamics.

III. FORMULATION OF THE NEAR MINIMUM TIME TRAJECTORY PROBLEM FOR FLEXIBLE MANIPULATORS

Recall that

$$\frac{d..}{dt} \equiv \frac{d..}{ds}\frac{ds}{dt} = \dot{S}\ \frac{d..}{ds} = Z\ \frac{d..}{ds}$$

where $\dot{S}$ is the speed along the path can be varied as a function of S. That suggests that every variable can be expressed as function of independent variable S, distance along the path. Let $\dot{S}(S)=Z(S)$ in all the following. Initial and final states along the path would normally be given, $Z_0(S_0)$ and $Z_f(S_f)$. The optimum trajectory problem may be stated, using the path variable S as the independent variable rather than time, as follows:

Optimality criterion:

$$\text{Minimize} \quad J = \int_0^{t_f} dt = \int_{so}^{s_f} \frac{ds}{\dot{s}} \tag{3-1}$$

Subject to initial and final states of the path variables:

$$Z(S_0)=Z_0 \qquad Z(S_f)=Z_f$$

System dynamics, expressed in path variables:

$$C_{1i}(s,\underline{\delta}).Z.Z' = T_i(s,Z)- C_{2i}(s,Z,\underline{\delta}) \quad i=1,2$$

$$Z^2\begin{bmatrix} \delta_1'' \\ \delta_2'' \end{bmatrix} + Z.\ Z'.\begin{bmatrix} \delta_1' \\ \delta_2' \end{bmatrix} = J^{-1}_{2x2} \begin{bmatrix} f_1(s,Z,\delta) \\ f_2(s,Z,\delta) \end{bmatrix} \tag{3-2}$$

Actuator constraints:

$$T_{i\ min.}(s,Z) \le T_i \le T_{i\ max.}(s,Z) \tag{3-3}$$

Dynamic inequality constraints on flexible modes:

$$a_i(s) \le \delta_i(s) \le b_i(s) \qquad i=1,2 \tag{3-4}$$

The constraints (3-4) naturally arise in flexible structures. If such a constraint is not imposed there is no guarantee on the accuracy of the end point along the path. Following the rationale expressed in the introduction, one would solve the problem without the constraint (3-4). The problem reduces to the one solved in [1],[2],[3].

The solution method we use closely follows Bobrow et.al.'s method with some modifications for flexible manipulators. The solution of the above stated optimization problem follows: for any path S(x,y,z) with given $Z(S_0)$,$Z(S_f)$ to minimize J, Z should be as large as possible while satisfying the system dynamics and actuator constraints. In order to do so at any state on the path one should use maximum acceleration or deceleration. Then, the problem is reduced to finding the maximum accelerations and decelerations associated with each state of interest. It can be seen from equation (2-6a) that for each (S_i,Z_i)

$$\ddot{S}_d \le \ddot{S} \le \ddot{S}_a$$

$$\ddot{S}_a = \min \left\{ \ddot{S}_{ai} \right\} \quad (3\text{-}5)$$
$$\ddot{S}_d = \max \left\{ \ddot{S}_{di} \right\}$$

There may be some range of speeds associated with every point on the path that system can no longer satisfy all conditions (the Z range that above inequality is violated). The collection of these ranges defines the forbidden region on (S,Z) plane. The boundary between allowed and forbidden regions is constant for a given rigid manipulator for a given task. In the case of flexible manipulators, due to the coupling between equations (2-6a) and (2-6b) this boundary is also a function of flexible modes, not only (S, Z). So, depending on the time history of the flexible modes and unpredictable disturbances the boundary will vary. This is not true in the rigid case where the true extreme can be found. At this point the problem is to find when to use maximum accelerations and when maximum decelerations (i.e. to find the switching point(s)).See Fig. 3a-3b.

Finding switching points for near optimal performance of flexible manipulators then proceeds as follows:

1. Integrate $\ddot{S}=\ddot{S}(x,y)$ from the final state backward in time until it crosses forbidden region or initial position, using maximum deceleration.

2. Integrate $\ddot{S}(x,y)$ forward in time from initial conditions (S_0,Z_0) with maximum acceleration until the boundary is reached or the two curves cross each other. If the two curves cross each other before they enter forbidden region, then find that point. This is the last switching point and terminates the search. If not, then

3. Backup on the last forward integrated curve and integrate forward with maximum deceleration until the trajectory intersects:

   a. the boundary of the forbidden region. If the intersection is not tangent within some tolerance, repeat 3.

   b. or the line Z = 0. In this case the distance backed up in 3 was too great. Reduce the amount of backup and repeat 3.

4. Then using the tangent point as new starting point go to step two.

Notice that the last switching point is not the exact switching point, because the flexible modes will not match at this point. That will cause one to miss the final state somewhat. Also, when searching for the switching points one has to move in a continuous manner in order to keep track of the flexible mode histories accurately. In that sense, the algorithm given in [1] has been modified for flexible robotic manipulators.

IV. TRAJECTORY MODIFICATION AND FLEXIBLE MODES

Once the near optimal trajectory Z(S) of the previous problem is found, one may consider modifying the trajectory in such a way that the constraints on the flexible modes are satisfied too. For any modified Z(S) which is affordable by actuators the equation (3-2b) can be integrated forward using the initial conditions of flexible modes at the beginning of the task.

$$\underline{\delta}(s_0) = \underline{\delta}_0 \quad (4\text{-}1)$$

In fact regardless of the affordability of any trajectory in (S,Z) plane, the flexible mode history along the path can be found by an integration along that trajectory.

A number of practical trajectory modifications using the cubic spline functions have been tried by the authors. Trajectories are modified in a smoothing fashion so that abrupt changes of torques at the switching points are avoided, expecting that the modified trajectory will result in less excited flexible modes. To some extent that is true, but since the dynamics of the flexible modes are highly complicated and nonlinear, not only the torques but also the coupling between states are important, particularly in the case of a minimum time problem. The initial trajectory modifications have not resulted in a favorable dynamic behavior and may not be generalized for all paths, because the shape of the path is also part of the dynamics and this is not explicitly mapped in to (S,Z) plane. Some simulation results are shown in Fig. 8 - 10.

The trajectory modification problem is currently being formulated as an optimum control problem with dynamic constraints. A generalized quasilinearization algorithm is applied iteratively starting with the unconstrained solution and iteratively approaching to the solution of the problem with dynamic constraints [10],[11],[12].

V. SIMULATION RESULTS AND DISCUSSION

The two-link flexible manipulator model for task one (shown in Fig. 4a) was simulated for the two different cases in order to show the performance improvement achieved due to a light-weight system. In both cases actuators have the same capabilities. It is found that weight reduction by a factor of 2 results in approximately 60 % time reduction (Fig. 5a and 6a). This improvement, of course, varies depending on the task. Joint actuator histories are shown in Fig. 5b-6c and flexible mode responses are shown in Fig. 5c-6d.

Task 2 (Shown in Fig. 4b) was simulated for light-weight manipulator and results are shown Fig 7 a-d. The final trajectory is shown in Fig. 7b. One interesting point in this simulation is the fact that as soon as the manipulator end point enters the curvature the system must accelerate along the path in order to obey the constraints. In Fig. 7a the curve ab shows that immediately before the curvature the system is able to decelerate (aa' curve), but as end point enters the curvature the sudden appearance of a normal acceleration term in the dynamics of the system appears and end of the manipulator has to accelerate in order to stay on the path. This indicates how sensitive a trajectory modification would be in this part of the trajectory. The other point in the case of flexible arms is that at the last switching point flexible modes are not same, since they have different histories. This will cause error in the final state reached. See Fig. 6a, 7a. The last switching point needs to be varied from the original result of the above algorithm.

VI. CONCLUSION AND FURTHER WORK

In this paper we showed ways to improve performance and productivity of Robotic manipulators with flexible arms. One way was to use light-weight structures and the other was to incorporate the dynamics of manipulators in to trajectory planning level and make optimum utilization of given manipulator. Some practical trajectory modifications are presented. The sensitivity of the trajectories on (S,Z) plane is very high. Any small change in the slope may end up with quite different flexible mode history depending on the path and the speed along the path. The slope of the trajectory at the beginning of the task should be carefully modified if the execution time is of any interest, for small slopes where speed is small will take long execution time. Application of the method requires the manipulator dynamics, geometric path in work space, and actuator capabilities. Obviously as trajectory gets closer to the forbidden region boundary system capabilities are being used to the limits and any disturbance or uncertainty can easily put the system in to forbidden region and end of the manipulator will leave the desired path. This situation is more clear in the case of flexible robotic manipulators. While this analysis is nice in terms of knowing the maximum capabilities, in practice there will be some safety factor that will require to keep the trajectory away from the forbidden region boundary certain amount.

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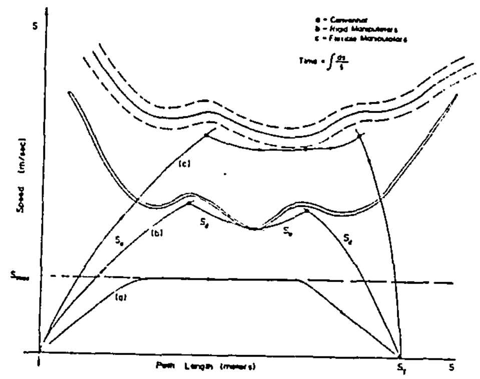

Fig.1 Different trajectory plans.

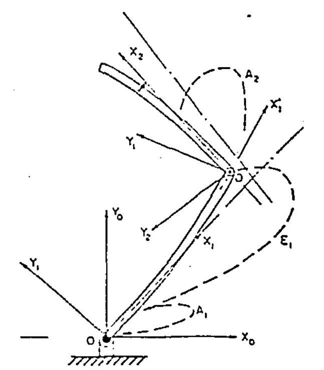

Fig.2 Manipulator Model.

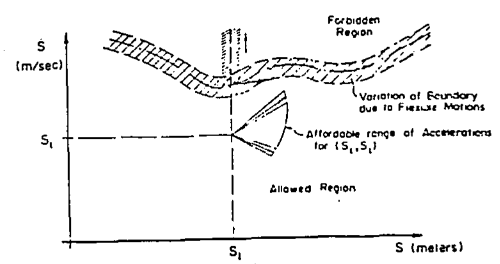

Fig. 3.a $(S,\dot{S})$ Plane

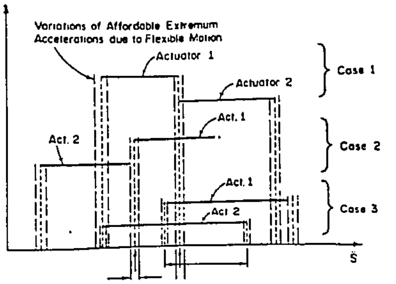

Fig. 3.b Different possible cases during a task.

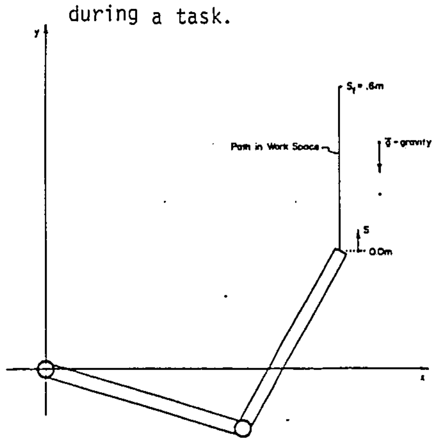

Fig. 4a. Task 1 in (x,y) plane.

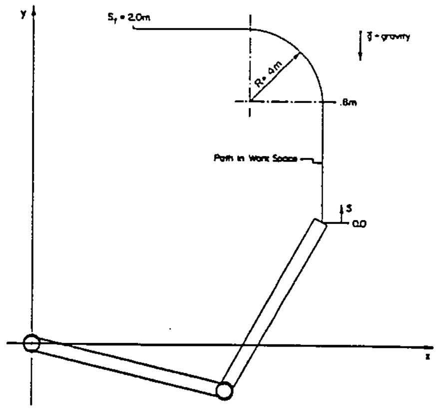

Fig. 4b Task 2 in (x,y) plane.

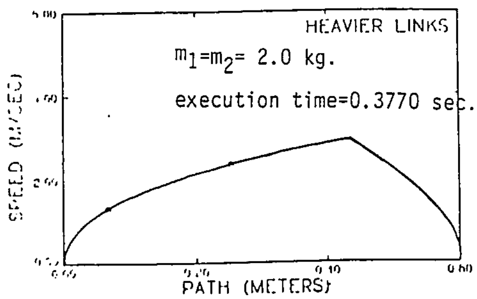

Fig. 5.a Trajectory for Path 1 of heavy links.

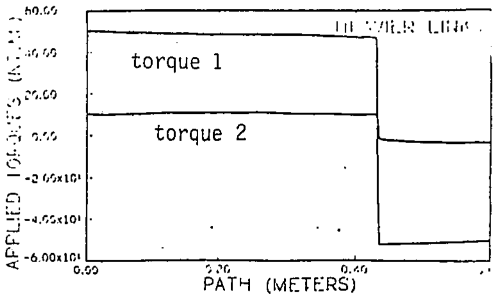

Fig. 5b. Torques along the path 1

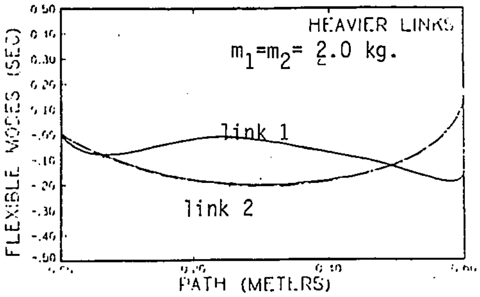

Fig. 5.c Flexible modes time variables

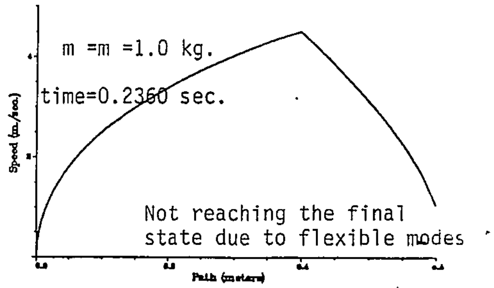

Fig. 6.a Trajectory of lightweight arms

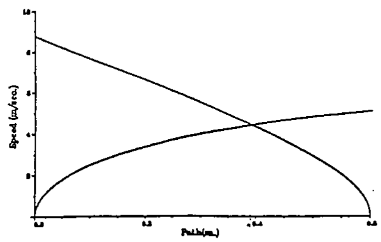

Fig. 6.b Finding the switching poin

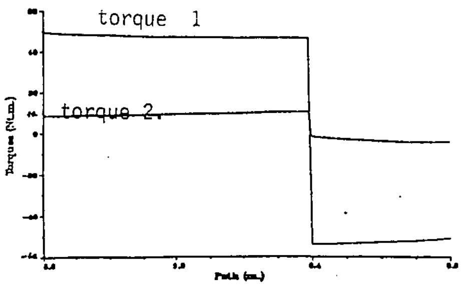

Fig. 6.c Torque histories of lightweig
arms along path 1.

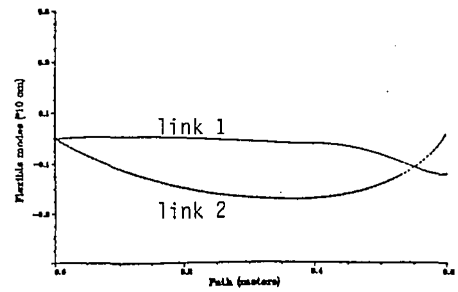

Fig. 6d. Flexible modes along path 1.

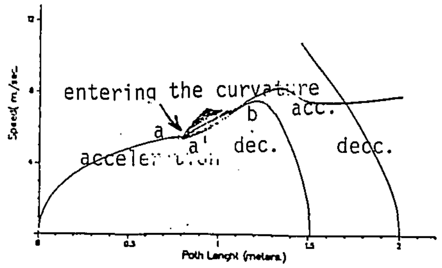

Fig. 7a. Finding the switching points for path 2.

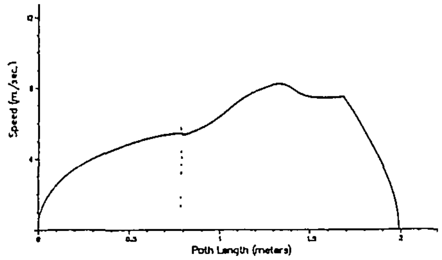

Fig. 7b. Trajectory for path 2.

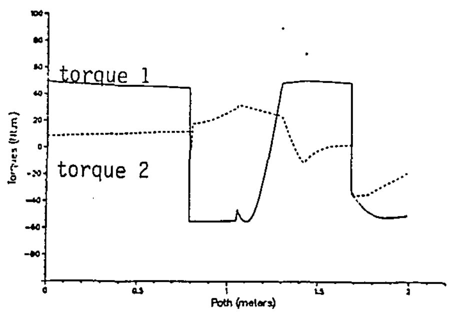

Fig. 7c. Torques along path 2.

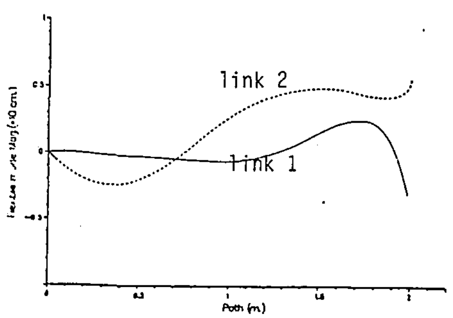

Fig. 7d. Flexible modes along path2.

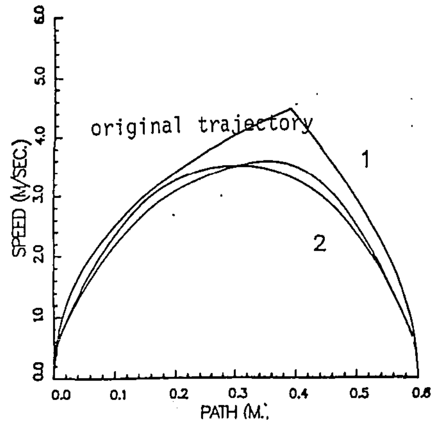

Fig. 8 Original and Modified Trajectories

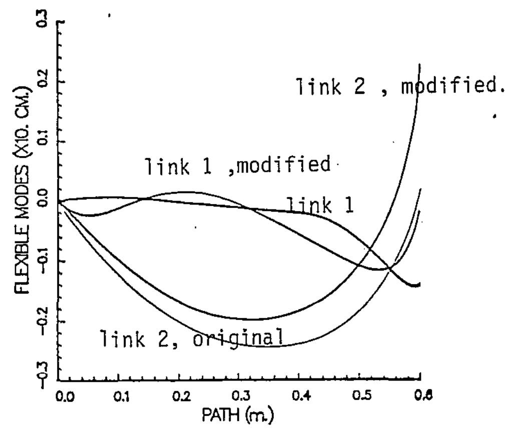

Fig. 9a. Flexible modes along original and modified trajectory 1.

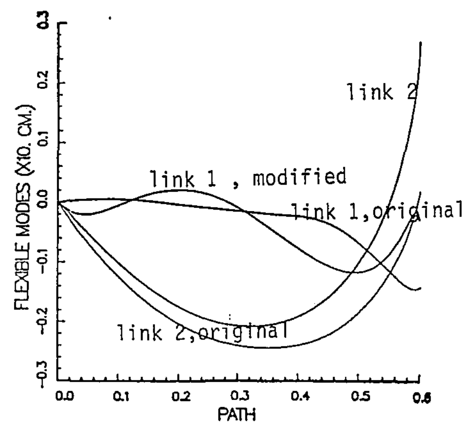

Fig. 9b. Flexible modes along original and modified trajectory 2.

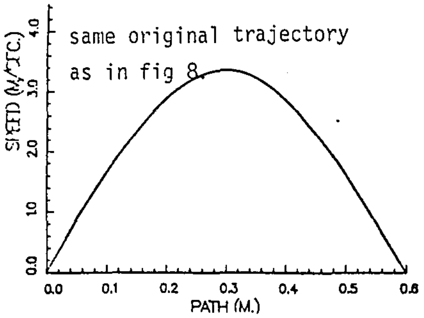

Fig. 10a. Modified trajectory 3.

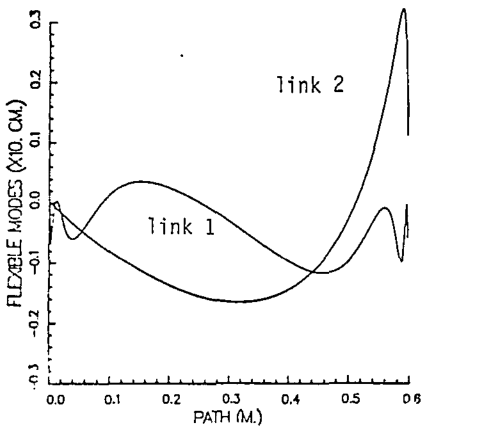

Fig. 10b. Flexible modes along modified trajectory 3.

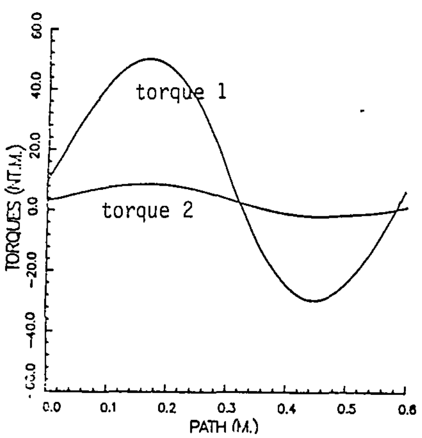

Fig. 10c. Torque histories along modified trajectory 3.