GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT INITIATION

adg

Date: July 22, 1977

Project Title: Preliminary Design of a 150 kWe Solar Powered Deep Well Irrigation Facility

Project No: A-1945

Project Director: Dr. Steve H. Bomar, Jr.

Sponsor: Black & Veatch Consulting Engineers; Kansas City, Missouri 64114

Agreement Period: From 2/1/77 Until 5/10/77

Type Agreement: Letter Contract No. 7510-1-77, dated 4/19/77 (Under ERDA Prime Contract No. EG-77-C-04-3916)

Amount: $10,000

Reports Required: Monthly Progress Reports; Final Report

Sponsor Contact Person (s):

Technical Matters	Contractual Matters (thru OCA)
Dr. J. C. Grosskreutz	Mr. Hans W. Strohm
Project Manager	Project Administrator
Black & Veatch Consulting Engineers	Black & Veatch Consulting Engineers
1500 Meadow Lake Parkway	1500 Meadow Lake Parkway
P. O. Box 8405	P. O. Box 8405
Kansas City, Missouri 64114	Kansas City, Missouri 64114
(913) 967-2000	(913) 967-2000
TELEX 42-6263	TELEX 42-6263

NOTE: *Since a definitive contract will not be issued, this will constitute the entire initiation package. Final billing instructions will be given to EES Accounting as soon as total estimated costs are submitted to Jay Wilson for authorization by Black & Veatch and Final Report is submitted.*

Defense Priority Rating: N/A

Assigned to: Applied Sciences Laboratory (School/Laboratory)

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CA-3 (3/76)

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GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION

Date: November 3, 1977

Project Title: Preliminary Design of a 150 KWe Solar Powered Deep Well Irrigation Facility

Project No: A-1945

Project Director: Dr. Steve H. Bomar, Jr.

Sponsor: Black & Veatch Consulting Engineers; Kansas City, Missouri 64114

Effective Termination Date: 5/10/77

Clearance of Accounting Charges: All have cleared, including close-out costs.

Grant/Contract Closeout Actions Remaining:

None.

_ Final Invoice and Closing Documents
_ Final Fiscal Report
_ Final Report of Inventions
_ Govt. Property Inventory & Related Certificate
_ Classified Material Certificate
_ Other ______________

Assigned to: Applied Sciences Laboratory/SE (School/Laboratory)

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A-1945

ENGINEERING EXPERIMENT STATION
GEORGIA INSTITUTE OF TECHNOLOGY • ATLANTA, GEORGIA 30332

April 11, 1977

Black and Veatch
P. O. Box 8405
Kansas City, Missouri 64114

Attention: Dr. S. L. Levy
Project 7510

Subject: Preliminary Design of a 150 KWe Solar Powered Deep Well Irrigation Facility, Monthly Technical Progress Report No. 2 covering the Period March 1 through March 31, 1977 (Georgia Tech Project A-1945).

Gentlemen:

Program activities this month have been somewhat delayed by late delivery of kinematic motions and mirror mounts which were needed for tracking error tests. However, progress has been made in several of Georgia Tech's areas of program responsibility.

1. During February inquiries were made of ANSALDO concerning its willingness to license the construction of Francia-type solar systems in the United States. Verbal response was obtained from Dr. Floris of ANSALDO that the company would be willing to selling these rights, and this fact was reported in our Monthly Technical Progress Report for February. A further response from Dr. Beer of ANSALDO was received by Mr. Poulos of Georgia Tech in a telephone call on March 15. ANSALDO proposes to sell manufacturing rights in the United States for "$25,000 for each system built, plus 5 per cent of the installed value of the kinematic motions and receiver, except that the $25,000 will be applied to the first part of the 5 per cent of the installed value." Stated more clearly, the royalties will be 5 per cent of the installed value of the kinematic motions and receiver, with a minimum of $25,000 per system. Presumably if a receiver other than the Francia design were used, that receiver would not be subject to royalties. The exact definition of a kinematic motion for royalty purposes is not clear, but presumably would include only the mirror supporting arm and tracking mechanism, but not the glass, steel beams, or motor drives since these are not claimed in Francia's U. S. Patent.

2. The crates from ANSALDO containing the kinematic motions and mirror mounting rings arrived at Georgia Tech on March 16, and several mirrors were mounted during the week of March 21. One of these was placed on a kinematic motion and installed on a stand on the roof of the Hinman Research Building to permit focussing tests for this Deep Well Irrigation System design program. Tests are underway to observe the following parameters by photographic recordings using this test apparatus:

   (a) Focussed image size and shape as a function of time of day for a heliostat located due south of the target at a range of 100 feet,

   (b) Focussed image size and shape as a function of time of day for a heliostat located southeast of the target at a range of approximately 100 feet,

   (c) The parameters described in (a) and (b) for at least one other mirror and frame assembly.

   From this information the size of a receiver aperture required to collect energy from a <u>perfectly tracking</u> collector field can be determined. These measurements will thus give information on the optical imperfection associated with two typical mirror and frame assemblies. Measurements using a laser scanning system are believed to be superfluous.

3. Discussions have been conducted with ANSALDO personnel concerning mechanical tracking errors and procedures we might use for measuring these errors. (Two engineers from the Italian company are at Georgia Tech to assist in installation of the facility.) They believe that the roof-mounted test apparatus is not steady enough for accurate alignment and tracking, and that the test results would not warrant spending the effort required to install a motor drive on this heliostat. Since the mechanical parts of the Georgia Tech facility are being assembled in the field at this time, they recommend tests on the real equipment for tracking measurements. I am trying to set up this test as quickly as possible, preferably before the entire heliostat field is aligned. The ANSALDO engineers stated that their company has a computer program to optimize aperture size and that the program was used to design the Georgia Tech receiver. This receiver will generate steam with a saturation pressure and temperature of 2,200 psia and 650 ^{o} F, and superheat the steam to 1,200 ^{o} F; its aperture is 1,200 mm diameter. From these data we might infer that a smaller aperture would be appropriate for a receiver operating at higher temperatures in order to reduce heat losses.

4. Mr. Scott Hodges of Black and Veatch has contacted Georgia Tech about engineering data and operating experience on Heat Transfer Salt (HTS) for possible use in the Deep Well Irrigation Facility. The requested information has been provided and Mr. Hodges has been invited to visit Georgia Tech to discuss this application with the engineers who built the Thermal Storage Subsystem Research Experiment; that experiment used HTS as one of its storage media.

Work on this program during April will continue to be concerned with determination of tracking errors and image properties. Assembly of the Solar Test Facility (on another contract) is providing opportunities for better understanding of the Francia collector system in preparation for starting the Preliminary Design Task on this program.

Respectfully submitted,

Steve H. Bomar, Jr.
Project Director

dr

PRELIMINARY DESIGN OF A 150 KWe SOLAR POWERED DEEP WELL IRRIGATION FACILITY

FINAL TECHNICAL REPORT

PRELIMINARY DESIGN OF A 150 KWe SOLAR POWERED DEEP WELL IRRIGATION FACILITY

FINAL TECHNICAL REPORT

Prepared for

Black and Veatch, Consulting Engineers
Kansas City, Missouri
(B&V Project 7510)

By

Engineering Experiment Station
Georgia Institute of Technology
Atlanta, Georgia 30332

Authors

Steve H. Bomar, Jr.
John H. Murphy

September 1977

Georgia Tech Project A-1945

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I. INTRODUCTION

This report describes work performed by the Engineering Experiment Station of the Georgia Institute of Technology for Black and Veatch on the design of a solar energy collector system. Black and Veatch was conducting a program for the Energy Research and Development Administration to develop a "Preliminary Design of a 150 kWe Solar Powered Deep Well Irrigation Facility" to be constructed on a site near Coolidge, Arizona; three competing preliminary designs were funded by ERDA. The Black and Veatch plant concept envisioned a solar collector system based on the solar tracking mechanism developed by Professor Giovanni Francia at the University of Genoa in Italy. At the time the deep well irrigation facility design was underway, the Engineering Experiment Station of Georgia Tech was constructing a 400 kWth Solar Thermal Test Facility designed by Francia and major portions of which were supplied by ANSALDO S.p.A. of Genoa. Black and Veatch placed a subcontract with the Engineering Experiment Station to provide design services for a similar solar collector system to be incorporated into the irrigation facility.

Georgia Tech proceeded through approximately five months of a planned seven-month contract until it became clear that the Francia collector design presented severe difficulties for the irrigation system application. These difficulties were (1) that we were unable to demonstrate that the Francia tracking mechanisms at Georgia Tech could meet the tracking accuracy requirements desired by Black and Veatch, and (2) that the desired scale-up of mirror sizes led to a massive tracking mechanism which appeared impractical to build. At that point in the program, Black and Veatch instructed Georgia Tech to discontinue work on the program and to close out the contract in an orderly manner.

This report is organized by subtasks and describes the activities conducted and the results obtained within each major block of work.

II. COST ESTIMATE FOR THE GEORGIA TECH SOLAR THERMAL TEST FACILITY

In order to furnish a cost baseline for the collector subsystem of the deep well irrigation facility, cost estimates for the Georgia Tech 400 kWth Solar Thermal Test Facility were furnished to Black and Veatch. These estimates were itemized to permit isolation of inapplicable costs such as the steam receiver at Georgia Tech. Data for Georgia Tech man-hours were directly attributable to construction of the facility and did not include subsidiary efforts such as facility testing and characterization, development of facility utilization plans, attendance at ERDA program reviews, etc.

Under its contract for construction of the GT/STTF, Georgia Tech subcontracted with ANSALDO S.p.A. of Genoa, Italy for a facility design and purchase of many key components. The design was generated under the supervision of Professor Francia and the components supplied by ANSALDO were produced and shipped to the United States. Georgia Tech also procured certain parts in the United States where this procedure appeared to be advantageous; in particular the mirror glass and the steel for the heliostat field framework were purchased locally. Georgia Tech then assembled the 400 kWth STTF in accordance with the design supplied by ANSALDO. This cost estimate is given in Table I and is believed to represent the costs for building a similar facility beginning with the experience we now possess.

TABLE I

COST ESTIMATE FOR GT/STTF

ANSALDO SUBCONTRACT		
Purchased Parts and Materials:		
550 Kinematic Motion Devices including mirror bending devices	$110,000	
1 Solar Energy Receiver equipped with a removable antiradiating structure	30,000	
1 Receiver Support Tower	20,000	
1 Thermal Cycle Set consisting of condenser, air extracting pump, feedwater pump, attemperator, attemperator pump, pressure regulating valve, one-way valve, gate valve, piping and joints, and control board	30,000	
1 Heliostat Driving System and Control	12,000	
1 Erection Instrument Set	6,000	
Total Parts and Materials Purchased by ANSALDO		$208,000
Materials Overhead at 5 percent of $208,000		10,000
Engineering:		
Framework	8,000	
Kinematic Motions	4,000	
Receiver	6,000	
Thermal Cycle Set	4,000	
System Engineering and Management	12,000	
Total Engineering by ANSALDO		34,000
Total Direct Cost and Overhead		252,000
General and Administrative at 5 percent of $252,000		12,600
Royalties to Professor Francia		5,400
TOTAL ANSALDO SUBCONTRACT		270,000
Packing for Sea Shipment		20,000
Start-Up Assistance at Georgia Tech		5,000
TOTAL ANSALDO CHARGES		295,000

(Continued)

TABLE I (Continued)

COST ESTIMATE FOR GT/STTF

Site preparation including survey, grading and crushed stone		$ 4,000
Fence with two 16-foot wide access gates		5,000
Concrete installed cost:		
Heliostat support columns and footings	$ 4,500	
Receiver tower footing	1,500	
Access channels, thermal cycle pad, drains	5,000	
Total installed cost for concrete		11,000
Heliostat supporting framework:		
North-south I-beams	3,800	
East-west square tubing	10,200	
Fabricated steel brackets	9,400	
Bolts and fittings	400	
Paint and painting contract	1,600	
Total cost for purchased parts of heliostat supporting framework		25,400
Cooling tower:		
Cooling tower assembly (retail value $5,000)	1,600	
Concrete footing, pump, valves, accessories	1,000	
Total cost for cooling tower		2,600
Mirror glass, 560 mirrors at $10.38 each		6,000
Utilities (water, sewage, electrical service and distribution)		5,000
Travel		3,000
Freight and express		12,000
Miscellaneous materials and supplies		2,000

(Continued)

TABLE I (Concluded)

COST ESTIMATE FOR GT/STTF

Georgia Tech salaries and wages directly attributable to facility engineering and installation	$60,000
Georgia Tech overhead and retirement at 77 percent of salaries and wages	46,000
TOTAL ESTIMATED FACILITY COST	477,000
MIRROR AREA (550 mirrors, 111 cm diameter)	532 m^2

III. PATENT AND LICENSING INVESTIGATIONS

Since Professor Francia was known to hold a United States Patent on his design for solar energy collector system (Multiple Mirrored Apparatus Utilizing Solar Heat, U. S. Patent No. 3,466,119 issued September 9, 1969) and he was known to have granted ANSALDO the right to employ his design for the Georgia Tech 400 kWth STTF, it was presumed that some license arrangement might be required for Black and Veatch or other parties to use the Francia solar collector system in the United States. Two lines of inquiry into this issue were undertaken: (1) an approach to ANSALDO concerning its willingness to sell the necessary rights for construction of systems in the United States, and (2) an investigation of the strength of Francia's patent position in the United States.

A telephone inquiry was made to Dr. Beer at ANSALDO on February 21, concerning ANSALDO's willingness to license the manufacture of Francia systems in the United States. Dr. Floris of ANSALDO replied on March 1 that ANSALDO is agreeable to selling these rights, but he did not know what the royalties might be; he promised to provide more information on this subject. A further response from Dr. Beer of ANSALDO was received by Mr. Poulos of Georgia Tech in a telephone call on March 15. ANSALDO proposed to sell manufacturing rights in the United States for "$25,000 for each system built, plus 5 percent of the installed value of the kinematic motions and receiver, except that the $25,000 will be applied to the first part of the 5 percent of the installed value." Stated more clearly, the royalties will be 5 percent of the installed value of the kinematic motions and receiver, with a minimum of $25,000 per system. Presumably if a receiver other than the Francia design were used, that receiver would not be subject to royalties. The exact definition of a kinematic motion for royalty purposes is not clear, but probably would include only the mirror supporting arm and tracking mechanism, but not the glass, steel beams, or motor drives since these are not claimed in Francia's U. S. Patent.

ANSALDO was requested to confirm these verbal messages by telegraph with a precise definition of the parts of the system to which the royalty payments would apply. At this stage we were told that Drs. Beer and Floris were not authorized to discuss royalties further and that we should conduct further negotiations with the legal authorities in the company. Contacts with the appropriate persons were attempted, but no response was obtained and no commitment was ever received in writing. During June, Mr. J. D. Walton of Georgia Tech was in Europe and could have visited Genoa to pursue this subject, but by that time the strength of Francia's patent was subject to questions. A decision was made by Black and Veatch and Georgia Tech that Mr. Walton would not be asked to visit ANSALDO.

The investigation of Francia's U. S. patent position was begun by conducting a patent search in the Georgia Tech Library. The search of the U. S. Patent records was conducted by checking under the names "Francia" and "ANSALDO" in the Patentee Index from 1969 through 1975, and in the Patent Gazette for 1976 and through April 1977. Three Francia patents were found: the Patent No. 3,466,119 mentioned previously and two others which covered mechanical devices not related to solar energy. The applicable patent covers the supporting framework, the mechanism which causes the mirrors to track the sun, and a device for adjusting mirror declinations.

The Georgia Tech Library has French Patent Abstracts from 1970 to the present, but these were not searched. The Library does not have Italian patents or abstracts.

The patent law firm Newton, Hopkins and Ormsby of Atlanta, which is retained by Georgia Tech for cases in this field of law, was consulted for its professional advice on the strength of Francia's patent. Messrs. Newton and Ormsby attended a meeting at Georgia Tech in which the questions concerning this program were discussed, the known Francia patent was examined, and the Georgia Tech STTF was inspected. It was decided that their first task would be to determine whether the Georgia Tech facility was covered by the Francia patent. After the tracking mechanism designs for this program were completed, their judgment could then be extrapolated to the specific collector system proposed for the irrigation facility.

After a lengthy search, Newton, Hopkins and Ormsby reached the conclusion that the Francia patent (3,466,119) does not cover the present Georgia Tech installation because the mechanical linkages used at Georgia Tech are substantially different from those described in the patent. Furthermore, review of the Patent Office file on the Francia patent showed that his original claims were extensively modified during prosecution of the patent, and what might have been a strong patent was reduced to a very limited one. The only area which might potentially impact the irrigation facility collector system is patents which might be pending at this time. The attorneys cautioned that pending applications may eventually result in a patent covering the Georgia Tech facility design and others developed on the irrigation facility program; patent application files are confidential and cannot be searched.

IV. MIRROR SLOPE ERROR AND TRACKING MECHANISM POINTING ERROR TESTS

In order to select the optimum height and aperture size for the solar receiver supported above the collector field, it was necessary that the errors associated with the positioning of the reflected beams at the receiver be understood. These errors arise from two sources: (1) mirror slope errors (deviations of real mirror surfaces from ideal mirror surfaces), and (2) tracking the mechanism pointing errors (deviations of the real aiming point from the desired aiming point). The most expeditious way to collect quantitative data on these errors for a Francia collector field was to conduct tests on components of the Georgia Tech STTF. Three separate series of experiments were conducted to measure the required data.

A. Observation of Reflected Image Characteristics

The first mirror tests consisted of observation of image sizes and shapes throughout the day for four different mirror and frame assemblies. These tests were conducted on the roof of the Hinman Research Building at Georgia Tech before the tower had been erected at the 400 kWth STTF. A single kinematic motion (mirror support mechanism) was mounted on a stand so that solar images could be reflected from a mirror onto a plywood target at a range of 100 feet. Four mirrors were attached to mirror-support frames, focused to obtain the smallest possible images on the target, and photographs of the images on the target were made each hour throughout the day. Since the kinematic motion was not mechanically driven it was necessary to manually position the solar image on the target for each photograph.

As expected, the image size and shape for each mirror changed throughout the day as the angle of incidence changed. However, an unexpected pattern variation among individual mirrors was also observed. The images from two mirrors at solar noon are shown in Figures 1 and 2. It is seen that the focussed spot is distinctly smaller and sharper for one mirror than for the other. Interchange of the mirror glasses and frames demonstrated that the different spot characteristics were functions of the glass. Figures 1 and 2 illustrate extreme cases; most images had the general appearance of Figure 1.

B. Measurement of RMS Surface Errors for Mirror and Frame Assemblies

An optics specialist at Georgia Tech was consulted regarding techniques for measuring the surface slope errors associated with mirror glass and mirror support frames. He strongly recommended that the mirror and support assembly be evaluated rather than the mirror glass alone because it is impossible to restrain the glass in a reproducable manner unless it is mounted on a support ring as shown in Figure 3.

Mirror slope error tests were conducted using the experimental arrangement illustrated in Figure 4. Mr. W. M. Bohon of Black and Veatch helped to conduct the experimental measurements and reduced the resulting data. An optical range was set up in a hallway so that a mirror and frame assembly could be supported at one end and a target placed at the other end. Using a Foucault test with a light bulb behind the aperture in the target, a test mirror was focussed so that its radius of curvature approximated the length of the range. (The Foucault test is described by Russell W. Porter in "Mirror Making for Reflecting Telescopes," Amateur Telescope Making, Book One, Scientific American, Inc., 1948.) The measurement of surface slope error was then made using a modification of the Hartmann test for grading the performance of

Figure 1. Photograph of Reflected Image at Solar Noon (Target Grid Marked in One-Foot Squares).

Figure 2. Photograph of Reflected Image at Solar Noon (Target Grid Marked in One-Foot Squares).

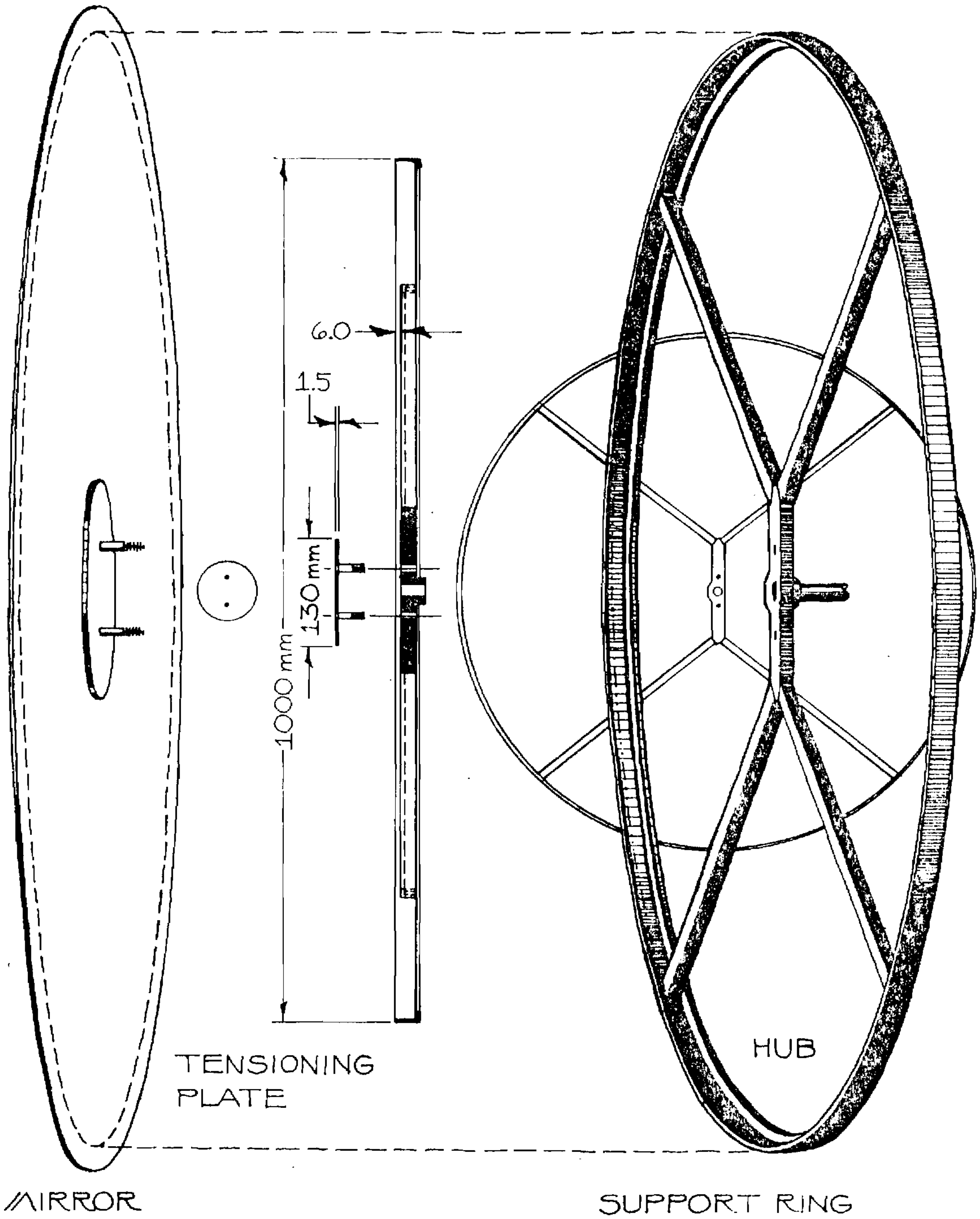


Figure 3. Mirror and Frame Assembly for Georgia Tech STTF.

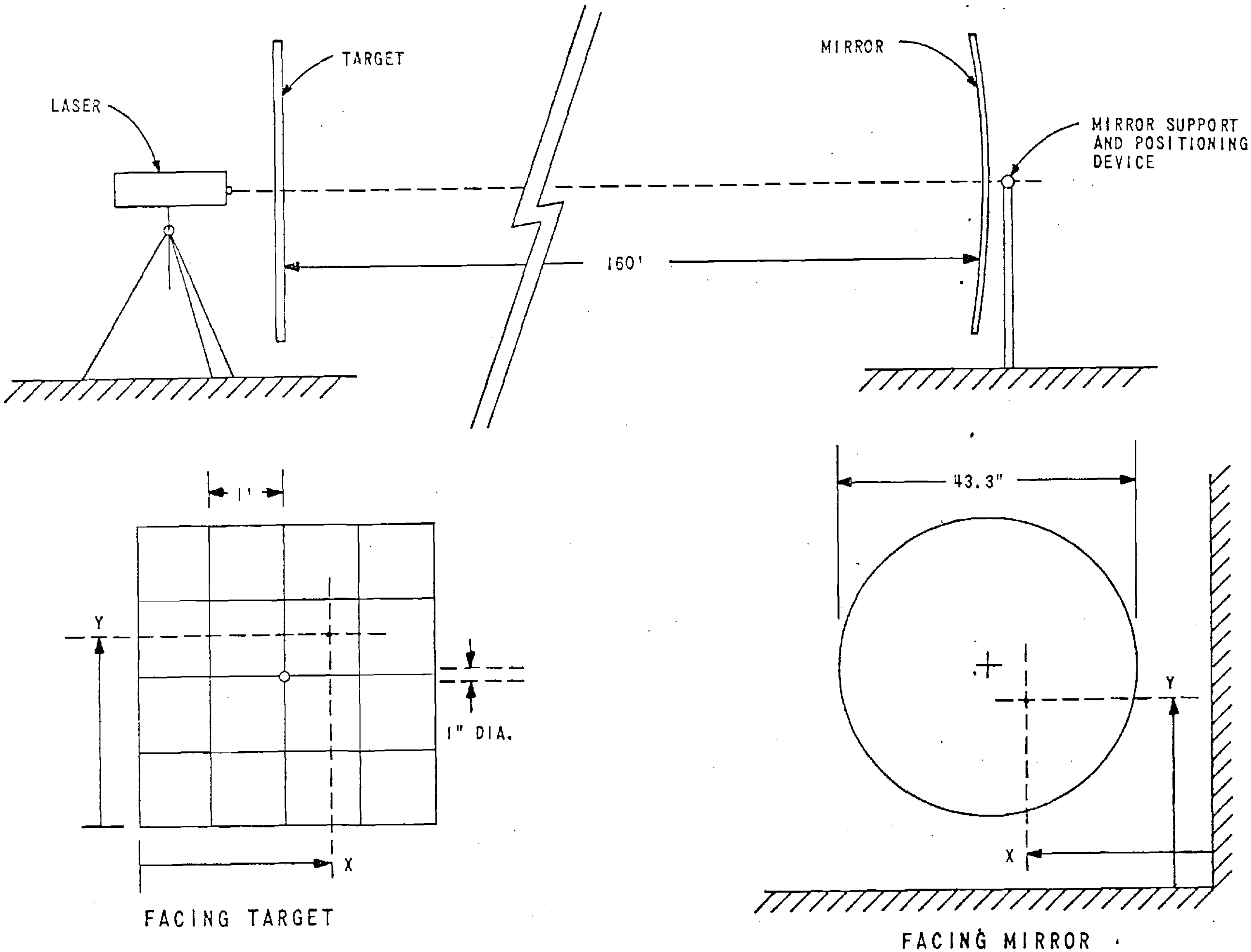


Figure 4. Schematic of Test Equipment for Mirror Surface Slope Error Tests.

large mirrors. (The Hartmann test is described by William A. Calder in "The Hartmann Test," Amateur Telescope Making Advanced, Munn and Company, Inc., 1946.)

A helium-neon laser was placed at the target aperture and its beam reflected off the mirror and back to the target. A perfectly spherical mirror would have reflected the beam back to the laser aperture, but the curvature of a real mirror is, in general, imperfect. Thus, the reflected laser beam usually arrived at the target some distance from the laser aperture. The coordinates of the beam positions on the mirror and target were recorded for 20 positions on the mirror surface and from these data the slope errors of the mirror surface were estimated.

Ten combinations of mirrors and frames were tested using drawn glass mirrors and eleven combinations of mirrors and frames were tested using float glass mirrors. For each mirror and frame combination tested, the average slope error in the X and Y directions were calculated for the 20 test points. Also, the standard deviations in the X and Y directions were computed. From these two standard deviations, an overall standard deviation of slope error was computed. For drawn glass mirrors the overall standard deviations ranged from 1.05 to 3.69 milliradians and for float glass mirrors these values ranged from 2.12 to 3.01 milliradians. From these and other analysis the following conclusions were drawn:

(1) The slope errors for drawn glass and float glass mirrors were not substantially different.

(2) Distortion of the focussed mirrors due to flexure of the mirror support frames was imperceptible in comparison to the inherent imperfection of the mirrors.

C. Measurement of RMS Pointing Errors for Tracking Mechanisms

Upon installation of the test tower at the Georgia Tech STTF, it became feasible to accurately align several kinematic motions (mirror support mechanisms) and focus heliostats on a target attached to the tower. This permitted the actual performance of the tracking mechanisms to be observed for a whole day. A kinematic motion with support frame and mirror is shown in Figure 5.

A small electric motor was fitted with suitable reduction gears to drive three kinematic motions, and the drive and kinematic motions were installed in the STTF heliostat field. At the recommendation of ANSALDO engineers who were at Georgia Tech during this period of time, the drive motor was controlled by a duty-cycle timer so that the tracking speed could be varied to obtain small adjustments in the rate of mirror movement. The kinematic motions were aligned in accordance with the procedure specified by ANSALDO and operated for a complete day to observe tracking performance. The position of the reflected image from each mirror was recorded photographically every hour. Subsequently, the centers of the image positions were plotted from the photographic negatives; a typical plot for the three kinematic motions is shown in Figure 6.

A total of 13 sets of tracking data were collected; each set represented one day of operation although some days were incomplete because of interference by clouds. The kinematic motions were realigned between some

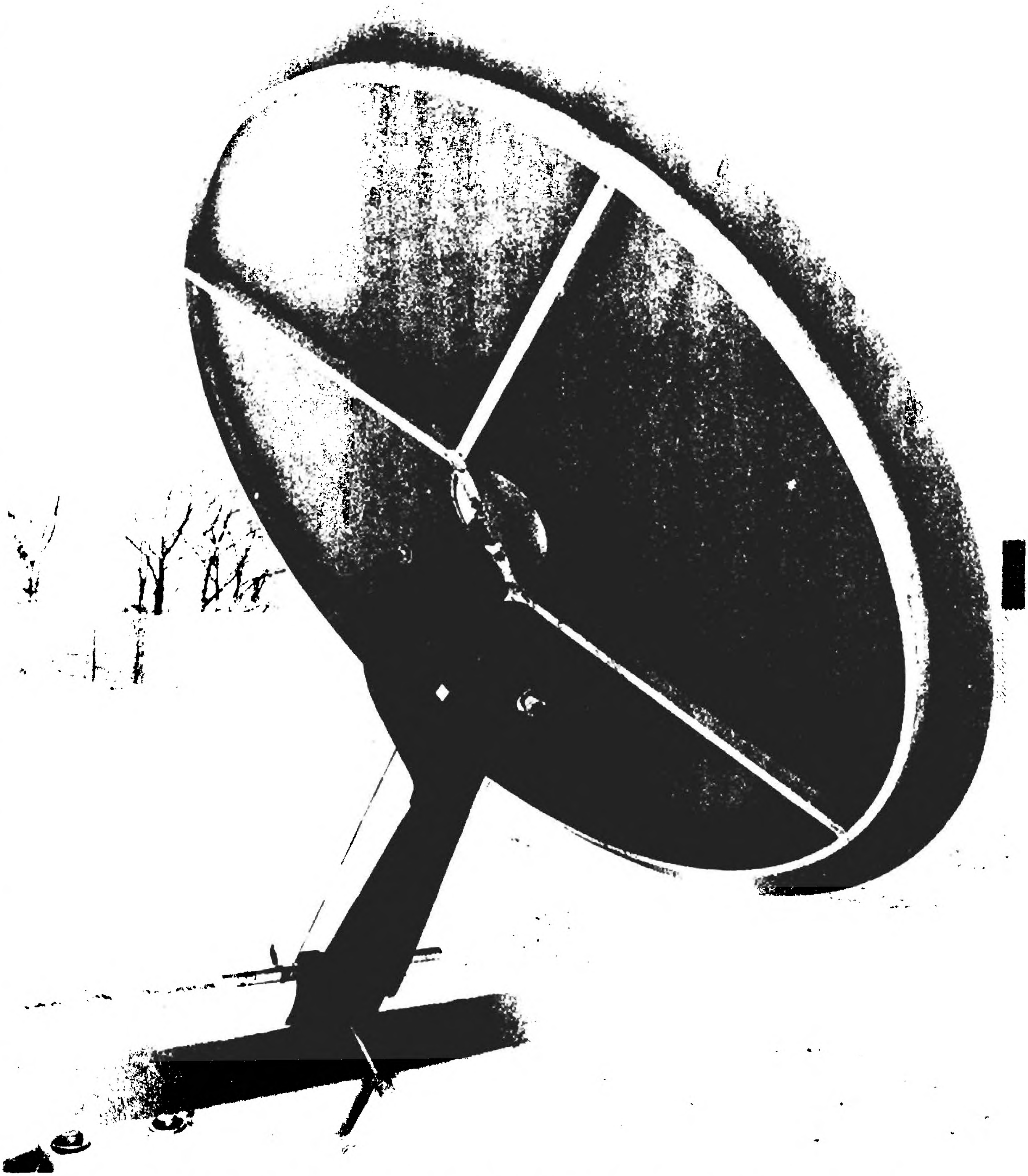


Figure 5. Georgia Tech Kinematic Motion with Mirror and Mirror-Support Frame.

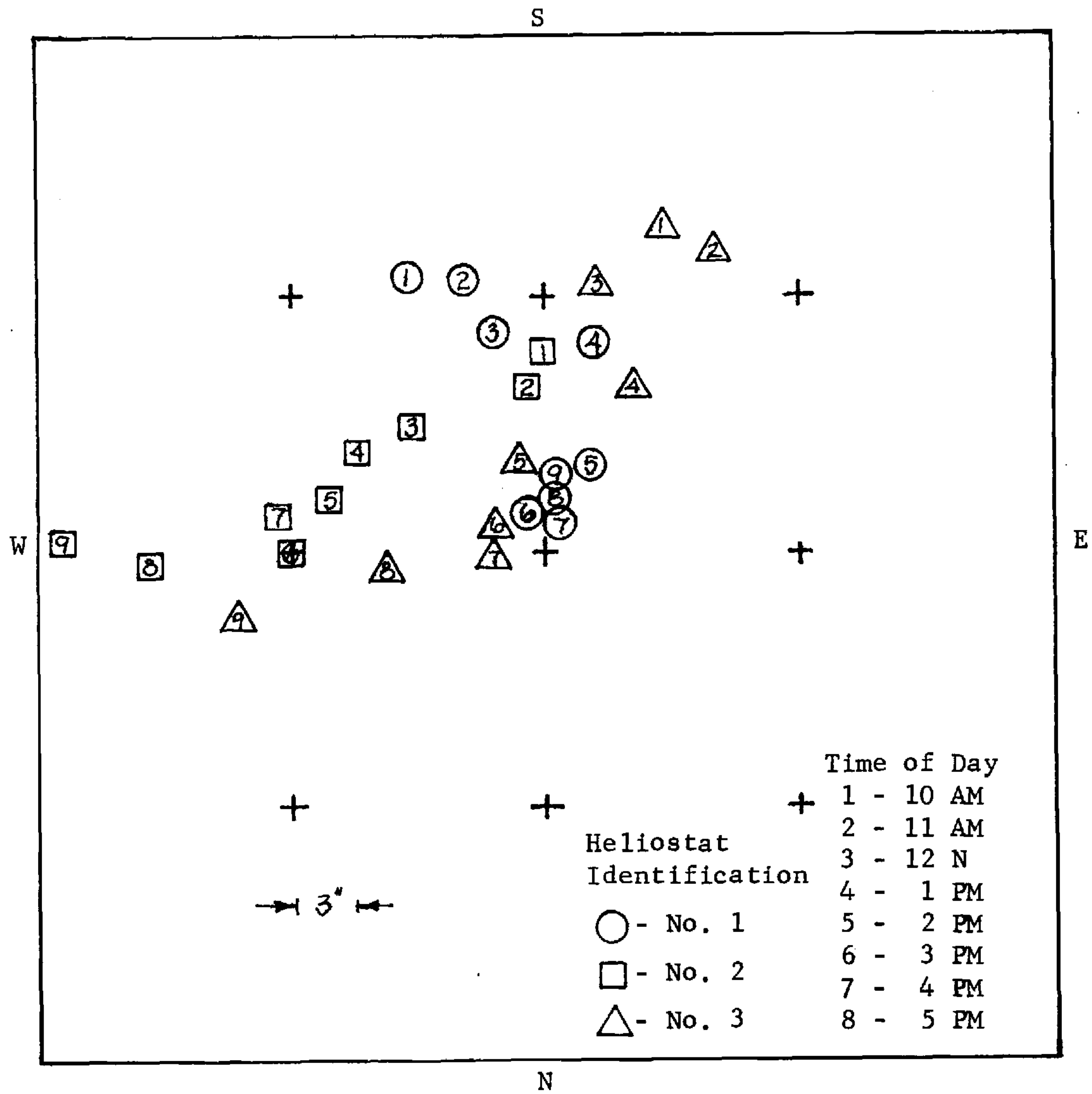


Figure 6. Typical Plot of Tracking Heliostat Images for One Day (Solar Noon Occurs at 1:30 PM).

data sets in order to measure operator variability in alignment. After the first five data sets the image movements on the target were observed to have been in the range of 10 to 30 milliradians from the beginning to the end of the day. Since the tracking mechanism error budget had been established at four to six milliradians, an effort was undertaken to identify the source of the unacceptably high errors and to improve tracking performance.

The tracking error problem was discussed by telephone with Professor Francia, and he reported that the tracking errors of 8 to 10 milliradians have been experienced at St. Ilario, but that errors of 2 to 3 milliradians are more typical.

It was suspected that the source of error might be misalignment of the equatorial axes of the kinematic motions. This alignment was checked using the star Polaris as a reference; true north is a position in the sky approximately 50 minutes of arc from Polaris at a circumferential position which varies with time. It was concluded that the Georgia Tech equatorial axes were out of alignment by 0.5 to 1 degree, but after new alignments were made no substantial improvement in tracking accuracy was observed. These difficulties remained unresolved when work on the program was discontinued.

D. Calculation of Tracking Errors as a Function of Heliostat Misalignment

The alignment of a kinematic motion is subject to errors arising from human operator variability as well as systematic errors in the tools employed. ANSALDO has devised an alignment procedure for the Georgia Tech system which consists of three major steps:

(1) Placing of the equatorial axis parallel to the earth's axis of rotation--This is accomplished by first positioning the kinematic motion arm perpendicular to the east-west supporting beams which have been carefully aligned. Then a leveling platform, which has the appropriate latitude angle incorporated into its structure, is attached to the kinematic motion and a spirit level is placed on the platform; four shim bolts are adjusted to level the platform.

(2) Positioning of the fixed pivot point on a line from the kinematic motion to the receiver--This is accomplished using a tool which incorporates a rifle telescope to establish the required line between the kinematic motion and receiver; the tool also sets the fixed pivot point at the correct radius from the center of the kinematic motion.

(3) Synchronization of the hour angle--This is accomplished using a tool which locks the rotating equatorial axis in the solar noon position. When all kinematic motions in the field have been placed at the noon hour angle, they will subsequently move together to the hour angle required to track the sun at any time of day.

A geometric analysis was accomplished to express the position of the reflected beam from any heliostat on the target, given the position of the heliostat in the field, the error in equatorial axis alignment, the error in placement of the fixed pivot point, the time of day, and the position of the target (receiver). Because the mathematical computations to perform the calculation were quite laborious and it was desired that many cases be examined (many values of time, heliostat position and alignment parameters), the analytical expressions were programmed to be run on a PdP/8A computer. A description of the calculations performed by the program, a program listing, and a diagram of the coordinate systems is given in Appendix A.

The first cases examined by the computer analysis were for a heliostat position matching one of the tracking test heliostats and equatorial axis misalignments of 0.5 and 1 degree east of true north. The target spots were predicted to move across the target from north to south, which generally agreed with behavior observed during the tracking tests; however, the tests also showed a west to east component of motion which was not predicted by the computer program. Trials of other misalignment cases were interrupted because the tracking tests themselves had not shown improvement when the equatorial axes were aligned using Polaris.

If the computer analyses had been carried further, the planned approach was to search for misalignment conditions in which predicted behavior of the target spots was consistent with the observed behavior. If this could have been achieved, then the exact character of the alignment errors could have been inferred. It should also have been possible to identify those alignment errors which had the most critical effect on heliostat performance.

V. CONCEPTUAL DESIGN OF SOLAR COLLECTOR TRACKING MECHANISMS

The design of the collector field was approached by first developing four tracking mechanism concepts, then selecting the most promising of these for further design and structural analysis. The proposed tracking principle was the equatorial-mount, constant-drive speed system first constructed by Giovanni Francia at the University of Genoa in Italy. This system of heliostat control is used in the Georgia Tech 400 kW Solar Thermal Test Facility and offers certain advantages in mechanical simplicity in comparison to two-axis heliostat tracking methods. To document the principle of operation, a brief description of the mechanism is given below. Following this description, the conceptual design effort performed for the deep well irrigation plant is reported.

A. Heliostat Installed at Georgia Tech

The heliostat is equipped with a drive mechanism known as a "kinematic motion" developed by Professor Francia and first demonstrated experimentally at St. Ilario. Figure 7 (a) shows the principle of operation of the "kinematic motion." Point A is used as a reference point. Line AB is the extension of a line drawn from the sun through point A. Line CA is the extension of a line drawn from the receiver through point A. Lines CA and AB are of equal length and form the equal sides of the equilateral triangle ACB. The line CM is an extension of the side of the triangle ACB. A mirror is placed at point M perpendicular to line MCB. Since line MCB is parallel to the bisector of angle SAR, the mirror surface will reflect the light from the sun (point S) onto the receiver (point R). To maintain this

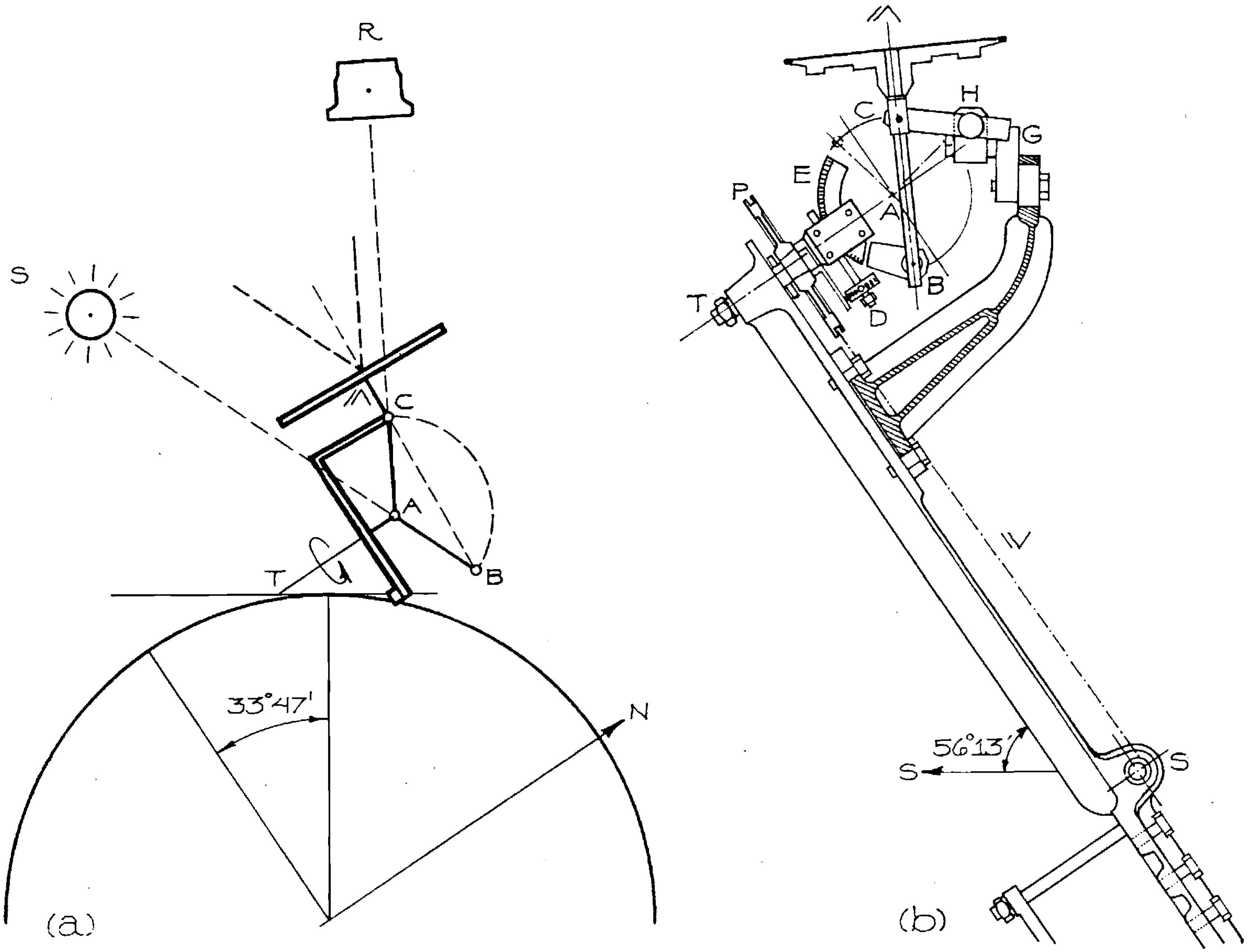


Figure 7. Schematic and Drawing of Kinematic Motion for Georgia Tech Solar Thermal Test Facility.

relationship as the sun moves through the day, point B must rotate about axis TA, (parallel to the earth's axis) at 15^{0}/hr and MCB must rotate around point C. This figure is oriented so that axis TA is parallel to the earth's axis when located at the latitude of Atlanta (33^{0} 47'). Figure 7 (b) is a drawing of the kinematic motion and support arm for the Georgia Tech test facility. As illustrated in this figure the support arm will be mounted at an angle of 56 degrees 13 feet from the horizontal facing south. The axis of rotation is shown by line AT which is located parallel to the earth's axis. Rotation is provided by a cable W around the pulley at P and driven through the shaft S. Alignment with the sun (line AB) is provided by a worm gear at D acting on the circumferential gear arm E. Declination adjustments also are provided through D. Alignment of the receiver (line AC) is provided through point H attached to a movable collar on the rod G. The kinematic motion support arm is attached to the east-west beams of the heliostat supporting structure as shown in Figure 7 (b). These beams are 4 inch x 4 inch square tubes with a 1/8 inch thick wall mounted so that the south facing side is inclined at an angle of 33 degrees 47 feet from the vertical.

B. Development of Heliostat Design Concepts

The heliostat mechanisms designed by Professor Francia and installed at St. Ilario and at Georgia Tech were believed to have several aspects which might be improved by additional design effort. The disadvantages recognized in Francia's designs were high capital cost and a requirement for labor-intensive installation and maintenance procedures.

The high capital costs of the tracking mechanisms result from the use of numerous precision machined parts, designed and manufactured specifically for this application. ANSALDO showed in its contract for the Georgia Tech 400 kW Solar Thermal Test Facility that its procurement cost for the tracking mechanisms was $200 each. Georgia Tech contacted several machine parts manufacturers for independent cost estimates to produce the Georgia Tech mechanisms in lots of 550, and received estimates ranging from $1,200 to $2,000 per unit. Thus, procurement costs for duplicates of the Georgia Tech mechanisms were not known with a high level of precision, but were probably in excess of $200 each. Since the collector system of any solar thermal apparatus is a dominant cost item, manufacturing cost was a prime concern in the conceptual design studies of the irrigation system tracking mechanism. The design effort was planned to take advantage of mass production manufacturing methods and to incorporate standard commercial components wherever possible.

The Francia tracking mechanism was not believed to possess the structural rigidity and strength which would be required for the irrigation system heliostats. Another troublesome feature of the Francia mechanism was the manner in which the mirror support point (point C of Figure 7) was located and secured. The linkage connecting point C and the frame was, for some mirror positions, difficult to arrange and tighten securely. It was intended to simplify this design feature. Finally, declination adjustment in the Francia mechanism must be accomplished manually for each individual mechanism, it is highly desirable that this be accomplished automatically or at least remotely.

Four conceptual designs were developed. The layouts for these designs were made to conform to the requirements of Figure 8. In the upper left

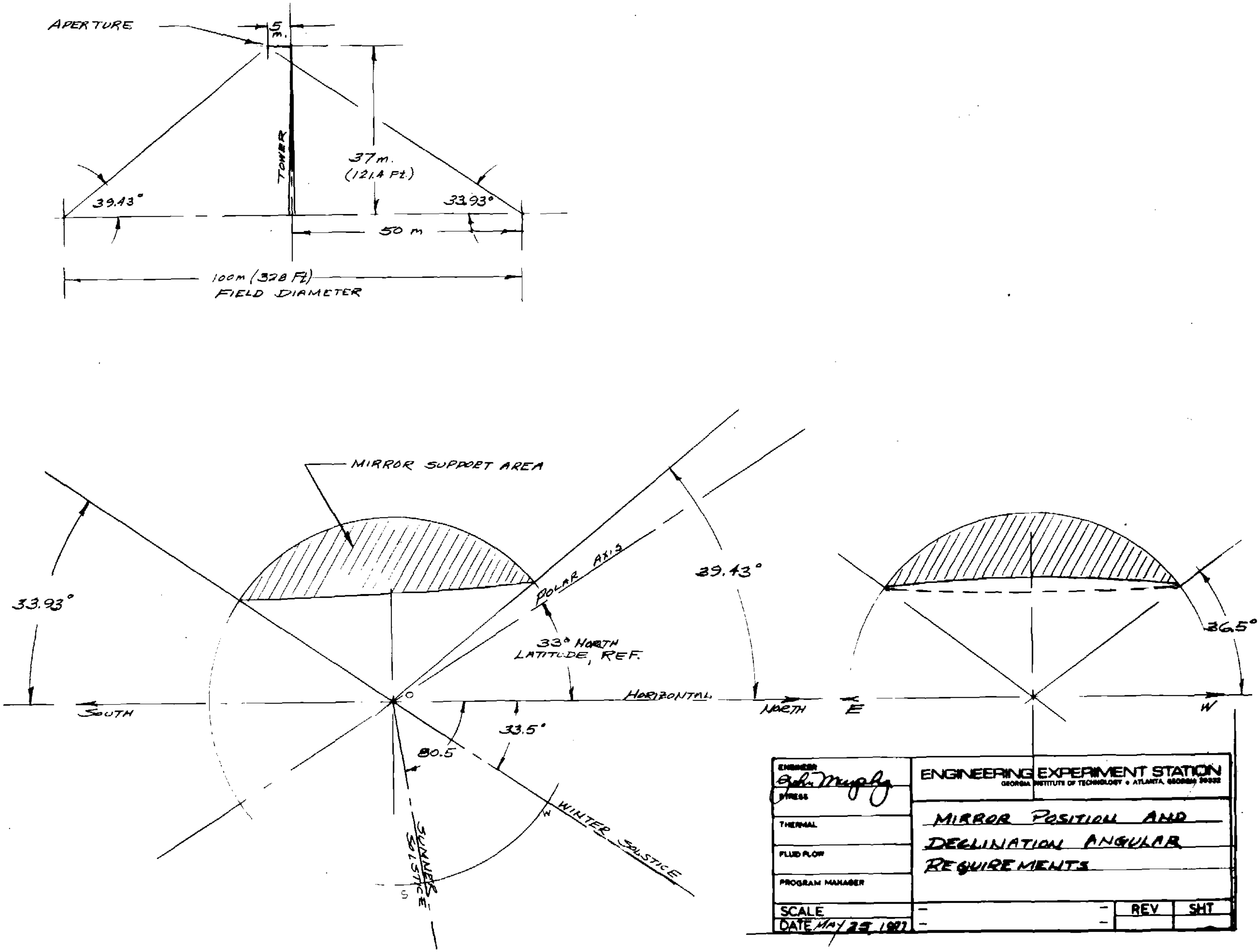


Figure 8. Mirror Position and Declination Angular Requirements.

corner of Figure 8 is shown the mirror field diameter and target position. This fixes the range of angles which are described by the line drawn from the reference point A through the mirror support point C and pointing to the target (see Figure 7); therefore, the required positions for the mirror support point C will be on the shaded cap of the sphere as shown at the bottom of Figure 8. The mechanism drive axis (polar axis) was drawn in Figure 8 for a 33 degree north lattitude reference. Positions of the mirror drive point (point B of Figure 7) must be adjusted between summer solstice and winter solstice as shown in Figure 8.

The four conceptual designs are shown in Figures 9 through 12. In Figure 9, the reference point A is a real point on the drive shaft of the mechanism. Drive point B is connected to A by a rigid link. Positioning of point B for declination requirements is accomplished by a worm and worm wheel. This makes positioning of point B simple and accurate. On the other hand, support of the mirror at point C is by means of a structure which is not integral with the rest of the mechanism. This means that location of point C is somewhat more difficult and would require special tooling. However, whereas point B must be continuously repositioned throughout the year, point C must be located only once. In order to maintain proper clearances and prevent interference during the motion of the mechanism, drive point B is connected to the mirror by an off-set link. Freedom of motion of the off-set linkage while maintaining rigidity was the main concern of this design.

In the design of Figure 10, the positions of the support and drive points were reversed, that is, drive point B is now above the polar axis

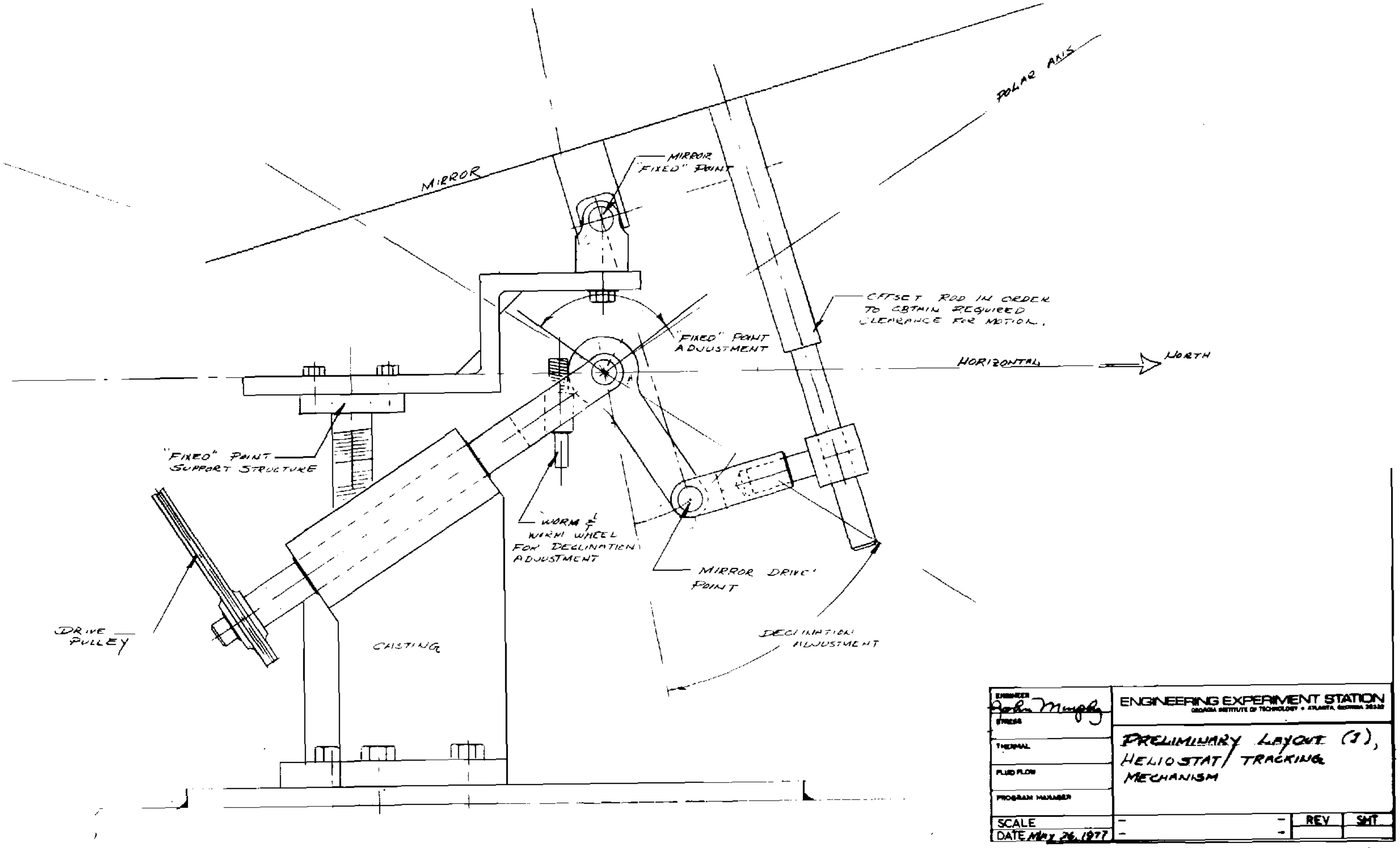


Figure 9. Preliminary Layout No. 1, Heliostat Tracking Mechanism.

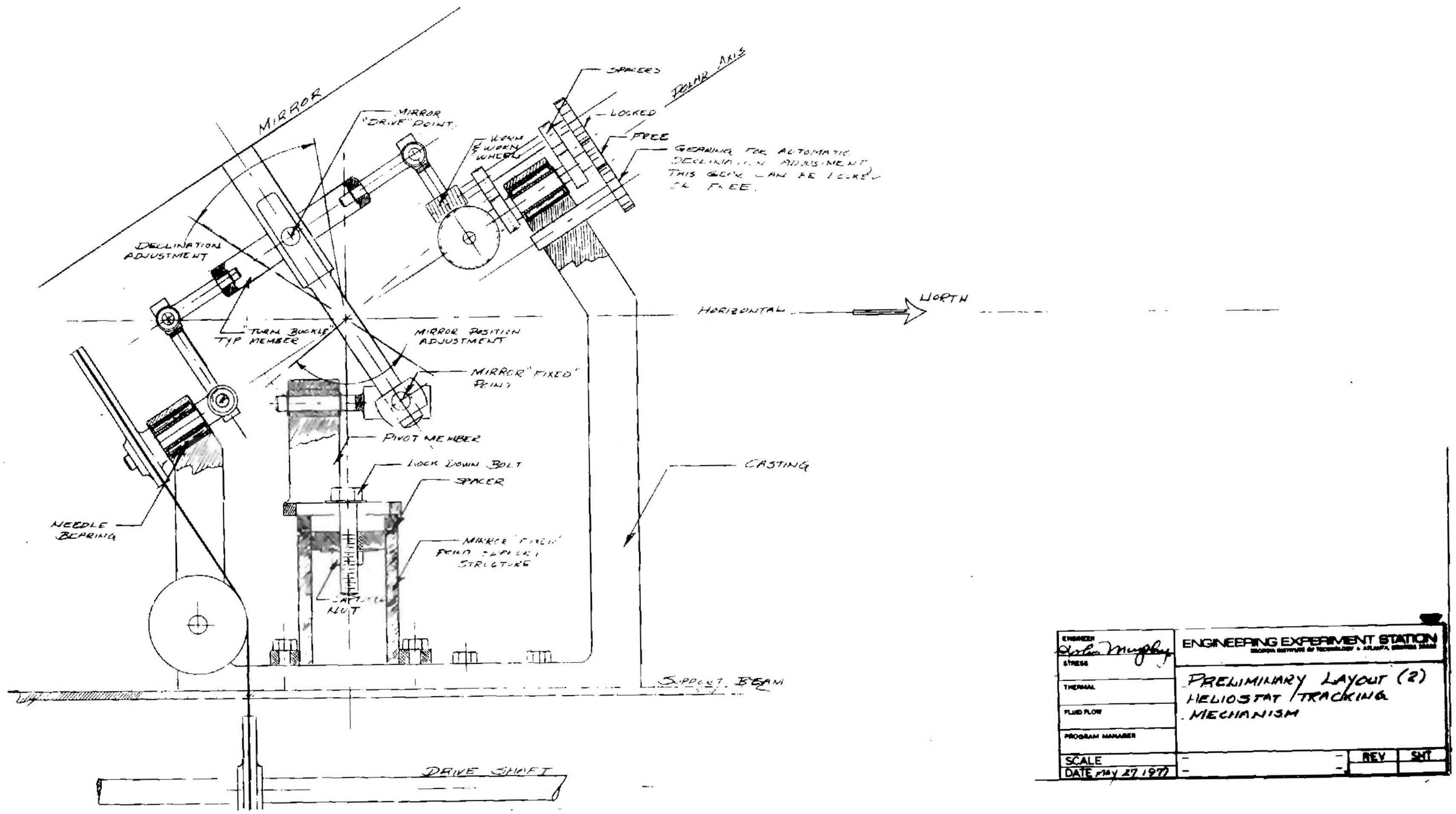


Figure 10. Preliminary Layout No. 2, Heliostat Tracking Mechanism.

and support point C is below. There are two advantages to be gained in placing the support point on the underside: (1) more room is available and it is unnecessary to reach over the top of the mechanism to support the mirror, and (2) the problem of interference between the mirror support rod and the mechanism drive shaft is largely eliminated. The mirror drive point is above the polar axis and declination adjustment is accomplished by a parallel linkage which keeps the drive point B at the correct radial distance from the center of the sphere (reference point A). By means of a solenoid and gear train, the declination angle can be automatically changed during the daily rotation (or nightly return) of the mechanism. The weakness of this design appeared to be in the complexity of the drive linkage as well as its probable lack of adequate strength and rigidity.

Figure 11 shows a design concept similar to Figure 10. In this design, an attempt was made to overcome the lack of rigidity and strength of the parallel linkage by replacing it with a more rigid member and gear sector. In addition, the drive shaft has been moved from the lower south side of the mechanism to the upper north side. This results in more room being available for positioning the support point C. A disadvantage in the designs of Figures 10 and 11 is that placing the support point of the mirror on the lower side and the drive point on the upper side will produce more swing of the mirror during the daily tracking.

The fourth conceptual design is shown in Figure 12. Here the mirror support and drive points are in the original positions of the Georgia Tech installed mechanism and the design of Figure 9. Both points are attached to gear sectors which in turn can be rotated about the drive shaft axis. Hence,

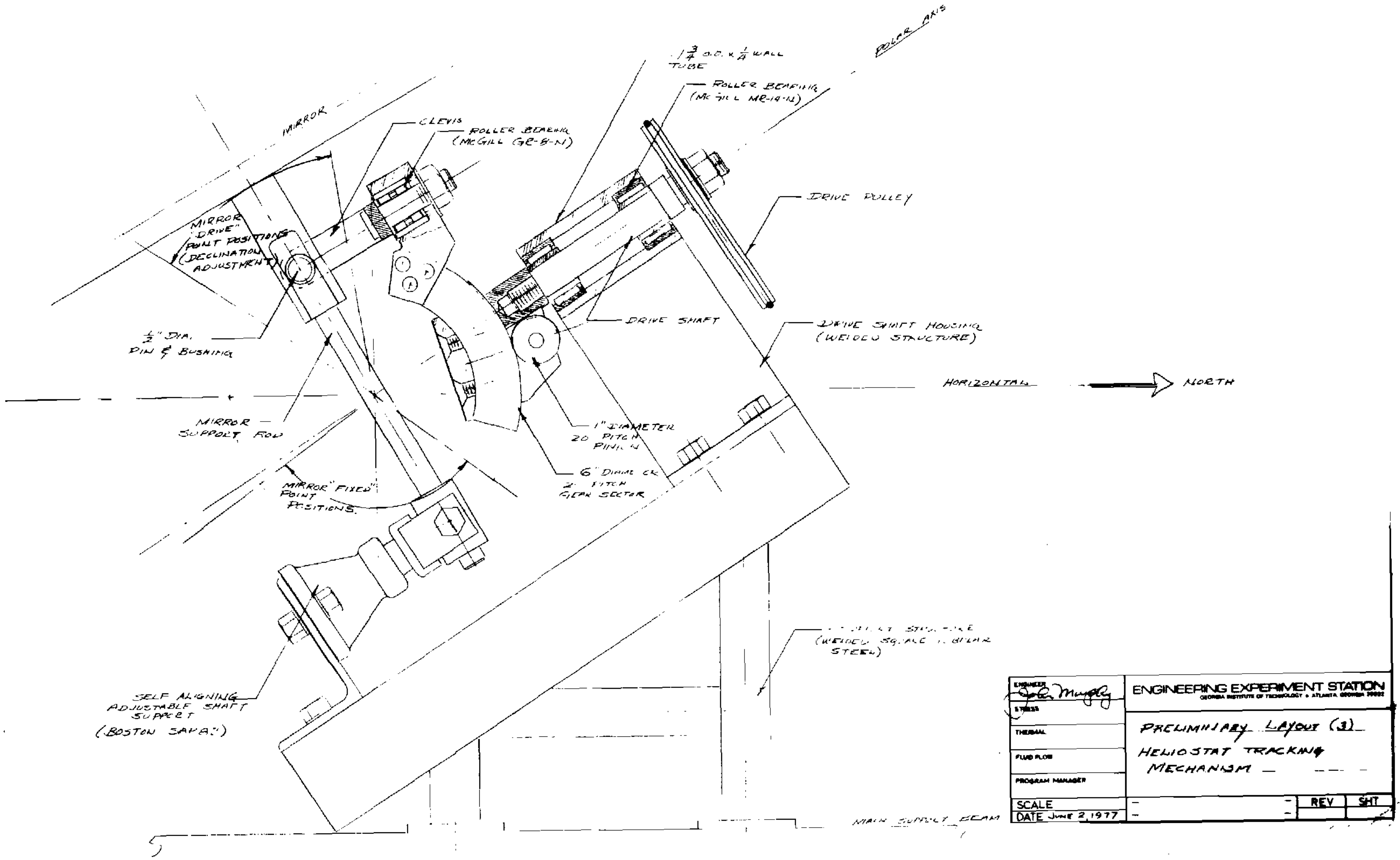


Figure 11. Preliminary Layout No. 3, Heliostat Tracking Mechanism.

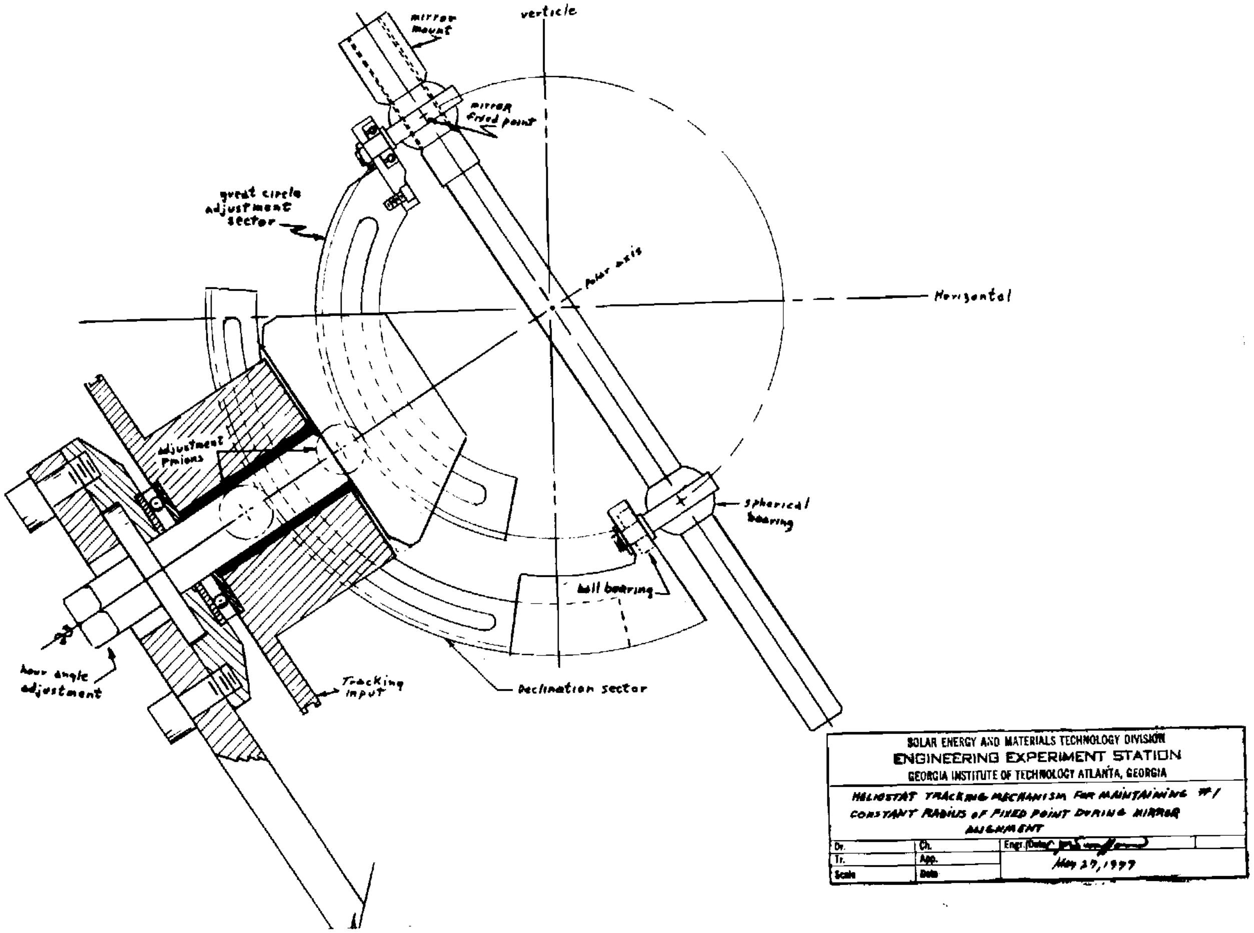


Figure 12. Preliminary Layout No. 4, Heliostat Tracking Mechanism.

the support point can be quickly and accurately located for the various heliostats and the drive point is easily adjusted to the required declination setting.

A manufacturing analysis of the four designs was made in order to establish their relative cost. In this analysis it was assumed that the mechanisms were to be produced in large quantities and, therefore, special tooling and manufacturing techniques, such as die casting, stamping, etc., were assumed to be employed. Also, wherever possible, commercially available components were found and used in the individual mechanisms. Thus, the price of a particular design reflects the material (or component) cost plus man-hour cost under large quantity production conditions. The results of this analysis are presented in Table II. Cost data were assembled from two local machine parts manufacturers and from technical literature on the design of mechanical devices.*

At the conclusion of the conceptual design study, a conference was held between Black and Veatch and Georgia Tech personnel in which the design of Figure 9 was selected for further development. This design was believed to offer the most rigid and durable structure and would be the most cost effective.

C. Investigation of Drive Motors for Tracking Mechanisms

A search has been conducted for motor and bearbox assemblies suitable for driving individual tracking mechanisms, both hour angle and

*Herbert F. Rondeau, "The 1-3-9 Rule for Production Cost Estimation," Machine Design, August 21, 1975, pp 50-53.

TABLE II

COSTS OF CONCEPTUAL TRACKING MECHANISMS

Design	Material and/or Component Cost ($)	Labor Cost, at $12.00/hr ($)	Total Cost ($)
Figure 9	50.00	125.00	175.00
Figure 10	70.00	130.00	200.00
Figure 11	90.00	120.00	210.00
Figure 12	75.00	150.00	225.00

declination movements. Engineering consideration applicable to this selection were: cost, accuracy of drive speed, ability of many units to be operated synchronously, power consumption, control complexity and cost, ability to rapidly move heliostats to the sunrise position or off target, drive power and torque. The devices considered included synchronous motors, solenoids, and stepping motors.

On the basis of synchronization and high-speed movements, the class of synchronous motors was eliminated from consideration. After starting, a group of synchronous motors will run at the same speed but the length of the time required to start and stop them is variable. Minor variations in speed would require the use of a power supply whose frequency could be varied. A separate motor or gearbox would be required to furnish rapid-movement capability. Small synchronous motors are inefficient users of electric power.

Rotary solenoids would require a design and manufacturing effort especially for this application. A ratchet mechanism would be required, as would a separate solenoid for reversing. The power supply would need staggered pulsing in order to spread the large current drain which occurs when the solenoids are moved.

Stepping motors appear to be suitable for application on the tracking mechanism. Several manufacturers have been contacted and a variety of designs are available; the most acceptable from a cost standpoint is manufactured by Philips Electronics in the Netherlands and marketed in the United States by North American Philips, Incorporated. A unit has been identified which has suitable output capacity and would cost about $10 per assembly in quantities of 100 or more. The assembly consists of a motor, gearbox and electronic control module; electric power must be supplied at all times to the motor and a one-volt control pulse must be supplied to the control module when stepping is desired. Other manufacturers of stepping motors were eliminated from consideration because of high costs and complex control systems.

D. Selection of Mirror Glass for Collector Field

Gardner Mirror Corporation of North Wilkesboro, North Carolina was contacted regarding the availability of mirror glass in large sizes. In 6 mm (0.25 inch) thickness, the largest size Gardner has made is 84 x 130 inches and the largest standard size is 72 x 144 inches. In 3 mm (0.125 inch) thickness, the largest size considered feasible is 48 x 84 inches. These maximum sizes would be very difficult to handle during field assembly and selection of smaller mirrors was strongly recommended.

For planning purposes, costs of about $0.70 per square foot for 3 mm and $1.00 per square foot for 6 mm mirrors were suggested. Laminated mirrors cost about twice as much as unlaminated mirrors.

VI. PRELIMINARY DESIGN OF A SOLAR COLLECTOR SYSTEM

Four candidate heliostat design concepts had been generated, and one of these was selected for further development in the preliminary design task of the program. Since the sizing of structural parts was an integral part of preliminary design, it was also necessary to select a mirror size in order that mechanical loads could be specified. It had been determined that glass mirrors could be purchased with linear dimensions up to seven feet, although it was recognized that mirrors of this size would be difficult to handle during field installation. However, mirror area and the power collected per mirror increase in proportion to the square of the diameter. Therefore, as a first approximation, the largest feasible mirror size would lead to the smallest number of heliostat assemblies and the most economical system.

The mirror size influences the system cost through other considerations, particularly the physical scale of the tracking mechanisms. Larger mirrors impose higher structural loads and require more massive structural supports than smaller mirrors. The effect of this consideration on the optimum mirror size could not be determined until specific preliminary designs had been completed.

The heliostat concept shown previously in Figure 9 was chosen for preliminary design. It was planned that the heliostats would be supported on individual cylindrical concrete footings and that the equatorial axis alignment would be made by adjustment of the bolts which attached the heliostats to the footings. The power drive for the heliostats was to be by individual motor drives or by a single motor driving several heliostats;

one common power station, as used at Georgia Tech, was considered impractical in a field as large as anticipated for the irrigation system. Similarly, declination adjustment was to be accomplished remotely for each heliostat or for the group which were coupled to a common drive.

The structural requirements of the heliostats were considered to be governed by wind loading. The collector field was to survive a 100 mile-per-hour wind and be operational during a 20 mile-per-hour wind. A considerable effort was made to find a reliable prediction method for the determination of wind loads. The load acting on a flat plate subjected to wind is given by:

$$\text{Load} = C\ q\ s$$

where

Load is either a force or moment.

C is a coefficient depending on angle of attack.

$q = \frac{1}{2}\ \rho V^2$

s = surface area of one side of the plate.

Two references were employed which were somewhat contradictory. "Marks Handbook" was used to determine the maximum normal wind force on the plate where the value of C is given as 1.7.* No information is given in the handbook for evaluation of moments or other loading conditions. The paper by Brown and McKee** gives a smaller C value for the maximum normal load and

*Baumeister and Marks, Standard Handbook for Mechanical Engineers, McGraw Hill, 7 Ed., p 12-19.

**J. S. Brown and K. E. McKee, "Wind Loads on Antenna Systems," The Microwave Journal, September 1964.

gives a value for the maximum moment. The value of C for the maximum moment is ½ d, where d is the plate diameter. The 1.7 value was used for calculation of the normal wind load on the mirror and the ½ d value for calculation of the moment, however these two loading conditions occur at different angles of attack and therefore were assumed not to act simultaneously.

Before starting the preliminary design, a computer program was written which calculated the angular positions and dimensions of all heliostat components during its motion. This was done in order to determine the most critical heliostat configuration, so that adequate clearances could be maintained. This computer program is given in Appendix B.

First, consideration was given to a seven foot diameter mirror. A mounting ring for support of the mirror was designed first and is shown in Figures 13 and 14. This ring was proportioned to give the required strength for mirror survival under the 100 mph wind condition and to give the required stiffness under all operating conditions. The heliostat mechanism for the seven foot mirror is shown in Figure 15. Design calculations were made and the mirror support arm and off-set link were proportioned for survival strength and operating stiffness. Other parts of the linkage were detailed, but design calculations were not completed. At this point it was evident that the mechanism for the seven foot mirror was massive; therefore, it was decided to postpone completion of the design and to consider a smaller mirror size.

A design was initiated on a heliostat mechanism to support a five foot diameter mirror. The mirror support ring is shown in Figure 16 and the mechanism is shown in Figure 17. Results from the computer program showed

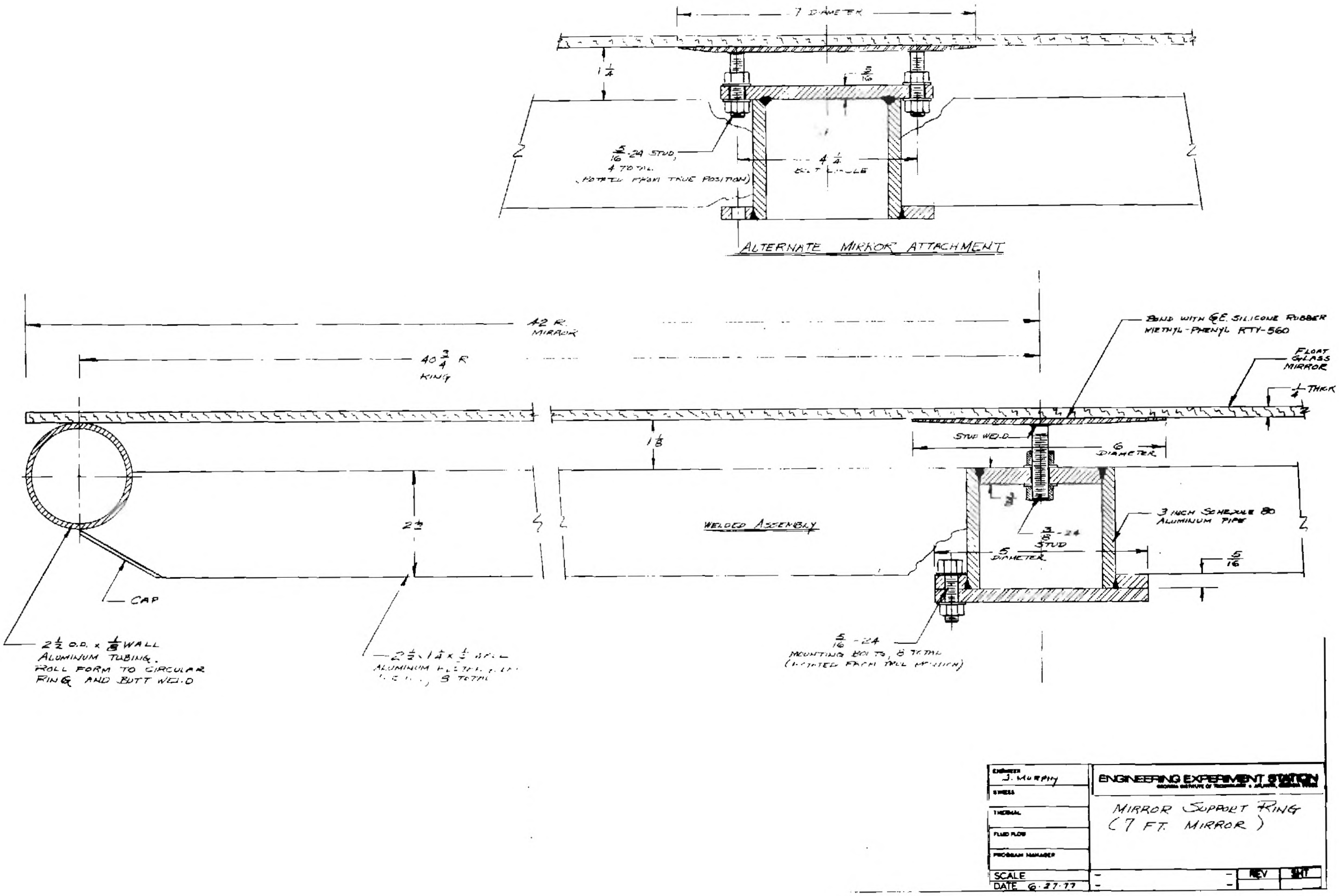


Figure 13. Mirror Support Ring for Seven Foot Mirror.

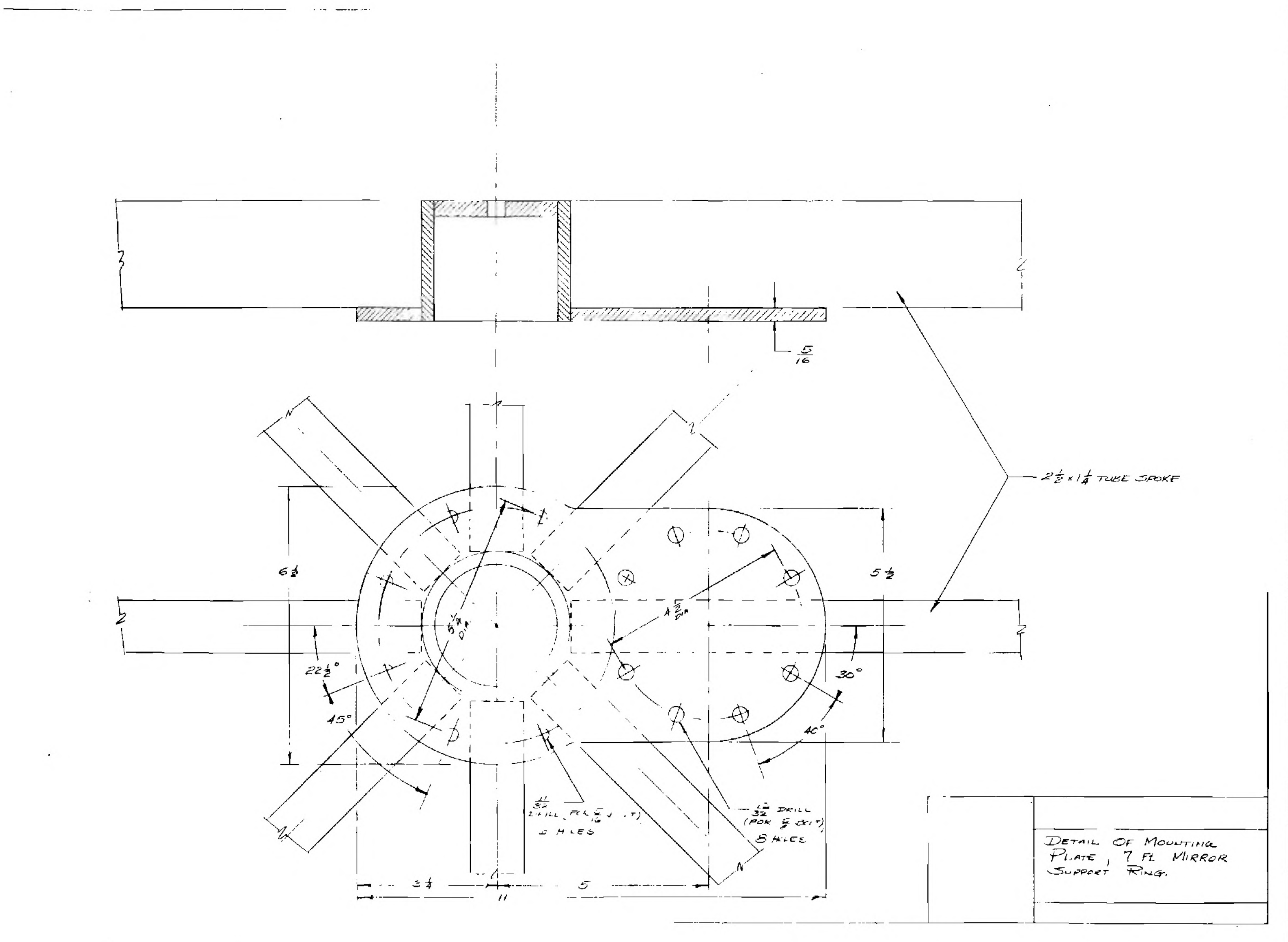


Figure 14. Mounting Plate Detail for Seven Foot Mirror Support Ring.

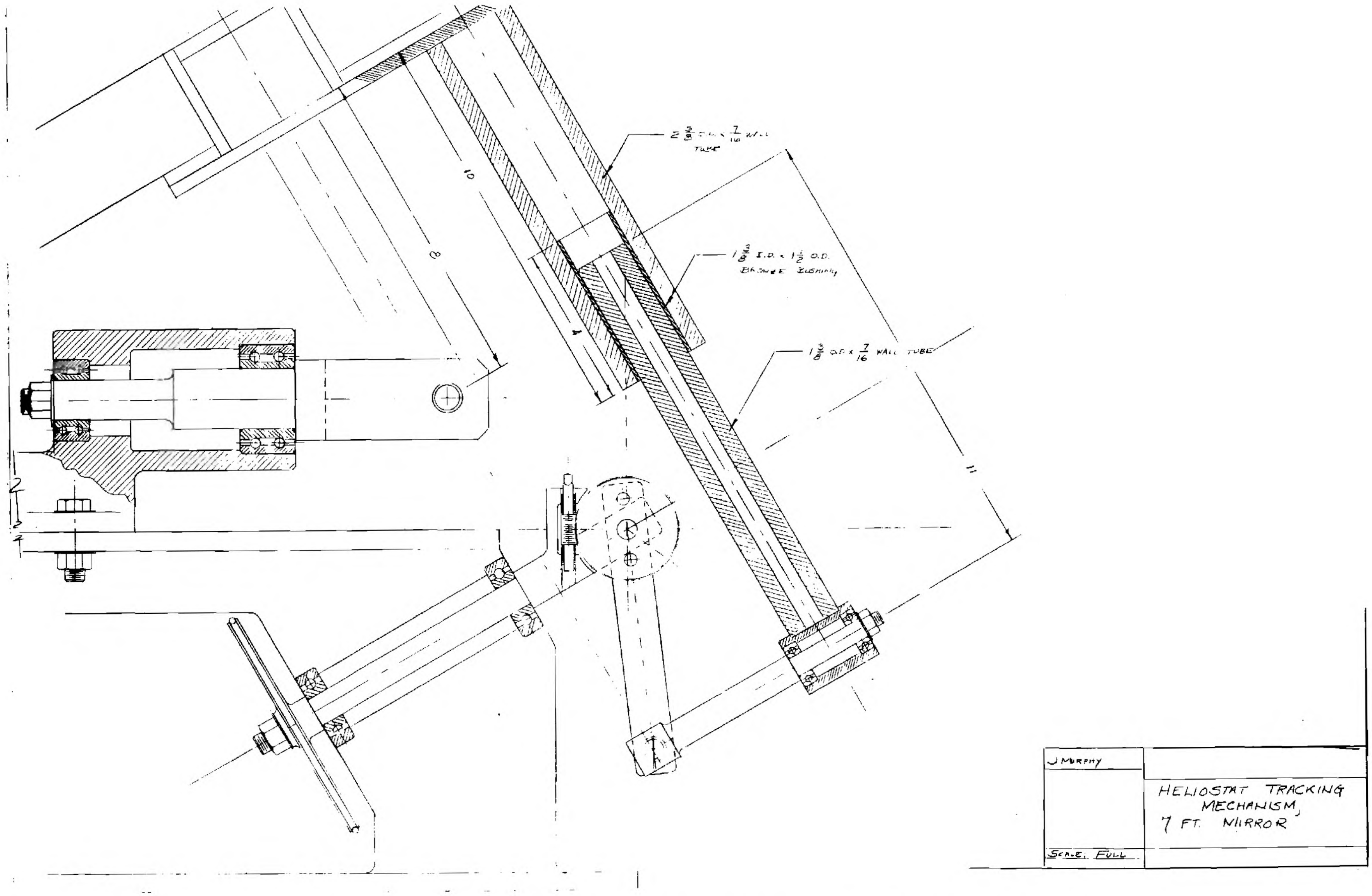


Figure 15. Heliostat Tracking Mechanism for Seven Foot Mirror.

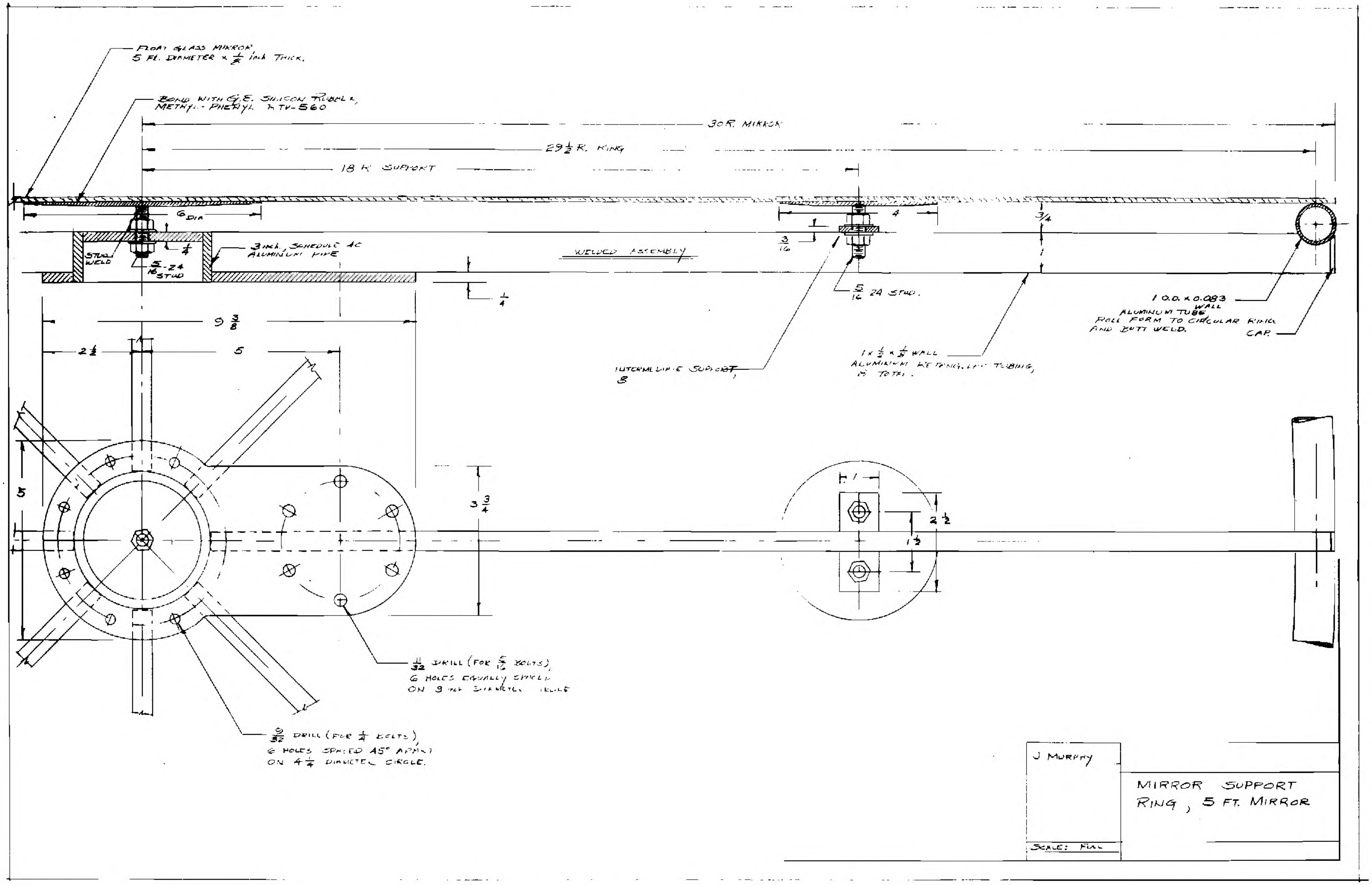


Figure 16. Mirror Support Ring for Five Foot Mirror.

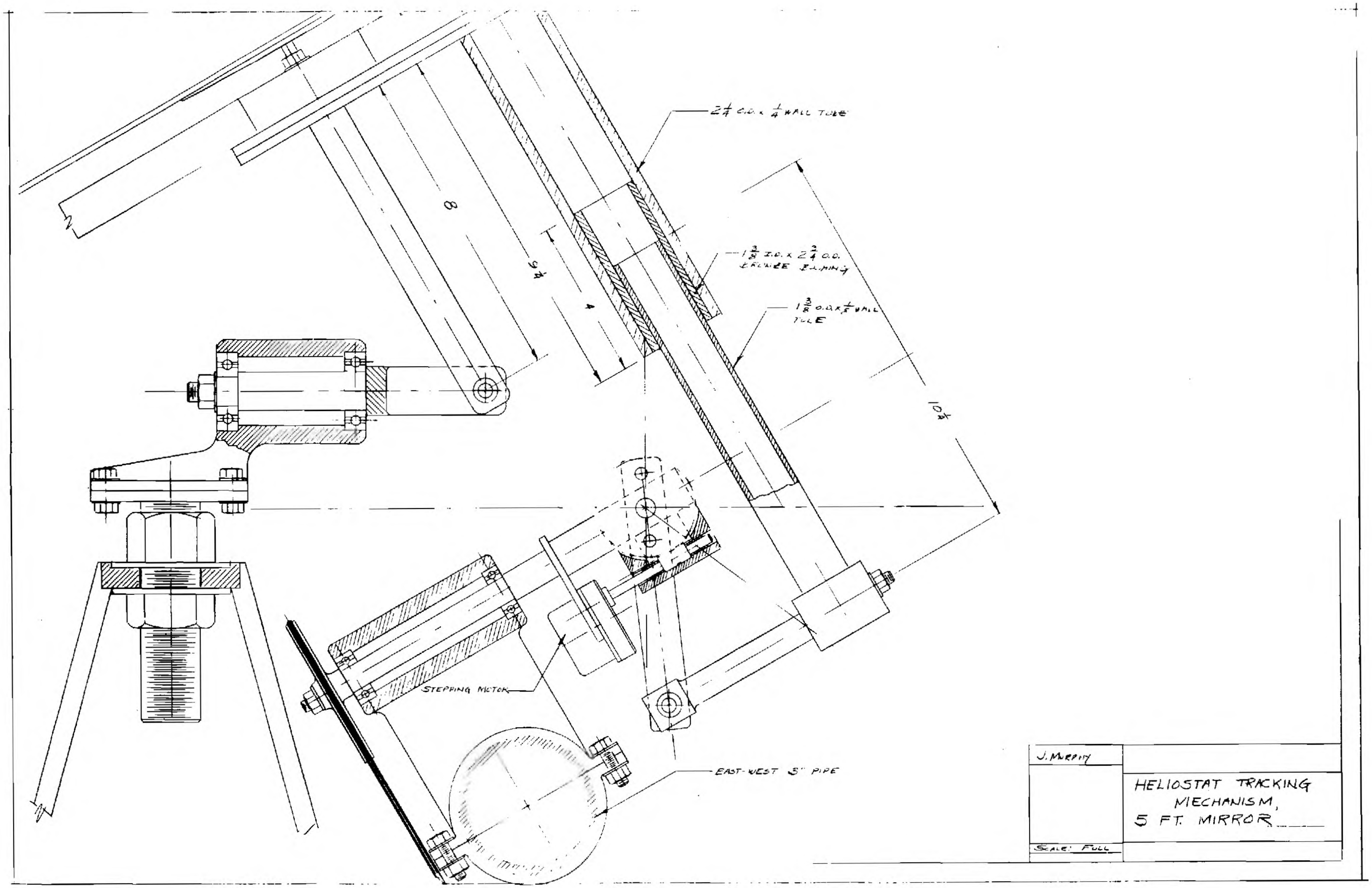


Figure 17. Heliostat Tracking Mechanism for Five Foot Mirror.

that the required basic overall dimensions of the mechanism were similar to those for the seven foot mirror heliostat. However, design calculations resulted in mechanism components subtantially lighter than the components of the seven foot heliostat. In this design, the drive shaft, declination adjustment method and drive shaft housing were detailed. Declination adjustment is accomplished by a worm and worm wheel driven by a stepping motor. The stepping motor is manufactured by Philips Electronics in the Netherlands and marketed in the United States by North American Philips. The stepping motor system was selected over other drive methods, such as synchronous motors and rotary solenoids, because it offers more positive control and could be easily integrated into the mechanism. The motor manufactured by Philips Electronics was found to be more economical than those of other manufacturers.

The design effort was terminated prior to completion of the detail of the main drive arrangement and the mirror support structure. Although considerable work remained, it was believed that the five foot mirror heliostat was the more realistic and could be developed into an economical and reliable design. In his design for the Georgia Tech system, Professor Francia had selected a mirror diameter of 111 cm (43.7 inches), an increase from 78 cm (30.7 inches) over his most recent installation at Genoa. Francia's decision to increase the mirror diameter to about 3.5 feet and our conclusion that five feet is more practical than seven feet, all suggest that the optimum mirror size is in the range between three and five feet. However, the question can be answered only by performing heliostat designs in sufficient detail to evaluate system costs, including manufacturing costs of the heliostat mechanisms and costs of installation and alignment.

At the conclusion of the program, all design work was discontinued and this report was prepared to document the results of studies to date.

APPENDIX A

COMPUTER PROGRAM FOR DETERMINATION OF MECHANISM TRACKING ERROR

This program determines the horizontal displacement of the reflected beam from the intended target. It allows for several errors being present in the mechanism. The program assumes that the mirror fixed support point (point C) is located correctly such that a line from the center of the sphere through this point is pointing to the intended target. However, the mirror drive point (point B) may be set at a declination angle which is in error from the true declination angle by an amount DE^o. The radial position of the fixed point, Rt, and the radial position of the drive point, Rs, may be set differently; indicating that one of these points does not lie on the required spherical surface. Finally, the axis of the mechanism may be set such that it is not aligned with the true polar axis by assigning angles ERX^o, ERY^o and ERZ^o, which the mechanism X, Y, Z axis makes with the polar X, Y, Z axis.

The mirror position is established by assigning it's X and Y coordinates and the height of the target as shown in Figure A-1. The program immediately establishes a new coordinate system located at the mirror and oriented with the polar axis as shown in Figure A-2. All calculations are made in the new coordinate system except that the final X and Y displacements (XD and YD) of the reflected beam from the intended target are given in a horizontal plane.

Following is given the step by step input and calculation procedure. The calculation used to determine the true declination angle for a given time of year is also given.

The computer program was written to run on a PdP-8 computer using a modified version of the language FOCAL. A listing of the program along with the results of a typical computer run is given.

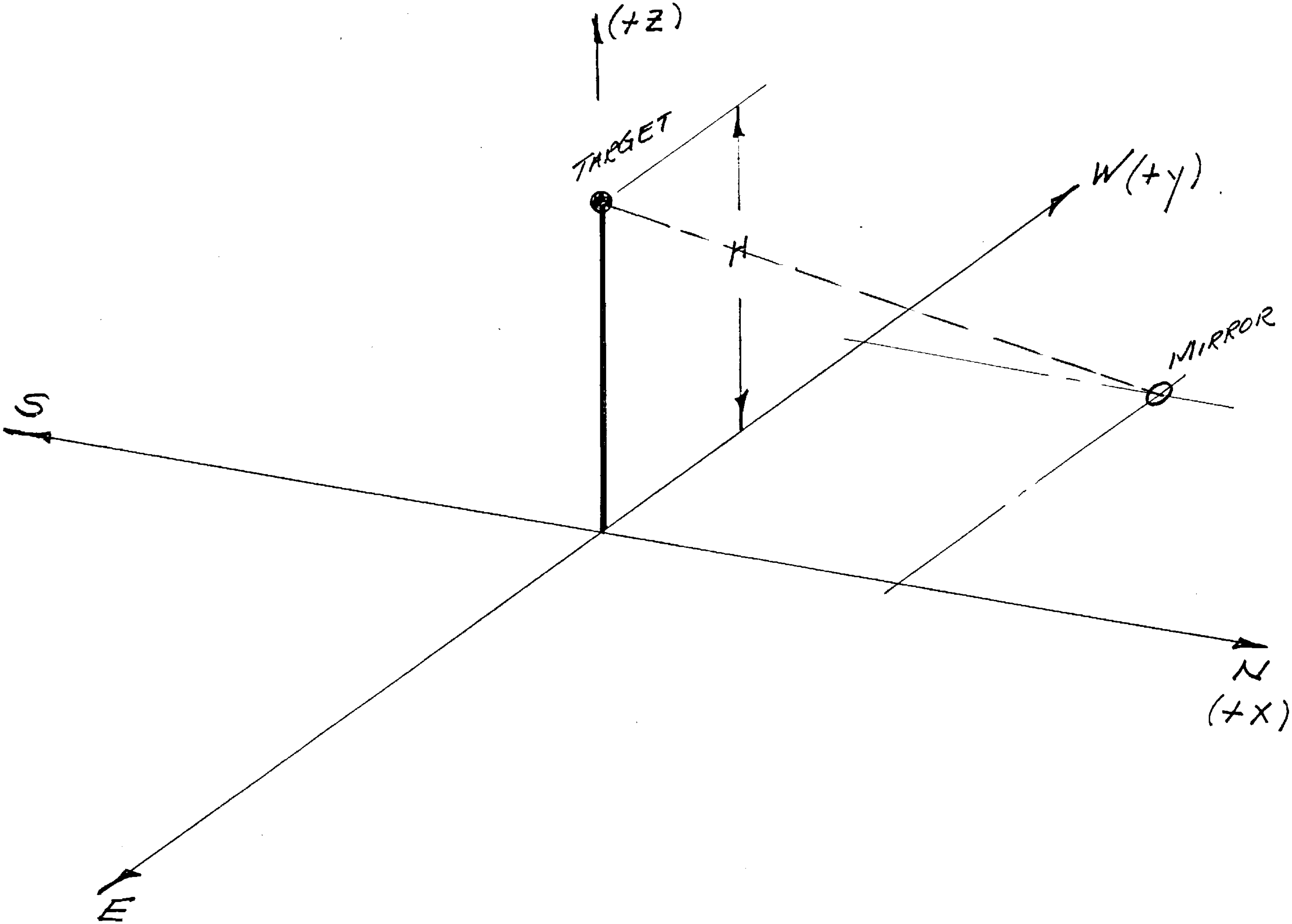


FIGURE A.1 INITIAL COORDINATE SYSTEM FOR MIRROR LOCATION

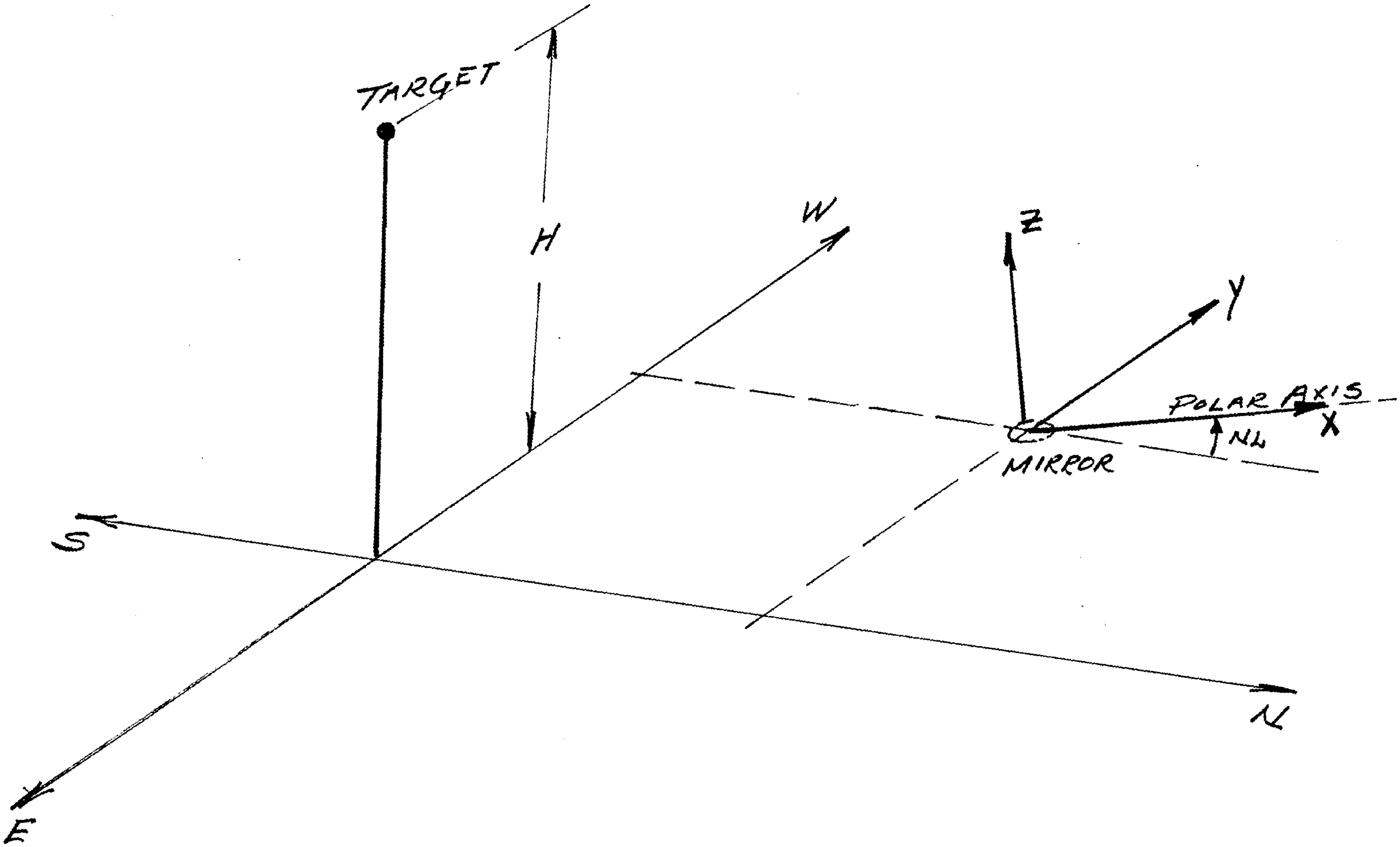


FIGURE A·2 COORDINATE SYSTEM FOR COMPUTATIONAL PROCEEDURE

FRANCIA TRACKING ERROR ANALYSIS

INPUT DATA

1. Position of Mirror
   X & Y (Ft.)

2. Height of Target, above mirror
   H (Ft.)

3. Time of Year
   EP, an angle (°)
   EP=0, Winter solstice
   EP=180, Summer solstice

4. Degrees North Lattitude
   NL, an angle (°)

5. Length of ~~Mirror~~ Mechanism Link pointing to Target
   RT (inches)

6. Length of Mechanism Link pointing to sun
   RS (inches)

7. ANGLES That Drive Axis makes with True polar axis.

(EX) ERX , angles (°)
(EY) ERY
(EZ) ERZ

8. Set Declination Error

DE , angles (°)

CALCULATIONS

1. Set up new coordinates, located at mirror and oriented with true polar axis. Calculate coordinates of target in this system.

$$X = H \sin(NL) - X \cos(NL)$$

$$Y = -Y$$

$$Z = (X - H \tan(NL)) \sin(NL) + \frac{H}{\cos(NL)}$$

Calculate this first

2. Calculate distance from mirror to target

$$DT = \sqrt{X^2 + Y^2 + Z^2}$$

3. Calculate direction cosines of ray pointing to target

$$DXT = X/DT$$
$$DYT = Y/DT$$
$$DZT = Z/DT$$

4. Calculate coordinates of fixed point C.

$$XT = (RT)(DXT)$$
$$YT = (RT)(DYT)$$
$$ZT = (RT)(DZT)$$

5. Calculate true declination ~~error~~ angle.

$$TDEC = \text{ArcCos}\left[-\text{Sin}(23.5)\,\text{Cos}(EP)\right]$$

6 Calculate declination angle set on mechanism

$$SDEC = TDEC + DE$$

Remaining calculations are made for any given time of day.

We must set time of day (actually an input)

Set

TIDAY, an angle (°)

TIDAY = 0, morning

TIDAY = 180, afternoon.

7. Calculate "tentative" coordinates of drive point B.

$$XST = (RS)\ \cos(SDEC)$$
$$YST = (RS)\ \sin(SDEC) \cos(TIDAY)$$
$$ZST = -(RS)\ \sin(SDEC)\ \sin(TIDAY)$$

8. Calculate correct coordinates of drive point B because of misalignment.

Calculate following:

(Note, these could be calculated prior to setting TIDAY)

$$LX = \cos(EX)$$
$$MX = \cos(EY)$$
$$NX = \cos(EZ)$$

$$LY = -\frac{MX}{\sqrt{(LX)^2 + (MX)^2}}$$

$$MY = \frac{LX}{\sqrt{(LX)^2 + (MX)^2}}$$

$$NY = 0$$

$$LZ = -\frac{(LX)(NX)}{\sqrt{(LX)^2 + (MX)^2}}$$

$$MZ = -\frac{(MX)(NX)}{\sqrt{(LX)^2 + (MX)^2}}$$

$$NZ = \sqrt{(LX)^2 + (MX)^2}$$

Now calculate correct coordinates.

$$OXS = (XST)(LX) + (YST)(LY) + (ZST)(LZ)$$

$$OYS = (XST)(MX) + (YST)(MY) + (ZST)(MZ)$$

$$OZS = (XST)(NX) + (YST)(NY) + (ZST)(NZ)$$

9. Calculate direction cosines of normal to mirror

$$L = \sqrt{(XT-OXS)^2+(YT-OYS)^2+(ZT-OZS)^2}$$

$$D1 = \frac{XT-OXS}{L}$$

$$D2 = \frac{YT-OYS}{L}$$

$$D3 = \frac{ZT-OZS}{L}$$

10 Calculate cosine of angle between sun and mirror.

$$COTH = -(D1)\cos(TDEC) - (D2)\sin(TDEC)\cos(TID\ldots + (D3)\sin(TDEC)\sin(TIDAY)$$

11. Calculate direction cosines of reflected ray.

$$D4 = 2(D1)(COTH) + \cos(TDEC)$$

$$D5 = 2(D2)(COTH) + \sin(TDEC)\cos(TIDAY)$$

$$D6 = 2(D3)(COTH) - \sin(TDEC)\sin(TIDAY)$$

12. Calculate angle between reflected ray and target.

$$CRT = (DXT)(D4) + (DYT)(D5) + (DZT)(D6)$$

$$ANG = \mathrm{ArcCos}\,(CRT)$$

13. Calculate direction cosines of reflected ray in a coordinate system located at mirror but with X Y plane horizontal and Z vertical.

$$D7 = (D4)\,\mathrm{Cos}(NL) - (D6)\,\mathrm{Sin}(NL)$$

$$D8 = D5$$

$$D9 = (D4)\,\mathrm{Sin}(NL) + (D6)\,\mathrm{Cos}(NL)$$

14. Calculate distance to spot on target (Z coordinate equal H)

$$DST = H / (D9)$$

15. Calculate corresponding X + Y coordinates of spot at Z = H

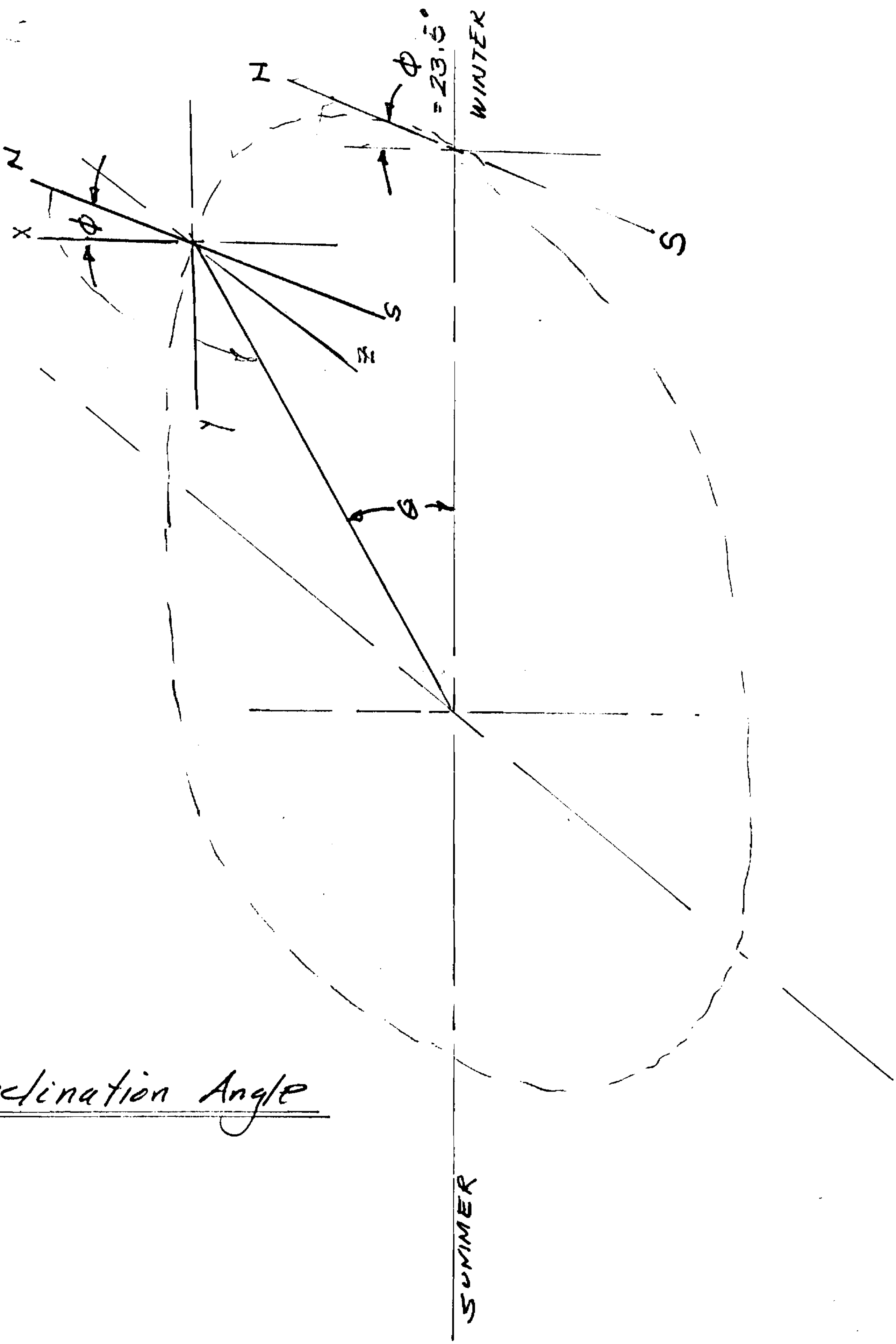
$$XR = (DST) \cdot (D7)$$
$$YR = (DST)(D8)$$

16 Calculate X coordinate of target in horizontal-vertical coordinate system.

$$XX = X \cos(NL) - Z \sin(NL)$$

17. Calculate deviations

$$XD = XR - XX$$
$$YD = YR - Y$$



Declination Angle

Direction Cosines of N-pole.

$$\cos\phi, \quad \cos(90+\phi), \quad \cos 90$$

or

$$\cos\phi, \quad -\sin\phi, \quad 0$$

Direction Cosines of direction to sun.

$$\cos 90, \quad \cos\theta, \quad \cos(90-\theta)$$

or

$$0, \quad \cos\theta, \quad \sin\theta$$

Therefore angle between N-pole and sun, "declination angle Δ" is

$$\cos\Delta = \cos\phi\,(0) + (-\sin\phi)\cos\theta + (0)\sin\theta$$

$$\boxed{\cos\Delta = -\sin\phi\cos\theta}$$

$$\Delta = \cos^{-1}\left[-\sin\phi\cos\theta\right]$$

```
C [illegible] 7/12/77

01.10 [illegible]
01.[illegible] S [illegible]FOUT(27)[illegible] Z=FOUT(72);S Z=FOUT(27);S Z=FOUT(74)
01.[illegible] V 1.25[illegible] 0,0,[illegible]
01.50 [illegible] !,"X,Y,H,EP,NL,RT,RS,ERX,ERY,EPZ,DE,TIDAY(START),TIDAY(STOP)",!
01.[illegible] F I=1,13;A Q(I)
01.65 D 1.5;F I=1,13;T !,I,Q(I)
01.70 T !,"CHANGES?(1 IF YES,0 IF NO)";S Z=FIN();IF(Z-177)1.75,,1.75
01.72 T !!,"ENTER INDEX,VALUE";A I,Q(I);G 1.65
01.75 S X=Q(1);S Y=Q(2);S H=Q(3);S EP=Q(4);S NL=Q(5);S RT=Q(6);S RS=Q(7);$"_$
      S [illegible]Q(8);S [illegible]Q(9);S EZ=Q(10);S [illegible]=Q(11)
01.80 S RC=.01745329;S EP=EP*RC;S NL=NL*RC;S EX=EX*RC;S EY=EY*RC;S EZ=EZ*RC;$"_$
      S DE=DE*RC     2
01.[illegible] [illegible] 1.9;T !!,"CALCULATING...",!!
01.[illegible] GO LPT;L O,F
01.92 T !!,"[illegible]",!;D 1.[illegible]
01.93 T !!;L O,C
01.[illegible] D 8
01.[illegible] F I=Q(12),10,Q(13);S TIDAY=I*RC;D 2;D 3;D 4
01.[illegible] G [illegible]

02.10 S DT=FSQT([illegible])
02.30 S [illegible]=X/DT;S [illegible]=Y/DT;S DZT=Z/DT
02.40 S XT=RT*[illegible];S YT=RT*[illegible];S ZT=RT*DZT
02.50 S TDEC=FSIN(.41[illegible])*FCOS(EP);$"_$
      S TDEC=(3.1[illegible])[illegible]FATN(TDEC/FSQT(1-TDEC*TDEC))
02.[illegible] S SDEC=TDEC+DE
02.[illegible] S X[illegible]=RS*FCOS(SDEC)[illegible];S YST=RS*FSIN(SDEC)*FCOS(TIDAY);$"_$
      S Z[illegible]=[illegible]FSIN(SDEC)*FSIN(TIDAY)

[illegible] FCOS([illegible]);S [illegible]FCOS([illegible]);S NX=FCOS(EZ)
02.[illegible] [illegible] LX[illegible]FSQT(LX*LX[illegible]);S NY=0
02.[illegible] S [illegible]=-[illegible]*NX);S NZ=-MX*NX/FSQT([illegible]*MX*NX);$"_$
      S [illegible]FSQT(LX*LX+[illegible])
02.[illegible] S DXB=X[illegible]*LX+[illegible]*LY+ZST*LZ
02.[illegible] S [illegible]ZST*MZ
02.77 S [illegible]ZST*NZ
02.80 S S1=XT-[illegible];S S2=[illegible];S S3=ZT-DZS;S L=FSQT(S1*S1+S2*S2+S3*S3);$"_$
      S [illegible]=S1/L;S D2=S2/L;S D3=S3/L
[illegible] S [illegible]=D[illegible]*FCOS(TDEC)-D2*FSIN(TDEC)*FCOS(TIDAY)+D3*FSIN(TDEC)*FSIN(TIDAY)

03.[illegible] [illegible]FCOS(TDEC);S [illegible]D2*COTH*FSIN(TDEC)*FCOS(TIDAY);$"_$
      S [illegible]COTH-FSIN(TDEC)*FSIN(TIDAY)
03.[illegible] S CRT=[illegible]*D3;S ANG=(3.1[illegible])[illegible](CRT/FSQT(1-CRT*CRT))
03.[illegible] S [illegible]FSIN(NL);S [illegible](NL)+D8*FCOS(NL)
03.40 S DST=H/D9;S [illegible];S YR=DST*D8
03.[illegible] S XR=X*FCOS(NL)-Z*FSIN(NL)
03.50 S XD=XR-X[illegible];S YD=YR-Y

04.[illegible] GO LPT;L O,D
04.10 T [illegible]"TIDAY",I,"     ANG",ANG,"     XD",[illegible]
04.[illegible] [illegible]

0[illegible] GO [illegible] O,F
0[illegible].20 [illegible]FOUT(12)
0[illegible] L C,C
0[illegible] [illegible]10[illegible]
0[illegible] GO [illegible] O,F
0[illegible] [illegible]FOUT(12)
[illegible]
```

[illegible]

?, H,EF,NL,RT,RS,ERX,ERY,ERZ,DE,TIDAY(START),TIDAY(STOP)

1.0000	33.5000
2.0000	30.5000
3.0000	76.0000
4.0000	160.0000
5.0000	33.0000
6.0000	-3.0000
7.0000	3.0000
8.0000	0.5000
9.0000	90.3535
10.0000	90.3535
11.0000	0.0000
12.0000	0.0000
13.0000	180.0000

TIDAY	0.0000	ANG	0.0072	XD,YD	-0.6140	0.3110
TIDAY	10.0000	ANG	0.0061	XD,YD	-0.4727	0.3092
TIDAY	20.0000	ANG	0.0050	XD,YD	-0.3526	0.3132
TIDAY	30.0000	ANG	0.0042	XD,YD	-0.1992	0.3217
TIDAY	40.0000	ANG	0.0037	XD,YD	-0.0371	0.3336
TIDAY	50.0000	ANG	0.0036	XD,YD	0.1787	0.3478
TIDAY	60.0000	ANG	0.0043	XD,YD	0.2914	0.3630
TIDAY	70.0000	ANG	0.0051	XD,YD	0.4474	0.3779
TIDAY	80.0000	ANG	0.0060	XD,YD	0.5915	0.3917
TIDAY	90.0000	ANG	0.0072	XD,YD	0.7193	0.4030
TIDAY	100.0000	ANG	0.0077	XD,YD	0.8769	0.4111
TIDAY	110.0000	ANG	0.0082	XD,YD	0.9112	0.4151
TIDAY	120.0000	ANG	0.0086	XD,YD	0.9700	0.4141
TIDAY	130.0000	ANG	0.0088	XD,YD	1.0017	0.4075
TIDAY	140.0000	ANG	0.0088	XD,YD	1.0057	0.3947
TIDAY	150.0000	ANG	0.0086	XD,YD	0.9825	0.3753
TIDAY	160.0000	ANG	0.0082	XD,YD	0.9336	0.3488
TIDAY	170.0000	ANG	0.0077	XD,YD	0.8617	0.3147
TIDAY	180.0000	ANG	0.0072	XD,YD	0.7712	0.2726

APPENDIX B

CALCULATIONS FOR DETERMINING LENGTHS AND ANGULAR POSITIONS OF MECHANISM LINKAGES

A program was written to run on a PdP-8 computer for calculating the dimensions and angular positions of mechanism linkages during its daily tracking motion. Data input can be assigned which represents any desired mirror position in the field, the north latitude location of the field and the time of year. Output of the program is the length of the mirror support rod between drive point B and support point C and the angles, in a horizontal rectangular coordinate system, of the rod at these two points.

In the following is given the step by step input and calculations used in the program, the basis for the calculations and the program listing along with a typical output.

PROGRAM FOR DETERMINING LINKAGE LENGTH AND ANGULAR CLEARENCE REQUIREMENTS

INPUT

1. Fixed point C position
   A an angle (°)
   B an angle (°)

2. North Latitude
   NL an angle (°)

3. Declination angle

   DEC an angle (°)
   [DE]

4 Radius of magic sphere

   R (inches)

5. Start, Stop, Increment TD angles (°)

CALCULATE

1. Direction Cosines and coordinates of point C.

$$CX = \cos(A)\ \sin(B)$$
$$CY = \cos(A)\ \cos(B)$$
$$CZ = \sin(A)$$

$$XC = (R)(CX)$$
$$YC = (R)(CY)$$
$$ZC = (R)(CZ)$$

2. Direction Cosines of Polar axis

$$LX = \cos(NL)$$
$$NX = \sin(NL)$$

$$MY = 1$$

$$LZ = -\sin(NL)$$
$$NZ = \cos(NL)$$

3, Now for each stepped value of TD (time of day, an angle in degrees - usually from 0 to 180) Calculate direction cosines and coordinates of drive point B

$$BX = (LX)\cos(DEC) - (LZ)\sin(DEC)\sin(TD)$$
$$BY = (MY)\sin(DEC)\cos(TD)$$
$$BZ = (NX)\cos(DEC) - (NZ)\sin(DEC)\sin(TD)$$

$$XB = (R)(BX)$$
$$YB = (R)(BY)$$
$$ZB = (R)(BZ)$$

4. Calculate length of mirror support rod between points C and B.

$$L = \sqrt{(XC-XB)^2 + (YC-YB)^2 + (ZC-ZB)^2}$$

$$\{\ S1 \qquad S2 \qquad S3\ \}$$

5. Calculate direction Cosines of mirror support rod.

$$RX = \frac{XC - XB}{L} = \frac{S1}{L}$$
$$RY = \frac{YC - YB}{L} = \frac{S2}{L}$$
$$RZ = \frac{ZC - ZB}{L} = \frac{S3}{L}$$

6. Calculate angles mirror support rod makes with support at point C

$$ACX = A1 = \cos^{-1}(RX)$$

$$ACXZ = A2 = \tan^{-1}\left(\frac{RZ}{RY}\right)$$

$$ACYZ = A3 = \tan^{-1}\left(\frac{RZ}{RX}\right)$$

7. Calculate angles mirror support rod makes with support at point B.

$$B1 = \tan^{-1}\left(\frac{BZ}{BY}\right)$$

$$B2 = \tan^{-1}\left(\frac{BZ}{BX}\right)$$

$ABXZ = A4 = A2 - B1$

$ABYZ = A5 = A3 - B2$

8. Print out

TD

$ACX = A1$

$ACXZ = A2$ — all angles (°)

$ACYZ = A3$

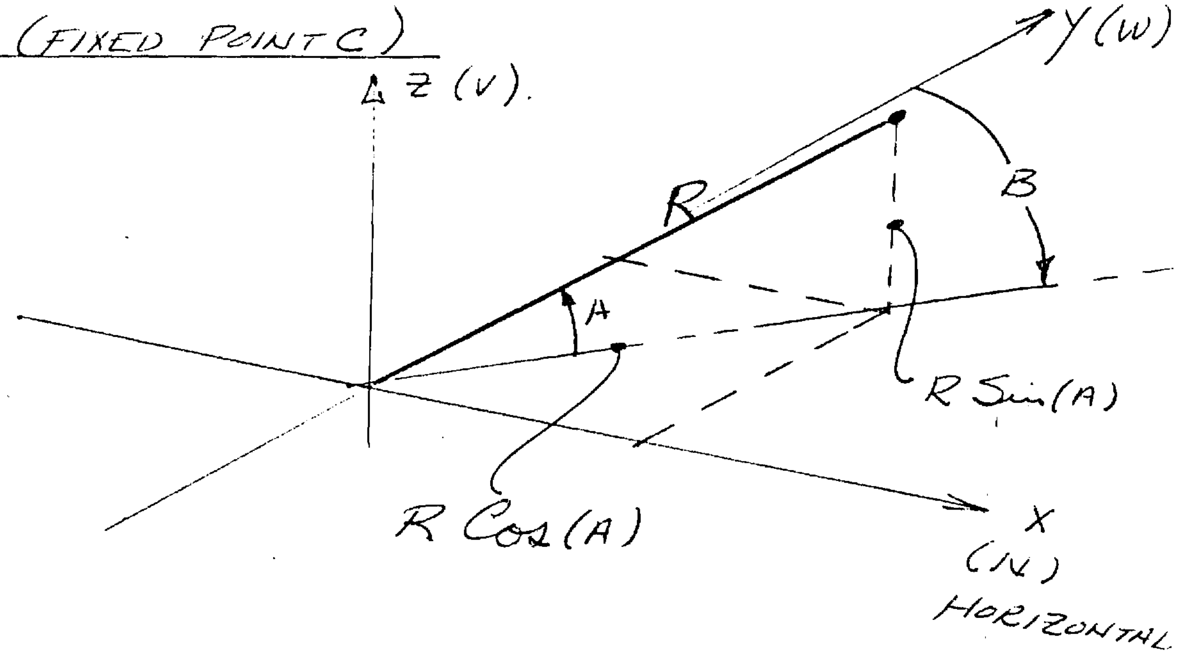
$ABXZ = A4$

$ABYZ = A5$

L — (inches)

BASIS FOR CALCULATIONS

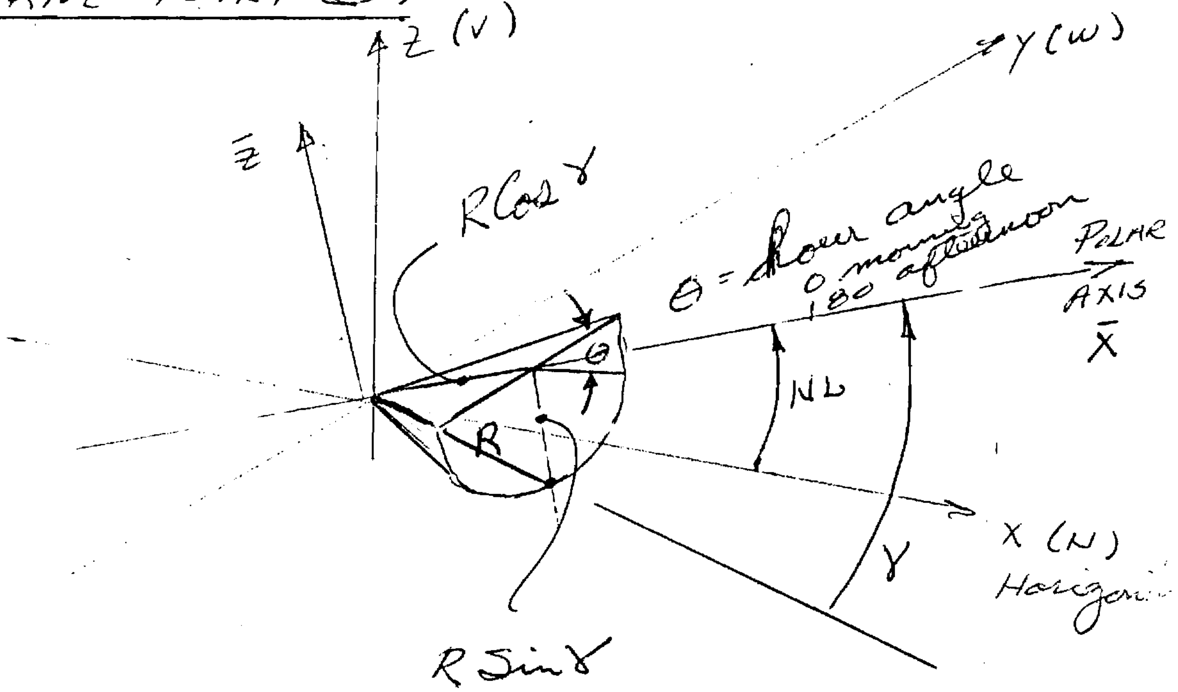
TARGET (FIXED POINT C)



$$X = R \cos(A) \sin(B)$$
$$Y = R \cos(A) \cos(B)$$
$$Z = R \sin(A)$$

$$\cos X = \frac{X}{R} = \cos(A) \sin(B)$$
$$\cos Y = \frac{Y}{R} = \cos(A) \cos(B)$$
$$\cos Z = \frac{Z}{R} = \sin(A)$$

SUN (DRIVE POINT B)



γ = declination angle
= 66.5 Winter
= 113.5 Summer.

Direction Cosines of a set of axis oriented with polar axis with respect to X Y Z coordinates.

Polar $\bar{X}$:

$$\begin{cases} D\ \cos X = \cos(NL) \\ D\ \cos Y = 0 \\ D\ \cos Z = \cos(90-NL) = \sin(NL) \end{cases}$$

Polar $\bar{Y}$:

$$\begin{cases} D\ \cos X = 0 \\ D\ \cos Y = 1 \\ D\ \cos Z = 0 \end{cases}$$

$$\cos X = \cos(NL + 90) = -\sin(NL)$$
$$\cos Y = 0$$
$$\cos Z = \cos(NL)$$

Coordinates with respect to polar axis ($\bar{X}\ \bar{Y}\ \bar{Z}$)

$$\bar{X} = R \cos\gamma$$
$$\bar{Y} = R \sin\gamma \cos\theta$$
$$\bar{Z} = -R \sin\gamma \sin\theta$$

Coordinates in $X\,Y\,Z$

$$X = \bar{X}\cos(NL) + \bar{Y}(0) + \bar{Z}(-\sin(NL))$$
$$Y = \bar{X}(0) + \bar{Y}(1) + \bar{Z}(0)$$
$$Z = \bar{X}\sin(NL) + \bar{Y}(0) + \bar{Z}\cos(NL)$$

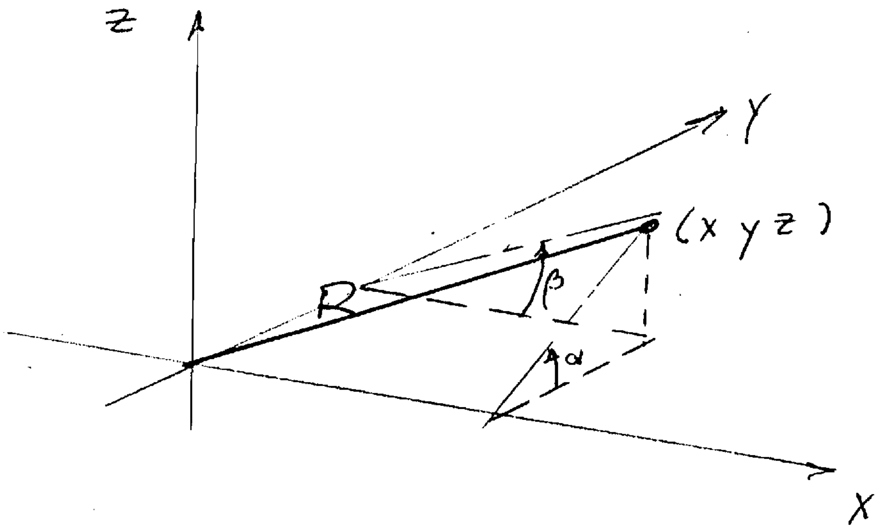
or

$$X = R\cos\gamma\cos(NL) + R\sin\gamma\sin\theta\sin(NL)$$
$$Y = R\sin\gamma\cos\theta$$
$$Z = R\cos\gamma\sin(NL) - R\sin\gamma\sin\theta\cos(NL)$$

con'd

$$\cos X = \cos\gamma\cos(NL) + \sin(\gamma)\sin(\theta)\sin(NL)$$
$$\cos Y = \sin(\gamma)\cos(\theta)$$
$$\cos Z = \cos(\gamma)\sin(NL) - \sin(\gamma)\sin(\theta)\cos(NL)$$

ANGLES



$$\tan\alpha = \frac{Z}{Y} = \frac{R\,D \cos z}{R\,D \cos y}$$

$$\tan\alpha = \frac{D \cos z}{D \cos y}$$

$$\alpha = \tan^{-1}\left(\frac{D \cos z}{D \cos y}\right)$$

$$\tan\beta = \frac{Z}{X} = \frac{R\,D \cos z}{R\,D \cos x}$$

$$\beta = \tan^{-1}\left(\frac{D \cos z}{D \cos x}\right)$$

```
C OMB ANGLES .JM  2/05/77

01.01 V K
01.02 S Z=FOUT(27)$S Z=FOUT(72)$S Z=FOUT(27)$S Z=FOUT(74)
01.10 V L,25$V 0,0,1
01.50 T !,"A,B,NL,DEC,R,TIDAY(START),TIDAY(STOP),TIDAY(INCREMENT)",!
01.60 F I=1,8$A Q(I)
01.65 D 1.5$F I=1,8$T !,I,Q(I)
01.70 T !,"CHANGES?(1 IF YES,0 IF NO)"$S Z=FIN()$IF(Z-177)1.75,,1.75
01.72 T !!,"ENTER INDEX,VALUE"$A I,Q(I)$G 1.65
01.75 S RC=.0174533$F I=1,4$S Q(I+10)=Q(I)*RC
01.80 S A=Q(11)$S B=Q(12)$S NL=Q(13)$S DE=Q(14)$S R=Q(5)
01.90 D 1.02$T !!,"CALCULATING...",!!
01.91 OU LPT:$L 0,F
01.92 T !!,"INPUT",!$D 1.65
01.93 T !!$L 0,C

02.10 S CX=FCOS(A)*FSIN(B)$S CY=FCOS(A)*FCOS(B)$S CZ=FSIN(A)
02.20 S XC=R*CX$S YC=R*CY$S ZC=R*CZ
02.30 S LX=FCOS(NL)$S NX=FSIN(NL)$S MY=1$S LZ=-FSIN(NL)$S NZ=FCOS(NL)

03.10 F I=Q(6),Q(8),Q(7)$S TD=I*RC$D 4$D 5
03.20 OU LPT:$L 0,B
03.30 S Z=FOUT(12)
03.40 L 0,C
03.50 T !!,"WOULD YOU LIKE A VARIABLE LIST?(1 IF YES,0 IF NO)"$A VV$*^_$
      IF(VV),6,1
03.60 OU LPT:$L 0,B
03.70 T !,"   VARIABLE LIST",!,$
03.80 L 0,C
03.90 D 6$G 6.1

04.10 S BX=LX*FCOS(DE)-LZ*FSIN(DE)*FSIN(TD)$*^_$
      S BY=MY*FSIN(DE)*FCOS(TD)$*^_$
      S BZ=NX*FCOS(DE)-NZ*FSIN(DE)*FSIN(TD)
04.20 S XB=R*BX$S YB=R*BY$S ZB=R*BZ
04.30 S S1=XC-XB$S S2=YC-YB$S S3=ZC-ZB
04.40 S L=FSQT(S1*S1+S2*S2+S3*S3)
04.50 S RX=S1/L$S RY=S2/L$S RZ=S3/L
04.60 S A1=(3.14159/2)-FATN(RX/FSQT(1-RX*RX))$*^_$
      S A2=FATN(RZ/RY)$S A3=FATN(RZ/RX)
04.70 S B1=FATN(BZ/BY)$S B2=FATN(BZ/BX)
04.80 S A1=A1/RC$S A2=A2/RC$S A3=A3/RC$S B1=B1/RC$S B2=B2/RC
04.90 S A4=A2-B1$S A5=A3-B2

05.10 OU LPT:$L 0,B
05.20 T !!,"TD=",I,%20," ACX=",A1,"    ACXZ=",A2,"    ACYZ=",A3,!,%20,"ABXZ=",A4,$*^_$
      "    ABYZ=",A5,$*^_$
      "       L=",L,%1
05.30 L 0,C

06.10 OU LPT:$L 0,B
06.20 S Z=FOUT(12)
06.30 L 0,C

07.10 Q
```

[illegible] TDAY(STOP), TDAY(INCREMENT)

1.0000	55.0000
2.0000	0.0000
3.0000	30.0000
4.0000	64.5000
5.0000	5.0000
6.0000	0.0000
7.0000	180.0000
8.0000	15.0000

TD=	0.0000	ACX=	130.0140	ACXZ= -	74.0255	ACYZ= -	48.8708
		ABXZ= -	86.2911	ABYZ= -	78.8708	L=	2.9539
TD=	15.0000	ACX=	127.4150	ACXZ= -	82.2346	ACYZ= -	52.3301
		ABXZ= -	81.8349	ABYZ= -	51.5672	L=	4.2003
TD=	30.0000	ACX=	125.9400	ACXZ=	89.3018	ACYZ= -	54.0581
		ABXZ=	103.2820	ABYZ= -	35.0692	L=	5.3843
TD=	45.0000	ACX=	124.6280	ACXZ=	80.7772	ACYZ= -	55.0226
		ABXZ=	109.9630	ABYZ= -	26.4108	L=	6.4806
TD=	60.0000	ACX=	123.1440	ACXZ=	72.3183	ACYZ= -	55.5743
		ABXZ=	119.1260	ABYZ= -	22.2346	L=	7.4684
TD=	75.0000	ACX=	121.4620	ACXZ=	64.0210	ACYZ= -	55.8627
		ABXZ=	131.3310	ABYZ= -	20.0977	L=	8.3300
TD=	90.0000	ACX=	119.2430	ACXZ=	55.9527	ACYZ= -	55.9527
		ABXZ= -	34.0472	ABYZ= -	19.4538	L=	9.0501
TD=	105.0000	ACX=	116.7960	ACXZ=	48.1524	ACYZ= -	55.8627
		ABXZ= -	19.1601	ABYZ= -	20.0977	L=	9.6164
TD=	120.0000	ACX=	114.0520	ACXZ=	40.6326	ACYZ= -	55.5743
		ABXZ= -	6.1753	ABYZ= -	22.2346	L=	10.0188
TD=	135.0000	ACX=	111.0550	ACXZ=	33.3836	ACYZ= -	55.0226
		ABXZ=	4.1974	ABYZ= -	26.4108	L=	10.2504
TD=	150.0000	ACX=	107.8550	ACXZ=	26.3789	ACYZ= -	54.0581
		ABXZ=	12.5987	ABYZ= -	35.0691	L=	10.3072
TD=	165.0000	ACX=	104.5060	ACXZ=	19.5801	ACYZ= -	52.3301
		ABXZ=	19.1805	ABYZ= -	51.5672	L=	10.1883
TD=	180.0000	ACX=	101.0650	ACXZ=	12.9414	ACYZ= -	48.8708
		ABXZ=	25.2070	ABYZ= -	78.8709	L=	9.8957