Bio-Inspired Porous Network Topology for Optimal Injection and Withdrawal Processes in Soils

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ABSTRACT: Bronchi, arteries and veins, tree branches and roots, exhibit a fractal topology, i.e. networks formed by channels that successively split into smaller channels. A thorough literature review shows that self-similar topologies justify most empirical power laws encountered in nature and engineering design. Fractal models match but do not explain observations. Is the fractal topology optimal for all transport processes taking place between a porous system and a host medium? According to the constructal theory, the topology of a flow system should optimize an energy potential. The underlying assumption is that any network should have a purpose, a configuration and constraints. The main theoretical assumptions and developments of the constructal theory are presented. The thermal efficiency of an isolated heat exchanger pile is analyzed for different topologies. Simulations show that slender network components are preferable to isotropic topologies only if the contrast between soil and pile thermal conductivities is between 1 and 2 orders of magnitude. The orientation of fragmentation of the heat exchanger should also depend on potential variations of thermal properties across soil layers. The applicability and limitations of the constructal theory to optimize injection and withdrawal processes in soils is discussed.

1. INTRODUCTION

Impedance matching between different system components is required to optimize energy transfer. This is the case in dynamic systems including wave propagation (mechanical, or electromagnetic). Similarly, advective and/or diffusive transport must be optimized in many applications that involve fluid extraction or injection, or heat transport. Natural systems such as lungs, kidneys, tree roots and canopy have evolved to optimize transfer. The design of heat exchangers, radiators and other industrial systems seeks a similar goal albeit with a highly constrained geometry often dictated by manufacturing limitations.

The goal of this research is to explore the importance of incorporating environmental properties and network-environment interactions in optimization techniques used to predict/design flow network topologies. Natural systems are reviewed first, followed by the analysis of isolated thermal piles of different topologies.

2. FRACTAL POROUS NETWORK TOPOLOGY

A coastline has the same geometric appearance when observed at 0.1km, 1km or 10km. Similarly, bronchi, blood capillaries, plant ducts and sedimentary basins exhibit similar topologies at different scales. The concept of fractals was introduced to capture the scale-independence of fragmented topologies observed in nature (Mandelbrot, 1989). A set of objects obeys a fractal distribution if the number of objects N with a characteristic linear dimension greater than r satisfies:

$$N(r) = \frac{\alpha}{r^D}$$

where α is a constant and D is the fractal dimension; for example, the fractal dimension is less than three for completely fragmented media such as soils and jointed rock masses.

The fractal model has been extensively applied throughout the sciences, including geography (Goodchild & Mark, 1987), hydrology (Thompson et al., 1987; Nolte et al., 1989), geology (Turcotte, 1989; Ghosh & Daemen, 1993; Bonnet et al., 2001), geophysics (Davy et al., 1990; Silberschmidt, 2000; Dieterich & Smith, 2009), planetary sciences (Hartmann, 1969), biology (West et al., 1999) and medicine (Goldberger & West, 1987). The fractal representation is particularly attractive in geomechanics since it provides a theoretical foundation to the power-law constitutive relationships established empirically for permeability and retention properties (Tyler & Wheatcraft, 1989; 1990), the evolving grain size distribution in crushing (Einav 2007), and the surface roughness of rock joints (Power & Tullis, 1991; Poon et al., 1992).

Fractal length parameters in soils have been associated to packing parameters (Tyler & Wheatcraft, 1990). Similarly, the soil porous space has been analyzed as a fragmented fractal network, i.e. as intra-aggregate fractal networks connected by self-similar cracks. The model was improved to account for the bridges (i.e. contacts) that cement aggregates together (Rieu & Sposito, 1991): a soil-clustering factor is introduced to model the probability of particle fragmentation for each particle size present in the soil. The resulting bulk fractal dimension is larger than the fractal dimension of the intra-aggregate fractal network. Retention properties in the fractal porous network are deduced following standard Laplacian capillarity in porous networks: channels smaller than the saturation radius are filled with the wetting fluid. Similarly, macroscale flow properties can be obtained by upscaling pore-scale Hagen-Poiseuille flow through the fractal porous network. In a fractal pore topology, the probability of channel intersection is the same in all directions, so it is possible to compute the permeability tensor from the topology of only one cross section.

Limitations of Fractal Models. Power laws and fractal topologies are frequently encountered in nature. However, scales of observations available to verify the self-organization of natural topologies are limited (Avnir, 1998; Bonnet et al., 2001). Furthermore, very few attempts have been made to explain the “fractal nature of nature” on the bases of fundamental physical laws (Voss, 1992; Berkowitz, 2002; Brown et al., 2002; Stumpf & Porter, 2012). In other words, why do porous networks reflect power laws? Does it lead to energy minimization?

It is important to highlight that the fractal description of optimal network topologies (natural or man-made) fails to represent network self-organization at very small scales, i.e. the "inner cut-off" of natural networks (Bejan, 1997). In fractal network models, the size of the smallest network component needs to be postulated otherwise it is impossible to close the formulation.

3. THE CONSTRUCTAL THEORY

Limitations in fractal-based theories inherently follow from observations above: assumed fractal topology, the need to specify the smallest scale and lack of energy-minimization validation. The constructal theory was proposed as an alternative path (Bejan, 1997; Bejan & Marden, 2009), by establishing a purpose and constraints. A field variable (e.g. temperature difference, shear stress distribution) is optimized (e.g. by minimizing entropy production) within predefined boundary conditions (e.g. flow rate or temperature, deformation or stress), under given design specifications and constraints (e.g. relative size of the system compared to the volume under study; cost; conservation laws).

The constructal theory has been used to optimize the topology of heat exchangers (Bejan, 1997; Zimparov et al., 2006), and urban transportation networks (Bejan & Ledezma, 1998). It has successfully explained the partition of hot and cold temperatures in the atmosphere (Reis & Bejan, 2006), the size of living organisms with respect to their life "style" (e.g. swimming vs. running vs. flying animals), and tree-shaped flow networks observed in nature such as bronchi, arteries, trees, river basins (Bejan, 2005; Bejan & Marden, 2009).

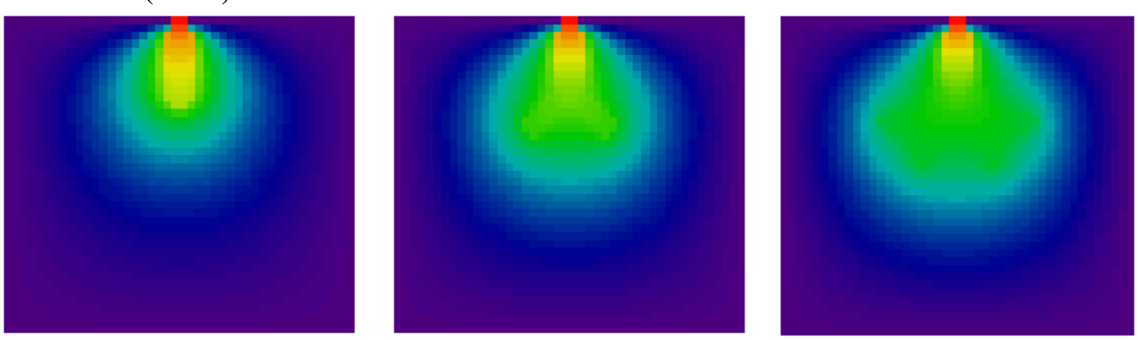
The constructal theory is an optimization method for finite flow systems (Bejan, 1998; Bejan & Lorente, 2004; Lorente & Bejan, 2005; Bejan & Lorente, 2006; Bejan, 2007). Topological features such as channel length, number of fragmentations per channel, channel opening size, are optimized for each construction step of the assembly. The first assembly (comprising a large duct and smaller branches) is used as a branch in the optimization of the second assembly. The construction of the second assembly involves the optimization of the size of the main duct relatively to the size of a branch (i.e. the overall size of the assembly obtained in the first step), given the total volume of the network. Therefore, the topology of the optimal network is fully determined by the successive optimization of assemblies, starting with the smallest channels. The resulting optimum is not necessarily a fractal. For instance, optimal tree-shaped networks exhibit successive bifid bifurcations, except for the first assembly (which contains more than two branches).

A more general optimization procedure, based on minimization of entropy production, couples the constructal theory with the thermodynamics of irreversible processes (Tescari et al., 2011). Entropy production is written as the product of the square of flux (heat flux for a heat propagation problem) and entropy impedance. The flux is given as part of the boundary conditions, and impedance depends on network topology parameters according to homogenization theoretical formulas. The optimum network topology is defined by the geometric parameters that minimize impedance. Note that another optimum is obtained if global network optimization is used instead of constructal optimization, i.e. if the network as a whole is optimized instead of each assembly.

4. HEAT EXCHANGE PILES

A numerical simulation study was conducted to explore the advantages of a tree-shaped topology with successive bifid network fragmentations as heat exchangers in soil (steady state - finite difference solution) The contrast between thermal conductivities is assumed not to exceed two orders of magnitude. We consider Dirichlet constant-temperature boundary conditions. To facilitate the analysis of results, we define an energy efficiency index *E* as the ratio between the heat input injected at the pile head and the heat input when there is no pile but only a footing of the same cross-sectional area at the surface.

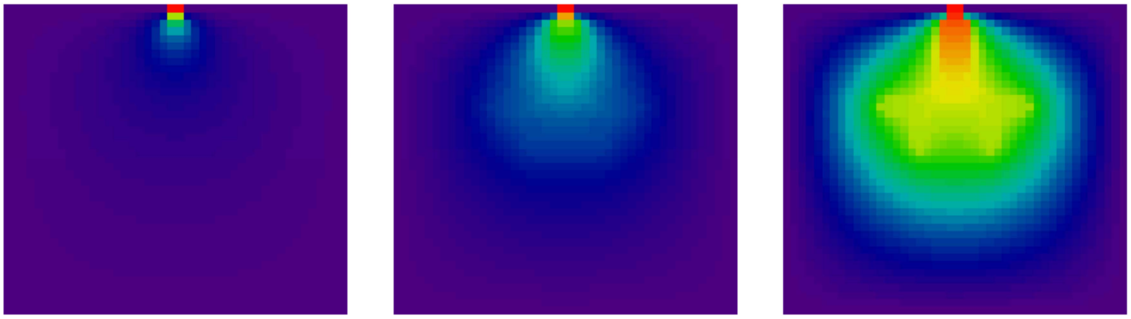
The first parametric study explores the effect of topology in a medium with homogeneous soil thermal conductivity. Results in Figure 1 show a 12% increase in heat transfer efficiency when the standard cylindrical pile geometry is modified to include a single-level bifurcation (“1Y”), and a 22% improvement in heat transfer with a two-level bifurcation (“2Y”).



Cylinder (0Y). E=6.4 **1 Fragmentation (1Y). E=7.2** **2 Fragmentations (2Y). E=7.8**

Figure 1. Influence of network topology on heat injection efficiency (k_p/k_s=40).

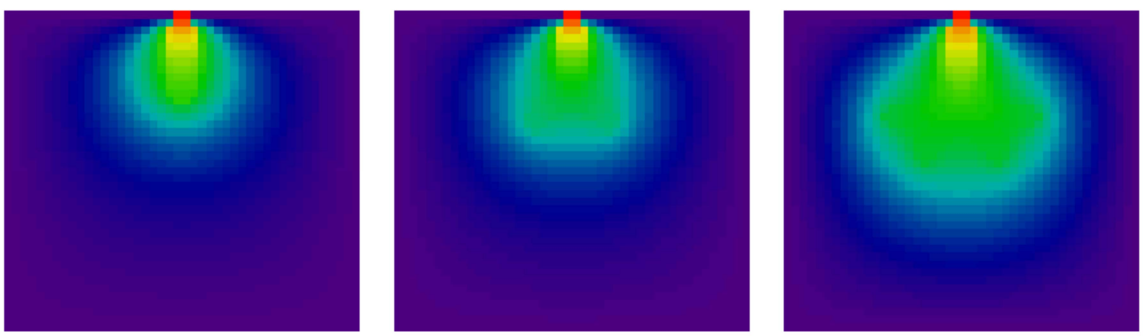
The thermal conductivity ratio k_p/k_s between the pile and the soil mass has a profound effect on the system performance (Figure 2). For the selected double-bifurcation 2Y topology: (1) heat transfer efficiency increases proportional to the square root of the conductivity ratio $E \approx (k_p/k_s)^{0.5}$, (2) heat is transferred at shallow depth for low k_r/k_h ratios, (3) the advantages of the bifurcation are realized when the conductivity contrast approaches one order of magnitude.



k_p/k_s=2 E=1.7 | k_p/k_s=10 E=4.4 | k_p/k_s=100 E=9.6

Figure 2. Effect of thermal conductivity contrast on heat injection efficiency (2Y topology).

The thermal conductivity of soils and rock masses increases with depth as a result of the increase in effective stress (and water saturation in some cases). Simulation results in Figure 3 where computed for a linear gradient such that the thermal conductivity ratio decreases from k_p/k_s=40 at the top to k_p/k_s=20 at the bottom of the main cylindrical core. There is a marked increase in heat injection efficiency as compared to results in Figure 1. It is anticipated that the orientation of bifurcations from the main branch of the heat exchanger should reflect the spatially varying thermal properties in the soil mass.



Cylinder (0Y). E=8.6 | 1 Fragmentation (1Y). E=9.7 | 2 Fragmentations (2Y). E=10.2

Figure 3. Influence of the gradient in soil thermal conductivity on heat transfer efficiency. The soil thermal conductivity increases linearly with depth, from k_p/k_s=40 at the pile head to k_p/k_s=20 at the bottom of the main cylinder. Note: refer to results in Figure 1.

5. CONCLUSIONS

Self-similar topologies and empirical power laws are common in nature and engineering designs. Yet, nature's fractality deserves careful, fundamentals-driven analysis. If accepted, fractals allow for the systematic reconstruction of networks with a minimal set of parameters; in real systems, an ending point must be selected a priori.

The constructal theory helps identify the system topology by optimizing an energy potential. The underlying assumption is that any network should have a purpose, a configuration and constraints.

The numerical analysis of a root-inspired heat exchanger topology shows the benefits of selecting pile materials with significantly higher thermal conductivity than the surrounding soil (1 to 2 orders of magnitude), the benefits of bifurcated pile geometry, and the implications of increasing soil thermal conductivity with depth. The orientation of bifurcations can be optimized to reflect the spatial variability in soil thermal properties.

Design constraints imposed in the constructal theory can become limiting assumptions in problems of network growth and adaptation encountered in nature, such as environment spatial heterogeneity and complex multi-scale coupled TCHM problems that are common in geomechanics.

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