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A COMPARISON OF THERMODYNAMIC LOSS MODELS SUITABLE FOR GAS TURBINE PROPULSION: THEORY AND TAXONOMY

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Abstract

The objective of this paper is to describe several figures of merit for estimation of loss in work potential based on the second law of thermodynamics and to evaluate their relative merits for propulsion system analysis and design. The loss figures of merit examined are exergy, gas horsepower, stream thrust, and thrust work potential. Definitions and simplified expressions for evaluating each are presented, and the relationships between these four metrics are expressed via contours on a T-S diagram. A general taxonomy classifying the various work potential figures of merit is suggested. The results indicate that that each method is well suited to a particular type of application, with the most appropriate choice of loss figure of merit depending on the particular application. Finally, thrust work potential is shown to be a special case of gas horsepower, which is in turn a special case of exergy.

Nomenclature

Note: lower case letters denote mass-specific quantities
A = Cross-Sectional Area (ft^2)
c_p = Constant Pressure Specific Heat (0.24 BTU/lbm-R)
Ex = Exergy (BTU)
g = Gravitational Acceleration (ft/sec-sec)
GHP = Gas Horsepower (BTU)
H = Enthalpy (BTU)
I = Impulse Function (lbf)
J = Work Equivalent of Heat, 778 ft-lb/BTU
M = Mach Number
$\dot{m}$ = Mass Flow Rate (lbm/s)
P = Pressure (atm)
R = Gas Constant (0.069 BTU/lbm-R for air)
S = Entropy (BTU/R)
Sa = Stream Thrust (lbf/lbm)
T = Temperature (R)
V = Gas Velocity (ft/s)
W_P = Thrust Work Potential (HP or ft-lb/s)
$\dot{W}_{out}$ = Power Output (HP)
γ = Ratio of Specific Heats (1.4)
ρ = Gas Density (slug/ft^3)
Subscripts
amb = Ambient Conditions
exp = Isentropically Expanded to Ambient Pressure

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Introduction

A truly good engine design is always a compromise between all competing aspects of design performance including thermodynamic performance, weight, cost, maintainability, etc. A necessary prerequisite to achieving this balance is an understanding of the *fundamental nature* of the trades involved and knowledge of the exact cost (in terms of performance, weight, and dollars) of every decision made during the design process. In particular, one would like to know the magnitude of the work loss incurred in each thermodynamic process inside the propulsion system such that the most significant sources of loss can be identified and targeted for improvement. This is especially true for high-speed propulsion systems where the losses associated with high-speed flow processes can easily become exorbitant if not properly addressed.

The need to accurately calculate loss of flow work potential relative to a thermodynamic ideal has led to interest in methods employing the second law of thermodynamics as a basis for loss estimation. This approach is appealing because it provides an unambiguous definition of an ideal against which the actual process can be compared. Thus, whereas conventional cycle analysis gives information as to the flow of *energy*, a second law-based method enables calculation of *work potential*. This capability will enable the creation of loss management models to identify and track all sources of thermodynamic loss in a propulsion system. Such an approach would make it possible to estimate the *absolute* loss associated with each loss mechanism in terms of a single figure of merit (FoM) applicable to all engine components and processes.

Several models for evaluation of loss in work potential have appeared in the past several decades, each different from the others in subtle ways. Most of the published work in this area focuses on a single model in isolation from the others. As a result, the relationships amongst these various figures of merit are ambiguous and the literature on the subject is somewhat disjointed. The purpose of this paper is to clarify this situation by examining the utility of the various loss estimation methods for gas turbine propulsion applications. This paper defines each method separately, discusses the historical context and compares the relative merits of each method for the purposes of jet propulsion. The loss models examined are exergy, gas horsepower (isentropic expansion work potential), stream thrust, and thrust work potential. These four were selected based on their promise as a universal loss metric for jet propulsion applications.

This paper approaches the comparison of work potential methods from a purely theoretical aspect. However, it is closely related to a second paper that is focused on demonstrating each of the four methods as clearly and concisely as possible on a simplified J-79 turbojet example.[1,2] In the second paper, the results of each analysis method are discussed in detail, summarized, and compared to draw further inferences beyond those given in this paper as to their potential usefulness as a loss figure of merit (FoM) for gas turbine engines.

Background

A substantial body of work has appeared in the past several decades dealing with second-law approaches to measuring loss in gas turbine engines. One such approach is the exergy concept, which has been applied to the gas turbine cycle by several authors, notably Clarke and Horlock,[3] who applied it to a simple turbojet example and showed where the most significant exergy losses were occurring. It is the best-known and most formalized method to estimate the magnitude of losses relative to a thermodynamically ideal process,[4,5] and first appeared in the United States due largely to the work of Keenan in the 1940s.[6] A considerable body of literature exists describing the theory and application of exergy analysis, and references 7, 8, 9, and 10 are standard texts on the subject. More recently, there has been a great deal of interest in applying exergy concepts to combined cycle power generation, of which El-Masri[11] gives an excellent example wherein he is able to identify in detail all sources of exergy loss occurring within a gas turbine topping cycle. In addition, he shows the impact of cycle changes on total exergy produced and destroyed. Another approach that has been proposed in the past is gas horsepower (of isentropic expansion), which is used by Nichols[12] as a universal figure of merit for combustor loss. It is also used extensively as a figure of merit for gas generator power output, but has received little attention beyond this limited application. A third figure of merit was proposed by Curran and Craig[13] based not on energy, but force (thrust), known as the stream thrust concept. This involves calculation of stream thrust potential (also known as specific thrust) at each flow station and optimizing the cycle to deliver the highest stream thrust potential. Later, Riggins[14] extended this concept by introducing the lost thrust method which allows accurate calculation of stream thrust loss due to inefficiencies. In addition, he introduced the thrust work potential and lost thrust work potential figures of merit and showed that optimization of exergy output does not necessarily lead to the best propulsive cycle from a thrust production point of view. Finally, Riggins suggested a modified definition of exergy, which he termed "engine-based exergy," and showed that this modified definition yielded results identical to those obtained through stream thrust methods.

A great deal of important work has been published over the past several decades in addition to that mentioned here, notably in the developing field of *entropy generation minimization*. An excellent discussion of this work and its applications is given in the recent textbook by Bejan.[15] For the present discussion, however, the authors will confine themselves to the four loss figures of merit previously mentioned and their direct application to jet propulsion.

Exergy

Exergy is a thermodynamic state describing the maximum theoretical (Carnot) work that can be obtained from a substance in taking it from a given chemical composition, temperature, and pressure to a state of chemical, thermal, and mechanical equilibrium with the environment. The general definition of exergy is given by:

$$Ex \equiv H - H_{amb} - T_{amb}\left(S - S_{amb}\right) + \left(\text{Other Terms}\right) \quad (1)$$

In this case, the "other terms" are used to denote exergy due to kinetic energy, potential energy, chemical potential, radiation, heat transfer, etc. Note that while energy is a conserved quantity, exergy is *not*, and is always destroyed when entropy is produced. Note also that the definition of exergy depends on the ambient environment.

The physical significance of the thermodynamic quantity in Eq. 1 is best described in terms of a Mollier diagram as shown in Figure 1. The dashed line with slope equal to the ambient temperature is the zero exergy reference line that represents the locus of points from which no work can be extracted through heat transfer. All points above this line have the potential to do work via heat transfer from a high temperature reservoir into the environment. Points below the reference line have potential to do work via heat transfer from the environment into a low temperature (perhaps cryogenic) reservoir. Also shown are isobaric contours for the reference[‡] pressure and some arbitrarily higher pressure. Exergy is depicted as the difference between the enthalpy delta from the point of

[‡] For the purposes of this paper, ambient conditions are taken to be the reference conditions.

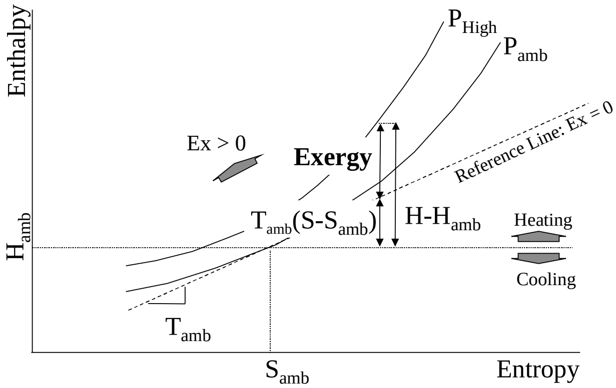

Figure 1: The Definition of Exergy Plotted on a Mollier Diagram [From 17].

interest to the zero exergy reference line. Since the exergy concept relates every state to the Carnot reference of work, *change in exergy is a measure of the loss in absolute work potential* at every station in an engine. It is a comprehensive measure of loss in work potential that captures the impact of all sources of loss.[16]

Contours of constant exergy can be plotted on a T-S diagram as shown in Figure 2. Note that the contours of constant exergy are straight lines with the zero exergy contour passing through the dead state. Also, it is clear from this figure that the zero exergy contour is tangent to the ambient pressure contour at ambient conditions.

For the case of calorically perfect air where chemical potential, kinetic energy, and potential energy are negligible it is a simple matter to obtain an equation for mass-specific exergy as a function of ambient conditions and gas conditions at a given engine station by noting that:

$$h - h_{amb} = c_p\left(T - T_{amb}\right) \tag{2}$$

and using the integrated form of the second TdS relation:

$$s - s_{amb} = c_p \ln\left(T/T_{amb}\right) - R\ln\left(P/P_{amb}\right) \tag{3}$$

Substitution of Eq. 2 and Eq. 3 into Eq. 1 yields:

$$ex = c_p\left(T - T_{amb}\right) - c_p T_{amb} \ln\left(T/T_{amb}\right) + RT_{amb} \ln\left(P/P_{amb}\right) \tag{4}$$

Rote application of this equation to every engine station yields the exergy at each station. Clearly, Eq. 4 is of limited value for propulsion applications where vitiation, vibrational excitation, chemical reactions, and other effects are important. Fortunately, it is relatively simple to obtain accurate estimates for flow exergy including these effects using modern thermodynamic properties software packages. The losses associated with each component can then be calculated based on the idea that the difference between the exergy fluxes into and out of a component must be equal to the sum of the power output and the exergy loss rate:

$$\dot{E}x_{in} - \dot{E}x_{out} = \dot{W}_{out} + \dot{\mathrm{E}}\mathrm{x}_{\mathrm{Loss}} \tag{5}$$

Exergy is the most general of the work potential figures of merit investigated here, and gives an estimate

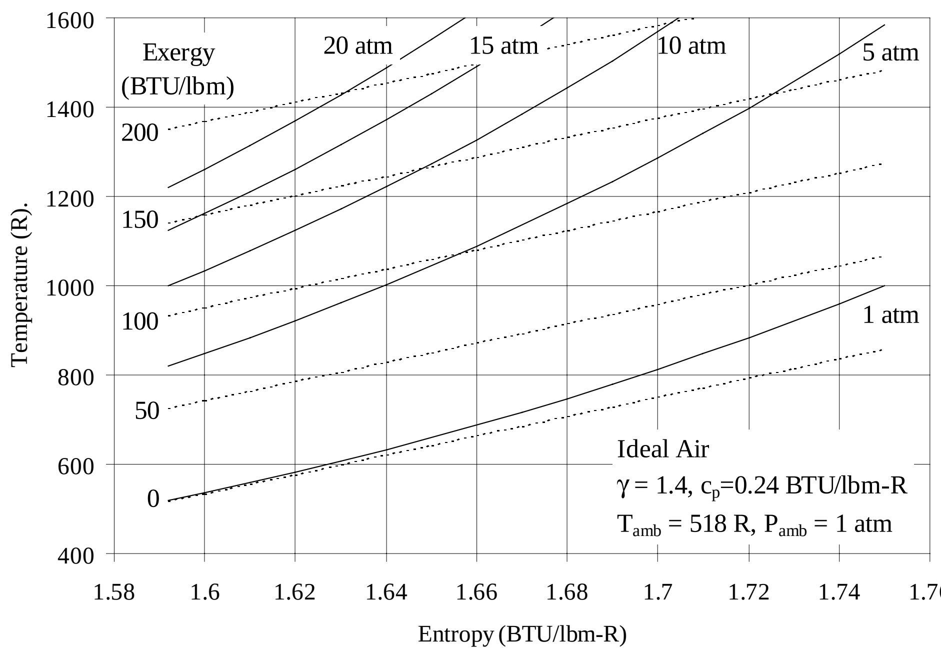

Figure 2: T-S Diagram Showing Contours of Constant Exergy (Solid Lines) and Isobaric Lines (Dashed Lines) for Ideal Air.

of the absolute work potential that could be obtained by *any heat engine* operating under specified conditions. The work potential estimates obtained using exergy assume that the working substance is taken to thermal and mechanical equilibrium with the environment, the first of which cannot be enforced using the simple Brayton cycle. As a result, even the perfect (no component loss) Brayton cycle will have exergy losses due to non-equilibrium combustion and exhaust heat loss. Moreover, when the objective is to produce jet thrust, a portion of the work done on the working fluid must appear as exhaust residual kinetic energy as viewed in the earth-fixed reference frame. Thus, there is a portion of the exergy content of the fuel that is *inherently unavailable* to the Brayton cycle and appears as a loss. In general, these inherent losses are far larger than exergy losses due to component inefficiencies for gas turbine engines. Consequently, optimization of a thrust-producing device to produce maximum exergy output may yield a less-than-optimal result if the objective is to produce thrust for propulsion, as observed by Riggins.[14]

However, for some applications such as combined cycle power generation, it appears that there is justification for optimizing exergy output in order to obtain maximum power output. The reason for this is that the steam bottoming cycle is able to extract the exhaust exergy of the gas turbine topping cycle, and therefore, all of the exhaust exergy becomes inherently available. One would thus expect that optimization of the topping cycle for maximum exergy output (in the form of shaft power and exhaust exergy) will naturally lead to more efficient combined-cycle plants.

Gas horsepower (Work Potential)

Gas horsepower is defined as the work that would be obtained by isentropically expanding a gas at a specified temperature and pressure to a prescribed reference pressure (usually taken to be local atmospheric). Thus, the temperature at the imaginary expanded condition is a fall-out of the isentropic expansion process. Expressed mathematically:

$$\text{Gas Horsepower} = GHP \equiv H(T_i, P_i) - H(P = P_{ref}, S = S_i) \tag{6}$$

where i is used to denote the thermodynamic state of the gas at point i. Note that gas horsepower is a function of ambient pressure only, and is independent of ambient temperature, unlike exergy. Gas horsepower is commonly used to measure the theoretical power output of core engines and gas generators. A simple expression for gas horsepower of a calorically perfect gas is easily derived by noting that:

$$ghp = c_p \left(T_i - T(P = P_{amb}, S = S_i)\right) \tag{7}$$

where the temperature after isentropic expansion to reference (ambient) pressure is:

$$T(P = Pamb, S = const) = T\left(P_{amb}/P\right)^{\frac{\gamma-1}{\gamma}} \tag{8}$$

combining Eq. 7 and Eq. 8 yields:

$$ghp = c_p T\left[1 - \left(P_{amb}/P\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{9}$$

Rote application of Eq. 9 to the results of a standard first law cycle analysis yields the gas horsepower at each station in the engine. These results can then be used to calculate the loss in gas horsepower in each component of the engine based on a "conservation of gas horsepower" principle similar to that used for calculation of exergy losses:

$$G\dot{H}P_{in} - G\dot{H}P_{out} = \dot{W}_{out} + G\dot{H}P_{loss} \tag{10}$$

One can obtain a physical feel for the meaning of Eq. 6 by comparison to the definition of exergy, as expressed in terms of a Mollier diagram, as shown in Figure 3. Note that whereas the contour of zero exergy is a line passing through the reference (dead) state, the contour of zero gas horsepower is the isobaric line corresponding to the reference pressure. The space between the zero exergy line and the zero gas horsepower contour is labeled as "thermal exergy" and constitutes the difference between gas horsepower and exergy. Clearly, gas horsepower and exergy are identical at the reference entropy, but diverge as entropy increases. This idea is further explained by plotting lines of constant gas horsepower on a T-S diagram, as shown in Figure 4. Note that as temperature and pressure increase above the ambient value, the contours of constant gas horsepower diverge

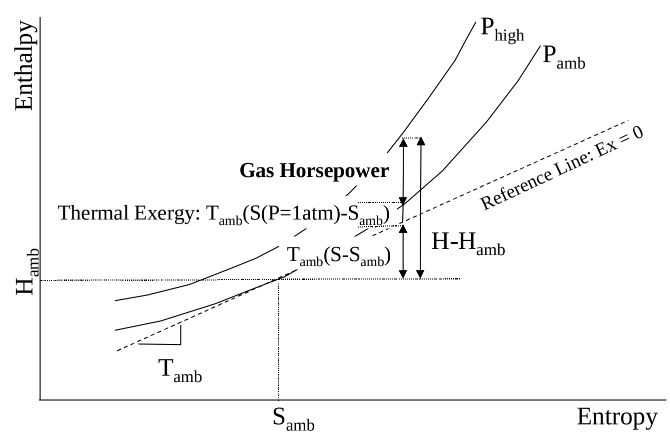

Figure 3: Gas Horsepower Plotted on a Mollier Diagram. Gas Horsepower is the Quantity Lying between the Isobaric Contours [From 17].

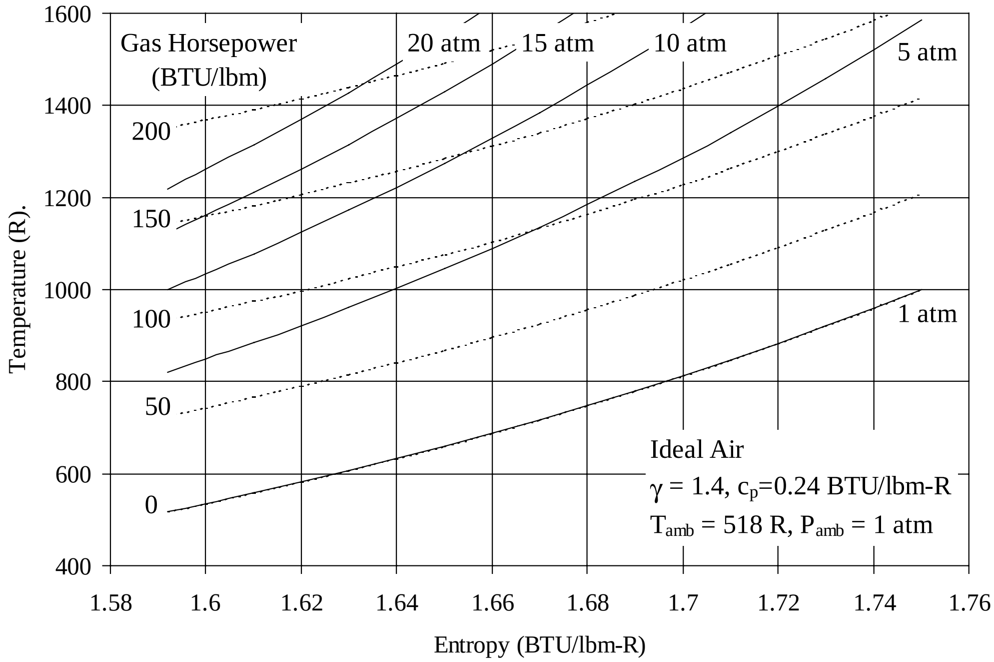

Figure 4: T-S Diagram Showing Contours of Constant Gas Horsepower (Dashed Lines) and Isobaric Lines (Solid Lines) for Ideal Air.

from the isobaric contours. Also, it is clear from Eq. 9 that *the gas horsepower is directly proportional to the gas temperature for a given pressure ratio* (for the calorically perfect gas model only).

It was pointed out previously that gas horsepower and exergy are thermodynamically identical quantities at the reference entropy, and diverge with increasing entropy. This is also reflected in the T-S diagram shown in Figure 5, which depicts lines of constant exergy and gas horsepower superimposed on the same plot for ideal air. The fundamental difference between these two quantities is that gas horsepower requires only mechanical (pressure) equilibrium with the environment, while exergy requires *both* mechanical and thermal equilibrium. Thus, *gas horsepower is a*

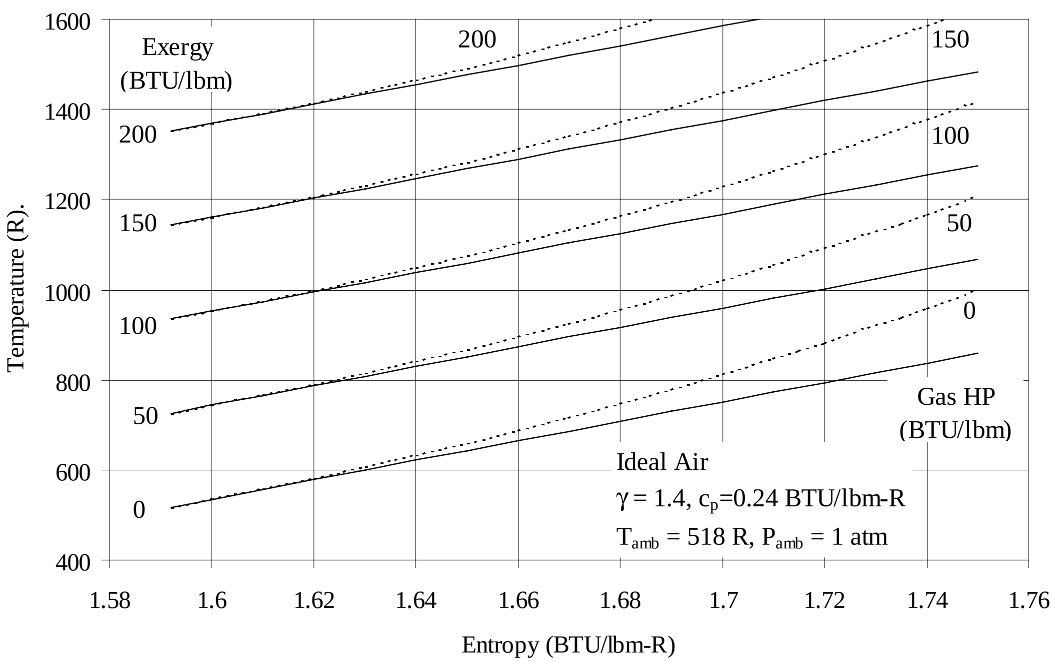

Figure 5: T-S Diagram Showing Contours of Constant Exergy (Solid Lines) and Constant Gas Horsepower (Dashed Lines) for Ideal Air.

special case of exergy for which work extraction through mechanical equilibrium is assumed while thermal equilibrium is not enforced.

Note that *gas horsepower losses in a given component will always be larger than their corresponding exergy losses*. This is because the loss in work will appear as an increase in heat (by the first law) and temperature of the exhaust gas. A portion of this exhaust heat is recoverable and can be used to produce work in a bottoming cycle, and is thus not seen as an irretrievable loss using exergy, but is an unrecoverable loss in gas horsepower.

The primary difference between gas horsepower and exergy for analysis of gas turbine (Brayton) cycles is that exhaust heat does not appear as a gas horsepower loss, but is instead transparent. Thus, a perfect Brayton cycle will appear to have no losses in gas horsepower.[§] Therefore, the distribution of losses is considerably different between the exergy and gas horsepower methods. In particular, the role of component losses and useful work production is considerably magnified, as non-equilibrium combustion and exhaust heat losses no longer appear in the loss stack-up. However, exhaust residual kinetic energy still appears as a large loss when the gas turbine is used to produce jet thrust.

Stream Thrust

Whereas exergy and gas horsepower concepts are based on work potential, the stream thrust concept is based on thrust potential at each flow station in the engine. *Thrust potential is defined as the thrust that would be obtained in expanding a flow from a given temperature and pressure to atmospheric pressure*. Stream thrust is based on the impulse function, which is defined in compressible fluid mechanics as:[18]

$$I \equiv PA + \rho AV^2 = PA\left(1+\gamma M^2\right) \qquad (11)$$

The impulse function is nothing more than a form of the momentum equation and can be used to find the net force (drag or thrust) exerted on a fluid stream between arbitrarily specified inlet and exit planes via evaluation of the impulse function at the inlet and exit planes:

$$F_{Net} = I_{Exit} - I_{Inlet} \qquad (12)$$

Stream thrust is defined as the impulse per unit mass of flow, more commonly known as specific thrust.[19] It is therefore related to impulse function by:

$$Sa \equiv I/\dot{m} \qquad (13)$$

It is generally inconvenient to work in terms of velocity, Mach number, and area when analyzing the thermodynamic cycle of an engine. Instead, it is preferable to evaluate the stream thrust in terms of temperatures and pressures at each flow station in the engine. This is done by calculating the velocity that would be obtained by an imaginary isentropic expansion from the conditions of interest to atmospheric pressure, where the expanded fluid velocity is related to the stream thrust via:

$$Sa = V_{\text{expanded}}/g \qquad (14)$$

This expanded velocity can be evaluated based on the gas horsepower at each flow station by noting that:

$$V^2/2 = J(ae)g \Rightarrow V = \sqrt{2(ae)Jg} \Rightarrow Sa = \sqrt{\frac{2(ae)J}{g}} \qquad (15)$$

or, simplifying slightly:

$$Sa = 6.955\sqrt{ae} \qquad (16)$$

where stream thrust is in lbf/lbm and gas horsepower is in BTU/lbm. An appealing attribute of stream thrust as a figure of merit is that it is a force-based quantity, and therefore independent of the observer's frame of reference (though Eq. 16 implicitly assumes that gas horsepower is measured with respect to the vehicle-fixed reference frame). This is in contrast to exergy and gas horsepower, wherein the measured value of these quantities depends on the reference frame. In addition, it is directly linked to what is arguably the ultimate figure of merit for jet propulsion applications: jet thrust.

A disadvantage of stream thrust is that it has no "conservation property" analogous to Eqs. 5 and 10 that allows direct estimation of stream thrust loss due to irreversibilities. That is to say:

$$Sa_{\text{lost}} \neq Sa_{\text{out}} - Sa_{\text{in}} \qquad (17)$$

for the general case where there are work interactions with other components, as in the compressor or turbine. Therefore, one must resort to the "lost thrust" method described by Riggins[14] and demonstrated in the second paper of this series.[1]

Thrust Work Potential

Thrust work potential is defined as the thrust work that would be obtained in expanding a flow at a given temperature and pressure to ambient pressure such that the thrust work obtained is equal to the thrust produced multiplied by the flight velocity of the aircraft.[20] This

[§] The authors are unaware of any treatment by which the maximum gas horsepower of a fuel has been calculated, and this appears to be a topic worthy of further investigation.

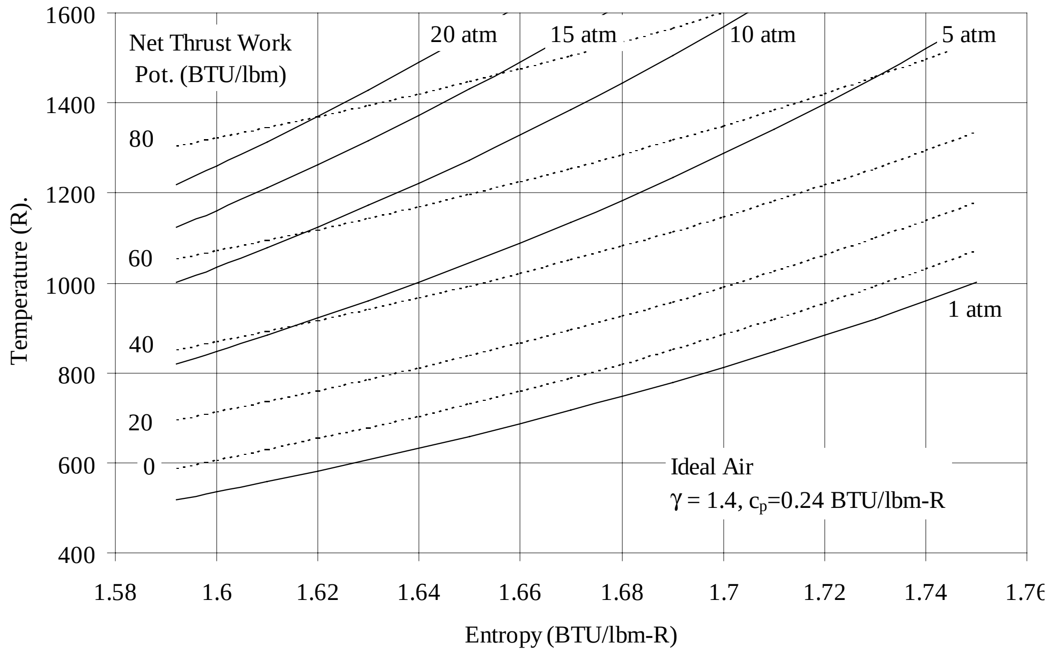

Figure 6: T-S Diagram with Lines of Constant Thrust Work Potential at M0.9, 20,000 ft (Solid Lines) and Isobaric Lines (Dashed Lines).

can be normalized by airflow rate to give specific thrust work potential at each station:

$$Wp \equiv {}^{Sa(u)}\!\big/_{J} \tag{18}$$

For the purposes of air-breathing propulsion, thrust work potential is inherently anchored in the Earth-fixed observer's frame of reference because it is based on the velocity of the vehicle relative to the Earth. Note that the thrust work potential is always less than the gas horsepower of the gas stream due to the fact that some of the gas horsepower must necessarily emerge as residual kinetic energy of the exhaust gasses (as viewed by the stationary observer). Thrust work potential is therefore linked to the gas horsepower through propulsive efficiency, which is in turn a function of exhaust velocity and flight velocity. In this regard, *thrust work potential can be viewed as a special case of gas horsepower that measures only work produced with respect to a particular reference frame*. By extension then, thrust work potential is a special case of exergy.

Lines of constant thrust work potential can be plotted on a T-S diagram as shown in Figure 6. Note that the contours are shaped the same as gas horsepower contours, but with two differences: their spacing is not constant, and the zero thrust work potential line does not coincide with the zero gas horsepower (atmospheric pressure) line. In fact, the spacing of thrust work potential contours is proportional to the *square* of the temperature because of its relationship to stream thrust, which also varies as the square of temperature. The displacement of the zero point physically corresponds to the thrust work required to offset the ram work of inlet compression. Thus, the zero thrust work potential line will move further upwards as flight velocity increases.[**] Finally, note that there is far less thrust work potential available than gas horsepower for a given flow temperature and pressure, especially at high temperature and pressure. This is due to increasing exhaust residual kinetic energy (lower propulsive efficiency), which is characteristic of the high specific thrust produced by high enthalpy flows.

Since thrust work potential is proportional to the stream thrust at each station, it does not yield any information beyond that which is obtained from the stream thrust analysis, and has the disadvantage that it is not a meaningful FoM for comparison of engines at static operation. However, because losses are expressed in terms of power rather than force, it can be directly compared against exergy and gas horsepower methods. In addition, thrust work potential does not count exhaust residual kinetic energy as being available for propulsive purposes. Therefore, it is not accounted as a loss, unlike exergy and gas horsepower.

[**] Note that thrust work potential for a rocket is the same as shown in the figure, except that the zero thrust potential line is equal to the zero gas horsepower line for all flight velocities.

Taxonomy of Work Potential Figures of Merit

Based on the discussion up to this point, it appears profitable to construct a rough taxonomy of the various loss figures of merit and classify them relative to one another, as shown in Table I. An "x" is placed in the appropriate matrix cells to indicate which sources of work potential are accounted for by each method. This table is by no means exhaustive, and is only intended to cover those concepts that appear to have the most use as propulsive work FoMs. Since this is a relatively new and maturing field, the definitions given in this table are the authors' interpretation of each FoM, and other authors have offered alternative definitions to those given here. Note that none of the work potential FoMs discussed here have been extended to account for the potential available in nuclear and subnuclear bonds. The authors are unaware of any figures of merit that capture this aspect of work potential, and this is apparently an area for future theoretical exploration in the physics, thermodynamics, and power generation communities.

Based on the discussion thus far, the relationship between exergy, gas horsepower, and thrust work potential becomes relatively clear. Thrust work potential is nothing more than a measure of the propulsion system's ability to project thrust work into another arbitrary (usually Earth-fixed) reference frame. It is a special case of gas horsepower to which it is related through propulsive efficiency. Gas Horsepower is merely a special case of exergy wherein only mechanical equilibrium with the environment is enforced. Exergy is a special case of what Evans[16] refers to as essergy, which can presumably be generalized to include atomic and subatomic work potential, both of which are quite beyond the realm of the authors' practical experience.

Comparison of Methods

Table II summarizes the relative strengths and weaknesses of each loss figure of merit. Both exergy and gas horsepower are physically intuitive and possess a "conservation" property that allows direct estimation of losses via Eqs. 5 and 10. However, neither produces results that are reflective of the true "costs" due to component losses in jet propulsion applications because they book-keep work potential sources that are inherently unavailable to jet propulsive machines. This point is discussed in the context of a simple optimization problem in Ref. 14 and is further elaborated upon in Ref. 1.

Table I: A Taxonomy of Prominent Work Potential Figures of Merit.

	Thermal Equilib.	***Mechanical Equilib.***	***Mixture Equilib.***	***Chemical Equilib.***	***Nuclear Equilib.***
Essergy[16]	X	X	X	X	
Exergy	X	X		X	
Gas Horsepower		X			
Thrust Work Potential		X			
Gibbs Free Energy				X	
Energy Source	Internal Energy Contained in Ensembles of Particles		Molecular Diffusion	Chemical Bonds	Nuclear Bonds
General Scale of Effect	Macroscopic Ensembles of Particles			Molecular Scale	Atomic Scale

Table II: Advantages and Disadvantages of Various Loss Figures of Merit for Jet Propulsion Applications.

	Advantages	***Disadvantages***
Exergy A Combined Cycle Figure of Merit	+Very General (Comprehensive) +Requires only Temp. & Press. to Calculate +"Conservation Law" Simplifies Loss Calcs. +Physically Intuitive Quantity (Power Loss)	-Counts Exhaust Heat, Irrev. Comb., Residual KE as Chargeable -Dependent on Reference Frame
Gas Horsepower A Turboshaft Figure of Merit	+Realistic Loss Estimate for Turbomachines +Requires only T & P to Calculate +"Conservation Law" Simplifies Loss Calcs. +Physically Intuitive Quantity (Power Loss)	-Counts Residual KE as Chargeable -Dependent on Reference Frame
Stream Thrust A Jet Thrust Figure of Merit	+Force-Based; Independent of Ref. Frame +Physically Intuitive Quantity (Thrust Loss) +Doesn't Count Residual KE as Chargeable	-No "Conservation Law" Applies -Not Directly Comparable to Available Energy or Exergy
Thrust Work Potential A Jet Work Figure of Merit	+Physically Intuitive Quantity (Power Loss) +Doesn't Count Residual KE as Chargeable +Directly Comparable to Exergy, Avail. Ener.	-No "Conservation Law" Applies -Dependent on Earth-Fixed Reference Frame -Work Potential = f(Flight Condition) -Not Meaningful for Static Operation

Stream thrust and thrust work potential account for jet propulsive losses in a physically realistic way, but do not possess a conservation property analogous to Eqs. 5 and 10 for direct estimation of loss. Thus, losses due to non-equilibrium combustion, exhaust heat, and exhaust residual kinetic energy are transparent (non-chargeable) for the thrust work potential model. Exclusion of these losses can be likened to the idea of "sunk costs" in economics wherein a cost that is irretrievably spent is ignored when making a decision as to allocation of scarce resources. Likewise, if the propulsion system cycle is specified, then one could view irreversible combustion, exhaust heat, and residual kinetic energy as sunk costs incurred in producing jet thrust.

Conclusions

Exergy can be thought of as a Carnot figure of merit in that a Carnot cycle will appear to have no losses when analyzed using exergy methods, while any departure from a Carnot cycle will appear as a loss in exergy. It is the most comprehensive and consistent loss FoM examined in that it can be shown to capture the effect of all losses relevant to contemporary propulsive cycles, including non-equilibrium combustion, exhaust heat, and exhaust residual kinetic energy.

Gas horsepower can be thought of as a Brayton figure of merit because a perfect Brayton cycle will have no loss of gas horsepower. It appears to be most useful for analysis of gas turbine power generation units and turboshaft engines. It is measured relative to a specified ambient pressure but not ambient temperature. However, it counts exhaust residual kinetic energy as a loss even though this portion of the exhaust gas horsepower is *inherently unavailable* to jet propulsion applications if the cycle is taken as given. Gas horsepower was shown to be a special case of exergy wherein only mechanical equilibrium is enforced. It was also mentioned that gas horsepower losses are always greater than exergy losses in a given component due to inability to recover exhaust heat exergy.

Stream thrust and thrust work potential produce results suggesting that it is appropriate to think of these as jet propulsion figures of merit. Stream thrust work potential is truly a measure of the thrust work that can be obtained by expansion from given conditions to ambient pressure, and is referred to a specified ambient pressure and flight velocity. Thus, thrust work potential is a special case of gas horsepower, and by extension, a special case of exergy.

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