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Final Report for Period: 09/2008 - 08/2009 Principal Investigator: Shapiro, Alexander . Organization: GA Tech Res Corp - GIT Submitted By: Shapiro, Alexander - Principal Investigator Title: Multistage Stochastic Convex Optimization

**Contribution to Project:** 

**Project Participants** 

# Name: Shapiro, AlexanderWorked for more than 160 Hours:YesContribution to Project:Name: Kleywegt, AntonWorked for more than 160 Hours:Yes

Post-doc

**Senior Personnel** 

Gradua	e Student
	Name: Narayanan, Vijay
	Worked for more than 160 Hours: Yes
	Contribution to Project:
	Started his Ph.D. research and was supported for one semester
	Name: Chun, So Yeon
	Worked for more than 160 Hours: Yes
	Contribution to Project:

**Undergraduate Student** 

**Technician**, **Programmer** 

**Other Participant** 

**Research Experience for Undergraduates** 

**Organizational Partners** 

# **Other Collaborators or Contacts**

# Activities and Findings

Shapiro, A., ``Optimization of Risk Measures' (plenary talk) and 'Stochastic Programming - Modeling and Basic Properties' (tutorial),

International Conference on Optimization under Uncertainties, Heidelberg, September 28 -30, 2005.

Shapiro, A., ``Complexity Analysis of Two and Multistage Stochastic Programming Problems',SJOM 2005 - The 3rd Sino-Japanese Optimization Meeting, Singapore, October 31-November 2, 2005 (plenary talk).

Shapiro, A., ``Stochastic Programming Approach to Optimization under Uncertainty', 19-th International Symposium on Mathematical Programming, Rio de Janeiro, Brazil, July 30- August 4, 2006 (plenary talk).

Shapiro, A., "Risk Averse Approach to Multistage Stochastic Programming', RISK DAY 2006 - Mini-Conference on Risk Management in Finance and Insurance, ETH Zurich, October 26, 2006 (invited talk).

Kleywegt, A.J., 'Effects pf Modeling Error', 11th International Conference on Stochastic Programming, Vienna, Austria, August 2007 (semi-plenary talk).

Shapiro, A., 'Computational Complexity of Two and Multistage Stochastic Programming Problems', 11-th Workshop on Well-Posedness of Optimization Problems and Related Topics, University of Alicante, Spain, September 10-14, 2007 (plenary talk).

Shapiro, A., 'Two and Multistage Risk Averse Stochastic Programming', Ninth International Conference Approximation and Optimization in the Caribbean, San Andres Island, Colombia, March 2-7, 2008 (plenary talk).

Shapiro, A., 'Some Recent Developments in Stochastic Programming', Efficient Monte Carlo: From Variance Reduction to Combinatorial Optimization, Sonderborg, Denmark, July 14-18, 2008 (plenary talk).

Shapiro, A., 'Some Recent Advances in Computational Approaches to Stochastic Optimization Problems', The 4th Sino-Japanese Optimization Meeting, National Cheng Kung University, Taiwan, August 27-30, 2008 (plenary talk).

### Findings: (See PDF version submitted by PI at the end of the report)

There are many issues associated with the use of multistage stochastic programs to model and solve applied problems, including computational complexity, and the determination of a good model to use, including selection of input parameter values for the stochastic program. We study the use of stochastic programs with modeling error and estimation of input parameter values, in the context of a revenue management problem. Recently it has been proposed to control a revenue management process by solving a certain multistage stochastic program. There are strong reasons to suspect that the proposed stochastic program has modeling errors. In addition, the probabilities associated with the scenario tree have to be chosen, presumably estimated based on observed data. We study the dynamic behavior of a process consisting of a sequence of estimated scenario trees and the associated optimal solutions, in the presence of modeling error. This is ongoing work with Vijay Narayanan, who was supported with this grant.

We studied problems involving a seller and buyers who are formulating and solving stochastic programs. The buyers do not know the input parameters of the stochastic programs, such as the probability distributions of future prices, or the probabilities that products will be available for purchase in future time periods. The buyers estimate these input parameters using observed data. However, the observed data are affected by the previous decisions of the buyers, which in turn depended on the previous estimates. We study the convergence of the estimates and chosen decisions, and compare the outcomes with settings in which the correct models are known.

This work was presented (by A. Kleywegt) at the 11th International Conference on Stochastic Programming in Vienna in August 2007.

#### **Training and Development:**

Two graduate students were trained in stochastic optimization.

#### **Outreach Activities:**

Not yet.

#### Journal Publications

Qian, Z. and Shapiro, A., "Simulation-based approach to estimation of latent variable models", Computational Statistics and Data Analysis, p. 1243, vol. 51, (2006). Published,

Shapiro, A. and Huifu Xu, "Uniform Laws of Large Numbers for Set-Valued Mappings and Subdifferentials of Random Functions", Journal of Mathematical Analysis and Applications, p. 1390, vol. 325, (2007). Published,

Ruszczynski, A. and Shapiro, A., "Optimization of convex risk functions", Mathematics of Operations Research, p. 433, vol. 31, (2006). Published,

Ruszczynski, A. and Shapiro, A., "Conditional Risk Mappings", Mathematics of Operations Research, p. 544, vol. 31, (2006). Published,

Nemirovski, A. and Shapiro, A., "Convex approximations of chance constrained programs", SIAM J. Optimization, p. 969, vol. 17, (2006). Published,

Shapiro, A., "On complexity of multistage stochastic programs", Operations Research Letters, p. 1, vol. 34, (2006). Published,

Linderoth, J., Shapiro, A., Wright, S.,E., "The empirical behavior of sampling methods for stochastic programming", Annals of Operations Research, p. 215, vol. 142, (2006). Published,

Shapiro, A., "Stochastic programming with equilibrium constraints", Journal Optimization Theory and Applications, p. 223, vol. 128, (2006). Published,

Shapiro, A., "Worst-case distribution analysis of stochastic programs", Mathematical Programming, Series B, p. 91, vol. 107, (2006). Published,

Blomvall, J. and Shapiro, A., "Solving multistage asset investment problems by the sample average approximation method", Mathematical Programming, Series B, p. 571, vol. 108, (2006). Published,

Ahmed, S., Cakmak, U. and Shapiro, A., "Coherent risk measures in inventory problems", European Journal of Operational Research, p. 226, vol. 182, (2007). Published,

Shapiro, A., "Stochastic programming approach to optimization under uncertainty", Mathematical Programming, Series B, p. 183, vol. 112, (2008). Published,

Alon, G., Beenstock, M., Hackman, S., Passy, U. and Shapiro, A., "Entropic nonparametric estimation of concave production technologies", Journal of Applied Econometrics, p. 795, vol. 22, (2007). Published,

Trindade, A., Uryasev, S., Shapiro, A. and Zrazhevsky, G., "Financial prediction with constrained tail risk", Journal of Baking and Finance, p. 3524, vol. 31, (2007). Published,

Shapiro, A. and Huifu Xu, "Stochastic mathematical programs with equilibrium constraints, modeling and sample average approximation", Optimization, p. 397, vol. 57, (2008). Published,

Shapiro, A., "Asymptotics of minimax stochastic programs", Statistics & Probability Letters, p. 150, vol. 78, (2008). Published,

Ruckmann, J.J. and Shapiro, A., "Augmented Lagrangians in semi-infinite programming", Mathematical Programming, Series B, p. 499, vol. 116, (2009). Published,

Nemirovski, A., Juditsky, A., Lan, G. and Shapiro, A., "Robust stochastic approximation approach to stochastic programming", SIAM J. Optimization, p., vol., (2008). Accepted,

Shapiro, A., "Asymptotic normality of test statistics under alternative hypotheses", Journal of Multivariate Analysis, p., vol., (2008). Accepted,

Shapiro, A., "Semi-infinite programming, duality, discretization and optimality conditions", Optimization, p., vol., (2008). Invited survey article,

#### **Books or Other One-time Publications**

#### Web/Internet Site

#### URL(s):

 $http://www2.isye.gatech.edu/people/faculty/Alex\_Shapiro/SPbook.pdf$ 

#### **Description:**

This is a (downloadable) preliminary version of the monograph "Lectures on Stochastic Programming" by A. Shapiro and A. Ruszczynski. We have worked on that monograph for several years. Currently we negotiate with the SIAM publisher for publication of this monograph.

#### **Other Specific Products**

#### Contributions

#### **Contributions within Discipline:**

In many situations there is a need to make, hopefully an optimal, decision under conditions of uncertainty. Everybody would agree with this statement. There is a disagreement, however, with how to deal with such situations. Uncertainty can come in many different forms, and hence there are various ways how it can be modelled. In a mathematical programming approach one formulates an objective function which should be optimized (say minimized) subject to specified constraints. Typically the objective function is subject to an error (say a round-off error) or, even worse, is uncertain. In

such cases fixing the involved parameters to a nominal value and then solving the corresponding optimization problem, could lead to a poor solution. The classical approach to dealing with such uncertain optimization problems is suggested by stochastic programming where the objective is optimized on average. Recently results related to computational complexity and risk averse approaches to stochastic programming were developed. This may have a significant impact on practical ways of dealing with optimization under uncertainty.

#### **Contributions to Other Disciplines:**

Not yet,

# Contributions to Human Resource Development:

Not yet.

**Contributions to Resources for Research and Education:** Not yet.

**Contributions Beyond Science and Engineering:** Not yet.

**Conference Proceedings** 

#### Categories for which nothing is reported:

Organizational Partners Any Book Any Product Any Conference

#### Discrete Choice Models with Multistage Decision Making

In classical discrete choice models, one considers a population of decision makers who have to choose among a discrete set of alternatives. Each decision maker n considers a set  $A_n$  of alternatives and chooses alternative  $a_n \in A_n$ . The random utility model of decision making supposes that each decision maker n associates a utility  $U_{an}$  with each alternative  $a \in A_n$ , and chooses the alternative  $a_n \in A_n$  with the largest utility, that is, the chosen alternative  $a_n$  satisfies  $U_{a_nn} \ge U_{a_n}$  for all  $a \in A_n$ . We would like to model the decisions made as a function of the attributes of the alternatives and the decision makers. Let  $x_{an}$  denote a vector of attribute values of alternative aand decision maker n, let  $\theta$  denote a vector of parameters to be estimated with data, let  $\xi_{an}$  denote an unobservable part of  $U_{an}$ , and let  $f(x, \theta, \xi)$  denote a chosen function. A popular choice is  $f(x, \theta, \xi) = \theta^T x + \xi$ . If  $\theta$  is the same for all decision makers and alternatives, and  $\xi$  has a distribution over the population of decision makers independent of x, then based on the distribution of  $\xi$  one can consider the probability that a decision maker, or the fraction of decision makers who, if presented with a set A' of alternatives, with attribute vector  $x_a$  for each  $a \in A'$ , will choose alternative  $a' \in A'$ , that is,  $P[U_{a'} \ge U_a \text{ for all } a \in A'] = P[f(x_{a'}, \theta, \xi_{a'}) \ge f(x_a, \theta, \xi_a) \text{ for all } a \in A'],$ Given a set of observed choices, one can consider the problem of choosing a value of the parameter  $\theta$  that best fits the data. Similar models in which  $\theta$ has a distribution over the population of decision makers can also be considered. Discrete choice models of the type above have been thoroughly studied in the literature, and have been widely applied.

The classical discrete choice model described above assumes that decision maker n knows the values of  $x_{an}$  for each  $a \in A_n$ , of  $\theta$  (or  $\theta_n$ ), and of  $\xi_{an}$ . However, often alternatives have attribute values that are not known by decision makers at the time the decision is made. Such a situation is typical in multistage decision problems. We are interested in developing discrete choice models that incorporate multistage decision making.

Our current project in this line of research is to consider settings in which decision makers have to either choose an alternative that is currently available, or to postpone a decision, anticipating a set of alternatives to choose from later. Some attribute values of the future alternatives are not known when the initial decision is made, and are forecasted by the decision makers based on historical data. The parameter  $\theta$  includes both parameters that enter into the utility function, as well as parameters of the forecasting method.