A DYNAMIC MODEL FOR DISCRIMINATION

BETWEEN INCAPACITATION AND DETERRENCE

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SUMMARY

A model is presented which simulates the criminal justice
system over a 25 year horizon. The simulation is run over discrete
one month periods incorporating the court, corrections and law
enforcement systems. The model is used to determine the effect of
various sentencing strategies involving the certainty and severity
of punishment. Those sentence policies which correspond to the
greatest estimated crime control effect are identified as optimal
solutions. Extensions of the analysis are developed to specify the
incapacitative and deterrent effects embodied in prevailing and
optimal solutions. In addition, results of the model are presented
for data bases originating in different geographical areas and a
detailed comparison is performed. Finally, methods for the validation
of the model are presented and validation results are outlined.

CHAPTER I

INTRODUCTION

The alarming rise in the level of crime experienced by our
society in the past decade indicates a definite need for re-examining
the criminal justice system. From 1965 to 1976, the crime rate in the
United States rose by over 140%. Although the theories explaining this
increase are varied, most experts will agree that growth of the absolute
level of deviance in society stems mainly from a breakdown in the
committment to conformity on an individual level. The impressive
magnitude of this breakdown has led many researchers to believe that
the individual's decision to engage in illegal activities is a function
of his perception of the economic costs and rewards for doing so in
combination with the criminal's idiosyncratic utility structure.

In the earliest years of the rapidly growing crime rate, the
popular strategy for dealing with the problem was to try and modify the
behaviors and character of the individual during incarceration. This
trend led to a large increase in expenditures for experimental programs
such as training and work therapy centers within correctional
institutions. Perhaps, due to its outwardly rehabilitative appearance,
this approach for dealing with offenders held great popular appeal for
many until about 1974. Around that time, a significant body of
literature appeared which presented evidence showing the failure of
rehabilitation. Most such studies showed recidivism rates for

correctional institutions to be roughly the same whether or not rehabilitative programs were present. As a result, the emphasis, at least in the research area of criminal justice, changed to more objective resource management problems within the system.

Much of the analytical research from 1975 to the present, in the area of resource constrained crime rate modeling, has addressed the question of allocation for a fixed resource rather than the absolute level of that allocation. In addition, concentration has switched from attacking the problem on an individual level to viewing the system in the aggregate. Here, judicial policy is seen as the controllable variables as opposed to "in-house" corrections policy. Specifically, attention has been directed toward determining what judicial policy within a stated expenditure level is associated with the lowest possible crime rate. A judicial policy consists of some trade-off combination of the certainty of imprisonment and the length of imprisonment meted out to convicted criminals.

Most of the research addressing this question up until now has been unable to distinguish between the effects of these two variables in quantitative terms. This has made it a difficult task to clarify the nature of "optimal" solutions. In addition, recent modeling has been static in nature and unable to account for changes in either indogenous and exogenous variables including time.

1.1 Research Objectives

The objective of this research is to extend the approach taken in previous research to circumvent some of the shortcomings mentioned

in the previous section. In particular, a dynamic model of the criminal justice system, integrating the courts, corrections and law enforcement bodies, is constructed. The formulation of the model addresses the questions of increasing crime rates, limits on corrections expenditures and judicial policy simultaneously, in order to account for the global impact of manipulating judicial policy. Also, the model maintains the flexibility to deal with trends in any variable present in the system over time. By this, we are able to forecast the effects of these changes into the future, as well as pinpoint optimal policies at any given time during the analysis under a given set of hypothetical conditions.

1.2 Overview of the Research

The literature survey of the previously mentioned research and findings is presented in Chapter II. The model by Blumstein and Nagin [1976] was of particular interest since its formulation was a fundamental building block for explicitly modeling judicial policies. The univariate time series modeling of crime rates by Deutsch [1976] also received greater attention because it deals with modeling crime rates independent of controllable factors. This approach to crime rate modeling eventually serves as the driving mechanism of the model through time. Finally, the research of Blumstein, Cohen, and Nagin [1975] is presented in some detail since their approach to the modeling of the behavior of criminal populations also appears in the final model.

Chapter III primarily discusses the formulation of the analytical model. This chapter illustrates how the past research is integrated to

arrive at the analytical form and discusses the necessary assumptions

for interpreting the results of the model. Finally, the third chapter

presents some closed form analysis possible when using the model, and

outlines the extensions beyond previous research which are present in

the model.

Chapter IV outlines a range of relevant experimentation

pertaining to the model. In addition, procedures for developing input

data to the model are discussed, and the technique for resolving the

model to delineate quantities of theoretical and practical interest are

presented. Chapter IV provides an item by item description of the input

and output of the computerized model. Sources for the development of

data and input parameters are discussed, and output of the computer

program is outlined and described. In addition, the sequence and flow

of logic within the computerized model is detailed.

Chapter V embodies the experimentation and simulation mentioned

in the proceeding three chapters. In this section, quantitative results

are presented and analyzed for the state of Georgia in considerable

detail. As well as providing detailed computational experience for the

basic model in Georgia, Chapter V presents results for the major

extensions of the model which are developed, and lays the groundwork

for conclusions stemming from the analysis. Finally, Chapter V

illustrates results from the model for the states of Missouri and Texas

and prepares a detailed comparison.

Chapter VI is a summary of the conclusions and recommendations

following from the previous five chapters. It also provides direction

for extensions of the research and offers a retrospective critique of

the procedures employed in our research.

CHAPTER II

LITERATURE SURVEY

The approach taken in the following literature survey is to
develop the notion of what is meant by the crime rate, what is involved
in its measurement, and how it can be characterized. This first section
reflects the views of contemporary students in criminal justice, and
some traditional ideas. This section is intended to develop the
motivation for modeling, and illustrate current thinking in the area.
The next two sections develop the concepts of deterrence and
incapacitation, respectively. Since these concepts are highly relevant
to the goals of our research in modeling, understanding of their nature
is essential.

The fourth section of the literature survey addresses some works
in contemporary criminal justice modeling. Each sub-section covers a
different approach appearing at one time or another in the literature
on modeling static systems. The final sub-section is of particular
importance, since it deals with very recent, "state of the art,"
techniques. Overall, the fifth and final section is material of a more
recent vintage, which develops the need and methodology of dynamic
modeling. Some very recent and extremely innovative research is
presented, which is among the most recent literature dealing with
stochastic modeling.

The intent of structuring the literature in the above format was

to prepare the reader for considering the plausibility of dynamic modeling with deterrent and incapacitative effects present. This would be the next logically coherent trend in the field, and is consistent with the stated goals of our research.

2.1 Crime in Society

Durkheim [1964] has proposed the interesting and provocative notion that some degree of crime is characteristic to a normal, healthy society. Durkheim states that the presence of crime in society is natural and emanates from those processes which preserve internal social stability. Erickson [1972] explains that the phenomenon of crime originates as the cultural integrity of a sub-culture is specified and reinforced. In addition, he contends that social reaction to crime also contributes to internal cohesion and serves the useful purpose of strengthening the essential and defining norms of society.

These ideas have given rise to the increasingly popular notion of some normal or "optimum" level of the crime rate. While the actual levels across various social subgroups may vary, Durkheim suggests that the approximate level within a specific group will rarely experience extreme fluctuation from its normal level. While these ideas may appeal to an observer as logically consistent or even plausible, a question arises concerning the bounds on our classification of deviant behavior. Specifically, a distinction must be made between that categorization of acts which include all deviant behaviors, or only those which are reacted to by society.

2.1.1 Measurement of the Crime Rate

The fact that the criminal justice system does not react (i.e., record, punish, arrest, etc.) to all violation of legal statute, enormously complicates the measurement of levels of crime. Clearly, the absolute crime rate does not lend itself to rigorous quantification and can only be approached through estimation of aggregate costs to society as a whole. Further, gross inconsistencies in the recording and classification of crimes by authorities and victims confuses the notion of a stable crime rate to an even greater extent. This suggests that apparent empirical evidence for dynamic stability (Blumstein and Cohen, 1973) within the crime rate is actually an indication of chronic stability in the level of societies reaction to crime. Consequently, stability in the level of punishment delivered by society over the latter part of the century, as observed by Durkheim and others, appears to bear no direct or obvious implication for the level of deviance existing during that period. Rather, the implication is continuous reformation and flexibility in the norms of society, which define the bounds on acceptable behavior.

2.1.2 Notion of a Behavior Distribution

A great deal of the recent literature relating to explanation and modeling of the crime rate posits a statistical distribution of crime related behavior (Cavan, Wilkins). Structurally, the general approach is to use a normal distribution to characterize the diversity of behavior in a society. The extremities of this scale include behavior that is severely deviant to that which is, "compulsively moralistic," (Blumstein, 1973). Although an obvious oversimplification,

the behavior distribution has proven to be a useful tool for modelers and appears repeatedly in the current literature. Somewhere near the criminal tail of the scale, analysts denote a limiting value which defines the threshold on behavior deemed as socially acceptable. Moreover, the behavior distribution is specified in some general form $g_B(x)$, while the level of punishment delivered by society, α, can be described using the general integral form:

$$\alpha = \int_{\beta_0}^{\infty} f(x)\ c(x)\ g_B(x)dx.$$

Where, $c(x)$, is the probability of arrest and conviction of an individual who has engaged in behavior x, (Blumstein, Cohen and Nagin, 1975).

During social stability, α, remains relatively stable reflecting a homeostatic punishment process. Perturbation of the threshold parameter, (β_0), through less permissive redefinition of social norms and subsequent punitive operations to a "lenient" society, might generate a short run increase of α. This would be followed by adjustment in the behavior distribution (left shifting) due to deterrence. This could be demonstrated, for example, by an increase in a societies certainty and in some cases, severity of enforcement.

2.2 Deterrence

While the purpose of isolating of criminal offenders strictly to confine their harm to society is obvious, the purpose of punishment is slightly more complex. Punishment of criminal offenders has the dual purpose of retribution for a wrongful act, and deterrence for would-be

offenders. Many social scientists today argue that not only does our correctional system fail to reform, but also fails to deter. Over the past few years, several efforts have been made to assess the deterrent effects of sentencing. These are mainly non-experimental studies based on often inaccurate police reports and diverse comparison of sentencing behavior among states, which do not show what happens when one deliberately changes sentencing patterns (other things equal). Despite this, most studies have produced consistent conclusions and the statistical techniques used make results due to chance unlikely. The thrust of these studies is revealing.

Analysis and Summary of most such studies through 1972 is provided by Hunt and Antunes, who conclude that "certainty" of punishment has a significant deterrent effect on crime rates, while "severity" of punishment has a deterrent effect only on murder. Certainty is measured by dividing the number of persons sent to prison in each state for a given year by the number of reported crimes in the preceeding years. The larger the ratio, the greater the certainty of punishment. Severity is simply the median sentence length in a given year for a given crime. Capital punishment was not considered. This would imply that a low conviction rate may lead to a higher crime rate and vice versa. An alternate explanation of these results may be that high crime rates lead to low conviction rates due to court overcrowding or corrections capacity constraints.

In short, the evidence is suggestive, (though not conclusive) that some penalties deter some crimes. Many researchers in criminal

justice, often overly concerned about the causes of crime, have
convinced themselves that the average criminal is radically different
from an ordinary person. That he or she is a compulsive and totally
irrational individual, indifferent or (incognizant) of the risks
involved in committing crimes. Apart from a small sub-class of extreme
deviants, there is little evidence to support this conclusion in regards
to the average or would-be criminal. While it may be true that the
average criminal's utility for risk may differ from that of the average
middle class citizen, it would not be surprising to find the marginal
criminal engaging in less crime if the costs were to sharply increase
relative to the benefits.

Tullock, Becker, and Erlich have attempted to explain the
behavior of the crime rate through the use of economic theory. Within
his model, Becker holds that any violation can be conceived as yielding
an increase in the offenders pecuinary wealth and/or psychic well being.
Simultaneously, in violating the law one also risks a decrease in
wealth and well being, for conviction entails paying a penalty, aquiring
a record, and other disadvantages. As an alternative one can engage in
a legal wealth or consumption generating activity. From this, a simple
economic model of choice between legal and illegal activity is
formulated. Erlich [1972] states, "the existence of a deterrent effect
can be inferred from empirical estimates of the absolute and relative
values of the elasticity of crime rates with respect to the average
offenders subjective probability of punishment and expected time served."
This of course, is extremely difficult to measure because variables that

affect deterrence are the same as those that affect incapacitation.

Wilson has illustrated the notion that the effective application of penalties, even modest ones, will deter certain forms of behavior. He cites the example, "Everyone who travels to Los Angeles from the east coast observes with awe, the extent to which traffic laws are obeyed." His explanation is that these laws have been enforced with enough vigor to make individuals feel that the risks of breaking them are sufficiently great and the costs of obeying them worthwhile. Other similar examples include studies where drunken driving in some European countries, and passing of bad checks in some states were found to be highly correlated to the intensity of enforcement efforts. Although these results are interesting, most serious students of crime would be reluctant to extend the inference to more extreme forms of crime, since these are generally associated with a different class of citizens. Differences in social class not withstanding, it is still not unreasonable to expect changes in the magnitude of risk to produce some affect on the crime rate, even though the studies do not address the importance of severe penalties. It is not clear, however, that these cases are of direct consequence when one considers that; in any rational system of criminal justice some very severe penalties will be necessary even if they have no general deterrent effect. This is because the extreme deviance of certain offenses precludes societies tolerance of small penalties, and there must always be a penalty to impose upon those already serving the maximum sentence. For example, the life sentence convict who may have "nothing to lose" by murdering another

inmate. Finally, the threat of severe penalties can be used as a
resource for investigators seeking to obtain criminal informers.
Moderating factors interacting with these arguments include the
obviously diminishing returns of severity and the relationship between
the severity of a penalty and its likelihood of imposition.

2.3 Incapacitation

As a function of imprisonment, camparitively little is known
about the effects of incapacitation. While recent research has started
to produce at least some sound empirical knowledge about deterrent
effects, quantitative intelligence of the size of incapacitative effects
is not readily available. For some time, we have understood that
physical segregation of prisoners preclude their participation in
criminal activity, however, questions on the quantitative implication
of this effect have gone unanswered.

Greenberg [1975] has presented some quantitative estimates of
the incapacitative effect of imprisonment on the rate at which seven
F.B.I. index offenses are committed. In developing these estimates,
Greenberg has provided a new interpretation of parole recidivism data.
Subject to data base limitations and a number of assumptions employed,
his model calculations provide order of magnitude estimates of the
collective incapacitative effect of imprisonment, clear of any deterrent
effect. These results will be summarized shortly.

To understand the meaning of incapacitation, it is useful to make
a distinction between "selective" and "collective" incapacitation. By
collective incapacitation is meant, crime reduction accomplished through

physical restraint regardless of the objective of the confinement.

Whether the goals of confinement happen to be incapacitative,

rehabilitative, deterrent or so on, decisions concerning who is to be

imprisoned need not necessarily relate to predictions as to future

conduct. For example, the continued incarceration of non-recidivists

can, in many cases, be termed "collective incapacitation." Collective

incapacitation is the most apparent dimension of overall incapacitation

and is referred to by Von Hirsh as, "the visible tip of the iceberg."

In contrast, selective incapacitation is defined to be, " the prevention

of crime through the physical restraint of persons selected for

confinement on the basis of a prediction that they, and not others, will

engage in forbidden behavior in the absence of confinement," (Goldfarb

and Singer, 1973). Therefore, selective incapacitation concerns those

offenders who pose a threat of serious danger to the public.

Theoretically, the optimum operating policy of our imprisonment system

would call for incarceration of only this category of deviant. As a

matter of fact, in October of 1973, the National Council on Crime and

Delinquency made the recommendation, "For all non-dangerous prisoners,

who constitute the great majority of offenders, the sentence of choice

should be one or another of a wide variety on non-institutional

dispositions," (Board of Directors, 1973).

Of course, the suggestion for determining which offenders are

to be institionalized on the basis of their violent or dangerous

propensities could be arrived at only through prediction of an

individuals behavior upon release. This issue is addressed in our

discussion concerning recent studies in recidivism and the art of its prediction, the feasibility of which is still a controversal subject.

Greenberg's findings relating to incapacitative estimates found that the amount of index crime prevented by incarceration of the present prison population amounted to less than eight percent of the total. Further, his estimates suggest that an increase of one year in the average length of time served (sentencing) could be expected to raise this figure to only 12 percent. In addition, Greenberg concludes that the unpredictability of prisoner behavior upon release precludes selective incapacitation as an effective direction for improving the incapacitative effectiveness of incarceration. While the rate of return to prison is high, most returns were not found to be the result of new convictions. This implies somewhat of a dilemma, since no method is presently known to reduce repeated crime through effective rehabilitation.

2.4 Basic Methods in Crime Rate Modeling for Static Systems

In considering the mathematical modeling of the crime rate, the analyst must bear in mind that it is not easy to specify an objective statement or the permissible means for optimization. Essentially, we want to minimize the total social costs of crime. Unfortunately, most of the components of this cost cannot be viewed in economic terms, and scientific methods don't necessarily guarantee the best balance between them. Inherent in any such balance is a weighting policy which is dependent upon subjective value judgement. In most cases, the application of operations research to criminal justice is not intended to produce optimal decisions, but rather to elucidate the implications of various

alternatives. Consequently, precise optimization of a deterministic

objective function is not highly meaningful.

Up until the present time, most modeling in criminal justice has

been deterministic in nature and must be considered as rough

approximation at best. In addition, a number of assumptions must always

be made, some of which do not have substantial direct support by data.

Consider the simple example of calculating the probability that a person

living in some area will be affected by crime in a given year. First

suppose that N crimes were committed during a given year in a small area

with population K. If we impose the assumptions that all members of the

population are equally likely to become victims and that a single victim

and criminal are associated with any given crime, then the probability

of a given person not being affected is:

$$(1 - \frac{1}{K})^N.$$

For very large populations, it is generally assumed that $N \longrightarrow \alpha K$,

where α is the average local crime rate. Consequently;

$$(1 - \frac{1}{K})^N \longrightarrow e^{-\alpha} \quad \text{(Avi-Itzhak, 1973)}$$

as $K \longrightarrow \infty$. The probability of being affected over a lifetime can

then be determined from,

$$1 - e^{-n\alpha}$$

where n is the assumed average longevity within the population.

2.4.1 Offender Behavior Modeling

Much recent work in criminal justice modeling is oriented to

describing the behavior of the individual offender. In his work,
Shinnar [1974] assumes that at the outset of his career the new criminal
commits offenses at a Poisson rate λ_o. The probability that the
offender is prosecuted and convicted subsequent to the commission of a
crime is denoted by q_o. If we assume S_1 to be the length of his first
sentence, we can associate a probability, $(\theta)_1$, to be associated with
his return to criminal activity after serving S_1. Similarly, S_1, θ_1,
λ_i, q_i, can be defined following the offender's surviving his or her
i^{th} conviction and sentence. With the additional assumptions that all
S_i are statistically independent for i = (1,2,3,...), and that the
length of the criminal career is exponentially distributed, the return
probabilities can be stated as:

$$\theta_i = \int_o^\infty e^{-s} \, dF_{S_i}(s) \quad \text{(Shinnar, 1974)}$$

where $F(S)$ is the distribution function of S_i.

2.4.2 Conviction Probability Modeling

Another technique, proposed by Avi-Itzhak and others, is criminal
justice directed toward formulating the probability of an offender
never being convicted. If we define the number of convictions during a
complete criminal career to be D (a random variable), the probability
of a criminal never being convicted can be given by:

$$P_o = P(D=0) = \sum_{x=0}^{\infty} (\frac{\lambda_o}{\lambda_o+N})^x (1-q_o)^x \frac{N}{\lambda_o+N}$$

$$= \frac{N}{N+\lambda_o q_o} \quad \text{(Avi-Itzak, 1973)}$$

where N is the inverse of the expected length of the criminal career. The probability that a criminal is not convicted, again having survived the ith conviction and sentence, is similarly defined as:

$$P_i = \sum_{x=0}^{\infty} \frac{\lambda_i}{N+\lambda_i} (1-q_i)^x \frac{N}{\lambda_i+N}$$

$$= \frac{N}{N+\lambda_i q_i} \quad \text{(Avi-Itzhak, 1973)}$$

We can further specify the distribution D in relation to P and θ as follows:

$$P(D > 0) = 1 - P_o$$

$$P(D > i(>0)) = \prod_{j=1}^{i} \theta_j (1-P_j) \, P(D > 0)$$

$$= (1-P_o) \prod_{j=1}^{i} \theta_j (1-P_j)$$

To find the expected value of D (expected number of conviction during a career):

$$E(D) = \sum_{n=0}^{\infty} (1-P_o) \prod_{j=1}^{n} \theta_n (1-P_n)$$

Similarly, the expected number of convictions can be stated as:

$$\Phi = E(D|D \geq 1) = \frac{E(D)}{1-P_o} = \sum_{n=0}^{\infty} \prod_{j=1}^{n} \theta_n (1-P_n) \quad \text{(Avi-Itzhak, 1973)}$$

Under static or "steady state" conditions, where we assume that the criminal population is constant, ϕ can be estimated from the proportion

of convictions where the offender has no prior record (virgin). If we

let Y denote the number of convictions recorded, then:

$$P(Y=1) = P_{virgin} = \frac{1}{\phi} \quad \text{(Shinnar, 1974)}$$

In addition, Shinnar has shown that

$$P(Y=y) = \frac{P(D \geq y | D \geq 1)}{E(D | D \geq 1)} = \frac{P(D \geq y | D \geq 1)}{\phi}$$

$$P(Y=1) = \frac{P(D \geq 1 | D \geq 1)}{\phi} = \frac{1}{\phi} \ .$$

Still other approaches to estimating ϕ can be found in the literature.

For example, Avi-Itzhak (1973) has estimated ϕ through repeat

probabilities. Letting R_i denote the probability that an offender with

i convictions will at least once more be convicted, we can write:

$$R_i = (1-P_i)\theta_i \quad i = 1,2,3...$$

by substituting into the equation, we have:

$$\phi = 1 + \sum_{n=1}^{\infty} \prod_{i=1}^{n} R_i.$$

If we could assume that prior record had no effect on an offender's

behavior (e.g., $R_1 = R_2 = R_3 = ... = R$),

$$\phi = \sum_{n=0}^{\infty} R^n = \frac{1}{1-R}$$

$$\text{or} \quad R = \frac{\phi-1}{\phi} \quad \text{(Belkin, Blumstein, Glass, 1973)}$$

2.4.3 Aggregate Offense Modeling

Yet another approach found in the literature toward effective modeling is aimed at predicting the number of total offenses which are expected to be committed by an individual during his or her criminal career. The expected value of this quantity and the size of the criminal population are the determinants of the level of crime in a society. If we let E(x) determine this expected value, and define a set of random variables, $\mu_0, \mu_1, \mu_2, \ldots$, where μ_j represents the number of violations committed by an offender between convictions j and (j+1), we can write:

$$E(x) = \sum_{n=0}^{\infty} E(\sum_{i=0}^{n} \mu_i | D=n) \ P(D=n).$$

Shinnar has shown that

$$P(\mu_i = x | D = n > i) = (\frac{\lambda_i(1-q_i)}{n+\lambda_i})^{x-1}(1 - \frac{\lambda_i(1-q_i)}{n+\lambda_i}) \quad x = 1,2,3\ldots$$

and

$$E(\mu_i | D = n > i) = E(\mu_i | D > i) = \frac{n+\lambda_i}{n+\lambda_i q_i}$$

substitution to the above in terms of our original variables will yeild:

$$E(X) = \sum_{n=0}^{\infty} E(\mu_n | D > n) \ P(D > n) + \sum_{n=0}^{\infty} E(\mu_n | D=n) \ P(D=n)$$

where:

$$E(\mu_n \mid D = n) = \frac{(1-P_o)\theta_n \, P_n \prod_{i=1}^{n-1}(1-P_i) \, \theta_i}{P(D = n)} \quad \frac{\lambda_n(1-q_n)}{N + \lambda_n q_o}$$

consequently,

$$E(X) = \sum_{n=0}^{\infty} \frac{\lambda_n}{n+\lambda_n q_n} \frac{P(D > n)}{1-P_n} = \sum_{n=0}^{\infty} \frac{P(D > n)}{q_n}$$

where $\dfrac{\lambda_n}{n+\lambda_n q_n}$ can be shown to be the expectation of the number of

offenses committed between the offenders n^{th} release and his or her

next conviction (or termination of the criminal career).

If it were assumed that the conviction probabilities were equal

for each time an individual were to commit an offense (i.e., $q_1 = q_2 =$

$q_3 = \ldots = q$), we could rewrite:

$$E(X) = \frac{1}{q} \sum_{n=0}^{\infty} P(D > n) = \frac{E(D)}{q}$$

which is not an unreasonable result. Using the result, $E(D) = (1-P)\phi$

and $0 \leq P_o \leq 1$, (by the definition of probability), we can make the

claim that :

$$(\frac{1-P_o}{q})\phi \leq \frac{\phi}{q} \ .$$

Imposing the additional assumption that $1-P_o \leq 1-P_i$ for $i \geq 1$, we can

make the additional claim that :

$$1-P_o \ \phi \geq \phi - 1$$

---

thus, bounding the value of E(X) between:

$$\frac{\phi-1}{q} \leq E(X) \leq \frac{\phi}{q} \ .$$

Studies by Sellin, Wolfgang and others have provided evidence that the repeat probability $(1-P_i)$ tends to increase with the severity of prior record of the offender, serving to make our additional assumption reasonable.

2.4.4 N^{th} Time Out Models

Some contemporaries in criminal justice modeling used still another approach oriented toward a hypothetical, "N^{th} time out" system. In such a system, an offender will experience his first N-1 convictions without being sentenced to prison. If a criminal is convicted an Nth time, he is assumed to serve a prison sentence infinite in length. If we again allow the number of convictions to equal:

$$\phi_N = 1 + \sum_{n=1}^{N-1} \prod_{i=1}^{n} (1-P_i)$$

maintaining the assumption that all q_i are equal, we can write the expected number of offenses committed during a criminal's lifetime as:

$$E(X)_N = \frac{1-P_o}{q} \phi_N = E(X).$$

2.4.5 Current Work in Crime Rate Modeling

Blumstein and Nagin [1976] develop a model that estimates total

crime rate as a function of imprisonment policies, incorporating

estimated incapacitative and deterrent effects. Formulation is within

an optimization framework minimizing crime rate and is utilized to

investigate the implications of alternative incarceration policies.

Within their model, the historically observed stability of the

imprisonment rate is incorporated as a constraint in the long term.

Consequently, they pose an allocation of limited resource problem to

achieve maximum crime reduction through deterrent and incapacitative

effects.

As direct policy variables, Blumstein and Nagin employ the

probability of imprisonment given conviction (or certainty = Q), and

the average time served by imprisoned offenders (or severity = S).

Other variables appearing in their model include the probability of

conviction given a crime (q), total crime rate (C), the rate at which

free criminals commit crimes (λ), the maximum per capita imprisonment

rate (μ), the proportion of time a criminal is free (N), and upperbounds

on Q and $S(Q_m, S_m)$. The formulation of the crime rate then follows the

logic of multiplying the number of criminals by their rate of crimes

committed by years free, and dividing this quantity by the total

population multiplied by the total number of crimes. In terms of our

variables :

$$C = (\lambda\ N)(P)$$

where P represents the criminal portion of the population. Thus, the

constrained optimization model can be stated as:

$$\text{Min:} \quad C = \lambda\, N(Q,S)\, P(Q,S)$$

$$\text{Subject to:} \quad I = qQSC = qQS\lambda N(Q,S)\, P(Q,S)$$

$$0 \leq Q \leq Q_m \leq 1$$

$$0 < S \leq S_m$$

Here, $P(Q,S)$ can be viewed as the decision to engage in criminal activities and is assumed in the model to be of the mathematical form:

$$P(Q,S) = \frac{e^{\mu(Q,S)}}{1 + e^{\mu(Q,S)}} \; .$$

The behavior of this function is specified by the determination of $\mu(Q,S)$ of the form:

$$\mu\,(Q,S) = \gamma_0 + \gamma_1\, Q + \gamma_2\, QS^n.$$

This represents the expected utility for incapacitation (negative), $E[D(S)]$, where:

$$E[D(S)] = aqQ + dqS^n$$

$$= \gamma_1 Q + \gamma_2 S^n$$

In addition, γ_0 is included to allow for adjustment in the size of the criminal population.

With the functional form of the criminal population specified, Nagin and Blumstein next characterize the proportion of time that criminals are free. Because λqQ is the expected imprisonment rate, it

follows that the inverse can be viewed as the expected time between commitments. Since S is the time served upon imprisonment, then the total time a criminal exists is:

$$(\lambda qQ)^{-1} + S$$

and the proportion of time free becomes:

$$(\lambda qQ)^{-1} \left[(\lambda qQ)^{-1} + S\right]^{-1}$$

$$= \frac{1}{1 + \lambda qQS} \quad \text{(Shinnar, 1973)}$$

The mathematical form of the Nagin, Blumstein optimization model can now be restated as:

$$\text{Min:} \quad C(Q,S) = \frac{1}{1 + \lambda qQS} \cdot \frac{e^{\mu(Q,S)}}{1 + e^{\mu(Q,S)}}$$

$$\text{Subject to:} \quad I(Q,S) = \frac{\lambda qQS}{1 + \lambda QqS} \frac{e^{\mu(Q,S)}}{1 + e^{\mu(Q,S)}} \leq \mu$$

$$0 \leq Q \leq Q_m \leq 1$$

$$0 < S \leq S_m$$

2.5 Basic Methods for Non-Stationary Systems

The overwhelming evidence of growth in the crime rate over the past decade is indicative of the need for dynamic modeling of the crime rate. The major difficulty is to quantitatively identify the parameter changes contributing to the sudden upswing in recorded crime rates. The evidence suggests that the main factors are the increasing size of

the criminal population, sentencing, parole policy, and the behavior characteristics of criminals. In particular, the dominating factor has been cited to be the increasing size of the criminal population and specifically, the input rate of new offenders versus the attrition rate of old offenders. This can complicate the formulation of crime rate models, particularly since the criminal career, in most studies, is assumed to start with the first arrest or conviction. In actuality, the criminal career starts with the first offense, and it is possible that many offenders are never convicted, confusing the notion of the size of the criminal population to an even greater extent.

One technique, based on the belief that non-stationarity in the crime rate has resulted from the increasing rate of new offenders, has been proposed by Shinnar [1973]. By assuming criminal career lengths constant (equaling T), uniform intensity of all offenders over time, and every criminal career starting at age 18-t, the proportion of the general population engaging in criminal acts was identified. If the number embarking on a criminal career at some point in time were Δ, and it had been increasing at an average rate, "a," until that time, then the number of offenders at that time should number:

$$F_0 = \Delta + \frac{\Delta}{a} + \frac{\Delta}{a^2} + \ldots \frac{\Delta}{a^{T-1}} = \Delta \frac{a^{T-1}}{(a-1)a^{T-1}}$$

and the number of offenders under 18 would be:

$$\Delta + \frac{\Delta}{a} + \frac{\Delta}{a^2} + \ldots \frac{\Delta}{a^{t-1}} = \Delta \frac{a^{t-1}}{(a-1)a^{t-1}}$$

from this, the proportion of offenders below 18 years of age at that time can be given by:

$$p = a^{T-t} \frac{a^{t-1}}{a^{T-1}} \ .$$

If we define, b, as the average rate at which the number embarking on a criminal career is increasing, following the year in question, we can write the offender population size as:

$$F_n = \left(\frac{b(b^n - 1)}{b - 1} \quad F \frac{1-a^{T-n}}{(1-a)a^{T-n-1}} \right) \Delta. \quad \text{(Shinnar, 1974)}$$

If we were able to assume the number of crimes committed by each offender k, we can give:

$$E(D) = KTq$$

and

$$\phi = \frac{E(D)}{1-P_0} = \frac{KTq}{1 - (1-q)^{KT}} \ .$$

Consequently, the expected number of convictions for the base year is given by:

$$G_n = F_n K_q = \left(\frac{b(b^n - 1)}{b - 1} + \frac{1-a^{T-n}}{(1-a)a^{T-n-1}} \right) \Delta Kq.$$

The probability that a criminal will experience his first conviction after j years is given by:

$$\beta = (1-q)^{Kj-K}(1 - (1-q)^K) \qquad j = 1,2,\ldots T$$

The expected number of virgin convictions n years following the base
year can be given by:

$$Z_n = \sum_{i=0}^{n} \Delta b^{(i)} \beta_{n-i+1} + \sum_{i=1}^{T-n-1} \frac{\Delta \beta_n + i + 1}{a^i} .$$

Consequently, the probability of a given conviction involving no prior
record in that (base +n) year can be stated as:

$$P_{virgin} = \frac{Z_n}{G_n} .$$

Despite its versatility, models of this type are still deterministic,
and cannot be utilized in describing changes in the parameters through
time.

2.5.1 Stochastic Empirical Models

More recently, empirical-stochastic models have been advanced by
Deutsch [1976] and others. In one recent study, Deutsch has
characterized the arrivals or "occurrences," of crimes as a stochastic
process, where eight index crimes are viewed in ten major U.S. cities.
In this article, a three stage procedure in model construction is
proposed, involving identification of a model form appropriate to the
particular data base, numerical assignment of tentative model parameters,
and statistical testing to update model form, for re-identification (as
well as adequate fit). By utilizing the Box-Jenkins multiplicative
autoregressive-moving average models, Deutsch has succeeded in developing
time dependent point forecasts of crime rates, which transmit seasonal
trend and fluctuation as well as linear trend and non-seasonal

fluctuation. In that study, rape and homicide were fit to lead-time

dependent forecasts of the form:

$$\hat{Z}_t(\ell) = \hat{Z}_t(\ell-1) = Z_t - \theta_1 a_t. \quad \text{(Deutsch, 1976)}$$

Where $\hat{Z}_t$ is the estimated level of reported crime at time t, ℓ is the

lead time, θ is a seasonal moving average, and a_t is an NID$(0,\sigma^2)$

error term. For six other index crimes studied, a similar but slightly

more complicated forecast of the form:

$$\hat{Z}_t(\ell) = \hat{Z}_t(\ell-1) + \hat{Z}_t(\ell-12) - \hat{Z}_t(\ell-13) \quad \ell \geq 15 \quad \text{(Deutsch, 1976)}$$

was developed. These stemmed from functions describing the reported

level of crime at time t, developed directly from the Box-Jenkins

model:

$$Z_t = Z_{t-1} + a_t - \theta_1 a_{t-1} \quad \text{(Deutsch, 1976)}$$

for rape and homicide, and

$$Z_t = Z_{t-1} + Z_{t-12} - Z_{t-13} + a_t - \theta_1 a_{t-1} - \theta_{12} a_{t-12}$$

$$+ \theta_{12}\theta_1 a_{t-12} \quad \text{(Deutsch, 1976)}$$

for the other six index crimes studied. Deutsch's forecasting results

were found to be highly efficient, and behaved in an intuitively

reasonable manner for estimates far into the future. Overall, each

step-ahead forecast reflected the previous forecast updated for error

and trend, and as such, characterized a simple Markov Chain process.

In addition, Deutsch's algorithm for model generation precludes "apriori bias" of the analyst by structuring the model in direct consequence to the individual data base.

2.5.2 Dynamic Modeling of the Criminal Population

Of substantial interest to our research objectives, Blumstein, Cohen and Nagin [1975] have proposed three way partitioning of the total population. Specifically, Blumstein (et. al.) considers the prison, free criminal and law abiding populations, and has made an attempt to characterize the flow rates between individual segments. Such an approach offers strong promise when one considers the potential of associating flow rates with their respective heuristic interpretations. The approximate form of the model is as follows:

Prison Population = P(t)

$K_2(t)$

$\theta K_1(t)$

$(1-\theta)K_1(t)$

$K_3(t)$

Free Criminals = C(t)

$K_4(t)$

Law-abiding Population = L(t)

$K_5(t)$

(Blumstein, et al., 1975)

Of particular interest is the flow between the law abiding and free criminal populations. Within these flows operates the social processes of redefinition in criminal behavior, the central theme of the homeostatic notion. In other words, the model is constructed such

that any purturbation in a flow will be followed by adjustment back toward an equilibrium state, in agreement with the observed stability in the level of punishment delivered by society. This, of course, is an oversimplification of the process.

The formal description of the model can be expressed as the time rate of change (or first derivative) of the respective populations.

$$\dot{P}(t) = K_1(t)P(t) + K_2(t)C(t)$$

$$\dot{C}(t) = \theta K_1(t)P(t) - K_2(t)C(t) - K_4(t)C(t) + K_3(t)L(t)$$

$$\dot{L}(t) = (1-\theta)K_1(t)\,P(t) + K_4(t)C(t) - K_3(t)L(t) + K_5(t)L(t)$$

where

$K_1(t)$ = release from prison

$K_2(t)$ = imprisonment rate of criminals

$K_3(t)$ = rate of which law abiders become criminals

$K_4(t)$ = rate at which criminals become law abiders

$K_5(t)$ = birth rate

θ = rate at which released prisoners return to criminal activity.   (Blumstein, et.al., 1975)

The dynamic character of the model was then explored under the assumption that all $K_i(t)$ flow terms are constant. Under this assumption, a differential equation governing the behavior of $P(t)$ was developed and compared with the dynamics of the actual time series for imprisonment rates. It is worthwhile noting at this point that the only known values of the system are K_1, using the results of Gottfredson

[1959] and a number of other follow-up studies on released prisoners. Other flow parameters were calculated in the following approximate manner:

$$K_2 = \text{Prison receptions (known)} \div C(t)$$

$$K_4 = 1 \div \text{(average length of the criminal career)}$$
$$\text{(guestimated value)}$$

$$K_3 = \frac{-\theta K_1 P(t) + (K_2 + K_4)\, C(t)}{T(t) - P(t) - C(t)}$$

Statistical testing of results with the actual data have proven to be encouraging, and some interpretation of flow rates was provided.

According to Blumstein, the K_1 and K_2 flow parameter of the model represent the two aspects of severity and certainty of punishment. Since increasing severity of punishments (average sentence length) would decrease the release rate, k is seen as an inverse measure of severity. Alternatively, the more criminals imprisoned, (K_2), the greater the certainty of punishment in a society will become. Since K_3 and K_4 represent flows between criminals and law abiders, they characterize what Durkheim refers to as the "level of conformity" in a society. In particular, the magnitude of flow from law abiders to criminals, (K_3), measures the "commitment to conformity." This is a complex interaction between general deterrence, internalization of social norms and other elusive factors contributing to the motivations of individual members of society. Analogously, the flow from criminals to law abiders, (K_4), represents what Blumstein has called, the "endurance of the criminal

role." This flow consists of a combination of the individual's disincentives to remain an offender, and opportunities associated with engaging in legitimate behavior.

2.5.3   Fluctuations in Criminal Population as a Markov Process

Blumstein's characterization has been extended into a three stage Markov process, where flow rates serve as transition probabilities and population segments are states of the system. Again, imposing the restriction that all $K_i(t)$ are constant over time, Blumstein has proposed the following transition matrix for his model:

$$
M = \begin{array}{c} P(t) \\ C(t) \\ L(t) \end{array}
\begin{array}{ccc} P(t+1) & C(t+1) & L(t+1) \end{array}
\begin{bmatrix}
1-K_1 & \theta K_1 & (1-\theta)K_1 \\
K_2 & 1-K_2-K_4 & K_4 \\
0 & K_3 & 1-K_3
\end{bmatrix}
$$

Examination of the matrix will reveal its positive recurrent nature, and the subsequent existence of a stationary distribution. This enables the analyst to observe system behavior in terms of a steady-state or "equilibrium" condition. In fact, this analysis was used to investigate the effects of each parameter (see Blumstein, et.al., 1975) in the model. The major findings were that relatively little could be done to reduce the proportion of criminals, but to the extent that opportunities to engage in legitimate activities and deterrence were operating, "more reasonable attempts could be made to reduce criminality."

Although Blumstein's model appears to provide satisfactory accounting of observed imprisonment rates, it is severely limited by

the assumption of constant flow rates. This precludes the integration of adaptive behavior into the model. In addition, the model fails to explicitly characterize deterrent effects, and is thus restricted in its generality.

2.6 Summary

Like many social science applications of operations research, criminal justice modeling is plagued by uncertainty in the estimation of model parameters and ambiguity in interpretation of results. Evidence from the most recent research, however, has tended to de-emphasize the importance of the relationship between judicial behavior and public safety, in light of the relative magnitude of the criminal population and society's capacity for incarceration. Similar results have been obtained in the area of feasible limits on law enforcement policy. Apparently, the results so far indicate that there is little that can be done in the way of implementable policy to control crime. Apart from accepting this grim conclusion, we fully recognize the limitations and lack of generality characteristic to current research in the field, and can only seek to enrich the substance of results through further research.

CHAPTER III

MODEL DESCRIPTION

In this chapter the basic model for relating judicial sanctions to levels of crime is developed and summarized. The first section presents the components and form of the basic equation. The following three sections develop the major building blocks appearing in the formulation. The approach taken in these sections is to represent results from the literature and illustrate the modification procedures necessary for integration of these results into our model. The first of these sections develops the underlying driving mechanism behind the model. This stems from an application of Box-Jenkins models to crime rates first presented by Deutsch [1976]. The second of these sections develops the adaption of the three-way model of society (first presented by Blumstein, Cohen and Nagin, 1975) to our model. The last of the three sections describes the optimization process within the model borrowing heavily from the work of Nagin [1976]. The final section summarizes the model and provides an in-depth comparison of the model to its predecessors.

3.1 Form of the Model

In retrospect we have found that state of the art analysis has focused primarily on the three basic elements of the criminal justice system. These elements are; law enforcement, the corrections system and the courts. In addition, it has been shown that leading research

for each of these elements is embodied in the models of Deutsch,

Blumstein, Cohen and Nagin, respectively.

The overall objective stated for our research was to develop

a model characterizing the relationship between these basic elements.

Figure 1 represents an abstraction of the criminal justice system where

arrows between boxes represent linkages between the various bodies

within the system.

```
┌─────────────┐      ┌─────────────┐      ┌─────────────┐
│     Law     │ ←──→ │   Courts    │ ───→ │ Corrections │
│ Enforcement │      │             │      │             │
└─────────────┘      └─────────────┘      └─────────────┘
      ↑                                          │
      └──────────────────────────────────────────┘
```

Figure 1.   The Criminal Justice System

The specific means by which each of the elements is modeled has

been presented in the previous chapter.   To summarize, it was shown

that law enforcement is modeled frequently in terms of its response

variable, i.e., the crime rate.   Modeling of corrections has primarily

focused on the flow of individuals through the system which for most

purposes stem from activities within the courts.   From this framework,

we would like to develop a model which will tie activities from within

the courts to the crime rate.   That is, we want to assess the activities

in the courts in terms of their impact on the rate of crime.   Such an

analysis should invariably involve the corrections system in a way that

will account for the moderating constraints and impacts embodied in this element, which are inherent to the system as a whole. That is, we cannot facilitate a change in any one element which will not have ramifications for the remaining components of the system.

In addition to the general conditions stated for the models, we also need to impart to the formulation characteristics which enhance its usefulness. Specifically, the analysis should provide insights for improving controllable policies and extrapolating for results into the future. This would be done to predict the future behavior of the prevailing system and evaluate the potential results of policy improvements which may be suggested. Finally, since the situation suggests an integrative model of its predecessors, it should combine their virtues and extend their capabilities. In the following section, a formulation representative of the criminal justice process is proposed. The model features representations of each of the elements embodied in a basic equation defining their interrelationship. Although this formulation represents only one of numerous possible approaches, state of the art research is such that satisfactory means are available for adequately modeling each of the individual components.

Figure 2 represents the logical relationships used to model crime and links each to its respective element of the system. This representation was developed first by Blumstein and Nagin [1976], who employed this logic for static modeling of the criminal justice system. With appropriate notation we can rewrite the entries from Figure 2 using this equation:

$$Z_t = \lambda_t \cdot k_t \cdot D_t$$

where,

Z_t = crime rate at time t

λ_t = average rate at which offenders commit crimes

D_t = proportion of the population engaging in crime during t

k_t = proportion of time a criminal is free

$$\begin{pmatrix} \text{Crimes in} \\ \text{Period t} \end{pmatrix} = \begin{pmatrix} \text{Offenses Per} \\ \text{Offender in} \\ \text{Period t} \end{pmatrix} \times \begin{pmatrix} \text{Proportion of} \\ \text{Offenders Free} \\ \text{to Commit Crime} \\ \text{in Period t} \end{pmatrix} \times \begin{pmatrix} \text{Proportion of the} \\ \text{Population Choosing} \\ \text{to Engage in Crime} \\ \text{During Period t} \end{pmatrix}$$

| Law Enforcement | Corrections | | Courts |

Figure 2. Summary of the Logical Relationship

This equation is heretofore referred to as the basic equation of the model. In using the basic equation, we will attempt to determine the optimal Q and S policy embodied in the D_t component of equation #1, optimal in the sense that the specified policy will result in the lowest possible level of expected crimes. To recall from the previous chapter, Q was the probability of imprisonment given conviction for a crime prevailing in the court system of interest, while S was the average sentence length meted out in that court system. These, of course, are the controllable variables within the system, subject to

certain feasibility requirements.

The procedure for using the model will be to characterize the prevailing levels of crime using the formulation of Deutsch [1976]. Corrections activities for the same periods of interest will be modeled simultaneously using a modified version of the dynamic markovian model of Blumstein, Nagin and Cohen [1975]. Integration of these two models will enable us to specify three of the four components of the basic equation modeled for prevailing conditions in the past, present and future. The remaining unknown component, D_t, is then solved for and analyzed in terms of the model of Nagin, Blumstein [1976], incorporating controllable variables, Q and S. Finally, analysis is employed to determine optimal values of Q and S in terms of minimizing Z_t and the significance of this change in policy is evaluated. Figure 3 is a summary of the components of the model in terms of the three building blocks.

$$Z_t = \lambda \cdot K \cdot D$$

Deutsch [1976]   Blumstein [1975]   Nagin [1976]

Figure 3.   Summary of Model Components in Relation
to the Three Building Blocks

The following sections of this chapter develop the specifics of each component of the basic equation, outlining the necessary modifications for their implementation and developing the mechanics for executing the model.

3.2 Specification of Z_t

The idea of characterizing crime rates with Box-Jenkins models was advanced by Deutsch during 1976. In this section, part of this work is represented in a form useful for our modeling purposes. For a more in-depth treatment of the subject, (see [11]).

In our model, the random variable, Z_t, represents the occurrence of reported index crimes in period t. For the dual purpose of precision (in using time series) and meaningful transient analysis, a period duration of one month has been chosen. Since monthly data for occurrences of index crimes becomes less available with each level of aggregation, annual figures for individual states have been transformed using monthly index crime occurrences in metropolitan areas. The actual data goes form 1966 to 1975, and represents the total number of reported offenses for each month. This data was obtained from the "Uniform Crime Reports," an annual publication of the FBI.

Following the work of Deutsch [1976], a multiplicative autoregressive moving-average model, originally proposed by Box-Jenkins, was utilized to characterize the level of crime. The model is initially loaded using the first twelve monthly estimates, and subsequent forecasts are developed using the form:

$$Z_t = Z_{t-1} + Z_{t-12} - Z_{t-13} + a_t - \theta_1 a_{t-1} - \theta_{12} a_{t-12} + \theta_{12}\theta_1 a_{t-13}$$

for $t > 12$, where a_t is calculated from W_t using the relation:

$$W_t = Z_t - Z_{t-1} - Z_{t-12} + Z_{t-13}$$

for t > 12. Finally,

$$a_t = W_{t-12} + \theta_1 a_{t-1} + \theta_{12} a_{t-12} + \theta_1 \theta_{12} a_{t-13}$$

where the values of a_t for t < 12, were defined to be zero, and the remainder solved for recursively. θ_1 and θ_{12} are parmeter estimates for the fitted model. The actual past data and forecasted levels of crime reveal an alarming growth in the rate of crime experienced by society over the past decade. Despite a recent slowdown in the rate of growth (possibly related to increasing prison populations and corrections activity), the most recent data still exhibit an increase in the level of reported offenses. Although some have argued that the increases in crime of recent years is partly attributable to inconsistencies in reporting practices and increasing willingness of the general public to report crimes, these factors alone are not sufficient to discount real expansion in the criminalistic faction of our society.

Assuming our time series approach adequately models occurrences of crime on a monthly basis, we can forecast to obtain estimates of the Z_t random variable far into the future. Previous research with this procedure suggests that is indeed an adequate representation of past data and has proven it highly efficient for short and long range forecasting purposes (see [11]). We are now in a position to specify values of Z_t in (equation #1). The following section develops our adaptation of the Blumstein, Cohen and Nagin model for specification of K.

3.3 Specification of K

For a very long duration, it is not extremely difficult to estimate the proportion of time that a criminal is free. This is because viewing this proportion over a very long period is equivalent to assuming it is constant. Such an approach does not account for changes in the magnitude and mix of the criminal population and is insensitive to whatever prevailing trend may exist in corrections activity at a given time. In order to preclude these difficulties, we would like to arrive at an estimate of the proportion of time an offender is free, based on information that is current. Current in the sense that our estimate reflects both the immediate inmate population and the prevalence of the criminal role at any instant. Through this procedure, we can formulate a dynamic mechanism to generate unique estimates for each period of interest.

If we think of the proportion of time that an offender is free as the proportion of offenders that are free over time, we can approximate this quantity by the following ratio:

$$K = C_t \Big/ C_t + P_t$$

where C_t = the criminal population at time t,

P_t = the inmate population at time t.

With a very few exceptions, this ratio characterizes the proportion of potential deviants who have the capacity to violate the law outside of prison.

At the very heart of the model lies the dynamic process by which estimates of C_t and P_t are produced uniquely for every one month time period. In the remainder of this section, we illustrate our motivation and procedure for employing the model of Blumstein, Nagin and Cohen [1975] in developing monthly estimates of the C_t and P_t random variables. In doing so, we develop the second of the four components in the basic equation.

3.3.1 Motivation for Using Time Series Modeling of P_t

Recent findings in criminal reform suggest that only a small fraction of the criminal population is responsible for the majority of crimes committed and, therefore, the most effective approach to the control of crime is incarceration of this small fraction of extreme deviants. This belief has carried over in contemporary judicial behavior.

The results have been severe overcrowding in prison facilities and unprecedented pressure on the constraints of the economic resource that society is prepared to allocate for corrections. The incidence of violent crimes, however, has leveled off somewhat during this time. This leveling off is attributable partly to the isolation of highly active criminals and partly to the deterrent effect associated with increasing prison disposition of criminal cases. The model being presented has been designed to provide estimates of this deterrent effect.

For the purpose of modeling the growth of the prison and criminal populations, we can postulate a descriptive model anchored by

the time series description of the actual growth in the imprisonment rate. To do this, an approach to modeling P_t analogous to that for modeling Z_t is employed, and provides us with a time series description of monthly prison populations for the past, present and future. In their model, Blumstein, Nagin and Cohen have shown that such a process can be generated by following a first order linear differential equation, possibly with time varying coefficients. To illustrate this connection, Blumstein, Nagin and Cohen (see [5]) describe the derivative of P_t by its corresponding difference equation using the form:

$$\dot{P}_t = P_t - P_{t-1}.$$

The general first order differential equation with constant coefficients is:

$$\dot{P}_t + dP_t = F$$

and approximating with the difference equation, we have:

$$\dot{P}_t + dP_t = (P_t - P_{t-1}) + dP_t = F$$

Putting this in the form of our original equation, the following second order autoregressive function was derived by Blumstein (see [5]):

$$P_t = \frac{P_{t-1}}{1+d} + \frac{F}{1+d}.$$

Thus, the differential equation #2 is the mathematical characterization of a dynamic process that would generate the time series that were observed. As it now stands, a process has been developed which could

represent any of numerous physical mechanisms. We would like to develop
a model which will allow the adaptation of this flow process to
contemporary corrections activity. Development of such a model will
allow us to estimate the size of the prison or criminal population
during any time period using the Blumstein model.

3.3.2 Application of the Blumstein, Nagin and Cohen Model

In their research, Blumstein, Nagin and Cohen show that for such
a dynamic description of the imprisonment rate to be useful, a model of
the social mechanism generating imprisonment rates must be formulated.
This model must also generate flows which are consistent with the trend
observed in the actual time series, and be plausible from an intuitive
standpoint. The basis of their model is the partitioning of the total
population into subgroups, one of which will be the prison population.
The rate of exchange between groups is then explored with each group
defined in terms of its own time rate of change.

3.3.3 Blumsteins Three-Way Partitioning

Assuming that each member of society can be classified as either
a legitimate citizen, criminal or prison inmate, Blumstein postulates
the description of society presented in Figure 4.

C_t represents those individuals whose behavior in total is
defined by society to be criminally deviant and eligible for
imprisonment. Conversly, L_t represents those individuals whose
behavior is considered to be socially acceptable, and P_t are members
of society at time t who are confined in penal institutions, isolated
from the remainder of society. The arrows between nodes characterize

Figure 4.   Blumstein's Descriptive Model of Society

the exchange rate between groups.   They are defined by Blumstein to be:

f_{1t} – The rate at which inmates are released from

prison in period t.

f_{2t} – The rate at which criminal cases are disposed

of through prison sentences during period t.

f_{3t} – The rate at which juveniles become delinquent,

and formerly legitimate citizens enter into

criminal activity.

f_{4t} – The rate at which criminals cease their

illegitimate activities and re-enter normal

society during period t.

f_{5t} – The rate of growth in the total population

during period t.

γ – rate of prisoner recidivism (to be distinguished

from rehabilitation).

If we think of the behavior of P_t as governed by an equation of

the form:

$$\dot{P}(t) = -f_{1t} \, P(t) + f_{2t} \, C(t)$$

and for C_t:

$$\dot{C}(t) = \gamma f_1 \, P(t) - f_2 C(t) - f_4 \, C(t) + f_3 \, L(t).$$

Blumstein has shown that we can observe the behavior of the model under the assumption of constant f_i. Specifically, we are interested in the accuracy with which the model predicts the behavior of P_t, since our earlier analysis has provided us with apriori information regarding this phenomenon. In short, P_t is considered the only known value output by the model and available for diagnosing the results predicted by the model. Clearly, the effectiveness of this formulation is tied directly to the estimation of flow parameters.

3.3.4 Blumstein's Three-Way Model as a Markov Process

According to Blumstein, if we consider each of the nodes in Figure 4 to be a state of the system, we can represent the figure with the following transition matrix.

$$M = \begin{array}{c} \\ P_t \\ C_t \\ L_t \end{array} \begin{array}{ccc} P_t & C_t & L_t \\ \begin{bmatrix} 1-f_1 & \gamma f_1 & 1-\gamma f_1 \\ f_2 & 1-f_2-f_4 & f_4 \\ 0 & f_3 & 1-f_3 \end{bmatrix} \end{array}$$

(Blumstein, et. al.)

Note the positive recurrent nature of this matrix and the subsequent

existence of a stationary distribution. This feature of the M matrix

permits the use of simulation techniques to examine the transient and

stationary distribution for different values of f_i and γ. Systematic

adjustment of the f_i can be used to investigate the effect of each

individual parameter on the equilibrium condition. Parameters which

are unknowns are then manipulated in order to obtain agreement with

the mechanism governing the imprisonment rate time series.

3.3.5 Discussion of Flow Variables in the Markov Process

At this point, the importance of policy variables should be

introduced. Within the D formulation of the general model (for the

total crime rate, not previously discussed), two controllable

variables appear. They are defined as, Q and S, the certainty and

severity of punishment, respectively. Certainty of punishment refers

to the likelihood of a prison sentence, given conviction for a crime.

Severity is a measure of the average length of sentences meted out.

Both quantities relate directly to judicial procedure constrained by

the limits of legal statute. For example, certain offenses are

subject to minimum and/or maximum sentence lengths within which,

presiding officials have relatively complete autonomy. The variables

Q and S relate directly to two of the four flow variables appearing

in the transition matrix for the process.

Since S involves the amount of time that individuals remain

incarcerated, it can be viewed as an inverse measure of the rate at

which they are released. This is precisely the f_1 flow parameter

appearing in the model. The relationship between f_1 and S can be

viewed as follows:

$$f_1: \quad S^{-1}.$$

Consequently, the value of f_1 could theoretically be related to a given value of S. It is worth noting, however, that other alternatives exist for calculating f_1 independent of S, such as the time rate of change in prison populations over a one year period. Similarly, Q is a variable regulating the number of convicted criminals entering prison. The relationship between f_2 and Q can be summarized, theoretically, by a relationship of the approximate form:

$$f_2: \quad CZ_t(Q)$$

where c is the proportion of reported crimes punishable by imprisonment, and Z_t is the level of reported offenses.

The only remaining parameters to be discussed for the Markov process are f_3 and f_4. These are the flows between the legitimate citizen and criminal populations. This probably represents a complex product of a number of different contributing factors, among them, the level of heterogeneity of society, the degree of internalization of social norms, and the deterrent effects associated with penalties. These factors all operate on different dimensions of an individual's motivation and utility structure. Rather than attempting to guesstimate these values outright, we can postulate reasonable bounds on their exact values, and simulate to solve for values which force agreement with the prison population time series. For example, if we

consider f_4, the rate at which offenders desist with criminal activities, as inversely related to the length of the average criminal career, one way to characterize the relationship is as follows:

$$f_4: \quad 1/T_t$$

where T_t represents the average duration of the criminal career at time t. In summary, when using Blumstein's Markovian model, we see that the relation between the flow variables is maintained in logical order through interrelationships defined by the Markov chain. Table 1 is a summary of plausible bounds for each of the flow parameters presented by Blumstein, Cohen and Nagin (see [5]).

3.3.6 Closed Form Transient Results

Once adequate solutions for each of the four flow variables in Blumstein's model has been found, we would like to be able to specify transient behavior of the system at any instant in time. Closed form, transient results would enhance the model's predictive powers not restricted to discrete time periods, and facilitate clean analytical results of flow variable changes, without taking successive powers of the M matrix. These results can be obtained using a simple application of geometric transform analysis.

It is possible to obtain the generating function of matrices and vectors by taking the generating function of each entry in a given matrix or vector. Consider the general relationship:

$$\Pi(n+1) = \Pi(n) \, M$$

Table 1. Plausible Bounds on Flow Parameters
(Blumstein, Nagin and Cohen, 1975)

f_{1t} – Release rate from prison $\Longrightarrow .2 < f_{1t} < 1.0$.

f_{2t} – Imprisonment rate $\Longrightarrow .01 < f_{2t} < .1$.

These bounds encompass the minimum proportion of

the total population in prison and the maximum

feasible capacity of the prison system.

f_{3t} – Rate of law abiders entering crime $\Longrightarrow .001 < f_{3t} < .01$.

These limits similarly reflect the observed minimum

proportion of the population in prison and maximum

corrections capacity given the existing values of Q

and S.

f_{4t} – Rate at which criminals reform $\Longrightarrow .2 < f_{4t} < 1.0$.

This interval accounts for the upper bound of f_{3t} and

the extreme case of total rehabilitation.

where $\Pi(n)$ is a vector of state probabilities. By taking the generating function of this equation, we obtain:

$$Z^{-1} [G(Z) - \Pi(0)] = G(Z) M.$$

Rearranging terms yields:

$$G(Z) - Z G(Z) M = \Pi(0)$$

$$G(Z) (I - ZM) = \Pi(0)$$

$$G(Z) = \Pi(0) (I - ZM)^{-1} \quad \text{(equation \#3)}$$

where $G(Z)$ is the generating function of $\Pi(n)$, and I is the identity matrix. Equation #3 suggests that the transform of the state probability vector is equivalent to the apriori state probability vector post-multiplied by $(I - ZM)^{-1}$, where $(I - ZM)^{-1}$ exists. In order to obtain a solution to a transient problem, we can weight the rows of $(I - ZM)^{-1}$ by the initial state probabilities, sum, and then take the inverse transform of each element in the result.

To illustrate this application for our purpose, consider a hypothetical two-population flow process described by:

$$M = \begin{array}{cc} & \begin{array}{cc} P_t & G_t \end{array} \\ \begin{array}{c} P_t \\ G_t \end{array} & \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{5} & \frac{3}{5} \end{bmatrix} \end{array}$$

where P_t is the prison population at time t, and G_t is the general

population at time t for these parameters,

$$(I - ZM)^{-1} = \begin{bmatrix} 1 - \dfrac{1}{2}Z & \dfrac{-1}{2}Z \\[3ex] \dfrac{-2}{5}Z & 1 - \dfrac{3}{5}Z \end{bmatrix}$$

and

$$(I - ZM)^{-1} = \begin{bmatrix} \dfrac{1 - \dfrac{3}{5}Z}{(1-Z)(1-\dfrac{1}{10}Z)} & \dfrac{\dfrac{1}{2}Z}{(1-Z)(1-\dfrac{1}{10}Z)} \\[5ex] \dfrac{\dfrac{2}{5}Z}{(1-Z)(1-\dfrac{1}{10}Z)} & \dfrac{1 - \dfrac{1}{2}Z}{(1-Z)(1-\dfrac{1}{10}Z)} \end{bmatrix}$$

Using partial fraction expansion on an element by element basis, we obtain:

$$(I - ZM)^{-1} = \begin{bmatrix} \dfrac{\frac{4}{9}}{(1-Z)} + \dfrac{\frac{5}{9}}{(1-\frac{1}{10}Z)} & \dfrac{\frac{5}{9}}{(1-Z)} - \dfrac{\frac{5}{9}}{(1-\frac{1}{10}Z)} \\[5ex] \dfrac{\frac{4}{9}}{(1-Z)} - \dfrac{\frac{4}{9}}{(1-\frac{1}{10}Z)} & \dfrac{\frac{5}{9}}{(1-Z)} + \dfrac{\frac{4}{9}}{(1-\frac{1}{10}Z)} \end{bmatrix}$$

$$(I - ZM)^{-1} = \dfrac{1}{1-Z}\begin{bmatrix} \dfrac{4}{9} & \dfrac{5}{9} \\[3ex] \dfrac{4}{9} & \dfrac{5}{9} \end{bmatrix} + \dfrac{1}{1-\frac{1}{10}Z}\begin{bmatrix} \dfrac{5}{9} & \dfrac{-5}{9} \\[3ex] \dfrac{-4}{9} & \dfrac{4}{9} \end{bmatrix}$$

Upon taking the inverse of the generating function, we obtain:

$$T(n) = \begin{bmatrix} \dfrac{4}{9} & \dfrac{5}{9} \\[3ex] \dfrac{4}{9} & \dfrac{5}{9} \end{bmatrix} + \left(\dfrac{1}{10}\right)^{n}\begin{bmatrix} \dfrac{5}{9} & \dfrac{-5}{9} \\[3ex] \dfrac{-4}{9} & \dfrac{4}{9} \end{bmatrix}$$

The usefulness of this approach becomes clear if we consider the inverse of the generating function in equation #3, yielding:

$$\Pi(n) = \Pi(0) \ T(n)$$

which becomes the exact analytical description of transient behavior. Specifically, the $(ij)^{th}$ element of $T(n)$ represents the distribution of the population in group j at time n, given that i was the resulting equilibrium condition of a previous policy.

3.4 Specification of λ

λ represents the average rate at which offenders commit crimes and implies a fixed "level of arrivals" for offenses in any given monthly period. One approach to determining a value of λ, would be to speculate about the number of criminal acts committed by the average offender in any given month. Such a procedure would inevitably involve some assessment of the motivations behind the decision to commit a serious violation of legal statute. Up until recently, a large body of criminal justice research for decision making has entangled itself in the causes of crime. Given the overwhelming evidence for the failure of offender rehabilitation and the urgent need for policy reform, contemporary criminal justice modeling has focused more objectively on the behavior in aggregate measures of social well being. Recent findings suggest that prisons with intensive rehabilitation and training programs have (with a few exceptions) roughly equivalent recidivism rates as those where idle incapacitation of inmates is practiced. Consequently, more energy is

now being directed toward improving the mechanism of controlling

offenders and less to modifying the deviant behaviors of individuals.

Rather than attempting to estimate the number of offenses

committed by the average criminal in each month, we can approximate

this quantity by the appropriate ratios. To do this, the difference

between the criminal population and deterrent effect must be fully

understood. The deterrent effect embodies all those individuals who

in some way, engage in illegal activities during a given period. This

includes career criminals, marginal criminals, and any person somehow

connected with violation of legal statute in the period of interest.

The criminal population consists only of those offenders who remain

active criminals six months or longer. Typically, this career criminal

would tend to see his illegal activities as his livelihood and hence,

have the tendency to view crime in the same light as the typical worker

would view his occupation. As a result of this, we would not expect

the average career criminal to commit crime in a seasonal fashion.

Rather, we would expect there to be a consistent pattern in monthly

per offender crimes as opposed to the overall crime rate which is

seasonal in nature. Seasonality, as in the overall level of crime,

also appears in the deterrent effect, since we believe the proportion

of the population engaging in criminal acts to be a seasonal

phenomenon (i.e., more people who are not career criminals shoplift

during the Christmas shopping season).

Using this argument, we can approximate λ for any period using

a seasonally corrected rate of crime. That is, λ_t, is equal to the

ratio of seasonally corrected offenses in period t, the average

criminal population prevailing in period t. Symbolically;

$$\lambda_t = \frac{Z'_t}{\overline{C}_t} .$$

In order to de-seasonalize the crime rate, we employ a conversion

of the $(0,1,1)(011)_{12}$ form to $(0,1,1)$. This is done as follows:

$$(1-B)(1-B^{12})Z_t = (1-\theta B)(1-\Theta B^{12})a_t$$

$$(1-B)\frac{(1-B^{12})Z_t}{(1-\Theta^{12})} = (1-\theta B)a_t$$

or

$$(1-B)Z'_t = (1-\theta B)a_t.$$

In other words, Z'_t reduces to the MA(1) model of the form:

$$Z'_t = a_t + \theta a_{t-1}.$$

In the denominator of the λ_t formulation, $\overline{C}_t$ represents the 12 month

running average of the criminal population. The motivation for using

a 12 month running average figure relates to the potential error in

estimating C_t. The error in estimating C_t over any 12 month period

will tend to represent errors over and under for individual months

as the optimization process compensates toward minimizing the deviation

from the actual prison time series. As a result, any individual

estimate may represent a larger error in one direction than the 12 month

running average of the estimates. In addition, Deutsch [1976] has shown the Z_t forecasts to be highly efficient, suggesting the error embodied within an individual estimate of Z_t' to be of acceptable magnitude.

3.5 Specification of D

In this section, the analysis for the D component of the basic equation is developed and described. We proceed by viewing D in the same perspective as Nagin and Blumstein, and employ their methods of analysis. Recall from previous sections that all the remaining building blocks from the basic equation were estimated in some way, except D. In our analysis, we will solve for a numerical value of D in each period using the results described. We then perform analysis of D using the model of Nagin and Blumstein to determine the precise problem for finding optimal values of Q and S, which would minimize the total value of D (and subsequently reduce the level of offenses). The remainder of this section outlines the specifics for implementing Nagin's formulation within our model.

In the basic equation, D represents the proportion of the general population choosing to engage in illegitimate activities. This component is meant to capture the extent to which individuals respond to the costs associated with the penalty structure. As such, this can be viewed as a general deterrence effect, varying over time with the disincentives for remaining a criminal. In the context of our model, D is explicitly a function of the certainty and severity of punishment, used to explore the implications of alternative

incarceration policies. Essentially, the problem becomes one of allocating the limited resource of man-years of imprisonment.

3.5.1 The Deterrent Formulation

Our model follows directly from the successful work of Warner [1962] and more recently Mundell [1976], in modeling individual choice behavior in the areas of transportation mode and college choice. Here we assume that the choice to engage in criminal activities, similarly, follows a logistic function. Under this assumption, D, as a function of Q and S, takes the following form adopted from the model of Blumstein and Nagin [1976]:

$$D(Q,S) = \frac{e^{g(Q,S)}}{1 + e^{g(Q,S)}}$$

where g(Q,S) is the disutility function associated with a prison sentence. The specification of the function g(Q,S) determines the behavior of the logistic function. Nagin has shown the form of g(Q,S) to be the following:

$$g(Q,S) = a + bQ + cQS \quad [\text{Blumstein, Nagin, 1976}]$$

Here the assumption that both Q and S deter criminal activities constrains the values of b and c to be negative. This is because values of b and c which are non-negative would require that

$$\frac{\partial g}{\partial Q} > 0$$

and

$$\frac{\partial g}{\partial S} > 0$$

a logical contradiction. Also, by constraining the value of, a, to be strictly negative, we can accommodate the idea that prison sentences are inherently undesirable independent of their duration. The curvature of the $g(Q,S)$ function reflects the nature of the individual's distaste for incarceration. We can summarize the form of D by the following:

$$D = \frac{\exp[a + bQ + cQS]}{1 + \exp[a + bQ + cQS]}$$

With the values of D, Q and S known, we can experiment with the functional form of D, in order to obtain estimates of, a, b and c. Consequently, experimentation with the model could provide insight into the aggregate nature of the disutility associated with various incarceration policies.

3.5.2 Description of the Policy Space

Within the model, we must recognize feasible limits on implementable policy. Since Z_t is monotonically decreasing with Q and S (an intuitively reasonable result), the best possible sanction would make Q and S large without bound. This, of course, is not possible given the restrictions on the economic resource that society is prepared to allocate toward the prevention of crime. We must, therefore, impose upper and lower bounds on the values for Q and S. The bounds on S are of the form: $0 < S \leq S_{max}$, where S_{max} is the maximum average sentence length allowable, given the level of permissiveness in our society. Since Q is a probability, its bounds are subject to the same restraints that confine the maximum value of S, as well as the definitional requirements of a probability. The

constraints of Q are of the form: $0 \leq Q \leq 1$. Finally, we must impose the constraint

$$Q*S* \leq QS$$

where Q* and S* represent the optimal values of Q and S, respectively. This constraint reflects the fixed nature of short run corrections expenditure. Since the purpose of this analysis is to address the question of allocation for a fixed resource, and not the absolute level of that resource, the above form of the prison capacity constraint (adapted from Blumstein, Nagin, 1976) was regarded as appropriate.

3.6 Summary of the Primary Model and Optimization

We have now developed the comprehensive model for describing the overall level of crime, and are prepared to summarize its form as follows:

$$Z_t = \lambda \cdot K \cdot D$$

substituting, we obtain:

$$Z_{t-1} + Z_{t-12} - Z_{t-13} + a_t - \theta_1 a_{t-1} - \theta_{12} a_{t-12} + \theta_{12}\theta_1 a_{t-13} = \lambda \cdot P \cdot D$$

where

$$\lambda \cdot P \cdot D = \frac{Z'_t}{\sum\limits_{i=1}^{12} \frac{C_i}{12}} \cdot \left[\frac{C_t}{C_t + P_t} \right] \left[\frac{e^{[a + bQ + cQS]}}{1 + e^{[a + bQ + cQS]}} \right]$$

Note that every value of Z_t, C_t, P_t and D are unique for each monthly estimate of the process. This is due to the empirical stochastic mechanism of Z_t, and the underlying dynamic process for generating

estimates of C_t and P_t.

Once the simulation has produced satisfactory experimental estimates of, a, b and c, we can proceed with the constrained optimization of Q and S over the decision-theoretic logistic function. The optimization will take the following form:

$$\text{Min:} \quad \frac{z'_t}{\sum\limits_{i=1}^{6} \dfrac{C_i}{6}} \cdot \left[\frac{C_t}{C_t + P_t}\right] \cdot \left[\frac{\exp[a + bQ + cQS]}{1 + \exp[a + bQ + cQS]}\right]$$

$$\text{s.t.} \quad Q*S* \leq QS$$

$$0 \leq Q \leq 1$$

$$0 < S \leq S_{max}$$

Constraining S to be strictly greater than zero, precludes the logical contradiction of a policy where Q is at its maximum value, while sentence lengths are held to zero. This function can be optimized for the two variables using line search or any number of non-linear optimization methods.

3.6.1 Comparison of Models

Table 2 compares the formulation developed in this chapter with three other closely related models from which this work stems. A number of relevant characteristics in the areas of effeciency and comprehensiveness are considered. With the possible exception of a need for computational experimentation, the model integrates several of the important virtues of its predecessors, and hopefully will avert some of their shortcomings. The comparison offered in Table 2 reflects

Table 2.   Partial List of Comparative Attributes

	Blumstein, Cohen and Nagin [1975]	Blumstein and Nagin [1976]	Deutsch [1976]	Deutsch, Malmborg [1977]
	(1)	(2)	(3)	(4)
Dynamic Model - Model 4 is time dependent in nature and provides analysis for changing conditions over discrete periods.	✓		✓	✓
Estimation of Deterrent Effects - Model 4 isolates the policy variables of the model within the deterrent formulation. Consequently, the responses from perturbing policy variables can be analyzed in terms of their deterrent effects.				✓
Accounts for Population Dynamics - By utilizing the descriptive formulation of Model 1 to characterize the fluctuations within subgroups of the overall population, Model 4 can relate the flows to the overall rate of crime.	✓			✓
Forecasting Ability - Following from the approach of Model 3, our model uses an efficient forecasting approach independent of the right-hand side formulation. These forecasts constrain the logistic formulation of the crime rate in order to obtain forecasted behavior of the model parameters.			✓	✓

Table 2 (cont'd)

	(1)	(2)	(3)	(4)
Accordance with Observed History – The behavior of Model 4 is moderated by both the historical notion of a stable imprisonment rate and contemporary explosion in the growth rate of reported crimes. These ideas are incorporated as mathematical constraints or are inherent in the formulation of the descriptive model.			✓	✓
Provides Insight for Decision-Making – Model 4 combines the forecasting approach of Model 3 with the relative functional structures in Models 1 and 2, used in sensitivity studies. Specifically, the Model 4 can forecast overall levels of crime for gross resource type planning or illuminate functional relationships useful in resource allocation planning.		✓	✓	✓
Estimation of Incapacitative Effects – By relating sentence lengths and imprisonment flows within a crime rate formulation, Model 4 could be used to study this relationship quantitatively.				✓
Provides Transient Analysis – Since Model 4 describes a markovian process analogous to Model 1, discrete transform analysis can be employed to obtain closed form expressions describing time lag associated with a given policy change. This approach will also aid in determining the time necessary to reach the equilibrium condition.			✓	✓
Optimization – Analogous to the constrained optimization framework for static conditions found in Model 2, the structure of Model 4 will permit static optimization for policy variables, as well as an iterative solution for an optimal policy in the dynamic case.		✓		✓
Estimation of Criminal Population – By associating a unique estimate of the size of the criminal population with each short run measurement of crime levels, Model 4 will offer a functional relation between crime and number of criminals over time.	✓			✓

Table 2 (cont'd)	(1)	(2)	(3)	(4)
<u>Makes Extensive Use of Previous Research</u> – Model 4 is an extension from Models 1, 2 and 3, with a few minor innovations and re-arrangement of form.	✓	✓	✓	✓
<u>Useful in Short Run Analysis</u> – Similar to Model 3, Model 4 is based on month by month analysis over a period of several years.			✓	✓
<u>Ability to Adapt Analysis to Smaller Scale</u> – With each parameter meaningful and available on a state or local basis, Model 4 can be utilized for analysis of a less aggregate nature.	✓		✓	✓
<u>Mathematical Tractability</u> – As an integrated combination of several mathematically trac-table models, Model 4 will most likely prove analytically feasible, although cumbersome.	✓	✓	✓	✓

the integrative nature of our model. In fact, our model contains only

two totally new approaches. These are, the method for determining λ,

and the use of the $^{C}t/C_t + P_t$ ratio, to characterize the amount of

time offenders are free to commit crimes. As such, the models

greatest virtue is that it combines the best research in each of the

three basic elements of the criminal justice system, and brings

together a description of the whole, in which every basic element is

accounted for.

3.7 Summary

In formulating our model of the criminal justice system, a

large body of literature is integrated in an effort to extract the

advantages of each approach and minimize the shortcomings of the model

as a whole. In addition, new approaches are taken toward estimating

parameters, such as λ_t and f_i, in order to further enhance the model's

capability. Perhaps the most significant innovation is the model's

ability to simulate the system over time and predict behavior far into

the future. By combining the forecasting efficiency of time series

analysis and the logical interrelationships of earlier developments,

our formulation has imparted a dynamic character to formulations which

previously could be analyzed under static conditions only. Results

stemming from the model should provide insights into prevailing judicial

policies, as well as suggest new directions for policy improvement.

CHAPTER lV

MODEL EXECUTION

The purpose of this chapter is to aquaint the reader with the

necessary logic for integrating the three major components of the

model. Namely, the Z_t, K, and D formulations. The approach taken in

this chapter is to illustrate how each of the individual components

is constructed from the data and to develop the role of each component

in relation to each of the remaining components. This is done to

clarify the procedures necessary for simulating the model. The first

and second section treat the procedure for offenses and prison data,

respectively. The third section illustrates the procedure for deter-

mining an optimal policy in terms of Q and S, following Nagin's

functional characterization of deterrence. The final section is a

brief summary followed by a concise outline of experimentation

conducted in the following chapter.

As seen in Chapter III, the form of the basic model is embodied

in the basic equation,

$$Z_t = \lambda_t \; K_t \; D_t$$

In this chapter, we will discuss how to go about utilizing this form.

Figure 5 is a macro flow diagram summarizing the procedure for

executing the basic equation. In this chapter, a step by step

discussion proceeds through the nodes of this diagram, with possible

Input Initial Observations

Generate Z_t and Z_t' forecasts

Input Prisons Data

Search for optimal markovian parameters

Generate P_t forecasts

Generate C_t values

Generate $\dfrac{C_t}{C_t+P_t}$ ratios

Solve for deterrent values

Exponent Value subroutine

Solution for γ_i subroutine

Evaluate Q*S*

Nonlinear Optimization Q*S*

STOP

Figure 5. The Macro Flow Diagram

extensions discussed in the final section.

4.1 Utilizing the Offenses Modeling Technique of Deutsch

In representing the overall level of crime, a statistic referred
to as "the total number of reported offenses" is employed. This con-
stitutes the Z_t component of the basic equation. A discrete period
duration of one month is appropriate for employing the modeling
procedure of Deutsch [1976]. Total reported offenses by month have
been compiled by Deutsch for ten major metropolitan areas. If we were
to assume that the monthly trends in the occurrence of crime for the
metropolitan areas were similar to those for the entire state of
interest, we could transform the mean of the metro data to the state
total. To do this, the proportion of each monthly figure to the annual
total, would be calculated. Since annual totals are available for all
the states in recent years, the proportion of that figure attributable
to any one given month in the year would be calculated using the
monthly proportion of the total (in the metropolitan data). In
analysis provided by Deutsch (see [11]), modeling of total reported
offenses in metropolitan areas suggested a seasonal component, as well
as a trend in most such series. This indicated a $(011)_1$ x $(011)_{12}$ fit.
The appropriate values of θ_1 and θ_{12} are calculated using the ESTM
iterative parameter estimation routine.

For the purposes of simulating the model, our procedure is to
take transformed monthly offenses figures for each state from January
1974 until December 1976. Using these 36 observations, an additional
265 observations are forecasted and collected in the vector of Z_t

realizations for each period of analysis. From these values, the deseasonalized Z_t values are generated and stored providing one sub-component of right-hand side values in the basic equation. The next section details the procedure for determining the vector of $[C_t/C_t + P_t]$ ratios enabling us to solve for the values of D_t in each monthly period.

4.2 Executing Blumstein's Criminal Population Model

In estimating the flow variables of the Markov process governing the size of the criminal population, the most severe shortage of data exists. To overcome the limited availibility of data in this area, we can employ a numerical approach toward generating estimates of the size of the criminal population in the short run. To do this, each of the unknown flow variables of the transition matrix will be allowed to vary within their plausible bounds. Simultaneous variation of f_{2t}, f_{3t}, and f_{4t} will be conducted systematically, until the combination which is most consistent with the observed behavior in the prison population time series is found. This procedure is repeated for every six month interval, until the entire series for the prison population is described. For future forecasting, transition patterns are calculated using the same procedure with forecasted levels of the prison population, provided by the time series description of the imprisonment process.

4.2.1 Recidivism in Blumstein's Populations Model

The value of γ represents the recidivism rate of released prisoners. A reasonable estimate of γ is one-third [Blumstein, 1975]. Our definition of the criminal population implies that γ includes all

those releases who commit at least one crime within twelve months of their release. In a 1971 study by Robinson and Smith, it was found that 51% of released prisoners returned to prison during the three years immediately following their release. In another study by Gottfredson [1959] it was reported that during a two-year follow-up period, 38% of released prisoners returned to prison. Since recidivism rates decline each additional year following release, and not all releases who return to criminal activity are apprehended, Blumstein claims that a reasonable value of γ for this model is one-third (see [5]). In addition, Blumstein has provided computational experience showing the relative insensitivity of the Markov Chain to variations in γ (see [5]). To see clearly how γ appears in the formulation, Figure 6 represents the transition matrix underlying the prisons and criminal populations model of Blumstein.

$$
\begin{array}{c}
 & C_t & P_t & L_t \\
C_t & \begin{bmatrix} 1-f_{1_t} & \gamma f_{1_t} & (1-\gamma)f_{1_t} \\ f_{2_t} & 1-f_{2_t}-f_{4_t} & f_{4_t} \\ 0 & f_{3_t} & 1-f_{3_t} \end{bmatrix} \\
\end{array}
$$

Where the states are as follows:

C_t = criminal population at time t

P_t = prison population at time t

L_t = law abiding population at time t

Figure 6. Transition Matrix Underlying the Markov Chain

In adapting Blumstein's Markov Model to our analysis, we are applying his estimate of γ, on a national level, to analysis of individual states. Since there is no evidence that recidivism is highly variable between states (see [3]), this is a reasonable simplification. Furthermore, the relative insensitivity of γ (see [5]), discounts the potential for error in this procedure. The sections immediately following describe the steps involved in simulating Blumstein's Markov formulation.

4.2.2 Simulation of the Markov Chain Underlying Blumstein's Model

In this section, the computational procedure for utilizing Blumstein's Populations Model is outlined. Essentially, there are two basic phenomena which are integrated in order to drive the Markov Chain. These are, the time series description of the prisons population and the transition matrix description of the social process, presented in the previous section. Figure 7 is an illustration of the general procedure.

Figure 7. Procedure for Executing the Markov Process

Computational experience with flow variable values, offered by
Blumstein, revealed the process to reach a steady state on the average
in six trasitions. Since the matrix is positive recurrent, we know
it will always attain a steady state, and in this case, the average
duration of transient behavior is six transitions. A steady state
within Blumstein's Process, however, is highly counter intuitive.
Our procedure for executing the process precludes this difficulty by
analyzing the system in six period intervals. That is, we evaluate
the output of the transition matrix for each six period interval,
using the corresponding six elements of the P_t time series. After
this, the procedure is performed again with the next six transitions
and next six elements of the P_t time series, and so on. This approach
is based on the idea that a Markovian description of the flow of
individuals through the corrections system and society is more appro-
priate for capturing the randomness component of the phenomenon, as
opposed to a period by period determination of flow variables (see
Blumstein, Cohen and Nagin, 1975). For this reason, the six month
procedure is utilized when executing the model.

4.2.2.1 The Iterative Procedure for Executing the Markov Chain.
Specific steps involved in the procedure for executing the Markov
process are vector-matrix arithmetic over six periods, adjustment of
pattern search for flow variables, and re-evaluation of the current
solution. Each of these steps is now described. The vector matrix
operation involves multiplying the initial 3x1 distribution;
$[C_t, P_t, L_t]$, by the underlying transition matrix. Each multiplication
produces an additional 3x1 vector of the above form. The resulting

vector provides our estimate of the components of the $[C_t/C_t + P_t]$ ratio (present in the basic equation) stemming from this intermediate solution for the f_{1_t} values.

Adjustment of the pattern search for flow variables involves the point search aspect of the discrete version Hooke and Jeeves algorithm. For executing the model, an IBM-16 Double Precision Code was adapted to produce and evaluate flow variable solutions. Figure 8 is a figurative summary, integrating adjustment of pattern search for flow variables to the other two steps involved in the execution of Blumstein's three-way Markov model.

```
 ┌──────────────────────────────────────────────┐
 │ Input parameters to generate Markov Chain     │
 │                                                │
 │   a.  Perform transitions; generate           │
 │       population estimates                     │
 │                                                │
 │   b.  Evaluate sum of squares error            │
 └──────────────────────────────────────────────┘
                       │
                       ▼
 ┌──────────────────────────────────────────────┐
 │ Input sum of squares error                     │
 │                                                │
 │   a.  Pattern search to find improving         │
 │       directions                               │
 │                                                │
 │   b.  Determine new base point combina-        │
 │       tion of flow variables                   │
 └──────────────────────────────────────────────┘
```

Figure 8. Figurative Summary of the Search Process

The re-evaluation of the solution step involves the channeling of data to an objective value evaluation subroutine. This final step is the direct link between the Markov Process and time series description of actual prison populations. That is, the objective subroutine compares

the current periods P_t value from the 3x1 vector solution with the time

series value of a states prison inmate population, corrected for per

capita. The per capita correction is performed using estimates of

total population statistics for the state of analysis (see [28]). At

that point, the difference is taken and squared. The squared differ-

ence is then accumulated in the summation of the prevailing six periods

series of squared differences. The optimal solution for flow variables

corresponds to the square difference summation which is a minimum, thus

evaluating solutions on the basis of least squares.

4.2.2.2 Computation of K_t. Once an optimal flow variable

solution is found, the values of C_t and P_t are available for six

periods. Consequently, the K component of the basic equation is

obtained for six periods, where;

$$K = \frac{C_t}{P_t + C_t}$$

We thus have obtained estimates of the proportion of time a

criminal is expected to be free on a monthly basis, in a given six

months for which the integrative model is simulated.

This process is in turn repeated until the analysis runs through

all six month periods contained in the prison data. The routine is

then extended for the length of the simulation, substituting forecasted

values of monthly prison populations. In the following section, the

procedure for obtaining the D_t values of the basic equation;

$$Z_t = \lambda_t \cdot K_t \cdot D_t$$

is outlined, and the necessary extensions for relating policy variables, Q and S to D_t, are developed.

4.3 Utilization of Nagin's Deterrence Formulation

In this section, the procedure for obtaining the value of the only component of the basic equation, D_t, for which a value has not been derived, is outlined. In addition, this section illustrates the procedure for obtaining an optimal Q, S policy, stemming from our analysis of D_t. Our discussion of D_t proceeds through each of the boxes pictured in the flow diagram.

Figure 9. Flow Diagram Summary of D_t Analysis

4.3.1 Solution for D_t

Since a procedure to specify each of the components of the basic equation has been developed, we can proceed to solve for D_t from the basic equation. Recall, the basic equation was written originally as;

$$Z = \lambda_t \cdot K_t \cdot D_t$$

Rearranging terms, we can specify D_t as follows;

$$D_t = Z_t [\lambda_t \cdot K_t]^{-1}$$

for any period t. Using this procedure in every period, we can obtain a vector containing the values of monthly deterrent effects for each month that the model is simulated.

4.3.2 Solution for Prevailing Q and S

To perfrom analysis appropriate for each D_t, we must have available the prevailing Q and S for the state of interest. If we recall the values of Q_t and S_t, for each period they were estimated using the relations of Nagin [1976], they were;

$$Q_t = \frac{\text{State prison population in month t}}{\text{State prison admissions in month t}}$$

where S_t represents the prevailing average sentence length, and;

$$Q_t = \frac{\text{State prison admissions in month t}}{\text{Average monthly convictions in month t}}$$

where Q_t represents the prevailing probability of imprisonment given conviction. Using estimates of this form, the values of Q and S are available in each period.

4.3.3 The Optimization Procedure

Once the values of D_t, Q_t, and S_t are obtained, we can proceed to employ Nagin's characterization of deterrence. This is done by equating our value of D_t to the functional form of deterrence;

$$D_t = \frac{e^{g[Q_t S_t]}}{1 + e^{g[Q_t S_t]}}$$

where the form of $g[Q,S]$ is rewritten as;

$$g[Q_t S_t] = \gamma_0 + \gamma_1 Q_t + \gamma_2 Q_t S_t$$

where the γ_i are negative constants reflecting the disutility of a prison sentence. To summarize, we obtain a numerical value of D_t for each monthly period and equate it to deterrence as a function of policy variables Q_t and S_t, giving;

$$D_t = \frac{e^{[\gamma_0 + \gamma_1 Q_t + \gamma_2 Q_t S_t]}}{1 + e^{[\gamma_0 + \gamma_1 Q_t + \gamma_2 Q_t S_t]}}$$

where $\gamma_i < 0$, for all i.

4.3.3.1 Solution for γ_0 Values. In solving for the γ_i, we have the immediate problem of trying to estimate three parameters, γ_0, γ_1, γ_2, with only one known value. To overcome this, we must estimate γ_0 by speculating what the value of D_t would be in the absence of sanctions (i.e., $Q = S = 0$). This can be done by extrapolating from $g(Q,S)$ to $g(0,0)$, assuming an appropriate value of $g(0,0)$, [Nagin, 1976]. In our analysis, we will define a sensitivity parameter, β, to characterize the $g(0,0)$ state (see Chapter V). Given that a value of $g(0,0)$ is attainable, we can immediately solve for γ_0, since we are left with;

$$(1+\beta)D_t = \frac{e^{\gamma_0}}{1 + e^{\gamma_0}}$$

giving one equation in one unknown. In simulating the model, this procedure is repeated in each monthly period providing the vector of γ_0 values utilized in a later stage of the analysis.

4.3.3.2 Solution for γ_1 and γ_2 Values.

When determining the values of γ_1 and γ_2, an analogous problem exists to that posed in determining the value of γ_0. That is, we are left with the problem of determining two unknowns from only one known value. Up to this point, we have developed the relation;

$$D_t = \frac{e^{[\gamma_0 + \gamma_1 Q_t + \gamma_2 Q_t S_t]}}{1 + e^{[\gamma_0 + \gamma_1 Q_t + \gamma_2 Q_t S_t]}}$$

where the values of D_t and γ_0 are now known, as well as the values of Q and S. The problem reduces to solving for γ_1 and γ_2, where the value of the expression;

$$[\gamma_1 Q_t + \gamma_2 Q_t S_t]$$

is known. Clearly, there are an infinite number of possible combinations of γ_1 and γ_2, which could satisfy such a relation.

In his analysis, Nagin claims that the $\gamma_1 Q_t$ portion of the above expression represents the stigmatization component of the disutility associated with a prison sentence, and $[\gamma_2 Q_t S_t]$ represents the disutility of actual time served (see 22). In our analysis, we will assume that a proportion, ε, of the disutility is attributed to $\gamma_1 Q_t$, and a proportion, $(1-\varepsilon)$, is a sensitivity variable (see Chapter V). Specifying a value of ε will enable us to determine specific values of

γ_1 and γ_2.

As a result of the preceeding procedures, we can specify the values of γ_0, γ_1, and γ_2, prevailing in each monthly period of the simulation. In the following section, it is seen how these values of γ_i are utilized in formulating the problem to determine the values of Q^* and S^*, which minimize D_t, and consequently, will minimize $\dot{Z}_t$, the expected number of total offenses.

4.3.4 Formulating the Optimization to Determine Q* and S*

Once this phase of the analysis of D_t is reached, the policy variables, Q_t and S_t, are now treated as unknown quantities. Up to this point, we have specified the values of γ_0, γ_1, and γ_2. We can, therefore, state the problem of obtaining the values of Q and S to minimize D_t as follows;

$$\text{Min; } D_t = \frac{e^{[\gamma_0 + \gamma_1 Q + \gamma_2 QS]}}{1 + e^{[\gamma_0 + \gamma_1 Q + \gamma_2 QS]}}$$

where Q and S are the variables. Since γ_0, γ_1 and γ_2 are all negative constants the problem is equivalent to;

$$\text{Min; } [\gamma_0 + \gamma_1 + \gamma_2 QS].$$

This is also the equivalent to solving the problem;

$$\text{Min; } Z_t = \lambda_t \cdot K_t \cdot \frac{e^{[\gamma_0 + \gamma_1 Q + \gamma_2 QS]}}{1 + e^{[\gamma_0 + \gamma_1 Q + \gamma_2 QS]}}$$

in terms of the optimal Q, S solution. Thus, by finding the values of

Q and S to minimize $[\gamma_0 + \gamma_1 + \gamma_2 QS]$, we have also found the Q, S values to minimize Z_t.

In the previous chapter, the following three conditions on Q and S were required to maintain feasibility:

$$0 < S* < S_{max}$$

$$0 \leq Q* \leq 1$$

$$Q*S* \leq Q_t S_t$$

As a result, the minimization problem can now be restated in its final form as;

$$\text{Minimize:} \quad \gamma_0 + \gamma_1 Q + \gamma_2 QS$$

$$\text{Subject to:} \quad 0 \leq S* < S_{max}$$

$$0 \leq Q* \leq 1$$

$$Q*S* \leq Q_t S_t$$

For performing the above nonlinear optimization, the same package utilized in executing Blumstein's markovian model is accessed from a different subroutine in the coded model.

4.4 Summary

In this chapter, the basic steps for executing our model, integrating the work of Deutsch, Blumstein and Nagin, have been outlined. Essentially, it was shown how to go about computing optimal values of Q and S, which will result in a minimum number of expected offenses.

In the next chapter, applications of the model are documented in cases where the actual data is employed from three states. Comparative and sensitivity studies are provided, contrasting the three data bases of the different states, and comparing behavior within our model to predecessor models. In addition, extensions of the analysis, presented in this chapter, are outlined, performed and documented.

CHAPTER V

COMPUTATIONAL EXPERIENCE

This chapter embodies the procedures and considerations necessary to simulate the model and provides a detailed example for illustration. The first section presents the solution procedure and assumptions necessary to obtain the desired output. The second section demonstrates the uses of the model through a detailed example using the Georgia Data Base. The third section considers the possibilities for modeling the input policy parameters and its subsequent implications for use of the model. The fourth section is a presentation of sensitivity studies in the Q, S policy space, attacking the problem via a series of approaches. The fifth section demonstrates the analysis necessary to de-confound the effects of incapacitation and deterrence, and provides supporting computational experience. Finally, the sixth and last section involves a comparison of judicial policies between Georgia, Missouri and Texas, with results of the model for these states.

5.1 Using the Model

Citing the developments of Chapter Three, the basic equation which underlies the model is given below.

$$z_t = \lambda_t \left(\frac{c_t}{c_t + P_t} \right) d(QS)$$

In simulating this relationship over discrete time intervals of one month duration, the main driving mechanism within the model is the Z_t forecasting mechanism. This controls the behavior of the left-hand side throughout the analysis. Imbedded within the right-hand side is an independent forecasting model characterizing the behavior of prison populations over time. The $C_t/C_t + P_t$ ratio is derived indirectly from these prison population forecasts using a three way Markov process as discussed in Chapter Three. The parameter, λ_t, is both a function of the left-hand side and right-hand side forecasting submodels. If we recall from Chapter Three, λ_t is the ratio of seasonally adjusted monthly offenses (indirectly a function of Z_t), to the six month running average of the criminal population (indirectly a function of the prison population).

As the preceeding paragraph would imply, d(Q,S), remains the only unknown quantity within the basic equation. As a result of this, d(Q,S) is solved for in each period using the relation given below.

$$d(Q,S) = \frac{Z_t}{\lambda_t} \left(\frac{C_t + P_t}{C_t} \right)$$

Consequently, each building block of the basic equation can be quantified and examined separately. The remainder of this section develops the important extensions of the basic solution procedure.

5.1.1 The Structure of d(Q,S)

The structure of the deterrent formulation was developed and presented in Chapter Three. In that section, the deterrent impact of

sanctions explicitly incorporated Q and S, the conrollable judicial
policy variables, into the functional form given below.

$$d(Q,S) = \frac{\exp[\gamma_0 + \gamma_1 Q + \gamma_2 QS]}{1 + \exp[\gamma_0 + \gamma_1 Q + \gamma_2 QS]}$$

Since it is possible to solve for $d(Q,S)$ numerically, the left-hand
side of the above relation can be treated as a known quantity. If the
prevailing judicial policies regarding average sentence length and
probability of imprisonment given conviction are known, the only
remaining unknowns are the intercept and coefficients within the expo-
nential. These, of course, are γ_0, γ_1 and γ_2.

5.1.2 Motivation for the Necessary Assumptions to Solve γ_i

It is known by definition of the above choice behavior function
that the γ_i are all negative constants. Unfortunately, only knowing
the values of Q and S on the right-hand side leaves us in a position
of trying to estimate three unknowns, γ_0, γ_1, γ_2, with only two known
values, (Q,S). As a result of this, some assumptions will be neces-
sary in order to determine the value of the γ_i parameters. These are
in addition to the more basic assumption that the effect of sanctions
is to reduce crime. The latter, of course, is supported by the
argument that by confining offenders, they are unable to inflict
offenses upon society, thereby reducing crime, at least to the extent
of the individual's capacity.

5.1.3 Determination of γ_0

To approach the problem, consider a situation where no sanctions

are present, i.e., $Q = S = 0$. By imposing the assumption that this will result in some increase in the proportion of crimes committed over the current level, we can estimate γ_0. If we call this proportion β, where $\beta > 0$, we can proceed with the following development.

Under prevailing policy: $d(Q,S) = d(Q,S)$

Under zero sanctions: $d(0,0) = (1+\beta)\, d(Q,S)$

Consequently, we have:

$$(1+\beta)\, d(Q,S) = \frac{\exp[\gamma_0 + \gamma_1(0) + \gamma_2(0)]}{1 + \exp[\gamma_0 + \gamma_1(0) + \gamma_2(0)]}$$

$$(1+\beta)\, d(Q,S) = \frac{\exp[\gamma_0]}{1 + \exp[\gamma_0]}$$

$$\gamma_0 = \log_e[(1+\beta)\, d(Q,S)/(1 - (1+\beta)\, d(Q,S))]$$

where $\beta > 0$.

5.1.4 Determination of γ_1 and γ_2

Finally, to estimate γ_1 and γ_2, we must impose an assumption regarding the proportion of the disutility associated with imprisonment that is attributable to the actual severity of a sentence, and that which is attributable to the stigmatization associated with a prison sentence. If we let ε be that proportion of disutility which is associated with the stigmatization of a prison sentence, we can proceed to estimate γ_1 and γ_2 through the following developments. Taking the log of the deterrent effect, we have:

$$\log_e[d(Q,S)/(1 - d(Q,S))]$$

Using a previous result for γ_0, we can write:

$$\gamma_1 Q + \gamma_2 QS = \log_e[d(Q,S)/(1 - d(Q,S))] - \gamma_0$$

thus,

$$\gamma_1 Q + \gamma_2 QS = \log_e[d(Q,S)/(1 - d(Q,S))] -$$

$$\log_e[(1+\beta) \, d(Q,S)/(1 - (1+\beta) \, d(Q,S))].$$

Consequently,

$$\gamma_1 = \frac{\varepsilon(\log_e[d(Q,S)/(1 - d(Q,S))] -}{Q}$$

$$\frac{\log_e[(1+\beta) \, d(Q,S)/(1 - (1+\beta) \, d(Q,S))]}{Q}$$

and

$$\gamma_2 = \frac{(1-\varepsilon)(\log_e[d(Q,S)/(1 - d(Q,S))] -}{QS}$$

$$\frac{\log_e[(1+\beta) \, d(Q,S)/(1 - (1+\beta) \, d(Q,S))]}{QS}$$

where, $\beta > 0$ and $0 \le \varepsilon \le 1$.

It is worth noting that the values of γ_0, γ_1 and γ_2 are

determined uniquely for each one month period.

5.1.5 Solution for the Optimal Policy

Once values for the γ_i are obtained for each period, the model proceeds to solve for those values, Q^* and S^*, which will result in the greatest deterrent impact. This is done by finding a policy which is feasible in terms of the corrections capacity constraint and minimizes the proportion of the population which engage in illegal activities during period t. Since the corrections capacity constraint can be stated for any period t, as:

$$Q_t^* \, S_t^* \leq Q_t \, S_t$$

the optimization problem can then be stated as:

$$\text{Minimize:} \quad \gamma_{0_t} + \gamma_{1_t} Q + \gamma_{2_t} QS$$

$$\text{subject to:} \quad Q_t^* \, S_t^* \leq Q_t \, S_t$$

$$\text{and:} \quad 0 < Q_t \leq 1$$

$$\text{and:} \quad 0 < S_t \leq S_{t_{max}}$$

5.1.6 Limiting Cases of Q and S

The purpose of this section is to investigate the theoretical behavior of the model for limiting cases of Q and S. Namely, the zero and infinite sanction level cases. Computationally, this could be done by adding a constraint of the form: $Q = S = 0$, for the zero

sanction case or relaxing the S_{max} and product constraint to allow
infinite values of S in the infinite sanction level case. Effectively,
this would remove convicted felons permanently from the system and
deter all others.

   5.1.6.1 The Case of Zero Sanctions. As we can see from the
basic equation, the zero sanction level will inevitably perturb the
entire system. In fact, it can be shown that to allow the zero
sanction situation in our model would shift the expected number of
total offenses in period t, from Z_t to $(1+\beta)Z_t$.

   To see this, consider the basic equation:

$$Z_t = \lambda_t \left(\frac{C_t}{C_t + P_t} \right) d(Q,S)$$

If we assume the zero sanction situation, the deterrent effect will
increase by some proportion β. Consequently, we can rewrite the basic
equation as:

$$Z_t' = \lambda_t \left(\frac{C_t}{C_t + P_t} \right) (1+\beta) \, d(Q,S)$$

$$Z_t' = (1+\beta) \lambda_t \left(\frac{C_t}{C_t + P_t} \right) d(Q,S)$$

Thus, $Z_t' = (1+\beta)Z_t$ and, therefore, we could expect the total number of
offenses to increase by an amount βZ_t under the zero sanction condi-
tion.

   5.1.6.2 The Case of Infinite Sanctions. The effect of the
infinite sanction case can be seen if we consider the implications

of infinite sanction for the deterrent effect and subsequent ramifica-
tions in the basic driving equation of the model. Consider first, the
limit given by the expression below.

$$\lim_{QS \longrightarrow \infty} [\gamma_0 + \gamma_1 Q + \gamma_2 QS] = -\infty$$

Since the γ_i are by definition negative constants, if we define a
variable x, where:

$$x = -[\gamma_0 + \gamma_1 Q + \gamma_2 QS]$$

we can rewrite d(Q,S) in the following form:

$$d(Q,S) = \frac{e^{-x}}{1 + e^{-x}}$$

Taking the limit of x:

$$\lim_{x \longrightarrow \infty} \frac{e^{-x}}{1 + e^{-x}} = \frac{0}{1} = 0.$$

Substituting into the basic equation at taking limits, we have:

$$\lim_{QS \longrightarrow \infty} [Z_t] = \lambda \left(\frac{C_t}{C_t + P_t} \right) [0] = 0.$$

Thus, we see that the functional form of the model necessarily
predicts zero reported offenses for the case of infinite sanctions.
The thrust of this section has been to explore the behavior of the
model for the limiting cases of judicial policy variables Q and S.

Here it was seen that the behavior of the model for the zero sanction level reflects our assumption regarding social behavior in the absence of sanctions. Similarly, the results illustrated for the case of infinite sanctions embody the more general assumption that the presence of sanctions tends to decrease crime.

5.1.7 Sensitivity of the Model to the Necessary Assumptions

In this section, the potential impact on the results of the two basic assumptions necessary to solve for optimal values of Q and S is investigated. In a preceeding section, it was shown that statements regarding the zero sanction state and the utility distribution between stigmatization and the actual incarceration experience (inherent in the perception of a prison sentence) must be made in order to develop the γ_1 parameters. In effect, these statements impose the necessary constraints facilitating the solution of γ_0, γ_1, and γ_2.

5.1.8 The Implications of the β Assumption

We first focus on the solution procedure for γ_0, since it is performed independently of the procedure for γ_1 and γ_2. Recall that it was necessary to determine a value of β, representing the increase in criminal activity in the absence of sanctions. Clearly, the final (Q,S) solution of the optimization problem will not be affected by the choice of β. This can be seen if we consider the effect of increasing the expected level of crime anticipated in the absence of sanctions above some current level of crime, Z_t, by an amount, β. This leads to the following development.

$$Z_t = \lambda_t \left(\frac{C_t}{C_t + P_t}\right) e^{[\gamma_0 + \gamma_1 Q + \gamma_2 QS]} \qquad \gamma_i < 0 \forall$$

increasing Z_t by an amount β, gives:

$$(1+\beta)Z_t = \lambda_t \left(\frac{C_t}{C_t + P_t}\right) (1+\beta) e^{[\gamma_0 + \gamma_1 Q + \gamma_2 QS]} \qquad \gamma_i < 0 \forall$$
$$\beta > 0$$

Knowing for this condition that: $Q = S = 0$

we have:

$$e^{[\gamma_0]} = (1+\beta) e^{[\gamma_0 + \gamma_1 Q + \gamma_2 QS]}. \qquad \gamma_i < 0 \forall$$
$$\beta > 0$$

At this point, the functional relationship between β and γ_0 becomes apparent if we consider the limit:

$$\lim_{\beta \longrightarrow \infty} (1+\beta) e^{[\gamma_0 + \gamma_1 Q + \gamma_2 QS]} = \infty$$

as a result, we can express the limit:

$$\lim_{\beta \longrightarrow \infty} e^{[\gamma_0]} = \infty .$$

Consequently, we can see the effect of increasing β is to increase γ_0, as a result, the objective value of the optimization problem:

$$\text{Min:} \quad \gamma_0 + \gamma_1 Q + \gamma_2 QS$$

$$\text{s.t.} \quad Q_t^* S_t^* \leq Q_t S_t$$

$$\text{and:} \quad 0 < Q \leq 1$$

$$\text{and:} \quad 0 < S_t \leq S_{t_{max}}$$

will also increase with β. It then follows that the value of the deterrent formulation under optimal condtions:

$$e^{[\gamma_0 + \gamma_1 Q^* + \gamma_2 Q^* S^*]}/1 + e^{[\gamma_0 + \gamma_1 Q^* + \gamma_2 Q^* S^*]}$$

will appear larger, the larger that we assume the increase in reported crimes, β, will be (if sanctions are eliminated). Moreover, the optimal values of the sanction variables (Q^*, S^*) will not be affected by changing the value of β, since the effect of increasing (or decreasing) the value of the entire expression:

$$(1+\beta)K = \gamma_0 + \gamma_1 Q + \gamma_2 QS$$

will not affect the ratio γ_1/γ_2. This in turn, will result in an optimization procedure for the same linear combination at Q and S, regardless of the value of β.

The results of the preceding analysis has been to prove that our assumption regarding the zero sanction state necessary to solve for the value of γ_0 will not affect the optimal solution for Q and S. It will, however, affect what the model predicts the savings in reported offenses stemming from optimization will be. In fact, it was shown that the effect of assuming too large a value of β, would be to overestimate the effectiveness of (Q,S) sanctions and subsequently overstate the impact of optimization. Analogously, to assume too

small a value of β, would have the effect of understating the impor-
tance of optimizing judicial sanction variables. In any case, it is
the significance of optimization, rather than the correctness, which
is affected by the choice of β.

5.1.9 The Implications of the ε Assumption

In addition to deciding on the appropriate value of β, an
assumption regarding the disutility associated with a prison sentence
must be imposed. Essentially, there are two aspects of this disutility.
One involves the actual time incarcerated in the prison environment,
which is presumed to be in itself an onerous experience. The other
aspect of the disutility associated with a sentence is related to the
stigmatization perceived by an offender, that is, the disutility of
aquiring a prison record and its associated consequences.

Since the latter is unrelated to the actual sentence imposed,
it relates exclusively to the sanction variable Q. On the other hand,
the disutility of the actual prison sentence relates directly to both
Q and S. In terms of the deterrent formulation, this implies that γ_1
relates to the stigmatization component and γ_2 to the sentence compo-
nent. Hence, the relation:

$$\gamma_0 + \gamma_1 Q + \gamma_2 QS.$$

As explained earlier in this section, the approach taken in allocating
the total disutility to each of the components is to define a parameter,
ε, representing that proportion of the disutility attributable to
stigmatization. As a result, we define the disutility, ds, as:

$$ds = \gamma_1 Q + \gamma_2 QS$$

and that component attributable to stigmatization as:

$$\varepsilon ds = \gamma_1 Q \qquad 0 < \varepsilon < 1$$

and that component attributable to the actual sentence as:

$$(1-\varepsilon)ds = \gamma_2 QS \qquad 0 < \varepsilon < 1.$$

We can see from the above that the effect of overstating ε, would be to inflate the absolute value of γ_1 (a negative constant) and consequently, favor Q in the optimization process. Alternatively, to underestimate ε, would discount the importance of Q in the optimization process, consequently, favoring the value of S.

In addition to altering the optimal values of Q and S, the choice of ε, will also influence the value of the objective function within the optimization subproblem. This in turn, will influence the results for the importance of the optimization process. To see this, consider the deterrent formulation as a function of ε.

$$d(Q,S) = e^{[\gamma_0 + \varepsilon ds + (1-\varepsilon)ds]}$$

where $\qquad \varepsilon ds = \gamma_1 Q$

and $\qquad (1-\varepsilon)ds = \gamma_2 QS.$

Clearly, the behavior of the optimization process is dependent on whether the value of the S exceeds unity and the ratio γ_1/γ_2.

Specifically, if we have the condition:

$$S > \frac{\gamma_1}{\gamma_2}$$

the optimization process will favor the value of S. Alternatively,
Q will be favored if the opposite is true. In addition, if the value
of γ_1 exceeds γ_2 and $S^* < \gamma_1/\gamma_2$, overstating the value of ϵ will
inflate the value of $d(Q,S)$ and overestimate the impact of sanction
variables, Q and S. It is, therefore, necessary to know the value of
the individual γ_1 and γ_2 parameters in order to assess the affect of
the ϵ assumption. This question is addressed in sensitivity studies
later in the chapter.

5.2 Analysis Using Georgia Data

The purpose of this section is to illustrate the application of
the model to a data base from the state of Georgia. In order to
clarify the use of this example, we impose direct assumptions regard-
ing the values of β and ϵ. Specifically, we will assume that the
crime rate experienced by society in the absence of sanctions will be
20% greater than otherwise for each period, (i.e., $\beta = 0.20$). Further-
more, we will assume that 75% of the disutility of a prison sentence
will, on the average, be attributable to the actual sentence and 25%
of the disutility will be attributable to stigmatization, (i.e., $\epsilon =$
.25). Moreover, the choice of these values of β and ϵ, will facili-
tate a meaningful comparison of our own results with another model by
Blumstein and Nagin [1976], who provide limited computational experi-
ence for the same assumptions.

5.2.1 Input Policy Variables for Georgia

Other information which is pertinent to interpreting the
results contained in this section is that an average sentence length
of 1.67 years and a probability of imprisonment of .30606 is used
throughout the analysis in this section. These were found to be the
mean values for average sentence length and imprisonment probability
in Georgia courts during the period from January 1974 until December
1976. We, therefore, will assume that these values will remain
constant for the duration of the simulation, so as not to obscure the
behavior of other important parameters in presenting the Georgia
example. A closer look at the development and ramifications of this
data is taken in a later section. It must be remembered, however,
that the values, $S = 1.67$ and $Q = .30606$ represent prevailing policy
in Georgia, assumed to be unchanged over the 25 year simulation. This
somewhat unrealistic condition is imposed only to facilitate the
clear illustration of an exmaple run, and is relaxed somewhat in a
subsequent section.

5.2.2 Total Reported Offenses in Georgia

In order to interpret the behavior of the model over a 25 year
simulation, an appropriate point of departure is to examine the
behavior of the Z_t forecasting mechanism, which is the driving force
behind the model. When the model was simulated from the present to
the year 1994, a rapid growth in the level of reported offenses was
predicted. This was due to the nature of the original Z_t time series
for the months from January 1976 until December 1976. The data is
presented in Table 3.

Table 3.   Total Reported Offenses for Georgia

Month	1974	1975	1976
Jan.	15837	15456	16991
Feb.	14484	15517	17237
Mar.	14641	15822	17028
Apr.	14817	15566	17342
May	15033	16073	17150
June	15243	16171	17563
July	15439	16332	17421
Aug.	15491	16291	17397
Sept.	15597	16516	17411
Oct.	15493	16981	18091
Nov.	15512	16452	17960
Dec.	15482	16470	18114

The Georgia total reported offenses data was identified as a non-stationary series, appropriately modeled by the $(011)(011)_{12}$ Box-Jenkins formulation. Iterative estimation of the parameters suggested a value of $\theta = .2695$. As a result, the forecasts were characterized by a steady growth over the 25 year simulation. To illustrate, Table 4 shows the Z_t values predicted for seven sample periods covering the analysis.

From examining the values in Table 4, we see that the value for the left-hand side of the basic equation is increasing with time. Just as the value of Z_t increases, the value of $\hat{Z}_t$, the seasonally corrected offense rate, will also increase temporally. As a result,

Table 4. Total Reported Offenses Forecasted for Seven Sample Periods

Period		Z_t (forecasted)
March	1975	15823
September	1979	21721
January	1983	25905
May	1987	32652
July	1990	36216
November	1994	41228
December	1997	45098

we would now expect at least one component on both sides of the basic equation to be increasing over time. In order to clarify the effect of the increasing Z_t on the other parameters of the basic equation, a control run, where Z_t is held constant, was also performed. The result of this run is referred to continually in the following analysis.

5.2.3 Prison Populations in Georgia

Another forecasting submodel appearing in the basic equation is the forecasting mechanism for predicting prison populations in the state of Georgia over the next 25 years. Like the total reported offenses data, the behavior of the state institution inmate population totals predicted for Georgia during the simulation, will be related to the input data. Table 5 shows the state institution inmate population totals in Georgia for the months from January 1974 until December 1976. Statistical analysis of this series again suggested the $(011)(011)_{12}$ Box-Jenkins forecasting model as appropriate. This time, however, the

Table 5. Georgia State Institution Inmate Population Totals

Month	1974	1975	1976
Jan.	9832	10891	11421
Feb.	9904	11072	11613
Mar.	10171	11323	11453
Apr.	10392	11172	11548
May	16262	11341	11481
June	10780	11360	11587
July	10955	11459	11537
Aug.	11050	11305	11521
Sept.	11128	11326	11469
Oct.	11045	11513	11756
Nov.	11061	11422	11423
Dec.	10985	11389	11350

parameters were estimated to be: θ_1 = .6279 and θ_2 = .2028.

Unlike the total reported offenses series, the growth predicted for the state institution inmate population totals was gradual. Table 6 illustrates the behavior of the forecasts for seven periods of interest covered in the analysis. This was also true of the simulation run for the deterministic Z_t situation, since the two submodels are developed independently of each other.

5.2.4 Criminal Population Movement in Georgia

The contradictory growth rates of the total reported offenses and the prison populations submodels explain a number of key relationships between parameters of the model. For example, the ratio:

$$\left(\frac{C_t}{C_t + P_t}\right)$$

was found to remain fairly constant over the 25 year simulation for both the non-stationary Z_t case and the deterministic Z_t. In fact, the results for this parameter were identical for both cases, since the above ratio relates only to the prison populations forecasting submodel and the three way markovian search pattern for C_t. A sample of the results for seven periods of interest, along with their corresponding general deterrent effect, is presented in Table 6.

Table 6. Illustration of the Growth in
Georgia State Prison Populations

Period		Prison Population	λ_t	d_t	C_t
March	1975	11321	0.29	1.32%	1.15%
September	1979	11660	0.37	1.38%	1.16%
January	1983	11756	0.43	1.42%	1.16%
May	1987	12180	0.53	1.43%	1.17%
July	1990	12416	0.59	1.39%	1.19%
November	1994	12643	0.67	1.41%	1.18%
December	1997	12903	0.73	1.38%	1.18%

The data appearing in Table 7, clearly suggests that state institutions house between 16% and 17% of the offender population at any one time. Although these results are somewhat high in comparison to analogous results presented in the Blumstein, Nagin, Cohen [1975]

Table 7. Criminal and Prison Population Percentages

Month		C_t	P_t	C_t/C_t+P_t	$d_t(Q,S)$
March	1975	1.15%	0.2305%	83.3%	1.32%
September	1979	1.16%	0.2312%	83.4%	1.38%
January	1983	1.16%	0.2311%	83.4%	1.42%
May	1987	1.17%	0.2331%	83.4%	1.43%
July	1990	1.19%	0.2314%	83.8%	1.39%
November	1994	1.18%	0.2337%	83.5%	1.41%
December	1997	1.18%	0.2313%	83.7%	1.38%

model, they are within a reasonable order of magnitude. Their results suggest that this percentage averages between 12% and 13% on a national level.

5.2.5 Behavior of λ_t in Georgia

Also apparent from Table 7 is the fact that the absolute magnitude of the criminal population in the state of Georgia is predicted to rise only slowly over the next 24 years. If we consider the non-stationary Z_t situation, we see that the parameter on the right-hand side of the basic equation, which adjusts for the large increases in Z_t (the left-hand side), is λ_t, the average number of offenses committed by the average criminal in period t. To illustrate, C_t (criminal population), d_t (deterrent effect) and λ_t are presented in Table 8. Here, the units of λ_t are crimes per offender per month. The figures from the table suggest that the average number of crimes committed by an individual career criminal will grow from about 3.5

Table 8. Sample Results for Seven Periods

Period		Z_t	λ_t	d_t	C_t
March	1975	15823	0.29	1.32%	1.15%
September	1979	21721	0.37	1.38%	1.16%
January	1983	25905	0.43	1.42%	1.16%
May	1987	32652	0.53	1.43%	1.17%
July	1990	36216	0.59	1.39%	1.19%
November	1994	41228	0.67	1.41%	1.18%
December	1997	45098	0.73	1.38%	1.18%

in 1975 to about 8.8 in 1997. This result did not hold for the case

of deterministic Z_t, where λ_t fluctuated only very narrowly with

changes in C_t.

5.2.6 General Deterrent Effects in Georgia

One other interesting result,which can be observed from Table 8,

is the relative stability of the general deterrent effect, d(Q,S), over

the 25 year simulation. This result was true of both deterministic and

non-deterministic Z_t forecasting models. A qualitative explanation and

discussion of this and the previously described behavior in the model

is offered in a later section.

5.2.7 Optimization Process for the Georgia Data Base

Recall that the procedure for obtaining optimal values of Q and

S was first to determine numerical values for d(Q,S) in each period.

This done, the values of the coefficients (γ) for the objective function

are obtained, given the necessary assumptions. For example, the values

of γ_0, γ_1 and γ_2, for the month of March 1975, would be obtained as follows:

$$\gamma_0 = \log_e\{[1.2(.0132068)]/[1 - 1.2(.0132068)]\} = -4.129$$

$$\gamma_1 = .25\{\log_e[.0132068/1 - .0132068] - \gamma_0\}/.30606 = -.15109$$

$$\gamma_2 = .75\{\log_e[.0132078/1 - .0132068] - \gamma_0\}/(1.67)(.30606) = -.2716$$

where, $.0132068 = d(Q,S)$ for March 1975.

In simulation studies covering the months from January 1974 to December 1998, some typical values obtained for γ_0, γ_1 and γ_2 (using data for the state of Georgia) appear in Table 9.

Table 9. γ_i Values for Seven Periods

Period		Deterrent Effect	γ_0	γ_1	γ_2
March	1975	1.32%	-4.129	-.15109	-.2716
September	1979	1.38%	-4.077	-.15127	-.2720
January	1983	1.42%	-4.055	-.15129	-.2718
May	1987	1.43%	-4.045	-.1531	-.2749
July	1990	1.39%	-4.070	-.1516	-.2724
November	1994	1.41%	-4.072	-.15120	-.2718
December	1997	1.38%	-4.081	-.1510	-.2713

Throughout this simulation, the values of Q and S were held at the constant levels of .30606 and 1.67 years, respectively. Once values for the γ_i are obtained for every period, the model proceeds to solve

for those values of Q* and S* which will result in the greatest deter-
rent impact. This is done by finding a policy which is feasible in
terms of the corrections capacity constraint, and minimizes the per-
centage of the population who engage in illegal activities during
period t (the deterrent effect). The corrections capacity constraint
can be stated for any period t as:

$$Q^*_t \ S^*_t \leq Q_t \ S_t$$

The optimization problem for Q and S can then be summarized for
period t as:

$$\text{Min:} \quad [\gamma_{0_t} + \gamma_{1_t} \ Q + \gamma_{2_t} \ QS]$$

$$\text{s.t.} \quad Q^*_t \ S^*_t \leq Q_t \ S_t$$

$$\text{and} \quad 0 < Q \leq 1$$

$$\text{and} \quad 0 < S \leq S_{max}$$

The procedure was performed for each month from January 1974
to December 1998. The results for the seven periods mentioned in
Table 9 are presented in Table 10. It is worth noting that since the
input values of Q and S were constant, their optimal values over time
remain constant within four decimanl places. The relatively stable
behavior in the value of the deterrent effect explains why this is so.
In addition, the optimization procedure (being a discrete version
pattern search, developed by Hooke and Jeeves) is only as accurate as

Table 10. Optimization Results for Georgia

Period		Q	S	Q*	S*	ΔQ	ΔS
March	1975	.3016	1.67	.4605	1.11	.15	-.56
September	1979	.3061	1.67	.4605	1.11	.15	-.56
January	1983	.3061	1.67	.4605	1.11	.15	-.56
May	1987	.3061	1.67	.4605	1.11	.15	-.56
July	1990	.3061	1.67	.4605	1.11	.15	-.56
November	1994	.3061	1.67	.4605	1.11	.15	-.56
December	1997	.3061	1.67	.4605	1.11	.15	-.56

the number of step-size reductions performed. For executing the above optimization, the initial step-sizes were:

$$\Delta Q = Q/3$$

$$\Delta S = S/2 \text{ years}$$

with eight step-size reductions performed in each optimization. Consequently, the search was accurate to within about 10^{-8} units for each variable.

5.2.8 Motivation for Using Constant Input Policy Variables

The rationale in using values of Q and S, which are held constant over time, is that the best model for predicting each is the mean of the actual series. This would imply a stationary sentencing policy from month to month. (In fact, when the actual series of imprisonment probabilities was analyzed, it was found to be non-stationary. This case is discussed in a subsequent section.)

When conditions of static judicial administration were
simulated over a 24 year period, the gain in policy effectiveness
stemming from optimization was similarly stable over time. For each
monthly period, the optimal values of Q and S were identical, since
the constraint set was unchanged throughout the simulation and the
objective function coefficients (γ_i) did not vary significantly. With
the optimization process in each period a repetition of the process
in every other period, the percentage gain in effectivenss by switch-
ing to optimal levels of Q and S would be expected to be constant.
To verify that this was in fact the case, the number of crimes saved
in each period was measured and its percentage of the total was
computed.

5.2.9 Evaluation of the Impact of Optimization

In order to determine how many crimes would be prevented in a
month by going from prevailing to optimal values of Q and S, the basic
equation was reconstructed using Q* and S*. The deterrent effect in
period t, under optimal policy, was obtained by the substitution.

$$d_t^*(Q,S) = \frac{\exp[\gamma_0 + \gamma_1 Q^* + \gamma_2 Q^*S^*]}{1 + \exp[\gamma_0 + \gamma_1 Q^* + \gamma_2 Q^*S^*]}$$

and the expected number of crimes under optimal policy can then be
stated as:

$$Z_t^* = \lambda_t \left(\frac{C_t}{C_t + P_t}\right) d^*(Q,S).$$

The expected number of crimes saved during t, is:

$$Z_t - Z_t^*$$

and the percentage savings through optimization:

$$100[(Z_t - Z_t^*)/Z_t].$$

The numerical results for seven typical periods over the twenty-four year horizon are presented in Table 11.

Problems with this approach of measuring the impact of optimization can arise if a new policy was such that its effect were to enlarge or reduce the prison population to a significant degree. Such a shift would thereby change the value of the $[C_t/C_t + P_t]$ ratio and perturb the system. This, however, was not considered a serious problem, since earlier analysis by Blumstein and Cohen [1975] and Greene [1974] have shown that prison populations account, at most, for about 17% of the criminal population. (In fact, the prison populations accounted for about 20% of the criminal population at its maximum level in our study.) Furthermore, such shifts in state institution populations tend to be gradual and moderated by the presence of the uniform corrections capacity constraint. Analogous results for the case of deterministic Z_t were identical with the exception that the saving in numbers of crimes did not change over time.

5.2.10 Discussion of Sample Results

The results presented in Table 7 suggests that the proportion of society which comprise institutionalized offenders should be expected to demonstrate remarkable stability over time. This implication tends to reinforce the fundamental notion underlying the

Table 11.   Expected Total Offenses in Georgia

Period		Prevailing Policy	Optimal Policy	Saving	Percentage
March	1975	15823	15463	360	2.278%
September	1979	21721	21226	495	2.278%
January	1983	25905	25315	590	2.278%
May	1987	32652	31908	744	2.278%
July	1990	36216	35391	825	2.278%
November	1994	41228	20289	939	2.278%
December	1997	45098	44071	1927	2.278%

Blumstein, Nagin, Cohen model [1975], namely, "the stability of punishment." Blumstein claimed that the punishment process was a homeostatic phenomenon, which remained stable over time, regardless of the level of deviance present in society. This notion was originally offered by Durkheim [1964] in a manuscript entitled, "The Rules of the Sociological Method." Durkheim suggested that the presence of crime in society is natural and emanates from the same processes which preserve internal social stability.

Blumstein used Durkheim's ideas as a stepping stone in developing the concept of a behavior distribution. Blumstein believed that the level of deviance present in society at any one time appeared to bear no direct relation to the level of punishment meted out by the society. In other words, if the level of crime were to suddenly experience a sharp rise, society would redefine the limits on acceptable behavior rather than expand the punishment process. This would

be interpreted as a shift to the left. Alternatively, if the level of
crime were to decrease sharply, the society would respond with more
vigorous enforcement of existing laws as corrections, judicial and
law enforcement resources became free to press for greater effective-
ness. This would be represented by a shift to the right in Figure 10.
A classic example of this phenomenon would be the development of legal
off-track betting in several states which gave legitimacy to the
behavior of thousands of individuals previously considered as criminals.

Socially Unacceptable
Behavior

Socially Acceptable
Behavior

ℓ

Severely Deviant

Compulsively Moralistic

ℓ = the current limit on socially acceptable behavior.

Figure 10. The Behavior Distribution

Blumstein, Nagin and Cohen used this concept of equilibrium in the
social order as the basic theme of their 1975 model. Despite the
modifications within our own model, which uses math programming
techniques to determine the markovian driving parameters, the implica-

tion of the results is astoundingly similar, that is, punishment is a basically homeostatic process.

Even though stability was present to a large extent in corrections activity, throughout the 25 year simulation, criminal activity rose sharply. At least, what we now consider criminal activity experienced a significant upward trend. We have previously shown that the criminal population is expected to rise only slowly in Georgia. Also, the proportion of the overall population which in some capacity (not necessarily career criminals) engages in illegal activities (deterrent effect) is likewise expected to fluctuate very slowly upward in the next 25 years. Consequently, we must turn to other causes to explain the alarming rise in total offenses which are expected over this period.

The explanation offerred by the model is a growth in λ_t, the average number of offenses committed by the individual criminal in period t. One possible explanation for this phenomenon lies in the fact that the growth in prison populations lags far behind the growth in the crime rate. As a result, criminals may view the risks involved in committing an offense as remaining stable, while the benefits for doing so are enhanced. That is, a criminal perceives his probability of imprisonment given conviction as increasing only marginally with each additional offense. The benefits associated with committing that additional offense, however, may increase linearly or exponentially, given the risk-amenable utilities thought to be characteristic of many criminals. At the same time, the criminal population would only account for a very small proportion of the total population, thereby,

tending to impart only a marginal shift in the deterrent effect.

5.3 The Effects of Nonstationary and/or Correlated Sentencing Practices

If we were to re-examine our actual data from which the pre-vailing policy variables, Q and S, are derived, it would be of con-siderable interest in any evidence of growth or seasonality were present. In fact, any pattern recurring in either series could illuminate a trend present in current policy which may have serious implications for the effectiveness of the criminal justice system, today or in the future. For example, falling sentence lengths over time with stationary probabilities of imprisonment would necessarily be followed by a period of rising sentence lengths, due to the systems relentless tendency toward the equilibrium condition. This is underscored by the concept of the behavior distribution underlying the formulation of the model. If such a phenomenon were in fact observed, it could betray the presence of a pendulum of justice effect operating within our system, lending itself to explicit modeling in further analysis.

5.3.1 Average Sentence Length

Recall that the average sentence length was determined by dividing the total prison population in some month by the number of prison receptions in that same month. This procedure was undertaken for the months between January 1974 and December 1976. Table 12 presents the figures obtained for these months and Figure 11 shows a plot of the time series. Units in the table are years.

Figure 11.   Average Sentence Time Series for Georgia 1974 - 1976

Table 12.   Average Sentence Lengths for Georgia

Month	1974	1975	1976
Jan.	1.58	1.51	1.83
Feb.	1.79	1.61	2.00
Mar.	1.55	1.25	1.59
Apr.	1.48	1.28	1.74
May	1.52	1.46	1.76
June	1.78	1.56	2.09
July	1.59	1.74	2.02
Aug.	1.83	1.64	1.76
Sept.	1.66	1.88	1.83
Oct.	1.31	1.57	1.61
Nov.	2.14	2.04	1.63
Dec.	1.49	1.35	1.75

Subsequent statistical analysis of the data identified the set as a stationary, non-seasonal series.   This can be verified qualitatively by inspection of Figure 11.   Consequently, the mean was used to forecast the series which had a value of 1.67 years.

5.3.2   Probability of Imprisonment Given Conviction

The monthly probability of imprisonment given conviction was computed by taking the ratio of prison receptions for a given month, and the average monthly convictions total.   The results calculated for the months from January 1974 to December of 1976 are presented in Table 13.   Also, Figure 12 shows a plot of the time series.

When the data was analyzed, it was found to be non-seasonal and

Table 13. Monthly Probability of Imprisonment for Georgia

Month	1974	1975	1976
Jan.	.221	.295	.260
Feb.	.361	.263	.246
Mar.	.365	.304	.326
Apr.	.348	.326	.325
May	.366	.326	.290
June	.366	.277	.276
July	.283	.301	.252
August	.343	.269	.269
Sept.	.373	.299	.237
Oct.	.401	.379	.281
Nov.	.341	.229	.217
Dec.	.348	.329	.326

non-stationary. Inspection of Figure 12 provides a quick verification of this analysis. Diagnosis of the time series through a statistical identification routine suggested the (011)(0,0,0) Box-Jenkins forecasting model as appropriate.

The (001)(000) Box-Jenkins model proceeds using the following definitions:

$$w_t = Q_t - Q_{t-1} \qquad t = 2.....36$$

$$a_t = w_t + \Phi_1 a_t \qquad t = 2.....36$$

$$Q_t = Q_{t-1} - \Phi_1 a_{t-1} + a \qquad t = 2.....36$$

$$\hat{Q}_t = Q_{t-1} - \Phi_1 a_{t-1} \qquad t = 37$$

Figure 12.   Probability of Imprisonment Time Series for Georgia
              (1974-1976)

$$\hat{Q}_t = \hat{Q}_{t-1} \qquad\qquad t = 37...$$

where $\qquad a_1 = w_1 = 0$

and, Φ_1 is the iteratively estimated growth parameter found to equal
.7706 for the probability of imprisonment given conviction time series.
Figure 13 shows a plot of how the forecasted series will behave over
time.

When the model was simulated with the Box-Jenkins fit of Q
imbedded in the simulation, the first 36 periods were characterized by
constantly changing Q, and consequently, a constantly changing capacity
constraint. The value of S was constant at 1.67 years. For periods
beyond December 1976, the value of Q (and subsequently Q*) remains
constant at .2751. Given our previous sensitivity studies of the
systems, we would expect the optimization process to lean much more
strongly toward Q, given the capacity constraint was tightened by
about 10%. This was, in fact, what was observed as an initital policy
of (.2751, 1.67 years), led to an optimal policy of (.7575, .61 years).

Under the initial (.2751, 1.67) policy, about 20% of the savings
in reported offenses due to sanctions was attributable to incapacita-
tion. When the optimal (.7575, .61) policy was in effect, only about
7% of the impact of sanctions was due to incapacitation on average.
Despite this, the (.7575,.61) policy was responsible for about a 7.7%
rise in the overall impact of sanctions. This result suggested the
possibility that the optimization of imprisonment policy was even more
critical at the lower values of the capacity constraint.

Figure 13.   Behavior of Forecasted Q Values

The main idea stemming from this experiment has been one stressing the importance of stability in judicial policy. Specifically, if the judicial system tends to oscillate over short periods producing a pendulum of justice-type effect, it can expect wide variation in the per dollar return of corrections allocations over time. In addition, the system should develop a dynamic (Q,S) policy synchronized with its schedule of expenditures in order to maintain uniform per dollar effectiveness over time. As we have demonstrated, this policy would tend to emphasize the imprisonment option with shorter sentences in lean years and the reverse in prosperous years. The ethics question of such a practice would, of course, be a moderating factor present in the system, but has not been treated in our analysis.

5.4 Sensitivity Studies in the Q,S Policy Space

In their analysis of the implications of alternative sentencing policies, Blumstein and Nagin [1976] emphasize the importance of the value of trade-offs implicit in formulating a Q,S policy. They view the debate over the volume of imprisonment as destined to be a stand-off, given the clear demonstration of the existence of a stable imprisonment rate. This demonstration brought to light the major issue of their thesis, which was the problem of allocating a fixed prison resource, rather than deciding on absolute allocation level.

Similar to Blumstein and Nagin, our model explores the implications of alternative imprisonment policies. In this section, the model estimates the crime-control potential of imprisonment deriving from a combination of deterrent and incapacitation effects, while

incorporating due process and resource constraints explicitly. In much the same manner as Blumstein and Nagin, we consider a homogeneous criminal population committing a single aggregate crime type. The last part of this section is a comparative analysis between the two models.

In the period between 1960 and the first year analyzed by the model, the reported index crime rate in the United States rose by 157%. The rise of the reported index crime rate in the state of Georgia during this time was 173%. Although the trend in reported offenses may to some extent reflect a growth in reporting rates, there is no doubt that the actual crime rate has risen sharply. This section focuses on different strategies aimed at reversing, or at least moderating, this trend through the use of prison. Such an approach has attracted considerable attention in the last two or three years, given the failure of rehabilitative approaches, despite the fact that it is overtly punitive.

The first step in investigating the impact of various forms of imprisonment policy is to identify those variables subject to direct manipulation. These, of course, are Q, the probability of imprisonment given conviction, and S, the average sentence length. The purpose of this analysis is then to evaluate the impact of incremental changes in Q and S relative to each other, in order to determine which of these is the more effective reducer of crime. This knowledge would enable the decision maker to design a policy resulting in the optimal utilization of his scarce resource, namely, available man-years of imprisonment. In addition, such an analysis could provide insight for the

absolute level of allocation question by bringing to light the impli-
cations for incremental tightening or relaxation of the resource
constraint. Specifically, we would like to determine if the expected
rate of offenses behaves linearly or non-linearly with respect to Q
and/or S, and at what level of the policy space will incremental
changes in either variable be most effective.

5.4.1 Design of a Factorial Experiment

Since $d_t(Q,S)$ is monotonically decreasing in Q and S, then the
crime rate, Z_t, is also monotonically decreasing in Q and S, which is
of course, consistent with the incapacitative and deterrent effects of
the two sanction variables. As we have shown in the previous section,
without other constraints, the optimum sanction would make Q and S
large without bound. By imposing an upper limit on the average
sentence length, we assume that sanctions beyond S_m are precluded.
Similarly, constraining Q to be less than unity can reflect limitations
on universal imprisonment of all convicted persons through practices
like diversion of first offenders. Despite this, it is the resource
limit on imprisonment which has proven to be the binding constraint
for all practical calculations.

Blumstein and Nagin [1976] have shown the imprisonment resource
constraint to be operative in the form of a limit on the product of
policy variables, Q and S. The computational results mentioned thus
far, would suggest that the state of Georgia might better deter
criminal activity by more frequent prison disposition of criminal
cases with somewhat shorter sentences than is the current policy.

To explore this hypothesis more explicitly, a factorial
experiment at three levels of Q and S is proposed. The dual purpose
of this experiment was to determine the effect of the resource
constraint and initial values of Q and S on the optimal solution. In
other words, at what point does the effectiveness of Q tail off in
favor of increasing S? Is the relation between Z_t and the policy,
linear or nonlinear? And at what levels?

Figure 14 illustrates the nine starting points at which obser-
vations were taken in the Q,S policy space.

S

	1	2	3	4	5	6	7

(A)	(B)		(C)
Q = .306	Q = .306		Q = .306
S = 1.67	S = 3.35		S = 6.69
(D)	(E)		(F)
Q = .612	Q = .612		Q = .612
S = 1.67	S = 3.35		S = 6.69
(G)	(H)		(I)
Q = .918	Q = .918		Q = .918
S = 1.67	S = 3.35		S = 6.69

Q values: .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0

Figure 14. Starting Points for the Factorial Experiment

For each of the points appearing in the figure, Table 14 gives the
initial values, the optimal values, and the changes in Q and S, as

Table 14. Optimization Results for the Nine Starting Points

Point	Solution (Q,S) Starting	Optimal	ΔQ	ΔS	% Savings
A	(.306, 1.67)	(.461, 1.11)	.155	-.56	2.278%
B	(.306, 3.35)	(.620, 1.65)	.314	-1.70	4.579%
C	(.306, 6.69)	(.850, 2.41)	.544	-4.28	7.792%
D	(.612, 1.67)	(.620, 1.65)	.008	-.02	0.061%
E	(.612, 3.35)	(.850, 2.41)	.238	-.94	1.756%
F	(.612, 6.69)	(.997, 4.11)	.385	-2.58	2.831%
G	(.918, 1.67)	(.863, 1.78)	-.005	.11	0
H	(.918, 3.35)	(.995, 3.09)	.077	-.24	0.381%
I	(.918, 6.69)	(.995, 6.17)	.077	-.52	0.381%

well as the percentage savings due to optimization of sanction levels.
Although comparison in terms of absolute numbers of crimes is not
meaningful due to different spending levels, the percentage change
reflects the benefit from reallocating prison resources given the
absolute level of resource allocation and political disposition
implicit in the starting policy.

5.4.2 Analysis of Experimental Results

Table 14 would indicate that once a policy provides for a
probability of imprisonment that equals or exceeds about .918,
optimization for the given capacity constraint will not result in a
significant gain unless the average sentence length is below about
1.67 years. This can be verified if we examine the small percentage
savings experienced when the initial value of Q is about .918 for
three levels of S, and the similarly small savings when the starting
policy is (.612 , 1.67). In addition, when Q is small, about .306,
the optimization process tends to increase in Q for all three levels
of S, doing so in a linear fashion in Q and a negative linear fashion
in S. This suggests that Q is clearly dominating at the lower levels
of the capacity constraint. Finally, Table 14 suggests an analogous
result for Q in the range of about .612, although both the increase
in Q and the decrease in S are found to be less dramatic. Figures 15
and 16 are plots of the percentage increases in Q and S, suggested by
the optimization procedure against the capacity constraint for the
three levels of Q and S. Tables 15 and 16 illustrate the procedure
from which the plots are derived.

Table 15. Percentage Changes in Q

Low Level of Q = .306

Prison Capacity Constraint (QS)	Percentage Change in Q ($\Delta\%$ Q)
.511	50.6%
1.02	103.0%
2.06	178.0%

Medium Level of Q = .612

Prison Capacity Constraint (QS)	Percentage Change in Q ($\Delta\%$ Q)
1.02	1.3%
2.05	39.0%
3.08	63.0%

High Level of Q = .918

Prison Capacity Constraint (QS)	Percentage Change in Q ($\Delta\%$ Q)
1.53	-6.0%
3.07	8.4%
6.14	8.4%

Table 16.   Percentage Changes in S

Low Level of S = 1.67

Prison Capacity Constraint (QS)	Percentage Change in S ($\Delta\%$ S)
.511	−33.0%
1.02	−64.0%
1.53	6.5%

Medium Level of S = 3.35

Prison Capacity Constraint (QS)	Percentage Change in S ($\Delta\%$ S)
1.02	−51.0%
2.05	−28.0%
3.08	7.1%

High Level of S = 6.69

Prison Capacity Constraint (QS)	Percentage Change in S ($\Delta\%$ S)
2.05	−64.0%
4.1	−39.0%
6.14	−8.4%

Figure 15.   Percentage Change Plot of Q

Figure 16.   Percentage Change Plot of S

The patterns for Q illustrate a dramatic change in the slope for the three different levels as we observed previously. The patterns for S indicate a bowl shape whose vertex angle decreases as the levels of S increase. The percentage change readings for the low, medium and high levels of S have a maximum range of about 45% occuring near the prisons capacity constraint of 1.53. The maximum range for percentage change readings in Q also occurs near the prisons constraint reading of 1.53, but is about 190%. This would indicate that the optimization process is far more sensitive to Q up to a capacity constraint of about 3.0, after which Q is maintained at its maximum level and S is monitonically increased. The degree of "sharpness" in the bowl of S reflects the relative proportion of its contribution to the capacity constraint, which is diverted to Q in the optimal solution. Naturally, as S becomes larger, a smaller proportion of S is required to establish Q at its optimum level, and thus, the vertex angle of the bowl decreases.

5.4.3 Analysis for a Linear System

Intuitively, the apparent sensitivity of the optimization process to Q, would suggest to the casual observer that the level of Q is probably more of an indication of the potential impact of optimization on the crime rate, than the level of S. One way to ascertain this, would be to perform a complete analysis of variance on our experimental data and compare F-ratio values.

This, of course, would require the assumption of a linear system of the (percentage savings through optimization) response in

Q and S. Figures 17 and 18 illustrate a graphical test of the linear assumption. Off hand, a linear fit does not appear unreasonable in Q or S. We, therefore, assume the linear statistical model:

$$\begin{pmatrix} \text{percentage savings} \\ \text{through optimization} \end{pmatrix} = \begin{bmatrix} k & Q_i + S_j + (QS)_{ij} + E_{ij} \end{bmatrix} \begin{cases} i = 1, 2, 3 \\ j = 1, 2, 3 \end{cases}$$

where k is some unknown constant. At this point, it should be noted that this analysis should be viewed with a high degree of skepticism. This is mainly due to the fact that the experiment has only nine widely scattered observations. Clearly, any strongly nonlinear behavior of the response in the region between our observations would invalidate the analysis.

Table 17 is a convenient summary of the experimental data in analysis of variance format.

Table 18 illustrates the prelimary analysis of variance for the single observation per cell, 3x3 factorial experiment. The symbol y is used to denote the percentage savings through optimization in the table. Using this formulation, we can proceed to calculate the appropriate sums of squares as follows.

$$SS_Q = [(14.649)^2 + (5.197)^2 + (0.762)^2]/3 - \frac{(20.59)^2}{9}$$

$$= (71.531 + 9.003 + .1935) - 47.105 = 33.622$$

$$SS_S = [(2.88)^2 + (6.716)^2 + (11.004)^2]/3 - \frac{(20.59)^2}{9}$$

$$= (2.7648 + 15.035 + 40.363) - 47.105 = 11.057$$

Figure 17.   Graphical Test of the Linear Assumption in Q

Figure 18.   Graphical Test of the Linear Assumption in S

Table 17. Summary of Responses in ANOVA Format
for Varying Resource Constraint Values

Probability of Imprisonment given Conviction (Q)	Average Sentence Length (S)		
	1.67	3.35	6.69
0.306	2.278%	4.579%	7.792%
0.612	0.610%	1.756%	2.831%
0.918	0.00%	0.381%	0.381%

Table 18. Preliminary ANOVA

Source of Variation	SS	df	MS	$E(MS_Q)$
(Q)	$\sum_{i=1}^{3} \dfrac{yi.^2}{3} - \dfrac{y..^2}{9}$	2	MS_Q	$\sigma^2 + \dfrac{\Sigma t_i^2}{2}$
(S)	$\sum_{i=1}^{3} \dfrac{y.j^2}{3} - \dfrac{y..^2}{9}$	2	MS_S	$\sigma^2 + \dfrac{\Sigma \beta_j^2}{2}$
Capacity Constraint, QS (Interaction)	(sub.)	4	$MS_{Res.}$	$\sigma^2 \dfrac{\Sigma\Sigma(\sigma P)_{ij}^2}{4}$
Total	$\sum_{i=1}^{2} \sum_{j=1}^{2} y_{ij}^2 - \dfrac{y..^2}{9}$	8		

$$SS_T = 98.632307 - 47.105 = 51.5273$$

$$SS_{Residual} = 51.5273 - 33.622 - 11.057 = 6.848307.$$

In order to test for QS significance, the sum of squares for non-additivity is computed as:

$$SS_N = \frac{\left[\sum\limits_{i=1}^{3} \sum\limits_{j=1}^{3} y_{ij} y_{i.} y_{ij} - y.. (SS_Q + SS_S + \frac{y..^2}{9}) \right]^2}{9(SS_Q \cdot SS_S)} \quad \text{(Tukey, 1961)}$$

$$= \frac{(2040.1158 - 1889.8326)^2}{3345.8261} = 6.7502$$

Consequently, we can estimate the experimental error sum of squares by:

$$SS_F = SS_{Residual} - SS_N = 6.848307 - 6.7502 = .0981$$

Table 19 summarizes the final ANOVA results.

Table 19. Final ANOVA

Source of Variation		SS	df	MS	F_0
Q	Probability of Imprisonment	33.622	2	16.811	514.1
S	Average Sentence Length	11.057	2	5.5285	169.1
Non-additivity (Prison Capacity)		6.7502	1	6.7502	206.4
Error		.0981	3	.0327	
Total		51.5273	8		

As expected, the resulting analysis for the linear system suggests the level of Q as highly significant in determining the potential for optimization. Although the level of S is also significant, the F-ratio of S is only one-third as large as that for Q. In addition, the level of the capacity constraint was also found to be significant in the region analyzed, the F-ratio of which was about 20% greater than the F-ratio of S.

The indication most apparent from this experiment is that any marginal increase in the prisons capacity constraint for the state of Georgia will be reflected exclusively through a rise in Q, if optimal policies are pursued. This is certain for any expansion of the corrections resource, up to a constraint corresponding to 1.01. The current corrections resource in Georgia corresponds a constraint value of .511. (This result is apparent from Table 15.) Since it is not likely that this state (or any other) is willing to expand its corrections resources almost two-fold in the near future, changes in policy should focus exclusively on Q. Indeed, the interval of Q, in which the relative benefit in the expected rate of crime is greatest, is between 0 and .612 (see Figure 15). This would indicate that the greatest hope for controlling crime in our society today, lies in the deterrent impact of using the imprisonment option more frequently in the cases of convicted offenders. If this is truly the case, it would be intuitive to think that Q is responsible for the majority of crime prevention due to sanctions already under current policy. In fact, we show this to be the case in a later section.

The most general conclusion stemming from this analysis, is

one that is strikingly consistent with most research in the field from
recent years. That is, it is the certainty of punishment as opposed
to its severity that most effectively deters offenders from committing
crimes. Whether this stems from patterns in the average criminal's
utility structure or the onerous nature of prison life is not the
issue in this research. The implications, however, for the control of
social deviance could hardly be more explicit.

5.4.4 Experimentation Within Plausible Limits of Current Corrections Capacity

Since the results obtained from the previous sections pertained
to sanction levels which due to their magnitude are mainly of theoret-
ical interest, a series of experimental runs at "affordable" levels of
corrections expenditure were performed. In this experiment, 50 simula-
tions of the system, each at a different level of the capacity
constraint, ranging from 0.1002 to .9185, were performed. The current
level of the corrections capacity constraint in the state of Georgia
is about .511. Since previous analysis has been suggestive of the
system being most highly sensitive to Q, sharp rises in the value of
this variable were anticipated as the capacity constraint was relaxed.

The approach of this experiment was to fix S at its current
level of 1.67 years and vary the starting value of Q, from 106 to 155,
by increments of .01. Figure 19 is a plot of the 50 values of QS and
Q*, obtained by the experiment.

It is equivalent to a reverse image (except for scale) of the
S vs. S* plot, since the only way possible to increase Q in the final
solution would be to decrease S. As a result, this plot provides a

clear picture of the behavior in the system when corrections expendi-
tues are varied in regions of practical interest.

We can see from Figure 19 that optimal strategy for corrections
allocation around the current level of expenditure would indeed focus
primarily on Q as previous analysis has suggested. In fact, the
situation would not be likely to change until the system were at a
corrections capacity level of about .75. This means that all increases
in corrections allocation, up to a level corresponding to 50% more
spending than the current tab, should be directed toward increasing
the certainty of imprisonment for convicted offenders, that is, if
we are to follow a policy associated with maximum "per dollar" crime
prevention. Specifically, the system would increase Q from .306 to
.45, with S held constant at 1.67 before any increase in the average
sentence length would be considered.

From Figure 19, we can also observe the behavior of the system
in regions corresponding to extremely depressed spending. Here the
slope of the plot is significantly steeper again in favor of increasing
Q. In addition, four significant "break points" in the line corre-
sponding to capacity constraint values of; .125, .210, .75 and .801,
can be observed. These points are of particular interest, since they
represent values where the nature of optimization seems to shift its
emphasis. For example, if S is held constant at 1.67 years and Q is
between .000 and .075, the optimization appears almost indifferent
toward Q or S until the value of Q reaches about .08 (S is still held
constant at 1.67 years). At this time, Q would be extremely dominating.

Figure 19.   Q* versus (QS)

This situation prevails until Q reaches about .14, where the optimization still favors Q at a less extreme rate until it reaches about .45, at which time S becomes the dominating variable. The emphasis again shifts toward Q at about Q = .49, where the system again appears nearly indifferent. Clearly, the system is highly nonlinear through most of the observable QS policy space.

5.4.5 Comparison of Results Between Models

In order to gain greater insight into the validity of our interpretations from the model, it is of interest to compare its output with other models designed to perform similar analysis. Moreover, we would expect results between analogous models to yield like results for like inputs within the same order of magnitude. A logical candidate model for this comparison was presented by Blumstein and Nagin [1976] in an article entitled, "On the Optimum Use of Incarceration for Crime Control." Their formulation provided a fundamental building block in the structure of our own model.

Fortunately, in their thesis, Blumstein and Nagin provide some limiting case computational experience, which serves to articulate the behavior of their model for stated inputs. The structure of our own model is such that the inputs could be identically reproduced and outputs compared with minimal modification of their interpretations.

5.4.5.1 An Illustrative Example. In their modeling of the criminal justice system on a national level, Blumstein and Nagin estimate λ_t for the year of 1970 to be five crimes per year per offender. This estimate corresponds to a λ_t of .4167 crimes per month in our own model, which although not available for that year,

is reasonably close to λ values generated within our model for other years. They also estimate the 1970 national (QS) policy at .25, 2.6, which is within an order of magnitude of our current estimate of Q,S policy for the state of Georgia, which is .306, 1.67. These similarities greatly enhance the possibilities for comparison between the two models, at least in terms of input parameters.

In their subsequent analysis to determine optimal policy, Blumstein and Nagin propose the optimal policies of 1, 2.6 and 1, 1. The latter of which was considered feasible and would presumably reduce the crime rate by about 25%. The 1, 2.6 policy was anticipated to reduce the crime rate by 50%, but was in violation of the prisons capacity constraint.

Our approach to a comparison was then to simulate our own model, inputing .25, 2.6 for starting values of Q,S, switching to the 1, 1 and 1, 2.6 policies, and observing their respective savings in percentage of crimes averted. The results are compiled in Table 20.

Table 20. Model Comparison Summary

Q,S Policy		Percent Savings Via Optimization	
Starting	Optimal	Blumstein, Nagin	Deutsch, Malmborg
(.25, 2.6)	(1, 1)	$\Rightarrow$ 25%	18.6%
(.25, 2.6)	(1, 2.6)	$\Rightarrow$ 50%	42.3%

5.4.5.2 Example Discussion. The results provided in Table 20

reveal the strong similarities between the two models and emphasizes

their common formulation of the deterrent mechansim. Whatever differ-

ence is present relates directly to the different approaches for

estimating λ (as described in previous chapters) and our own model's

more complex development for estimating the criminal population.

5.4.6 Development of a Q-S Nomogram

Figure 20 is a chart for determining optimal levels of sanction

variables Q and S for incremental percentage changes in corrections

expenditures. The acceptable range for this chart is from -80% to +80%.

That is, for aggregate changes in corrections expenditure between -80%

and +80%, Figure 20 can be used to determine the most efficient QS

policy. To use the figure, determine what the shift in corrections

allocation will be, then enter the figure on the curve corresponding

to this percentage. The point at which the curve intersects another

curve gives the optimal values of Q and S. For example, if it were

decided that the state of Georgia would boost its corrections

allocation by 20%, the optimal levels of Q and S from Figure 20 would

be .35 and 1.54 years, respectively.

5.5 Separating Incapacitation from General Deterrence

One question which the Blumstein, Nagin model did not address

involved the determination of the relative impacts of incapacitation

and general deterrence for a given imprisonment policy. This distinc-

tion has potentially important implications for the effectiveness of

a policy. On the one hand, deterrence tries to reduce crime by posing

a threat of punishment, thereby discouraging criminality, on the other

Figure 20.   Q* – S* Nomogram for Incremental Changes in
Corrections Expenditure

hand, incapacitation reduces crime by isolating the criminal from the rest of society through imprisonment.

Deterrence operates to reduce criminality,in those not directly imprisoned, by posing a threat of punishment for any crimes they might commit. It may operate by reducing the number of new entries into criminal activity, or by shortening the careers or lowering the crime rates of criminals not yet punished.

We also know that imprisonment can reduce crime through incapacitation. This isolates imprisoned individuals from the remainder of society, preventing them from committing crimes. In our model, the magnitude of the incapacitative effect is directly related to λ_t, the rate at which offenders commit crimes while free in period t. In the following section, we will use this relation to evaluate the incapacitative effect inherent in the imprisonment policy of the state of Georgia over a 24 year period.

5.5.1 Formulation of the Incapacitative Effect

Since λ_t is a measure of the free criminal's propensity to commit offenses in period t, if we knew the number of periods (n) an offender was incarcerated, the product, $\lambda_t n$, would estimate the potential savings realized by imprisonment of that individual for n periods. It follows that if we knew the number of individuals who were incarcerated in each period, r_t, we would estimate the number of crimes averted in the future through incapacitation from prevailing policy in period k as:

$$\sum_{i=k}^{k+n} \lambda_i \, r_i \, .$$

Similarly, the number of crimes in period k which could have occured, but were avoided through the incapacitative effect stemming from prevailing policy in previous periods, can be determined by recursively accumulating the portion of the incapacitative effect in those previous periods, which was operative in period k. This relation is derived as follows:

$$\sum_{j=k-n}^{k-1} \lambda_j \, r_j \qquad \{k = n, \; n+1, \; n+2 \ldots \ldots \infty\}$$

Obviously, this quantity is not estimable for those periods, k, in which k < n, where k-n forms the limit on historical data.

5.5.2 Simulation of the Incapacitative Effect

Given the proposed formulation of the previous section, the only information not previously input or estimated by the model is r_t. To determine r_t we consult the annual publication of the Georgia State Office of Offender Rehabilitation. This publication provides monthly figures of releases and admissions of inmates to and from Georgia state institutions. From this, we were able to obtain prison receptions figures for the state from January of 1974 to December of 1976. These figures are presented in Table 21. Subsequent statistical analysis of the data determined the appropriate modeling mechanism to the series mean. The reader can qualitatively verify this hypothesis by examining Figure 21, which is a plot of the time series. The mean of the series was 643.03 prisoners per month.

The results of the 24 year simulation were entirely consistent with other results from the model. That is, they seemed to suggest

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Figure 21.   Monthly Prison Receptions Time Series (1974-1976)

Table 21.   Monthly Georgia State Prison Receptions

Month	1974	1975	1976
Jan.	589	651	596
Feb.	526	614	553
Mar.	608	814	711
Apr.	652	811	641
May	650	726	646
June	553	689	544
July	621	631	566
Aug.	537	672	643
Sept.	597	593	619
Oct.	757	703	714
Nov.	457	543	804
Dec.	657	814	650

that the incapacitative effect of the current sanction level was significant, yet clearly a subordinate effect to general deterrence which comprised the residual 100-S' percent. Where S' is the percentage of crimes averted through overall sanctions, Q and S, specifically, due to incapacitation. The results of the simulation for seven periods of interest are presented in Table 22. Inspection of the table will reveal that the March 1975 period has been replaced in the tables by September 1975. This is due to the fact that an average sentence length of 1.67 years required a recursion of 21 monthly periods before the first incapacitative effect could be accumulated. Consequently, the first period for which the percentage of crimes due to incapacitation could be estimated was September 1975, 21 months

following the start of the simulation.

The figures which appear in Table 22 suggest incapacitation
to entail about 20% of the effect of sanctions under current policy.
This result is roughly consistent with the result of our linear
analysis of variance, which suggested about 25% of the effect of
sanctions is attributable to incapacitation. This result is
encouraging, since these analyses drew from completely independent
sources within the model and subsequently serves to enhance our
confidence in the model's formulation of the criminal justice system.

5.5.3 The Effect of Optimization

Given that previous analysis for our model implied that optimi-
zation of QS policy within the stated resource constraint would tend
to emphasize the certainty as opposed to the severity of punishment,
we would expect the incapacitative effect under optimal policy to
decrease in favor of the deterrent effect. In fact, when the model
was simulated over a 24 year period under optimal policy, there was
an average effect due to operating under the more Q intensive policy
and about a 2.3% savings realized in total expected offenses. Table
23 presents the results for seven periods of interest.

Although the results in Table 23 would seem obvious, given we
shifted from a (.306, 1.67) to a (.4605, 1.11) policy, they do serve
to illuminate the incremental effect of a shift in Q and S, within
the resource constraint. In addition, the results provide another
intuitive validation of the correctness of our simulation of the
analytical model.

Table 22. Percentage of Overall Effect of Sanctions Q and S Due
to Incapacitation for Policy: Q = .306, S = 1.67 Years

Period	Incapacitative Effect	Deterrent Effect	λ_t	d_t
September 1974	20.93%	79.07%	.2814	1.15%
September 1979	21.01%	78.99%	.3717	1.38%
January 1983	20.81%	79.19%	.4361	1.42%
May 1987	20.34%	79.66%	.5330	1.43%
July 1990	20.23%	79.77%	.5954	1.39%
November 1994	19.83%	80.17%	.6724	1.41%
December 1997	19.60%	80.40%	.7326	1.38%

Table 23. Percentage of Overall Effect of Sanctions Q and S Due
to Incapacitation for Policy: Q* = .4605, S* = 1.11 Years

Period	Incapacitative Effect	Deterrent Effect	λ_t	d_t
September 1975	15.43%	84.57%	.2814	1.15%
September 1979	13.74%	86.26%	.3717	1.38%
January 1983	13.41%	86.59%	.4361	1.42%
May 1987	13.14%	86.86%	.5330	1.43%
July 1990	13.05%	86.95%	.5954	1.39%
November 1994	12.79%	87.21%	.6724	1.41%
December 1997	12.83%	87.17%	.7326	1.38%

5.5.4 Comparison Between Models

Attempting to compare the results of this section with analogous results from other models is hampered by a lack of analytical research in this area. In their analysis of the effect of the (1, 2.6) policy, Blumstein and Nagin state that they believe the switch from (.25, 2.6) policy resulted in a savings in reported offenses of about 50%. The analogous figure from our analysis was about 42%. Of the 50% of crimes averted, Blumstein and Nagin speculated in their thesis that about 30% of that savings was due to incapacitation and about 70% due to deterrence.

In fact, we performed the precise determination of this percentage. This was done by determining the percentage of crimes averted due to incapacitation under the (.25, 2.6) policy and the (1, 2.6) policy. For the former, incapacitation was responsible for about 31% of the entire savings due to sanctions. For the latter, the savings attributable to incapacitation, on average, was about 33%, an increase of 2%. Consequently, the proportion of the increase in total savings was about 5%. This estimate was in sharp contrast to the estimate offered by Blumstein and Nagin, who believed this figure would be near 30%. Part of the discrepancy, of course, is attributable to the fact that their analysis was on a national level and we performed the experiment for the state of Georgia only. Table 24 presents a sample summary of the results for the experiments. The first month for which this result was available was August 1976, 32 months following the start of the simulation.

Table 24.   Percent of Savings Due to Incapacitation

Period		Policy (Q = .25, S = 2.6)	Policy (Q = 1, S = 2.6)
August	1976	31.98%	37.69%
September	1979	31.85%	33.44%
January	1983	30.88%	35.10%
May	1987	30.00%	31.96%
July	1990	30.14%	31.31%
November	1994	30.12%	31.99%
December	1997	30.06%	31.17%

5.6   Introduction to the Comparison Procedure

The purpose of this section is to demonstrate the application of the model to data bases originated in states other than Georgia. Specifically, analysis is presented for the states of Missouri and Texas, with comparisons offered. Within this section, it is shown how judicial policies differ greatly between geographical areas, and consequently, the implications for policy improvement may not be the same from state to state. The comparison proceeds by first discussing the data base for each state and ultimately discussing the results obtained from the model using each individual data base.

5.6.1   Input Requirements

If we recall, the data required to execute the model for any individual state consists of 36 monthly estimates of the following quantities:

a.   Total reported offenses.

  b. Total state institution inmate population totals.

  c. Total state institution admissions total.

  d. Average sentence length prevailing in each month.

  e. Probability of imprisonment prevailing in each month.

Fortunately, we were able to develop complete data bases for Missouri and Texas, with the aid of the appropriate law enforcement and corrections agencies of those states. In the following sections, each of these data bases is discussed individually and in relation to each other.

5.6.2 Total Offenses for Missouri

In order to obtain monthly figures of total reported offenses in the state of Missouri (for the period from January 1974 until December 1976), a procedure analogous to that used in developing similar data for the state of Georgia was employed. In this case, the monthly behavior of total reported offenses for the city of St. Louis was imparted to the state's annual total, in order to obtain the monthly state totals. The results of this calculation are presented in Table 25.

From Table 25, it can be seen that the mean value of 19658 for the Missouri total offenses time series is significantly larger than the mean value of 16449, which is the analogous result in the Georgia data. The population totals of Georgia and Missouri are approximately 4.95 and 4.70 million, respectively, suggesting the per capita rate of crime to be greater in Missouri than Georgia. Figure 22 illustrates the behavior of the Missouri series. The non-stationary nature of the

Figure 22.   Total Reported Offenses Time Series 1974 - 1976

Table 25.   Total Reported Offenses in Missouri

Month	1974	1975	1976
Jan.	16079	21038	19658
Feb.	16952	20155	20285
Mar.	16689	21396	20403
Apr.	17026	20145	20370
May	17293	20228	19987
June	17997	22171	20356
July	20697	23881	21557
Aug.	22336	23575	22951
Sept.	21330	21883	19675
Oct.	21973	22467	19697
Nov.	19444	19185	17735
Dec.	20908	20972	17940

offenses series was analyzed as a $(0,1,1)(0,1,1)_{12}$ empirical stochastic model with iterative estimation of the parameters, resulting in the values: $\theta_1 = 0.399$, $\theta_{12} = 0.694$.   The model was then run with the preceding imbedded as the Z forecasting mechanism.

5.6.3 Total Reported Offenses in Texas

In obtaining monthly figures of total reported offenses for Texas, the standard procedure was implemented.   For the state of Texas, monthly crime rates from the city of Dallas, Texas were used to develop the total offenses time series for that state.   The results of the calculations pertaining to the months from January 1974 until December 1976 appear in Table 26.

As would be expected, the mean of the above series exceeds the

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Table 26.   Total Reported Offenses in Texas

Month	1974	1975	1976
Jan.	43644	54740	61108
Feb.	40056	46038	53471
Mar.	44796	51702	55301
Apr.	44268	51114	55389
May	47233	54674	55395
June	27730	55037	58820
July	51929	61979	63585
Aug.	54453	60302	61586
Sept.	48296	56189	56102
Oct.	51992	56332	55543
Nov.	50626	53272	51889
Dec.	51809	60309	15150

analogous values for Missouri and Georgia by an order of magnitude. This relates to the fact that the population of Texas is considerably larger (about 12.7 million) than that of either Missouri or Georgia. The Texas series was identified as a $(0,1,1)(0,1,1)_{12}$ model with $\theta_1 = .320$, $\theta_{12} = .288$. Figure 22 shows the behavior of the Texas series.

5.6.4 Missouri Prison Populations

The second requirement in executing the model for the state of Missouri was 36 monthly observations of state inmate population totals corresponding to the monthly total reported offenses for that state. These observations are presented in Table 26.

Interestingly, the mean of the series is slightly over one-third of

the analogous series for the state of Georgia, despite the fact that

the populations of the two states differ only slightly. This result

provides considerable insight into the judicial practices of state

courts as discussed in a subsequent section.

Statistical analysis of the Missouri prison populations time

series suggested the process to be appropriately modeled by the

non-stationary (0,1,0) Box-Jenkins forecasting model. A graphical

representation of the series is presented in Figure 23, whose forecasts

are generated by the form:

$$Z_t = Z_{t-1} + a_t.$$

5.6.5 Texas Prison Populations

Monthly observations of Texas state inmate population totals,

utilized to execute the model for that state, are presented in Table

28. Once again, these values pertain to the months from January 1974

until December 1976. The ratio of the mean of the Texas prison popu-

lation time series to the state population was found to be intermediate

with respect to the analogous ratios for Georgia and Missouri. That

is, the per capita prison population was found to be highest in Georgia

followed by Texas and Missouri, respectively. The significance of

this relationship, in terms of the results from the model, is discussed

in a later section.

Statistical analysis of the Texas prison time series identified

the process to be non-stationary and required a second difference of

Table 27.   Missouri Prison Populations

Month	1974	1975	1976
Jan.	3514	3867	4513
Feb.	3531	3948	4553
Mar.	3547	4126	4565
Apr.	3535	4141	4627
May	3598	4193	4702
June	3640	4201	4732
July	3690	3985	4753
Aug.	3698	4120	4744
Sept.	3709	4138	4784
Oct.	3720	4242	4795
Nov.	3735	4300	4759
Dec.	3754	4368	4809

Table 28.   Texas Prison Populations

Month	1974	1975	1976
Jan.	17129	17059	19099
Feb.	17210	17365	19383
Mar.	17251	17501	19857
Apr.	17340	17652	20032
May	17144	17544	20281
June	17121	17721	20616
July	17014	17912	20748
Aug.	16956	18151	20976
Sept.	16995	18357	20572
Oct.	17059	18516	20641
Nov.	16985	18724	20568
Dec.	16833	16833	20717

Figure 23.   Prison Populations Time Series 1974 - 1976

the form $(0,2,1)$. Consequently, the process can be written as:

$$Z_t = Z_{t-2} + a_{t-1} = \theta a_{t-2} \, .$$

Graphical illustration of the series behavior is presented in Figure 23.

5.6.6 Missouri Prison Admissions

Monthly observations of prison admissions in Missouri state prisons during the period from January 1974 until December 1976 were consistent with their corresponding prison population observations. Consistent in the sense that Missouri had the lowest per capita rate of prison receptions and releases of the three states analyzed, and the lowest per capita prison population. The 36 month Missouri prison admissions time series is presented in Table 29. Analysis of the series was suggestive of the non-stationary $(0,1,1)$ Box-Jenkins forecasting model to be imbedded into the model which can be written in the form:

$$Z_t = Z_{t-1} + a_t + \theta a_{t-1}$$

where the value of θ was estimated to be 0.7489. The behavior in the time series can be observed in Figure 24. The mean of the series was found to be, 181, over the thirty-six historical observations.

5.6.7 Texas Prison Admissions

The 36 monthly state prison admission series for the state of Texas demonstrated the lowest per capita prison reception rate of the three states for which the analysis was performed. The series is

Table 29.   Missouri Prison Admissions

Month	1974	1975	1976
Jan.	121	173	196
Feb.	140	171	187
Mar.	134	251	254
Apr.	199	214	188
May	147	172	200
June	165	201	247
July	184	192	176
Aug.	135	160	177
Sept.	145	146	151
Oct.	186	195	201
Nov.	144	221	177
Dec.	146	246	175

similar to the analogous series for the state of Missouri, in that it represents a non-stationary (0,1,1) process. The θ parameter for the Texas data, however, was estimated to be 0.6889. The data is presented in Table 30 with a graphical illustration presented in Figure 24. The per capita state prison admission rate in Texas was considerably closer to the same figure for the state of Missouri than for Georgia. This means that the monthly prison turnover is much higher in Georgia than in either Missouri or Texas, indicating that Georgia prisons process more individuals (per capita) in a given time period than the other two states. The implication of this for judicial policy is considered in a subsequent section.

Figure 24.   Prison Receptions Time Series 1974 - 1976

Table 30.   Texas Prison Admissions

Month	1974	1975	1976
Jan.	621	815	751
Feb.	624	810	761
Mar.	608	748	945
Apr.	679	814	842
May	605	721	720
June	506	740	850
July	579	689	748
Aug.	647	715	848
Sept.	614	760	773
Oct.	697	791	823
Nov.	670	778	782
Dec.	520	832	816

5.6.8 Average Sentence Lengths in Missouri

To obtain the 36 month time series of average sentence length observations for the state of Missouri, the procedure is to divide the prison population time series by the prison admission time series. This was done in order to obtain the time series presented in Table 31. Clearly, the average sentence length in the state of Missouri is many times greater than the corresponding average sentence length figures from the state of Georgia.   Specifically, the mean of the Missouri series is 23.51 years, as opposed to 1.67 years for the state of Georgia.

This result implies that judicial policy in the state of Missouri is oriented largely toward the severity of punishment.   As

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Table 31.   Average Sentence Lengths for Missouri

Month	1974	1975	1976
Jan.	29.04	22.35	23.03
Feb.	25.22	23.09	24.35
Mar.	26.47	16.44	17.97
Apr.	17.76	19.35	24.61
May	24.48	24.38	23.51
June	22.06	20.90	19.16
July	20.05	20.76	27.01
Aug.	27.39	25.75	26.80
Sept.	25.58	28.34	31.68
Oct.	20.00	21.75	23.86
Nov.	25.94	19.46	26.89
Dec.	25.71	17.76	27.48

a result, we would expect that individuals admitted would remain
incarcerated for many periods, thus contributing to the extremely
low turnover which was observed. In fact, later analysis of imprison-
ment probabilities for the state of Missouri will show that judicial
behavior in that state imposes prison sentences only infrequently, yet
tends to delegate severe sentences when the imprisonment option is
exercised. In our analysis for the state of Georgia on the other hand,
it was found that more frequent prison disposition of criminal cases
was practiced, yet sentences tended to be of shorter duration.

The time series appearing in Table 31 was identified as a
stationary (0,0,0) process, best characterized by its mean values.
As a result, a constant value of S = 23.51 was used in simulating the

model for the Missouri data base.

5.6.9 Average Sentence Lengths for Texas

Like Missouri, the time series of average sentence lengths for
the state of Texas was an order of magnitude larger than the Georgia
series. The average of the Texas series equalling 25.62 years was
the largest among the three states considered in the analysis. The
series was obtained using procedures identical to those used in
Georgia and Missouri, and is listed in Table 32. In addition, a
graphical illustration of the 36 historical observations is presented
in Figure 25.

As was the case for Missouri, judicial policy in the state of
Texas is orientated strongly toward the severity of punishment, as
opposed to its certainty. Indeed we will find this to be the case
when imprisonment probabilities in the state of Texas are considered.
This conclusion is reinforced by the low turnover rates observed for
Texas prisons, suggesting fewer sentences of greater duration.

Analysis of the Texas average sentence length series suggested
the stationary (0,0,0) model, leading us to use a constant value of
S = 25.62 in simulating for the Texas data base.

5.6.10 Probability of Imprisonment for Missouri

In determining the second component of judicial policy for the
state of Missouri, (i.e., Q), it was found that the series represented
a departure from the imprisonment probabilities obtained for Georgia
in two respects. First, the magnitude of monthly imprisonment
probabilities for Georgia was nearly ten times the magnitude of those

Figure 25.   Average Sentence Length Time Series 1974 – 1976

Table 32.   Average Sentence Lengths for Texas

Month	1974	1975	1976
Jan.	27.58	20.93	25.43
Feb.	27.58	21.44	25.47
Mar.	28.43	23.40	21.01
Apr.	25.54	21.69	23.79
May	28.34	24.33	28.17
June	33.84	23.95	24.25
July	29.39	26.00	27.79
Aug.	26.21	25.39	24.74
Sept.	27.68	24.15	26.61
Oct.	24.47	23.41	25.00
Nov.	25.35	24.07	26.30
Dec.	32.27	22.76	25.39

for Missouri. This result provides convincing evidence that judicial

behavior can differ greatly from state to state, especially in terms

of sentencing practices. In fact, these results suggest that the

character of judicial policy in Missouri has an almost opposite

emphasis from judicial policy in Georgia. Namely, while the severity

of punishment is emphasized in Missouri, it is the certainty of pun-

ishment which is predominant in the judicial priorities of Georgia.

The other respect in which the imprisonment probabilities

series of Missouri and Georgia differ is the actual generating process

underlying the time series. While it was shown in section 5.3 that

the time series for Georgia imprisonment probabilities was a non-

stationary (0,1,1) process, the analogous series for Missouri was a stationary (0,0,0) process, suggesting the mean as a forecasting mechanism. Since the mean of the Missouri imprisonment probability series presented in Table 33 was .04504, this constant value of Q was used in simulating for the Missouri data base. A graphical illustration of the series is presented in Figure 26.

One possible interpretation of the stationary imprisonment probabilities for the state of Missouri is that judicial policy has remained relatively stagnant over the past several years. That is, the current policy has remained unchanged from past years, while in Georgia a more dynamic judicial process prevails. Alternatively, the prisons capacity in that state may be crippled by its obligation to fufill numerous sentences of long duration imposed in past years. In any case, our analysis could be helpful in evaluating the Missouri policies as possibly suggesting ways for improving the situation.

5.6.11   Probability of Imprisonment in Texas

The time series of imprisonment probabilities from the state of Texas was computed similarly for the months of January 1974 until December 1976, and is presented in Table 34. The mean value of these 36 observations was found to be (0.06855), of the same order of magnitude as the series of imprisonment probabilities obtained for the state of Missouri. The Texas series was also found to resemble the Missouri result, in that the underlying process was a stationary (0,0,0) and best forecasted by its mean value. A graphical illustration of

Table 33. Imprisonment Probabilities in Missouri

Month	1974	1975	1976
Jan.	.03763	.04112	.04985
Feb.	.04129	.04242	.04609
Mar.	.04015	.05866	.06225
Apr.	.05844	.05311	.04615
May	.04250	.04252	.05003
June	.04584	.04533	.06067
July	.04445	.04020	.04082
Aug.	.03022	.03393	.03856
Sept.	.03399	.03336	.03837
Oct.	.04232	.04340	.05102
Nov.	.03703	.05760	.04490
Dec.	.03491	.05865	.04877

Table 34. Imprisonment Probabilities in Texas

Month	1974	1975	1976
Jan.	.07114	.07444	.05145
Feb.	.07789	.08797	.07116
Mar.	.06786	.07234	.08544
Apr.	.07669	.07963	.07601
May	.06404	.06594	.06497
June	.05301	.06723	.07225
July	.05575	.05558	.05882
Aug.	.05941	.05928	.06885
Sept.	.06357	.06763	.06889
Oct.	.06703	.07022	.07409
Nov.	.06617	.07302	.07535
Dec.	.05018	.06898	.07535

Figure 26.   Imprisonment Time Series

the series is presented in Figure 26.

Overall, the imprisonment probability results for Texas were very similar to the analogous Missouri results. This suggests that judicial practices in these states are quite similar and in sharp contrast to the situation existing in Georgia. In the next section, the ramifications of these results are explored in greater depth by simulating the model for the Texas and Missouri data bases.

5.6.12 Discussion of Corrections Capacity in Texas and Missouri

At this point, it is worth discussing one counter-intuitive condition which exists within the input data of our model. That is, if we examine closely, it is apparent that while Georgia may have the highest "per capita prison population," it also has the lowest prisons capacity constraint, i.e., $QS = 0.511$, (while $QS = 1.05$ for Missouri and $QS = 1.76$ for Texas). This is the lowest capacity of the three states considered in the analysis. Although this does not appear to make sense immediately, it actually reflects the high cost of long term incarceration. This stems from the fact that individuals who receive very long sentences tend to be severe deviants, requiring expensive high security facilities designed strictly for corrections. On the other hand, offenders receiving very short sentences tend to impose lower security requirements costing far less per unit time of incarceration. As a result, it is possible for one state to expend less in detaining a large number of low risk offenders, than it is for another state to detain a much smaller number of severe deviants.

5.6.13 Results of Model Simulation for the Texas and Missouri Data Bases

To explore for trends in the analysis for Missouri and Texas, similar to those which were predicted by the model to evolve in Georgia, analogous results for the aforementioned states were calculated and examined. Specifically, the average monthly number of offenses per criminal, the prison populations and the criminal populations were explored for Missouri and Texas over the 25 year horizon.

5.6.13.1 Results for λ_t.

In this analysis, it was found that the behavior of λ_t in Missouri was expected to behave in a manner similar to λ_t for Georgia. On the other hand, λ_t for Texas was found to grow only slightly. This result can be explained by the slow growth behavior of the prison population forecasting model for Missouri, and the near stationary behavior of the prison populations in Georgia. This behavior of the prison populations is in contrast to the behavior of crime rates which were predicted to rise sharply in both Georgia and Missouri. During this same period, Texas prison populations are expected to grow considerably along with the crime rate, thereby, moderating the growth of λ_t. In all three cases, the proportion of the criminal population which remains at large, $C_t/C_t + P_t$, is expected to remain nearly stable. Tables 35 and 36 present sample results for λ_t and $C_t/C_t + P_t$, respectively, for each of the three states.

5.6.13.2 Deterrent Effects in Texas and Missouri.

In order to illustrate the impact on the prevailing judicial policy in Georgia, Missouri and Texas, clear of any factors relating to the size and population of the individual states, it is appropriate to examine their

Table 35. λ_t For Five Sample Periods

Period	Georgia	Missouri	Texas
March	.2938	.2568	.2813
January	.4361	.3293	.3063
May	.5330	.3238	.3117
July	.5954	.4231	.3340
November	.6724	.5643	.4062

Table 36. $\dfrac{C_t}{C_t + P_t}$ For Five Sample Periods

Period	Georgia	Missouri	Texas
March	.8330	.8231	.8403
January	.8340	.8307	.8486
May	.8340	.8258	.8605
July	.8380	.8283	.8585
November	.8350	.8285	.8486

deterrent effects. This is because the deterrent effect represents
a proportion of the population in each state and as such, is dimension-
less. The deterrent effects for five periods of interest during the
24 year simulation are presented in Table 37 for each of the three
states.

The results in Table 37 suggest that expenditures for corrections
in Texas, ultimately produces the smallest deterrent effect of the
three states. The most apparent reason behind this result is that

Table 37. Deterrent Effects For Five Sample Periods

Period		Georgia	Missouri	Texas
March	1975	1.32%	1.12%	1.09%
January	1983	1.42%	1.11%	1.07%
May	1987	1.43%	1.31%	1.16%
July	1990	1.39%	1.13%	1.19%
November	1994	1.42%	1.20%	1.13%

Texas also allocates the largest resource in terms of its corrections capacity constraint and therefore, would expect to receive a higher return. This reasoning also extends to Missouri, which bankrolls the second largest corrections system, followed by Georgia which allocates the smallest resource to obtain the smallest deterrent impact. This analysis, of course, says nothing about the per dollar efficiency of the corrections allocation within each state, which is addressed in a subsequent section.

5.6.13.3 Comparison of the Effect of Optimization. The most astounding contrast between the three states existed within the optimization process. Table 38 is a summary of the resulting optimal judicial policy for each period for each state. Bear in mind that this result is strictly for constant input values of decision variables Q and S, and as such, the results apply for every monthly period within the 25 year horizon.

Checking back to section 5.4 will show the results for Texas and Missouri to be totally consistent with sensitivity studies

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Table 38. Summary of the Optimization Process
for Decision Variables Q and S

		Georgia	Texas	Missouri
prevailing Q		.30606	.0686	.0450
prevailing S		1.11 yrs.	25.62 yrs.	23.51 yrs.
	Q*	.4605	.6753	.6024
	S*	1.67 yrs.	2.60 yrs.	1.76 yrs.
	ΔQ	+.15	+.61	+.56
	ΔS	-.56 yrs.	-23.02 yrs.	-21.75 yrs.

performed for the Georgia data base. That is, for relaxation of the
Georgia capacity constraint corresponding roughly to the existing
Missouri and Texas capacity constraints, the optimal policy is found
to be very close to the same form. This would lead us to conclude
that despite differences in the nature of corrections resource alloca-
tion between states, the social mechanisms underlying the deterrent
effect are essentially the same. Consequently, the prescription for
judicial policy should also be roughly consistent. Given the present
magnitude of this allocation, a more efficient strategy for controlling
crime within existing corrections capacity is to insure a higher level
of imprisonment probability with shorter sentences, i.e., increase the
flow rate of individuals within prison system without increasing
capacity.

5.6.13.4 Comparison of Savings Through Optimization. To
further illustrate the significance of potential improvement through
policy adjustment, Table 39 illustrates the number of crimes saved in
each of the states for five sample periods during the simulation, and
the corresponding percentage savings. Clearly, the potential improve-
ment in crime control for the state of Georgia is the lowest, due to
the fact that Georgia maintains the lowest corrections capacity of
the three states. Also, Georgia's prevailing judicial policy is
closest to the theoretically correct policy, further narrowing the
margin for improvement.

Table 39. Crime Saving Percentages

Period		Georgia crimes saved%	Missouri crimes saved%	Texas crimes saved%
March	1975	360...2.28%	4372...43%	11875...33%
January	1983	590...2.28%	5606...43%	13056...33%
May	1987	744...2.28%	6152...43%	15123...33%
July	1990	825...2.28%	7720...43%	16617...33%
November	1994	939...2.28%	9561...43%	18121...33%

The most important result from Table 39 is that the states of
Missouri and Texas stand to realize a substantial improvement in the
efficiency of their corrections system without allocating additional
funds. The model suggests that these two states can upgrade their
crime control effectiveness by redistributing the dollars they are now
using for long term incarceration and maintenance of high security

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institutions. Texas and Missouri represent prime examples of the
predominance of the certainty of punishment as opposed to its severity
within the feasible region of spending.

5.6.13.5 Separating Incapacitation from Deterrence. One
additional result obtained from the model relates to the separation of
deterrence and incapacitation. Table 40 is a summary of the average
distribution of the crime control effect stemming from general deter-
rence and incapacitation under current and optimal policies for each
of the three states involved in the analysis. From the table, it can
be seen that the redistribution of these measures is far more pro-
nounced in Texas and Missouri than in Georgia. This stems from the
nature of the shift in policy brought about by the optimization
process. It is also evidence of the relatively small impact of inca-
pacitation as compared with deterrence under optimal conditions, once
again emphasizing that it is effectively the threat of punishment, as
opposed to the actual punishment, which is most correlated with control-
ling crime. As a final note, it should be mentioned that the averages
appearing in Table 40 represent a much smaller sample under current
policy for Missouri. This is because the recursive accumulation pro-
cedure for calculating the incapacitative effect in that state required
a much larger start-up period than for Georgia, due to high average
sentence lengths under prevailing policy. Consequently, this quantity
could be determined for only a small number of periods.

The incapacitative effect in the state of Texas under prevailing
policy could not be obtained, due to the fact that the average sentence

Table 40. Distribution of Crime Control Effect

	Georgia		Missouri		Texas	
	optimal policy	prevailing policy	opt. pol.	prev. pol.	opt. pol.	prev. pol.
Incapacitation:	13%	24%	8%	93%	6%	98% (est.)
Gen. Deterrence:	87%	76%	92%	7%	94%	2% (est.)

length under prevailing policy 26.52 years exceeded the duration of
the simulation. As a result, the value in Table 40 was estimated by
assuming the unit percentage relation between incapacitative effect
and sentence length in Texas was the same for Missouri.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The conclusions reached from this research can be divided into
three categories; (1) those relating to policy implications within a
constrained resource situation, (2) those relating to policy variation
amongst geographical areas, and (3) those relating to sensitivity
studies investigating assumptions underlying the models execution.
The following sections treat each category of conclusions individually
and describes their relationship.

Section 6.2 presents recommendations for furthering the
research in this thesis and other related areas of potential interest.
Recommendations for furthering the resarch embody the areas of
transient analysis, higher resolution of crime type, forecast revision
with changing sentencing levels and the potential of integrative model-
ing techniques extended to other areas.

6.1 Conclusions

The results in section 5.6 indicate a similarity of results
that is common to each data base for which analysis was provided. In
all cases it was the certainty of punishment, as opposed to its
severity, which exhibited the greatest crime control potential relative
to the prevailing policy.

In section 5.4, it was shown that this characteristic endures

until the system capacity constraint is relaxed to three times its presently feasible limit. Since it is not realistic to expect that society's allocation for criminal justice will triple anytime soon, we can conclude that the evidence stemming from this analysis strongly supports the current popular notion favoring the dominance in the certainty of punishment. The implication of this for judicial policy is that court authorities exercise the imprisonment option for criminal offenses with greater frequency and milder sentences. The impact of such a generalized policy change will be to increase turnover rates in corrections facilities without affecting population sizes at any given instant. Although this would inevitably incur some increase in administrative costs, the results of section 5.4 suggest the resulting increases in per dollar crime control effectiveness to be more than offsetting.

In addition to determining the relative importance of the certainty of punishment, the model is able to estimate the effectiveness of a judicial policy in terms of general deterrence and incapacitation. The results stemming from these experiments served to illustrate the decreased incapacitative effect and increased general deterrent effect characteristic of optimal policies. This notion is consistent with the remainder of results, again reinforcing the importance of the certainty of punishment as opposed to its severity.

The results in section 5.6 indicate extreme variation in judicial practice between states. Specifically, judicial policies in Missouri and Texas were an order of magnitude different from judicial policy in Georgia. In addition, it was determined that the state of

Georgia spends less money per capita than either Texas or Missouri
for corrections which have comparable per capita crime control
potential.

Further evidence of this is present in state prison turnover
levels presented for each state in Chapter V. Essentially, these
results serve to support the conclusions of the preceding section
and illuminate the inconsistent pattern existing in sentencing
practices between states. Finally, in states where prevailing
judicial policy is highly sub-optimal, the potential returns for
optimization of the current policy are the greatest in terms of
crime control effectiveness.

Two assumptions,for which no previous investigation is offered
in the literature, were involved in specifying the values of beta and
epsilon. In the analysis offered in section 5.1, the sensitivity of
our model to the values of beta and epsilon was explored. In choosing
a value of beta, it was found that the form of the optimal solution
was not affected. Overstating the value of beta, however, would tend
to understate the importance of the optimzation. Specifying a value
of epsilon did tend to alter the form of the optimal solution, as well
as having an affect on the impact of optimization. In Chapter V, it
was shown that the impact of epsilon could be explained in terms of
the prevailing Q and S values. Specifically, it was shown that for
states characterized by large average sentence lengths, the optimiza-
tion process would tend to favor the value of S more than otherwise.
The magnitude of this affect was shown to be insufficient to discount

the overall dominance of Q when analysis was performed for the states of Texas and Missouri.

6.2   Recommendations

In developing descriptions of transient behavior for the model, closed form results were obtained only within the populations submodel. A significant extension of this research would embody the derivation of a closed form transient expression for the basic equation. This undertaking would involve the development of a time oriented formulation of lambda. This extension would enable the analyst to derive results for the model without the aid and expense of computer simulation.

Besides providing only partial transient analysis, the research offers no computational experience for investigating the validity of the six month iterative cycle for determining flow variables. A worthwhile effort would be to explore the impact of alternative iterative cycle lengths and provide a comparison. In addition, a different approach to the determination of flow variables may be suggested. For example, a time series description approach for determining flow variable values within Blumstein's Markov Chain description might prove more effective than the pattern search approach employed in our research.

Aside from the Criminal Justice System, the integrative modeling technique presented in this thesis may be appropriate to model building efforts in many unrelated areas. In most situations where a system can be separated into distinct subdivisions, the

potential for integrative modeling exists. Usually, integrative modeling requires only the specification of the logical interrelation between each subsystem and the development of the appropriate mathematical characterization of that relationship. For this reason, the technique has considerable potential for extensions in many disciplines.

One approach frequently taken in other criminal justice modeling is to resolve crime rates into specific categories. This has been used largely in time series modeling of crime rates. A meaningful extension to our analysis would be to adapt the model to a higher resolution of crime type. This would circumvent the limiting assumption of a single criminal type committing a single crime type. For example, the model might be utilized in modeling only the occurrence of burglary or armed robbery and consider the criminal population as consisting of only burglars or armed robbers.

A final extension to the model, and perhaps the most difficult to implement, would involve the rebuilding of the forecasting mechanisms within the model with each policy change. By this, we mean that each time an optimal policy is formulated, the expected number of crimes which result under that optimal policy is used as the estimate of total offenses in the next period. This would lead to an optimization which is truly unique for each period and dependent exclusively on the events of previous periods. This extension would greatly enhance the dynamic character of the model and make forecasting the system more meaningful.

APPENDIX

The appendix lists the fortran code of the model used to generate
the results in chapter V and some example output. The input to the pro-
gram is as follows:

     1st card: 36 monthly observations of prevailing imprisonment
                 probabilities

     2nd card: 36 monthly observations of total reported offenses

     3rd card: 36 monthly observation of the population of the area
                 of interest

     4th card: 36 monthly observations of the state of prison
                 population

     5th card: 36 monthly observations of the prevailing average
                 sentence length

A brief description of the routines is as follows:

1. MAIN.   This program is the main-line routine which calls all
    subsequent subroutines.

2. FRCST.   This subroutine forecasts the values of Q, S and the
    prison population which are used throughout the simulation.

3. SLIP.   This subroutine determines the proportion of the crime
    control effect which is attributable to incapacitation.

4. PLOTTER.   This subroutine is the main-line routine which
    calls the subroutines used for plotting total offenses and
    deterrent effect time series.

5. RANGE.   This subroutine compiles the limits of a data set
    which is to be plotted.

6. INITIAL.   This subroutine laods the data into the appropriate
    array for plotting.

7. SCALE.   This subroutine scales the raw data prior to printing
    the plots.

8. GRAF. This subroutine orders the data within the plotting array.

9. OUTPUT. This subroutine actually prints the plot of total offenses or deterrent effects.

10. OPT. This subroutine solves for the coefficient values given the deterrent effect, and the necessary assumptions.

11. PTRN. This subroutine executes the pattern search step in determining optimal flow and policy variables.

12. FOBJ. This subroutine evaluates the sum of squares objective function corresponding to the current flow variable solution.

13. LIMS. This subroutine determines the feasibility of the current flow variable solution.

14. OBJ. This subroutine evaluates the current Q, S solution.

15. LIMS1. This subroutine determines the feasibility of the current Q, S solution.

16. CONTOU. This subroutine evaluates deterrent effect solutions for a range of Q and S values and is used to plot contour lines of objective function values in the Q and S policy space.

```
100010    PROGRAM MAIN(INPUT,OUTPUT,TAPE1,TAPE2,TAPE3,TAPE4,
     *    TAPE5,TAPE6,TAP
0001*    COMMON DGRID(42,72),IDATA(2,72),ISCALE(42),DET(300),
     *    OPTS(300),OPT
0001*+         SSQ(300),PMAT(9),ESTPR(6,50),ESTCR(6,50),
     *    PRIS(300),GAM0(30
0001*+         GAM1(300),GAM2(300)
0001*    COMMON/BLOCKD/LO
0001*    COMMON/BLOCK/STA11,STA22,STA33,NN,QCOUNT,TPOP,LLCON
0001*    COMMON/BLOCKA/TPO(300),QQQ(300),C
0001*    COMMON/BLOCKB/SSS(300),DET1(300),XAM(300),XAM1(300),
     *    ZDIF(300)
0001*    COMMON/BLOCKC/INUM,NFOR
0001*    COMMON/BLOCKE/Z(300),ZB(300)
0003*    DIMENSION RAT(300),RATA(300)
0003*    DIMENSION ZP(300),YRAVG(300),TERMS(300)
0003*    DIMENSION X(3),STEP(3),B1(3),B2(3),B3(3),B4(3)
0004*    DIMENSION CAVG(300),CINT(300),TER(300)
0004*    DIMENSION A(300),W(300),WP(300)
0004*    DIMENSION DIFS(300),DIFQ(300),DEP(300)
0004*    DIMENSION DETB(300),AXX(62),AXY(62)
0004*    DIMENSION FN(60,60),CLEV(7),IBUF(512)
0004*    DIMENSION T1(300),T2(300),T3(300),T4(300),T5(300),
     *    T6(300)
0005*    DIMENSION ZPOPT(300),YRMIN1(300),TERMS1(300)
0005*    DIMENSION DE(300),DE1(300),PERC(300),PERC1(300)
0005*    DIMENSION CAPAC(300),CAPA1(300)
0007*    INTEGER XNAME,YNAME
0007*    INUM=36
0007*    IYEAR=1973
0007*    KYEAR=IYEAR
0007*    RECEP=643.02778
0013*    READ(4,*),(QQQ(I),I=1,36)
0014*    102 READ(7,*),(Z(KA),KA=1,INUM)
0017*    READ(8,*),(TPO(I),I=1,36)
0017*    TH1=0.2695
0017*    TH12=0.1530
0022*    A(1)=0.0
0023*    WP(1)=0.0
0024*    W(1)=0.0
0025*    DO 106 I=2,INUM
0026*    W(I)=Z(I)-Z(I-1)
0027*    WP(I)=W(I)
0028*    A(I)=WP(I)+TH1*A(I-1)
0029*    106 CONTINUE
0029*    NFOR=252
0033*    Z(INUM+1)=Z(INUM)+Z(INUM-11)-Z(INUM-12)-TH1*A(INUM-1)
0033*    Z(INUM+1)=Z(INUM+1)-TH12*A(INUM-12)+TH1*TH12*A(INUM-
     *    12)
0034*    DO 108 J=2,12
```

```
0035*   Z(INUM+J)=Z(INUM+J-1)+Z(INUM+J-12)-Z(INUM+J-13)-
    *     TH12*A(INUM-12+J)
0036*   108 CONTINUE
0037*   IF(NFOR.LE.12)GO TO 109
0038*   Z(INUM+13)=Z(INUM+12)+Z(INUM+1)-Z(INUM)+
    *     TH12*TH1*A(INUM)
0039*   ICOUNT=INUM+14
0040*   DO 110 K=ICOUNT,300
0041*   Z(K)=Z(K-1)+Z(K-12)-Z(K-13)
0042*   110 CONTINUE
0043*   109 CONTINUE
0046*   IEXAM=1
0046*   IWON=INUM+NFOR
0047*   IF(IEXAM.EQ.1)GO TO 112
0048*   PRINT 113
0049*   113 FORMAT(*THE ORIGINAL Z DATA WAS*)
0050*   PRINT 114,(Z(IJ),IJ=1,INUM)
0051*   114 FORMAT(/4F10.1)
0052*   LDUM=INUM+1
0053*   PRINT 115
0054*   115 FORMAT(*THE SUBSEQUENT FORECASTS ARE*)
0055*   PRINT 116,(Z(JI),JI=LDUM,IWON)
0056*   116 FORMAT(/4F10.1)
0057*   112 CONTINUE
0058*   LPRIM=IWON/12
0059*   PRINT 117,LPRIM
0060*   117   FORMAT(*THE DATA COVERS,*I4*,YEARS*)
0060*   NPRIS=36
0060*   READ(2,*),(PRIS(KKK),KKK=1,NPRIS)
0060*   CALL FRCST
0061*   MCOUNT=0
0062*   SUM=C
0063*   DO 118 L=1,LPRIM
0063*   SUM=C
0064*   DO 119 LL=1,12
0065*   MASS=Z(MCOUNT+LL)
0066*   SUM=SUM+MASS
0067*   IF(LL.EQ.12)YRMEAN=(SUM/12)
0068*   IF(LL.EQ.12)FIRST=Z(MCOUNT)
0069*   IF(LL.EQ.12)LAST=Z(MCOUNT+LL)
0071*   IF(LL.EQ.12)TERM=((LAST-FIRST)/12)
0072*   119 CONTINUE
0073*   YRAVG(L)=YRMEAN
0074*   TERMS(L)=TERM
0074*   MCOUNT=MCOUNT+12
0075*   118 CONTINUE
0076*   NCOUNT=0
0081*   ZP(1)=Z(1)
0081*   DO 54321 I=2,288
0081*   ZP(I)=Z(I-1)+A(I)-TH1*A(I-1)
```

```
0081*  54321 CONTINUE
0092*  NPRIS=36
0109*  IA=0
0111*  QCOUNT=0
0111*  NTIMS=(NPRIS+NFOR)/6
0115*  STA11=.001
0115*  STA22=.01
0115*  STA33=.989
0115*  139 CONTINUE
0116*  DO 141 NN=1,NTIMS
0116*  IF(NN.GE.2)STA11=ESTPR(6,NN)
0116*  IF(NN.GE.2)STA22=ESTCR(6,NN)
0117*  STA33=1-STA11-STA22
0117*  NV=3
0117*  STEP(1)=0.045
0117*  STEP(2)=0.0045
0117*  STEP(3)=0.34
0117*  X(1)=0.05
0118*  X(2)=0.005
0118*  X(3)=0.6
0118*  NRD=8
0118*  IPR=0
0119*  LLCON=0
0120*  CALL PTRN(NV,X,STEP,NRD,YMAX,B1,B2,B3,B4,IPR)
0121*  F2=X(1)
0121*  F3=X(2)
0121*  F4=X(3)
0125*  PMAT(1)=.5
0126*  PMAT(2)=(1./3.)*.5
0126*  PMAT(3)=1./3.
0127*  PMAT(4)=F2
0128*  PMAT(5)=1-F2-F4
0129*  PMAT(6)=F4
0130*  PMAT(7)=0.0
0131*  PMAT(8)=F3
0132*  PMAT(9)=1-F3
0132*  STA1=STA11
0132*  STA2=STA22
0132*  STA3=STA33
0133*  DO 142 NON=1,6
0134*  ESTPR(NON,NN)=STA1*PMAT(1)+STA2*PMAT(4)+STA3*PMAT(7)
0135*  X1=ESTPR(NON,NN)
0136*  X2=STA1*PMAT(2)+STA2*PMAT(5)+STA3*PMAT(8)
0137*  X3=STA1*PMAT(3)+STA2*PMAT(6)+STA3*PMAT(9)
0137*  STA1=X1
0137*  STA2=X2
0137*  STA3=X3
0138*  142 CONTINUE
0139*  SSQ(JBB)=0.0
0140*  DO 143 NON=1,6
```

```
0141*   SS=(ESTPR(MON,NN)*TPO(MON+QCOUNT)-PRIS(MON+QCOUNT))
    *   **2
0142*   SSQ(JBB)=SSQ(JBB)+SS
0143*   143 CONTINUE
0153*   STA1=STA11
0153*   STA2=STA22
0153*   STA3=STA33
0154*   DO 147 KUP=1,6
0155*   ESTCR(KUP,NN)=STA1*PMAT(2)+STA2*PMAT(5)+STA3*PMAT(8)
0156*   Y2=ESTCR(KUP,NN)
0157*   Y1=STA1*PMAT(1)+STA2*PMAT(4)+STA3*PMAT(7)
0158*   Y3=STA1*PMAT(3)+STA2*PMAT(6)+STA3*PMAT(9)
0159*   STA1=Y1
0160*   STA2=Y2
0161*   STA3=Y3
0162*   147 CONTINUE
0170*   151 CONTINUE
0171*   QCOUNT=QCOUNT+6
0172*   141 CONTINUE
0175*   PRINT 155
0176*   155 FORMAT(*THE POPULATION ROUTINE IS COMPLETE*)
0177*   LC=0
0178*   DO 156 JLN=1,NTIMS
0180*   DO 157 LIP=1,6
0181*   RAT(LIP+LC)=ESTCR(LIP,JLN)/(ESTCR(LIP,JLN)+ESTPR(LIP,
    *   JLN))
0182*   157 CONTINUE
0182*   LC=LC+6
0183*   156 CONTINUE
0184*   NCO=0
0185*   DO 158 LAM=1,NTIMS
0187*   DO 159 JFK=1,6
0188*   CINT(JFK+NCO)=ESTCR(JFK,LAM)
0189*   159 CONTINUE
0189*   NCO=NCO+6
0190*   158 CONTINUE
0193*   DO 160 IT=1,NTIMS
0195*   DO 161 IB=1,6
0196*   AVER=CINT(IB*IT)
0197*   SOS=SOS+AVER
0198*   161 CONTINUE
0199*   TER(IT)=SOS/6
0199*   SOS=0.0
0200*   160 CONTINUE
0201*   ID=0
0202*   DO 162 IE=1,NTIMS
0204*   DO 163 IC=1,6
0205*   CAVG(IC+ID)=TER(IE)
0206*   163 CONTINUE
0206*   ID=ID+6
```

```
0206*  162 CONTINUE
0207*  NOR=NPRIS+NFOR
0208*  DO 165 IO=1,NOR
0209*  RATA(IO)=((ZP(IO)/CAVG(IO))/TPO(IO))
0210*  DET(IO)=((Z(IO)/TPO(IO))/(RATA(IO)*RAT(IO)))
0211*  165 CONTINUE
0212*  LETE=0
0217*  PRINT*,"THE VECTOR OF MONTHLY DETERRENT EFFECTS"
0218*  PRINT*,"FROM JANUARY,1974 UNTIL DECEMBER,1998"
0218*  PRINT*,"IS READ FROM LEFT TO RIGHT"
0218*  PRINT 4011,(DET(I),I=1,288)
0218*  4011 FORMAT(///3F20.12)
0219*  PRINT 40189
0219*  40189 FORMAT(//////*PLOT OF THE TOTAL OFFENSES TIME
      *  SERIES*)
0219*  PRINT*,"FROM JANUARY,1974 THROUGH DECEMBER,1976"
0231*  IFILE=1
0231*  DO 1133 I=1,36
0231*  1133 DET(I)=DET(I)*1000000.
0234*  INU=36
0235*  DO 175 IQ=1,INU
0236*  IDATA(1,IQ)=Z(IQ)
0237*  175 CONTINUE
0237*  CALL PLOTTER(INU,IFILE)
0237*  PRINT 41089
0237*  41089 FORMAT(//////*PLOT OF DETERRENT EFFECT TIME
      *  SERIES*)
0237*  PRINT*,"FROM JANUARY,1974 THROUGH DECEMBER,1976"
0238*  DO 913 IQ=1,INU
0238*  IDATA(1,IQ)=DET(IQ)
0239*  913 CONTINUE
0239*  CALL PLOTTER(INU,IFILE)
0239*  DO 1134 I=1,36
0239*  1134 DET(I)=DET(I)/1000000.
0242*  CALL OPT(IWON)
0250*  DO 1135 J=1,IWON
0251*  DIFS(J)=OPTS(J)-SSS(J)
0252*  DIFQ(J)=OPTQ(J)-QQQ(J)
0253*  1135 CONTINUE
0254*  PRINT 1136
0255*  1136 FORMAT(1X,*ACCORDING TO OUR MODEL,OPTIMIZATION*,
0256**  OF POLICY VARIABLES GIVES THE FOLLOWING RESULTS*)
0258*  PRINT 1137
0259*  1137 FORMAT(////1X,*PERIOD            PREVAILING AND
      *  OPTIMAL*,
0259**   POLICY VARS. *)
0260*  PRINT 1138
0261*  1138 FORMAT(20X,*Q*,10X,*QSTAR*,10X,*S*,10X,*SSTAR*)
0262*  PRINT 1139
0263*  1139 FORMAT(70(*-*))
```

```
0264*  NRE=(INUM+NFOR)/12
0264*  LLLC=0
0265*  DO 1140 I=1,NRE
0266*  DO 1141 J=1,12
0266*  T1(J+LLLC)=OPTQ(J+LLLC)
0266*  T2(J+LLLC)=SSS(J+LLLC)
0266*  T3(J+LLLC)=OPTS(J+LLLC)
0266*  T4(J+LLLC)=DIFQ(J+LLLC)
0266*  T5(J+LLLC)=DIFS(J+LLLC)
0267*  T6(J+LLLC)=QQQ(J+LLLC)
0268*  1141 CONTINUE
0269*  LLLC=LLLC+12
0270*  IYEAR=IYEAR+1
0271*  1140 CONTINUE
0271*  MONTH=9
0271*  NI2=NRE*12
0271*  DO 5002 I=1,NI2
0271*  MONTH=MONTH+1
0272*  IF(MONTH.GT.12)MONTH=1
0272*  J33=I/12-1/20000
0272*  IYEAR=1974+J33
0272*  IF(MONTH.EQ.12)IYEAR=IYEAR-1
0272*  PRINT 5003,MONTH,IYEAR,T6(I),T1(I),T2(I),T3(I)
0272*  5003 FORMAT(///1X,I2,*/*,I4,2(10X,F5.4),2(5X,F9.2))
0272*  5002 CONTINUE
0272*  8007 CONTINUE
0273*  DO 1144 I=1,IWON
0274*  DETB(I)=GAM0(I)+GAM1(I)*OPTQ(I)+GAM2(I)*OPTQ(I)
     *  *OPTS(I)
0275*  ZB(I)=(EXP(DETB(I)))/(1+EXP(DETB(I)))
0275*  DETB(I)=ZB(I)
0275*  ZB(I)=DETB(I)*RATA(I)*RAT(I)*TPO(I)
0276*  1144 CONTINUE
0277*  DO 1145 I=1,IWON
0278*  ZDIF(I)=Z(I)-ZB(I)
0279*  1145 CONTINUE
0280*  PRINT 1146
0281*  1146 FORMAT(1X,*SAVINGS IN REPORTED CRIMES PER
     *  PERIOD THRU*,
0282*++ OPTIMIZATION HERE AS FOLLOWS*)
0283*  PRINT 1147,(ZDIF(J),J=1,IWON)
0284*  1147 FORMAT(1X,6F10.0)
0285*  DO 8001 I=1,288
0286*  DEP(I)=ZDIF(I)/Z(I)
0287*  8001 CONTINUE
0288*  PRINT*,"THE RATIOS OF THE SAVINGS IN TOTAL OFFENSES
     *  FORECASTED"
0288*  PRINT*,",(STEMMING FROM OPTIMIZATION),TO THE TOTAL
     *  OFFENSES"
0288*  PRINT*,"FORECASTED FOR PERIODS  1/75 ,TO, 12/98 BY
     *  THE MODEL"
```

```
0288*  PRINT*,"IS READ ACROSS FROM LEFT TO RIGHT"
0289*  PRINT 8002,(DEP(I),I=1,300)
0290*  8002 FORMAT(6F8.5)
0296*  KU=48
0297*  DO 1173 J=1,60
0298*  AXY(J)=J*.3
0299*  1173 CONTINUE
0300*  DO 1174 IJ=1,20
0301*  AXX(IJ)=IJ*.05
0302*  1174 CONTINUE
0303*  DO 1175 IQ=1,20
0304*  DO 1176 JQ=1,60
0305*  FN(IQ,JQ)=GAM0(KU)+GAM1(KU)*AXX(IQ)
0305*  FN(IQ,JQ)=FN(IQ,JQ)+GAM2(KU)*AXX(IQ)*AXY(JQ)
0306*  FN(IQ,JQ)=(EXP(FN(IQ,JQ))/(1+EXP(FN(IQ,JQ))))
0306*  FN(IQ,JQ)=FN(IQ,JQ)*RATA(KU)*RAT(KU)*TPO(KU)
0306*  FN(IQ,JQ)=ZDIF(KU)/FN(IQ,JQ)
0307*  1176 CONTINUE
0308*  1175 CONTINUE
0309*  GO TO 1149
0313*  PRINT 1178
0314*  1178 FORMAT(*THE REGION NEAR THE OPTIMUM IS AS
       *    FOLLOWS FOR*)
0315*  NEW=KU/12
0316*  KKYR=KYEAR+NEW
0317*  MONT=KU-(12*NEW)
0318*  BB1=GAM0(KU)+GAM1(KU)*OPTQ(KU)
0318*  BB1=BB1+GAM2(KU)*OPTQ(KU)*OPTS(KU)
0319*  BB1=(EXP(BB1)/(1+EXP(BB1)))
0319*  BB1=BB1*RATA(KU)*RAT(KU)*TPO(KU)
0320*  AA1=(EXP(GAM0(KU))/(1+EXP(GAM0(KU))))
0320*  AA1=AA1*RATA(KU)*RAT(KU)*TPO(KU)
0321*  CC1=GAM0(KU)+GAM1(KU)+7*GAM2(KU)
0321*  CC1=(EXP(CC1)/(1+EXP(CC1)))
0322*  CC1=CC1*RATA(KU)*RAT(KU)*TPO(KU)
0322*  DELTA1=(BB1-AA1)/4
0322*  DELTA2=(CC1-BB1)/4
0323*  PRINT 1180,MONT,KKYR
0324*  1180 FORMAT(1X,I2,*/*,I4///)
0325*  PRINT 1181,((FN(I,J),I=1,20),J=1,60)
0326*  1181 FORMAT (10F10.6)
0327*  1149 CONTINUE
0328*  CALL PLOTS(IBUF,512,1,0)
0329*  CALL PLOTMX(100.)
0330*  NUM=6H H-H
0331*  NCON=7
0331*  CLEV(1)=.05
0332*  CLEV(2)=.02278
0332*  CLEV(3)=.20
0333*  CLEV(4)=.30
```

```
0333*  CLEV(5)=.40
0334*  CLEV(6)=.50
0334*  CLEV(7)=.01000
0335*  NX=20
0336*  NY=60
0337*  XNAME=6HQQQQQQ
0338*  YNAME=6HSSSSSS
0339*  HT=0.07
0340*  CALL CONTOU(NUM,NCON,CLEV,FN,NX,NY,AXX,AXY,XNAME,
       *  YNAME,HT)
0341*  CALL PLOT(0,0,999)
0342*  MCOU1=0
0343*  SUM1=0.0
0344*  DO 190 L=1,LPRIM
0345*  SUM1=0.0
0346*  DO 191 LL=1,12
0347*  MASS1=ZB(MCOU1+LL)
0348*  SUM1=SUM1+MASS1
0349*  IF(LL.EQ.12)YRAVG1=(SUM1/12)
0350*  IF(LL.EQ.12)FIRST1=ZB(MCOU1)
0351*  IF(LL.EQ.12)LAST1=ZB(MCOU1+LL)
0352*  IF(LL.EQ.12)TERM1=((LAST1-FIRST1)/12)
0353*  191 CONTINUE
0354*  YRMIN1(L)=YRAVG1
0355*  TERMS1(L)=TERM1
0356*  MCOU1=MCOU1+12
0357*  190 CONTINUE
0358*  NCOUN=0
0367*  A(1)=0
0367*  ZPOPT(1)=ZB(1)
0367*  DO 430 I=2,300
0367*  A(I)=ZB(I)-ZB(I-1)+TH1*A(I-1)
0367*  ZPOPT(I)=ZB(I-1)+A(I)-TH1*A(I-1)
0367*  430 CONTINUE
0368*  DO 195 I=1,288
0369*  XAM(I)=(ZP(I)/CAVG(I))/TPO(I)
0370*  XAM1(I)=(ZPOPT(I)/CAVG(I))/TPO(I)
0370*  CAPAC(I)=XAM(I)*RECEP
0371*  CAPA1(I)=XAM1(I)*RECEP
0371*  DE(I)=(CAPAC(I)/TPO(I))/(RATA(I)*RAT(I))
0371*  DE1(I)=(CAPA1(I)/TPO(I))/(RATA(I)*RAT(I))
0371*  PERC(I)=DE(I)/DET(I)
0371*  PERC1(I)=DE1(I)/DETB(I)
0371*  195 CONTINUE
0373*  DO 12345 I=1,288
0373*  PERC(I)=PERC(I)*100.
0373*  12345 CONTINUE
0373*  DO 12354 I=1,288
0373*  PERC1(I)=PERC1(I)*100
0374*  12354 CONTINUE
```

```
0377* CALL SLIP
0379* END
0400* SUBROUTINE FRCST
0401* COMMON DGRID(42,72),IDATA(2,72),ISCALE(42),DET(300),
0401*+      OPTS(300),OPTQ(300),
0402*+      SSQ(300),PMAT(9),ESTPR(6,50),ESTCR(6,50),
0402*+      PRIS(300),GAM0(300),
0403*+      GAM1(300),GAM2(300)
0404* COMMON/BLOCKA/TPO(300),QQQ(300),C
0404* COMMON/BLOCKB/SSS(300),OET1(300),XAM(300),XAM1(300),
      *  ZDIF(300)
0405* COMMON/BLOCKC/INUM,NFOR
0405* DIMENSION A(300),W(300),WP(300)
0406* DO 1 I=1,300
0407* TPO(I)=4900000+2200*(I)
0408* 1 CONTINUE
0409* DO 2 I=1,300
0410* QQQ(I)=.30606
0411* SSS(I)=1.67
0411* 2 CONTINUE
0417* TH1=.6279
0417* TH12=.2028
0417* A(1)=0.0
0417* W(1)=0.0
0417* WP(1)=0.0
0418* DO 77 I=2,INUM
0419* W(I)=PRIS(I)-PRIS(I-1)
0420* WP(I)=W(I)
0421* A(I)=WP(I)+TH1*A(I-1)
0422* 77 CONTINUE
0423* PRIS(INUM+1)=PRIS(INUM)+PRIS(INUM-11)-PRIS(INUM-12)-
      *  TH1*A(INUM-1)
0424* PRIS(INUM+1)=PRIS(INUM+1)-TH12*A(INUM-12)+
      *  TH1*TH12*A(INUM-12)
0425* DO 3 I=2,12
0426* PRIS(INUM+I)=PRIS(INUM+I-1)+PRIS(INUM+I-12)-
      *  PRIS(INUM+I-13)
0426* PRIS(INUM+I)=PRIS(INUM+I)-TH12*A(INUM-12+I)+
      *  TH12*TH1*A(INUM-13+I)
0427* 3 CONTINUE
0428* IF(NFOR.LE.12)GO TO 66
0429* PRIS(INUM+13)=PRIS(INUM+12)+PRIS(INUM+1)-PRIS(INUM)+
      *  TH12*TH1*A(IN
0430* IFE=INUM+14
0431* DO 4 I=IFE,300
0432* PRIS(I)=PRIS(I-1)+PRIS(I-12)-PRIS(I-13)
0433* 4 CONTINUE
0434* 66 CONTINUE
0434* GO TO 11
0435* DO 5 I=2,INUM
```

```
0435*  TH1=.7706
0436*  W(I)=QQQ(I)-QQQ(I-1)
0437*  A(I)=W(I)+TH1*A(I-1)
0438*  QQQ(I)=QQQ(I-1)-TH1*A(I-1)+A(I)
0438*  5 CONTINUE
0439*  QQQ(INUM+1)=QQQ(INUM)+TH1*A(INUM)
0440*  IJP=INUM+2
0441*  DO 6 I=IJP,300
0442*  QQQ(I)=QQQ(I-1)
0443*  6 CONTINUE
0443*  11 CONTINUE
0444*  RETURN
0445*  END
0448*  SUBROUTINE SLIP
0449*  COMMON DGRID(42,72),IDATA(2,72),ISCALE(42),DET(300),
0449*+        OPTS(300),OPTQ(300),
0449*+        SSQ(300),PMAT(9),ESTPR(6,50),ESTCR(6,50),
0449*+        PRIS(300),GAM0(300),
0449*+        GAM1(300),GAM2(300)
0450*  COMMON/BLOCKB/SSS(300),DET1(300),XAM(300),XAM1(300),
       *   ZDIF(300)
0450*  COMMON/BLOCKE/Z(300),ZB(300)
0450*  DIMENSION N(300),SSAVE(300),SSAV1(300),SSOI(300),
       *   PRS(300)
0450*  DIMENSION PERCP(300),PER1P(300)
0450*  RECEP=643.02778
0451*  DO 1 I=1,288
0452*  N(I)=SSS(I)*12
0453*  1 CONTINUE
0454*  M=N(1)+1
0455*  DO 2 I=M,288
0456*  AADD=0.0
0457*  NF=N(I)
0458*  DO 3 J=1,NF
0459*  ADD=RECEP*XAM(I-J)
0460*  AADD=ADD+AADD
0461*  3 CONTINUE
0462*  SSAVE(I)=AADD
0463*  2 CONTINUE
0464*  DO 4 I=1,288
0465*  N(I)=OPTS(I)*12
0466*  4 CONTINUE
0467*  MM=N(1)+1
0468*  DO 5 I=MM,288
0469*  AADD=0.0
0470*  NF=N(I)
0471*  DO 6 J=1,NF
0472*  ADD=RECEP*XAM1(I-J)
0473*  AADD=ADD+AADD
0474*  6 CONTINUE
```

```
0475* SSAV1(I)=AADD
0476* 5 CONTINUE
0477* IF(MM.LE.M)KU=M
0478* IF(M.LT.MM)KU=MM
0479* DO 7 I=KU,288
0480* SSDI(I)=SSAV1(I)-SSAVE(I)
0481* IF(SSDI(I).LE.0.0)GO TO 8
0482* PRS(I)=SSDI(I)/ZDIF(I)
0483* GO TO 9
0484* 8 PRS(I)=0.0
0485* 9 CONTINUE
0486* 7 CONTINUE
0488* DO 13 I=KU,288
0489* PERCP(I)=SSAVE(I)/Z(I)*100.
0490* PER1P=SSAV1(I)/ZB(I)*100.
0491* 13 CONTINUE
0492* PRINT*,"STARTING WITH PERIOD"
0492* PRINT*,KU
0492* PRINT*,"AND ENDING WITH PERIOD 288, INCAPACITATION
     *  HAS"
0492* PRINT*,"RESPONSIBLE FOR THE FOLLOWING PERCENTAGES OF
     *  CRIMES"
0492* PRINT 14,(PERCP(I),I=KU,288)
0492* 14 FORMAT(///6F11.3)
0494* RETURN
0495* END
0500* SUBROUTINE PLOTTER(INU,IFILE)
0501* COMMON DGRID(42,72),IDATA(2,72),ISCALE(42),OET(300),
     *  OPTS(300),OPT
0501*+      SSQ(300),PHAT(9),ESTPR(6,50),ESTCR(6,50),
     *  PRIS(300),GAM0(30
0501*+      GAM1(300),GAM2(300)
0502* DATA IMIN/99999/,IMAX/-99999/
0704* CALL RANGE(IMIN,IMAX,INU,IFILE)
0705* CALL INITIAL(INU)
0706* CALL SCALE(IMIN,IMAX,IINCR,ILIM)
0707* CALL GRAF(IMIN,INU,IFILE,IINCR)
0708* CALL OUTPUT(INU,ILIM)
0709* RETURN
0710* END
0711* SUBROUTINE RANGE(IMIN,IMAX,INU,IFILE)
0712* COMMON DGRID(42,72),IDATA(2,72),ISCALE(42),OET(300),
     *  OPTS(300),OPT
0712*+      SSQ(300),PHAT(9),ESTPR(6,50),ESTCR(6,50),
     *  PRIS(300),GAM0(30
0712*+      GAM1(300),GAM2(300)
0713* DO 100 I=1,IFILE
0714* DO 100 J=1,INU
0715* IF(IDATA(I,J).LT.IMIN) IMIN=IDATA(I,J)
0716* IF(IDATA(I,J).GT.IMAX) IMAX=IDATA(I,J)
```

```
0717* 180 CONTINUE
0718* RETURN
0719* END
0720* SUBROUTINE INITIAL(INU)
0721* COMMON DGRID(42,72),IDATA(2,72),ISCALE(42),DET(300),
    *  OPTS(300),OPT
0721*+        SSQ(300),PMAT(9),ESTPR(6,50),ESTCR(6,50),
    *  PRIS(300),GAM0(30
0721*+        GAM1(300),GAM2(300)
0722* DO 181 I=1,42
0723* DO 181 J=1,INU
0724* DGRID(I,J)=1H
0725* 181 CONTINUE
0726* RETURN
0727* END
0728* SUBROUTINE SCALE(IMIN,IMAX,IINCR,ILIM)
0729* COMMON DGRID(42,72),IDATA(2,72),ISCALE(42),DET(300),
    *  OPTS(300),OPT
0729*+        SSQ(300),PMAT(9),ESTPR(6,50),ESTCR(6,50),
    *  PRIS(300),GAM0(30
0729*+        GAM1(300),GAM2(300)
0730* IINCR=(IMAX-IMIN)/41+1
0731* ILIM=(IMAX-IMIN)/IINCR+2
0732* ISCALE(1)=IMIN
0733* DO 100 I=2,ILIM
0734* ISCALE(I)=ISCALE(I-1)+IINCR
0735* 100 CONTINUE
0736* RETURN
0737* END
0738* SUBROUTINE GRAF(IMIN,INU,IFILE,IINCR)
0739* COMMON DGRID(42,72),IDATA(2,72),ISCALE(42),DET(300),
    *  OPTS(300),OPT
0739*+        SSQ(300),PMAT(9),ESTPR(6,50),ESTCR(6,50),
    *  PRIS(300),GAM0(30
0739*+        GAM1(300),GAM2(300)
0740* DIMENSION DALPHA(2)
0741* DATA DALPHA/1HD,1HR/
0742* DO 100 I=1,IFILE
0743* DO 100 J=1,INU
0744* IPLOT=(IDATA(I,J)-IMIN)/IINCR+1
0745* IF((IMIN+IINCR*(IPLOT-1)).LT.IDATA(I,J)) IPLOT=IPLOT+
    *  1
0746* IF(DGRID(IPLOT,J).NE.1H ) DGRID(IPLOT,J)=1H*
0747* IF(DGRID(IPLOT,J).EQ.1H ) DGRID(IPLOT,J)=DALPHA(I)
0748* 100 CONTINUE
0749* RETURN
0750* END
0751* SUBROUTINE OUTPUT(INU,ILIM)
0752* COMMON DGRID(42,72),IDATA(2,72),ISCALE(42),DET(300),
    *  OPTS(300),OPT
```

```
0752*+         SSQ(300),PMAT(9),ESTPR(6,50),ESTCR(6,50),
    *  PRIS(300),GAM0(30
0752*+         GAM1(300),GAM2(300)
0753* DATA DLINE/1H*/
0754* PAUSE
0755* ISKIP=(42-ILIM)/ILIM
0755* PRINT 090
0755* 090 FORMAT(1H1)
0756* DO 130 I=1,ILIM
0757* PRINT 100, ISCALE(ILIM-I+1),(DGRID(ILIM-I+1,J),J=1,
    *  INU)
0758* 100 FORMAT (1H ,I6,1X,1H*,1X,72A1)
0759* IF(ISKIP.EQ.0) GO TO 130
0760* DO 120 J=1,ISKIP
0761* PRINT 110
0762* 110 FORMAT (1H ,7X,1H*)
0763* 120 CONTINUE
0764* 130 CONTINUE
0765* PRINT 140, (DLINE,I=1,INU)
0766* 140 FORMAT (1H ,7X,2H**,72A1)
0767* RETURN
0768* END
0800* SUBROUTINE OPT(IWON)
0801* COMMON DGRID(42,72),IDATA(2,72),ISCALE(42),DET(300),
    *  OPTS(300),OPT
0801*+         SSQ(300),PMAT(9),ESTPR(6,50),ESTCR(6,50),
    *  PRIS(300),GAM0(30
0801*+         GAM1(300),GAM2(300)
0801* COMMON/BLOCKA/TP0(300),QQQ(300),C
0801* COMMON/BLOCKB/SSS(300),DET1(300),XAM(300),XAM1(300),
    *  ZDIF(300)
0801* COMMON/BLOCKC/INUM,NFOR
0801* COMMON/BLOCKD/LO
0801* COMMON/BLOCK/STA11,STA22,STA33,NN,QCOUNT,TPOP,LLCON
0801* DIMENSION X(3),STEP(3),B1(3),B2(3),B3(3),B4(3)
0802* DIMENSION SUBSUM(300)
0813* DO 1 I=1,288
0814* GAM0(I)=ALOG((1.2*DET(I))/(1-1.2*DET(I)))
0815* SUBSUM(I)=ALOG(DET(I)/(1-DET(I)))
0815* SUBSUM(I)=SUBSUM(I)-GAM0(I)
0816* GAM1(I)=(.25*SUBSUM(I))/QQQ(I)
0817* GAM2(I)=(.75*SUBSUM(I))/(QQQ(I)*SSS(I))
0818* 1 CONTINUE
0820* DO 5 LO=1,IWON
0821* NV=2
0822* STEP(2)=.5
0823* STEP(1)=.15
0824* X(2)=.6
0825* X(1)=.385
0826* NRD=8
```

```
0827*  IPR=1
0827*  LLCON=1
0828*  CALL PTRN(NV,X,STEP,NRD,COST,B1,B2,B3,B4,IPR)
0830*  OPTQ(LO)=X(1)
0831*  OPTS(LO)=X(2)
0832*  5 CONTINUE
0833*  DO 10 I=1,IWON
0834*  DET1(I)=GAMO(I)+GAM1(I)*OPTQ(I)+GAM2(I)*OPTQ(I)
    *    *OPTS(I)
0835*  DET1(I)=EXP(DET1(I))/(1+EXP(DET1(I)))
0836*  10 CONTINUE
0837*  RETURN
0838*  END
1013*  SUBROUTINE PTRN(NP,P,STEP,NRD,COST,B1,B2,T,S,IO)
1013*  COMMON DGRID(42,72),IDATA(2,72),ISCALE(42),DET(300),
    *    OPTS(300),OPT
1013**+       SSQ(300),PMAT(9),ESTPR(6,50),ESTCR(6,50),
    *    PRIS(300),GAMO(30
1013**+       GAM1(300),GAM2(300)
1013*  COMMON/BLOCK/STA11,STA22,STA33,NN,QCOUNT,TPOP,LLCON
1013*  DIMENSION P(3),STEP(3),B1(3),B2(3),T(3),S(3)
1014*  999 FORMAT("PATTERN SEARCH"//)
1015*  1000 FORMAT(5X"X1,...,XN,Y=",1P7D15.8/(18X7D15.8))
1016*  1001 FORMAT("ITERATION ",I3,5X,"BASE POINT B AND Y(B)
    *    ARE")
1017*  1002 FORMAT(" COMMENCE LOCAL EXPLORATION"/)
1018*  1004 FORMAT(//2X"Y(MIN)=",1PD15.8//I5,"FUNCTION
    *    EVALUATIONS RQ
1019**+ ED"//2X"X1,...,XN=",7D15.8/(13X7D15.8))
1020*  1005 FORMAT(35HINITIAL PARAMETERS OUT OF BOUNDS
    *    )
1021*  1008 FORMAT(2X"STEP SIZES=",1P7D15.3/(15X7D15.3))
1022*  1009 FORMAT(2X"EXTRAPOLATION FAILED. GO BACK TO OLD
    *    BASE POINT."/
1023*  1010 FORMAT(2X"EXTRAPOLATED POINT T AND Y(T)ARE"  /
    *    5X"X1,...,XN,
1024**+Y =",1P7D15.8/(18X7D15.8))
1025*  1011 FORMAT(2X"EXTRAPOLATION VIOLATES CONSTRAINTS.
    *    STAY WITH GIVE
1026**+"ASE POINT.")
1027*  IF(IO.GT.1) WRITE(3,999)
1028*  IF(LLCON.EQ.0)CALL LIMS(P,IOUT)
1028*  IF(LLCON.EQ.1)CALL LIMS1(P,IOUT)
1029*  IF(IOUT.LE.0)GO TO 7
1030*  WRITE(3,1005)
1031*  6 STOP
1032*  7 ITTER=0
1033*  DO 8 I=1,NP
1034*  B1(I)=P(I)
1035*  B2(I)=P(I)
```

```
1036*   T(I)=P(I)
1037*  8 S(I)=STEP(I)*10.
1038*    IF(LLCON.EQ.0)CALL FOBJ(P,C1)
1038*    IF(LLCON.EQ.1)CALL OBJ(P,C1)
1039*   L=1
1040*   ICK=2
1041*   C1B=99999999999999999999999.0
1042*   IF(IO.LE.0)GO TO 11
1043*   WRITE(3,1001) ITTER
1044*   WRITE(3,1000)(P(J),J=1,NP),C1
1046*   11 DO 99 INRD=1,NRD
1047*   DO 12 I=1,NP
1048*   12 S(I)=S(I)/10.
1049*   IF(IO.GE.2) WRITE(3,1008)(S(J),J=1,NP)
1050*   20 IFAIL=0
1052*   IF(IO.EQ.3) WRITE(3,1002)
1053*   DO 30 I=1,NP
1054*   IC=0
1055*   21 P(I)=T(I)+S(I)
1056*   IC=IC+1
1057*   IF(LLCON.EQ.0)CALL LIMS(P,IOUT)
1057*   IF(LLCON.EQ.1)CALL LIMS1(P,IOUT)
1058*   IF(IOUT.EQ.0)GO TO 215
1059*   IF(IO.LT.3) GO TO 23
1060*   C2=99999999999999999999999.0
1061*   WRITE(3,1000)(P(J),J=1,NP),C2
1062*   GO TO 23
1063*   215 CONTINUE
1063*   IF(LLCON.EQ.0)CALL FOBJ(P,C2)
1063*   IF(LLCON.EQ.1)CALL OBJ(P,C2)
1064*   L=L+1
1065*   IF(IO.LT.3) GO TO 22
1066*   WRITE(3,1000)(P(J),J=1,NP),C2
1067*   22 IF(C1-C2)23,23,25
1068*   23 IF(IC.GE.2) GO TO 24
1069*   S(I)=-S(I)
1070*   GO TO 21
1071*   24 IFAIL=IFAIL+1
1072*   P(I)=T(I)
1073*   GO TO 30
1074*   25 T(I)=P(I)
1075*   C1=C2
1076*   30 CONTINUE
1077*   IF(IFAIL.EQ.NP) GO TO 50
1078*   IF(C1.LT.C1B) GO TO 32
1079*   C1=C1B
1080*   GO TO 60
1081*   32 DO 35I=1,NP
1082*   35 B2(I)=T(I)
1083*   IF(IO.LT.2) GO TO 40
```

```
1084*  ITTER=ITTER+1
1085*  WRITE(3,1001) ITTER
1086*  WRITE(3,1000)(P(J),J=1,NP),C1
1088*  40 ICK=1
1089*  SJ=1.0
1090*  DO 43II=1,10
1091*  DO 42 I=1,NP
1092*  42 T(I)=B2(I)+SJ*(B2(I)-B1(I))
1093*  SJ=SJ-.1
1094*  IF(LLCON.EQ.0)CALL LIMS(T,IOUT)
1094*  IF(LLCON.EQ.1)CALL LIMS1(T,IOUT)
1095*  IF(IOUT.LT.1) GO TO 45
1096*  43 CONTINUE
1097*  IF(IO.GE.2) WRITE(3,1011)
1098*  ICK=2
1099*  DO 44 I=1,NP
1100*  44 T(I)=B2(I)
1101*  GO TO 46
1102*  45 C1B=C1
1103*  IF(LLCON.EQ.0)CALL FOBJ(T,C1)
1103*  IF(LLCON.EQ.1)CALL OBJ(T,C1)
1104*  L=L+1
1105*  46 DO 47 I=1,NP
1106*  P(I)=T(I)
1107*  47 B1(I)=B2(I)
1108*  IF(IO.GE.2.AND.ICK.EQ.1) WRITE(3,1010)(T(J),J=1,NP),
       *  C1
1109*  GO TO 20
1110*  50 GO TO (55,90),ICK
1111*  55 C2=C1
1112*  C1=C1B
1113*  IF(C1-C2) 60,60,70
1114*  60 ICK=2
1115*  IF(IO.GE.2)WRITE(3,1009)
1116*  DO 65 I=1,NP
1117*  B1(I)=B2(I)
1118*  P(I)=B2(I)
1119*  65 T(I)=B2(I)
1120*  GO TO 20
1121*  70 C1=C2
1122*  DO 75 I=1,NP
1123*  75 B2(I)=T(I)
1124*  IF(IO.LT.2)GO TO 40
1125*  ITTER=ITTER+1
1126*  WRITE(3,1001) ITTER
1127*  WRITE(3,1000) (P(J),J=1,NP),C2
1128*  GO TO 40
1129*  90 DO 91 I=1,NP
1130*  91 T(I)=B2(I)
1131*  99 CONTINUE
```

```
1201*  COMMON/BLOCK/STA11,STA22,STA33,NN,QCOUNT,TPOP,LLCON
1201*  DIMENSION X(3)
1202*  IOUT=0
1203*  IF(X(1).LT.0.010)IOUT=1
1204*  IF(X(1).GT.0.10)IOUT=1
1205*  IF(X(2).LT.0.001)IOUT=1
1206*  IF(X(2).GT.0.010)IOUT=1
1207*  IF(X(3).LT.0.200)IOUT=1
1208*  IF(X(3).GT.1.000)IOUT=1
1209*  RETURN
1210*  END
1250*  SUBROUTINE OBJ(X,Y)
1251*  COMMON DGRID(42,72),IDATA(2,72),ISCALE(42),DET(300),
    *    OPTS(300),OPT
1251*+      SSQ(300),PMAT(9),ESTPR(6,50),ESTCR(6,50),
    *  PRIS(300),GAM0(30
1251*+      GAM1(300),GAM2(300)
1251*  COMMON/BLOCK/STA11,STA22,STA33,NN,QCOUNT,TPOP,LLCON
1251*  COMMON/BLOCKD/LO
1251*  DIMENSION X(3)
1252*  Y=GAM0(LO)+GAM1(LO)*X(1)+GAM2(LO)*X(1)*X(2)
1253*  RETURN
1254*  END
1270*  SUBROUTINE LIMS1(X,IOUT)
1271*  COMMON DGRID(42,72),IDATA(2,72),ISCALE(42),DET(300),
    *    OPTS(300),OPT
1271*+      SSQ(300),PMAT(9),ESTPR(6,50),ESTCR(6,50),
    *  PRIS(300),GAM0(30
1271*+      GAM1(300),GAM2(300)
1271*  COMMON/BLOCK/STA11,STA22,STA33,NN,QCOUNT,TPOP,LLCON
1271*  COMMON/BLOCKA/TPO(300),QQQ(300),C
1271*  COMMON/BLOCKB/SSS(300),DET1(300),XAM(300),XAM1(300),
    *    ZDIF(300)
1271*  COMMON/BLOCKD/LO
1271*  DIMENSION X(3)
1272*  IOUT=0
1273*  IF(X(2).GT.50)IOUT=1
1274*  IF(X(2).LE.0)IOUT=1
1275*  IF(X(1).GE.1.0)IOUT=1
1276*  IF(X(1).LE.0.0)IOUT=1
1277*  IF((X(1)*X(2)).GT.((6/7)*SSS(LO)*QQQ(LO)))IOUT=1
1278*  RETURN
1279*  END
1500*  SUBROUTINE CONTOU(NUMB,NCON,CLEV,FN,NX,NY,AXX,AXY,
    *    XLABEL,YLABEL,H
1501*  DIMENSION FN(60,60),CLEV(1),AXX(1),AXY(1)
1502*  DIMENSION HP(62),VP(62),NSYM(62),HC(1000),VC(1000),
    *    NSC(1000)
1503*  DIMENSION LST(7)
1504*  INTEGER XLABEL,YLABEL
```

```
1505*  DATA NCHARS/6/
1506*  DATA  (LST(I00000),I00000=1,  7)/1,0,4,11,5,2,3/
1507*  NS=NY
1508*  EPS=.0001
1509*  XOROBO=0.0
1510*  YOR=0.0
1511*  IM=1
1512*  IHVSW=2
1513*  NTRANS=1
1514*  ZAP= 1.00000000E+00
1515*  ILL=0
1516*  JM=NS
1517*  JX=NX
1518*  IX=NY
1519*  IF((8-NCON)*NCON) 70, 70, 10
1520*  10 IF((61-NY)*(NY-3)) 70, 70, 20
1521*  20 IF((61-NX)*(NX-3).LE.0) GO TO 70
1522*  GO TO 90
1523*  70 WRITE(6,80) NUMB
1524*  80 FORMAT("0ERROR IN CALL TO CONTOUR FOR GRID ",A6)
1525*  GO TO 1160
1526*  90 CONTINUE
1530*  WRITE(6,422)
1531*  422 FORMAT("1",///,"EXAMINING COLUMN BY COLUMN
      *  PRODUCED ",
1532*+"THESE ESTIMATED CONTOURS",///," I",4X,"X(I)",6X,"J-
      *  1",
1533*+4X,"Y(J-1)",4X,"F(I,J-1)   J",5X,"Y(J)",7X,"F(I,J)",
      *  7X,
1534*+"YHAT",10X,"CL",//)
1535*  IHVSW=1
1536*  DO 150 I=1,NX
1537*  ZAP=-ZAP
1538*  KPT=0
1539*  DO 170 NC=1,NCON
1540*  CL=CLEV(NC)
1541*  A3=FN(I,1)
1542*  YY=-1.
1543*  IF(A3.GT.CL) YY=1.
1544*  DO 160 J=2,NY
1545*  A4=FN(I,J)
1546*  IF((A4-CL)*YY.GT.0) GO TO 180
1547*  IF(A4.EQ.CL) GO TO 185
1548*  JJ=J-1
1549*  IF(ABS(CL-A3).GT.ABS(CL-A4)) JJ=J
1550*  ISW=0
1551*  IF(JJ.EQ.1) ISW=1
1552*  IF(JJ.EQ.NY) ISW=-1
1553*  B1=FN(I,JJ-1+ISW)
1554*  B2=FN(I,JJ+ISW)
```

```
1555*   B3=FN(I,JJ+1+ISW)
1556*   C1=AXY(JJ-1+ISW)
1557*   C2=AXY(JJ+ISW)
1558*   C3=AXY(JJ+1+ISW)
1559*   CALL APPR(CL,B1,B2,B3,C1,C2,C3,X1,X2)
1560*   XX=X2
1561*   IF(X1.GE.AXY(J-1)-EPS.AND.X1.LE.AXY(J)+EPS) XX=X1
1562*   WRITE(6,111) I,AXX(I),J-1,AXY(J-1),FN(I,J-1),J,AXY(J)
        *   ,FN(I,J),XX,
1563*   111 FORMAT(1X,I3,1X,E10.3,1X,I3,1X,E10.3,1X,E10.3,1X,
        *   I3,1X,
1564**+E10.3,1X,E12.4,1X,E12.4,1X,E12.4)
1565*   KPT=KPT+1
1566*   HP(KPT)=AXX(I)
1567*   VP(KPT)=XX
1568*   NSYM(KPT)=LST(NC)
1569*   A3=A4
1570*   YY=1.
1571*   IF(A3.LT.CL) YY=-1.
1572*   GO TO 160
1573*   185 KPT=KPT+1
1574*   HP(KPT)=AXX(I)
1575*   VP(KPT)=AXY(J)
1576*   NSYM(KPT)=LST(NC)
1577*   WRITE(6,112) I,AXX(I),J,AXY(J),FN(I,J),XX,CL
1578*   112 FORMAT(/,1X,I3,1X,E10.3,1X,26X,I3,2X,E10.4,2X,
        *   E10.4,1X,
1579**+E12.4,1X,E12.4)
1580*   YY=-YY
1581*   A3=A4
1582*   GO TO 160
1583*   180 A3=A4
1584*   160 CONTINUE
1585*   170 CONTINUE
1586*   IF(KPT.LE.40) GO TO 570
1587*   PRINT 520,NUMB
1588*   520 FORMAT("0TOO MANY POINTS IN A ROW OR COLUMN FOR
        *   GRID ",A6)
1589*   DO 540 IXXX=1,KPT
1590*   540 WRITE(6,560) KPT,IXXX,HP(IXXX),VP(IXXX),
        *   NSYM(IXXX)
1600*   560 FORMAT(" KPT,IXXX,HP,VR,NSYM ",2I10,2E20.10,I10)
1601*   570 CONTINUE
1602*   CALL REAR(KPT,HP,VP,NSYM,IHVSW,ZAP)
1603*   CALL STORP(KPT,ILL,HP,VP,HC,VC,NSC,NSYM)
1604*   150 CONTINUE
1605*   WRITE(6,423)
1606*   423 FORMAT("1",///,"EXAMINING BY ROWS PRODUCES
        *   THESE",
1607**+" ESTIMATED CONTOURS",//,"   J",4X,"Y(J)",6X,"I-1",4X,
        *   "X(I-1)",
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```
1608*+4X,"F(I-1,J)",2X,"I",5X,"X(I)",7X,"F(I,J)",9X,"XHAT",
1609*+9X,"CL",//)
1610* IHVSW=2
1611* DO 950 J=1,NY
1612* ZAP=-ZAP
1613* KPT=0
1614* DO 970 NC=1,NCON
1615* CL=CLEV(NC)
1616* A3=FN(1,J)
1617* YY=-1.
1618* IF(A3.GT.CL) YY=1.
1619* DO 960 I=2,NX
1620* A4=FN(I,J)
1621* IF((A4-CL)*YY.GT.0.) GO TO 960
1622* JJ=I-1
1623* IF(ABS(CL-A3).GT.ABS(CL-A4)) JJ=I
1624* IF(A4.EQ.CL) GO TO 985
1625* ISW=0
1626* IF(JJ.EQ.1) ISW=1
1627* IF(JJ.EQ.NY) ISW=-1
1628* B1=FN(JJ-1+ISW,J)
1629* B2=FN(JJ+ISW,J)
1630* B3=FN(JJ+1+ISW,J)
1631* C1=AXX(JJ-1+ISW)
1632* C2=AXX(JJ+ISW)
1633* C3=AXX(JJ+1+ISW)
1634* CALL APPR(CL,B1,B2,B3,C1,C2,C3,X1,X2)
1635* XX=X2
1636* IF(X1.GE.AXX(I-1)-EPS.AND.X1.LE.AXX(I)+EPS) XX=X1
1637* WRITE(6,111) J,AXY(J),I-1,AXX(I-1),FN(I-1,J),I,AXX(I)
1638*+ FN(I,J),XX,CL
1639* KPT=KPT+1
1640* HP(KPT)=XX
1641* VP(KPT)=AXY(J)
1642* NSYM(KPT)=LST(NC)
1643* A3=A4
1644* YY=1.
1645* IF(A3.LT.CL) YY=-1.
1646* GO TO 960
1647* 985 KPT=KPT+1
1648* HP(KPT)=AXX(I)
1649* VP(KPT)=AXY(J)
1650* NSYM(KPT)=LST(NC)
1651* WRITE(6,112) J,AXY(J),I,AXX(I),FN(I,J),XX,CL
1652* YY=-YY
1653* A3=A4
1654* GO TO 960
1655* 980 A3=A4
1656* 960 CONTINUE
```

```
1657*  970 CONTINUE
1658*  IF(KPT.LE.40) GO TO 3040
1659*  PRINT 3020,NUMB
1660*  3020 FORMAT("0TOO MANY POINTS IN A ROW OR COLUMN FOR
      *  GRID ",A6)
1661*  DO 3070 IXXX=1,KPT
1662*  3070 WRITE(6,560) KPT,IXXX,HP(IXXX),VP(IXXX),
      *  NSYM(IXXX)
1663*  3040 CONTINUE
1664*  CALL REAR(KPT,HP,VP,NSYM,IHVSW,ZAP)
1665*  CALL STORP(KPT,ILL,HP,VP,HC,VC,NSC,NSYM)
1666*  950 CONTINUE
1667*  IF(ILL.GT.0) GO TO 810
1668*  PRINT  780,NUMB
1669*  780 FORMAT("0NO CONTOURS FOUND OF SPECIFIED LEVELS
      *  FOR GRID ",A6)
1670*  GO TO 1160
1671*  810 CONTINUE
1672*  CALL SYMBOL(0.,2.,.21,NUMB,90.,6)
1673*  CALL PLOT(1.25,.25,-3)
1674*  CALL GRID(0.,0.,1.,1.,8,8)
1675*  DELTAX=(AXX(NX)-AXX(1))/8
1676*  DELTAY=(AXY(NY)-AXY(1))/8
1677*  CALL AXIS(0.,0.,XLABEL,-6,8.,0.,AXX(1),DELTAX)
1678*  CALL AXIS(0.,0.,YLABEL,6,8.,90.,AXY(1),DELTAY)
1679*  WRITE(6,127) DELTAX,DELTAY
1680*  127 FORMAT(//," SCALE FACTORS FOR FOR X AND Y ARE ",
      *  2E20.10,//)
1681*  WRITE(6,815)
1682*  815 FORMAT("1",///," SUMMARY OF POINTS PLOTTED",//,
1683*+11X,"X",12X,"Y",9X,"PLOT COORDINATES IN INCHES ",2X,
1684*+"SYM #",//)
1685*  DO 1070 I=1,ILL
1686*  XC=(HC(I)-AXX(1))/DELTAX
1687*  IF(XC.GT.36) WRITE(6,103) XC,I,HC(I),DELTAX
1688*  103 FORMAT(F10.3,I7,2F10.3," XC,I,HC(I),DELTAX ")
1689*  YC=(VC(I)-AXY(1))/DELTAY
1690*  WRITE(6,861) I,HC(I),VC(I),XC,YC,NSC(I)
1691*  861 FORMAT(1X,I3,2X,E12.4,1X,E12.4,3X,E12.4,1X,E12.4,
      *  3X,I3)
1692*  IF(XC.LT.0) GO TO 1101
1693*  CALL SYMBOL(XC,YC,HT,NSC(I),0.,-1)
1694*  1070 CONTINUE
1695*  1160 RETURN
1696*  1101 PRINT*," NEGATIVE XC VALUE "
1697*  GO TO 1160
1698*  END
1700*  SUBROUTINE REAR(KPT,HP,VP,NSYM,IHVSW,ZAP)
1701*  DIMENSION HP(1),VP(1),NSYM(1)
1702*  IF( KPT .LE.1) RETURN
```

```
1703*  KPTM1=KPT+1
1704*  DO 10 I=1,KPTM1
1705*  CHUMP=VP(I)
1706*  IF(IHVSW.GE.2) CHUMP=HP(I)
1707*  KK=I+1
1708*  DO 20 K=KK,KPT
1709*  QQ=VP(K)
1710*  IF(IHVSW.GE.2) QQ=HP(K)
1711*  IF((CHUMP-QQ)*ZAP.LE.0) GO TO 30
1712*  T1=HP(K)
1713*  T2=VP(K)
1714*  IT3=NSYM(K)
1714*  HP(K)=HP(I)
1714*  VP(K)=VP(I)
1714*  NSYM(K)=NSYM(I)
1715*  HP(I)=T1
1716*  VP(I)=T2
1717*  NSYM(I)=IT3
1718*  30 CONTINUE
1719*  20 CONTINUE
1720*  10 CONTINUE
1721*  RETURN
1722*  END
1723*  SUBROUTINE STORP(KPT,ILL,HP,VP,HC,VC,NSC,NSYM)
1724*  DIMENSION HP(1),VP(1),HC(1),VC(1),NSC(1),NSYM(1)
1725*  IF(KPT.LT.1) GO TO 10
1726*  DO 20 I1=1,KPT
1727*  IF(ILL.GE.1000) GO TO 30
1728*  ILL=ILL+1
1729*  HC(ILL)=HP(I1)
1730*  VC(ILL)=VP(I1)
1731*  20 NSC(ILL)=NSYM(I1)
1732*  10 RETURN
1733*  30 WRITE(6,100)
1734*  RETURN
1735*  100 FORMAT(/,"1000 POINTS - LIMITED TO THESE ",/)
1736*  END
1740*  SUBROUTINE APPR(CL,B1,B2,B3,C1,C2,C3,X1,X2)
1741*  B32=(B3-B2)/(C3-C2)
1742*  B21=(B2-B1)/(C2-C1)
1743*  B321=(B32-B21)/(C3-C1)
1744*  A=B321
1745*  B=B32-B321*(C3+C2)
1746*  C=B3-C3*B32+B321*C3*C2-CL
1747*  DIS=B**2-4.*A*C
1748*  IF(DIS.LT.0) GO TO 90
1749*  DIS=SQRT(DIS)
1750*  X1=(-B+DIS)/(2.*A)
1751*  X2=(-B-DIS)/(2.*A)
1752*  RETURN
1753*  90 WRITE(6,100)
1754*  100 FORMAT(" ERROR IN APPR, DIS NEGATIVE")
1755*  RETURN
1756*  END
```

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```
18133 *                                           D  D
18 44 *                                             D
17955 *
17866 *
17777 *
17688 *
17599 *                                      D
17510 *
17421 *                                  D    DDD
17332 *
17243 *                               D
17154 *                                    D
17 65 *                            D   D D
16976 *
16887 *
16798 *
16709 *
16620 *
16531 *                         D  DD
16442 *
16353 *                       DD
16264 *
16175 *
16 86 *                     D
15997 *                    D
15908 *                  D
15819 *
15730 *
15641 *              D        D
15552 *             D DDD  D
15463 *           D         D
15374 *
15285 *         D
15196 *
15107 *       D
15 18 *
14929 *
14840 *     D
14751 *
14662 *   D
14573 *
14484 *  D
       *******************************************
```

PLOT OF THE TOTAL OFFENSES TIME SERIES
FROM JANUARY,1974 THROUGH DECEMBER,1976

```
.018145  *
.018120  *
.017895  *
.017770  *
.017645  *
.017520  *
.017395  *
.017270  *
.017145  *
.017020  *
.016895  *
.016770  *
.016645  *
.016520  *
.016395  *
.016270  *
.016145  *
.016020  *
.015895  *
.015770  *
.015645  *
.015520  *
.015395  *
.015270  *
.015145  *
.015020  *
.014895  *
.014770  *
.014645  *
.014520  *
.014395  *
.014270  *
.014145  *
.014020  *
.013995  *
.013770  *                                            OOOOOOOOOOOOO
.013645  *                                    OOOOOO
.013520  *                            OOOOOO
.013395  *
.013270  *              OOOOOO
.013145  *
.013020  *  OOOOOO
         ********************************************************************
```

PLOT OF DETERRENT EFFECT TIME SERIES
FROM JANUARY,1974 THROUGH DECEMBER,1976

ACCORDING TO OUR MODEL.OPTIMIZATION OF POLICY VARIABLES

GIVES THE FOLLOWING RESULTS

PERIOD	PREVAILING AND OPTIMAL POLICY VARS.			
	Q	QSTAR	S	SSTAR
1/1974	.3061	.4601	1.67	1.11
2/1974	.3061	.4601	1.67	1.11
3/1974	.3061	.4601	1.67	1.11
4/1974	.3061	.4601	1.67	1.11
5/1974	.3061	.4601	1.67	1.11
6/1974	.3061	.4601	1.67	1.11
7/1974	.3061	.4601	1.67	1.11
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
9/1997	.3061	.4601	1.67	1.11
10/1997	.3061	.4601	1.67	1.11
11/1997	.3061	.4601	1.67	1.11
12/1997	.3061	.4601	1.67	1.11

STARTING WITH PERIOD 21

AND ENDING WITH PERIOD 288.

INCAPACITATION WAS

RESPONSIBLE FOR THE FOLLOWING PERCENTAGES OF CRIMES
SAVED
READ ACROSS FROM LEFT TO RIGHT

22.159	21.541	22.354	22.461	21.772	21.532
21.869	21.535	21.657	21.402	21.671	21.794
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
19.544	19.477	19.595	19.498	19.602	19.654
19.692	19.496	19.587	19.575		

THE VECTOR OF MONTHLY DETERRENT EFFECTS
FROM JANUARY,1974 UNTIL DECEMBER,1998
IS READ FROM LEFT TO RIGHT

.013020328152	.013020328163	.013020328201
.013020328211	.013020328216	.013020328219
.	.	.
.	.	.
.	.	.
.013446379680	.013446379651	.013446379632
.013446379621	.013446379615	.013446379612

THE RATIOS OF THE SAVINGS IN TOTAL OFFENSES FORECASTED
,(STEMMING FROM OPTIMIZATION).TC THE TOTAL OFFENSES
FORECASTED FOR PERIODS 1/75 ,TO, 12/98 BY THE MODEL
IS READ ACROSS FROM LEFT TO RIGHT

```
.02271    .02271    .02271    .02271    .02271    .02271
.02271    .02271    .02271    .C2271    .02271    .02271
.02271    .C2271    .02271    .C2271    .02271    .02271
.02271    .02271    .02271    .02271    .02271    .02271

  .         .         .         .         .         .
  .         .         .         .         .         .
  .         .         .         .         .         .

.C2271    .02271    .02271    .C2271    .02271    .02271
.02271    .02271    .02271    .C2271    .02271    .02271
.02271    .C2271    .02271    .02271    .02271    .02271
```

SAVINGS IN REPORTED CRIMES PEF PERIOD

THRU OPTIMIZATION

WERE AS FOLLOWS

```
360.      329.      332.      336.      341.      346.
351.      352.      354.      352.      352.      352.
351.      352.      359.      353.      365.      367.
371.      370.      375.      386.      374.      374.
385.      391.      387.      394.      389.      394.
396.      395.      395.      411.      408.      411.
423.      427.      423.      429.      425.      433.
430.      430.      430.      443.      440.      443.
455.      459.      455.      461.      457.      465.
462.      461.      462.      475.      472.      474.
486.      491.      486.      492.      488.      496.

  .         .         .         .         .         .
  .         .         .         .         .         .
  .         .         .         .         .         .

929.      934.      929.      935.      931.      939.
936.      936.      936.      949.      946.      949.
961.      965.      961.      967.      963.      971.
968.      968.      968.      981.      978.      981.
993.      997.      992.      999.      995.     1003.
1000.      999.      999.     1013.     1009.     1012.
1024.     1028.     1024.     1030.     1026.     1034.
1031.     1031.     1031.     1044.     1041.     1044.
1056.     1060.     1056.     1062.     1058.     1066.
1063.     1062.     1063.     1076.     1073.     1076.
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