THE DEVELOPMENT AND APPLICATION OF PROBABILITY DISTRIBUTIONS OF AIRCRAFT ENGINE REMOVALS

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ABSTRACT

The purpose of this study is twofold. First, it is desired to develop a functional relationship between the age of an aircraft engine and its probability of being removed for major overhaul maintenance or for minor (hospital) maintenance. Second, it is desired to use these functional relationships to forecast the number of future removals, and to test these forecasts against those currently in use.

Data were collected on the R-3350 engines of the Delta Airlines DC-7 fleet, from which the following functional relationships were developed:

$$f_i = 10^{-5}\left[(816.49)-(25.72)(i)+(1.04)(i)^2\right]$$

for $i = 1,2,3,4, \ldots, 59$

and $f_i = 1.00000 : i = 60$

$$h_i = 10^{-5}\left[(705.55)+(12.84)(i)\right]$$

for $i = 1,2,3,4, \ldots, 50$

and $h_i = 0.00000 ; i = 51, 52, 53, \ldots, 60$

In the above expressions, f_i is an estimate of the probability that an engine will receive a removal for major overhaul during the i^{th} age interval, and h_i is an

estimate of the probability that an engine will receive a removal for hospital maintenance during the i^{th} age interval.

The forecasting method developed in this study is compared to the method used by Delta Airlines for major overhaul maintenance. The standard deviations of the forecasting errors for these two methods are 3.0 and 3.6 respectively. Since the airline made no forecast of hospital maintenance removals, no comparison of forecasting methods could be made in this area.

It is recommended that this type of analysis is appropriate for any type of item which has a rate of failure dependent upon its age, on which sufficient records are kept to provide the necessary data, and whose value is sufficient to economically justify this analysis.

The recommendation is made that Air Force experience with the J-57 engines be utilized by the airlines in obtaining estimates of the functions for f_i and h_i for use in planning the forthcoming commercial jet airliner programs. It is further recommended that the results of this study be applied to an economic analysis of the optimum maximum time at which hospital maintenance will be prescribed.

CHAPTER I

GENERAL INTRODUCTION

In progressive organizations there has been a generation of ideas and effort directed towards efficient operation. The commercial airlines and military air forces of the world have been leaders among these progressive groups. They have had to work towards minimum cost and safe operation, two objectives not always compatible. Small margins of profit, increased equipment cost, and increased cost of maintenance have forced the airlines towards progressive action in operational efficiency. Since equipment and maintenance represent major costs, it seems that these areas are fruitful ones in which to realize savings. This study was concerned with aircraft engine management, the planning for the handling of engine inventories. One engine assembly, type R-3350, for the DC-7 aircraft costs about $105,000. One engine assembly, type J-57, for the commercial jet airliner costs about $250,000. It follows from the above estimates that sizable savings can be realized by purchasing an optimum quantity of spare engines and providing adequate overhaul facilities.

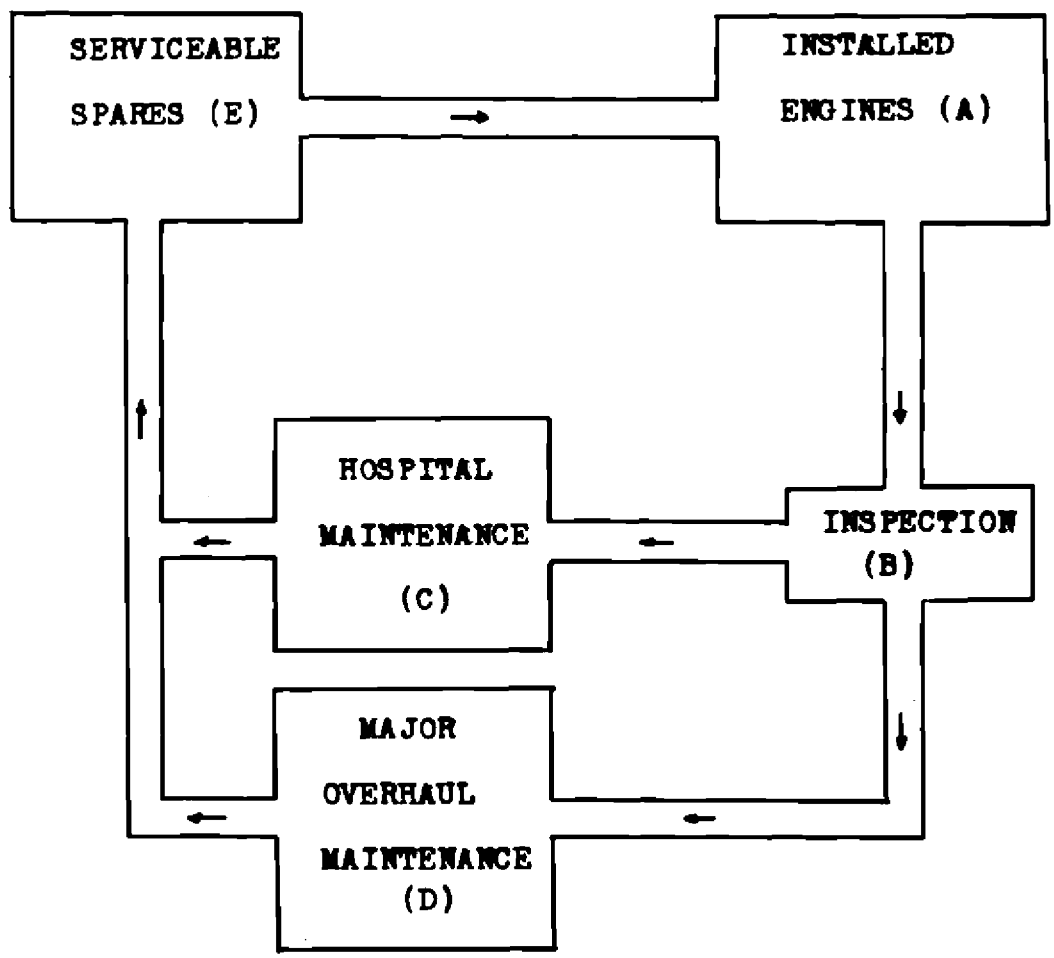

FIGURE 1. THE COMMERCIAL AIRLINES "ENGINE PIPELINE"

The "engine pipeline" of the commercial airlines is shown in Figure 1. The model of Figure 1 may be thought of as a queuing model, where engines are waiting to get into maintenance. There is: (a) an input process, which is the removal distribution, (b) a service policy which is determined by the results of the inspection, (c) a service rate which is the maintenance rate on each line, and (d) an output process. It is feasible that cost figures can be associated with: (a) increasing the rate of maintenance, (b) buying additional spares, and (c) not being able to fly an aircraft for the lack of a serviceable engine to replace a removal. With the above cost figures and mathematical models which could be developed for the system behavior, an optimum policy for the rate of maintenance and spare engine procurement can be approximated. This statement of the "system" is perhaps an over simplification, but it does lend itself to analysis.

This study was not a consideration of the entire system referenced above, but it is related to the system. Airlines have experienced extreme difficulty in forecasting maintenance requirements, which are directly related to engine removal rates.

The two basic problems that were investigated in this study were: (a) the development of mortality rates for the R-3350 aircraft engines of the DC-7 fleet of Delta Airlines,and (b) the application of these rates to installed inventories to forecast future removals.

CHAPTER II

LITERATURE SURVEY

Apparently the model of Figure 1 has not been considered in its entirety. In a paper, An Application of the Birth and Death Process to the Provision of Spare Machines, published by J. Taylor and R. R. P. Jackson[1] of the Analysis Branch of the British Overseas Airways, a similar engine system is considered. Taylor and Jackson are, however, concerned with only one type of maintenance. They make the following statement:

> While it is true to say that safety must condition all thinking of aircraft maintenance safety engineers, one of their main day to day problems is how to plan the work in the hanger so that unscheduled removals of unserviceables cause as little disturbance to routine operations as possible. Unfortunately, failures are nasty creatures whichever way they are looked at, and although some allowance for their incidence can be made in routine planning of work, they tend to be very will o' the wisp in behavior. In scientific or statistical circles, this behavior is know as random.

Based upon this randomness, differential probability expressions are developed and steady state expressions follow.

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[1]Numbers in parenthesis refer to references listed in the bibliography.

D. A. Stoddart, Jr. (2), in a Master of Science thesis, has described a method for forecasting automatic transmission failure in a specific make of automobile. His data are of three basic types: complete early failure data, monthly sales data (which gives an estimate of population age and size), and maximum miles per month traveled in terms of per cent of total sales. In many ways Stoddart's work is similar to other mortality studies; however, one very cumbersome point in his work is that his studies are extremely involved and estimates and assumptions must be made. This is, of course, partly due to the fact that there is no complete reporting system under which each automobile is reported on periodically.

In searching for work done in the area of failure data analysis it was found that there have been a number of studies concerned with the analysis of failure data. These studies all seem to be characterized by the following approach: A number of operating parts are installed into operation as new parts and then they are allowed to operate or age until they have all failed. Histograms of failures by age intervals and the corresponding density functions are developed. D. J. Davis (3) of the Rand Corporation developed three functions which he compares for three theories of failure, namely, the normal theory of failure, human mortality, and exponential theory of failure. His functions are probability density function, cumulative distribution functions, and conditional probability functions.

In the analysis of the data obtained during this study, there will be considered functions similar to those above although these are not the functions to be used in forecasting. It is primarily desired to arrive at a mortality rate or death rate function.

During the period of this study, engines were received, flown, overhauled, and reinstalled. The inventory from which the data were taken was a constantly changing inventory. Some engines received as many as ten overhauls while some of the engines received late in the program had received none. This situation is similar to the situation with which an actuary deals. It would be impractical to study a large group of say one thousand newly born babies until they all die and develop a mortality curve; instead, the actuaries study death by age interval and exposure by age interval, and thus arrive at a mortality rate. It would be impractical in this case to take a large number of engines and study them until they all fail. The actuarial approach will thus be used in the analysis of these data.

An analogy between a human being and an aircraft engine might be drawn as follows: a human is a collection of working parts which function together, any one of which might fail and cause hospitalization or death. An aircraft engine is a collection of working parts which work together, and again the failure of one part might cause a need for engine repair or replacement.

In 1950, the U. S. Air Force started working towards a method of engine performance information analysis using some of the principles of actuarial science. By 1954 the method had developed to the point where an average life factor called the Engine Life Expectancy (ELE) was being developed for each type of engine and these values were being used for forecasting future removals. These forecasts were better than forecasts previously obtained, yet they still left much to be desired. The method used by the U. S. Air Force (4) is explained in U. S. Air Force Technical Order 25-128.

An interesting method of estimating mortality rates is given in Rand Corporation Report RM-1807 (5). Reference is made to the Air Force "Actuarial Program" on engines, and it is assumed that crude failure rates are available. The method consists of fitting an exponential curve of the following type to the crude rates:

$$P\ (t) = (C)^{a + bt}$$

where P (t) equals the probability of failure in operating from t to (t + 1) hours; and C, a, and b, are parameters to be estimated from the crude rates.

George W. Brown (6) published a very interesting article on tumbler mortality. Brown's methods are similar in many ways to those employed in this work; however, Brown is concerned with the fitting of an incomplete gamma function distribution to his data, and this study is not.

CHAPTER III

DEVELOPMENT OF ENGINE MORTALITY RATES AND RELATED VALUES

Introduction

The engine "system" of the Commercial Airlines may be thought of in terms of the model shown in Figure 1. Installed engines at (A) are actively engaged in the flying program. Engines are removed for two primary reasons, either because they have reached maximum allowable operating time, which is established by the Civil Aeronautics Administration, or they have some type of malfunction. All removals are inspected at (B). The maximum operating time removals are fed into the major overhaul maintenance at (D). Engines removed because of some type of malfunction are inspected at (B) and sorted according to the seriousness of the malfunction. If the difficulty is minor in nature, the engines are sent to hospital maintenance (C), otherwise they go into major overhaul maintenance. Engines going into hospital maintenance are classified as hospital removals. Engines other than maximum time removals going into major overhaul maintenance are classified as failure removals. There are thus three classifications of removals: hospital, failure, and time removals.

Hospital removals go into hospital maintenance. Failure and time removals go into major overhaul maintenance. Major maintenance consists of complete teardown, inspection of each component part using X-ray, magnaflux, and other devices, re-assembly, and test. All defective parts are discarded or repaired, and replaced by new or newly repaired parts. An engine receiving a major overhaul leaves with zero hours or time since overhaul (TSO). Hospital maintenance is a small facility engaged in repair work not necessitating complete disassembly. An engine leaving minor repair leaves with the same TSO as it had when it entered hospital maintenance. Engines leaving both hospital and major maintenance go into a pool of serviceable spare engines (E). As engines are removed at (A) they are replaced by engines taken from (E). A single engine might pass through a loop twice in one month; however, a single engine will go through major overhaul on the average of once every four months. In this chapter, the problem treated is that of developing some mortality distributions for aircraft engines.

Hypothesis I

The probability of an aircraft engine being removed for major or hospital maintenance is a function of the time on the engine.

The Data

Data Collection

The data were collected on the R-3350 engines of Delta Airlines DC-7 Fleet. The first of these aircraft were received in the latter part of March, 1954. The data (see Appendix Sample 1) were taken for each month, March 1, 1954, to December 15, 1957. Each row corresponds to one particular engine. An engine will be referred to in general as the k^{th} engine. The rows are numbered, and on the first sheet of each set, the engine serial numbers are given. Data were collected on 105 engines for 46 months; however, the illustrative sample shown is of 28 engines for the first eight months of operation. Aircraft were received at various times all through the 46 month period up to November 26, 1957.

An explanation of the data sheet follows. There are three columns for each month (A, B, and C). In Column A are two figures, $O_{\alpha jk}/O_{\Omega jk}$. The symbol $O_{\alpha jk}$ is the number of hours on the k^{th} engine the first day of the j^{th} month, and $O_{\Omega jk}$ is the number of hours on the k^{th} engine the last day of the j^{th} month. "Hours on the engine" is defined to be the flying time on the engine since the last major overhaul or since new (TSO). All engines receiving a major overhaul leave with zero time. New engines have zero time. Column B is an extra column used for overflow from C and will be explained in connection with Column C which is an activity column. There are letters and two numerical figures in this column where entries occur. They are "code/day/TSO," where the codes are as follows:

ac--An incoming engine on a newly received aircraft, or a new spare being installed for the first time.

a---An engine coming out of serviceable spares and being installed on an aircraft.

F---A failure removal.

T---A time removal.

H---A hospital removal.

The day refers to the day of the month on which the activity occurred. The TSO is the time on the engine since last major overhaul at the time of the activity. When an engine changes status two or more times in one month Column B is used for the overflow. For example, an engine might be installed, hospital removed, and reinstalled in the same month. Arrows are used to indicate the order of the activity. In the Appendix Sample 1, engine Serial Number 548360 (k = 10) was received April 1, 1954, installed in the number two position of aircraft Number 703. On May 22, 1954, it received a hospital removal with 377 hours and was reinstalled on June 10 in aircraft Number 702, in the number one position with 377 hours. On August 4, 1954, it received a hospital removal with 825 hours and it was reinstalled in the number two position of aircraft number 703 on September 7. The engine received a failure removal on September 16, 1954, with 895 hours. It was installed in aircraft Number 701 in the number two position on October 7, 1954. The reason for showing the aircraft and position in the circle by each installation activity was that the Column A data were available by aircraft number and position and not by engine serial numbers.

Data Classification and Summarization

Classification of failures and hospital removals.--These removals were classified for each month (j) according to the age interval in which they occurred where age intervals are defined below. In Appendix Sample 2, a sample classification sheet of the removals occurring during months, 20, 21, 22, 23, 24, 25, 26, 27, and 28 for intervals 1-28 is given.

The maximum allowable operating time for the engines under study was 1200 hours. It was decided to use intervals of age having a twenty hour span. Thus, 0-20, 20-40, 60-80, 80-100, . . . , 1180-1200 are the intervals. They will be numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, . . . , i, . . . , 60 and referred to by their numbers; therefore, the 4th interval is the interval from 80-100 hours. An interval will be referred to in the general case as the i^{th} interval. The first month M_1 is March, 1954. The subscript j is given to months. Thus, M_j is the j^{th} month. Engines carry subscript k.

F_{ijk} is a failure removal of the k^{th} engine, occurring in the i^{th} interval during the j^{th} month. H_{ijk} is a hospital removal for the k^{th} engine occurring in the i^{th}

interval during the j^{th} month. The data were then summarized as follows:

$$F_{ij.} = \sum_{k=1}^{105} F_{ijk} \text{ ; and } H_{ij.} = \sum_{k=1}^{105} H_{ijk}$$

$$F_{i..} = \sum_{j=1}^{46} F_{ij.} \text{ ; and } H_{i..} = \sum_{j=1}^{46} H_{ij.}$$

The summarized data are shown in Table 1.

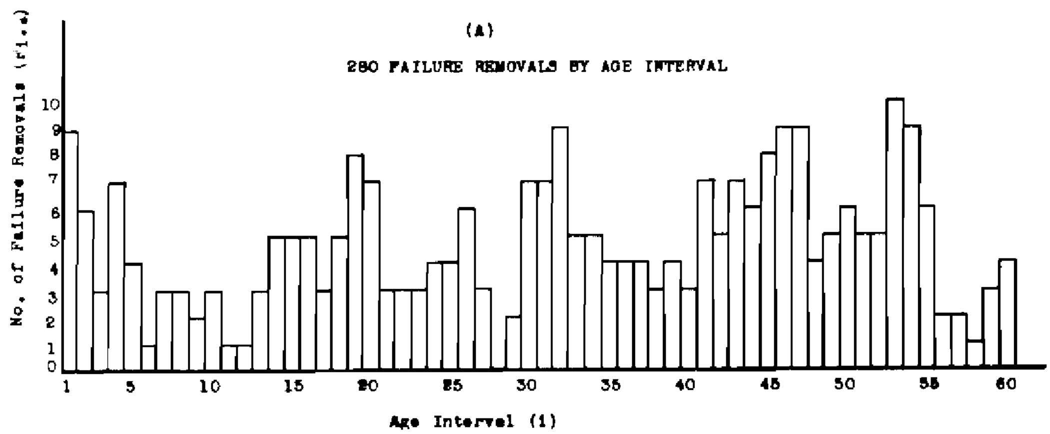
There were 280 failure removals and 242 hospital removals during the 46 month period. Histograms of $F_{1..}$ and $H_{1..}$ as functions of i are shown in Figure 2. The use of the twenty hour age intervals for this study will be explained in Chapter IV; however, it was decided that if the data were compressed into wider intervals (100 hours for example) which carry TSO subscripts of the first of the interval, that some pattern of failure might be recognized. This was done and the results are shown in Table 2.

For the 100 hour interval histograms, see Figures 3A and 3B. It is clear that no evident pattern of failure is present.

<u>Classification of flying hours and definition of exposure</u>.--Large summary work sheets were prepared having a row for each age interval (60 rows), and a column for each month (46 columns); thus each cell carries an ij subscript. The cells were large enough physically to contain many figures.

Table 1. The Number of Failure and Hospital Removals in Twenty Hour Age Intervals

i	F_i	H_i	i	F_i	H_i
1	9	9	31	7	4
2	6	8	32	9	4
3	3	6	33	5	5
4	7	1	34	5	5
5	4	4	35	4	5
6	1	6	36	4	10
7	3	3	37	4	4
8	3	5	38	3	1
9	2	4	39	4	5
10	3	3	40	3	5
11	1	1	41	7	1
12	1	10	42	5	6
13	3	5	43	7	4
14	5	5	44	6	7
15	5	4	45	8	2
16	5	5	46	9	9
17	3	2	47	9	7
18	5	9	48	4	4
19	8	4	49	5	2
20	7	4	50	6	3
21	3	0	51	5	0
22	3	9	52	5	0
23	3	4	53	10	1
24	4	4	54	9	1
25	4	4	55	6	0
26	6	5	56	2	0
27	3	2	57	2	0
28	0	4	58	1	0
29	2	7	59	3	0
30	7	9	60	4	0



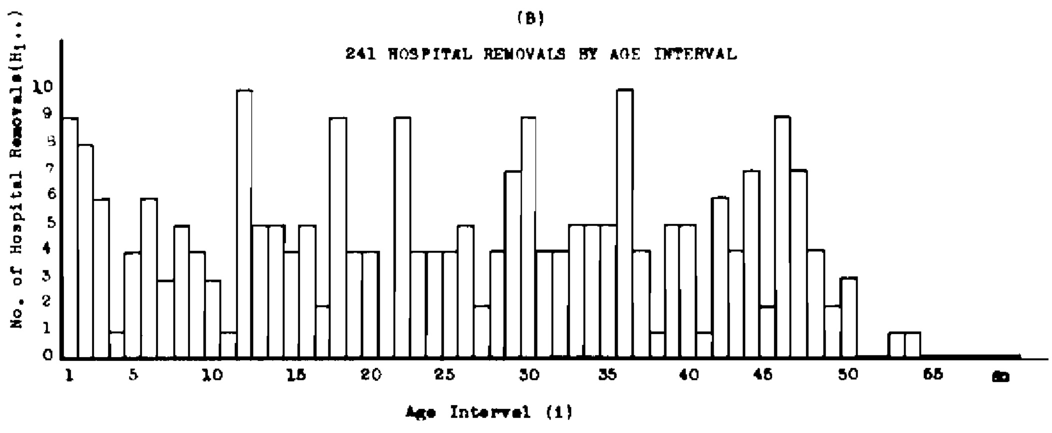


FIGURE 2. HISTOGRAMS OF REMOVALS
IN TWENTY HOUR AGE INTERVALS

The method of classification can best be explained by means of an illustration. The column A entry of the j^{th} month for the k^{th} engine $(O_{\alpha jk}/ O_{\Omega jk})$ is 22/401. The engine has 22 hours TSO at the first of the month j, and 401 hours at the last of the j^{th} month. On the large summary work sheet these (401 - 22) = 379 hours are distributed as follows: 18 is recorded in the second row (i = 2), and 20 is recorded in each interval up to i = 21, which has a 01 recorded. Thus, the flying hours for each engine, k, are recorded for each month, j, by each age interval, i. The total within the cell is

$$P_{ij.} = \sum_{k=1}^{105} P_{ijk}$$

where P_{ijk} is the number of hours flown in the i^{th} interval, during the j^{th} month by the k^{th} engine. At this point another definition is advisable. A unit of engine exposure is defined as twenty flying hours, and is denoted by

$$E_{ijk} = \frac{P_{ijk}}{20}$$

Exposures were summarized by listing by intervals and months as shown in the Appendix Sample 3 which is a summary for months 20-28 and interval 1-28. These data were summarized as follows:

$$E_{i..} = \sum_{j=1}^{46} \sum_{k=1}^{105} E_{ijk}$$

Table 2. The Number of Failure and Hospital Removals in 100 Hour Age Intervals

100 hour interval	TSO subscript	F_{TSO}	H_{TSO}
000-100	000	29	28
100-200	100	12	21
200-300	200	15	25
300-400	300	28	24
400-500	400	17	21
500-600	500	18	27
600-700	600	30	23
700-800	700	18	25
800-900	800	33	20
900-1000	900	33	25
1000-1100	1000	35	2
1100-1200	1100	12	0

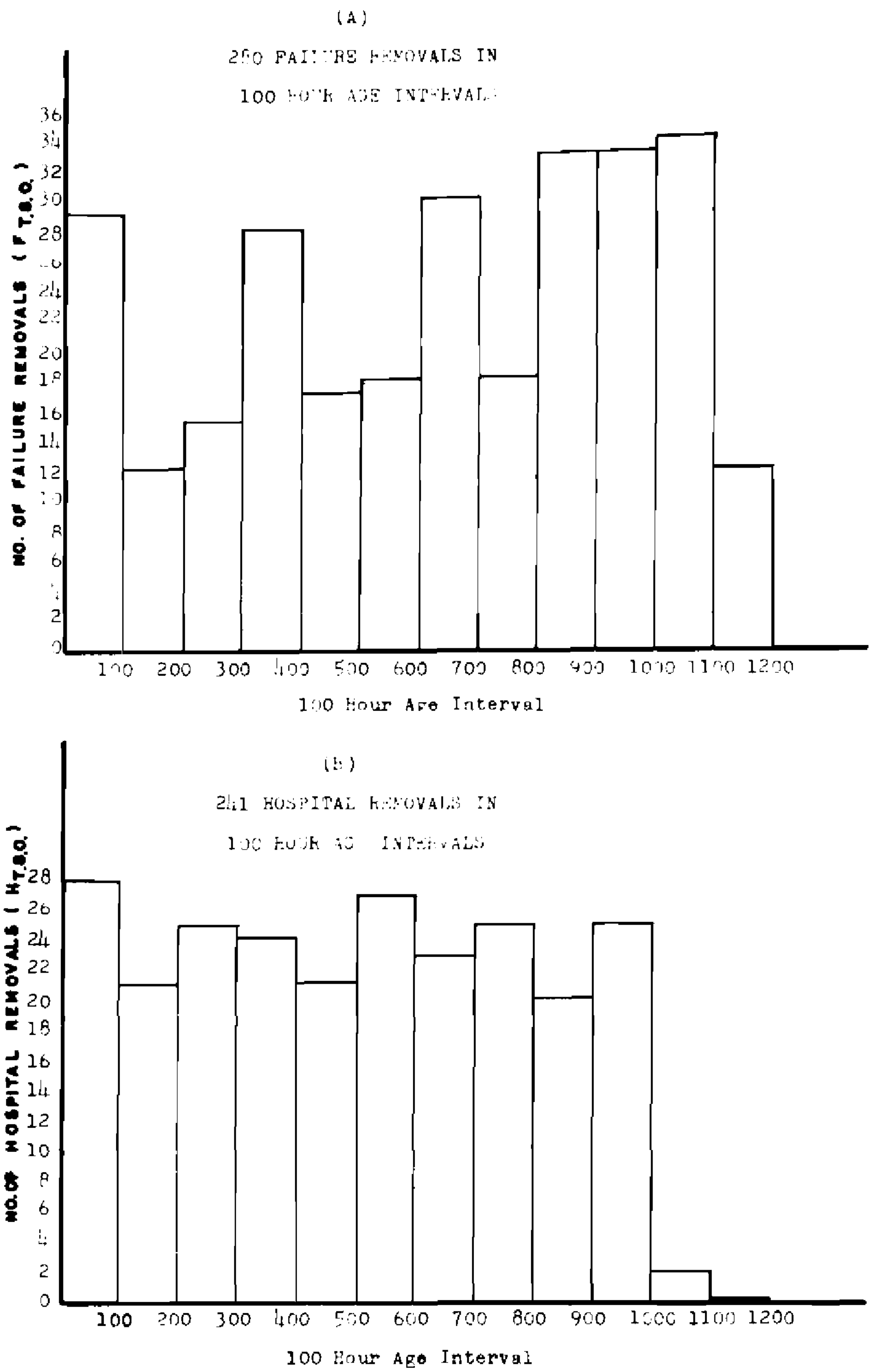


FIGURE 3. HISTOGRAMS OF REMOVALS IN
100 HOUR AGE INTERVALS

where $E_{i..}$ is the total number of exposures in the i^{th} interval for the 46 months and for all engines studied. The results are shown in Table 3, and a histogram of exposures by interval is shown in Figure 4.

Histograms of failure removals, hospital removals, time removals, and exposures by month are shown in Figures 5A, 5B, 5C, and 5D, respectively. The data were summarized by months as follows:

$$F_{.j.} = \sum_i \sum_k F_{ijk}$$

$$H_{.j.} = \sum_i \sum_k H_{ijk}$$

$$T_{j.} = \sum_k T_{jk}$$

$$E_{.j.} = \sum_i E_{ij.}$$

Analysis of Data

As was stated earlier, $F_{i..}$, $H_{i..}$, and $E_{i..}$ are the basic data for the analysis. A set of values called crude failure rates are calculated for all i, as follows:

$$f'_i = \frac{F_{i..}}{E_{i..}}$$

The units for f'_i are failures/exposure and f'_i is an estimate of the probability that an engine in interval i will fail before reaching interval $(i+1)$. The f'_i values, given in

Table 3. The Number of Exposures by Age Interval

i	E_i	i	E_i	i	E_i
1	631.30	21	510.30	41	391.65
2	627.40	22	502.35	42	379.80
3	614.00	23	496.00	43	376.50
4	606.70	24	489.95	44	366.10
5	589.90	25	487.00	45	362.75
6	595.95	26	477.50	46	343.85
7	592.95	27	472.45	47	335.75
8	580.15	28	471.00	48	327.80
9	587.15	29	459.55	49	319.35
10	579.85	30	460.20	50	303.40
11	570.10	31	454.25	51	290.35
12	568.20	32	441.20	52	278.90
13	564.55	33	434.40	53	268.55
14	563.60	34	428.90	54	252.05
15	558.85	35	427.08	55	174.50
16	537.10	36	422.70	56	102.70
17	536.95	37	418.68	57	98.30
18	538.60	38	402.70	58	91.85
19	528.10	39	405.55	59	76.45
20	516.15	40	396.55	60	29.21

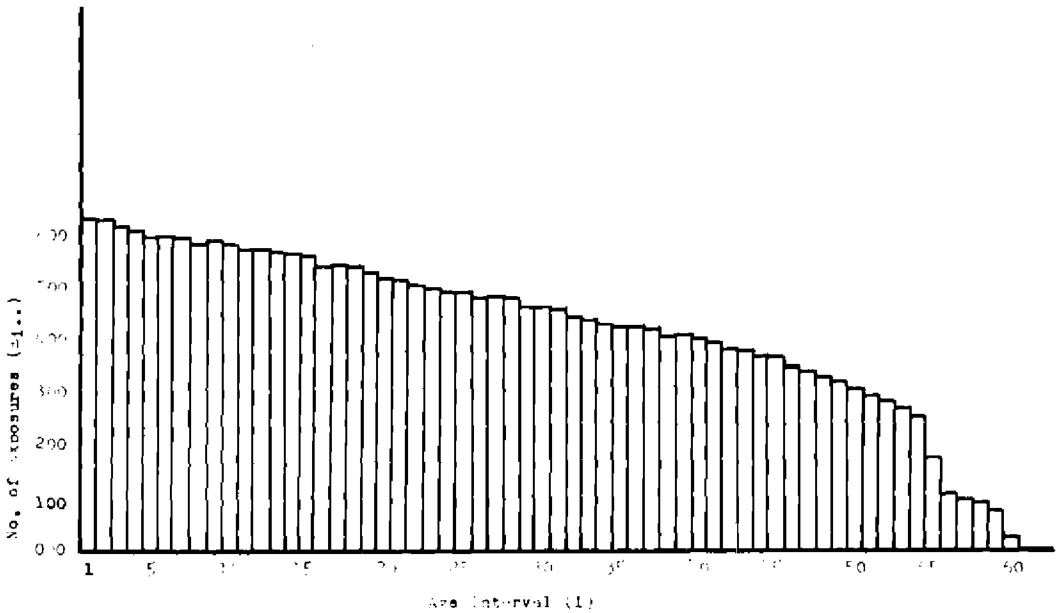


FIGURE 4. HISTOGRAM OF 25,715 EXPOSURES
IN TWENTY HOUR AGE INTERVALS

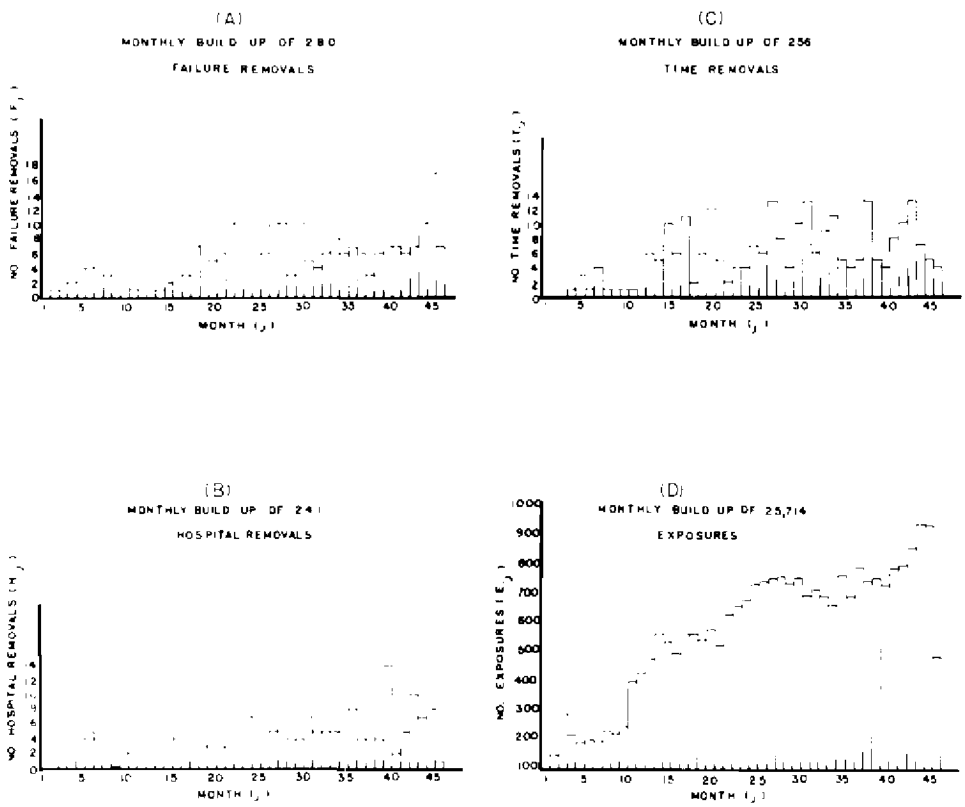


FIGURE 5. HISTOGRAMS OF REMOVALS AND EXPOSURES BY MONTHS

Table 4, are plotted as a frequency polygon in Figure 6A. In Table 4, the expression l'_i gives the per cent surviving; and for $i = 1$, l'_i is called the radix of the table and is selected to be 100.00 engines or 100.00% of any number of engines. These engines will be "aged" against the f'_i values, and the d'_i (number of engines failed) will be calculated in the following manner.

$$(f'_i)\ (l'_i) = d'_i$$

where

$$l'_i = 100; \quad i = 1$$

and

$$l'_i = (l'_{i-1} - d'_{i-1}) \ ; \ i = 2, 3, \ldots , 60$$

Since 100.00 was selected as the radix of the table; the l'_i column of Table 4 will be called the per cent surviving in the $(i-1^{th})$ interval, and d'_i the per cent failing in the i^{th} interval. It will be noted that there are a total of 60 intervals. The sum,

$$\sum_{i=1}^{59} d'_i = 59.81$$

is the per cent failed before maximum time. It will be noted that if there were no maximum time, the remaining 100.00-59.81 = 40.19 engines would eventually all fail. Now since both

Table 4. Crude Failure Rates and Calculated Values

i	f'_i	l'_i	d'_i	$D'_{i100.00}$	$D'_{59.81}$
1	0.01425	100.00	1.42	0.0142	0.0237
2	0.00956	98.58	1.05	0.0105	0.0176
3	0.00489	97.63	0.47	0.0047	0.0079
4	0.01153	97.16	1.12	0.0112	0.0187
5	0.00678	96.04	0.66	0.0066	0.0110
6	0.00168	95.38	0.16	0.0016	0.0027
7	0.00506	95.22	0.48	0.0048	0.0080
8	0.00517	94.74	0.50	0.0050	0.0084
9	0.00340	94.24	0.32	0.0032	0.0054
10	0.00517	93.92	0.49	0.0049	0.0082
11	0.00175	93.43	0.16	0.0016	0.0027
12	0.00176	93.27	0.17	0.0017	0.0028
13	0.00531	93.10	0.49	0.0049	0.0082
14	0.00887	92.61	0.82	0.0082	0.0137
15	0.00895	91.79	0.82	0.0082	0.0137
16	0.00931	90.97	0.85	0.0085	0.0142
17	0.00559	90.12	0.50	0.0050	0.0084
18	0.00928	89.62	0.83	0.0083	0.0139
19	0.01515	88.79	1.35	0.0135	0.0226
20	0.01356	87.44	1.18	0.0118	0.0197
21	0.00584	86.26	0.50	0.0050	0.0083
22	0.00597	85.76	0.52	0.0052	0.0087
23	0.00605	85.24	0.51	0.0051	0.0085
24	0.00816	84.73	0.70	0.0070	0.0117
25	0.00821	84.03	0.68	0.0068	0.0114
26	0.01256	83.35	1.05	0.0105	0.0176
27	0.00635	82.30	0.53	0.0053	0.0088
28	0.00000	81.77	0.00	0.0000	0.0000
29	0.00435	81.77	0.34	0.0034	0.0057
30	0.01521	81.43	1.24	0.0124	0.0207
31	0.01541	80.19	1.24	0.0124	0.0207
32	0.02041	78.95	1.61	0.0161	0.0269
33	0.01151	77.34	0.89	0.0089	0.0149
34	0.01166	76.45	0.89	0.0089	0.0149
35	0.00937	75.56	0.71	0.0071	0.0119
36	0.00946	74.85	0.71	0.0071	0.0119
37	0.00955	74.14	0.72	0.0072	0.0120
38	0.00745	73.42	0.54	0.0054	0.0090
39	0.00986	72.88	0.72	0.0072	0.0120
40	0.00756	72.16	0.55	0.0055	0.0092

Table 4. Crude Failure Rates and Calculated Values (Continued)

i	f'_i	l'_i	d'_i	$D'_{i100.00}$	$D'_{i59.81}$
41	0.01787	71.61	1.29	0.0129	0.0216
42	0.01316	70.32	0.92	0.0092	0.0154
43	0.01859	69.40	1.29	0.0129	0.0216
44	0.01639	68.11	1.12	0.0112	0.0187
45	0.02205	66.99	1.47	0.0147	0.0246
46	0.02617	65.52	1.72	0.0172	0.0287
47	0.02680	63.80	1.71	0.0171	0.0287
48	0.01220	62.09	0.76	0.0076	0.0127
49	0.01566	61.33	0.96	0.0096	0.0161
50	0.01978	60.37	1.19	0.0119	0.0199
51	0.01722	59.18	1.02	0.0102	0.0171
52	0.01793	58.16	1.04	0.0104	0.0174
53	0.03724	57.12	2.13	0.0213	0.0356
54	0.03571	54.99	1.96	0.0196	0.0327
55	0.03438	53.03	1.83	0.0183	0.0306
56	0.01947	51.20	0.99	0.0099	0.0166
57	0.02035	50.21	1.02	0.0102	0.0171
58	0.01099	49.19	0.54	0.0053	0.0090
59	0.03924	48.65	1.91	0.0191	0.0319
60	0.13793	46.74	6.45	0.4674	0.1078
			59.81	1.0000	1.0000

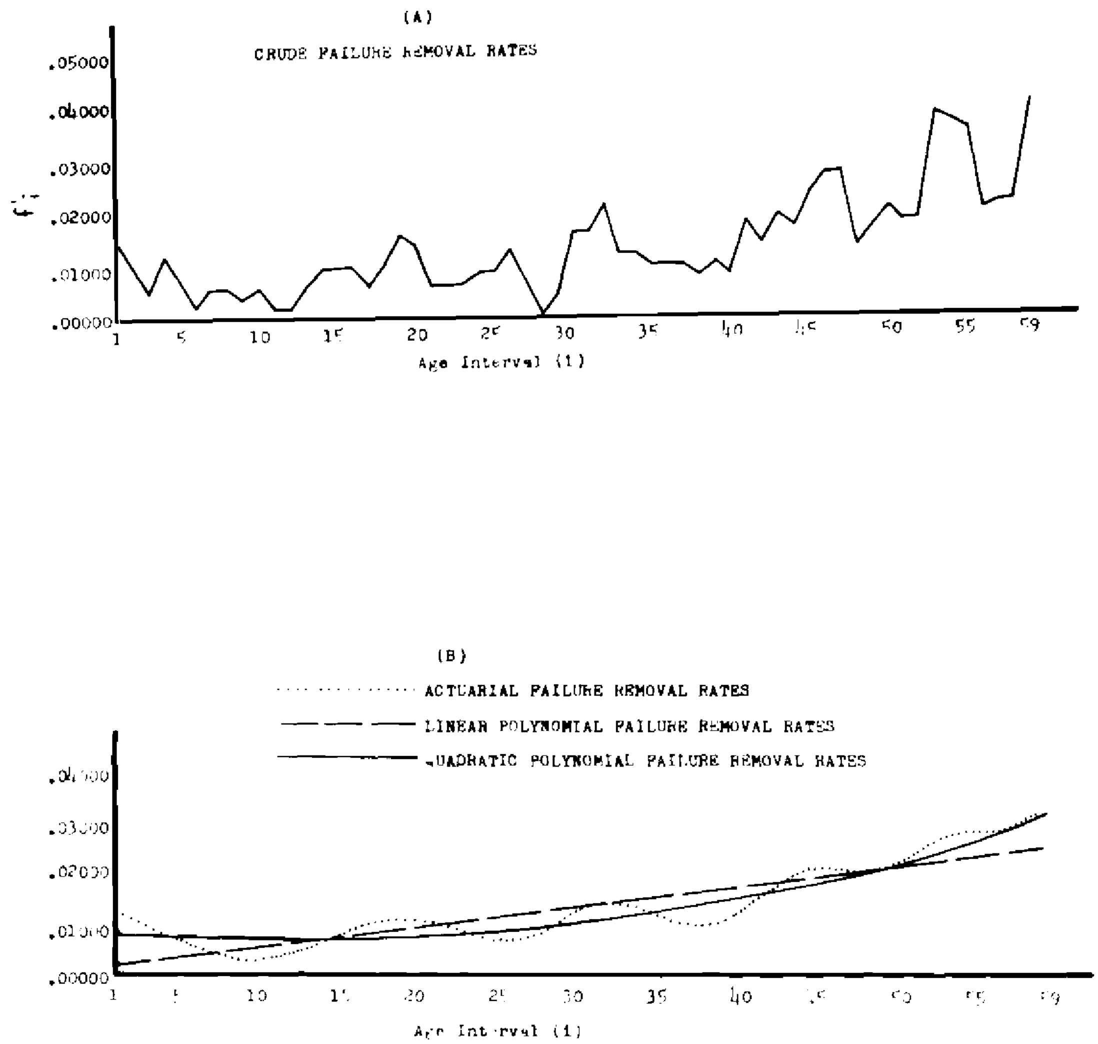


FIGURE 6. ENGINE FAILURE RATES

failure and time removals are needed in forecasting for major overhaul maintenance requirements, it can be assumed that the 40.29 engines removed at the end of interval 60 are considered as removals in the 60th interval and added to d'_{60}. This can be accomplished by adjusting the f'_{60} to equal 1.00000 (or a removal rate of one in the last interval). After making this adjustment the d'_i column shows the distribution of removals of 100.00 engines which are flown until they all receive major overhaul. The expression $\frac{d'_i}{100.00}$ is a density function of these removals and will be called $D'_{i100.00}$. The $D'_{i100.00}$ values are listed in Table 4. If we consider $\frac{d'_i}{59.81}$ as the density function of failure removals only, the results are shown in Table 4 as $D'_{i59.81}$, where $i < 60$.

At this point it would be a simple task to plot $D'_{i100.00}$ or $D'_{i59.81}$ as a function of i and develop a mathematical formula to fit the D'_i values which would be used to develop the same type of functions developed by Davis; however, it is the failure rates that are to be used in Chapter IV of this thesis; so consider again the graph of f'_i of Figure 6A. It will be noted that the f'_i values are irregular in nature with considerable variance. It is interesting to note that human mortality rates have rather large irregularities. Morton D. Miller (7) of the Actuarial

Society of America states "Despite the elimination of factors which might in themselves introduce irregularities, a series of observed mortality rates always remains irregular." It is desired to obtain an adequate representation of the basic pattern which f'_i is believed to follow. The f'_i values may be thought of as having two components $f'_i = f_i + e_i$. The first is a smooth underlying series f_i, and the second e_i is a random array of positive and negative terms which account for the variation in f'_i. Two methods of obtaining f_i were employed in this study.

The first method is an adjusted average or linear compound method. Consider the i^{th}, f'_i, then

$$f_i = a_8 f'_{i-8} + \dots + a_1 f'_{i-1} + a_0 f'_i + a_1 f'_{i+1} + \dots a_8 f'_{i+8}$$

The a's are used which satisfy two assumptions. The first one as stated by Miller is "Over a limited range, most regular series met with by the actuary may be closely approximated by a third degree polynomial, and this condition will ordinarily be made to reproduce such a polynomial without modification." The second assumption is that the third degree polynomial fits the data from which it came. In this study, the seventeen point formula was used which gives:

$a_0 = .1892$

$a_1 = .1764$

$a_2 = .1411$

$a_3 = .0923$

$a_4 = .0421$

$a_5 = .0025$

$a_6 = -.0186$

$a_7 = -.0204$

$a_8 = -.0100$

For $i < 9$ and for $i > 50$ the formula given above is not satisfactory since there must be eight f' values on each side of the f'_i being smoothed. This difficulty was overcome by letting $f'_1 = f'_0 = f'_{-1} = f'_{-2} = \ldots f'_{-8}$ and $f'_{59} = f'_{60} = f'_{61} = f'_{62} = \ldots f'_{68}$. The results of this method are labeled actuarial rates and are shown graphically in Figure 6B. The e_i's are classified by order of magnitude and shown graphically in Figure 7. It is noted that this histogram appears normal.

The second method used to obtain f_i is the method of least squares. Consider the following:

$$f_i = b_0 + b_1 (i) + b_2 (i^2) + \ldots + e_i$$

then

$$e_i = f_i - \left[b_0 + b_1 (i) + b_2 (i^2) \ldots\right]$$

and

$$Q = \sum_i (e_i)^2$$

The method of least squares gives estimates of the values for the b_i's, which minimize Q. An IBM 650 Computer Routine was employed to give the best fitting polynomials of degree 1, 2, 3, 4, and 5. It was found that there was considerable difference in the linear and the quadratic fit; however, the 3^{rd}, 4^{th}, and 5^{th} degree polynomials vary only slightly from the quadratic. The graphs of the linear and quadratic are shown in Figure 6B. The f'_{60} value was handled as a separate point and was not considered in the fitting routine. The f'_{60} value was adjusted to equal to 1.00000. The best fitting polynomials are given for $i < 60$ as follows:

(a) $$f_i = 10^{-5}\left[185.157 + 36.383(i)\right]$$

(b) $$f_i = 10^{-5}\left[816.492 - 25.715(i) + 1.035(i)^2\right]$$

(c) $$f_i = 10^{-5}\left[886.025 - 39.064(i) + 1.586(i)^2 - 0.00613(i)^3\right]$$

(d) $$f_i = 10^{-5}\left[947.384 - 58.082(i) + 2.985(i)^2 - 0.042(i)^3 + 0.0003(i)^4\right]$$

(e) $$f_i = 10^{-5}\left[1562.191 - 333.087(i) + 33.999(i)^2 - 1.403(i)^3 + 0.026(i)^4 - 0.00017(i)^5\right]$$

The problem now arises as to which set of f_i values to accept, the set of actuarial rates, the f_i from the linear or the f_i from the quadratic polynomial. It was decided that higher than second degree polynomials will not be considered since they vary only slightly from the quadratic.

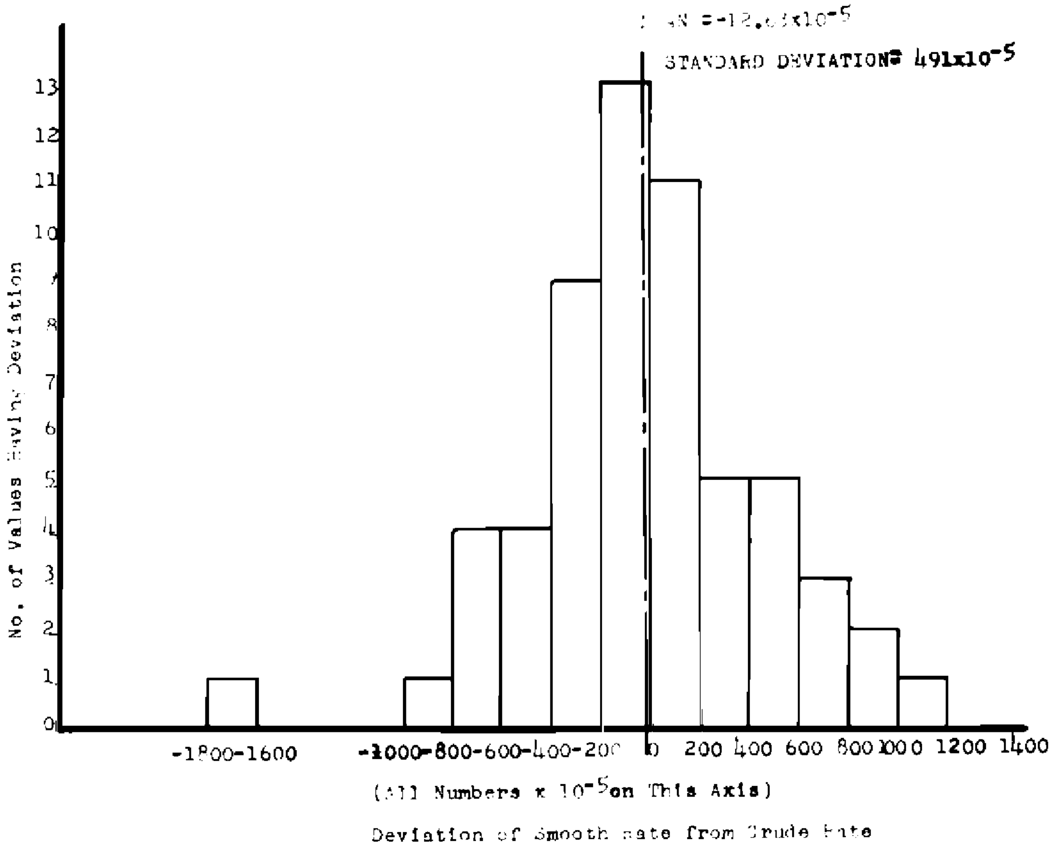


FIGURE 7. DISTRIBUTION OF FAILURE RATE SMOOTHING ERROR BY ACTUARIAL METHOD

In thinking of mortality, it is general to think of relatively high infant mortality, decreasing young adult mortality, and increasing middle age and old age mortality. This of course will rule out the linear fit and leaves both the actuarial rates and the quadratic for consideration. Now the problem is whether or not it is desired to accept the waves in the actuarial rate curves as significant. From conversations with responsible maintenance personnel at the Delta Airlines it was found that there was no inspection cycle coinciding with these humps in the actuarial curve and that these men could not justify the humps. After considerable deliberation it was decided to accept the quadratic fit which behaves according to the general mortality statements made above. The f_i values from the quadratic fit are listed in Table 5. Calculations for l_i, d_i, and D_{i100}, and $D_{i53.040}$ are made in the manner described earlier for l'_i, d'_i, $D'_{i100.00}$ and $D'_{i59.81}$. The results are shown in Table 5.

In addition some cumulative values for $D_{i100.00}$ and $D_{i53.040}$ are calculated as follows:

$$D_{r100.00} = \sum_{i=1}^{r} D_{i100.00} \ ; \text{ for } r \leq 60,$$

and

$$D_{r53.040} = \sum_{i=1}^{r} D_{i53.040} \text{ for } r < 60$$

It is felt that the following comments regarding the handling of interval 60, and the functions $D_{i100.00}$, $D_{i53.040}$, and their cumulatives should be made.

First, it is pointed out that when the data were collected, it was doubtful whether the removals classified as failure removals in the last interval were actually failure removals or should have been classified as time removals. Second, the writer assumes, by making $f_{60} = 1.00000$, and treating all d_{60} as time removals, that all failure removals occur before interval 60 and all engines remaining at the end of $i = 59$ are removed as time removals during the 60^{th} interval. If interest is in a density function of major overhaul removals (i.e., maximum time and failure removals), it can be obtained by dividing d_i by 100.00 for all values of i since all 100.00 engines either fail or receive a time removal. This is done for all values of i, $i = 1$ to $i = 60$, and is called $D_{i100.00}$. If however, it is desired to obtain a density function of failure removals only,this can be accomplished by dividing d_i by 53.040 for all values of i, $i = 1$ to $i = 59$. This does not account for any "failures" in the last interval. The 46.950 engines surviving $i = 59$ are all assumed to receive a time removal in $i = 60$. Graphs of l_i, $D_{i100.00}$, $D_{i53.040}$, $D_{r100.00}$, and $D_{r53.040}$ are shown in Figures 8A, 8B, 8C, 9A, and 9B, respectively.

Table 5. Engine Failure Rates and Calculated Values

i	f_i	l_i	d_i	D_{i1100}	$D_{i153.04}$	$\sum_{i=1}^{r} D_{i1100}$	$\sum_{i=1}^{r} D_{i153.04}$
1	.00792	100.000	.792	.00792	.01493	.00792	.01493
2	.00769	99.208	.763	.00763	.01439	.01555	.02932
3	.00748	98.445	.736	.00736	.01388	.02291	.04320
4	.00730	97.709	.713	.00713	.01344	.03004	.05664
5	.00713	96.995	.692	.00692	.01305	.03696	.06969
6	.00699	96.304	.673	.00673	.01269	.04369	.08238
7	.00687	95.631	.657	.00657	.01238	.05026	.09476
8	.00677	94.974	.643	.00643	.01212	.05669	.10688
9	.00669	94.331	.631	.00631	.01189	.06300	.11877
10	.00662	93.700	.620	.00620	.01169	.06920	.13046
11	.00662	93.079	.612	.00612	.01154	.07532	.14200
12	.00656	92.467	.607	.00607	.01144	.08139	.15344
13	.00656	91.860	.607	.00603	.01137	.08742	.16481
14	.00658	91.258	.600	.00600	.01131	.09342	.17612
15	.00663	90.056	.601	.00601	.01133	.09943	.18745
16	.00669	90.657	.603	.00603	.01137	.10546	.19882
17	.00677	89.454	.606	.00606	.01143	.11152	.21025
18	.00687	88.848	.610	.00610	.01150	.11762	.22175
19	.00700	88.238	.618	.00618	.01165	.12380	.23340
20	.00714	87.620	.626	.00626	.01180	.13006	.24520
21	.00731	86.994	.636	.00636	.01199	.13642	.25719
22	.00749	86.358	.647	.00647	.01220	.14289	.26939
23	.00770	85.712	.660	.00660	.01244	.14949	.28183
24	.00793	85.051	.674	.00674	.01271	.15623	.29454
25	.00817	84.377	.689	.00689	.01299	.16312	.30753
26	.00844	83.688	.706	.00706	.01331	.17018	.32084
27	.00873	82.981	.724	.00724	.01365	.17742	.33449
28	.00904	82.257	.744	.00744	.01403	.18486	.34852
29	.00937	81.513	.764	.00764	.01440	.19250	.36292
30	.00972	80.750	.785	.00785	.01480	.20035	.37772
31	.01009	79.968	.807	.00807	.01521	.20842	.39293
32	.01048	79.157	.830	.00830	.01565	.21672	.40858
33	.01090	78.328	.854	.00854	.01610	.22526	.42468
34	.01133	77.457	.878	.00878	.01665	.23404	.44133
35	.01178	76.597	.902	.00902	.01701	.24306	.45834
36	.01226	75.694	.928	.00928	.01750	.25234	.47584
37	.01275	74.766	.953	.00953	.01797	.26187	.49381
38	.01327	73.813	.979	.00979	.01846	.27166	.51227

Table 5. Engine Failure Rates and Calculated Values (Continued)

i	f_i	l_i	d_i	D_{i100}	$D_{i53.04}$	$\sum_{i=1}^{r} D_{i100}$	$\sum_{i=1}^{r} D_{i53.04}$
39	.01380	72.834	1.005	.01005	.01895	.28171	.53122
40	.01436	71.829	1.031	.01031	.01944	.29202	.55066
41	.01494	70.797	1.058	.01058	.01995	.30260	.57061
42	.01554	69.739	1.084	.01084	.02044	.31344	.59105
43	.01615	68.656	1.108	.01108	.02089	.32452	.61194
44	.01679	67.547	1.134	.01134	.02138	.33586	.63332
45	.01745	66.413	1.159	.01159	.02185	.34745	.65517
46	.01813	65.254	1.183	.01183	.02230	.35928	.67747
47	.01883	64.071	1.206	.01206	.02274	.37134	.70021
48	.01955	62.864	1.229	.01229	.02317	.38363	.72338
49	.02029	61.635	1.251	.01251	.02359	.39614	.74697
50	.02105	60.385	1.271	.01271	.02396	.40885	.77093
51	.02184	59.114	1.291	.01291	.02434	.42176	.79527
52	.02265	57.823	1.310	.01310	.02470	.43486	.81997
53	.02347	56.513	1.326	.01326	.02500	.44812	.84497
54	.02432	55.187	1.342	.01342	.02530	.46153	.87027
55	.02518	53.844	1.356	.01356	.02552	.47510	.89579
56	.02607	52.489	1.368	.01368	.02579	.48878	.92158
57	.02697	51.120	1.378	.01378	.02595	.50256	.94753
58	.02790	49.741	1.388	.01388	.02615	.51644	.97368
59	.02885	48.354	1.396	.01396	.02632	.53040	1.00000
60	(1.00000)	46.960	46.960	.46960		1.00000	
				1.00000	1.00000		

It is to be emphasized that this study is not particularly concerned with any set of values other than mortality rates f_i, and hospital rates h_i which will be discussed in detail at this time.

Hospital removal rates similar to those for major overhaul were calculated as follows:

$$h'_i = \frac{H_{i..}}{E_{i..}}$$

A listing of h'_i values is shown in Table 6. A graph of h'_i values is shown in Figure 10A. It will be noticed that there is considerable variation in h'_i which is an estimate of the probability that an engine in the i^{th} interval will receive hospital removal before reaching interval $(i + 1)$. It is believed that these probability values do not follow such an erratic pattern by nature, but that there is some underlying h_i which are more "well behaved".

The same two general methods that were used to obtain f_i from f'_i were employed to obtain h_i from h'_i. The results obtained from the seventeen point smoothing formula are labeled actuarial hospital rates and are shown graphically in Figure 10B. The distribution of error, e_i, is shown in Figure 11.

Best fitting polynomials of degrees one through five were fitted to the data using the least squares routine.

Graphs of the first two of these polynomials are shown in Figure 10B. The higher degrees differ by a negligible amount from the linear, so the linear function was selected. The polynomials are as follows:

(a) $h_i = 10^{-5}\left[705.554 + 12.839(i)\right]$

(b) $h_i = 10^{-5}\left[864.837 - 5.899(i) + 0.375(i)^2\right]$

(c) $h_i = 10^{-5}\left[1059.609 - 50.412(i) + 2.578(i)^2 - 0.029(i)^3\right]$

(d) $h_i = 10^{-5}\left[1223.575 - 10.547(i) + 7.864(i)^2 - 0.193(i)^3 + 0.002(i)^4\right]$

(e) $h_i = 10^{-5}\left[1582.468 - 299.173(i) + 33.215(i)^2 - 1.524(i)^3 + 0.031(i)^4 - 0.0002(i)^5\right]$

The values of h_i calculated from the linear polynomial are given in Table 7. It is noted that the curves were fitted to the first fifty intervals only, since no engine having more than one thousand hours receives hospital maintenance unless some emergency situation exists.

Calculations similar to those for the failure rates are made. The values are: h_i = an estimate of the probability that an engine in the i^{th} interval fails before reaching the $(i + 1)^{th}$ interval. S_i = the per cent surviving at the beginning of the i^{th} interval. $R_{i40.512}$ = the density function of the 40.512 removals, and

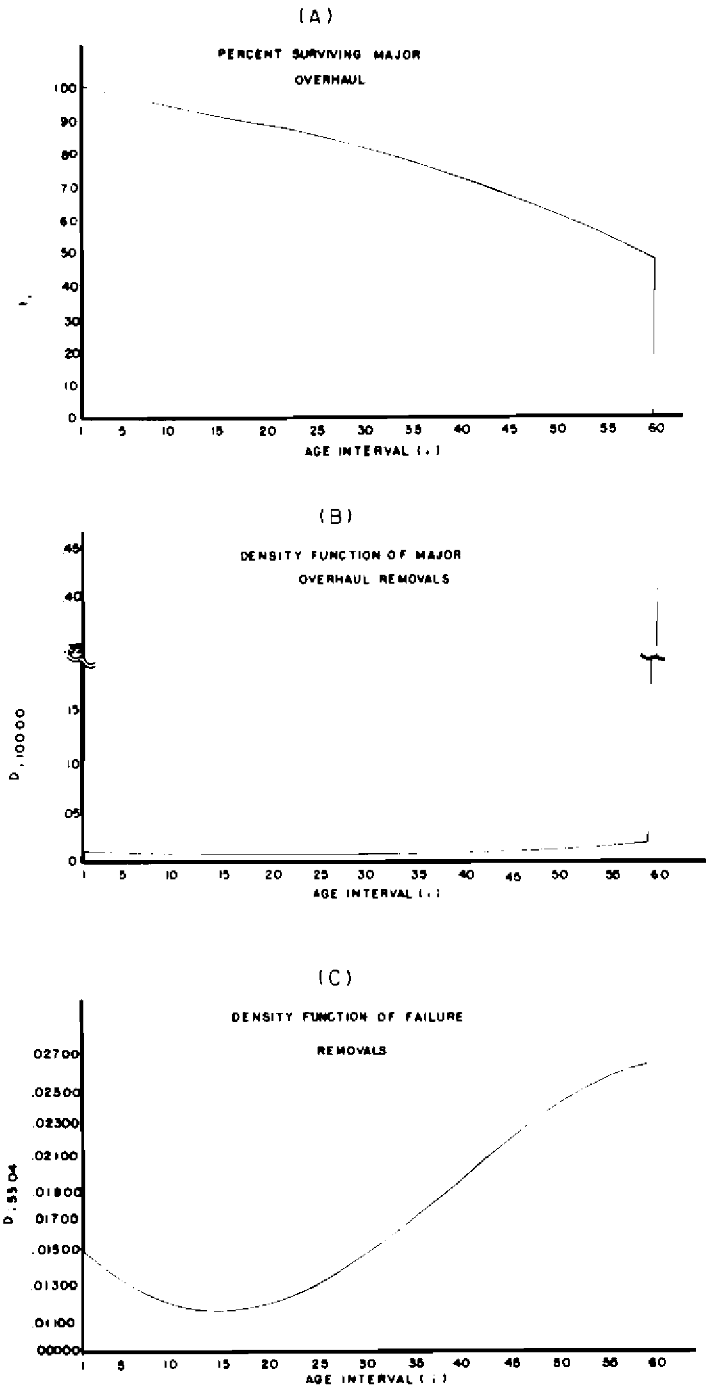


FIGURE 8. FUNCTIONS CALCULATED FROM THE MAJOR OVERHAUL REMOVAL RATES

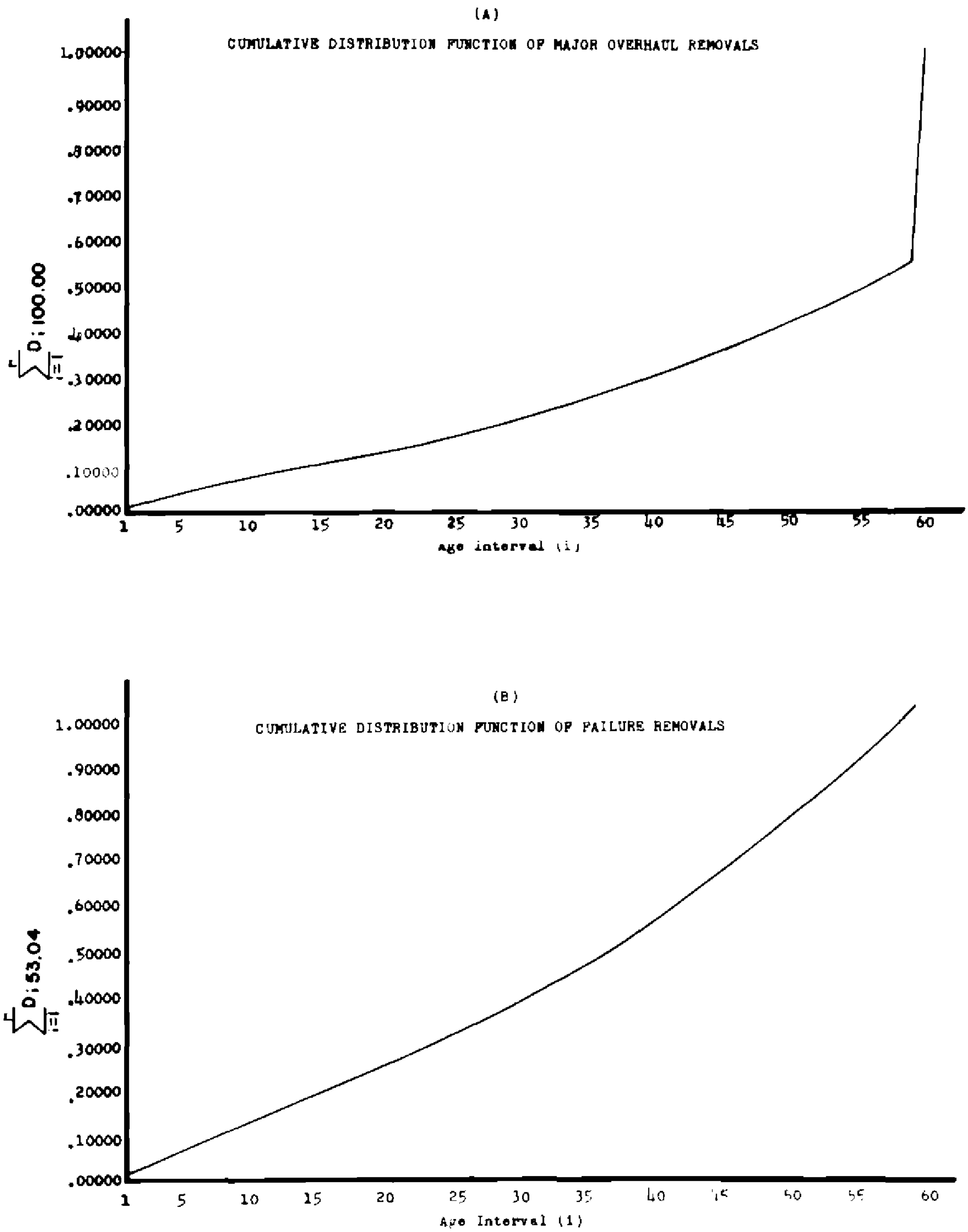


FIGURE 9. FUNCTIONS CALCULATED FROM THE MAJOR OVERHAUL REMOVAL RATES

Table 6. Crude Hospital Removal Rates by Age Interval

i	h'_i	i	h'_i	i	h'_i
1	.01425	21	.00000	41	.00255
2	.01275	22	.01793	42	.01580
3	.00977	23	.00806	43	.01062
4	.00165	24	.00816	44	.01912
5	.00678	25	.00821	45	.00551
6	.01007	26	.01047	46	.02617
7	.00506	27	.00423	47	.02085
8	.00861	28	.00849	48	.01220
9	.00681	29	.01523	49	.00626
10	.00517	30	.01955	50	.00989
11	.00175	31	.00881	51	.00000
12	.01760	32	.00907	52	.00000
13	.00866	33	.01151	53	.00372
14	.00887	34	.01166	54	.00397
15	.00716	35	.01171	55	.00000
16	.00931	36	.02366	56	.00000
17	.00372	37	.00955	57	.00000
18	.01617	38	.00248	58	.00000
19	.00757	39	.01233	59	.00000
20	.00775	40	.01261	60	.00000

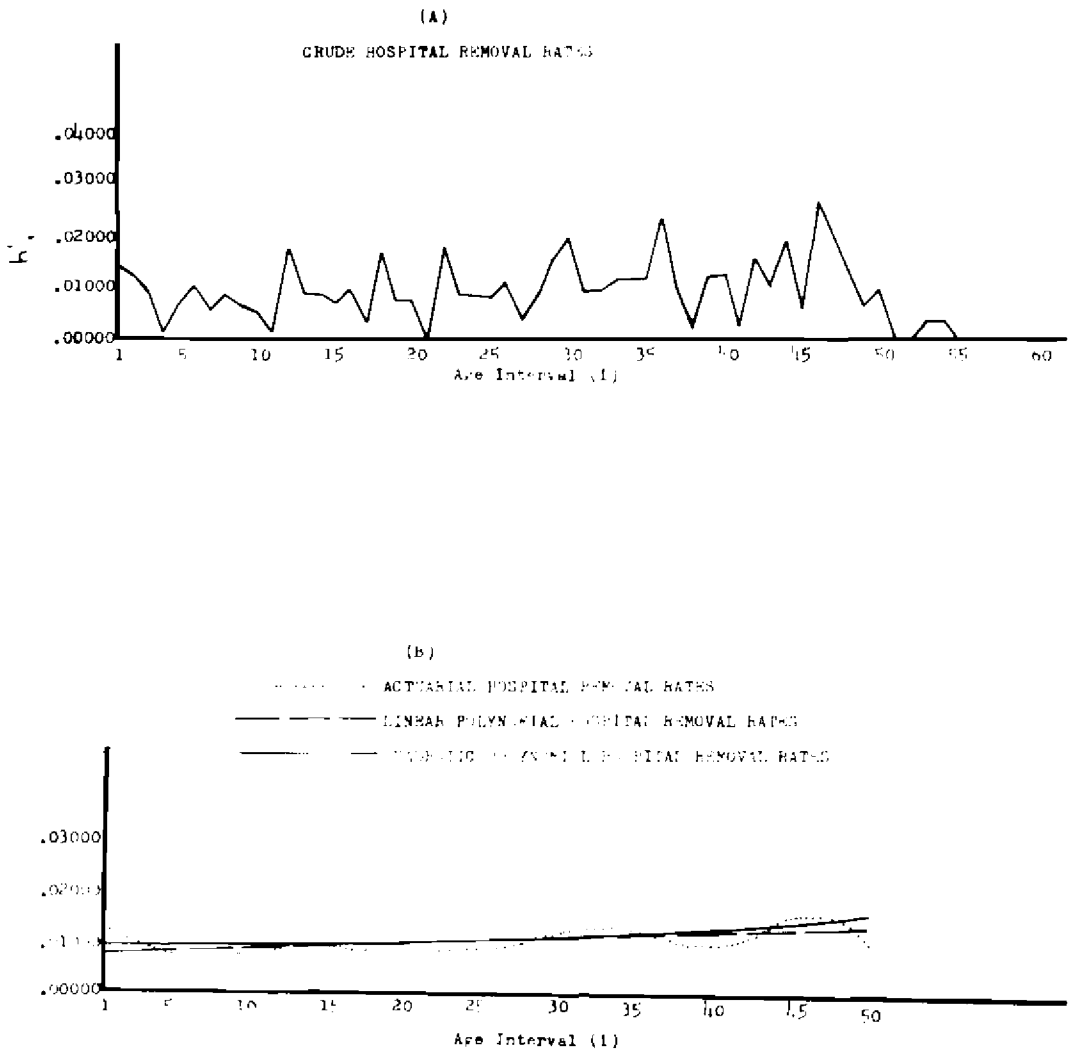


FIGURE 10. ENGINE HOSPITAL REMOVAL RATES

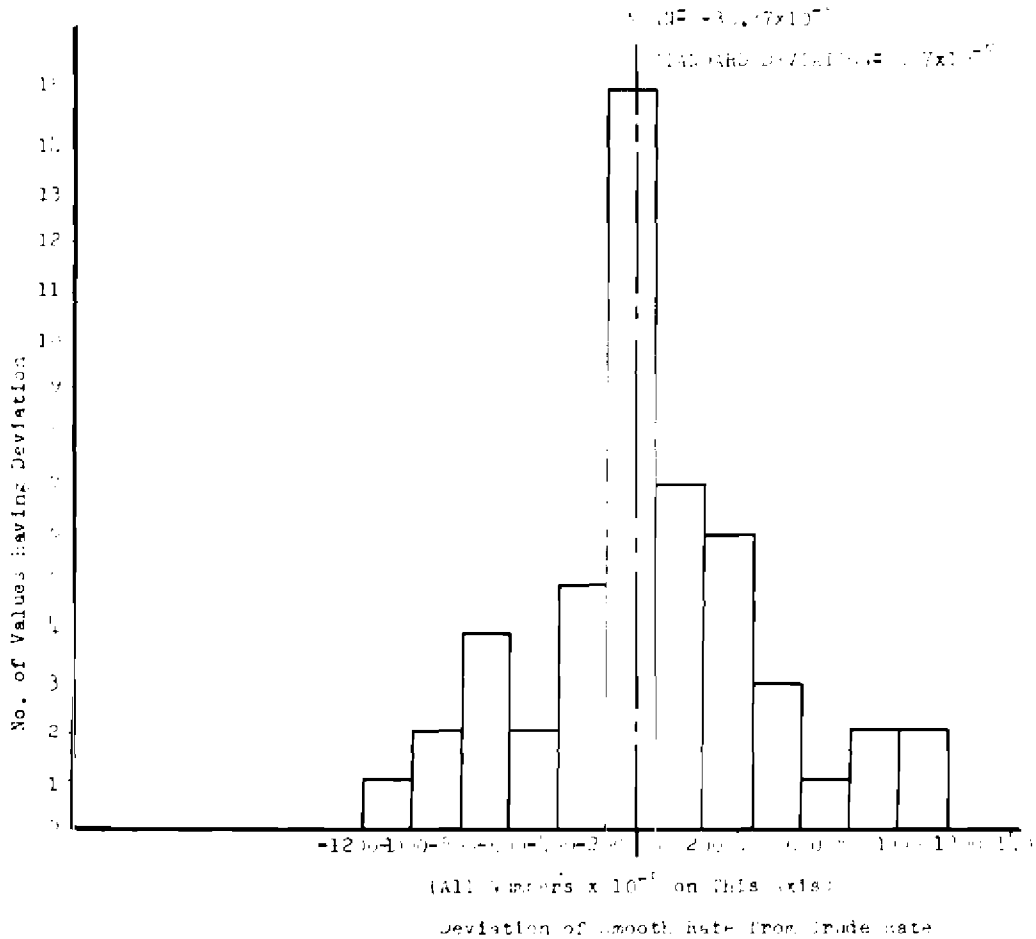


FIGURE 11. DISTRIBUTION OF HOSPITAL REMOVAL RATE SMOOTHING ERROR BY ACTUARIAL METHOD

$$\sum_{i=1}^{r} R_{140.512}$$

is the cumulative $R_{140.512}$ function. Calculations are as follows:

$$(h_i)(S_i) = o_i$$

where

$$S_i = 100.00 \text{ ; for } i = 1,$$

and

$$S_i = S_{(i-1)} - o_{(i-1)} \text{ ; for } i = 2,3, \ldots , 60$$

Again it was decided to use 100.00 engines at the start of the program. It is assumed for the purpose of developing the functions that an engine hospital removed is not reinstalled. Values for S_i, $R_{140.512}$, and the

$$\sum_{i=1}^{r} R_{140.512}$$

values are shown in Table 7, and their graphs are given in Figures 12A, 12B, and 12C.

Conclusions

The probability an engine in the i^{th} age interval will receive a major overhaul removal in aging to the $(i + 1)^{th}$ age interval is estimated by f_i where

$$f_i = 10^{-5} \left[816.492 - 25.715(i) + 1.035(i)^2\right]$$

Table 7. Hospital Removal Rates and Calculated Values

i	h_i	S_i	O_i	$R_{140.512}$	$\sum_{i=1}^{r} R_{140.512}$
1	.00718	100.000	.718	.01772	.01772
2	.00731	99.282	.726	.01792	.03564
3	.00744	98.556	.733	.01809	.05373
4	.00757	97.823	.741	.01829	.07202
5	.00770	97.082	.748	.01846	.09048
6	.00783	96.335	.754	.01861	.10909
7	.00795	95.581	.760	.01876	.12785
8	.00808	94.821	.765	.01889	.14674
9	.00821	94.055	.773	.01908	.16582
10	.00834	93.282	.778	.01920	.18502
11	.00847	92.504	.783	.01933	.20435
12	.00860	91.721	.789	.01948	.22383
13	.00872	90.932	.793	.01957	.24340
14	.00885	90.139	.798	.01970	.26310
15	.00898	89.341	.802	.01980	.28290
16	.00911	88.539	.807	.01992	.30282
17	.00924	87.732	.810	.01999	.32281
18	.00937	86.922	.815	.02012	.34293
19	.00950	86.107	.818	.02019	.36312
20	.00962	85.289	.820	.02024	.38336
21	.00975	84.469	.824	.02034	.40370
22	.00988	83.645	.826	.02039	.42409
23	.01001	82.819	.826	.02046	.44455
24	.01014	81.990	.832	.02054	.46509
25	.01027	81.158	.833	.02056	.48565
26	.01039	80.325	.835	.02061	.50626
27	.01052	79.490	.846	.02088	.52714
28	.01065	78.644	.838	.02069	.54783
29	.01078	77.806	.839	.02071	.56854
30	.01091	76.967	.840	.02073	.58927
31	.01104	76.127	.840	.02073	.61000
32	.01116	75.287	.840	.02073	.63073
33	.01129	74.447	.841	.02076	.65149
34	.01142	73.606	.840	.02073	.67222
35	.01155	72.766	.841	.02076	.69289
36	.01168	71.925	.840	.02073	.71371
37	.01181	71.085	.839	.02071	.73442
38	.01193	70.346	.838	.02069	.75511
39	.01206	69.408	.837	.02066	.77577
40	.01219	68.571	.836	.02064	.79641

Table 7. Hospital Removal Rates and Calculated Values (Continued)

i	h_i	S_i	o_i	$R_{140.512}$	$\sum_{i=1}^{r} R_{140.512}$
41	.01232	67.735	.835	.02061	.81702
42	.01245	66.900	.833	.02056	.83758
43	.01258	66.067	.831	.02051	.85809
44	.01271	65.236	.829	.02046	.87855
45	.01283	64.407	.826	.02039	.89894
46	.01296	63.581	.824	.02034	.91928
47	.01309	62.757	.822	.02029	.93957
48	.01322	61.935	.819	.02022	.95979
49	.01335	61.116	.816	.02014	.97993
50	.01348	60.300	.812	.02007	1.00000
51	.00000	59.488	.000	.00000	1.00000
52	.00000	59.488	.000	.00000	1.00000
53	.00000	59.488	.000	.00000	1.00000
54	.00000	59.488	.000	.00000	1.00000
55	.00000	59.488	.000	.00000	1.00000
56	.00000	59.488	.000	.00000	1.00000
57	.00000	59.488	.000	.00000	1.00000
58	.00000	59.488	.000	.00000	1.00000
59	.00000	59.488	.000	.00000	1.00000
60	.00000	59.488	.000	.00000	1.00000

for all $i < 60$, and $f_i = 1.00000$ for $i = 60$.

The probability an engine in the i^{th} age interval will receive a hospital removal in aging to the $(i + 1)^{th}$ age interval is estimated by h_i, where

$$h_i = 10^{-5} \left[705.554 + 12.839(i)\right] \ ; \ i \leq 50$$

and

$$h_i = 0 \text{ for } i > 50$$

These functions, f_i and h_i, will be the primary input for Chapter IV; however, it should be pointed out that density functions and cumulative distribution functions for hospital, failure, and all major overhaul engines have been developed and are of academic interest.

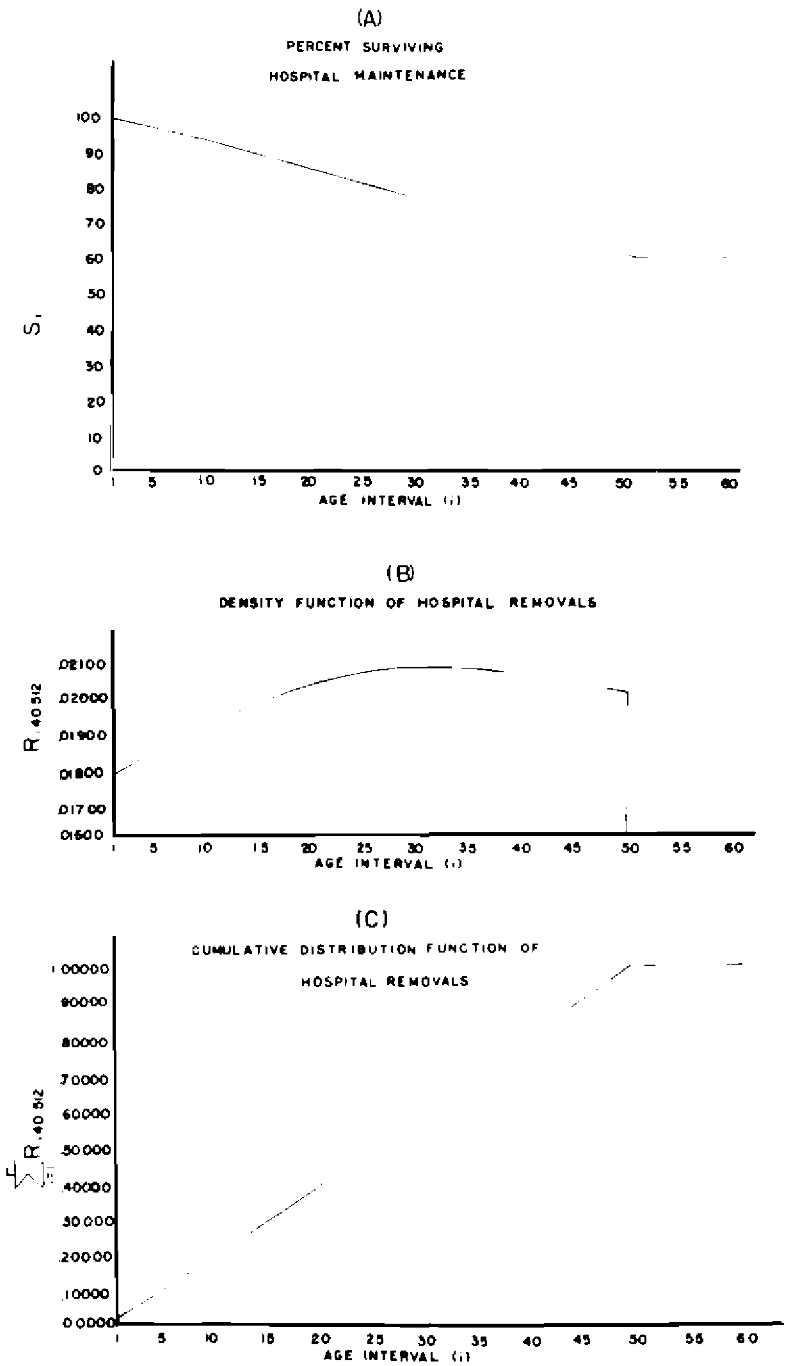


FIGURE 12. FUNCTIONS CALCULATED FROM HOSPITAL REMOVAL RATES

CHAPTER IV

DEVELOPMENT AND TEST OF A FORECASTING MODEL

Introduction.--One of the major problems with which the commercial airlines and military air forces have dealt has been that of forecasting engine removal quantities. Such forecasts are essential in providing adequate overhaul facilities, equipment, and spare parts. The following method is presently used in planning for removals or forecasting by the commercial airlines. A factor is computed for each month of the flying program, for failure, all major overhaul, and hospital removals. For the purpose of this thesis, the factor will be called the upper curve (UC). Three upper curve values were calculated monthly for months 1-45. The values were first $(UCF)_j$, second $(UCMO)_j$, and third $(UCH)_j$. These expressions are defined as follows:

$$(UCF)_j = \frac{\sum_i \sum_k P_{ijk}}{\sum_i F_{ij.}} = \frac{P_{.j.}}{F_{.j.}}, \text{ where } F_{.j.} \neq 0$$

$$(UCMO)_j = \frac{\sum_i \sum_k P_{ijk}}{\sum_i (F_{ij.} + T_{j.})} = \frac{P_{.j.}}{F_{.j.} + T_{j.}}$$

where $F_{.j.} + T_{j.} \neq 0$

$$(UCH)_j = \frac{\sum_i \sum_k P_{ijk}}{\sum_i \sum_k H_{ijk}} = \frac{P_{.j.}}{H_{.j.}} \text{ where } H_j \neq 0$$

In cases where $F_{.j.}$, $(F_{.j.} + T_{j.})$, or $H_{.j.}$ are equal to zero the corresponding UC is indeterminate. The three UC values were calculated for each month and are shown in Table 8. These values are presented graphically in Figures 13A, 13B, and 13C.

It will be noted that the graphs of $(UCF)_j$ and $(UCH)_j$ fluctuate over a wide range of values, while the $(UCMO)_j$ are relatively stable in nature. The forecasting factor, usually made up from about six previous months' data, is calculated from the $(UCMO)_j$. Suppose for example that the present time is January 1, 1957, the 35th month of the program, and it is desired to calculate a forecasted number of major overhaul removals for the months January, 1957 through August, 1957. A forecasting factor is computed as follows:

$$\frac{\sum_{j=28}^{j=34} P_j}{\sum_{j=28}^{34} F_j + \sum_{j=28}^{34} T_j} = \text{Forecasting Factor}$$

It will be noted that about six months experience data are used to attempt to smooth out the fluctuations present in the $(UCMO)_j$. For the period mentioned above, the Forecasting

Table 8. Upper Curve Values by Months

j	$(UCF)_j$	$(UCMO)_j$	$(UCH)_j$
1	-	-	-
2	2825	2825	-
3	2803	2803	1121
4	2126	1417	531
5	929	530	929
6	974	780	780
7	634	380	951
8	1505	1129	2258
9	428	389	-
10	787	674	2360
11	7868	3934	2623
12	1658	1659	4146
13	2353	941	1569
14	11031	1839	1103
15	5220	870	2088
16	1395	751	2442
17	3664	785	1570
18	1576	1226	2207
19	1336	764	2000
20	2281	671	1901
21	1714	935	3427
22	1244	1037	1777
23	925	720	1619
24	1345	961	1921
25	1799	959	1599
26	2443	1221	2094
27	1479	643	2956
28	1495	831	2492
29	4812	1203	3609
30	1493	746	3732
31	2743	762	1959
32	3518	704	2814
33	2268	907	2722
34	1634	688	2615
35	2507	1367	1671
36	2273	1240	1705
37	2593	1414	3890
38	4883	916	2442
39	2485	1356	3728
40	2055	1308	1027
41	2210	1032	7736
42	2812	1758	3374
43	2411	1687	1687

Table 8. Upper Curve Values by Months (Continued)

j	$(UCF)_j$	$(UCMO)_j$	$(UCH)_j$
44	1857	1093	2653
45	1086	839	2308

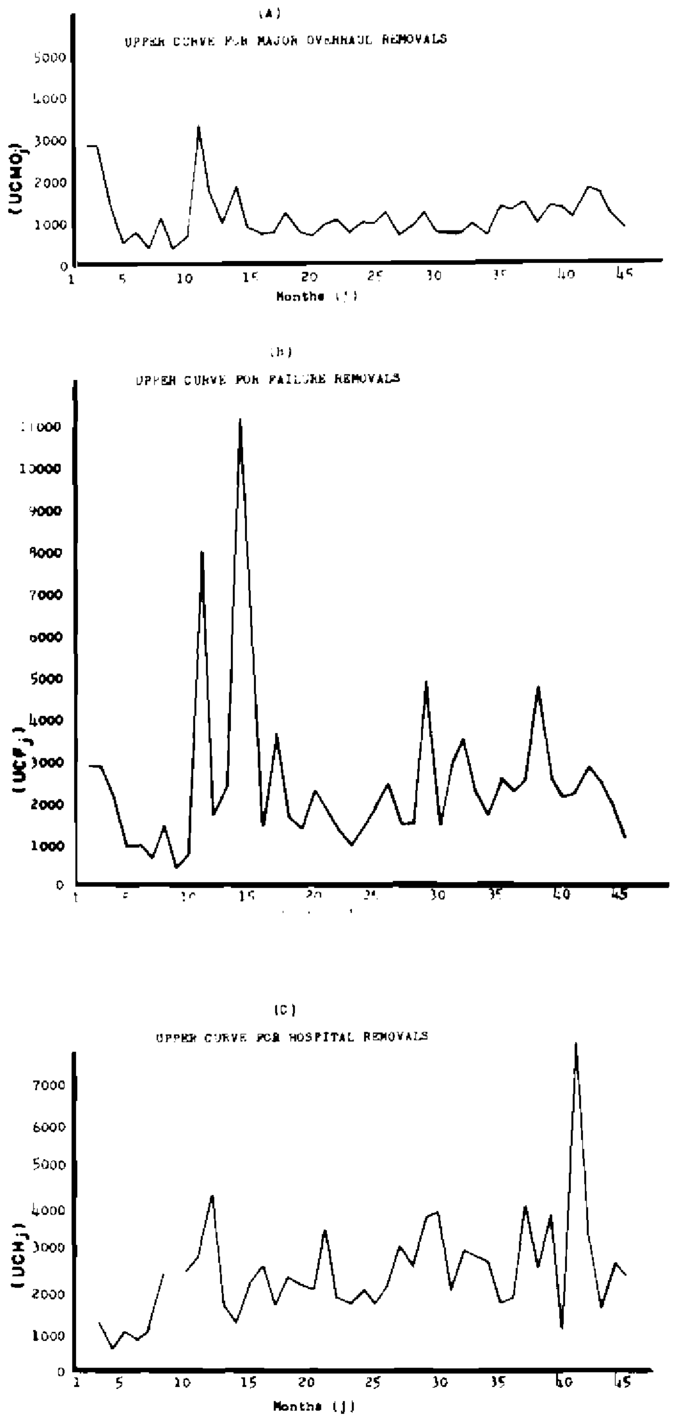


FIGURE 13. GRAPH OF UPPER CURVE VALUES BY MONTHS

Factor = 943 hours/removal. The forecast is used as follows for the j^{th} month.

$$\left[(\text{No. engines})(\text{Hrs./Day Utilization})(\text{Days/Mo.})\right] \div \left[\text{Forecasting Factor}\right] = \text{Forecasted Removals}$$

The Forecasting Factor is periodically revised as data are obtained from experience. The removals for major overhaul are seldom forecast for more than six months in the future. This is not because of the lack of the need for a long range forecast, but rather the unstable nature of the forecasting factor. Delta Airlines made several forecasts during 1957; however, only two of the basic forecasts are shown in Table 9. It will be noted that an eight months' forecast was the longest range forecast made.

There is little attempt made to forecast for the hospital maintenance requirements because of the large variation of the $(UCH)_j$ values.

The Air Force has for several years been engaged in forecasting using a method which utilizes mortality distributions similar to those developed in Chapter III of this thesis. The method has proved especially useful in long range forecasting, and in planning for new programs.

<u>Purpose</u>.--The purpose of this chapter is to develop a procedure for an accurate forecast of major overhaul maintenance requirements, and a usable forecast of hospital maintenance requirements.

Table 9. Forecast Data for Delta Airlines
1957 Forecasts

Date of Forecast	For the Month	Forecast Factor Used	Proposed Daily Utilization	Forecasted Number of Maj. O'Hls
Dec 56	Jan 57	943 hours	11 hours	15.5
Dec 56	Feb 57	943	11	15.5
Dec 56	Mar 57	943	11	15.5
Dec 56	Apr 57	943	11	15.5
Dec 56	May 57	943	11	15.5
Dec 56	Jun 57	943	11	15.5
Dec 56	Jul 57	943	11	15.5
Dec 56	Aug 57	943	11	17.0
Aug 57	Sep 57	992	11	18.5
Aug 57	Oct 57	992	11	20.0
Aug 57	Nov 57	992	11	20.0
Aug 57	Dec 57	992	11	20.0

In order to compare the results of this proposed procedure against some actual experience, the supposition is made that time is turned back to January 1, 1957. The forecast will be made for the year 1957. Since the first eight months of 1957 was the longest period for which Delta Airlines made a single forecast, the results will only be compared for this period.

Hypothesis II.--The mortality rates f_i, and h_i of Part I can be applied against installed inventories of engines, to give a better estimate of future removals than existing methods.

Procedure.--In order to forecast for 1957, the data available at January 1, 1957 were used. At this time the installed inventory of engines (I) was distributed among the intervals as shown in Table 10. It is noted that

$$\sum_{i} I_i = 44$$

engines were on hand at this time. The proposed monthly engine utilization rate is given by: (Proposed utilization, Hours/day)(Days/Month). These values for 1957 are given in Table 11. The delivery schedule of new aircraft for 1957 was as shown in Table 12.

The method of forecasting can best be explained in connection with Table 13. Certain approximations are made

Table 10. Inventory of Engines by Age Interval for January 1, 1957

i	I'$_i$	i	I'$_i$	i	I'$_i$
1	5	21	0	41	0
2	1	22	0	42	0
3	3	23	3	43	0
4	0	24	0	44	2
5	0	25	0	45	0
6	0	26	1	46	0
7	1	27	0	47	1
8	1	28	3	48	0
9	0	29	0	49	1
10	2	30	1	50	0
11	1	31	1	51	1
12	2	32	0	52	0
13	1	33	1	53	1
14	1	34	0	54	0
15	4	35	0	55	0
16	0	36	1	56	0
17	1	37	1	57	0
18	0	38	0	58	0
19	2	39	0	59	0
20	0	40	1	60	0
				Grand Total	44

Table 11. Engine Utilization Rates and Other Computation Factors

Month	Proposed Daily Utilization Rate	Days per Month	Hours/Month/ Engine (Rounded)
Jan.	11 hours	31	341
Feb.	11	28	308
Mar.	11	31	341
Apr.	11	30	330
May	11	31	340
Jun.	11	30	330
Jul.	11	31	341
Aug.	11	31	340
Sep.	11	30	330
Oct.	11	31	340
Nov.	11	30	330
Dec.	11	31	341

Table 12. Delivery Schedule of New Aircraft

Aircraft Number	Scheduled Date of Delivery
712	June 22, 1957
713	July 27, 1957
714	August 17, 1957
715	September 26, 1957
716	November 15, 1957
717	December 26, 1957

in making the forecast. First, it is assumed at the beginning of the forecast period that all engines in interval one have a zero TSO. Second, it is assumed that all engines removed for major overhaul maintenance will be replaced by engines having zero TSO. Calculations are as follows: Multiply $(f_i)(I_i)^{(1)}$ for i = 1 to i = 60. The result $f_i I_i^{(1)}$ is the number of failures in the i^{th} age interval after artificially aging the inventory twenty hours. This leaves $I_i^{(1)} - f_i I_i^{(1)}$ engines surviving major overhaul in the i^{th} age interval, after the above referenced aging.

The total number of major overhaul removals after aging each engine twenty hours as of January 1, 1957 is the sum of $f_i I_i^{(1)}$ over all i values. The

$$I_i^{(1)} - f_i I_i^{(1)}$$

engines surviving the i^{th} age interval are now twenty hours older. The number of engines that receive major overhaul maintenance,

$$\sum_{i=1}^{60} f_i I_i^{(1)},$$

are replaced by engines from major overhaul maintenance so that

$$I_1^{(2)} = \sum_{i=1}^{60} f_i I_i +$$

new engines received.

$$I_i^{(2)} = I_{i-1}^{(1)} \quad \text{for } i = 2, 3, 4, \ldots, 60$$

The $I^{(1)}$ values are now replaced by the $I^{(2)}$ values and the second replication of the loop is started. During 1957 there are 4012 hours per engine scheduled to be flown or 4,012/20 = 200.6 replications of the above cycle. Replications are denoted by the superscript (n).

In phasing in the new engines received it is necessary to align calendar time and replication. For example, four engines are received on an aircraft on June 22, 1957. The projected utilization to this date is 1,860 hours per engine. 1,860/20 = 93 replications. After 93 replications, four engines + the sum of $f_iI_i^{(93)}$ over all i values for the 93rd replication are placed in $I_1^{(94)}$.

In calculating removals per month, the monthly utilizations are converted to replications and the sums of $f_iI_i^{(n)}$ over all i values are added for these replications. For example, consider the first month, January, 1957. There is a monthly utilization of 341 hours for January, 1957. 341 divided by 20 gives 17.05 replications. Forecasted major overhaul removals for January, 1957 are given as follows:

$$\left(\sum_i f_iI_i\right)^{(1)} + \left(\sum_i f_iI_i\right)^{(2)} + \ldots + \left(\sum_i f_iI_i\right)^{(17)}$$
$$+ .05 \left(\sum_i f_iI_i\right)^{(18)}$$

Table 13. Explanation of Forecasting Calculations

i	f_i	h_i	$I_i^{(1)}$	$f_iI_i^{(1)}$	$h_iI_i^{(1)}$	$I_i-f_iI_i^{(1)}I_i^{(2)}$
1	f_1	h_1	I_1	f_1I_1	h_1I_1	$I_1-f_1I_1$
2	f_2	h_2	I_2	f_2I_2	h_2I_2	$I_2-f_2I_2$
3	f_3	h_3	I_3	f_3I_3	h_3I_3	$I_3-f_3I_3$
4	f_4	h_4	I_4	f_4I_4	h_4I_4	$I_4-f_4I_4$
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
60	f_{60}	h_{60}	I_{60}	$f_{60}I_{60}$	$h_{60}I_{60}$	$I_{60}-f_{60}I_{60}$

$$\sum_i f_iI_i + \text{ new engines } = I_1^{(2)}$$

The following calculations utilizing the h_i values are made at the same time the above calculations are made. For $I_i^{(1)}$, the value $h_i I_i^{(1)}$, the number of hospital removals in the i^{th} interval, is calculated for all i; and the sum of the $h_i I_i^{(1)}$ over all i values is calculated which is the total hospital removals after twenty hours of artificial flying. Forecasted hospital maintenance removals for January, 1957 are given as follows:

$$(\sum_i h_i I_i^{(1)} + (\sum_i h_i I_i)^{(2)} + \ldots + (\sum_i h_i I_i)^{(17)}$$
$$+ .05\ (\sum_i h_i I_i)^{(18)}$$

The procedure outlined will require about 24,000 multiplication operations, 36,000 additional operations, and 12,000 subtraction operations in completing the 201 replications necessary for the year 1957. Because of the time element involved in making these calculations, an IBM 650 Computer Bell System Routine was written to calculate the sum of $f_i I_i^{(n)}$, and the sum of $h_i I_i^{(n)}$ over all i values for each replication and adjust the $I_i^{(n)}$ values for aging the inventory and new arrivals at the proper place. The orders for the Bell General Purpose type program are given in Table 16 which is given in the Appendix, and the flow diagram of the program is as shown in Figure 15 in the Appendix.

<u>Results</u>.--The program was successfully run and the routine by replication for the sum of $f_i I_i^{(n)}$ and the sum of $h_i I_i^{(n)}$

over all i values are shown in Table 17 which is in the Appendix. The major overhaul forecast by this method, the major overhaul forecast by the UC method, and actual major overhaul removals for the year 1957 are shown in Table 14. These same results are shown graphically in Figure 14A.

The hospital removals forecasted by this method together with the actual occurrence of hospital removals for the first eleven months of 1957 are shown in Table 15. These results are shown graphically in Figure 14B.

Analysis of Results.--In the case of the major overhaul forecast, it is desirable to compare the Delta Airlines forecast and the forecast accomplished by the methods developed in this study. To accomplish this, both forecasts will be compared against the actual removals that occurred. Forecasted removals for the first eight months of each forecast are all that are statistically compared.

Two measures of the goodness of forecast will be used. The first is the mean deviation of the forecast errors. This factor is given as follows:

$$M.\ D. = \frac{\sum_{j=1}^{N} \left| (F + T)_j - X_j \right|}{N}$$

where X_j is the forecasted number of removals by the method in question in the j^{th} month, and $(F + T)_j$ is the number of major overhaul removals in the j^{th} month. The expression N is the number of months being used. For the comparison of major overhaul removals, $N = 8$.

Table 14. Monthly Major Overhaul Removals and Forecasted Quantities from Present and Proposed Methods of Forecasting for 1957

Month	Actual Major Overhaul Removals	Upper Curve Forecast	Thesis Routine Forecast
Jan	11.0	15.5	12.25
Feb	11.0	15.5	12.27
Mar	11.0	15.5	15.82
Apr	16.0	15.5	15.72
May	11.0	15.5	14.49
Jun	11.0	15.5	15.98
Jul	15.0	15.5	17.16
Aug	16.0	17.0	17.38
Sep	20.0	18.5	17.10
Oct	17.0	20.0	21.51
Nov	22.0	20.0	21.22
Dec	18.0	20.0	21.91

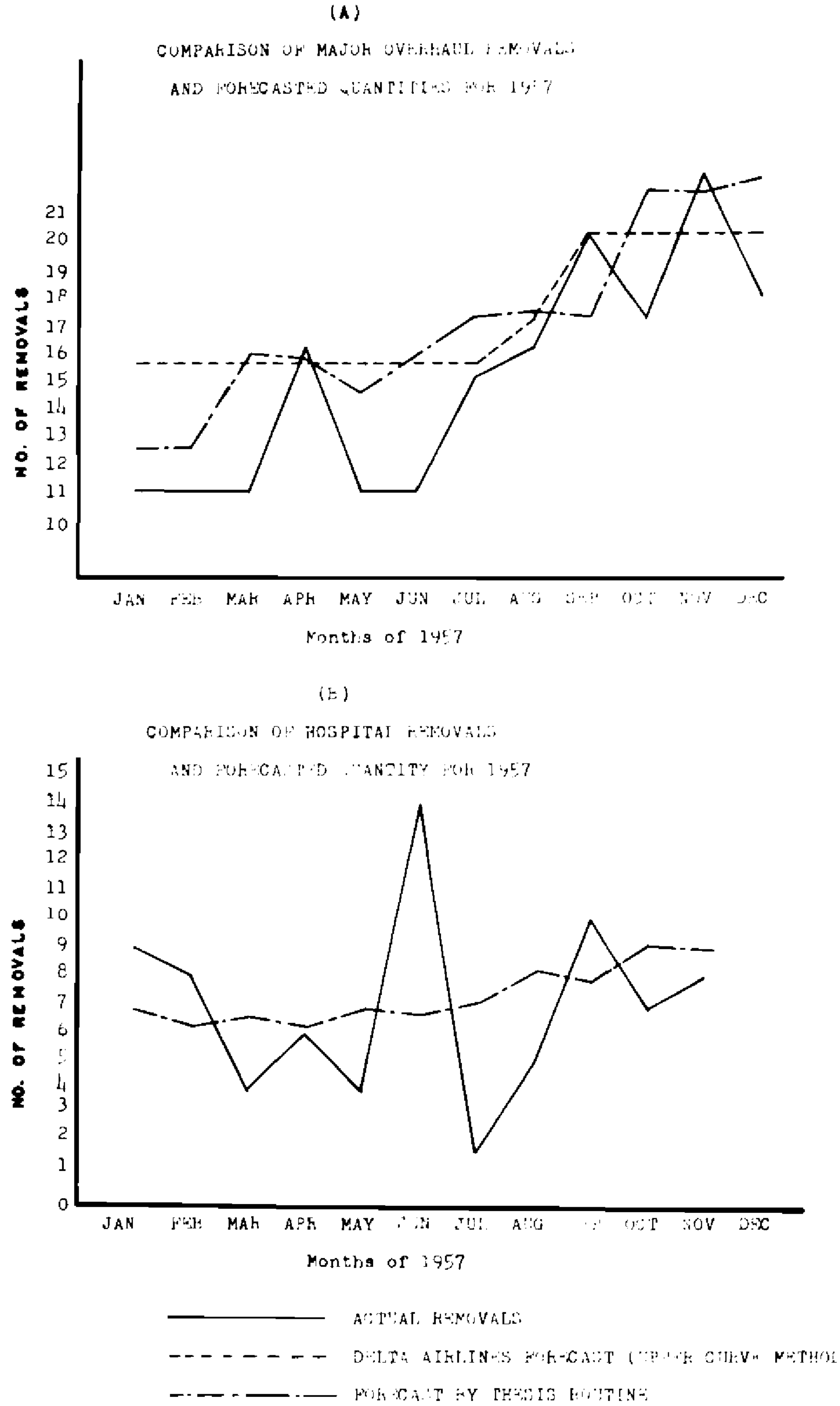


FIGURE 14. COMPARISON OF FORECASTED REMOVAL QUANTITIES

The second measure that will be used is the estimate of the standard deviation of the forecast errors. This factor is given as follows:

$$S.\ D. = \sqrt{\frac{\sum_{i=1}^{N} \left[(F + T)_j - X_j\right]^2}{N}}$$

where the symbols have the same meaning as in the case of the mean deviation.

The first calculations were made for the upper curve forecast made by personnel of the airline. The mean deviation was 3.1 and the standard deviation was 3.6.

The second calculations were made for the forecast using the routine developed in this thesis. The results showed a mean deviation of 2.4, and a standard deviation of 3.0. For the first eight months, the computer routine forecast 121 removals, the upper curve method 125.5 and there actually occurred 102 removals. In the case of the hospital removals, there were no formal forecasts made and it is thus impossible to compare the method of this thesis to any other method of forecasts; however, the same two measures of deviation as employed above will be used to make statements about the deviation of the forecast from the actual occurrence. In this case, values will be calculated over eleven months. The calculations give a mean deviation of 2.8 and a standard deviation of 3.4. For the eleven

Table 15. Monthly Hospital Removals and Hospital Removal Forecast for the Months January 1957 through November 1957

Month	Actual Hospital Removals	Forecasted Removals
Jan	9	6.83
Feb	8	6.32
Mar	4	6.60
Apr	6	6.31
May	4	6.85
Jun	14	6.69
Jul	2	7.18
Aug	5	8.21
Sep	10	7.86
Oct	7	9.12
Nov	8	8.97

CHAPTER V

GENERAL CONCLUSIONS AND RECOMMENDATIONS

Conclusions.--Conclusions can be drawn for only the R-3350 engines of Delta Airlines DC-7 fleet; however, there is every reason to believe that all statements can be extended to cover the entire range of equipment which fails at a rate that can be expressed as a function of its age.

For the engines of this study, the following statements can be made.

(a) An estimate of the probability that an engine in age interval i will receive a major overhaul removal in aging to interval (i + 1) is given by f_i, where f_i is developed in Chapter III as a function of i.

(b) An estimate of the probability that an engine in age interval i will receive a hospital removal in aging to interval (i + 1) is given by h_i, where h_i is developed in Chapter III as a function of i.

(c) The computer routine developed in Chapter IV gave a superior forecast of removals during the period for which the methods were compared. The hospital removal forecast although not compared to any other forecast, seems to give a usable forecast of this type of maintenance.

Recommendations.--It is recommended that this study be extended to include the failure of component parts such as pistons, valves, etc. It is further recommended that this approach be taken to planning for initial procurement of spare engines for the jet programs of the future. Although no airline experience data will be available for calculation of f_i and h_i values in the early part of these programs, it is felt that the Air Force experience with the J-57 engines in the B-52 series aircraft can be used to obtain estimates of the f_i and h_i values.

This type of analysis can be extended to cover any item on which sufficient records are kept to obtain the necessary data, and which has a value sufficient to justify the cost of this analysis.

It is further recommended that the probability distributions, together with overhaul and hospital maintenance cost figures, be applied to economic analysis directed towards optimizing the maximum TSO at which hospital maintenance will be accomplished.

A P P E N D I X

Sample Data Sheets

			M1			M2			M3			M4		
	Engine Serial No.	Aircraft & Pos. as Rec'd	A1	B1	C1	A2	B2	C2	A3	B3	C3	A4	B4	C4
1	548319	701-2			ac/29/0	0/367			367/586	a/24/541 (701-1) ←	H/20/541	586/795		T/27/795
2	548320	701-3			ac/29/0	0/189		F/7/189	0/0			0/112		a/17/0 (701-[illegible])
3	548321	701-1			ac/29/0	0/367			367/390		F/2/390	0/20		a/22/0 (702-[illegible])
4	548322	701-4			ac/29/0	0/317		H/24/317	51/317			317/500		a/17/317 (701-1)
5	548339	702-2						ac/31/0	0/495			495/531		F/3/531
6	548340	702-3						ac/31/0	0/495			495/736		
7	548341	702-1						ac/31/0	0/495			495/584		H/10/584
8	548342	702-4						ac/31/0	0/344		F/14/344	0/0		
9	548359	704-1				0/137		ac/14/0	137/398			398/639		
10	548360	703-2				0/207		ac/1/0	207/377		H/22/377	377/529		a/10/377 (703-1)
11	548361	703-1				0/207		ac/1/0	207/377			377/468	A/21/468 [illegible] (703-[illegible])	← H/1/468
12	548362	703-3				0/131	H/19/131 ←	ac/1/0	131/280		a/14/131 ([illegible])	280/467		H/21/467
13	548363	704-4				0/105		ac/14/0	105/368			368/566	a/30/566 ([illegible])	← H/21/566
14	548364	704-2				0/137		ac/14/0	137/398			398/639		
15	548365	704-3				0/137		ac/1/0	137/353		H/24/353	353/473	F/30/473	← a/14/353 (703-1)
16	548369	703-4				0/207			207/468			468/708		
17	548399	701-2						ac/7/0	0/86		ac/20/0	86/349		
18	548400	701-3				0/171		ac/19/0	171/431			431/697		
19	548413	703-3				0/75			75/336			336/576		
20	548414	701-1							0/367		ac/2/0	367/387	a/17/387 (704-1)	← H/17/387
21	548429	711-4												
22	548430	707-2												
23	548431	711-3												
24	548432	711-2												
25	548433	707-4												
26	548434	711-1												
27	548435	702-2						ac/24/0				0/417		ac/3/0
28	548436	701-4				0/51			51/224		H/20/224	224/370	← H/[illegible]/[illegible] a/[illegible] 701-[illegible]	← a/1/224 ([illegible])

SAMPLE 1

	M_5			M_6			M_7			M_8		
	A_5	B_5	C_5	A_6	B_6	C_6	A_7	B_7	C_7	A_8	B_8	C_8
1	0 / 16		a/28/0 (704-3)	16 / 239		H/29/239	239 / 448		a/2/239 703-3	448 / 683		
2	112 / 273	a/31/273 (704-3) ←	H/21/273	273 / 515			515 / 929		F/26/929	0 / 0		
3	20 / 209	a/9/20 (702-1) ←	H/1/20	209 / 463			463 / 596		F/19/596	0 / 0		
4	500 / 800			800 / 1000			1000 / 1119		F/22/1119	0 / 0		
5	0 / 126		a/15/0 (701-3)	126 / 342		F/24/342	0 / 27	H/27/27 ←	a/24/0 (702-2)	27 / 27		
6	736 / 776		T/7/776	0 / 8	H/5/8 ←	a/2/0 (703-3)	8 / 107		a/16/8 (703-2)	107 / 332	a/5/131 (704-2) ←	S/8/131
7	584 / 596	F/15/596 ←	a/12/584 (701-3)	0 / 153		a/19/0 (701-3)	153 | 0 / 10	a/26/0 (704-2) ←	F/12/174	10 / 44		H/5/44
8	0 / 208		a/7/0 (702-3)	208 / 375		F/19/375	0 / 0			0 / 105		a/21/0 (704-4)
9	639 / 861		F/27/861	0 / 19		a/29/0 (704-3)	19 / 245	a/28/245 (701-2) ←	S/26/245	245 / 876		a/8/356 (703-2)
10	829 / 778			778 / 825		H/4/825	825 / 895	F/16/895	a/7/825 (703-4)	0 / 197		a/7/0 (701-2)
11	468 / 690		a/9/500 (703-4)	690 / 763	a/20/460 (702-4) ← F/21/763	H/10/760	0 / 87		a/19/0 (703-1)	87 / 102		F/8/102
12	467 / 600	H/13/547 ← a/8/500 (703-4)	a/1/467 (701-4)	600 / 674		F/5/674	0 / 49		a/22/0 (704-4)	49 / 361	a/8/142 (703-1) ←	S/7/142
13	566 / 646		F/9/646	0 / 73		a/21/0 (704-4)	73 / 167		H/10/167	167 / 217	F/13/217	← a/5/167 (704-3)
14	689 / 885		F/31/885	0 / 0		a/30/0 (703-4)	0 / 181	a/12/19 (702-3) ←	H/2/19	181 / 423		
15	0 / 20		a/27/473 (704-1)	20 / 263			263 / 489			263 / 746		
16	708 / 789		T/8/789	0 / 204		a/5/0 (701-4)	204 / 370		H/19/370	370 / 370		a/28/370 (702-4)
17	349 / 580			580 / 831			831 / 908		T/9/908	0 / 197		a/7/0 (701-1)
18	697 / 794		T/12/794	0 / 172	H/30/172 ←	a/10/0 (703-4)	0 / 331	F/28/331 ←	a/9/172 (701-2)	0 / 0		
19	576 / 847			847 / 868		T/2/868	0 / 129		a/10/0 (704-4)	129 / 279		F/21/279
20	387 / 649		H/28/649	649 / 870		a/5/649 (703-3)	870 / 988	a/27/0 (702-2) ←	T/2/988	0 / 273		
21												
22												
23												
24												
25												
26												
27	417 / 633			633 / 750			750 / 897		T/24/897	0 / 0		
28	370 / 680			680 / 812		F/20/812	0 / 95		a/19/0 701-4	95 / 378		

SAMPLE 1 (Cont'd)

Sample Sheet of
Removal Summary

X Failure removal

✓ Hospital Removal

i	M_{20}	M_{21}	M_{22}	M_{23}	M_{24}	M_{25}	M_{26}	M_{27}	M_{28}
1		X			X	X			
2	✓	✓			XX		✓	X	✓
3					✓				
4									
5			X						
6									
7				X					
8									X
9					✓				
10		✓	✓						
11									
12				✓				✓	
13				✓✓			X		
14			X					✓	
15	✓			X	✓				✓
16			✓						
17	✓			XX ✓					
18	✓								✓
19						X		X	
20			X	✓			X		XX
21									
22			✓		X				
23			X						
24									
25					X	✓✓		X	
26			X	✓			X		
27						X ✓			
28									

SAMPLE 2

SAMPLE SHEET OF
EXPOSURE SUMMARY

i	M_{20}	M_{21}	M_{22}	M_{23}	M_{24}	M_{25}	M_{26}	M_{27}	M_{28}
1	14.60	13.75	14.00	18.00	20.85	16.10	13.00	15.00	22.90
2	14.70	13.50	13.65	18.85	19.40	17.70	13.00	14.30	22.65
3	13.60	14.75	13.95	19.05	17.30	18.70	12.35	12.95	23.60
4	12.65	14.75	14.25	19.40	17.60	18.80	11.20	14.00	23.65
5	12.80	13.95	14.65	19.45	18.90	16.10	13.90	14.00	22.85
6	11.65	15.00	14.90	18.75	20.35	13.80	15.40	14.30	21.45
7	12.50	15.70	13.50	18.90	20.00	13.25	15.10	15.05	20.55
8	10.75	15.50	15.15	18.35	20.05	13.60	14.05	13.30	21.05
9	11.45	16.50	14.25	17.25	21.25	13.60	13.45	13.95	22.85
10	12.90	20.00	13.15	16.20	19.55	17.85	12.90	14.90	24.00
11	13.00	17.00	13.00	15.00	18.75	16.25	12.33	15.65	22.60
12	12.65	17.35	12.45	14.70	18.75	17.25	12.00	15.90	20.80
13	11.10	16.80	14.05	14.50	18.50	17.00	12.25	15.00	20.45
14	10.00	15.80	14.80	13.20	17.35	17.70	13.35	15.15	19.45
15	9.65	14.35	14.95	13.05	17.20	17.80	13.15	15.00	16.75
16	9.80	13.15	14.10	10.90	17.10	10.50	12.40	15.00	15.70
17	11.75	13.00	13.00	11.50	17.00	10.00	15.00	15.00	14.20
18	12.30	12.80	13.50	11.35	16.15	19.05	16.45	12.70	13.65
19	11.20	12.00	14.80	11.25	15.70	19.45	16.35	11.80	4.20
20	10.30	11.35	15.60	10.00	16.00	19.00	15.65	11.65	13.40
21	12.00	8.65	17.40	8.70	16.60	20.40	14.15	12.30	12.55
22	10.75	7.50	18.15	9.10	14.75	20.55	14.00	12.95	13.00
23	10.00	7.60	16.95	10.40	13.55	20.25	13.60	13.55	14.60
24	9.60	8.40	16.00	10.55	14.45	20.00	12.35	14.65	15.00
25	7.20	10.90	16.00	9.40	14.60	19.80	12.55	14.05	15.80
26	6.90	11.10	15.10	9.50	12.50	20.00	14.00	11.20	13.45
27	6.00	11.90	15.10	8.00	12.15	19.35	13.75	11.45	12.66
28	6.00	10.00	15.75	8.40	11.25	16.75	14.15	12.10	13.85

SAMPLE 3

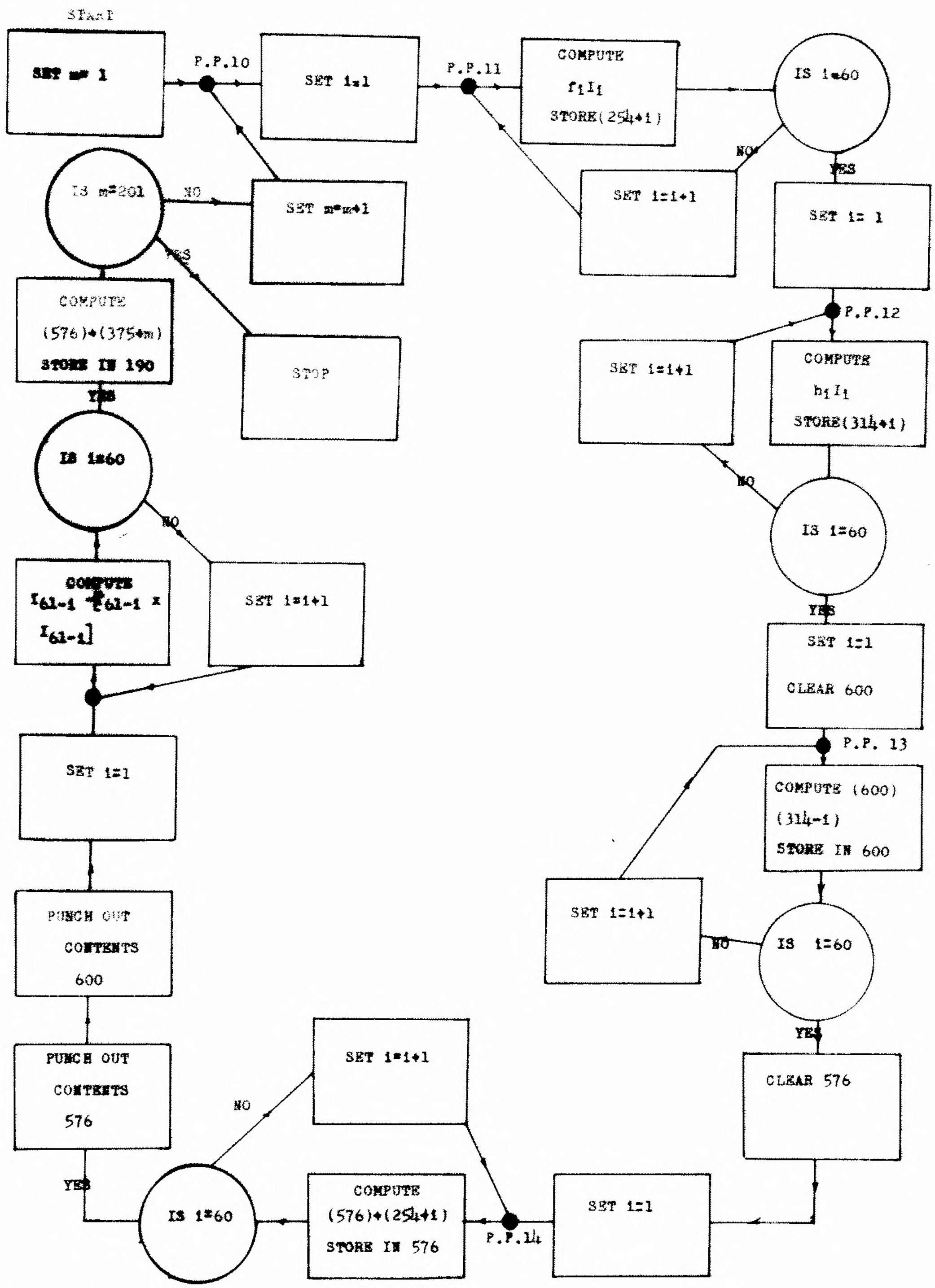


FIGURE 15. FLOW CHART FOR THE IBM 650 COMPUTER ROUTINE
(BELL SYSTEM)

Table 16. Bell System Routine for the IBM 650 Computer

Number	Order
1	+9800011000
2	+9100111000
3	+2070190255
4	+8101060011
5	+9800012000
6	+9100111000
7	+2130190315
8	+8101060012
9	+2600900600
10	+9800013000
11	+9300100000
12	+1315600600
13	+8301060013
14	+2900576576
15	+9800014000
16	+9400100000
17	+1255576576
18	+8401060014
19	+7300576576
20	+7300600600
21	+9800015000
22	+9500000111
23	-1249314250
24	+8501060015
25	+9800016000
26	+9200010000
27	+1576375190
28	+8201201011
29	+0000000000

Table 17. Computer Calculations Giving the Hospital and Failure Removals by Replication of the Computer Forecasting Procedure

Replication	$\sum_i f_i I_i^{(n)}$	$\sum_i h_i I_i^{(n)}$	Replication	$\sum_i f_i I_i^{(n)}$	$\sum_i h_i I_i^{(n)}$
1	0.423	0.399	41	0.554	0.374
2	0.429	0.403	42	1.597	0.361
3	0.436	0.395	43	0.551	0.373
4	0.443	0.399	44	1.071	0.369
5	0.450	0.392	45	0.560	0.369
6	0.458	0.396	46	0.580	0.374
7	0.467	0.401	47	1.034	0.393
8	1.282	0.382	48	1.028	0.401
9	0.467	0.392	49	1.515	0.385
10	1.245	0.397	50	1.006	0.388
11	0.466	0.407	51	1.487	0.356
12	1.214	0.400	52	0.501	0.364
13	0.468	0.410	53	0.989	0.366
14	1.188	0.415	54	0.983	0.370
15	0.470	0.413	55	0.505	0.375
16	0.480	0.407	56	0.513	0.376
17	1.839	0.411	57	0.522	0.377
18	0.470	0.425	58	1.920	0.378
19	0.480	0.419	59	0.969	0.383
20	0.490	0.423	60	2.785	0.387
21	1.136	0.418	61	0.657	0.398
22	0.498	0.416	62	0.664	0.400
23	0.508	0.421	63	0.670	0.396
24	1.129	0.396	64	0.677	0.399
25	1.119	0.405	65	0.685	0.395
26	0.514	0.403	66	0.692	0.397
27	0.525	0.408	67	0.701	0.399
28	1.118	0.412	68	1.077	0.391
29	0.535	0.393	69	0.702	0.396
30	1.116	0.397	70	1.061	0.398
31	1.109	0.406	71	0.703	0.403
32	0.543	0.414	72	1.048	0.399
33	2.216	0.400	73	0.706	0.403
34	0.528	0.416	74	1.039	0.406
35	1.083	0.411	75	0.710	0.405
36	0.537	0.419	76	0.719	0.402
37	0.547	0.388	77	1.343	0.404
38	2.153	0.384	78	0.710	0.410
39	0.533	0.391	79	0.720	0.407
40	0.543	0.378	80	0.730	0.408

Table 17. Computer Calculations Giving the Hospital and Failure Removals by Replication of the Computer Forecasting Procedure (Continued)

Replication	$\sum_i f_i I_i^{(n)}$	$\sum_i h_i I_i^{(n)}$	Replication	$\sum_i f_i I_i^{(n)}$	$\sum_i h_i I_i^{(n)}$
81	1.029	0.406	121	0.849	0.469
82	0.738	0.405	122	0.854	0.470
83	0.748	0.406	123	0.860	0.470
84	1.036	0.395	124	0.867	0.472
85	1.030	0.398	125	0.873	0.471
86	0.754	0.398	126	0.880	0.473
87	0.764	0.400	127	0.887	0.474
88	1.039	0.401	128	0.094	0.500
89	0.772	0.392	129	0.923	0.504
90	1.043	0.394	130	1.089	0.506
91	1.038	0.398	131	0.925	0.509
92	0.779	0.401	132	1.087	0.510
93	1.546	0.395	133	0.930	0.512
94	0.764	0.401	134	1.086	0.515
95	1.054	0.428	135	0.937	0.516
96	0.803	0.432	136	0.947	0.515
97	0.812	0.418	137	1.235	0.517
98	1.547	0.416	138	0.944	0.521
99	0.796	0.420	139	0.954	0.520
100	0.804	0.415	140	0.953	0.522
101	0.813	0.413	141	1.105	0.552
102	1.292	0.408	142	0.974	0.522
103	0.810	0.414	143	0.984	0.524
104	1.050	0.413	144	1.120	0.519
105	0.815	0.414	145	1.120	0.490
106	1.740	0.417	146	0.997	0.491
107	1.019	0.426	147	1.007	0.492
108	1.015	0.430	148	1.138	0.493
109	1.235	0.423	149	1.019	0.491
110	0.997	0.426	150	1.147	0.493
111	1.215	0.411	151	1.148	0.496
112	0.760	0.417	152	1.064	0.527
113	0.987	0.417	153	1.418	0.526
114	0.983	0.420	154	2.880	0.530
115	0.766	0.423	155	1.153	0.544
116	0.805	0.453	156	1.040	0.547
117	0.813	0.455	157	1.048	0.541
118	1.454	0.457	158	1.386	0.542
119	1.010	0.459	159	1.040	0.545
120	1.837	0.463	160	1.049	0.543

Table 17. Computer Calculations Giving the Hospital and Failure Removals by Replication of the Computer Forecasting Procedure (Continued)

Replication	$\sum_i f_i I_i^{(n)}$	$\sum_i h_i I_i^{(n)}$	Replication	$\sum_i f_i I_i^{(n)}$	$\sum_i h_i I_i^{(n)}$
161	1.056	0.544	181	1.099	0.558
162	1.279	0.543	182	1.105	0.560
163	1.059	0.548	183	1.111	0.561
164	1.172	0.548	184	1.119	0.563
165	1.068	0.549	185	1.127	0.564
166	1.493	0.520	186	1.133	0.566
167	1.160	0.525	187	2.963	0.568
168	1.161	0.529	188	1.197	0.581
169	1.264	0.526	189	1.121	0.584
170	1.156	0.529	190	1.199	0.585
171	1.258	0.523	191	1.127	0.588
172	1.051	0.527	192	1.204	0.591
173	1.160	0.529	193	1.136	0.591
174	1.162	0.531	194	1.211	0.592
175	2.888	0.534	195	1.146	0.593
176	1.048	0.548	196	1.155	0.593
177	1.087	0.578	197	1.291	0.595
178	1.383	0.549	198	1.160	0.597
179	1.177	0.551	199	1.169	0.597
180	1.556	0.554	200	1.178	0.598
			201	1.279	0.627

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