

METHODOLOGY FOR PROBABILITY OF KILL
AGAINST A MOVING TARGET IN
AIR-TO-GROUND GUNNERY

A THESIS

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METHODOLOGY FOR PROBABILITY OF KILL
AGAINST A MOVING TARGET IN
AIR-TO-GROUND GUNNERY

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SUMMARY

The present gun effectiveness methodology used by the Air Force is only capable of assessing the probability of kill on still point and area targets. This research paper is the result of an improved methodology designed to fill the gap that currently exists in air-to-ground gunnery methodology. This new methodology is designed to assess the probability of kill on a moving ground target in the air-to-ground close air support role.

The methodology is developed in seven areas of study: mission setup, target tracking, flight dynamics, gun/projectile parameters, target model, projectile/target encounter, and probability of target hit/kill. The mathematics are developed in the respective areas and programmed in FORTRAN IV.

The mission setup assumes a flat, nonrotating earth with an aircraft located some distance from the target in three-dimensional space with five degrees of freedom. The aircraft flies toward the target from some specified slant range, airspeed, and dive angle. The target begins at some specified X, Y coordinate on the ground; it attempts to evade the attacker by traveling in some direction or arc at a specified velocity. The aircraft sight system uses first

order lead prediction by looking at the future position of the target at a projectile time of flight later and then attempts to maneuver to that position. When the aircraft reaches some specified open fire slant range, the gun is turned on. It will fire until the specified burst time is reached if the target remains within three standard deviations of the specified aim error, and the aircraft remains above some specified safe pullup altitude. The gun has been programmed as a seven-barrel Gatling, General Electric GAU-8. Its time-to-rate has been incorporated into the computer program. This gun will accommodate other rounds as data input, but a gun change will require a change in the FORTRAN program. The probability of target hit/kill is cumulative for each round fired; therefore, at the end of the burst time, the expected hits and probability of kill are known. The probability of kill is calculated by using a nine-point Hermite-Gauss approximation. The computer program results were compared to the effectiveness of the Operation Evaluation Group (OEG) Program, which has been accepted by the Joint Technical Coordinating Group (JTCCG) and documented in the Joint Munitions Effectiveness Manual (JMEM). The effectiveness results were very close with negligible differences. The new methodology will enable the analyst to evaluate the effectiveness of a gun/aircraft unit against a moving target; however, it should be noted that vulnerability data for moving targets does not exist at this time. The

computer output results are printed in four areas, which are sight tracking, aircraft, gun projectile, and target data, with pertinent header information for each. This study has revealed several major ideas, some of which will require future research. Probably the most obvious discovery shows the evasive tactics for ground targets. Also the program could be used to generate sight depression/lead compensation tables for a pilot teaching aid in air-to-ground fixed gun sight systems. Another major finding is the need for lead computing sight systems even in air-to-ground guns if the target is moving.

Several areas of this study could be improved. First, a more realistic pilot model could be incorporated into the model; however, maximum limits, aim error, g limit, alpha limit, range check, altitude check, stall rate, and various other limits attempt to simulate the pilot in this study. Second, the program could be used to evaluate the effectiveness against helicopters with a minimum program change. Third, the more important variables could be computer plotted, and fourth, a more realistic target could be employed if it was not predictable but random.

CHAPTER I

INTRODUCTION

Description of the Problem

The present gun effectiveness methodology used by the Air Force is only capable of assessing the damage inflicted on non-moving point and area targets [9]. This assessment is accomplished through the use of a computer simulation program which uses a Monte Carlo technique to determine the probability of hitting a rectangular target with one or more rounds from a single burst. It assumes that high rate-of-fire gunnery is a stationary Markov process and that the aiming and ballistic dispersions are independent in the range (along the flight path of the aircraft) and deflection (normal the flight path in the horizontal plane) coordinates. The effect of correlation of successive aim points is considered, and bi-variate normal aiming error and ballistic dispersion are assumed [1 and 2]. The strafing aircraft flies at a constant airspeed and dive angle from a specified open fire slant range.

It was not until recently that the correlation gun model was actually validated. In 1975, the Air Force conducted test firings at Nellis Air Force Base, Nevada, against potential ground targets. The results from the

test were compared to the correlation gun model simulations. The model was found to be somewhat conservative and was updated to produce more accurate simulations.

In actual combat the target will most likely be moving, and the aircraft will not be able to strafe with wings level and a constant dive for every encounter. There are gun simulation models currently in use that employ some of these tactics but they are limited to certain targets, such as trains, and the aircraft is limited to specific maneuvers associated with the target [4].

The role of air-to-ground gun effectiveness has been one of the major areas of concern during all of our recent past wars and conflicts. With the new technological advances being made in armament development we face the problem of developing simulation models that resemble actual combat engagements.

Research Objective

This research paper is the result of an improved methodology which is designed to fill in the gap that currently exists in air-to-ground gunnery methodology. It will assess the probability of kill on a moving ground target for high rate-of-fire guns in the air-to-ground role. The attacking aircraft will have complete maneuverability within the capabilities of the airframe. The ground target will have a given velocity, direction, and turn rate

specified at some initial engagement position.

The improved gun effectiveness mathematics will be developed and programmed in FORTRAN. The effectiveness results will be compared to the previously discussed validated air-to-ground gun model against similar targets. It has only been speculation in the past that ground targets cannot travel fast enough to evade an attacking aircraft. If this speculation should become a reality the mathematics and computer program will be designed so that it can be used to simulate effectiveness on targets such as helicopters or other aircraft flying near the ground in a level plane. The speed of these low flying targets could range from zero to some value which approaches the velocity of the pursuing aircraft.

Summary

This research paper is the result of an improved methodology to assess the probability of kill on a moving target for high rate-of-fire guns in the air-to-ground close air support role. The state of the art gun effectiveness methodology is only capable of assessing the damage inflicted on still point and area targets [9]. Chapter I begins by discussing the present gun methodology versus the improved methodology and ends with the research objective. Chapter II contains a description of two popular and widely accepted air-to-ground gun models. In Chapter III the mathematics

are developed for the new methodology described in Chapter I. The gun/aircraft unit is depicted in a dynamic and typical combat role of pursuing the evading ground target. In the final part of the chapter the probability of target hit/kill is calculated based on the strafing attack dynamics. Chapter IV describes the FORTRAN program that produces the effectiveness results, and Chapter V discusses the results. The final chapter discusses the new model with its limitations, validation, and recommendations for future research.

CHAPTER II

REVIEW OF EFFECTIVENESS STUDIES
AND COMPUTER MODELS

One of the most popular air-to-ground gun effectiveness models currently in use by the Air Force was developed by the Center for Naval Analyses, Operations Evaluation Group, Washington, D.C. [2]. This model was briefly described in the introduction in Chapter I. It employs a Monte Carlo technique which determines the probability of killing a stationary ground target from an aircraft equipped with a gun firing a single burst of a given number of rounds. The individual aimpoints cannot be specified in advance, therefore the program assumes that successive aimpoints are correlated. These aimpoints are normally distributed about the center of the target. Ballistic dispersion is also present, therefore the i^{th} round does not impact at the aimpoint, but at some point nearby. The stationary target is in the form of a rectangular projection of the real target on the plane which is normal to the line of flight of the attacking aircraft. The actual target will seldom or never be rectangular in shape, but the projected target can always be approximated by a rectangle. The aircraft flies directly at the target at some predetermined slant range, air speed,

and dive angle. During this time the gun is being fired at the target for some specified burst time in seconds. Most of the input parameters are explicitly defined and printed in the output. The probability of target kill is printed for each projectile fired. The end result is a cumulative probability of target kill [9]. An analyst must be thoroughly familiar with this program in order to properly interpret the results. First of all, the accuracy of the results depends upon the number of Monte Carlo iteration that has been specified. Some compromise must be made here between computer run time and desired accuracy. Second, the aircraft will fly near or into the ground if the burst time is too long for a short slant range. In this case the user must exercise caution in setting up the input parameters so that the aircraft will have sufficient altitude to pull up safely after the specified burst time has elapsed.

Another popular air-to-ground gun simulation program is the RAND model [4]. This air-to-ground model was developed by the Rand Corporation. It computes kill probabilities for forward-firing air-to-ground guns and rockets versus stationary targets. It employs two types of kill mechanisms: blast-fragmentation and penetration. When high explosives (HE) projectiles are simulated the blast-fragmentation portion of the program is used. The interaction between projectile upon impact and target is described by the Carlton damage function. If penetration is considered, the weapon-

target interaction is described by a kill probability conditional upon impacting the target presented area or vulnerable area. This program considers two types of targets: rectangular point targets, and rectangular area targets. Fixed guns are considered, but the aircraft can be assumed to be pulling up at a constant g force in order to allow the rounds to be aimed at different points in the ground plane. There is an option in the program to slew the guns in the azimuth plane. The strafing aircraft is assumed to dive or pull up at a constant angle, or constant g force. The aim and ballistic errors are assumed to be normally distributed. The simulation of the ballistic trajectories are straight line segments which are acceptable for rounds fired with high muzzle velocity from a high speed aircraft over slant ranges less than 5000 feet. This model is not as versatile as the previous air-to-ground gun model because: it does not have the capability to simulate a gun time-to-rate curve; it cannot evaluate a mixed round belt; i.e., an (HEI) high explosive incendiary, and an (API) armour piercing incendiary round mix in a single pass. It uses a constant conditional kill probability for an entire burst as opposed to one that changes as the aircraft/gun unit gets closer to the target. However, both models have the option to evaluate the effectiveness of a fuel sensitive target. An analyst desiring to evaluate gun effectiveness for point targets could use either model; however, area target effectiveness can only be evaluated by using the RAND model [4].

CHAPTER III

DEVELOPMENT OF METHODOLOGY

This air-to-ground gun effectiveness model depicts an aircraft in three dimensional space with the ability to maneuver in a dynamic role of pursuing an evading ground target. The ability to maneuver in an aerial engagement is very important along with many other parameters such as muzzle velocity, aim error, ballistic dispersion, fire control, gun rate of fire, projectile size, drag, and pilot performance. These and other parameters will be considered within the following methodology.

Mission Set-Up

The initial conditions for the engagement problem can be selected or computed. If the conditions are selected, time must be allowed for the pilot to fly through the sight setting conditions. This is a standard mission planning procedure. Assume the initial sighting conditions are 300 knots, 30 degree dive angle, 2500 feet slant range, and 5 seconds allowed for tracking time. The aircraft velocity in feet per second is

$$V_A = VACK * 1.6878$$

where V_{ACK} is the speed of the aircraft in knots. The initial airspeed [11] assuming thrust equals drag is given by integrating the following first order differential equation. (Since the independent variable is time, dot notation will be used to determine the order of the equation. This notation will be consistent throughout the report.)

$$\dot{V}_A = -g \sin\theta \quad [11]$$

where g is acceleration due to gravity only (32.1687 ft/sec), and θ is the dive angle, then

$$dv = -g \sin \theta dt.$$

Integrating both sides of the equation gives:

$$\int_{V_0}^{V_A} dv = -g \sin \theta \int_0^t dt$$

$$V_A - V_0 = -(g \sin \theta)t$$

$$V_0 = V_A + (g \sin \theta)t$$

which is the initial airspeed for a planned airspeed of V_A at open fire. For the planned conditions cited the initial

airspeed is

$$V_0 = 506.34 - 16.08 * 5.00$$

$$V_0 = 425.94 \text{ feet per second.}$$

Note that θ is negative indicating a dive angle. Then

$$R_0 = R_I + V_A * T_T$$

$$R_0 = 2500.0 + 506.34 * 5.0$$

$$R_0 = 5031.7 \text{ feet}$$

where R_0 is the initial range in feet, R_I is the input line of sight range in feet, and T_T is the tracking time in seconds. Therefore, the initial range of the aircraft to the target is the velocity of the aircraft times the tracking time allowed for nulling tracking errors, plus the input line of sight range.

Target Tracking Assessment

The traditional depressed reticle sight used in air-to-ground gunnery is a simple form of impact predictor. It displays to the pilot where a round will impact on the ground if:

1. Aircraft altitude relative to target is equal to a predetermined value
2. Dive angle is equal to a predetermined value
3. Roll angle is zero
4. Speed and weight are equal to a predetermined value
5. Lift acceleration is slightly less than one g
6. Side slip is zero
7. Cross wind is zero
8. Gun boresight errors are zero.

A skilled pilot develops a significant negative correlation among the various error sources. He is, for example, able to adjust his firing altitude to approximately compensate for the deviation from planned dive angle. The sight system, commonly called the "pipper" is offset laterally to compensate for non-zero roll angle. Sensitivities to deviations in angle-of-attack and side slip are known to the pilot and the aimpoint is shifted off the target by an amount proportional to his estimate of these deviations. Many other factors such as crosswind can be estimated with considerable effectiveness by a well trained pilot, however, due to the nature of this study crosswind will not be considered in this report. The sources of error are so numerous that accurate air-to-ground gunnery with a fixed sight system seems impractical, and yet skilled pilots can consistently put fifty percent or more of their rounds in a 400-square foot target traveling at speeds

up to 450 knots. Only two kinds of errors will be considered in this study: aim error, and ballistic dispersion. These two errors alone constitute an entire study and will not be derived or proven mathematically. These two errors will be inputs to the simulation program and based on actual test results.

When operating in a combat environment with modest defenses, the lengthy period of time required to set up the shot for practice range accuracy can be taken only at great risk to pilot and aircraft. This may, however, be the appropriate tactic to avoid the necessity for a repeated pass at the same target. Subsequent passes can be used to improve the pilot's accuracy, but has a similar, if not greater, benefit to the ground defenses. Accuracy cannot be realistically separated from the risk that is incurred in achieving that accuracy. Air-to-ground gunnery improvements should therefore result in longer range accuracy, as well as a reduced amount of time that must be committed to a specific target, a reduced amount of time spent on a predictable path, and a flexibility in approach tactics.

The location of the pipper on the sighting system is given by the sum of the gun bore axis angle, trajectory shift, gravity drop, and the correction for the vertical displacement of gun. See Figure III-1 for a description of the sight depression geometry.

The sight depression for the planned firing condition

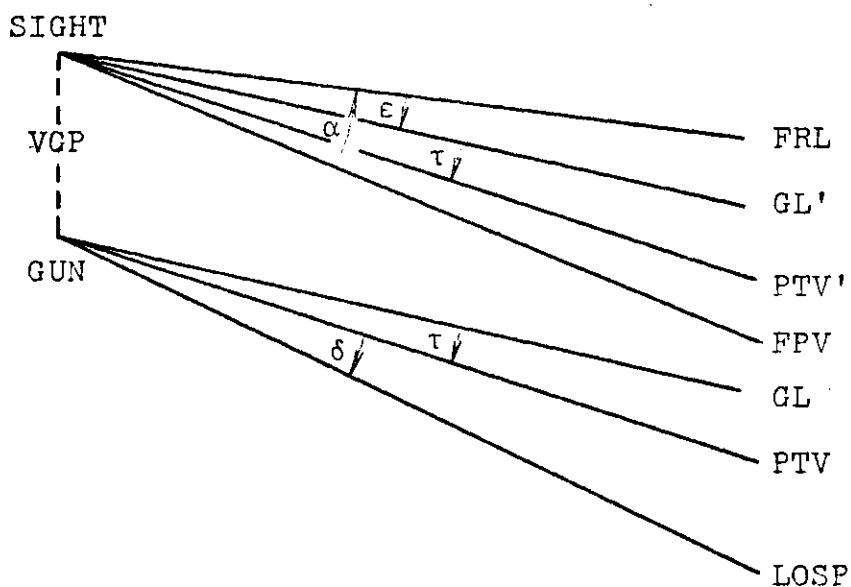


Figure III-1. Sight Depression Geometry [6]

FRL - fuselage reference line

GL - gun bore axis

PTV - projectile total velocity vector

FPV - aircraft flight path vector

GL' - line parallel to gun bore axis

VGP - vertical gun position W.R.T. sight in feet

PTV' - line parallel to the projectile total velocity vector

LOSP - line of sight to projectile at impact

is

$$SD = \epsilon + \tau + \delta + CVD$$

where

ϵ = angle between FRL and GL' (rad)

τ = trajectory shift (rad)

δ = angular gravity drop (rad)

CVD = correction for vertical displacement of gun (feet)

and $CVD = VGP/R_0$.

The angular gravity drop δ is calculated by computing a projectile trajectory for the planned conditions. Delta is calculated in the gun/projectile parameters section. The trajectory shift due to gun bore angle of attack is

$$\tau = V_A(\alpha - \epsilon) / (V_A + V_M) \quad [13].$$

The angle τ is the angle between the gun bore and the initial velocity vector of the projectile. V_A is the velocity for the planned firing condition, V_M is the gun muzzle velocity, and α is the angle of attack of the FRL. Assuming the aircraft is in non-accelerating flight; i.e., Force [thrust] = Force [drag] then α is calculated as follows:

$$\alpha = \frac{W \cos \theta - QSA C_{L\alpha} \alpha_{OL}}{QSAC_{L\alpha} + T} \quad [6]$$

where

$$Q_{SA} = 0.5 \rho V_A^2 S$$

W = weight of the aircraft (pounds)

S = aerodynamic reference area for aircraft (ft^2)

C_{La} = coefficient of lift versus α slope (per degree)

α_{OL} = angle of attack at zero lift (degrees)

ρ = air density (slugs)

The air density is given by the following [3],

$$\rho = 0.00237692 \left[\frac{518.688 - 0.00356616 |Z_A|}{518.688} \right]^{4.2561}$$

and $|Z_A|$ is the absolute value of the altitude of the aircraft.

The thrust to counteract drag at the initial airspeed is given by

$$T = D = 0.5 \rho V_0^2 S C_D \quad [3]$$

where C_D is the aircraft coefficient of drag.

In attacking the target the aircraft makes a tracking pass at the target with a bank to the right or left depending on the azimuth position of the target. While tracking the target through the sight system the angular position of the target in sight or body coordinates is determined. The line of sight range to the target is

$$R = [(X_T - X_A)^2 + (Y_T - Y_A)^2 + (Z_T - Z_A)^2]^{1/2}$$

where X_T , Y_T , Z_T and X_A , Y_A , Z_A are the coordinates of the target and aircraft respectively. The line of sight range will be calculated periodically at some small time increment into the engagement. This iterative process will be updated by a specified time increment and used as a check to determine the gun open-fire range. This range check will be discussed later in more detail.

The orientation in space is given by the transformation expressing body-fixed coordinates in terms of earth-fixed coordinates. The line of sight vector (\hat{r}) is as follows:

$$\hat{r} = r_1 \hat{i} + r_2 \hat{j} + r_3 \hat{k}$$

where

$$r_1 = \frac{X_T - X_A}{R}$$

$$r_2 = \frac{Y_T - Y_A}{R}$$

$$r_3 = \frac{Z_T - Z_A}{R}$$

and the line of sight unit vector in the aircraft body axis system is,

$$\hat{\mathbf{r}}_A = X_L \hat{\mathbf{i}}_B + Y_L \hat{\mathbf{j}}_B + Z_L \hat{\mathbf{k}}_B$$

where X_L , Y_L , and Z_L are transformed from r_1 , r_2 , r_3 by

$$\begin{vmatrix} X_L \\ Y_L \\ Z_L \end{vmatrix} = \begin{vmatrix} AX_1 & AX_2 & AX_3 \\ AY_1 & AY_2 & AY_3 \\ AZ_1 & AZ_2 & AZ_3 \end{vmatrix} \begin{vmatrix} r_1 \\ r_2 \\ r_3 \end{vmatrix} \quad [13]$$

where

$$AX1 = \cos(\alpha) \cos(\theta) \cos(\psi) - \sin(\alpha) (\cos(\phi) \sin(\theta) \cos(\psi) + \sin(\phi) \sin(\psi))$$

$$AX2 = \cos(\alpha) \cos(\theta) \sin(\psi) - \sin(\alpha) (\cos(\phi) \sin(\theta) \sin(\psi) - \sin(\phi) \cos(\psi))$$

$$AX3 = -\cos(\alpha) \sin(\theta) - \sin(\alpha) \cos(\phi) \cos(\theta)$$

$$AY1 = \sin(\phi) \sin(\theta) \cos(\psi) - \cos(\phi) \sin(\psi)$$

$$AY2 = \sin(\phi) \sin(\theta) \sin(\psi) + \cos(\phi) \cos(\psi)$$

$$AY3 = \sin(\phi) \cos(\theta)$$

$$AZ1 = \sin(\alpha) \cos(\theta) \cos(\psi) + \cos(\alpha) (\cos(\phi) \sin(\theta) \cos(\psi) + \sin(\phi) \sin(\psi))$$

$$AZ2 = \sin(\alpha) \cos(\theta) \sin(\psi) + \cos(\alpha) (\cos(\phi) \sin(\theta) \sin(\psi) - \sin(\phi) \cos(\psi))$$

$$AZ3 = -\sin(\alpha) \sin(\theta) + \cos(\alpha) \cos(\phi) \cos(\theta)$$

Next the target angular coordinates in the field of view along the aircraft X axis is

$$EL = \text{ASIN}(Z_L) \text{ and } AZ = \text{ATAN} \left[\frac{Y_L}{X_L} \right], [12]$$

when EL and AZ are elevation and azimuth respectively.

The azimuth and elevation as computed in the previous section are graphically displaced in Figure III-2, and the target may be displaced from the pipper as shown.

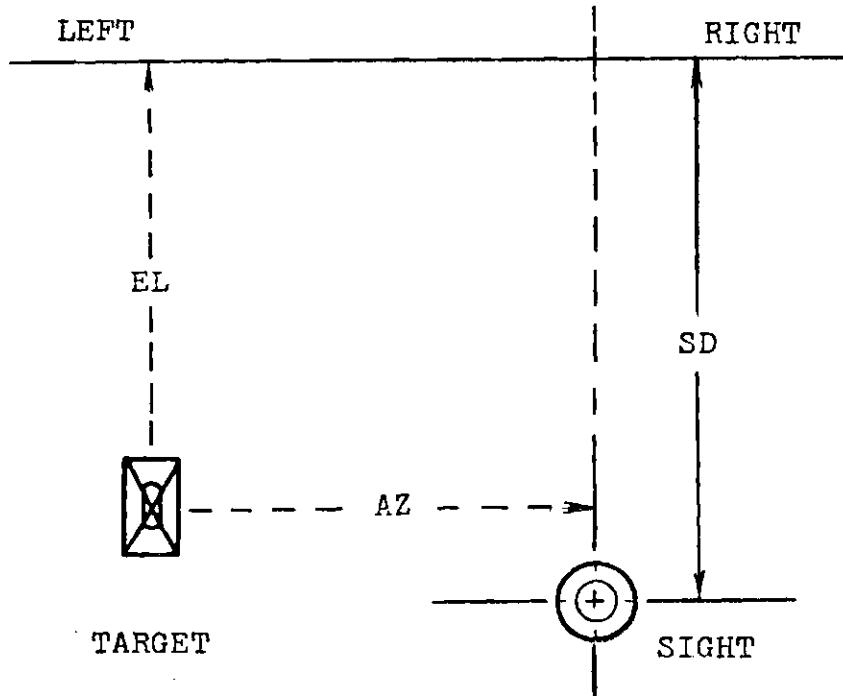


Figure III-2. View of Target Through Sight

The aircraft bank angle is used to null the azimuth error. The amount of the bank angle increment should be proportional to the azimuth error. The maximum bank angle should be limited by the maximum bank rate which can be

assumed to be on the order of 180° per second. The maximum roll or bank increment is given by,

$$\Delta\phi_{MAX} = \dot{\phi}_{MAX} * \Delta t,$$

where Δt is the time increment of the engagement, and the bank increment is,

$$\Delta\phi = C_1 * AZ$$

C_1 is the proportionality or bank angle gain constant. The commanded bank angle is,

$$\phi_i = \phi_{i-1} + \Delta\phi$$

where i = present value, and $i-1$ is the previous value. The bank increment ($\Delta\phi$) is set so that it cannot exceed the value $\Delta\phi_{MAX}$.

The tracking task in elevation is accomplished by commanding an angle of attack differential proportional to the elevation tracking error. In accomplishing this task the aircraft cannot exceed angle of attack limitations as defined by structural load factor and aerodynamic stall. The following equations are used to determine the angle of attack limits. Lift is defined as

$$L = 1/2 \rho V_A^2 S C_L$$

and the coefficient of lift C_L is

$$C_L = C_{L\alpha} * \alpha + C_{L0}$$

where C_{L0} is the coefficient of lift at $\alpha = 0$. The aircraft load factor η is

$$\eta = \frac{L + T*\alpha}{W} [13].$$

The angle of attack at aerodynamic stall is defined as

$$\alpha_s = \frac{C_{LMAX}}{C_{L\alpha} + \alpha_{OL}} .$$

The structural load factor (η_s) is an input quantity which the maximum load factor must not exceed to insure structural integrity of the airframe. The maximum allowable angle of attack (α_{ST}) for structural limitations is

$$\alpha_{ST} = \frac{\eta_s * W}{1/2 \rho V_A^2 S (C_{L\alpha} + C_{L0})}$$

The commanded angle of attack cannot exceed either α_s or α_{ST} .

The commanded angle of attack is defined as

$$\alpha_i = \alpha_{i-1} + \Delta\alpha$$

where $\Delta\alpha$ is the commanded increment determined by

$$\Delta\alpha = SD - EL.$$

If the engagement integration interval is small the change in α must be limited by a maximum angle of attack rate ($\dot{\alpha}_{MAX}$). The maximum rate is

$$\dot{\alpha}_{MAX} = \eta * W / [1/2 \rho V_A^2 S (C_{L\alpha} + C_{LO})].$$

Thus $\Delta\alpha$ cannot exceed $\Delta\alpha_{MAX}$ defined by

$$\Delta\alpha_{MAX} = \dot{\alpha}_{MAX} * \Delta t.$$

Aircraft Flight Dynamics

The maneuvering stability is the static stability of an aircraft undergoing normal accelerations. In order to fully define the problem of stability two reference axes will be introduced, the body axes and the wind axes systems. These two axis systems form the basis of a system of notation used to describe the motions of an airplane. Each system consists of three mutually perpendicular axes passing through the center of gravity of the airplane, which adequately covers most of the aerodynamic problems in stability

considerations.

The body axes system is fixed in the airplane and is the system of mutually perpendicular axes passing through the aircraft's center of gravity and whose X-axis is parallel to the thrust axis, the wing mean aerodynamic chord, or some other longitudinal reference and is positive in the direction of the nose of the aircraft. Figure III-3 shows this reference system.

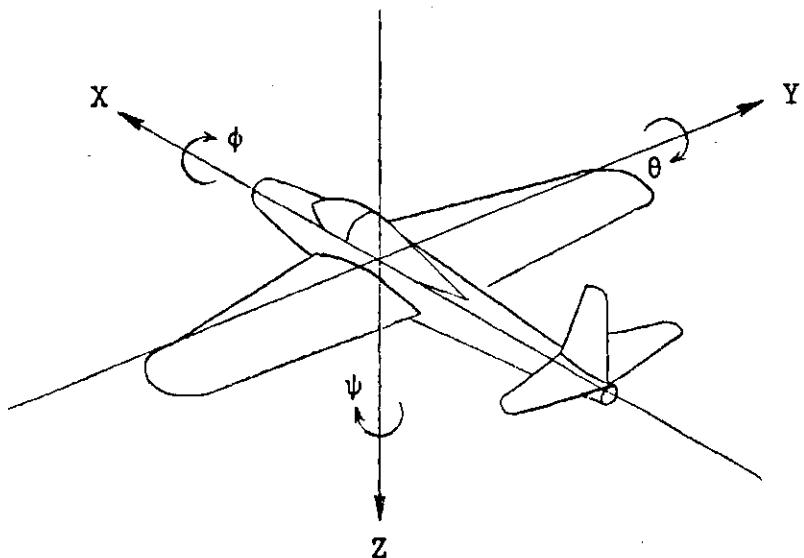


Figure III-3. Airplane Body Axes System

The wind axes system differs from the body axes system in that the X-axis is parallel to the relative wind, positive backward. This Z-axis is positive down, and the

Y-axis is positive to the right; and the Y-force, as before, is the side force. Now the X, Y, Z velocity components represent the components of the relative wind. The wind axes system is the one used for basic aerodynamic performance work. In most cases the body axes system is used in stability work. The wind axes system is the one used for basic aerodynamic performance work. In most cases the body axes system is used in stability work. The wind axes system can be seen in Figure III-4.

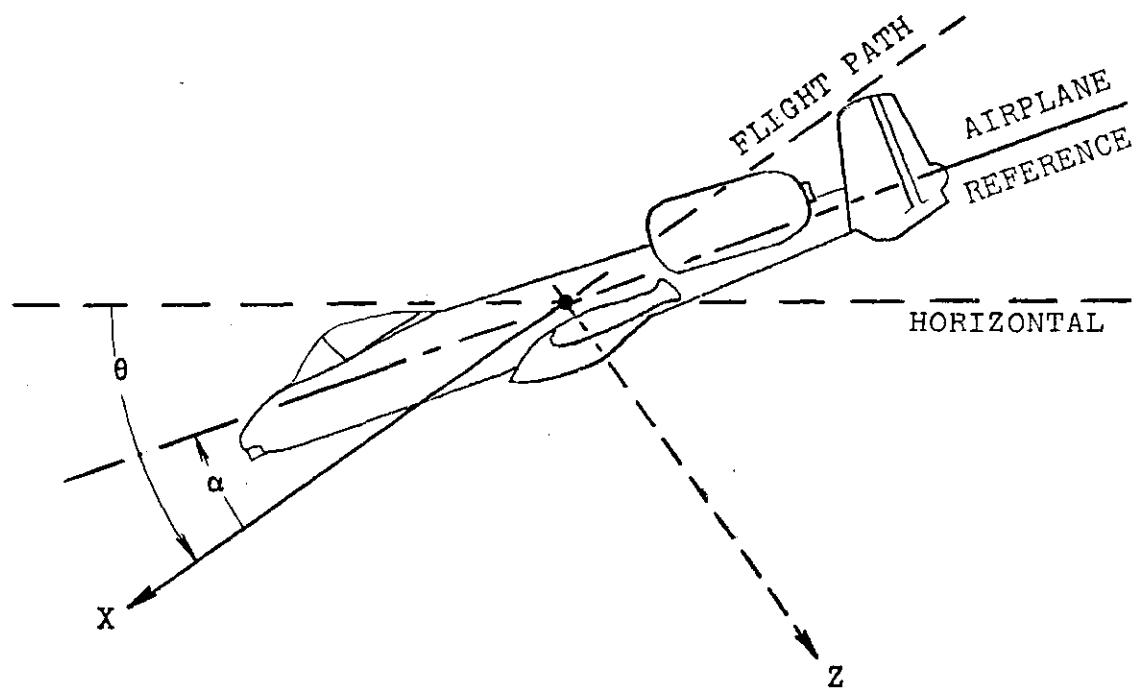


Figure III-4. Wind Axes System

The wind axes equations [5] are as follows:

$$T_X - D - mg \sin\theta = m\dot{V}_A$$

$$T_Y - C + mg \cos\theta \sin\phi = mV_A(\dot{\psi} \cos\theta \cos\phi - \dot{\theta} \sin\phi) \quad [5]$$

$$T_Z - L + mg \cos\theta \cos\theta = mV_A(\dot{\theta} \cos\phi + \dot{\psi} \cos\theta \sin\phi) \quad [5]$$

where T_X , T_Y , T_Z are the components of thrust in the aircraft body axes, and D is the drag force, C is side force, L is lift force, and $m = W/g$.

Note: For most purposes T_Y and C are zero, and for unaccelerated flight $T_X = D$.

For the first wind axes equation

$$\dot{V}_A = -g \sin\theta$$

which is the acceleration due to gravity only. θ is negative for dives so \dot{V}_A has a positive acceleration. From the second and third wind axes equations we have,

$$\dot{\theta} = \frac{G}{V_A} \left\{ \left[\frac{(C_{L\alpha} + C_{L0})(.5\rho V_A^2 S)}{W} \right] \cos\phi - \cos\theta \right\}$$

which is the first derivative for the aircraft dive angle.

And

$$\dot{\psi} = \frac{ng \sin\phi}{V_A \cos\theta},$$

which is the first derivative for the aircraft heading. n is the g's that the aircraft will pull during the attack maneuver and was calculated in the target tracking section.

The aircraft velocity vector components are

$$\dot{x} = V_A \cos\theta \cos\psi$$

$$\dot{y} = V_A \cos\theta \sin\psi$$

$$\dot{z} = -V_A \sin\theta$$

Then integrating $\dot{\theta}$, $\dot{\psi}$, \dot{x} , \dot{y} , and \dot{z} will yield θ , ψ , x_A , y_A , and z_A after one time interval. The motion of the center of gravity is described in terms of changes in heading, vertical inclination, and magnitude of the airplane's velocity vector relative to the ground. The magnitude of this vector is V_A . θ and z_A should be checked against a dive recovery table which shows the altitude lost during a recovery maneuver from a given dive angle. If the current altitude z_A is less than the value in the table, then terminate the engagement; i.e., cease fire and determine the probability of target kill at this point in time.

Time is updated by the following,

$$t_i = t_{i-1} + \Delta t$$

where t_i gives the present time, t_{i-1} is the previous time value, and Δt is the time increment in seconds.

Gun-Projectile Parameters

The gun muzzle velocity vector in the aircraft body axes is

$$\vec{v}_M = V_M \cos(\epsilon) \hat{i}_B + V_M \sin(\epsilon) \hat{k}_B$$

where ϵ is the gun bore angle below the fuselage reference line. Now we need the components of the muzzle velocity vector \vec{v}_M in the earth axes. The transformation for this is,

$$\begin{vmatrix} v_{MX} \\ v_{MY} \\ v_{MZ} \end{vmatrix} = \begin{vmatrix} AX1 & AY1 & AZ1 \\ AX2 & AY2 & AZ2 \\ AX3 & AY3 & AZ3 \end{vmatrix} \begin{vmatrix} V_M \cos(\epsilon) \\ 0 \\ V_M \sin(\epsilon) \end{vmatrix} \quad [13]$$

where the components AX1 through AZ3 were defined in the target tracking section. The projectile initial velocity components are,

$$V_{PT} = (V_{PX}^2 + V_{PY}^2 + V_{PZ}^2)^{1/2}$$

where

$$v_{Px} = \dot{x}_A + v_{Mx}$$

$$v_{Py} = \dot{y}_A + v_{My}$$

$$v_{Pz} = \dot{z}_A + v_{Mz}$$

The gun position is described in terms of the sight system in the target tracking assessment section of this report. Therefore, the initial position of the projectile must be shifted by an amount proportional to the vertical gun placement. This is accomplished by

$$x_p = x_A + VGP * AZ1$$

$$y_p = y_A + VGP * AZ2$$

$$z_p = z_A + VGP * AZ3$$

when AZ1, AZ2, and AZ3 are defined in the target tracking section. Now x_p , y_p , z_p is the corrected initial position of the projectile. Geometrically this is represented in Figure III-5.

The projectile trajectory equations are as follows,

$$\ddot{x}_p = -\frac{\pi}{8} \rho v_p d^2 \dot{x}_p \quad [12]$$

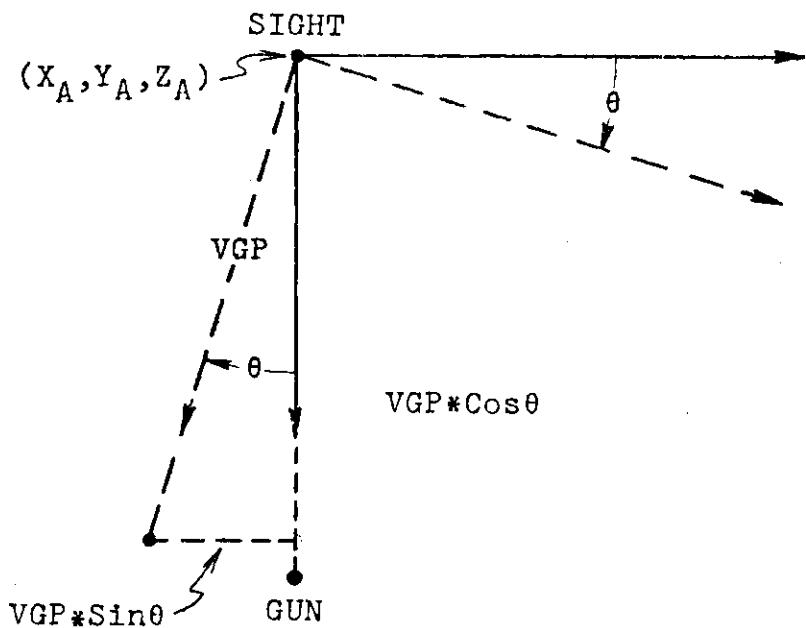


Figure III-5. Projectile Initial Position, $\psi=0$, $\phi=0$

$$\ddot{Y}_P = -\frac{\pi}{8} \rho V_P d^2 \dot{Y}_P \quad [12]$$

$$\ddot{Z}_P = -\frac{\pi}{8} \rho V_P d^2 \dot{Z}_P + g \quad [12]$$

where d is the diameter of the projectile in feet. Now integrate the three preceding equations until Z_P is equal to zero. The corresponding (X_P, Y_P) is the projectile impact point. The velocity of the projectile at impact is,

$$V_P = (\dot{X}_P^2 + \dot{Y}_P^2 + \dot{Z}_P^2)^{1/2}$$

and the slant range is,

$$SR = [(X_A - X_P)^2 + (Y_A - Y_P)^2 + (Z_A - Z_P)^2]^{1/2}$$

and the projectile gravity drop is defined as

$$\delta = ACOS \left[\frac{V_{PX}X_P + V_{PY}Y_P + V_{PZ}Z_P}{V_{PT} * SR} \right] \quad [12]$$

X_P, Y_P is an ideal aimpoint or impact point. The location of the coordinate (X_P, Y_P) is a function of the dynamics of the engagement as modeled. In general the coordinates $(X_{P1}, Y_{P1}) \dots (X_{Pn}, Y_{Pn})$ could be generated as indicated in Figure III-6.

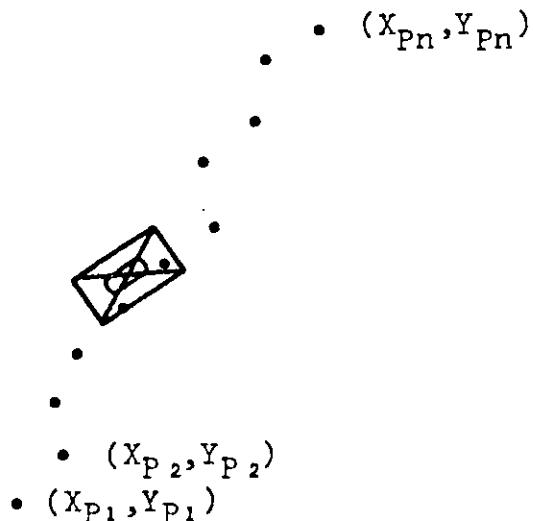


Figure III-6. Generated Impact Points

Target Model

When the attack engagement begins the target is placed anywhere in the X, Y plane with one of two options for evasive tactics. The first option is exercised by giving the target a constant velocity and direction. V_T is the velocity of the target, and θ_T is the direction of the velocity. Orientation of the target is determined by θ_T which starts at any specified coordinate position and progressively increases to the angle desired. This option is graphically explained in Figure III-7.

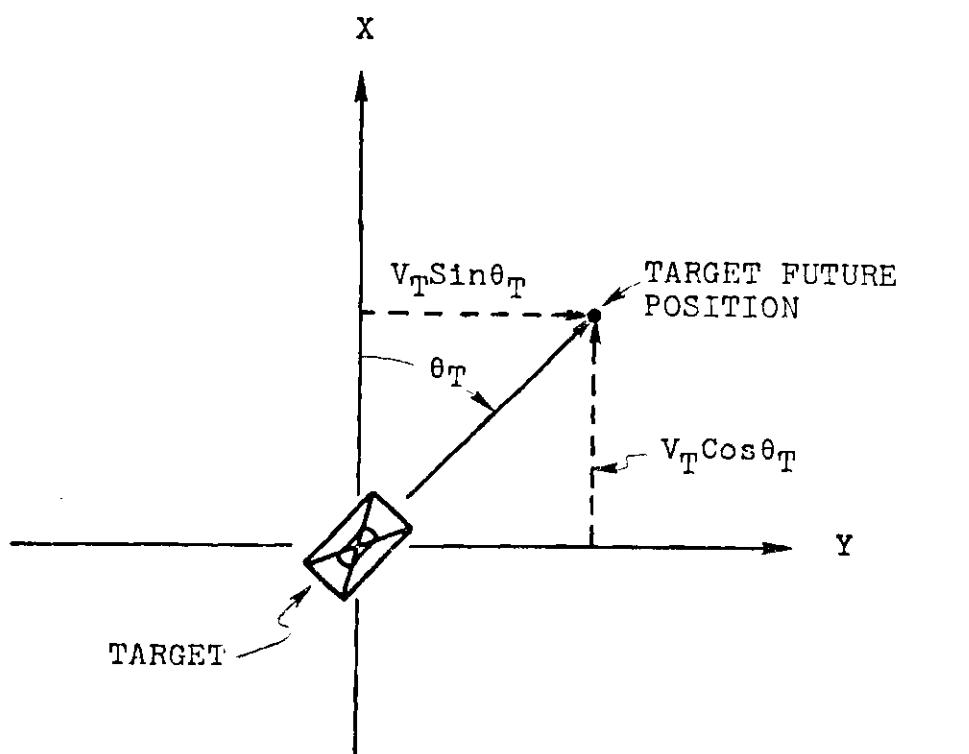


Figure III-7. Linear Target Model

The initial position of the target is defined as (X_{T0}, Y_{T0}) , and any position thereafter can be defined by knowing the direction (θ_T), velocity (V_T), and time into the engagement. Mathematically, velocity is constant and is defined as below,

$$\dot{X}_T = V_T \cos \theta_T$$

$$\dot{Y}_T = V_T \sin \theta_T .$$

Then the initial position to some time in the future the target can be located by integrating the above equations to give,

$$X_T = X_{T0} + V_T \cos \theta_T * t$$

$$Y_T = Y_{T0} + V_T \sin \theta_T * t .$$

The second option allows the target to turn in any direction from any orientation. The initial position of the target is also defined as (X_{T0}, Y_{T0}) , and ω_T is the turn rate. V_T is the velocity, and θ_T is the direction of the target velocity. The orientation in the X, Y coordinate system is the same as the previously described linear model. This is shown in the following Figure on Target Turn Geometry.

The mathematics which enables one to determine the target position is developed as follows,

$$\dot{\theta}_T = \omega_T \quad [11]$$

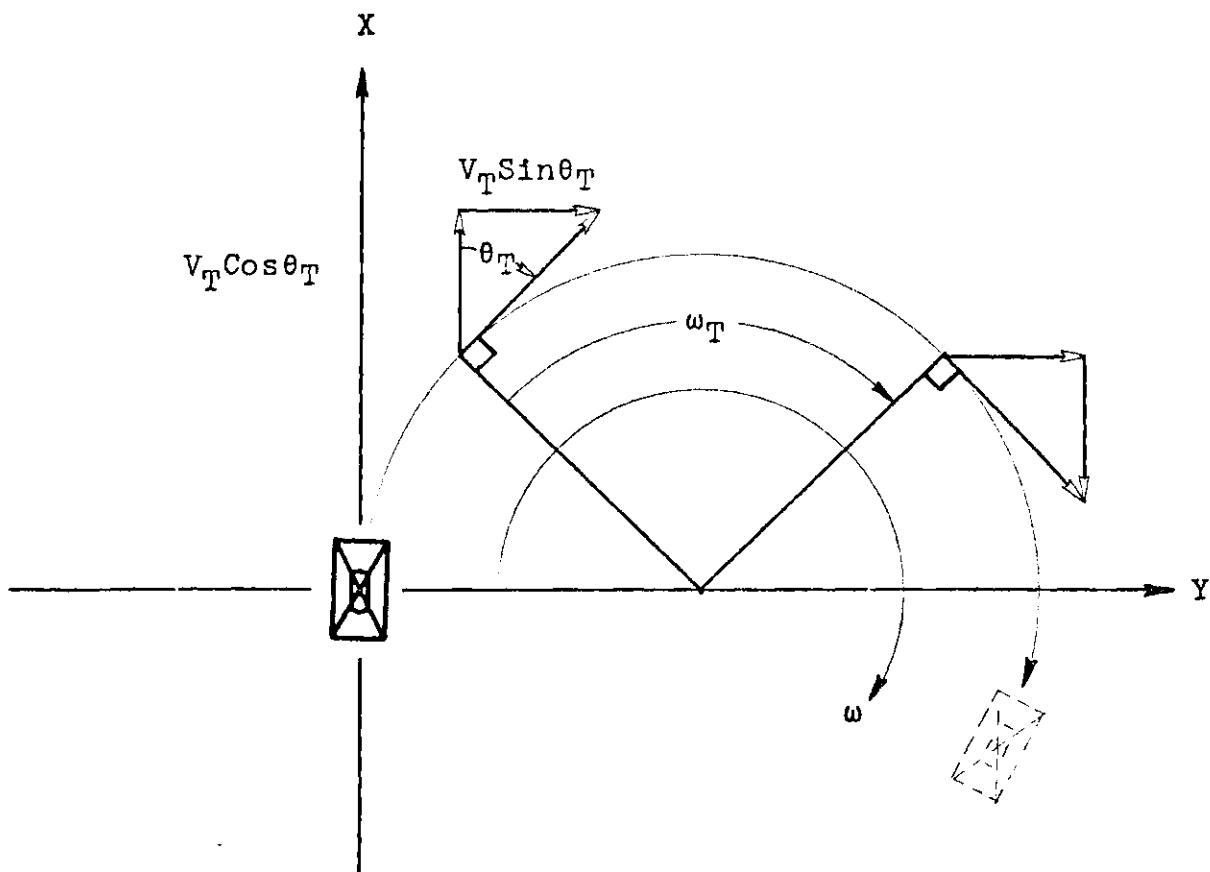


Figure III-8. Target Turn Geometry

where ω is defined as the angular velocity. If θ_T is measured in radians and t in seconds, ω_T is expressed in radians per second. When the angular velocity is constant, the angular displacement of the target in time (t) is given by integrating both sides of the previous equation as follows,

$$d\theta_T = \omega_T dt$$

$$\int_{\theta_{TO}}^{\theta_T} d\theta_T = \omega_T \int_{t=0}^t dt$$

$$\theta_T - \theta_{TO} = \omega_T t$$

$$\theta_T = \theta_{TO} + \omega_T t,$$

and the time rate of change for the X coordinate is

$$\dot{x}_T = V_T \cos(\theta_{TO} + \omega_T t).$$

Now this equation can be integrated to find the X position of the target for any time into the engagement by

$$\int_{x_{TO}}^{x_T} dx = \int_0^t V_T \cos(\theta_{TO} + \omega_T t) dt$$

$$x_T - x_{TO} = \frac{V_T}{\omega_T} \sin(\theta_{TO} + \omega_T t) \Big|_0^t$$

$$x_T - x_{TO} = \frac{V_T}{\omega_T} [\sin(\theta_{TO} + \omega_T t) - \sin \theta_{TO}]$$

$$x_T = x_{TO} + \frac{V_T}{\omega_T} [\sin(\theta_{TO} + \omega_T t) - \sin \theta_{TO}]$$

The Y position is found similarly using the definition of \dot{x}_T from the linear model with time and velocity incorporated.

$$\dot{y}_T = V_T \sin(\theta_{TO} + \omega_T t)$$

Integrate both sides of the equation to find the Y position of the target for any time into the engagement by

$$\int_{Y_{TO}}^{Y_T} dy = \int_0^t V_T \sin(\theta_{TO} + \omega_T t) dt$$

$$Y_T - Y_{TO} = -\frac{V_T}{\omega_T} \cos(\theta_{TO} + \omega_T t) \Big|_0^t$$

$$Y_T - Y_{TO} = \frac{V_T}{\omega_T} \left[\cos \theta_{TO} - \cos(\theta_{TO} + \omega_T t) \right]$$

$$Y_T = Y_{TO} + \frac{V_T}{\omega_T} \left[\cos \theta_{TO} - \cos(\theta_{TO} + \omega_T t) \right].$$

Projectile/Target Encounter

Miss Distance with Respect to Target Normal to Trajectory

The mathematical flow returns to the target section to determine the target position at projectile impact.

Where the target coordinate is (X_T, Y_T) , and the projectile coordinate is (X_P, Y_P) and the miss distance is,

$$X_{MISS} = X_T - X_P$$

$$Y_{MISS} = Y_T - Y_P$$

The X and Y projectile miss coordinates are rotated to a plane normal to the line of sight plane as described in Figure III-9.

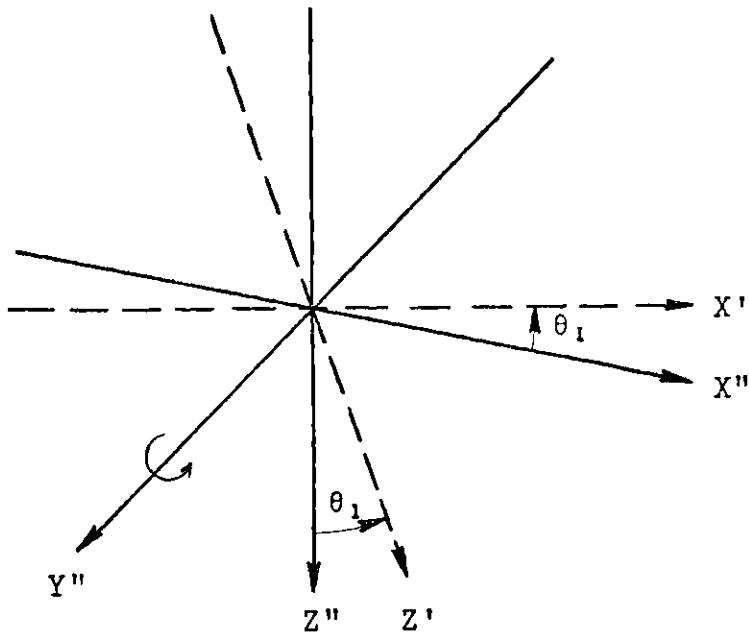


Figure III-9. Projectile Impact Position Rotated Normal to the Dive Angle Theta

The new angle of rotation is described as

$$\theta_1 = \pi/2 - \text{ASIN} \left[\frac{\dot{z}_P}{v_P} \right]$$

then in matrix form

$$\begin{vmatrix} X' \\ Y' \\ Z' \end{vmatrix} = \begin{vmatrix} \cos\theta_1 & 0 & -\sin\theta_1 \\ 0 & 1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 \end{vmatrix} \begin{vmatrix} X'' \\ Y'' \\ Z'' \end{vmatrix}$$

After the dive angle rotation we must consider the heading angle (ψ), by rotating it to the normal. Graphically this is shown in Figure III-10. The new heading angle of rotation is described as

$$\psi_1 = \text{ATAN} \left[\frac{\dot{Y}_P}{\dot{X}_P} \right]$$

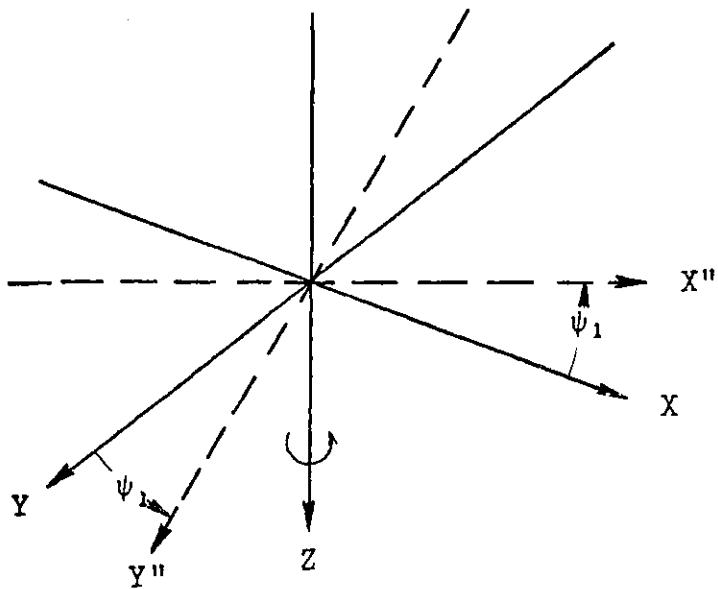


Figure III-10. Projectile Impact Position Rotated Normal to the Heading Angle (ψ)

then written in matrix notation

$$\begin{vmatrix} X'' \\ Y'' \\ Z'' \end{vmatrix} = \begin{vmatrix} \cos\psi_1 & \sin\psi_1 & 0 \\ -\sin\psi_1 & \cos\psi_1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} X \\ Y \\ Z \end{vmatrix}$$

The matrix multiplication is then carried out as follows,

$$\begin{vmatrix} X' \\ Y' \\ Z' \end{vmatrix} = \begin{vmatrix} \cos\theta_1 & 0 & -\sin\theta_1 \\ 0 & 1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 \end{vmatrix} \begin{vmatrix} \cos\psi_1 & \sin\psi_1 & 0 \\ -\sin\psi_1 & \cos\psi_1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} X \\ Y \\ Z \end{vmatrix}$$

$$\begin{matrix} XMISS' \\ YMISS' \\ ZMISS' \end{matrix} = \begin{vmatrix} X' \\ Y' \\ Z' \end{vmatrix} = \begin{vmatrix} \cos\theta_1 \cos\psi_1 & \cos\theta_1 \sin\psi_1 & -\sin\theta_1 \\ -\sin\psi_1 & \cos\psi_1 & 0 \\ \sin\theta_1 \cos\psi_1 & \sin\theta_1 \sin\psi_1 & \cos\theta_1 \end{vmatrix} \begin{matrix} XMISS \\ YMISS \\ ZMISS \end{matrix}$$

Then the transformation is complete yielding X' , Y' , Z' as follows,

$$\begin{aligned} X' &= XMISS \cdot \cos\theta_1 \cos\psi_1 + YMISS \cdot \cos\theta_1 \sin\psi_1 \\ Y' &= -XMISS \cdot \sin\psi_1 + YMISS \cdot \cos\psi_1 \\ Z' &= XMISS \cdot \sin\theta_1 \cos\psi_1 + YMISS \cdot \sin\theta_1 \sin\psi_1. \end{aligned}$$

Now X' , Y' , Z' is the projectile impact point rotated normal to the target.

Target Presented Area

The target presented area is the area of the target viewed from any direction normal to the projectile velocity at impact. The areas of the front side and top of the target will be A_F , A_S , A_T , respectively. The unit vector components which define the presented area are as follows

$$\mu_1 = \frac{\dot{x}_P \cos(\text{TGHD}) + \dot{y}_P \sin(\text{TGHD})}{v_P}$$

$$\mu_2 = \frac{\dot{y}_P \cos(\text{TGHD}) - \dot{x}_P \sin(\text{TGHD})}{v_P}$$

$$\mu_3 = \frac{\dot{z}_P}{v_P}$$

where the target heading is TGHD, and is defined as

$$\text{TGHD} = \omega_T * t + \theta_{TO}.$$

Then the presented area of the target is,

$$A_P = |A_F * \mu_1| + |A_S * \mu_2| + |A_T * \mu_3| \quad [12]$$

Target Vulnerable Area

The vulnerable area of a target is the area that, if struck, with a projectile of sufficient velocity will yield

a target kill. The ratio of vulnerable area to presented area is defined as the probability of kill, given a hit, as follows,

$$P_{K/H} = \frac{A_V}{A_P}.$$

The vulnerable area of a target in most cases is defined as a function of the projectile impact angle, and the projectile impact velocity. These data can be obtained from tables for any specified target. Due to the nature and classification of these data none of the vulnerable area tables will be published with this report in order to keep it unclassified.

Probability of Target Hit/Kill

The single shot hit probability is determined by using the classical probability equations which assume a bi-variate normal distribution. The mean point of projectile impact is located at (X_{AI}, Y_{AI}) relative to the target with a standard deviation $(\sigma_{XB}, \sigma_{YB})$. The probability that a single round will hit the target given (X_{AI}, Y_{AI}) is:

$$\text{PHSSA} = \frac{1}{2\pi\sigma_{XB}\sigma_{YB}} \int_{-WD}^{WD} \int_{-LE}^{LE} \exp \left\{ -\frac{1}{2} \left[\left(\frac{X_T - X_{AI}}{\sigma_{XB}} \right)^2 + \left(\frac{Y_T - Y_{AI}}{\sigma_{YB}} \right)^2 \right] \right\} dx dy \quad [7]$$

where WD and LE are the half target width and length.

The previous probability equation can be integrated by using the error function, defined as,

$$\text{ERF}(X) = \frac{2}{\sqrt{\pi}} \left(X - \frac{X^3}{1!3} + \frac{X^5}{2!5} - \frac{X^7}{3!7} + \dots - \dots \right) [8]$$

therefore using the ERF function PHSSA is given by

$$\begin{aligned} \text{PHSSA} = & \frac{1}{4} \left\{ \left[\text{ERF} \left(\frac{\text{LE}-X_{AI}}{2\sigma_{XB}} \right) + \text{ERF} \left(\frac{\text{LE}-X_{AI}}{2\sigma_{XB}} \right) \right] \right. \\ & \left. \left[\text{ERF} \left(\frac{\text{WD}-Y_{AI}}{2\sigma_{YB}} \right)^2 + \text{ERF} \left(\frac{\text{WD}-Y_{AI}}{2\sigma_{YB}} \right)^2 \right] \right\} \end{aligned}$$

and the probability density function for determining whether the burst pattern mean point of impact falls between X and Y + dx and Y and Y + dy is

$$\begin{aligned} f_{AI}(X_{AI}, Y_{AI}) = & \frac{1}{2\pi\sigma_{X_{AI}}\sigma_{Y_{AI}}} \exp \left\{ -\frac{1}{2} \left[\left(\frac{X_{AI}-X'}{\sigma_{X_{AI}}} \right)^2 \right. \right. \\ & \left. \left. + \left(\frac{Y_{AI}-Y'}{\sigma_{Y_{AI}}} \right)^2 \right] \right\} [7] \end{aligned}$$

This equation is also called the aiming error probability

density function, which is bi-variate normal, and uncorrelated with a mean at (X', Y') . These coordinates locate the center of the distribution with respect to the target and are components of minimum miss.

The probability (PHSS) of a single round impacting the target when aiming error and ballistic dispersion are involved is

$$\text{PHSS} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{AI}(X, Y) \text{PHSSA}(X, Y) dx dy.$$

This equation can be solved by using the Hermite-Gauss quadrature method [7]. This technique is used to evaluate integrals in the form

$$\int_{-\infty}^{\infty} e^{-X^2} f(X) dx$$

by approximating the integral as

$$\sum_{i=0}^{M} w_i f(x_i)$$

The function $f(x_i)$ is evaluated with x_i values which are the roots of the Hermite polynomial of degree $n + 1$. This numerical technique is used to evaluate PHSSA for each aimpoint (x_i) weighting (w_i) by the probability of the aimpoint occurring, then summing the products for all aimpoints [7].

PHSS can be expressed in the Hermite-Gauss integral form by separating the variables as,

$$\begin{aligned}
 \text{PHSS} = & \frac{1}{8\pi\sigma_X \sigma_{Y_{AI}}} \int_{-\infty}^{\infty} \left[\text{ERF}\left(\frac{LE-X_{AI}}{\sqrt{2}\sigma_{XB}}\right) + \text{ERF}\left(\frac{LE+X_{AI}}{\sqrt{2}\sigma_{XB}}\right) \right] \\
 & \text{EXP}\left[-\frac{1}{2}\left(\frac{X_{AI}-X'}{\sqrt{2}\sigma_{X_{AI}}}\right)^2\right] dx_{AI} \quad \int_{-\infty}^{\infty} \left[\text{ERF}\left(\frac{WD-Y_{AI}}{\sqrt{2}\sigma_{YB}}\right) \right. \\
 & \left. + \text{ERF}\left(\frac{WD+Y_{AI}}{\sqrt{2}\sigma_{YB}}\right) \text{EXP}\left[-\frac{1}{2}\left(\frac{Y_{AI}-Y'}{\sqrt{2}\sigma_{Y_{AI}}}\right)^2\right] \right] dy_{AI} \quad [7]
 \end{aligned}$$

Now apply the Hermite-Gauss approximation to the above equation. This will yield

$$\text{PHSS} = \frac{1}{\pi} \sum_{j=1}^n \sum_{k=1}^n w_j w_k \text{PHSSA}(x_j, y_k).$$

PHSSA is multiplied by the probability of target kill, given a hit to yield the single shot probability of kill given an aimpoint.

$$\text{PKSSA} = \text{PHSSA} * P_{K/H}$$

For a given number of rounds (NR) the probability of survival (PSA) is,

$$\text{PSA} = (1 - \text{PKSSA})^{NR}$$

and for all aimpoints the probability of survival is

$$PS = \frac{1}{\pi} \sum_{j=1}^n \sum_{k=1}^n w_j w_k PSA(x_j, y_k);$$

and the probability of target kill is

$$P_K = 1 - PS$$

CHAPTER IV

ATOG COMPUTER PROGRAM

The mathematics developed in Chapter III has been programmed in FORTRAN IV in order to produce effectiveness results for comparison and validation. Results from the validated OEG air-to-ground gun simulation model discussed in Chapter II will be compared to the ATOG simulation results in the next chapter.

Program Description

The program begins by reading in all of the constants and input variables needed to produce effectiveness results. These data are read in under six different categories: program control and miscellaneous, projectile characteristics, target data, aircraft data, gun systems characteristics, and the target vulnerable area tables. The main program calls subroutine SIGHT to determine the sight depression, initial angle of attack based on altitude, range to target, projectile time of flight and gravity drop. There upon the flow returns again to the main program which calls subroutine TRACK which determines the range from the aircraft to the target, and relative position of target in the sight system in elevation and azimuth. The flow returns again to the main program which calls subroutine FLIGHT. This subroutine

determines the new velocity and position of the aircraft after some small time increment. Next subroutine GOTGT is called to determine the new target position after a specified time increment. At this point in time the main program attempts to make any corrections in roll, pitch and yaw that is necessary to keep the aircraft in pursuit of the evading, or still target. After some specified delta time increment there is a test to determine if the target is within the predetermined open fire slant range. If the target is within this specified range it will check to see if the target is within three standard deviations of aim and ballistic dispersions. If these conditions are not met the program flow returns to the TRACK subroutine and continues to fly the aircraft until the two previous checks determine that the target is possible to hit. At this time subroutine GUN is called to determine projectile position and velocity, impact velocity, and transforms the gun muzzle velocity components to earth axes components. After each round has been fired the probability of target kill is determined by calling subroutine PK1. This routine uses the nine point Hermite-Gauss quadrature method for generating aim points, which are used in determining if the projectile hit and/or killed the target. After the specified burst time or total time into the engagement has elapsed the flow returns to the main program where arrays of output data have been stored as it was generated. The output is printed in four sections:

the sight/tracking data, aircraft data, target data, and the gun/projectile data. The program will continue to read in and execute data for as many cases as desired.

Overview Program Flow Chart

The following flow chart graphically displays the flow of the computer program ATOG described in the previous section.

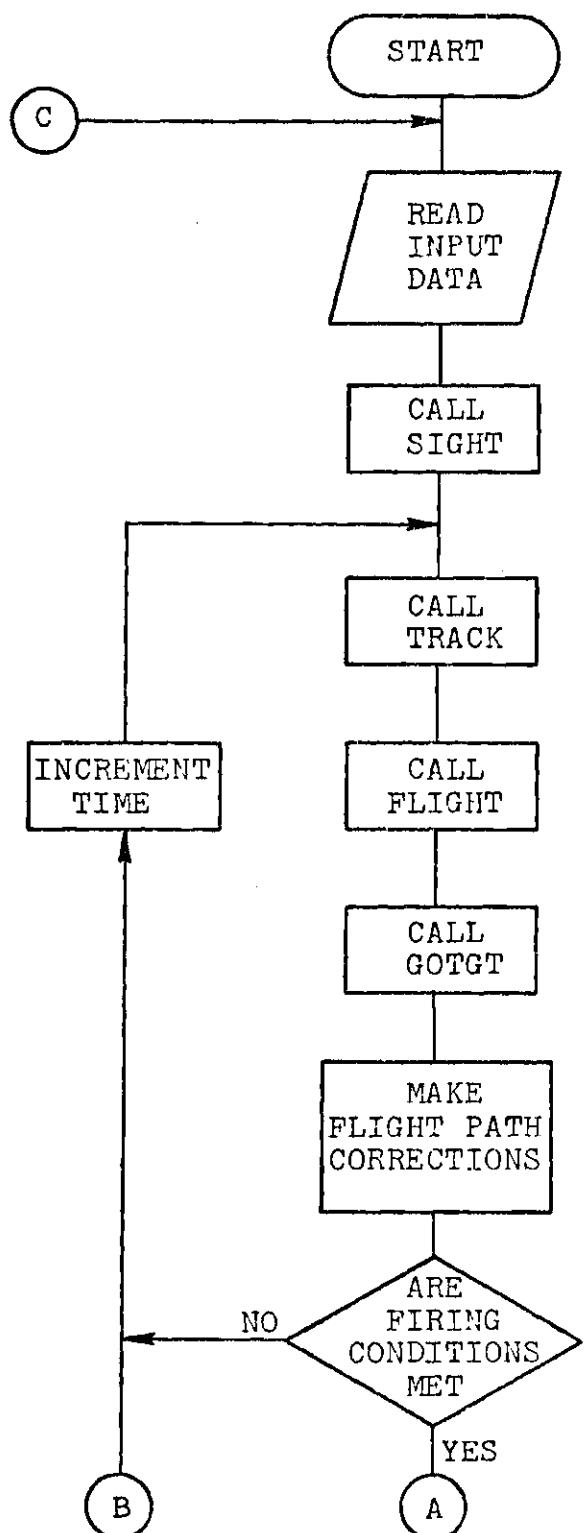


Figure IV-1. Flowchart Overview of ATOG Computer Program

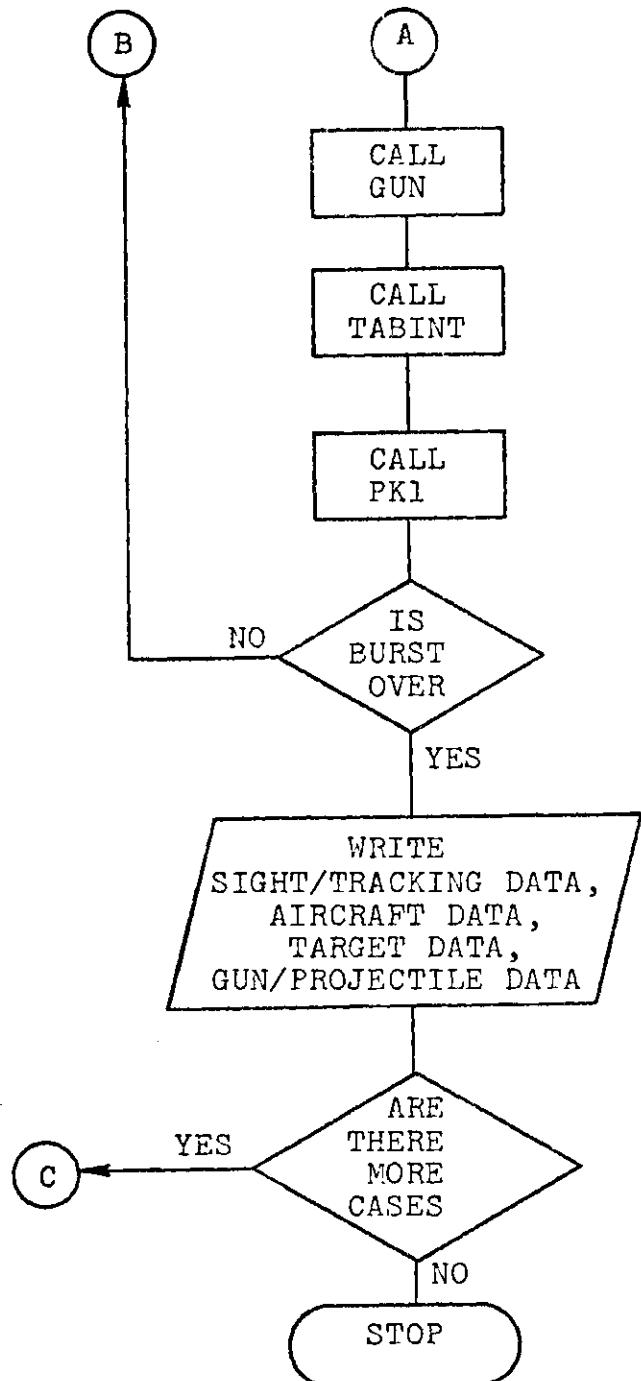


Figure IV-1. Flowchart Overview of ATOG Computer Program
(continued)

CHAPTER V

DEMONSTRATION OF RESULTS

The results obtained from the computer program described in Chapter IV are limited to one particular aircraft, one gun, and one target. The aircraft characteristics used in this simulation closely resemble that of the A-10 in dynamic performance. The gun system is a 30 millimeter, seven barrel, Gatling which mimics the General Electric GAU-8. The target is defined only as an area in square feet that is still or moving a specified velocity and direction. The size target used for this analysis does not represent any potential ground target for the gun/aircraft unit. This program is capable of accepting inputs from any aircraft, gun, target combination desired. The aim error and ballistic dispersion used was representative of the gun/aircraft unit based on actual test results. The aim error used for this analysis was 5.887 mils sigma, and the ballistic dispersion was 1.4 mils CEP as noted on the top of Tables VI-1 and VI-3, respectively.

Tables V-1 through V-4 exhibit various target maneuvers with the gun effectiveness results recorded. Can an aircraft with a fixed sight system hit a moving target? This question frequently arises in air-to-ground gunnery

analysis. Table V-5 shows the effectiveness results for both fixed, and lead predicting sight systems. The results (not shown) for a still target using both sight systems yield approximately the same effectiveness results. The lead prediction sight system uses first order lead, based on projectile time of flight and future position of the target.

Table V-1. Effectiveness Against a Moving Ground Target
for 250 Knots, 3 Degree Dive Angle

MOVING GROUND TARGET										
VELOCITY (MPH)	HEADING (DEGREES)					TURN RATE (DEGREES/SEC)				
	0	45	90	135	180	5	10	15	20	30
15	0.09	0.05	0.05	0.07	0.10	0.07	0.07	0.07	0.06	0.06
30	0.09	0.04	0.04	0.05	0.11	0.07	0.06	0.06	0.06	0.05
45	0.09	0.03	0.03	0.04	0.11	0.06	0.05	0.06	0.05	0.05
60	0.08	0.02	0.02	0.04	0.12	0.06	0.05	0.05	0.05	0.05

Table V-2. Effectiveness Against a Moving Ground Target
for 250 Knots, 15 Degree Dive Angle

MOVING GROUND TARGET										
VELOCITY (MPH)	HEADING (DEGREES)					TURN RATE (DEGREES/SEC)				
	0	45	90	135	180	5	10	15	20	30
15	0.11	0.09	0.10	0.10	0.12	0.10	0.09	0.10	0.10	0.08
30	0.11	0.09	0.09	0.10	0.12	0.09	0.09	0.09	0.09	0.07
45	0.11	0.08	0.09	0.10	0.12	0.09	0.08	0.09	0.08	0.07
60	0.11	0.08	0.09	0.11	0.12	0.09	0.08	0.08	0.08	0.05

Table V-3. Effectiveness Against a Moving Ground Target
for 250 Knots, 30 Degree Dive Angle

MOVING GROUND TARGET										
VELOCITY (MPH)	HEADING (DEGREES)					TURN RATE (DEGREES/SEC)				
	0	45	90	135	180	5	10	15	20	30
15	0.16	0.14	0.16	0.18	0.21	0.14	0.13	0.13	0.14	0.14
30	0.15	0.12	0.15	0.20	0.24	0.13	0.12	0.13	0.13	0.11
45	0.14	0.12	0.14	0.22	0.25	0.12	0.12	0.12	0.12	0.09
60	0.14	0.11	0.13	0.22	0.26	0.11	0.11	0.11	0.11	0.06

Table V-4. Effectiveness Against a Moving Ground Target
for 250 Knots, 45 Degree Dive Angle

MOVING GROUND TARGET										
VELOCITY (MPH)	HEADING (DEGREES)					TURN RATE (DEGREES/SEC)				
	0	45	90	135	180	5	10	15	20	30
15	0.46	0.42	0.45	0.46	0.50	0.42	0.41	0.41	0.43	0.42
30	0.43	0.39	0.44	0.47	0.52	0.38	0.37	0.38	0.40	0.32
45	0.38	0.36	0.42	0.42	0.52	0.34	0.34	0.35	0.36	0.23
60	0.33	0.32	0.40	0.37	0.50	0.31	0.31	0.32	0.36	0.18

Table V-5. Sight System Comparison for Aircraft Speed of 250 Knots, 30 Degree Dive Angle

SIGHT SYSTEM COMPARISON/GUN EFFECTIVENESS											
TARGET VELOCITY (MPH)	TARGET HEADING (DEGREES)										
	0		45		90		135		180		
	YES	NO	YES	NO	YES	NO	YES	NO	YES	NO	
15	0.16	0.13	0.14	0.11	0.16	0.10	0.18	0.12	0.21	0.14	
30	0.15	0.10	0.12	0.05	0.15	0.02	0.20	0.06	0.24	0.10	
45	0.14	0.05	0.12	0.01	0.14	0.00	0.22	0.02	0.25	0.04	
60	0.14	0.02	0.11	0.00	0.13	0.00	0.22	0.00	0.26	0.01	
80	0.12	0.01	0.10	0.00	0.12	0.00	0.22	0.00	0.28	0.00	
100	0.12	0.01	0.09	0.00	0.11	0.00	0.23	0.00	0.29	0.00	
TARGET TURN RATE (DEGREES)											
	5		10		15		20		30		
	YES	NO	YES	NO	YES	NO	YES	NO	YES	NO	
15	0.14	0.12	0.13	0.09	0.13	0.08	0.14	0.08	0.14	0.10	
30	0.13	0.08	0.12	0.03	0.13	0.02	0.13	0.02	0.11	0.06	
45	0.12	0.03	0.12	0.01	0.12	0.00	0.12	0.00	0.09	0.04	
60	0.11	0.00	0.11	0.00	0.11	0.00	0.11	0.00	0.06	0.02	
80	0.10	0.00	0.10	0.00	0.11	0.00	0.11	0.00	0.04	0.00	
100	0.10	0.00	0.10	0.00	0.11	0.00	0.12	0.00	0.04	0.00	

NOTE: YES, means the pilot is using lead prediction
 NO, means the pilot is not using lead prediction

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The objective of this research was to develop the mathematics for an improved air-to-ground gun model, designed to assess the probability of kill on a still, or moving ground target. In order to validate the mathematics it was necessary to program the model in FORTRAN, and then compare the results to the Operations Evaluations group (OEG) model effectiveness for similar engagements.

Results

The computer simulation model ATOG produces results in four areas of concern: (1) sight/tracking, (2) aircraft, (3) target, (4) gun/projectile. Each respective topic has its associated variables printed for each tenth of a second into the engagement. Other non-time related data for each subject can be found at the top of Tables VI-1 through VI-4.

A typical attack engagement begins by observing the target on a flat, non-rotating earth at an X, Y coordinate. It may be still, or traveling at some constant velocity and direction, or turning at a constant rate and velocity. The aircraft is introduced into the system at some specified coordinate relative to the target with a predetermined velocity and dive angle. The aircraft tracking system is

Table VI-1. Sight Tracking Data

SIGHT/TRACKING DATA						
		THE SIGHT DEPRESSION IN MILS IS.....		42.8724		
		THE RANGE AIM ERROR IN MILS IS.....		5.8870		
		THE DEFLECTION AIM ERROR IN MILS IS.....			5.8870	
TIME	AZI MUTH	ELEVATION	S-DIF	EL LEAD	AZ LEAD	TOT LEAD ANGLE
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.1000	0.0000	-0.890	-0.245	-0.461	-0.131	.0490
0.2000	-0.373	-0.243	-0.303	-0.106	-0.107	.0810
0.3000	-0.061	-0.164	-0.241	-0.159	-0.107	.0605
0.4000	-0.061	-0.070	-0.239	-0.362	-0.138	.0378
0.5000	-0.061	-0.329	-0.01656	-0.105	-0.108	.0191
0.6000	-0.061	-0.491	-0.01656	-0.046	-0.107	.0122
0.7000	-0.061	-0.495	-0.01656	-0.058	-0.115	.0118
0.8000	-0.061	-0.492	-0.01656	-0.054	-0.102	.0113
0.9000	-0.061	-0.488	-0.01649	-0.049	-0.099	.0108
1.0000	-0.061	-0.484	-0.01649	-0.044	-0.097	.0104
1.1000	-0.061	-0.481	-0.01649	-0.039	-0.094	.0100
1.2000	-0.061	-0.478	-0.01635	-0.035	-0.091	.0096
1.3000	-0.061	-0.476	-0.01631	-0.028	-0.088	.0093
1.4000	-0.061	-0.474	-0.02253	-0.025	-0.083	.0089
1.5000	-0.061	-0.472	-0.02253	-0.023	-0.080	.0086
1.6000	-0.061	-0.471	-0.02253	-0.021	-0.076	.0082
1.7000	-0.061	-0.470	-0.0219	-0.019	-0.073	.0079
1.8000	-0.061	-0.469	-0.0219	-0.016	-0.070	.0075
1.9000	-0.061	-0.468	-0.01616	-0.014	-0.067	.0072
2.0000	-0.061	-0.468	-0.01614	-0.013	-0.063	.0068
2.1000	-0.061	-0.468	-0.01612	-0.012	-0.060	.0065
2.2000	-0.061	-0.468	-0.01611	-0.010	-0.053	.0057
2.3000	-0.061	-0.469	-0.01609	-0.009	-0.050	.0053
2.4000	-0.061	-0.469	-0.01609	-0.008	-0.046	.0047
2.5000	-0.061	-0.470	-0.01608	-0.008	-0.043	.0044
2.6000	-0.061	-0.471	-0.01607	-0.007	-0.040	.0040
2.7000	-0.061	-0.471	-0.01607	-0.007	-0.033	.0037
2.8000	-0.061	-0.472	-0.01606	-0.006	-0.030	.0034
2.9000	-0.061	-0.473	-0.01606	-0.006	-0.025	.0031
3.0000	-0.061	-0.474	-0.01605	-0.005	-0.020	.0028
3.1000	-0.061	-0.475	-0.01605	-0.005	-0.016	.0023
3.2000	-0.061	-0.476	-0.01605	-0.005	-0.013	.0019
3.3000	-0.061	-0.477	-0.01604	-0.004	-0.012	.0015
3.4000	-0.061	-0.478	-0.01604	-0.004	-0.010	.0013
3.5000	-0.061	-0.478	-0.01604	-0.004	-0.010	.0012
3.6000	-0.061	-0.479	-0.01604	-0.004	-0.010	.0011
3.7000	-0.061	-0.480	-0.01604	-0.003	-0.009	.0010
3.8000	-0.061	-0.481	-0.01604	-0.003	-0.010	.0010
3.9000	-0.061	-0.482	-0.01603	-0.003	-0.011	.0011
4.0000	-0.061	-0.483	-0.01603	-0.003	-0.012	.0012
4.1000	-0.061	-0.484	-0.01603	-0.003	-0.010	.0010
4.2000	-0.061	-0.485	-0.01603	-0.003	-0.009	.0009
4.3000	-0.061	-0.486	-0.01603	-0.003	-0.010	.0010
4.4000	-0.061	-0.487	-0.01603	-0.003	-0.011	.0011
4.5000	-0.061	-0.488	-0.01603	-0.003	-0.012	.0012
4.6000	-0.061	-0.489	-0.01603	-0.003	-0.013	.0013
4.7000	-0.061	-0.490	-0.01603	-0.003	-0.012	.0012
4.8000	-0.061	-0.491	-0.01603	-0.003	-0.010	.0010
4.9000	-0.061	-0.492	-0.01603	-0.003	-0.011	.0011
5.0000	-0.061	-0.493	-0.01603	-0.003	-0.012	.0012
5.1000	-0.061	-0.494	-0.01603	-0.003	-0.013	.0013
5.2000	-0.061	-0.495	-0.01603	-0.003	-0.014	.0014
5.3000	-0.061	-0.496	-0.01603	-0.003	-0.016	.0016
5.4000	-0.061	-0.497	-0.01603	-0.003	-0.018	.0018
5.5000	-0.061	-0.498	-0.01603	-0.003	-0.020	.0020
5.6000	-0.061	-0.498	-0.01603	-0.003	-0.021	.0021
5.7000	-0.061	-0.499	-0.01603	-0.003	-0.020	.0020
5.8000	-0.061	-0.499	-0.01603	-0.003	-0.019	.0019
5.9000	-0.061	-0.500	-0.01603	-0.003	-0.018	.0018
6.0000	-0.061	-0.501	-0.01603	-0.003	-0.017	.0017
6.1000	-0.061	-0.501	-0.01603	-0.003	-0.016	.0016
6.2000	-0.061	-0.501	-0.01603	-0.003	-0.015	.0015
6.3000	-0.061	-0.501	-0.01603	-0.003	-0.014	.0014
6.4000	-0.061	-0.501	-0.01603	-0.003	-0.013	.0013
6.5000	-0.061	-0.501	-0.01603	-0.003	-0.012	.0012
6.6000	-0.061	-0.501	-0.01603	-0.003	-0.011	.0011
6.7000	-0.061	-0.501	-0.01603	-0.003	-0.010	.0010
6.8000	-0.061	-0.501	-0.01603	-0.003	-0.009	.0009
6.9000	-0.061	-0.501	-0.01603	-0.003	-0.008	.0008
7.0000	-0.061	-0.501	-0.01603	-0.003	-0.007	.0007
7.1000	-0.061	-0.501	-0.01603	-0.003	-0.006	.0006
7.2000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
7.3000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
7.4000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
7.5000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
7.6000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
7.7000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
7.8000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
7.9000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
8.0000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
8.1000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
8.2000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
8.3000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
8.4000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
8.5000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
8.6000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
8.7000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
8.8000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
8.9000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
9.0000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
9.1000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
9.2000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
9.3000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
9.4000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
9.5000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
9.6000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
9.7000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
9.8000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
9.9000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005
10.0000	-0.061	-0.501	-0.01603	-0.003	-0.005	.0005

Table VI-3. Gun Projectile Data

+++++ GUN/PROJECTILE DATA +++++

GUN RATE IN ROUNDS PER MINUTE.....	4140.0000
BURST TIME IN SECONDS.....	2.0000
PROJECTILE SIZE IN MILLIMETERS.....	30.0000
BALLISTIC DISPERSION IN RANGE (MILS)....	1.4000
BALLISTIC DISPERSION IN DEFLECTION (MILS)	1.4000
NUMBER OF EXPECTED HITS ON TARGET.....	5.1399
PROBABILITY OF TARGET KILL.....	.0349

TIME	ROUND	X-MISS	Y-MISS	RANGE	IMP VEL	E(HITS)	PK
5.4800	1	3.6	-6.1	2865.6	3034.3	.1	.0007
5.5044	2	3.6	-5.8	2788.9	3034.4	.2	.0015
5.5289	3	3.6	-5.5	2778.5	3034.5	.3	.0023
5.5533	4	3.6	-5.1	2768.1	3041.1	.4	.0032
5.5778	5	3.6	-4.8	2757.7	3041.2	.6	.0040
5.6022	6	3.5	-4.5	2747.2	3041.3	.7	.0049
5.6267	7	3.5	-4.2	2736.8	3048.0	.8	.0057
5.6511	8	3.5	-3.8	2726.3	3048.1	.9	.0066
5.6756	9	3.5	-3.5	2715.9	3048.2	1.1	.0075
5.7000	10	3.4	-3.2	2705.4	3054.9	1.2	.0085
5.7145	11	3.3	-3.0	2695.0	3055.0	1.3	.0094
5.7290	12	3.3	-2.7	2688.7	3055.0	1.5	.0103
5.7435	13	3.3	-2.5	2682.5	3055.1	1.6	.0113
5.7580	14	3.2	-2.2	2676.3	3061.8	1.7	.0122
5.7725	15	3.2	-2.0	2670.1	3061.8	1.9	.0132
5.7870	16	3.1	-1.8	2663.9	3061.9	2.0	.0141
5.8014	17	3.1	-1.5	2657.7	3061.9	2.2	.0151
5.8159	18	3.1	-1.3	2651.5	3068.6	2.3	.0161
5.8304	19	3.0	-1.1	2645.2	3068.7	2.5	.0170
5.8449	20	3.0	-.9	2639.0	3068.8	2.6	.0180
5.8594	21	3.0	-.7	2632.8	3068.8	2.7	.0190
5.8739	22	2.9	-.5	2626.5	3068.9	2.9	.0200
5.8884	23	2.9	-.3	2620.3	3075.6	3.0	.0210
5.9029	24	2.9	-.1	2614.1	3075.7	3.2	.0220
5.9174	25	2.8	0	2607.8	3075.7	3.3	.0230
5.9319	26	2.8	.2	2601.6	3075.8	3.5	.0239
5.9464	27	2.8	.4	2595.4	3075.9	3.6	.0249
5.9609	28	2.8	.5	2589.1	3082.6	3.8	.0259
5.9754	29	2.7	.7	2582.9	3082.7	3.9	.0269
5.9899	30	2.7	.9	2576.6	3082.7	4.1	.0279
6.0043	31	2.7	1.0	2570.4	3082.8	4.2	.0289
6.0188	32	2.7	1.1	2564.1	3082.9	4.4	.0299
6.0333	33	2.6	1.3	2557.9	3089.6	4.5	.0309
6.0478	34	2.6	1.4	2551.6	3089.7	4.7	.0319
6.0623	35	2.6	1.5	2545.4	3089.8	4.8	.0329
6.0768	36	2.6	1.6	2539.1	3089.8	5.0	.0339
6.0913	37	2.6	1.7	2532.9	3089.9	5.1	.0349

Table VI-4. Target Data

TARGET DATA	
VELOCITY IN MILES PER HOUR.....	45.0000
INITIAL DIRECTION OF TARGET VEL (DEGREES)	90.0000
ANGULAR TURN RATE IN DEGREES.....	0.0000

TIME	X-POS	Y-POS	IMP-VEL	PRES AREA	VUL AREA
5.4800	1000.0	361.7	3034.3	215.7	1.528
5.5044	1000.0	363.3	3034.4	215.7	1.528
5.5289	1000.0	364.9	3034.5	215.7	1.584
5.5533	1000.0	366.5	3041.1	215.7	1.597
5.5778	1000.0	368.1	3041.2	215.7	1.543
5.6022	1000.0	369.7	3041.3	215.7	1.532
5.6267	1000.0	371.4	3048.0	215.7	1.533
5.6511	1000.0	373.0	3048.1	215.7	1.521
5.6756	1000.0	374.6	3048.2	215.7	1.509
5.7000	1000.0	376.2	3054.9	215.7	1.513
5.7145	1000.0	377.2	3055.0	215.7	1.503
5.7290	1000.0	378.1	3055.0	215.7	1.495
5.7435	1000.0	379.1	3055.1	215.8	1.487
5.7580	1000.0	380.0	3061.8	215.8	1.494
5.7725	1000.0	381.0	3061.8	215.8	1.487
5.7870	1000.0	381.9	3061.9	215.8	1.481
5.8014	1000.0	382.9	3061.9	215.8	1.476
5.8159	1000.0	383.9	3068.6	215.8	1.485
5.8304	1000.0	384.8	3068.7	215.8	1.481
5.8449	1000.0	385.8	3068.8	215.8	1.476
5.8594	1000.0	386.7	3068.8	215.8	1.472
5.8739	1000.0	387.7	3068.9	215.8	1.469
5.8884	1000.0	388.6	3075.6	215.8	1.479
5.9029	1000.0	389.6	3075.7	215.8	1.476
5.9174	1000.0	390.5	3075.7	215.8	1.473
5.9319	1000.0	391.5	3075.8	215.8	1.470
5.9464	1000.0	392.5	3075.9	215.8	1.468
5.9609	1000.0	393.4	3082.6	215.8	1.479
5.9754	1000.0	394.4	3082.7	215.8	1.477
5.9899	1000.0	395.3	3082.7	215.8	1.474
6.0043	1000.0	396.3	3082.8	215.8	1.472
6.0188	1000.0	397.2	3082.9	215.8	1.470
6.0333	1000.0	398.2	3089.6	215.8	1.482
6.0478	1000.0	399.2	3089.7	215.8	1.480
6.0623	1000.0	400.1	3089.8	215.8	1.478
6.0768	1000.0	401.1	3089.8	215.8	1.476
6.0913	1000.0	402.0	3089.9	215.9	1.475

mathematically designed to pursue the target within certain limitations. Pertinent constraints on aircraft capabilities and turn rates are read in as a limiting factor which allows the mathematical model to perform as a live model.

The aircraft continues to pursue the target for a specified engagement time. If the open fire range (specified in the input) is achieved before the engagement time elapses a check will be made to determine if it is possible to hit the target based on the aim error. After these two conditions are met the gun will commence to fire for a specified number of seconds, or until the engagement time elapses. Then the probability of target hit/kill is determined.

Comparison and Validation

One of the best ways to validate the new methodology is to check it with another model known to produce accurate results. The OEG model methodology and effectiveness results are documented in the Joint Munitions Effectiveness Manual (JMEM) and accepted by the armed services Joint Technical Coordinating Group (JTCA) [9]. Due to the OEG's popularity it became a prime subject for comparison to the new model. Table VI-5 shows four areas of comparison using a still target.

The data in Table VI-5 was collected from both models under similar attack conditions. The target, gun and aircraft were all the same. The OEG dive angle remains constant

Table VI-5. Gun Model Effectiveness Comparison

AIRCRAFT DIVE ANGLE (DEGREES)		AIRCRAFT SLANT RANGE (FEET)		PROJ IMPACT VEL (FT/SEC)		PROBABILITY OF KILL	
OEG	ATOG	OEG	ATOG	OEG	ATOG	OEG	ATOG
15.0	16.1	2000	1951	3210	3212	0.32	0.27
15.0	16.3	3000	2951	3010	2998	0.13	0.11
15.0	16.4	4000	3951	2820	2797	0.05	0.05
30.0	31.1	2000	1946	3210	3221	0.42	0.40
30.0	31.3	3000	2946	3010	3012	0.17	0.18
30.0	31.4	4000	3946	2820	2816	0.08	0.10
45.0	45.8	2000	1942	3210	3221	0.87	0.79
45.0	46.0	3000	2942	3010	3017	0.53	0.50
45.0	46.1	4000	3942	2820	2827	0.33	0.34

for all cases while the ATOG dive angle increases slightly. This slight increase is due to the aircraft's dynamic characteristic to 'bunt' as it draws nearer to the target. The aircrafts' open fire slant ranges differ slightly between the two models. This difference is due to the way the open fire slant range is selected. In the OEG model it is a fixed input. In the ATOG program it is calculated based on the input sight harmonization range; then the mean gun burst time is equally divided on each side of this range. The OEG model uses projectile trajectories calculated from a trajectory program based on a constant altitude of four thousand feet. The ATOG's trajectory characteristics are all calculated within the program, and are representative of the altitude at which each round is fired. This should account for the projectile impact velocity differences. The vulnerable area of a target (not shown in the table) is a function of the dive angle, slant range, and projectile impact velocity. Since there are slight differences in all three areas between the two models it would be difficult to achieve the exact probability of target kill. However, the effectiveness results are very close and the differences are negligible.

The ATOG program is simpler to run, requires one tenth the set up time, and executes each case in three seconds less than that required for the OEG model. However, the OEG model has a plot option but the ATOG model does not. ATOG

has three times more information printed out than the OEG model. The largest and most obvious difference between the two models is the potential capability of the ATOG program to evaluate the damage inflicted on a moving target.

Recommendations and Conclusions

This new simulation model has many and varying advantages over all of the air-to-ground gun models known to be in use at this time. There are several areas in the model that could be topics for future research. First, the target model could be improved upon by adopting a completely random target [10]. Second, sight depression/lead compensation tables could be generated for use as a teaching aid in air-to-ground fixed gun sight systems. Third, graphic output could be incorporated into the program to display each engagement. Fourth, the program can be modified with a minimum effort to be used to assess the probability of kill against low flying aircraft such as helicopters.

The ATOG computer program should prove to be a valuable tool in future air-to-ground gun studies. It is conservative, yet it uses some of the most sophisticated and accurate mathematical techniques known.

APPENDIX I

MATHEMATICAL VARIABLE LIST

- A_F - Front presented area of target (square feet)
- A_P - Target presented area (square feet)
- A_S - Side presented area of target (square feet)
- A_T - Top presented area of target (square feet)
- A_V - Target vulnerable area (square feet)
- $AX1$ - $C\alpha C\theta C\psi - S\alpha(C\phi S\theta C\psi + S\phi S\psi)$
- $AX2$ - $C\alpha C\theta S\psi - S\alpha(C\phi S\theta S\psi - S\phi C\psi)$
- $AX3$ - - $C\alpha S\theta - S\alpha C\phi C\theta$
- $AY1$ - $S\phi S\theta C\psi - C\phi S\psi$
- $AY2$ - $S\phi S\theta S\psi + C\phi C\psi$
- $AY3$ - $S\phi C\theta$
- AZ - Azimuth of target angular coordinates in the field of view along the aircraft X axis
- AZA - Target vulnerable area as a function of azimuth
- $AZ1$ - $S\alpha C\theta C\psi + C\alpha(C\phi S\theta C\psi + S\phi S\psi)$
- $AZ2$ - $S\alpha C\theta S\psi + C\alpha(C\phi S\theta S\psi - S\phi C\psi)$
- $AZ3$ - - $S\alpha S\theta + C\alpha C\phi C\theta$
- C - Side force on the aircraft in the wind axis equation
- C_D - Aircraft coefficient of drag
- C_L - Aircraft coefficient of lift
- $C_{L\alpha}$ - Coefficient of lift versus α slope
- C_{L0} - Coefficient of lift at $\alpha = 0$

- CVD - Correction for vertical displacement of gun (feet)
- Cl - Proportionality roll angle gain constant
- d - Diameter of projectile (feet)
- D - Drag force
- EL - Elevation of target angular coordinates in the field of view along the aircraft's X axis
- ELA - Target vulnerable area as a function of elevation
- ERF - A function name used to describe a numerical evaluation technique
- FPV - Aircraft flight path vector
- FRL - Fuselage reference line (horizontal plane of the aircraft about which all angular measurements of the aircraft are made)
- g - Acceleration due to gravity
- GL - Gun bore axis
- GL' - Line parallel to the gun bore axis
- GR - Gun rate of fire (rounds/minute)
- \hat{i}_B - i component in the line of sight vector of the aircraft body axis system
- \hat{i}_T - Vector component of the transformed velocity vector
- \hat{j}_B - j component in the line of sight vector of the aircraft body axis system
- \hat{j}_T - Vector component of transformed velocity vector
- \hat{k}_B - k component in the line of sight vector of the aircraft body axis system
- \hat{k}_T - Vector component of the transformed velocity vector
- L - Lift force
- LE - Length of target (feet)

LOSP - Line of sight to projectile at impact

m - Mass = weight/gravity

MDX - Target miss distance in X coordinate rotated to the normal (feet)

MDY - Target miss distance in Y coordinate rotated to the normal (feet)

NR - A specified number of rounds fired in a burst

PHSS - The probability of a single round impacting the target when aiming error and ballistic dispersion are involved

PHSSA - The probability of hitting the target with a single round

PKSSA - Single shot probability of kill given an aimpoint

P_{K/H} - Probability of target kill/given a hit

P_K - Probability of target kill

PSA - Probability of survival

PS - Probability of survival for all aimpoints

PTV - Projectile total velocity vector

PTV' - Line parallel to the projectile total velocity vector

QSA - An intermediate variable used to calculate alpha

R - Line of sight range to the target (feet)

\hat{r} - The line of sight unit vector

\hat{r}_A - Line of sight unit vector in the aircraft body axis system

R_I - Input line of sight range to target from aircraft (feet)

R_O - Initial range of aircraft to target (feet)

r_1	- i component of the line of sight unit vector \hat{r}
r_2	- j component of the line of sight unit vector \hat{r}
r_3	- k component of the line of sight unit vector \hat{r}
S	- Aerodynamic reference area of aircraft (square feet)
SD	- Sight depression (mils)
SR	- Projectile slant range (feet)
T	- Thrust of the aircraft
t	- Time (cumulative)
t_B	- Engagement duration time (seconds)
t_f	- Projectile time of flight (seconds)
TGHD	- Target heading (radians)
t_i	- Present time
T_T	- Tracking time (seconds)
T_X	- Component of thrust in the aircraft X-axis
T_Y	- Component of thrust in the aircraft Y-axis
T_Z	- Component of thrust in the aircraft Z-axis
V_A	- Velocity of the aircraft (feet/second)
VACK	- Velocity of the aircraft (knots)
VGP	- Vertical gun position with respect to the sight (feet)
V_M	- Gun muzzle velocity (feet/seconds)
V_{MX}	- Earth axis transformation for the gun muzzle velocity in the X direction
V_{MY}	- Earth axis transformation for the gun muzzle velocity in the Y direction

- V_{MZ} - Earth axis transformation for the gun muzzle velocity in the Z direction
- V_0 - Initial airspeed for a planned airspeed of V_A at open fire (feet/seconds)
- V_P - Velocity of the projectile at impact (feet/second)
- V_{PT} - Projectile initial velocity (feet/second)
- V_{PX} - Projectile initial velocity component in the X direction
- V_{PY} - Projectile initial velocity component in the Y direction
- V_{PZ} - Projectile initial velocity component in the Z direction
- V_T - Velocity of the target (feet/second)
- W - Weight of the aircraft (pounds)
- WD - Width of the target (feet)
- W_i - Weighting factor used in determining the probability of the aimpoint occurring
- \dot{X} - Change in the X aircraft position after delta time (feet/second)
- X_A - The X coordinate position of the aircraft (feet)
- X_{AI} - The mean aimpoint of projectile impact position in the X coordinate
- X_i - Aimpoints
- X_L - Line of sight coordinate in the aircraft body axis system
- X_{MISS} - Projectile X miss distance from target (feet)
- X_P - Projectile impact point in the X coordinate (feet)
- \dot{X}_P - Projectile velocity in the X direction (feet/second)
- \ddot{X}_P - Projectile acceleration in the X direction (feet/second)²

- X_{PT} - X coordinate of projectile impact point relative to the target (feet)
- X_T - The X coordinate position of the target at any time t (feet)
- \dot{X}_T - The target velocity in the X direction (feet/second)
- X_{TB} - X coordinate in the target body
- X_{TO} - Initial X position of the target (feet)
- X' - The transformed projectile impact X coordinate (feet)
- Y_A - The Y coordinate position of the aircraft (feet)
- Y_{AI} - The mean aimpoint of projectile impact position in the Y coordinate
- \dot{Y} - Change in the Y position of the aircraft after delta time (feet/second)
- Y_L - Line of sight coordinate in the aircraft body axis system
- Y_{MISS} - Projectile Y miss distance from target (feet)
- Y_P - Projectile impact point in Y coordinate (feet)
- \dot{Y}_P - Velocity of the projectile in the Y direction (feet/second)
- \ddot{Y}_P - Acceleration of the projectile in the Y direction (feet/second)²
- Y_{PT} - Y coordinate of projectile impact point relative to the target (feet)
- Y_T - The Y coordinate position of the target at any time t (feet)
- \dot{Y}_T - The target velocity in the Y direction (feet/second)
- Y_{TB} - Y coordinate in target body
- Y_{TO} - Initial Y position of the target (feet)
- Y' - The transformed projectile impact Y coordinate (feet)

- \dot{z} - Change in the Z position of the aircraft after Δt (feet/seconds)
- z_A - The Z coordinate position of the aircraft, or altitude (feet)
- z_L - Line of sight coordinate in the aircraft body axis system
- z_{MISS} - Projectile Z miss distance from the target (feet)
- z_p - Projectile impact point in the Z coordinate (feet)
- \dot{z}_p - Velocity of projectile in the Z direction (feet/seconds)
- \ddot{z}_p - Acceleration of the projectile in the Z direction (feet/second)
- z_T - The Z coordinate position of the target (feet)
- z' - The transformed projectile impact Z coordinate (feet)
- α - Aircraft angle of attack (radians)
- α_i - Present commanded angle of attack (radians)
- α_{MAX} - Maximum allowable angle of attack (radians)
- α_{OL} - Angle of attack at zero lift (radians)
- α_s - Aircraft angle of attack at aerodynamic stall (radians)
- α_{ST} - Maximum allowable angle of attack for structural limitations (radians)
- δ - Projectile angular gravity drop (radians)
- $\Delta\alpha$ - Increment angle of attack (radians/ second)
- Δt - Time increment (seconds)
- $\Delta\phi$ - Roll or bank increment (radians/second)
- $\Delta\phi_{MAX}$ - Maximum roll angle (radians/second)
- ϵ - Angle between the FRL and the GL (radians)

- θ - Aircraft dive angle or pitch (radians)
- $\dot{\theta}$ - The relative change in the aircraft dive angle after Δt (radians/second)
- θ_T - Linear direction of the target velocity (radians)
- $\dot{\theta}_T$ - The target velocity change with time
- θ_1 - The transformed angle of projectile impact for the aircraft dive angle
- η - Aircraft load factor
- η_S - Aircraft structural load factor
- ρ - Air density (slugs)
- σ_{XB} - Standard deviation of the ballistic dispersion in range (mils CEP)
- σ_{YB} - Standard deviation of the ballistic dispersion in deflection (mils CEP)
- σ_{AR} - Standard deviation of the aim error in range
- σ_{AD} - Standard deviation of the aim error in deflection
- τ - Trajectory shift angle - deflection of the bullet due to the gun being at an angle of attack with respect to the aircraft flight path (radians)
- ϕ - Roll or bank angle of aircraft (radians)
- $\dot{\phi}_{MAX}$ - Maximum bank angle rate (radians/second)
- ϕ_i - Commanded roll angle (radians)
- ψ - Yaw or heading of aircraft (radians)
- $\dot{\psi}$ - Change in the heading of the aircraft after Δt (radians/second)
- ψ_T - Heading angle between the target axis and earth axis
- ψ_1 - The new projectile rotation angle of impact for the aircraft heading angle
- π - Pi (a constant approximately equal to 3.1415926)

- ω_T - The target angular turn rate (radians/second)
- μ_1 - Component in the direction of the velocity of the projectile at impact
- μ_2 - The second component in the direction of the velocity of the projectile at impact
- μ_3 - The third component in the direction of the velocity of the projectile at impact

APPENDIX II**ATOG FORTRAN COMPUTER PROGRAM LISTING**

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PROGRAM ATOG(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
COMMON/GOT/VT,XTO,YTO,THETO,OMEGT
COMMON/SIG/VGP,ALPOL,EPSI,VM,RHO,SD,RI,V
COMMON/TRA/XT,YT,ZT
COMMON/FLI/RAD,G,T,ACD,TII,TIME,PNEU,CLALPO,AP
COMMON/MACH/MACH(20),CD(20)
COMMON/VALUE/WPROJ,DIA,PIE
COMMON/DOTAP/XDOT,YDOT,ZDOT,XDOTP,YDOTP,ZDOTP
COMMON/PK1/SIGAR,SIGAO,SIGBP,SIGBD,GR,VULAT,PK
COMMON/ALL/RTHETA,VA,XA,YA,ZA,EL,FPSI,R,ALPHA,PHI,AZ,S,CLALP,W
DIMENSION XTT(200),YTT(200),XAC(200),YAC(200),ZAC(200),TIMEAT(200)
DIMENSION FLA(200),AZA(200),RA(200),DALP(200),ALP(200),PHIA(200)
DIMENSION SI(200),YO(6),XMIS(200),YMTS(200),ZMIS(200),VPI(200)
DIMENSION DIVET(4),VELT(8),VULT(4,8),VAT(200),TIM(200)
DIMENSION XTPOS(150),YTPOS(150),ZTPOS(150),VAA(200),GEES(200)
DIMENSION THE(200),PKS(200),EHT(200),ARFAP(200),GUNR(200)
DIMENSION XPRI(200),YPP(200),VPIM(200),ELL(200),AZL(200),TLA(200)
DATA PIE/3.141592654/,G/32.1687/,RAD/0.0174532925/
C***** PROGRAM CONTROL AND MISCELLANEOUS *****
7776 READ(5,1) ILEAD,TB,TII,ZMIN
  IF(EOF(5).NE.0.0) GO TO 7777
  1 FORMAT(I2,3F10.0)
C***** PROJECTILE CHARACTERISTICS *****
  2 FORMAT(4F10.5)
  3 FORMAT(10F5.4)
C***** TARGET DATA *****
  4 FORMAT(8F10.0)
  5 FORMAT(3F10.0)
C***** AIRCRAFT DATA *****
  6 FORMAT(8F10.1)
  7 FORMAT(8F10.1)
  8 FORMAT(8F10.0)
C***** GUN SYSTEM CHARACTERISTICS *****
  9 FORMAT(8F10.0)
C***** VULNERABLE AREA TABLES *****
  10 FORMAT(4F10.0)
  11 FORMAT(8F10.1)
  12 FORMAT(8F10.0)
  DO 53 J=1,4
  13 FORMAT(8F10.0)
  52 FORMAT(8F10.0)
  53 CONTINUE
  IF(VT.EQ.0.0) VT=0.1
C THIS IS CHARACTERISTIC OF THE GAU-8 GUN ONLY
  RNDFR=10.0+(TBURST-0.4)*68.0
  IBURST=IFIX(RNDFR)+1
  JUMP=0
  PROD=1.0
  EHITS=0.0
  PNEUMX=5.0
  DOTN=3.0
  CLMAX=0.75
  XMIS(1)=0.0
  YMIS(1)=0.0
  ZMIS(1)=0.0
  VPI(1)=0.0
  VT=VT*1.466667
  EPSI=FPSI*RAD
  RPSI=PSI*PAD
  SI(1)=0.0
  PHI=PHI*PAD
  PHIMAX=PHIMX*RAD

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THETO=THETO*RAD
OMEGT=OMEGT*RAD
ALPOL=1.0*RAD
CLALPO=-ALPOL*CLALP
TIME=0.0
J=0
L=1
XTT(L)=XT
YTT(L)=YT
TIMEAT(L)=TIME
DALP(1)=0.0
ELA(L)=0.0
AZA(L)=0.0
ELL(L)=0.0
AZL(L)=0.0
TLA(L)=0.0
C TRACKING TIME (IN SECONDS)
TT=4.0
C INITIAL AIRSPEED
V=VACK*1.6878
C INITIAL FIRING RANGE
RSHOT=(TR-TT)*V/2.0+RI
RTHETA=THETA*RAD
VDOT=-SIN(RTHETA)*G
VO=V-VDOT*TT
VA=VO
C INITIAL RANGE (IN FEET)
R0=PI*(VO+V)*0.5*TT
R=R0
RA(L)=R
THE(L)=-RTHETA/RAD
C INITIAL AIRCRAFT ALTITUDE AND AIR DENSITY
ZA=RI*SIN(RTHETA)
RHO=.002377*((518.688-.0035662*ABS(ZA))/518.688)**4.2561
TSIGHT=.5*RHO*S*VO*VO*ACD
C SIGHT SETTINGS
XA=-R0*COS(RTHETA)*COS(RPSI)
YA=-R0*COS(RTHETA)*SIN(RPSI)
ZA=R0*SIN(RTHETA)
XAC(L)=XA
YAC(L)=YA
ZAC(L)=ZA
VAA(L)=VA
CALL SIGHT(YO,TO,TF,6,IFLAG,H,TSIGHT)
CL=ALPHA*CLALP+CLALPO
ALIFT=CL*0.5*RHO*VA*VA*S
ALP(1)=ALPHA
PHIA(1)=PHI
C SET THRUST TO COUNTERACT DRAG AT INITIAL AIRSPEED
T=0.5*RHO*S*ACD*VO*VO
PNEU=(ALIFT+T*ALPHA)/W
GEES(L)=PNEU
C WRITE(6,204) XA,YA,ZA,T
204 FORMAT(3X,4F10.3)
10 CONTINUE
L=L+1
TIME=TIME+TII
TIMEAT(L)=TIME
C TRACK DETERMINES TGT LOCATION IN HUO COOR AND SR TO TGT
CALL TRACK
OMEGA=THETO+OMEGT*TIME
XTOOT=VT*COS(OMEGA)
YTOOT=VT*SIN(OMEGA)
ZTOOT=0.0
ELA(L)=FL
AZA(L)=AZ
RA(L)=R
RSAV=R
XTSAV=XT

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YTSAV=YT
IF(ILLEAD.EQ.0) GO TO 22
XT=XTSAV+XTDOT*TF
YT=YTSAV+YTDOT*TF
CALL TRACK
22 CONTINUE
C LEAD ANGLE ARRAYS, ELEVATION, AZIMUTH, TOTAL LEAD ANGLE
ELL(L)=(EL-SD)
AZL(L)=AZ
TLA(L)=ACOS(COS(AZL(L))*COS(ELL(L)))
XT=XTSAV
YT=YTSAV
R=RSAV
C ROLL OR BANK INCREMENT WHERE C1 IS THE BANK ANGLE GAIN CONSTANT
DELPHM=PHIMAX*TII
DELPHI=C1*AZ
IF(ABS(DELPHI).GT.DELPHM) DELPHI=DELPHM*SIGN(1.0,DELPHI)
C COMMANDED BANK ANGLE
PHI=PHI+DELPHI
PHIA(L)=PHI/RAD
C WRITE(6,206) DELPHM,PHIMAX,TII,DELPHI,C1,AZ,PHIA(L)
206 FORMAT(3X,7F10.3)
CALL FLIGHT
THE(L)=-RTHTETA/RAD
SI(L)=RPSI/RAD
XAC(L)=XA
YAC(L)=YA
ZAC(L)=ZA
C IF(ZAC(L).LT.ZMIN) GO TO 11
VAAL(L)=VA
CALL GOTGT
XTT(L)=XT
YTT(L)=YT
DALPHA=SD-EL
CL=ALPHA*CLALP+CLALPO
ALIFT=CL*0.5*RHO*VA*VA*S
PNEU=(ALIFT+T*ALPHA)/W
GEES(L)=PNEU
C ANGLE OF ATTACK AT MAX LIFT
AFSTAL=CLMAX/CLALP
C AIRCRAFT LOAD FACTOR LIMITED BY STRUCTURAL LOAD FACTOR
AFGLIM=PNEUMX*W/(0.5*VA*VA*S*RHO*CLALP)
C MAX ANGLE OF ATTACK NOT TO EXCEED STALL RATE
AFLIM=AMIN1(AFGLIM,AFSTAL)
ALDOMX=DOTN*W/(0.5*VA*VA*S*RHO*CLALP)
DALPTI=ALDOMX*TII
C INCREMENT ANGLE OF ATTACK NOT TO EXCEED MAX LIMITS
IF(ABS(DALPHA).GT.DALPTI) DALPHA=DALPTI*SIGN(1.0,DALPHA)
DALP(L)=DALPHA
C THE COMMANDED ANGLE OF ATTACK
AFNEW=ALPHA+DALPHA
ALPHA=AMIN1(AFNEW,AFLIM)
ALP(L)=ALPHA
C CHECK TO SEE IF AZIMUTH IS IN LIMITS
RT=(AF+AS+AT)/3.0
SIGAZ=SORT(SIGAD*SIGAD+SIGBD*SIGBD)*0.003
SIGEL=SORT(SIGAR*SIGAR+SIGBR*SIGBR)*0.003
AZMIN=SIGAZ+RT/R
ELMIN=SIGEL+RT/R
IF(ABS(AZ).GT.AZMIN.OR.ABS(EL).GT.ELMIN) GO TO 44
C CORRECTION FOR VERTICAL DISPLACEMENT OF GUN
YO(4)=XA+VGP*(SIN(ALPHA)*COS(RTHETA)*COS(RPSI)+COS(ALPHA)*(SIN(PHI
1)*SIN(RTHETA)*SIN(RPSI)+SIN(PHT)*COS(RPSI)))
YO(F)=YA+VGP*(SIN(ALPHA)*COS(RTHETA)*SIN(RPSI)+COS(ALPHA)*(COS(PHI
1)*SIN(RTHETA)*SIN(RPSI)-SIN(PHI)*SIN(RPSI)))
YO(6)=ZA+VGP*(-SIN(ALPHA)*SIN(RTHETA)+COS(ALPHA)*COS(PHI)*COS(RTHE
1TA))
C NOW CHECK RANGE TO SEE IF GUN SHOULD BE CALLED
IF(R.GT.PSHOT) GO TO 44

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JUMP=1
IF(JUHSAV.EQ.0.AND.JUMP.EQ.1) TII=.18
IF(JUHSAV.EQ.0.AND.JUMP.EQ.1) GO TO 44
TII=60.0/GR
TO=0.0
TF=0.1
IFLAG=1
J=J+1
C 0.22 SECONDS/9 ROUNDS = 0.024444
IF(J.GT.9.AND.J.LT.10) TII=.0244444
CALL GUN(YO)
25 CALL RKCUT(YO,TO,TF,6,IFLAG,H)
TF=TO+H
IF(YO(6).GT.-400.0) IFLAG=2
IF(YO(6).GT.0.0) GO TO 30
GO TO 25
30 IFLAG=1
C THIS FINDS THE VALUES WHERE THE PROJ IMPACTS GROUND
DTR=-YO(6)/YO(3)
YO(4)=YO(4)+YO(1)*DTR
YO(5)=YO(5)+YO(2)*DTR
YO(6)=YO(6)+YO(3)*DTR
TF=TF+DTR
TIME=TIME+TF
C WRITE(6,103) XT,YT,ZT,TF,YO(4),YO(5),YO(6),TIME
103 FORMAT(4X,8F8.3)
CALL GOTGT
C WRITE(6,103) XT,YT,ZT,TF,YO(4),YO(5),YO(6),TIME
TGHD=OMEGT*TIME+THETO
XTDOT=VT*COS(TGHD)
YTDOT=VT*SIN(TGHD)
ZTDOT=0.0
VPIM(J)=SQRT((YO(1)-XTDOT)**2+(YO(2)-YTDOT)**2+(YO(3)-ZTDOT)**2)
TIME=TIME-TF
TIM(J)=TIME
C NOW CALCULATE MISS DISTANCE
XMIS(J)=YO(4)-XT
YMIS(J)=YO(5)-YT
ZMIS(J)=YO(6)-ZT
C PROJECTILE IMPACT POSITIONS
XTPOS(J)=YO(4)
YTPOS(J)=YO(5)
ZTPOS(J)=YO(6)
XPT=XMIS(J)*COS(TGHD)+YMIS(J)*SIN(TGHD)
YPT=-XMIS(J)*SIN(TGHD)+YMIS(J)*COS(TGHD)
ZPT=ZMIS(J)
VPI(J)=SQRT(YO(1)*YO(1)+YO(2)*YO(2)+YO(3)*YO(3))
C PROJECTILE IMPACT POSITIONS ROTATED TO THE NORMAL
THETA1=PIE/2.0-ASIN(YO(3)/VPI(J))
SI1=ATAN(YO(2)/YO(1))
XPRI(J)=XMIS(J)*COS(THETA1)*COS(SI1)+YMIS(J)*COS(THETA1)*SIN(SI1)
YPRI(J)=-XMIS(J)*SIN(SI1)+YMIS(J)*COS(SI1)
UNITV1=(YO(1)*COS(TGHD)+YO(2)*SIN(TGHD))/VPI(J)
UNITV2=(YO(2)*COS(TGHD)-YO(1)*SIN(TGHD))/VPI(J)
UNITV3=YO(3)/VPI(J)
AP=ABS(AF*UNITV1)+ABS(AS*UNITV2)+ABS(AT*UNITV3)
AREAP(J)=AP
GUNR(J)=R
CALL TABINT(THE(J),VPIM(J),VULAT,CIVET,VELT,VULT,4,8)
CALL PK1(XPRI(J),YPRI(J),AP,VULAT,R,PKG,1.0,PHG)
PROD=PROD*(1.0-PKG)
EHITS=EHITS+PHG
EHT(J)=EHITS
PK=1.0-PROD
PKS(J)=PK
VAT(J)=VULAT
CALL GOTGT
IF(J.EQ.IBURST) GO TO 11
44 CONTINUE

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JUMSAV=JUMP
TB=TB-TII
IF(TB) 11,10,10
11 IFLY=L
IFIREF=J
VT=VT*0.68181818
OMEGT=OMEGT/RAD
THETO=THETO/RAD
SD=SD*1000.0
THETA=-THETA
C **** SIGHT/TRACKING DATA ****
      WRITE(6,12) SD,SIGAP,SIGAO
12 FORMAT(1H1, 9X,*+++++++) SIGHT/TRACKING DATA ++++++++
$+*,/
110X,*THE SIGHT DEPRESSION IN MILS IS.....*,F10.4,/
210X,*THE RANGE AIM ERROR IN MILS IS.....*,F10.4,/
310X,*THE DEFLECTION AIM ERROR IN MILS IS.....*,F10.4,/
410X,*TIME*,5X,*AZIMUTH*,2X,*ELEVATION*,3X,*S-DIF*,5X,*EL LEAD*,  

52X,*AZ LEAD*,3X,*TOT LEAD ANGLE*,/)
      WRITE(6,13) (TIMEAT(I),AZA(I),ELA(I),DALP(I),ELL(I),AZL(I),TLA(I),
1I=1,IFLY)
13 FORMAT(9X,F6.4,4X,F6.4,4X,F6.4,4X,F6.4,4X,F7.4,3X,F7.4,5X,F7.4)
C **** AIRCRAFT DATA ****
      WRITE(6,14) VACK,THETA
14 FORMAT(1H1, 9X,*+++++++) AIRCRAFT DATA ++++++++
$+*,/
110X,*SPEED IN KNOTS.....*,F10.4,/
210X,*INPUT DIVE ANGLE.....*,F10.4,/
305X,*TIME*,2X,*VELOCITY*,2X,*X-POS*,4X,*Y-POS*,4X,*Z-POS*,3X,  

4*RANGE*,4X,*QDLL*,2X,*PITCH*,5X,*YAW*,5X,*GEES*,3X,*ALPHA*)
      WRITE(6,15) (TIMEAT(I),VAA(I),XAC(I),YAC(I),ZAC(I),RA(I),PHIA(I),
1THE(I),SI(I),GEES(I),ALP(I),I=1,IFLY)
15 FORMAT(4X,F6.4,2X,F5.1,2X,F7.1,2X,F7.1,2X,F6.1,2X,F6.3,2X,
1F6.3,2X,F6.3,2X,F6.3,2X,F6.3)
C **** TARGET DATA ****
      WRITE(6,16) VT,THETO,OMEGT
16 FORMAT(1H1, 9X,*+++++++) TARGET DATA ++++++++
$+*,/
110X,*VELOCITY IN MILES PER HOUR.....*,F10.4,/
210X,*INITIAL DIRECTION OF TARGET VEL (DEGREES)*,F10.4,/
310X,*ANGULAR TURN RATE IN DEGREES.....*,F10.4,/
410X,*TIME*,7X,*X-POS*,5X,*Y-POS*,5X,*IMP-VEL*,3X,*PRES AREA*,3X,  

5*VUL AREA*)
      WRITE(6,17) (TIM(I),XTT(I),YTT(I),VPIM(I),AREAP(I),VAT(I),I=1,
1IFIREF)
17 FORMAT(9X,F6.4,6X,F6.1,4X,F6.1,6X,F6.1,4X,F6.1,4X,F6.3)
C **** GUN/PROJECTILE DATA ****
      WRITE(6,18) GR,TRURST,DIA,SIGBR,SIGRD,EHT(IFIRE),PKS(IFIRE)
18 FORMAT(1H1, 9X,*+++++++) GUN/PROJECTILE DATA ++++++++
$+*,/
110X,*GUN RATE IN ROUNDS PER MINUTE.....*,F10.4,/
210X,*BURST TIME IN SECONDS.....*,F10.4,/
310X,*PROJECTILE SIZE IN MILLIMETERS.....*,F10.4,/
410X,*BALLISTIC DISPERSION IN RANGE (MILS)*,F10.4,/
510X,*BALLISTIC DISPERSION IN DEFLECTION (MILS)*,F10.4,/
610X,*NUMBER OF EXPECTED HITS ON TARGET.....*,F10.4,/
710X,*PROBABILITY OF TARGET KILL.....*,F10.4,/
810X,*TIME*,4X,*ROUND*,3X,*X-MISS*,3X,*Y-MISS*,4X,*RANGE*,3X,  

9*IMP VEL*,3X,*E(HITS)*,4X,*PK*)
      WRITE(6,19) (TIM(I),I,XPRI(I),YPRI(I),GUNR(I),VPIM(I),EHT(I),PKS(I
1),I=1,IFIREF)
19 FORMAT(9X,F6.4,5X,I3,5X,F4.1,4X,F4.1,4X,F6.1,4X,F6.1,3X,F4.1,4X,
1F5.4)
      GO TO 7776
7777 CALL EXIT
      END

```

```

SUBROUTINE SIGHT(YO,TO,TF,N,IFLAG,H,TSIGHT)
COMMON/SIG/VGP,ALPOL,EPSI,VM,RHO,SD,PI,V
COMMON/DOTAP/XD0T,YD0T,ZD0T,XD0TP,YD0TP,ZD0TP
COMMON/ALL/RTHETA,VA,XA,YA,ZA,EL,PSI,R,ALPHA,PHI,AZ,S,CLALP,H
DIMENSION YO(6)
PHOI=0.002377*((518.688-0.0035662*ABS(ZA))/518.688)**4.2561
QSA=0.5*PHOI*VA*VA*S
ALPHA=(W*COS(RTHETA)+QSA*CLALP*ALPOL)/(QSA*CLALP+TSIGHT)
QSA=0.5*RHO*V*V*S
ALFIR=(W*COS(RTHETA)+QSA*CLALP*ALPOL)/(QSA*CLALP+TSIGHT)
TAU=(V*(ALFIR-EPSI))/(V+VM)
CVD=VGP/RI
IFLAG=1
N=6
TO=0.0
TF=0.1
YD0TP=0.0
YP=0.0
THEHSH=RTHETA+ALFIR-EPSI-TAU
XD0TP=(V+VM)*COS(THEHSH)
ZD0TP=-(V+VM)*SIN(THEHSH)
XP=RI*COS(RTHETA)
ZP=RI*SIN(RTHETA)
YO(1)=X0OTP
YO(2)=Y0OTP
YO(3)=Z0OTP
YO(4)=XP
YO(5)=YP
YO(6)=ZP
10 CALL RKCUT(YO,TO,TF,6,IFLAG,H)
TF=TO+H
IF(YO(6).GT.-400.0) IFLAG=2
IF(YO(6).GT.0.0) GO TO 15
GO TO 10
15 IFLAG=1
DTR=-YO(6)/YO(3)
YO(4)=YO(4)+YO(1)*DTR
YO(5)=YO(5)+YO(2)*DTR
YO(6)=YO(6)+YO(3)*DTR
DELTA=-ATAN(ZP/(YO(4)-XP))+THEHSH
SD=EPSI+TAU+DELTA+CVD
RETURN
END

```

```

SUBROUTINE TRACK
COMMON/TRA/XT,YT,ZT
COMMON/ALL/RTHETA,VA,XA,YA,ZA,EL,RPSI,R,ALPHA,PHI,AZ,S,CLALP,W
R=SQRT((XT-XA)**2+(YT-YA)**2+(ZT-ZA)**2)
C LINE OF SIGHT UNIT VECTOR
R1=(XT-XA)/R
R2=(YT-YA)/R
R3=(ZT-ZA)/R
STHETA=SIN(RTHETA)
CTHETA=COS(RTHETA)
SPSI=SIN(RPSI)
CPSI=COS(RPSI)
SPHI=SIN(PHI)
CPHI=COS(PHI)
SALPHA=SIN(ALPHA)
CALPHA=COS(ALPHA)
AX1=CALPHA*CTHETA*CPSI-SALPHA*(CPHI*STHETA*CPSI+SPHI*SPSI)
AX2=CALPHA*CTHETA*SPSI-SALPHA*(CPHI*STHETA*SPSI-SPHI*CPSI)
AX3=-CALPHA*STHETA-SALPHA*CPHI*CTHETA
AY1=SPHI*STHETA*CPSI-CPHI*SPSI
AY2=SPHI*STHETA*SPSI+CPHI*CPSI
AY3=SPHI*CTHETA
AZ1=SALPHA*CTHETA*CPSI+CALPHA*(CPHI*STHETA*CPSI+SPHI*SPSI)
AZ2=SALPHA*CTHETA*SPSI+CALPHA*(CPHI*STHETA*SPSI-SPHI*CPSI)
AZ3=-SALPHA*STHETA+CALPHA*CPHI*CTHETA
XL=AX1*R1+AX2*R2+AX3*R3
YL=AY1*R1+AY2*R2+AY3*R3
ZL=AZ1*R1+A22*R2+A23*R3
C TARGET ANGULAR COORDINATES IN FIELD OF VIEW ALONG AIRCRAFT X AXIS
EL=ASIN(ZL)
AZ=ATAN(YL/XL)
RETURN
END

```

```

SUBROUTINE FLIGHT
COMMON/FLI/RAD,G,T,ACD,TII,TIME,PNEU,CLALPO,AP
COMMON/DOTAP/XDOT,YDOT,ZDOT,XDOTP,YDOTP,ZDOTP
COMMON/ALL/RTHETA,VA,XA,YA,ZA,EL,RPSI,R,ALPHA,PHI,AZ,S,CLALP,W
AMASS=W/G
RPHI=PHI
RALPHA=ALPHA
AP=0.002377*((518.688-0.0035662*ABS(ZA))/518.688)**4.2561
VDOT=(T-(0.5*AP*VA*VA*S*ACD))/AMSS-(G*SIN(RTHETA))
THETDOO=(G/VA)*(((CLALP*RALPHA+CLALP0)*(0.5*AP*VA*VA*S))+T*
1RALPHA)*COS(RPHI)/W-COS(RTHETA))
SIDOT=PNEU*G*SIN(RPHI)/(VA*COS(RTHETA))
VA=VA+VDOT*TII
RTHETA=RTHETA+THETDOO*TII
RPSI=PPSI+SIDOT*TII
XDOT=VA*COS(RTHETA)*COS(RPSI)
YDOT=VA*COS(RTHETA)*SIN(RPSI)
ZDOT=-VA*SIN(RTHETA)
XA=XA+XDOT*TII
YA=YA+YDOT*TII
ZA=ZA+ZDOT*TII
IF(ZA.GE.0.0) GO TO 75
56 THETA=RTHETA/RAD
SI=RPSI/RAD
ALT=-ZA
75 CONTINUE
RETURN
END

```

```
SUBROUTINE GOTGT
COMMON/GOT/VT,XTO,YTO,THETO,OMEGT
COMMON/FLI/RA0,G,T,ACD,TII,TIME
COMMON/TRA/XT,YT,ZT
IF(OMEGT.EQ.0.0) GO TO 4
XT=XTO+(VT/OMEGT)*(SIN(OMEGT*TIME+THETO)-SIN(THETO))
YT=YTO+(VT/OMEGT)*(COS(THETO)-COS(OMEGT*TIME+THETO))
GO TO 5
4 XT=XTO+VT*COS(THETO)*TIME
YT=YTO+VT*SIN(THETO)*TIME
5 CONTINUE
RETURN
END
```

```

SUBROUTINE PKCUT(Y0,T0,TF,N,IFLAG,H)
C*****4TH ORDER INTEGRATION ROUTINE*****
C
REAL K
DIMENSION Y0(N),YOI(600),K(600,4),B(4),A(4,4)
DATA B/.5,.5,.5,1.0/
DATA A/.5,0.,0.,.166666,0.,.5,0.,.333333,0.,0.,1.,.333333,0.,0.,
     0.,.166666/
IF(IFLAG.EQ.2) GO TO 1
H=.1
GO TO 2
1 H=.01
2 CONTINUE
5 IF(T0.GE. TF-.001) GO TO 70
DO 6 M=1,N
6 YOI(M)=Y0(M)
TI=T0
DO 25 KK=1,N
DO 25 KKK=1,4
25 K(KK,KKK)=0.
DO 50 I=1,4
TO=TI+B(I)*H
CALL VALUE(Y0,K(1,I),N)
DO 60 M=1,N
SUM=0.
DO 80 II=1,4
SUM=SUM+A(I,II)*K(M,II)*H
80 YO(M)=SUM+YOI(M)
60 IF(I.EQ.4) GO TO 5
50 CONTINUE
70 CONTINUE
RETURN
END

```

```

SUBROUTINE VALUE(YO,K,N)
C***** APPROPRIATE DERIVATIVES FOR INTEGRATION *****
C* COMMON/VALUE/WPROJ,DIA,PIE
C* COMMON/FLI/RAD,G,T,ACD,TII,TIME,PNEU
C* REAL K
C* DIMENSION YO(N), K(N)
C* CALCULATE PROJECTILE DRAG
C* DO 13 M=1,N,6
C* RHOP=.0023769*(1.-(.0035683*ABS(YC(M+5)))/518.688)**4.2561
C* VSPROJ=1117.0-.004*ABS(YO(M+5))
C* VLPROJ=SQRT(YO(M)**2+YO(M+1)**2+YO(M+2)**2)
C* AMACHP=VLPROJ/VSPROJ
C* CALL CRINT(AMACHP,CD)
C* CALCULATE COMPONENT ACCELERATION AND VELOCITY OF EACH PROJECTILE *
C* CONST3=(3500.*G*RHOP*VLPROJ*CD*(PIE*(DIA/25.4)**2)/576.)/WPROJ)
C* K(M)=CONST3*YO(M)
C* K(M+1)=CONST3*YO(M+1)
C* K(M+2)=CONST3*YO(M+2)+G
C* K(M+3)=      YO(M)
C* K(M+4)=      YO(M+1)
C* K(M+5)=      YO(M+2)
13  CONTINUE
RETURN
END

```

```
SUBROUTINE CDINT(PMACH,CDPAG)
C***** THIS SUBROUTINE INTERPOLATES IN A CD VS MACH TABLE TO OBTAIN CD ****
C***** COMMON/MACH/MACH(20),CD(20)
      REAL MACH
      NOST=1
      NOLT=20
      IF(PMACH.LE.MACH(NOST)) GO TO 1
      NOST=NOST+1
      DO 2 I=NOST ,NOLT
      IF (MACH(I).EQ.0.) GO TO 3
      IF(PMACH.LT.MACH(I)) GO TO 4
      IF(PMACH.EQ.MACH(I)) GO TO 5
 2 CONTINUE
      CDRAG=CD(NOLT)
      GO TO 6
 1  CDRAG=CD(NOST)
      GO TO 6
 3  CDRAG=CD(I-1)
      GO TO 6
 4  OLIMT=MACH(I-1)
      HILIMT=MACH(I)
      FACTOR=(PMACH-OLIMT)/(HILIMT-OLIMT)
      CDRAG=(CD(I)-CD(I-1))*FACTOR
      CDRAG=CDPAG+CD(I-1)
      GO TO 6
 5  CDRAG=CD(I)
 6  CONTINUE
      RETURN
      END
```

```

SUBROUTINE GUN(Y0)
COMMON/DOТАP/XDOT,YDOT,ZDOT,XDOTP,YDOTP,ZDOTP
COMMON/ALL/RTHETA,VA,XA,YA,ZA,EL,EPST,R,ALPHA,PHI,AZ,S,CLALP,W
COMMON/SIG/VGP,ALPOL,EPSI,VM,RHO,SD,PI,V
DIMENSION Y0(6)
STHETA=SIN(RTHETA)
CTHETA=COS(RTHETA)
SPSI=SIN(EPSI)
CPSI=COS(EPSI)
SPHI=SIN(PHI)
CPHI=COS(PHI)
SALPHA=SIN(ALPHA)
CALPHA=COS(ALPHA)
AX1=CALPHA*CTHETA*CPSI-SALPHA*(CPHI*STHETA*CPSI+SPHI*SPSI)
AX2=CALPHA*CTHETA*SPSI-SALPHA*(CPHI*STHETA*SPSI-SPHI*CPSI)
AX3=-CALPHA*STHETA-SALPHA*CPHI*CTHETA
AY1=SPHI*STHETA*CPSI-CPHI*SPSI
AY2=SPHI*STHETA*SPSI+CPHI*CPSI
AY3=SPHI*CTHETA
AZ1=SALPHA*CTHETA*CPSI+CALPHA*(CPHI*STHETA*CPSI+SPHI*SPSI)
AZ2=SALPHA*CTHETA*SPSI+CALPHA*(CPHI*STHETA*SPSI-SPHI*CPSI)
AZ3=-SALPHA*STHETA+CALPHA*CPHI*CTHETA
VMX=(AX1*VM*COS(EPSI))+(AZ1*VM*SIN(EPSI))
VMY=(AX2*VM*COS(EPSI))+(AZ2*VM*SIN(EPSI))
VMZ=(AX3*VM*COS(EPSI))+(AZ3*VM*SIN(EPSI))
VPX=XDOT+VMX
VPY=YDOT+VMY
VPZ=ZDOT+VMZ
VPT=SORT(VPX*VPX+VPY*VPY+VPZ*VPZ)
VP=SORT(XDOTP*XDOTP+YDOTP*YDOTP+ZDOTP*ZDOTP)
Y0(1)=VPX
Y0(2)=VPY
Y0(3)=VPZ
RETURN
END

```

```

C SUBROUTINE TABINT(X,Y,Z,XT,YT,ZT,M,N)
C
C      DIMENSION XT(M),YT(N),ZT(M,N)
C
C      X AND Y ARE INPUTS
C      Z IS OUTPUT
C      XT, YT, AND ZT ARE TABLE PARAMETERS
C      M AND N ARE DIMENSIONS OF TABLE ARRAYS
C      ILESS=0
C      IF(M.EQ.1) GO TO 300
C      IF (X.EQ.XT(1)) GO TO 300
C      IF (X.GT.XT(1)) GO TO 210
C      ILESS=1
C      GO TO 500
210    DO 220 J=1,M
C      IF(XT(J).GE.X) GO TO 230
220    CONTINUE
C      ILESS=2
C      GO TO 500
230    IF(Y.EQ.YT(1)) GO TO 380
C      IF (Y.GT.YT(1)) GO TO 240
C      ILESS=3
C      GO TO 500
240    DO 250 K=1,N
C      IF(YT(K).GE.Y) GO TO 260
250    CONTINUE
C      ILESS=4
C      GO TO 500
260    P=(XT(J-1)-X)/(XT(J)-XT(J-1))
C      Q=(YT(K-1)-Y)/(YT(K)-YT(K-1))
C      Z=(1.-P)*(1.0-Q)*ZT(J-1,K-1)+P*(1.-Q)*ZT(J,K-1)+Q*(1.-P)*ZT(J-1,K)
1+P*Q*ZT(J,K)
C      RETURN
300    IF(Y.EQ.YT(1)) GO TO 370
C      IF (Y.GT.YT(1)) GO TO 340
C      ILESS=5
C      GO TO 500
340    DO 350 K=1,N
C      IF(YT(K).GE.Y) GO TO 360
350    CONTINUE
C      ILESS=6
C      GO TO 500
360    Q=(YT(K-1)-Y)/(YT(K)-YT(K-1))
C      Z=(1.-Q)*ZT(1,K-1)+Q*ZT(1,K)
C      RETURN
370    Z=ZT(1,1)
C      RETURN
380    P=(XT(J-1)-X)/(XT(J)-XT(J-1))
C      Z=(1.-P)*ZT(J-1,1) +P*ZT(J,1)
C      RETURN
500    CONTINUE
GO TO (1,2,3,4,5,6), ILESS
1    CONTINUE
11   WRITE (6,11) X
FORMAT(1H ,10X,E16.8,3X,33HX INPUT VALUE TOO LOW - 3-D TABLE)
GO TO 510
2    CONTINUE
12   WRITE (6,12) X
FORMAT(1H ,10X,E16.8,3X,34HX INPUT VALUE TOO HIGH - 3-D TABLE)
GO TO 510
3    CONTINUE
13   WRITE (6,13) Y
FORMAT(1H ,10X,E16.8,3X,33HY INPUT VALUE TOO LOW - 3-D TABLE)
GO TO 510
4    CONTINUE
14   WRITE (6,14) Y
FORMAT(1H ,10X,E16.8,3X,34HY INPUT VALUE TOO HIGH - 3-D TABLE)
GO TO 510
5    CONTINUE

```

```
15  WRITE (6,15) Y
      FORMAT(1H ,10X,E16.8,3X,33HY INPUT VALUE TOO LOW - 2-D TABLE)
      GO TO 510
6   CONTINUE
16  WRITE (6,16) Y
      FORMAT(1H ,10X,E16.8,3X,34HY INPUT VALUE TOO HIGH - 2-D TABLE)
      CONTINUE
510 TRACEB= ASIN (1.1)
      RETURN
      END
```

```

SUBROUTINE PK1(BX,BY,AREAT,AV,RP,PKG,RNDINC,PHG)
C***** THIS SUBROUTINE CALCULATES PK PER SURST *****
C***** DIMENSION ZX(9),WF(9),ZNE(9),EPS(9)
C***** DATA WF/.72023522,.47265156,.47265156,.08847453,.08847453,
C***** .00494362,.00494362,.00003961,.00003961/
C***** DATA ZX/0.,.72355102,-.72355102,1.46855329,-1.46855329,2.26658058,
C***** -2.26658058,3.19099320,-3.19099320/
C***** DATA SBX/1.4/,SBY/1.4/,SAX/5.887/,SAY/5.887/
C***** SORT2=1.414
C***** PHG=0.0
C***** PLIVE=n.
C***** ZWID=SORT(AREAT)*.5
C***** ZLEN=ZWID
C***** SAFY=SAY*RP/1000.0
C***** SAFX=SAX*PP/1000.0
C***** SX=SORT2*SBX*(RP/1000.)
C***** SY=SORT2*SBY*(RP/1000.)
C***** AIM POINTS GENERATED USING 9 POINT HERMIT GAUSS
C***** DO 100 JI=1,9
C***** EPS(JI)=SORT2*SAFX*ZX(JI)+BX
100 ZNE(JI)=SORT2*SAFY*ZX(JI)+BY
DO 200 J=1,9
DO 200 K=1,9
DUM1=EPRFCN((ZLEN-EPS(J))/SX)
DUM2=FPRFCN((ZLEN+EPS(J))/SX)
DUM3=FRRFCN((ZWID-ZNE(K))/SY)
DUM4=EPRFCN((ZWID+ZNE(K))/SY)
PKG=(0.25*(DUM1+DUM2)*(DUM3+DUM4))*AV/AREAT
PLTVA=(1.-PKG)**RNDINC
PLIVE=PLTVA+WF(J)*WF(K)*PLIVA/3.14159
PHGA=0.25*(DUM1+DUM2)*(DUM3+DUM4)
PHGA=(1.0-PHGA)**RNDINC
200 PHG=PHG+WF(J)*WF(K)*PHGA/3.1415926
PHG=1.0-PHG
PKG=1.-PLIVE
RETURN
END

```

```
FUNCTION ERFCN(T)
C***** THIS SUBROUTINE EVALUATES THE ERROR FUNCTION GIVEN BY, Y=ERF(T), *
C* WHERE T IS THE ARGUMENT.
C***** A=1.0
1 IF(T) 2,1,3
1 ERRFCN=0.
RETURN
2 A=-1.
3 T=A*T
4 IF(T>4.2) 5,4,4
4 ERRFCN=A
RETURN
5 FRAC=T
DUM=FRAC*A
G=1.5
TSQ=T**2
6 FRAC=(FRAC*TSQ)/G
IF(FRAC<1.E-8) 8,7,7
7 DUM=DUM+FRAC*A
G=G+1.
GO TO 6
8 ERRFCN=1.1283792*DUM*EXP(-TSQ)
RETURN
END
```

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