Georgia Tech | Air Transportation Laboratory

Algorithms for Economically and Environmentally Efficient Terminal Area Transition Metering

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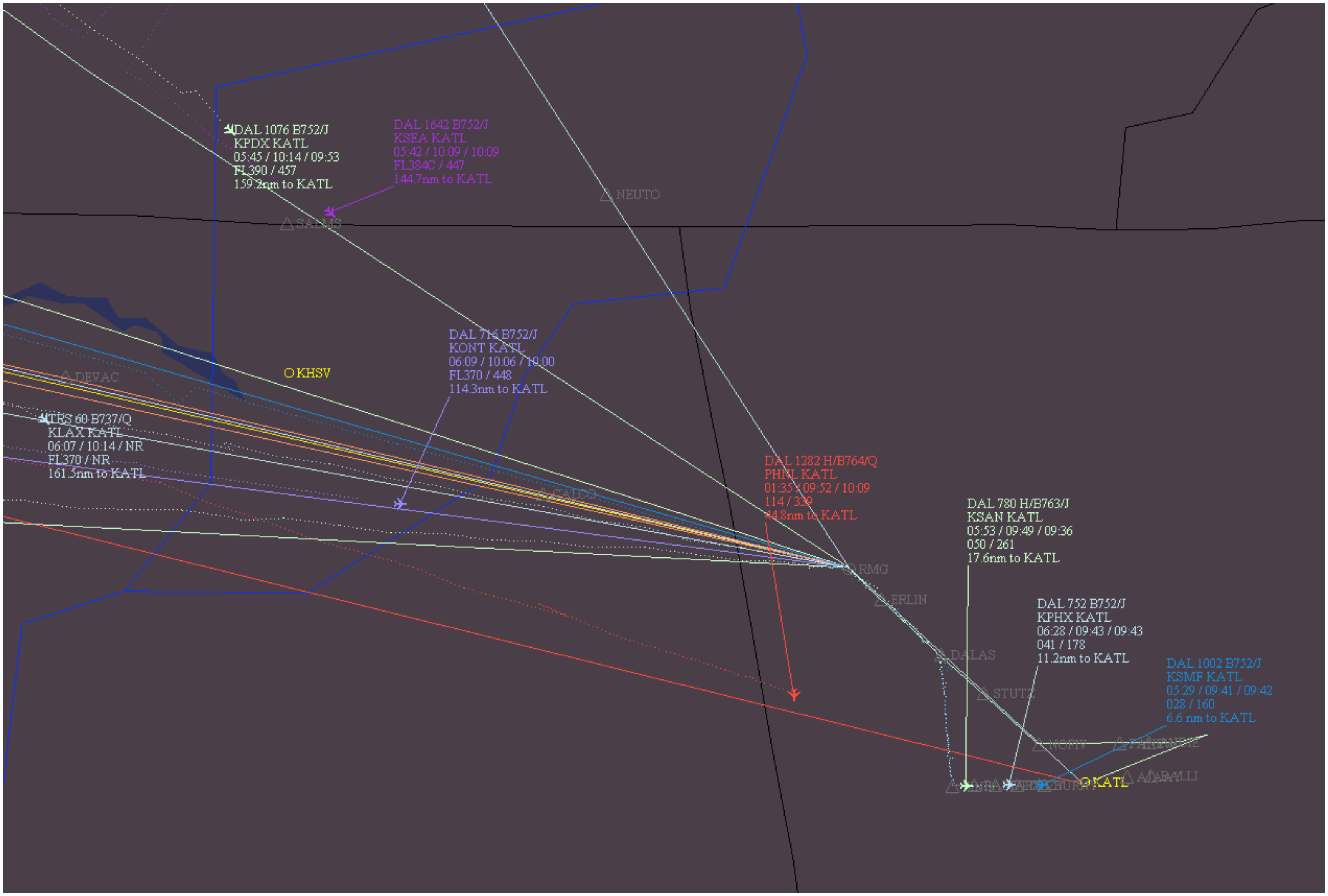
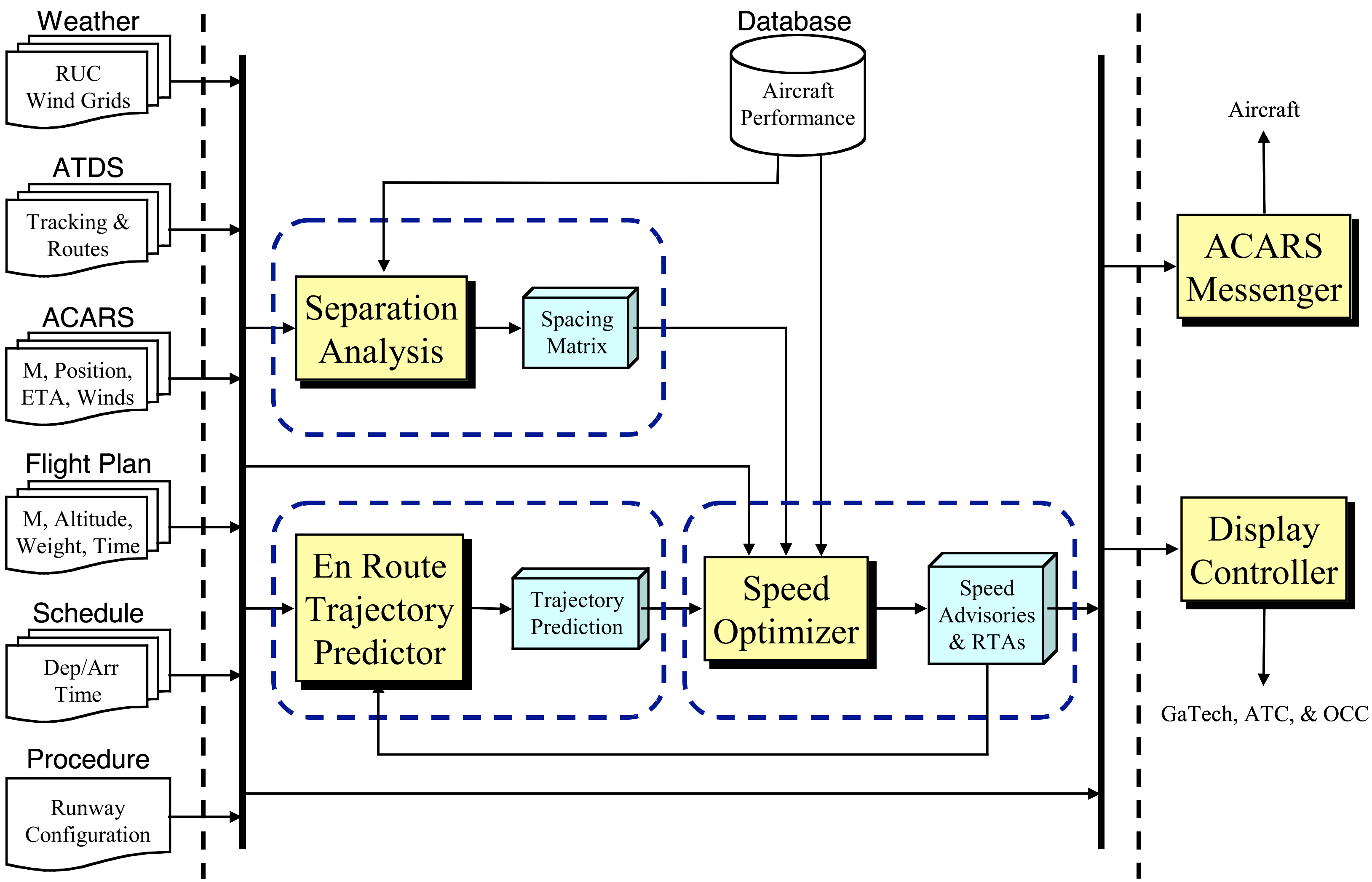
Achieving the Desired Spacing



Optimization Overview



Objective Function: Minimizing Fuel Over Given Distance

$$\dot{f}_i \geq a_{i,1} M_x + b_{i,1}$$
$$\dot{f}_i \geq a_{i,2} M_x + b_{i,2}$$
$$\vdots$$
$$\dot{f}_i \geq a_{i,m} M_x + b_{i,m}$$

$$\min Z = \sum_{n=1}^{N} \dot{f}_{M_{d_i}} (T_i - \Delta t_i)$$

Objective Function: Minimize Fuel Burn Over Given Distance

- **Inclusion of dt in objective function is a simple addition, but necessitates additional constraints [1]**
- **Must now approximate bilinear term with a grid**
- **Further constraints at right limit selection of grid points to four adjacent planes**

$$r_i = \dot{f}\big|_{M_{d_i}} \Delta t_i$$

$$\lambda_{kl_i} \approx F_k \Delta T_l$$

Fuel burn value equal to sum of fuel grid points

$$\dot{f}\big|_{M_{d_i}} = \sum_k \sum_l F_k \lambda_{kl_i}$$

$$\Delta t_i = \sum_k \sum_l \Delta T_l \lambda_{kl_i}$$

Combined bilinear term equal to sum of grid points

$$r_i = \sum_k \sum_l (F_k \Delta T_l) \lambda_{kl_i}$$

Sum of grid points must be equal to a complete term

$$\sum_k \sum_l \lambda_{kl_i} = 1$$

$$\mu_{k_i} = \sum_l \lambda_{kl_i}$$

$$\eta_{l_i} = \sum_k \lambda_{kl_i}$$

$$\lambda_{kl_i} \geq 0$$

Use SOS variables to limit number of adjacent planes

$$\mu_{k_i} \in SOS2$$

$$\eta_{l_i} \in SOS2$$

Can have a positive or negative term (speed increase or decrease)

$$r_i \rightarrow free$$

[1] D.A. Babayev. Piece-Wise Linear Approximation of Functions of Two Variables. Journal of Heuristics, 2: 313-320. 1997. Kluwer Academic Publishers.

Constraints: Sequence and Spacing

- Necessary for aircraft to rearrange scheduled arrival times
- Allows algorithm to examine all possible arrival sequences

- Separation constraints for a pair of aircraft

$$T_2 - T_1 + \alpha_{1,2} \leq P z_1$$

$$2\alpha_{2,1} - (T_2 - T_1 + \alpha_{2,1}) \leq P(1 - z_1)$$

$$T_3 - T_1 + \alpha_{1,3} \leq P z_2$$

$$2\alpha_{3,1} - (T_3 - T_1 + \alpha_{3,1}) \leq P(1 - z_2)$$

$$T_3 - T_2 + \alpha_{2,3} \leq P z_3$$

$$2\alpha_{3,2} - (T_3 - T_2 + \alpha_{3,2}) \leq P(1 - z_3)$$

Conditions to satisfy separation, based on aircraft type

- Example for three aircraft, variable sequence constraints will create additional constraints

$$\sum_{n=1}^{N} i$$

Constraints: Speed Changes

$$\delta_i \geq \frac{|\Delta t_i|}{\mathrm{M}_i}$$
$$\delta_i \leq \mathrm{M}|\Delta t_i|$$
$$\delta_1, \delta_2, \ldots \delta_n \text{ binary}$$

- Maximum one speed change per aircraft

$$\sum_{j=1}^{J} \delta_i \leq j$$

- Limit number of aircraft able to make a change

$$\Delta t_i = T_i \frac{\Delta M_i}{M_i}$$

- Mach-Time Derivation

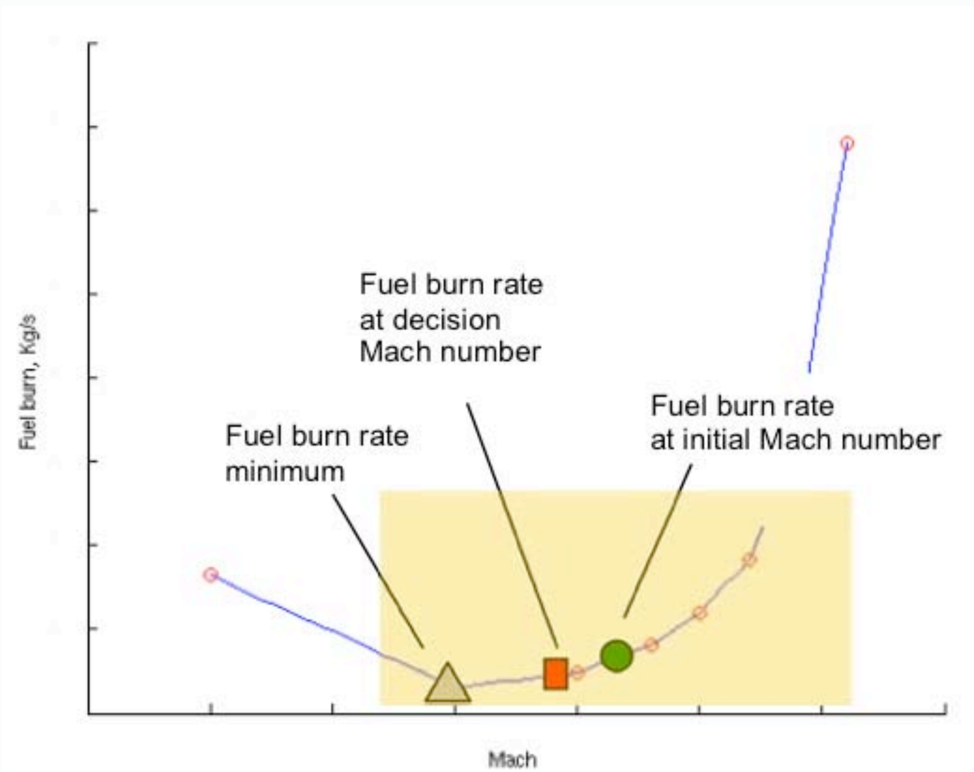
$$M_{d_i} = M_i + \Delta M_i$$
$$t_{f_i} = t_i - \Delta t_i$$

- Calculation of decision Mach and final ETA

Bounds
$$-\infty \leq \Delta t_i \leq \infty$$
$$-0.02 \leq dM_i \leq 0.02$$

- Bounds on decision variables

Constraints: Fairness



$$P_{f_i} = \frac{\dot{f}_i\big|_{M_d}}{\dot{f}_{\min}} \cdot 100$$

$$P_{f_i} - P_{f_{i+1}} \leq |tolerance|.$$

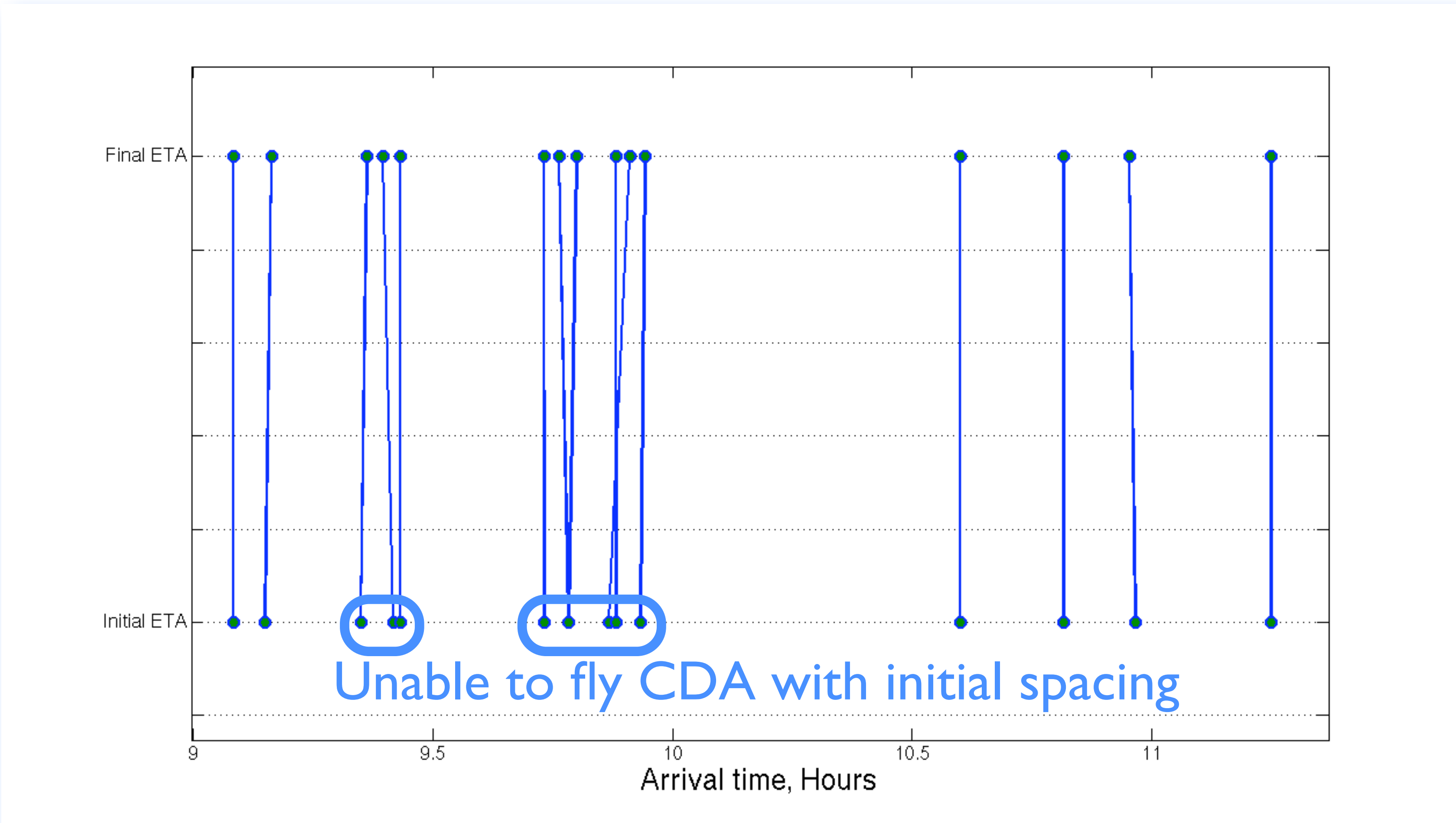
❖**Equate percentage increase in fuel burn for every group of aircraft belonging to an individual airline**

Sample Scenario

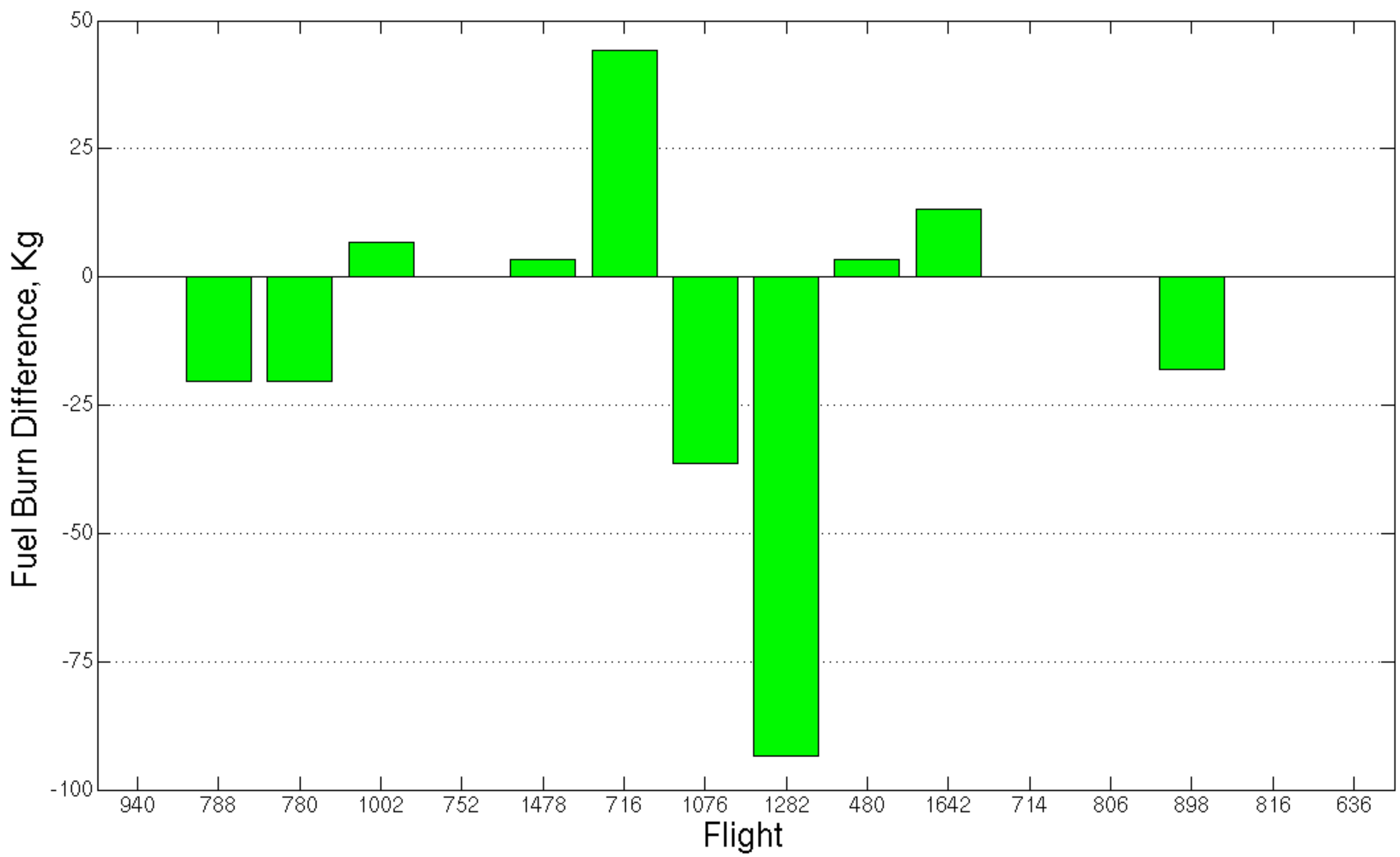
Flight Number	Aircraft Type	Initial Mach	Flight Departure Time	Initial ETA	Required Sep. (s)	Initial Sep. (s)
940	752	0.78	3:35 AM	9:05 AM	131.1	240
788	763	0.785	3:39 AM	9:09 AM	107.2	720
780	763	0.785	3:51 AM	9:21 AM	115	240
1002	752	0.78	3:55 AM	9:25 AM	135	60
752	752	0.78	3:56 AM	9:26 AM	131.1	1080
1478	763	0.785	4:14 AM	9:44 AM	115	180
716	752	0.78	4:17 AM	9:47 AM	135	0
1076	752	0.775	4:17 AM	9:47 AM	107.2	300
1282	764	0.79	4:22 AM	9:52 AM	107.2	60
480	763	0.785	4:23 AM	9:53 AM	115	180
1642	752	0.78	4:26 AM	9:56 AM	135	2400
714	752	0.78	6:06 AM	10:36 AM	131.1	780
806	763	0.78	6:19 AM	10:49 AM	115	540
898	752	0.775	6:28 AM	10:58 AM	135	1020
816	752	0.78	6:45 AM	11:15 AM	135	1500
636	752	0.78	7:10 AM	11:40 AM		

- **Flights in RED would be unable to fly the CDA as initially spaced. Aside from the obvious spacing conflicts, there are clusters of aircraft that would be affected by isolated Mach change decisions**

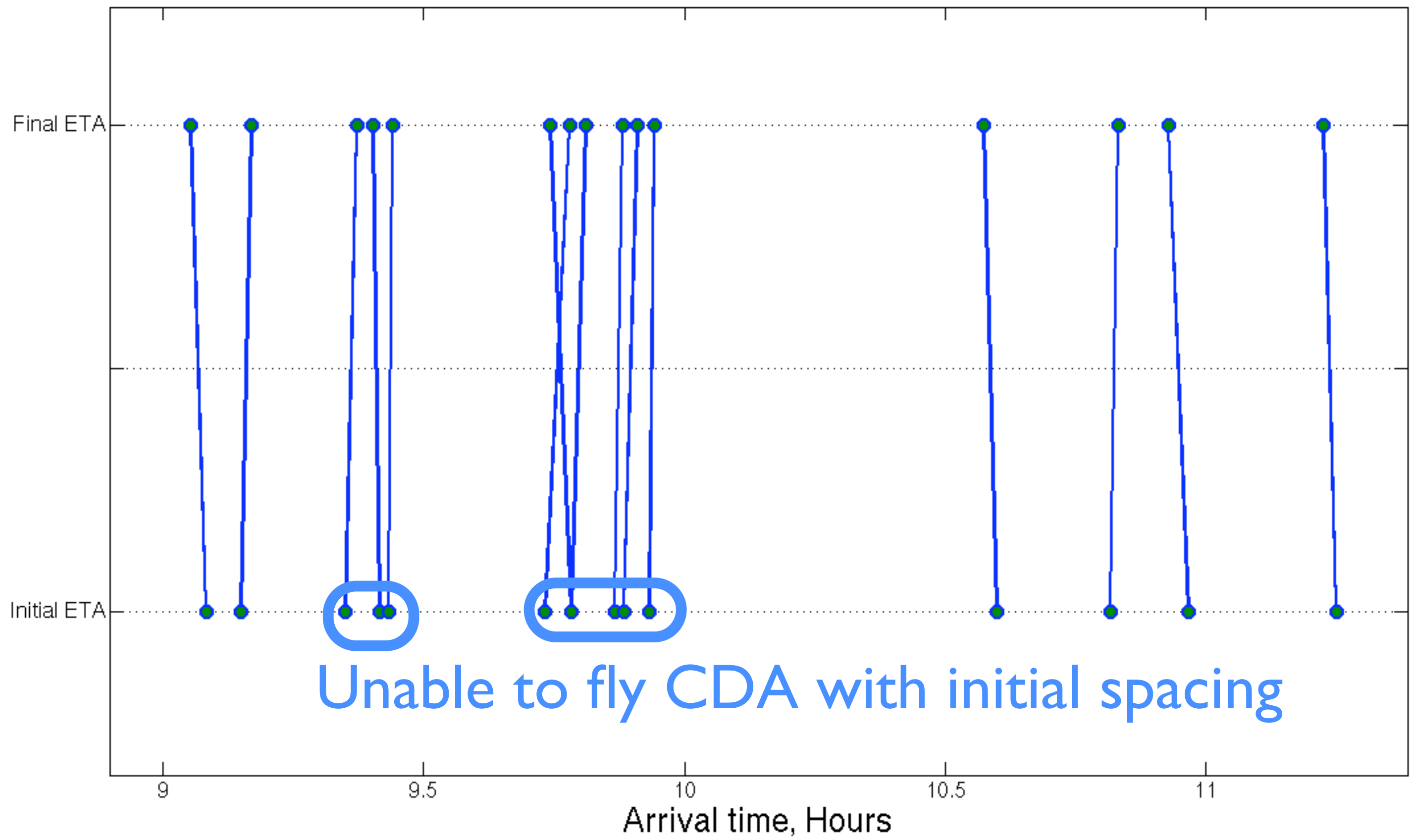
Results Without Fairness: Initial and Final ETA Separation



Results Without Fairness: Fuel Burn Change



Results With Fairness: Initial and Final ETA Separation



Results With Fairness: Fuel Burn Change