

Informal Final Report  
NIA Task 6322;  
NASA # NNL12AB26T

# On-Demand Aircraft Conceptual Design and Development

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# Introduction

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- ❖ This presentation describes the work conducted at Georgia Tech for NIA Task 6322
- ❖ The presentation is divided into four parts, corresponding to major tasks and their chronological progression:
  1. Modeling of the Cirrus SR-22 as a baseline aircraft and a simple retrofit of this aircraft with an electric propulsion system. This task produced models to serve as a basis of comparison to proposed electric aircraft concepts.
  2. An approach for sizing electric aircraft concepts and exploring the sensitivity of the sizing results to battery, aerodynamic, and weights technology assumptions.
  3. Development and sizing of a “low risk” electric aircraft concept similar to the NRL Ion Dasch design.
  4. Revisiting of the SR-22 baseline model based on new information and lessons learned. This task improved our calibration of the baseline models.
- ❖ An appendix to the report provides a detailed description of the cooling drag model developed as a part of the baseline modeling task



# Part 1: Baseline Modeling and Performance Analysis of Electric Aircraft Concepts

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# Introduction: Zip Aviation

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## ❖ Aircraft

- Fleet of one-, two-, and four-place
- General aviation
- Electric propulsion systems
- High degree of autonomy



## ❖ Potential for low vehicle operating costs

- High utilizations
- High vehicle reliability

## ❖ Georgia Tech task:

- Part of larger, NASA-led on-demand air transportation system study
- Study performance and design implications of fully electric propulsion



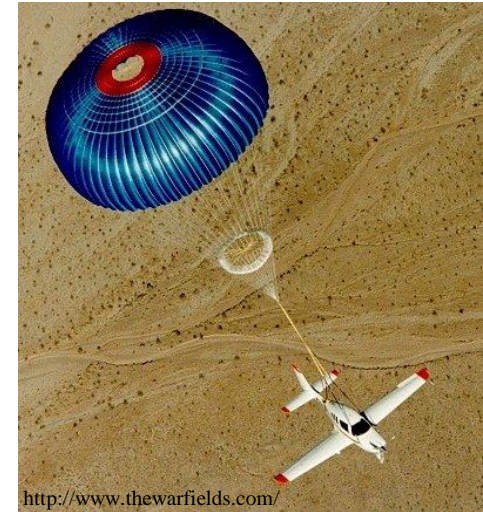
# Baseline Aircraft Modeling

## ❖ Purpose of baseline models

- Serve as comparison point for new designs
  - Illustrate differences in propulsion systems
  - Benchmark aircraft performance
- Validation and refine modeling process

## ❖ Cirrus SR-22 baseline

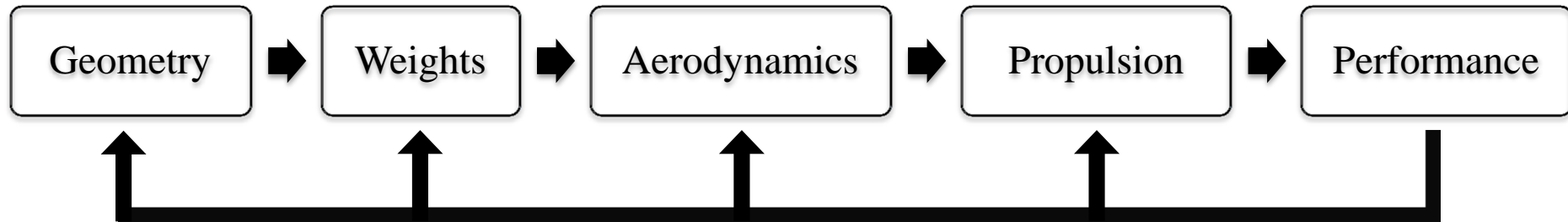
- Four-place, single-engine, GA aircraft
- Used by several “air taxi” carriers
  - SATSair, ImagineAir, Skyway Air Taxi, OpenAir
- State-of-the-art for GA technologies
- Data available from POH and other sources



Cirrus SR-22



# Modeling Procedure

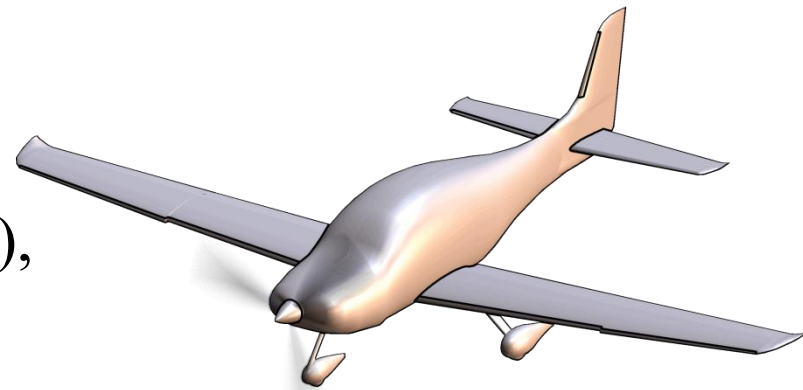


## ❖ Geometry

- VSP
- Pilot's Operating Handbook (POH), 3-views, photographs

## ❖ Weights

- FLOPS
- Delivered weight and equipment list



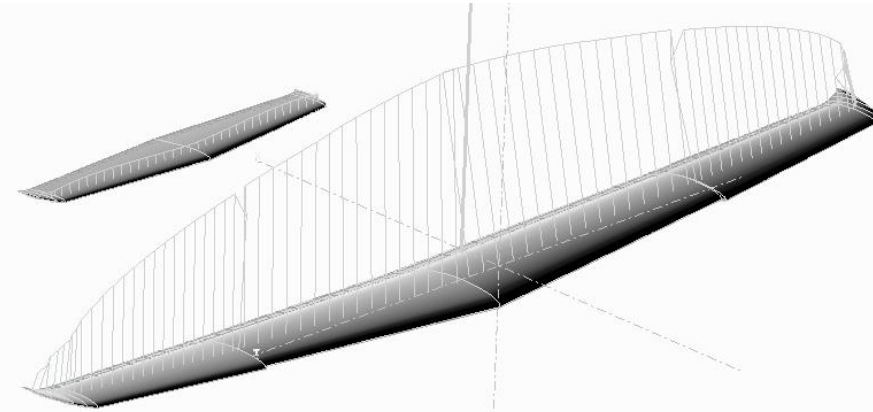
SR-22 VSP Model



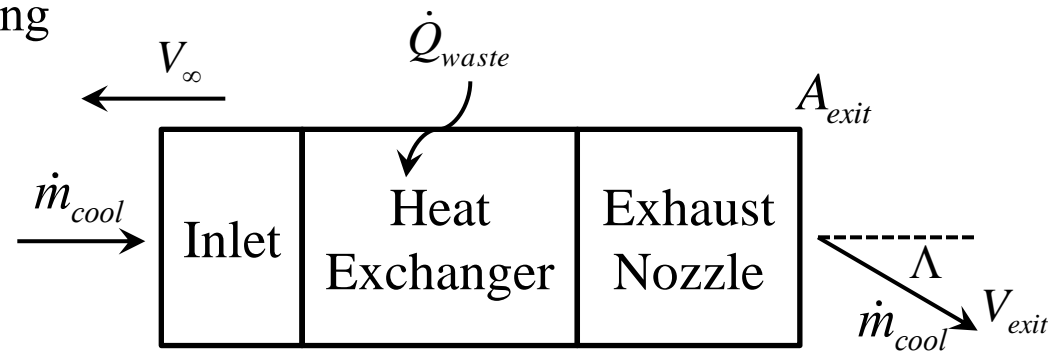
# Modeling Procedure

## ❖ Aerodynamics

- Parasite drag buildup
- XFLR5 for induced drag and profile drag due to lift
- Cooling drag
  - Internal combustion engines vs. electric motor
  - Model treats cooling system as a ramjet-like propulsion system
  - Determine “thrust” of cooling system
  - Requires estimates of
    - Total pressure loss
    - Cooling system exit  $C_p$
  - Model as constant “corrected drag”



SR-22 XFLR5 Model



# Modeling Procedure

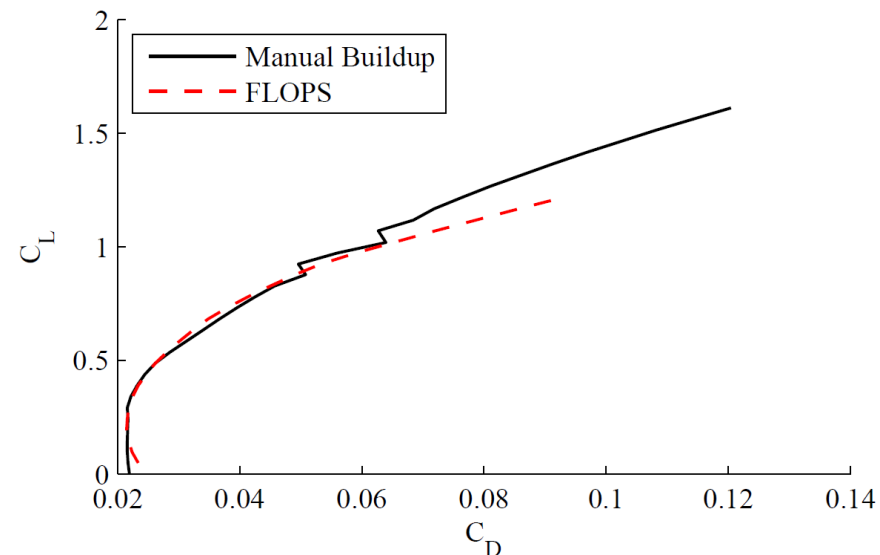
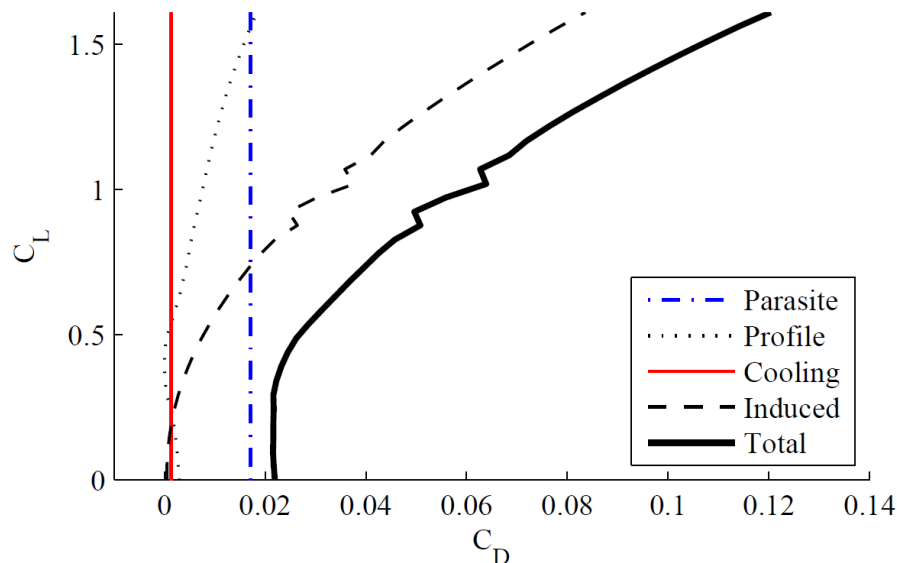
## ❖ Aerodynamics (cont)

- Scrubbing drag
  - Modifies parasite drag buildup
  - Increased velocity over components in slipstream
    - Reynolds number effects
    - Dynamic pressure increase

$$V_{\text{slipstream}} = \frac{V_{\infty}}{2} + \sqrt{\left(\frac{V_{\infty}}{2}\right)^2 + \frac{P\eta_{\text{prop}}}{2(A - A_{\text{blockage}})\rho_{\infty}V_{\infty}}}$$

$$C_{D0,c} = \frac{(C_{f,c})(FF_c)(Q_c)(S_{\text{wet},c})}{S} \left( \frac{V_{\text{slipstream}}^2}{V_{\infty}^2} \right)$$

## ■ FLOPS calibration





# Modeling Procedure

## ❖ Propulsion

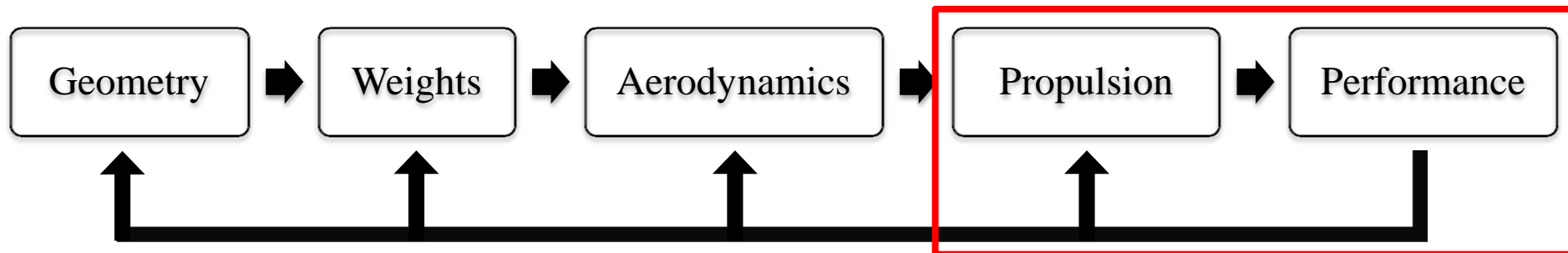
- FLOPS
- POH, Teledyne Continental engine installation and operation manual

FLOPS engine and propeller models iterated to match unaccelerated rate of climb from POH data

$$ROC = \frac{V(T - D)}{W}$$

## ❖ Performance

- FLOPS
- POH



# Baseline Model Validation

## ❖ Point Performance

- Rate of climb
- Fuel flow rates

## ❖ Mission Performance

- Predicted range

Altitude (ft)	FLOPS ROC (ft/min)	POH ROC (ft/min)	Percent Difference
0	1412	1398	1.00%
2000	1282	1279	0.24%
4000	1151	1160	-0.78%
6000	1028	1041	-1.27%
8000	910	922	-1.28%
10000	794	803	-1.17%
12000	681	684	-0.39%
14000	571	565	1.10%

Example Mission	POH Range (nmi)	FLOPS Range (nmi)	Percent Difference
High Altitude	600	621.9	3.65%
Long Low Altitude	250	255.5	2.20%
Short Low Altitude	87	93.6	7.59%

Altitude (ft)	Airspeed (knots)	POH FFR (lb/hr)	FLOPS FFR (lb/hr)	Percent Difference
2,000	186	139.8	144	2.92%
2,000	167	102.0	103	0.97%
6,000	184	120.6	121	0.33%
6,000	162	84.6	87	2.76%
10,000	182	103.8	102	-1.76%
10,000	158	72.6	73	0.55%
14,000	178	88.8	83	-6.99%
14,000	159	67.2	70	4.00%



# Active Sizing Constraint Determination

- ❖ Use FLOPS model in optimization mode
  - Objective: minimize takeoff gross weight
  - Variables: gross weight, thrust, wing area
- ❖ Specified altitude, velocity, range, payload weight
- ❖ Constraints:
  - Takeoff distance from POH
  - ROC at altitude from POH
  - Stall speed from federal aviation regulations (FARs): 61 knots

	Optimized FLOPS SR-22	Baseline SR-22	Percent Difference
Gross Weight (lb)	3468	3400	2.0%
Wing Area (ft <sup>2</sup> )	147.2	144.9	1.6%
Thrust (lb)	1120	1090	2.7%
ROC at 10,000 ft (fpm)	807	803	0.4%
Stall Speed (knots)	<b>59.1</b>	<b>59</b>	0.2%
Takeoff Field Length (ft)	1593	1594	-0.1%



# How would an “Electric SR-22” perform?

- ❖ What is the practicality of replacing the engine with a battery-electric propulsion system?
- ❖ What is the range with different levels of battery, controller, and motor technologies?



Beyond Aviation Cessna 172



# Technology Assumptions

	Technology Year		
	2015	2035	2050
Motor Nominal Specific Power (hp/lb)	3	4.5	7.5
Motor Efficiency (with Gearbox)	0.925	0.95	0.97
Controller Specific Weight (lb/hp)	0.05	0.05	0.05
Controller Efficiency	0.98	0.99	0.99
Battery Energy Density (W-hr/kg)	200	600	1200
Battery Efficiency	0.98	0.98	0.99



# Electric SR-22-like Aircraft

## ❖ Exchange propulsion systems

- Remove engine, fuel tanks, etc.
  - Engine: 661 lb
  - Miscellaneous Systems: 18 lb
  - Fuel system: 11 lb
  - Unusable fuel + engine oil: 33 lb
  - Propeller (still needed): -83 lb
  - Cowl (still needed): -23 lb
  - TOTAL: 617 lb
- Add motor and controller
  - Motor (310 hp at 3 hp/lb): 103 lb
  - Controller (0.05 lb/hp): 15.5 lb
- Net empty weight reduction: 498 lb



<http://whycirrus.com>



# Electric SR-22-like Aircraft

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## ❖ Power differences

- Electric motors have negligible power lapse with altitude
- Tradeoffs between low and high altitude performance
  - Size to low altitude?
    - Increased performance at high altitude compared to conventional engine
    - 310 hp required to match maximum power of SR-22 engine
    - Net empty weight reduction from SR-22: 498 lb
  - Size to high altitude?
    - “Lose” performance at low altitude compared to conventional engine
    - 230 hp required to match SR-22 ROC at 8,000ft
    - Net empty weight reduction from SR-22: 529 lb

## ❖ Batteries

- ~500 lb available from OEW decrease plus typical fuel weight
- Maintain SR-22 maximum takeoff weight





# Electric SR-22-like Aircraft

## ❖ Drag Reduction

- Cooling
  - Original: ~6% of drag at cruise
  - Electric variant: ~0.1% of drag at cruise
  - $C_D$  decrease of ~13 counts
- Wetted area reduction
  - $C_D$  decrease of ~4 counts
- Maximum L/D increase
  - From 18.7 to 19.9
  - 6.4% increase



Electric SR-22 VSP Model





# Electric “Breguet” Equations

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## ❖ Assumes:

- Constant vehicle weight
- Constant propeller and electrical system efficiencies
- Flight at constant L/D

## ❖ Range:

$$R = \eta \frac{L}{D} \frac{W_{bat}}{W} \frac{uK}{g}$$

## ❖ Endurance:

$$N = \frac{R}{V} = \frac{\eta}{V} \frac{L}{D} \frac{W_{bat}}{W} \frac{uK}{g}$$



# Range Considerations

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## ❖ Can we exchange “fuel” weight and payload weight?

- Possible in conventional aircraft
- Uncertain in battery-electric aircraft
  - Requires removal / replacement of batteries

## ❖ How do we handle reserves?

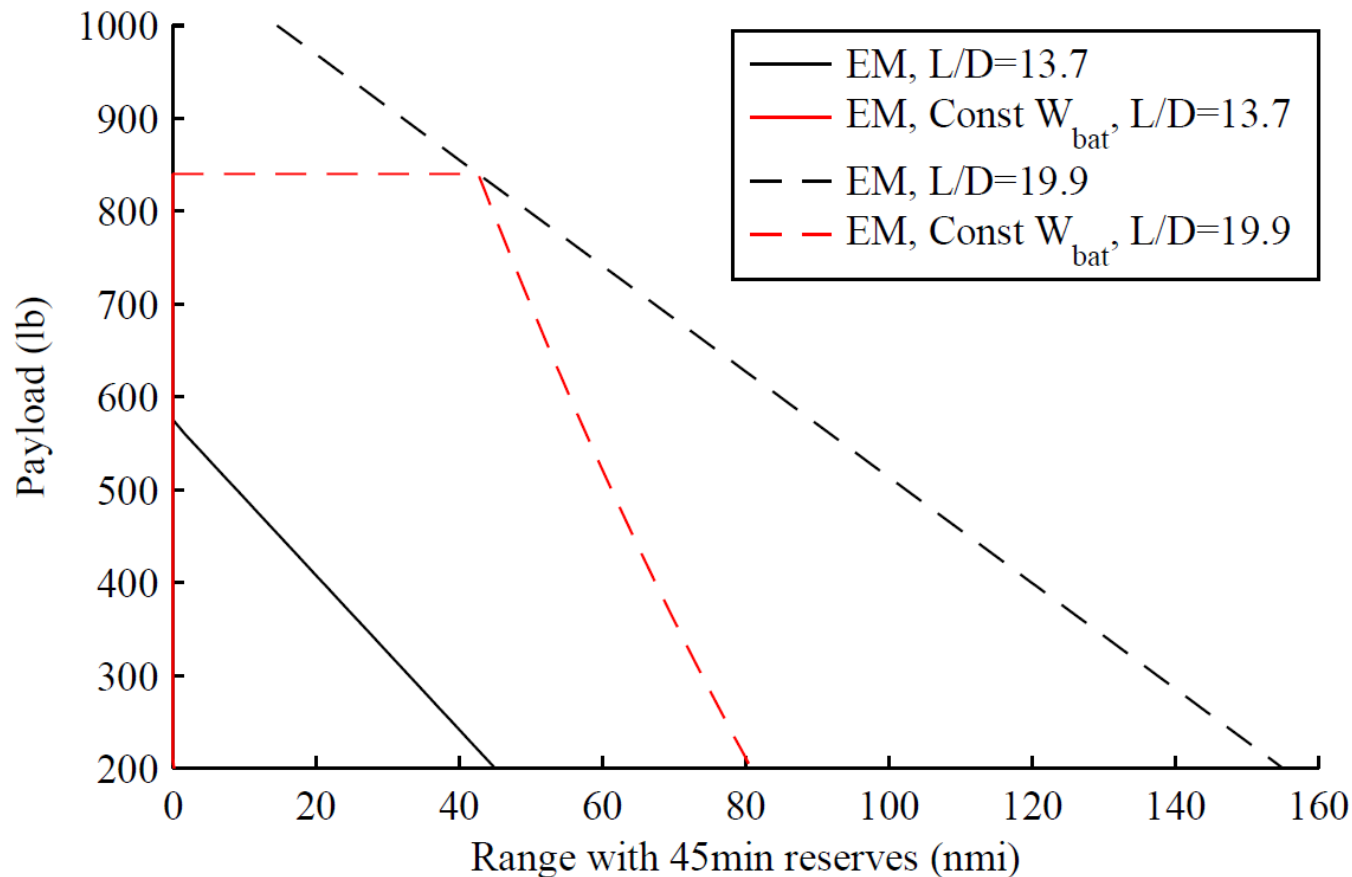
- Federal aviation regulations require reserve “fuel”
  - Practical range analysis must include reserves
- Battery health reserves
  - Important for sizing
  - Ignored for the immediately following analyses
  - We’ll come back to this issue...



# Electric SR-22 Payload-Range

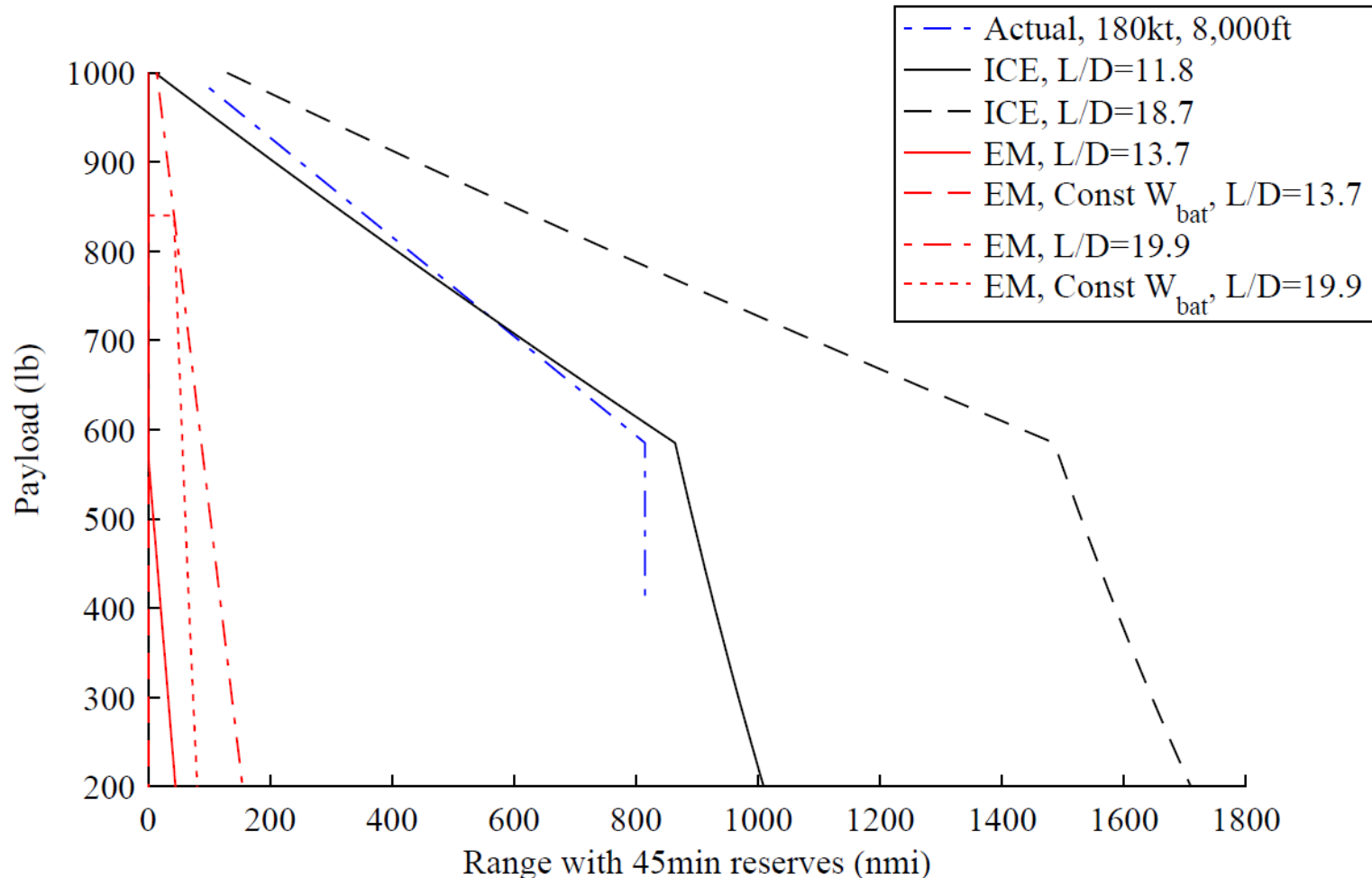
## ❖ 2 conditions:

- (L/D) max, 128 knots at 8,000 ft
- 180 knots at 8,000 ft



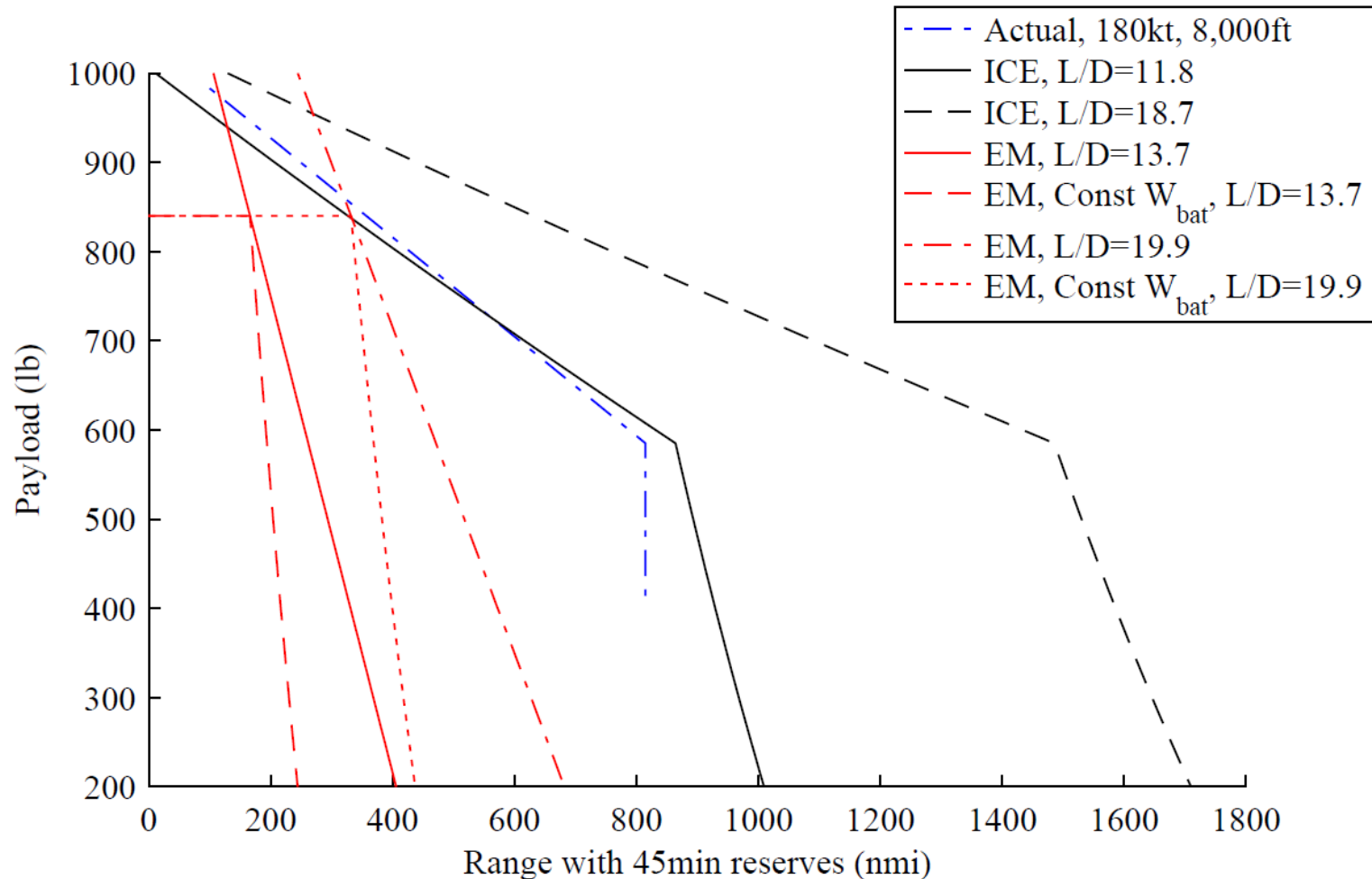
# Payload-Range Comparisons

## ❖ 2015 Technology Assumptions



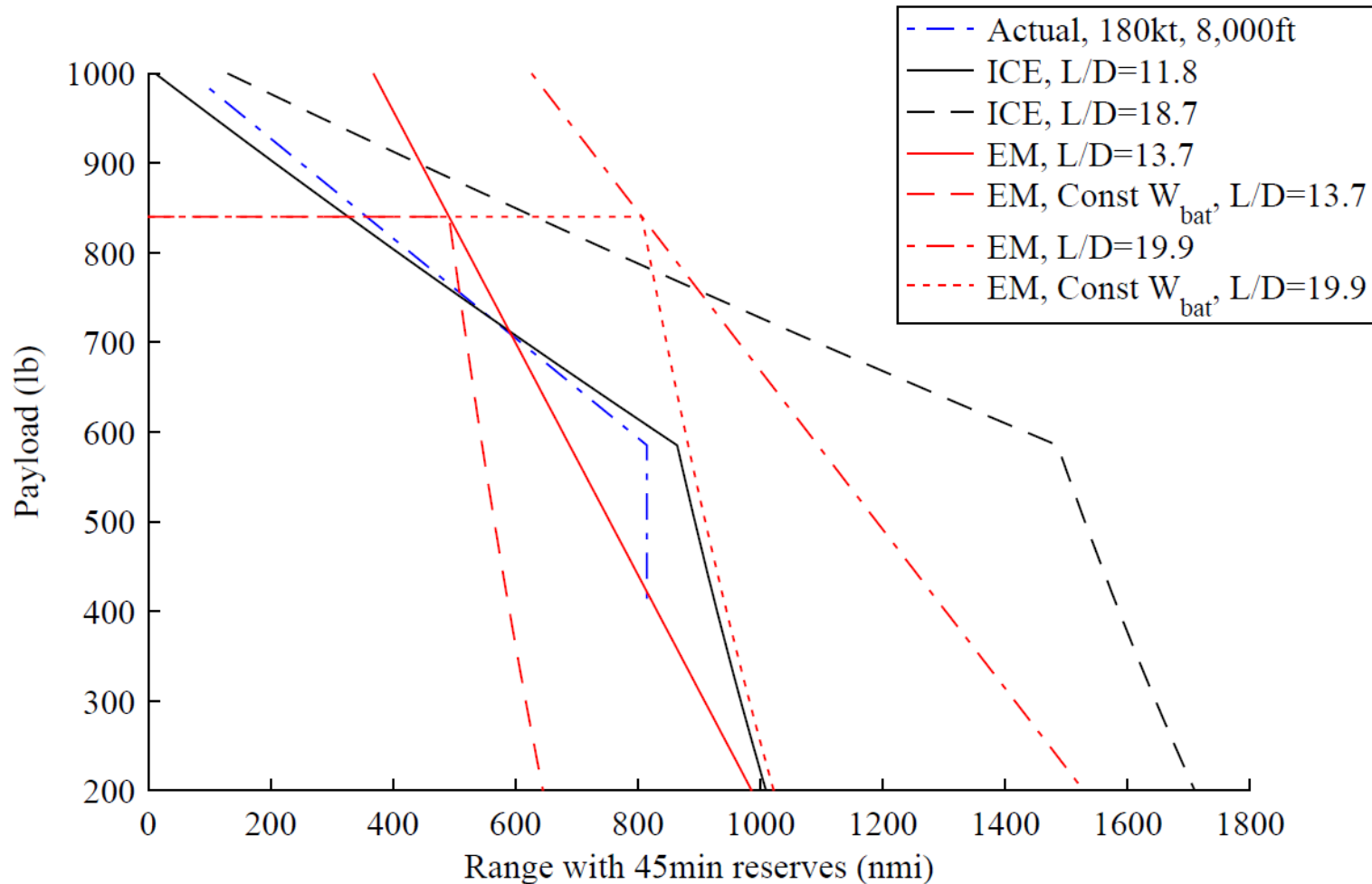
# Payload-Range Comparisons

## ❖ 2035 Technology Assumptions



# Payload-Range Comparisons

## ❖ 2050 Technology Assumptions

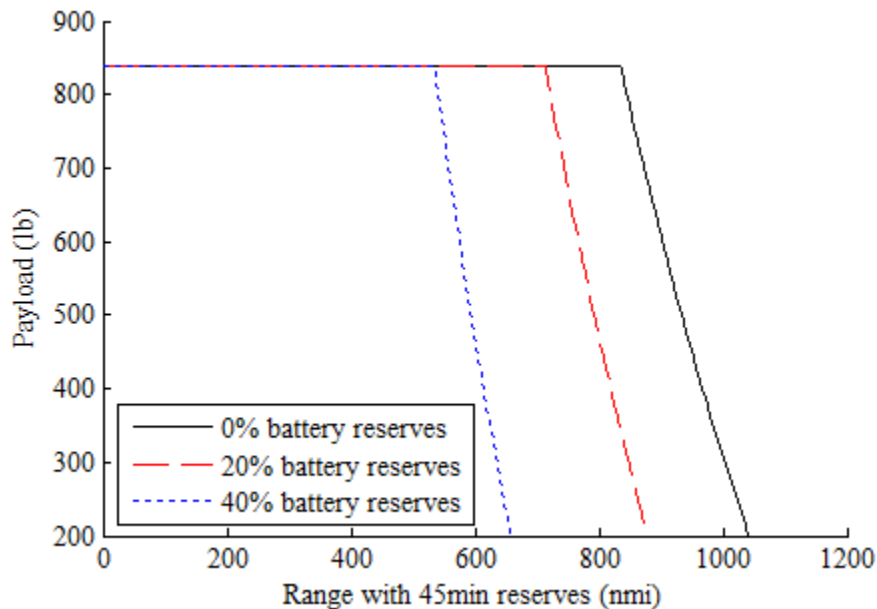


# Battery Health Reserves

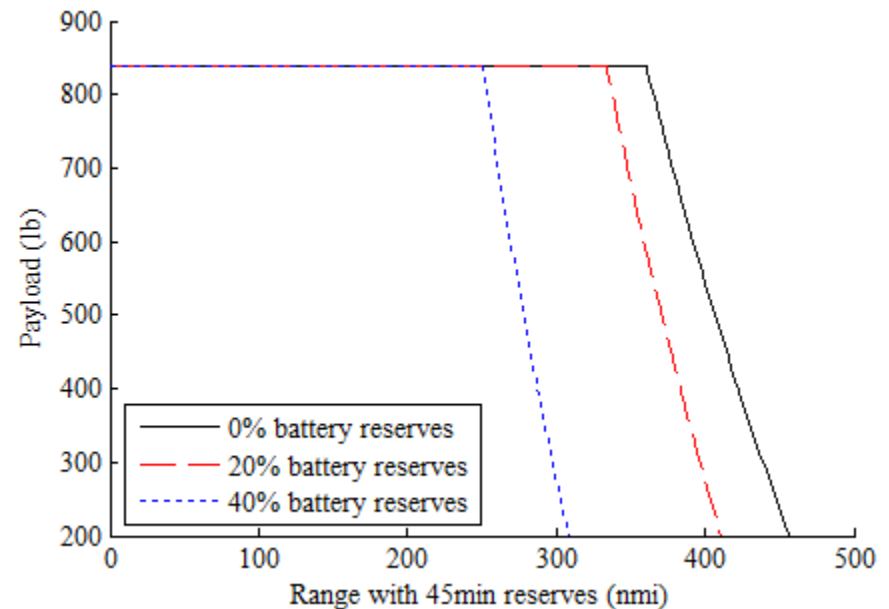
- ❖ Previous charts showed range using entire battery charge
- ❖ What if we do not use the last  $(1-\kappa)\%$  of the battery for range performance?

- Recall: 
$$R = \eta \frac{L}{D} \frac{W_{bat}}{W} \frac{u\kappa}{g}$$

- For a constant battery weight:



2050 Assumptions

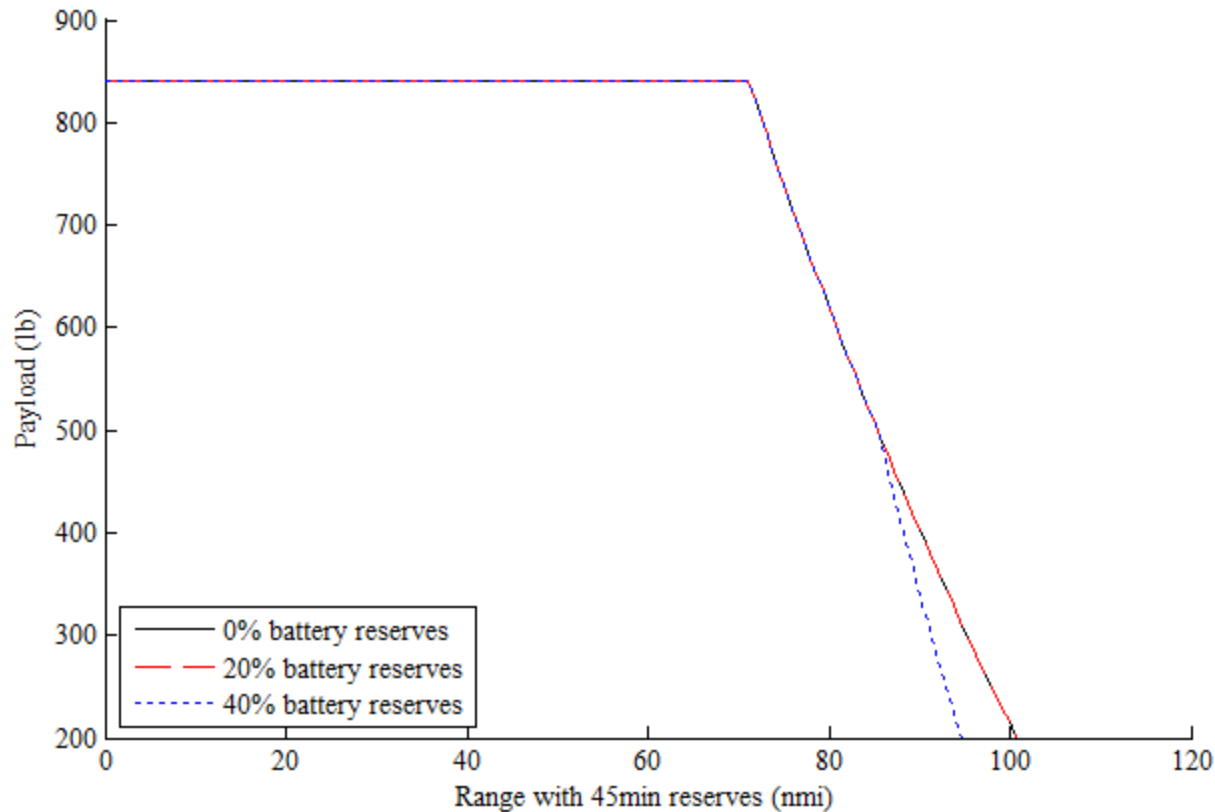


2035 Assumptions



# Battery Health Reserves

## ❖ 2015 Assumptions:



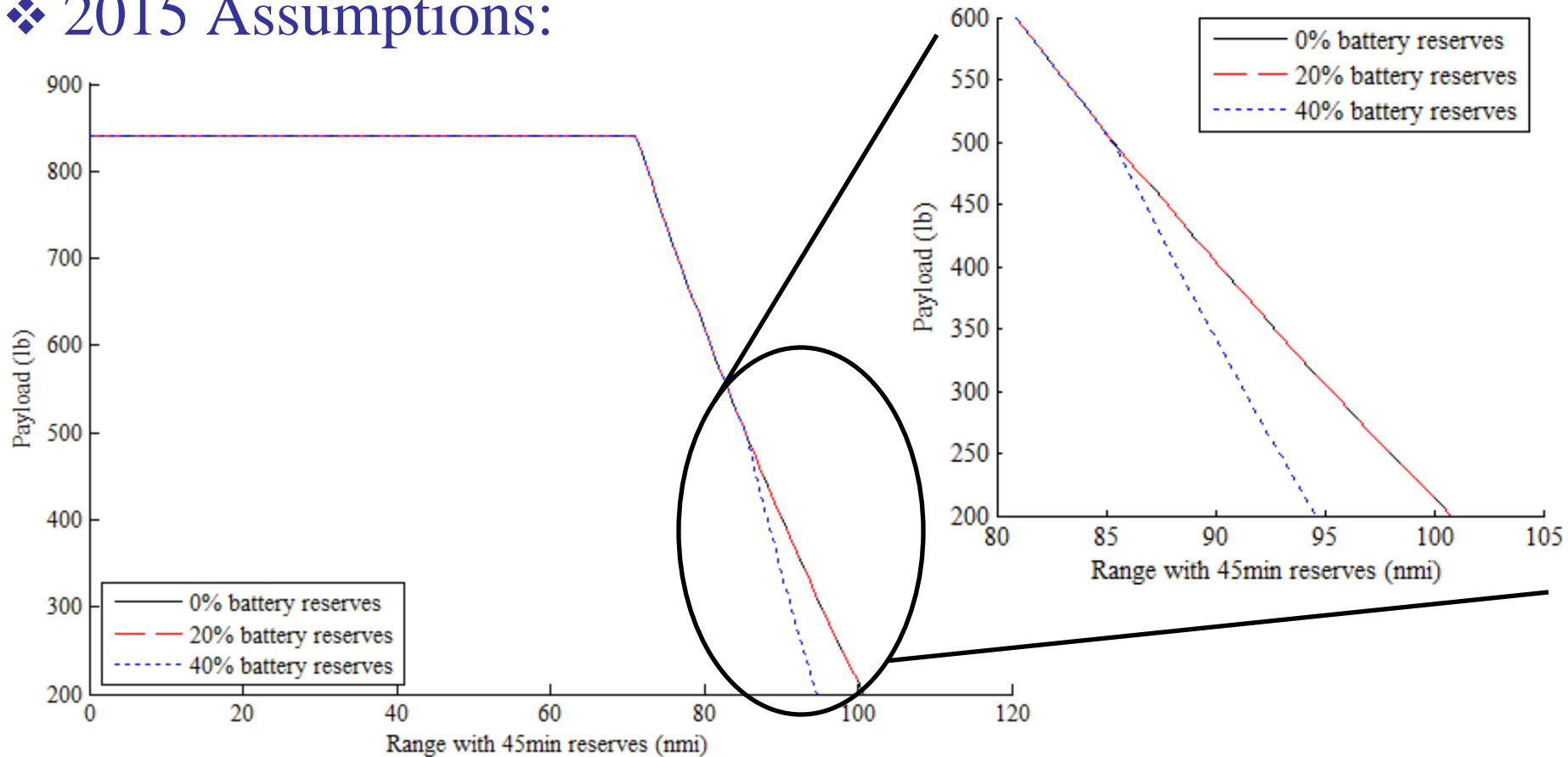
- Why are the 0% and 20% reserves ranges the same?
- Why is there no difference between the 3 different reserves above ~500 lbs?
- Why is the 40% reserve different below ~500 lbs?





# Battery Health Reserves

## ❖ 2015 Assumptions:

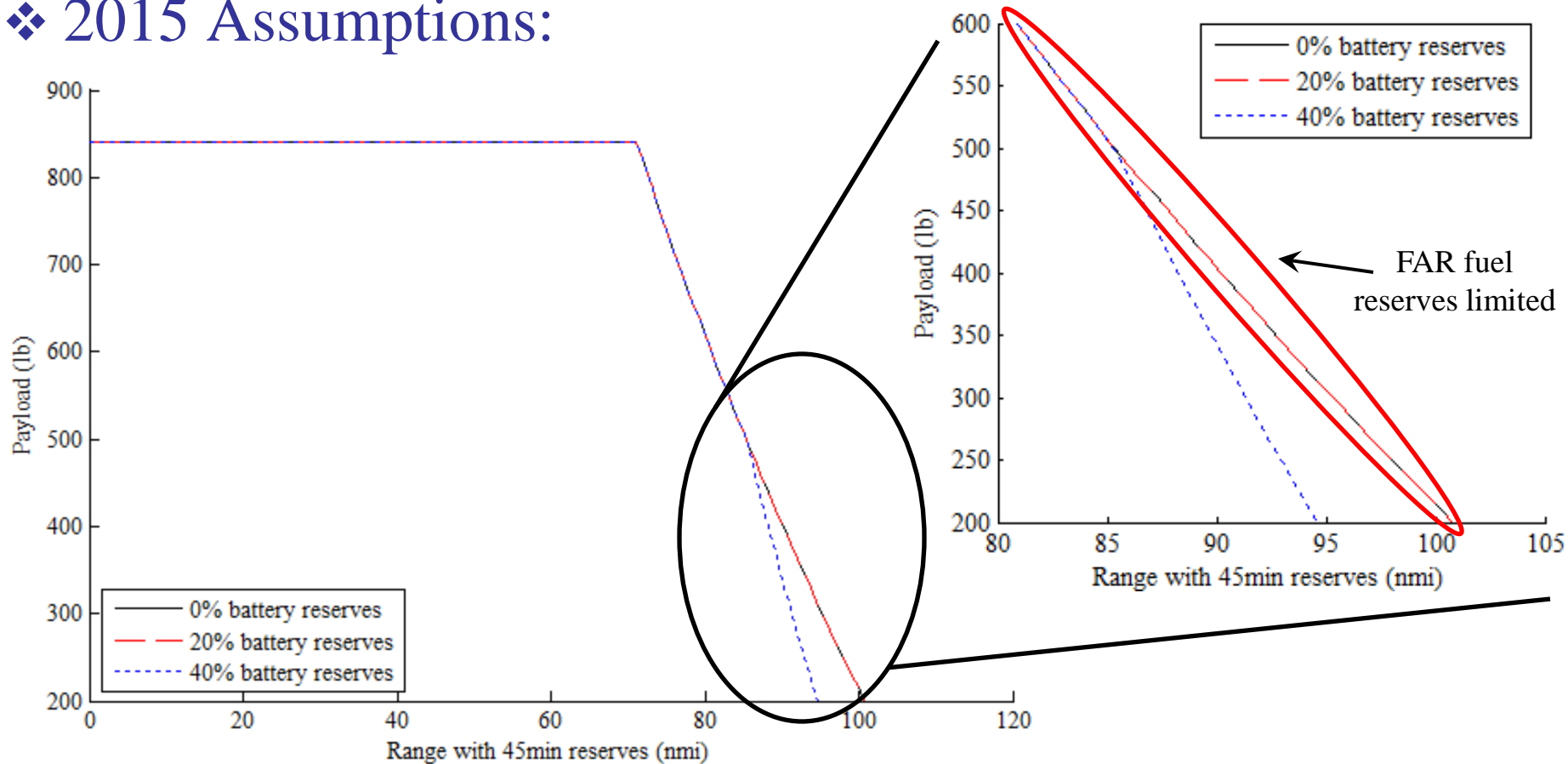


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# Battery Health Reserves

## ❖ 2015 Assumptions:

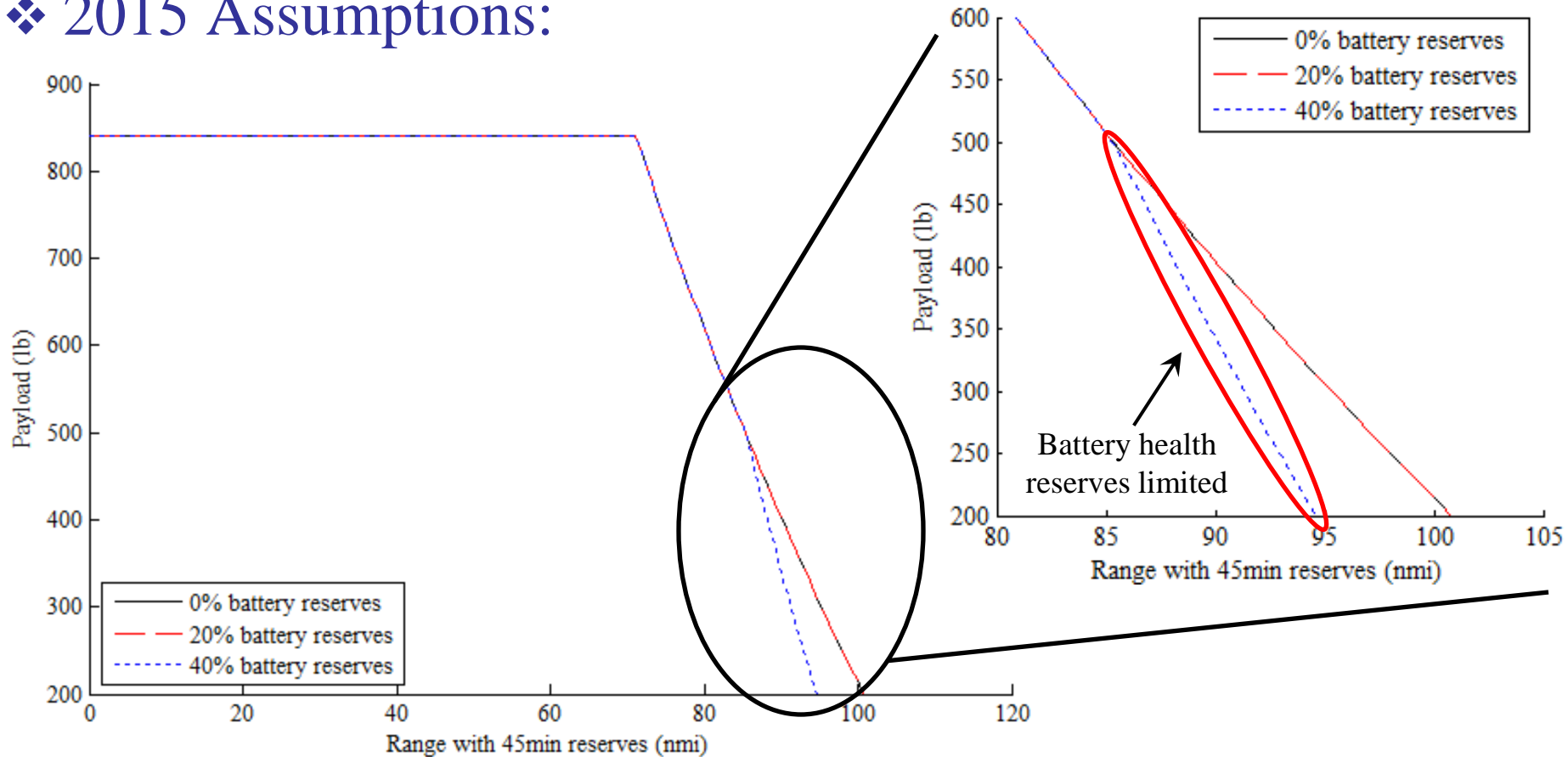


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# Battery Health Reserves

## ❖ 2015 Assumptions:

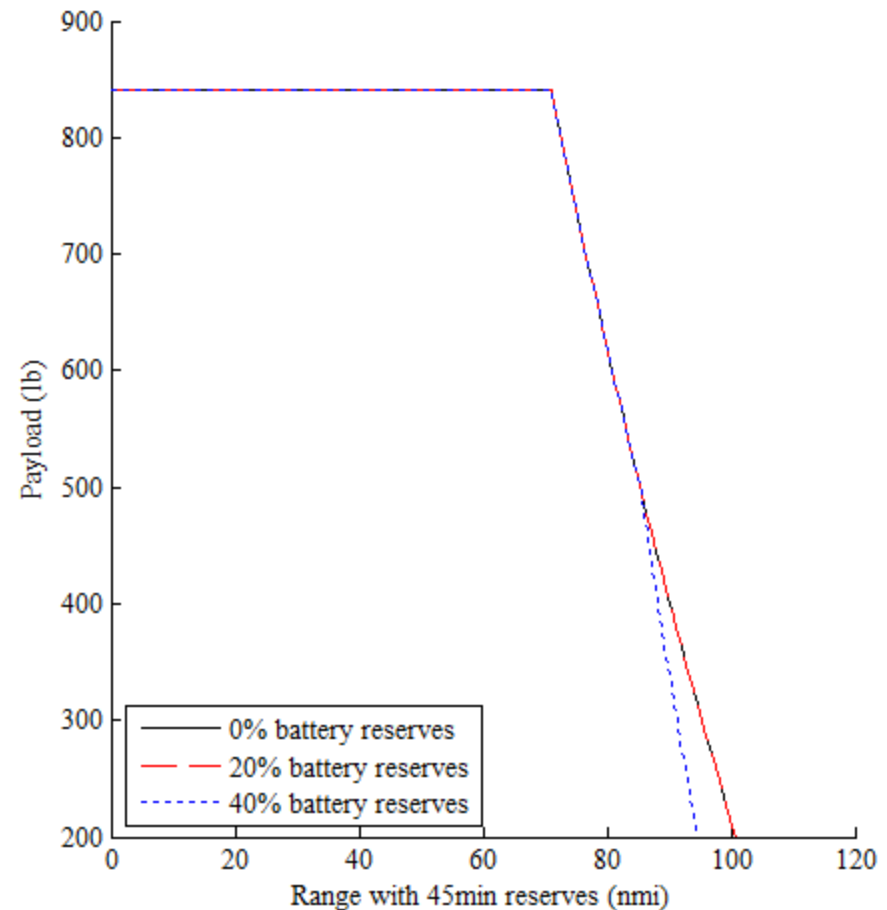


- Why are the 0% and 20% reserves ranges the same?
- Why is there no difference between the 3 different reserves above ~500 lbs?
- Why is the 40% reserve different below ~500 lbs?



# Battery Health Reserves and FAR Fuel Reserves

- ❖ Different battery health reserve percentages do not change resulting range because of the need to satisfy FAR fuel reserves
- ❖ FAR fuel reserves
  - Require more than 20% of battery in all cases
  - Require more than 40% of battery for payloads above ~500 lbs
  - Require between 20% and 40% of the battery from 200 - ~500 lbs
- ❖ Somewhat synergistic, especially in near-term



# Part 1 Conclusions

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## ❖ Performance considerations for electric propulsion

- Negligible power lapse with altitude
  - Low altitude vs. high altitude point-performance tradeoffs
  - May change active sizing constraints
- Constant vehicle weight in cruise
  - No cruise climbs required for max range
- Can we trade payload and battery weight?
- Battery health and FAR fuel reserves
  - Potentially synergistic, especially in near-term
  - Do we allow FAR reserves to use all charge in the battery?
  - Should we use dedicated reserve batteries?

## ❖ Electric aircraft that are simple modifications to existing airframes not likely to be practical in near-term

- Need increased L/D, efficiencies, and/or battery technologies



# Part 2:

## Electric Aircraft Sizing and Design Space Exploration

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# Electric Aircraft Sizing

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❖ How do we determine size of battery-electric aircraft?

$$W = W_{empty} + W_{bat} + W_{payload}$$

$$W = \frac{W_{payload}}{1 - (W_{empty} / W) - (W_{bat} / W)}$$

❖ How do we estimate empty weight fraction ( $W_{empty}/W$ )?

- Use traditional aircraft regressions
  - Difference in engine weights vs. motor weights
- Develop new regressions with electric aircraft

❖ What about the “battery weight fraction” ( $W_{bat}/W$ )?

- Not exactly analogous to fuel weight fraction ( $W_f/W$ )



# Electric Aircraft Sizing

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## ❖ Battery weight fraction

$$\frac{W_{bat}}{W} = \frac{R_{total}g}{\eta(L/D)u\kappa}$$

- Must consider practical operations:
  - FAR fuel reserves
  - Battery health
- FAR fuel reserves
  - Use increased range in sizing calculations:  $R_{total} = R_{design} + N_{design} V_{design}$
- Battery health reserves
  - Several possible approaches
  - Approach implemented in this work:
    - Ensure design mission flown without infringing battery health
    - Allow FAR fuel reserves to encroach on battery health reserves





# Electric Aircraft Sizing

## ❖ Battery weight fraction calculation considering reserves

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**Algorithm 1** Battery Weight Fraction Calculation

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1:

$$\left(\frac{W_{\text{bat}}}{W}\right)_1 = \frac{(R_{\text{design}} + N_{\text{design}} V_{\text{design}}) g}{\eta (L/D) u (1)}$$

2: For  $\kappa < 1.0$ :

$$R_1 = \eta \frac{L}{D} \left(\frac{W_{\text{bat}}}{W}\right)_1 \frac{u \kappa}{g}$$

3: if  $R_1 \geq R_{\text{design}}$  then

4:

$$\frac{W_{\text{bat}}}{W} = \left(\frac{W_{\text{bat}}}{W}\right)_1$$

5: else

6:

$$\left(\frac{W_{\text{bat}}}{W}\right)_2 = \frac{R_{\text{design}} g}{\eta (L/D) u \kappa}$$

7:

$$\frac{W_{\text{bat}}}{W} = \left(\frac{W_{\text{bat}}}{W}\right)_2$$

8: end if

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# Electric Aircraft Sizing

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## ❖ Wing sizing condition

- FAR required stall speed (61 knots)
- $C_{L\max}$  estimate: 1.99 (from SR-22)

## ❖ Power sizing condition

- ROC at altitude
- 800 ft/min at 10,000 ft (to match SR-22)

## ❖ Sizing mission

- 2015: 200 mile range at 150 mph
- 2035: 300 mile range at 200 mph
- 2050: 500 mile range at 250 mph

## ❖ Technology assumptions

- $L/D = 18.75$
- Propeller efficiency = 0.85



# Initial Sizing Estimates

	Technology Year		
	2015	2035	2050
Gross Weight (lb)	11,170	3,575	3,035
Empty Weight (lb)	4,924	1,935	1,691
Empty Weight Fraction	0.441	0.541	0.557
Battery Weight (lb)	5,406	801	503
Wing Area (ft <sup>2</sup> )	445.5	142.6	121.0
Motor Power (hp)	548	175	149

- ❖ Near term technologies pushing weight certification limits
- ❖ Mid-term and far-term sizes are SR-22-like (3400 lb)



# Sizing Sensitivity

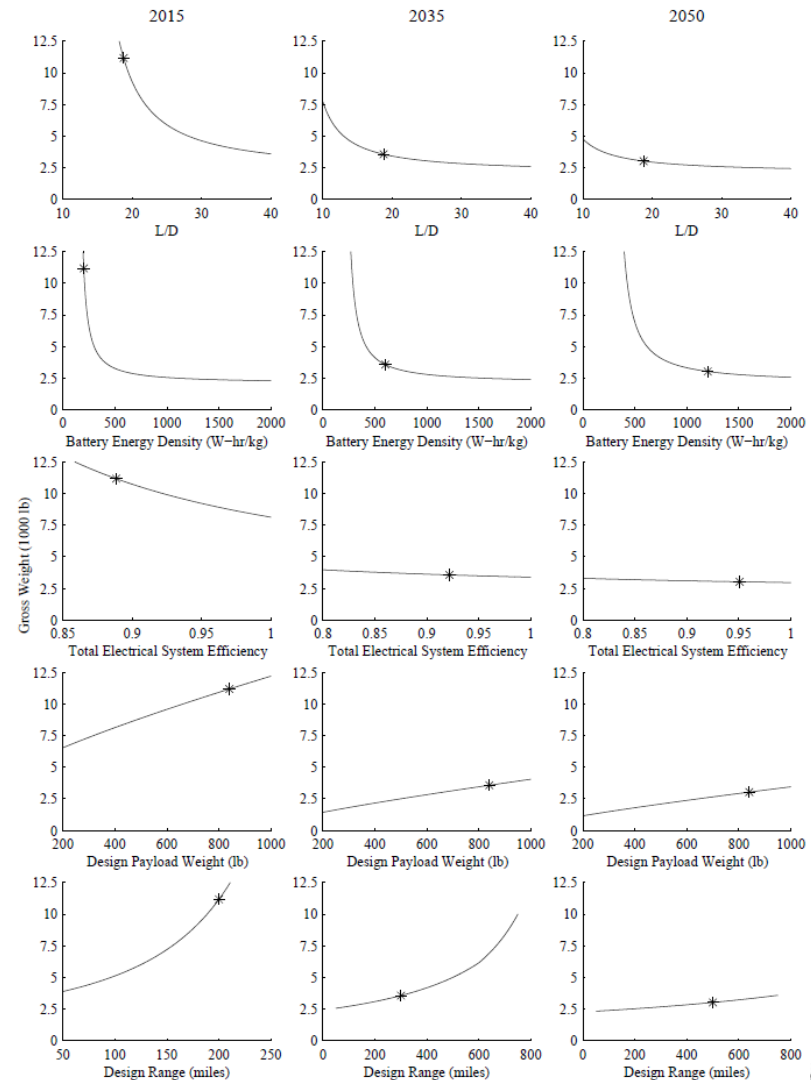
❖ How sensitive is aircraft size to design and mission parameters?

❖ Sensitivity to

- L/D
- Battery energy density
- Electrical system efficiency
- Propeller efficiency
- Payload weight
- Design range
- Battery health reserve fraction

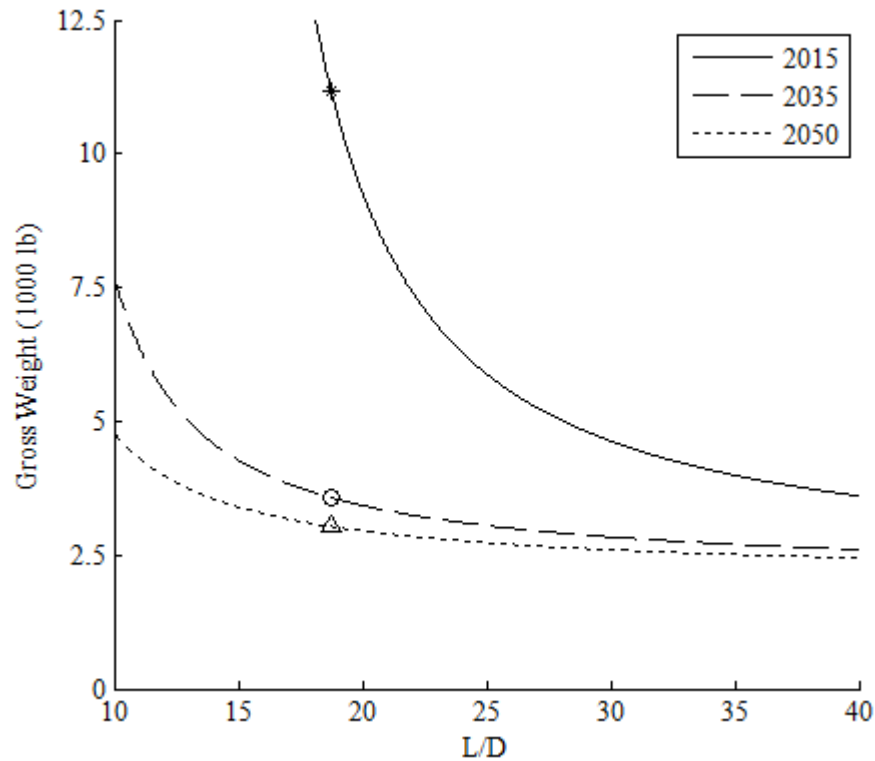
❖ Baseline Values

- Technologies / missions set by standard assumptions
- $L/D = 18.75$
- Propeller efficiency = 0.85

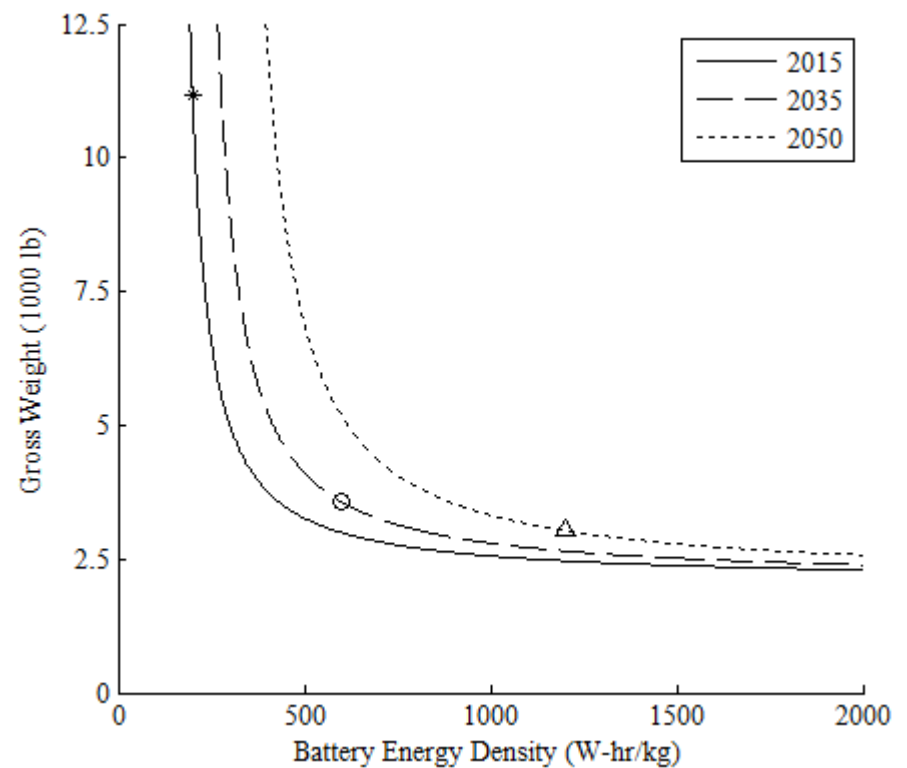


# Sizing Sensitivity

## ❖ Lift-to-drag ratio

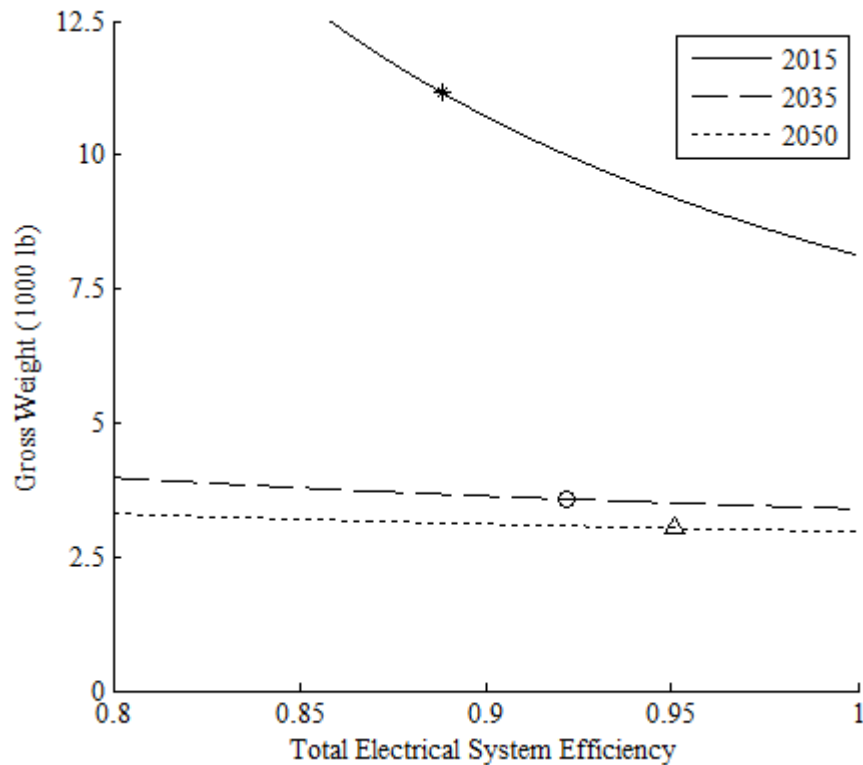


## ❖ Energy density

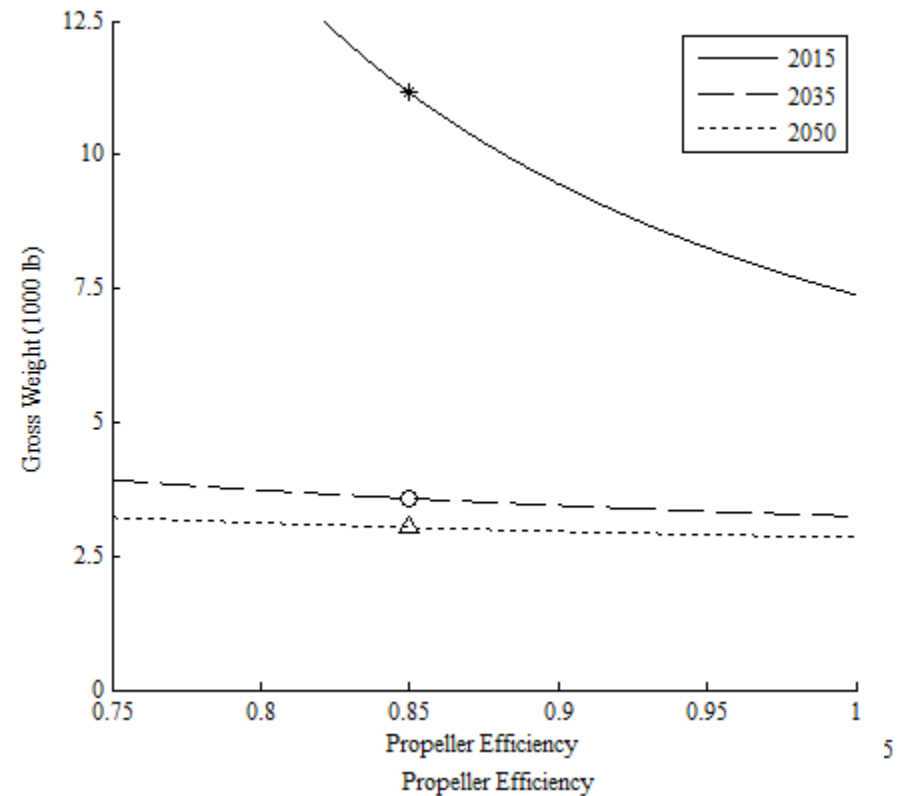


# Sizing Sensitivity

## ❖ Electrical system efficiency



## ❖ Propeller efficiency

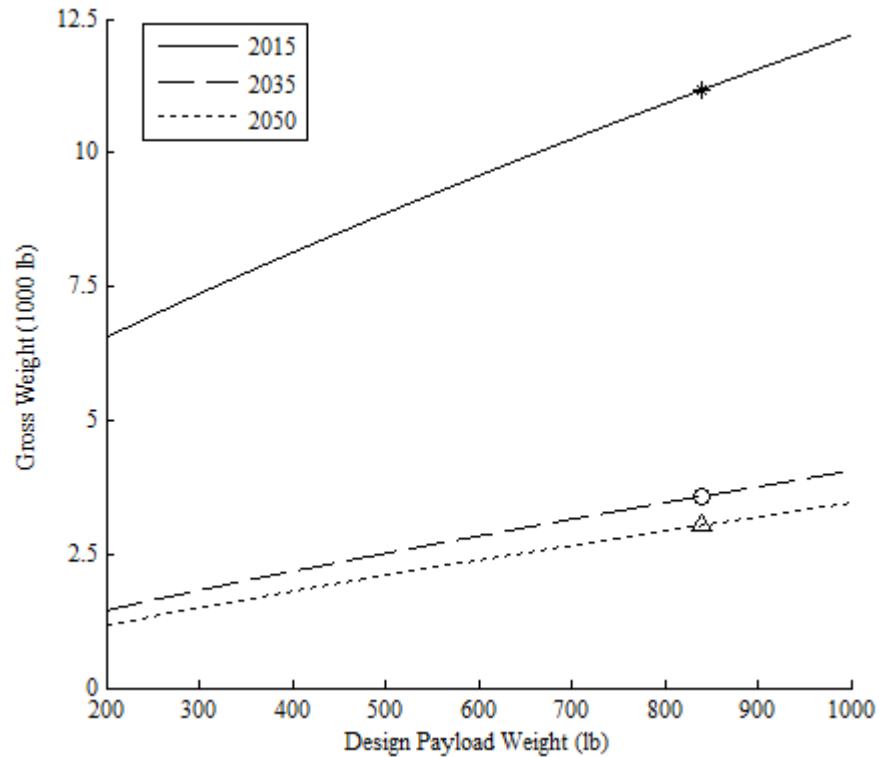


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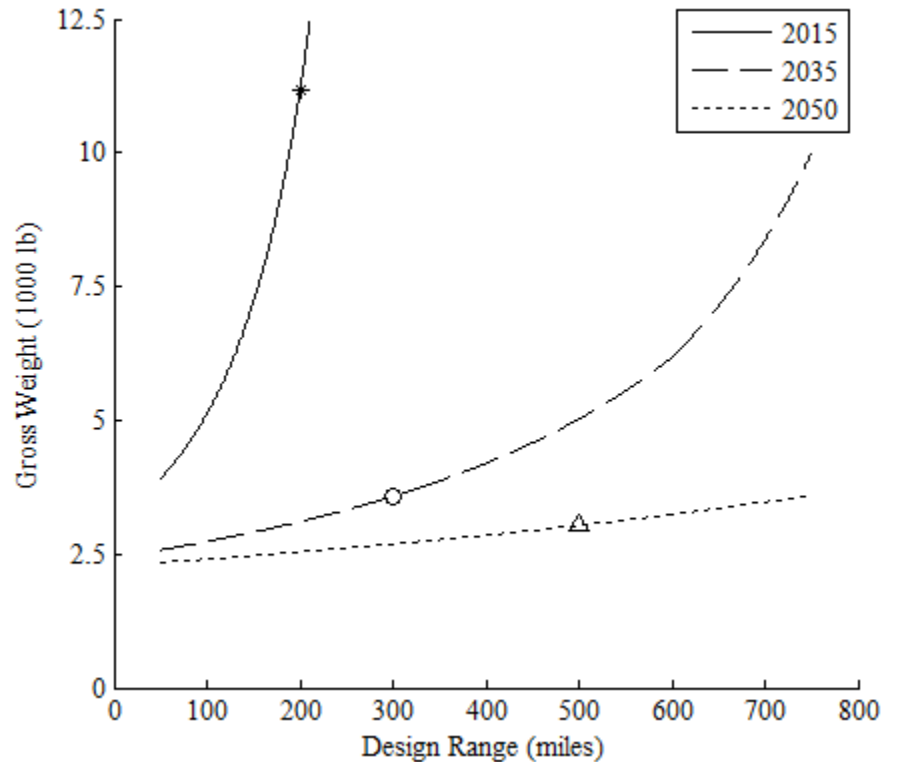


# Sizing Sensitivity

## ❖ Payload Weight

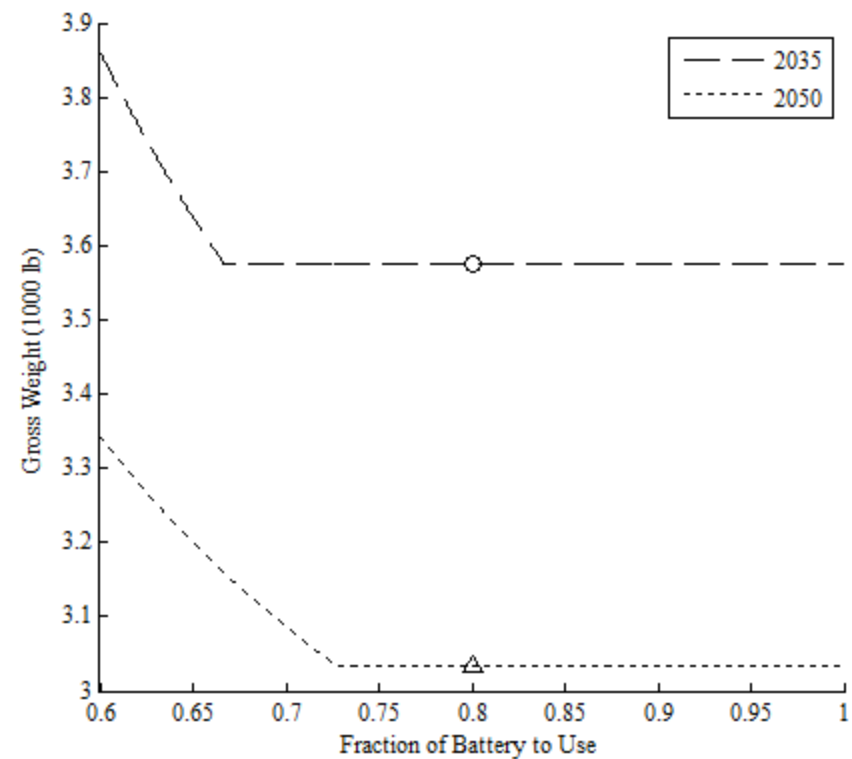
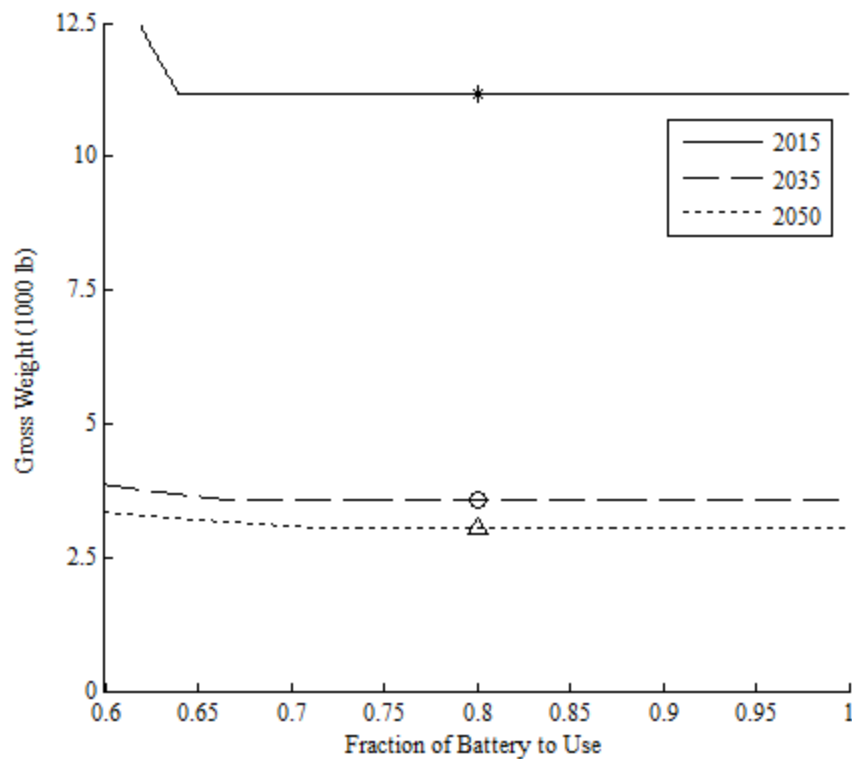


## ❖ Design range



# Sizing Sensitivity

## ❖ Battery health reserve fraction



Zoom on 2035 and 2050 curves

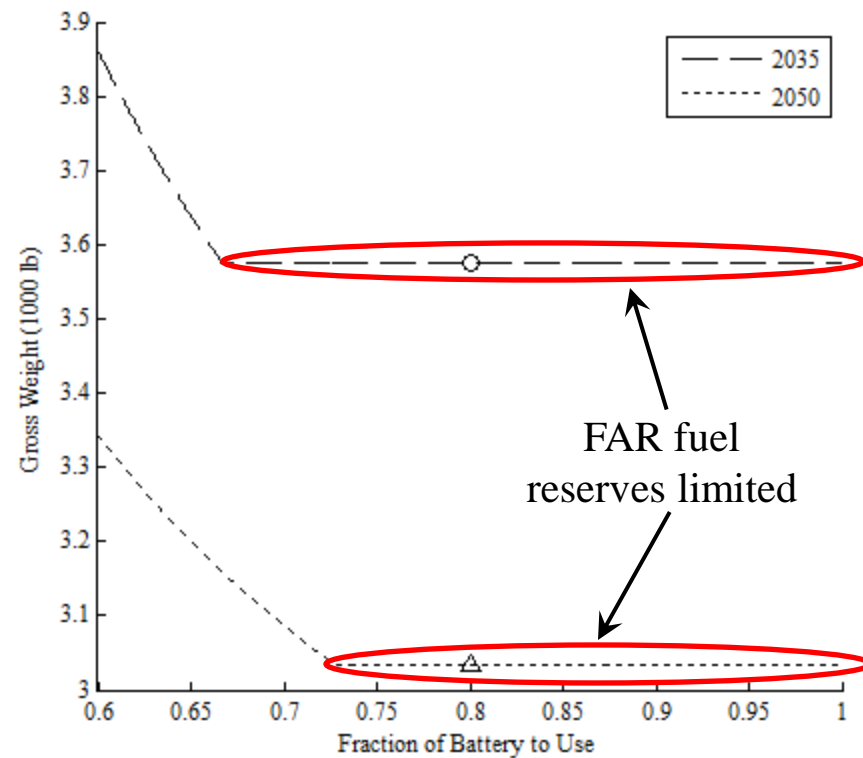
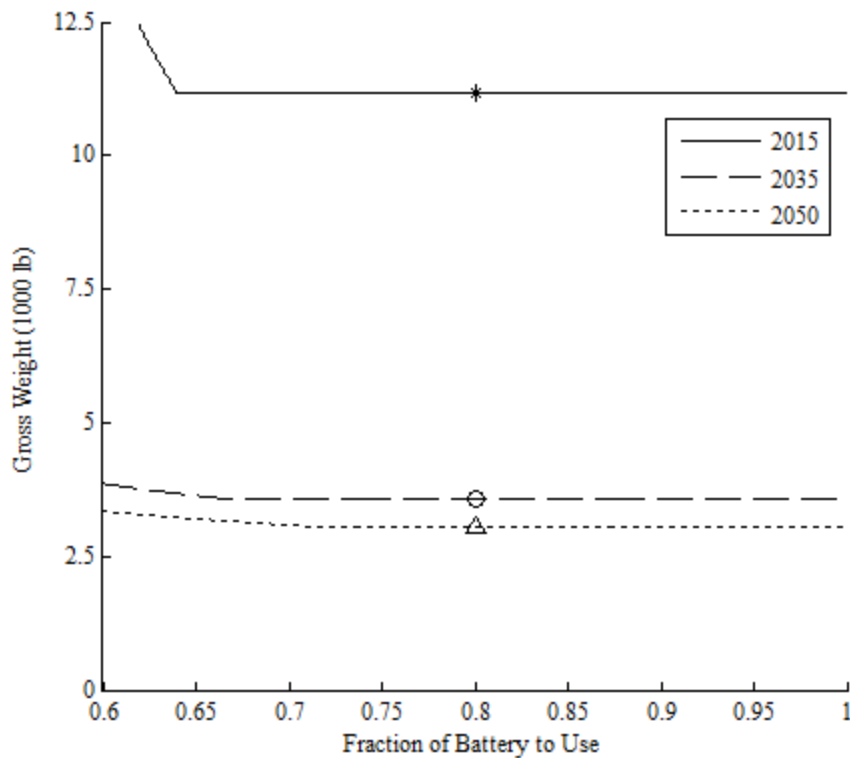




# Sizing Sensitivity

## ❖ Battery health reserve fraction

$$\left( \frac{W_{\text{bat}}}{W} \right)_1 = \frac{(R_{\text{design}} + N_{\text{design}} V_{\text{design}}) g}{\eta (L/D) u (1)}$$



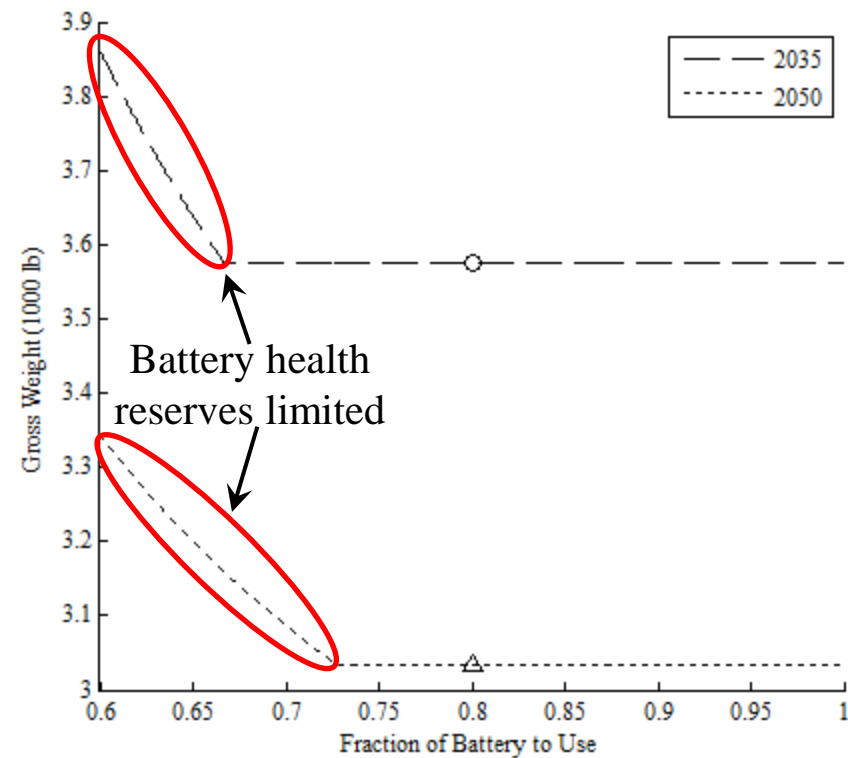
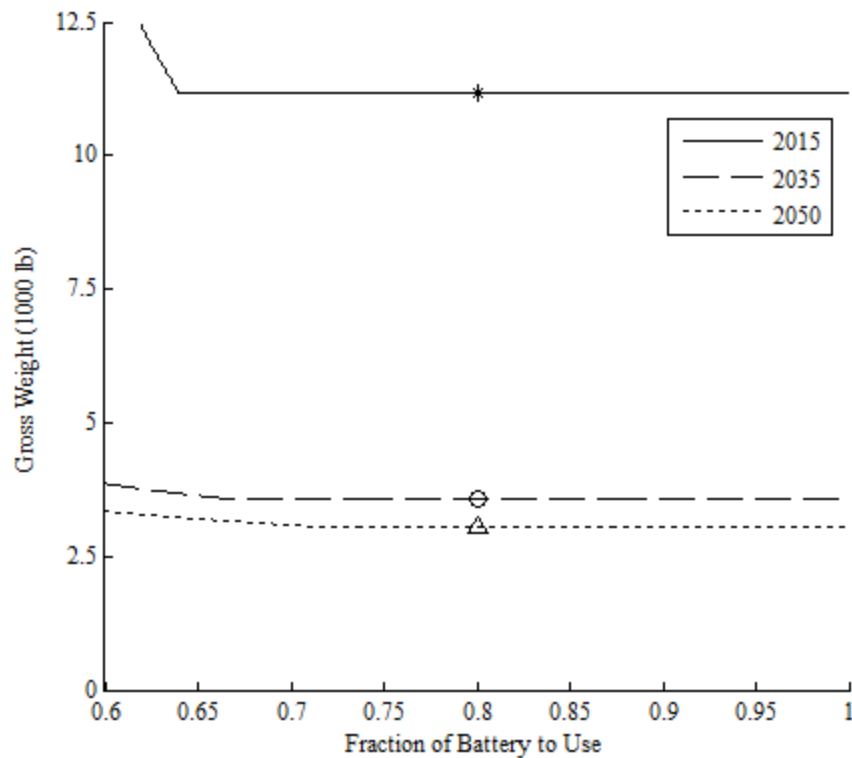
Zoom on 2035 and 2050 curves



# Sizing Sensitivity

## ❖ Battery health reserve fraction

$$\left(\frac{W_{\text{bat}}}{W}\right)_2 = \frac{R_{\text{design}} g}{\eta (L/D) u \kappa}$$



Zoom on 2035 and 2050 curves



# Battery Health & Fuel Reserves

	Technology Year		
	2015	2035	2050
Gross Weight (lb)	11,170	3,575	3,035
Empty Weight (lb)	4,924	1,935	1,691
Battery Weight (lb)	5,406	801	503
% Energy Required for Cruise	64.0%	66.6%	72.8%
Reserve Time Using 80% Batteries (min)	20.0	24.1	19.9
Reserve Distance Using 80% Batteries (mi)	50.0	60.2	49.7

❖ Synergy between FAR fuel reserves and battery health reserves



# Sizing Sensitivity

## ❖ Second sizing sensitivity

## ❖ Multiple baselines

### ■ L/D

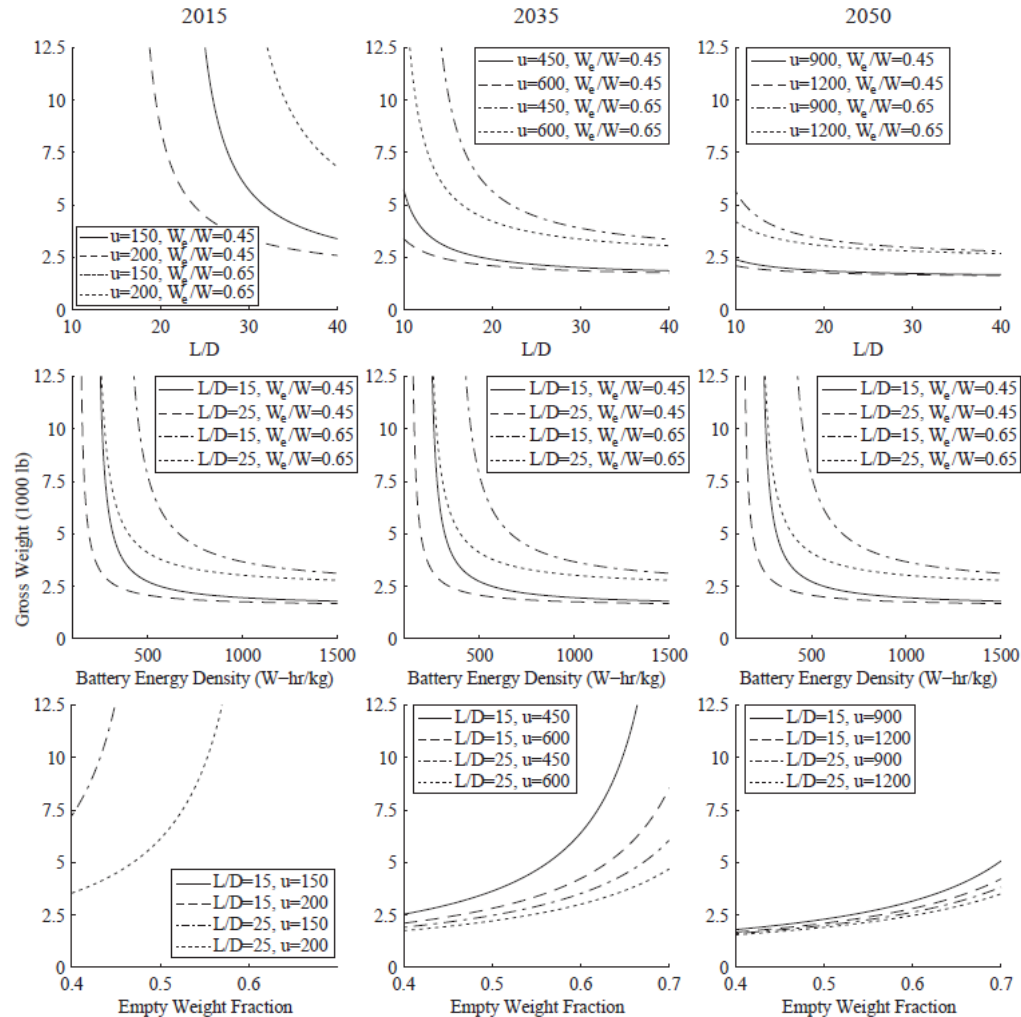
- 15
- 25

### ■ Energy density

- 2015: 150, 200
- 2035: 450, 600
- 2050: 900, 1200

### ■ Empty weight fraction

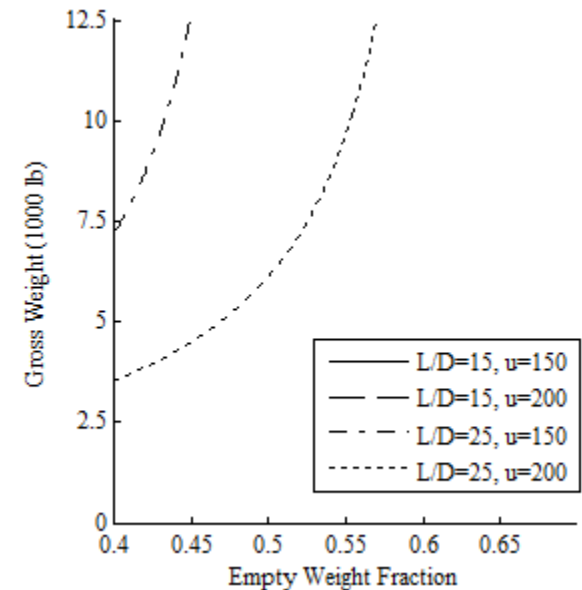
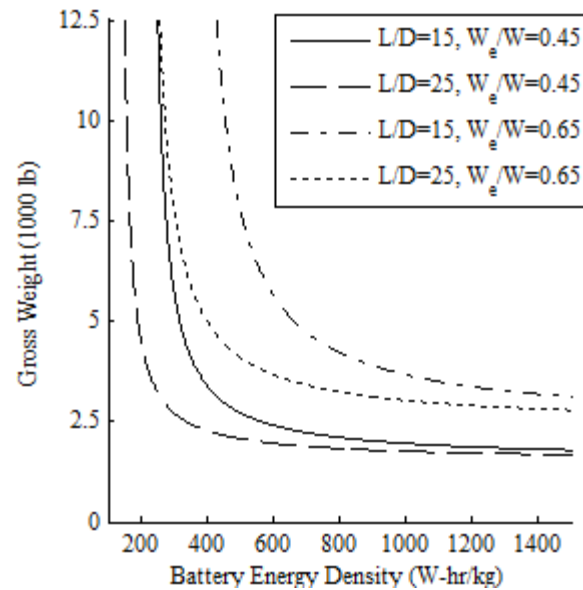
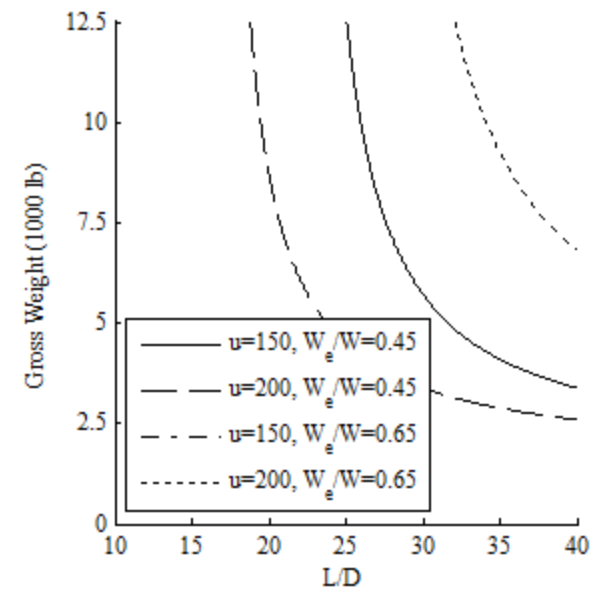
- 0.45
- 0.65



# Sizing Sensitivity

## ❖ 2015

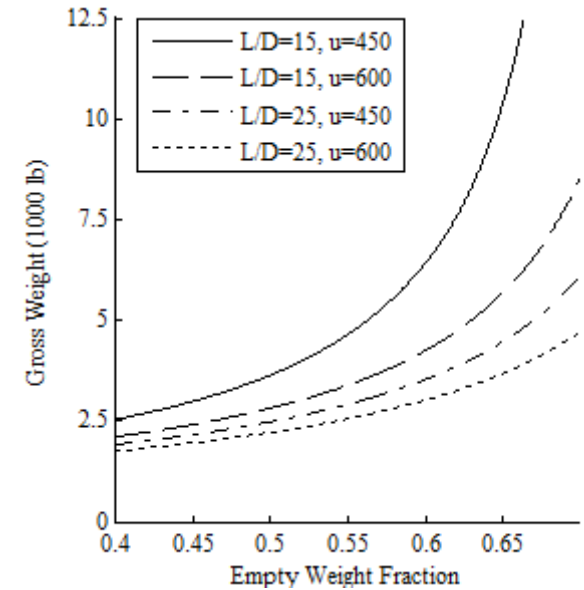
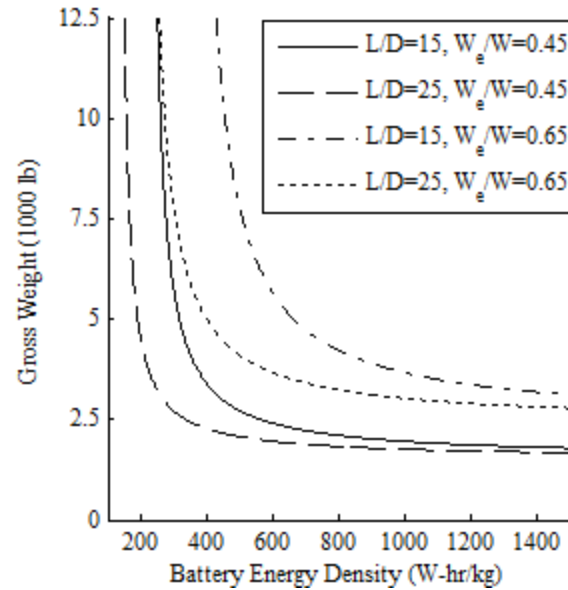
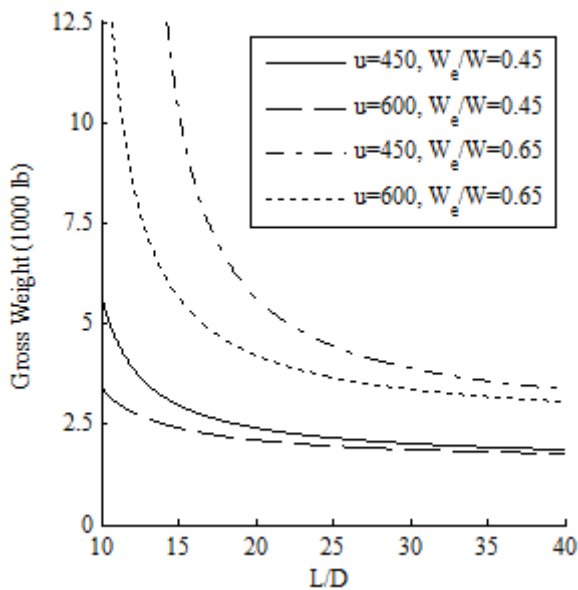
- High sensitivity to all 3 parameters
- Noticeable “knees” in energy density
  - Some L/D curves



# Sizing Sensitivity

## ❖ 2035

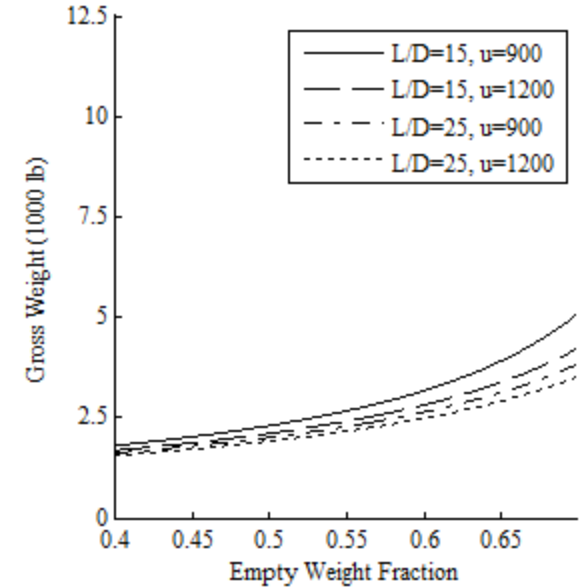
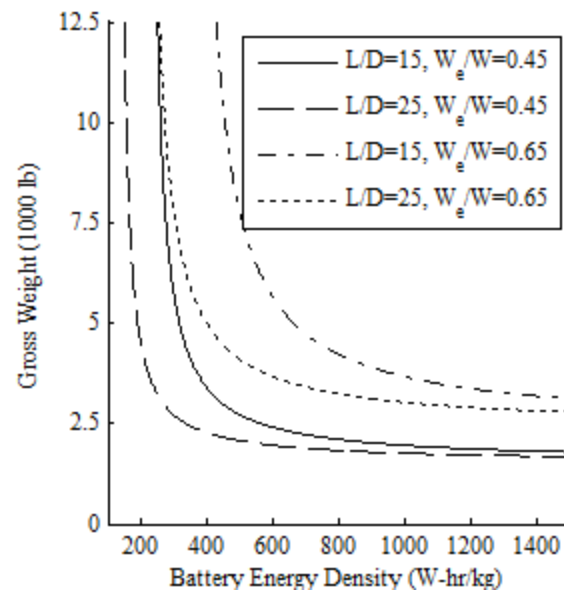
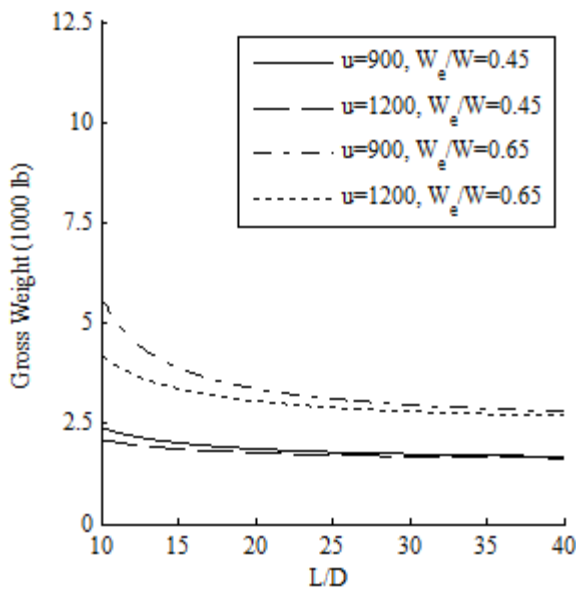
- Noticeable “knees” in L/D and energy density
  - Less sensitivity to L/D than 2015
- Grouping of empty weight fraction levels



# Sizing Sensitivity

## ❖ 2050

- Grouping of empty weight fraction levels
- Much less sensitivity to L/D, empty weight fraction
- Noticeable “knees” in energy density only



# Part 2 Conclusions

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- ❖ New concepts required in near-term to obtain practical on-demand aircraft
  - Higher lift-to-drag ratios
  - Propeller efficiency increases
  - Lower empty weight fractions
- ❖ Battery energy density
  - Slightly higher energy densities can lead to practical aircraft
  - Most important driver of aircraft size at current technology levels
    - Technology investments here will have biggest payoff
- ❖ FAR fuel reserves and battery health reserves somewhat synergistic
- ❖ With more advanced technologies missions in excess of 500 miles will be practical





Part 3:

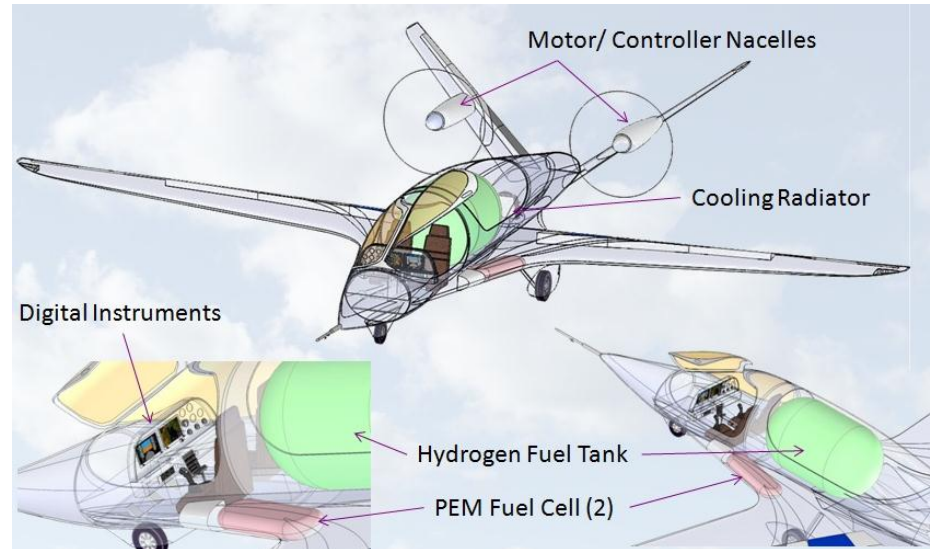
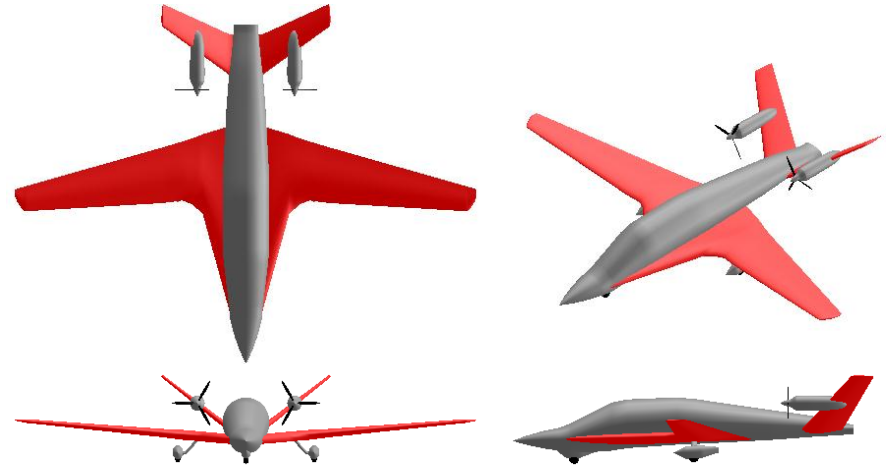
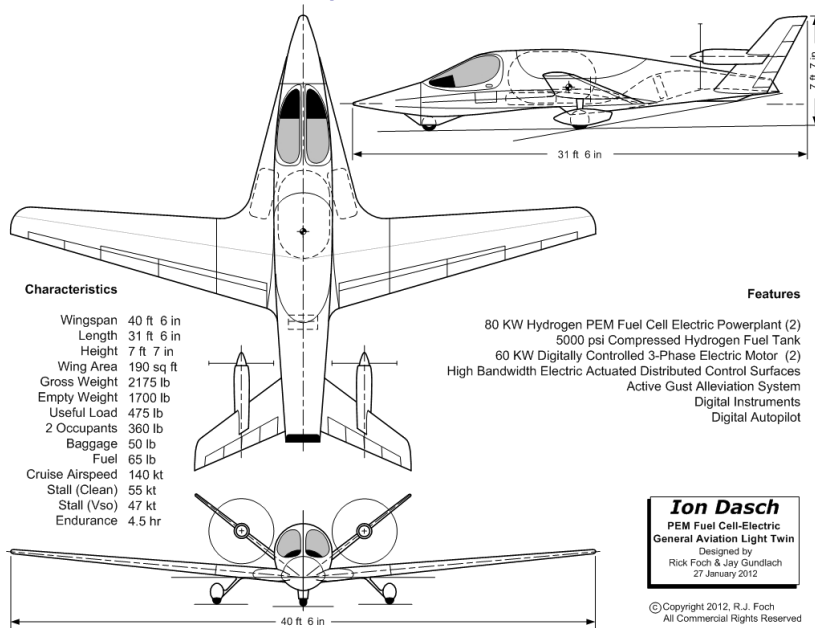
“Low Risk” Advanced Concept  
Inspired by the NRL Ion Dasch

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# NRL Concept: Ion Dasch

- ❖ Advanced concept being actively promoted that utilizes electric propulsion
- ❖ Hydrogen fuel cell / electric hybrid



# Concept Development Philosophy

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## ❖ Begin with Ion Dasch

- NRL seeking to make an Ion Dasch demonstrator

## ❖ Understand benefits/penalties of design

## ❖ Modify Ion Dasch concept as appropriate

- Adapt to “Zip” mission
- Take advantage of “first order” effect of potential increase in propeller efficiency
- Limit technological risk

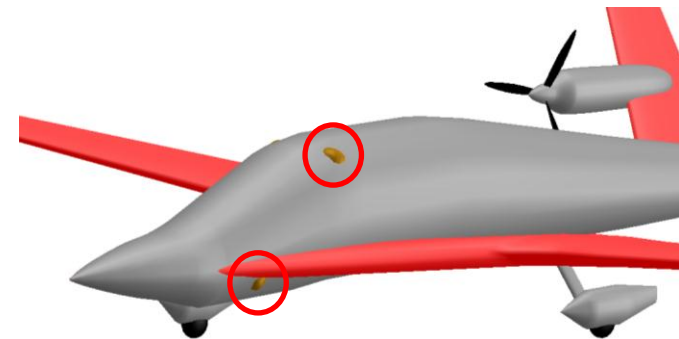
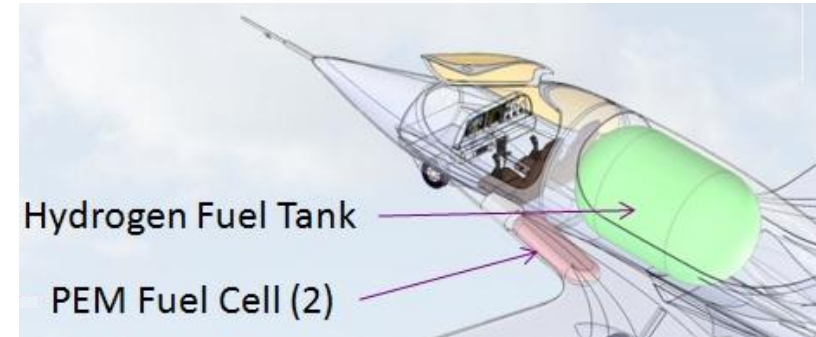
## ❖ Compare concept to baseline SR-22 and electric variant



# Ion Dasch to Georgia Tech Concept

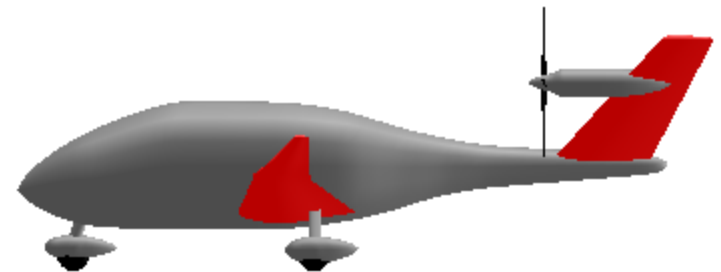
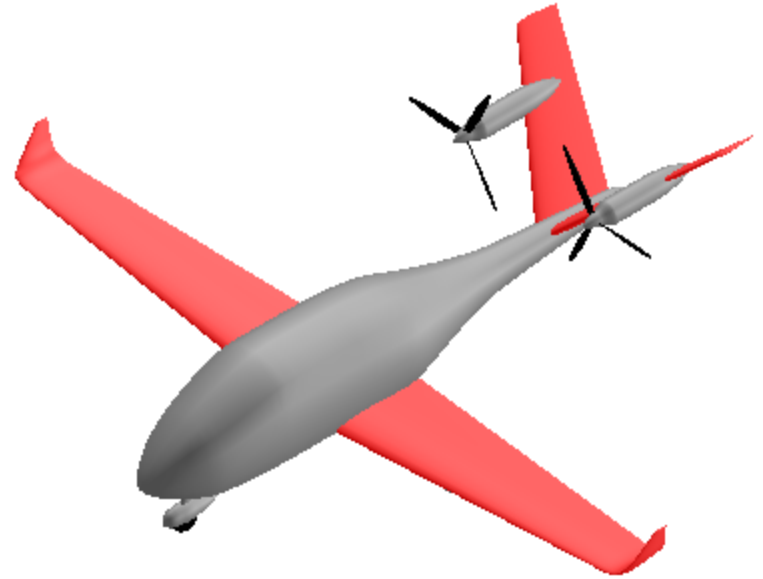
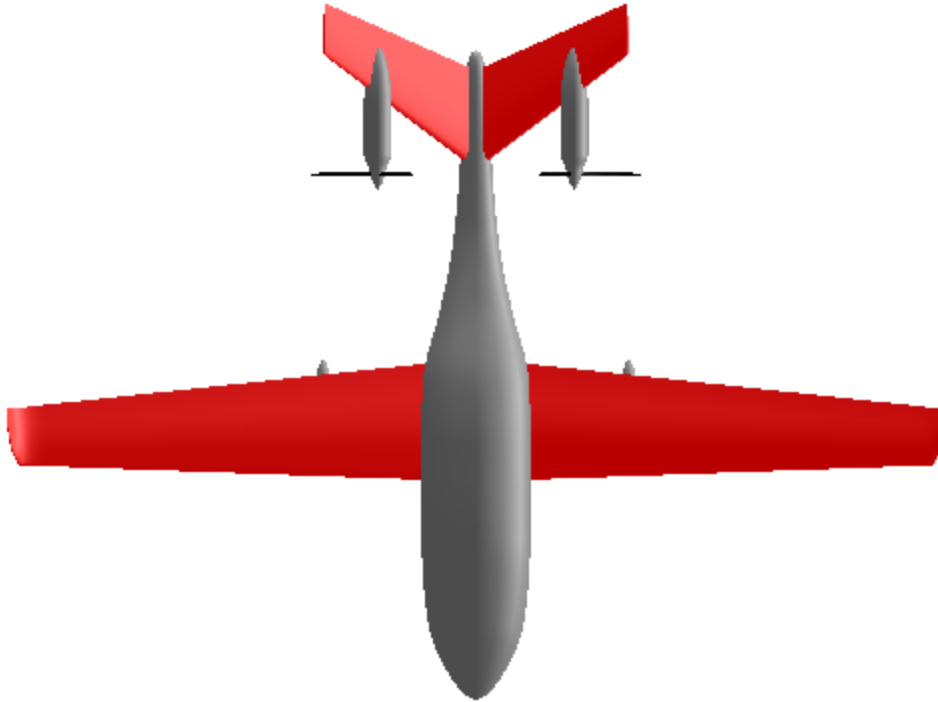
## ❖ Concept changes:

- Wing strakes removed
  - Space not required for fuel cells
  - Excess wetted area
  - Potential for larger interference drag
- Forward sweep removed
  - Unnecessary for CG placement and drag rise
  - Simple, tapered wing for reduced complexity and manufacturing costs
- Fuselage
  - Increased cabin size for standard size side-by-side passengers
  - No hydrogen tank required, space utilized for passengers/baggage
  - Laminar flow nose section
- Winglets added for anticipated higher CL operation



# Georgia Tech Concept

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# Propeller Efficiency

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❖ First order effect in range

$$R = (\eta_{prop})(\eta_{elec}) \frac{L}{D} \frac{W_{bat}}{W} \frac{uK}{g}$$

❖ Concept offers potential to substantively increase propeller efficiency from traditional designs

- Small, lightweight motors allow installation on V-tails
- V-tail placement allows for two, larger propellers
- Propeller disk loading reduced



# Propeller Model

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- ❖ Based on the “Advanced General Aviation Propeller Study” (AGAP) done by Hamilton Standard in 1972
- ❖ Defines propeller parametrically in terms of
  - Number of blades
  - Blade activity factor (AF)
  - Blade integrated design lift coefficient (CLi)
  - Diameter (D)
- ❖ Operating conditions input/output via
  - Advance ratio,  $J = V / (n D)$
  - Thrust coefficient,  $C_T = T / (\rho n^2 D^4)$   
where  $T = W / (L/D) / (\# \text{ props})$
  - Power coefficient,  $C_P = 550 \text{ SHP} / (\rho n^3 D^5)$
- ❖ Used in FLOPS (ENGGGEN)



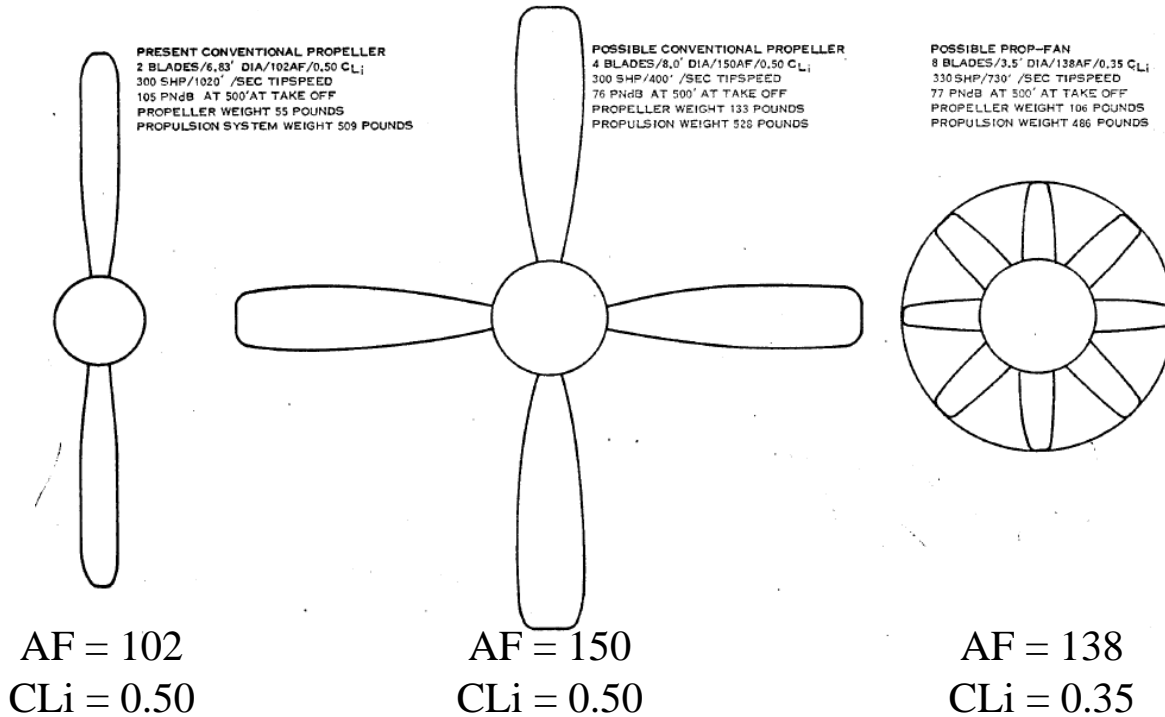
# Propeller Geometry Definition

## ❖ Activity factor

- $b$  = blade section width, ft
- $D$  = propeller diameter, ft
- $AF = \frac{100,000}{16} \int_{0.15}^{1.0} \left( \frac{b}{D} \right) x^3 dx$

## ❖ Integrated design lift coefficient

- $C_{L_D}$  = blade section design lift coefficient
- $C_{L_i} = 4 \int_{0.15}^{1.0} C_{L_D} x^3 dx$



[NASA-CR-114289]





# SR-22 Propeller Efficiency

## ❖ Baseline SR-22 propeller

- 3 blades, 6.333 ft diameter,  $AF = 125$
- At high speed cruise condition (180 knot, 8,000 ft)
  - POH quotes 2600 RPM
  - Optimum  $CL_i = 0.3021$

## ❖ SR-22 propeller comparisons:

Aircraft	Number of Propellers	Velocity (knots)	Engine Speed (RPM)	Advance Ratio	Thrust Coefficient	Efficiency	% Difference from Baseline
SR-22	1	180	2600	1.11	0.041	0.845	--
SR-22	1	130	2600	0.80	0.030	0.584	-30.9%
Electric SR-22	1	130	1364	1.53	0.099	0.889	5.2%
GT Concept	2	130	2429	0.86	0.015	0.916	8.4%

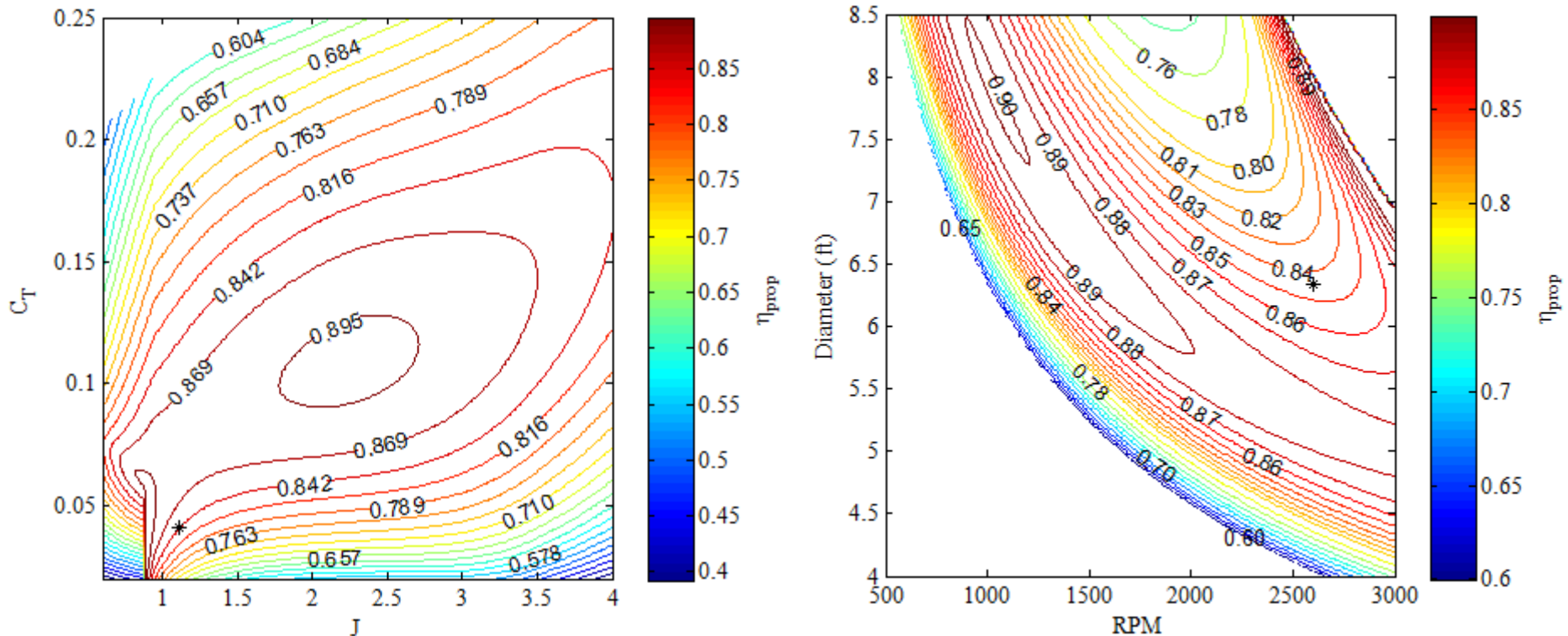
Calculations performed at a weight of 2900 lb and 8000 ft altitude

## ❖ Efficiency can be gained by

- Increasing number of propellers (more lightly loaded props)
- Reducing RPM



# SR-22 Propeller Efficiency Trades



\* Indicates baseline point with 3 blades,  $AF=125$ ,  $D=6.333$  ft,  $RPM=2600$ ,  $CLi=0.3021$

Recall:  $J = V / (n D)$  and  $C_T = T / (\rho n^2 D^4)$

❖ Efficiency gained by increasing propeller diameter, reducing RPM



# Optimal Propeller Design

## ❖ What if the propeller design changes?

- 6.5 ft diameter fixed for V-tail installation
- 3 blades
- Vary RPM
- Vary CLi
- Vary AF
  - $\pm 20\%$  of baseline:  $100 \leq AF \leq 150$
  - Full valid range for code:  $80 \leq AF \leq 200$

Number of Propellers	CLi	Activity Factor	Engine Speed (RPM)	Advance Ratio	Thrust Coefficient	Efficiency	% Difference from Baseline
2	0.4106	100	866	2.35	0.104	0.928	9.8%
2	0.4278	80	930	2.18	0.090	0.969	14.7%

- Efficiencies are uninstalled



# Installation Losses

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## ❖ Scrubbing drag from propeller slipstream

- Removal of propeller from nose makes laminar flow feasible
- Slipstream acts over reduced wetted area compared to SR-22
- Propeller slipstream velocity reduced compared to SR-22
- Reduction in scrubbing drag accounted for in current aerodynamic buildup for concept

## ❖ Installation losses due to blockage

- Not yet quantified
- Related to the blockage area
- Reduced blockage in new concept vs. SR-22 may allow significant reduction in losses

## ❖ Installation losses should be less significant in new concept compared to SR-22



# Initial Sizing

- ❖ Size to specified range, FAR reserve, battery health reserve with Breguet-like range and endurance equations
- ❖ Wing sized with 61 knot stall speed
- ❖ Power sized with most constraining of:
  - 750 ft/min ROC at 8,000 ft
  - 1.5% climb gradient with one motor out at 5,000 ft pressure altitude
    - FAR 23.49 and FAR 23.67
  - 2,000 ft balanced field length
    - Uses FLOPS equations with slope factor FTOFL = 0.85
      - FTOFL = 0.7612 to match SR-22 POH value
      - FTOFL = 0.8343 to match detailed FLOPS analysis
- ❖ Empty weight fraction:

$$\frac{W_e}{W_0} = \frac{(W_e)_{SR-22} - (W_{engine})_{SR-22} + W_{motor} + W_{controller}}{(W_0)_{SR-22}}$$



# Initial Sizing

## ❖ Technology assumptions:

Propeller Efficiency	0.85 - 0.95
Motor Nominal Specific Power (hp/lb)	3
Motor Peak Specific Power (hp/lb)	4
Motor Efficiency (with Gearbox)	0.925
Controller Specific Weight (lb/hp)	0.05
Controller Efficiency	0.98
Battery Specific Energy (W-hr/kg)	200 - 400
Battery Efficiency	0.98

- Use nominal specific power assumption for all sizing
  - May be overly conservative
  - May be able to use peak power for takeoff or other power-sizing constraints
- Assume 20% of battery charge must be maintained for battery health
  - Allow reserves to use the last 20% of the battery



# Initial Sizing

- ❖ Results for 200 mile mission at 150 mph, 840 lb payload,  $L/D=20$ , and  $FTOFL=0.85$

Propeller Efficiency	0.85	0.90	0.95	0.85	0.90	0.95
Battery Energy Density (W-hr/kg)	200	200	200	400	400	400
Gross Weight (lb)	19,868	12,258	9,227	3,193	3,047	2,937
Empty Weight (lb)	9,965	6,137	4,620	1,618	1,544	1,491
Empty Weight Fraction	0.502	0.501	0.501	0.507	0.507	0.508
Battery Weight (lb)	9,015	5,253	3,746	724	653	596
Wing Area (ft <sup>2</sup> )	792.5	488.9	368.1	127.4	121.5	117.1
Total Power Required (hp)	970	565	427	206	197	197
Power per Motor (hp)	485	283	213	103	98	98

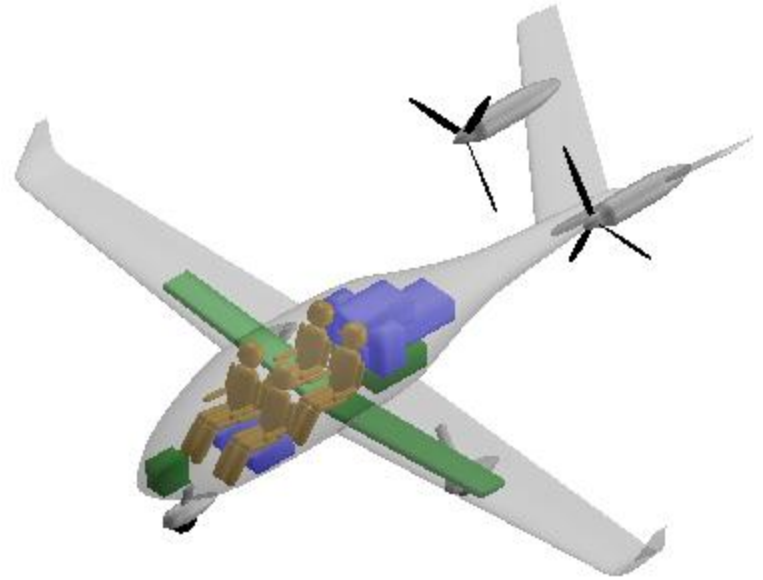
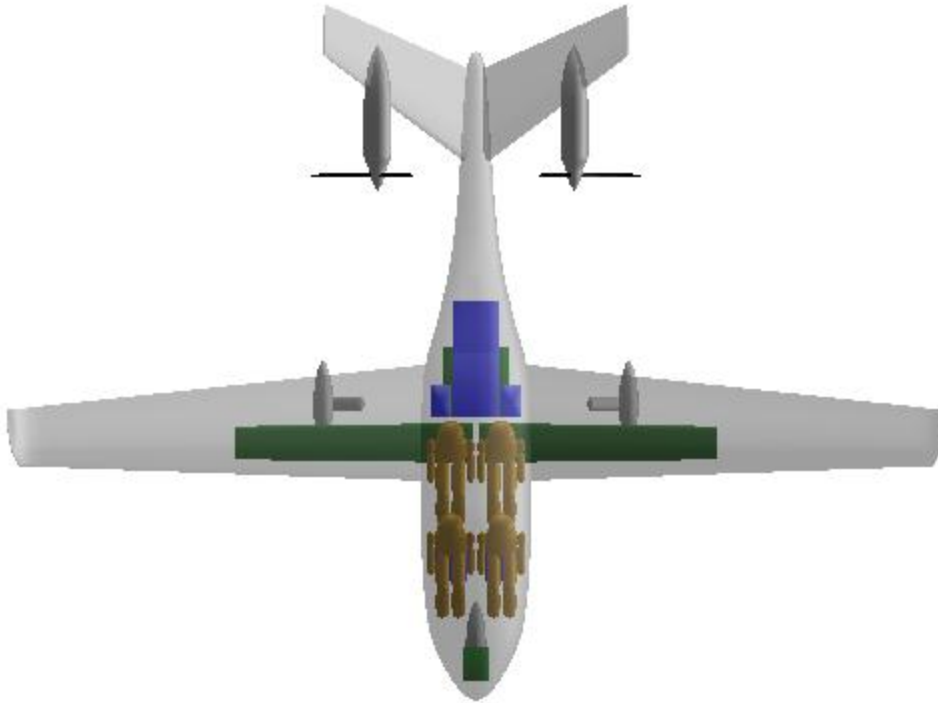
## ❖ Iteration

- Drag buildup for new concept
- Iterated until  $L/D$  assumption matched calculated value



# Georgia Tech Concept

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# Georgia Tech Concept

## ❖ Sizing Results

- Breguet analyses
- Zip mission (200 mi at 150 mph)

	Georgia Tech Concept	Electric SR-22	Baseline SR-22	Baseline SR-22
Gross Weight (lb)	2,975	3,400	3,400	3,400
Operating Empty Weight (lb)	1,507	1,831	2,329	2,329
Operating Empty Weight Fraction	0.507	0.538	0.685	0.685
Battery (Fuel) Weight (lb)	618	749	136	95
Zip Mission Payload	840	821	935	976
Propeller Efficiency	0.928	0.889	0.584	0.845
Wing Area (ft <sup>2</sup> )	123.1	144.9	144.9	144.9
Wing Span (ft)	35.4	38.3	38.3	38.3
Zip Mission Cruise L/D	20.0	18.9	17.0	17.0
Total Power Required (hp)	190	310	310	310
Power per Motor (hp)	95	310	310	310

SR-22 SFC = 0.509 lb/hr/hp

Propeller efficiencies are uninstalled



# Concept Features

## ❖ Natural laminar flow nose

- 15% laminar flow assumed
- 6.2% (12 count) CD0 savings (compared to fully turbulent)

## ❖ Natural laminar flow airfoils

- NASA NLF(1)-0414F
- 60% laminar flow assumed
  - Same as baseline SR-22

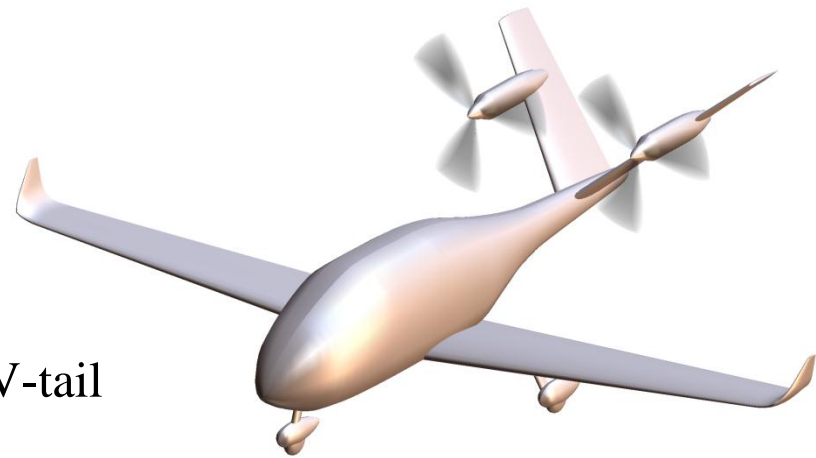
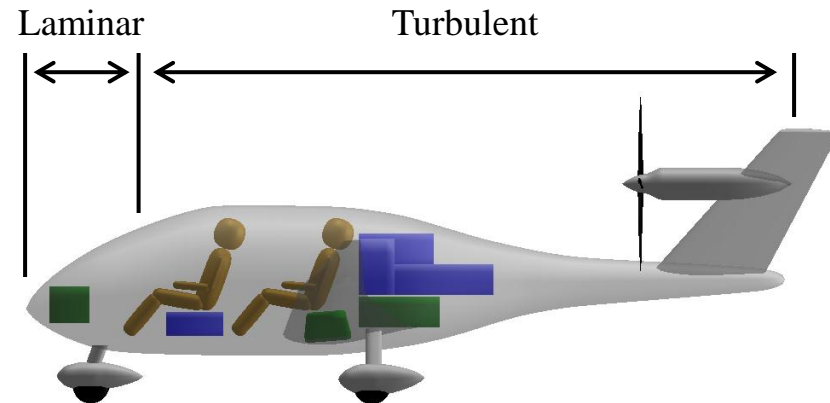
## ❖ Fuselage wetted area reduction behind baggage compartment

## ❖ Winglets for reduced induced drag

- Cruise L/D increase of 1.8%
- Aid in takeoff distance

## ❖ Scrubbing drag reduction

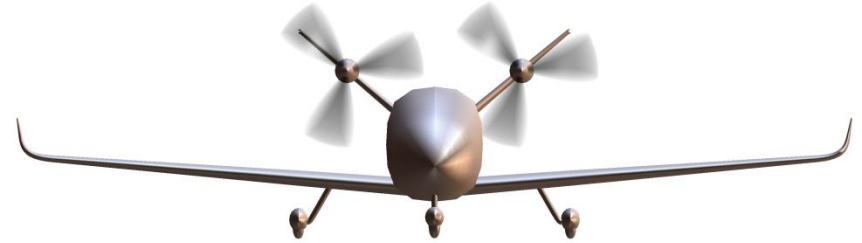
- 7.5% CD0 reduction with 2 propellers on V-tail (compared to a single propeller on nose)
  - 0.9% reduction with 15% fuselage laminar flow



# Concept Features

## ❖ Reduced cooling drag

- 0.3% of total drag at cruise
- Baseline SR-22
  - 6% of total drag at high speed cruise
  - 9% of total drag at Zip mission cruise



## ❖ Increased propeller / propulsion efficiencies

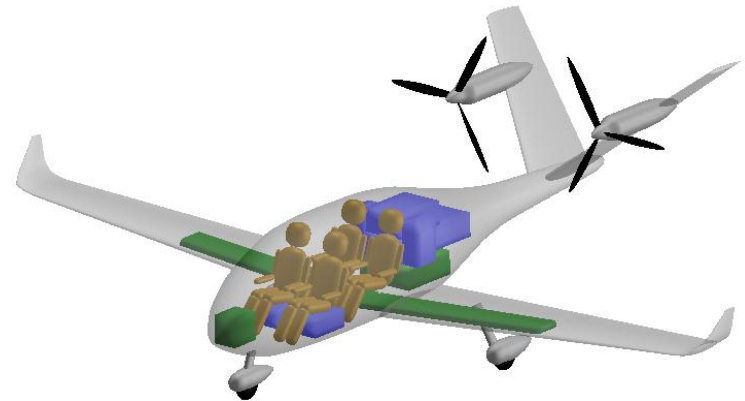
- $\eta_{\text{prop}} > 90\%$  possible
  - Larger, more lightly loaded propellers
  - Lower RPMs
  - Less blockage behind propellers
- Baseline SR-22  $\eta_{\text{prop}} < 85\%$

## ❖ Sufficient volume for 4 passengers and baggage

- Larger cabin than SR-22

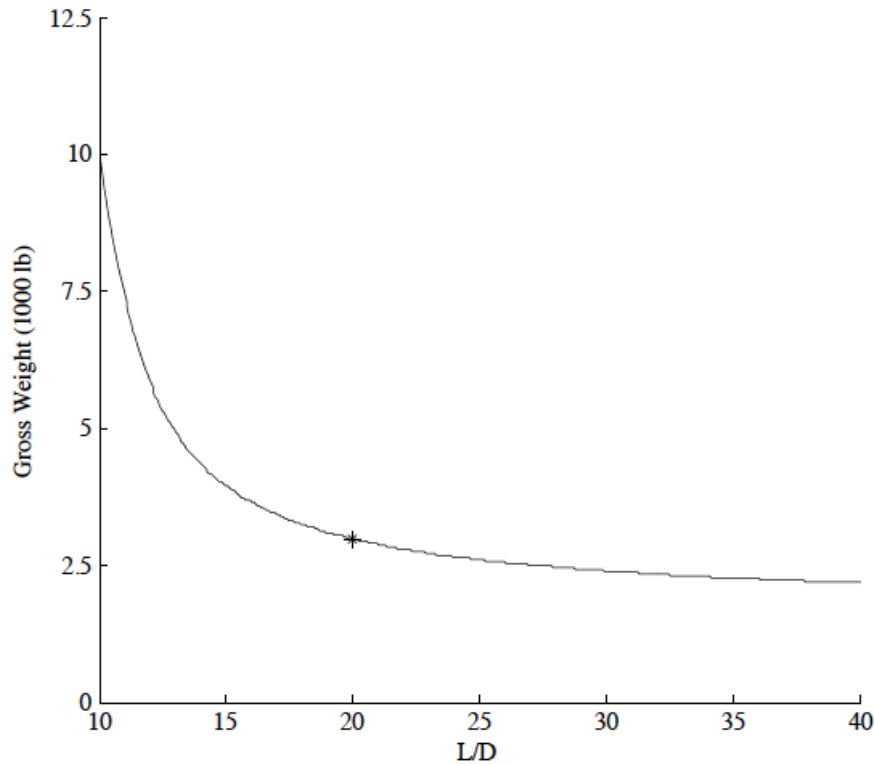
## ❖ Propellers removed from passenger operations

- Less cabin noise and vibration
- Increased safety for untrained passengers

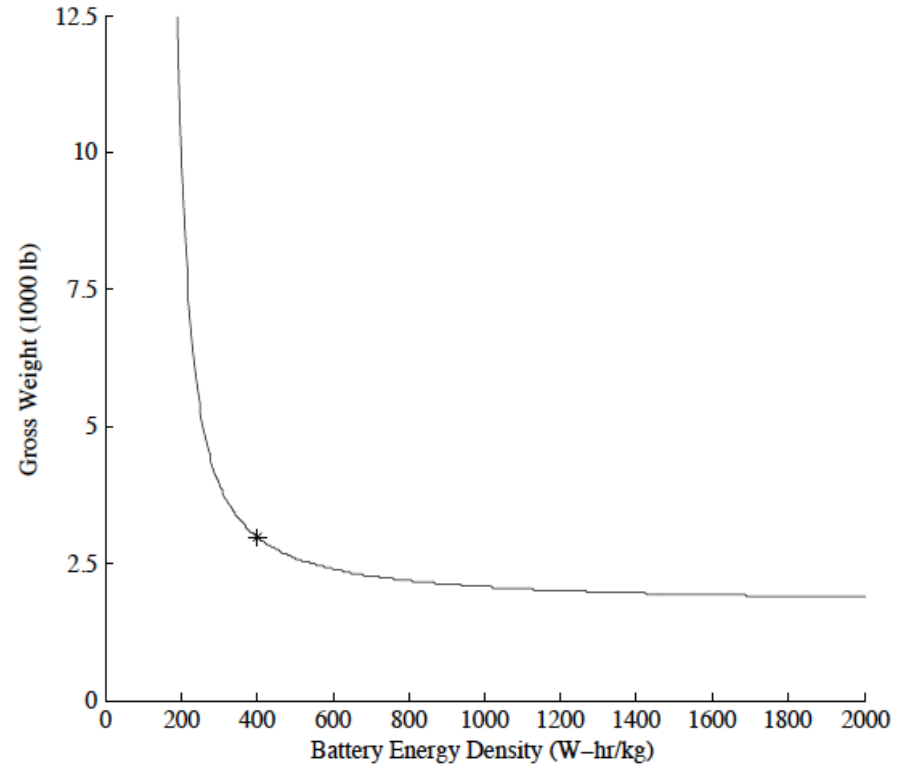


# Sensitivity Study

## ❖ Lift-to-drag ratio

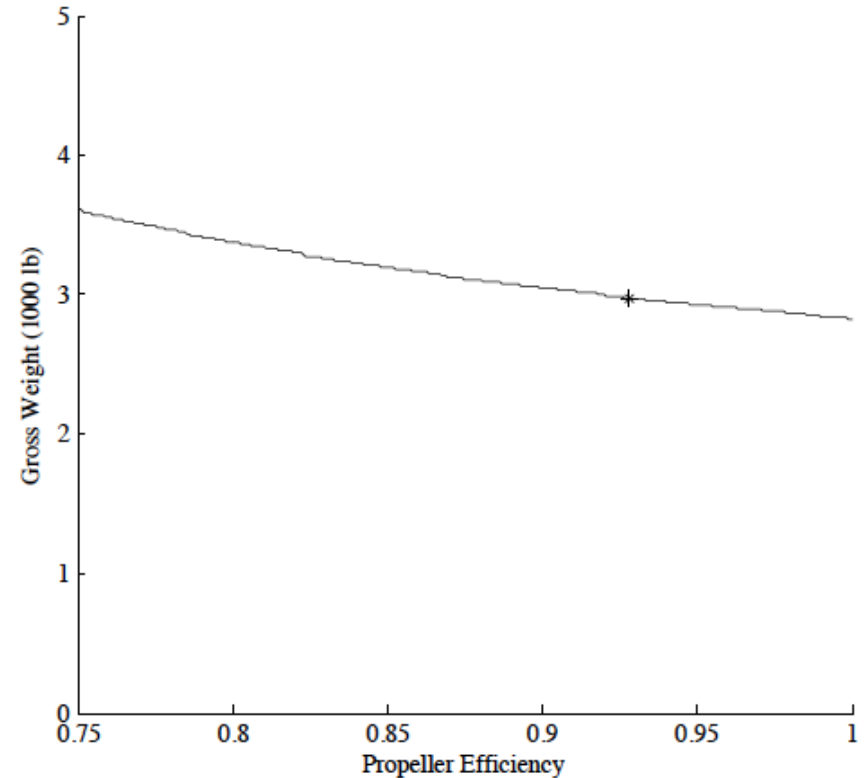
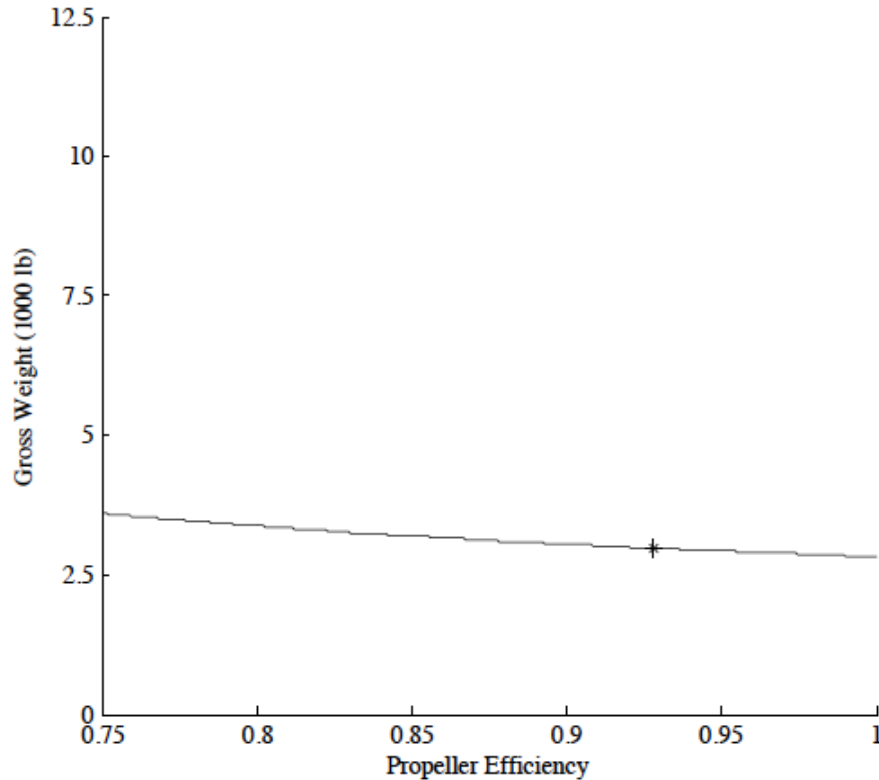


## ❖ Battery energy density



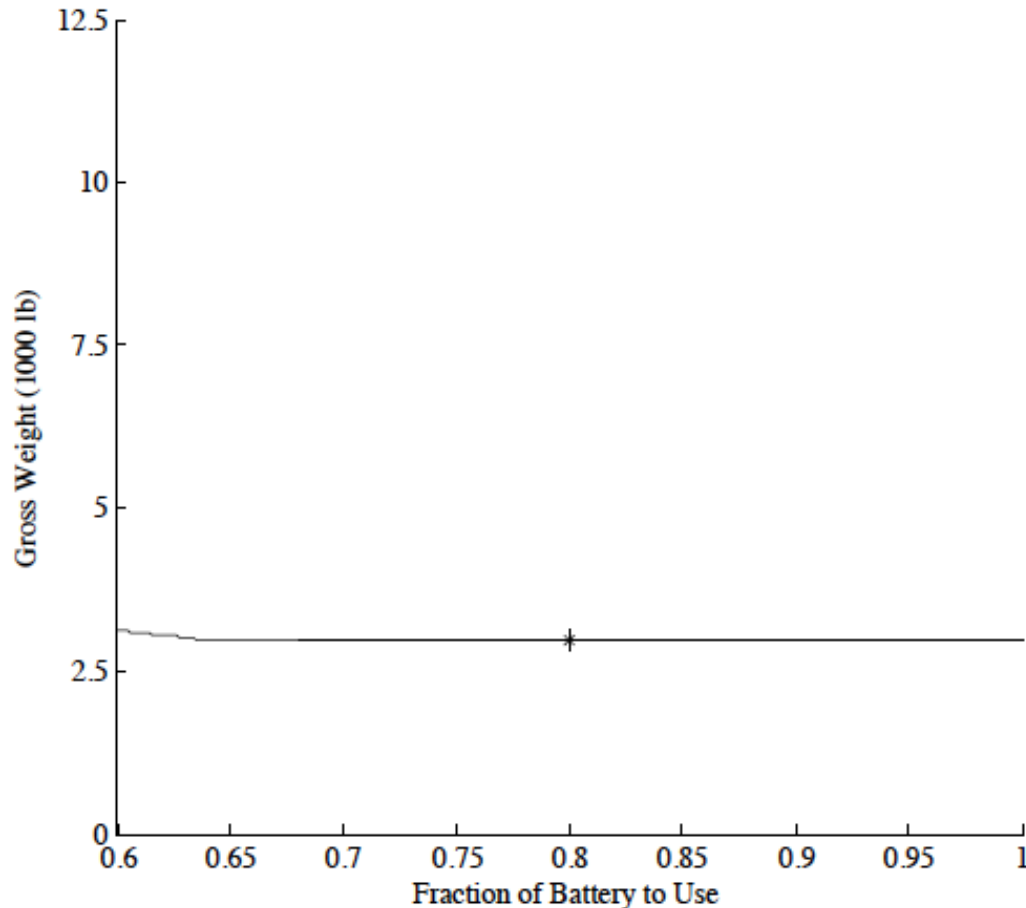
# Sensitivity Study

## ❖ Propeller Efficiency



# Sensitivity Study

## ❖ Available energy fraction

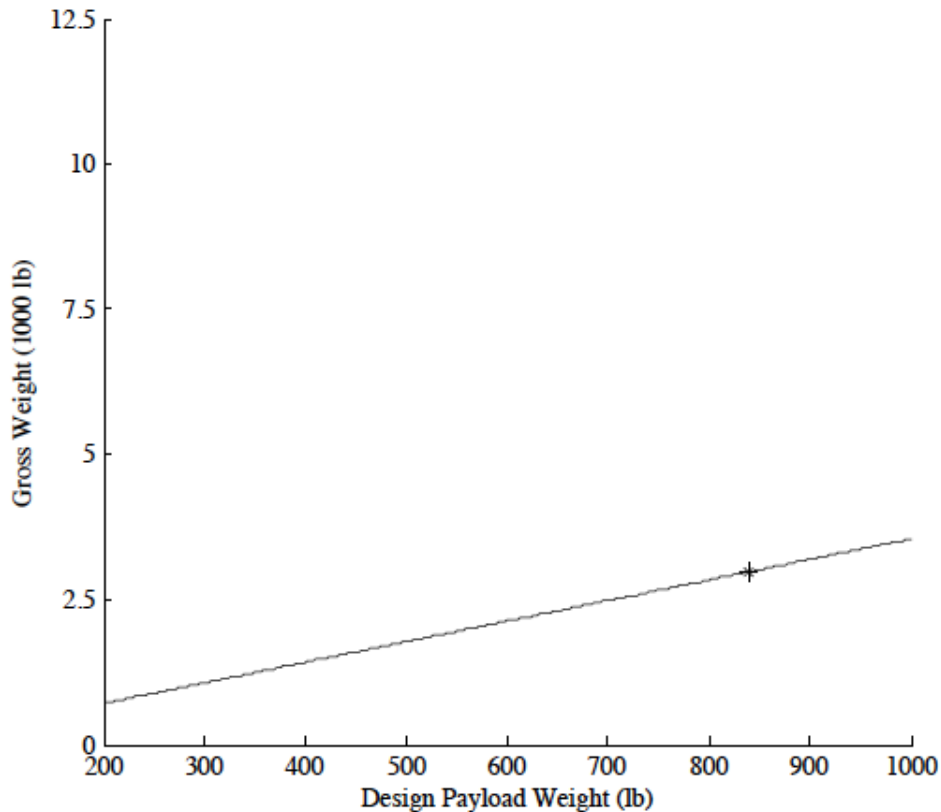


- FAR fuel reserve limited
- ~64% of total energy needed for cruise
- ~20 mins of reserve time / ~50 miles reserve distance possible while using 80% of batteries

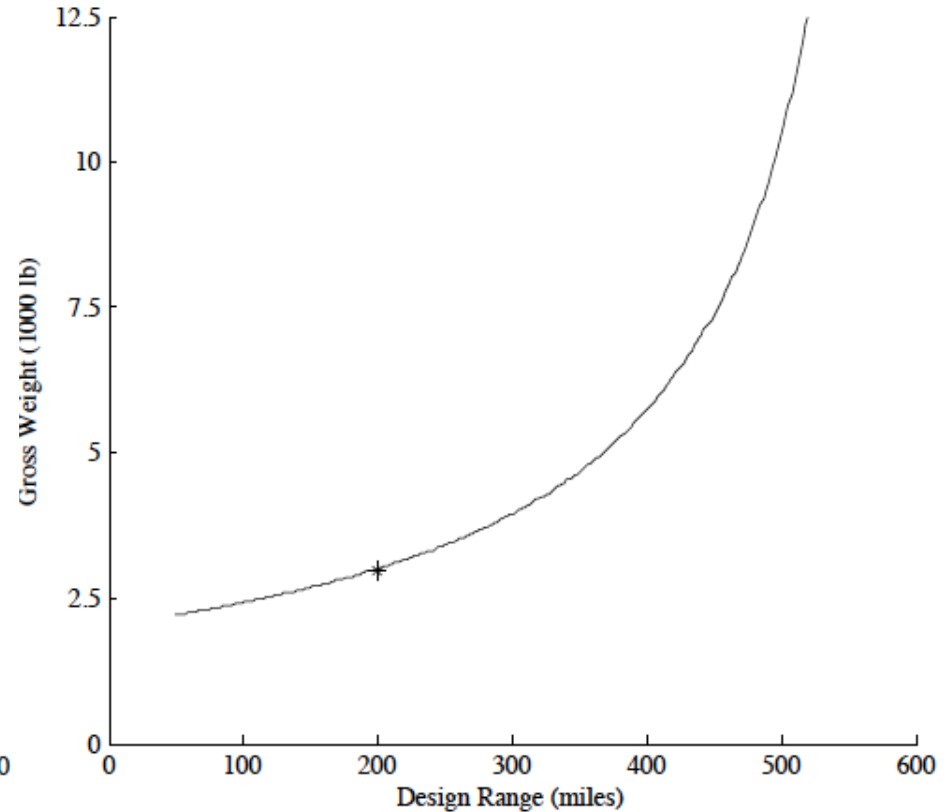


# Sensitivity Study

## ❖ Payload weight



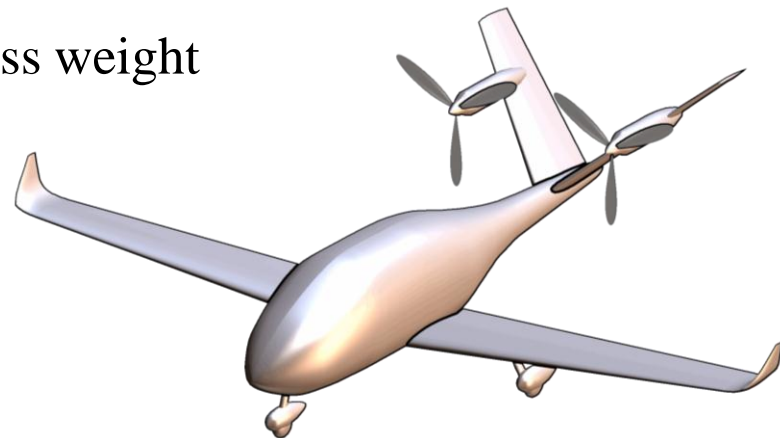
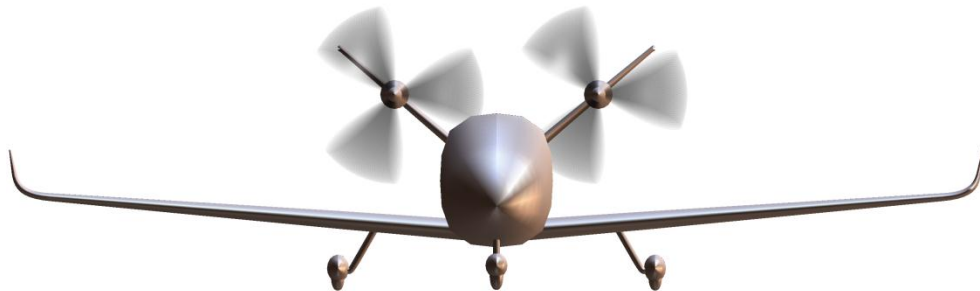
## ❖ Design range



# Part 3 Conclusions

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- ❖ Realistic “low risk” concept possible with 400 W-hr/kg batteries
- ❖ Potential for improvement in “first order” effects over SR-22-like baseline
  - Propeller efficiency increases
  - Lift-to-drag ratio increases
- ❖ Battery health reserves and FAR fuel reserves synergistic
  - ~20 min reserve time while using 80% of battery or less
- ❖ Longer-range variants feasible
  - 400 mi range achievable with 5760 lb gross weight





# Part 4:

## SR-22 Validation Revisited and Comparisons of Conventional and Electric Variants

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# SR-22 POH Drag Calibration

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- ❖ Goal: Determine cruise drag estimate using data from the POH, and use this result to calibrate drag buildup
- ❖ Drag estimation procedure
  - Assume  $D = T$  (steady, level flight with thrust aligned with velocity vector)
  - $T = P_{\text{available}} / V$ 
    - $V$  can be found for a given operating condition from the POH cruise tables
  - $P_{\text{available}} = P_{\text{shaft}} * \eta_{\text{prop}}$ 
    - $P_{\text{shaft}}$  can be found for a given operating condition from the POH cruise tables
    - Propeller efficiency ( $\eta_{\text{prop}}$ ) is unknown initially
- ❖ Use an iterative procedure to find the propeller efficiency
  - Use the Hamilton Standard Model to determine uninstalled efficiency
  - Use a correction for installation loss taken from Torenbeek's "Synthesis of Subsonic Airplane Design" to determine installed efficiency (from uninstalled)



# SR-22 POH Drag Calibration (cont.)

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## ❖ Iterative procedure for propeller efficiency determination:

- Estimate a propeller efficiency
- Determine drag from POH cruise tables
- Determine implied L/D
- Determine new optimal propeller efficiency from implied L/D
- Repeat until propeller efficiency converges

## ❖ For the SR-22 in cruise at 180 knots, 8000 ft, 2900 lb, this iterative procedure gives a propeller efficiency of 85.6%, which implies

- Cruise L/D = 8.05
- Cruise CD = 0.0288
- Cruise Drag = Cruise Thrust = 360 lb
  - Note that our analysis gives the same result as Peter Garrison's source at Hartzell for the installed thrust for the SR-22 propeller

## ❖ Calibrate drag buildup at this condition



# SR-22 Drag Buildup Calibration

- ❖ Create geometry model in openVSP
- ❖ Perform component drag buildup for parasite drag
  - Flat plate skin friction coefficients corrected with form factors
  - Wetted areas from VSP
  - Estimates of interference factors using guidelines from Raymer
  - Estimate of “scrubbing drag” from propeller slipstream by modifying velocities in component drag buildup
  - Estimates of percentage of laminar flow over each component
  - Excrescence drag estimation
- ❖ Estimate cooling drag using GT-developed model
- ❖ Estimate induced drag and profile drag due to lift with XFLR5
  - Wing and horizontal tail modeled; fuselage not modeled
  - Assume Cirrus twists wing to achieve a high span efficiency factor (perhaps untrue; no data available on wing twist distribution)
  - Account for “trim drag” at only one reference flight condition

Higher confidence values

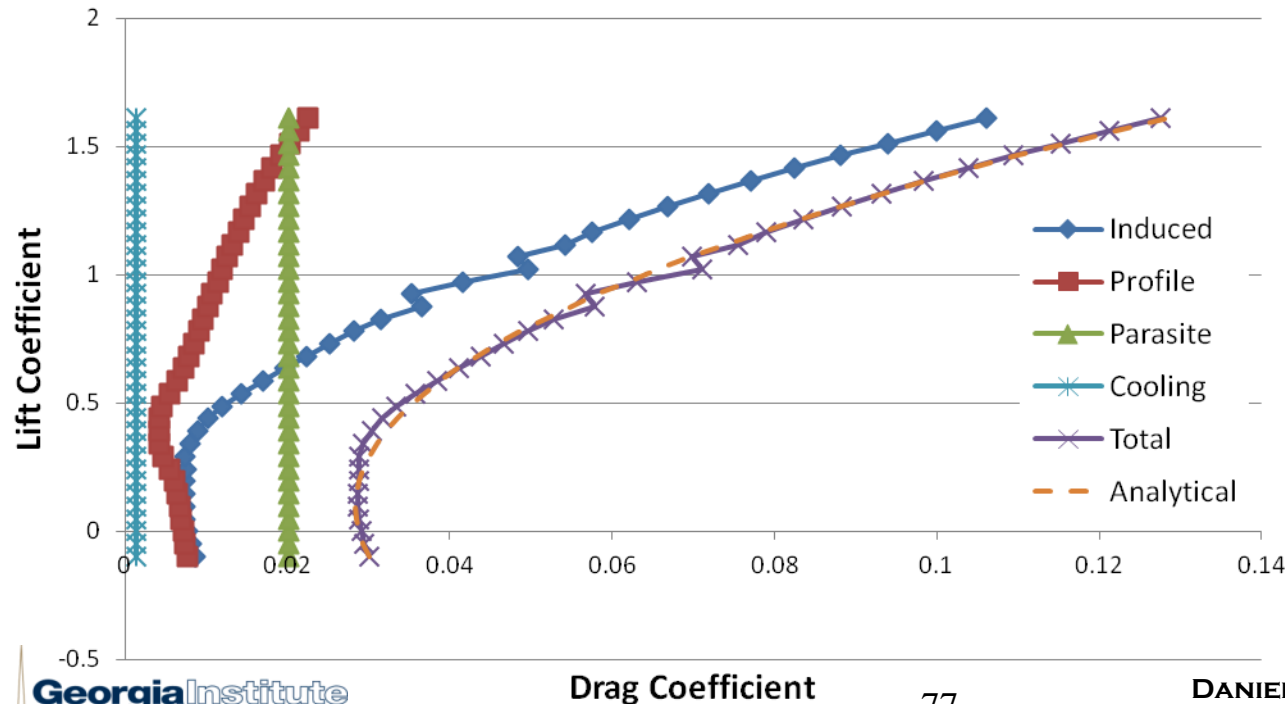
Calibrated to match POH results



# SR-22 Drag Buildup

## ❖ Drag Polar:

- Analytical model,  $CD = CD_0 + CL^2/(\pi e_0 AR) - CL_{offset}CL$ 
  - $CD_0 = 0.0288$
  - $e_0 = 0.71$  (Oswald efficiency; includes profile drag due to lift)
  - $AR = 10.26$
  - $CL_{offset} = 0.0085$
- Numerical buildup:



Assuming constant zero-lift parasite drag coefficient:

- Max L/D with no trim drag  
16.3 (numeric)  
16.0 (analytical)
- Max L/D with trim drag  
15.9 (numeric)



# L/D at Different Cruise Weights

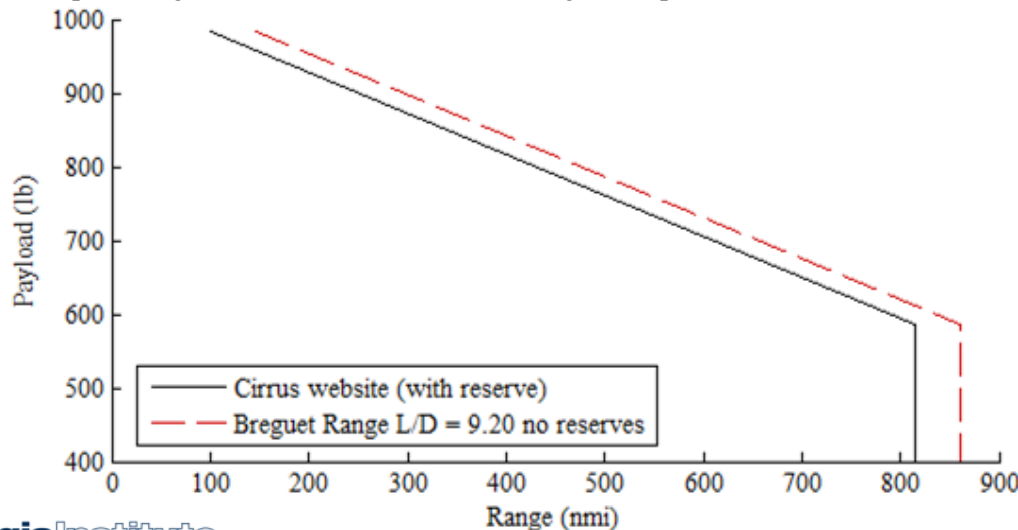
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- ❖ The drag buildup was calibrated to the POH at the CL corresponding to 2900 lbs at 180 kts, 8000 ft
- ❖ As aircraft weight changes at the same (V, h) flight condition, CL and the resulting CD and L/D must change
  - MTOGW of 3400 lbs implies  $L/D = 9.46$
  - (MTOGW – Max Usable Fuel) of 2914 lbs implies  $L/D = 8.09$
- ❖ We therefore expect that values of L/D for missions along the MTOGW limit line on a payload range diagram will be between 8.09 and 9.46



# Payload-Range Diagram Match

- ❖ Cirrus quotes payload-range diagram at 8000 ft, 180 knots
- ❖ Goal is to estimate the payload range diagram with Breguet equation
- ❖ Breguet equation presumes flight at constant  $L/D$ ,  $\eta_{\text{prop}}$ , and SFC; Cirrus POH and payload-range diagram corresponds to constant velocity, constant altitude profile
- ❖ How do we make a payload-range diagram created with the Breguet range equation match Cirrus's?
- ❖ Assume the following are “known” quantities:
  - 85.6% propeller efficiency (from analysis above; presumed constant over the range of missions on the MTOGW line on the payload-range diagram)
  - $\text{SFC} = (17.8 \text{ GPH}) * (6 \text{ lb/gal}) / (.75 * 310 \text{ hp}) = 0.4594 \text{ lb/(hp hr)}$  (from POH; presumed constant at relevant power settings for the range of missions on the MTOGW line on the payload-range diagram)
  - Maximum takeoff weight = 3400 lb (from POH)
  - Max fuel weight =  $(81 \text{ gal}) * (6 \text{ lb/gal}) = 486 \text{ lb}$  (from POH)
- ❖ Solve for the unknown “Breguet equivalent  $L/D$ ” to match the slope of the MTOGW constraint line
  - Resulting  $L/D = 9.20$
  - As expected, this value is between the minimum weight value of 8.09 and the maximum weight value of 9.46 noted in the previous slide, providing additional confidence in the drag buildup



- Why does the Breguet range curve indicate longer ranges?
  - No reserve fuel accounted for
  - Assumes all fuel is used in cruise



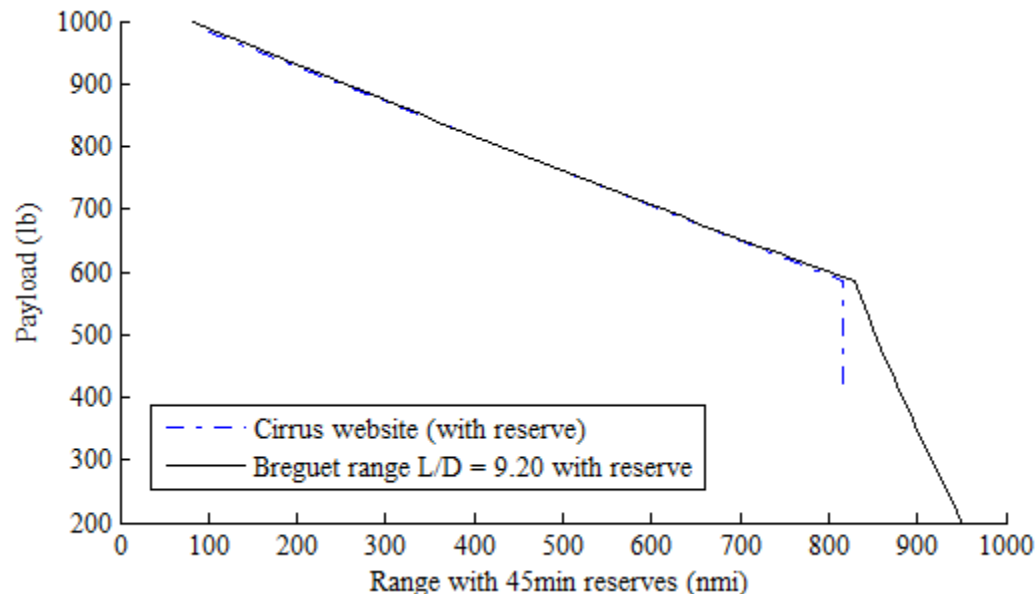
# Payload-Range Diagram Match (cont.)

## ❖ Payload-range diagram with reserve fuel presumptions

- Breguet range equation for cruise segment
- Breguet endurance equation to estimate reserve fuel requirements
  - Use analytical polar to determine CL condition for  $\max C_L^{3/2}/C_D$  in order to set speed for reserve segment

## ❖ Assumptions:

- Breguet equivalent  $L/D = 9.20$  (from analysis above)
- 85.6% propeller efficiency (from analysis above)
- $SFC = (17.8 \text{ GPH}) * (6 \text{ lb/gal}) / (.75 * 310 \text{ hp}) = 0.4594 \text{ lb/(hp hr)}$  (from POH)
- Takeoff weight = 3400 lb (from POH)
- Max fuel weight =  $(81 \text{ gal}) * (6 \text{ lb/gal}) = 486 \text{ lb}$  (from POH)





# Maximum L/D

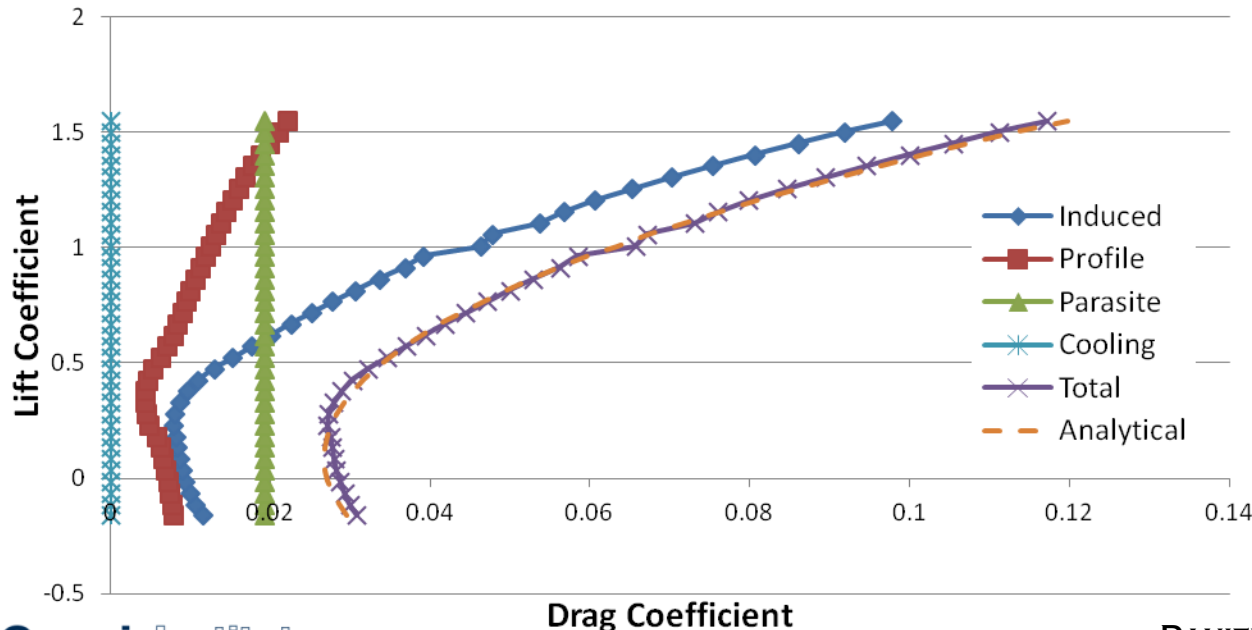
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- ❖ Using the baseline drag polar, the maximum L/D is estimated as 16.3; however, this polar is trimmed at only a single CL
- ❖ Analysis was re-run to trim aircraft at the CL for maximum L/D
  - Resulting L/D max trimmed is 15.9 (2.0% lower)
- ❖ These values of  $(L/D)_{\max}$  are more reasonable than original predictions, but they may still remain slightly high. Sources of uncertainty:
  - Oswald efficiency estimate (0.71) presumes that the wing is nearly “optimally twisted” and that fuselage effects on Oswald efficiency are not significant
  - $CL_{\text{offset}}$  calculation



# “Electric SR-22”

- ❖ Increase in installed propeller efficiency to 87.4%
  - Propeller RPM can vary over wider range (with gearbox)
  - Maintain same diameter and activity factor, but optimize integrated design lift coefficient
  - Smaller blockage area behind propeller due to reduced cowling size required to house electric motor. Reduces installation losses.
- ❖ Drag buildup of electric SR-22 concept
  - Reduced cowling wetted area
  - Reduced cooling drag
- ❖ Drag polar (trimmed to 180 kts, 8000ft):



- Analytical drag polar:  
 $CD_0 = 0.2678$   
 $e_0 = 0.71$   
 $AR = 10.26$   
 $CL_{offset} = 0.008065$

Assuming constant zero-lift parasite drag coefficient:

- Max L/D with no trim drag  
16.8 (numeric)  
16.6 (analytical)
- Max L/D with trim drag  
16.5 (numeric)



# $(L/D)_{\max}$ Drag Buildup

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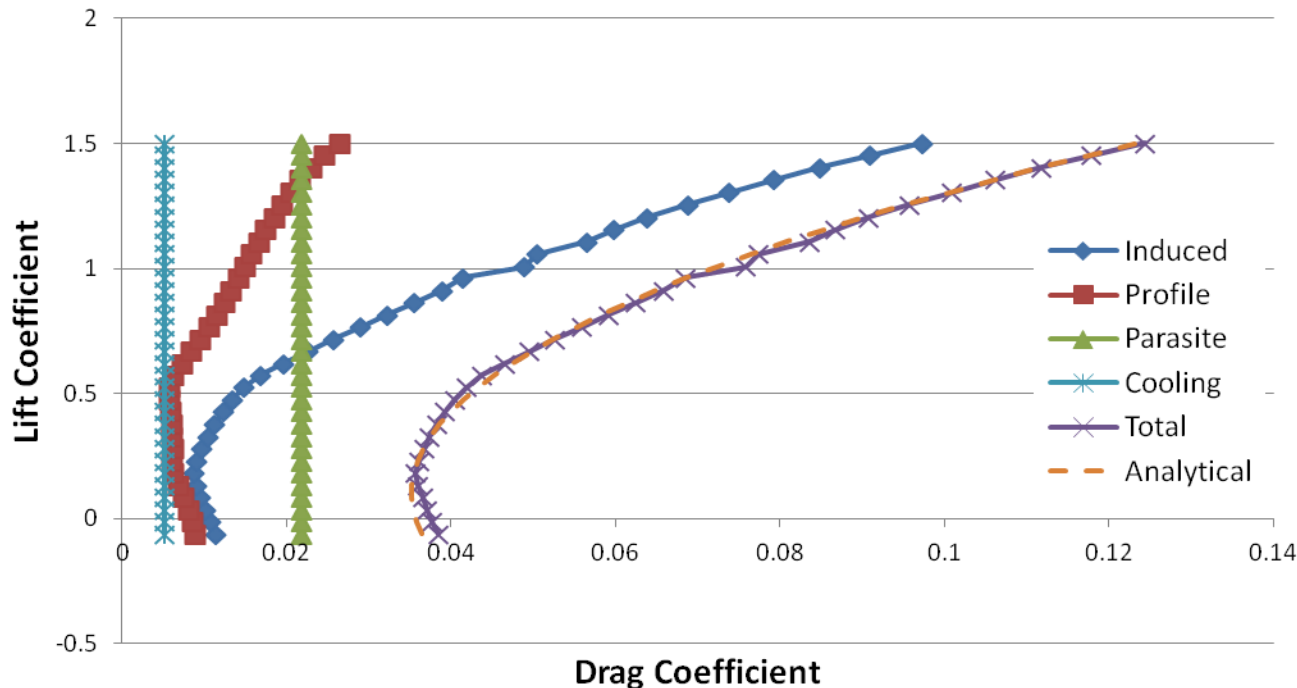
- ❖ It was previously assumed that the zero-lift parasite drag coefficient was nearly constant across operating conditions
- ❖ However, analysis shows that the zero-lift parasite drag coefficient changes significantly from 180 knots to  $(L/D)_{\max}$  velocity (at 8,000 ft)
- ❖ GT performed new drag buildups at lower velocities:
  - 2900 lbs at  $(L/D)_{\max}$  velocity
  - 3400 lbs at  $(L/D)_{\max}$  velocity



# $(L/D)_{\max}$ Drag Buildup

## ❖ Drag Polar for 2900 lbs, 90.8 knots, 8000 ft:

- Analytical model,  $CD = CD_0 + CL^2/(\pi e_0 AR) - CL_{\text{offset}} CL$ 
  - $CD_0 = 0.03566$
  - $e_0 = 0.69$  (Oswald efficiency; includes profile drag due to lift)
  - $AR = 10.26$
  - $CL_{\text{offset}} = 0.009$
- Numerical buildup:



Max  $L/D = 13.9$

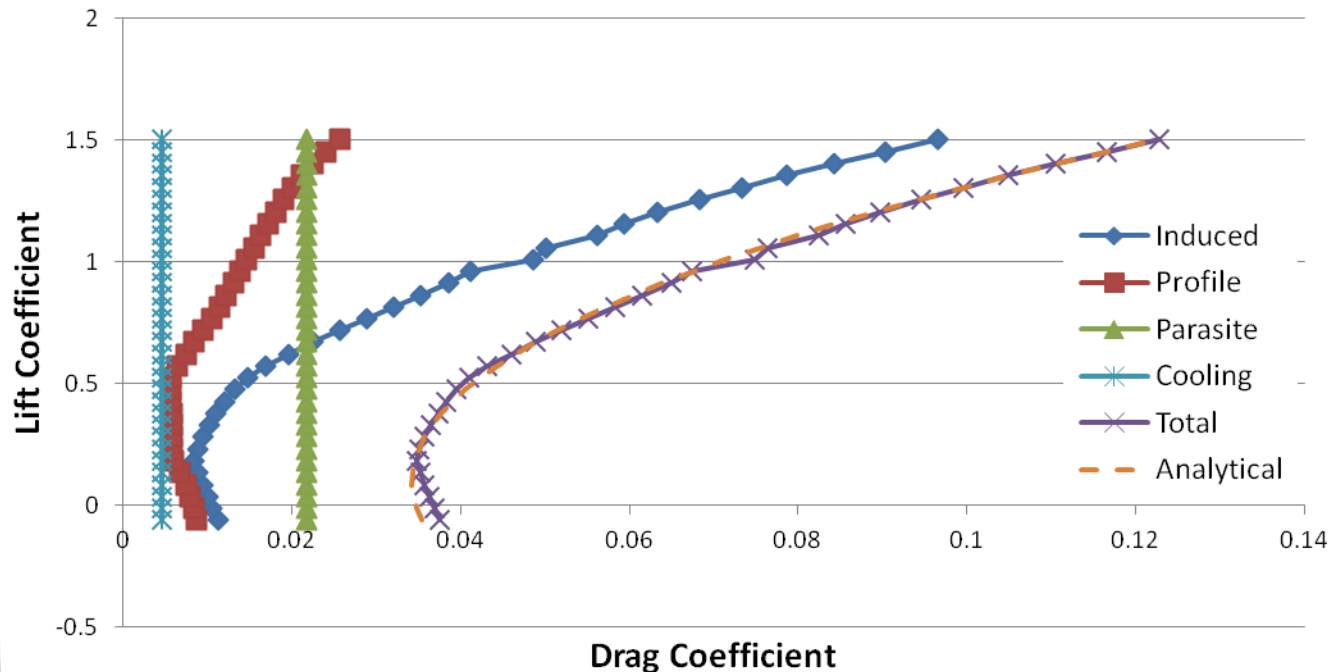
$\eta_{\text{propeller}} = 68.5\%$



# $(L/D)_{\max}$ Drag Buildup

## ❖ Drag Polar for 3400 lbs, 98.3 knots, 8000 ft:

- Analytical model,  $CD = CD_0 + CL^2/(\pi e_0 AR) - CL_{\text{offset}} CL$ 
  - $CD_0 = 0.0347$
  - $e_0 = 0.69$  (Oswald efficiency; includes profile drag due to lift)
  - $AR = 10.26$
  - $CL_{\text{offset}} = 0.009$
- Numerical buildup:



Max L/D = 14.1  
 $\eta_{\text{propeller}} = 77.4\%$



# $(L/D)_{\max}$ Results

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❖ Comparing results at 2900 lb to results at 3400 lb:

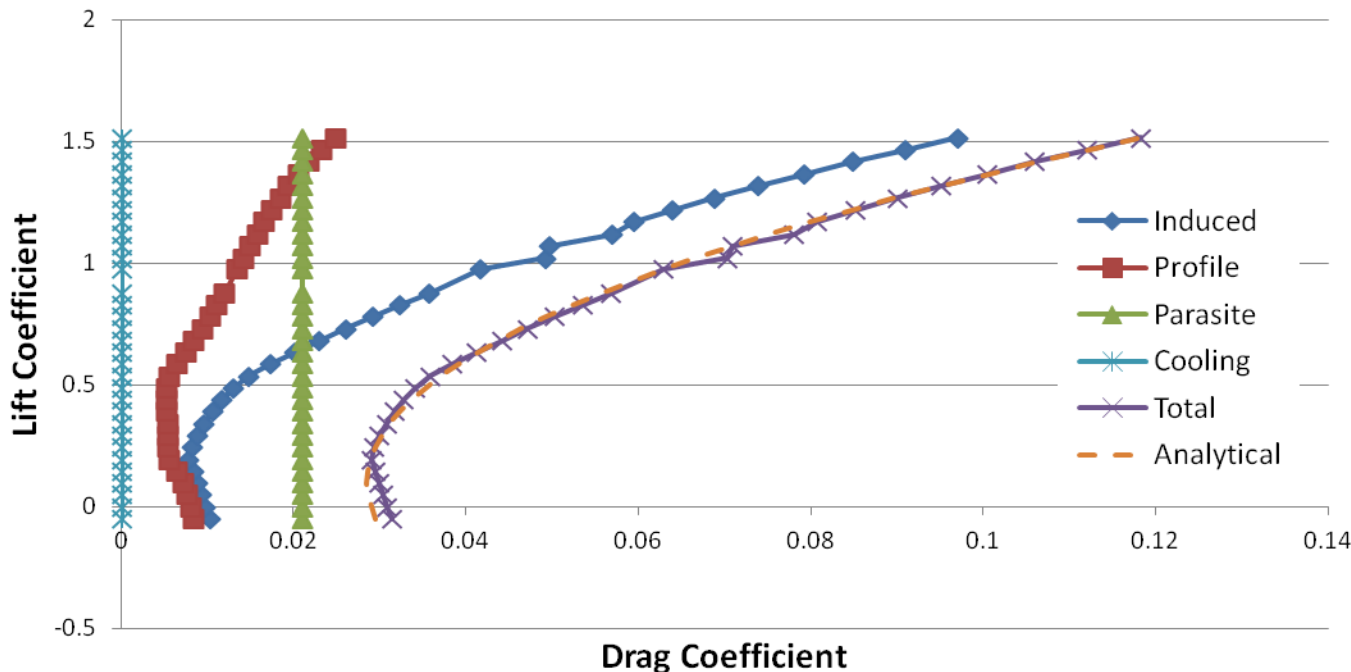
	3400 lb	2900 lb	% Difference
True Airspeed (knots)	98.3	90.8	8.3%
Calibrated Airspeed (knots)	87.1	80.5	8.3%
Drag (lb)	241.8	209.2	15.5%
Thrust HP Required	64.6	51.7	25.1%
Propeller Efficiency	0.774	0.685	13.0%
BHP Required	83.5	75.4	10.7%
% Power	26.9	24.3	10.7%
Fuel Flow Rate (gal/hr)	6.88	6.36	8.3%
Fuel Flow Rate (lb/hr)	41.3	38.1	8.3%
SFC (lb/hp/hr)	0.495	0.506	-2.2%
Specific Range (nmi/gallon)	14.27	14.28	0.0%
L/D max	14.06	13.86	1.5%
CL	0.913	0.913	0.0%
CD	0.0649	0.0659	-1.5%
Parasite/Induced Drag Ratio	1.539	1.576	-2.4%



# “Electric SR-22” $(L/D)_{\max}$ Drag Buildup

## ❖ Drag Polar for 3400 lbs, 109.9 knots, 8000 ft:

- Analytical model,  $CD = CD_0 + CL^2/(\pi e_0 AR) - CL_{\text{offset}}CL$ 
  - $CD_0 = 0.0290$
  - $e_0 = 0.69$  (Oswald efficiency; includes profile drag due to lift)
  - $AR = 10.26$
  - $CL_{\text{offset}} = 0.009$
- Numerical buildup:



Max  $L/D = 15.5$

$\eta_{\text{propeller}} = 88.1\%$



# “Electric SR-22” Payload-Range Diagram

- ❖ Use “electric Breguet” range equation
- ❖ Study high speed cruise and maximum L/D profiles
- ❖ Study constant battery weight and variable battery weight cases
- ❖ Assume cruise at maximum takeoff weight, which implies  $L/D = 10.1$
- ❖ Other assumptions:
  - Propeller efficiency of 87.4%
  - Analytical drag polar used to determine optimal reserve speed (for 45min endurance segment)
  - Standard technology assumptions:

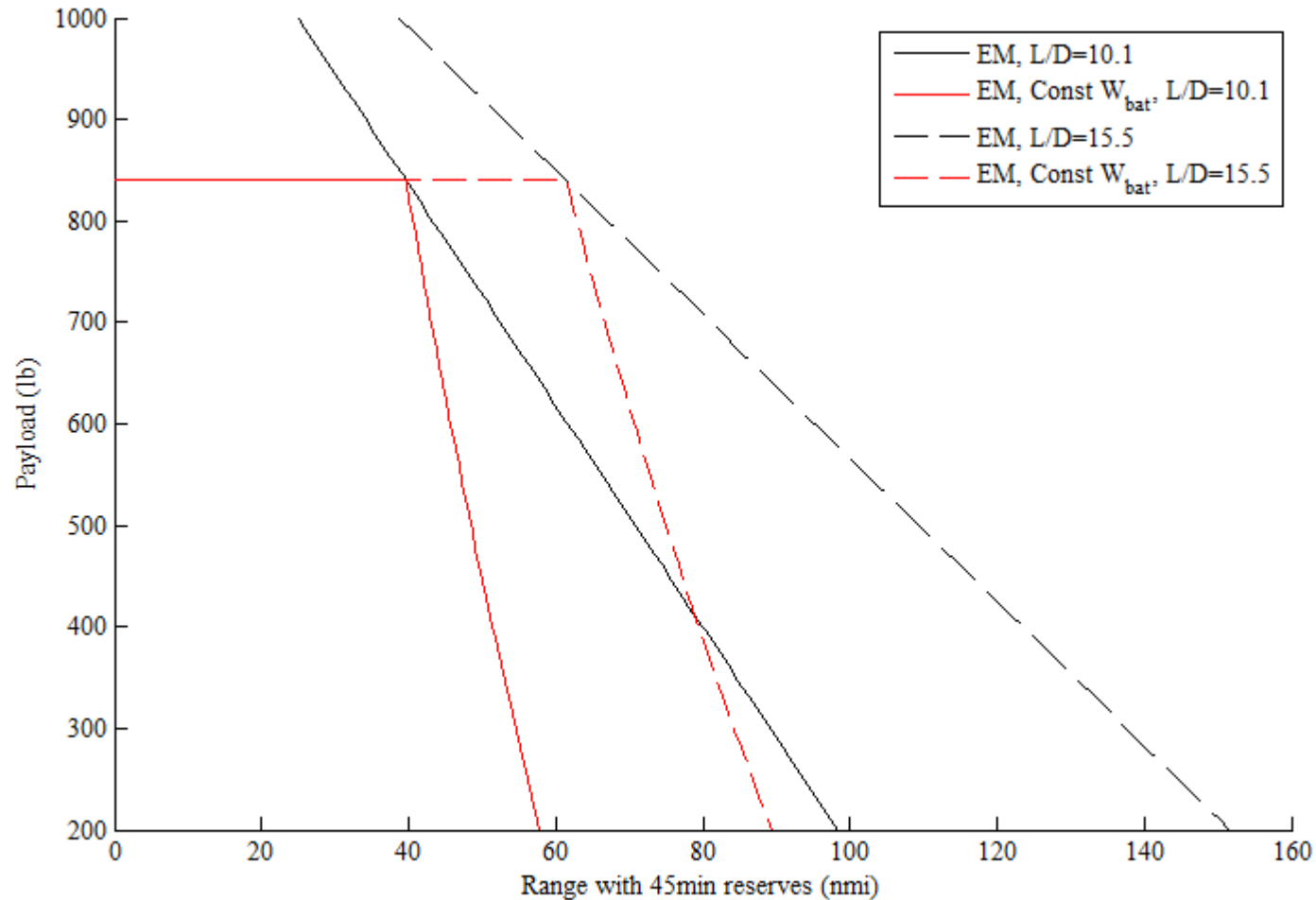
	Technology Year		
	2015	2035	2050
Motor Nominal Specific Power (hp/lb)	3	4.5	7.5
Motor Efficiency (with Gearbox)	0.925	0.95	0.97
Controller Specific Weight (lb/hp)	0.05	0.05	0.05
Controller Efficiency	0.98	0.99	0.99
Battery Energy Density (W-hr/kg)	200	600	1200
Battery Efficiency	0.98	0.98	0.99





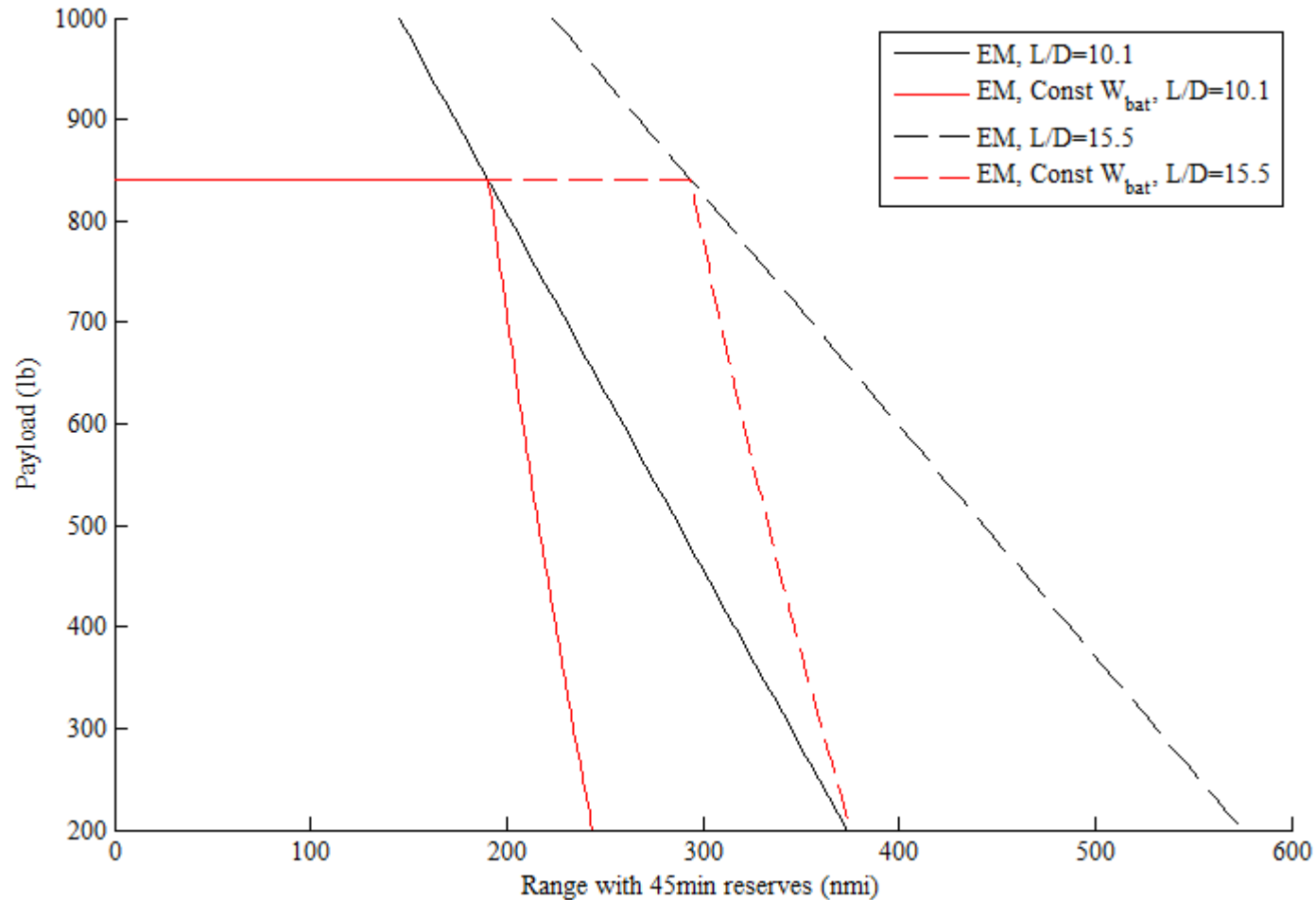
# “Electric SR-22” Payload-Range Diagram

❖ For 2015 technology assumptions:



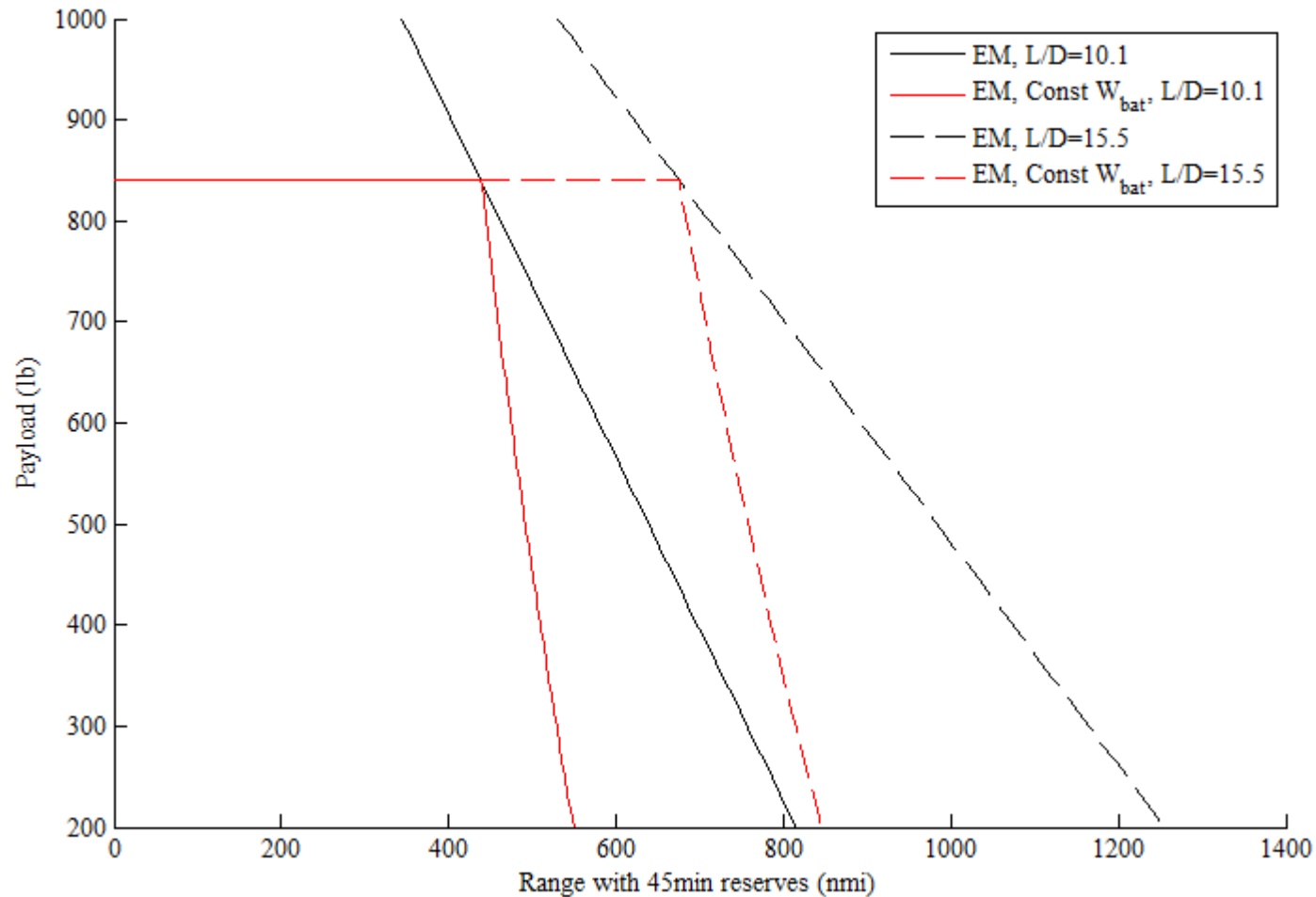
# “Electric SR-22” Payload-Range Diagram

❖ For 2035 technology assumptions:



# “Electric SR-22” Payload-Range Diagram

❖ For 2050 technology assumptions:



# Payload-Range Diagram Comparisons

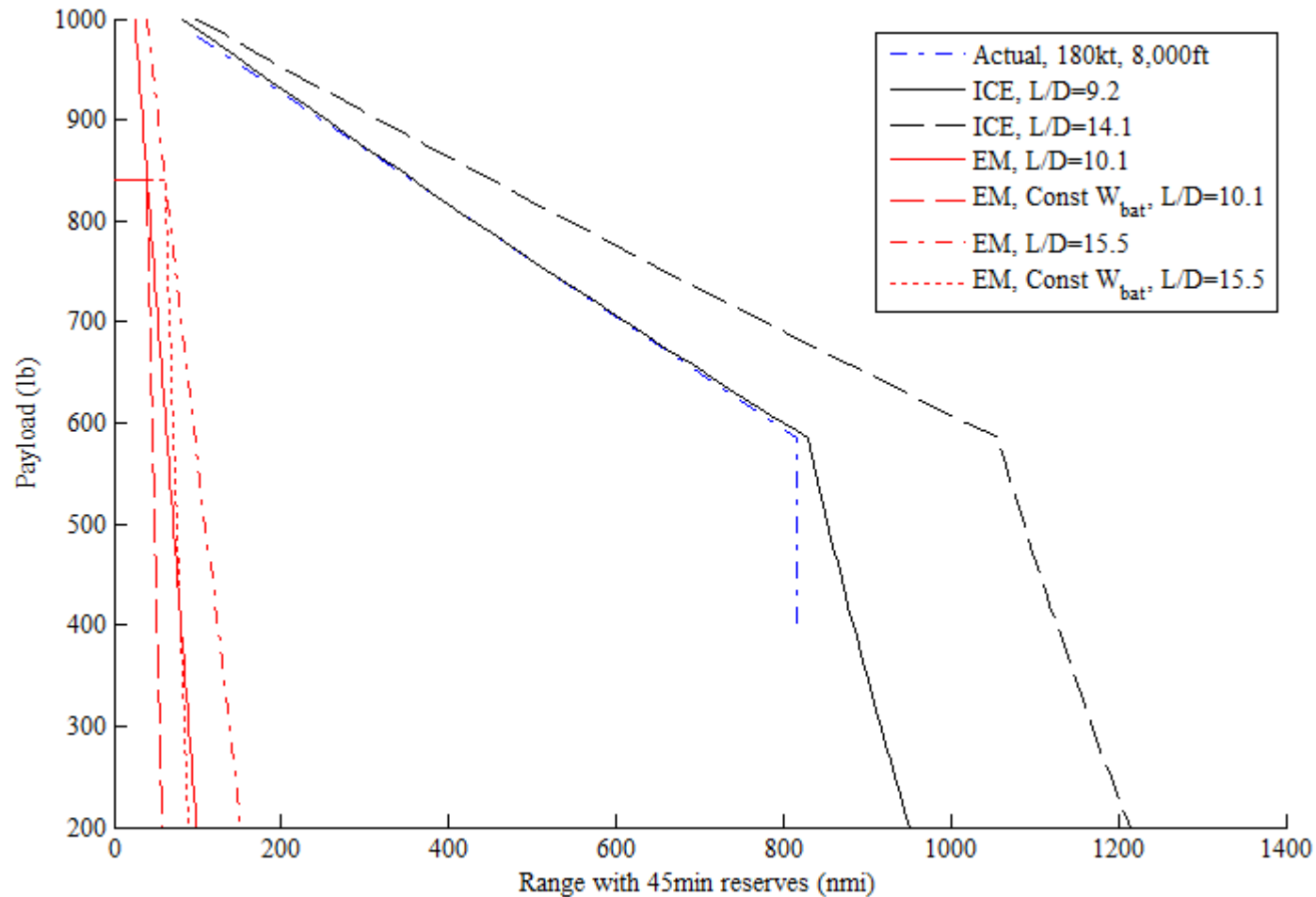
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- ❖ Now study a comparison of the “electric SR-22” to the conventional SR-22 (both Cirrus data and our Breguet analysis)
- ❖ Compare three baseline technology year assumptions of “electric SR-22” to the conventional SR-22



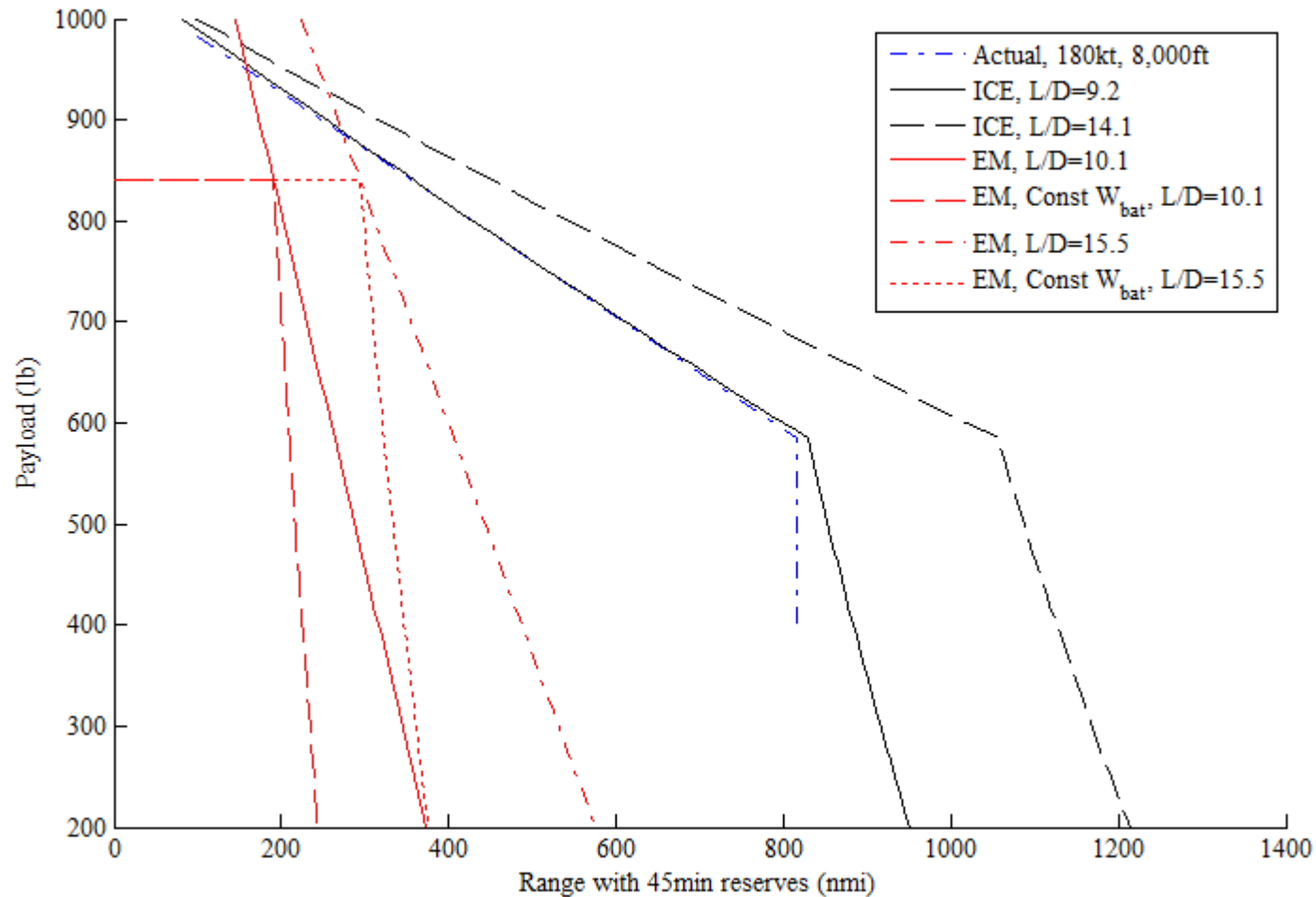
# Payload-Range Diagram Comparisons

❖ For 2015 technology assumptions:



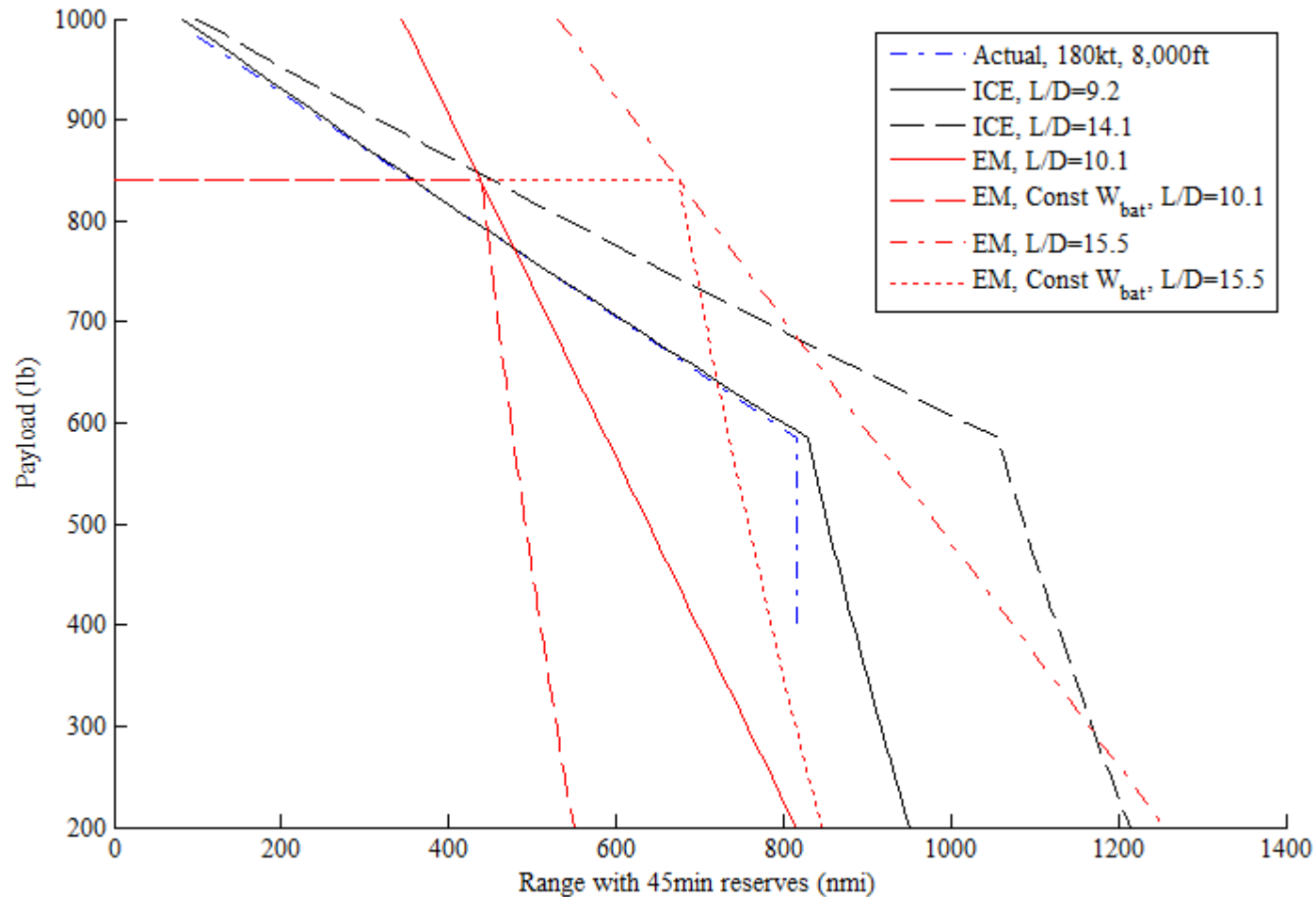
# Payload-Range Diagram Comparisons

❖ For 2035 technology assumptions:



# Payload-Range Diagram Comparisons

❖ For 2050 technology assumptions:



# Part 4 Conclusions

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Although the exact range performance predicted previously (for the SR-22 and “Electric SR-22”) differs from the results of this analysis, the major conclusions do not change:

- ❖ In the near-term, aircraft that are simple modifications to existing airframes will likely not be capable of operating at practical ranges
- ❖ Innovative new concepts will be required to achieve practical range performance by:
  - Increasing the cruise lift-to-drag ratio (e.g. “right-sizing” the wing for cruise at CLs near L/D max. Could be achieved by innovative propulsion-airframe integration to increase CL<sub>max</sub>.)
  - Reducing aircraft empty weight
  - Increasing propeller efficiency (larger, slower turning props, or lightly-loaded distributed props)
- ❖ Most benefit in the long term will be achieved by increases in battery energy density





# Appendix: Cooling Drag Model

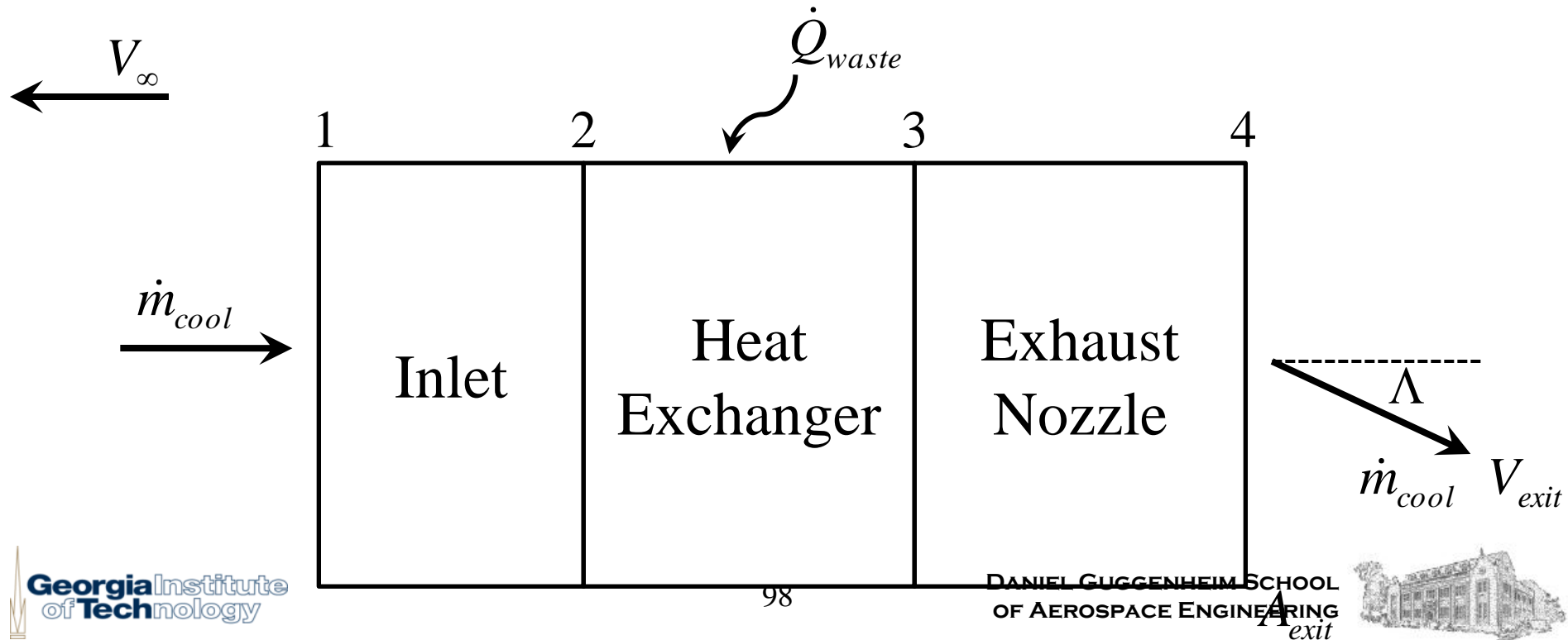
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# Cooling Drag

- ❖ Treat cooling system as a generic propulsion device (i.e., a ramjet)
- ❖ Determine the “thrust” of the device:

$$T = \dot{m}_{cool}(V_{exit} \cos(\Lambda) - V_{\infty}) + (p_{exit} - p_{\infty})A_{exit} \cos(\Lambda)$$



# Cooling Drag

---

## ❖ Assume a drop in total pressure

- Can look at drop over each component, but we will assume isentropic everywhere except over heat exchanger

$$\frac{p_{t_3}}{p_{t_2}} = \pi$$

## ❖ The waste heat ( $\dot{Q}_{waste}$ ):

- Total energy in fuel less the shaft power and energy in engine exhaust gases

$$\dot{Q}_{waste} = \dot{Q}_{fuel} - \dot{W}_{shaft} - \dot{m}_{exhaust} c_p (T_{t_{EGT}} - T_{\infty})$$



# Cooling Drag

---

❖ Total temperature of flow after heat exchanger ( $T_{t,3}$ )

- Assumed to be either cylinder head temperature or material temperature limit

❖ Required mass flow to maintain  $T_{t,3}$  determined:

$$\dot{m}_{cool} = \frac{\dot{Q}_{waste}}{c_p \left[ T_{t3} - \left( T_{\infty} + \frac{V_{\infty}^2}{2c_p} \right) \right]}$$

❖ Actual mass flow rate may be greater than required due to leakage:

$$\dot{m}_{actual} = \kappa \dot{m}_{cool}$$



# Cooling Drag

---

## ❖ Static pressure at exit

- Assume an exit pressure coefficient to determine

$$p_4 = p_\infty + C_{p,exit} q_\infty$$

## ❖ Static temperature at exit

$$\frac{T_{t_4}}{T_4} = \left( \frac{p_{t_4}}{p_4} \right)^{(\gamma-1)/\gamma}$$

## ❖ Exit density

$$\rho_4 = \frac{p_4}{RT_4}$$

## ❖ Exit velocity

$$V_{exit} = \sqrt{(T_{t_4} - T_4) 2c_p}$$



# Cooling Drag

---

## ❖ Exit area

$$A_{exit} = \frac{\dot{m}_{actual}}{\rho_4 V_{exit}}$$

## ❖ Total cooling drag

- Angularity of exhaust flow,  $\Lambda$ , considered

$$D_{cool} = (p_{\infty} - p_{exit}) A_{exit} \cos(\Lambda) + \dot{m}_{actual} (V_{\infty} - V_{exit} \cos(\Lambda))$$

## ❖ For a given exit pressure coefficient, nature will enforce that only a certain loss can physically occur

- Loss cannot make  $p_4 > p_{t,4}$



# Cooling Drag

---

## ❖ The above procedure

- Sizes the cooling system exit area
- Should be performed on a critical cooling design point

## ❖ If analyzing condition other than critical cooling point

- The mass flow rate through the system will be set by the exit area
- Must change calculation methods
- Iterative procedure necessary to determine new heat exchanger exit temperature ( $T_t$ , 3)



# Cooling Drag

---

## ❖ Off-design analysis

- Initial guesses (from previously described procedure) required for
  - $T_{t,3}$
  - $T_4$
  - $\rho_4$
  - $V_{\text{exit}}$
  - $\dot{m}_{\text{cool}}$

- Find new  $T_{t3}$ :

$$T_{t3} = \frac{\dot{Q}_{\text{waste}}}{\dot{m}_{\text{cool}} c_p} + \left( T_{\infty} + \frac{V_{\infty}^2}{2c_p} \right)$$

- Iterate





# Cooling Drag

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- ❖ The cooling drag is dependent on the flight condition (particularly for aircraft without variable exit areas)
- ❖ Sensitivity study performed to study dependence of 4 parameters on various cruise speeds, altitudes, and pressure losses for the SR-22:
  - Cooling drag (lb)
  - Cooling drag coefficient
  - Cooling drag area ( $D/q$ ,  $\text{ft}^2$ )
  - Corrected cooling drag (lb)



# Cooling Drag

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## ❖ Corrected Drag

- Analogous to “corrected thrust” in propulsion system design
- “Dimensional analysis identifies correlating parameters that allow data taken under one set of conditions to be extended to other conditions.” (Mattingly)
- Corrected thrust has “become a standard in the gas turbine industry” (Mattingly)
- Collapses variation of drag (thrust) with flight condition

- Formula: 
$$D_c = \frac{D}{\delta_0}$$

- where, 
$$\delta_0 = \frac{p_t}{p_{ref}}$$



# Cooling Drag Sensitivity Study

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## ❖ Varied altitude, velocity, pressure loss

- 8 altitudes (2,000 to 16,000 ft)
- 3 velocities used for given altitude (based on values in POH cruise tables)
- 6 pressure losses studied (0.97 to 0.995) for each of the 18 altitude/velocity pairs

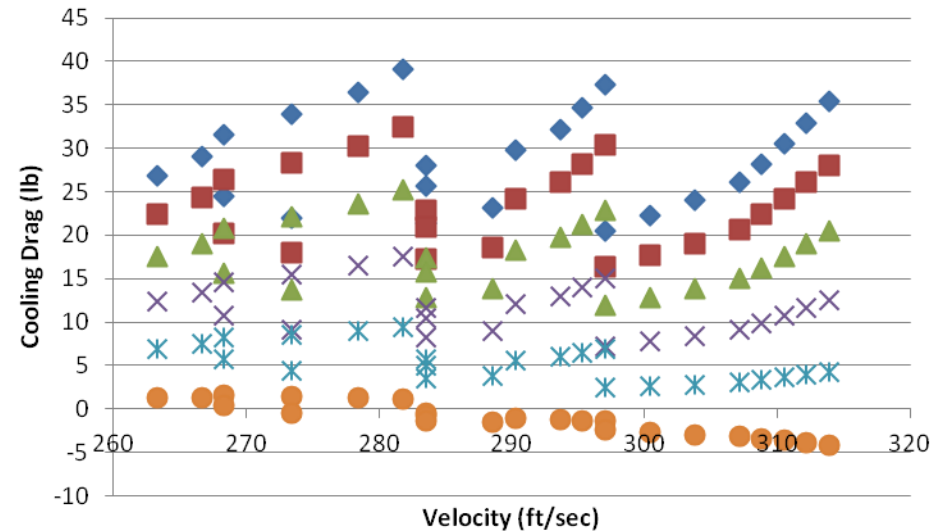
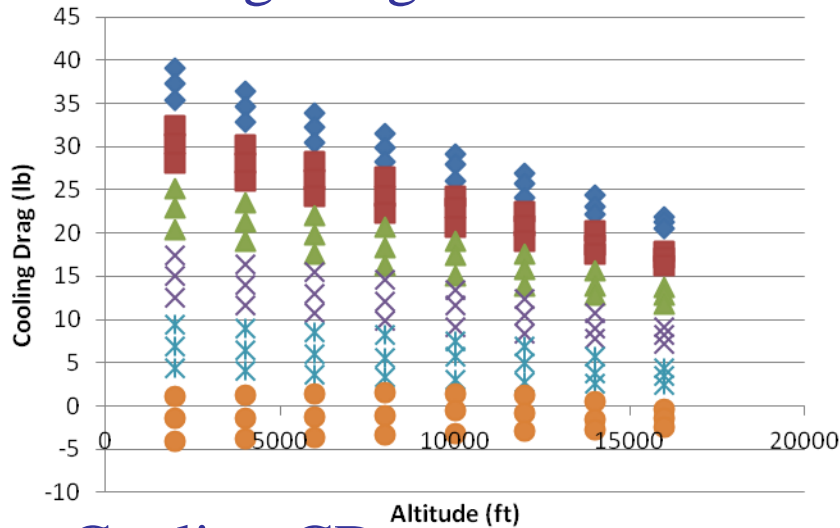
## ❖ Important notes:

- For a specified, single on-design cooling condition:
  - Altitude = 0 ft, Velocity = 170.47 ft/sec,  $\Delta$  Temperature = +41°F
  - RPM = 2700, Shaft Power = 310 hp, Fuel Flow Rate = 25.7 GPH, EGT = 1600°F, CHT = 400°F
  - Mass Flow Rate multiplier = 1, Exit  $C_p$  = -0.4,  $\pi_{2-3}$  = 0.985
- Studying cruise off-design cooling conditions
- Assumed linear increase in EGT with % power, but constant CHT

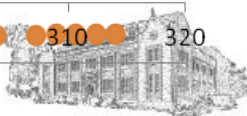
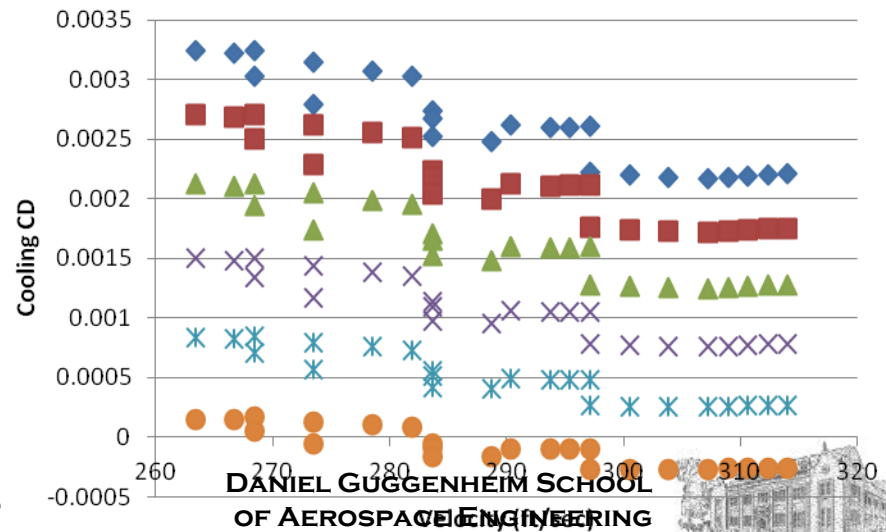
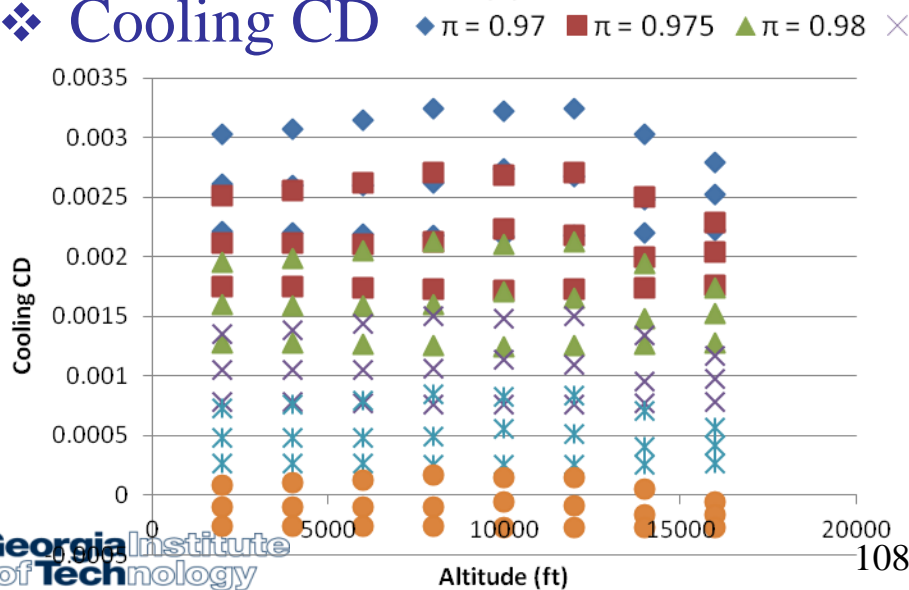


# Cooling Drag Sensitivity Study

## ❖ Cooling Drag

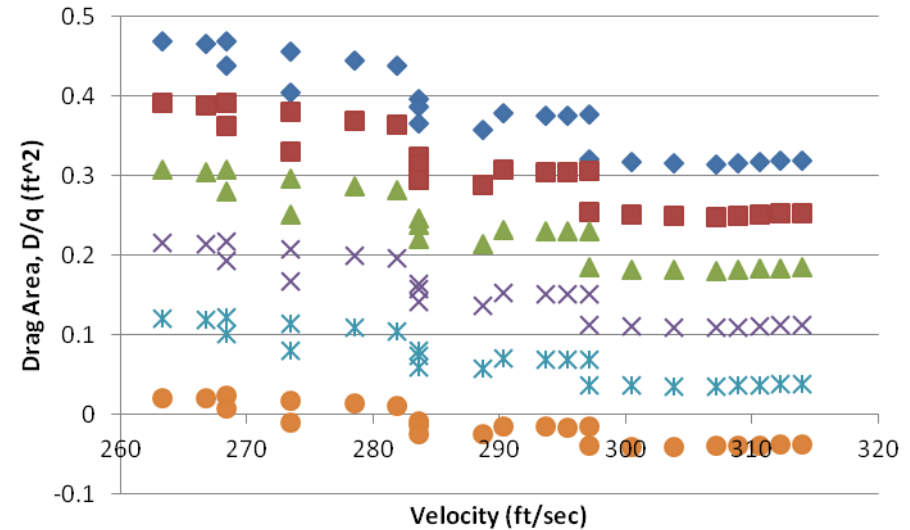
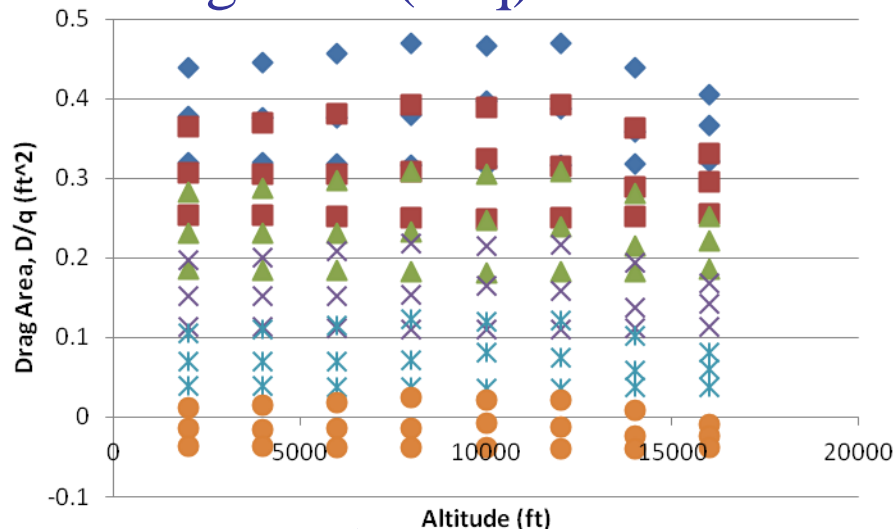


## ❖ Cooling CD

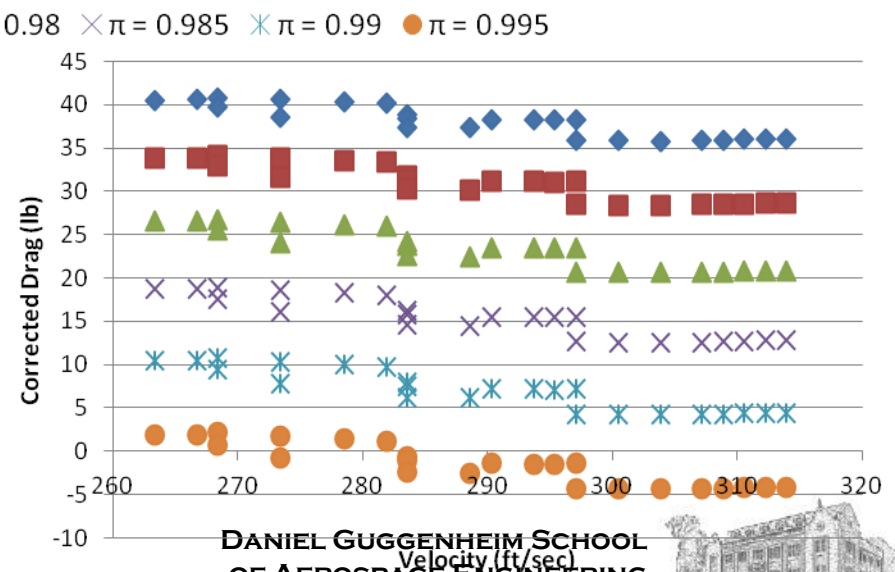
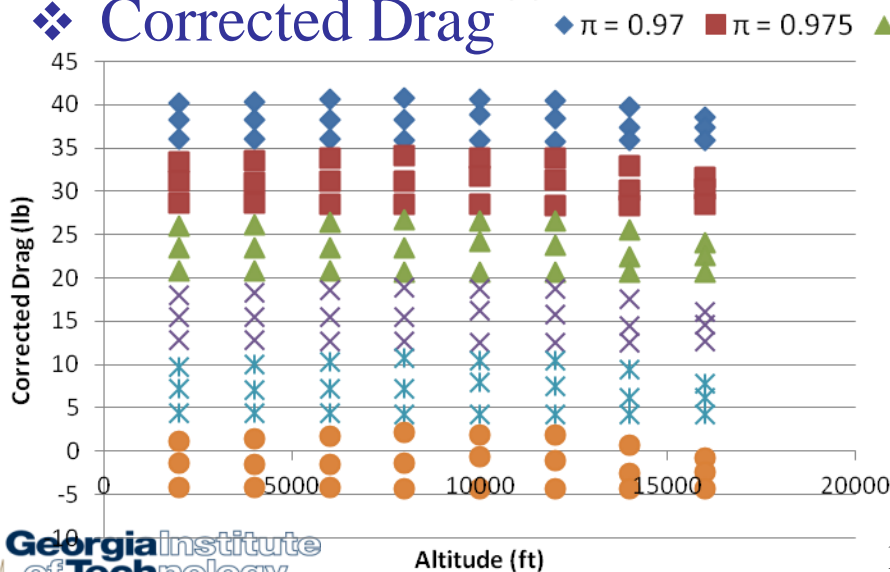


# Cooling Drag Sensitivity Study

## ❖ Drag Area ( $D/q$ )



## ❖ Corrected Drag



# Cooling Drag Sensitivity Study

## ❖ Variation of parameters for different pressure losses

$\pi = 0.97$

	Drag (lb)	CD	D/q (ft <sup>2</sup> )	Corrected Drag (lb)
Minimum	20.542	0.00217	0.3148	35.776
Maximum	39.096	0.00324	0.4701	40.743
Average	28.972	0.00263	0.3814	38.040
Standard Deviation	5.485	0.00039	0.0562	1.842
Normalized Std Dev	0.1893	0.1475	0.1475	0.0484
Range	18.554	0.00107	0.1553	4.966
Max Percent Difference	47.46	33.03	33.03	<b>12.19</b>

$\pi = 0.98$

	Drag (lb)	CD	D/q (ft <sup>2</sup> )	Corrected Drag (lb)
Minimum	11.854	0.00125	0.1810	20.574
Maximum	25.171	0.00213	0.3080	26.737
Average	17.764	0.00162	0.2349	23.314
Standard Deviation	3.720	0.00032	0.0462	2.273
Normalized Std Dev	0.2094	0.1966	0.1966	0.0975
Range	13.317	0.00088	0.1270	6.164
Max Percent Difference	52.91	41.23	41.23	<b>23.05</b>

$\pi = 0.99$

	Drag (lb)	CD	D/q (ft <sup>2</sup> )	Corrected Drag (lb)
Minimum	2.450	0.00025	0.0367	4.154
Maximum	9.447	0.00085	0.1229	10.667
Average	5.371	0.00050	0.0724	7.028
Standard Deviation	2.136	0.00022	0.0314	2.398
Normalized Std Dev	0.3978	0.4333	0.4333	0.3412
Range	6.997	0.00059	0.0862	6.513
Max Percent Difference	74.07	70.12	70.12	<b>61.06</b>

$\pi = 0.975$

	Drag (lb)	CD	D/q (ft <sup>2</sup> )	Corrected Drag (lb)
Minimum	16.303	0.00172	0.2494	28.352
Maximum	32.408	0.00271	0.3928	34.060
Average	23.546	0.00214	0.3105	30.911
Standard Deviation	4.616	0.00036	0.0520	2.108
Normalized Std Dev	0.1960	0.1674	0.1674	0.0682
Range	16.105	0.00099	0.1434	5.708
Max Percent Difference	49.70	36.50	36.50	<b>16.76</b>

$\pi = 0.985$

	Drag (lb)	CD	D/q (ft <sup>2</sup> )	Corrected Drag (lb)
Minimum	7.228	0.00076	0.1100	12.493
Maximum	17.491	0.00150	0.2178	18.908
Average	11.690	0.00107	0.1553	15.331
Standard Deviation	2.852	0.00027	0.0392	2.363
Normalized Std Dev	0.2440	0.2526	0.2526	0.1541
Range	10.263	0.00074	0.1078	6.415
Max Percent Difference	58.68	49.51	49.51	<b>33.93</b>

$\pi = 0.995$

	Drag (lb)	CD	D/q (ft <sup>2</sup> )	Corrected Drag (lb)
Minimum	-4.112	-0.00027	-0.0390	-4.410
Maximum	1.616	0.00017	0.0240	2.084
Average	-1.156	-0.00009	-0.0132	-1.549
Standard Deviation	1.856	0.00016	0.0228	2.390
Normalized Std Dev	-1.6057	-1.7296	-1.7296	-1.5430
Range	5.728	0.00044	0.0630	6.494
Max Percent Difference	354.48	262.54	262.54	311.55



# Cooling Drag Sensitivity Study

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## ❖ Conclusions

- No well-defined trend for all pressure losses with any of the four parameters
- Least amount of variation in corrected drag
  - Can consider this to be approximately constant across a wide range of operating conditions

## ❖ Find corrected cooling drag at design cruise point and input this as a constant into FLOPS

- Keeps with analogy of cooling system as a ramjet
- Relatively little variability in total drag:
  - ~15% variability in ~10% of drag ( $\pi=0.975$ ) is only ~1.5% variability in overall drag
  - ~25% variability in ~7% of drag ( $\pi=0.98$ ) is only ~1.75% variability in overall drag
  - ~35% variability in ~5% of drag ( $\pi=0.985$ ) is only ~1.75% variability in overall drag



# Constant Corrected Drag

- ❖ The variation of the drag coefficient due to a constant corrected drag:

$$\begin{aligned}
 C_{D_{cool}} &= \frac{D_{cool}}{qS} \\
 &= \left( \frac{D_{corrected} (p_{t,0}/p_{ref})}{qS} \right) \\
 &= \left( \frac{D_{corrected}}{qS} \right) \frac{p \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{\gamma/(\gamma-1)}}{p_{ref}} \\
 &= \left( \frac{D_{corrected}}{0.5 \rho V^2 S} \right) \delta \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{\gamma/(\gamma-1)} \\
 &= \left( \frac{D_{corrected}}{0.5 \left( \frac{\delta}{\theta} \rho_{ref} \right) (Ma)^2 S} \right) \delta \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{\gamma/(\gamma-1)} \\
 &= \left( \frac{D_{corrected} \theta}{0.5 \rho_{ref} M^2 a^2 S} \right) \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{\gamma/(\gamma-1)} \\
 &= \left( \frac{D_{corrected} (T/T_{ref})}{0.5 \rho_{ref} M^2 (\sqrt{\gamma RT})^2 S} \right) \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{\gamma/(\gamma-1)} \\
 &= \left( \frac{D_{corrected}}{0.5 \rho_{ref} M^2 \gamma RT_{ref} S} \right) \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{\gamma/(\gamma-1)}
 \end{aligned}$$

- Only a function of
  - Mach number
  - Atmospheric properties
  - Wing area

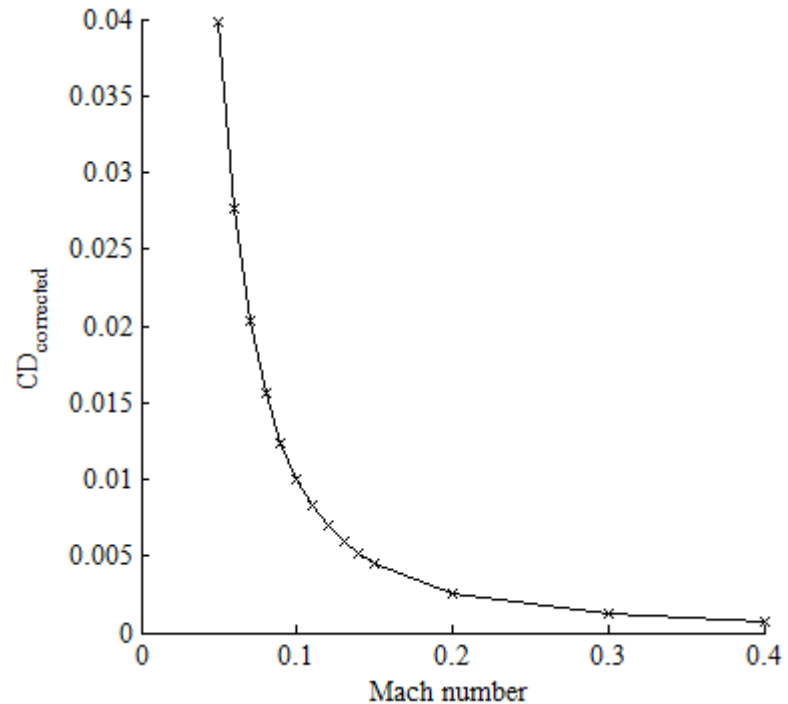
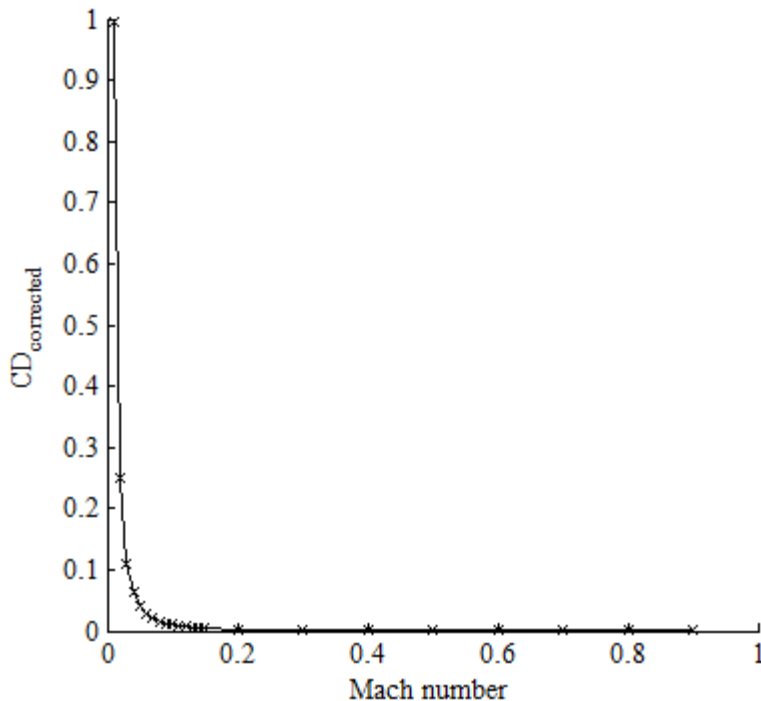




# Constant Corrected Drag

❖ The variation of the drag coefficient due to a constant corrected drag:

- As Mach number  $\rightarrow 0$ ,  $CD \rightarrow \text{infinity}$



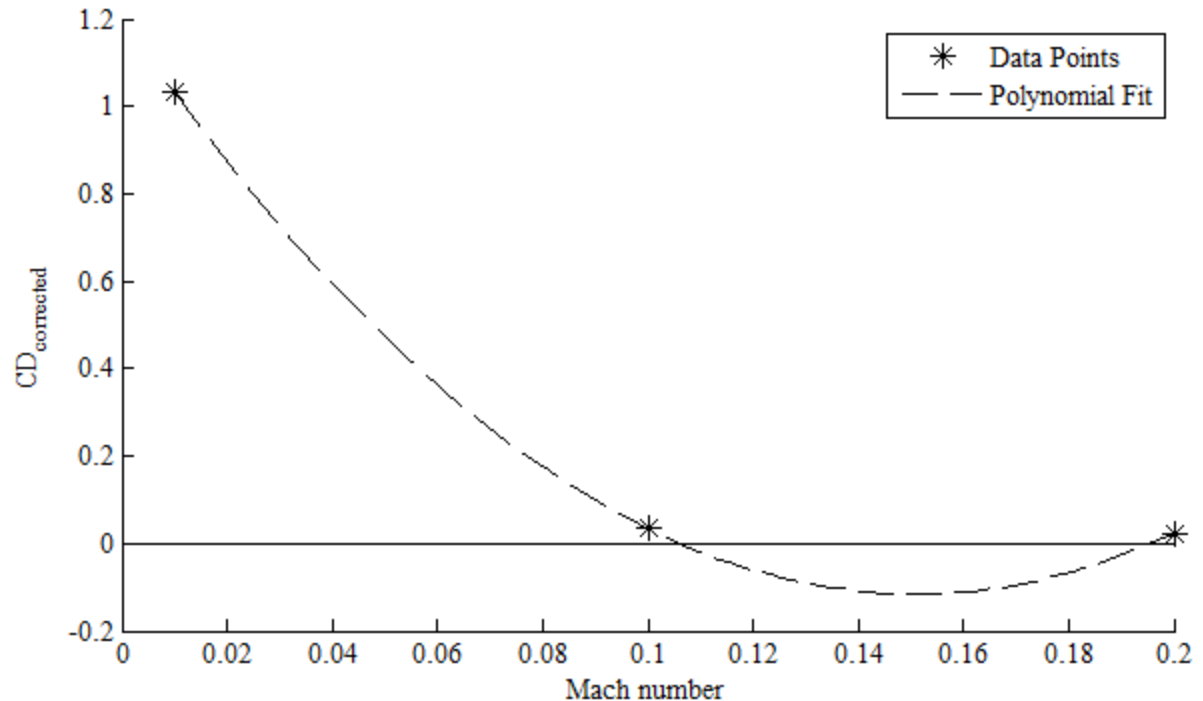
# Constant Corrected Drag

## ❖ FLOPS Implementation:

- FLOPS does performance calculations by interpolating drag tables to determine CD
- If there is too great a rise between values, FLOPS can interpolate a negative drag (thrust)!

Table 1: FLOPS Drag Table with Constant Corrected Drag of 21.36 lb

MACH	CDF
0.010	1.03198
0.100	0.03406
0.200	0.02403
0.300	0.02129
0.400	0.01988
0.500	0.01893
0.600	0.01820
0.700	0.01759
0.725	0.01745
0.750	0.01732
0.775	0.01719
0.800	0.01706
0.825	0.01693



# Constant Corrected Drag

## ❖ FLOPS Implementation:

- Change minimum Mach number in table to  $M=0.05$
- Use variation of  $CD$  as determined by functional relationship down to  $M=0.15$
- At Mach numbers below  $M=0.15$ , hold  $CD$  constant at the value for  $M=0.15$

