## OPTIMIZATION OF BRAYTON-RANKINE BINARY CYCLES

## A THESIS

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## Studies and Research

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Robert Lyutak Kim

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## OPTIMIZATION OF BRAYTON-RANKINE BINARY CYCLES

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Ropald L. KardYn; Co~Chalrman

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Rogef W. Carlson

Ratib A. Karam
Date approved by Chairman Jan. 20, 1978

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## SUMMARY

A binary cycle is a Brayton cycle which rejects waste heat to a Rankine cycle. Binary cycles can offer high overall plant efficiencies for reasonable gas turbine inlet temperatures which results in fuel conservation and reduced waste heat rejection requirements.

The objective of this research is to develop techniques for determining optimum plant configurations and operating conditions for complex binary cycles. The system employs feedwater heaters, moisture separators, multi-stage compressors, and a regenerator and includes the effects of superheating. A model which describes the efficiency of the binary cycle is established in a form suitable for optimization. The efficiency is then optimized with respect to operating conditions and parameters. As part of the optimization procedure, the system is partitioned into Brayton and Rankine portions by fixing coupling variables, and then for fixed values of coupling variables the two cycles are optimized separately. In the Rankine portion of the binary cycle, dynamic programming is used; in the Brayton cycle, a modified search technique is used.

Several computer subroutines describing thermodynamic properties of steam are constructed to evaluate the Rankine cycle portion of the binary cycle. For the Brayton cycle, ideal gas law is employed to describe thermodynamic properties of helium.

For each assigned reactor exit temperature, optimization runs
are made with boiler saturation tenperatures ranging frou $400{ }^{\circ} \mathrm{F}$ to $700^{\circ} \mathrm{F}$ in $100{ }^{\circ} \mathrm{F}$ increments. Different amounts of superheat are employed to raise the boiler exit temperature up to $1000^{\circ} \mathrm{F}$. Compressor stages are varied from 1 to 4 and feedwater heater stages from 1 to 9 . For given reactor exit temperatures, boiler saturation temperatures, and amount of superheat, binary cycle efficiencies are optimized with respect to the following variables: number of feedwater heaters, number of compressor stages, compression ratios, feedwater heater inlet saturation temperatures, and regenerator effectiveness. Optimum values for these variables are tabulated for reactor exit temperatures of $1900{ }^{\circ} \mathrm{R}$, $2100{ }^{\circ} \mathrm{R}$, and $2300{ }^{\circ} \mathrm{R}$, respectively. For given reactor exit temperatures, binary cycle efficiencies are optimized with respect to all variables including boiler saturation temperatures and amount of superheat. For the range of reactor outlet and boiler saturation temperature examined, and for boiler exit temperature up to $1000^{\circ} \mathrm{F}$, binary cycle efficiencies become optimum at $700^{\circ} \mathrm{F}$ boller saturation temperature, $200{ }^{\circ} \mathrm{F}$ to $300{ }^{\circ} \mathrm{F}$ superheat, no regeneration for reactor exft temperatures below $2100{ }^{\circ}{ }_{R}$ and appreciable amount of regeneration above $2100{ }^{\circ} \mathrm{R}$. For these conditions the optimum number of feedwater heaters is 9 and the optimum number of compressor stages is 1 . The optimum binary cycle efficiencies are 0.506 at $1900{ }^{\circ} \mathrm{R}, 0.533$ at $2100^{\circ} \mathrm{R}$, and 0.560 at $2300^{\circ} \mathrm{R}$, respectively.

CHAPTER I.

## INTRODUCTION

## Purpose and Objective of This Work

With emphasis in the United States on fuel conservation and minimum impact on the enviroment by future power generating systems, there is considerable interest in using binary cycles that can raise overall efficiencies into the range of $50 \%$ or more. A binary cycle is a Brayton cycle which rejects its waste heat to a Rankine cycle. Binary cycles can offer high overall plant efficiencies for reasonable gas turbine inlet temperatures which results in fuel conservation and reduced waste heat rejection requirement.

Several parametric studies have been performed on binary cycles using high temperature gas-cooled reactors as the heat source. 1,2,3,4 These studies have shown that high efficiencies can be achieved with binary cycles; however, optimum plant configurations and operating conditions for maximizing overall efficiencies have not been found. One parametric study indicated that the optimum plant configuration may be fairly simple; ${ }^{4}$ however, this conclusion remains to be proven.

The objective of this research is to develop techniques for detemining optimum plant configurations and operating conditions for complex binary cycles using optimization techniques from the field of operations research. Because models of binary cycles include highly nonlinear, nonconvex equations and integer variables, it is necessary to develop specialized algortthms for the optimization procedure.

## Literature Survey of Binary Cycles

In an early analysis of binary cycles, Bammert compared efficlencies of helium-steam binary cycles with high performance gas turbine cycles and concluded that no advantage in efficiency could be obtained with the binary cycle. This was an erroneous conclusion because his system configuration for the binary cycle was far from being optimum.

Later Kilaparti and Nagib ${ }^{2}$ performed thermodynamic analyses on closed-cycle, helium-stean binary systems in which heat rejected from the gas turbine cycle was used for feedwater heating. They concluded that the efficiency of such a combined cycle was higher than the efficiency of the gas turbine or steam cycle alone. McCracken and Rust ${ }^{3}$ performed parametric studies on helium-steam binary cycles which showed that the system analyzed by Kilaparti and Nagib was not arranged in optimum manner. By employing two-stage compression on the gas cycle and using its reject heat to produce saturated steam, higher efficiencies were obtained. Additional studies by McCracken, Rust, and Miller ${ }^{4}$ showed that even higher cycle efficienctes could be obtained by simplifying the system through eliminating multi-stage compression on the gas turbine cycle and feedwater heating on the steam cycle.

Morgan and David ${ }^{5}$ analyzed binary cycles using organic fluids in the Rankine bottoming cycle and concluded that the combined system with the organic fluid provided a significant improvement in efficiency over the gas turbine cycle. For example, a regenerative gas turbine
cycle with an efficiency of $36.6 \%$ can be improved to an efficiency of 47.1\% when an organic Rankine cycle is added to the gas turbine cycle. For most of the selected organic fluids, the combined cycle showed better efficiencies than systems using steam in the Rankine cycle. The efficiencies of gas turbines are one of the major factors which determine the efficiencies of combined cycles. Thus improvements in gas turbines over the past few years have made binary cycles more attractive. ${ }^{6}$

A recent study of HTGR binary cycles is found in the paper presented by Sager, Robertson, and Schoene. ${ }^{7}$; According to this paper, the HTGR binary cycle power plant provided a potential of substantial improvement in overall plant efficiency and reduced unit capital cost compared with other available systems. By replacing the precooler of the gas turbine HTGR with a vaporizer and installing a secondary cycle using a suitable low temperature supercritical fluid, it is possible to extract approximately $33 \%$ more work from the system than is obtainable from the gas turbine cycle alone.

The Gas Turbine High Temperature Gas Cooled Reactor (GT-HTGR) with a supercritical Rankine cycle employing amonia as the working fluid was under development by General Atomic Company. Schoene, et.al. ${ }^{8}$ presented information on a binary plant that used ammonia as a secondary cycle fluid. More recently, Vrable and Schuster ${ }^{9}$ described the systems aspects of a conceptual design for an ammia binary GT-HTGR plant. The designs of ammonia turbine and pump are presented by McDonald and Vepa. 10

In all of the studies of binary cycles, cotaputer codes were developed which contained mathematical models of the system under consideration. Parametric studies were thus performed to investigate changes in helium compressor inlet temperature, compressor pressure ratio, regenerator efficiency, etc. None of these codes were designed to optimize plant configurations and operating characteristics in order to maximize plant overall efficiencies. Consequently, for a specified gas turbine inlet temperature (reactor exit temperature), the optimum plant for maximizing the overall efficiency has yet to be found.

## Mathematical Programming and Related Applications

Since World War II the fiteld of mathematical programing or operations research has developed rapidly. Optimal operating conditions and design configurations for numerous systems have been derived from appropriate application of optimization techniques.

Mathematical programing approaches are categorized in terms of the nature of the objective function to be maximized (or minimized), the nature of the decision variables over which the optimization is to be accomplished, and the constraints which restrict the decision. If both the objective function and the constraints are linear, the problem is referred to as a linear program. If either the objective functity or the constraints are not linear, the problem is a nonlinear program. If some decision variables are restricted to take on integer values, the problems become integer Inear programs or integer nonlinear programs, respectively. General purpose optimization procedures exist for linear programs and limited classes of nonlinear programs. However, many
nonlinear programs and nearly all integer programs must be approached by specialized algorithms designed for particular problems. Preliminary models of the binary cycle reactor system include both highly nonlinear equations and some integer variables. Thüs the problem falls into the class requiring specialized algorithms.

The general solution strategy employed in nearly all the successful algorithms for complex nonlinear and integer programs is one of enumeration. Examples of such approaches include Reeves, 11 Falk and Soland, ${ }^{12}$ Soland, ${ }^{13}$ Ritter, ${ }^{14}$ and Rosen and Ornea, 15 In each case the authors essentially proceed by dividing the solution space into a number of regions, each associated with a relatively more tractable nonlinear or linear program. Standard nonlinear programming techniques are then employed to obtain optimal solutions for each of the regions. The overail optimum solution is selected from the optimal regional solutions.

In many cases this approach is augmented by a branch-and-bound scheme which uses a bound on the best solution which could be produced in a region. The bounds are chosen so that they are relatively efficient to calculate. Thus, for many of the solution regions it is not necessary to undertake the relatively more complex process of determining an exact solution to the region problem. In such cases, the bound demonstrates that the region problem cannot possibly produce an overall optimuan.

Optimization techniques have been fairly widely applied to engineering design problems. Application has been particularly common in design of chemical plants, electric power distribution systems, and
several other fields. However, practically no literature has been found which deals with optimization of thermodynamic cycles like the Rankine or the Rankine-Brayton binary cycle. The three most related studies are discussed below.

Stone, Sudde, and Freedman ${ }^{16}$ devised computer codes "CROCK" and "SHOCK," which minimize the weight of a heat rejection system (condenser) for space power systems. In the first code, the waste heat is radiated directily to space from the condenser; in the second, the sensible heat is radiated from a single phase fluid. In both codes, the weights of the condensers are described by a series of equations. The weights are then minimized with respect to a few optimization variables by either total enumeration or a cyclic coordinate search.

A chemical process called the William-Otto process is optimized by Sriram and Stevens. ${ }^{17}$ In this process, the plant consists of a chemical reactor and the product-separation system which involves a cooler, a decanter, and a distillation column. The objective of the problem is to adjust flow rates and system operating conditions under existing physical limitations such that the net profit obtained from the product is maximum.

Dynamic programming has been successfully employed in a number of engineering problems: optimization of chemical reactors, ${ }^{18}$ crosscurrent extractors, ${ }^{19}$ mass transfer and separation processes, ${ }^{20}$ and evaporators. 21 The 1ast application appears to be similar to the evaporator and multi-feedwater heater aspects of the Rankine cycle. For this reason, this application is discussed in greater detail.

Itahara and Stiel 21 applied dynanic programing to a saline water conversion system to minimize total area of multiple-effect evaporators. In this analysis an evaporator was partitioned into many stages. Then the mass flow rate and temperature of the $n+1$ th stage outlet vapor stream were assigned as input variables; heat and mass balance equations over the $n$th stage were assigned as transformation equations. The minimum evaporator surface area for the $n$th stage was determined by the optimum decision on the outlet vapor temperature of the n th stage. This process is repeated for the preceding stage ( $\mathrm{n}-1$ th stage) and the optimum decision is made up to date. The minimum total area for stages 1 through $n$ is obtained when the optimum decision for the first stage is reached. Thus for multiple-effect evaporators, the minimum total area is obtained by employing a backward recurrsive solution strategy of dynamic programming.

## A Parametric Study of Helium-Steam Combined Cycles

Figure 1-1 illustrates a schematic diagram for a combined cycte on which parametric studies were made by McCracken, Rust, and Miller, 4 In the gas cycle, the compressor and turbine efficiencies were $90 \%$. An upper limit of $90 \%$ was taken for the regenerator efficiency. Reactor exit temperatures were examined from the present operating temperature of the Fort St. Vrain HTGR of $1444{ }^{\circ} \mathrm{F}$ to a maximum of $2400^{\circ} \mathrm{F}$, which was the operating temperature of the Ultra High Temperature Reactor Experiment (UHTREX). The reactor exit pressure was the same as at Fort St. Vrain, 700 psi. Pinch-point temperature differences in helium-steam heat exchangers were assigned a minimum value of $50^{\circ} \mathrm{F}$.

Component pressure losses in the gas cycle were assigned the values of the fort St. Vrain reactor, except for the regenerator and intercoolers, which were assigned pressure drops of 4 psi each. On the steam side of the binary plant, efficiencles of $85 \%$ were assigned to the steam turbine and boiler feed pump. A pinch-point temperature difference of $10{ }^{\circ} \mathrm{F}$ was assigned to the feedwater heater and, when feedwater heating was employed, the efficiency for the pre-extraction stages of the steam turbine was assigned a value of $90 \%$. The efficiency of the remaining stages was then normalized so that the total turbine efficiency would still be 85\% . The condenser pressure was assigned a value of 1 psi and the condensate subcooled $10^{\circ} \mathrm{F}$.


Figure 1-1. Combined cycle plant schematic diagram ${ }^{4}$

Figure 1-2 illustrates combined cycle efficiency for a plant with two compressors and a feedwater heater. In this diagram combined cycle efficiency increaseswith reactor exit temperature and boiler exit pressure.


Figure 1-2. Combined cycle efficiency as a function of reactor ${ }^{3}$ exit temperature (feedwater heating optimized)

Mathematical formulation and derivation of the basic equations are presented in Chapter II and optimization methods used to obtain their solution are detailed in Chapter III. Chapter IV presents results obtained from the binary cycle optimization and Chapter $V$ concludes this work and gives suggestions and recommendations.

## CHAPTER II

PROBLEM FORMUEATION

Figure 2-1 illustrates a schematic diagram for a generalized combined cycle in which optimum operating conditions and plant configurations will be determined in this stady. The system employs $M$ feedwater heaters, L moisture separators, N intercoolers, a regenerator, and includes the effect of superheating. Thus the number of feedwater heaters, molsture separators, and intercoolers employed in the system is an unknown. These numbers will be determined in the process of optimization and model formulation.

In modeling of binary cycles, certain assumptions are necessary. For the Brayton cycle, compressor and turbine efficiencies must be assumed. For the Rankine cycle, boiler feed pump and turbine efficiencies must be given. In feedwater heaters and steam-helium heat exchangers, minimum pinch point temperature differences must be specified. These assumptions will be discussed in the following sections.

Once these assumptions are made, a model which describes the binary cycle efficiency is established in terms of the following set of undetermined variables: (a) gas cycle compressor ratio; (b) number of stages of intercooling; (c) regenerator efficiency; (d) boiler exit pressure; (e) superheated steam exit temperature; (f) number of feedwater heaters; and (g) feedwater heater inlet saturation temperatures. The binary cycle efficiency is then optimized with respect to the


Figure 2-1. Combined Cycle Plant Schematic Diagram
optimization variables in the presence of several constraints.
Substantial improvements in model formulation are achieved in this work relative to previously reported models. ${ }^{3,4}$ The improvements are due to (a) assignment of steam-turbine efficiency as function of steam quality, (b) enployment of moisture separators to improve steam quality, and (c) development of analytical expressions to account for pressure drop in each heliua cycle component.

## Rankine Cycle

A multi-stage Rankine cycle employing many moisture separators and feedwater heaters is shown in Figure 2-2. The corresponding temperature-entropy diagram is shown in Figure 2-3.

Assumptions in the steam side of the binary cycle are as follows:
(1) In feedwater heaters, pinch point temperature differences of $10^{\circ} \mathrm{F}$ are assumed.
(2) In steam turbines, moisture separators are introduced whenever steam qualities become lower than $95 \%$. When this happens the steam qualities are raised to $99 \%$. The saturated water collected from the separators is sent back to the zero stage feedwater heater to heat up the feed from the pump. It should be noted that the zero stage feedwater heater is a unit entirely separate from the rest of the feedwater heater system. In addition, selection of moisture separators and their arrangements in the system precedes the optinization process of the feedwater heaters. Thus the choices of optimum number of feedwater heaters and their respective temperatures in the later


Feedwater Heaters

Hifre 2-2. Rankine Cycle with Multi-Feedwater Heaters


Figure 2-3. T-S Diagram of a Rankine Cycle with MultiFeedwater Heaters
optimization stages do not alter the basic arrangenents of moisture separator units.
(3) The efficiency of a steam turbine is a function of the steam quality. For superheated steam, turbine efficiencies are assumed 92\%. For unsaturated steam, turbine efficiencies gradually decrease with increasing moisture content. An equation describing turbine efficiencies for unsaturated steam is assumed as follows:

Turbine Efficiency $=0.92-(1-$ Quality $)(0.17 / 0.12)$

Equation (2-1) is based upon examining actual Mollier diagrams for several large commercial steam turbines.
(4) The upper limit for the temperature of saturated steam exiting from the boiler is assumed 700 of. The temperature of the condenser is $100^{\circ} \mathrm{F}$. Consequently, the steameycle is operable between $700^{\circ} \mathrm{F}$ and $100^{\circ} \mathrm{F}$. The operating temperature range of the steam cycle is divided into 30 discrete $20^{\circ} \mathrm{F}$ temperature intervals. Then ft is assumed that the steam turbine efficiency is a constant within the interval, but varied as a function of steam quality in each mesh point.
(5) The maximum allowable steam temperature was set at $1000{ }^{\circ} \mathrm{F}$ because of the metallurgical problems with higher temperatures.

Computation of the Rankine cycle efficiency begins with the construction of the TABLE subroutine which generates and stores fluid
enthalpies and entropies of all the stages of turbine expansion and moisture separation, and stores the frequency and location of the moisture separation. The subroutine also computes flow rates of saturated liquid exiting from the separators. The rest of the calculations in the Rankine cycle are carried out based on information and properties provided from TABLE.

Figure $2-4$ illustrates a T-S diagran for a Rankine cycle with an index of LS th moisture separator. Let $\chi$ (LSS) and $\pi$ (LS) be defined as follows:

$$
\begin{aligned}
\chi(L S)= & \text { Amount of saturated water collected from the LS th } \\
& \text { moisture separator and sent back to the zero stage } \\
& \text { feedwater heater. } \\
\pi(L S)= & \text { Amount of the steam passing through the turbine } \\
& \text { located between }(L S-1) \text { th moisture separator and LS } \\
& t h \text { separator. }
\end{aligned}
$$

The relations between $\chi(L S)$ and $\pi(L S)$ are given by the following equations:

$$
\begin{aligned}
& x(L S)=\pi(L S)\left(H_{\text {out }}(L S)-H_{\text {in }}(L S)\right) /\left(H_{\text {out }}(L S)-H_{\text {sat }}(L S)\right) \\
& \pi(1)=1
\end{aligned}
$$

$$
\pi(L S+1)=\pi(L S) \div \chi(L S)
$$

where
$H_{i n}(L S)=$ enthalphy per unit mass of steam entering the, LS ${ }^{4}{ }^{\text {Wh }}$


Entropy
Figure 2-4. T-S Diagram of a Rankine Cycle with Several Moisture Separators and no Feedwater Heater.

$$
\begin{aligned}
& \text { moisture separator. } \\
H_{o u t}(L S)= & \text { enthalphy per unit mass of steam exiting the LS th } \\
& \text { moisture separator. } \\
H_{s a t}(L S)= & \text { enthalphy per unit mass of saturated water corres- } \\
& \text { ponding to the temperature of } L S \text { th moisture sep- } \\
& \text { arator. }
\end{aligned}
$$

When the system employs many feedwater heaters, the amount of steam fed into each feedwater heater has to be known before the computation of the turbine work starts. Let $\omega_{i}$ be the amount of steam fed into the $i$ th feedwater heater. To describe $\omega_{i}$ in terms of $\omega_{1-1}$ and other known properties in the previous stages, a heat balance around the $i$ th stage feedwater heater is required.

Figure 2-5 shows the inlet and outlet conditions of the $i$ th stage feedwater heater.


Figure 2-5. Energy Balance on th Feedwater Heater

An energy balance on the 1 th feedwater heater gives

$$
h_{1} \omega_{i}+h_{200+1}\left(\sum_{j=1}^{i} \omega_{j-1}\right)+h_{100+1}=h_{200}+(1+1)\left(\sum_{j=1}^{i} \omega_{j}\right)+h_{100+(i-1)}
$$

This leads to

where

$$
\begin{gathered}
\omega_{0}=0 \\
h_{100+M}=\left\{\begin{array}{l}
h_{c}+\text { pumping work or } \\
\text { enthalpy of the heated stream } \\
\text { leaving the zero stage feedwater } \\
\text { heater. }
\end{array}\right.
\end{gathered}
$$

where $h_{c}$ is the enthalpy of the condenser exit stream.
For $i=1$, Eq. (2-6) reduces to

$$
\begin{equation*}
\omega_{1}=\frac{h_{100}-h_{101}}{h_{1}-h_{202}} \tag{2-7}
\end{equation*}
$$

For $1=2$, Eq. (2-6) becomes

$$
\begin{equation*}
\omega_{2}=\frac{\left(h_{101}-\mathrm{h}_{102}\right)+\left(w_{1}\right)\left(h_{203}-\mathrm{h}_{202}\right)}{h_{2}-h_{203}} \tag{2-8}
\end{equation*}
$$

For systemis with many moisture separators and fedotrer

Furthermore，a general expression which gives the overall turbine work for all possible combinations of moisture separators can not be found． For a given region of the turbine expansion，several moisture separations can take place．Since addition of moisture separators reduces the flow rate in the turbine；flow rates at different points of the expansion may not be the same．These differences in flow rates have to be carefully considered in the computation of turbine works．

There are many different ways of arranging the moisture separ－ ators and feedwater heaters．Figure 2－6 1llustrates three examples of such arrangements．In these examples，the $i$ th feedwater heater begins at the state corresponding to $T_{s(i)}$ and the region for computing the turbine work ends at $\mathrm{T}_{\mathrm{s}(\mathrm{i}+1)}$ ．Figure 2－7 illustrates how a region of the steam turbine is partitioned into the following three subregions：top，center；and bottom subregions．In case $I$ ，the bottom subregion does not exist；in case III，center subregion is missing．


Case I


Case II

Case III

Figure 2－6．Examples of Different Moisture Separator Arrangements in a Turbine Expansion from $\mathrm{T}_{\mathrm{s}(\mathrm{i})}$ to $\mathrm{T}_{\mathrm{s}}(\mathrm{i}+1)$

存名

Bottom $=0$
Case I

$$
\text { Cop } \begin{aligned}
& \text { Topt } \\
& \text { Bottom } \\
& \text { Center }=0
\end{aligned}
$$

Case II
Case III

Figure 2-7. Partitioning of a Steam Turbine Region into Three Subregions

To illustrate how the turbine work is computed, the author has chosen case II of Figure 2-6 as an example, and presented it again in Figure 2-8.

Let $W_{\text {top }}(1), W_{\text {center }}$ (i), and $W_{\text {bot }}$ (i) be the turbine works corresponding to the top, center, and bottom subregions of the 1 th region in the turbine, respectively. Let LSIN and LSOUT be the indexes of the moisture separation stages, where 1 SIN corresponds to the inlet of turbine expansion and LSOUT cerresponds to the outlet. The amount of work generated in the turbine expansion from $T_{s(i)}$ to $T_{s(i+1)}$ is computed from the following equations:

$$
\begin{align*}
W_{\text {top }}(i) & =(\pi(\text { LSIN })-\text { OSUM })\left(H_{H}(i)-H_{\text {in }}(\text { LSIN })\right)  \tag{2-9}\\
W_{\text {center }}(i) & =\left[(\pi(k)-\text { OSUM })\left(H_{\text {out }}(k-1)-H_{\text {in }}(k)\right)\right]_{k=L S I N+1}  \tag{2-10}\\
& +\left[(\pi(k)-\text { oSUM })\left(H_{\text {out }}(k-1)-H_{\text {in }}(k)\right)\right]_{k=L S O U T-1}
\end{align*}
$$



Entropy
Figure 2-8. T-S Diagram of a Rậnkine Cycle with Two Feedwater Heaters and Three Molsture Separators. (This diagram corresponds Case II of F19:2-6)

$$
\begin{align*}
& W_{\text {bot }}(i)=(\pi(L S O U T)-O S U M)\left(H_{\text {out }}(\text { LSOUT }-1)-H H(i+1)\right)  \tag{2-11}\\
& \\
& \text { Turbine } \operatorname{Work}(i)=W_{t o p}(i)+W(i)_{\text {center }}+W(i)_{\text {bot tom }}
\end{align*}
$$

$\mathrm{HH}(\mathrm{i})=$ enthalpy of steam sent to the 1 th feedwater heater. OSUM $=$ total amount of steam extracted from the turbines and sent to the feedwater heaters before it reaches ith stage, $\omega_{1}$.

## Brayton Cycle

Assumptions in the modeling of the Brayton cycle are as follows:
(1) Ideal gas laws can be usedsto model the turbine and compressor works. This assumption is quite good because helium behaves as an ideal gas for the temperatures and pressures found in this system.
(2) Gas turbine and compressoriefficiencies were assumed $90 \%$; although projections indicate that efficiencies as high as 92 and $93 \%$ might be obtained in the near future. 22
(3) In the regenerator the temperature differences between the inlet and outlet hot (or cold) streams are specified. Since the specifled regenerator temperature difference is proportional to the amount of heat exchanged, the specification eventually leads to information on the regenerator effectiveness.
(4) The economizer is an intercooler. Each intercooler transfers the same amount of heat.
(5) The reactor exit pressure is the same as the Fort St. Vrain HTGR, 700 psia. The pressure loss across the reactor is 10 psi , which
is the same as the pressure loss across the Fort St. Vrain reactor.
(6) Instead of using a fixed pressure loss for each of the heat exchangers, the pressure losses were determined separately for each component and varied as functions of either the gas compression ratio or the amount of heat exchanged.

The frictional pressure loss in a plpe is described as follows:

$$
\begin{equation*}
\Delta p=f_{w}\left(\rho v^{2} / 2 g_{c}\right)(L / D e)=f_{w}\left(G^{2} / \rho\right)(L / D e)\left(1 / 2 g_{c}\right) \tag{2-12}
\end{equation*}
$$

where

$$
\begin{aligned}
& f_{w}=\text { Darcy-Weisbach friction factor } \\
& G=\rho v=\text { mass velocity } \\
& \rho=\text { fluid density } \\
& \text { De }=\text { hydraulic diameter } \\
& L=\text { pipe length } \\
& V=\text { fluid velocity }
\end{aligned}
$$

Raising the pressure on a gas in a pipe will cauce the density to increase. If the mass flow rate or $G$ is kept constant in the system, according to Eq. (2-12) the pressure loss in the pipe will decrease. In addition, the pipe pressure loss is linearly proportional to the pipe length.

Pressure losses for heat exchangers in the gas cycle are assumed to behave according to the discussion in the preceding paragraph. In addition, the total pressure loss across all intercoolers increases with increasing number of intercoolers or compression stages.

However, for a fixed number of compression stages, the pressure drop across each intercooler becomes smaller as the corresponding stage pressure increases.

The assumed pressure losses in psi are as follows:

$$
\begin{align*}
& \Delta P_{r}=10  \tag{2-13}\\
& \Delta P_{b}=3 r_{p}^{N} \quad \text { (shell side) }  \tag{2-14}\\
& \Delta P_{e}=1.0 r_{p}^{N} \quad \text { (sheil side) }  \tag{2-15}\\
& \Delta P_{i}=\left(1.0 r_{p}^{N}\right) /\left(r_{p}^{n-1}\right)  \tag{2-16}\\
& \Delta\left(P_{r e g}\right)_{c}=(1 / 100) X \quad \text { (tube side) }  \tag{2-17}\\
& \Delta\left(P_{r e g}\right)_{h}=(1 / 100) X \quad \text { (shell side) } \tag{2-18}
\end{align*}
$$

where
$X=$ temperature differences between the inlet and outlet hot (or cold) stream of regenerator $\Delta P_{r}=$ pressure loss across the reactor $\Delta \mathrm{P}_{\mathrm{b}}=$ pressure loss across the boiler
$\Delta P_{e}=$ pressure loss across the economizer $\Delta P_{1}=$ pressure loss across the intercooler $\Delta\left(p_{r e g}\right)_{c}=$ pressure loss across the regenerator in the cold stream $\Delta\left(P_{r e g}\right)_{h}=$ pressure loss across the regenerator in the hot stream $\mathrm{N}=$ total number of compression stages employed $\mathfrak{n}=$ index of $n$th intercooler (where the 1 st intercooler
is the economizer )
A schematic diagram for a Brayton cycle with three stage compression is shown in Figure 2-9. The corresponding T-S diagram is in Figure 2-10.


Figure 2-9. Brayton Cycle with Three Compression Stages.


Figure 2-10. T-S Diagram for The Brayton Cycle with Three Compression Stages.

Using the standard formulas for an ideal gas, the turbine work for the three compression stage gas cycle shown in Figure 2-9 is

$$
\begin{equation*}
\text { Turbine Work }=c_{p} n_{t} T 1\left[1-\beta\left(\frac{1}{r_{p}}\right)^{3\left(\frac{\gamma-1}{\gamma}\right)}\right] \tag{2-19}
\end{equation*}
$$

where,
$\mathrm{Tl}=$ turbine inlet temperature
$\mathrm{n}_{\mathrm{t}}=$ turbine efficiency
$c_{\mathrm{p}}=$ specific heat
$\mathrm{r}_{\mathrm{p}}=\frac{P_{5}}{P_{4}}=\frac{P_{6}}{P_{M_{2}}}=\frac{P_{7}}{P_{M_{3}}}$
$\gamma=$ specific heat ratio, $c_{p} / c_{v}$
It was noted that $\left(\frac{\mathrm{P}_{9}}{\mathrm{P}_{4}}\right),\left(\frac{\mathrm{P}_{5}}{\mathrm{P}_{\mathrm{M}_{2}}}\right),\left({ }_{\mathrm{P}_{6}}\right)$ are pressure ratios across the first, second, and third intercoolers, respectively. The turbine work for $N$ stages of compression is

$$
\begin{aligned}
& \text { Turbine Work }(\mathrm{N} \text { stage })=c_{p} \eta_{t} T 1 \quad\left(1-\beta\left(\frac{1}{r_{p}}\right)^{N\left(\frac{\gamma-1}{\gamma}\right)}\right) \\
& \left.B=\left(\frac{P_{8}}{P_{1}}\right)\left(\frac{P_{7}}{P_{8}}\right)\left(R_{N}\right) \ldots\left(R_{2}\right)\left(R_{1}\right)\left(\frac{P_{3}}{P_{9}}\right)\left(\frac{P_{2}}{P_{3}}\right)\right)^{\left.\frac{(\gamma-1}{\gamma}\right)}
\end{aligned}
$$

$$
\mathrm{R}_{\mathrm{N}}=\text { pressure ratio across } \mathrm{N} \text { th intercooler }
$$

When the assumed pressure losses of all the heat exchangers are introduced into Eq. (2-21), $\beta$ and the turbine work can be expressed as functions of $r_{p}, X$, and $N$. The derivation of $B$ as a function of $r_{p}, X$, and N is given in Appendix A.

## Construction of the Binary Cycle

For a given reactor outlet and condenser exit condition, the gas turbine portion of the binary cycle shown in Figure 2-1 can be described by the following equations:

$$
\begin{align*}
& T T 2=T T 1-W T\left(r_{p}\right) / C_{p} \\
& X=T T 2-T T 3 \\
& T T 2-T T 3=T T 8-T T 7 \\
& T T 4=T T 5-W C\left(r_{p}\right) / C_{p} \\
& m\left(H_{o}-H_{b}\right)=C_{p}(T T 3-T T 9)+N C_{p}(T T 9-T T p) \\
& T T_{p}=T_{b}+\Delta T P_{p i n}+460 \\
& m\left(H_{b}-H_{6}\right) / N=C_{p}\left(T T T_{p}-T T 4\right) \\
& T T 6=T T 4 \tag{2-29}
\end{align*}
$$

$T T 7=T T 5$
$\mathrm{TT} 5=\mathrm{TT} 9$

$$
\begin{align*}
& \text { KT }\left(r_{p}\right)=\eta_{t} C_{P} T T 1\left[1-B\left(\frac{1}{r_{p}}\right)^{\left.N\left(\frac{\gamma-1}{\gamma}\right)\right]}\right.  \tag{2-31}\\
& B=B^{\frac{\gamma-1}{\gamma}} \\
& B=\text { Product of Pressure Ratios }
\end{align*}
$$

$$
\begin{equation*}
W C\left(r_{p}\right)=\frac{c_{p}}{\pi_{c}} \operatorname{TT}_{4}\left[r_{p}^{\frac{\gamma-1}{\gamma}}-1\right] \tag{2-34}
\end{equation*}
$$

where

$$
\begin{aligned}
T T & =\text { symbol for temperatures in gas turbine cycle } \\
T & =\text { symbol for temperatures in Rankine cycle } \\
\eta_{t} & =\text { turbine efficiency of Brayton cycle } \\
\eta_{c} & =\text { compressor efficiency of Brayton cycle } \\
r_{p} & =\text { compression ratio across a compressor stage } \\
Y & =\text { ratio of specific heats } \\
C_{p} & =\text { specific heat at constant pressure } \\
N & =\text { number of compression stages } \\
H_{0} & =\text { enthalpy of steam exiting from boiler } \\
H_{6} & =\text { enthalpy of water exiting the last feedwater heater } \\
H_{b} & =\text { enthalpy of saturated liquid corresponding to boiler satura- } \\
& \text { tion temperature } \\
T_{s i} & =1 \text { th feedwater heater inlet saturation temperature } \\
T_{b} & =\text { boiler saturation temperature } \\
T T 1 & =\text { reactor exit temperature } \\
\Delta T_{p i n} & =\text { pinch point temperature differences at economizers } \\
m & =\text { ratio of steam to helium mass flow rates } \\
W C & =\text { compressor work } \\
W T & =\text { turbine work }
\end{aligned}
$$

Equations (2-22) through (2-31) can be recombined into the following seven equations:

$$
\begin{equation*}
\operatorname{TT} 2=\operatorname{TT} 1-W T\left(r_{p}\right) / C_{p} \tag{2-35}
\end{equation*}
$$

$$
\begin{equation*}
X=T T 2-T T 3 \tag{2-36}
\end{equation*}
$$

$$
\begin{align*}
& T T 2-T T 3=T T 8-T T 5  \tag{2-37}\\
& T T 4=T T 5-W C\left(r_{p}\right) / C_{p}  \tag{2-38}\\
& m\left(H_{0}-H_{b}\right)=C_{p}(T T 3-T T 5)+N C_{p}\left(T T 5-T T T_{p}\right)  \tag{2-39}\\
& T T_{p}=T_{b}+\Delta T T_{P I n}+460  \tag{2-40}\\
& m\left(H_{b}-H_{6}\right) / N=C_{p}(T T P-T T 4) \tag{2-41}
\end{align*}
$$

For given reactor outlet and condenser exit conditions, if one assigns a value for each of $N, m, r_{p}, T_{b}, S H$ (the amount of steam superheat in $\left.{ }^{\circ} \mathrm{F}\right), \mathrm{X}, \mathrm{T}_{\mathrm{S} 1}, \mathrm{~T}_{\mathrm{S} 2}, \ldots, \mathrm{~T}_{\mathrm{SM}}$, the combined cycle is entirely specified. The parameters for every stream in the cycle are thas determinable through the seven equations just presented and the additional information on the Rankine cycle.

## Modeling of the Optimization Problem

The system is to be modeled in form sultable for optimization; that is, the objective function and constrafints will be formulated in such a way that the mathematical model becomes a true representation of the system and at the same time a representation which is suitable for mathematical programming aptimization.

In the present model, the combined cycle efficiency, which is the objective functionfor the system, has been implicitly expressed as a function of optimization varifables through many coupled equations. These equations are presented in the previous section.

The system includes some integer variables, as well as continuous variables, and is restricted by both linear and nonlinear constraints.of Physical bounds on each variable appear as simple linear constraints.

[^0]The more complex, nonlinear constraints arise in the boiler and the regenerator. The temperature of the superheated steam exiting from the boiler is restricted to be lower than the inlet helium stream by at least some specified amount. In addition, there exists minimum temperature restrictions between the hot and cold streams across the regenerator.

In brief, symbolic form, the mathematical optimization problem resulting from these constraints is as follows:

$$
\begin{align*}
& \text { Maximize } \eta=\frac{\left(W_{N E T}\right)_{B}+\left(W_{N E T}\right)_{R}}{Q}  \tag{2-42}\\
&\left(W_{N E T}\right)_{B}=f_{1}\left(N, r_{p}, X, T_{b}, S H, T_{s l}\right)  \tag{2-43}\\
&\left(W_{N E T}\right)_{R}=f_{2}\left(T_{b}, S H, M, T_{s l}, T_{s 2}, \ldots, T_{s M}\right)  \tag{2-44}\\
& Q=f_{3}\left(N, r_{p}, X, T_{b}, T_{s l}\right) \tag{2-45}
\end{align*}
$$

These equations are subject to $N$ and $M$ being integer values where $N \geqslant 1$ and $M \geqslant 0$ and $r_{p}, X, T_{b}, S H, T_{s 1}, T_{s 2}, \ldots, T_{s M}$ satisfying linear bounds and nonlinear constraints from the boiler and regenerator. The meaning of the terms in Equations $(2-42),(2-43),(2-44)$ and $(2-45)$ are as follows:

$$
\begin{aligned}
\eta & =\text { efficiency of combrned system } \\
\left(W_{N E T}\right)_{B} & =\text { net work produced by the Brayton cycle } \\
\left(W_{N E T}\right)_{R} & =\text { net work produced by the Rankine cycle } \\
Q & =\text { heat input from the reactor } \\
N & =\text { number of compressor stages } \\
M & =\text { number of feedwater heaters } \\
\mathbf{r}_{\mathbf{P}} & =\text { compression ratio across a compressor stage }
\end{aligned}
$$

```
T
    heater
    X = the temperature difference between the inlet and outlet
    hot (or cold) stream of the regenerator
Tb}=\mathrm{ temperature of saturated steam corresponding to the
    boiler exit pressure
SH = the amount of superheat in OF
    The nonlinear constraints are
\[
\begin{aligned}
& \mathrm{TT} 3-(\mathrm{TO}+460) \geqslant 50^{\circ} \mathrm{F} \\
& \mathrm{TT} 3-\mathrm{TT} 7 \geqslant 50^{\mathrm{o}_{\mathrm{F}}} \\
& \mathrm{TT} 4-(\mathrm{T} 6+460) \geqslant 50^{\circ} \mathrm{F}
\end{aligned}
\]
```

(Constraint 1)
(Constraint 2)
(Constraint 3)

Constraint 1 means that the boiler inlet temperature of the gas should be greater than the steam exit temperature by at least $50{ }^{\circ} \mathrm{F}$. Constraint 2 implies that there should be at least a $50{ }^{\circ} \mathrm{F}$ temperature difference between the hot and cold streams in the regenerator. Constraint $\mathbf{j}$ imposes restrictions on the compressor inlet temperature, such that it is always at least $50{ }^{\circ} \mathrm{F}$ above the temperature of the water exiting the last feedwater heater.

The nonlinearity of Constraint 1 can be shown in the following derivations:

$$
\begin{align*}
& T T 3-(T 0+460) \geqslant 50  \tag{2-46}\\
& X=T T 2-\operatorname{TT} 3 \tag{2-47}
\end{align*}
$$

Substitution of Eq. (2-47) into Eq. (2-46) yields

$$
\begin{equation*}
\mathrm{TT} 2-\mathrm{X}-\mathrm{T} 0 \geqslant 50+460 \tag{2-48}
\end{equation*}
$$

Since

$$
\begin{equation*}
T T 2=T T 1-\left(T u r b i n e \text { Work } / C_{p}\right) \tag{2-49}
\end{equation*}
$$

and

Turbine Work $=C_{P} \operatorname{TTl} \eta_{t}\left(1-\beta\left(\frac{1}{r_{P}}\right)^{\frac{1}{} \frac{\gamma-1}{\gamma}}\right)$

Substitution of Eqs. (2-49) and (2-50) into Eq. (2-48) results in
$\mathrm{TT1}-\operatorname{TTln}_{t}\left[1-8\left(\frac{1}{\mathrm{r}_{p}}\right)^{\frac{1}{2}}\right)-\mathrm{X}-\mathrm{T} 0 \geqslant \mathrm{~S} 10$

The nonlinear relations between $X, r_{p}$, and $N$ are shown in Eq. (2-51).

## Derivation of Binary Cycle Efficiency

Binary cycle efficiencies can be expressed in terms of Rankine cycle and Brayton cycle efficiencies. Figure 2-11 illustrates a schematic diagram for the simplest binary system. This diagram will be used to identify some of the components in the following analyses:

Let

$$
\begin{aligned}
Q & =\text { reactor heat input to Brayton cycle } \\
Q_{R} & =\text { boller heat input to the Rankine cycle } \\
\eta & =\text { binary cycle efficiency } \\
n_{B} & =\text { Brayton cycle efficiency }
\end{aligned}
$$



Figure 2－I1．Simple Binary Cycle

$\eta_{R}=$ Rankine cycle efficiency
$\left(\mathrm{W}_{\mathrm{NET}}\right)=$ net work generated in the binary cycle $\left(W_{N E T}\right)_{B}=$ net work generated in the Drayton cycle $\left(\mathrm{W}_{\mathrm{NET}}\right)_{\mathrm{R}}=$ net work generated in the Rankine cycle The net work from the Brayton cycle is given by

$$
\begin{equation*}
\left(W_{\mathrm{NET}}\right) \mathrm{B}=\eta_{\mathrm{B}} Q^{2} \tag{2-52}
\end{equation*}
$$

and

$$
\begin{equation*}
Q_{R}=Q-\left(W_{N E T}\right)_{B} \tag{2-53}
\end{equation*}
$$

Substituting Eq. (2-52) into Eq. (2-53) leads to

$$
\begin{equation*}
Q_{R}=Q\left(1-r_{B}\right) \tag{2-54}
\end{equation*}
$$

Substituting of Eq. (2-54) into Eq. (2-55) results in

$$
\begin{equation*}
\left(W_{N E T}\right)_{R}=\eta_{R} Q\left(1-\eta_{B}\right) \tag{2-56}
\end{equation*}
$$

The efficiency of a binary cycle is defined by

$$
\begin{equation*}
n=\left[\left(W_{\mathrm{NET}}\right)_{\mathrm{B}}+\left(\mathrm{W}_{\mathrm{NET}}\right)_{\mathrm{R}}\right] / \mathrm{Q} \tag{2-57}
\end{equation*}
$$

Substitution of Eq. (2-52) and Eq. (2-56) into Eq. (2-57) yields

$$
\begin{align*}
n & =\left[\eta_{B} Q+Q\left(1-n_{B} n_{R}\right] / Q\right.  \tag{2-58}\\
& =n_{B}+\left(1-n_{B}\right) n_{R}
\end{align*}
$$

Equation (2-58) is the objective function which is to be maximized.

## CHAPTER III

OPTIMIZATION METHODS AND SOLUTION

## Partitioning of the Binary cycle

The concept of convexity has great importance in the optimization of nonlinear systems. When every line connecting two points in the set falls entirely in the set, then the set is a convex set. If any point on a line connecting two points on the surface of a function is always greater than or equal to the corresponding functional value, then the function is a convex function. When a function is convex the set of points falling above the functional surface is a convex set.

When a nonlinear system is convex, a local optimum is a global optimum, i.e., if a solution cannot be tmproved in the neighborhood of a current value, it is an overall optimum. When the system is convex it can be optimized using well-known nonlinear programming techniques.

For a nonlinear, nonconvex system, there is no unique way of optimizing the system. In most of the literature, a nonconvex system is partitioned into several convex systems. Then the global optimum is searched from the local optimum of the individual convex systems.

Careful analyses of binary cycles indicate that the system is nonlinear, nonconvex. Nonconvexity of the system is mainly due to the existence of a strong coupling between the Rankine and Brayton portions of the combined cycle, and partially due to the introduction
of $r_{p}$ and $X$ in the expressions for component pressure losses in the Brayton cycle.

Since the major source of nonconvexity is known, decoupling of the two cycles was chosen as a means of converting the nonconvex system Into two more tractable systems. Studies of the binary cycle model indicate that for fixed values of $\mathrm{SH}, \mathrm{T}_{\mathrm{B}}$ and $\mathrm{T}_{\mathrm{sl}}$, the Drayton and Rankine portions of the combined cycle can be decoupled into two independent cycles. Once separated, many of the original complexities are eliminated; remaining variables can be optimized by treating the two cycles independently.

An explanation of optimization by decoupling is given with the following example:

Let $F(x, y, z)$ be a function of arbitrary variables $x, y$ and $z$. Then maximization of $F(x, y, z)$ is equivalent to the following two procedures: First maximize $F(x, y)$ with $z$ fixed at certain values. Then select the best point among the maximums of $F(x, y)$ already established in the previous procedures. A mathematical expression of the statement is as follows:

$$
\operatorname{Max} F(x, y, z)=\operatorname{Max}_{\operatorname{all} z}\left\{\begin{array}{c}
\operatorname{Max} F(x, y)  \tag{3-1}\\
\text { fixed } z
\end{array}\right\}
$$

Equation (2-58) of Chapter II is introduced here again.

$$
\begin{equation*}
\eta=n_{B}+n_{R}\left(1-n_{B}\right) \tag{2-58}
\end{equation*}
$$

In terms of independent variables, $\eta_{,} \eta_{B}$ and $\eta_{R}$ can be expressed as follows:

$$
\begin{equation*}
n=n\left(N, r_{P}, X, T_{b}, S H, M, T_{S 1}, T_{s 1}, i=2, \ldots, M\right) \tag{3-2}
\end{equation*}
$$

$$
\begin{equation*}
\eta_{B}=\eta_{B}\left(N, r_{P}, X, T_{b} ; S H, T_{s 1}\right) \tag{3-3}
\end{equation*}
$$

$$
\begin{equation*}
\eta_{R}=\eta_{R}\left(T_{b}, S H, M, T_{s l}, T_{s i}, i=2 ; \ldots, M\right) \tag{3-4}
\end{equation*}
$$

Substituting Eqs. (3-2), (3-3), and (3-4) into Eq. (2-58) and using Eq. (3-1), one obtains

Since $0<\eta_{B}<1$, it follows that $0<\left(1-\eta_{B}\right)<1$. Also $0<\eta_{R} \leqslant 1$, consequently, Eq. (3-5) yields

$$
\begin{aligned}
& {\left[\begin{array}{lll}
1-\left(\begin{array}{cc}
\text { Max } & \left.n_{B}\right) \\
& \text { fixed } T_{B 1}: S H, T_{b}
\end{array}\right]
\end{array}\right]}
\end{aligned}
$$

$$
\begin{align*}
& \text { Max } \eta=\text { all }_{T_{s 1}}^{\text {Max }}, S H, T_{b} \quad\left\{\begin{array}{l}
\text { Max } \eta \\
\text { fixed } T_{s 1}
\end{array}, S H, T_{b}\right\}  \tag{3-5}\\
& =\operatorname{all}_{T_{s 1}}^{\text {Max }}, S H, T_{b} \quad\left\{\begin{array}{l}
\text { fixed } T_{s I} \\
\text { Max } \\
S H
\end{array} T_{b}^{n_{B}+\eta_{R}\left(1-\eta_{B}\right)}\right\}
\end{align*}
$$

It should be noted from Eq. (3-6) that optimum conditions for the binary cycle do not generally coincide with optimum conditions for the Brayton or Rankine cycles alone. But for a fixed set of coupling variables, the optimum conditions for the binary cycle and the two separated cycles coincide.

Based on the information presented, the two cycles in the combined system are separated by assuming fixed values for the coupling variables. The optimization strategies of the individual cycles are then studied in the following sections.

## Optimization of Brayton Cycle

As shown in Eq. (3-3) the Brayton portion of the binary cycle is described as a function of $N, r_{p}, X, T_{b}, S H$, and $T_{s 1}$. For fixed $T_{s 1}, T_{b}$ and $S H$, the cycle still depends upon $r_{p}, X$, and $N$.

Figures 3-1, 3-2, and 3-3 illustrate equal contour maps of Brayton cycle efficiency as functions of $r_{p}$ and $X$ for various final feedwater heater inlet saturation temperatures and number of compression stages. In these figures, the upper right corner of the $X$ and $r_{p}$ domains are restricted by Constraints 1 and 2 deseribed in Chapter II. The system analyzed in Fig. 3-1 has a reactor outlet temperature of $1900{ }^{\circ} \mathrm{R}$ ( $1440^{\circ} \mathrm{F}$ ), boiler saturation temperature of $599^{\circ} \mathrm{F}$, feedwater heater inlet saturation temperature of 120 oF , one stage of compression, and no superheating. For these conditions, the maximum Brayton cycle efficiency occurs at point ${ }^{*}$ when the cycle has no regeneration.

The operating conditions used for Fig. 3-2 are the same as in Fig. 3-1 except that the feedwater heater saturation temperature is $350^{\circ} \mathrm{F}$.


Inlet and Outlet Temp. Difference in Regenerator, ${ }^{\circ}$ F

Figure 3-1. Contour Map of Brayton Cycle Efficiency


Figure 3-2. Contour Map of Brayton Cycle efficiency

Reactor extr temp. $=1900^{\circ} \mathrm{R}$
Boiler exit saturation temp. $=599{ }^{\circ} \mathrm{F}$
Final feedwater heater sat temp, $=350{ }^{\circ}{ }^{\circ} \mathrm{F}$
Number of compression stages $=3$
No superheat employed.


Figure 3-3. Contour Map of Brayton Cycle Efficiency

It should be noted that the surface in Fig. 3-2 has two local maximums (at points $\stackrel{\star}{\mathrm{P}}_{1}$ and $\stackrel{\star}{\mathrm{P}}_{2}$ ). These two maxipums resulted from the desirability of large compression ratios associated with no regeneration, or large amount of regeneration with sinaller compression ratios. The existence of more than one local maximin for Brayton cycie efficiencies creates serious problems in optimization. If a search begins at $\stackrel{\star}{P}_{1}$, the solution must become worse before the global optimum value $\mathrm{P}_{2}$ is reached.

Figure 3-3 differs ifrom Fig. 3-4 in that the system employs three compression stages instead of one.

The contour maps of Constraints 1,2 , and 3 are $f 11$ ustrated in Figs. 3-4, 3-5, and 3-6. To construct these contour maps, nonequality expressions of Constraints 1, 2, and 3 shown in Chapter II are rearranged into equality equations by defining functions $C_{1}, C_{2}$, and $C_{3}$ as follows:

$$
\begin{equation*}
C_{1}=T T 3-(T 0+460+50) \tag{3-7}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{C}_{2}=\mathrm{TT} 3-(\mathrm{TT} 7+50) \tag{3-8}
\end{equation*}
$$

$$
\begin{equation*}
c_{3}-\operatorname{TT} 4-(T 6+460+50) \tag{3-9}
\end{equation*}
$$

The values of $C_{1}, C_{2}$, and $C_{3}$ are plotted as functions of $r_{p}$ and $X$ in Figs. 3-4, 3-5, and 3-6, respectively. Shaded areas show positive values of these functions where the constraints are not violated.

These figures show that all three constraints form reasonably


Figure 3-4. Contour Map of Constraint 1


Figure 3-5. Contour Map of Constraint 2

$$
\begin{aligned}
& \text { Reactor exit temp. }=1900^{\circ} \mathrm{F} \\
& \text { Boiler exit saturation temp. }=599^{\circ} \mathrm{F} \\
& \text { Final feedwater heater sat. temp. }=120 \text { of } \\
& \text { Number of compression stage }=1 \\
& \text { No superheat employed. }
\end{aligned}
$$



Figure 3-6. Contour Map of Constraint 3
smooth contours in the $X$ and $r_{p}$ domanin However, one serious problem in optimization arises from the fact that the contours in Figs, 3-4 and 3-5 are curved into unfavorable directions. Because of the unfavorable orientations in constraints, the feasible region defined by Constraints 1 and $2, r_{p}>1$ and $X \geqslant 0$ does not form a convex set. The dashed line in Fig. 3-4 is a hypothetical linear objective function. Even if the linear objective function is employed, as long as the curvatures are oriented in the direction shown it is possible that the system can have local optimums with different objective function values. In this figure $\stackrel{*}{P}_{1}$ and $\stackrel{*}{P}_{2}$ are suchin local optinums for the linear function defined by the dotted line.

Since our model does not fit convexity assumptions, some special search technique is required. Careful examination of Figs. 3-1, 3-2, and 3-3 indicate that when $X$ is fixed to a jeertain value and $r_{p}$ varied, there is a unique optimum for each given value of $X$. It is also noted that when $X$ is partitioned into reasonably small mesh points, the opti- . mum compression ratio corresponding to a spectific mesh point of $X$ does not differ much from the optimum compression ratio corresponding to the adjacent mesh point. Let $X_{i}$ be the $i$ th mesh point in $X$. Let $r_{p i}^{*}$ be the optimum compression ratio at $X_{i}$. The latter statement implies that once $r_{p 1}^{*}$ is established, $r_{p 2}^{*}$ will be found within one or two steps away from $\mathbf{r}_{\mathrm{p} 1}^{*}$; Consequently for $1>2$, one does not have to make optimization searches over the entire range of $\mathbf{r}_{\mathbf{p}}$. A search over a range covering one or two steps away from the optimal compression ratio corresponding to the preceding $X$ mesh point is adequate.

These ideas lead to the following Brayton cycle optimization scheme:

## Step I.

The $X$ coordinate is divided into many equal mesh points with a $25^{\circ} \mathrm{F}$ difference between any two adjacent points. Let $X_{i}$ be the value of $X$ at 1 th mesh point. Let $X_{1}=0$.

Step II.
With $X$ fixed at $X_{1}$, a simple line search is made along the increasing direction of $\mathbf{r}_{\mathbf{p}}$ to establish the optimuin Brayton efficiency * The optimum gas compression ratio is recorded as $\mathbf{r}_{\mathrm{pl}}^{*}$. It should be noted that the optimum search along $r_{p}$ is limited in the feasible region, bounded by Constraints 1 and $2, X \geqslant 0$, and $r_{p}>1$. In addition, whenever Constraint 3 is encountered, a penalty is imposed in the efficiency (objective function) such that the optimum search is 1imited to the region where Constraint 3 is not violated.

## Step III.

With X at $\mathrm{X}_{2}$, a feasible base point is established. The base point is $\left(X_{2}, r_{p l}^{*}\right)$ if that point is feasible, and otherwise $r_{p}$ is reduced until a feasible point is encountered. This becomes the base point. Starting. from the base point, the optimum search along $r_{p}$ is pursued one or two steps away from the base point. The optimum gas compression ratio is recorded as $r^{*} \mathbf{p}^{*}$.

Step IV.
With $X$ at $X_{1}$, the optimization process of Step 3 is repeated but each time starting at the new base point. The base point is ( $X_{i}$, $r_{P(i-1)}$ ) if it is feasible, otherwise $r_{p}$ is reduced until the first feasible point is encountered. The optimum gas compression ratio obtained is recorded as $\mathrm{r}_{\mathrm{pi}}^{\boldsymbol{*}}$.

## Step V.

When the entire range of $X_{i}$ is searched, the global optimum is selected from the individual optimums corresponding to $\mathbf{r}_{\mathrm{pi}}^{\mathbf{*}}$.

The optimization procedure for the Brayton cycle portion of the binary cycle is illustrated in Fig. 3-7. Black dots on the figure represent optimum Brayton cycle efficiencies for the corresponding $X_{i}$. The upper right region of the diagetmat festricted by Constraints 1 and 2 . In this diagram the distance for each step in the $r_{p}$ direction is 0.083 . The distance between two adjacent mesh points for the regenerator is $25^{\circ} \mathrm{F}$. In the actual optimization procedures, however, somewhat smailer steps in changes of $r_{p}$ were used. The global: optimum Brayton efficiency seems to be 0.117 . The optimum point occurs either at $\left(X_{6}=275\right.$, and $\left.r_{p}=1.75\right)$ or at $\left(X-300\right.$, and $\left.r_{p}=1.67\right)$.

## Optimization of Rankine Cycle

In Eq. (3-4), the Rankine cycle efficiency was expressed as

$$
\begin{equation*}
n_{R}=\eta_{R}\left(T_{b}, S H, M, T_{s 1}, T_{s i}, \quad i=2, \ldots, M_{1}\right)^{p} \tag{3-10}
\end{equation*}
$$



Figure 3-7. An Example of Optlimization Procedure for The Brayton Cycle Part of The Binary Cycle.
where all the symbols in the expression have been defined previously． The coupling variables between the Rankine and Brayton cycle were $T_{b}$ ， SH，and $\mathrm{T}_{\mathrm{sI}}$ ．For fixed values of the coupling variables；the Rankine cycle efficiency is still a function of $M$ and $T_{s i}$ ，where $i=2, \ldots, M$ ．To optimize the Rankine portion of the binary cycle as functions of $M$ and $\mathrm{T}_{\text {si }}$ ，dynamic programming has been used．

Dynamic prograuming is very useful in the optimization of Rankine cycles because it can reduce the total number of computations of turbine work necessary for optimization．This can be explained through the following examples．Suppose a Rankine cycle employs $M$ feedwater heaters． The total number of feedwater heater temperatures to be optimized is M． Assume that there are 30 temperature mesh points on which feedwater heaters can be assigned to operate，and suppose one totally enumerates the alternatives for this system．Then there will be 30 possibilities of locating first feedwater heater， ature of first feedwater heater，the inlet saturation temperatures for the second feedwater heater mist be searched．Then for each given temperature of first and second feedwater heater，the inlet saturation temperature of third feedwater heater is searched This procedure will be continued ＊ until the $M$ th feedwater heater inlet saturation temperature is searched．

When the total number of feedwater heaters enployed in the system increases from $M$ to $M+1$ the entire procedure and feedwater heater inlet saturation temperature search will be repeated．Thus as the number of feedwater heaters employed in the system varies from $M$ to $M+10$ ，there will be substantial repetition in the search of feedwater heater temperatures
and, consequently, a lot of repetitions in the computations of turbine work.

Dynamic programming is used for a sequential multi-stage decision process. In dynamic programing formulations, a multi-stage system is partitioned into several single stages. Then at each stage, decisions for every given input value (or event) are made; These decisions and the corresponding input values are stored. It should be realized that even though the decision made on this stage is based on the information provided from the preceding stage and present stage, the best decisions made at this stage represent the optimal decisions from the first stage to this stage.

Therefore, when decisions in the next stage are made, one does not have to go through the information of all the preceding stages, but based on the information at this stage and the next stage, optimal decisions for the next can be made.

The procedure continues until the final stage is reached. A best decision on the final stage determines optimal values and conditions for the problem.

The advantage of using dynamic programing in the Rankine cycle model is that through the division of the feedwater heater system into several single feedwater heater stages and through the use of stage-bystage decision "procedures, one can minimize the repetition of feedwater heater inlet saturation temperature search. Consequently the total number of turbine work calculations is reduced.

Formulation of dynamic programing for the Rankine cycle is as follows:

1) Each stage of the feedwater heater is the stage of dynamic programming.
2) Let $i$ be the index of the $i$ th stage, then input variables are $T_{s(i-1)}$ and $T_{s(i+1)}$.
3) The decision variable is $T_{s i}$.
4) The recursive equations are a set of equations describing the i th region turbine work in terms of enthalpies and flow rates corresponding to the (i-1) th stage.
5) Immediate returns are mass flow rate, $w_{i}$, and turbine work at the $i$ th stage. Total return is the sum of total turbine work generated by the optimal decision all the way through the $i$ th stage.
6) The optimal policy of the dynamic program is to generate maximum steam turbine work over the entire region of the turbine expansion.

A schematic diagram of the dynamic programing formulation is illustrated in Fig. 3-8.

Figure 3-9 illustrates a multi-feedwater heater Rankine cycle. The Rankine cycle in dynamic programming form is solved as follows:

1) For a given $T_{s(i+1)}$ and from known information of $T_{s(i-1)}$, an optimum value of $T_{s(i)}$ is searched in the temperature range from $T_{s(i+1)}$ to $T_{s(i-1)}$. Due to the formulation of the Rankine cycle in Chapter II, the feedwater heater temperatures are allowed only on temperature mesh points.
2) Procedure (1) is repeated for all possible values of $\mathrm{T}_{\mathrm{s}}(\mathrm{i}+1)$;


Figure 3-8. A Schematic Diagram of Dynamic Programming Formulation


Figure 3-9. Multi-Feedwater Heacer Rankine Cycle
and optimum $T_{S(1)}$ and 1 th region turbine work are recorded. When $T_{s}(i+1)=100{ }^{\circ} \mathrm{F}$ (or condenser temperature), the optimum $T_{s(1)}$ becomes the actual optimum $i$ th feedwater heater inlet saturation temperature of the system employing $i$ stages of feedwater heaters.
3) As the number of feedwater heaters increases from $i$ to $i+1$, the number of feedwater heater inlet saturation temperatures to be searched increases. Since information about optimum $T_{s(i)}$ is available through procedures 1 and 2 , searches on $T_{s(i)}$ are not repeated. However, to determine optimum values of $T_{s(i+1)}$ which maximize the ( $1+1$ ) th region turbine work, the following procedures are pursued: For a given $T_{s(i+2)}$ and with the information on optimum $T_{s(i)}$, a search for the $T_{s(i+1)}$ is made in the temperature range from $T_{s(i+2)}$ to $T_{S(i)}$. When $T_{s(i+2)}=100{ }^{\circ} \mathrm{F}$, the optimum $T_{S}(i+2)$ becomes the actual optimum $(i+2)$ th feedwater heater inlet saturation temperature.
4) Procedure (3) is repeated for all possible values of $\mathbf{T}_{\mathbf{s}}(i+2)$, and appropriate optimum values are recorded.

## CHAPTER IV

## RESULTS AND DISCUSSION

The mathematical model of the binary cycle described in Chapter II was analyzed with the optimization techniques described in Chapter III to determine the optimum conditions and plant configurations to maximize binary cycle efficiencies. For given reactor exit temperatures, boiler saturation temperatures, and amount of superheat, binary cycle efficiencies have been optimized with respect to the following variables: number of feedwater heaters, number of compressor stages, compression ratios, final feedwater heater inlet saturation temperature, and regenerator effectiveness (expressed by temperature differences across the regenerator). Optimum values for these variables are tabulated In Tables 4-1 to 4-3 for reactor exit temperatures of $1900{ }^{\circ} \mathrm{R}, 2100^{\circ} \mathrm{R}$, and $2300{ }^{\circ} \mathrm{R}$, respectively.

For each assigned reactor exit temperature, optimization runs were made with boiler saturation tempetatures ranging from 400 o to $700^{\circ} \mathrm{F}$ in $100{ }^{\circ} \mathrm{F}$ increments. Different amounts of superheat were enployed to raise the boiler exit temperature up to $1000{ }^{\circ} \mathrm{F}$, except the case where the boiler saturation temperature was $400^{\circ} \mathrm{F}$. For this case too much superheat was found unfavorable and, consequently, the boiler exit temperature was limited to 900 OF.

For each optimization run, optimization searches were made with compressor stages varying from 1 to 5 and feedwater heater stages

Table 4-1. Efficienciea of Binary Cycles Corresponding to Reactor Exit Temperature of $1900{ }^{\circ}{ }^{\circ}$

| Boller <br> Saturation <br> Temperature <br> in ${ }^{\circ} \mathrm{F}$ | Anount of Superheat in ${ }^{\circ} \mathrm{F}$ | Number of Feedwater Heatera | Final * <br> Feediäater <br> Heater <br> Tenp: in or | Number of Compressor Stages | Compression Ratio | Temperature Difference Across Regenerator | Efficiency of Rankine Cycle | Efficiency of Brayton Cycle | ```Efficiency of Binary. Cycle``` |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |
| 700 | 0 | 0 |  | 1 | 2.10 | 0 | . 3485 | .1603 | . 4529 |
| 700 | 100 | 5 | 340 | 1 | 1.95 | 0 | . 4332 | .1240 | . 5034 |
| 700 | 200 | $g$ | 380 | 1 | 1.80 | 0 | . 4456 | . 1034 | . 5029 |
| 700 | 300 | 9 | 380 | 1 | 1.75 | 0 | . 4549 | . 0944 | . 5063 |
| 600 | 0 | 2 | 300 | 1 | 1.70 | 300 | . 3963 | . 1252 | . 4720 |
| 600 | 100 | 7 | 340 | 1 | 1.65 .. $\because$ | 300 | . 4059 | . 1122 | . 4725 |
| 600 | 200 | 9 | 360 | 1 | 1.65 | 275 | . 4160 | . 1039 | . 4767 |
| 600 | 300 | 9 | 380 | 1 | 1.65 | 200 | . 4259 | . 0928 | . 4792 |
| 600 | 400 | 9 | 360 | 1 | 1.80 - | 50 | . 4336 | . 0895 | . 4844 |
| 500 | 0 | 6 | 320 | 3 | 1.20 . | 475 | . 3629 | . 1611 | . 4636 |
| 500 | 100 | 9 | 360 | 2 | 1.30 | 450 | . 3686 | . 1566 | . 4675 |
| 500 | 200". | 9 | 340 | 1 | 1.60 | 400 | .3753 | . 1492 | . 4685 |
| 500 | 300 | -9 | 340 | 1 | 1.90 | 225 | . 3831 . | . 1328 | . 4650 |
| 500 | 400 | 9 | 340 | 1 | 1.90 | 125 | . 3914 | .1163 | . 4622 |
| 500 | 500 | 8 | $320$ | 1 | 1.95 | 0 - | . 4001 | . 1074 | . 4645 |
| 400 | 0 | 5 | $280$ | 3 | 1.20 | 575 | . 3049 | . 2204 | . 4581 |
| 400 | 100 | 5 | $260$ | 3 | 1.20 | 575 | +3083 | . 2204 | . 4608 |
| 400 | 200 | 6 | $280$ | 1 | 1.65 | 475 | . 3156 | . 2033 | . 4547 |
| 400 | 300 | 7 | 300 | 1 | 1.90 | 325 | . 3264 | + 1855 | . 4514 |
| 400 | $400^{\circ}$ | 6 | -280 - | 1 | 2.00 | 200 | . 3344 | .1596 | . 4406 |
| 400 | 500 | 5 | 240 | 1 | 2.20 | 50 | . 3403 | .1455 | .4363 |

* This temperature is saturation temperature for steam entering the feedwater heater.


[^1]

* This temperature is saturation tenperature for steam entering the feedwater heater.
from 1 to 9. Feedwater heater temperatures (including temperature of the last feedwater heater) ranged from the lower limit restricted by the zero stage feedwater heater tenperature to an upper limit constrained by the boiler saturation temperature. However, the boundary temperatures (the boiler saturation temperature and zero stage feedwater heater temperature) were excluded from the feedwater heater tenperature search. The incremental temperature used in the search was $20^{\circ} \mathrm{F}$. Gas compression ratios were searched with steps of 0.05 and temperature differences across the regenerator were in steps of $25^{\circ} \mathrm{F}$. For different boiler exit conditions, each table lists optimum binary efficiencies, efficiencies of the Rankine and Brayton portions of the binary cycle, and optimum number of feedwater heaters and compressor stages. The tables also include optimum compression ratios and optimum temperature differences across the regenerators.

It is noted from the tables that for most boiler exit conditions; optimum binary cycles favor single stage compression and large numbers of feedwater heaters. There are a few cases in which the optimum occurs at a larger number of compression stages. Even for these cases, the optimum efficienctes do not differ much from efficiencies found with single stage compression. For example, when no superheat is employed with a $500^{\circ} \mathrm{F}$ boiler saturation temperature, the optimum number of compression stages required for a binary cycle with a $1900{ }^{\circ} \mathrm{F}$ reactor exit temperature is 3 . The optimum efficiency 1s 0.4656 . When an equivalent system employing a single compressor is optimized, the maximum efficiency obtained is 0.4632 . Thus the
two efficiencies differ by $0.68 \%$.
Employment of large numbers of feedwater heaters were generally preferred in optimum binary cycles because improvements in the Rankine cycle due to the addition of feedwater heaters raised combined cycle efficiencies in spite of reduction in gas cycle efficiencies. However, there are cases in which optimum cycles did not have large numbers of feedwater heaters. These cases are found when the temperature range between the last feedwater heater and the zero stage feedwater heater (or pump outlet in case a zero stage feedwater heater is not employed) is not wide enough to insert large numbers of feedwater heaters. It should be noted that due to the formulation of the model, adjacent feedwater heaters must be at least $20^{\circ} \mathrm{F}$ apart.

Binary cycles with boiler saturation temperatures of $700{ }^{\circ} \mathrm{F}$ and no superheating are examples in which optimum cycles are found with no feedwater heaters. When the boiler exit consists of $700{ }^{\circ} \mathrm{F}$ saturated steam, the turbine requires 16 moisture separators. Due to the employment of many moisture separators, the amount of steam flowing through the turbine is reduced and, consequently, addition of feedwater heaters which will induce further reduction of steam in the turbine is not desirable. Consequently, optimum efficiencies are found without feedwater heaters.

Optimum inlet gaturation temperatures of the last feedwater heaters varied from $220^{\circ} \mathrm{F}$ to $400{ }^{\circ} \mathrm{F}$ depending on the boiler and reactor exit conditions. It is also found that optimum exit tenperatures of
the last feedwater heaters for binary cycles are generally lower than the corresponding temperatures obtained through the optinizations of Rankine cycles alone.

Optimum temperature differences across regenerators were varied from 0 to $825{ }^{\circ} \mathrm{F}$ depending on the reactor and boiler exit conditions, gas compression ratios, and inlet saturation temperatures of last feedwater heaters.

For given reactor exit temperatures, binary cycle efficiencies have been optimized with respect to all variables including the boiler saturation temperatures and amount of superheat. Optimum values for these variables and optimum cycle efficiencies are tabulated in Table 4-4.

As is observed in Table $4-4$, foythe ratge of reactor outlet temperatures examined, binary cycle effictencies become optimum at $700{ }^{\circ} \mathrm{F}$ boiler saturation temperature and $200 \mathrm{o}^{\mathrm{F}}$ to $300{ }^{\circ} \mathrm{F}$ superheat. For these conditions the optimum number of feedwater heaters is 9 and the optimum number of compressor stages is 1 . The optimum binary cycle efficiencies are 0.5063 at $1900{ }^{\circ} \mathrm{R}, 0.533$ at $2100{ }^{\circ} \mathrm{R}$, and 0.560 at $2300{ }^{\circ} \mathrm{R}$, respectively.

Optimum remperatures for individual feedwater heaters were also investigated, Tn , this research. These results are presented in Appendix B. In most of the cases, optimum conditions are found when temperatures of individual feedwater heaters are spaced more or less uniformly In the region bounded by the inlet saturation temperature of the final feedwater heater and the zero stage feedwater heater.

Table 4-4. Optimum Efficiencies of Binary Cycles


* This cemperature $1 s$ saturation temperature for stean entering the feedwater heater.

Figure 4-1 11lustrates binary cycle efficiencies at several boiler saturation temperatures as functions of reactor exit temperatures. Each point in the line represents optimal binary cycle efficlency corresponding to a given reactor and boller saturation temperature. At each point, the amount of superheat is also optimized. Binary cycle efficiencies increase almost linearly with reactor exit temperature and increase with increasing boiler saturation temperature. It is also noted that effects of boiler saturation temperature on binary cycle efficiencies are greater when temperatures are increased from $600^{\circ} \mathrm{F}$ to $700^{\circ} \mathrm{F}$ than from $400^{\circ} \mathrm{F}$ to $500^{\circ} \mathrm{F}$.

Figures 4-2, 4-3, and 4-4 illustrate binary cycle efficiencies as functions of boller exit temperatures for various saturation temperatures. The boiler exit temperature is the sum of the boiler saturation temperature and amount of superheat. These figures illustrate the effects of both boiler saturation temperature and amount of superheat on binary cycle efficiencies.

From Fig. 4-2 it is seen that for the first $100^{\circ} \mathrm{F}$ of superheat, binary cycle efficiencies increase with boilier exit temperature. Then, depending on the boiler saturation temperature, the efficiency either continuously increases or begins to decrease with further increases in superheating.

A sma11 degree of non-unimodal behavtor in the binary cycle efficiency is observed in Fig. 4-2. When $500^{\circ}$ F boiler saturation temperature and $500^{\circ} \mathrm{F}$ of superheat is employed, the binary cyche efficiency is slightly higher than the efficiency needed to fom a




Figure 4-3. Binary Cycle Efficiency Versus
Boiler Exit Temperature.


Figure 4－4．Binary Cycle Efficiency Versus
Boiler Exit Temperature．
smooth unimodal efficiency fünction. The higher value in efficiency for these conditions may be explained by examining the behavior of Brayton, Rankine, and binary cycle efficiencies on superheat shown in Fig. 4-5. As expected, Brayton cycle efficiencies decrease and Rankine cycle efficiencies increase with increasing superheat. For superheats near $500^{\circ} \mathrm{F}$, the Brayton cycle efficiency does not decrease as rapidly as at lower superheats. This causes the combined cycle efficiency to start increasing with increasing superheat.

Binary cycle efficiencies presented in Fig. 4-3 are greater than the corresponding binary efficiencies in Fig. 4-2. Likewise efficiencies in Fig. 4-4 are greater than efficiencies in Fig. 4-3. The differences in efficiencies of these figures are due to the Brayton portion of the combined cycle efficiency increasing with increasing reactor exit temperature.

Small degrees of fluctuations in binary cycle efficiencies are observed in the original data of Figs. 4-2, 4-3, and 4-4. These fluctuations may have been caused for different reasons. But the magnitudes of some of these fluctuations are probably comparable to gross errors accumulated throughout the computations. Thus small perturbations in the original data were neglected and the best lines through the points in Figs. 4-2, 4-3, and 4-4 are presented.

In all of these figures, binary cycle efficiencies corresponding to boiler saturation temperatures of $700^{\circ} \mathrm{F}$ and no superheat are comparatively low. As mentioned previously, at these particular coiditions the amount of water extracted from steam turbines for moisture separation


Figure 4-5. Binary, Rankine and Brayton Cycle Efficiency Versus Amount of Superheat
is extremely large. Consequently, the amount of steam sent to the turbines is reduced and so is the turbine work.

Binary cycle efficiencies corresponding to boiler saturation temperatures of $400^{\circ} \mathrm{F}$ and large amounts of superheat ( $500{ }^{\circ} \mathrm{F}$ ) are also very poor. The reason can be explained as follows: Increasing superheat at low boiler saturation temperatures causes a reduction in the amount of regeneration and a lowering of the compressor inlet temperature. The lowered compressor inlet temperature reduces the compressor work and thus increases the Brayton cycle efficiency. However, due to the lowered compressor inlet temperature and reduced regenerator effectiveness, the temperature of the stream leaving the regenerator is reduced which increases the heat input. The combined effect is that the increased heat input overshadows the advantage gained through the reduction in compressor work, Consequently, Brayton cycle efficiencies decrease rapidly with high superheats at low boiler saturation temperatures.

Sensitivity Aralyses
Perturbations of variables from cofibined cycle optimum conditions may or may not produce significant differences in binary cycle efficiencies. In this portion of Chapter IV, the responses of binary cycle efficiencies with respect to changing variables are studied. The results are shown in Figs. 4-6 through 4-22.

Figures $4-6,4-7$, and 4-8 illustrate contour maps of Brayton cycle efficiencies as functions of $r_{p}$ and $X$ for $300^{\circ} \mathrm{F}$ superheat and reactor exit temperatures of $1900{ }^{\circ} \mathrm{R}, 2100^{\circ} \mathrm{R}$, and $2300{ }^{\circ} \mathrm{R}$, respectiyely.

$:$ ？
多里：


Inlet and Outhet Teinp. Difference in Regenerator, ${ }^{\circ} \mathbf{F}$ 3

Figure 4-7. Contour Map of Brayton Cycle Efficiency


Figure 4-8. Contour Map of Brayton Cycle Efficiency

The global optimum points found in these three figures are ( $r_{p}=1.75$, $X=0)$, ( $\left.r_{p}=2.15, X=0\right)$, and $\left(r_{p}=1.98, X=450\right)$ respectively. When gas compression ratios increase from 1.1 for a given value of $X$, Brayton cycle efficiencies increase until they reach a maximum and then decrease with a further increase in compression ratio. Due to the existence of constraints in the system, all values of regeneration for certain ranges of compression ratios aré not allowed. Thus for some regions of these diagrams it seems as if Brayton cycle efficiencies increase or decrease monotonically with compression ratios.

The sensitivity of Brayton cycle efficiencies with respect to perturbations of $X$ and $r_{p}$ from the optimum conditions of combined cycles are also observed from these figures. Obviously, sensitivity on $r_{p}$ is found by observing changes in Braytion cycle efficiencies along the vertical $r_{p}$ axis, while maintaining $X$ at the initial optimum values. Likewise, sensitivities on $X$ are found from information along the horizontal axis.

Brayton cycle efficiencies are reduced by $4 \sim 5 \%$ when compressor ratios are deviated from optimum values by $\pm 0.125$. The efficiencies are also reduced by 3~5\%, when temperature differences across regenerators are deviated from the optimum value by $50{ }^{\circ} \mathrm{F}$.

It is worth noting that constraints on the system impose restrictions on $X$ and $r_{p}$ such that large deviations in one variable from the optimum conditions without readjustment of another variable change the system into physically impossible situations. Thus it is necessary in the Brayton part of the binary cycle to adjust $X$ and $r_{p}$ simultaneously
so that large deviations in $X$ or $r_{p}$ can be tolerated.
Figures 4-9, 4-10, and 4-11 illustrate Brayton, Rankine, and binary cycle efficiencies as functions of final feedwater heater inlet saturation temperature. For the specified reactor temperatures, boiler saturation temperatures, and amounts of superheat; binary cycle efficiencies were optimized for each value of final feedwater heater inlet saturation temperature.

As expected, Brayton cycle efficiencies decrease with increasing final feedwater heater inlet saturation temperatures because increasing final feedwater heater inlet saturation temperatures raise Brayton cycle compressor inlet temperatures. The abrupt change in the curvature of Brayton cycle efficiencies at $600^{\circ} \mathrm{F}$ of Figs. 4-9 and 4-10 and at 660 ${ }^{\circ} \mathrm{F}$ of Fig. 4-11 are due to sudden changes in the number of compressor stages employed in the system. Rankine cycledefletepcies. increase with increasing final feedwater heater inlet saturation temperatures until they reach an optinum and then decrease with further increases in temperatures. When final feedwater heater inlet saturation temperatures are raised, both the heat input to the boiler and work output from the steam turbine are reduced. The reduction in the turbine work is caused by the extraction of steam from the turbine to be sent to the feedwater heaters. When the final feedwater heater inlet saturation temperature is low, the advantage gained from the reduction of heat input to the boiler overcomes the disadvantage caused by the reduction of turbine work. Thus the Rankine cycle efficiency is raised. At extremely high saturation temperatures, the situation is reversed and Rankine cycle efficiencies decrease with


Efficiency: of Binary, Rankine, and Brayton Cycles Versus Final Feedwater Heater Saturation Temperature.
4-9. Efficiency of Binary, Rankine, and Brayton Cycles
Versus Final Feedwater Heater, Saturation Temperature.



Figure 4-11. Efficiency of Binary, Rankine, and Brayton Cycles Versus Final Feedwater Heater Saturation Temperature.
increasing final feedwater heater inlet saturation temperatures.
The influence of final feedwater heater inlet saturation temperature on binary cycle efficiencies is that an optimum temperature is achieved at lower temperatures than would be required to maximize the efficiency of the Rankine cycle. In Figs. 4-9, 4-10, and 4-11, optinum final feedwater heater inlet saturation temperatures are $380{ }^{\circ} \mathrm{F}, 360{ }^{\circ} \mathrm{F}$, and $400^{\circ} \mathrm{F}$, respectively. When final feedwater heater inlet saturation temperatures are deviated from the optimum points by $\pm 100{ }^{\circ} \mathrm{F}$, binary cycle efficiencies are reduced by $1 \%$.

Figures 4-12 and 4-13 illustrate Rankine cycle efficiencies as a function of the number of feedwater heaters for several final feedwater heater inlet saturation temperatures. The final feedwater heater inlet saturation temperature is constant for each curve. When the final feedwater heater inlet saturation temperature is low (below $300^{\circ} \mathrm{F}$ ) only a limited number of feedwater heaters is allowed in the system. For few feedwater heaters, Rankine cycle efficiencies generally increase rapidly with increasing number of feedwater heaters. However, as the number of feedwater heaters increases, the rate of increase in efficiency decreases and eventually a point is reached where improvements are negligible with further addition of feedwater heaters.

Figures $4-12$ and $4-13$ also 111ustrate how Rankine cycle efficlencies vary with final feedwater heater inlet saturation temperatures. As seen, optimum final feedwater heater inlet saturation temperatures vary according to the number of feedwater heaters employed and the amount of superheat used in the system.


Figure 4-12. Rankine Cycle Efficiency Versus Number of Feedwater Heaters fifor Different Final Feedwater Heater Saturation Temperatures.


Figures 4-14, 4-15, and 4-16 illustrate Brayton, Rankine and binary cycle efficiencies as a function of the number of compressor stages. For the specified reactor temperatures, boiler saturation temperatures, and amount of superheat, binary cycle efficiencies were optimized with respect to all other variables except the varied number of compressor stages. Brayton cycle efficiencies decrease with increasing number of compressor stages with the rate of decrease greater when the system enploys a small number of compressor stages. The main reason for the reduction in efficiency is that compressor inlet temperatures increase with increasing number of compressor stages. Some comments on compressor inlet temperatures will be made in a later part of this chapter. Another reason for the reduction in Brayton cycle efficiencies is that the total pressure loss across intercoolers increases as the number of compressor stages increase.

Rankine cycle efficiencies are not directly affected by changes in the number of compressor stages. However, in the combined systempere is an optimal final feedwater heater inlet saturation temperature for each set of compressor stages which varies with the number of compressor stages selected. Consequently, Rankine cycle efficiencies are varied because the optimal feedwater heater inlet saturation temperature changes with the number of compressor stages. Due to the effect on the Brayton cycle efficiency, binary cycle efficiencies decrease with increasing number of compressor stages. The rate of reduction in binary cycle efficiency per addition of a compressor stage is greater when the system employs a small number of compressor stages. The reduction in binary cycle ${ }^{\text {es }}$ : efficiency is $5 \sim 6 \%$ when compressor stages increase from one to two.


Number of Compressor Stages, N

Figure 4-14. Rankine, Brayton, and Binary Cycle Efficiency Versus Number of Compressor Stages.


Figure 4-15. Rankine, Brayton, and Binary Cycle Efficiency Versus Number of Compressor Stages.


Number of Compressor Stages, $N$

Figure 4-16. Rankine, Brayton, and Binary Cycle Efficiency Versus Number of Compressor Stages.

Figures 4-17, 4-18, and 4-19 illustrate Rankine, Brayton, and binary cycle efficiencies as a function of the amount of superheat for a $700^{\circ} \mathrm{F}$ boiler saturation temperature. In each of these figures, for given reactor and boiler saturation temperatures all other variables are at their optimum values for a given amount of superheat. The addition of superheat raises Rankine cycle efficiencies and lowers Brayton cycle efficiencies. For a boiler saturation temperature of $700^{\circ} \mathrm{F}$, the initial addition of $100^{\circ} \mathrm{F}$ of superheat increases the binary cycle efficiency by 7~11\%. However, further additions of superheat beyond $100^{\circ} \mathrm{F}$ do not significantly improve binary cycle efficiencies.

The reason for the sudden rise in Rankine cycle efficiency (and binary cycle efficiency) for an initial $100^{\circ} \mathrm{F}$ of superheat for a boiler saturation temperature of $700{ }^{\circ} \mathrm{F}$ is that the severe moisture problems during the expansion in the steam turbine for the saturated steam cycle significantly reduces the turbine output work. The optimum amount of superheat in Figs. $4-17$ and $4-18$ is $300{ }^{\circ} \mathrm{F}$ and the optimum amount in Fig. 4-1-9 is $2000^{\circ}$. As long as reasonable amounts of superheat ( $100{ }^{\circ} \mathrm{F}$ or more) are employed at a $700{ }^{\circ} \mathrm{F}$ boiler saturation temperature, binary cycle efficiencies are Insensitive to the choice of superheat.

For botler saturat iont temperatures other than $7000^{\circ}$ (1.e., 600 , 500 , and $400^{\circ} \mathrm{F}$ ), the moisture problems for the saturated steam cycle are not as severe. In fact, with a $600^{\circ} \mathrm{F}$ boiler saturation temperature the addition of superheat does not change binary cycle efficiencies significantly. At $500^{\circ} \mathrm{F}$ boiler saturation temperatures and highy


Figure 4-17. Cycle Efficiency Versus Amount of Superheat


Figure 4-18. Cycle Efficiency Versus Amount of Superheat


Figure 4-19. Cycle Efficiency Versus Amount of Superheat
superheats ( $400{ }^{\circ} \mathrm{F}, 500^{\circ} \mathrm{F}$ superheat), binary cycle efficiencies begin to be lowered. For $400^{\circ} \mathrm{F}$ boiler saturation temperatures and high superheats, binary cycle efficiencies are significantly reduced. The reduction in binary cycle efficiencies at the lower boiler saturation temperatures and high superheat is due to the considerable reduction in Brayton cycle efficiencies.

Figure 4-20 illustrates binary cycle efficiencies as a function of boiler saturation temperature with all other variables optimized. As expected, binary cycle efficiencies increase with increasing boiler saturation temperature and reactor exit temperatures. Figure 4-21 shows binary cycle efficiencies as a function of boiler pressure. As is seen, binary cycle efficiencies increase with boiler pressure, but with a greater increasing rate at lower pressures. The difference in the behavior of the two figures (Figs. 4-20 and 4-21) originated from the fact that saturation pressure increases with the saturation temperature, but with greater increasing rate at higher temperatures.

Figure 4-22 illustrates compressor inlet temperatures and Brayton cycle efficiencies as a function of the number of compressor stages. Previously it was mentioned that Brayton cycle efficiencies decrease with increasing number of compression stages because compressor inlet temperatures increase. The reason for the increase in compressor inlet temperatures is as follows: When the number of compressor stages increase, the ratio of steam to helium flows increase which, in turn, causes a lower compressor inlet temperature. However, increasing the number of intercoolers causes a reduction in steam flow rate through each inter-


Figure 4-20. Binary Cycle Efficiency Versus Boiler Saturation Temperature.



Figure 4-21. Binary Cycle Efficiency Versus Boiler Pressure.

```
Reactor exit temp. \(=1900{ }^{\circ}{ }_{\mathrm{R}}\) Boiler saturation temp \(=700{ }^{\circ} \mathrm{F}\) Amount of superheat \(=300{ }^{\circ} \mathrm{F}\)
Final feedwater heater sat. temp. \(=380^{\circ} \mathrm{F}\) Amount of regeneration and compression ratio optimized
```



Figure 4-22. Compressor Inlet Temperature and Brayton Cycle Efficiency Versus Number of Compressor Stages.
cooler which in turn causes a higher compressor inlet temperature. The net effect is that the reduction in steam flow through each intercooler overshadows the increase in total steam-to-helfum flow ratio. Consequently, compressor inlet temperatures increase with increasing number of compression stages.

## Computation Time

The numerical scheme for optinizing binary cycles have been programmed for use on the CDC Cyber 70 / Model 74 at the Georgia Institute of Technology. This program, written in FORTRAN 4 and named BINARY, is designed for optimizing helium-steam binary cycles. The program listing for BINARY is given in Appendix C.

The average computing time required for optimizing a binary cycle for a given reactor outlet temperature is 353 seconds. The Rankine portion of the binary cycle takes an average of 244 seconds and the Brayton portion takes 109 seconds.

## GHAPTER V

## CONCLUSIONS AND RECOMMENDATIONS

## Conclusions

Over the range of temperatures examined, optimum binary cycle efficiencies are very sensitive to the choice of reactor exit and boiler saturation temperatures, reasonably sensitive to final feedwater heater inlet saturation temperatures and number of compression stages; and insensitive to the amount of superheat as long as a minimum of 100 of or more of superheat is employed. Optimum binary cycle efficiencies are also insensitive to the choice of individual feedwater heater inlet saturation temperatures, provided the number of feedwater heaters used are large and all feedwater heater inlet saturation temperatures are spaced reasonably apart from each other. Thus assigninent of equal temperature spacings between feedwater heater inlet saturation temperatures is a good choice for optimum design.

For the conditions studied; optimum binary cycles require a boiler saturation temperature of $700^{\circ} \mathrm{F}$, superheat of $100^{\circ} \mathrm{F}$ or more, and single stage compression. Even though nine or more stages of feedwater heaters is numerically the optimum number, employment of such a large number of feedwater heaters is not justified because a sufficiently high binary cycle efficiency can be obtained through the use of few feedwater heaters. Besides, there is no economic advantage for employing large numbers of féedwater heaters.

In general, optimum conditions for a binary cycle do not coincide with optimized conditions for Rankine or Brayton cycles alone. For example, the final feedwater heater inlet saturation temperature of an optimal binary cycle is much lower than the corresponding temperature of an optimum Rankine cycle, and the optimal gas compression ratio for the Brayton cycle portion of an optimum binary cycle is much lower than the optimum compression ratio of a Brayton cycle. High efficiency Brayton cycles require regeneration and multi-stage compression while optimal binary cycles prefer single stage compression with or without regeneration. If reactor, exit tenperatures approach $2300{ }^{\circ} \mathrm{R}$, regeneration improves binary cycle efficiencies.

Comparing previous results by McCracken, ${ }^{3}$ improvements in binary cycle efficiencies of $7 \sim 8 \%$ are made. Four to five percent of these improvenents are due to finprovements in the Rankine cycle from effective employment of many feedwater heaters (nine feedwater heaters). The rest are due to the effective use of high quality steam and assignment of turbine efficiencies as functions of steam qualities which leads to higher overall turbine efficiencies.

A final conclusion is that binary cycles optimized to maximize overall plant efficiency tend to have simple plant configurations.

## Suggested Future Work

The optimization procedure and resulting computer code have been designed so that future modsfications, enlargements, and applications to other working fluids are possible. In the process of optimization, the Rankine and Brayton cycle portions of the binary cycle are treated
separately. This allows a freedom to experiment and modify one portion of the binary cycle without too much concern about the other portion of the cycle.

One area for future work is to try fluids different from water in the Rankine cycle, i.e., ammonia or freon. The use of bottom cycle fluids other than steam is accomplished by replacing the steam tables with thermodynamic tables of the other fluids. However, substantial experimentation and refinements are necessary with the new thermodynamic tables to preserve the accuracies of interpolation as high as possible.

It should be also realized that for another fluid to be applied to this program successfully, convexity of the system should be maintained when the system is formulated into a dynamic programming form. For most fluids this requirement is likely to be met.

Helium is a perfect gas and, thus, ideal gas laws have been used in the Brayton portion of the binary cycle. To apply this program to a gas other than helium, reassessments of component pressure losses as well as modifications or replacement of ideal gas laws are necessary. In addition, an analysis similar to the one used for the Brayton portion of the binary cycle should be repeated to make sure that the optimization strategy applies to that system satisfactorily.

APPENDICES




## APPENDIX A

## EXPRESSION FOR $\beta$

## General expression for $\beta$ is

$$
\begin{equation*}
\beta=\left[\left(\frac{P_{8}}{P_{1}}\right)\left(\frac{P_{7}}{P_{8}}\right)\left(\left(R_{N}\right) \ldots\left(R_{2}\right)\left(R_{1}\right)\right)\left(\frac{P_{3}}{P_{9}}\right)\left(\frac{P_{2}}{P_{3}}\right)^{\frac{\gamma-1}{\gamma}}\right. \tag{A}
\end{equation*}
$$

where all the symbols used are illustrated in Chapter II and Figs. 2-9 through 2-10. For $N=3$, and $P_{1}=700 P_{\text {sia }}, \beta$ yields

$$
\begin{equation*}
B=\left[\left(\frac{P_{8}}{P_{1}}\right)\left(\frac{P_{7}}{P_{8}}\right)\left(\left(\frac{P_{6}}{P_{M 3}}\right)\left(\frac{P_{5}}{P_{M 2}}\right)\left(\frac{P_{9}}{P_{4}}\right)\right)\left(\frac{P_{3}}{P_{9}}\right)\left(\frac{P_{2}}{P_{3}}\right)\right]^{\frac{\gamma-1}{\gamma}} \tag{A-2}
\end{equation*}
$$

Component pressure losses are as follows:

$$
\begin{align*}
& \Delta P_{r}=10  \tag{A-3}\\
& \Delta P_{b}=3 r_{p}^{3}  \tag{A-4}\\
& \Delta P_{e}=1.0 r_{p}^{3}  \tag{A-5}\\
& \Delta \mathbf{P}_{i}=\left(1.0 r_{p}^{3}\right) /\left(r_{p}^{n-1}\right) \tag{A-6}
\end{align*}
$$

Where $\mathrm{n}=\mathrm{n}$ th stage compression.

$$
\begin{align*}
& \Delta\left(\text { Preg }_{c}=(1 / 100) x\right.  \tag{A-7}\\
& \Delta\left(P_{r e g}\right)_{h}(1 / 1000) \tag{A-8}
\end{align*}
$$

Each term in the expression of $\beta$ is expressed as follows:

$$
\begin{equation*}
P_{1}=700 \tag{A-9}
\end{equation*}
$$

$P_{8}=710$
$\left(\frac{P_{8}}{P_{1}}\right)=\frac{710}{700}$

Since
$P_{7}=P_{1}+\Delta\left(P_{\text {reg }}\right)_{c}=710+\left(\frac{1}{100}\right) x$
The ratio of $\mathrm{P}_{7}$ to $\mathrm{P}_{8}$ is
$\left(\frac{P_{7}}{P_{8}}\right)=\frac{P_{8}+\left(\frac{1}{100}\right) X}{P_{8}}=1+\frac{\left(\frac{1}{100}\right) X}{P_{8}}$
From the relation between $\mathrm{P}_{\mathrm{M}_{3}}$ and $\mathrm{P}_{7}$ and from Eq ( $\mathrm{A}-12$ )
$P_{M 3}=\frac{1}{r_{p}} P_{7}=\frac{710+\left(\frac{1}{100}\right) x}{r_{p}}$
From the relation between $\mathrm{P}_{6}$ and $\mathrm{P}_{\mathrm{M}_{3}}$ and from Eq. (A-5)
$P_{6}=P_{M 3}+\frac{\Delta P_{e}}{r_{p}{ }^{2}}=P_{M 3}+1.0\left(\frac{r_{p}^{3}}{r_{p}^{2}}\right)$
The ratio of $\mathrm{P}_{6}$ and $\mathrm{P}_{\mathrm{M}_{3}}$ is

$$
\begin{equation*}
\left(\frac{P_{6}}{P_{M 3}}\right)=\frac{P_{M 3}+\frac{1.0^{r_{p}}}{r_{p}^{2}}}{P_{M 3}}=1+\left(\frac{1.0}{P_{M 3}}\right)\left(\frac{r_{p}^{3}}{r_{p}^{2}}\right) \tag{A-16}
\end{equation*}
$$

From the relation between $\mathrm{P}_{\mathrm{M}_{2}}$ and $\mathrm{P}_{6}$ and from Eq. (A-15)
$P_{M 2}=\frac{P_{6}}{r_{p}}=\frac{1}{r_{p}}\left(P_{M 3}+1.0\left(\frac{r_{p}^{3}}{r_{p} 2}\right)\right)$
Since $P_{5}=P_{M_{2}}+\Delta P_{i}$ and $n=2$, therefore
$P_{5}=P_{M 2}+1.0\left(\frac{r_{p}{ }^{3}}{r_{p}}\right)$

From Eq. ( $\mathrm{A}-18$ ), the ratio of $\mathrm{P}_{5} / \mathrm{P}_{\mathrm{M} 2}$ is
$\left(\frac{\mathrm{P}_{5}}{\mathrm{P}_{\mathrm{M} 2}}\right)=\frac{\mathrm{P}_{\mathrm{M} 2}+1.0\left(\frac{r_{p}{ }^{3}}{r_{\mathrm{p}}}\right)}{\mathrm{P}_{\mathrm{M} 2}}=1+1.0\left(\frac{r_{p^{3}}}{r_{\mathrm{p}}}\right)\left(\frac{1}{\mathrm{P}_{\mathrm{M} 2}}\right)$
From the relation between $\mathrm{P}_{4}$ and $\mathrm{P}_{5}$ and from Eq. (A-18)
$P_{4}=\frac{1}{r_{p}} P_{5}=\frac{1}{r_{p}}\left(P_{M 2}+1.0\left(\frac{r_{p^{3}}}{r_{p}}\right)\right)$

Since $P_{9}=P_{4}+\Delta P_{e}$, therefore
$P_{9}=P_{4}+1.0 r_{p}{ }^{3}$
The ratio of $P_{3}$ to $P_{4}$ is

$$
\begin{equation*}
\left(\frac{P_{9}}{P_{4}}\right)=\frac{P_{4}+1.0 r_{p}^{3}}{P_{4}}=1+\frac{1.0}{P_{4}} \tag{A-23}
\end{equation*}
$$

Since $P_{3}=P_{9}+\Delta P_{b}$, therefore
$P_{3}=P_{9}+3 r_{p}{ }^{3}$

The ratio of $\mathrm{P}_{3}$ to $\mathrm{P}_{9}$ is
$\left(\frac{P_{3}}{P_{9}}\right)=\frac{P_{9}+3 r_{p}{ }^{3}}{P_{9}}=1+\frac{3 r_{p}^{3}}{P_{9}}=1+\frac{3 r_{p}^{3}}{\left(P_{4}+1.0 r_{p}{ }^{3}\right)}$

Since $P_{2}=P_{3}+\Delta\left(P_{r e g}\right)$, therefore
$P_{2}=P_{3}+\left(\frac{1}{100}\right) x$
The ratio of $\mathrm{P}_{2}$ to $\mathrm{P}_{3}$ is

$$
\begin{align*}
\left(\frac{P_{2}}{P_{3}}\right)=\frac{P_{3}+\left(\frac{1}{100}\right) X}{P_{3}} & =1+\frac{\left(\frac{1}{100}\right) x}{P_{3}}  \tag{A-29}\\
& =1+\frac{\left(\frac{1}{100}\right) x}{P_{9}+3 r_{p}} \\
& =1+\frac{\left(\frac{1}{100}\right) x}{P_{4}+1.0 r_{p}^{3}+3 r_{p}^{3}} \\
& =1+\frac{\left(\frac{1}{100}\right) x}{P_{4}+4 r_{p}^{3}}
\end{align*}
$$

As seen, each term in Eq. (A-2) is found fromequations (A-11), (A-13), (A-16), (A-19), (A-23), (A-26), and (A-29), respectively.

## APPENDIX B

OPTIMUM FEEDWATER HEATER INEET SATURATION TEMPERATURES

Table B-1. Optimum Feedwater Heater Inlet Temperatures at $1900{ }^{\circ} \mathrm{R}$

| Boiler | Amount | Number of | Mesh Point Indices of Feedwater |
| :---: | :---: | :---: | :---: |
| Saturation | of | Feedwater | Heater Inlet Saturation Temperature |
| Temp. <br> in ${ }^{\circ} \mathrm{F}$ | Superheat $\text { in }{ }^{\circ} \mathrm{F}$ | Heaters' | Heater Inlet Saturation Temperature |



Feedwater Heater
Inlet Saturation $* 700-20.0 *$ Mesh point temp. index.
Temp.

Table B-2. Optimum Feedwater Heater Inlet Saturation Temperatures at $2100^{\circ}{ }^{\circ}$

| Boiler | Amount | Number of | Mesh Point Indices of Feedwater |
| :--- | :--- | :--- | :--- |
| Saturation | of | Feedwater |  |
| Temp. | Superheat |  |  |
| in | Heaters |  |  |$\quad$| Heater Inlet Saturation Temperature |
| :--- |



Feedwater Heater Inlet Saturation Temp.

Table B-3. Optimum Feedwâter Heater Inlet Saturation Temperatures

| Boiler | Amount | Number of | Mesh Point Indices of Feedwater |
| :--- | :--- | :--- | :--- |
| Saturation | of | Feedwater | Heater Inlet Saturation Temperature |
| Temp. | Superheat | Heaters |  |



Feedwater Heater $=700-20.0 *$ Mesh point temp. index.
Inlet Saturation Temp.

APPENDIX C

COMPUTER PROGRAMMING

This program, written in FORTRAN 4 and named BINARY, is designed for optimizing helium-steam binary cycle using DATA as input. DATA stores input information, output information, and thermodynamic properties of steam. Input information is recorded on the first card of DATA and output information is specified on the second card. The rest of the cards store themodynamic properties of steam and, therefore, the user should not alter these properties unless a modification of the code is intended.

In this program the Rankine cycle is operable between temperature limits of 100 of and $1000{ }^{\circ} \mathrm{F}$ and the bolve daturation temperature is varied from $400{ }^{\circ} \mathrm{F}$ to $700^{\circ} \mathrm{F}$ in $100{ }^{\circ} \mathrm{F}$ intervals. The amount of superheat can be varied from $0{ }^{\circ} \mathrm{F}$ to the upper temperature limit in $100{ }^{\circ} \mathrm{P}$ Intervals. The upper limit is decided from the fact that the boiler exit temperature should not exceed $1000{ }^{\circ} \mathrm{F}$.

## Inputispecifications

The reactor outlet temperture, the range of boiler saturation temperatures, and the range of superheat to be searched is specified as input. The input format is as follows:

## Card 1: Columns 1-10

Reactor exit temperatures in ${ }^{\circ} \mathbf{R}$ are specified as (F10.0) format.

## Columns 10-12

Index of highest boiler saturation temperature to be searched is specified as ( I2 ) format.
$\left.\begin{array}{l}\text { Index of Highest } \\ \text { Boiler Sat. Temp. }\end{array}=\frac{\left(800 . \text { Highest Boiler Sat. Temp., }{ }^{\circ}{ }^{\circ} \text { ) }\right.}{100}\right)$

Columns 12-14
Index of lowest boiler saturation temperature to be searched Is specified as ( I2) format.

Index of Lowest
Boiler Sat. Temp. $\left.=\frac{\left(800 .- \text { Lowest Boiler Sat. Temp., }{ }^{\circ}{ }^{\circ} \text { F }\right)}{100}\right)$

Columns 14-16
Index of smallest amount of superheat selected is specified as ( I2) format.
$\begin{aligned} & \text { Index of } \\ & \text { Lowest Superheat }\end{aligned}=1+\frac{\text { Amount of Superheat, }{ }^{\circ}{ }^{\circ} \text { F }}{100}$

Columns 16-18
Index of greatest amount of superheat selected is specified as ( I2) format.

Index of Greatest Superheat


## Output Specifications

The output of the program is controlled through appropriate specifications in the second card of DATA. By assigning either 0 or 1 to the appropriate columns of the second card, the amount of print out is adjusted. If a user does not want to control the amount of print out, he should leave the second card as is and neglect the following instructions:

Card 2 Columns 1-2
To print out the input, enter " 1 " in ( I2) format. Otherwise enter "0".

Columns 2-4
To print out the indices of optimum feedwater heater inlet saturation temperatures, enter " 1 " in ( I2) format. Otherwise enter " 0 ".

Columns 4-6
To print out thermodynamic properties of steam along the turbine expansion and moisture separation stages, enter "1" in ( I2 ) format. Otherwise enter " 0 ".
$\quad$ Important Symbols in output
LTO $=$ Index of boiler saturation temperature*
SH $=$ Amount of superheat in $\circ_{F}$
ETAM $=$ Optimum efficiency of binairy cycle
LTSM $=$ Index of optimum final feedwater heater inlet saturation

## temperature**

EREFI = Efficiency of Rankine portion of optimum binary cycle

EBEFI = Efficiency of Brayton portion of optimum binary cycle
LOPT $=$ Optimum number of feedwater heaters +1

NB $\quad=$ Optimum number of compressor stages

RB = Optimum compression ratio

IXB $=$ Index of optimum amount of regeneration across the regenerator***

TSUMR = Net computation time in seconds required to optimize Rankine portion of the binary. cycle for a specified boiler saturation temperature and amount of superheat.
$T S U M B=$ Net computation time in seconds required to optimize Brayton portion of the binary cycle for a specified boiler saturation temperature and amount of superheat.

* LTO is the integer value of the following equation:

LTO $=\frac{(700-\text { Boiler Sat Temp. in op })}{20 .}$
** LTSM is the optimum LTS; where LTS is the integer value of the following equation:
$L T S=\frac{\left(700 .- \text { Final Feedwater Heater Inlet Sat. Temp. in }{ }^{\circ} \mathrm{F}\right)}{20 .}$
*** IXB is the optimum IX, where IX is the integer value of the following equation:
$I X=1+\frac{x}{25}$
$X$ is the temperature difference across the regenerator.

```
    PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
    COMMON/DINA/ STS(40,40),SHH(40,40),SSSS(40,40),
        STHORK(40,40).
    * SOSUM(40,40),STSUM(40,4[C),SOMEGA(40,40),JN,II,KK,FJF,
    * OTS(40).
    * DSS(40), DTNORK(40), DOSUM(40),DTSUM(40),DOMEGA(4E),
    * DHH(40)
    *,SQUAL(46,40),SLTS(40,40),OLTS(40),LTS(40);JS.
        SREFI(30,40).
    * REFI(40),Q,HC
    COMMON/RANKIN/ WTOP(20),WCEN(20),H&20),WBOT(20).
        THORK(2E),
    * OSUM(20), HH(220),SS(220).OMEGA(20)
    * TSUM(20),TS(20)
    *, PP(20),HO,LTO,LAST,SO ,III,LCRIT
        COMMON/VAKI/MFEED,LOPT(5[),H6,LTEMP,TT1,LTSONE
        COMMON/CONT/TG,SH,TB,EREFI(50),EFSTAR,EBEFI(50).
    * ETA(50)
        COMMON/STAR/NSTAR,IXSTAR,RSTAR,RB(50),IXB{50),NB(50),
    * ETAM
    * gLTSM
    COMMON/PRINT/LPRINTI,LPRINT2,LPRINT3,LPRINT4,LPRINT5,
    * LPRINTG,LPRINTT,LPRINTE,LPRINT9,LPRINIO,LPRIN11;
    * LPRIN12
    LPRIN10=0
    LPRINTG=0
    LPRINT8=0
    LPRINT T=0
    LPRINT5=0
    LPRINT6=0
    LPRINT 4=0.
    REAO(5,100) TT1,JBEGIN,JEND,IBEGIN, IEND
    100 FORMAT( FIO.0, 4I2)
    READ(5,110) LPRINTI,LPRINTZ,LPRINT3
110. FORMAT( 3I2)
    CALL STEAM
    IF (LPRINT1 -LT. 1) GO TO 200
    WRITE (6,150) TT&,J日EGIN,JEND,IBEGIN,IEND,LPRINTI,
        LPRINTZ,LPRINT3
```



```
    * *IBEGIN=*,I2," IEND= ",I2," LPRINT1= ",I2;*
    * LPRINT2= *.I2,
    * "LPRINT3=*,I2)
206
                00 800 J=JBEGIN;JEND
    LTO=(J:1)*5
    DD 7OG I=ISEGIN,IEND
    SH=10G0O*(I-1)
    TEMPLIM= SH + (700 - LTC 20.0)
    IF (TEMPLIM &tT. 1000.) GO TO 800
    CALL BINARY
```

```
    WRITE ( 6,500) LTO,SH,ETAM,LTSM,EREFI(LTSM),
    * EBEFI(LTSM),
    * LOPT (LTSM),NB(LTSM), KE(LTSM) ,IXB(LTSM)
```

```
    FORMATIIX,"LTO SH ETAM LTSM EREFIM EBEFIM LOPT *.I4;
    * 2F1C.4.
    * I4,2F8.4,I4,/,1X,**NBM RBN IXBM =**,I5,F10.4,I5.)
        CONT INUE
    CONTINUE
    END
```

    SUBROUTINE BINARY
    COMMON/DINAR STS (4t,4C),SHH(40,40),SSSS (40.40),
    * STWORK (40,40).
    * SOSUM(40,40), STSUM, (40, 40), SOMEGA(40,40), JJ,II,KK,FJF.
    * DTS(40).
* DSS (40), OTHORK (40), DOSUM (40), DTSUM(40), DOME GA (40) .
- OHH (40)
* , SQUAL (40, 4t ), SLTS(40, 40), DLTS (40), LTS (40), JS.
* SREFI(30.4E),
- REFI (40), Q, HC
COMMON/RANKIN/ WTOP(20), WCEN(20), W(20),WBOT(20),
- THORK(20).
* OSUM(20), HH(220),SS(220), OMEGA(20)
- TSUM(20),TS(20)
*, PP(20), HO,LTO,LAST,SO . III,LCRIT
COMMON/VARI/MFEED,LOPT (5L), H6,LTEMP,TTI,LTSONE
COMMON/CONT/TG,SH,TE,EREFI(50),EFSTAR,EBEFI(50),
* ETA(50)
COMMON/STAR/NSTAR, IXSTAR,RSTAR,RB(50), IXB(50), NE(50),
* ETAM
* LLTSM
COMMON/PRINT/LPRINT1,LPRINT2,LPRINT3,LPRINT4,LPRINT5,
- LPRINT6,LPRINT7,LPRINT8,LPRINT9,LPRIN1D. LPRIN11,
- LPRIN12
DIMENSION JSTAR(40.40), TMESH(40,40), DREFI(40), LKK(40,
- 401
DIMENSION LTEMPM(40)
TSUMR=0
ISUMB $=0$
DO $10 \mathrm{~L}=1,30$
ETA(L)=
LTEMPM(L) $=0$
$005 L N=1.30$
LKK (LN,L) $=0$
CONTINUE

```
    10 CONTINUE
    LTEMP=0
    I\I=1
    LTSUP=LTO +1
    DO 3000 KFIR=LTSUP,29
    CALL SECOND(R1)
    LTEMP=KFIR
    LTS(1)=LTEMP
20 TS(1)=700.*2C.*LTS(1)
    T=TS(i)
    HE=HL (T)-10
    TB=700.-20.*LT0
    LAST=30
    TU =SH+TB
    I=1
    II=1
    CALL RANK
    MFEED=LCRIT-LTS(1)
    IF (MFEED •LT. 10, GO TO 31
    MFEEO = 10
30 II=2
    III=2
    IF (LTS(1) .GE. (LCRIT-1)) GD TO 4000
    JS=1TS(1)+1
    DO 100 J=JS,30
    JJ=J
    LTS(2)=J
    TS(2)=700.-2t.*J
    IF(II .GT. 2) GO TO 150
    CALL RANK
    JSTAR(LTS(1),1)=LTO
    JSTAR(JJ,2) =LTS(1)
    SLTS(JJ;2)=J
    STWORK (JJ;2) =TWORK(2)
    SOSUM(JJ,2)=OSUH(2)
    STSUM(JJ,z)=TSUM(2)
    SOMEGA(JJ,1)=ONEGA(1)
    100 CONTI'NUE
    JJ=30.
    D0. 12C K=1,2
    IF (K.EQ.i) GO TO 110
    JJ=KK
110 L=*K+2+1
    KK=JSTAR(JJ,L)
    TMESH(\tilde{L}L)=KK
120:CONTINUE
    DLTS(1) =LTEMP
    "OTS(1)=TS(1)
    DHH(1)=HH(1)
    OSS(1)=SS(1)
```

```
            DTHORK(1)=TWORK(1)
            DLTS(2)=TS(2)
            DTS(2)=TS(2)
            OHH(2)=HH(2)
            DSS(2)=SS(2)
            DOMEGA(1) =ONEGA(1)
            OTHORK(2) =TWORK(2)
            DOSUM(2)=OSUM(2)
            OTSUM(2)=TSUM(2)
            DOSUM(1)=OSUM(1)
            DTSUM(1)=TSUM(1)
            DOMEGA(1)=OMEGA(1)
            DNEFI(2)=\hat{REFI(2)}
            MM=2
            IF (MFEED EQ, 2) GO TO 315
150 OO 30% M=3,MFEED
            II=M
            JI=(II-1)+LTS(1)
            00 200 J=JI,30
            LTS(M)=J
            TS(M)=700.-20**LTS(N)
            JJ=\
            CALL FOPT
            JSTAR (JJ,M) = KK
            SLTS(JJ,M) =LTS(M)
            STMORK(JJ,M) #THORK(M)
            SOSUM (JJ,M) =OSUM (M)
            STSUM (JJ*M)=TSUM (M)
            SOMEGA(JJ,M-1)=OMEGA(M=1)
200 CONTINUE
            JJ=30
            DO 160 KF1,M
            IF (K EQ.1) GO.T0 310
            JJ=KK
310 L=-K +M +1
            KK=JSTAR(JJ&L)
            TMESH(M,L)=KK
            IF(LPRINT5 - LT. 1) GO TAS60%
```



```
            * STSUM(JJ&L!,
            * L.JJっKK
180
            FORHAT(IX, "STWORK(I) SOMEGA(IEY):SOSUM(JJ.I)
            * STSUM(JJ,I) L**
```



```
160 CONTINUE
            OLTS(M)=LTS(M)
            OTS(M)=TS(M)
            DHH (M) =HH (M)
            :OSS(M)=SS(M)
            DTHORK(H) =TWORK(M)
```

```
    DOSUM (H) =OSUM (H)
    DTSUM(M) =TSUM(M)
    DOMEGA(M-1) = OMEGA (M-1)
    DREFI (M)=REFI (M)
    MM=M
    IF(DTSUM(M). LT. DTSUM(M-1)) GO TO 360
        CONTINUE
            MFZERO=MFEEO +1
            DO 35! L=MFZERO,11
            OTHORK(L)=0.
            DOMEGA(L)=0.
            DOSUM (L)=U..
            OTSUM(L)=L.
            OREFI(L)=0.
            CONTINUE
            GO TO 400
360 EREFI(LTS(1))=0REFI( MM-1)
            LOPT(LTS(1))=(MM-1)
            MMINUS=MM-1
            DO 355 L=1,MMINUS
            LKK(LTS(1),L)=TMESH(MMINUS,L)
            IF (LPRINTS -ET.LTGOTO 355
            WRITE (6,354) LKK(LTS (1),L),L,LTS(1)
                    354 FORMAT(1X;"LKK(LTS(I)+L)=",I3,*.L=",I3** LTS(1)=
            * ",I3)
355 CONTINUE
    DO 370 L=NM. 11
    DTHORK(L)=0.
    DOMEGA(L)=0.
    DOSUM(L)=6.
    OTSUM(LNFO.
    DREFI (L)=[.
370 CONTINUE
400. CALL, SECOND(R2)
    TIMER=R2 - R1
    TSUMR =TSUMR+TIMER
    IF LPRINT6 LT. 1) GO T0 550%
    WRITE (6,450) EREFI(LTS(1)IMLOPPTILTS(1))
450 FORMAT(1X,"EREFI(LTS(1)) LOPT(LTS(1)) = ".F12.4.I5 )
    HRITE(6,500)
    * (OTNORK(L),L=1,10),(OOSUM(L);L=1,10),(DTSUM(L),L=1,
    * 10).
```

$$
\operatorname{LTS}(1)=30
$$

$$
\operatorname{TS}(1)=700,-20 . * \operatorname{LTS}(1)
$$

$$
I=1
$$

$$
I I=1
$$

CALL RANK
DTHORK(LTS(1))=TWORK(1)
DOSUM (LTS (1) OOOSUM(1)
OTSUM(LTS(1) =TSUM(1)
DREFI(LTS(1))=REFI(1)
DOMEGA(LTS(1))=0
EREFI(LTS(1))=DREFI(LTS\{1)
LOPT(LTS(1)) $=1$
IF (LPRINT6 -LT. 1 : 60 TO 5600
WFITE(6,500) DTWORK(30), DOSUM(30). DTSUM(30).

- DOMEGA(30).
- DREFI(30)

5000 FORMAF(1X,


* ${ }^{*}$ DTSUM (30) $={ }^{\circ}$ 。

* Fi2.4,/, )

5600 LTSONE $=1 T S(1)$
CALL BRMAX
RB(LTS(1))=RSTAR

```
        IXB(LTS(1))=IXSTAR
        NE(LTS(1))=NSTAR
        EBEFI(LTS(1))=EFSTAR
        ETA(LTS(&))=EREFI(LTS(1)) +EBEFI(LTS(1)) -
    * EREFI(LTS(1))
    * *EBEFI(LTS(1))
        IF(LPRINT4 -LT. 1 ) 60 TO 5550
        HRITE(6,5500) LTS(1),LOPT(LTS(1)),EREFI(LTS(1)).
    * EbEFI(LTS(1)).
    * ETA(LTS(1)),RB(LTS(1)),IXB(LTS(1)),NB(LTS(1))
5500 FORMATIIX;"LTSI1) LOPT EREFI EBEFI ETA**2I3 *3F7.4.
    * /,IX,*RB,IXB,NB=**F7.4,2I4)
5550 ETAOLD=-1
    LTSUP=LTO+1
    LTOSTOP=LCRIT - 2
    DO 6500 L=LTSUP,LTOSTOO
    ETANEH=ETA(L)
    LTS(1)=L
    LTSNEN=LTS(1)
    IF(ETANEW GT. ETAOLD GO TO 6400
    ETAM=ETAOLD
    LTSM=LTSOLD
    GO TO 650C
6400 ETAM=ETANEH
    LTSA=LTSNEW
    ETAOLO=ETANEW
    LTSOLD=LTSNEN
6500 CONTINUE
    IF (ETAM,GT.ETA(30)) GO TO 66DO
    ETAM=ETA(30)
    LTSM=SO
    6600 IF (LPRIN10.LTO1)GO TO }720
    WRITE (6,70OD) ETAM: LTSM'
7000 FORMAT (1X,"ETAM=**F12.4,"LTSM=*,15)
    00 7555 L=2,10
    LTEMPM(L) =LKK(LTSM,L)
    LFEAL=L-1
    IF (LPRINTZ -LT. I (G0 TO 7555
    WKITE (6,7550) LTEMPM(L). LREAL
7550 FORMAT (1X, "LTEMPM(L)=*,I5,* LREAL=*.I5)
7555 CONTINUE
    WFITE (6,850G) TSUMR,TSUMB
850日 FORMAT(1X,*TSUMR=*,F1E.2;* TSUMB=*,F1E.2)
    RETURN
    END
```

```
    SUBROUTINE FOPT
    COMMON/DINA/ STS(40,40),SHH(40.40),SSSS(40,40).
        STWORK(40,40),
    * SOSUM(40,4C),STSUM(40,40),SOMEGA(4D,40),JJ,II,KK,FJF,
    * DTS(40).
    * DSS(40), DTWORK(40),DOSUM(40),DTSUM(40),00MEGA(40) .
    * DHH(40)
    *,SQUAL(40,40),SLTS(40,40),DLTS(40),&TS(40),JS.
    * SREFI(36,40),
    * REFI(40),Q,WC
    COMMON/RANKIN/ WTOP(20),WCEN(20),W(20),WBOT(20);
    * TWORK(20).
    * OSUM(20), HH(220),SS(220),OMEGA(20)
    * TSUM(20).TS(20)
    *,PP(20),HO,LTO,LAST,SO ,III,LCRIT
        COMMON/VARI/HFEED,LOPT (50),H6,LTEMP,TTI,LTSONE
        COMMON/CONT/TO,SH,TB,EREFI(50),EFSTAR,EBEFI(5C),
    * ETA(50)
        LTS(II-1)=(II-2)+LTS(1)
        TS(II-1)=70G.-20.*LTS(II-1)
        KK=(II-2)+LTS(1)
        CALL SET
        CALL RANK
        TSUMB=TSUM(II)
        KI=II-1+LTS(1)
        KF=JJ-1
        00 200 K=KI,KF
        KK=K
        LTS(II-1)=KK
        TS(II-1)=700.-20*LTS(KK)
        CALL SET
        CALL RANK
        TSUMF=TSUM(II)
        IF(TSUMF.LT.TSUMB) GO TO 300
        TSUMB=TSUMF
    20D CONTINUE
        GO TO 500
300 KK=KK-1
    400 LTS(II-1)=KK
            TS(II-1)=700.-20.*L7S(KK)
            CALL SET
            GALL RANK
500 RETURN
            ENO
```

```
    SUBROUTINE SET
    COMMON/DINA, STS(40,40),SHH(40,40),SSSS(40,40),
        STHORK(40,40),
    SOSUM(40,40),STSUM(40,40),SOMEGA(40,40),JJ,II,KK,FJF,
        DTS(40).
        DSS(40), DTHORK(40), DOSUM(40),DTSUM(40),DOMEGA(40),
        DHH(40)
        ,SQUAL(40,40),SLTS(4L,40),OLTS(40),LTS(40),JS,
        SREFI(30,40).
* REFI (4J), Q,WC
    COMMON/RANKIN/ WTOP(20),WCEN(20),W220),HBOT(20),
        THORK(20).
* OSUM(20), HH(22f),SS(220).OMEGA(20)
        *
* TSUM(20),TS(20)
*,PP(20),HU,LTO,LAST,SO,III,LCRIT
    LTS(II-1)=SLTS(KK,II-1)
    TSUM(II-1) =STSUM(KK,II-1)
    OMEGA(II-\overline{Cl}=\mathrm{ SOMEGA(KK,II-2)}
    OSUM(II-1)=SOSUM(KK_II-1).
    TWORK(II-1)=STWORK(KK,II-1)
    RETURN
    END
    SUBROUTINE RANK
    CCMMON/LIST/ TSS(40),SSISO(40),PPP&40),HHH(240).
* SSS(240).
* QUIS(40),HIS(40),TIS(40),QU(40),TEMP(40), QUNEW(40),
* HHHNEW(40)
*SSSNEW(40), LSAI(20),PAİ(20) ,HIN(20),HOUT(20.)
* SAI(20).HSAT(20),HWAT(20) .HSEP.SAISUM
    COMMON/RANKIN/ WTOP(20),WCEN(20), W(20),HBOT(20),
* THORK(20).
* OSUM(20). HH(220),SS(220),OMEGA(20)
* TSUM(20),TS(20)
*, PP(2O),HO,LTO,LAST,SO ,III,LCRIT
    COMMON/DINA/ STS(40,40),SHH(40,40),SSSS (40,40):
* STHORK(40,40).
* SOSUM(40,40),STSUM(40,40),SOMEGA(40,40),JJ,II,KK,FJF,
* DTS(40).
* DSS(40),DTWORK(40), DOSUM(40),DTSUM(40),DOMEGA(40).
* DHH(4J)
*,SQUAL(40,4(0),SLTS(40.40),DLTS(40),LTS(40),JS,
* SREFI(30,40).
*. REFI (40),Q&WC
    COMMON/CONT/TO,SH,TB,EREFI(50),EFSTAR,EBEFI(50),
* ETA(50)
```

```
    COMMON/PRINT/LPRINT1,LPRINT2,LPRINT3_LPRINT4,LPRINT5.
    * LPRINT6,LPRINT7,LPFINTB,LPRINT 9,LPRIN10,LPRIN11,
    * LPRIN12
    LAST=30
    TB=70C.-2C.*LTO
        EFC=0.85
    T=TB
    P=PL(T)
    PB=O
    WC=0.0161.37*((PB-1,%)*144.4778.16)/EFC
    IF(SH.LE. D.D) GOTO 10
    TO=TB+SH
    PG=PB
    T=TU
    P=P|
    HI=HV(T,P)
    SO=SV(T,P)
    GOTO 13
        HO=HG(T)
        Si=SG(T)
        TO=TB
    HTOP=HB
        IF(III.GT. 1) GOTO i5
        CALL TABLE
    15TS(1)=700.-20.*LTS(1)
        T=TS(1)
        HH(100)=HL(T)-10.
        FLOW=1.0
        I=II-1
        IF(I.GE. i) GO TO 22C
        IF (SAISUM LT. D.0000001) GO TO 55
        NI=14
        DO 20 K=1,NI
        KIN =K
        IF(LTO. LTE LSAI(K)) GO TO 30
        CONTINUE
        LSIN=KIN
        OO 40 K=6SIN,NI
        KOUT=K
        IF( (LTS(1).LT.LSAI(K)).OR.(LSAI(K....LT. 0.OOOCCOOI)):
            * GO TO 50
    40 CONTINUE
50 LSOUT =KOUT
            60 TO 60
55. LSIN=1
            LSOUT=1
            PAI(1)=1
            60 HH(1)=NHH(LTS(1))
            OSUM(1)=0.
            IF(LSIN.EQ,LSOUT) GOTO 200
```

```
    100. WTOP(1)=(PAI(1)-OSUM(1))=(HC - HIN(1)) FLOW
    IF((LSQUT-1).LE. LSIN) GO TO 150
    HCEN(1)=0.0
    LSINP=LSIN+1
    LSOUTME LSOUT -1
    DO 120 K=LSIND,LSOUTM
    H(1)=(PAI(K) -OSUM(I))F(HOUT(K-1)-HIN(K))*FLOK
    HCEN(1) =WCEN(1) +H(1)
    CONTINUE
    GO TO 180
    WCEN(1)=0.0
    180. WBOT(1)=( PAI(LSOUT) -OSUM(1))*(HOUT(LSSOUT -1) - HH(1)
    J&FLOW
        TWORK(1)=HTOP(1)+WCEN(1)+WBOT(1)
        GO TO 210
    20G TWORK(1)=\ PAI(1) -OSUM(1))*( HC -HH(1))*FLOW
210
    TSUM(1)=TWORK(1)
        IF (LPRINTT &LTE 1) GO TO 212
        WRITE(6,7000) PAI({),OSUM{1),HD,HH(1),THORK(1),
    * TSUM(1)
7000 FORMAT(1X, "PAI(1) OSUM(1) HO HH(1) TWORK(1) TSUM(1)",
    4F1(.3)
    212 IF(LTS(1).EQ.3C) GD TO 215
        Q=HO-HH(1CO)
        GO TO 217
    215 HH(100)=WC+69.73
        IF(HH(10日).GT.HSEP) GO TO 216
        HH(10C)=NC+69.73+SAISUH* (HSEP= (WC+69.73+10))
    216 HH(20C+I+1)=HH(10C) +10
        Q=H0-HH(1.00)
217.EFI=(TSUM(I+1)-WC)/Q
    REFI(I+1)=EFI
    GO TO 500:
    220 rS(I)=700.-20.*LTS(I)
    NI=14
    IF (SAISUM .LT. O.00000001) GO TO 528
        DO 250 K=I,NI
        KIN=K
        IF(fLTS(I).LT. LSAI(K)).OR.I LSAI{K ).LT.O.0000081))
        * GOTO 300
    250. CONTINUE
    300 LSIN=KIN
        DO 400 K=LSIN,NI
        KOUT=K
        IF((LTS(I+I).LT.LSAI(K)).OR.( LSAIEK ).LT.0.000OOO1))
        - GO TO 500
    400 CONTINUE
    500 LSOUT =KOUT
        GO TO 540
520 LSIN=1
```

```
    LSOUT:E1
    PAI(LSIN)=1
        HH(I) = HHH(LTS(I))
    HH(I+I)=HHH(LTS(I+1)
    TS(I)=700-20%(LTS(I))
    T=TS(I)
    PP(I)=PL(T)
    P=PP(I)
    HH(200+I)=HL(T)
    HH(50 +I)=HG(T)
    SS(200+I)=SL(T)
    SS(50+I)=SG(T)
    HH(100+I-1)=HH(2C0+I)-10.
    TS(I+1)=750.-20.*LTS(I+1)
    T=TS(I+1)
    HH(200+I+1)=HL(T)
    IF(LTS(I+1).EQ. 3t) GO TO 560
    HH(100+I)=HH(200+I+1) -10.
    GO TO 565
560 HH(100+I)= WC+ 69.73
    IF( HH(100+I) GT. HSEP) GO TO 563
    HH(100+I)= NC+69.73 +SAISUH*( HSEP - (WC+69.73+10))
56
    HH(200+I+1) = HH(100+I) +10
565 OMEGA(I) =(1.0/( HH(I)-HH(200+I+1)))*(HH(100+I-1) -
        HH(100+I) +
    *(OSUM(I))* (HH(200+I+1) -HH(200+I)))
    OSUM(I+1)=OSUM(I) +OMEGA(I)
    IF( LSIN. EQ. LSOUT) GO TO 2000
1000 WTOP(I+1)=(PAI(LSIN) FOSUM(I+1))*(HH(I) - HIN(LSIN))
    * FLDW
    IF((LSOUT-13.LE. LSIN) GO TO 1150
    NCEN(I+1)=0.0
    LSINP=LSIN+1
    LSOUTMF LSOUT -1
    OO 1100 K=LSINP,LSOUTM
    W(I+1)=(PAI(K) OSUM(I+1))*(HOUT(K-1)-HIN(K ))*FLOW
    WCEN(I+1)=WCEN(I+1) +W(I+1)
1100 CONTINUE
    60 TO 120G
1150 WCEN(I+1)=0.0
1200 WBOT(I+1)=(PAI(LSOUT) -OSUM(I+1))&&HOUT(LSOUT-1)
    * -HH(I+1))*FLOW
    TWORK (I+1)=WTOP(I+1) +HCEN(I+1) +WBOT (I+1)
    GO TO 2500
2000 TWORK(I+1)=(PAI(LSIN) -OSUN(I+I))*(HH(I)-HH(I+1))
    * *FLOW
        WCEN(I+1) =0.0
        WTOP(I+1)=THORK(I+1)
        HBOT(I+1)=0.G
2500. TSUM(I+1) =TSUM(I) +TWORK(I+1)
```

```
    Q=HO -HH(100)
    EFI=(TSUM(Z+1)-NC)/Q
    REFI(I+1)=EFI
50B0 RETURN
    END
    SUBROUTINE TABLE
    COMMON/ACOM/EN(5C,50),ST(50,50),TM(53),PRH(50),PRS(50)
        ,ENL(130),EN
    *G(130),STL(130),STG(130),TAT(130),TSAT(230),PSAT(E30),
    * TSATT(370),P
    *SATT(37j),TF,TA,TAZ,HF
    COMMON/LIST/ TSS(40),SSISO(40),PPPA40),HHH(240),
        SSS(240),
    * QUIS(40),HIS(40) ,TIS(40),QU(4D),TEMP(4E).QUNEW(4C).
    * HHHNEW(40)
    * ,SSSNEW(40) LSAI(20),PAT(20) ,HIN(20),HOUT(20)
    * SAI(20),HSAT(2G),HWAT(2E) ,HSEP ,SAISUH
    COMMON/RANKIN/ HTOP(20),HCEN(20),W&20),W8OT(2G),
* TWORK(2U).
* OSUM(20), HH(220),SS(220),OMEGA(20)
* TSUM(20),TS(20)
*, PP(20),HO,LTO,LAST,SC ,III,LCRIT
    COMMON/PRINT/LPRINT1,LPFINT2,LPRINT3,LPRINT4,LPRINT5;
* LPRINTG,LPRINT7,LPRINT8,LPRINT9,LPRINIG,LPRIN11,
* LPRIN12
    DO 50 K=1. 15
    HIN(K)=0
    HOUT (K) =0
    HWAT(K)=0
    SAI(K)=0
    PAI (K)=0
    LSAI (K)=0
50 CONTINUE
    00 60 L=1,35
    SSSNEH(L)=0
    HHHNEW(L)=0
    QUNEW(L)=0
6 0 ~ C O N T I N U E ~
    LSTART = LTO+1
    LFINAL=LAST
        LS=0
    DO 500 L=LSTART,LFINAL
    T=7C0.-20.*L
    TSS(L)=T
```

```
    IFIL.EQ. LSTART) GOTO 250
    SSS(L-1)=S
    TSS(L)=T
    SSISO(L)=SSS (L-1)
    GO TO 260
    250 SSISO(L)=S0
    260 T=TSS(L)
    PPP(L)EPL(T)
    P=PPP(L)
    HHH(200+L)EHL(T)
    HHH(5C +1)=HG(T)
    SSS(200+L)=SE(T)
    SSS(504L)=SG(T)
    IF(SSISO(L) -SSS(50+L)) 350,350,300
    S=SSISO(L)
    QUIS(L)=1.0
    TA=TSS(L)
    HIS(L)=HSP(S,P)
    QU(L)=1.0
    EFT=0.92
    IF(L.GT.LSTART) GO T0. 320
    HHH(L) =HO -EFT* (HO-HIS(L))
    GO TO 330
    320 HHH(L)=HHH(L-1)-EFTF(HHH(L-1) -HIS&L))
    TIS(L)=TF
    H=HHH(L)
    TAZ=TIS(L.)
    TEMP(L)=THP(H,P)
    T=TEMF(L)
    SSS(L)ESV(T.P)
    T=TSS(L)
    S=SsS(L)
    60T0.400
350
    QUIS(L)=(SSISO(L) -SSS(200+L))/(SSS(50+L) -SSS(200+L))
    HIS(L)=QUIS(L)*(HHH(50+L)-HHH(200+L)) +HHH(200+L)
    IF (QUNEH(L-1).GT. QU(L-1))G0 T0 355
    EFT=0.92-(1 -QU(L-1))*(0.17)/(0.12)
    GOTO 358
355 EFT=0.92-(1 -QUNEW(L-1))N(0.17)/ 0.12
358 IF(L.GTE LSTARTJ GO TO 360
    EFT=0.92
    HHH(L)=H0-EFT*(H0=HIS (L))
    GO TO 365
    HHH(L) EHHH(L-1) -EFT* (HHH(Lm1) -HHLS(L))
365. TIS(L)=TSS(L)
    IF(HHH(L):HHH(50 +L)) 37L, 370,333
37日 QU(L)=(HHH(L) =HHH(20C+LD)\(HHH(50+L) #HHH(200+L))
SSS(L)=QU(L)*(SSS(50+L) -SSS(200+L)) +SSS(200+L)
IF (LEEQ. 30) GO TO 375
IF( QU(L).LT.0.95) G0 T0 360
```

```
    375 S=SSS(L)
        T=T.SS;(L)
    GO TO 400
```

    \(380 \quad \therefore\) QUNEW \((L)=689\)
        HHHNEW(L) \(=Q U N E H(L) A(H H B(50+L)-H H H(2 O D+L))+H H H(2 O C+L)\)
        SSSNEW(L) \(=\) QUNEW(L)P(SSS \((50+L)-S S S(200+L)+S S S(200+L))\)
        LS \(=\mathbf{L S}+1\)...
        LSAI (LS) =L
        HIN(LS) \(=\mathrm{HH} H(L)\)
        HOUT (LS) = HHHNEW (L)
        STVSSSNEWCL)
        HHH (L) =HHHNEW(4)
        T=TSS (L)
        HSAT (I) =HL(T):
        HWAT(LS) =HL (T)
        PAI (1) \(=1\)
    480 IF (LPRINTS LTT 1 GO TO 500
WRITE (6,2CQC) T,HHH(L),SSS(L), L, HHHNEW(L), SSSNEW(L)
* QU(L)
- QUNEW(L), EFT , PPP(L) , HHH(200 + L * HHH( $50+L)$.
* SSS (200+L).
- SSS(50 +L), SSISO(L), HIS(L) , LS,LSAI (LS), HIN(LS).
- HOUT(LS)

- F12.4,
* * $L=\cdots, I 4$ * HHHNEW(L) $=*, F 12.4, * \operatorname{SSSNEW}(L)=*$,
* F12.4./. 1X.
* "QU(L) =*,F12.4,*QUNEW(L) = *,F12.4, "EFT=*,F1E.4,
* $\quad$ PPP $(L)=*, F 12$

SSS $(200+L)={ }^{* 2}$.

* HIS(L) =",

F12.4.
* ${ }^{*}$ HCUT (LS) ${ }^{* *}$.F12.4 )
500 CONTINJE
$00.100 \mathrm{~K}=1, \mathrm{LS}$
IF ( HOUT (K). LT. G.C001) 60 TO 600
SAI (K) = (PAI (K) F( HIN(K) EHOUT (K): / HWAT(K) -HOUT (K))
- $\therefore 1$
PAI $(K+1)=\operatorname{PAI}(K)-S A I(K)$
100
CONTINUE
IF (LPRINII -LT. 1 ) 60 TO 610
WRITE( $6,100($ HIN $(K), K=1$, 8$)$, $($ HOUT $(K), K=1,8)$,
* (HWAT $(K), K=1,8)$
*, $(\operatorname{SAI}(K), K=1,8),(P A I(K), K=1,8),(L S A I(K), K=1,8)$

- $1.1 X$



```
    GOTO 200
    110 TAT(J)=690. +2.* (J-119)
    200 CONTINUE
    TAT(I27) =705.47
    OC 300 j=1,218
    IF(J.GT.126) GO TO 250
    TSAT(J)= 98.+2.0.J
    GO TO 300
    250 IF(J.GT.211) GO TO 260
    TSAT(J)=350+(J-126)*4*0
    GO TO 300
260 TSAT(J)=690. +(J-211)*2.0
    300 CONTINUE
        TSAT(219) =705.47
    00 350 J=1,355
    310 IF (J.GT. 23) GO TO 320
        PSATT(J)=5.0 +(J-23)*0.2
        GO TO 350
    320 IF(J.GT. 44) GO TO 330
        PSATT(J) =26.0 +(J-44)=1.0
        GO TO 350
330 IF(J.GT. 131) G0 T0 335
    PSATT(J) =200.0 +(J=131)*2.0
    GO TO 350
335 IF(J.GT. 161) G0 TO 340
    PSATT (J)=350.E+(J-j61)*5.0
    GO TO 350
340 IF(J.GT. 256) GO TC 343
    PSATT(J)=1300.0 +(J-256)*10.0
    GO TO 350
343 IF(J.GT. 347) G0 TO 345
    PSATT(J) =3120. +(J-347)*20.0
    GOTO 350
345 IE(J.GT. 355) GOTC 350
    PSATT(J) =3200.+(J-355)*10.0
    350 CONTINUE
    PSATT(356) =3208.2
    RETURN
    END
```

    FUNCTION HV (T,P)
    COMHON\&ACOM/EN(50,50),ST(50.50), TMK 53 ), PRH (50), PRS (50)
        ENLE(136), EN
    *G(130TRSTL(130), STG(130), TAT(130), TSAT(230), PSAT (230),

- TSATST370i.P

```
        *SATT(370I,TF,TA,TAZ,HF
            IF(T.LT. 200.) 60 70 48
            IF(T.GT.5CO) GO TC 5
            J=(T-50)/50
            G0 TO 15
5..IF (T:GT. 1C00) GO TO 10
    J=9 +(T-540./20
    60 TO 15
    10 IF(T. GT.1100) GO T0 48
        J=34
    15 IF(P.GY.0.121 GO TO 20
    I=1
    G0 TO 40
    20 IF (P.GT. 10) GO TO 25
        I=1 + (P-{. 12)/4.94
        GO TO 40
    25 IF(P.GT.5t) GO TO 30
        I=3 +(P-1W)/20
        G0 T0 40
    30 IF(P.GT.6C0) GO TO 35
        I=5 +(P-50)/50
        GO TO 40
    35 IF(P.GT.3400) GO TO 48
        I=16 +(P-600)/200
    40 IF((EN(I,J).EQ.10OC.O).OR.(EN(I+1,J)&EQ.100G.0)) GO
        - TO 45
            X1=(EN(I+1;J+1)-EN(I,J+1)}F{P-PRH(I))/(PRH(I+1)-PRH(I)
        * ) +EN(I,J+1)
        X2={EN(I+1,J) -EN(I,J)) -(P-RRH(I))/(PRH(I+{) -PRH(I)
            )+EN(I,J)
            HV=( X1 -X2) *( T-TH(J|)/(TM(J+1) -TM(J))+X2
            GO 10 50
45 TREAL =T
            TETG(P)
            IF(TOLT.TZ ) GO T0 48
            H1=EN(I,J+1) +( (EN(I+1;J+1) -EN(I;J+1))/(PRH(I+1) -
            PRH(I)\)*(PaPR
            *H(I))
            T=T2
            H2 =HG(T)
            T=TREAL
            IF( H1.LT. H2) GOTO 46
            HV=H2 +(H1 -H2)*(T-T2 )/(TM(J+1) -T2)):
            IF(AEN(I,J+1)\cdotEQ.1000.0)%OR.(EN(I+1,J+1).EQ.1C00.O))
            * GOTO47
            GO TO 50
    47.HI=EN(I,J+2) +((EN(I+1;)J+2)-EN(I,J+2)}/(PRH(I+1)-
        * PRH(I):)*(P-PR
        *H(IJ)
            HV=H2 +(HI -H2)*(IT-TE IN(TH(J+2) -TZJ)
```

601050
$46 \quad H V=H 2+(T-T 2) *(205) *(1,0 / 5)$
GO TO 50
48 HV=C.C
50. RETURN

END

FUNCTION SVCT,P)
COMMUN/ACOM/EN(50,50), ST (50, 50), TM(53), PRH (50), PRS(50)

* ENL(13L), EN
-G(136), STL(130), STG(130), TAT(130), TSAT(230), PSAT(230),
- TSATT(370), ${ }^{-1}$
*SATT(370), TF, TA, TAZ,HF
IFIT.LT. 160.1 GO TO 4 B
IF (T.GT.SHO) GO TO 5
$J=(T-50) / 50$
GO TO 15
5 IF (T.ET. 1000 ) 60 TO 10
$J=9+(T-50 \mathrm{G}) / 20$
GO 1015
10 IF (T. GT.1100) GOTO 48
$J=34$
15 IF(P.GT. C.12) GOTO 20
$1=1$
GO TO 40
20 IF(P.GT. 2t) GO 1025
$I=2+(P-4) / 4$
GO TO 40
IF (P.GT. 300 ) GO TO 30
$I=6+(P-20) / 20$
GO 1040

30. IF(P.GT. 600) G0 TO 35
$I=20+(P-300) / 50$
GO:TO 40
35 IF(P.GT.2000) GO 7037
$I=26+(P-600) / 1 C 0$
607040
37 IFPP. GT. 3400 GOTO 48
$I=40+(P-2000) / 200$
40 IF(IST(I,J).EQ.L.COE).OR.(ST(I+1,J\}.EQ. 1.000$))$ GO TO $+45$
$X_{1}=(S T(I+1, J+1)-S T(I, J+1))=(P-P R S(I)) /(P k S(I+1)-$

* PRS (I) $)+S T(I, J+1)$
$X_{2}=\{S T(I+1, J)-S T(I, J)) *(P-P R S(I)\} /(\operatorname{PRS}(I+1)-\operatorname{PRS}(I))$
*     + ST(I, J)

```
            SV =(X1-X2)*(T-TM(J))/(TM(J+1)-TM(J)) 4X2
            G0 TO 50
45 TREAL =T
            TZ=TL(P)
            IF(T&LT.T2) GO T0 48
            SI=ST(I,J+1)+(ST(I+1,J+1) -ST(I,J+1))+(P-PRS(I))/
            * {PRS(I+1) - PRS(
            * I)!
            T=T2
            S2=SG(T)
            T=TREAL
            IF(S1 eLTe S2 G0 T0 46
            SV=S2 +(Si -S2) क((T-T2)/(TM(J41) -T2))
            60 T0 50
                            SV=S2+(T-T2)*(0.004n)*(1.0/5)
                            GO TO 50
    48 SV=9.6
    50 RETURN
            END
        FUNCTION HGIT:
        COMMON/ACOM/EN(50,50),ST(50,50),TH153),PRH(50),PRS(50)
        - ENL(130),EN
        *G(130),STL(130),STG(130),TAT(130),TSAT(230), PSAT(230),
    * TSATE(370),P
    *SATT(370),TF,TA,TAZ,HF
        IF(T.LT. 100.) GO TO 60
        IF (T.GT. 690.) 60 T0 5
        J=(T-95)/5
        G0 T0 40
    5 IF (T.GT. 705.47) GO TO 60
    1 0
    j=119 4(T-690)/2
        ENG(127) =946.0
        TAT(127) =705.47
    4 0
    HG =( ENG(J+1) ENG(J))*(T-TAT(JJ)AGTAT(J+1) -TAT(J))
        * +ENG(J)
        GO TO 70
    60. HG=0.0
70. RETURN
    ENO
```

```
        FUNCTION HL(T).
        COMMON/ACOM/EN(5C,50),ST(50,50),TM(53),PRH(50),PRS(50)
    * ENL(136).EEN
    *G(130),STL(130),STG(130),TAT(130),TSAT(230),PSAT(230),
    * TSATT(370),P
    *SATT(370),TF,TA,TAZ,HF
        IF(T.LT. 100.) GO TO 60
        IF (T.GT. 690.) 60 T0 5
        J={T-95}/5
        60 TO 40
5
    1 0
    IF (T. GT. 705.47) GO TO 60
    J=119 +(T-69C)/2
        ENL(127) =996.0
        TAT(127) = 705.47
4 0
    HL=(ENL(J+1) -ENL(J))=(T -TAT(J))/(TAT(J+1) -TAT(J))+
        * ENL(J)
            GOTO 75
    60 HL=0.0
70 RETURN
    END
```

        FUNCTION SG(T)
        COMMON/ACOM/EN(50,50), ST(50,50), TH(53), PRH(50), PRS(50)
        - ENL(130),EN
        *G(130), STL (136), STG(130), TAT(130), TSAT(230), PSAT(230),
        - TSATT(370),P
        -SATT(370),TF,TA,TAZ,HF
        IFIT.LT. 100.) GO TO 60
        IF (T.GT. 690.) GO TO 5
        \(J=(T-95) / 5\)
        GO TO 40
    5 IF (T.GT. 705.47) GO TO 60
$10 \mathrm{~J}=119+(\mathrm{T}-690) / 2$
STG(127) $=1.0612$
TAT(127) $=705.47$
40 SGF( STG(J+1) -STG(J))*(T-TAT(J))/ATAT(J+1) ETAT(J)) +
* STG(J)
60 TO 70
60
$56=0.0$
70 RETURN
END
$42+$

```
    FUNCTION SL(T)
    COMMON/ACOM/EN(50,50),ST(50,50),TM453),PRH(50),PRS(50)
    * ENL(I3G),EN
    *G(130),STL(130),STG(130),TAT(130),TSAT(230), PSAT(230),
    * TSATT(370),D
    *SATT(370),TF,TA,TAZ,HF
    IF(T.LT. 10C.) GO TO EO
    IF (T.GT. 690.) GO TO 5
    J=(T-95)/5
    GO TO 40
    5 IF (T.GT. 7L5.47) 60 T0 60
    10 J=119 +(T*690)/2
    STL(127)=1.0612
    TAT(127) = 705.47
4 0
    SL =(STL(J+1) -STL(J))&(T-TAT(J))/&TAT(J+1)-TAT(J)) +
        * STL(J)
        GOTO 70
    6 0 ~ S L = 0 . 0 ~ \% ~
    70 RETURN
    END
        FUNCTION PL(T)
        COMHON/ACOM/EN(50,50),ST(50,50),TM(53),PRH(50),PRS(50)
        * : ENL(13C),EN
    *G(130),STL(130),STG(130), TAT(130),TSAT(230), PSAT(230),
    * TSATT(370),P
    *SATT(370):TF,TA,TA2,HF
    IF(T.LT. 100.) GOTC 60
    IF (T. GT. 350.) GO TO 5
    J=(T-98)/2
    60 50 40
    5 IF (T. GT. 690.) GOTTO 10
    J=126+(T-350)/4
    G0 T0"40
10... IF (T.GT. 705.47) GOFTO 60
    J=2.41+1T-6940/2
    PSAT(219)=320862
    #ESAT(219) =7C5.47
    40 PL=(PSAT(J+1) -PSAT(J))*I TPTSAT(J))/(TSAT(J+1) -
        * TSAT(J))+PSAT(J)
            GOTO 70
60 PL=0.t
    70. RETURN
        END
```

FUNCTEOA TL(P)
COMMON/ACOM/EN(50,50), ST (50, 50), TH(53), PRH(50), PRS(50)

* ENL (230) -EN
*G(130), STL(130), STG(130), TAT (130), TSAT(230), PSAT(230),
- TSATT(370), P
*SATT(370),TF,TA,TAZ,HF
IF (P.LT. $\mathrm{L}, 6$ ) GO TO 60
IF(P,GT.5.0) GO 105
$J=(P-(i \cdot 4) / G .2$
GO 70.40
5 IF(P. GT. 26.0) GO TO 10
$J=44+(P-26) / 1 . D$
GO TO 40
13.IF(P.GT. 200.0) G0 TO 15
$J=132+(P-2 C 0 \cdot 0) / 20 t$
GO TO 40
15.IF(P.GT. 350.0$)$ GO TO 20
$J=161+(P-350.0) / 5.0$
GO TO 40
20 IF(P.GT. 1306.0) GO TO. 25
$J=256 .+(P=1300-0) / 10.0$
GO TO 40
25 IF(P.GT. 312C.0) GO TO 30
$J=347 .+(P-3120.0) / 20 * 0$
GO TO 40
IF(P.GT. 3200.0) GO TO 35
$J=355 .+(P-3200.0) / 10.0$
35 IF(P,GT.3208.2) GO TO 60

45. TL=(TSATT(J*1)

* tSATT(J)

GO TO 70
$60 \quad T L=0,1$
76 RETURN
END

FUNCTION HSPIS.P)
COMMON/ACOM/EN(50,50), ST(50,50), TH(53), PRH(50), PRS (50)

* ENL (130), EN
* $G(130)$, STL (130), STG(130), TAT (130), TSAT (230), PSAT (230),
* TSATT(3701, P
*SATT (373), TF,TA,TA2, HF OIMENSION DT(4)
or (1) $=50$
DT(2) $=5$

```
    OT(3)=1
    DT(4)=0.1
    T=TA
500 DO 590 M=1,4
510 T=T+DT(M)
    Su =SV(T.P)
    IF(SO.LT.S) GO TO 510
    IF(ABS(S-S0),LE.0.OCDC3) GOTO 600
520 T=T-DT (N)
5 9 0 ~ C O N T I N U E ~
    T=T+DT(4)
    SG =SV(T,P)
    HSP=HV(T;P)
    TF=T
    RETURN
    ENO
```

        FUNCTION THP (H,P)
        COMMON/ACOM/EN(50,50),ST(50,50),TM(53),PRH(50), PRS(50)
            -ENLII3t.).EN
    *G(130), STL(130), STG(130), TAT(130), TSAT(230), PSAT(230),
    - TSATT(370), P
    *SATT(E7U), TF, TA,TAZ,HF
    DIMENSTON :DT (3):
    \(\operatorname{DT}(1)=1\)
    \(\operatorname{Dr}(2)=0.2\)
    OT \((3)=0.84\)
    T=TA2
    600
$610 \quad T=T+B T(M)$
D0 69C $M=1,3$
$H F=H V(T, P)$
IF (HF.LE.H) GO TO 610
IF (ABS (H-HF).LE.C.OO1) GOTO:700
$620 \mathrm{~T}=\mathrm{T} \subset \mathrm{D}(\mathrm{M})$
690 CONTINUE
$T=T+0 T(3)$
HF=HV (T,P)
700
THP=T
RETURN
END

```
        SUBROUTINE BFMAX
        COMMON/VARI/MFEED,LOPT(5(),H6,LTEMP,TTI,LTSONE
        COMMON/CONT/TO,SH,TB,EREFI(50),EFSTAR,EBEFI(5C),
            ETA(5)
        COMMON/BEिAYTO/TTZLIM,TT5LIM&X,TT3,DELIMR,NCOM,TT4*
            TJ4LIM
    * LTSM
        COMMON/STAR/NSTAR,IXSTAR,RSTAR,RB(50),IXB(50),NB(50).
    * ETAM
        COMMON/PRINT/LPRINT1,LPRINT2,LPRINT3,LPRINT4,LPRINT5,
    * LPRINTG,LPRINT7.LPRINTE,LPRINT9,LPRIN1L,LPRINII.
    * LPRINI2
            DIMENSION EF1(10),EF2(10),OIF(10),EFP(50),EF(10),
    * ENSTAR(10), IXNSTAR(1U),NNSTAR(14),RP(50)
        IXF=(TT1-560.-T0)/25.0
        DC 64C0 NC=1,E
        NCOM=NC
        AMINR=1.0
        K=0
        EFPEAK=-1
        DO 30C0 IX=1,IXF
        EFOLD=EFPEAK
        x=25.[*(IX-1)
        K=K+1
        J=0
        J=J+1
        IF (K-1) 10,10,18
            IF(J-1) 16,16;18
    RS=1.1
    R=RS
    DELRS=0.05
    DELR= DELRS
18. }\quadRS=
20 EF1(J)=F(R)
    IF (DEELIMR. LT. O.OCO1) GO TO 25,
    GO TO 30
        R=F-DELR
        GO TO 20
    R=R+DELR
    EF2(J)=F(R)
    OIF(J)=EF2(J) - EFI(J)
    IF (LPRINT8 .LT. 1) GO TO 45
    HFITE(6,60BC) EFI(J), EF2(J), DIF(J)
6000 FORMAT(1X, "EF1 EF2 DIF=`,3F12.4)
    45 IF(OIF(J)) 50,250,250
    50. R=R-DELR
    60 EFi(J)=F(\hat{N})
        65 IF(R-AMINR) 70,2200,8C
    70 R=R+DELR
        EF(J)=F(R)
```

```
    75 GOTO 220R
    80 R=R-DELR
    90. EF2(J)=F(R)
    95 DIF(J)=EF2(J)-EFI(J)
    100 IF(DIF(J)) 2000,2000.110
    110 EF1(J)=EF2(J)
    120 GD TO 65
    250 EFi(J)=EF2(J)
    251. IF(OELIMR) 252,2200,255
    252. R=R=OELR
    EF(J)=F(R)
    253 GO TO 220t
    255 R=R+OELR
    260 EF2(J) EF(R)
    265 DIF(J)xEFZ(J) -EFI(J)
    IF( LFRINTB &T: 1 GO TO 270
    WFITE (6,7E0G) EFI(J),EFZ(J).R
700C FORMAT(1X,2F12.4;"R CORESOND TO EF2=*,F12.4)
    270 IF,(DIF(J)) 1000,250.250
    1000 R=R-DELR
    EF(J)=F(R)
    1050 60 T0 2200
    2000 R=R+DELR
    EF(J)=F(R)
2200 EFP(IX)=EF(J)
    RP(IX)=R
    EFNEW=EFP(IX)
    IXNEW=IX
    IFIEFNEW.GT. EFOEDV GOTO 2220
    EFPEAK=EFOLO
    IXPEAK=IXOLD
    GO TO 2300
2220. EFPEAK=EFNEW
    IXPEAK=IXNEW
    EFOLD=EFPEAK
    IXOLO=IXPEAK
2300 IF (LPRINT8 LT. 1) GO TO 2203
            WRITE{6,2260) R,EF(J),J,DELR,DELIMR,RS,X
    2260 FORMAT(1X,"R= ",F12.5,"EF= ",F12.5,"J= ",I3,"DELR= ",
    * F12.5;
    * "DELIMR=*,F12.5,*RS= ",F12.5,* X=*,F8.2)
        URITE(6,2280) TT3;TT3LIM,TT5LIMTTT4;TT4LTM
2280 FORMAT(1X,5F12.4)
    2283 IF (EF(J) LE. D.OO1) GO TO 3500
    3000 CONTINUE
3500 RPEAK=RP(IXPEAK)
    IF (LPRINT9 &T. i) GO TO 5500
    WRITE(6,2285) RPEAK
2285 FORMAT(1X,"HE IS RPEAK=",F12,4)
    WRITE(6,4000) DELIMR
```

```
4000 FORMATILX, OELIMR IS OUT OF LIMIT =**FIE.4)
    WFITE(6,5000) EFPEAK%#
500!. FORMAT (1X, "EFPEAK=* F12.4)
5500 IXNSTAR (NC)=IXPEAK
    RNSTAR(NC)=RPEAK
    ENSTAR(NC) =EFPEAK
    IF(NC.EQQ &) GO T0 64D0
    IF(ENSTAR(NC=1). GE ENSTAR(NC)) GOTO 65%0
    CONTINUE
    NSTAR=NCON - 1
        EFSTAF=ENSTAR (NSTAR)
        IXSTAR=IXNSTAR(NSTAR)
        RSTAR=RNSTAR (NSTAR)
        R'ETURN
        END
```

            FUNCTION F (R)
        COMMON/VARI GMFEED,LOFT (5E).H6.LTEMP,TTI.LTSONE
        COMMON/CONT/TO,SH, TBGEREFT (50), EFSTAR,EBEFI(50),
        - ETA(50)
        COMMONVRANKIN/ WTOP(20), WCEN(20), WE20), WEOT(20).
            TWORK 20 ol
            OSUH(20), HH(220),SS(220), OMEGA(20)
            TSUMK20n,TS(20)
    *, PP (20), HO, LTO, LAST,SO, III
        COMMONGBRAYTO/TT3LIM,TTSLIM,X,TT3,DELIMR,NCOM,TT4,
            - TTLLIM
        DIMENSION PRES(10), PRATIO(10)
        H1 \(=\mathrm{HO}\)
        \(T=T B\)
        H16FHL (T)
        HB=H1L
        RATIO=R
        R=RATID
        \(\mathrm{N}=\mathrm{NCOM}\)
        TTZLIM \(=10+50+460\)
        GAMA \(=1\). 66
        CP=1. 24
        \(T E=0.9\)
        \(C E=0.9\)
        \(Y I=(G A M A, ~ 21) / G A M A\)
        RI \(=\) R**YI
        \(P P 7=710.4(0.01 * X)\)
        \(R 78=1 .+(0.01 * x) / 710\).
        RLOSS=1.
    ```
    IF (N,ERO1).60 FO 110
    PNES{N) =PP7/F
    DO 100'IS=2,N
    I =N+2-IS
    PRATIO(I)=1* +i,0*(R**N)/(PRES(I)*&R**(I-I))
    RLOSS=RLOSS*PRATIO(I)
    PRES(I-1) =(PRES(I) 41.0* (R**N|/(R**(I*1)))/R
    100 CONTINUE
    PP4=(PRES(2)+1*[*(R*#N)/R)/R
    GO TO 120
110 PP4=PP7/R
    120 R94=10 +1.0* (R**N)/PP4
```



```
    R23 =1. +0001*X: *(RP4+400*{R*4N3)
    TLOSS=RLOSS*(710%/7COE)*(R78)*(R94)*(R39)* (R23)
        B=TLOSS**YI
    NTTECP*TT1*TE*(1-E/(N1**N))
    TT2=TT1-WTT每(1/CP)
    TT3=TT2-X
    COMP= (1/CE):* (R1-1)
    DENOM=(HI-HB)+ (N*1)*(14COMP)*(HB-HG)*(1.E/N)
    FNUM1 =CP*TTK
    FNUM2 =CP* (N-1)* (TB+510)*(1+COMP)
    G=(FNUM1 +FNUMZ =N*CP* (TE+516))/DENOM
    TTP=TB+510
    TT4=TTP=(G/(CP*N))*(HE-H6)
    TT5FTT4 +TT4*COMP
    TT5LIM=TT5+50.
    IF((TTJ - TTSLIM) &T. (TT3 -TT5LIM) G0 T0 187
    GO TO 186
187 DELIMR=TT3 -TT3LIM
    60 TO 190
188 DELIMR=TT3 -TTSLIM
    190 TT8=TT5+TT2-TT3
    HCC=TT4*COMP*CP*N
    QBRAY=CP& (TTI-TTB)
    EFBRAY=(WTT-WCC)/OBRAY
    TS(1)=750.-20.*LTSONE
    T6=TS(1) #10.
    TT4LIM=T6 + 510.
    IF(TT4 ELTG TTHLIM, GOTO 250
    GOTO 300
    250 EFBRAY=0.01
    300 F=EFBRAY
    RETURN
    END
```




|  | 100000 | 1.0000 | 1.6005 | 1.0.040 | 1.0004 | 1.0000 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | 00 |  |  |
| 1.0000 | 1.00 | 1.0 | 00 | 1.0000 | 0000 | 1.0000 | 00 | 00 |  |
| 1.0000 | 1.0300 | 1.0600 | 1.0000 | 1.00000 | 1.0000 | 1.0000 | 1.0600 | 2.0090 | 00 |
| 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0C80 | 1.0000 | 1.0000 |  |
| 1.0400 | 1.03 .00 |  |  | 1.0000 | . 0000 | 1.0000 |  |  |  |
| 2.3181 | 1.9303 | 1.8526 | 1.8066 | 1.77315 | 1.747 | 1.0000 | 1.0 | 1. |  |
| 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | . 0000 |
| 1.0000 | 1. COOC | . 0000 | 0000 | 1.0000 | . 0000 | 1.0000 | 1.0000 | 1.9CO | 0 |
| 1.0000 | 1.0900 |  |  | . 00 | 00 | 1.000 | 1.000 | OCO | 0 |
| 1.0000 | 1.0000 | 1.00 | 1.0000 | 1.0.000 | 1.0029 | 1.0000 |  |  |  |
| 2.3492 | 1.9617 | 1. 8844 | 1. 0387 | 1.8062 | 1.78 c 5 | 1.6992 | 1.6 | . 0 |  |
| 1.0000 | 1.0000 | 1.0000 | 1.0C00 | 1.0000 | 1.0000 | 1.0090 | 1.000. | 1.0000 | 0005 |
| 1.0000 | 1.0300 | 1.000 C | 1.0600 | 1.0400 | . 0000 | 1.0000 | 1.0000 | 1.0600 | 003 |
| 1.0000 | 1.0000 | 1.0600 | 1.0000 | 1.6000 | 1.0003 | 1. 0000 | 1.0400 | 1.0000 | 00 |
| 1.0000 | 1.0000 | 1.0600 | 120600 | 1.0000 | 1.0000 | 1.0c00 |  |  |  |
| 2.3783 | 1.9912 | 1. 9141 | 1.6687 | 1.8364 | . 8111 | 2.7312 | 1.682 | 1.647 | 181 |
| 1.5943 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | . 2000 | 1.0000 | 1.C60 | 1.0000 | 000 |
| 1.0000 | 1.0000 | 1.0000 | 1.8000 | 1.ECOO | 1.0000 | 1.0000 | 1.ccoo | 1.ccen | 0 ? |
| 1.0000 | 1.0000 | 1.0600 | 1.0.00 | 1.0.000 | 1.0003 | 2.0000 | 1.0600 | 1.0000 | 00 |
| 1.0000 | 1.0000 | 1.0000 | 1.0.0t | 1.0000 | 1.0000 | 1.0060 |  |  |  |
| 2.4060 | 2.0190 | 9421 | 897 | 1.8648 | 1.8397 | 1.760 8 | 1.723 | 1.6790 | 16 |
| 1.6286 | 1.6085 | 1.5906 | . 5743 | 1.5593 | . 5453 | 1. 0100 | 1.0000 | 1.0000 | 00 |
| 1.0300 | 1.0300 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | . 0000 |
| 1.4000 | 1.0000 | 1.0800 | 0005 | 1.0000 | 1.0000 | 1.0000 | 1.000 | 1.0000 | 1.0cor |
| 1.00 | 1.0000 | 1.0600 | 00 | . 20 | . 0000 | 1.0600 |  |  |  |
| 2.43 | 2.0455 | 1.9687 | 9236 | 1.8 | 1.8665 | 1.783 | 1.7 | 1.7 | 1 |
| 1.6592 | 1.6480 | 1. 6231 | . 6678 | 1.5938 | 1.5806 | 1.5687 | 1.557 | 1.5464 | 61 |
| 1.5119 | 1.4894 | 1.0000 | 1.0008 | 1.000 C | 1.0000 | 1.0000 | 1.0000 | 1.0000 | ct |
| 1.0000 | 1.0304 | 1.0600 | 1.0000 | 1.0000 | 1.0005 | 1.0000 | 1.0000 | 1. |  |
| 1. | 1.0000 | . 0600 | 0000 | 1.0000 | - | 1.0000 |  |  |  |
| 2.45 | 2.0707 | 1.9940 | 1.9490 | 1.9 | . | 1.814 | 1.7 | 1. |  |
| 1.6872 | 1.6686 | 1.6522 | 1.6376 | 1.6242 | 1.6120 | 1. 6006 | 1.5699 | 1.5798 | 1.5703 |
| 1.5483 | 1.5282 | 1.5496 | 1.4921 | 1.4753 | 1.4590 | 1.6006 | $1 . C 000$ | 1.0600 | $0{ }^{\text {r }}$ |
| 1. | 1.0 | 1.0 | 1.0006 | 1 | 00 | 1.0000 | 1.0000 | 1. | 1.0000 |
| 1.000C | 1.0000 | 1.06C5 | 1.0005 | 1.0000 | 1.0000 | 1.0000 |  |  |  |
| 2.4672 | 2.0804 | 2.0038 | 1.9588 | 1.9268 | 1.9320 | 1.8243 | 1.77 .8 | 1.7 | 93 |
| 1.6978 | 1.6793 | 1.6632 | . 6487 | 1.6356 | 1.6235 | 1.6123 | $6 \subset 1$ | 1.5920 | 827 |
| 1.5613 | 1.5420 | 1.5241 | . 5074 | 15 | 1.4762 | 1.4468 | 1.4182 | 1.0000 | . 0000 |
| 1.0000 | 1.0000 | 1.0000 | 1.0600 | 1.0000 | 1.0000 | 2.0000 | 1.0000 | 1.300 | 1.000 |
| 1.0000 | 1.0000 | 1.0000 | 1.000c | 1.0000 | 1.0000 | 1.9COO |  |  |  |
| 2.4768 | 2.0900 | 2.0134 | 9684 | 65 | 1.9117 | 1.8341 | . 7882 | 1.755 | 94 |
| 1.7081 | 1.6898 | 1.6738 | 1.6595 | 1.6465 | 1.6346 | 1.623 | 1.6133 | 1.6037 | 1.5946 |
| 1.5737 | 1.5549 | 1.5377 | 1.5216 | 1.5668 | 1.4719 | 1.4644 | 1.4381 | 1.4120 | 00 |
| 1.0000 | 1.0606 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0E00 | 1.0000 | . 0000 |
| . 0600 | 1.0000 | 1.0000 | 1.0000 | 1. 1.000 | 2.0400 | 1.0000 |  |  |  |
| . 4862 | 2.0994 | 2.0228 | 1.9779 | 60 | . 9212 | 1.8437 | 9.7979 | 1.7651 | . 7394 |
| 1.7182 | 1.7000 | 1.6041 | 1.669 | 1.6571 | 1.6493 | 1. 6345 | 1.6244 | 1.6149 | . 6059 |
| 1.5855 | 1.5672 | 1.5505 | 1.5349 | 1.5203 | 1.5065 | 1.4.605 | 1.4558 | 1.4320 | 82 |
| 1.3840 | 1.0000 | 1.0000 | 1.0000 | 1.6000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | Cb |
| 1.0000 | 1.0000 | 1.0000 | 0096 | 1.0000 | - 0000 | 1.9000 |  |  |  |
| 2.4954 | 2.1087 | 2.0321 | 1.9872 | 1.9553 | 1.9365 | 1. 8.531 | 1.8675 | 1.774 | .7491 |
| 1.7280 | 1.7099 | 1.6941 | 1.6801 | 1.6674 | 1.6557 | 1. 6450 | 1.6350 | 1.6257 | . 6169 |
| 1.5968 | 1. 5789 | 1.5626 | 1.5475 | 1.5334 | . 5201 | 1.4952 | 1.4720 | 1.4498 | 4281 |
| 2.4 | 1.3843 | 1.3612 | . 0000 | 1.0000 | 1.0000 | 1.0000 | 1.00 | 1.0 | , 000 |
| 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |  |  |  |
| 2.5045 | 2.1178 | 2.0413 | 1.9964 | 1.9645 | 1.9397 | 1.8624 | 1.8268 | 1.784 | . 7586 |
| 1.7376 | 1.7196 | 1.7039 | 1.6906 | 1.6773 | 1.6658 | 126552 | 1.6453 | . 6361 | 74 |
| 1. 6077 | 1. 5901 | 1.5741 | 1.5595 | 1.5458 | 1.532 | 1.5030 | 1.4869 | 1.4659 | 1.4457 |
| 1.4259 | 1.4061 | 1.3860 | 1.3652 | 1.3431 | 1.0000 | 1.0000 | 1.6000 | 1.0000 | 1.8000 |
| 1.0000 | 1.0000 | 1.0006 | 1.0000 | 1.ector | 1.0000. | 1.0000 |  |  |  |
| 2.5135 | 2.1268 | 2.0503 | . 0054 | 1.9735 | 1.948 d | 1.8715 | 1.8260 | 1.7 | 1.7630 |
| 1.7470 | 1.7291 | 1.7135 | 1.6996 | 1.6871 | 1.6757 | 1. 6651 | 1.6554 | 1.46462 | 1.6376 |
| 1.6182 | 1.6009 | 1.5852 | 1.5709 | 1.5575 | 1.5458 | 1.5219 | 1.5007 | 1.4807: | 1.4617 |
| 1.4433 | 1.4252 | 1.4071 | 1.3888 | 1.3699 | 1.3503 | 1.3442 | 1.0000 | 1.0000 | 1.0000 |
| . 00 | 1.0000 | 1.0600 | 0800 | . 0000 | 1.0000 | - 060 |  |  |  |


| 2.5224 |  |  | 2.0143 |  | 1.9577 | 93 | 0 |  | 1.7771 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.7562 | 1. | 1 | 1. |  |  | 1.6748 |  |  | 76 |
| 1.6283 | 1.6113 | \%. 5959 | 1.5810 | 1.5687 | 1.5565 | 1.5341 | 1.5136 | 1.4945 | 1.4764 |
| 1.4590 | 1.4422 | 1.4255 | 1.4090 | 1.392 | 1. 3752 | 1.3575 | 1.3390 | 1.3192 | 1.2972 |
| 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.000c | 1.0000 | 1.0000 |  |  |  |
| 2.5311 | 2.1444 | 2160679 | 2.0236 | 169912 | 1.9664 | 1.8893 | 1.8439 | 1.8114 | 1.7861 |
| 1.7653 | 1.7475 | 147320 | 1.7183 | 1.7359 | 1.6946 | 1.6842 | 1.6746 | 1.6656 | 1.6572 |
| 1.6382 | 1. 6213 | 1.6062 | 1.5923 |  |  | 1.5457 | 1.5258 | 587 | 0 O |
| 1.47 | 1.4576 | 1.4421 | 1.4268 | 1.4116 | 1.3963 | 1.3808 | 1.3649 | 1.3 | 1.3309 |
| 1.2320 | 1.0000 | 1.0000 | 1.06C0 | 1.0000 | 1.00CO | 1.0000 |  |  |  |
| 2.5397 | 2.1530 | 2,06765 | 2.0316 | 1.9998 | 1.9751 | $1.898 i$ | 1.8526 | 1.8202 | -7949 |
| 1.7741 | 1.7.764 | 1.1741 C | 1.7273 | 1.7150 | 1. 70.38 | 1.6935 | 1.68839 | 1.6750 | 1.6665 |
| 1.6477 | 1.6311 | 1.6161 | 1.6025 | 1.5899 | . 5781 | 1.5567 | 1.5374 | 1.51 .95 | 1.5028 |
| 1.48 | 1.4716 | 1.4572 | 1.4428 | 1.4287 | 4147 | 1.4067 | 1.386 | 1.372 | 1.3574 |
| 1.3258 | 1.2900 | 1.2451 | 1.0000 | 1.0000 | 1.0009 | 1.060 |  |  |  |
| 2.54.81 | 2.1615 | 2.0850 | 2.0401 | 2.0063 | 1.9836 | 1.9665 | 1.0612 | 1.8289 | 36 |
| 1.7829 | 1.7652 | 1.7499 | 1.7362 | 1.7239 | 1.7120 | 1.7025 | 1.6930 | 1.6041 | 758 |
| 1.65 | 1.6406 | 1.6258 | 1.6123 | 1.5999 | 1.588 | 1.5673 | 1.5484 | 1.5311 | 1.5149 |
| 1.4996 | 1.4851 | 1.4711 | 1.4757 | 1.4443 | 1.4312 | 1.4183 | $1.4 C 54$ | 1.3925 | 1.3794 |
| 1.3523 | 1.3232 | 1.2948 | 1.2527 | 1.0000 | 1.0000 | 1.0000 |  |  |  |
| 2.5565 | 2. 1699 | 2.6933 | 2.9485 | 2.0167 | 1.9920 | 1.9250 | 1.8697 | 1.837 | 1.8322 |
| 1.7915 | 1.7739 | 1.7585 | 1.7449 | 1.7327 | 1.7216 | 2.7113 | 1.7019 | 1.6931 | 1.6848 |
| 1. 6662 | 1. 6499 | 1,6352 | 1.6219 | 1.6096 | 1.5982 | 1.5776 | 1.5596 | 2.5421 | 1.5263 |
| 1.5116 | 1.4975 | 1.4641 | 1.4711 | 1.4586 | 1.4463 | 1.4342 | 1.4222 | 1.4103 | 1.3984 |
| 37 | 1.3494 | 1.3228 | 1.2938 | 1.2618 | 1.2193 | 1.0006 |  |  |  |
| 2.5 | 2. 17 | 2.1616 | 2.0568 | 2.0250 | 2.0003 | 1.9233 | 1.8780 | 1.8458 | 1.864 .6 |
| 1.7 | 1.7824 | 1.9671 | 1.7535 | 1.7413 | 1.7302 | 1.7200 | 1.7 | 1.7019 | 1.6937 |
| 1.6752 | 1.6589 | 1.6444 | 1.6312 | 1.6190 | 1.6078 | 1.5874 | 1.5692 | 1.5526 | 1.5373 |
| 1.52 | 1.5693 | 1.4963 | 1.4839 | 1.4718 | 1.4501 | 1.4487 | 1.4374 | 1.4263 | 1.4253 |
| 1. | 1.3711 | 1.3482 | . 3241 | 1.2982 | 1. 2698 | 1.2373 |  |  |  |
| 0.6 | 2.1863 | 2.1697 | 2.0649 | 2.0331 | 2.0084 | 1.9315 | 1. 8863 | 1.854 | 1.8289 |
| 1.8083 | 1.7907 | 1.7755 | 1.7619 | 1.7498 | 1.7387 | 1.7286 | 1.7192 | 1.7105 | 1.7023 |
| 1.6839 | 1.6678 | 1.6533 | 1.6402 | 1.E283 | 1.6171 | 1.5970 | 1.5791 | 1.5628 | 1.5478 |
| 1.53 | 1.5205 | 1.5079 | 1.4959 | 1.4843 | 1. | 1.4621 | 1.4 | 1.4409 | 1.4306 |
| 1.41 | 1.3899 | 1. 3694 | 1.3484 | 1.3255 | 1.3033 | 1.2786 |  |  |  |
| 0.0 | 2.1983 | $2{ }^{1} 1178$ | 2.6734 | 2.0412 | 2.0165 | 1.9396 | 1.8944 | 1.8622 | 1.8371 |
| 1.8165 | 1.7990 | 1.4837 | 1.7702 | 1.7581 | 1.7471 | 1.7370 | 1.7276 | 1.7189 | 1.7108 |
| 1. 6925 | 1.6764 | 1.6621 | 1.6491 | 1.6372 | . 6262 | 1.6C64 | 1.5837 | 1.5726 | 79 |
| 1. 5441 | 1.5312 | 1.519 c | 1.5073 | 1.4961 | 1.4852 | 1.4747 | 1.464 | 1.4545 | 1.4447 |
| 1.42 | 1.4066 | 1.3678 | 1.3688 | 1.3495 | 1. 3297 | 1.3090 |  |  |  |
| 0.0 | 2. 2022 | 2.1257 | 2.0809 | 2.0491 | 2.0244 | 1.9476 | 1. 9024 | 1.8702 | 1.8451 |
| 1. 8246 | 1.8071 | 1.7919 | 1.7784 | 1.7663 | 1.7553 | 1.7452 | 1.7359 | 1.727 .3 |  |
| 1.7109 | 1.6850 | 1.6707 | 1.6578 | 1.6460 | 1.6351 | 1.6154 | 1. 5980 | 1.5822 | 1.5677 |
| 1.5542 | 1. 5415 | 1.5296 | 1.5182 | 1.5073 | 1.4968 | 1.4867 | 1.4768 | 1.4672 | 1.4578 |
| 1.4395 | 1.4217 | 14.4442 | 1.3867 | 1.36 .92 | 1.3515 | 1.3334 |  |  |  |
|  | 2.2101 | 2.1336 | 2.0888 | 2.0570 | 2.0323 | 1.9554 | 1.9103 | 1.8781 | 1.8531 |
| 1.8325 | 1.8151 | 4.7999 | 1.7865 | 1.7744 | 1.7634 | 1.7534 | 1.7441 | 1.7355 | 1.7274 |
| 1.7692 | 1.6933 | 1.6792 | 1. 6.663 | 1.6546 | 1.6430 | 1.6243 | 1.8070 | 1.591 .4 | 1.55771 |
| 1.5639 | 1. 5515 | 1.5598 | 1.5206 | 1.5180 | 1.5078 | 1.4980 | 1.4884 | 1.4792 | 1.4701 |
| 1.4526 | 1.4357 | 1.4191 | 1.4020 | 1.3866 | 1.370.5 | 2.3541 |  |  |  |
| O.1 | 2.2178 | 2.1413 | 2.0965 | 2.0648 | 2.0401 | 1.9632 | 1.9181 | 1.8860 | 1.8609 |
| 1.8404 | 1.8230 | 1. 8078 | 1.7944 | 1.7824 | 1.7714 | 1.7614 | 1.7521 | 1.7435 | 1.7355 |
| 1.7174 | 1.7016 | 14.6874 | 106747 | 1.6630 | 1.6523 | 1.6330 | 1.6159 | 1.6004 | 1.5863 |
| 1.4733 | 1. 5611 | 1. 1.5496 | 2.5387 | 1.5283 | 1.5184 | 1.5088 | 1.49 | 1.49 | 1.4818 |
| 1.4649 | 1.4487 | 1.4329 | 1.4175 | 1.4024 | 1.3873 | 1.3723 |  |  |  |
| 0.0 | 2.2255 | 2.1490 | 2.1042 | 2.0724 | 2.0477 | 1.9709 | 1.9658 | 1.8937 | 1.8687 |
| 1.8482 | 1.8308 | 1.8157 | 1.0023 | 1.7902 | 1.7793 | 137693 | 1.7604 | 1.7515 | 1.7435 |
| 1.7254 | 1.7096 | 1.6956 | 1.6829 | 1.671 .3 | 1.6607 | 1.6415 | 1.6245 | 1.6492 | 1.5953 |
| 1.5824 | 1.5704 | 1.5591 | 1.5.484 | 1.5382 | 1.5285 | 1.5192 | 1. 5101 | 1.5014 | 1.4929 |
| 1.4766 | 1.4610 | 1.4459 | 1.4312 | 1.4168 | 1.4026 | 1.3896 |  |  |  |
| 0.0 | 2. 2330 | 2.1.566 | 2.1118 | 2.0800 | 2.0553 | 1.9785 | 1.9334 | 1.9013 | 1.8764 |
| 1.8559 | 1.8385 | 10 | 1.0100 | 1.7980 | 1.787i | 1.7771 | 1.07679 | 1.7593 | 1.7513 |
| 1.7333 | 1.7176 | 1.7036 | 1.6910 | 1.6795 | 1.6689 | 1.6498 | 1. 6330 | $1.617 \%$ | 1.6040 |
| 1.5913 | 1.5795 | 1.5683 | 1.5578 | 1.5479 | 1.5383 | 1.5692 | 1.5204 | 1.5118 | 1.5036 |


| 1.487 |  | 14481 | 1.4440 | 1.4303 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6.0 | 2.2405 | 261640 | 2.1293 | 200875 | 2.0620 | 1.9.9860 | 1.9410 | 1.9089 | 1.8839 |
| 1.8635 | 1.8461 | 148310 | $1{ }^{*} .8176$ | 1.8057 | 1.7946 | 1.784 | 1.7756 | 1.7671 | 1.75 .91 |
| 1.7411 | 1.7255 | 1.7115 | 1.6990 | 1.6875 | 1.676 | 1.6580 | 1.6413 | 1.626 | 1.6126 |
| 1.6000 | 1.5883 | 1.5773 | 1.5670 | 1.5572 | 2.5478 | 1.5388 | 1.5302 | 1.5819 | 1.5136 |
| 1.4984 | 1.4837 | 1.4696 | 1.4561 | 1.44429 | 1.4308 | 1.4174 |  |  |  |
| 0.0 | $2.247 \%$ | 2.1715 | 2.1267 | 26949 | 2.0782 | 1.9935 | 1.9484 | 1.916 | 4 |
| 1.87 | 1.8536 | 1.8385 | 1.8252 | 1.8132 | 1.8024 | 92 | 1.7832 | 1.774 |  |
| 1.7488 | 1.7332 | 1.7193 | 1.7068 | 1.6954 | 1.6849 | 1.6660 | 1.6494 | 1.634 | 1.6210 |
| 1.6085 | $1.5970^{\circ}$ | 1.54861 | 1.5759 | 1.5662 | 1.5570 | 1.5482 | 1.5397 | 1.5316 | 1,5237 |
| 1.5186 | 1.4943 | 1.4807 | 1.4675 | 1.4548 | 1.4425 | 1.4384 |  |  |  |
| C.f | 2.2552 | 2.1788 | 2.1 .340 | 2.1022 | 2.0776 | 2.0003 | 1.955 | 1.7 | -878 |
| 1.8784 | 1.8610 | 1.8465 | 1.8327 | 1.8207 | 1.8099 | 1.7999 | 1.7908 | 1.7822 | 1.7743 |
| 1.7564 | 1.7409 | 1.7270 | 1.7145 | 1.7032 | 1.6927 | 1.6739 | 1.6574 | 1.6427 | 1.6292 |
| 1.6169 | 1.6054 | 1.5947 | 1.5846 | 1.5751 | 1.5660 | 1.5573 | 1.5490 | 1.541 | 1.5332 |
| 1.5185 | 11.5045 | 1.4913 | 1.4.785 | 1.4.4662 | 1.4543 | 1.4427 |  |  |  |
| 0.6 | 2. 2625 | 2.1860 | 2.1.413 | 2.1095 | 2.0948 | 2.0082 | 1.963 | 1.9310 | 1.9061 |
| 1.6857 | 1.8684 | 1.8533 | 1.8400 | 1.8281 | 1.8173 | 1.8073 | 1.7992 | 1.78 .97 | 1.7818 |
| 1.7640 | 1.7484 | 1.7346 | 1.7222 | 1.7108 | 2.7004 | 1.6517 | 1.6653 | 1.6506 | 6373 |
| 1.6250 | 1.6137 | 1.6631 | 1.5931 | 1.5837 | 1.5747 | 1.5662 | 1.558 c | 1.5521 |  |
| 1.5 | 1.5144 | 1. 5014 | 1.4890 | 1.4771 | 1.4655 | 1.4543 |  |  |  |
| 0.0 | 2.2697 | 2.1932 | 2.1484 | 2.1167 | 2.0920 | 2.0153 | 1. | 1.9383 | 34 |
| 1.8929 | 1.8756 | 1.8606 | 1.8473 | 1.8354 | 1. 8246 | 1.8147 | 1.8055 | 1.7971 | 1.7891 |
| 1.7714 | 1.7559 | 2.7421 | 1.7297 | 1.7184 | 1.7086 | 1.6894 | 1.6732 | 1.6585 | 6452 |
| 1.6331 | 1.62 | 1.61 | 1.6014 | 1.5921 | 1.5833 | 1.5748 | 1.5667 | 1.5590 | 1.5515 |
| 1.5373 | 1.5239 | 1.5113 | 1.4991 | 1.4875 | 1.4763 | 1.4654 |  |  |  |
| 1.1 | 2.2767 | 2.2603 | 2.1555 | 2.1238 | 2.0991 | 2. 0224 | 1.9 | 1.9 | . 9205 |
| 1.9001 | 1.8828 | 1.8678 | 1.8545 | 1.8426 | 1.8318 | 1.8219 | 1.8128 | 1.864 | 964 |
| 1.778 | 1.76 | 1.74 | 1.7371 | 1.7258 | 1.7155 | 1.6970 | 1.6807 | 1. 6662 | 30 |
| 1.6410 | 1.6298 | 1.6194 | 1.6096 | 1.6004 | 1.5916 | 1.5833 | 1.5753 | 1.5677 | 1.5603 |
| 1.5463 | 1.5332 | 1.5208 | 1.5089 | 1.4976 | 1.4866 | 1.4751 |  |  |  |
| 1.0 | 2.3122 | 2.2347 | 2.1.90e | 2.1532 | 2.1336 | 2.0569 | 2.t120 | 1.980 | 52 |
| 1.9680 | 1.9598 | 1.9627 | 1.8884 | 1.8776 | 1.8588 | 1.8570 | 1.6490 | 1.9395 | 1.8317 |
| 1.8141 | 1.7988 | 1.7852 | 1.7730 | 1.7619 | 1.7517 | 1.7335 | 1.7175 | 1.7c33 | 1.6905 |
| 1.6787 | 1.6679 | 1.6578 | 1.6484 | 1.6395 | 1.6312 | 1.6232 | 1.6156 | 1.5684 | 1.6014 |
| 1.58 a3 | 1.5761 | 1.5646 | 1.5537 | 1.5434 | 1.5335 | 1.5240 |  |  |  |
| 68.00 | $0 \quad 72.99$ | 977.98 | 382.97 | 87.97 | 92.96 | 197.96 | 102.95 | 167.95 | 112.9 |
| 117.95 | 122.95 | 5127.96 | 132.96 | 137.97 | 142.99 | 9148.00 | 153.02 | 158.04 | 163.0 |
| 168.099 | 173.12 | 2178.15 | 5183.19 | 188.23 | 173.28 | 198.33 | 203.39 | 208.45 | 213.5 |
| 218.59 | 223.67 | 7 228.76 | 633.85 | 238.95 | 244.05 | + 249.17 | 254.3 | 259.4 | 264.5 |
| 269.7 | 274.6 | 280.0 | 285.2 | 290.4 | 295.6 | 300.8 | 306.05 | 311.3 | 6. 5 |
| 321.8 | 327.05 | 5332.3 | 337.6 | 342.9 | 348.25 | 5353.6 | 358.95 | 364.3 | 369.7 |
| 375.1 | 380.5 | 386.0 | 391.4 | 396.9 | 402.4 | 407.9 | 413.5 | 417.0 | 424.5 |
| 430.2 | 435.85 | 5441.5 | 447.25 | 452.9 | 458.7 | 464.5 | 470.3 | 476.1 | $482 . \mathrm{C}$ |
| 487.9 | 493.9 | 499.9 | 545.9 | 512.0 | 518.1 | $5<4.3$ | 530.4 | 536.8 | 543.1 |
| 549.5 | 559.9 | 562.4 | 560.9 | 575.6 | 582.35 | 5 589.1 | 595.9 | 6t2.9 | . 8 |
| 617.1 | 624.4 | 631.8 | 639.3 | 646.9 | 654.7 | 662.7 | 670.75 | 679.1 | 7.65 |
| 696.4 | 705.55 | 5714.9 | 725.0 | 735.8 | 746.95 | 758.5 | 770.8 | 784.5 | 790.5 |
| 797.1 | 804.4 | 812.6 | 822.4 | 835.0 | 854.2 | 906.0 |  |  |  |
| 1105.1 | 1107.2 | 1109.3 | 1114.5 | 1113.6 | 1115.7 | 1117.8 | 1119.9 | 1122.0 | 1124.0 |
| 1126.1 | 1126.2 | 1136.2 | 1132.2 | 1134.2 | 1136.2 | 1130.2 | 1140.2 | 1142.1 | 1144.0 |
| 1146.0 | 1147.9 | 1149.7 | 1154.6 | 1153.4 | 1155.3 | 1157.1 | 1158.85 | 1160.6 | 1162.3 |
| 1154.0 | 1162.3 | 1167.4 | 11619.0 | 1170.6 | 1172.2 | 1173.8 | 1175.3 | 1176.8 | 1178.? |
| 1179.7 | 1181.1 | 1182.5 | 1183.8 | 1185.2 | 1186.4 | 1187.7 | 1188.9 | 1196.1 | 1193.4 |
| 1192.3 | 1193.4 | 1194.4 | 1195.4 | 1196.3 | 1197.2 | 1198.5 | 1198.8 | 1199.6 | 1200.3 |
| 1201.0 | 1201.6 | 1202.1 | 1202.7 | 1203.1 | 1203.5 | 1203.9 | 1204.1 | 1204.4 | 1204.6 |
| 1204.7 | 1204.8 | 1204.8 | 1204.7 | 1204.6 | 1204.3 | 1204.1 | 1203.7 | 1203.3 | 1202.8 |
| 1202.2 | 1201. 5 | 1200.0 | 1200.6 | 1199.0 | 1198.0 | 1196.9 | 1195.6 | 1194.3 | 1192.8 |
| 1191.2 | 1189.5 | 1167.7 | 1185.4 | 1183.6 | 1181.31 | 1179.0 | 1176.4 | 1173.7 | 1170.8 |
| 1167.7 | 1164.4 | 1164.9 | 1157.2 | 1153.2 | 1148.9 | 1144.2 | 1139.1 | 1133.7 | 1127.8 |
| 1121.4 | 1114.5 | 1147.0 | 1098.8 | 1089,8 | 1079.71 | 1068.5 | 1055.6 | 1040.6 | 1033.6 |
| 1625.9 | 1017.2 | 2007.2 | 995.2 | 979.7 | 956.2 | 906.1 | - |  |  |
| c. 1295 | 0.1384 | 0.1472 | 0.1559 | 0.1646 | 0.17320 | 0.1817 | 0.1 .901 | 0.1935 | 0.2068 |
| c. 2150 | 0.2232 | 0.2313 | 0.2393 | 0.2473 | 0.25520 | 0.2631 | 0.2709 | 0.2787 | 0.2864 |
| 0.2940 | 0.3016 | 0.3091 | 0.3166 | 0.3241 | 0.33150 | 0.3388 | 0.3461 | c.3533 | 0.3605 |
| 0.3677 | 0.3605 | 0.3819 | 0.3890 | 0.3960 | 0.4029 | 0.4098 | 0.4167 | 0.4236 | 0.4303 |


| . 4372 | 0.4439 | 0.4536 | 0.4573 | 0.4640 | 0.4705 | 0.4772 | 0.4837 | 0.4902 | 0.5097 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6.5 | 0.50i.9 | 0.5161 | 0.5225 | 0.5269 | 0.5352 | 0.5416 | 0.5478 | 0.554 | 0.5604 |
| 0.56 | C. 57 | 0.5791 | 0.5853 | 0.5915 | 0.5976 | 0.6038 | 0.6099 | 0.616 | 0.6222 |
| 62 | 0.63 | 0.640 | 0.6 | 0.65 | 0.65 | 0.654 | 0.67 | 0.67 | 0.6035 |
| 0.68 | 0.695 | 0.7612 | 0.767 | 0.7133 | 0.719 | 0.725 | 0.731 | 0.737 | 6. |
| c. 750 | 0.7562 | 0.7625 | 0.768 | 0.7750 | 0.781 | 0.787 | 0.79 | C. 36 | 0.0069 |
| 0.81 | D. 8200 | 0.826 | 0.833 | 0.840 | 0.847 | 0.854 | 0.861 | 0.96 | 0.8761 |
| c. 8837 | 0.0915 | D. 89.9 | 0.9082 | 0.9174 | 0.926 | 0.936 | 0.946 | 0.9 | 4 |
| 6.9589 | 6.9749 | 0.9818 | 0.9901 | 1.0006 | 1.016 | 1. 0612 |  |  |  |
| 1.9 | 1.970 | 1.957 | 1.945 | 1.933 | 1.922 | 1. 9112 | 1.90 | 1,8 | . |
| 1.86 | 1.858 | 1.8'4 | 1.839 | 1.8295 | 1.820 | 1.8111 | 1.802 | 1.79 |  |
| 1.77 | 1.7681 | 1.760 | 1.752 | 1.744 | 1.736 | 1.7290 | 1.721 | 1.71 | . 707 C |
| 1.7000 | 1. 7070 | 1.686 | 1.679 | 1.672 | 1.666 | 1.6599 | 1.653 | 1.64 | 1.6412 |
| 1.6351 | 1.6291 | 1.623 | 1.617 | 1.E116 | 1.605 | 1.6003 | 1.594 | 1.589 | 1.5731 |
| 1.57 | 1.5731 | 1.567 | 1.5626 | 1.5575 | 1.5523 | 1. 5473 | 1.542 | 1.537 | 1.5323 |
| 1.527 | 1.522 | 1.517 | 1.5127 | 1.5080 | 1.5032 | 1.498 | 1.493 | 1.48 | 1.4843 |
| 1.4797 | 1.475 | 1.470 | 1.465 | 1.4611 | 1.455 | 1.451 | 1.447 | 1.442 | 1.4379 |
| 1.433 | 1.4286 | 1.4240 | . 419 | 1.4146 | 1.409 | 1.405 | 1.46025 | 1.395 | 1.3905 |
| 1.385 | 1.3837 | 1.375 | . 370 | 1.3654 | 1.360 | 1.355 | 1.3496 | 1.344 | 1.3386 |
| 1.333 | 1.3272 | 1.361 | . 315 | 2.309 | . 302 | 1.296 | 1.2893 | 1.2821 | 1.2745 |
| 1.266 | 1.2584 | 1.249 | 1.240 | 1.2307 | 1.220 | 1.208 | 1.195 | 1.181 | 1.1744 |
| 1.167 | 1.1591 | 1.149 | 1.139 | 1.1252 | 1.164 | 1.8612 |  |  |  |
| 6.949 | 1.0079 | 1.069 | 1.134 | 1.2030 | 1.275 | 1.3505 | 1.42 | 1.5 | 1.6099 |
| 1.69 | 1.789 | 1.890 | 1.995 | 2.1068 | 2.223 | 2.3445 | 2.47 | 2.60 | 2.7436 |
| 2.889 | 3,041 | 3.19 | 3.3653 | 3.5381 | 3.718 | 3.9065 | 4.1025 | 4.366 | 4.5197 |
| 4.741 | 4.972 | 5.212 | 5.4623 | 5.7223 | 5.992 | 6. 2736 | 6. 5656 | 6.869 | 7.1945 |
| 7.5110 | 7.8500 | 8.283 | 8.568 | 8.947 | 9.340 | 9.747 | 10.16 | 10.6: | 11.058 |
| 11.526 | 12.511 | 12.512 | 13.031 | 13.568 | 14.12 | 14.69 | 15.289 | 15.90 | 16.533 |
| 17.18 | 17.85j | 18.556 | 19.274 | 20.015 | 20.77 | 21.56 | 22.379 | 23.21 | 24.079 |
| 24. | 25.893 | 26. 62 | 27.797 | 28.796 | 29.82 | 31.88 | 31.97 | 33.09 | 34.243 |
| 35.427 | 36.644 | 37.894 | 39.179 | 40.500 | 41.8 | 43.24 | 44.67 | 46.14 | 47.653 |
| 49.200 | 54.786 | 52.414 | 54.183 | 55.795 | 57.55 | 59.35 | 61.19 | $63 . C 8$ | 65.021 |
| 67.0 | 69.03.0 | 71.119 | 73.251 | 75.433 | 77.667 | 79.953 | 82,293 | 84.68 | 87.137 |
| 89.643 | 92.205 | 94.826 | 97.506 | 164.24 | 103.04 | 155.30 | 168.83 | 111. | 114.873 |
| 117.99 | 121.177 | 12.4.430 | 127.751 | 131.142 | 134.60 | 141 | 149.17 | 156.91 | 164.968 |
| 173.339 | 182.04 | 191.080 | 200.46 | 210.211 | 220.321. | 230.847 | 241 | 252.94 | 264.612 |
| 276.694 | 289.201 | 3t2.143 | 315.529 | 329.369 | 343.674 | 358.46 | 373.72 | 389.49 | 405.76 |
| 422.55 | 439.87 | 457.73 | 476.14 | 495.12 | 524.67 | 534.81 | 555.55 | 576.90 | 598.87 |
| 621.40 | 644.73 | 668.65 | 693.23 | 718.50 |  | 771.15 | 798.55 | 826.69 | 855.58 |
| 885.23 | 915.66 | 946.83 | 978.91 | 1011.75 | 1045.43 | 1679.96 | 1115.3 | 1152.63 | 1188.80 |
| 1226.88 | 1265.a9 | 1305.84 | 1346.7 | 1380.6 | 1431.5 | 1475.4 | 1520.4 | 156E. 3 | 1589.7 |
| 1661. | 1716.8 | 1761.2 | 1812.8 | 1865.6 | 1919 | 1974.7 | 2031. | 20 |  |
| 220 | 2270.3 | 2333.5 | 2398.2 | 2464.4 | 2532.2 | 2601.5 | 267ća | 274 |  |
| 2095. | 2934.5 | 2973.7 | 3013.4 | 3053.6 | 3094.3 | 3135.5 | 3177.2 | 3208.2 |  |
| 85.21 \% | 94.38 | 102.74 | 107.91 | 113.26 | 117.98 | 112.22 | 126.07 | 129.61 |  |
| 135.93 | 236.78 | 141.47 | 144.00 | 146.41 | 148.70 | 150.88 | 152.96 | 154.9 | 156.89 |
| 158.73 | 160.52 | 162.24 | 170.05 | 176.8 | 182.8 | 188.27 | 193.21 | 197.75 | 201.96 |
|  | 219.56 | 213.03 | 216.32 | 219.44 | 222. | 225 | 227. | 230 |  |
| 235.49 | 237.82 | 240.07 | 242.25 | 246.41 | 250.3 | 254.05 | 257 | 260 | 264.17 |
| 267.25 | 270.21 | 273.06 | 275.90 | 270.45 | 281.02 | 283.50 | 235.90 | 288.2 | 290.50 |
| 292.12 | 294.86 | 296. | 290.99 | 300.99 | 302. | 304.83 | 306.69 | 208.51 | 310.29 |
| 312.6 | 313.75 | 315.43 | 317.08 | 318.69 | 320.28 | 321.84 | 323.37 | 324.88 | 326.36 |
| 327.82 | 329.26 | 330.67 | 332.06 | 333.44 | 334.79 | 336.12 | 337.43 | 33 t .73 | 340.01 |
| 341.27 | 342.51 | 343.74 | 344.95 | 346.15 | 347.33 | 348.50 | 349.65 | 350.23 | 351.92 |
| 353.04 | 354.14 | 355.23 | 356.31 | 357.38 | 358.43 | 359.48 | 360.51 | 361.53 | 362.55 |
| 363.55 | 364.54 | 365.53 | 366.50 | 367 | 368. | 369 | 370.31 | 371.24 | 372.16 |
| 373.08 | 373.98 | 374.88 | 375.77 | 376.65 | 377.53 | 378.40 | 379.26 | 380.12 | 380 |
| 381.80 | 383.80 | 385.91 | 387.91 | 389.88 | 391.80 | 393.70 | 395.04 | 377.39 | 399.19 |
| 400.97 | 402.72 | 404.44 | 406.13 | 407.80 | 409.45 | 411.07 | 412.67 | 414.25 | 415.81 |
| 417.35 | 418.87 | 420.36 | 421.04 | 423.31 | 424.75 | 426.18 | 427.59 | 428.99 | 430.36 |
| 431.73 | 434.41 | 437.04 | 439.61 | 442.13 | 444.65 | 447.02 | 449.40 | 451.74 |  |
| 456.28 | 458.56 | 460.68 | 462.82 | 464.93 | 467.01 | 469.05 | 471.07 | 473.05 | 475.01 |
| 476.94 | 478.84 | 481.72 | 482.57 | 484.40 | 486.20 | 487.98 | 489.74 | 491.46 | 493.19 |
| 494.89 | 496.57 | 498.22 | 499.86 | 501.48 | 503.08 | SC4.67 | 506.23 | 507.78 | 509.32 |


| 84 | 512.34 | 513.83 | 515.030 | 516.75 | 21 | 64 | 521.06 | 522.46 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 525.24 | 526.60 | 527.96 | 529.30 | 530.63 | 531.95 | 533.26 | 534.56 | 535.85 | 537.13 |
| 538.39 | 539.69 | 540 -90 | 542.14 | 543.36 | 544.5 | 545.79 | 546.99 | 548.18 | 549.36 |
| 554.53 | 551.70 | 552.86 | 554.00 | 555.14 | 556.28 | 557.40 | 558.52 | 559.63 | 560.73 |
| 561.82 | 562.91 | 563.99 | 565.06 | 566.13 | 567.19 | 568.24 | 569.28 | 570.32 | 571.36 |
| 572.38 | 573.40 | . 574.42 | 5175.42 | 576.42 | 577.42 | 579.40 | 581.35 | 583.28 | 505.18 |
| 87.07 | 588.93 | 590.78 | 5920.61 | 594.41 | 596.20 | 597.9 | 599.72 | 601.4 | 603.17 |
| 604.87 | 606.55 | 608.22 | 609067 | 611.51 | 613.1 | 614.73 | 616.33 | 617.9 | 619.47 |
| 621.02 | 622.55 | 624.07 | 625.58 | 627.08 | 628.56 | 636.04 | 631.50 | 632.94 | 634.38 |
| 635.80 | 637.22 | 630.662 | 640.01 | 641.39 | 642.76 | 644.12 | 645.46 | 646.80 | 648.13 |
| 649.45 | 650.75 | 652.05 | 653.34 | 654.62 | 655.89 | 657.15 | 41 | 59.65 | 88 |
| 662.11 | 663.33 | 664.54 | 665.74 | 666.93 | 668.11 | 669.29 | 670.46 | 671.62 | 672.77 |
| 673.91 | 675.05 | 676.18 | 677.30 | 678.42 | 679.53 | 680.63 | 681.72 | 682.81 | 693.89 |
| 684.96 | 686.03 | 687.09 | 688.14 | 689.18 | 690.22 | 691.26 | 692.28 | 693.30 | 94.32 |
| 695.33 | 696.33 | 697:32 | 698.31 | 699.30 | 700.28 | 701.25 | 701.73 | 702.22 | 702.75 |
| 3. 18 | 703:6 | 704.13 | 704.61 | 705.08 | 705.47 | 0. | B. |  | 0.0 |

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Mr. Robert L. Kim was born on March 27, 1939 In Pohang, Korea. He attended Seoul Middle School in Seoul, Korea and graduated from Seoul High School in 1959. He served in the Korean Army for one year.

He came to the United States as a freshman at Cornell University, where he received a Bachelor of Scfence in Chemical Engineering in 1966.

Mr. Kim continued graduate study at the University of Michigan and received a Master of Science in the Department of Nuclear Engineering in 1968.

Between 1969 and 1972, he was enrolled in M.I.T. for further graduate study in the Department of Nuclear Engineering and met the course standard toward a Doctoral degree.

In 1973, Mr. Kim came to Georgia Institute of Technology to continue his study toward a Doctor of Philosophy in the School of Nuclear Engineering.

During the period of the Doctoral degree, Mr. Kim was supported by institutional funds and a NASA project. .Both were provided as graduate Research Assistantships.

He is married to Christine;Stimind they have two children, Jean and Lynn.


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[^1]:    * This temperature is saturation temperature for stan entering the feedwater heater.

