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SURVEY OF THE LITERATURE

ON DIFFUSERS

12T

A THESIS

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ABSTRACT

The purpose is to review and to abstract the literature concerning diffusers. The study has been restricted to the flow of incompressible fluid in staight wall diffusers, circular and two-dimensional. An attempt has been made to outline the effect of several variables on the performance of the diffuser and on the behavior of the flow.

The energy efficiency of the diffuser and the kinetic energy coefficient at the exit are selected as parameters featuring the quality of a diffuser. The first of these parameters is a measure of the energy conservation and the second is a measure of the exit velocity distribution. The variables influencing these parameters are: the angle between the diverging walls, the area ratio of the diffuser in case of circular diffusers or the length ratio in case of two-dimensional diffusers, the Reynolds number at the entrance section, the kinetic energy coefficient of the entering flow, and the roughness of the walls.

A review of the analytical studies of the flow evolution in an adverse pressure gradient is presented. The various methods of analysis are summarized and criticized. All of the analytical studies are based upon assumptions which lead to an expression for the velocity distribution as a function of a single dimensionless variable. The results of a recent experimental study indicate that a single parameter representation is improbable with the result that the value of these analytical studies is questionable.

A review of the experimental works is then presented. An attempt is undertaken to correlate the data of 10 series of experiments (5 on circular diffusers and 5 on two-dimensional diffusers). The influence of each variable upon the characteristics of the diffuser is pictured in several graphs.

The area ratio is shown to have a small influence on the energy efficiency of small angle, circular diffusers and of wide angle diffusers in which separation occurs at the entrance. If the value of the entrance Reynolds number is greater than about 1×10^5, the effect of this variable, *per se*, is minor. However the entrance kinetic energy coefficient is a major variable, the energy efficiency diminishing rapidly as the velocity distribution becomes less and less uniform.

The variation of the exit kinetic energy coefficient is shown as being a continuous increasing function of the angle, the area ratio, and the kinetic energy coefficient at the entrance. High turbulence, as evidenced by high Reynolds numbers, decreases this coefficient.

Similar results, although different in magnitude are shown for two-dimensional diffusers. A boundary curve between the non separated flow zone and the separated flow zone is presented as a function of the angle and the length ratio.

The review points out the lack of data and the poverty of our knowledge on the behavior of the diverging flow.

INTRODUCTION

For a long time an understanding of the flow characteristics in a diffuser has remained an unsolved problem. Some phenomena, such as the transformation of energy could be easily explained and even computed on the basis of the mechanics of a perfect fluid. However, the results obtained from experiments do not conform to this theory. The presence of a viscous fluid flowing along a diverging boundary explained this discrepancy. This involves the consideration of the behavior of the boundary layer in a significant adverse pressure gradient. A mathematical analysis, derived from Prandtl's mixing-length theory, did not result in a satisfactory solution. Only a qualitative explanation could be given.

The problem of diffuser flow is important. Diffusers have long been widely used in various technical fields. Water tunnels, draft tubes, airplane ducts, wind tunnels and a variety of other similar apparatus require the design of a conduit which will expand the flow most efficiently. A number of experimenters have investigated this problem since the beginning of this century. Several of them have made systematic tests in an attempt to find the effect of the different variables involved on the characteristics of the flow. Others have tried to improve those characteristics by adding special mechanical devices to the diffuser. A third category of researchers, with the help of test data, proposed semi-empirical methods with the view of predicting the evolution of flow through a diffuser.

The purpose of the present work is to review and to abstract the corresponding literature. The work has been restricted to the flow of incompressible fluid in straight-wall diffusers, circular and two-dimensional. An attempt has been made to summarize the available information concerning the effect of several characteristics of the diffuser and of the entering flow on the performance of the diffuser.

SYMBOLS

A	cross sectional area of diffuser
A_r	area ratio
D	diameter of circular diffuser
W	width of two-dimensional diffuser
θ	angle between diverging walls
L	length of diffuser side wall
l_a	length of exit duct required to get complete pressure recovery
$\underline{R}$	Reynolds number $\frac{VD}{\nu}$, for circular diffuser
	$\frac{VW}{\nu}$, for two-dimensional diffuser
ρ	density of the fluid
μ	dynamic viscosity of the fluid
ν	kinematic viscosity of the fluid
g	acceleration of gravity
p^*	piezometric pressure at a point
q	dynamic pressure at a point
P	total energy at a point
u	velocity of the fluid at a point along the x axis
U	velocity outside the boundary layer or maximum velocity at a section
V	mean velocity at a section
x	distance along the axis of the diffuser
x'	distance along the wall
y	distance from the wall perpendicular to the flow
δ	thickness of the boundary layer
δ^*	displacement thickness : $\int_0^\delta \left(1-\frac{u}{U}\right) dy$
δ^{**}	momentum thickness : $\int_0^\delta \left(1-\frac{u}{U}\right)\frac{u}{U}\, dy$
H	form parameter $= \frac{\delta^*}{\delta^{**}}$
α	kinetic energy coefficient $= \frac{1}{A}\int_A \left(\frac{u}{V}\right)^3 dA$
η_E	energy efficiency
τ	shear stress in a fluid at a point
τ_0	shear stress at the wall

Subscripts

1	entrance section of the diffuser
2	exit section of the diffuser

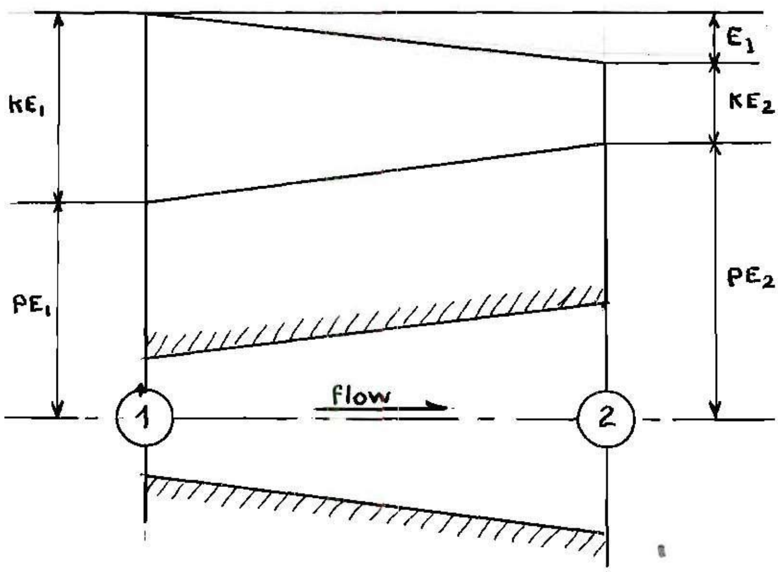
CHAPTER I

PRELIMINARY ANALYSIS

The only thing that a circular and a two-dimensional diffuser have in common is that they both play the same role. Available results show that there is no quantitative similarity between these two kinds of diffusers. The problems involved in each case will be treated separately.

Role of a Diffuser

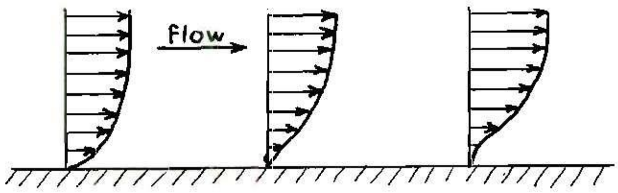
The main purpose of a diffuser is to transform into piezometric energy the kinetic energy of the entering flow. This transformation appears on the following sketch where the kinetic energy KE and the piezometric energy PE at section 1 (entrance) and section 2 (exit) are expres-



sed by the same units. A loss of energy E_1 results from this transformation.

This loss is due to the fact that the problem of diffusers deals with real fluid. This fluid must (1) expand between the diverging boundaris and requires a certain amount of energy to overcome its own viscous shear; (2) flow along walls whose friction retards the stratum of fluid adjacent to the boundary.

The profile formed by the velocity vertors across a section is flattened at the wall as indicated below. The shape of this velocity profile



depends on different variables that we will select later. It may be pointed out that the knowledge of the evolution of this shape is of much interest for various applications. For instance, the overall efficiency of a propeller placed at the exit of a diffuser is definitely affected by the velocity profile in which it works. In the same way, the efficiency of a radiator at the exit of an airplane diverging duct is related to this profile. The effectiveness of a diffuser at the outlet of a highway culvert or similar structure is also related to the velocity profile at th e outlet. Badly distorted velocity profiles may be conducive to channel erosion downstream from the diffuser; whereas, the likelyhood of downstream channel erosion is reduced with more uniform velocity profiles.

Therefore the function of a diffuser is to transfer the kinetic energy in an expanding section into piezometric energy. This function is best accomplished by both an efficient transformation and the production of a velocity distribution at the exit of the diffuser as uniform as possible.

Quantitative Characteristics

Various coefficients can be defined to measure the effectiveness of a diffuser.

Energy Efficiency

Transformation of energy leads to the definition of the energy efficiency. Ordinarily, the efficiency is expressed as the ratio of the quantity of energy received from a machine to the quantity of energy furnished to this machine. In the case of a diffuser, such definition would give:

$$\eta_E = \frac{(KE)_2 + (PE)_2}{(KE)_1 + (PE)_1} \tag{1}$$

But the purpose of a diffuser is more to transform as much as possible of the kinetic energy into piezometric energy than to perform this transformation with the least loss. The energy efficiency is commonly written:

$$\eta_E = \frac{(PE)_2 - (PE)_1}{(KE)_1 - (KE)_2} \tag{2}$$

If we call ρ, p^* and u, the density, the piezometric pressure and the velocity of the fluid at a point respectively, this expression becomes:

$$\eta_{E} = \frac{\int_{A_2} p^{*}u\,dA - \int_{A_1} p^{*}u\,dA}{\int_{A_1} \frac{\rho}{2}u^3 dA - \int_{A_2} \frac{\rho}{2}u^3 dA} , \qquad (3)$$

A_1 and A_2 being the area of the cross sections respectively at the entrance and at the exit.

Kinetic Energy Coefficient

There are several quantities which have been used to define a velocity distribution across a section; such lengths as the thickness of the boundary layer δ, the displacement thickness δ^*, the momentum thickness δ^{**}, or such dimensionless quantities, as the shape factor H or the kinetic energy coefficient α, may be used to get an idea of the evolution of velocity distribution in an expanding conduit. Unfortunately, it is still impossible to differentiate the velocity distribution from the numerical value of any of these quantities. However, in view of comparing data, α, the kinetic energy coefficient, has been chosen and used in the present work to give an approximate idea of the velocity distribution.

The dimensionless quantity α is defined by the expression:

$$\alpha = \frac{1}{A}\int_{A}\left(\frac{u}{V}\right)^3 dA , \qquad (4)$$

in which V is the mean velocity of the flow across the section A. It is obvious that with a perfectly uniform velocity distribution α is unity. As the velocity distribution becomes more and more nonuniform α increases. Of course, it still remains impossible to draw the profile of the velocity distribution which corresponds to a given value of α, but for the different cases encountered in the experiments reviewed there-after, this coefficient appears to be a good instrument of comparison.

In the following work, special attention has been given to the variation of the energy efficiency η_E and of the kinetic energy coefficient α_2 at the exit, because these two are considered characteristic coefficients of a diffuser.

Dimensional Analysis

The dimensional analysis allows us to find the quantities influencing the variation of the two characteristic coefficients. The variables involved are:

(1) for the diffuser, θ, the total angle between the diverging walls, A_1, the area of the entrance section, A_2, the area of the exit section and ε, the roughness height;

(2) for the flow, V_1, the entering mean velocity and α_1, the kinetic energy coefficient at the entrance; and

(3) for the fluid, ρ, the density and μ, the dynamic viscosity.

A possible dimensionless grouping is as follows:

$$\eta_E = \phi(\theta, A_r, \underline{R}_1, \alpha_1, \varepsilon/\lambda) \tag{5}$$

and:

$$\alpha_2 = \phi(\theta, A_r, \underline{R}_1, \alpha_1, \varepsilon/\lambda)\,. \tag{6}$$

A_r is the area ratio written as $A_r = \frac{A_2}{A_1}$; $\underline{R}_1$ is the Reynolds number at the entrance written as $\underline{R}_1 = \frac{V_1 \lambda_1}{\nu}$, where λ_1 is a characteristic dimension of the entrance section, (diameter D_1 for a circular diffuser, width W_1 for a two-dimensional diffuser).

It is reasonable to think that the roughness of the walls has some influence on the behavior of the flow, but the literature concerning diffusers shows a complete lack of data on this subject. This is quite unfortunate because some indication of the boundary roughness would probably explain discrepancies in results of similar experiments.

In fact, other variables should also be taken into consideration because all the conditions of an experiment on adiffuser are not fully defined by equations (5) and (6). Since it is impossible to compare data between different tests, great attention must be paid to the way perfectly similar tests have been run. For instance, experiments show that part of a complete pressure recovery is performed in the exit length of a diffuser. The pressure recovery computed between the entrance and a station in the exit length is higher than that computed between the entrance and the exit of a diffuser. A quantitative illustration in Fig. 1 for two two-dimensional diffusers with and without exit length. On Fig. 2 is shown the distance l_a required to get complete pressure recovery for different angles θ and different entrance kinetic energy coefficients.

It is interesting to note that all the following experiments were run at Reynolds numbers above 10^4. In this range the viscous effects of the fluid are negligible. It is then possible to compare without error data of tests run with air and water, the only fluids used. For air the only limitation is the Mach number which must be less than 0.4. Below this limit air can be considered as an incompressible fluid. Some experimenters instead of blowing the air through the diffuser, drained it by means of a vacuum chamber or by means of a propeller placed at the exit or in an exit length. In the latter case the results are different since the flow is less turbulent and since a backflow after separation impossible.

The same dimensional analysis expressions, (5) and (6), are valid for the two-dimensional diffusers. However the experiments in two-dimensional diffusers, generally run with air, used two diverging plates of constant length including a variable angle for the same series of tests. It appeared more practical to use as variables for the diffuser: θ, the total angle between the diverging plates, L, the length of these plates and W_1, the width of the entrance section. Thus equations (5) and (6) read:

$$\eta_\varepsilon = \phi(\theta , L/W_1 , \underline{R}_1 , \alpha_1 , \varepsilon/W_1) \qquad (7)$$

$$\alpha_2 = \phi(\theta , L/W_1 , \underline{R}_1 , \alpha_1 , \varepsilon/W_1) \qquad (8)$$

L/W_1 is the length ratio of the diffuser. The difference between expressions (5), (6) and (7), (8) does not matter since circular diffusers and two-dimensional diffusers are unrelated as far as quantitatives results are concerned.

It may be pointed out that the term two-dimensional diffuser pictures a theoretical diffuser. In the experiments reviewed, it seems that generally the dimensions of such diffusers are approximately two-dimensional at the entrance, but at the exit one can expect influences of the parallel side walls as the distance between them becomes smaller than that between the diverging ones.

Theoretical Analysis

To predict the variation of the energy efficiency and of the kinetic energy coefficient one may show by a theoretical analysis the influence of each variable.

On the basis of the theory of irrotational flow of a perfect fluid, a diffuser behaves as an ideal diffuser. The kinetic energy coefficient remains constant through the diffuser and is equal to 1. The energy efficiency is 1 since the change in kinetic energy is then exactly equal to the change in piezometric energy. This ideal picture is far from the real one.

It is believed that a better approach to the problem would be found in the solution of the boundary layer equations in an adverse pressure gradient. Unfortunately, up to date, such a problem has defied mathematical analysis and no purely theoretical solution exists.

The flow of a viscous fluid, according to Prandtl's theory, forms through the diffuser, at each section, a central core of constant maximum velocity U between two boundary layers of thickness δ.

First, in the central core, as long as the boundary layers do not join on the axis of the diffuser, there is no shear stress and therefore no loss of energy. Thus Bernoulli's equation is:

$$\frac{\rho}{2} U_1^2 + p_1^* = \frac{\rho}{2} U_2'' + p_2^* \qquad (9)$$

between the entrance and the exit of the diffuser, whereas the piezometric pressure p^* at any given section is constant acrossthe core.

Second, in the boundary layer, the velocity distribution is given by the solution of the Navier-Stokes equations. The dimensional analysis of these differential equations shows that the piezometric pressure across the boundary layer is constant and equal to the piezometric pressure in the central core. Using this notion, often checked by experiments, and furthermore, using expression (4) of the kinetic energy coefficients at the entrance and at the exit of the diffuser, expression (3) of the diffuser efficiency, in the case of an incompressible fluid, becomes:

$$\eta_E = \frac{p_2^* - p_1^*}{\alpha_1 \rho \frac{V_1^2}{2} - \alpha_2 \rho \frac{V_2^2}{2}} \qquad (10)$$

The numerator can be expressed in terms of maximum velocities, using equation (9) and finally the expression of the energy efficiency becomes:

$$\eta_E = \left(\frac{U_1}{V_1}\right)^2 \frac{1 - \left(\frac{U_2}{U_1}\right)^2}{\alpha_1 - \alpha_2 \frac{1}{A_r^2}} \qquad (11)$$

This means that, once the evolution of the velocity distribution is known in a diffuser, the efficiency can be computed as long as separation does not occur and as long as the boundary layers do not reach the axis. Therefore our knowledge of the velocity distributions in these diffusers allows us to solve the various problems involved by diffusers.

This demands the solution of the boundary layer equations; although approximate approaches have been presented by several experimenters, no no satisfactory solution exists. Some of these approximations are derived from theoretical boundary layer equations. It is possible to integrate these equations when they are simplified. Others investigators derive empirical methods based on experimental data. These various methods predict the velocity distribution at any station along the boundary as long as separation does not occur. When separation occurs, the jet of fluid flows between fluid boundaries and consequently the flow is very unsteady. Velocity distributions become hazardous functions of the time. Several analytical methods are summarized in the following chapter.

CHAPTER II

ANALYTICAL RESEARCH

Several researchers have tried to find approximate solutions by deriving simplified equations. Most of them based their calculations on the von Karman momentum equation:

$$\frac{\tau_o}{\rho U^2} = \frac{d\delta^{**}}{dx} - \frac{dp/dx}{\rho U^2}\left(2\,\delta^{**} + \delta^{*} \right) \tag{12}$$

or:

$$\frac{\tau_o}{\rho U^2} = \frac{d\delta^{**}}{dx} - \frac{dp/dx}{\rho U^2}\left(H + 2 \right) \delta^{**} . \tag{13}$$

The adverse pressure gradient occurs in many phenomena. The analytical researchers mentionned thereafter were most concerned with flow along wings and the methods are often valid for both positive and negative pressure gradients. They were often checked only by aeronautical experiments on airfoils but they are still valid in the case of flow in a diffuser.

One of the features of the various analyses is the parameter used to characterize the form of the velocity profile. One of the assumptions made by each investigators is that the velocity profiles form a single-parameter family of curves.

Here thirteen methods are summarized:

Gruschwitz Method (reference 1)

One of the first and also one of the most important papers in analytical research was presented by Gruschwitz in 1931. The parameter was the velocity u_1 measured at a distance from the wall equal to the momentum thickness δ^{**}. Plotting test data, Gruschwitz found that his parameter: $n = 1 - (u_1/U)^2$ was a single function of the form parameter H. Furthermore, he found that the dimensionless quantity $\frac{\delta^{**}}{q}\frac{dP_1}{dx}$ was a single linear function of n (q: kinetic energy: $\frac{\rho U^2}{2}$, and P_1: total energy = kinetic + piezometric pressure at a distance δ^{**} from the wall). Thus Gruschwitz writes a differential equation which, combined with the von Karman momentum equation and solved by trial and error, gives δ^{**} and H. Separation would occur for $n = 0.8$ which corresponds to $H = 1.86$.

This method has long been the only one used and has generally given good results, except to predict the separation point. But it requires lengthy computations since it is not only a trial and error method but also a step-by-step method.

Nikuradse Method (reference 2)

Just two years before Gruschwitz, Nikuradse studied the two-dimensional flow between diverging boundaries and worked out a method based on the use of two parameters T and L, related one to the other by a single curve, according to his experiments. T was a parameter of friction and L a parameter of friction and pressure. The latter is a function only of the product $\delta^{**}\underline{R}^{1/4}$. Assuming that the ratios δ^{*}/W and U/V are constant, T can be computed from L and gives τ_o.

The simplicity of this method makes it very attractive but it is obvious that the assumptions concerning δ^{*}/W and U/V do not check further experiments (see Fig. 19).

Buri's Method (reference 3)

Buri based his calculations on the results obtained from the flow on a flat plate without pressure gradient. Assuming that the velocity distribution follows a power-law of the form $u/U = (y/\delta)^{1/7}$, which gives a ratio δ^{**}/δ^{*} constant, he obtained the expression:

$$\frac{\tau_0}{\rho U^2} = C \left\{\frac{U \delta^{**}}{\nu}\right\}^{-1/4} , \tag{14}$$

where $C = 12.55 \times 10^{-3}$. In the case of a pressure gradient, Buri assumed that this expression is still valid but affected by a coefficient $\xi = (\delta^{**}/\delta^{*})^{1/4}$ which is no longer constant. This coefficient varies with the shape of the velocity profile. According to Buri, the internal shear stress is the main factor which affects the velocity distribution. The distribution of this internal shear may be written:

$$\tau/\tau_0 = 1 - B\, y/\delta - \ldots \tag{15}$$

From the coefficient B, Buri derived a form parameter F,

$$F = \frac{\delta^{**}}{U} \frac{dU}{dx} \left(\frac{U\delta^{**}}{\nu}\right)^{1/4} , \tag{16}$$

and he shows that H and are single functions of F. Finally Buri obtained the expression:

$$a - \frac{b}{U} \frac{dU}{dx} z = \frac{4}{5} \frac{dz}{dx} , \tag{17}$$

where $z = \delta^{**} R^{1/4}_{(\delta^{**})} . F$; $a = 14.75 \times 10^{-3}$; $b = 3.945$.

The error of this method is to assume that τ_o is only a function of $R_{(\delta^{**})}$. This is not true when separation is approached. On the other hand, the values of a and b computed from various experiments are not perfectly equal.

Method of Squire and Young (reference 4)

In order to solve the momentum equation, Squire and Young used two other equations, the Karman-Schoenherr equation giving the drag coefficient on a flat plate with zero pressure gradient and the momentum equation for this plate. Thus they obtained two relations between δ^{**}, U and τ_o and consequently a differential equation in terms of τ_o only, which can be integrated if the form parameter H is assumed constant with x.

Checking this method against test data, it seems that, using $H = 1.4$, the results are in good agreement with the experimentation as far as airfoils are concerned.

Fediaevsky's Method (reference 5)

To compute the development of a turbulent boundary layer, Fediaevsky attempted to compute first the shear distribution across this boundary layer. He made the fundamental assumption that the shear distribution is a power series of y/δ . Thus all the shear profiles, for laminar as well as for turbulent flow are a single-parameter family of curves, the parameter being the slope at the surface of the shear distribution curve. Then the velocity distribution can be computed from the relation:

$$\frac{dP}{dx} = \frac{d\tau}{dy} \quad , \tag{18}$$

applied along a stream line and starting from a known upstream distribution.

This method involves tedious computations of several points along several stream lines, but, in spite of good agreement with several test data, Fediaevsky's analysis is incorrect. It has been shown by other experimenters that there is no direct connection between the velocity distribution and the local shear distribution.

Lee's Method (reference 6)

As it is possible for a flat plate with zero pressure gradient to express the ratio δ^{**}/x as a function of the length Reynolds number by means of the expression:

$$\delta^{**}/x = A/R_x^n \, , \tag{19}$$

Lee writes a similar expression for a flat plate with a pressure gradient:

$$\delta^{**}/x = B/R_x^m \, , \tag{20}$$

B and m being functions of x. He shows that, according to his experiments, B and m are single functions of the parameter:

$$G(x) = \frac{x}{q} \frac{dq}{dx} \tag{21}$$

In the same way, H is a linear function of G(x). Thus Lee obtained an expression for the drag coefficient as a function of G(x) and $d\delta^{**}/dx$ and he derives $d\delta^{**}/dx$ assuming that G(x), and consequently B and m, remain constant. Then the drag coefficient is a function only of $G(x)$.

This method is doubtful for several reasons. First, the scatter of Lee's data does not justify a linear expression for H. Second, it is not logical to maintain $G(x)$ constant to derive $d\delta^{**}/dx$ because $G(x)$ is an unknown function of x. Third, the curves giving B and m as a function of $G(x)$ have been plotted by Lee for positive and negative pressure gradients. When $G(x) = 0$ (zero pressure gradient), results should be similar to those generally found for a flat plate with zero pressure gradient, but Lee's results are different.

Method of von Doenhoff and Tetervin (reference 7)

By plotting u/U as a function of the form parameter H for constant values of y/δ^{**}, von Doenhoff and Tetervin found that the collected data showed the velocity distributions as a function of H alone for a given value of y/δ^{**}. Then the assumption was made that the rate of change of H is related to the external forces τ_o/q and $\frac{\delta^{**}}{q}\frac{dq}{dx}$. In dimensionless form, this notion is expressed by the equation:

$$\delta^{**} \frac{dH}{dx} = e^{(aH - b)}\left[\frac{\delta^{**}}{q}\frac{dq}{dx}\frac{2q}{\tau_o} - c(H - H_o)\right], \qquad (22)$$

where H_o is the initial value of the form parameter and $\frac{2q}{\tau_o}$ a skin friction coefficient computed by the Squire and Young formula and making this equation (22) independent of $\underline{R}_{(\delta^{**})}$. However the expression found by von Doenhoff and Tetervin for $\delta^{**}\frac{dH}{dx}$ has been empirically obtained from experimental data which were largely scattered. This expression is complex and difficult to handle.

This method shows good agreement with the experimentation but consistent differences appear as the separation point is approached. One of the reasons is that the term $\frac{\delta^{**}}{q} \frac{dq}{dx} \frac{2q}{\tau_0}$ shows small values when separation occurs whereas τ_0 being zero this term should be infinite. This method is better than that of Gruschwitz who neglected the influence of $\underline{R}_{(\delta^{**})}$.

The value of the form parameter H has often been used as a criterion of separation. According to the foregoing investigation, it is possible that separation occurs when H is about 1.6 and may be considered to have certainly occured when H is 2.6. Schubauer and Klebanoff (reference 8), investigating an airfoil, found separation to occur when $H = 2.7$.

Kehl's Method (reference 9)

In 1943, Kehl analysed the flow between converging and diverging boundaries and found, as Gruschwitz in 1931, that the velocity profiles were a single function of Gruscwitz's parameter n. Kehl derived an equation for n which took care of the Reynolds number $\underline{R}_{(\delta^{**})}$ by adding a new term to an equation found by Gruschwitz.

This method checked Kehl's experiments but did not apply to other data at higher Reynolds numbers.

Garner's Method (reference 10)

Garner's method uses results from Buri's and Tetervin's methods with some slight differences. Separation was found to occur when $H = 2.0$.

Binder's Method (reference 11)

This method issued from studies made on two-dimensional diverging channels. Binder defined two dimensionless coefficients:

$$\Psi = \frac{d}{dx}\left[\delta^* \left(\frac{U\delta^*}{\nu}\right)^{1/4}\right] \quad \text{and} \quad \Gamma = \frac{\delta^*}{\rho U^2}\frac{dp}{dx}\left(\frac{U\delta^*}{\nu}\right)^{1/4} \tag{23}$$

and showed that experimental data checked the relation $\Psi = a + b\Gamma$. This relation can be partially integrated and then δ^* can be obtained by a graphical integration. All the characteristics of a two-dimensional diffuser can be computed step-by-step, but the computations are tedious as a trial-and-error calculation is necessary in each step.

This method gives good results but is long and limited to two-dimensional diverging flow without separation. When $\Gamma > 0.08$, there is a possibility of separation. Binder reported this method in 1947 and again in 1953 (reference 35) and in these two papers did not give exactly the same numerical values for a and b.

Method of Rubert and Persh (reference 12)

These two researchers tried to correlate all the methods attempted up to that time. They checked the accuracy of the various methods and worked out an empirical method for two cases: conical diffusers and two-dimensional diffusers. For each case, an equation is offered, giving the rate of change of the momentum thickness, δ^{**}, with x by a step-by-step method. The equation is complex and expressed in terms of the non-dimensional pressure gradient $\frac{\delta^{**}}{q}\frac{dq}{dx}$, the form parameter H and the Reynolds number

$\underline{R}(\delta^{**})$.

This method was applied by the writer to several conical diffusers and always gave smaller results than those obtained by experimentation. This was true even when small steps of computation were taken.

Method of Lin and Tetervin (reference 13)

This method developed in 1950 is probably the most complex of all. Lin and Tetervin started with the Prandtl boundary layer equation. The velocity distribution, as in Tetervin's previous paper, is assumed to be a function only of y/δ^{**} and H and it is also assumed that the shear stress is a function only of U, dU/dx, y/δ^{**} and H. After long calculations, an expression for $\delta^{**} \frac{dH}{dx}$ is obtained. This expression involves several graphical integrations.

Goethals' Method (reference 31)

This method was elaborated from tests run on circular diffusers. It is purely empirical and only valid for circular diffusers. The velocity distributions were found to follow the law:

$$u/U = 1 - K \log (y/\delta) , \tag{24}$$

while δ can be expressed as:

$$\delta/x = A - B\, x/D , \tag{25}$$

K, A and B being functions only of the angle θ.

This method is extremely simple but has been verified only for one entrance velocity distribution and one Reynolds number R_1. It gives only an approximate idea of the velocity distribution. Some inconsistancies appear in this method. The form parameter remains constant through the diffuser. If the value of H is accepted as a criterion of separation, it is curious to see that, according to the values of K given by Goethals, the form parameter reaches a maximum for $\theta = 11$ degrees and then decreases as θ keeps increasing above 11 degrees.

This last method, as Buri's method, assumes a certain law to represent the velocity distribution. Buri's power-law and Goethals' logarithmic law are the only two laws used. It is then possible to get the displacement thickness δ^*, the momentum thickness δ^{**}, and the kinetic energy coefficient α in terms of $\frac{\delta}{D/2}$ for circular diffusers, in terms of $\frac{\delta}{W/2}$ for two-dimensional diffusers. The kinetic energy coefficient which we are interested in, reads for circular diffusers:

$$\alpha = \frac{1 - 2\frac{\delta}{D/2}\frac{3n}{3n-1} - 4\left(\frac{\delta}{D/2}\right)^2\frac{3n}{3n-2}}{\left[1 - 2\frac{\delta}{D/2}\frac{n}{n-1} - 4\left(\frac{\delta}{D/2}\right)^2\frac{n}{n-2}\right]^3} , \qquad (26)$$

with the power-law $u/U = (y/\delta)^n$, and:

$$\alpha = \frac{\left(1 - \frac{\delta}{D/2}\right)^2 - 2\frac{\delta}{D/2}(1-3K'-6K'^2-6K'^3) - \left(\frac{\delta}{D/2}\right)^2(1-\frac{3}{2}K'-\frac{3}{2}K'^2-\frac{3}{4}K'^3)}{\left[\left(1 - \frac{\delta}{D/2}\right)^2 - 2\frac{\delta}{D/2}(1-K') - \left(\frac{\delta}{D/2}\right)^2(1-\frac{1}{2}K')\right]^3} , \qquad (27)$$

with the logarithmic-law $u/U = 1 - K \log (y/\delta)$, where $K' = 0.4343\ K$.

These formulae are rather complicated and involve long computations. The shortest way to get the value of the kinetic energy coefficient is still a graphical integration of:

$$\alpha = \frac{1}{V^3 W} \int_0^W u^3 \, dy \quad , \tag{28}$$

for the two-dimensional diffusers, and:

$$\alpha = \frac{1}{V^3 R^2} \int_0^W u^3 \, d(r^2) \quad , \tag{29}$$

for the circular diffusers. Furthermore, the results obtained from formulae (26) and (27) are not very accurate for several reasons: first, the thickness of the boundary layer cannot be accurately measured; second, neither the power-law, nor the logarithmic-law fit a velocity distribution perfectly. In addition, it is interesting to note that, unless n in the expression (26) and K' in the expression (27) are functions of x, the coefficient α becomes constant as soon as the boundary layers meet at the center of the diffuser.

The analytical research which has been reviewed here shows the general tendency to base the derivations on the use of a single parameter. Yet this assumption is not strictly valid. Besides the difficulty of featuring a two-dimensional flow which fits the two-dimensional momentum equation, it has been shown (reference 42) that none of the parameters used is independent of the local skin friction. The local skin friction coefficient is itself greatly affected by the pressure gradient. Also the single parameter assumption does not take into account the previous history of the fluid. Generally researchers checked their method against their own experiments or against few experiments where the range of variation of at least one variable was small. The review shows that the problem of diffusers still defies mathematical analysis and that the answer to this problem must be found in an extensive experimentation.

CHAPTER III

EXPERIMENTAL RESEARCH

We will now be concerned only with experimental research on diffusers.

Review of the Literature

A review of the literature shows that from 1909, date of the first experiments, up to date, April 1954, about twenty investigators have made experiments on diffusers (see references 14 to 35). Since the present work aimed to compare numerical data, it was important that the results given by different reports be transformed to get the values of the characteristic coefficients η_E and α_2. Unfortunately this was not always possible for the twenty series of experiments mentioned. In many cases, the entrance conditions of the flow are unknown or of such nature as to give doubtful results. In other cases, the results are given in a certain form, different from an energy efficiency. It is sometimes impossible to get back to this efficiency as the intermediate steps of computations are missing in the reports.

After examination, data of ten investigators appeared, first, to fit the requirements of our purpose and, second, to cover the range of the ten other series of experiments and their results, if these were available. This number of experiments is very small if one considers the importance of the problem and the very wide range of variation of the variables involved. In fact, subsequent results will show that complete knowledge of this question is still lacking.

Among these ten investigators, six studied circular diffusers and five studied two-dimensional diffusers, Gibson studying both of them. All give either the energy efficiency or a way to compute it from their results. Only eight give traverses of velocity distribution.

A pertinent review of these works can be found in reference 36. This review only summarized the various information given by published papers. No attempt was made to correlate the corresponding data. It will be attempted herein to give a brief summary of the experiments which lead to the data used in the present work. This involves some studies made and published since the publication of reference 36.

Summary of Publications

The early experimental work of Gibson (1913), (reference 15, 16, and 17), although one of the first in this field, remains one of the most complete studies of diffusers, in spite of some omissions. The pressure recovery was studied from the flow of water in circular and rectangular diffusers for angles from 3 up to 180 degrees with area ratios of 2.25, 4 and 9. Unfortunately, only the range of variation of the Reynolds number at the entrance is given and velocity distributions are lacking.

Lyon (reference 22) in 1921 ran experiments with water on conical diffusers of angles of 4, 6, 8, 10, and 12 degrees with area ratio of 4, and of angles of 8 and 10 degrees with an area ratio of 9. Energy efficiency and four velocity traverses (one at the entrance, two inside the diffuser, one at the exit) are given for each diffuser. To our knowledge, Lyon is the only experimenter who tested a concrete diffuser (4 degrees) with rough boundaries. His results are sometimes curious, as will be shown later, and instabilities seem to have occured in his experiments.

Vedernikoff (reference 24), in 1926, reported on a study of the flow of air in a two-dimensional channel with diverging walls of length ratio L/W_1 = 10 for angles from 0 to 24 degrees. The velocity distributions are very poorly defined and this lack of accuracy limits the usefulness of his work.

Further tests with air were run in 1931 by Peters (reference 26) on circular diffusers with angles from 5.2 to 180 degrees. The main purpose of this study was to investigate the influence of upstream conditions on the energy efficiency of the diffuser. Only the entrance velocity traverse is known.

Demontis (reference 27), in 1936, reported on a study of two-dimensional diffusers using a length ratio of 3.5 and angles from 0 to 31 degrees. The diffusers were tested both by sucking and by blowing the air through them. Energy efficiencies and velocity traverses for some of the diffusers are available.

Kalinske (reference 28), in 1944, undertook a rather revolutionary study with water of three diffusers with angles 7.5, 15 and 30 degrees and an area ratio of 2.98. The measurements made with great care give the energy efficiency and several velocity traverses only in 7.5 degree diffuser for two entrance Reynolds numbers.

In 1950, Goethals (reference 31) reported on experiments run on three circular diffusers of 7, 10, and 15 degree angles and an area ratio of 4.

The tests were run with air drained through the diffusers by a fan. Efficiency can be computed and several velocity traverses are given.

Systematic tests were later run, in 1952, by Robertson (reference 32) on circular diffusers with water. The influence of the entrance conditions on the flow was studied in three diffusers with area ratio 3.88 and angles of 5, 7.5, and 10 degrees. Unfortunately, the values of the kinetic energy coefficients at the entrance and at the exit of the diffusers can only be obtained by extrapolation of the values obtained from velocity traverses taken inside the diffusers.

In 1953, Reid (reference 34) studied the performance of two-dimensional diffusers with air and presented the most thorough work that has been made on two-dimensional diverging flow. However, only a few velocity traverses at the exit are given.

The same year, Binder (reference 35) presented a very complete study of a 10.2 degree, two-dimensional diffuser tested with air. Several velocity traverses in the diffuser itself and far downstream in an exit length are given. The pressure recovery is also given for this diffuser and a 8.14 degree diffuser with induced entrance boundary layer, both with length ratio of 12 and with or without an exit length.

More details and numerical values about these experiments are given in the following tables (Table I, page 30 and Table II, page 31).

Table I

Circular Diffusers

Column 2: A = air; W = water - Column 3: B = air is blown; S = air is sucked

1	2	3	4	5	6	7	8	9	10
Author and year	Fluid	Motion	Angle θ°	Area ratio A_R	Reynolds number R_1	Mach number M_1	based on p_2^*	Velocity traverses	Entrance conditions
Gibson 1913	W		10-180° 3-180° 10-180°	2.25 4 9	0.5 to 2.5×10^5		in exit length	none	$1/D_1 = 2.6$
Lyon 1921	W		4-12° 8 & 10°	4 9	2.4 to 5×10^5		at the exit	4 for each diffuser	rounded inlet
Peters 1931	A	B	5-180°	2.34	2×10^5	0.13	in exit length & at the exit	at the entrance	$1/D_1 = 0$ to 60
Kalinske 1944	W		7.5° 15° 30°	2.98	6.8 & 9.7×10^4		in exit length & at the exit	several in 7.5° diffuser	fully developed flow
Goethals 1950	A	S	7° 10° 15°	4	3.5 to 4.5×10^5	0.1	in exit length	several in the three diffusers	$1/D_1 = 1$
Robertson 1952	W		5° 7.5° 10°	3.88	0.5 to 2×10^6		at the exit	2 stations in each diffuser	$1/D_1 = 3.3$ to 10.4

Table II
Two-Dimensional Diffusers

Column 2: A = air; W = water; - Column 3: B = air is blown; S = air is sucked
Columns 6 & 7: P = distance between parallel walls

1	2	3	4	5	6	7	8	9	10	11	12
Author and year	Fluid	Motion	Angle $\theta°$	Length ratio L/W_1	P/W_1	P/W_2	Reynolds number $\underline{R1}$	Mach number M_1	based on p_2^*	Velocity traverses	Entrance conditions
Gibson 1913	W		10-30° 5-40° 10-90°	2.51-7.15 4.4-34.4 5.7-45.9	1.0 1.0 2.25	0.44 0.25 0.25	0.5 to 2.5×10^5		in exit length	none	$1/D_1$ = 2.6
Vedernikoff 1926	A	B	0-24°	10	1.0	1.0-0.13	5×10^5	0.25	at the exit	at the exit	rounded inlet
Demontis 1936	A	B S	0-31°	3.5	1.0	1.0-0.34	1.5×10^5	0.04	at the exit	entrance & exit	rounded inlet
Reid 1953	A	S	8.0-17.4° 6.0-18.9° 5.4-15.9° 3.8-15.2° 2.7-10.7°	5.50 7.75 11.00 15.25 21.75	8.0 8.0 8.0 8.0 8.0	4.57-3.04 4.45-2.63 4.0-2.0 4.0-1.6 4.0-1.6	5×10^5	0.21	at the exit	1 at the entrance 8 at the exit	rounded inlet
Binder 1953	A	B	8.14° & 10.2°	12					exit & ex.lg.	several in 10.2°	rounded inlet

As far as circular diffusers are concerned, columns 2 and 3 of Table I show that the various tests have been run in different ways. Only the works of Peters, Kalinske and Robertson are comparable to those of Gibson, although the latter gives no velocity traverses (column 9) and Robertson gives no results based on a complete pressure recovery (column 8) as the other experimenters do. This table points out that most of the experiments were confined to small angles (column 4) and to area ratios between 2.25 and 4 (column 5). Only Gibson studied an area ratio of 9 for several angles. Column 6 shows the various ranges of Reynolds numbers used, but only Robertson made a systematic study of the effect of this variable on the performance of a diffuser. Column 7 shows that when air has been used, the Mach number was low enough to consider this fluid as incompressible.

Table II for two-dimensional diffusers is also helpful. As has been shown by Demontis (see Fig 15), results depends largely on the way the fluid is set in motion. This points out that Reid's complete study is comparable only to Demontis' results (column 3). It is seen also that the information concerning the velocity distribution is poor (column 11). Divergencies in the results arise also from the fact that only Reid and perhaps Binder feature genuine two-dimensional flows. Columns 6 and 7 demonstrate this very vividly. The angles and the length ratios (columns 4 and 5) used by the experimenters provide a fairly wide range of values, but there is no systematical study of the influence of the Reynolds number (column 8).

Correlation and Discussion of Data

The comparison of the results of the previously mentionned investigations is pictured in several graphs where an attempt has been made to feature the influence of only one variable at a time on the energy efficiency η_E and the kinetic energy coefficient α_2 at the exit. First, we will be concerned with circular diffusers and then with two-dimensional diffusers.

Circular Diffusers

<u>Energy **Efficiency versus Angle** θ</u>.—In figure 3, efficiency has been plotted against the angle θ. Only the results corresponding to almost similar values of α_1 and $\underline{R}_1$ have been taken into consideration. The knowledge of α_1 and α_2 was necessary to compute the efficiency. For Gibson's results, the computations leading to the value of the energy efficiency were made assuming $\alpha_1 = 1.01$. This is justified by the conditions of the experiments ($l/D_1 = 2.6$). Using $\alpha_2 = 1.03$ is also justified by the fact that p_2^* was measured far enough downstream in the exit length to get a velocity distribution undisturbed by the diffuser. Lyon's data are not recorded here because they present such discrepancies that their validity seems doubtful. The smallest value of α_1 used by Kalinske is 1.11 and this definitely affects his results (curve (3) should lie between curves (5) and (6)). In the same way, the high Reynolds numbers used by Robertson and Goethals affect their results for small angles θ, where curves (1) and (7) separate from Gibson's curve (5), which corresponds to experiments under similar conditions. Curve (1), which is based on a partial pressure recovery, is

slightly below curve (7), which corresponds to similar tests with a complete pressure recovery. In spite of these discrepancies, this diagram is satisfactory. Peter's data, for instance, check fairly well Gibson's results for an area ratio of 2.25. It can be seen already from this graph that the area ratio has little influence when the angle is small.

Energy Efficiency versus Area Ratio.—This fact is outlined by figure 4 where it is seen that the energy efficiency does not drop appreciably when the area ratio increases for small angles. The curves have been plotted against Gibson's results for three area ratios of 2.25, 4 and 9. The main feature of this diagram is the complete lack of information for area ratios between 4 and 9. A close inspection of this figure shows that there are two kinds of curves: those corresponding to angles up to 15 degrees and those for angles above 15 degrees. For instance, it can be seen that, as it is possible to draw similar curves for angles between 15 and 20 degrees, performing the same operation between 10 and 15 degrees would indicate a relative change in the orientation of the curves, i.e., a change in the radius of curvature of these curves. This is due to the occurence of separation. When the flow separates from the wall, the formation of eddies increases the turbulence. This highly turbulent motion flattens the velocity distribution and the kinetic energy coefficient increases less an less rapidly in the direction of the flow. Consequently the energy efficiency decreases less rapidly. In addition, for wide angles diffusers the boundary layers rapidly reach the center and, for this reason, the kinetic energy coefficient increases less rapidly than when a central core exists. These reasons make the efficiency almost constant between the area ratios 4 and 9 for a 30 degree diffuser. The figure shows the data of Peters, Kalinske, Robertson and Goethals and their position with respect to Gibson's curves; as for the previous figure, the differences between these various results are explained by the differences in Reynold's numbers and entrance kinetic energy coefficients.

<u>Energy Efficiency versus Entrance Reynolds Number</u>.— The variation of the energy efficiency with the Reynolds number is shown in figure 5. This figure is the only one on which Lyon's results concerning efficiency appear. These results seem doubtful and probably instabilities occured in Lyon's experiments. Kalinske's results are definitely affected by the change by the Reynolds number of the entrance kinetic energy coefficient. Robertson is the only researcher who undertook a systematic study of the effect of the Reynolds number. The range tested is small and the kinetic energy coefficient α_1 remains almost constant in this range. Robertson's results must be compared with Goethals' data for $\underline{R}_1 = 9.1 \times 10^4$ obtained for a similar diffuser, although the coefficient $\alpha_1 = 1.05$ used by Goethals causes a decrease in the energy efficiency. The lack of data for the low Reynolds numbers makes difficult to understand the influence of this variable. However, it can be stated that high Reynolds numbers give better efficiency and it is believed that the performance of a diffuser is only affected slightly by the variation of this variable. In fact, such tests are not easy to run because the shape of the velocity distribution at the entrance and, therefore, α_1 are functions of the Reynolds number and the roughness of the wall as evidenced by the Karman-Prandtl equation.

Energy Efficiency versus Entrance Kinetic Energy Coefficient.—The influence of the coefficient α_1 on the energy efficiency was studied by Peters and his results are plotted on figure 6. It is seen that α_1 does not appreciably affect the energy efficiency. It is interesting to note that the effect becomes negligible when θ is greater than 30 degrees (not plotted on figure 6). This means that α_1 has no effect on the performance of a diffuser as soon as separation occurs, since it is reasonnable to think that the flow separates from the wall at the beginning of the diverging section when the angle is wide enough, i.e., wider than 30 degrees. However, the range tested by Peters is small because in his experiments he uses only smooth pipes. As can be seen on the following figure (Fig. 7), when the length l of pipes preceding the diffuser is greater than that defined by the ratio $l/D_1 = 70$, the coefficient α_1 remains approximately constant and is equal to 1.045. If the pipe is assumed smooth, this value of α_1 corresponds to a Reynolds number 2×10^5 which is that given by Peters. If rough pipes are used, α_1 can reach higher values, such as that obtained by Kalinske ($\alpha_1 = 1.42$). Then the efficiency drops as α_1 increases as shown by figure 8. The curves plotted in the upper diagram do not pass through the points obtained from the results of Robertson and Kalinske because a small correction was made to take care of the slight differences in area ratios used by these two experimenters.

Separation.—Separation may occur in a diffuser for two reasons; either the angle is too wide or the diffuser is to long. Then as explained by Prandtl, "the forward velocity of the fluid... may be insufficient for the fluid to force its way for very long against the adverse pressure gardient. It is then brought to rest, and further on, next to the wall a slow backflow sets in". It is curious to note that very few experimenters mention separation. It seems to occur at the end of a 10 degree diffuser of area ratio 4, according to Robertson.

If we look at figure 3, we see that only two experimenters investigated the velocity distribution of a 10 degree diffuser, namely Robertson and Goethals, and there is no information about the velocity distribution of diffusers of angles above 10 degrees. It is reasonnable to think that, if the angle increases above 10 degrees and the area ratio increases above 4, the point of separation goes upstream along the wall until it reaches the entrance of the diffuser. With such data, the plotting of the area ratio for which separation occurs versus the angle θ would make a very interesting curve which would indicate the boundary between the zone of non-separated flow and the zone of separated flow. On another hand, it is definite that the entrance Reynolds number and the entrance kinetic energy coefficient influence this phenomenon and, therefore, the position of this curve (see similar curve for two-dimensional diffusers: Fig. 16). As indicated by Robertson, high Reynolds numbers and low kinetic energy coefficients at the entrance delay separation.

Of course, the kinetic energy coefficient α_2, as other velocity distribution parameters, could not be computed in the conical diffusers when the flow is separated. In the following graphs, the angle is therefore limited to 10 degrees.

<u>Kinetic Energy Coefficient α_2 at the Exit versus Angle θ</u>.—The variation of α_2 versus angle θ is indicated by figure 9. The data of Robertson and

Lyon have been used to plot the lower curve corresponding to $\alpha_1 = 1.01$. Goethals' data corresponding to a higher value, $\alpha_1 = 1.045$, give the upper curve. The lower curve must be considered as an approximate curve. The points are scattered because Robertson's data were obtained by extrapolation and because Lyon's results are doubtful. On the contrary Goethals' data are accurate. These two curves do not cover all the available data concerning α_2, although there is not a large amount of information about velocity distributions. Curves are difficult to plot since each group of data correspond to a different set of experiments. Therefore, the four variables θ, A_r, $\underline{R}_1$, and α_1 differ in exact interpretation from one experiment to another. An extrapolation for the purpose of eliminating one variable and comparing two similar experiments is not always safely possible.

On figure 9, it is already noteworthy that the entrance kinetic energy coefficient α_1 has a definite influence on α_2. Although Goethals used a value of Reynolds number higher than Robertson and Lyon and therefore should get a relatively lower α_2, his results are much higher. This is due only to the slight difference in α_1 (1.01 to 1.045).

<u>Kinetic Energy Coefficient α_2 versus Area Ratio and Kinetic Energy Coefficient α_1</u>.—The strong influence of α_1 is again emphasized by figure 10 where α is plotted against the area ratio A_r. When the area ratio is 1, α is α_1. The results of 4 experimenters for angles between 7 and 8 degrees, show that a slight difference in α_1 produces a large spreading of the values of α_2. It must be pointed out that this graph does not intend to feature the variation of the kinetic energy coefficient through a diffuser,

from the entrance ($A_r = 1$) to the exit. This variation is shown by figure 11 for two angles, 7 and 10 degrees. The variation of the displacement thickness and the momentum thickness are also plotted against x'/L for the 10 degree diffuser. It can be seen that the coefficient α is quite valid as a criterion to define a velocity distribution, since all the curves show the same way of variation.

Kinetic Energy Coefficient α_2 versus Entrance Reynolds Number.—The influence of $\underline{R}_1$ on α_2 is shown on figure 12. Robertson's results are by far the most interesting. They show that the entrance Reynolds number has as α_1, a strong influence on α_2. It seems, however, that it is overestimated by this diagram since Lyon's curve would be horizontally levelled if the values of α_1 were the same. Robertson's data are obtained from velocity traverses taken inside the diffusers and, therefore, may not be quantitavely accurate.

Two-Dimensional Diffusers

Energy Efficiency versus Angle θ and Length Ratio L/W_1.—The two figures 13 and 14 feature, respectively, Reid's and Gibson's results. The energy efficiency is shown in level lines of constant value. For both experimenters this value had to be computed because neither one gave the energy efficiency as we previously defined it. For Gibson α_1 and α_2 were taken as in the case of circular diffusers for the same reasons. Reid gives a kinetic energy coefficient at the entrance and claims that the dimensions of the diffuser have no effect on it. α_2 was obtained for his diffusers by extrapolation of 8 values computed from velocity traverses given in 8 different diffusers. This operation may seem doubtful, but, as far as computations of the energy efficiency are concerned, the kinetic energy coefficient α_2 does not need to be accurately defined because the efficiency was computed by the expression:

$$\eta_E = \frac{p_2^* - p_1^*}{\rho \frac{V_1^2}{2} \left(\alpha_1 - \alpha_2 \frac{1}{A_r^2}\right)} , \qquad (30)$$

derived from formula (10). The area ratio ranges from 2 to 6 and its square from 4 to 36, which makes the term $\alpha_2 \frac{1}{A_r^2}$ small and even sometimes negligible comparatively to the term α_1. The two figures 13 and 14 are not similar because, first, Gibson ran tests for angles generally wider than Reid; second, the Reynolds numbers and the kinetic energy coefficients at the entrance are different, as indicated; third, Gibson's efficiency is computed on the basis of a complete pressure recovery. On figure 13, the dotted line approximately joins the points of maximum efficiencies for the greatest length ratio. An equation of this line would be a function of both θ and L/W_1.

Figure 15 shows the results of Vedernikoff and Demontis. Vedernikoff's experiments do not lead to very accurate results and the curve is doubtful above 14 degrees. Demontis' results show the difference resulting from testing diffusers in two different ways. Curve B indicates that the results came from diffusers in which the air was blown, while for curve S the air was drained. Vedernikoff and Demontis used almost similar Reynolds numbers and kinetic energy coefficients at the entrance although they both vary slightly with the angle.

Unfortunately, it is impossible to get quantitative results from the available data as far as influence of $\underline{R}_1$ and α_1 on the energy efficiency is concerned because no systematic tests were ever run in this sense. Comparison of the results of the various experimenters leads to poor or incredible results.

This outlines the lack of data in the field of two-dimensional diffusers. It will be seen later that the data concerning velocity distributions are also poor or doubtful.

Separation.—As for circular diffusers, separation cannot be accurately defined. However separation occurs for wider angles than for the circular diffusers. Vedernikoff found separation to occur when θ was 14 degrees ($L/W_1 = 10$), Demontis for $\theta = 19$ degrees ($L/W_1 = 3.5$) when the air is drained, $\theta = 22$ degrees ($L/W_1 = 3.5$) when the air is blown. Inspection of velocity distributions given by Reid shows That a 17.43 degree diffuser of length ratio 5.5 and a 10.70 degree diffuser of length ratio 21.75 were close to separation at the exit. The results of these three experimenters make figure 16 possible, where it is assumed that the Reynolds number does not affect the curve. The three experimenters used a rounded inlet at the entrance and had almost similar kinetic energy coefficients α_1.

Kinetic Energy Coefficient α_2 versus angle θ.—The kinetic energy coefficient α_2 has been computed for exit traverses until the flow separates. As could be expected, α_2 increases when the angle increases. The figure 17 shows this variation. From Reid's results two curves are available for two values of L/W_1. For $L/W_1 = 5.5$ the curve changes curvature when $\theta = 11$ degrees. It does not seem that this is correct because nothing can explain such a variation. In fact the last point of the curve ($\theta = 17.4$ degrees) is not sure since the flow was very unsteady and Reid gave two different velocity distributions at the exit of this diffuser; the flow switched continuously from one diverging wall to the other. On the figure, another curve is proposed for $L/W_1 = 5.5$ which does not take this last point into account.

Kinetic Energy Coeeficient α_2 versus Length Ratio L/W_1.—Figure 18 is similar to figure 10 for circular diffusers except that L/W_1 is used in place of the area ratio. The right hand graph (both scales proportional to those of the left hand graph) shows Demontis' results. Likewise for two-dimensional diffusers, a slight difference in α_1 produces a much larger difference in α_2. The real variation of the kinetic energy coefficient through a diffuser is shown by figure 19 plotted from Binder's data. Eight traverses in the diffuser and four in the exit length were used to compute the various parameters α, δ^*, δ^{**}, and H. These four parameters vary in the same way.

Figure 20 shows the variation of α_2 when the length ration increases. For an angle of about 10.7 degrees, the rate of change of α_2 with L/W_1 is approximately constant. This indicates that, if the 10.2 degree diffuser of figure 19 were longer than it is ($L/W_1 = 12$), the curve $\alpha = f(x'/L)$ between $x'/L = 0$ and $x'/L = 1$ would become steeper and steeper, and at $x'/L = 1$, α_2 would reach higher values. In the same way, the curve $H = f(x'/L)$ would become progressively steeper also; and finally, separation would occur for a sufficiently high value of H at $x'/L = 1$. In other words, this means that, even in a small angle diffuser, the flow may separate from the wall providing that the diffuser is long enough. Separation may require a great length of diverging wall to occur since, as it can be seen from the curve of figure 20, for $\theta = 8$ degrees the rate of change of α_2 with L/W_1 becomes smaller and smaller as the angle θ decreases and when L/W_1 increases.

There are no data to investigate the influence of the Reynolds number and of the kinetic energy coefficient at the entrance on α_2. Results concerning these variables can be expected to be qualitatively similar to those obtained for circular diffusers.

CONCLUSIONS

The results analysed in this work put the emphasis on the poor performances of diffusers. Good characteristics are obtained only by small angle diffusers but the best efficiency angle depends on the area ratio. To connect two sections of pipe, a small angle diffuser is long and if the area ratio is high, the gain in energy transformed becomes less than the amount of energy loss in the transformation.

The question arises as to whether there does not exist another way to transform kinetic energy into piezometric energy. To our knowledge, diffusers are the only devices which are able to perform this transformation without requiring external energy. In spite of their poor characteristics, the energy efficiency and the performance of diffusers can be greatly improved by several means which are mentioned below. They all tend to increase the optimum divergence angle and to delay separation.

(1) In diffusers of small area ratio, the installation of a thin, central, longitudinal partition augments the pressure recovery (reference 34).

(2) Even a short exit duct improves appreciably the pressure recovery (reference 34).

(3) Spiral flow at the entrance or induced in the diffuser by small wings perpendicular to the wall increases the efficiency by bringing momentum to the walls (reference 26).

(4) One or several screens placed across the diffuser have a sta-

bilizing effect on the flow and prevent separation in wide angle diffusers. Although overcoming the resistance of the screen requires a certain amount of energy, the screen increases the normal velocity gradient at the wall and decreases the pressure gradient along the wall. (references 39 and 40).

(5) By suction of the boundary layer through the wall, the efficiency of a diffuser can be greatly improved and the velocity distribution flattened. A pump is necessary to overcome the suction pressure. The pump, with a 75 per cent efficiency, requires 2 to 3 per cent of the power passing through the entrance section of the diffuser (reference 41).

(6) Another efficient process consists of feeding with fluid the boundary layer through slots in the walls. This device does not require any additional power since the pressure of the flow in the diffuser is always less than the pressure in the exit duct where the feeding fluid may be taken (reference 31).

It seems that diffusers are seldom used without one of these additional improvements. However, from the purely scientific standpoint, it is quite desirable that we know how the diverging flow behaves. The available experiments make our knowledge of this problem far from complete, mostly because of a lack of organization of the researches and standardization of the results. In addition, there are still ranges of variation of some variables involved in the problem on which information is not complete. Such information would be most helpful in completing the present work and in improving our understanding of the effect of an adverse pressure gradient on the behavior of the flow.

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ILLUSTRATIONS

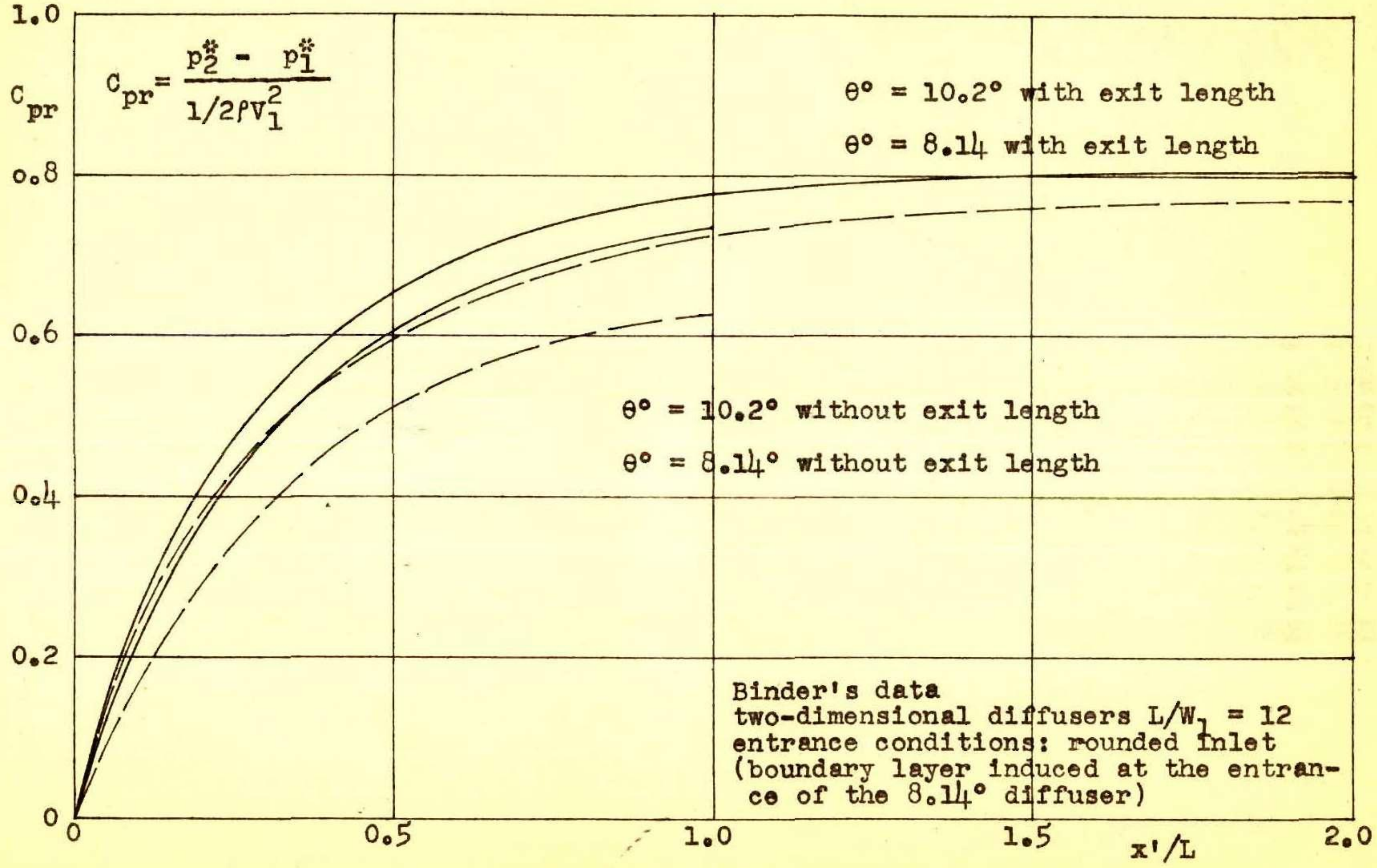


Fig. 1 - Pressure recovery with and without exit length

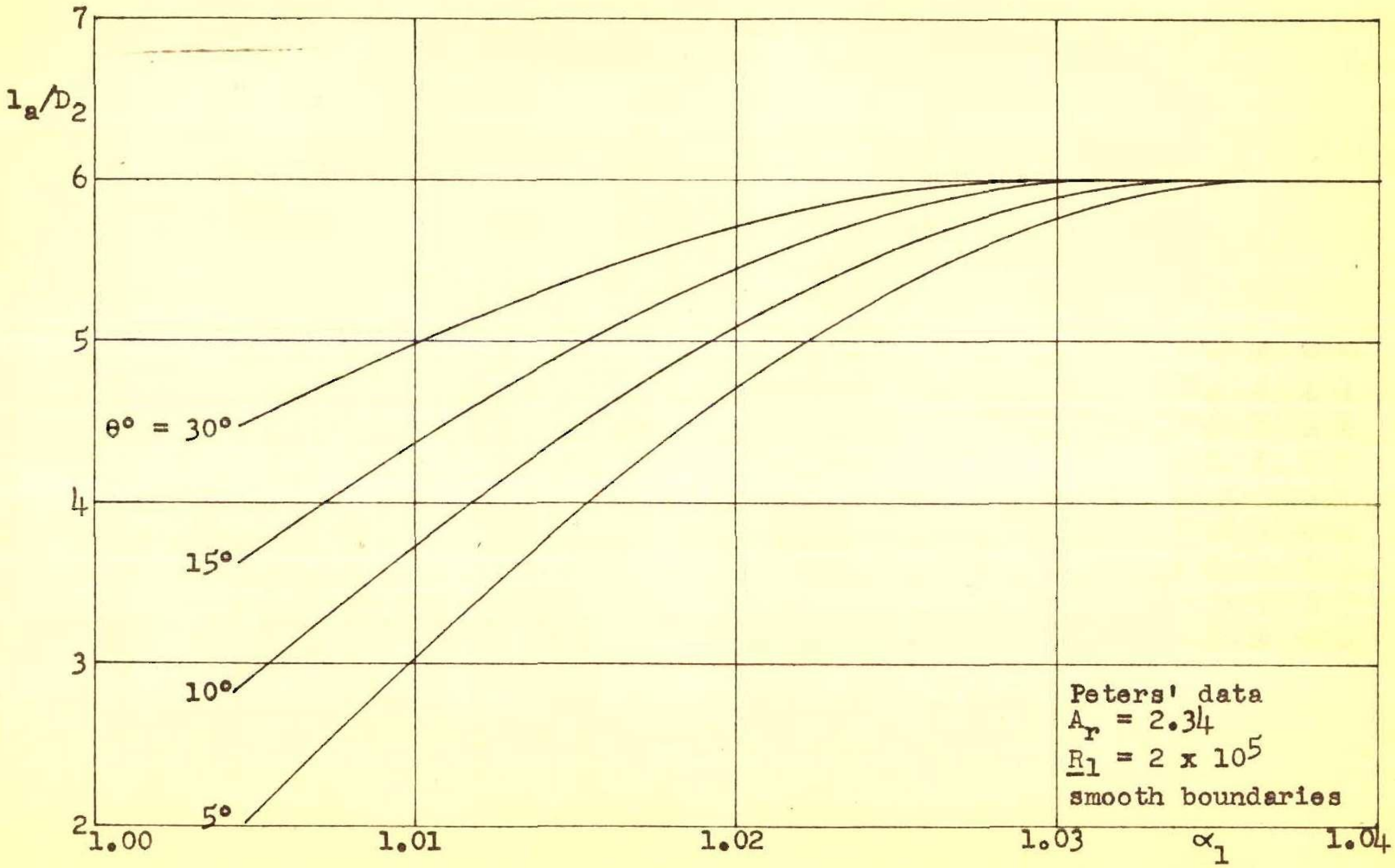
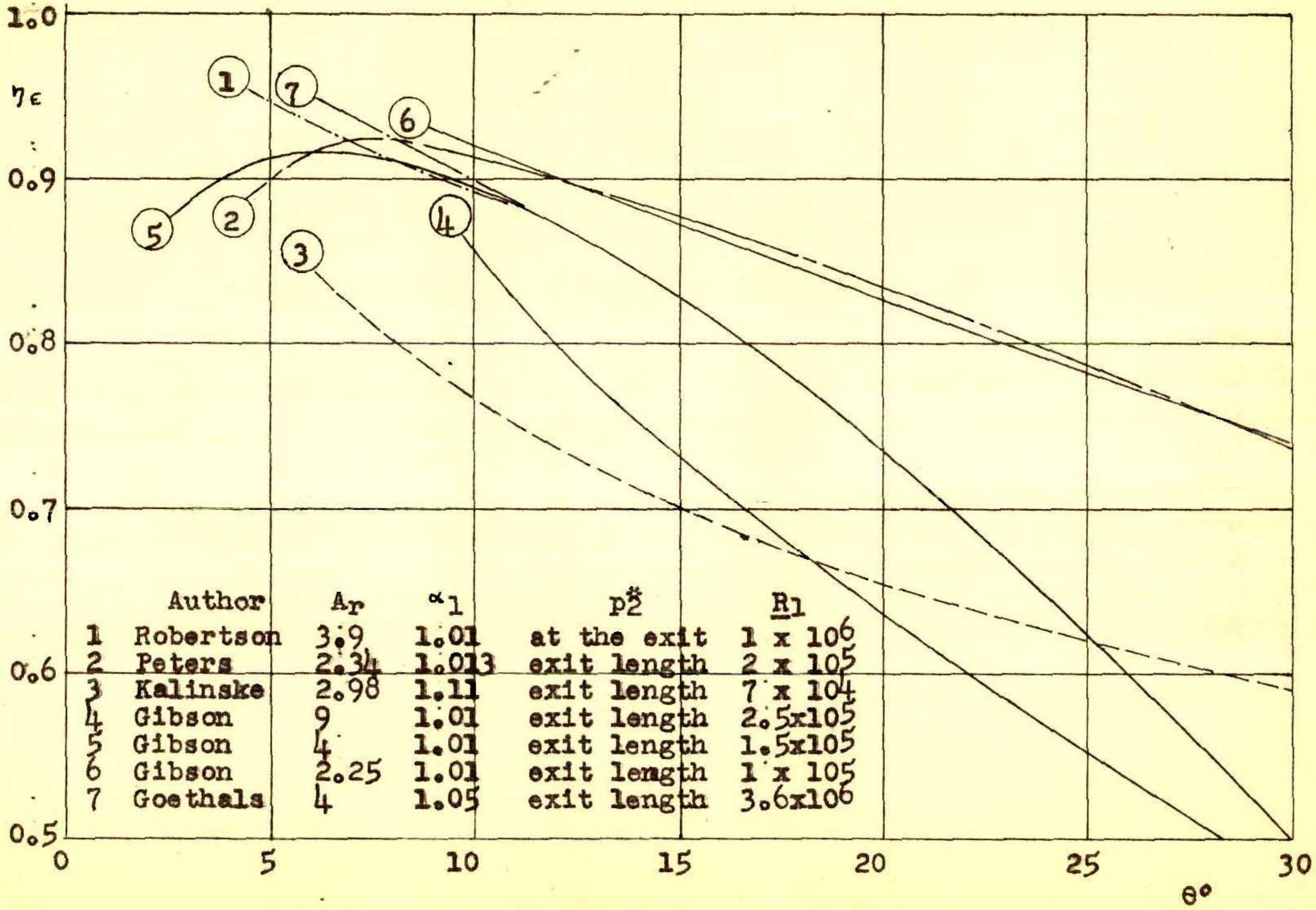


Fig. 2 - Length of exit conduit for complete pressure recovery



	Author	A_r	α_1	p_2^*	R_1
1	Robertson	3.9	1.01	at the exit	1×10^6
2	Peters	2.34	1.013	exit length	2×10^5
3	Kalinske	2.98	1.11	exit length	7×10^4
4	Gibson	9	1.01	exit length	2.5×10^5
5	Gibson	4	1.01	exit length	1.5×10^5
6	Gibson	2.25	1.01	exit length	1×10^5
7	Goethals	4	1.05	exit length	3.6×10^6

Fig. 3 - Energy efficiency versus angle θ°

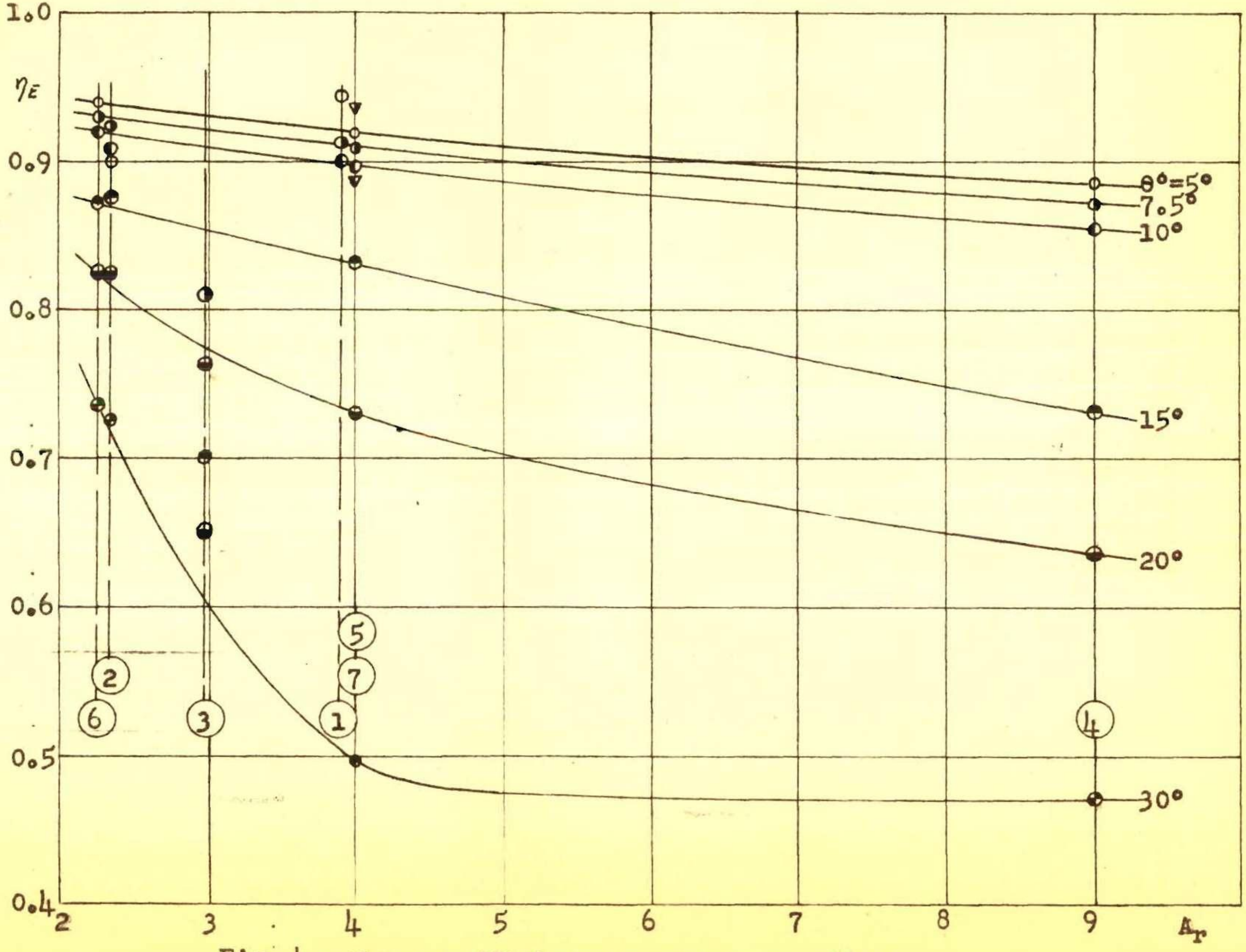
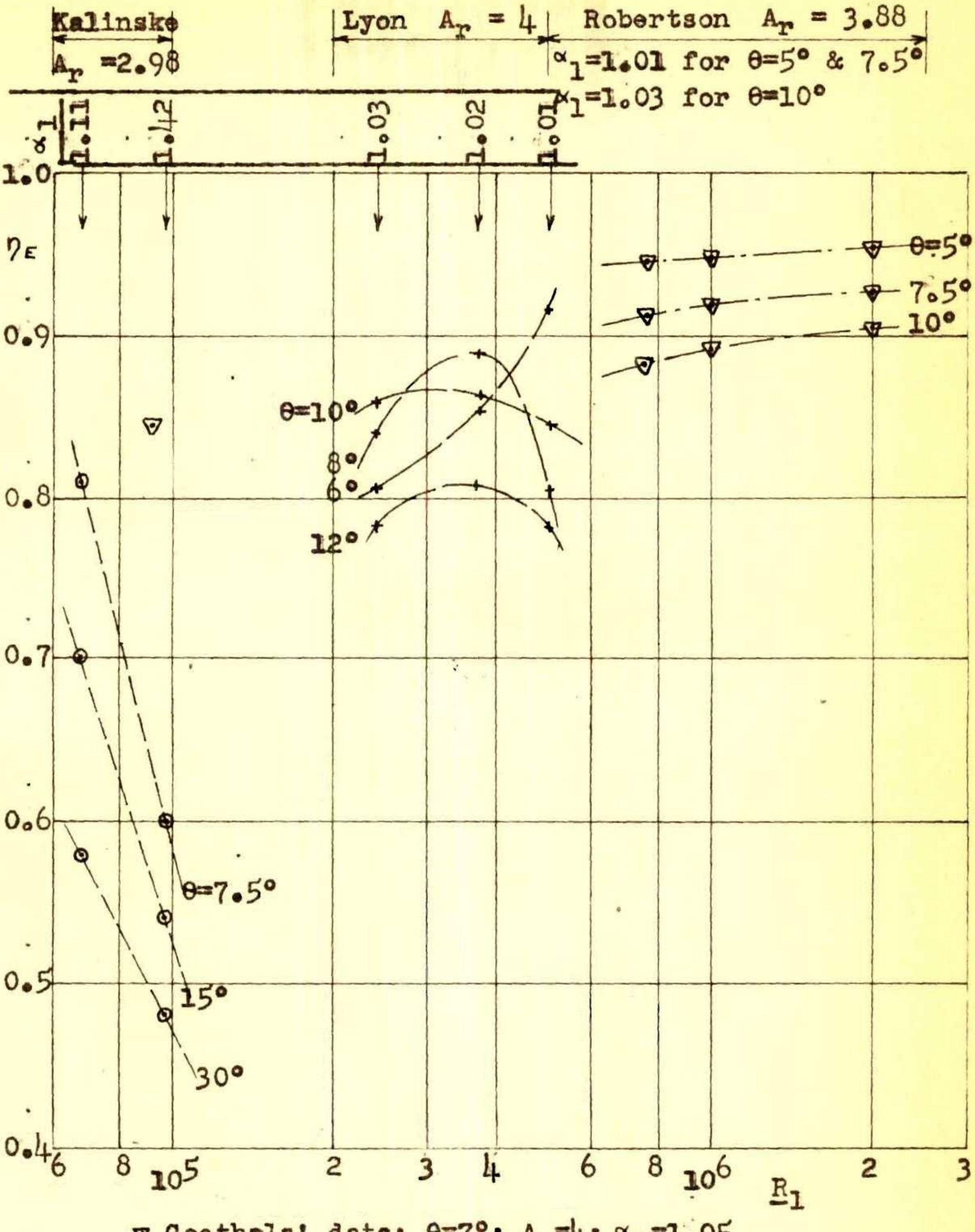


Fig. 4 - Energy efficiency versus area ratio



▽ Goethals' data: θ=7°; A_r=4; α_1=1.05

Fig. 5 - Energy efficiency versus entrance Reynolds number

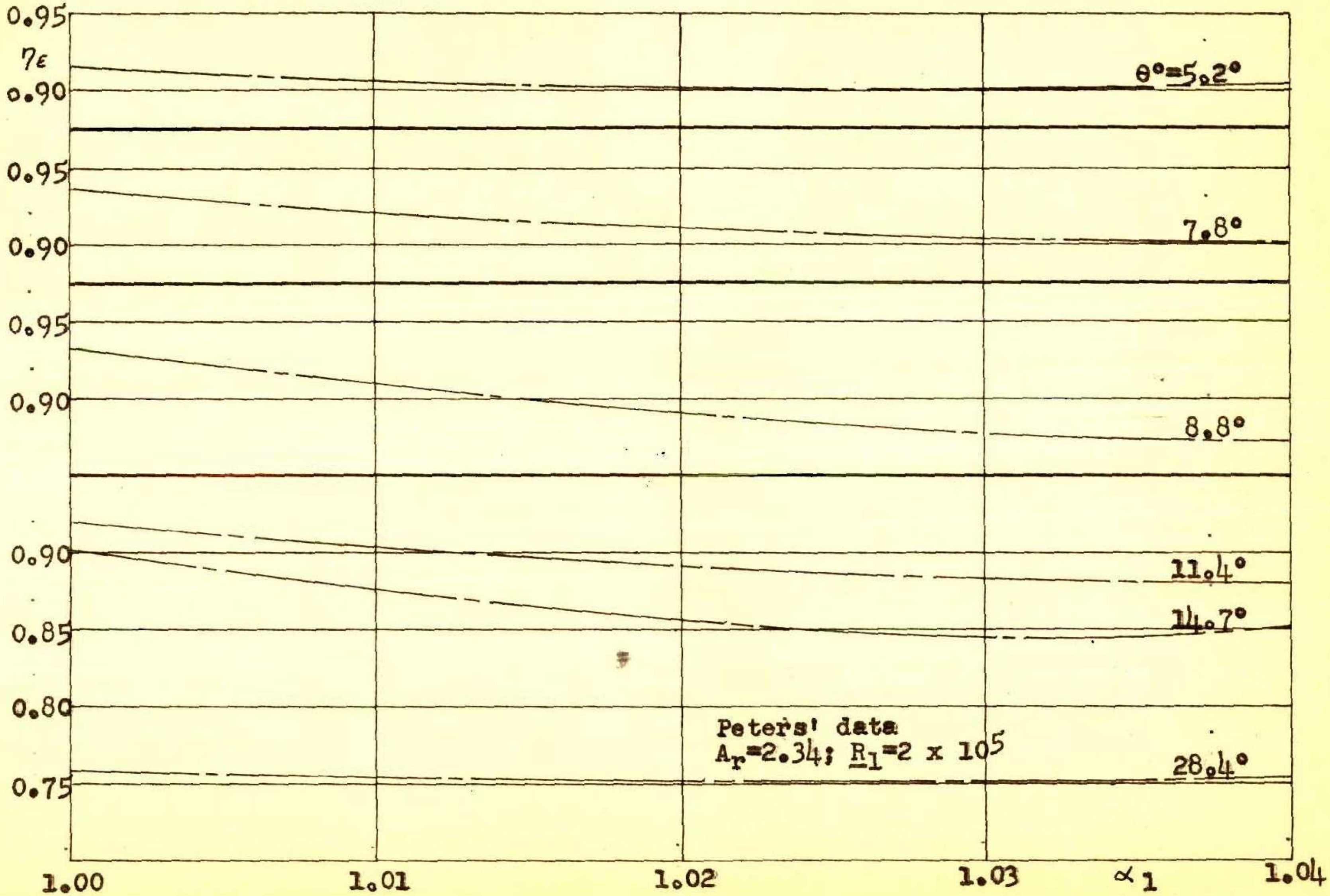


Fig. 6 - Energy efficiency versus entrance kinetic ene_rgy coefficient

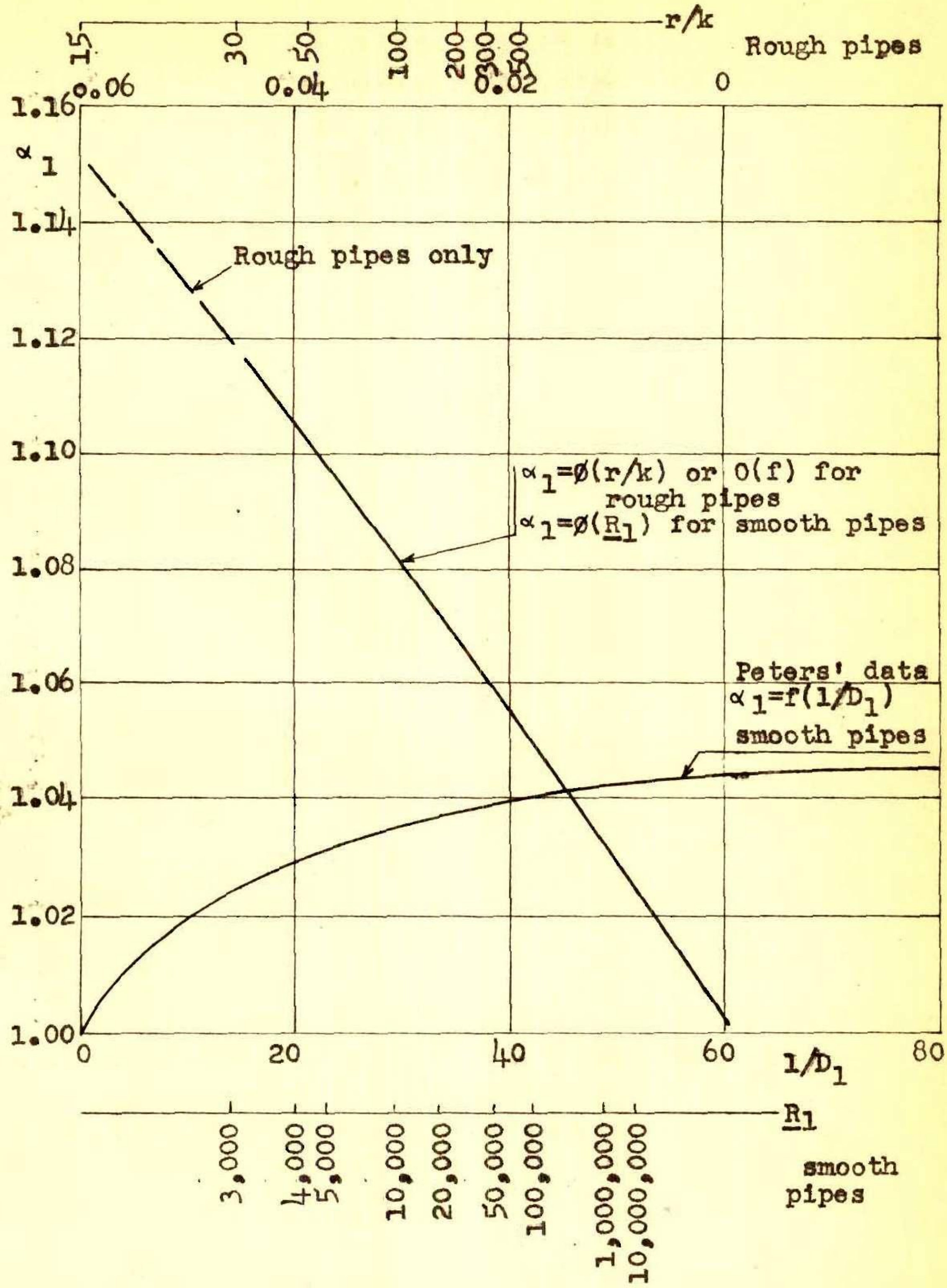


Fig. 7 - Variation of the entrance kinetic energy coefficient with the length of preceding pipes

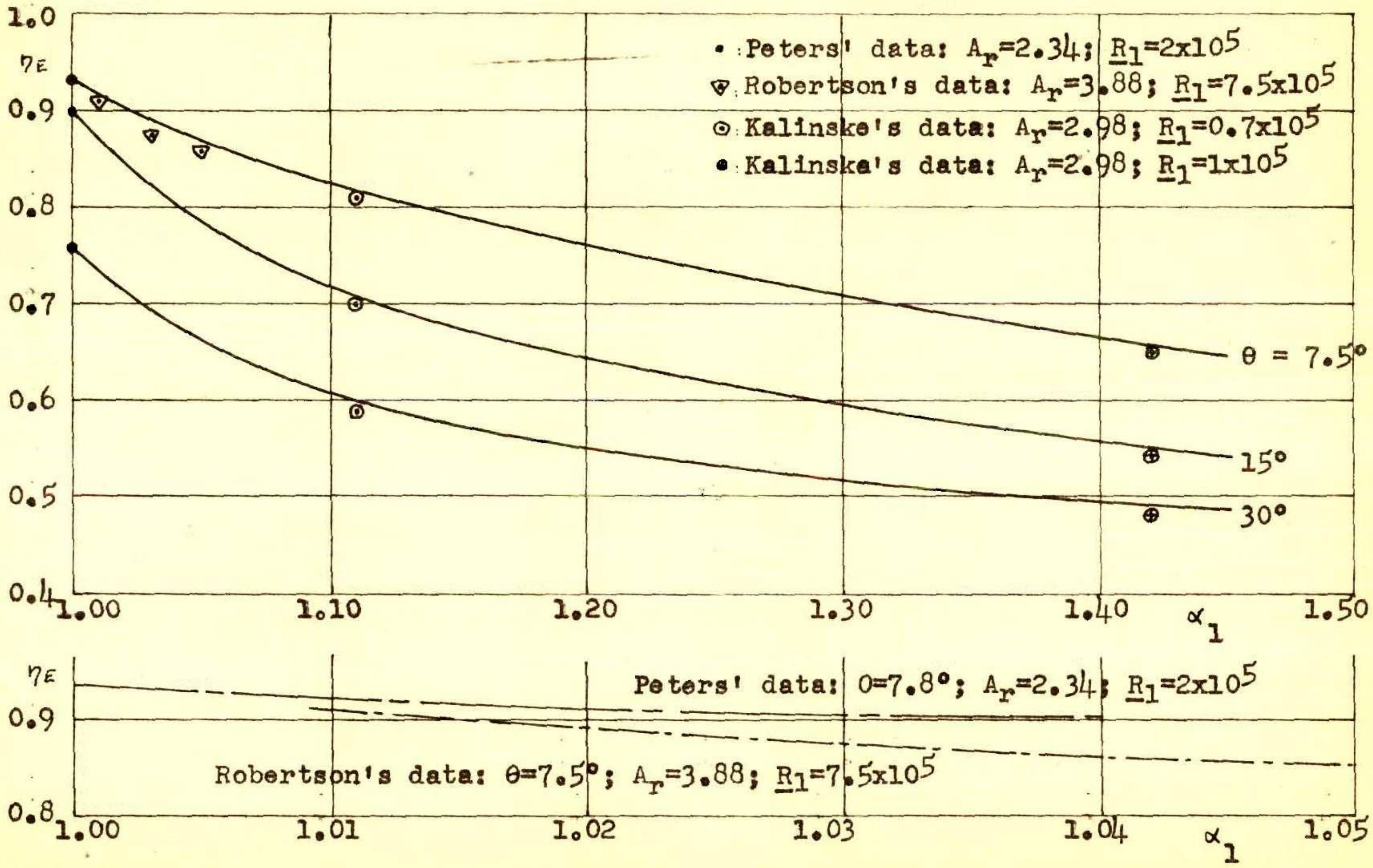
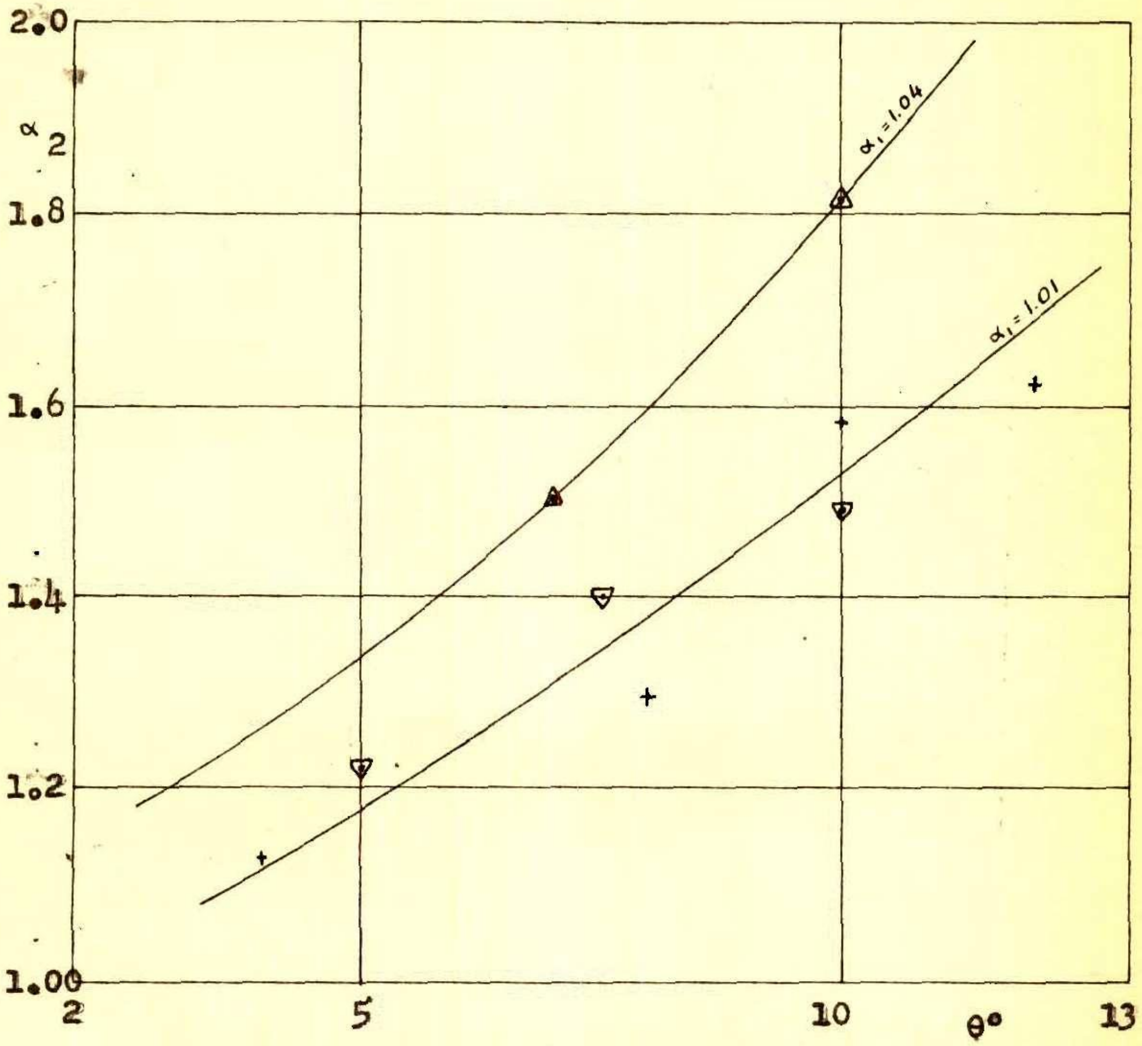


Fig. 8 - Energy efficiency versus entrance kinetic energy coefficient



+ Lyon's data: $A_r = 4$; $\underline{R}_1 = 5 \times 10^5$
▽ Robertson's data: $A_r = 3.88$; $\underline{R}_1 = 1.5 \times 10^6$
△ Goethals' data: $A_r = 4$; $\underline{R}_1 = 3.6 \times 10^6$

Fig. 9 - Exit kinetic energy coefficient versus angle θ°

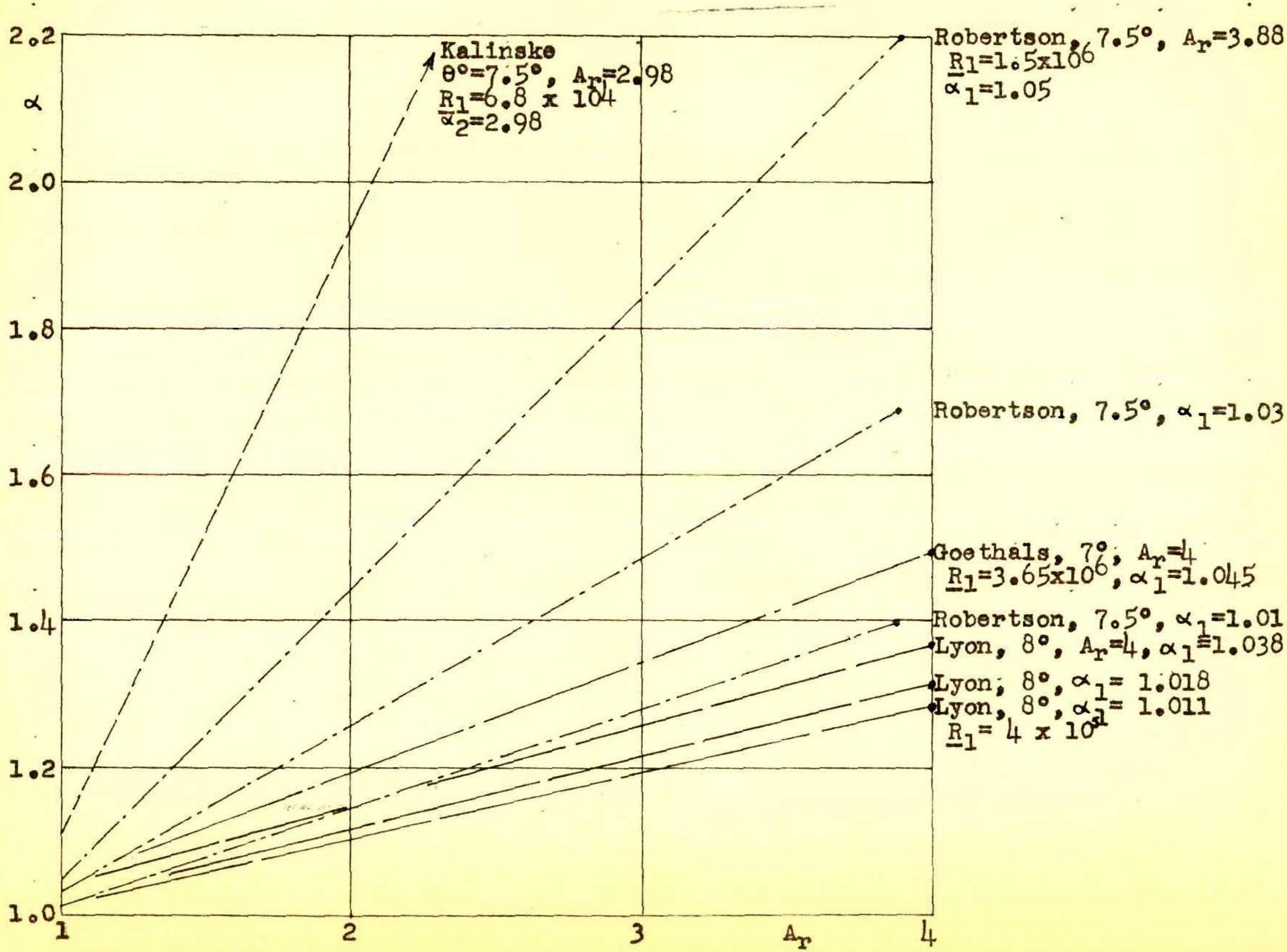


Fig. 10 - Kinetic energy coefficient versus area ratio

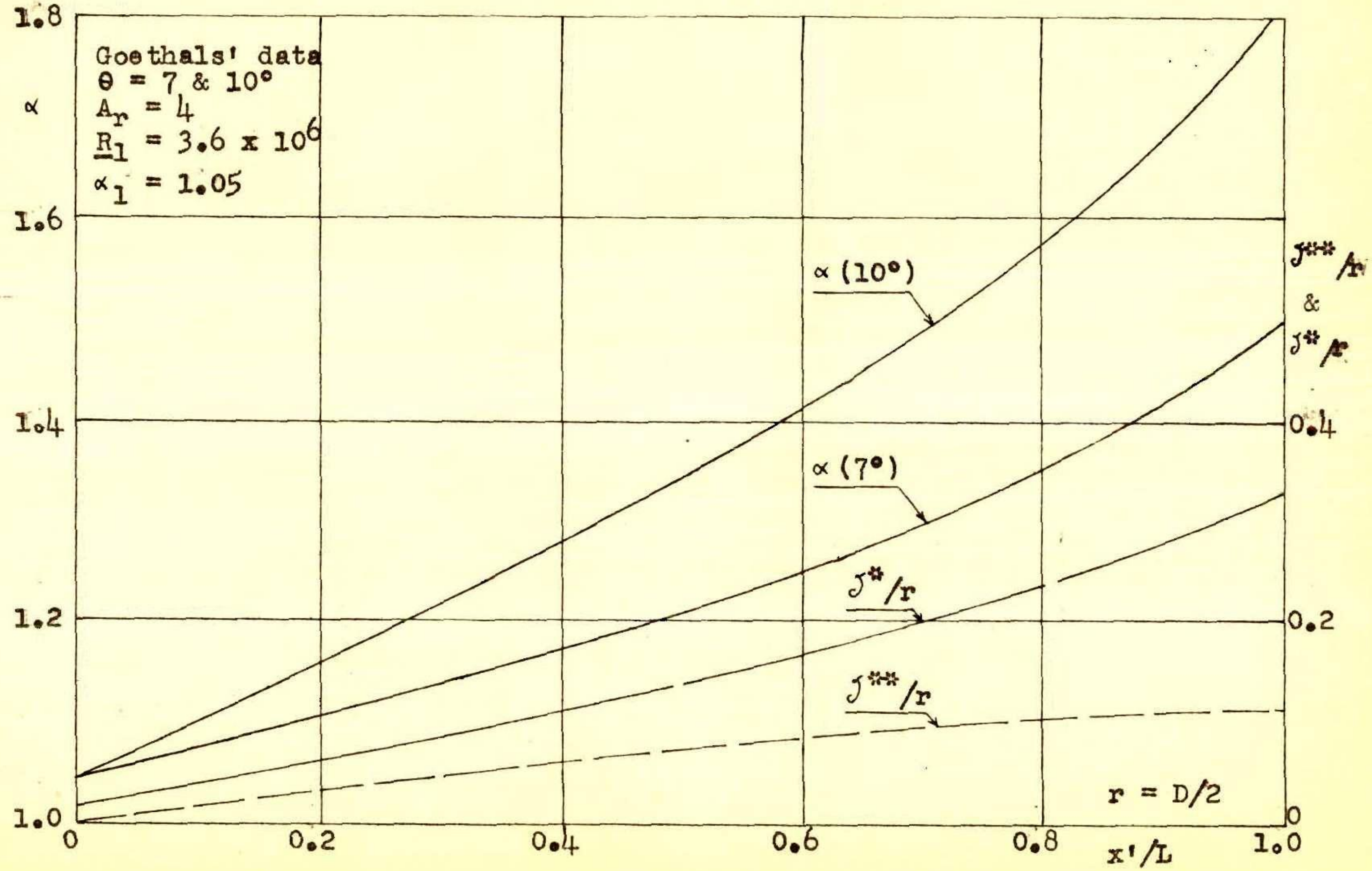
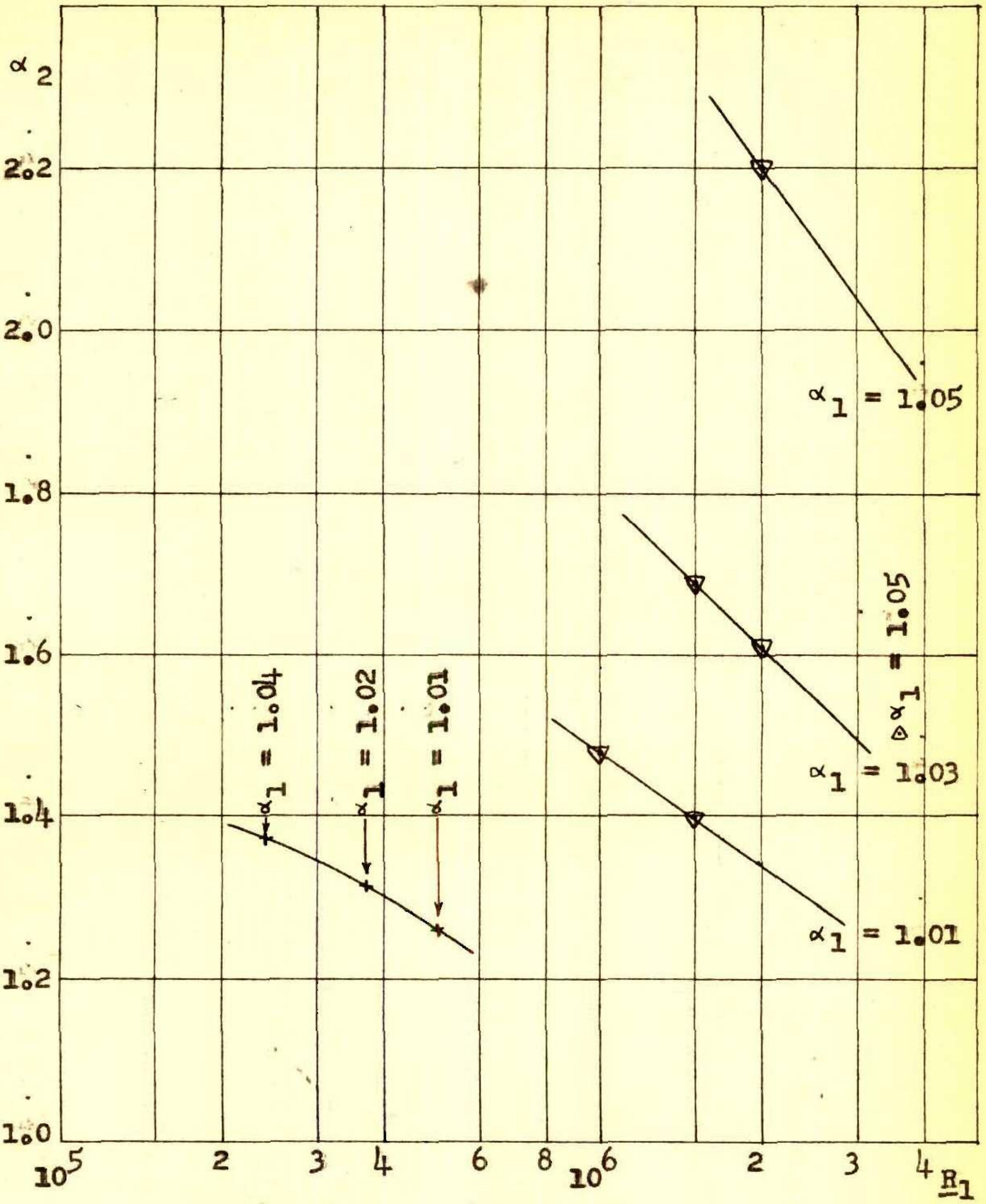


Fig. 11 - Variation of the velocity distribution parameters in a circular diffuser



\+ Lyon's data: $A_r = 4$; $\theta° = 8°$

▼ Robertson's data: $A_r = 3.88$; $\theta° = 7.5°$

▲ Goethals' data: $A_r = 4$; $\theta° = 7°$

Fig. 12 - Exit kinetic energy coefficient versus entrance Reynolds number

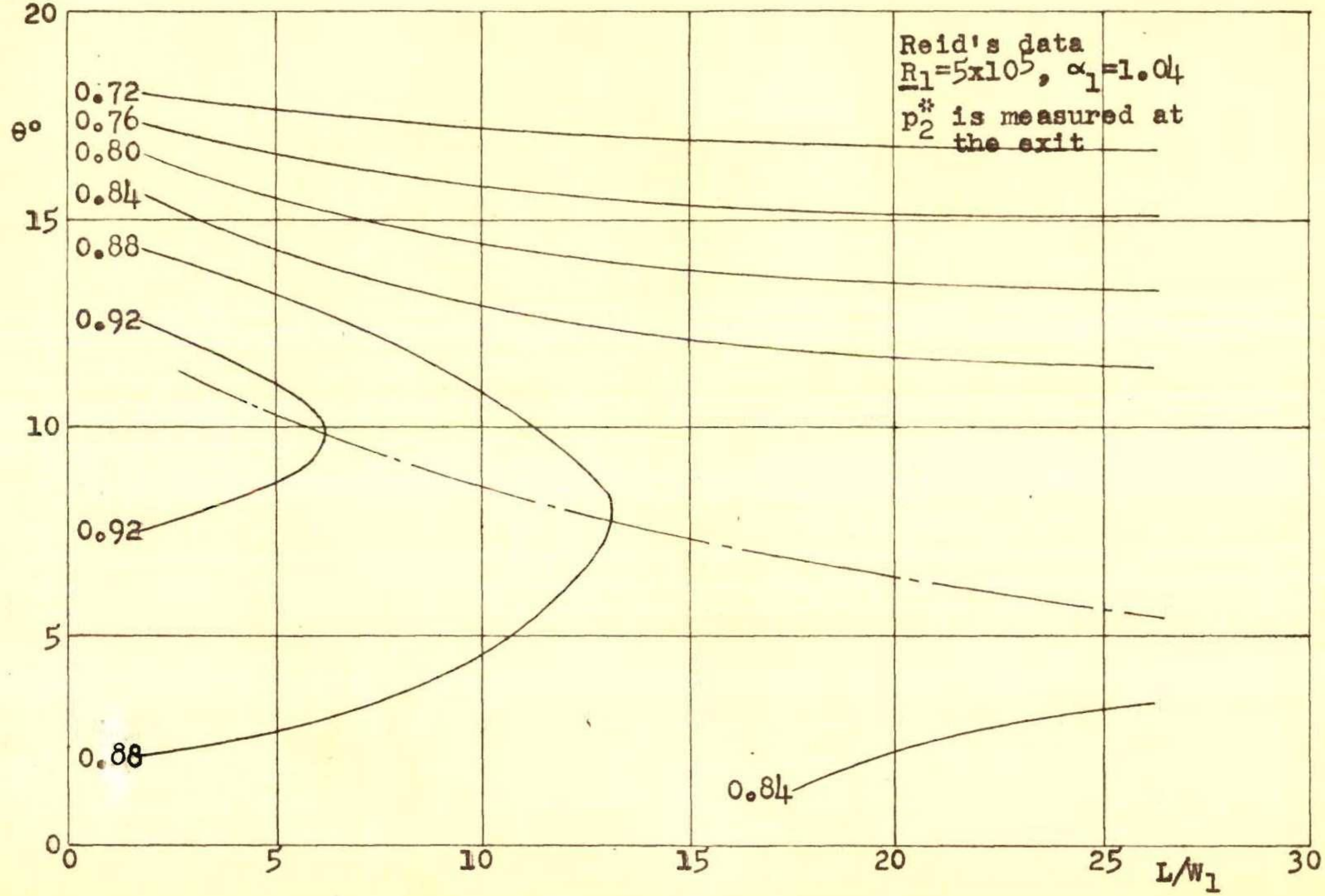


Fig. 13 - Energy efficiency versus angle θ° and length ratio

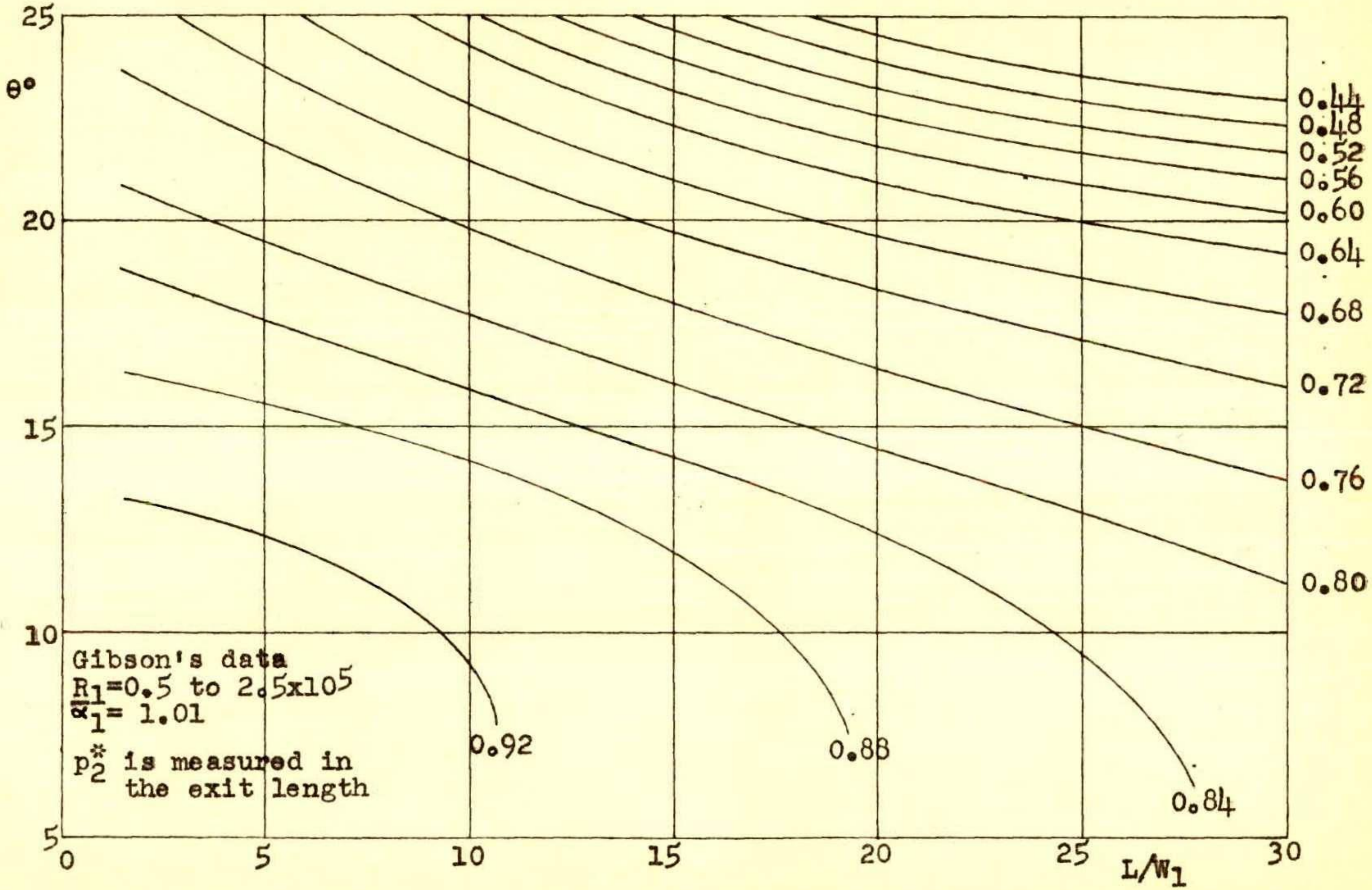


Fig. 14 - Energy efficiency versus angle θ° and length ratio

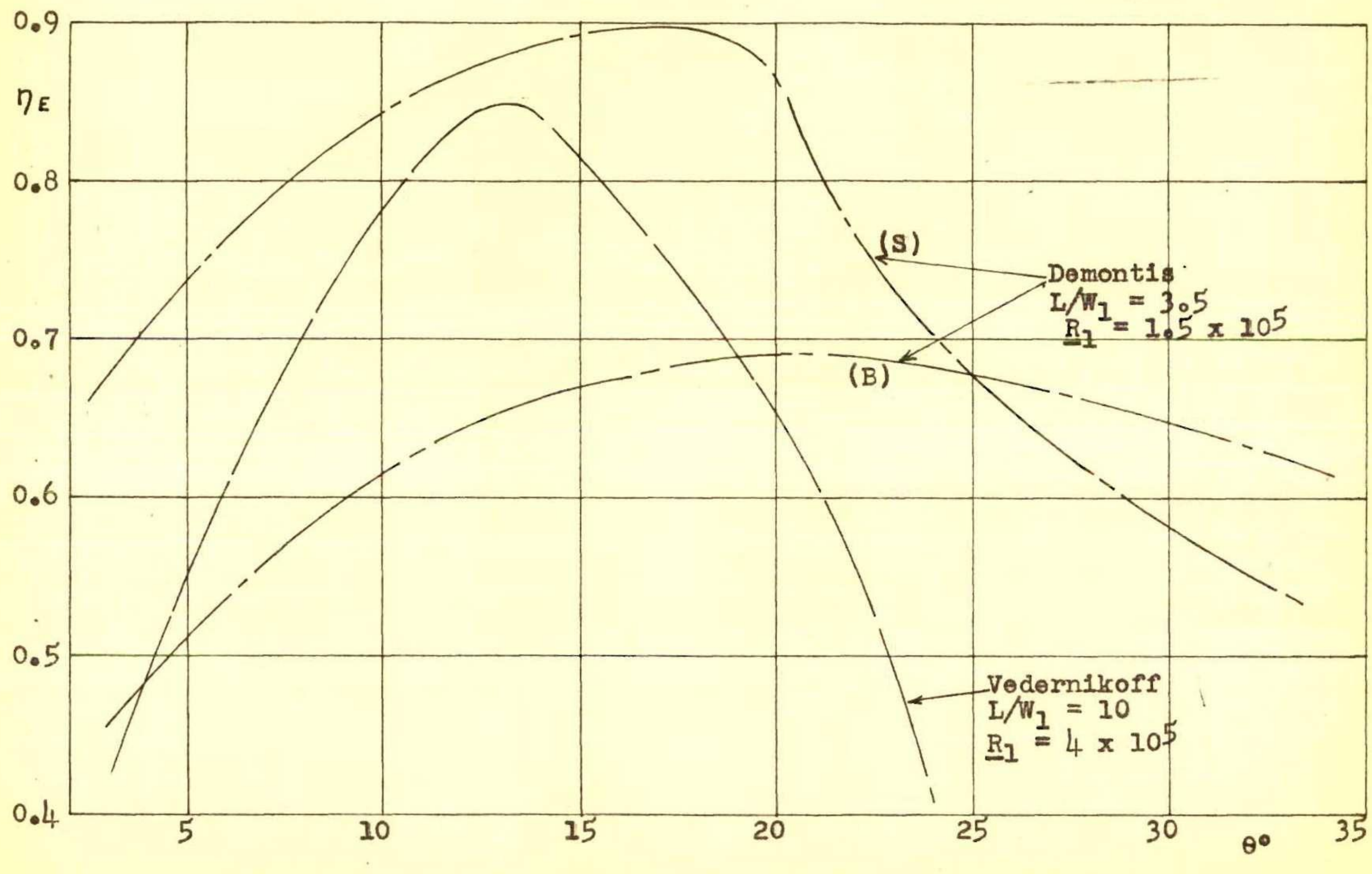


Fig. 15 - Energy efficiency versus angle θ°

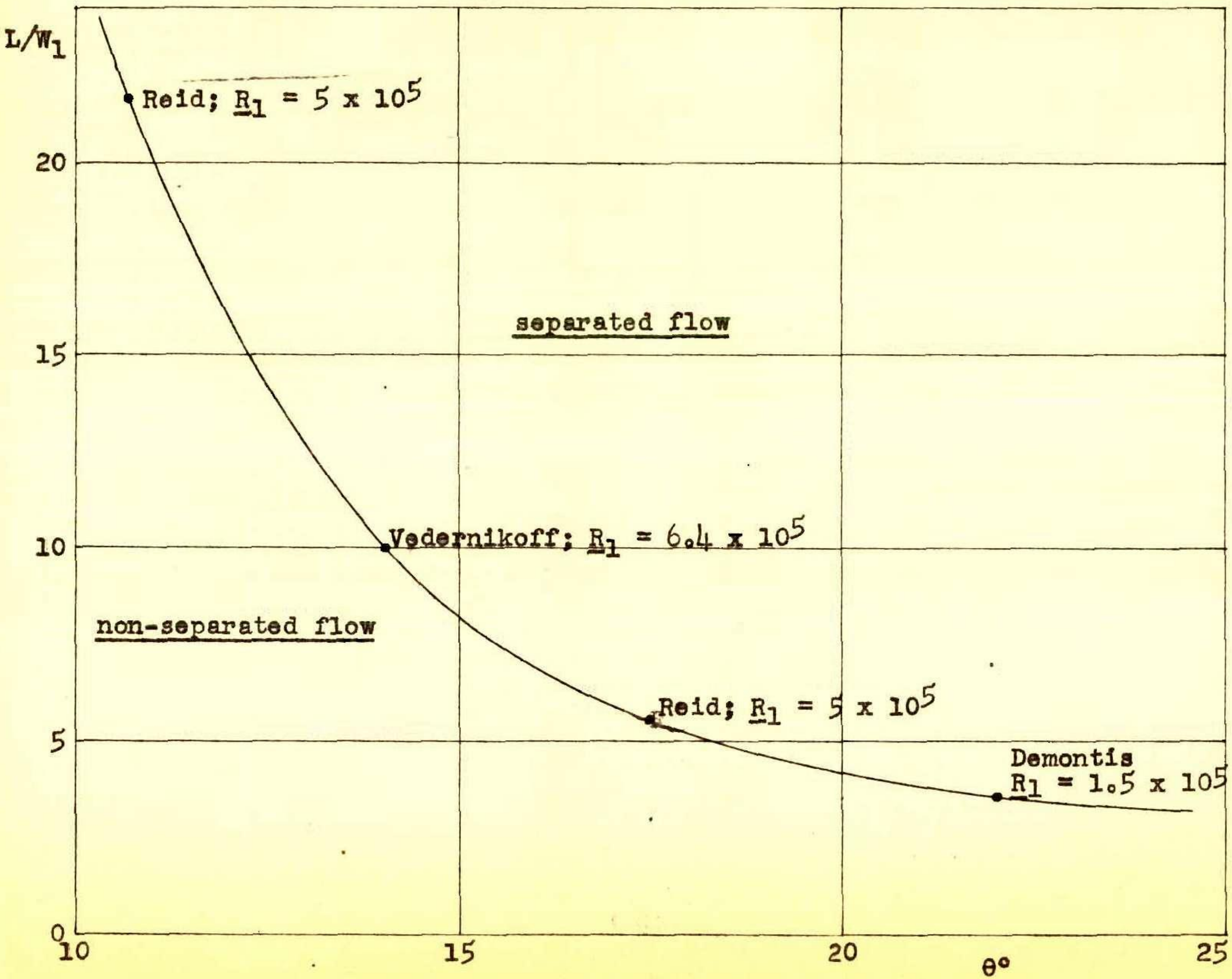
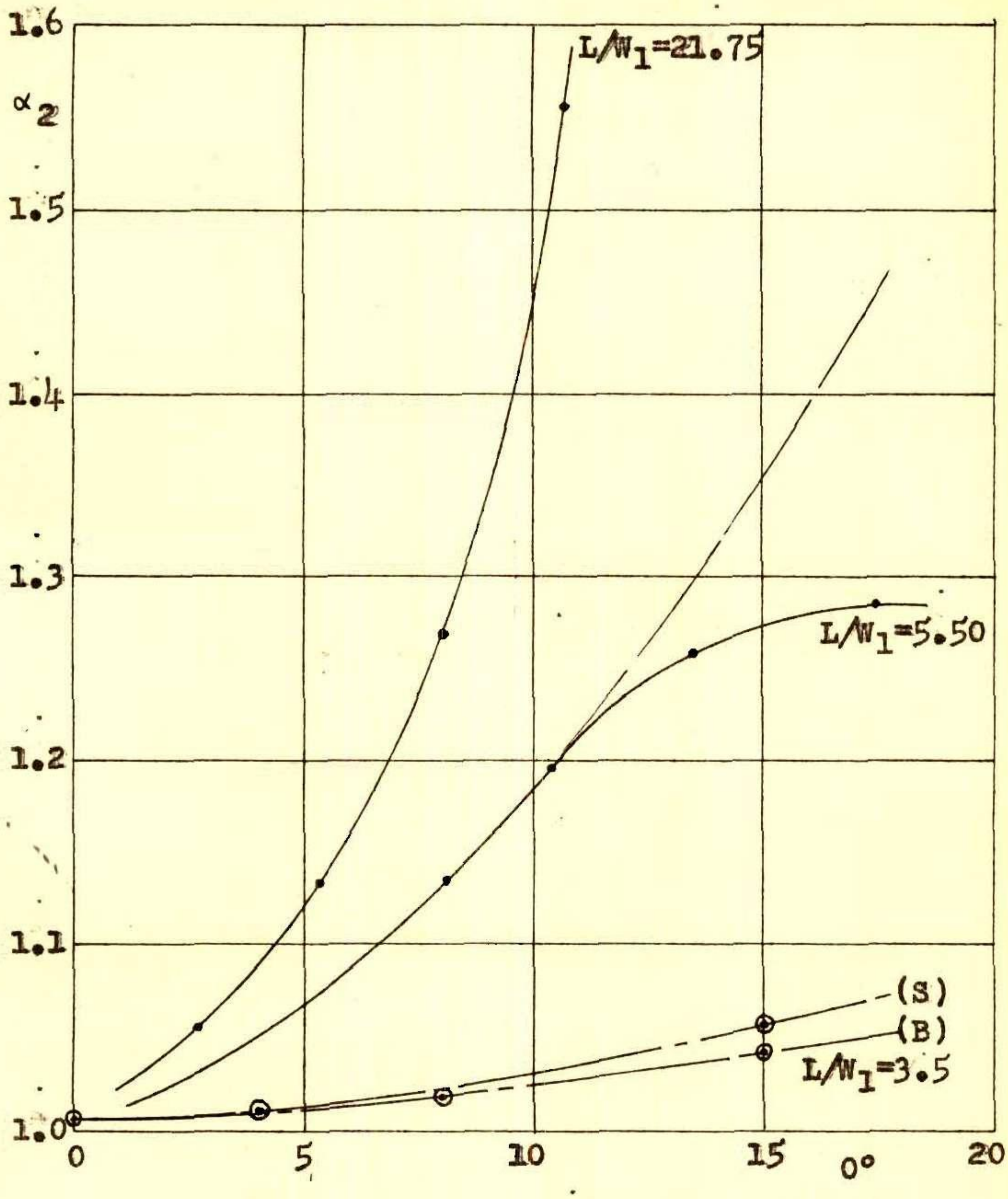


Fig. 16 - Separation in two-dimensional diffusers



• Reid's data: $\alpha_1 = 1.04$; $\underline{R}_1 = 5 \times 10^5$

⊙ Demontis' data: $\alpha_1 = 1.01$; $\underline{R}_1 = 2 \times 10^5$

Fig. 17 - Exit kinetic energy coefficient versus angle $\theta°$

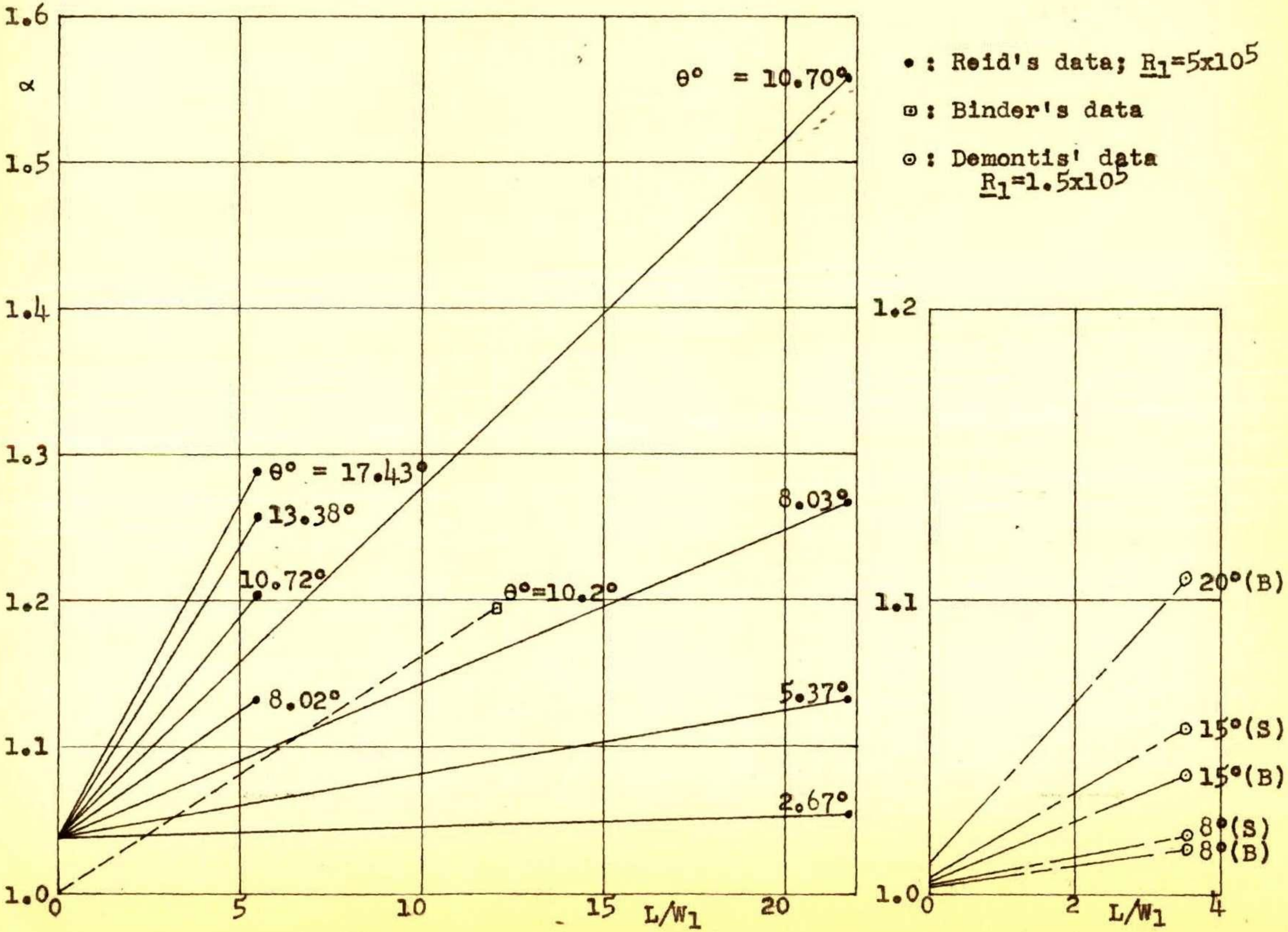


Fig. 18 - Kinetic energy coefficient versus length ratio

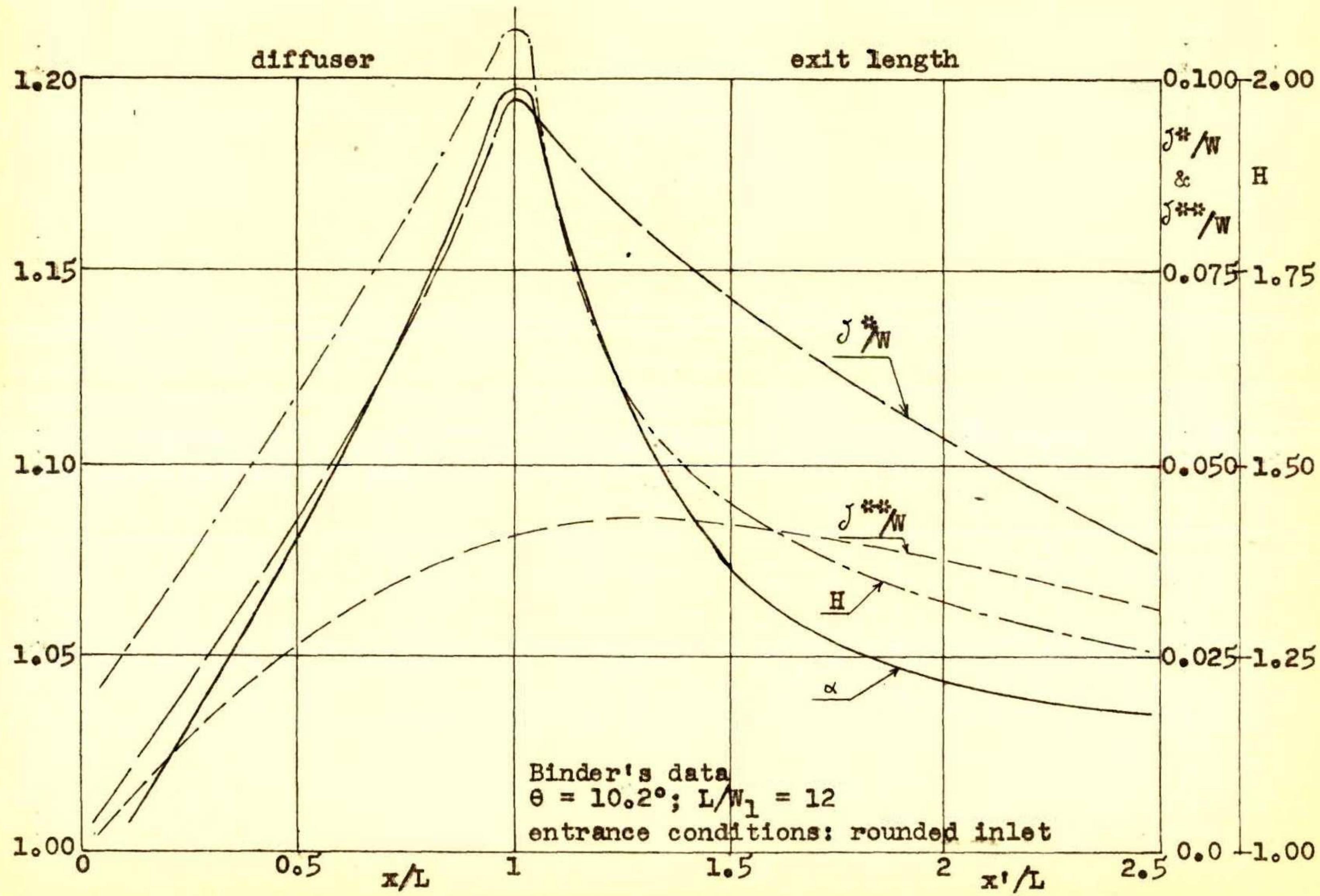


Fig. 19 - Variation of the velocity distribution parameters in a two-dimensional diffuser

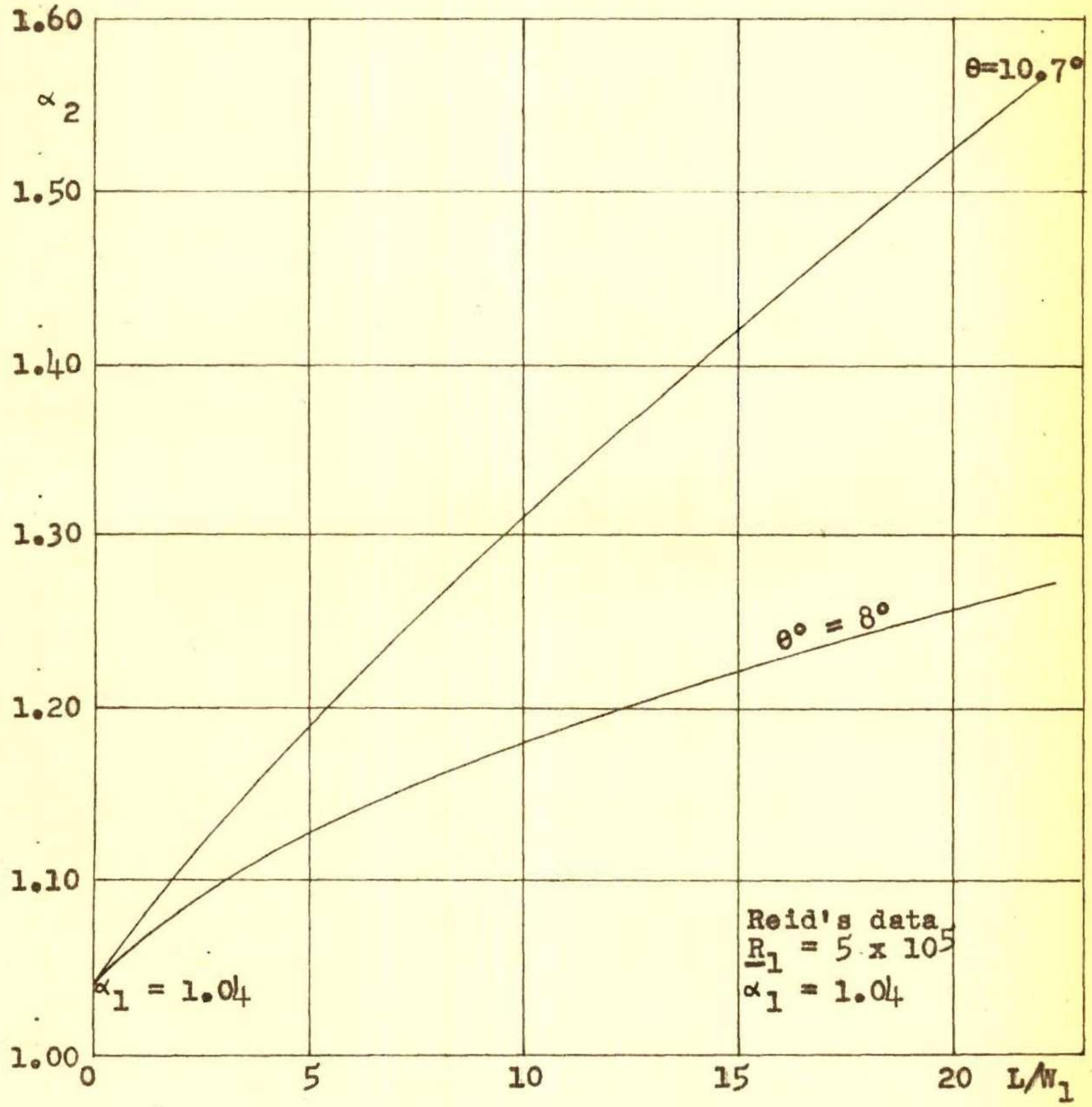


Fig. 20 - Exit kinetic energy coefficient versus length ratio