APPLICATION OF SINGULAR PERTURBATION THEORY TO THE CONTROL OF FLEXIBLE LINK MANIPULATORS

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Abstract: The control problem for robotic manipulators with flexible links is considered in this paper. The dynamic equations of motion can be derived by means of a recently developed Lagrangian-assumed modes method. In the case of flexibility at links it has been shown that there is no analogue of the well established computed torque method widely adopted for rigid arm control. Under the assumption that the flexible dynamics is faster than the rigid dynamics, singular perturbation theory provides an engineering tool for reduced order modeling. The resulting slow subsystem allows the determination of a tracking control as for rigid manipulators, since the number of control variables equals that of controlled variables. For the fast subsystem an additive control is in charge of stabilizing the deflections along the joint angle trajectory. The result is a composite control strategy which combines the advantages of rigidity ("controllability") with those of flexibility ("lightweight compliant structures").

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1. INTRODUCTION

Flexible link manipulators represent one successful strategy to improve today's robot performance. Control is one key to an effective use of lightweight flexible arms. The main drawback is represented by the much more complicated dynamics, as compared to the case of rigid links. As a consequence the control goal for flexible manipulators is to be properly reformulated. In fact a flexible manipulator is required not only to execute some motion, but also it is desired to stabilize the vibrations naturally excited along the motion and damp them out as fast as possible at the terminal point.

The basic step in designing a control system consists in disposing of a good dynamical model for the flexible manipulator. To this purpose the recursive Lagrangian-assumed modes method developed in (Book, 1984) apparently represents a complete and straightforward modeling technique. The generalized coordinates of the dynamic system are the joint variables and the amplitudes of the mode shapes derived by the modal analysis for link deflections.

A first research effort to control a one link flexible arm moving along a pre-defined trajectory is presented in (Siciliano et al., 1986); the approach is based on Model Reference Adaptive Control for the full order system, but it does not seem extensible to multi-degree-of-freedom arms with flexible links. A different strategy which aims at decoupling the joint angle motion from the flexible motion is reported in (Singh and Schy, 1985), but additional stabilizing end forces are required. As a matter of fact, whenever only as many control inputs as joint variables are available, nonlinear control of combined rigid body and flexible motion seems to remain an unresolved issue. In support of this statement, the recent work investigating the feedback linearizibility of manipulators with flexible joints (Cesareo and Marino, 1984) or links (Marino et al., 1986) has shown that the system does not satisfy the conditions for external linearization.

This paper presents an approach to the control of flexible manipulators moving along pre-defined trajectories, based on singular perturbation theory. A conceptually similar approach has proved successful for the case of flexibility concentrated at joints (Marino and Nicosia, 1984). The crucial assumption to be made is that the spectrum of the flexible modes is "well" separated from the spectrum of the rigid modes; a first order estimate of the perturbation parameter is given by the ratio of speeds of the slow vs the fast dynamics (Kokotovic, 1984). In this way two reduced order systems are identified: a slow subsystem which nicely turns out to be of the same order as that of a rigid manipulator, and a fast subsystem in which the slow state variables act as parameters. According to a composite control technique (Suzuki, 1981) a slow feedback tracking control is designed first, then a fast feedback control is added, whose purpose is to stabilize the fast subsystem along its equilibrium trajectory naturally set up by the slow subsystem under the slow control. If the fast subsystem is stabilizable for any trajectory of interest in the manipulator workspace, Tikhonov's theorem (Kokotovic, 1984) will assure that the orbits of the full order system will approach in the limit those derived by the two subsystems. The approach presented in this paper has been tested by means of simulations carried out for the one link flexible arm in the Flexible Automation Laboratory at Georgia Tech.

Last but not least it must be mentioned that full state availability is assumed for control synthesis. In reality the flexible deflections can be obtained from strain gage measurements (Hastings and Book, 1985), whereas their derivatives need to be reconstructed through either a Luenberger observer or a Kalman filter (Sangveraphunsiri, 1984).

2. THE DYNAMIC MODEL

The equations of motion for a manipulator with flexible links can be successfully derived using the recursive Lagrangian approach outlined in (Book, 1984). A solution to the flexible motion of a link i is obtained through modal analysis, under the assumption of small deflection of the link,

$$\mathbf{u}_i(\eta,t) = \sum_{j=1}^{m_i} \delta_{ij}(t)\phi_{ij}(\eta) \tag{1}$$

where ϕ_{ij} is the eigenvector expressing the displacement of mode j of link i's deflection, δ_{ij} is the time-varying amplitude of mode j of link i, and m_i is the number of modes used to describe the deflection of link i. As far as the external forcing terms in Lagrange's formulation, if the clamped-free assumption is adopted for each flexible link, there will be no displacements at joint locations and then the corresponding generalized forces will be zero. Thus, according to the derivation in (Book, 1984), the dynamic equations of motion for an n-degree-of-freedom manipulator with up to n flexible links can be written in the following form:

$$\mathbf{J}(\mathbf{q},\delta)\begin{bmatrix}\ddot{q}\\ \hline \ddot{\delta}\end{bmatrix} = \begin{bmatrix}\mathbf{f}_1(\mathbf{q},\dot{\mathbf{q}},\delta,\dot{\delta})\\ \hline \mathbf{f}_2(\mathbf{q},\dot{\mathbf{q}},\delta,\dot{\delta})\end{bmatrix} + \begin{bmatrix}\mathbf{u}\\ \hline \mathbf{0}\end{bmatrix} \tag{2}$$

where:

$\mathbf{J}$ is the inertia matrix,

$\mathbf{q} = (q_1\ q_2\ \dots\ q_n)^T$ is the vector of joint variables,

$\delta = (\delta_{11}\ \delta_{12}\ \dots\ \delta_{1m_1}\ \delta_{21}\ \dots\ \delta_{2m_2}\ \dots\ \delta_{nm_n})^T$ is the vector of deflection variables,

$\mathbf{f}_1$ includes all the remaining dynamic terms from the joint equations excluding the second derivatives of the generalized coordinates,

$\mathbf{f}_2$ includes all the remaining dynamic terms from the deflection equations excluding the second derivatives of the generalized coordinates,

$\mathbf{u} = (\tau_1\ \tau_2\ \ldots\ \tau_n)^T$ is the control vector of generalized forces applied at the joints;

in (2) appears also a null vector 0 which reflects the assumption of clamped-free vibrational modes.

The positive definite inertia matrix J can be inverted and partitioned as follows:

$$J^{-1} = H = \left[\begin{array}{c|c} H_{11[n\times n]} & H_{12[n\times m']} \\ \hline H_{21[m'\times n]} & H_{22[m'\times m']} \end{array}\right] \tag{3}$$

where $m' = m_1 + m_2 + \ldots + m_n$. Eqs. (2) then become

$$\ddot{q} = \mathbf{H}_{11}\mathbf{f}_1 + \mathbf{H}_{12}\mathbf{f}_2 + \mathbf{H}_{11}\mathbf{u} \tag{4a}$$

$$\ddot{\delta} = H_{21}\mathbf{f}_1 + H_{22}\mathbf{f}_2 + H_{21}\mathbf{u}. \tag{4b}$$

Let define the state variables

$$x = (\mathbf{q}^T \mid \dot{q}^T)^T \qquad z = (\delta^T \mid \dot{\delta}^T)^T. \tag{5}$$

Eqs. (4) become

$$\dot{\mathbf{x}} = \mathbf{g}_1(\mathbf{x},\mathbf{z}) + B_1(\mathbf{x},\mathbf{z})\mathbf{u} \tag{6a}$$

$$\mathbf{g}_1 = \left[\begin{array}{c} 0 \\ \hline H_{11}\mathbf{f}_1 + H_{12}\mathbf{f}_2 \end{array}\right] \qquad B_1 = \left[\begin{array}{c} 0 \\ \hline H_{11} \end{array}\right]$$

$$\dot{\mathbf{z}} = \mathbf{g}_2(\mathbf{x},\mathbf{z}) + B_2(\mathbf{x},\mathbf{z})\mathbf{u} \tag{6b}$$

$$\mathbf{g}_2 = \left[\begin{array}{c} 0 \\ \hline H_{21}\mathbf{f}_1 + H_{22}\mathbf{f}_2 \end{array}\right] \qquad B_2 = \left[\begin{array}{c} 0 \\ \hline H_{21} \end{array}\right]$$

The eqs. (6) of the full order system clearly show why the control problem for flexible link manipulators is a hard one to solve. Exact compensation techniques (inverse dynamics or computed torque method in the robotics literature (Luh et al., 1980)) which rely on the invertibility of the mapping between input torques and the accelerations of the generalized coordinates are not applicable. Similarly the state equations cannot be globally decoupled to design sliding mode controllers for each joint (Slotine, 1985). In addition the perfect model following conditions which underlie the model reference adaptive control approaches (Balestrino et al., 1983) cannot be satisfied. A different control strategy then must be pursued, with the goal of achieving a trade off between accurate joint trajectory tracking and stable vibration motion along the trajectory.

3. A SINGULARLY PERTURBED MODEL

Under the assumption that the spectrum of rigid body motion is well separated from the spectrum of flexible link deflections, the system can be considered a singularly perturbed one. Since the system is nonlinear, the procedure for identifying the perturbation parameter is not straightforward and may involve considerable effort, even if the flexible dynamics are known to be faster than the rigid dynamics. A first order estimate is given by the ratio of the highest frequency of the slow dynamics vs the smallest frequency of the fast dynamics; assuming that a perturbation parameter can be factored out of the flexible dynamics, the overall system in singularly perturbed form results

$$\dot{\mathbf{x}} = \mathbf{g}_1(\mathbf{x},z) + B_1(\mathbf{x},z)\mathbf{u} \tag{7a}$$

$$\mu\dot{z} = \mathbf{g}'_2(\mathbf{x},z) + B'_2(\mathbf{x},z)\mathbf{u}. \tag{7b}$$

At this point the typical steps of a singular perturbation technique can be taken. Because of the presence of μ, the system (7) exhibits a boundary layer phenomenon in the fast variables $\mathbf{z}$. Formally setting $\mu = 0$ accomplishes an order reduction from n + m' to n, because the differential equations (7b) degenerate into the algebraic equations

$$\mathbf{0} = \mathbf{g}_2(\bar{\mathbf{x}},\bar{\mathbf{z}}) + B_2(\bar{\mathbf{x}},\bar{\mathbf{z}})\bar{\mathbf{u}} \tag{8}$$

where the bar is used to indicate that the variables belong to the so-called slow subsystem with $\mu = 0$.

Under the assumption that in a domain of interest (8) has an isolated root, i.e. the system (7) is in standard form (Kokotovic, 1984),

$$\bar{\mathbf{z}} = \mathbf{h}(\bar{\mathbf{x}},\bar{\mathbf{u}}), \tag{9}$$

a well-defined n-dimensional reduced model can be obtained by substituting (9) in (7a), i.e.

$$\dot{\bar{\mathbf{x}}} = \mathbf{f}(\bar{\mathbf{x}}) + B(\bar{\mathbf{x}})\bar{\mathbf{u}}. \tag{10}$$

The conditions under which (8) admits a solution for $\bar{z}$ are discussed in (Siciliano and Book, 1986) for the specific case of flexible link manipulators. The model (10) is a quasi-steady-state model, because $\mathbf{z}$, whose velocity $\dot{\mathbf{z}}$ can be large when μ is small, may rapidly converge to the root (9), which is the quasi-steady-state form of (7b). Loosely speaking, the slow response is approximated by the reduced model (10), while the discrepancy between the response of the reduced model (10) and that of the full model (7) is the fast transient.

To derive the fast subsystem or the boundary layer system, it is assumed that the slow variables are constant in the boundary layer; that is $\dot{\bar{\mathbf{z}}} = \mathbf{0}$ and $\mathbf{x} = \bar{\mathbf{x}} =$ constant. Operating the state variable change around the equilibrium trajectory $\mathbf{z}_f = \mathbf{z} - \bar{\mathbf{z}}$, and correspondingly $\mathbf{u}_f = \mathbf{u} - \bar{\mathbf{u}}$, the fast subsystem of (7) results

$$\frac{d\mathbf{z}_f}{d\tau} = \mathbf{a}(\bar{\mathbf{x}},\mathbf{z}_f) + C(\bar{\mathbf{x}},\mathbf{z}_f)\mathbf{u}_f \tag{11}$$

where $\tau = t/\mu$ is the fast time scale. It must be emphasized that in (11), due to the time scale introduced, the slow variables $\bar{\mathbf{x}}$ act as parameters. Setting $\mathbf{u}_f = \mathbf{0}$ in (11) also gives an estimate of the natural frequencies excited by the rigid body trajectory under the effect of the slow control; this point may be conveniently exploited for trajectory planning purposes.

4. COMPOSITE CONTROL DESIGN ISSUES

As evidenced by the two reduced order subsystems (10) and (11), the design of a feedback control $\mathbf{u}$ for the full order system (7) can be split into two separate designs of feedback controls $\bar{\mathbf{u}}$ and $\mathbf{u}_f$ for the two reduced order systems, namely a two-time scale composite control (Suzuki, 1981)

$$\mathbf{u} = \bar{\mathbf{u}}(\bar{\mathbf{x}}) + \mathbf{u}_f(\bar{\mathbf{x}},\mathbf{z}_f) \tag{12}$$

with the constraint that $\mathbf{u}_f(\bar{\mathbf{x}},\mathbf{0}) = \mathbf{0}$, such that the fast control $\mathbf{u}_f$ is inactive along the solution (9).

As far as the slow control is concerned, the reduced system (10) allows the adoption of well established control techniques as developed for rigid manipulators, such as (Luh et al., 1980), (Slotine, 1985), (Balestrino et al., 1983) to mention only a few. More specifically, assume that a trajectory is specified for the joint variables, say $\hat{x}(t)$, as a result of an inverse kinematic computation from the end effector trajectory for the equivalent rigid body manipulator. The slow control can be generally thought of as

$$\bar{u}(\bar{x}) = B^{\dagger}(\bar{x})[-\mathbf{f}(\bar{x}) + \mathbf{v}(\bar{x},\hat{x})] \tag{13}$$

where the first term provides a precompensation of the nonlinear term $\mathbf{f}$, and $\mathbf{v}$ is an additional control input which allows the slow subsystem to track the desired trajectory and compensate for parameter uncertainties and modeling inaccuracies (Slotine, 1985), (De Maria et al., 1985).

Incidentally it might observed that the strategy of adaptively controlling the system (7a), by just dropping the flexible dynamics (7b) and considering $\mathbf{z}$ in (7a) as a disturbance to the system, is likely to fail, since no assumption on the boundedness of the disturbance can be made.

Another promising approach towards the synthesis of the slow control could be a one based on the concept of integral manifold for the slow system (Sobolev, 1984); it has already been proposed for the case of flexibility at joints (Spong et al., 1985). The application to flexible link manipulators is currently being investigated by the authors.

At this point the singular perturbation theory requires that the boundary layer system (11) be uniformly stable along the equilibrium trajectory $\bar{\mathbf{z}}$ given in (9). This can be accomplished only if the fast subsystem (11) is uniformly stabilizable for any slow trajectory $\bar{x}(t)$. If this assumption holds, a fast state feedback control of the type

$$\mathbf{u}_f(\bar{\mathbf{x}},z_f) = K_f(\bar{x})z_f \tag{14}$$

will serve the purpose. On the other hand $\bar{\mathbf{x}}$ can be seen as unknown parameters and $\mathbf{u}_f$ can be designed as an adaptive controller.

Under the above conditions Tikhonov's theorem (Kokotovic, 1984) assures that the states of the full order system can be approximated by

$$\mathbf{x} = \bar{\mathbf{x}} + \mathbf{O}(\mu) \tag{15a}$$

$$z = \bar{z} + z_f + \mathbf{O}(\mu). \tag{15b}$$

The goal of tracking the joint trajectory and stabilizing the deflections around the quasi-steady-state trajectory, naturally set up by the slow system under the slow control, is achieved by an $\mathbb{O}(\mu)$ approximation. This is the typical result of a singular perturbation approach.

5. A CASE STUDY

The one link flexible arm prototype in the Flexible Automation Laboratory at Georgia Tech (fig. 1) has been chosen to develop a case study. Its dynamic model can be found in (Siciliano and Book, 1986). Two modes have been chosen in (1) to expand the deflection of the single flexible link. It has been tested that the dynamic system is in the standard form (Kokotovic, 1984), that is (8) has an isolated root of the type (9). The slow control (13) can be chosen as a linear model following control. Since the fast subsystem is a linear system parametrized in the slow variables, the fast control (14) can be selected according to optimal control with a prescribed degree of stability. Several simulation studies are currently being carried out and will be described in details in the final version of the paper. Figs. 2-5 illustrate some preliminary results with a trapezoidal velocity profile commanded to the joint angle.

6. CONCLUSIONS

A singular perturbation approach has been proposed for control of lightweight flexible manipulators moving along predefined paths.

The drawback concerned with flexible arm tracking control, namely the number of control inputs is less than the number of controlled variables, has been turned around by means of a model order reduction which is characteristic of a two-time scale approach. Indeed, for the slow subsystem well established control strategies, as for rigid arms, can be adopted. For the fast subsystem a stabilizing control along the quasi-steady-state trajectory, set up by the slow subsystem as controlled by the slow control, is needed. The control goal is achieved by an $O(\mu)$ approximation, where μ represents the ratio of the speeds of the slow vs the fast dynamics. The stability of the full order system has not been directly addressed, and estimates of the domain of attraction and of an upper bound on the pertubation parameter are still being sought.

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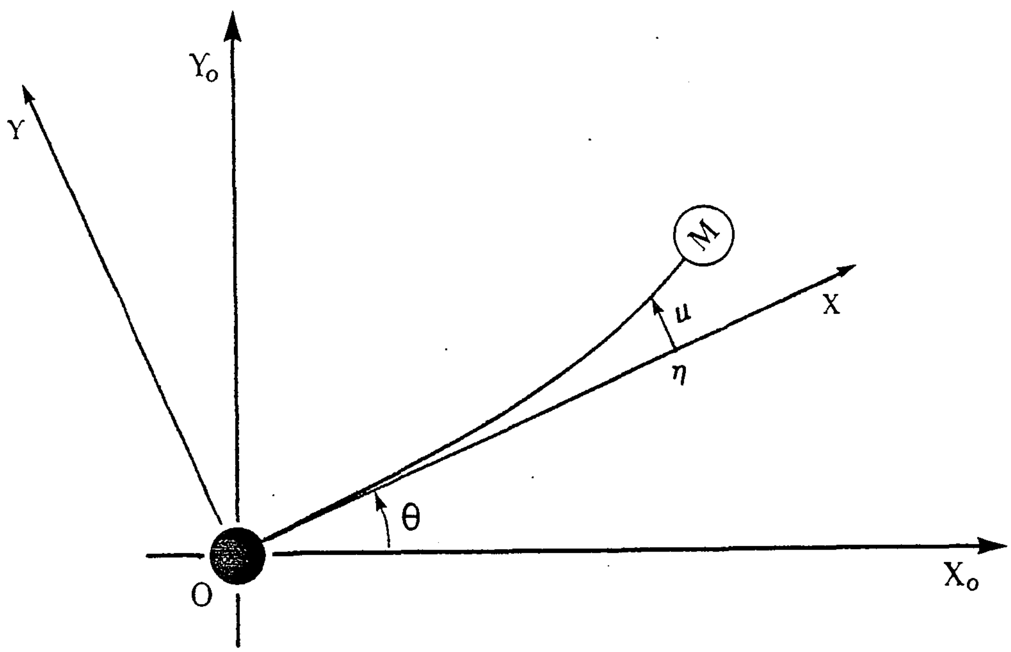
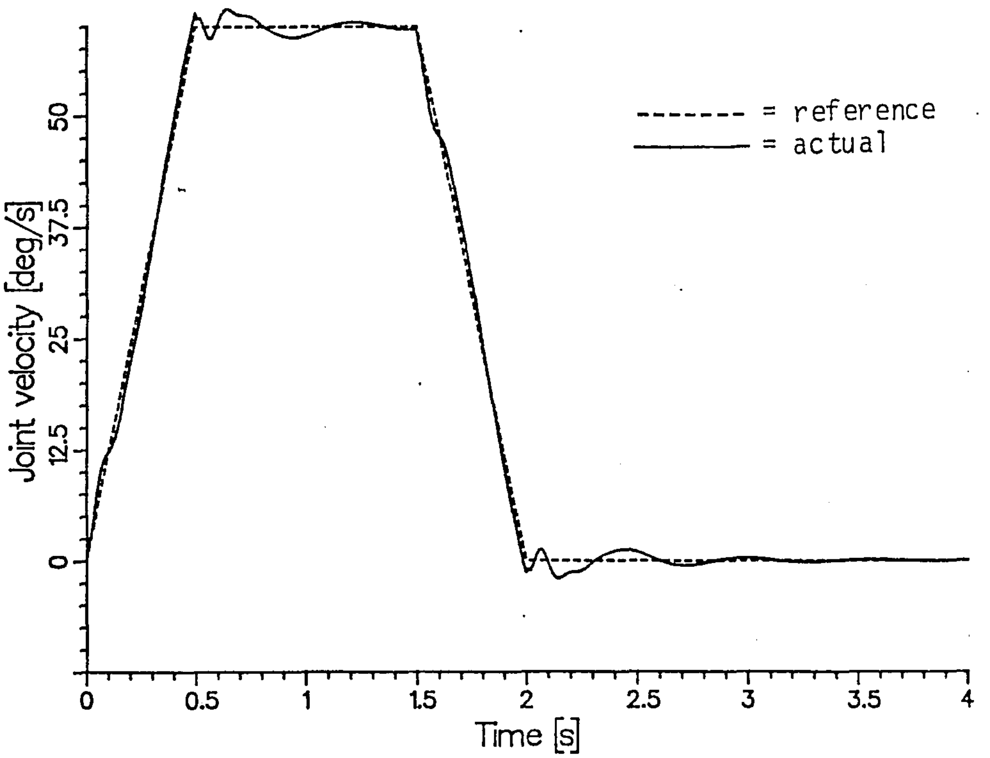
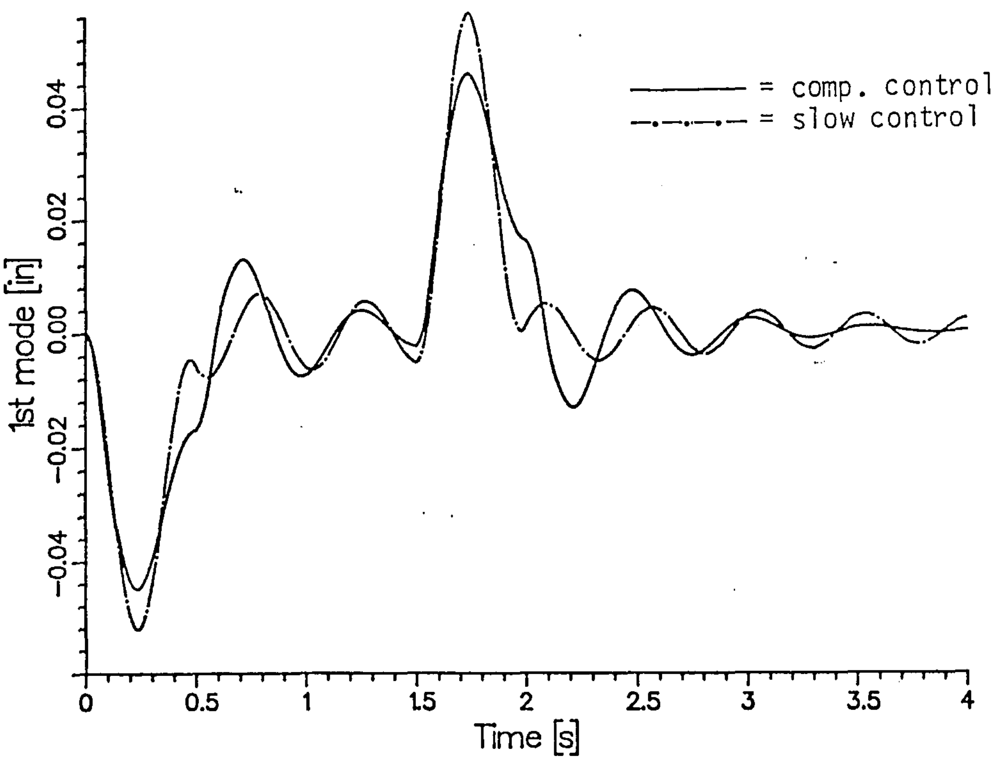


Fig. 1



= reference
= actual
Joint velocity [deg/s]
0
12.5
25
37.5
50
0
0.5
1
1.5
2
2.5
3
3.5
4
Time [s]

Fig. 2



= comp. control
= slow control
1st mode [in]
−0.04
−0.02
0.00
0.02
0.04
0
0.5
1
1.5
2
2.5
3
3.5
4
Time [s]

Fig. 3

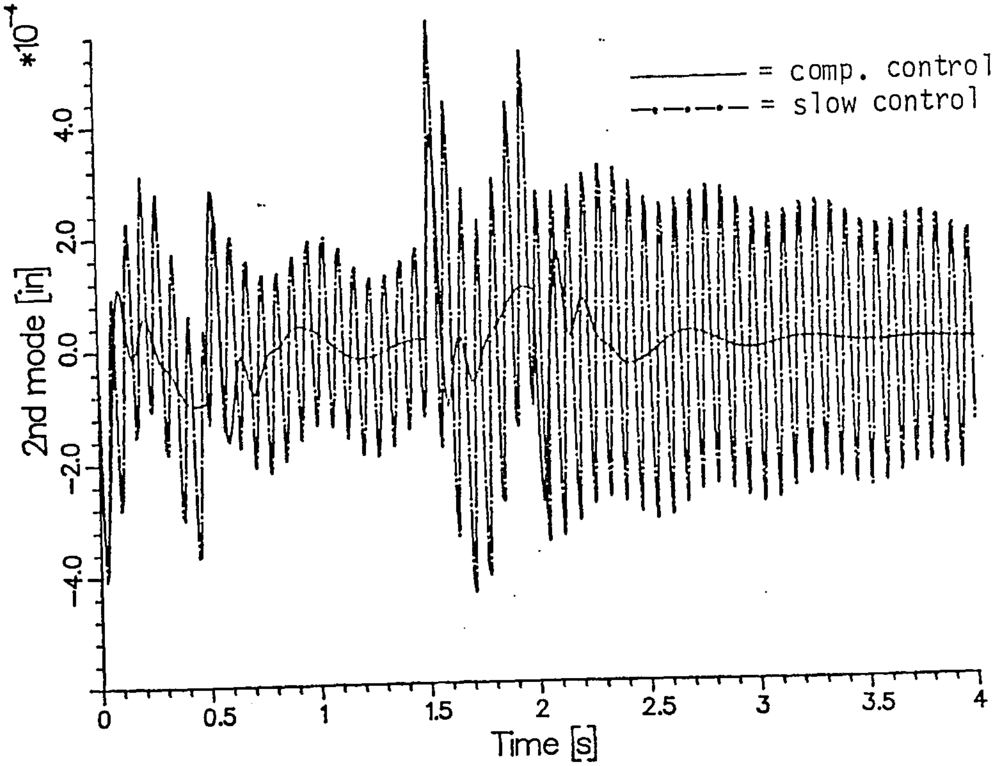

= comp. control
= slow control
*10^-4
2nd mode [in]
4.0
2.0
0.0
-2.0
-4.0
0
0.5
1
1.5
2
2.5
3
3.5
4
Time [s]

Fig. 4

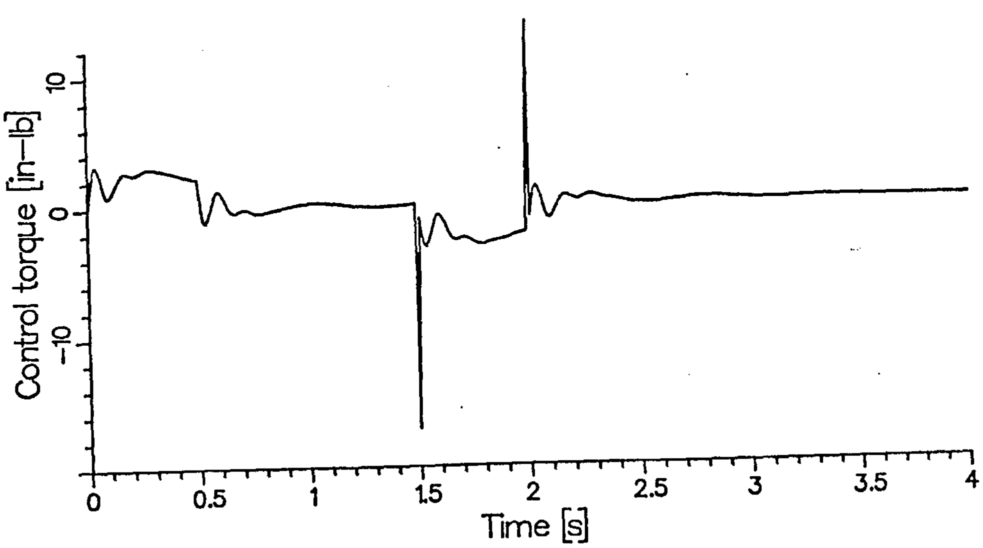

Control torque [in-lb]
10
0
-10
0
0.5
1
1.5
2
2.5
3
3.5
4
Time [s]

Fig. 5