A BI-LEVEL OPTIMIZATION APPROACH FOR TECHNOLOGY SELECTION

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Abstract

Technology selection is a crucial step in the process of aircraft design. If the performance and economic requirements are not fulfilled for any combination of the design variables, new technologies need to be infused in the design. Typically, the designer has a pool of technology options. The technologies to be infused in the new design are to be selected from this pool so as to achieve improvements such as increased performance, reduced risk, reduced cost etc. Thus, it is critical to be able to perform a quick and accurate assessment of the available technologies in the early stages of the design process. However, if the set of available technologies is large, the designer runs into a huge combinatorial optimization problem. To tackle the technology selection problem, a systematic approach called Technology Identification, Evaluation and Selection (TIES) has been developed to choose the best set of technologies and arrive at a feasible and viable design solution. However, the issue of dealing with large combinatorial problems still remains. A new approach for tackling the same problem of technology selection was inspired from the TIES methodology and is discussed in this paper. This approach is based on identifying an optimal point in an intermediate variable space, which later on serves as the target point for technology selection. The new approach, called 'Bi-level approach' provides additional insights and expedites technology selection, thus rendering efficiency to the preliminary design process. After describing the bi-level approach, its application to an aircraft design problem is presented.

Introduction and Background

One can think of the technology selection process in an abstract sense as being an exercise in constrained combinatorial optimization. The objective is to select an optimal set of technologies from a list of possible technology choices. Optimal in this context means those technologies that represent the best fit to a given set of conflicting requirements and program objectives. This situation is encountered by aircraft manufacturers during the process of designing an aircraft to meet a new requirement. The production company's objective is to find an aircraft design that will be sufficiently advanced to offer a significant competitive advantage but which can also be developed within allowable time and budget constraints. A typical aircraft manufacturer may have dozens of technology concepts under development at any given time, but only a few of these technologies are likely to be suitable for a given set of program objectives. Some of these technologies are incompatible with others or are dependent on others in complex ways. Finally, the number of permissible technology combinations grows exponentially with the number of technologies available. The computational analysis involved in such a case is prohibitively time consuming making it difficult to investigate every possible technology combination. It therefore becomes imperative for the manufacturing company to find a means of efficiently searching technologies in order to locate those few that are the best fit for the given requirements combinations while taking the compatibility constraints into account.

Roth, Mavris et al[1, 2] have shown that use of genetic Algorithms (GA) within the Technology Identification, Evaluation, and Selection (TIES) method is an effective means of solving the constrained combinatorial optimization problem. The method works by using TIES techniques to create a compact, generic model to represent the impact of any given technology in terms of certain kappa-factors[3], which later on are used to evaluate the system level responses. The functionality of the TIES method can be depicted as in Figure 1.

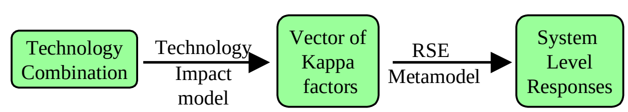

Figure 1: Technology Evaluation in the TIES Method

The GA uses this technology impact model to assess the fitness of the new design. The GA works by creating a pool of technology combinations and evaluating them with the TIES model to estimate the performance of the whole system. These combinations are then compared to one another and the superior ones are kept in the pool while the inferior are discarded. The surviving combinations are then used as 'parents' to create a new generation of combinations. This process is repeated over many generations until the population converges to an optimized set of technology combinations. The surviving technology combinations are

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taken to be the best solutions for the given objective function.

As stated in Ref. 2, the abovementioned technology selection approach is very useful for various reasons. It allows one to create a generic technology model that can easily be extended to include new technology options as they emerge. The model can be created at minimal expense and incorporates a combination of expert opinion and analytical data. While using GA, it is easy to incorporate many types of data into the objective function, including subjective data, non-numerical data, probabilistic data etc.

As seen in Figure 1, prior work done using this technique has involved application of the technology impact model on the candidate technology set, which is followed by application of the RSE metamodel[3] to obtain the final response values. DeLaurentis, Mavris[4] suggested a new modeling framework for the complex system design process. This framework is based on the traditional control system feedback architecture. Instead of choosing from a specific set of technologies, this control system framework aims at finding the levels of technological improvement (in terms of the k-factors) that would meet the commanded objectives. These levels are used as targets for the science and technology groups to achieve. The present work adopts this framework while addressing the technology selection problem.

TIES Methodology

The TIES (Technology Identification, Evaluation and Selection) methodology provides the decision maker/designer with the ability to easily assess and balance the impact of various technologies in the absence of sophisticated, time-consuming mathematical formulations. Detailed description and application of the TIES method is given in Refs. 3, 5 and 6.

In the TIES methodology, the impact of a given technology is qualitatively assessed through some technology metric 'k-factors'. These k-factors modify disciplinary technical metrics, such as specific fuel consumption or cruise drag, which are calculated within a synthesis tool as a vehicle is sized. The modification is essentially a change in the technical metric; either enhancement or degradation as the vehicle mission is simulated. In effect, the k-factors mimic the discontinuity in benefits and/or penalties associated with the infusion of a new technology.

To overcome the problem of infeasibility or non-viability in the design, first, the relevant k-factors are determined and technologies that affect these k-factors are chosen as the candidate technologies for the study. A compatibility matrix is formalized through Integrated Product Teams to establish physical compatibility rules between technologies. Once the compatibility matrix is determined, the potential system and sub-system level impact of each technology is established by creating the Technology Impact Matrix (TIM). The columns of the TIM correspond to candidate technologies and the rows depict the effect produced on the disciplinary metrics by these candidate technologies. The net impact of a given mix of technologies is obtained as summation of individual technology effects. This net impact is given in the form of a vector [to be called 'net k-vector' (**k**) henceforth] whose elements are the summed k-factors.

A metamodel RSE is created for each system metric via a Design of Experiments (DoE) by bounding the k-vector element ranges. The metamodels are second-order Response Surface Equations (RSE) of the form:

$$R = b_o + \sum_{i=1}^{k} b_i k_i + \sum_{i=1}^{k} b_{ii} k_i^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} b_{ij} k_i k_j \qquad (1)$$

where, R is a given system metric, b_i are the linear regression coefficients, b_{ii} are the quadratic coefficients, b_{ij} are the cross-product coefficients, k_i, k_j are the k-vector elements and $k_i k_j$ are the interactions

The technology evaluation is performed in terms of the system metrics by defining an overall evaluation criterion (OEC) or through the TOPSIS technique.[5]

If the computational expense of the analysis is manageable, a full-factorial investigation or an exhaustive evaluation of technology mixes is conducted; otherwise, an alternate method of evaluation is needed to downsize the problem. Heuristic optimization techniques like Genetic Algorithm (GA) serve this purpose.

Generic Optimization Problem with Two Mappings

Let us consider a generic optimization problem that involves two successive mappings as in the TIES technology optimization. Let T, K and R be the three spaces involved and let t, k and r be the variables in these domains respectively. Let f be the mapping function from space T to space K and g be the one from K to R as shown in Figure 2. So we have, r = g(k) = g(f(t)).

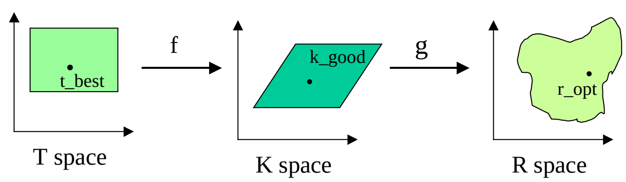

Figure 2: Optimization problem with three spaces

Consider the problem of optimizing over T space to obtain an optimum value, r_opt in the R space. Since two mappings are involved in the mapping from domain space T to the range space R, there are two possible ways of doing the optimization. The first method is to consider the composite function g *o* f that maps t in T to r in R directly, and find out the optimum values t_best and r_opt by merely searching in the T space. Second method takes into consideration the presence of the intermediate space K. Let us define k_good = f(t_best). So r_opt = g(f(t_best)) = g(k_good). k_good can be found out by optimization between the spaces K and R, and then t_best, the value point in T space for which f(t) = k_good can be determined, thus completing the problem.

The TIES approach follows the first method discussed here. It treats the system level response as a direct function of a given technology combination and so involves explicit evaluation of each combination during the optimization process. However, to achieve the same goal, a two-step process can be pursued based on the second method. In fact, the idiosyncrasies of the spaces K and R in the technology selection problem allow one to make use of the deterministic gradient based techniques for the first step. Since the T space consisting technology combinations has discrete points, it is not possible to find the pre-image of k_good in a general case. This transforms the second step into a combinatorial optimization problem with the goal of successfully approximating k_good that is obtained in step 1. In this investigation, the second method discussed above is pursued to solve the technology selection problem in the TIES framework. It is termed 'Bi-level approach' for the two steps involved that adopt different types of optimization methods.

The bi-level approach makes use of gradient based optimization for which fast and deterministic techniques are available. Moreover, the part of the technology evaluation process that is replaced by gradient based methods happens to be computationally more intensive. Although bi-level approach does not give the exact answer to the optimization problem, it can help the designer in getting moderate but quick solutions. If the dimensionality of the problem is very high, the combinatorial optimization employed by the GA-TIES approach gets very time consuming. In such cases, bi-level approach proves to be a useful tool.

The Method

The Bi-level Approach

In the TIES method, the technology mixes are compared explicitly in terms of their responses at the system level i.e. the system level metrics. For every set of compatible technologies, a net k-vector, **k** is determined and it is used to evaluate the RSEs. Depending on the dimensionality of the problem, all technology mixes are exhaustively evaluated and compared to find the best mix or else genetic algorithms are used. Thus, a single level optimization is carried out in which the optimization variables are the technology mixes and the objective function is the TOPSIS output or OEC.

For the bi-level approach, technology selection is treated as a bi-level optimization process. At the first level, the net k-vector, **k** is optimized to achieve desired responses while satisfying the compatibility constraints. Here, the optimization variables are the disciplinary metric k-factors. The k-factors that build **k** can take real values in the interval decided during the creation of TIM. After finding the optimum k-vector ($\mathbf{k}_{opt}$), a second optimization is carried out to find the mix of technologies that would produce a net k-vector **k** as close as possible to $\mathbf{k}_{opt}$. Since the k-factors can take all possible real values in their bounded intervals, they constitute continuous variables of the first level optimization. So, at the first level, gradient based methods can be employed, whereas heuristic techniques are needed at the second level as technology mixes are discrete variables.

The TIES method and the bi-level approach are compared in Figure 3.

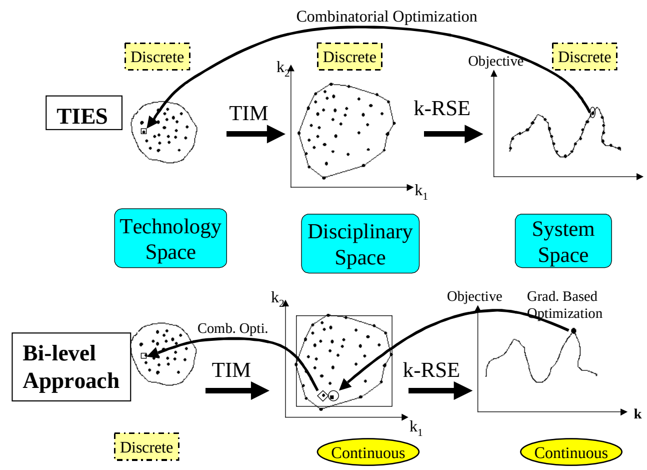

Figure 3: Comparison between the TIES approach and the bi-level approach

<u>Detailed discussion of the Bi-level Approach</u>

As seen in Figure 3, to start with, for both the approaches, there exists a finite bunch of technology mixes. Let us call the space which constitutes these technology mixes as 'Technology space'. The space containing all possible **k**s is the 'k-space' or the 'Disciplinary space' and the space containing the response values is called the 'System space'.

Since there are only finitely many technology mixes, both the approaches have discrete domains in the technology space. The TIES approach acts on this discrete space to obtain the net k-vectors using the Technology Impact Matrix [TIM] and then the system responses using the k-RSEs. In both these operations, only a finite number of domain points are involved, and so the resulting co-domains (images) are also finite sets. This implies that the TIES approach has discrete domains in the k-space and the system space.

On the other hand, in the bi-level approach, the start is not made from the Technology space but from the k-space. Here, the TIM is used to find out the bounds on all the k-factors. The region in the k-space restricted by these bounds [referred as 'k-pocket' henceforth] is used for the first optimization step. This k-pocket encloses all of the finitely many net k-vector (**k**) points of the TIES approach. The corner points of the same k-pocket are utilized during the k-DoE creation and k-RSE generation.

In fact, the same k-pocket is transformed to an m-dimensional cube of side 2 centered at the origin when the elements of **k** are normalized. Therefore, the metamodel RSEs are applicable through out this k-pocket. In the bi-level approach, this m-dimensional cube in the k-space is treated as the domain in the disciplinary space. This domain forms a continuous and bounded region in R^n. This region, when acted by k-RSEs, which are multivariable quadratic polynomials, will again generate a continuous and bounded co-domain in the system space. Thus, the bi-level approach looks at the k-space and the system space as continuous ones, whereas the TIES approach views them as discrete spaces.

Thus, the first level optimization of the bi-level approach is done in continuous domain and co-domain. If the objective function is also tailored to be a continuous one, gradient based optimization techniques can be applied. These techniques have a definite line of attack for finding the optimum and they are capable of dealing with constraints in a problem. Use of gradient based techniques like sequential linear programming (SLP), sequential quadratic programming (SQP), and method of usable and feasible directions helps save lot of time in the overall optimization process.

<u>Comparison of Computational Expense between the Bi-level Approach and TIES</u>

It is to be noted that the transition from the technology space to the disciplinary space is merely a linear transformation occurring through the TIM.

$$\mathbf{k}_{(m\times1)} = [\,\mathbf{TIM}\,]_{(m\times n)} \;\mathbf{*}\; [\,a_1\; a_2\; \ldots\ldots\; a_n\,]^T_{(n\times1)} \qquad (2)$$

where, $\boldsymbol{k}$ is the net k-vector generated, m is the number of k-factors involved, n is the number of candidate technologies in the pool each a_i is a discrete variable taking values 1 or 0 corresponding to ‘on’ and ‘off’ situations of the candidate technology Ti.

Because of this simple linear relationship, the transition from technology space to disciplinary space is quite easy. On the other hand, the transition from the disciplinary space to the system space is quite involved as it is based on second order k-RSEs. The matrix representation of the relationship between the disciplinary space and the system space looks like

$$\mathbf{R} = \mathbf{I} + \mathbf{k}_{(1\times m)}\,[\mathbf{L}]_{(m\times1)} + \mathbf{k}_{(1\times m)}\,[\mathbf{Q}]_{(m\times m)}\;\mathbf{k}^T_{(n\times1)} \qquad (3)$$

where, I is the equation intercept i.e. the constant term of the RSE, $[L]$ is an $(m\times1)$ vector containing linear regression coefficients, $[Q]$ is an $(m\times m)$ upper triangular matrix of quadratic regression coefficients.

It is important to understand how involved are the computations while going from the technology space to the system space as compared to those for going from the technology space to k-space.

From Eqn (2), it can be seen that for determining the m elements of **k**, m linear combinations with n terms need to be evaluated. Thus, n×m computations are involved. Similarly, from Eqn (3), it can be deduced that in order to evaluate p responses, the number of computations involved are $n\times m + [\,(\,m^2 + 2m\,) \times p\,]$. Here, the addition operations are ignored, as they do not affect the computers processing time much.

In the TIES method, all three domains are discrete, and so the whole process becomes a combinatorial optimization problem, whereas, in the bi-level approach, only the latter part, which is associated with Technology space and k-space, is to be dealt as combinatorial optimization. Heuristic methods like GAs are employed for handling the combinatorial optimization problems. However, these methods, being non-deterministic, are very time-consuming as compared to the gradient based methods.

The time required for heuristic optimization in the TIES method is greater than that required in the bi-level approach, and the ratio of these computational times can be approximated as:

$$\frac{t\_TIES}{t\_Bi-level} \approx \frac{nm + (m^2 + 2m)p}{nm} \qquad (4)$$

For a complex design problem, n = number of candidate technologies = 35, m = number of k-factors = 20, and p = number of system responses evaluated = 5. These values yield efficiency ratio of 4 according to Eqn (4).

Of course, the gradient based optimization in the first step of the bi-level approach amounts to some additional work, but it is negligible in comparison to the one calculated above. Thus, for huge combinatorial optimizations (high value of n), the bi-level approach can help the designer save lot of computational effort.

<u>Validity of the bi-level approach</u>

The bi-level approach works well when the TIM is not sparse. Higher the density of TIM, more will be the proximity of the best plausible solution k_best to k_opt, which is the ideal solution.

If there are two or more peaks in the objective function, and the plausible solutions close to the lower peak yield higher performance values, they cannot be searched because the GA in step 2 searches the neighborhood of the global optimum only.

Implementation

To employ the bi-level approach and to get a proof of the concept, a notional 600-passenger transport aircraft design problem is considered in this paper. The problem consists of 35 technologies and 20 k-factors. After deciding the objective function, the best mix of technologies was found using the bi-level approach as well as the GA-TIES approach and the results were compared.

Problem Definition

600 passenger aircraft comes under the category of very large transport [VLT] aircraft. While designing aircraft of this kind, program affordability is a main concern. Total Operating Cost (TOC), which is a system level metric happens to be a good measure of affordability of a VLT aircraft. The total operating cost includes the Total Aircraft Related Operating Cost (TAROC) and Indirect Operating Cost (IOC). The Total Airplane Operating Cost includes the direct operating cost plus interest, and the costs associated with ground handling, ground administration, ground maintenance and ground depreciation. For the airline, ticket price is related to the TOC plus some margin of profit, so TOC needs to be minimized.

The constraints for this design problem were obtained from the general FAR requirements. The technical and economic system metrics of interest in this problem are outlined in Table I. As seen from the table, the baseline configuration in this case does not meet the requirements imposed on CO_2, NOx emissions, \$/RPM and DOC+I. The problem tackled in this investigation is that of searching an evolutionary design (one obtained by infusion of new technologies in the baseline configuration) with the goal of minimizing TOC while satisfying all the constraints.

Metric	Target or Constraint	Base-line	Units
Performance			
Approach Speed (Vapp)	155	116.7	kts.
Design Block Fuel Weight (Wfuel)	minimize	497100	lbs.
Landing Field Length (LdgFL)	11000	5364	ft.
Takeoff Field Length (TOFL)	11000	8651	ft.
CO_2/ASM (CO_2)	0.15	0.3485	lbs./ASM
NOx Emissions (NOx)	8000	10667.8	lbs.
Takeoff Gross Weight (TOGW)	minimize	1284008	lbs.
Economics			
Acquisition Price (Acq \$)	minimize	249.578	\$Million
Research, Development, Testing, and Evaluation Costs (RDT&E)	minimize	17347.4	\$Million
Average Required Yield per Revenue Passenger Mile (\$/RPM)	0.1	0.10207	\$/RPM
Total Airplane Related Operating Costs (TAROC)	minimize	5.077	cents /ASM
Total Operating Costs per trip (TOC)	minimize	0.07055	\$/ASM
Direct Operating Costs per trip (DOC)	minimize	0.03721	\$/ASM
Indirect Operating Cost per trip (IOC)	minimize	0.03334	\$/ASM
Direct Operating Cost plus Interest (DOC+I)	3.1	4.161	cents /ASM
Miscellaneous			
Wing Aerial wt (WAWt)	minimize	21.3	lbs./ft^2
Wing Loading (W/S)	maximize	154.67	lbs./ft^2
Block Time (Time)	minimize	15.85	hrs.

Table I: System Metrics for 600 Passenger Aircraft

Modeling and Simulation

The aircraft sizing and analysis tasks in this study utilized the FLight OPtimization System, FLOPS[6], a multidisciplinary system of computer programs used for the conceptual and preliminary design and analysis of aircraft configurations. This tool was developed by the NASA Langley Research Center. FLOPS was linked to the Aircraft Life Cycle Cost Analysis, ALCCA[7], program used for the prediction of all life-cycle costs associated with commercial aircraft. ALCCA was originally developed by NASA Ames and further enhanced by Aerospace Systems Design Laboratory (ASDL).

Specifying Technology Alternatives

In order to meet the specified goals and to achieve the desired affordability in the design, 35 technologies were identified for this problem. The technologies were categorized according to their need as shown in Table II.

Not all the technologies under consideration were fully mature. Technology maturity is specified with a qualitative scale known as the Technology Readiness Level[3] (TRL). The TRLs describe the maturation and development process of a technology and provide a basis by which different technologies can be compared as they progress through the gates of maturation. In general, the impact of a technology is probabilistic in nature, possibly even stochastic. For the purpose of this investigation, the technological impacts were assumed to be deterministic.

Technology Impact Matrix

For each of the 35 technologies, the benefits and penalties associated were identified in terms of 20 kappa factors and the Technology Impact Matrix (TIM) was constructed as shown in Table IV. The TIM contains the predicted impact values obtained assuming that each technology has matured to the point of full-scale application. This situation may actually take place in year 2015. The beneficial impacts of technologies were found from various literature sources, whereas wise guesses were made for most of the economic impacts.

Technology Compatibility

Once the technologies were identified, physical compatibility rules between technologies were established through brainstorming activities and literature reviews, and the Technology Compatibility Matrix (TCM) was created as shown in Figure 4.

Counting the Compatible Alternatives

If all possible combinations of the 35 technologies were compatible, $2^{35} \sim 3.4 \times 10^{10}$ combinations would exist. However, in reality, the fraction of the compatible alternatives out of these 2^{35} combinations is very small. As seen in Table III, each of the 35 technologies belongs to some exclusive category. These categories are mutually independent and so in this particular problem, two technologies are incompatible if and only if they belong to the same category. For example, the three technologies, T2, T12 and T17 are mutually incompatible as all of them are intended for tail-skin improvement. If any one of these three is employed in the new design, the other two cannot be used. Thus all three pairs formed in this category (T2, T12), (T2, T17) and (T12, T17) are incompatible. At the same time, these three technologies are totally unrelated to the rest of the technology pool i.e. presence of any member from the group

Tech No.	Technology (Identifier)	Current TRL	TRL=9 Year
T1	Adaptive Performance Optimization	9	2000
T2	Stitched RFI Composite on Tail Skin	4	2006
T3	Stitched RFI Composite on Tail Structure	4	2006
T4	Stitched RFI Composite on Wing Skin	4	2006
T5	Stitched RFI Composite on Wing Structure	4	2006
T6	Airframe Methods	4	2007
T7	Fire Suppression	3	2007
T8	Low Cost Composite Manufacturing on Tail Structure	2	2009
T9	Low Cost Composite Manufacturing on Wing Structure	2	2009
T10	Propulsion System Health Management	2	2009
T11	Smart Nacelle-PAI	3	2009
T12	Emerging Alloy Tech & Forming on Tail Skin	3	2010
T13	Emerging Alloy Tech & Forming on Tail Structure	3	2010
T14	Emerging Alloy Tech & Forming on Wing Skin	3	2010
T15	Emerging Alloy Tech & Forming on Wing Structure	3	2010
T16	Superplastic Forming on Fuselage Skin	2	2011
T17	Superplastic Forming on Tail Skin	2	2011
T18	Superplastic Forming On Wing Skin	2	2011
T19	Russian Aluminum Lithium Fuselage Skin	4	2011
T20	Revolutionary Metallic Materials Systems on Fuselage Structure	2	2011
T21	Revolutionary Metallic Materials Systems on Landing gear	2	2013
T22	Revolutionary Metallic Materials Systems on Tail Structure	2	2013
T23	Revolutionary Metallic Materials Systems on Wing Structure	2	2013
T24	Composite Fuselage Shell (Fuselage Skin)	2	2013
T25	Living Aircraft	2	2013
T26	Active Load Alleviation on Tail	4	2013
T27	Active Load Alleviation on Wing	4	2013
T28	Antenna Systems	2	2014
T29	Adaptive Wing Shaping	3	2014
T30	Biologically Inspired Material Systems on Fuselage Structure	1	2015
T31	Biologically Inspired Material Systems on Tail Structure	1	2015
T32	Biologically Inspired Material Systems on Wing Structure	1	2015
T33	BIOSANT on Fuselage Structure	1	2015
T34	BIOSANT on Tail Structure	1	2015
T35	BIOSANT on Wing Structure	1	2015

Table II: Technologies identified for the problem

{T2, T12, T17} does not affect selection of other 32 technologies. This is because the 'tail-skin' category is independent of the other 16 categories. When the technologies are regrouped according to their categories identified in Table III, the rearranged TCM appears like a blocked identity matrix as shown in Figure 5. Each block of the rearranged TCM corresponds to a category and so there are altogether 17 blocks. While making a choice for a compatible technology combination, one can select atmost 1 technology from each block. Therefore, there can be atmost 17 technologies in a compatible combination. For block 12, there are 4 choices: choose T2, choose T12, choose T17 or choose none of these. In general, for the i^{th} block containing b_i technologies, there are $(b_i + 1)$ choices. This gives rise to $\prod_{i=1}^{17} (b_i + 1)$ possibilities. In the current problem, this value turns out to be $2^{11} \times 4^4 \times 7^2 \sim 2.5 \times 10^7$. For such a large number of compatible cases, the computational effort required to conduct the full factorial analysis is very high and so the use of heuristic methods is justified in this problem. Although this number is huge, it is worth-noting that it forms just the 1337^{th} fraction of 2^{35}, which is the total number of combinations as discussed above.

Category Number	Category Description	Group of Technologies
C1	Adaptive Performance Optimization	T1
C2	Airframe Methods	T6
C3	Fire Suppression	T7
C4	Propulsion System Mgmt.	T10
C5	Smart Nacelle	T11
C6	Landing Gear	T21
C7	Living Aircraft	T25
C8	Active Tail Load Alleviation	T26
C9	Active Wing Load Alleviation	T27
C10	Antenna System	T28
C11	Wing Shaping	T29
C12	Tail Skin	T2, T12,T17
C13	Wing Skin	T4, T14, T18
C14	Fuselage Skin	T16, T19, T24
C15	Fuselage Structure	T20, T30, T33
C16	Tail Structure	T3, T8, T13, T22, T31, T34
C17	Wing Structure	T5, T9, T15, T23, T32, T35

Table III: Classification of the technologies in exclusive categories

The important conclusions to be drawn from the above analysis were:

1. The final solution to the current technology selection problem will have at most 17 technologies, each belonging to a different category.
2. The probability that a randomly chosen set of technologies forms a compatible combination is 1/1337 = 0.00075. The probability that it belongs to the feasible space will be even lesser.

	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14	T15	T16	T17	T18	T19	T20	T21	T22	T23	T24	T25	T26	T27	T28	T29	T30	T31	T32	T33	T34	T35
T1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T2		0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T3			0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	0
T4				0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T5					0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1
T6						0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T7							0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T8								0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	0
T9									0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1
T10										0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T11											0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T12												0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T13													0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	0
T14														0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T15															0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1
T16																0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
T17																	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T18																		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T19																			0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
T20																				0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0
T21																					0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T22																						0	0	0	0	0	0	0	0	0	1	0	0	1	0
T23																							0	0	0	0	0	0	0	0	0	1	0	0	1
T24																								0	0	0	0	0	0	0	0	0	0	0	0
T25																									0	0	0	0	0	0	0	0	0	0	0
T26																										0	0	0	0	0	0	0	0	0	0
T27																											0	0	0	0	0	0	0	0	0
T28																												0	0	0	0	0	0	0	0
T29																													0	0	0	0	0	0	0
T30																														0	0	0	1	0	0
T31																															0	0	0	1	0
T32																																0	0	0	1
T33																																	0	0	0
T34																																		0	0
T35																																			0

Symmetric Matrix

Compatible (0)

Incompatible (1)

Figure 4: Technology Compatibility Matrix (TCM) for 35 technologies

		C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12			C13			C14			C15			C16						C17					
		T1	T6	T7	T10	T11	T21	T25	T26	T27	T28	T29	T2	T12	T17	T4	T14	T18	T16	T19	T24	T20	T30	T33	T3	T8	T13	T22	T31	T34	T5	T9	T15	T23	T32	T35
C1	T1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C2	T6		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C3	T7			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C4	T10				0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C5	T11					0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C6	T21						0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C7	T25							0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C8	T26								0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C9	T27									0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C10	T28										0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C11	T29											0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C12	T2												0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	T12													0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	T17														0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C13	T4															0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	T14																0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	T18																	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C14	T16																		0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	T19																			0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	T24																				0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C15	T20																					0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
	T30																						0	1	0	0	0	0	0	0	0	0	0	0	0	0
	T33																							0	0	0	0	0	0	0	0	0	0	0	0	0
C16	T3																								0	1	1	1	1	1	0	0	0	0	0	0
	T8																									0	1	1	1	1	0	0	0	0	0	0
	T13																										0	1	1	1	0	0	0	0	0	0
	T22																											0	1	1	0	0	0	0	0	0
	T31																												0	1	0	0	0	0	0	0
	T34																													0	0	0	0	0	0	0
C17	T5																														0	1	1	1	1	1
	T9																															0	1	1	1	1
	T15																																0	1	1	1
	T23																																	0	1	1
	T32																																		0	1
	T35																																			0

Symmetric Matrix

Compatible (0)

Incompatible (1)

Figure 5: Rearranged TCM for the grouped technologies

	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14	T15	T16	T17	T18	T19	T20	T21	T22	T23	T24	T25	T26	T27	T28	T29	T30	T31	T32	T33	T34	T35
Wing Wt				-3	-13				-20					-2	-8			-3					-15				-5		-5			-2			-22
Fuse Wt																-12			-7	-11				-10						-13			-13		
VT Wt		-3	-13					-20				-2	-8				-3					-15				-5					-30			-3	
HT Wt		-3	-13					-20				-2	-8				-3					-15				-5					-30			-3	
Cdi	-4										-1																		-9						
Cdo						-2					-1																	-1	-9						
Landing Gear Wt																					-21														
Avionics Weight	2																									2	5	-45	2						
Hydraulics Wt																									-50				5						
Furn. equip wt							-2																												
VT Area																													-15						
HT Area																													-15						
Engine Weight										5																									
Fuel Consump										-4																	0.5								
RDT&E Costs	1	1	1	1	1	-7	0.5	0.5	0.5	1	1	1	0.5	0.5	0.5	1	1	2	2	2	1	1	2	1	3	3	3	1	3	1	1	0.5	0.5	0.5	1
O&S Costs	-1	1	1	1	1		1	1	1	-2						1	1	1	2	1	1	1	1	1	1	2	2		2	1	1	1	1	1	1
Prod Costs	1	1	2	1	2		1	1	1	2		1	1	1	1	3	1	2	2	1	1	1	1	1	1				-3	1	1	1	1	1	1
Utilization		-2	-1	-2	-1			-1	-1															-3		-2	-2								
Wing Area				-2	-1				-2					-2	-1			-1					-3						-3						
T/W ratio								2														2	1							2	4	5	5	4	5

Table IV: Technology Impact Matrix (TIM) [values in %]

Assess Technology Alternatives

An RSE metamodel was created for each system metric defined in Table I via a Design of Experiments by bounding the impact vector element ranges in the TIM as listed in Table V. A '0' implies no change in the technical metric while a negative value denotes reduction and a positive value an increase from the baseline values. Once the k-factor RSEs were determined, they were used to rapidly evaluate the impact of the various technologies based on a particular impact vector setting in lieu of executing FLOPS/ALCCA directly. References 4, 7 provide a more detailed description of the use of Response Surface Methods for technology assessments.

Selection of Best Technology Combination

The optimization problem tackled as part of this exercise is that of minimizing the cost metric, Total Operational Cost, TOC. The GA-TIES approach and the Bi-level optimization approach were used to solve the same problem. The comparison of their results and efficiencies is made in the following section.

The Problem Statement

The single objective constrained optimization problem in this exercise can be stated as follows:

Minimize TOC in cents/ASM subject to 7 constraints

Φ_1 : Vapp ≤ 155 knots

Φ_2 : NOx ≤ 8000 lbs

Φ_3 : CO_2 ≤ 0.15 lbs/ASM

Φ_4 : TOFL ≤ 11000 ft

Φ_5 : LdgFL ≤ 11000 ft

Φ_6 : $/RPM ≤ 0.1

Φ_7 : DOC + I ≤ 3.1 cents/ASM

These 7 constraints are the forced constraints of the system. They can be evaluated only when the system level response is evaluated. Apart from these, the problem has additional constraints in the form of compatibility of technologies.

ki	Technology Impact Elements	Minimum	Maximum
k1	Wing Weight	-35%	0%
k2	Fuselage Weight	-25%	0%
k3	Vertical Tail Weight	-38%	0%
k4	Horizontal Tail Weight	-38%	0%
k5	Cdi	-14.1%	0%
k6	Cdo	-13.1%	0%
k7	Landing Gear Weight	-21%	0%
k8	Avionics Weight	-45%	11%
k9	Hydraulics Weight	-50%	5%
k10	Furnishing and equipment weight	-2%	0%
k11	Vertical Tail Area	-15%	0%
k12	Horizontal Tail Area	-15%	0%
k13	Engine Weight	0%	5%
k14	Fuel Consumption	-4%	0.5%
k15	RDT&E Costs	-7%	27.5%
k16	O&S Costs	-3%	16%
k17	Production Costs	-3%	17%
k18	Utilization	-13%	0%
k19	Wing Area	-8%	0%
k20	Thrust-to-weight ratio	0%	14%

Table V: Technology impact elements [kappa factors]

The GA Technique

Both the approaches employed in this investigation used genetic algorithms to perform the constrained combinatorial optimization. Various constraint-handling techniques have been proposed for combinatorial optimization. Most of these make use of some kind of penalty function. The method selected by the authors is the one suggested by Deb[8,9], which is particularly applicable while using GAs.

Constraint handling using feasibility

For the constrained problem, Deb[9] suggests to define the fitness function as :

$$\text{Fitness}(X) = \begin{cases} f(X) & \text{if}\ \phi_j(X) \geq 0, \forall j = 1,2,\ldots,m \\ f_{worst} + \sum_{j=1}^{m} \phi_j(X) & \text{otherwise} \end{cases}$$

where, f is the objective function to be maximized, f_{worst} is the objective function value of the worst feasible solution in the population, X is the optimization variable (vector), $\phi_j(X)$ is the jth inequality constraint which should be maintained non-negative for feasibility, m is the total number of constraints.

If there are no feasible solutions in the population, f_{worst} is set to zero. If the constraints are non-commensurable, they are normalized to avoid any sort of bias toward any of them.

Deb suggests making use of tournament selection method while implementing the above-mentioned fitness in the GA algorithm. The typical fitness function yields the following rules while comparing two members of the population :

1. A feasible solution is always preferred over an infeasible one.
2. Between two feasible solutions, the one having better objective function is preferred.
3. Between two infeasible solutions, the one having smaller constraint violation is preferred.

Thus, in case of feasible solutions, the fitness value equals the objective function value, and the use of constraint violation in the comparisons aims to push infeasible solutions towards the feasible region. Since the selection procedure only performs pair-wise comparisons, no penalty factor is required. This is the main advantage of this method as other penalty function methods involve problem specific fine-tuning of the penalty coefficient(s).

Problem Solving

GA-TIES Approach

In this approach, the technology space was searched for compatible technology combinations that satisfy the constraints on the system responses and minimize the TOC value. The fitness function in this approach was the negative of the TOC value of a particular technology combination. The members of the GA population were the technology combinations. Each technology combination was represented as a binary string of length 35 and the chosen population size was 5x35 = 175. Presence of '1' at the kth place in the binary string implies that the kth technology is 'on' i.e. present in the combination, whereas a '0' implies that the corresponding technology is 'off' i.e. absent. For example, '00101000101000000000000001000000100101' represents the technology combination {T3 T5 T9 T11 T23 T30 T33 T35}. The crossover method used was 'Double point crossover' and the crossover rate for the GA was 75%. For mutation, repetitive bitswaps were used with a mutation rate of 15%. That is, 4 out of the 35 bits were chosen randomly and a '0' was changed to '1' and vice versa.

The process was set to termination when no improvement was seen in 60 successive generations. While employing Deb's technique in this problem, all technology combinations that have no more than two incompatibilities were considered feasible. This minor deviation was made in order to include some infeasible members in the population to ameliorate the evolution process. This alteration was seen to improve the success rate of the GAs. Minimum value of TOC obtained using this approach was 5.299 cents per available seat mile (ASM). The technology combination corresponding to the best solution has the binary representation '10000100001000001100000111111010101'. The kappa factors corresponding to the best solution are listed in Table VI. Figure 6 shows the evolution of the GA for the GA-TIES approach. Here, the TOC is plotted on the log10 scale for better visualization.

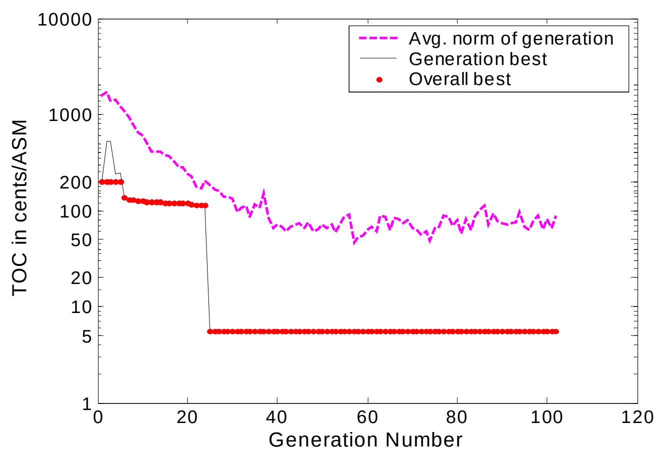

Figure 6: Progress of TOC minimization Algorithm for the GA-TIES approach

Bi-level Optimization Approach

Step 1 : Gradient Based Optimization

The metamodel RSEs were used to evaluate TOC and the other 7 system level responses that were selected as constraints. The optimization variable vector in this case is $\mathbf{k}$ = [k1 k2 …. k20],

where, $-1 \leq ki \leq 1$, for i = 1,2, …,20

The optimizer used in this problem was the 'fmincon' function of MATLAB that is based on the Sequential Quadratic Programming (SQP) technique. Several choices were made for the starting vector $\mathbf{k}$ and all the cases yielded the same answer $\mathbf{k_{opt}}$, which ensured the global nature of the optimum found. The elements of the optimum vector $\mathbf{k}_{opt}$ are listed in Table VI. The minimum value of TOC obtained in this step is 4.439 cents/ASM and it corresponds to the 'r_opt' discussed in the generic optimization problem. This is the best result possible for the technology impact model developed in this investigation. If there were a compatible technology combination whose net k-vector (**k**) equals $\mathbf{k}_{opt}$, then it would definitely be the best possible answer to this problem. This ideal case scenario is not possible because of the discrete nature of technology space as seen in Figure 3 and due to the incompatibilities existing among various technologies.

Step 2 : Genetic Algorithm

In this step, the technology space was searched using GA to find a compatible technology combination that will result in a **k**, which lies in the close neighborhood of $\mathbf{k}_{opt}$ in addition to satisfying the constraints put on the system level responses. The infrastructure of this GA is similar to the one mentioned above in the GA-TIES approach. The population size, crossover and mutation operators etc. were the same in both cases.

The objective function to be minimized in this problem was the closeness of the net k-vector (**k)** to $\mathbf{k}_{opt}$, which was measured using the Euclidean norm $\|\mathbf{k} - \mathbf{k}_{opt}\|_2$. In each generation, after evaluating the fitness of the population members, the technology combinations were sorted according to their fitness i.e. proximity to $\mathbf{k}_{opt}$. A list of top 500 technology combinations was maintained in this way, which was updated after every generation. When the GA search got over, the closemost 500 neighbors of $\mathbf{k}_{opt}$ were evaluated using the RSE metamodels to check the 7 compatibility constraints defined earlier. Among the feasible neighbors, the one yielding minimum value of TOC was taken as the best technology combination.

The minimum TOC value obtained using this approach was 5.333 cents/ASM, which corresponds to the technology combination with binary representation '10000100001000001000100111111010101'. Figure 7 presents the TOC values and the proximity to $\mathbf{k}_{opt}$ of the closemost 500 net k-vectors. The dots corresponding to the feasible **k** vectors are encircled and are of interest while comparing the TOC values. As seen from this figure, the minimum TOC solution is separated from $\mathbf{k}_{opt}$ by a Euclidean distance of 1.96, whereas the closest technology combination is at a distance of 1.93. It is to be noted that the closest combination does not possess the minimum TOC value, neither does it correspond to a feasible solution i.e. it does not satisfy all 7 compatibility constraints.

Result Comparison

Both the methods achieved practically the same value of the objective function i.e. TOC = 5.3 cents/ASM. A comparison of the binary strings of the best technology combinations obtained by the two approaches looks like:

GA-TIES : 10000100001000001100000111111010101
Bi-level : 10000100001000001000100111111010101

Both the approaches were seen to suggest selection of 14 technologies from the pool of 35 and 13 out of these were common, which are {T1, T6, T11, T17, T24, T25, T26, T27, T28, T29, T31, T33, T35}. Apart from these, the GA-TIES approach recommends T18, while the Bi-level approach suggests T21. Average runtime for the GA-TIES approach was found to be three times that of the Bi-level approach. This value is in concurrence with the prediction of Eqn (4). The first step of the Bi-level approach took around 25% of its total time.

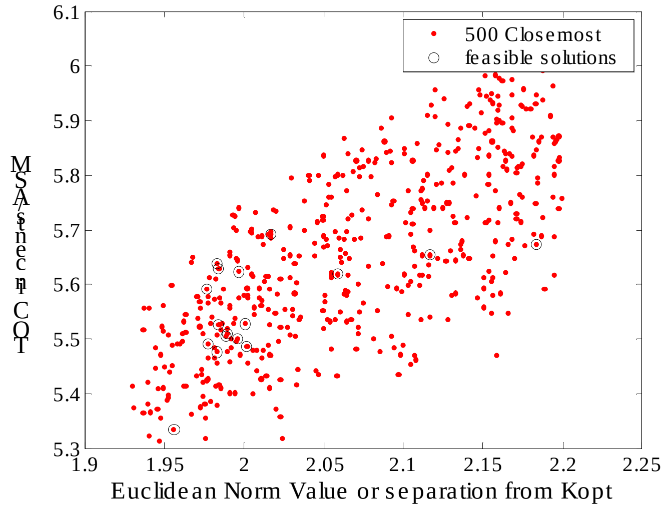

Figure 7: Obtaining feasible solutions using Bi-level approach

Observed Advantages of the Bi-level Approach

Based on the bi-level optimization analysis, the following deductions can be made in context to the particular problem under investigation:

While employing the Bi-level approach, some part of the problem is dealt by fast gradient-based optimization techniques instead of slow heuristic search methods as in GA-TIES approach. If the dimensionality of the problem is large, which is the case in the current problem, use of Bi-level approach can give satisfactory results while being thrice time-efficient as compared to the GA-TIES approach. Based on these initial results, it is possible to set minimum expectations for the objectives if a rigorous GA-TIES analysis is to be followed.

A second major advantage of the Bi-level approach has to do with the value of the intermediate step of determining $\mathbf{k}_{opt}$. For the specific set of system constraints, 4.439 cents/ASM is the least possible value of TOC that can be achieved irrespective of the choice of technologies. If this value of TOC is not satisfactory, it is essential for the designer to relax some other constraint(s) [DOC+I in this case]. This kind of contemplation is possible as soon as the first step i.e. the gradient based optimization gets over. The GA-TIES approach, on the other hand, does not provide any such estimate until the end of the GA optimization.

	Kappa factors and responses	Constraint Values	Bi-level Step 1	Step 2 GA	GA – TIES
kappa factors	k1		-1	-0.88	-1
	k2		-0.6973	-0.84	-0.84
	k3		-0.9	-0.9	-0.9
	k4		-0.9	-0.9	-0.9
	k5		-0.41	-0.41	-0.41
	k6		-0.31	-0.31	-0.31
	k7		-0.68	-0.68	1
	k8		0.196	-0.4754	-0.4754
	k9		-1	-0.8182	-0.8182
	k10		0.067	0.3333	0.3333
	k11		0.658	-0.989	-0.989
	k12		-1	-1	-1
	k13		0.9478	0.8	0.8
	k14		0.524	0.9524	0.9524
	k15		0.21	0.4105	0.4526
	k16		0.8	0.6	0.6
	k17		-0.15	0.25	0.3
	k18		-0.5135	-0.35	-0.35
	k19		-0.257	-0.48	-0.58
	k20		0.09	0.09	0.09
constraints	NOx (lbs)	8000	4514.83	6514.21	6797.30
	Vapp (kts)	155	91.13	101.13	100.91
	LdgFL (ft)	11000	4195.89	4643.83	4631.90
	TOFL (ft)	11000	4135.35	5818.94	5714.10
	CO_2/ASM (lbs)	0.15	0.0435	0.0702	0.0733
	$/RPM	0.1	0.0707	0.0853	0.0877
	ACQ ($Million)	200	112.07	154.80	146.12
	RDTE ($Million)	14000	10761.13	11837.69	12245.05
	DOC+I (¢/ASM)	3.1	2.2077	3.0740	3.0965
	TOC (¢/ASM)		**4.4395**	**5.3331**	**5.2998**
	Separation		0.0000	1.9555	2.5977

Table VI: Result comparison of the two optimization approaches

More broadly, it is possible to complete the first step of the Bi-level approach without dealing with the technologies. The bounds set on the k-factors (as shown in Table V) need to be obtained purely based on the FLOPS/ALCAA analysis. If this is achieved, the $\mathbf{k_{opt}}$ obtained can help the designer in making selection of technologies for the problem at hand. In addition, *such an effort will result in significant time-savings if the optimization process is to be done on a repetitive basis.* This scenario arises when new technologies are introduced during the course of a development process or if the R&D program of certain technologies gets terminated.

Conclusions

The paper introduces a new technique, called 'Bi-level approach', for tackling the technology selection problem. This approach can assist the designer in obtaining quick estimates of the minimum level of expectations from the rigorous GA-TIES approach. It also helps in predicting the best performance possible for the constrained optimization problem. Further, the technique entails additional insight to the designer while making choice of technologies. Finally, it has the potential to facilitate efficient re-evaluation of the optimal technology problem. The Bi-level procedure was demonstrated on a 600-passenger transport aircraft problem involving 35 potential technologies. It achieved the same objective function value as the GA-TIES approach while providing the added benefits described above.

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