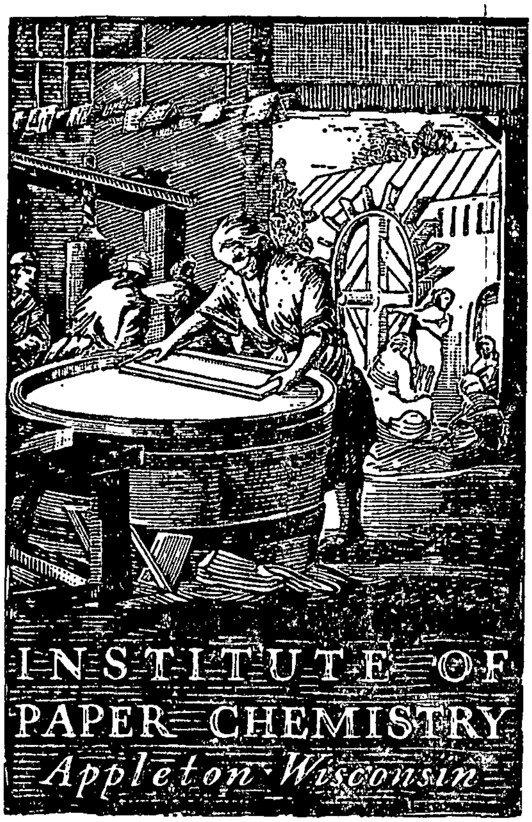

STRAIN PROPAGATION IN SACK PAPER AND SACKS DURING IMPACT

Project 2033

Report Thirty

A Progress Report

to

MULTIWALL SHIPPING SACK PAPER MANUFACTURERS

June 24, 1964

THE INSTITUTE OF PAPER CHEMISTRY

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SUMMARY

A fundamental study is in progress to determine the stress-strain behavior of multiwall sacks during impact. It is believed that basic work of this type is necessary for further improvement in understanding the performance of multiwall sacks.

In planning this fundamental study it was believed that attention should be given first to determining whether or not wave propagation effects are of great importance to sack impact behavior. In the present context, these effects are associated with the case of impact loading of such short duration that the time required for stress or strain to travel throughout the sack must be considered. The answer to this question may have ramifications in (a) interpretation of sack failure, (b) the nature of appropriate tests of sack paper, and (c) the likelihood of success in a theoretical analysis of sack impact.

Extensible multiwall sacks were strain gaged at four locations along a meridian of the face (from edge to center) to determine whether there was an appreciable time difference between locations with respect to maximum strain. A time difference was found, but on the average the greatest difference was only about 15% (cross direction) and 35% (machine direction) of the duration of the impact (i.e., zero to maximum strain). This result indicates that wave propagation effects are not of major importance to the behavior of the sack during impact because the impact strains build up at essentially the same time throughout the sack face. (Time differences equal to or greater than the duration of impact would have indicated the importance of wave propagation effects.)

It was also found that, in general, there was not an orderly progression of a strain pulse from the edge to the center of the sack. The data cast doubt on the assumption that the impact behavior is a simple thrust at the edges of the sack, followed by a predictable progression of strain toward the center of the sack face.

As a matter of background interest, the velocity of propagation of tensile strain in a sample of extensible sack paper (9% stretch) was measured by means of conductive-coating gages on a long tensile strip impacted at one end. The observed velocities, based on a number of trials, were 85,700 and 75,500 in./sec. for the machine and cross-machine directions, respectively. Estimates of velocity, based on modulus of elasticity and density were lower than the observed velocities by 19 and 3%, respectively. While a number of possible reasons may be suggested for the disparity between theory and experiment, it is believed that, on the whole, a reasonably good estimate of strain velocity is possible from theoretical considerations. [It may be remarked that these observed velocities are sufficiently high relative to the sack dimensions and the duration of sack impact that there is little reason to expect that wave propagation effects would be important to sack impact; this conclusion agrees with the observations cited above from sack impact tests.]

To verify the experimental techniques employed in this study, a long, thin steel bar was suspended as a ballistic pendulum and impacted at one end. The time of travel of the impact wave along the bar was measured with conductive-coating strain gages at four stations along the rod. The observed velocity of wave travel agreed with the theoretical velocity to 1/2 of one per cent, which was judged to be a favorable verification of the experimental technique.

It is recommended that future work on sack impact be directed to measuring the type, magnitude, and distribution of strain in the sack under various impact conditions. This work will require further development of an adequate strain gage for sack paper.

INTRODUCTION

Substantial progress has been made in recent years toward understanding the types of paper properties that govern the impact behavior of multiwall shipping sacks. Largely through the research efforts of the Multiwall Shipping Sack Paper Manufacturers, it has been shown that sack performance is dependent on paper properties which reflect the energy absorption and/or fatigue qualities of the paper (1).

While these results are interesting and helpful to the manufacturer and user of multiwall sacks, it is recognized that they do not constitute a complete description of sack behavior. Stemming from correlation studies of one basic style of sack and one commodity, the apparent importance of energy absorption and fatigue properties gives little information as to the mechanism of paper failure, or the effect of drop height, orientation, sack dimensions and style, and type of commodity on sack performance. To provide the answers to these latter questions, the technical committee has recommended that major attention be given to a fundamental study of sack impact behavior from the standpoint of the stress and strain induced in the paper as a result of impact. It is felt that significant improvement in the strength-to-cost ratio of sack paper might be achieved if such fundamental information were available.

There has been little progress made in a theoretical analysis of sack impact behavior because of the complex nature of impact phenomena. It appears that an understanding of sack impact is probably most accessible, at the present time, through experimental methods directed to measuring strains induced in the paper by impact. An exploratory study along these lines was performed several years ago as a part of Project 2033 using commercial strain gages at various locations on cement sacks (2). It was found, for example, that the rate of straining in a three-foot face drop is of the order of magnitude of one inch/inch/second — information which is of significance to the selection of testing instruments for evaluation of paper properties. It was also found that the strain induced on any one impact is only a modest fraction of the virgin stretch of the paper, indicating thereby the importance of the fatigue characteristics of sack paper. It should also be remarked that the study pointed out the difficulties of working with conventional strain gages on sack paper, spurring efforts subsequently to develop a suitable strain gage for use in research on paper sacks.

One question on sack impact behavior has seemed worthy of attention very early in the fundamental study now in progress. That is whether or not the sack ruptures by a wave propagation phenomenon. Stripped of the technical details, a wave propagation phenomenon may be described as a case where the time of application of the stress is short relative to the time required for the stress to travel throughout the entire body, and, therefore, differences in stress at various locations due to time effects need to be considered. The answer to this question may have consequences in a number of ways. For example, herein may lie an explanation for the large number of failures along the length of the face of the sack at or near the middle as a result of face drop. One characteristic of wave propagation phenomena (in terms of sack impact) is that pulses of stress emanating at the sides of the sack and traveling toward the center of the face as a result of face drop may reinforce each other when they meet and thereby double the stress level at that instant. In addition to helping explain observed phenomena, such information might find practical application in selecting sack dimensions which are best suited to the properties of the paper in its two directions, and _vice versa_.

Another consequence of wave propagation effects in sack behavior is in the testing of the sack paper. Quite apart from the change in material properties accompanying a gradual increase in strain rate, there are reportedly transition points in material behavior at high test rates where abrupt changes may occur in strength, elongation, or energy absorption (3, 4). It may be noted that the impulse tester (5) was developed on the premise that this type of behavior occurs in the impact of bags and sacks.

It should be mentioned that proof of the importance of wave propagation effects in sack impact would diminish substantially the prospect of formulating a theoretical analysis of sack behavior in the near future. A review of the literature on the mechanics of stress wave theory should convince one that only the simplest types of structures and material performance have been amenable to theoretical treatment (6).

The present study was undertaken to gain more insight into whether or not wave propagation phenomena are of importance in sack impact. The following criterion was adopted relative to the importance of wave propagation effects: If the time difference in the occurrence of peak strain at the edge and center of the sack is about the same as (or greater than) the time for strain to build up from zero to maximum at either location, then wave propagation is of importance (7). Inasmuch as this matter can be investigated by studying the time intervals at which strains appear at various locations in an impacted sack, the study was not dependent on perfecting a strain gage suitable for use on paper. For this reason electrical conductive coatings were used in this study as detectors of strain and it was not required to expend effort on calibrating the coatings to serve as gages for strain. It is hoped, however, that the coating approach to experimental strain measurement on sacks can be brought to a practical, working stage.

The study was comprised of the following three parts, presented in this order in the remainder of this report:

a) verification of experimental method for the case of impact of a steel bar,

b) measurement of the velocity of tensile strain propagation in extensible sack paper,

c) measurement of propagation of strain in extensible multiwall sacks during impact.

THEORETICAL CONSIDERATIONS

When a force or displacement is applied at one point of a body, a definite, usually small, interval of time passes before the effect is felt at a remote point in the body. The initial disturbance propagates as a wave from one point to another in the body. The rate at which the stress (or strain) wave travels is termed the velocity of propagation, C, which is dependent on the properties of the material. As long as the stress or strain is within the elastic range, the velocity of propagation for uniaxial stresses is, in theory, calculable from the following equation (8):

$$C = \sqrt{\frac{386.4E}{D}} \qquad C^2 = \frac{386.4E}{D} \qquad (1)$$

where C = velocity of propagation, in./sec. $E = \frac{Dc^2}{386.4}$

E = modulus of elasticity, lb./sq. in.

D = density, lb./cu. in.

If $D = 1.5 g/cm^3 = \frac{1.5}{454 g} \# \times (2.54)^3 \frac{cm}{in^3} =$

(The numerical factor 386.4 permits use of the weight density, D, rather than the mass density of the material.) The velocity of strain propagation is the same as the velocity of sound propagation in the material.

In a biaxial stress field, the velocity of propagation is greater than is given by Equation (1) because lateral contraction or expansion is suppressed and the material appears stiffer than in uniaxial stress (9). The biaxial effect depends on the magnitude of Poisson's ratio for the material; for example, the biaxial velocity is 16% greater than the uniaxial for a Poisson's ratio of 0.30 (9).

The velocity of wave propagation in steel, for example, is about 200,000 in./sec. Thus, a time interval of 5 millionths of a second is required for an applied stress or strain to travel one inch from the point of application. Because of differences in modulus of elasticity and density, the velocity of propagation in sack paper would be expected to be about 1/3 to 2/3 that of steel, based on Equation (1).

Wave propagation is of little or no concern in the slow-speed testing of materials or in structures which are loaded relatively slowly in service. Consider, for example, a conventional tensile test. The increments in the progressively increasing applied force occur at such a low time rate that the associated increment in stress or strain has more than ample time to travel throughout the specimen before the successive increment is applied. Thus, at any given instant the specimen may be considered as uniformly stressed over its length (barring clamp effects or changes in cross section).

On the other hand, in a very high-speed test the total time of application of the stress at, say, one end of the specimen may be of the same order of magnitude as the time for the stress to travel to the remote end of the specimen. In this event the stress is not uniform along the specimen and a consideration of wave propagation is required to describe the state of stress and strain at any position at a given instant.

Several distinct cases may be identified. Consider, for example, the classic case of an idealized long thin bar which is impacted by a compression force at one end. A sharp impact will start a pulse of compression stress σ, moving along the bar, as illustrated in Fig. 1a. At a later instant (Δt later), the pulse will have progressed a distance $C\Delta t$. If the end of the bar which is not struck is free, the pulse, upon reaching the end, will be reflected back as a tension pulse which proceeds back toward the struck end at the same velocity [see Fig. 1c (10)]. If the remote end had been fixed rather than free, the reflected pulse would be reflected back unchanged as a compression pulse. Each time the pulse arrives at an end of the bar it is reflected back as a tension or compression pulse depending on the fixity of the end concerned. Internal damping will progressively dissipate the pulse, however, and eventually the bar will reach equilibrium in a stress-free state.

The illustration in Fig. 1 refers to a single pulse traveling from one end of the bar to the other. A case of special interest is shown in Fig. 2a where tension stress pulses of equal magnitude start from each end of the bar at a given instant; that is, both ends are pulled suddenly and then released. At a particular later time (Fig. 2b) both pulses will have reached the midpoint of the bar, momentarily reinforcing each other and causing a stress intensity of twice the magnitude of either pulse (10). Subsequently the pulses separate (Fig. 2c), travel on to the extremities of the bar, are reflected back, reinforce, etc., and eventually damp out.

With reference to either Fig. 1 or 2, it may be noted that only limited portions of the bar are stressed at any given instant. An interesting aspect of Fig. 2 is that a small portion of the bar at midlength is momentarily stressed twice as severely as it had been slightly earlier or slightly later, and, for that matter, twice as severely as any other portion of the bar. It is interesting to speculate whether a similar mechanism may occur in the face of a sack. One may visualize, as in Fig. 3 for the cross section of an impacted sack, that the commodity exerts a short, sharp impact force at each edge of the sack, thereby sending tension pulses toward the center of the face, in direct analogy of the bar discussed above. The high incidence of paper failure at the center of the face of the sack is suggestive of the reinforcement of two pulses at that location with the consequent doubling of stress intensity.

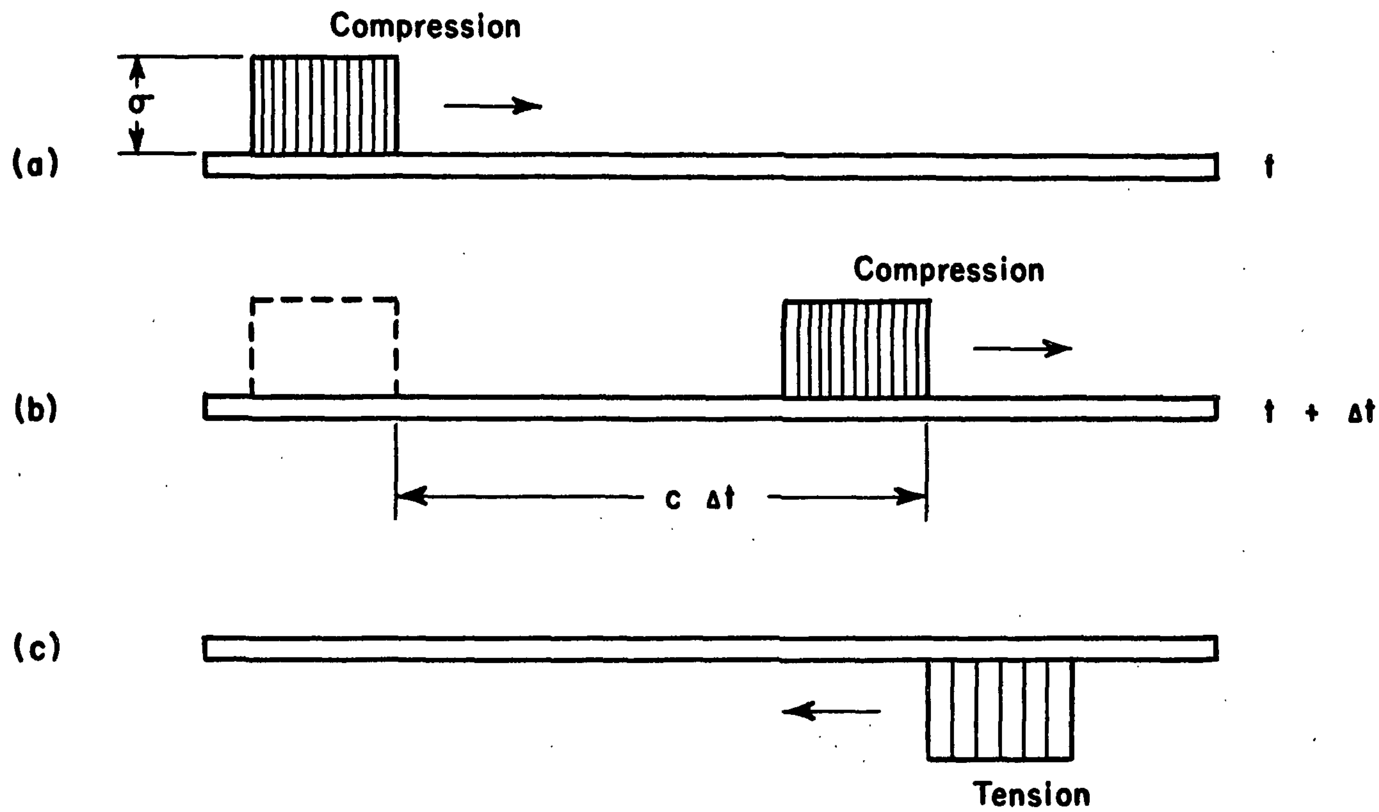

Figure 1. Impact Stress Pulse Moving Along a Long, Thin Rod

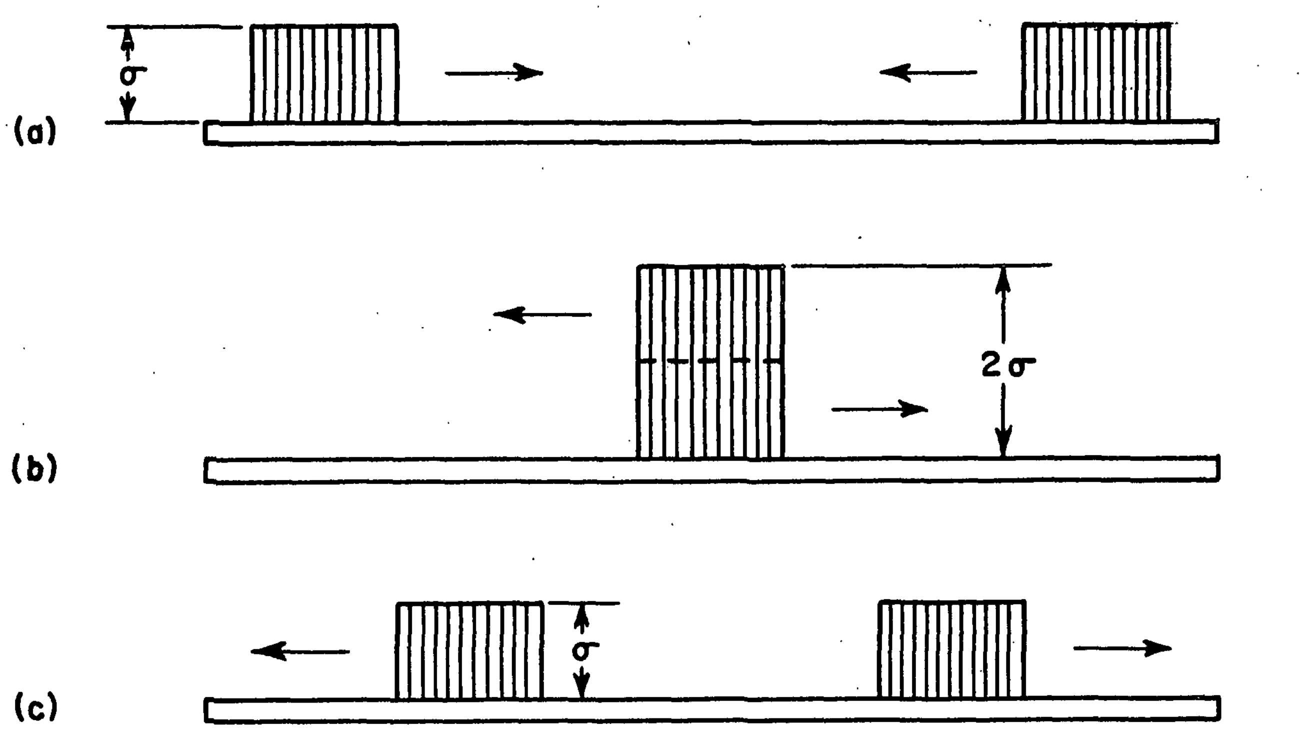

Figure 2. Impact Stress Pulses Moving in Opposite Directions Along a Long, Thin Rod

The impact stresses illustrated in Fig. 1 and 2 are highly idealized. The rectangular-shaped pulse is a special case in that it implies that the force builds up instantaneously to its peak value and later decreases instantaneously. In reality, a pulse is more likely to build up over some short time interval and likewise decrease in a finite time, as illustrated in Fig. 4, depending on the nature of the impact.

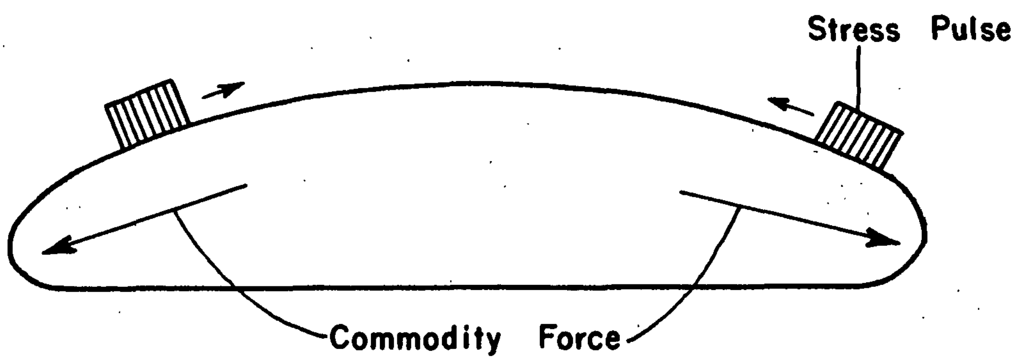

Figure 3. Cross Section of Filled Multiwall Sack During Impact

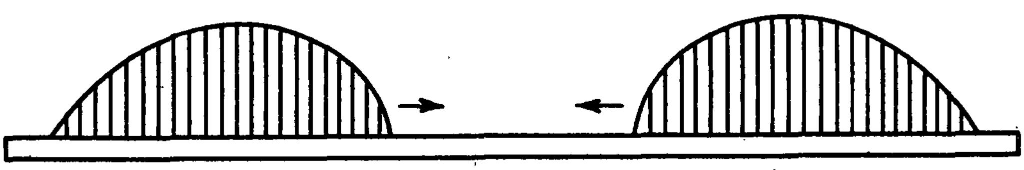

Figure 4. Nonrectangular Impact Pulses in Thin Rod

Figures 1, 2, and 4 have shown the spatial characteristics of impact pulses. An alternative representation is their timewise distribution (11). For example, a single, short, nonrectangular pulse of duration 2τ traveling a distance d from one end of a bar to the center may be represented in terms of the strain vs. time graph in Fig. 5, showing the state of strain at the impact end and at midlength as a function of time. The illustration is for the case of a relatively short duration pulse, such that the strain at the impact end has reached its maximum and diminished to zero before the leading edge of the pulse reaches midlength of the bar. One may visualize instances nearer the other extreme, as in Fig. 6, where the pulse duration is much longer and the difference in magnitude of strain between the two locations at a given instant is modest. In the extreme case of almost static application of force (as in conventional testing), the "pulse" length is so long that the two curves are virtually superimposed. This may be visualized in terms of Fig. 7 where the duration τ is very large relative to the time of travel, d/c, from end to midlength. (For purposes of illustration the time d/c has been foreshortened in Fig. 7 relative to the same interval in Fig. 5 and 6.) In this case the time for, say, the peak of the pulse to travel between the two locations is negligibly small compared with the duration of the pulse. Thus, it may be seen that the importance of wave propagation effects is mainly a matter of the duration of the impact pulse relative to the time required for the pulse to travel from one point to another in the impacted body, involving, therefore, the dimensions of the body and the velocity of wave travel in the material (6).

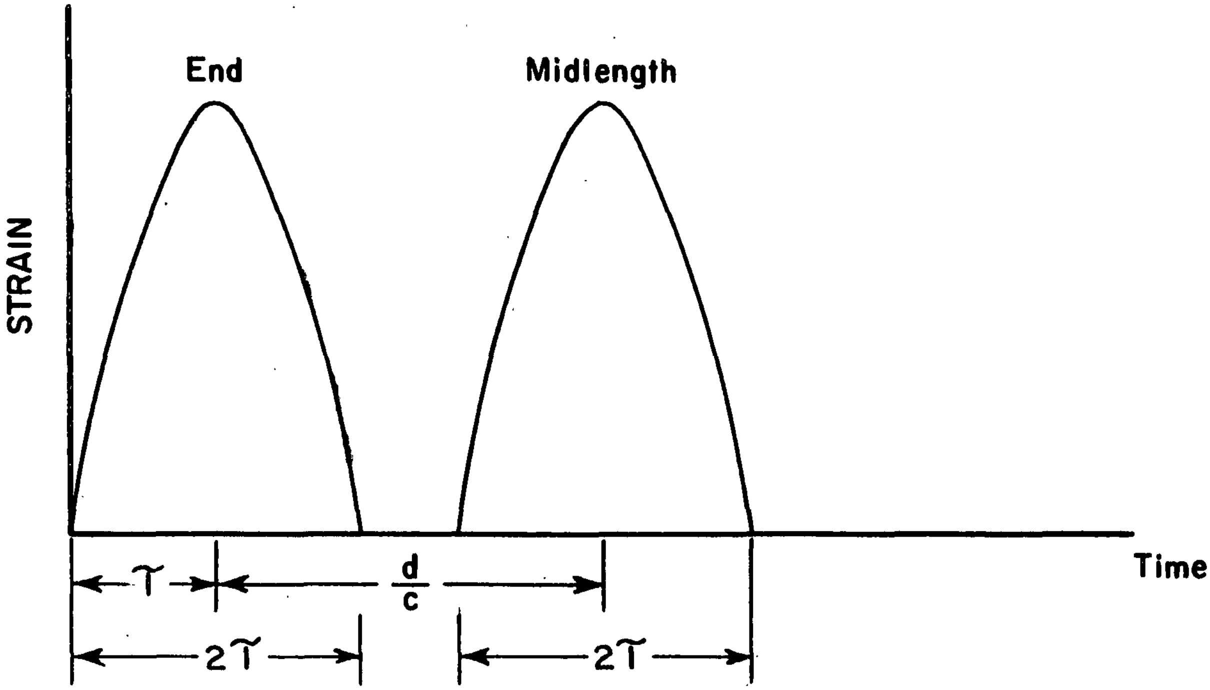

Figure 5. Time Representation of Short-Duration Impact Strain

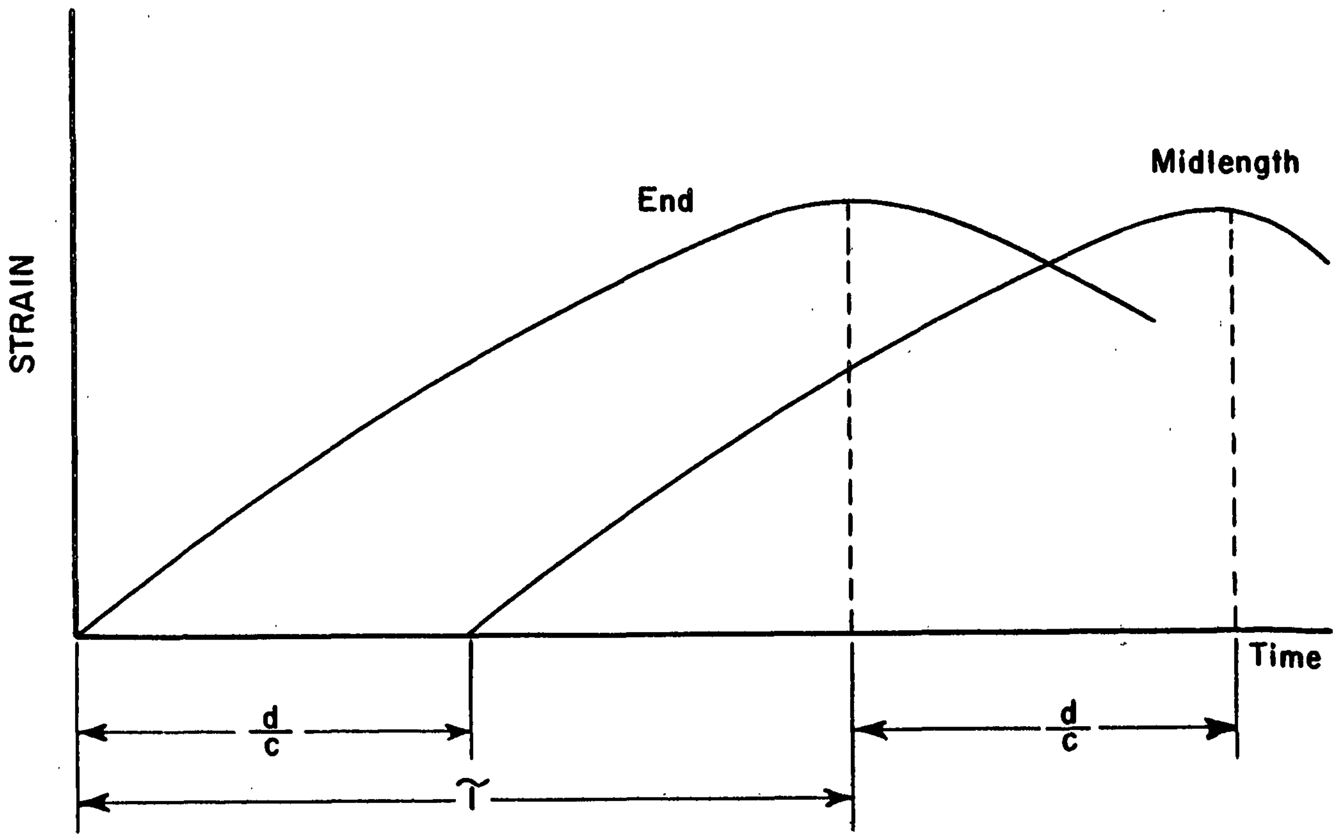

Figure 6. Time Representation of Long-Duration Impact Strain

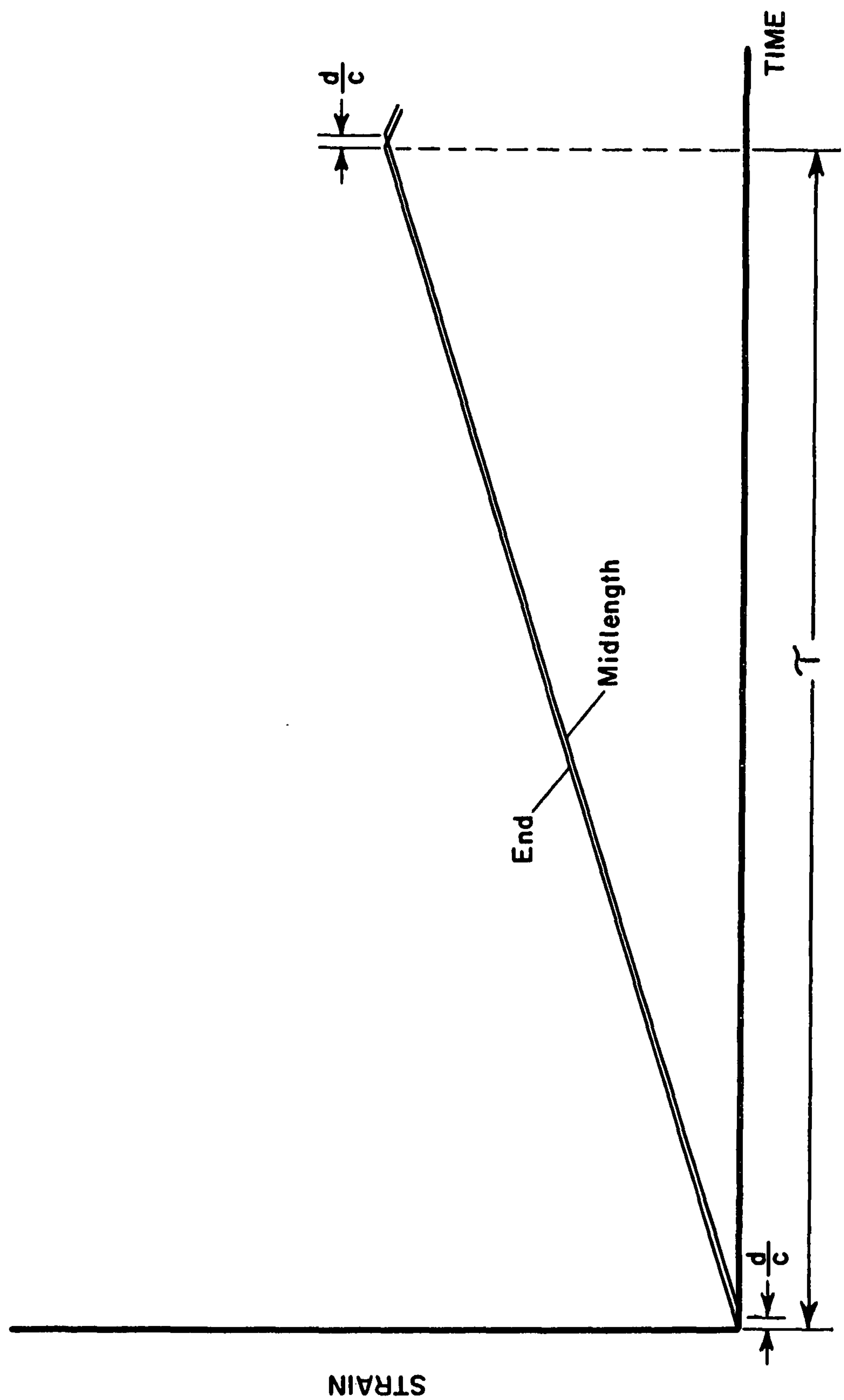

Figure 7. Time Representation of Slowly Applied Strain

INSTRUMENTATION

As pointed out previously, measurements were performed with respect to the following three types of phenomena:

a. propagation of compression impact strain in a steel bar;

b. propagation of tensile impact strain in long strips of sack paper; and

c. propagation of impact strain in multiwall sacks subjected to face drops. In all three types of experiments, four electrically conductive coatings were attached to the stressed body (bar, strip, or sack) to serve as detectors of strain.

The electric circuitry employed to record the signals from the strain gages was essentially the same in all experiments and is illustrated diagrammatically in Fig. 8. This circuit is similar to one reported in Reference (12). The three main parts of the circuit are:

a. four potentiometric circuits in parallel (one per gage), with a common 12-volt d.c. voltage supply;

b. two dual-beam Tektronix oscilloscopes (type 502) with Polaroid camera attachments; and

c. one trigger switch to actuate the sweep circuits of the oscilloscopes at the proper instant.

Each of the four potentiometric circuits consisted of a 50,000 ohm resistor (termed "ballast" resistor) in series with the strain gage, which generally had a resistance in the neighborhood of 500 ohms. A strain-induced change in the resistance of the gage appears as an approximately proportional change in the voltage across the gage (13). This voltage change is displayed as

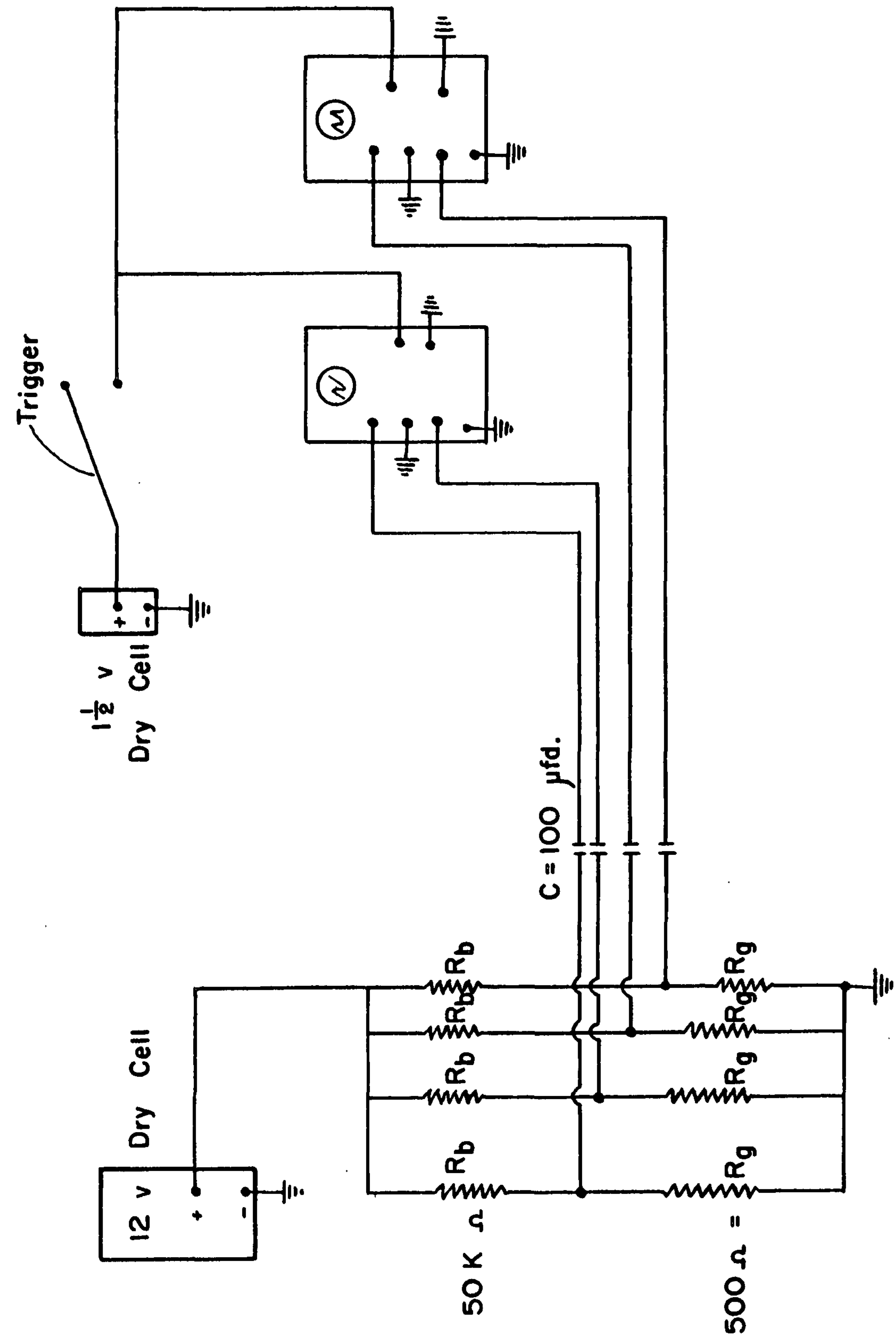

Figure 8. Electrical Circuit for Recording Impact Strain

a voltage vs. time curve on the face of the oscilloscope (two gages per oscilloscope) and is permanently recorded on Polaroid film.

An inherent drawback of the simple potentiometric circuit described above is that the change in voltage (due to strain) may be a very small fraction of the steady voltage which is necessarily displayed on the recording instrument. Accordingly, the precision of the measurement of strain suffers. This difficulty may be overcome by including a simple electric filter element in the potentiometric circuit, which suppresses the steady voltage and passes only the strain-induced change in voltage (13, 14). In the present work this was accomplished by placing a 100 μfd capacitor in series with the oscilloscope as indicated in Fig. 8. (The capacitor and oscilloscope are in parallel with the active gage.) This arrangement is known as a low-frequency (or high-pass) filter. Only those voltage changes which occur at a high rate, such as are due to impact strain, are faithfully passed to the oscilloscope. Slowly varying voltages, on the other hand, are attenuated — the slower the change, the more the attenuation; constant voltages are entirely suppressed.

The filter circuit for the present type of application must be designed so that the voltages which are attenuated are those of lower frequency than occur in the impact phenomenon under study. It may be calculated that the voltage change displayed through the filter circuit, illustrated in Fig. 7, is within 5% of the true value as long as the frequency is greater than about 10 cycles per second. This frequency corresponds to a rise time of 25 milliseconds or less in the strain signal. On the basis of past experience (2, 15, 16), the rise time of impact strain in a sack has been found to be of the order of one millisecond. Thus, there should be no appreciable attenuation of the "impact" voltage.

The sweeps of the two oscilloscopes were initiated through a switch in an external triggering circuit powered by a 1-1/2 volt dry cell. In the case of sack impact, the switch consisted of a four foot length of piano wire which was suspended tautly 3/4-inch above the impact base and connected in series with the dry cell and oscilloscope. The falling sack released one end of the trigger wire from a friction clamp, thereby interrupting the circuit and causing the oscilloscope to sweep.

In the experiments on bar impact and strip impact, the metallic impacting mass was itself a part of the trigger circuit. At the instant of impact the circuit was completed, causing the sweep to begin.

In one portion of the experimentation, a single commercial SR-4 wire-grid strain gage was employed on the steel bar. Because of its low output (relative to the conductive coatings), an Ellis BA-12 amplifier was inserted in the circuit in place of the potentiometric circuits. Also, when using the conductive coating gages on the steel bar, the resistance of the ballast resistors in the potentiometric circuits was reduced to increase the sensitivity of the circuit (because of the low strains induced in the bar). This was accomplished by shunting each ballast resistor with a low resistance element of variously 700 or 1500 ohms. Aside from these minor modifications, the electric circuits and recording equipment throughout the test program were as described above.

The experimentation described in this report was concerned almost exclusively with the measurement of elapsed time between various events occurring during impact, with durations in the millisecond and microsecond range. Inasmuch as two recording oscilloscopes were used, it was necessary to be assured that the time bases of both instruments were identical to within acceptably small error.

Thus, it was required to calibrate the oscilloscopes with respect to (a) time rate of sweep of the trace across the cathode ray tube face, and (b) the synchronization of the two oscilloscopes with respect to the beginning of the sweep.

The sweep rates were calibrated against a frequency generator at the outset of the experimentation and again about midway through the test program. It was found that the sweep rates of the two oscilloscopes agreed to within 1.8%, on the average, at the nominal sweep rate of 100 μsec./cm.; Oscilloscope A swept faster than Oscilloscope B. The actual sweep rate of either oscilloscope agreed to within 7% of the nominal rate (100 μsec./cm.), based on the frequency generator as a reference; however, the latter cannot be considered an absolute standard.

To check the synchronization of the beginning of the sweep in the two oscilloscopes, the impact strain from one SR-4 gage on the steel bar was displayed simultaneously on both oscilloscopes. The apparent times of an easily recognized point on the strain _vs._ time curves from the two oscilloscopes were compared. A difference in apparent time indicates a delay in triggering of one oscilloscope relative to the other (after allowing for the slightly differing sweep _rates_). This type of check was made from time to time during the test program.

At the outset of the program it was found that Oscilloscope A triggered slightly later than B. The delay was a reasonably constant per cent of the duration of the event being measured (which indicates that the delay was a constant per cent of the sweep speed). For example, A lagged B by about 5% at the beginning of the experimentation. Inasmuch as A swept faster than B by about 2% (see above), the actual delay was about 7%, although for purposes of correcting data for the delay, only the 5% differential need be considered.

During a portion of program, the behavior of the oscilloscopes reversed with respect to triggering differences. Oscilloscope B apparently lagged A by 2%, which, after allowing for differences in sweep rates, indicates both oscilloscopes were triggering simultaneously. A correction of data for this apparent delay involves adding 2% to the times read from Oscilloscope B.

In the latter phases of the test program, the oscilloscopes reverted to their initial difference in triggering.

It was felt that the differences in triggering behavior of the two oscilloscopes were not great enough to require correction of the data in this study. Nevertheless, as a matter of record, Table I lists the corrections for triggering which may be applied to the various phases of this study. These corrections would apply when the time of an event on one oscilloscope is compared with a time from the second instrument.

TABLE I

CORRECTION FACTORS TO ACCOUNT FOR TRIGGERING DIFFERENCE BETWEEN OSCILLOSCOPES

Phase of Test Program	Correction to Apparent Time	Gage No.
1. Impact of steel bar	Increase A by 5%	2, 4
2. Tensile impact of strip		
Machine direction	Increase B by 2%	1, 3
Cross direction	Increase A by 5%	2, 4
3. Sack impact		
Machine-direction strain	Increase A by 5%	2, 4
Cross-direction strain	Increase B by 2%	1, 3

It should be noted that when an <u>elapsed</u> time reading from one oscilloscope is compared with an elapsed time from the other oscilloscope, only the difference in sweep rates is pertinent. The correction factor here is: add 1.8% to Oscilloscope B (i.e., Gages 1 and 3) [or subtract 1.8% from Oscilloscope A (Gages 2 and 4)].

TEST PROCEDURE

IMPACT OF STEEL BAR

The impact behavior of a long steel bar of small cross section is well understood from both the theoretical and experimental standpoints. The present work was initiated with a brief experimental study of the impact of a steel bar in order to verify the experimental method to be used in subsequent phases on sacks and sack paper.

For this purpose a 36-inch long steel bar with 1-3/4 by 3/8-inch cross section was suspended at the quarter points of its length as a ballistic pendulum, as illustrated in Fig. 9. A striker bar was similarly suspended at one end of the test bar. The striker bar was 13 inches long, one-inch diameter, weighed 2.93 lb. and had a hemispherical striking head.

Electrical conductive coating gages were applied at four locations along the length of the bar. The center of Gage 1 was three inches from the impact end; the center-to-center distance of successive gages was ten inches. At a given location, gages were placed on both the top and bottom surfaces of the bar. Each pair was wired in series, and together comprised the active arm of one potentiometric circuit. This arrangement cancels out bending strains which may be induced by noncentral impact above or below the centerline of the bar cross section.

The technique of constructing the gages was as follows: The surface of the bar was sprayed with an electrical insulating solution. A paper overlay was

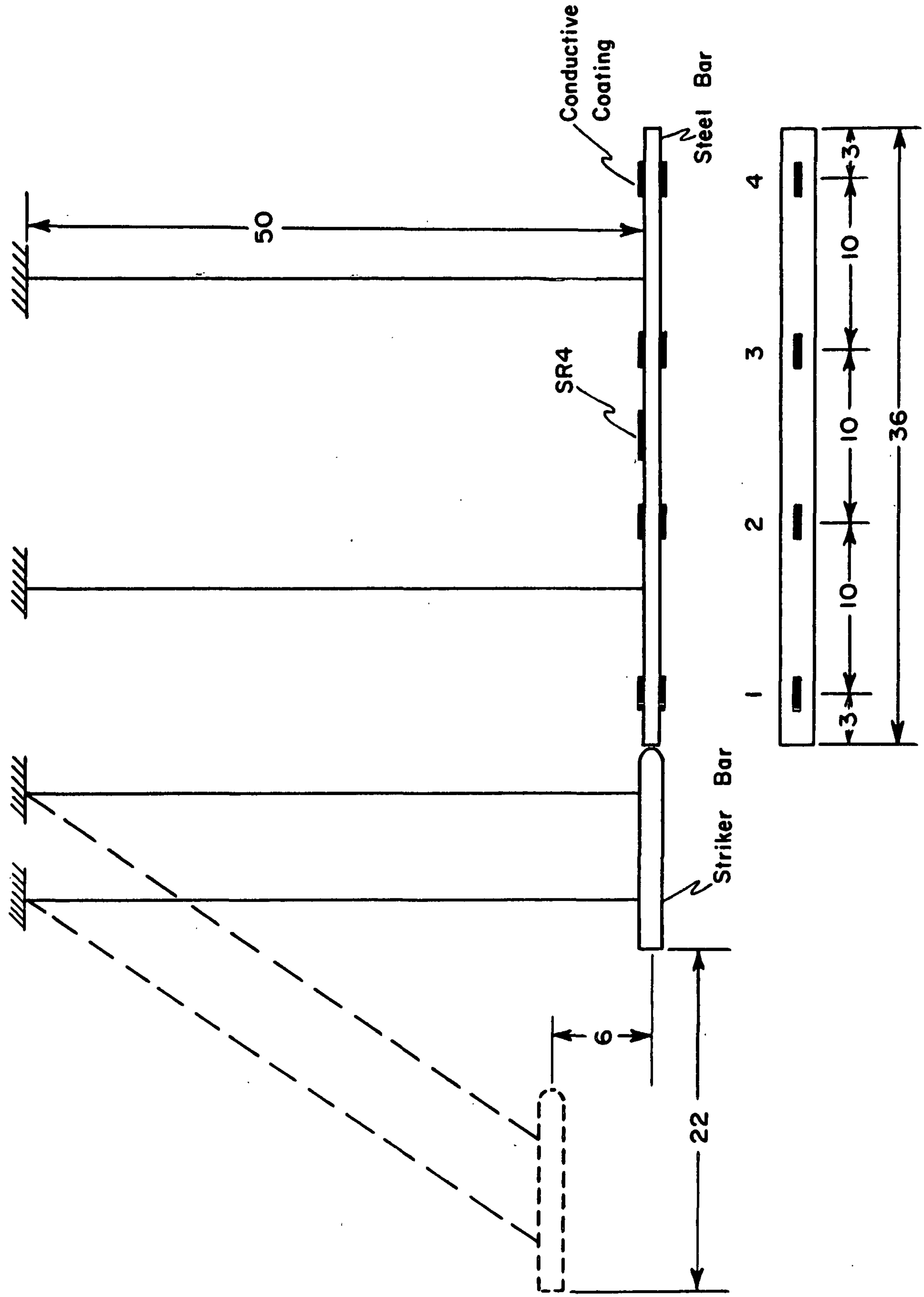

Figure 9. Experimental Set-up for Impact of Steel Bar

cemented to the upper and lower surfaces. A graphite dispersion [Aquadag (17)] was brushed onto the paper at a given gage location in a strip 1/4-inch wide and about 2-1/4 inches long. After the dispersion dried, a flat foil electrical lead was cemented to each end of the coating and silver paint was brushed onto the gage and foil lead; this provided electrical and mechanical connection between gage and lead and also defined the length of the gage (approximately 1.7 inches). The resistances of the individual gages ranged from 240 to 850 ohms.

A single SR-4 strain gage (Type A-6, 300 ohms, one inch gage length) was adhered directly to the bar surface at mid-length to serve in checking synchronization of the oscilloscopes, as discussed above.

A typical measurement of impact strain was as follows: The striker bar was manually swung away from the test bar and momentarily held about 22 inches ahead of, and six inches above, the test bar, as illustrated in Fig. 9. The striker bar was carefully aimed at the center point of the end cross section and then released. The striker bar and test bar were electrically a part of the triggering circuit, as discussed under Instrumentation. Thus, when the striker head impacted the test bar, each oscilloscope sweep commenced and recorded on Polaroid film the strain (vs. time) experienced at each gage location. The test bar was free to swing as a result of impact. After impact the striker bar was caught and held so there would not be successive impacts.

VELOCITY OF STRAIN PROPAGATION IN SACK PAPER

For purposes of measuring the velocity of wave propagation in sack paper, long tensile strips were cut from a roll of paper from Run YY (outer ply) of the 2nd fabrication program (18). This is an extensible sack paper with 9.0 and 4.8% stretch in the machine and cross directions, respectively. The tensile strengths of this sample are 20.8 and 15.0 lb./in. in the two directions, respectively. The basis weight is 51.8 lb./(24x36-500) and the thickness is 5.9 points.

Wave velocity in the machine direction was measured first. Two strips, twelve feet long in the machine direction and four inches wide, were prepared for the determination of velocity in the machine direction. Four Aquadag gages were applied at 40-inch intervals (center to center distance) along the strip — each gage at midwidth of the strip. The gage was one inch long in the machine direction of the paper and 1/4-inch wide. The Aquadag was brushed onto the paper in a liquid dispersion, after spraying the paper beneath the gage with insulating spray to prevent rippling of the paper by the aqueous Aquadag solution. After drying, foil leads were attached with cement and silver paint.

A strip was suspended vertically from the ceiling of the laboratory by means of a wooden clamp, as illustrated in Fig. 10. A hanger was attached to a wooden clamp at the bottom end of the strip, as illustrated in Fig. 11. On the hanger is the impact mass — a steel cylinder weighing 5.65 lb., which may be raised off the hanger pan and, upon release, slides freely down the hanger rod and applies tension impact force to the paper strip. Throughout the test program, the drop height was three inches. The weight pan and the impact mass were part of the triggering circuit (the mass was electrically insulated from the hanger rod by a nonmetallic sleeve). At the bottom of its fall the impact mass contacted the weight pan, completing the triggering circuit and causing the sweep in the oscilloscopes.

In the theoretical analysis of wave propagation there is a difference of opinion as to whether the velocity depends on the elastic modulus of elasticity irrespective of the stress level being applied. Some investigators contend that if the impact stresses are beyond the proportional limit, a tangent modulus should

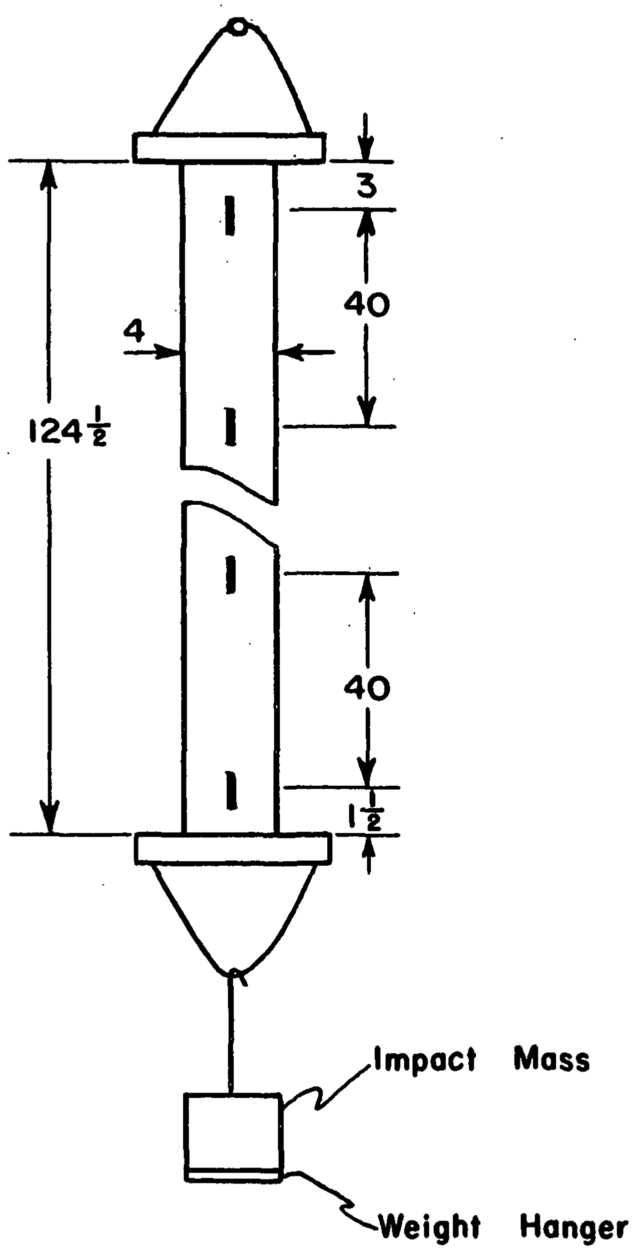

Figure 10. Tensile Strip of Sack Paper for Measurement of Machine-Direction Velocity of Propagation

be used in the equation for velocity, while others believe the elastic modulus suffices (6, 19). Stated another way, it is a controversial point whether the velocity at inelastic stress levels is less than or is equal to the elastic rate.

Figure 11. Photograph of Lower End of Tensile Strip with Impact Mass on Hanger

In consideration of this question, trials were performed at both elastic and inelastic levels. The first several trials with a given specimen were conducted with only the weight of the hanger and clamp (2.64 lb.) attached to the strip. Subsequent trials were performed with additional weights applied as dead loads to the strip, ranging from 20 to 40 lb. Heavier weights were tried but they resulted in paper failure on the first impact. These added weights were not impacted, but merely resided on the weight pan. Thus, the dead load on the strip was in the range of 25 to 50% of the tensile strength. At conventional test rates, the proportional limit of this sample of paper is 23 and 35% of the tensile strength for the machine and cross directions, respectively. It may be safely assumed, therefore, that the impact stress with the added weights was in the inelastic range of the stress-strain curve of the paper.

Early trials with the system described above revealed that the curve of strain vs. time was not fully satisfactory; the curve rose "slowly" from the base line, making it difficult to determine precisely the time of arrival of the strain wave at a given gage. There were two causes of this soft impact. One was the cable on the lower clamp to which the weight hanger was attached. This cable was stiff enough that the hanger did not draw it taut, but flexible enough that it cushioned the shock of the falling impact mass and gave a "soft" impact to the paper. The cable was eliminated by clamping the hanger to the wooden clamp on the specimen.

A second reason for the soft impact was that when the impact mass was lifted off the hanger to its three-inch drop height, the hanger weight was insufficient to prevent waviness in the strip. That is, the specimen was not taut just prior to impact. At impact, therefore, the shock was cushioned by the slack in the specimen. This difficulty was overcome by mechanically restraining the wooden clamp from moving upward when the impact mass was lifted off the hanger. With this method the strip remained taut and a sharp rise-time occurred in the strain signal.

Of course, when twenty to forty pounds of weight was added to the strip, waviness was no problem. On the contrary, creep of the specimen under load was very much in evidence. Whenever an increment of weight was added to the strip, it was desirable to wait a few minutes for the major part of the creep to occur before proceeding with the impact.

Two machine-direction strips were tested in the above manner. Repeat trials were made until the specimen broke.

Two cross-direction strips were also tested. The span of these specimens was about 34 inches; the total length of the strip was limited by the roll width, namely, 38 inches. Four Aquadag gages were spaced ten inches apart (center-to-center distance) on these specimens. Otherwise the test procedure was as described above for the machine direction.

IMPACT STRAIN IN MULTIWALL SACKS

To study the progression of strain across the face of a multiwall sack at the instant of impact, eight pasted valve sacks from Run YY of the second fabrication program were filled with 94 pounds of cement and strain gaged with Aquadag as illustrated in Fig. 12. In the case of the cross-direction gages (Fig. 12a), the four gages were arrayed on 2-1/8-inch centers from the center of face to the side of the sack. Four sacks were prepared with this pattern. In the machine-direction array (Fig. 12b), the gages were on 3-5/8-inch centers from the center of the face to the end of the sack.

In both cases the gages were one-inch long and one-quarter inch wide and were applied by brushing the Aquadag dispersion onto the paper (after filling and vibrating the sack) and later connecting foil leads with silver paint. As shown in Fig. 13, the foil leads were spot-cemented to the sack face about two inches from the gage, to resist breakage of the lead connection during impact. Heavier leads connected the foil to a terminal block on a drop platen, shown in Fig. 14. The purpose of the drop platen was to avoid handling the sack between drops and possibly creasing the gages in so doing. That is, the sack and platen were dropped together onto the impact base and then both were lifted to the drop test elevator for the subsequent drop. For reasons discussed later, the procedure was varied somewhat for the sacks with machine-direction gages. In this instance, the sack was dropped by itself onto a thin sheet of composition board lying on the impact base of the drop tester. To prepare for the next drop, the board and sack were lifted to the elevator and the bag was gently slid off the board onto the elevator, again avoiding direct handling of the sack. In these trials a small electrical terminal block was adhered directly to the sack face remote from the gages.

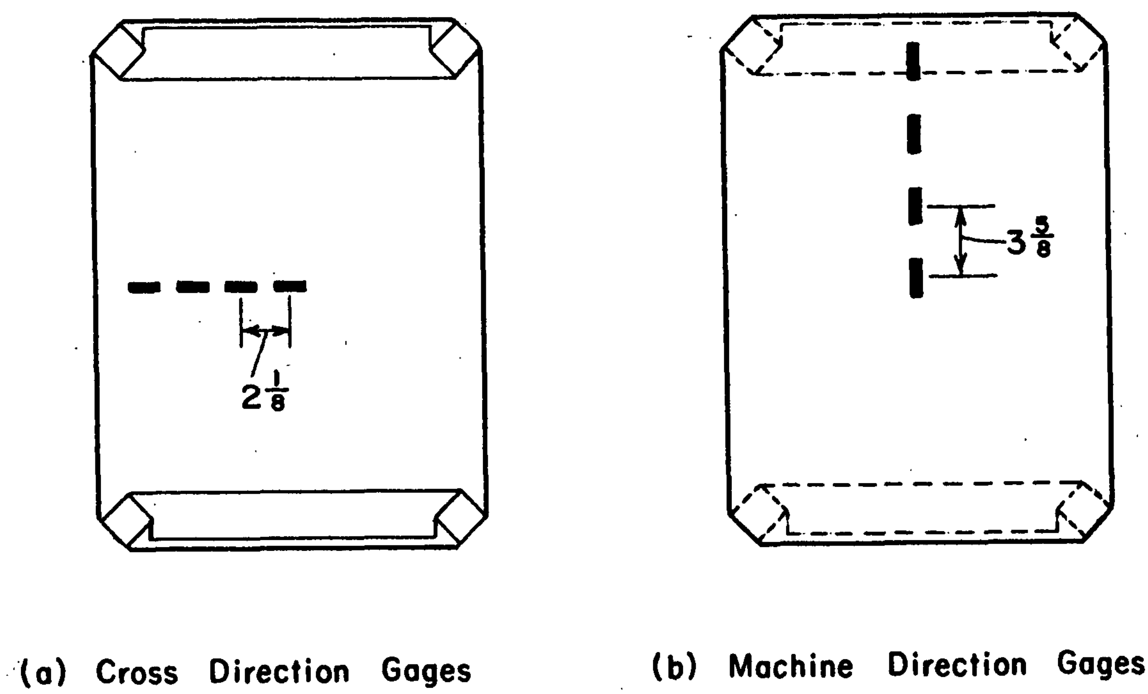

Figure 12. Location of Gages on Multiwall Sacks

The trials were conducted at drop heights of variously six and ten feet until a sack failed. All drops were face impacts, with the gaged face uppermost (away from the impact base).

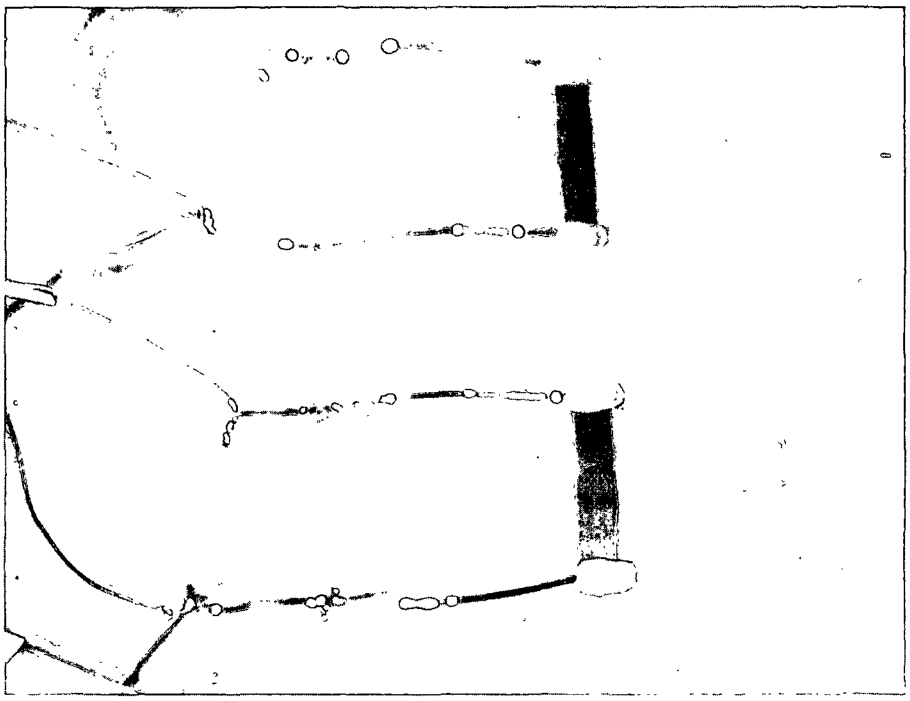

Figure 13. Aquadag Gages on Multiwall Sack

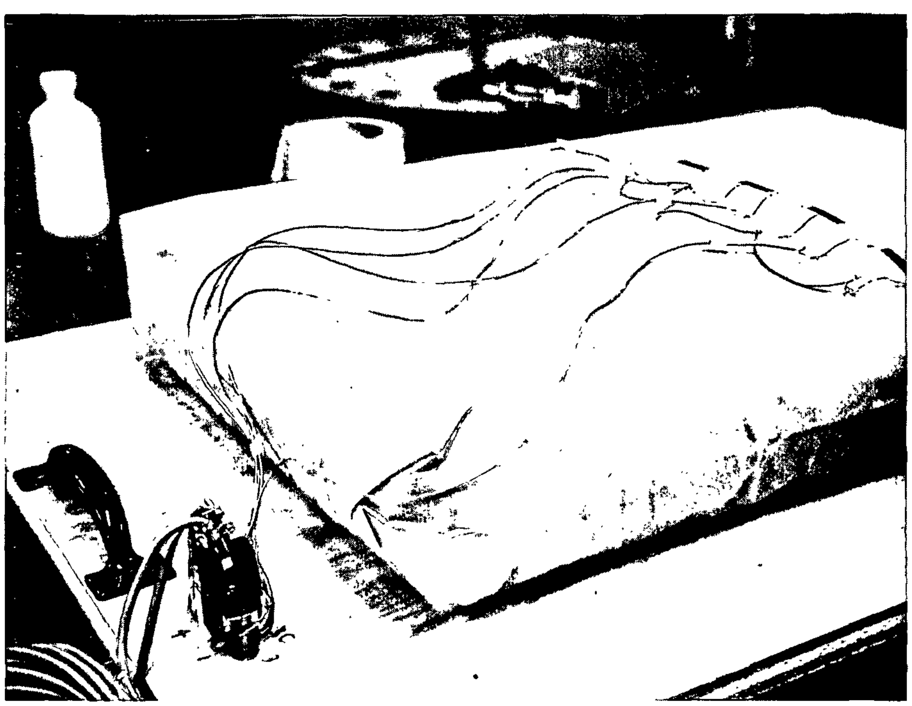

Figure 14. Multiwall Sack on Dropping Platen

DISCUSSION OF RESULTS

Three types of experiments were performed with regard to the propagation of strain during impact. The first involved impact of a steel bar for the purpose of validating the experimental method. The second experiment was performed on long tensile strips of extensible paper for the purpose of measuring the velocity of wave propagation in sack paper. Lastly, gaged multiwall sacks were filled with cement and face impacted for the purpose of studying the progression of impact strain across the face of the sack.

IMPACT OF STEEL BAR

The well-known behavior of an impacted steel bar was studied as a means of verifying the experimental technique and instrumentation to be employed in the subsequent study of the impact of sack paper. A 36-inch long steel bar suspended as a ballistic pendulum was impacted at one end by a striker pendulum. The compression wave traveling along the bar was sensed by four pairs of strain gages spaced ten inches apart.

A typical set of oscilloscope records for one impact are given in Fig. 15. Gage 1 (actually a pair of gages) was nearest the impacted end of the bar and successive gage locations along the bar are numbered consecutively. Gages 1 and 3 were recorded on Oscilloscope B (at the right-hand side of Fig. 15) and Gages 2 and 4 on Oscilloscope A. The horizontal axis of the photograph is the time axis, with <u>time increasing from right to left</u>. In the figure, one large division is 50 microseconds (50×10^{-6} sec.). Strain is measured along the vertical axis, with increasing compression strain in the upward direction. The conductive coating gages were not calibrated and therefore the scale factor is unknown. Previous experience has shown there is high variability in the sensitivity of this type of

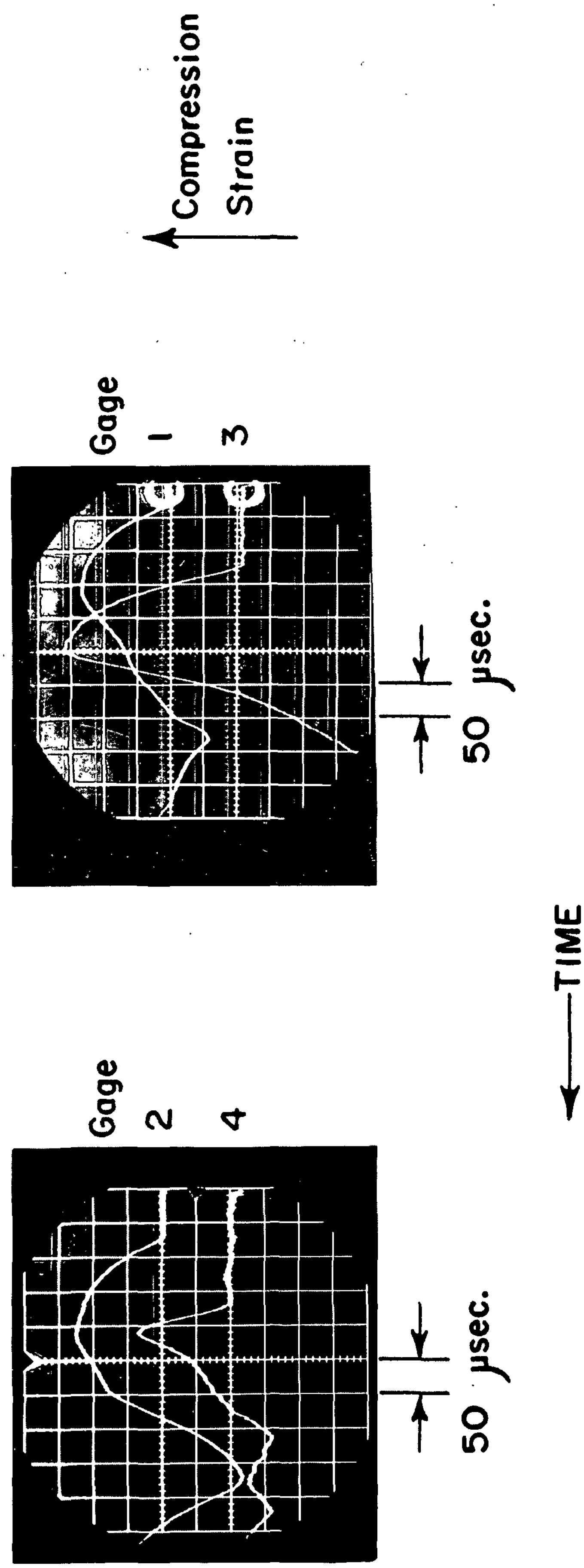

Figure 15. Oscilloscope Record of Impact Strain in Steel Bar

gage depending on gage structure and resistance (these gages varied from 240 to 850 ohms). Consequently, no particular significance should be attached to the differing magnitudes of peak strain between the several gages; these differences probably reflect differing sensitivities more than differing strain levels. The "halos" at the right edge of the record from Oscilloscope B are an electronic peculiarity of the particular cathode ray tube and are of no significance to this study.

It may be seen in Fig. 15 that the signal from Gage 1 was the first to rise above its base line. Approximately 40 μsec. later the signal from Gage 2 rises — this difference being the time required for the compression wave to travel the 10-inch distance between gages. The signals from Gages 3 and 4 follow in the anticipated order at approximately 50 μsec. intervals.

Returning to Gage 1, this location reached its peak compression strain about 135 μsec. after the first evidence of strain. This indicates that the wave front was not abrupt, but rather was a gradual increase (considering the time scale) from zero to maximum.

It may be noted from the figure that each successive gage location experienced peak strain at intervals of about 50 μsec., corresponding to the time required for a given point on the wave (in this case, the peak) to travel ten inches. An exception is Gage 4, which reached its peak strain at about the same time as Gage 2. The behavior of Gage 4 may be explained as follows: When a compression wave reaches the free end of a struck bar (the end which is not impacted) it is reflected back as a tension wave of nearly equal magnitude. In the case of Gage 4, which was three inches from the free end, the leading edge of the original compression wave had already been reflected back as a tension wave and made its effect felt at Gage 4 before the trailing peak of the original compression wave had arrived at Gage 4. The reflected tension component subtracts from the original compression component and at some particular instant there begins to be net decrease in signal and a peak appears in the signal (even though the peak of the original compression wave has not yet arrived at Gage 4).

Eventually, each of the other gage locations experience the effect of the reflected wave and of successive reflections from the ends of the bar which are both free. Of course, the waves finally damp out and the bar comes to rest at zero strain. But before this occurs there is a very complex pattern of reflected waves throughout the bar which augment and subtract from each other. For this reason, the strain signal from any given gage is very difficult to interpret after the first peak of the wave passes (and even before in the case of Gage 4). For this reason, the most reliable point of the curve for a study of wave propagation velocity is the first pick-up of strain at each gage, because at these instants no consideration need be given to reflected waves.

For the purpose of a systematic evaluation of the velocity of waves in the steel bar, photographs such as Fig. 15 were analyzed for 14 impact trials of the bar. In each instance, the time was noted at which the signal from each gage rose from its base line. The origin of time is taken as the right-hand edge of the oscilloscope grid. These readings are given in Table II. No correction has been made to these data to account for the slightly differing triggering response of the oscilloscopes; as discussed in Instrumentation, a 5% correction could be added to the times for Gages 2 and 4 to make them compatible with the readings for Gages 1 and 2. However, this correction is less than the precision of reading the photographs and is hardly warranted. Morever, for calculation of velocity it is appropriate and desirable to calculate the elapsed time for the wave to travel the 20 inches between Gages 1 and 3 and between 2 and 4. These time intervals are measured within a given oscilloscope and are, therefore, the most reliable. The extreme right-hand columns of Table II show these time intervals. As discussed in Instrumentation, a difference of 1.8% is expected between a time interval for Gages 1 and 3 relative to a time interval for Gages 2 and 4 because of a slight difference in sweep rates of the two oscilloscopes. It may be seen in Table II that the average time intervals for the two pairs of gages differ by 2%, which is compatible with the difference in sweep rates. The average time interval for the wave to travel 20 inches in the steel bar may be taken as 99 microseconds. Then the velocity of wave propagation in steel determined from this experiment is 20 in./99×10^{-6} sec. = 202,000 in./sec. The theoretical velocity, based on handbook values of $\underline{E} = 30 \times 10^6$ lb./sq. in. and density = 0.2823 lb./cu. in. for steel (8, 20), is

$$C = \sqrt{\frac{386.4E}{D}} = \sqrt{\frac{(386.4)(30\times10^6)}{0.2823}} = 203{,}000 \text{ in./sec.}$$

Thus, theory and experiment agree to 1/2 of one per cent.

This remarkably close agreement may be somewhat fortuitous inasmuch as (a) handbook values were used for modulus of elasticity and density of steel, (b) possible dependence of modulus on rate of straining has not been considered, and (c) the time base of the oscilloscopes was not calibrated against an absolute standard. Nonetheless, it seems clear that there is no reason to doubt the instrumentation, the experimental technique, or the method of interpreting the results from impact experiments conducted in the manner of this investigation.

TABLE II

TIME OF ARRIVAL OF COMPRESSION WAVE AT GAGES ON STEEL BAR

Time, microseconds[b]

Trial	Gage 1[a]	Gage 2	Gage 3	Gage 4	Time Interval[c] Gage 1 to Gage 3	Time Interval[c] Gage 2 to Gage 4
1	30	70	120	165	90	95
2	50	105	150	200	100	95
3	10	55	95	150	85	95
4	30	70	120	170	90	100
5	30	80	130	175	100	95
6	20	65	110	160	90	95
7	50	90	150	190	100	100
8	40	80	140	190	100	110
9	30	70	130	170	100	100
10	30	70	130	180	100	110
11	20	70	130	170	110	100
12	40	90	140	190	100	100
13	20	70	120	170	100	100
14	20	80	130	180	110	100
				Av.	98	100
				Composite av.	99	

[a]Gage 1 at impact end of bar; successive gages 10 inches apart.

[b]Zero time corresponds to right-hand edge of oscilloscope grid.

[c]Over 20 inches of bar.

[As a matter of general interest to wave propagation study, two impact trials with the SR-4 gage active at midlength indicated that the peak strain was 140 μin./in., which corresponds to a stress of about 4200 p.s.i. — well within the elastic range of steel. The duration of the pulse from zero to maximum was 145 μsec., on the average (based on SR-4 and Aquadag gages), giving an average strain rate of about one in./in./sec. The length of the pulse from zero to maximum strain was (145 μsec.)(2.02×10^5 in./sec.) = 29 inches. The trailing side of the wave cannot be analyzed easily because of the reflections set up at the ends of this relatively short bar; however, measurements of the duration of the impact (by means of the trigger circuit) indicated that the total length of the wave (zero to maximum to zero) was about 80 inches.]

VELOCITY OF STRAIN PROPAGATION IN SACK PAPER

The velocity of wave travel in a given material is of central importance to a consideration of wave propagation in a structure fabricated from the material. Whether or not strain propagation effects are of significance in a given instance of mechanically loading the structure is basically a matter of how long it takes for the impulse to traverse the body relative to the time during which the load is applied.

As far as is known, the velocity of wave travel in sack paper has not been determined experimentally. While the velocity may be readily estimated from the equation $C = \sqrt{E/D}$, which has been verified for metals, there is some question as to the appropriate modulus (sheet or fiber) and density (sheet or fiber) in the case of a nonhomogeneous material such as paper.

Initially it was believed that an experimental determination of wave velocity would be acquired as a by-product of the measurement of impact strains in an impacted sack. It soon became apparent, however, that there were a number of aberrations in the data from the sacks as to lead to large uncertainties in the velocity estimates. Accordingly, a more controlled type of experiment was performed on long tensile strips for the express purpose of measuring velocity of strain travel.

Two 12-ft. long strips of extensible sack paper (Run YY, outer ply) were suspended vertically and rapidly stressed in the machine direction by dropping a 5-1/2 lb. weight attached to the lower end of each strip. Aquadag gages spaced at 40-inch intervals indicated the passage of the impact wave from the bottom to the top of the strip. Determination of the time of travel between gages permits calculation of the velocity of wave propagation. A representative photograph of the curve of strain vs. time is given in Fig. 16. The oscilloscope gain and sweep speeds were adjusted to give a magnified view of the arrival of the wave at each gage, that is, the portion of the curve where the signal initially rises from its base line. The peak of the wave is purposely off-scale. Gage 4 is at the lower end of the tensile strip; that is, Gage 4 receives the impact first, followed by Gages 3, 2, and 1, in that order. The polarity of the signal was adjusted so that increasing tensile strain is downward for Gages 1 and 2 and upward for Gages 3 and 4.

The time of arrival of the wave at each gage is tabulated in Table III in the same format as Table II, presented above for impact of a steel bar. The origin of time is arbitrarily taken at the right-hand edge of the oscilloscope grid. Trials were performed at both low and high stress levels as indicated in the table by the magnitude of a dead weight suspended at the bottom end of the tensile strip. The two right-hand columns of the table show the time for the wave to travel 80 inches between alternate gages; these intervals are measured within an oscilloscope and therefore offer the greatest accuracy. The data from the first ten trials with Specimen No. 1 are of dubious accuracy because of the soft pick-up of strain due to slack in the system. This difficulty was rectified for the remaining trials of Specimen No. 1 and for all trials for Specimen No. 2, as discussed in Test Procedure.

The several averages in Table III are collected in Table IV according to specimen, gages involved, and stress level (low vs. high). The averages for the first ten trials of Specimen No. 1 are omitted because of their doubtful accuracy. The numeral in parentheses indicates the number of observations entering

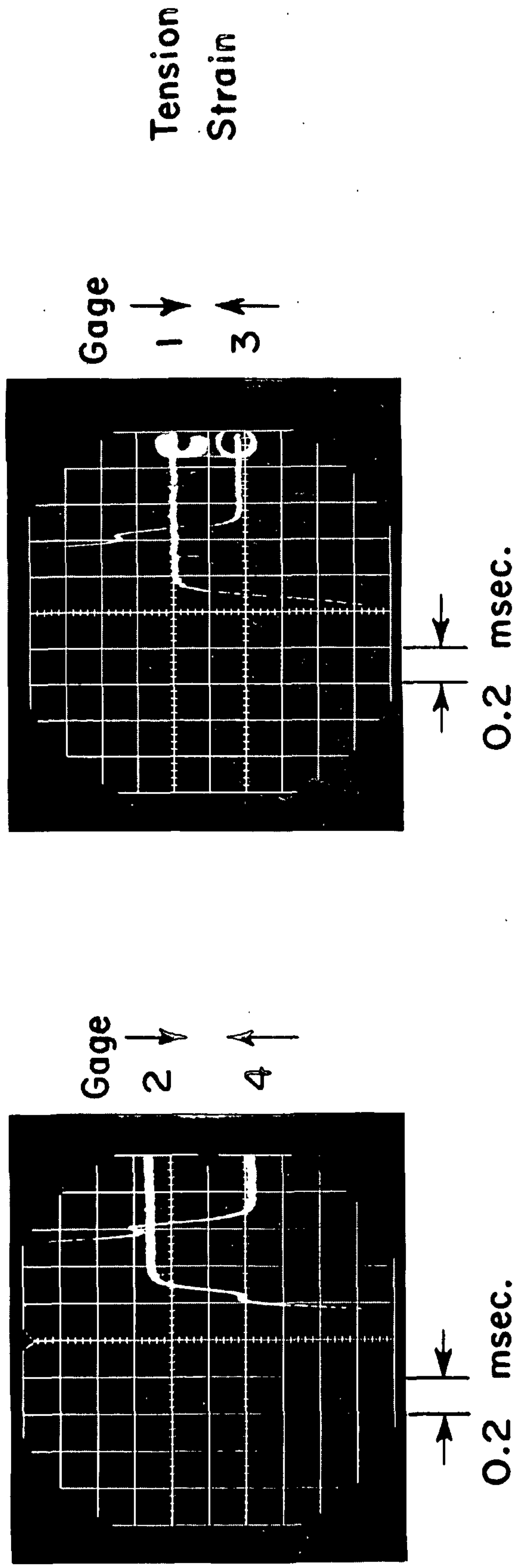

Figure 16. Oscilloscope Record of Impact Strain in Machine-Direction Tensile Strip of Extensible Sack Papers

TABLE III

TIME OF ARRIVAL OF TENSION WAVE AT GAGES IN
MACHINE DIRECTION TENSILE STRIP

Time, milliseconds[b]

Trial No.	Dead Weight	Gage 4[a]	Gage 3	Gage 2	Gage 1	Time Interval[c] Gages 4 to 2	Time Interval[c] Gages 3 to 1
				Specimen No. 1			
1	2.64	1.50	2.40	2.60	2.80	1.10	0.40
2	2.64	--	2.20	2.40	2.90	--	0.70
3	2.64	1.10	1.75	2.10	2.20	1.00	0.45
4	2.64	1.10	1.85	2.20	2.20	1.10	0.35
5	2.64	1.10	1.70	2.10	2.30	1.00	0.60
6	2.64	0.85	1.55	1.95	2.00	1.10	0.45
7	2.64	1.50	1.80	2.35	2.25	0.85	0.45
8	2.64	1.40	1.50	2.25	2.20	0.85	0.70
9	2.64	0.75	1.35	1.80	2.20	1.05	0.85
10	2.64	0.70	1.20	1.75	2.20	1.05	1.00
Av.						1.01	0.60
11	19.53	--[d]	0.90	1.40	1.80	--	0.90
12	19.53	--	--	1.35	1.90	--	--
13	19.53	--	0.95	1.45	1.85	--	0.90
14	19.53	--	0.95	1.40	1.80	--	0.95
Av.						--	0.88
15	29.57	--	1.05	1.50	1.90	--	0.85
16	29.57	--	1.05	1.50	1.90	--	0.85
Av.						--	0.85
17	39.51	--	0.90	1.45	1.85	--	0.95
18	39.51	--	1.05	1.55	1.85	--	0.80
Av.						--	0.87
				Specimen No. 2			
1	2.64	0.25	0.75	1.25	1.70	1.00	0.95
2	2.64	0.25	0.75	1.25	1.70	1.00	0.95
3	2.64	0.25	0.70	1.20	1.70	0.95	1.00
4	2.64	0.25	0.70	1.25	1.70	1.00	1.00
5	2.64	0.25	0.75	1.25	1.70	1.00	0.95
6	2.64	0.25	0.75	1.20	1.70	0.95	0.95
7	2.64	0.25	0.75	1.20	1.70	0.95	0.95
Av.						0.98	0.96
8	29.57	0.60	1.05	1.50	2.00	0.90	0.95
9	29.57	0.65	1.10	1.55	2.00	0.90	0.90
10	29.57	0.45	0.95	1.45	--[d]	1.00	--
11	29.57	0.45	0.95	1.40	--	0.95	--
12	29.57	0.60	1.00	1.45	--	0.85	--
13	29.57	0.50	0.95	1.40	--	0.90	--
Av.						0.92	0.92

[a]Gage 4 at impact end of strip; successive gages 40 inches apart.

[b]Zero time corresponds to right-hand edge of oscilloscope grid.

[c]Over 80 inches of length.

[d]Gage became inoperative.

the average. All composite averages in Table IV are weighted according to the number of observations.

TABLE IV

TIME INTERVAL FOR TENSION PULSE TO TRAVEL 80 INCHES IN MACHINE DIRECTION OF EXTENSIBLE SACK PAPER

Time, millisecond

	Low Stress			High Stress			
Specimen	Gages 4-2	Gages 3-1	Av.	Gages 4-2	Gages 3-1	Av.	Av.
1	--	--	--	--	0.87 (7)	0.87 (7)	0.87 (7)
2	0.98 (7)	0.96 (7)	0.97 (14)	0.92 (6)	0.92 (2)	0.92 (8)	0.95 (22)
Av.	0.98 (7)	0.96 (7)	0.97 (14)	0.92 (6)	0.88 (9)	0.90 (15)	0.933 (29)

$$\text{Average velocity} = \underline{C} = \frac{80 \text{ in.}}{0.933\text{x}10^{-3} \text{ sec.}} = 85{,}700 \text{ in./sec.}$$

It may be seen in Table IV that, on the average, the time of pulse travel over 80 inches of sack paper was 0.933 millisecond, based on 29 observations. Three-quarters of the number of observations come from Specimen No. 2, for which the time of travel was about 6% longer than in Specimen No. 1. The time interval at the high stress level was shorter than at low stress (0.90 vs. 0.97) which means that the velocity was greater at high stress by about 7%. This trend is the reverse of that reported for metals, where the velocity possibly may decrease at high stress because of the decrease in modulus. As shown at the bottom of the table, the average velocity in the machine direction of this sample of extensible paper was 85,700 in./sec.

For purposes of estimating velocity from $\underline{C} = \sqrt{\underline{E}/\underline{D}}$, the basis weight, thickness, and modulus of elasticity were measured for nine one-square foot samples of paper taken over a twelve-foot length of paper from the parent roll immediately following the paper from which the tensile strips were cut. The average values obtained were:

basis weight = 51.0 lb./(24 x 36-500)

thickness = 5.7 points

sheet density, D = 0.0207 lb./cu. in.

modulus of elasticity, E = 259,000 lb./sq. in.

Calculation of velocity from these measured sheet properties gives

$$C = \sqrt{\frac{386.4(259,000)}{0.0207}} = 69,500 \text{ in./sec.}$$

Thus, the estimated velocity is 18.9% lower than the observed velocity (85,700 in./sec.).

It is tempting to speculate that the disparity between estimated and observed velocities is attributable to the effects of rate of strain. That is, it has been found that the modulus of elasticity of paper increases with increase in rate of strain (21, 22). While the rate of strain was not evaluated in this experiment (because the strain is unknown), it is, of course, a good deal higher than the 0.133 in./in./min. rate used in the laboratory determination of modulus cited above. A dynamic modulus of elasticity, therefore, would be expected to increase the estimated velocity.

Before considering other possible factors which may contribute to the disparity between estimated and observed velocities, it may be well to examine the data for the cross-machine impact tests. These determinations of velocity of wave travel were conducted in a manner similar to the machine-direction tests except that the tensile strips were only 34 inches long and the four Aquadag gages were spaced ten inches apart. The time at which the wave reached each gage is tabulated in Table V for the two specimens tested in the cross direction. The average time interval for each pair of gages and each specimen at low and high stresses are collected in Table VI.

It may be seen in Table VI that both cross-direction specimens behaved nearly alike. There was no apparent difference in time of wave travel at high and low stress. On the average, the time of wave travel over a distance of 20 inches was 0.265 millisecond which corresponds to a velocity of 75,500 in./sec. — 12% lower than in the machine direction.

The cross-direction modulus of elasticity, measured at a conventional rate of strain, was 286,000 lb./sq. in. — somewhat greater than the machine-direction modulus (this was extensible paper). Using the aforementioned value of sheet density, the estimated velocity of wave travel is

$$\underline{C} = \sqrt{\frac{(386.4)(286,000)}{0.0207}} = 73,000 \text{ in./sec.}$$

This estimate is 3.3% lower than the observed velocity. The agreement between observed and estimated velocities is thus considerably closer for cross machine than for machine direction, although in both cases the estimated velocity is less than the observed.

It is not clear why the agreement between theory and experiment was markedly better in the cross direction than in the machine direction. In speculating on possible reasons, it is interesting to note that Andersson and Sjöberg (22) obtained load-elongation curves for flat kraft sack paper which suggest that the dependence of modulus of elasticity on strain rate may be greater in the machine direction than in the cross direction. Reproductions of load-elongation curves obtained by the above-mentioned investigators are given in Fig. 17. It may be noted that the strain rates varied by about 1000 fold which is of the

TABLE V

TIME OF ARRIVAL OF TENSION WAVE AT GAGES IN CROSS-DIRECTION TENSILE STRIP

Time, milliseconds[b]

Trial No.	Dead Weight	Gage 4[a]	Gage 3	Gage 2	Gage 1		Time Interval[c] Gages 4 to 2	Time Interval[c] Gages 3 to 1
				Specimen No. 1				
1	2.64	0.34	0.46	0.60	0.70		0.26	0.24
2	2.64	0.34	0.46	0.60	0.74		0.26	0.28
3	2.64	0.34	0.46	0.60	0.72		0.26	0.26
4	2.64	0.34	0.44	0.58	0.72		0.24	0.28
5	2.64	0.33	0.46	0.60	0.72		0.27	0.26
6	2.64	0.36	0.46	0.62	0.73		0.26	0.27
7	2.64	0.36	0.46	0.63	0.74		0.27	0.28
8	2.64	0.34	0.46	0.60	0.73		0.26	0.27
9	2.64	0.34	0.45	0.61	0.72		0.27	0.27
						Av.	0.26	0.27
10	19.88	0.42	--[d]	0.71	0.81		0.29	--
11	19.88	0.58	--	0.84	0.93		0.26	--
12	19.88	0.49	--	0.73	0.84		0.24	--
13	19.88	0.52	--	0.81	0.90		0.29	--
14	19.88	0.65	--	0.95	--		0.30	--
15	19.88	0.58	--	0.84	0.93		0.26	--
						Av.	0.27	
16	28.70	0.57	--	0.81	0.91		0.24	--
17	28.70	0.57	--	0.80	0.90		0.23	--
						Av.	0.24	--
				Specimen No. 2				
1	2.64	0.31	0.46	0.60	0.72		0.29	0.26
2	2.64	0.32	0.46	0.60	0.72		0.28	0.26
3	2.64	0.34	0.50	0.64	0.76		0.30	0.26
4	2.64	0.30	0.46	0.60	0.70		0.30	0.24
5	2.64	0.32	0.48	0.60	0.72		0.28	0.24
6	2.64	0.34	0.48	0.62	0.72		0.28	0.24
7	2.64	0.30	0.44	0.58	0.70		0.28	0.26
						Av.	0.29	0.25
8	19.88	0.40	0.54	0.68	0.82		0.28	0.28
9	19.88	0.48	0.58	0.72	0.82		0.24	0.24
10	19.88	0.50	0.62	0.76	0.88		0.26	0.26
11	19.88	0.48	0.60	0.73	0.85		0.25	0.25
						Av.	0.26	0.26

[a]Gage 4 at impact end of strip; successive gages 10 inches apart.

[b]Zero time corresponds to right-hand edge of oscillograph screen.

[c]Over 20 inches of length.

[d]Gage became inoperative.

general order of magnitude expected between the impact tests of this present study and conventional test rates. At low stress levels, where the curves are nearly straight, there appears to be a definite increase in modulus in the machine direction and, on the other hand, virtually no increase in cross-direction modulus. Based on these data, therefore, there is a possibility that the calculated velocity based on a modulus from a slow speed test would underestimate the observed velocity more severely in the machine direction than in the cross direction, as was experienced in the present study. In view of the scarcity of published data on rate effects on modulus, this discussion, while plausible, is largely speculative.

TABLE VI

TIME INTERVAL FOR TENSION PULSE TO TRAVEL TWENTY INCHES IN CROSS DIRECTION OF EXTENSIBLE SACK PAPER

	Time, millisecond						
	Low Stress			High Stress			
Specimen	Gages 4-2	Gages 3-1	Av.	Gages 4-2	Gages 3-1	Av.	Av.
1	0.26 (9)	0.27 (9)	0.26 (18)	0.26 (8)	--	0.26 (8)	0.26 (26)
2	0.29 (7)	0.25 (7)	0.27 (14)	0.26 (4)	0.26 (4)	0.26 (8)	0.26 (22)
Av.	0.27 (16)	0.26 (16)	0.27 (32)	0.26 (12)	0.26 (4)	0.26 (16)	0.265 (48)

$$\text{Average velocity} = \underline{C} = \frac{20 \text{ in.}}{0.265 \times 10^{-3}} = 75{,}500 \text{ in./sec.}$$

Other possible reasons for the difference between theory and experiment are (a) sampling and testing variations in modulus of elasticity and density, and (b) lack of an absolute calibration of the time base of the oscilloscopes. There is the added question of the correct determination of modulus of elasticity from the load-elongation curve of paper, which is seldom exactly straight for any appreciable distance from the origin. An error of this type, however, would

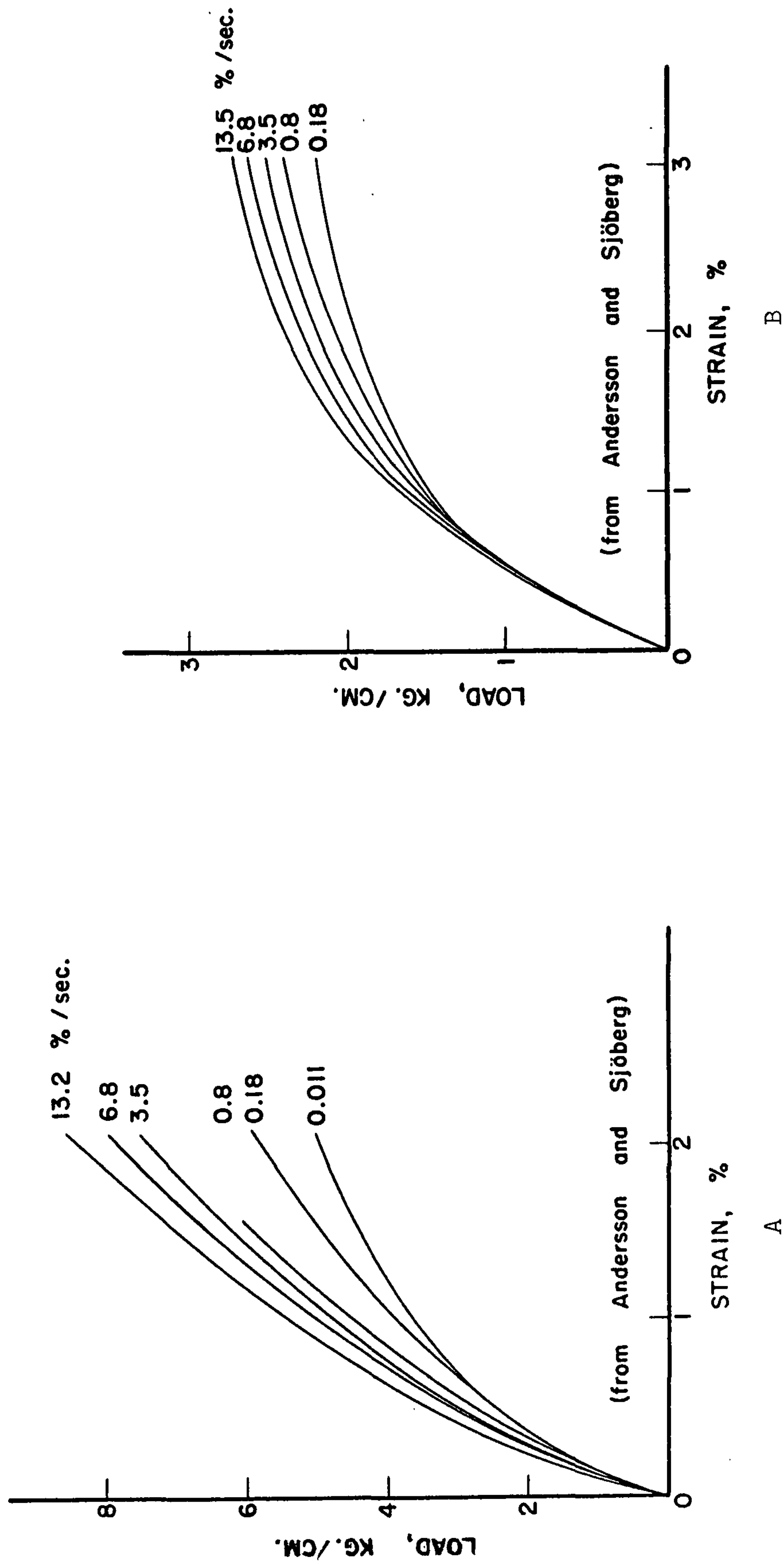

Figure 17. Effect of Strain Rate on Tension Load-Elongation Curves of Kraft Sack Paper. From Reference (22)

result, by itself, in overestimation rather than underestimation of velocity. It should be remarked that the density of the test strips was measured after the impact tests to determine whether an appreciable decrease may have occurred due to impact and creep of the specimen. The density decreased by about 1-1/2% in the machine-direction specimens and increased by 3% in the cross-direction specimens. These differences are probably mainly due to sampling and testing variability and, in any event, are too small by themselves to account for the disparities in velocities.

The modulus of elasticity of the tested strips was also determined after the impact tests. As was anticipated from the results of earlier studies (23), the moduli decreased, relative to the virgin sheet, as a result of previous strain history — by 16% in the machine direction and 28% in the cross direction. These changes are in the wrong direction to explain the difference between estimated and observed velocities.

However, the question may be raised whether these post-test moduli are more appropriate to the calculation of velocity than the virgin test values, on the basis that after the first impact trial a reduced modulus may be applicable. This viewpoint is strengthened by the summary of data in Table VII which indicates that the observed velocities for machine direction and cross direction are in the same relative order as the post-test moduli. More specifically,

$$\underline{C}_{M.D.}/\underline{C}_{C.D.} = 1.14$$

and, $\sqrt{\underline{E}_{M.D.}/\underline{E}_{C.D.}} = 1.03$, using post-test moduli.

While these ratios do not agree exactly, they are at least both greater than unity, whereas the virgin moduli ratio is $\sqrt{\underline{E}_{M.D.}/\underline{E}_{C.D.}} = 0.95$. The change in modulus from impact to impact (whether gradual or abrupt, for example) cannot be known from this experiment. If, however, one calculates velocities from the post-test values of modulus, the machine-direction velocity would be underestimated by about 26% (rather than the aforementioned 19%) and the cross-direction velocity by about 18% (rather than 3%).

TABLE VII

SUMMARY OF VELOCITIES AND MODULI OF ELASTICITY

	Machine Direction	Cross Direction
Observed Velocity, in./sec.	85,700	75,500
Modulus of Elasticity, lb./sq. in.		
Virgin paper	259,000	286,000
After impact	218,000	206,000

It seems unlikely that biaxial effects need to be considered (they would lead to somewhat higher estimates of velocity). The span-to-width ratios of the tensile strips were 31 and 8.5 for the machine- and cross-direction tests, respectively; these ratios are as great or greater than are ordinarily used in tensile testing for uniaxial properties of paper.

The question may also be asked whether the velocity depends on modulus and density of the fibers as opposed to the gross sheet properties, on the grounds that strain (or sound) propagates through the fibers rather than through the inter-fiber spaces. The question cannot be answered experimentally from this study. The opinion is held, however, that the sheet properties are the proper ones. The transmittance of stress or strain would be expected to depend on both the fiber characteristics <u>and</u> the geometry of the fibrous network; their combined effect should be reflected in the sheet modulus and apparent density. It is expected that the velocity of strain along the longitudinal axis of a single fiber would be quite different from the velocity through a random or semirandom network of the same type of fibers.

All things considered, it appears from this study that the velocity of strain propagation in extensible paper can be estimated with reasonable accuracy from $\underline{C} = \sqrt{\underline{E}/\underline{D}}$ — certainly better than an order of magnitude estimate.

With respect to the velocity of propagation of strain in sack papers other than the sample studied, it may be recalled that modulus of elasticity varies quite widely between regular and extensible papers and between principal directions of regular sack paper. There was little difference in the densities of regular and extensible papers in the second fabrication program, averaging 8.5 units for regular and 9.1 units for extensible papers (15). This is a difference of about 7%, which would give rise to a difference of 3-1/2% in the calculated velocity $\underline{C} = \sqrt{\underline{E}/\underline{D}}$.

Unfortunately, there was no occasion in the second fabrication program of Project 2033 to evaluate modulus of elasticity of the samples. However, in connection with the study of fatigue life, the tension stiffness of all samples was calculated. Tension stiffness is directly proportional to modulus of elasticity; however, these stiffnesses were defined in terms of a two-straight line approximation to the load-elongation curve (24) and are somewhat lower in all cases than the modulus. Nonetheless, as a first approximation, the ratio of two tension stiffnesses may be assumed to be about the same as the ratio of the moduli.

Table VIII lists the average tension stiffness for the samples of the second fabrication program, grouped according to regular and the three nominal stretch grades of extensible paper. The ratio of velocity of propagation of each class of paper relative to 9% extensible (based on $\underline{C} = \sqrt{\underline{E}/\underline{D}}$) is shown in the last column (ignoring differences in density). Thus, the velocity in the machine direction of regular paper may be anticipated to be 68% higher than that of 9% extensible. Taking $\underline{C}$ = 85,700 in./sec. for Run YY as being typical of 9% extensible, the expected velocity of an average regular paper is 144,000 in./sec. In the cross direction, the estimated velocity of regular paper is only 12% higher than that of 9% extensible, or 84,500 in./sec. for regular relative to 75,500 in./sec. for Run YY. The expected differences in velocity between classes of extensible papers is within 15%. In summary, it is expected that the only major departure from the propagation velocities measured in this study would be for regular sack paper in the machine direction, which is expected to be in the neighborhood of 140,000 in./sec. This is about two-thirds of the velocity in steel.

TABLE VIII

COMPARISON OF ESTIMATED VELOCITY OF STRAIN PROPAGATION OF REGULAR AND EXTENSIBLE PAPERS
(50-lb. kraft)

Type of Paper	Tension Stiffness[a]	Ratio, $\frac{\underline{C}}{\underline{C}_{9\%}}$[b]
	Machine Direction	
Regular	597	1.68
6% extensible	283	1.15
9% extensible	213	1.00
12% extensible	179	0.92
	Cross Direction	
Regular	305	1.12
6% extensible	262	1.03
9% extensible	249	1.00
12% extensible	279	1.06

[a]Proportional to modulus of elasticity.

[b]Based on $\underline{C} = \sqrt{\underline{E}/\underline{D}}$; density assumed constant.

IMPACT STRAIN IN MULTIWALL SACKS

The progression of strain across the face of a multiwall sack was measured during face drop of the sack from six- and ten-foot drop heights. On a given sack four Aquadag gages were arrayed in a line from the center of the face to an edge (see Fig. 12). For example, to measure cross-direction strain propagation, the gages were spaced from the center to the side of the sack (at mid-length). Machine-direction strain propagation was measured by four gages spaced from the center to one end (at midwidth). Photographic records were obtained of strain vs. time at each gage location. Four sacks were tested with each of the above-mentioned array of gages. A given sack was dropped repeatedly from either six or ten feet until it failed.

A representative record of strain vs. time is reproduced in Fig. 18 for a sack gaged in the cross direction and impacted from a ten-foot drop height. Gage 1 was at the edge of the sack (as in all other sacks tested) and Gage 4 was at the center. The curve for Gage 1 starts at the right-hand edge of the grid in the right-hand photograph and is the upper trace at the outset; the lower trace is Gage 3.. In the left-hand photograph Gage 2 is the upper curve at the beginning and Gage 4 is the lower. For Gages 1 and 2, increasing tension strain is in the downward direction, and for Gages 3 and 4 increasing tension is upward. The polarity of the signals was arranged in this way to employ the full screen height with a minimum of confusion between curves. Time increases from right to left in the photographs and each large grid division is one millisecond. As mentioned earlier, comparison of the ordinates of the four curves is not meaningful because it cannot be assumed that the four gages have equal sensitivities.

It may be noted that each curve undergoes a small rise and fall prior to the major pulse. [That the large pulse shown in these photographs is indeed

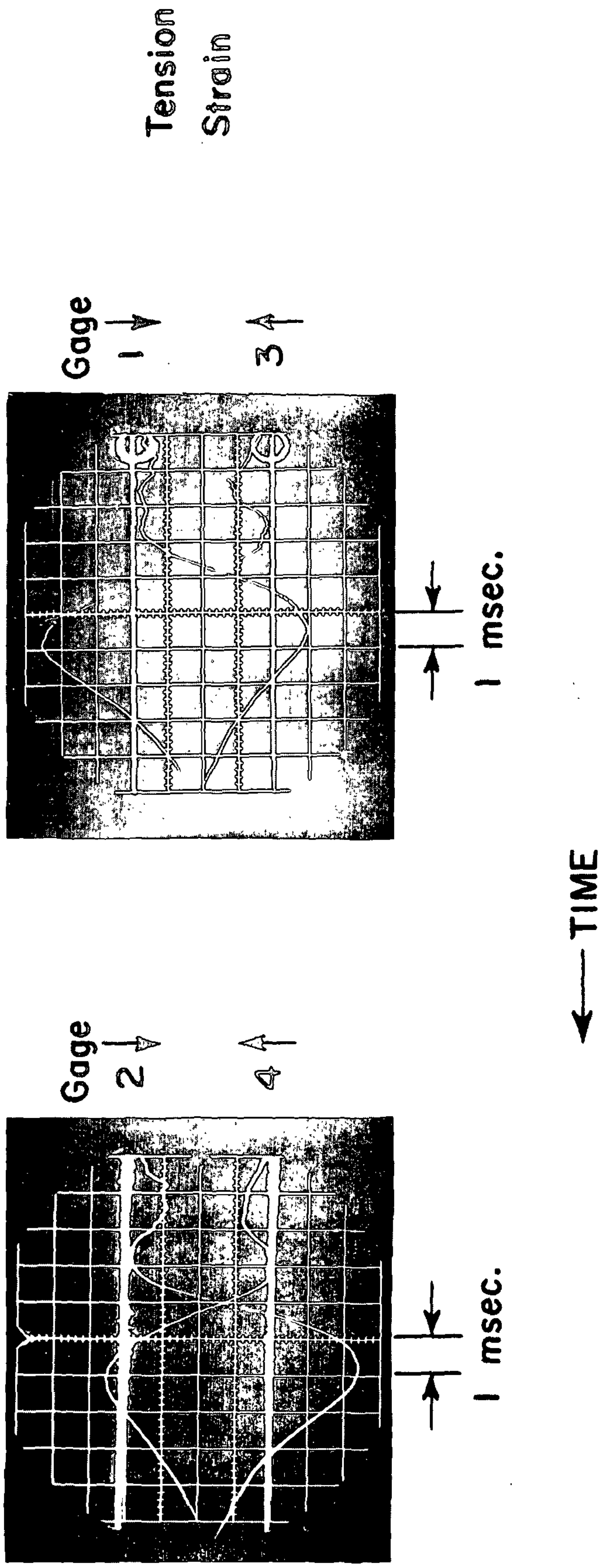

Figure 18. Oscilloscope Record of Impact Strain in Cross Direction of Face of Multiwall Sack

the major pulse was ascertained in other drops with a longer time span (20 milliseconds) on the oscilloscope.] This small initial pulse occurred throughout the experimentation. It is believed that it may be caused by a cushioning action of the air immediately beneath the sack just before it hits the base of the drop tester. This cushioning action may set up small stresses in the sack paper. It was felt that the cushioning action may have been exaggerated in the first series of tests by dropping the sack and wooden platen together. Therefore, the "machine direction" sacks were dropped without a platen. The small strain pulse persisted, however, with no appreciable diminution for having lessened the volume of trapped air beneath the falling body.

Turning attention to the major pulse, it may be seen in Fig. 18 that Gage 1 increased from zero strain (ignoring the cushioning strain) to maximum in 2.6 milliseconds. The rise times for the other gages were about the same. Successive gages (in the order 1 to 4) reached their respective maxima in consecutive order. That is, Gage 2 reached its peak value about a tenth of a millisecond later than Gage 1; Gage 3 about 0.2 millisecond after Gage 2; and Gage 4 about 0.2 millisecond after Gage 3. Thus, there is an indication of a progression of the strain from the edge of the sack (Gage 1) to the center of the sack (Gage 4). However, in this instance the time interval for the strain to progress from gage to gage is only a modest fraction of the duration of the strain (2.6 milliseconds, zero to maximum). It is evident in this case that there is only a minor difference in magnitude of strain between gages at any given instant, in so far as strain propagation effects are concerned.

It may be recalled that the observed velocity of strain propagation in cross-direction tensile strips from this sample of paper was 75,500 in./sec. Thus, the strain requires 1/75,500 = 0.013 millisecond to travel one inch (ignoring biaxial effects). Successive gages on the impacted sack were 2-1/8 inches apart (center-to-center distance). Thus, the anticipated time interval for strain to travel between successive gages is 2-1/8 by 0.013 = 0.03 millisecond. This estimate does not compare favorably with the observed intervals from Fig. 18 which are 0.10 or 0.20 millisecond. Consideration of biaxial effects would increase the disparity. In this particular case, therefore, it is questionable whether the strain mechanism in the sack is a simple propagation of strain from the edge to the center as in the tensile strip.

To systematically study the many strain records obtained for cross-direction impact strain along the lines discussed for Fig. 18, Tables IX, X, and XI have been prepared. Table IX lists the time of pick-up of strain at each gage and also the time of maximum strain. Zero time is taken arbitrarily at the right-hand edge of the oscilloscope grid. In reading the time of strain pick-up, the small pulse (believed to be due to air cushioning) has been ignored. A tangent line was "drawn" along the ascending slope of the major pulse, and the intercept of the tangent line and the base line of the particular gage was taken as the time of strain pick-up.

The duration of the strain pulse is defined for purposes of this study as the time interval from zero strain to maximum strain at a given gage location. Duration may be calculated readily from the data in Table IX; the results are listed in Table X. It may be seen that there was fairly good uniformity between gages and between sacks at a given drop height, although there appears to be a difference in pulse duration at the two drop heights (6 and 10 feet), as might be anticipated. The average duration in the 6-foot drop was 3.04 milliseconds, as shown at the foot of the table, while at 10 feet the average was 2.46 milliseconds. It is interesting to note that these average durations are nearly inversely proportional to the square root of the drop height, viz.,

TABLE IX

TIME OF PICK-UP OF STRAIN AND MAXIMUM STRAIN IN CROSS DIRECTION OF IMPACTED EXTENSIBLE SACKS

Drop Height, ft.	Trial	Time, milliseconds[a] At Pick-up of Strain				At Maximum Strain			
		Gage 1	Gage 2	Gage 3	Gage 4	Gage 1	Gage 2	Gage 3	Gage 4
					Sack No. 1				
6	1	3.5	3.9	3.9	3.9	6.2	6.6	6.4	6.7
	2	3.5	4.0	3.9	4.0	6.6	6.9	6.6	6.9
	3	3.3	3.5	3.7	3.5	6.3	6.8	6.5	6.9
10	1	2.5	2.9	3.0	3.2	5.0	5.1	5.2	--
	2	3.0	3.4	3.3	3.5	5.6	5.7	5.9	6.1
					Sack No. 2				
6	1	3.6	4.5	4.5	5.0	6.8	7.6	7.5	8.0
	2	2.4	3.0	2.8	3.3	--	--	--	--
	3	3.3	3.3	3.3	3.8	5.6	6.3	6.7	--
	4	4.0	3.8	3.8	4.0	--	7.6	--	7.6
	5	4.3	4.4	4.5	4.6	6.9	7.9	8.0	7.9
10	1	2.8	3.2	3.2	3.2	5.5	6.2	6.0	--
					Sack No. 3				
					(Failed on first drop from 10 feet)				
					Sack No. 4				
10	1	0.8	1.4	0.8	1.4	3.1	3.4	3.4	3.4

[a] Zero time arbitrarily taken as right-hand edge of oscilloscope grid.

TABLE X

DURATION OF STRAIN PULSE (ZERO TO MAXIMUM STRAIN) IN CROSS DIRECTION OF IMPACTED EXTENSIBLE SACKS

Drop Height, ft.	Trial		Time, milliseconds Gage 1	Gage 2	Gage 3	Gage 4	Av.
			Sack No. 1				
6	1		2.7	2.7	2.5	2.8	2.7
	2		3.1	2.9	2.7	2.9	2.9
	3		3.0	3.3	2.8	3.4	3.1
		Av.	2.9	3.0	2.7	3.0	2.9(12)[a]
10	1		2.5	2.2	2.2	--	2.3
	2		2.6	2.3	2.6	2.6	2.5
		Av.	2.6	2.2	2.4	2.6	2.4(7)
			Sack No. 2				
6	1		3.2	3.1	3.0	3.0	3.1
	2		--	--	--	--	--
	3		2.3	3.0	3.4	--	2.9
	4		--	3.8	--	3.6	3.7
	5		2.6	3.5	3.5	3.3	3.2
		Av.	2.7	3.4	3.3	3.3	3.2(13)
10	1		2.7	3.0	2.8	--	2.8(3)
			Sack No. 4				
10	1		2.3	2.0	2.6	2.0	2.2(4)

Average pulse duration for 6-foot drop = 3.04(25)
Average pulse duration for 10-foot drop = 2.46(14)

[a]Numeral in parentheses is number of observations determining the average.

TABLE XI

TIME INTERVAL FOR TRAVEL OF STRAIN BETWEEN GAGES
IN CROSS DIRECTION OF SACKS

Time, milliseconds

Sack	Drop Height, ft.	Trial No.	Gages 1 to 2	Gages 2 to 3	Gages 3 to 4	Gages 1 to 3	Gages 2 to 4
			At Pick-up of Strain				
1	6	1	0.4	0.0	0.0	0.4	0.0
		2	0.5	-0.1	0.1	0.4	0.0
		3	0.2	0.2	-0.2	0.4	0.0
	10	1	0.4	0.1	0.2	0.5	0.3
		2	0.4	-0.1	0.2	0.3	0.1
2	6	1	0.9	0.0	0.5	0.9	0.5
		2	0.6	-0.2	0.5	0.4	0.3
		3	0.0	0.0	0.5	0.0	0.5
		4	-0.2	0.0	0.2	-0.2	0.2
		5	0.1	0.1	0.1	0.2	0.2
	10	1	0.4	0.0	0.0	0.4	0.0
4	10	1	0.6	-0.6	0.6	0.0	0.0
		Av.	0.36	-0.05	0.22	0.31	0.18
			At Maximum Strain				
1	6	1	0.4	-0.2	0.3	0.2	0.1
		2	0.3	-0.3	0.3	0.0	0.0
		3	0.5	-0.3	0.4	0.2	0.1
	10	1	0.1	0.1	--	0.2	--
		2	0.1	0.2	0.2	0.3	0.4
2	6	1	0.8	-0.1	0.5	0.7	0.4
		2	--	--	--	--	--
		3	0.7	0.4	--	1.1	--
		4	--	--	--	--	0.0
		5	1.0	0.1	-0.1	1.1	0.0
	10	1	0.7	-0.2	--	0.5	--
4	10	1	0.3	0.0	0.0	0.3	0.0
		Av.	0.49	-0.03	0.23	0.46	0.12
		Composite av.	0.42	-0.04	0.22	0.38	0.16

$$\frac{3.04 \text{ msec. at six feet}}{2.46 \text{ msec. at ten feet}} = 1.24 \quad \text{and} \quad \sqrt{\frac{10 \text{ feet}}{6 \text{ feet}}} = 1.29.$$

The duration of strain measured in these tests agrees favorably with previous experience. For example, an earlier study with conventional strain gages revealed that the duration of strain in three-foot face drops of regular kraft sacks ranged from one to 3.6 milliseconds (these values are calculated from Table III of Reference (2). Moreover, photographic studies of sack rupture (15) revealed that the elapsed time between the instant of impact and evidence of paper failure in 10-foot drops ranged from one to 2.3 milliseconds. Rothman (16) reported durations of 2 to 3 milliseconds.

On the assumption that the strain pulse propagates from the edge to the center of the sack, Table XI lists the time interval for the pulse to travel from gage to gage in the cross direction of the test sacks. The time interval may be calculated with respect to any recognizable point on the strain *vs.* time curve which is common to all gages. The pick-up of strain and the maximum strain are the two definitely recognizable points, and the interval may be calculated readily from Table IX. The top half of Table XI refers to the pick-up of a strain and the lower half to maximum strain. The intervals based on maximum strain are more reliable because of some uncertainty in the tangent intercept value used for pick-up of strain. The two right-hand columns of Table IX show the time interval for strain travel from Gages 1 to 3 and from Gages 2 to 4. These intervals are measured within an oscilloscope and are more reliable than intervals between successive gages.

Table XI reveals that there was considerable variation in the time interval for both pick-up of strain and maximum strain. In isolated instances there is a suggestion of an orderly progression of strain from gage to gage (e.g., Sack 2 at 6 feet, Trial 5). By and large, however, there is little orderly pattern. The time interval between Gages 1 and 2 is generally the longest; this effect also appears in the interval for Gages 1 to 3 in the right-hand columns of the table. Possibly the location of Gage 1 on the curvature of the sack at the side edge is involved in some special way in this regard.

As noted above, tension strain would be expected to propagate in the cross direction of these sacks at a rate corresponding to approximately 0.013 millisecond per inch. Gages 1 and 3 or 2 and 4 were 4-1/4 inches apart and the expected time interval is, therefore, 4-1/4 x 0.013 = 0.056 millisecond. The observed intervals, however, are much longer: 0.380 millisecond for Gages 1 to 3 and 0.160 millisecond for Gages 2 to 4, on the average. Thus, the build-up of strain at successive locations across the sack face is not what would be expected from a consideration of strain propagation effects.

It may be suggested that the reason for longer time intervals is that the strains correspond to high stress levels and therefore a reduced modulus is effective, leading to lower velocities and hence longer time of travel. However, it may be recalled that the controlled experiments on wave propagation in tensile strips gave little indication that velocity depends on stress level, and, if anything, there is an increase in velocity at higher stress levels. Biaxial effects are in the wrong direction to explain the observed data because biaxial stress lead to somewhat greater velocities and hence shorter time intervals.

Irrespective of propagation effects, it may be noted that the timewise difference in strain at successive gages is modest relative to the duration of strain at each gage. On the average, the interval between successive gages was 0.14 millisecond (based on determinations from Gages 1-3 and 2-4). This is about 5% of the pulse duration (2.5 to 3.0 milliseconds) so that there cannot be very severe differences in strain between gage locations at a given instant due to time effects. An average set of strain vs. time curves for all gages may be constructed from these data to illustrate this point. Figure 19 shows the construction for the six-foot drop, assuming all gages reach the same maximum strain. (A somewhat greater interval between Gages 1 and 2 and a lesser interval between the remaining gages could be drawn, as mentioned above.) This diagram indicates that only modest differences in strain probably exist between gage locations due to time effects. The time difference in build-up of strain between center and edge is only about 0.42 millisecond, on the average, which is only 15% of the duration of the impact (zero to maximum strain).

A similar presentation of data for the four sacks gaged in the machine direction is given in Tables XII, XIII, and XIV. Table XII lists the time of pick-up of strain and maximum strain read from the oscilloscope photographs. The duration of strain (time, from zero to maximum strain), shown in Table XIII, was fairly uniform from gage to gage and sack to sack, except for Gage 1 (edge of sack) which was generally lower than the other locations. On the average, the duration of strain was 2.25 milliseconds for the 6-foot drops and 2.43 milliseconds for the 10-foot drops. These are of approximately the same magnitude as for the cross direction, but in the present instance the duration increased slightly with drop height, in contrast to the inverse square root effect cited earlier.

Table XIV lists the time interval between gages, calculated on the assumption that strain progresses from the edge to the center of the sack. It may be seen that many of the intervals are negative, indicating that the strain did not progress in the anticipated direction. The negative values are much in evidence in the "within" oscilloscope determinations and thus cannot be attributed to a difference in time base of the two oscilloscopes. Thus, there is little indication of an orderly propagation of strain across the sack face from end to center.

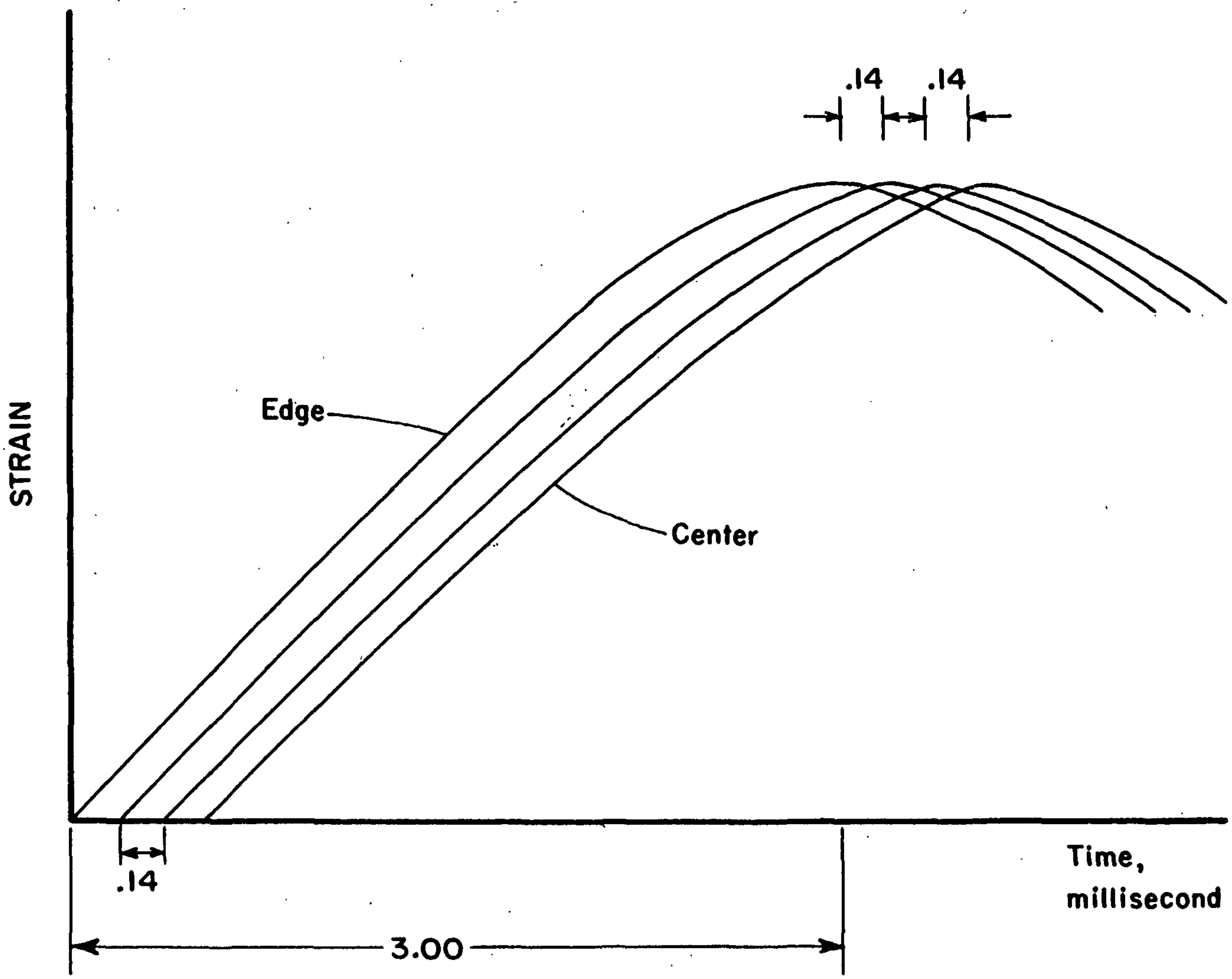

Figure 19. Schematic Representation of Strain Intensity at Four Points from Edge to Center of Sack Face (Cross Direction)

TABLE XII

TIME OF PICK-UP OF STRAIN AND MAXIMUM STRAIN IN MACHINE DIRECTION OF IMPACTED EXTENSIBLE SACKS

Drop Height, ft.	Trial	Time, milliseconds: At Pick-up of Strain, Gage 1	Gage 2	Gage 3	Gage 4	At Maximum Strain, Gage 1	Gage 2	Gage 3	Gage 4
					Sack No. 5				
6	1	1.50	1.90	2.30	2.40	3.70	4.40	4.80	5.00
	2	2.45	2.40	2.35	2.25	4.20	4.80	--	--
	3	2.25	2.10	2.20	2.05	3.45	4.40	4.50	4.50
	4	1.05	0.95	1.05	0.80	2.55	3.00	3.80	3.60
	5	1.95	1.50	1.75	1.40	3.10	3.35	4.10	3.60
10	1	1.95	1.85	1.90	2.05	--	--	--	--
					Sack No. 6				
6	1	1.80	1.60	1.90	--	4.30	4.25	4.55	--
	2	1.90	1.80	1.70	1.60	4.00	4.30	4.40	4.50
	3	2.80	2.55	2.50	2.20	4.70	4.75	4.65	4.20
10	1	1.00	1.00	1.00	1.10	2.95	--	3.60	3.80
	2	1.65	1.60	1.50	0.80	--	--	--	--
	3	0.90	1.00	1.75	2.40	2.80	4.00	4.25	4.95
	4	1.60	1.60	1.60	1.50	--	--	--	--
					Sack No. 7				
10	1	0.60	1.00	1.80	2.00	3.40	--	--	3.90
					Sack No. 8				
6	1	4.20	4.40	4.20	4.20	--	6.60	6.50	6.60

TABLE XIII

DURATION OF STRAIN PULSE (ZERO TO MAXIMUM STRAIN) IN MACHINE DIRECTION OF IMPACTED EXTENSIBLE SACKS

Drop Height, ft.	Trial		Time, milliseconds Gage 1	Gage 2	Gage 3	Gage 4	Av.
			Sack No. 5				
6	1		2.20	2.50	2.50	2.60	2.45
	2		1.75	2.40	--	--	2.08
	3		1.20	2.30	2.30	2.45	2.06
	4		1.45	2.05	2.75	2.80	2.26
	5		1.15	1.85	2.35	2.20	1.89
		Av.	1.55	2.22	2.48	2.51	2.16(18)
10	1		--	--	--	--	
			Sack No. 6				
6	1		2.50	2.65	2.65	--	2.60
	2		2.10	2.50	2.70	2.90	2.55
	3		1.90	2.20	2.15	2.00	2.06
		Av.	2.17	2.45	2.50	2.45	2.39(11)
10	1		1.95	--	2.60	2.70	2.42
	2		--	--	--	--	--
	3		1.90	3.00	2.50	2.55	2.49
	4		--	--	--	--	--
		Av.	1.92	3.00	2.55	2.62	2.46(7)
			Sack No. 7				
10	1		2.80	--	--	1.90	2.35(2)
			Sack No. 8				
6	1		--	2.20	2.30	2.40	2.30(3)

Average pulse duration for 6-foot drop = 2.25(32)
Average pulse duration for 10-foot drop = 2.43(9)

TABLE XIV

TIME INTERVAL FOR TRAVEL OF STRAIN BETWEEN GAGES IN MACHINE DIRECTION OF SACKS

Sack	Drop Height, ft.	Trial	Time, milliseconds, Gages 1 to 2	Gages 2 to 3	Gages 3 to 4	Gages 1 to 3	Gages 2 to 4
			At Pick-up of Strain				
5	6	1	0.40	0.40	0.10	0.80	0.50
		2	-0.05	-0.05	-0.10	-0.10	-0.15
		3	-0.15	-0.10	-0.15	-0.05	-0.05
		4	-0.10	0.10	-0.25	0.00	-0.15
		5	-0.45	0.25	-0.35	-0.20	-0.10
	10	1	-0.10	0.05	0.15	-0.05	0.20
6	6	1	-0.20	0.30	--	0.10	--
		2	-0.10	-0.10	-0.10	-0.20	-0.20
		3	-0.25	-0.05	-0.30	-0.30	-0.35
	10	1	0.00	0.00	0.10	0.00	0.10
		2	-0.05	-0.10	-0.70	-0.15	-0.80
		3	0.10	0.75	0.65	0.85	1.40
		4	0.00	0.00	-0.10	0.00	-0.10
7	10	1	0.40	0.80	0.20	1.20	1.00
8	6	1	0.20	-0.20	0.00	0.00	-0.20
		Av.	-0.02	0.15	-0.06	0.13	0.08
			At Maximum Strain				
5	6	1	0.70	0.40	0.20	1.10	0.60
		2	0.60	--	--	--	--
		3	0.95	0.10	0.00	1.05	0.10
		4	0.45	0.80	-0.20	1.25	0.60
		5	0.25	0.75	-0.50	1.00	0.25
	10	1	--	--	--	--	--
6	6	1	-0.05	0.30	--	0.25	--
		2	0.30	0.10	0.10	0.40	0.20
		3	0.05	-0.10	-0.45	-0.05	-0.55
	10	1	--	--	0.20	0.65	--
		2	--	--	--	--	--
		3	1.20	0.25	0.70	1.45	0.95
		4	--	--	--	--	--
7	10	1	--	--	--	--	--
8	6	1	--	-0.10	0.10	--	0.00
		Av.	0.49	0.28	0.02	0.79	0.27
		Composite av.	0.17	0.20	-0.03	0.38	0.15

The question may be raised whether the high frequency of negative time intervals in Table XIV for machine-direction strain may be due to the effects of cross-direction strain on the machine-direction gages. The gage at the center of the face, for example, is closer to the side edges of the sack than it is to the end of the sack. Early work in the development of conductive coatings in Project 2033 indicated that the transverse sensitivity of these gages is only about 5% of their axial sensitivity. Based on those results, there is no reason to suspect that the machine-direction gages are greatly influenced by cross-direction strain, although the pick-up of strain in a machine-direction gage could reflect the presence of cross-direction strain. Most of the negative time intervals in Table XIV are with respect to pick-up of strain. Thus, the time intervals corresponding to maximum strain are probably more reliable.

A more likely reason for the negative time intervals is believed to be the occurrence of nonflat drops. One may visualize that if the nongaged end of the sack hits the impact base before the gaged end, a strain may travel from gage to gage in the reverse order from what is anticipated. Or, less extreme, the apparent interval between say, Gages 3 and 4 may be less than between Gages 1 and 2 because of the arrival of an early strain pulse from the far end of the sack.

If only the time intervals for maximum strain are considered in Table XIV, the average time interval between successive gages in the machine direction is 0.27 millisecond. Thus, the time difference between the edge and center gages is 0.81 millisecond, which is about 35% of the pulse duration (zero to maximum strain) for the six-foot drop. This is considered to be a modest difference in so far as wave propagation effects are concerned. This time difference is about twice that of the cross-direction strains.

Considering both the cross-direction and machine-direction measurements of strain in impacted sacks, it appears that the concept of tensile strain propagation from the edge to the center (due to an impact thrust at the edge) is an oversimplification of the behavior of the sack. While there were differences in the time at which locations along a meridian picked up strain and reached maximum strain, these events did not usually occur in the orderly fashion of wave propagation and, on the average, the time interval was longer than would be anticipated from wave propagation theory. However, the timewise differences between gage locations with respect to attaining maximum strain were modest relative to the duration of the strain pulse, indicating that time effects in themselves would not be expected to lead to great disparities in strain between locations at a given instant during the impact.

The question may be asked: what properties of sack paper are of importance from a consideration of strain propagation effects? Theoretically, a high modulus of elasticity, E, and low density, D, are beneficial. These result in a high velocity of propagation which minimizes differences in strain at various locations in the sack due to time effects. Frequently, modulus and density are proportional to one another and it may be difficult in practice, therefore, to obtain a marked change in wave velocity. Also, it should be noted that an increase in modulus may be accompanied by a decrease in tensile energy absorption and stretch, which would be detrimental to the strength of the sack.

From a practical standpoint, the data of this experiment indicate that wave propagation effects are of little consequence to sack behavior. The velocity of propagation, C, would need to be an order of magnitude lower to be of any great concern. This would require, for example, two orders of magnitude reduction in modulus of elasticity (or two orders increase in density, or a combination giving two orders decrease in E/D). Such variations in these properties are probably far too great to be of any practical significance.

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THE INSTITUTE OF PAPER CHEMISTRY

C. N. Smith, Research Assistant

J. W. Gander, Research Aide

R. C. McKee, Chairman
Container Section