Control for Two Cooperative Disparate Manipulators

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ABSTRACT

The concept of a small arm mounted on the end of a large arm has been introduced to provide precise motion as well as a large workspace. However, the performance of such a serial configuration lacks payload capacity which is a crucial factor for handling a massive object. Also, this configuration induces more flexibility on the structure. To overcome these problems, the topology of bracing the tip of the small arm (not the large arm) and having an end effector in the middle of the chain is proposed in this paper. Also, simulation results are presented to show the possibility that the proposed control scheme provides the disparate manipulators with better accuracy than the large arm and more payload capacity than the small arm. In the near future, experimental studies on the existing arrangement (RALF and SAM) are scheduled to verify the feasibility of the proposed concepts.

1 INTRODUCTION

1.1 Motive

The proposed research seeks higher performance manipulators for large workspaces, particularly for those tasks that require precise positioning and mating of relatively massive payloads. Demand for these manipulators can be found in common space construction scenarios and more urgently in retrieval of nuclear waste from under ground storage tanks. The fundamental characteristics of such manipulators include their light weight and their flexibility. Also, there is the need to maintain a firm position constraint between the robot's base and the payload or workpiece. This requires a more complicated manipulator topology than a single serial link arm which cannot constrain two mating workpieces.

As a solution, the concept of a small arm mounted on the end of a large arm has been introduced to provide precise motion as well as a large workspace [1,2]. Such a configuration forms a redundant arm. Thus, the number of extra degrees of freedom can be used to brace on a stationary frame. Bracing is used to ensure accurate positioning of the end effector and to support the force at the end effector due to the change in structural stiffness. It has been proven that bracing reduces the positioning uncertainty and increases the stiffness of the manipulator by the closed kinematic constraint [3,4].

From real world experience with crane-human coordination when a heavy payload is unloaded (see Figure 1), we know that we can obtain precise positioning and high payload capacity. This crane-human configuration may be analogous to the topology of bracing at the tip of the small arm and having an end effector in the middle of the chain. In other words, the large arm provides the load capacity like a crane, and the small arm does the precise positioning like a human. This topology gives motivation to develop a coordination strategy for disparate large and small manipulators.

Application of the disparate manipulators approach in the nuclear waste industry include the retrieval of waste material from buried shell waste storage tanks. The operations that might use the disparate approach include cutting off and removing obstacles in the tank, dislodging the solid waste material using special tools, and placement, and maintenance of support equipment.

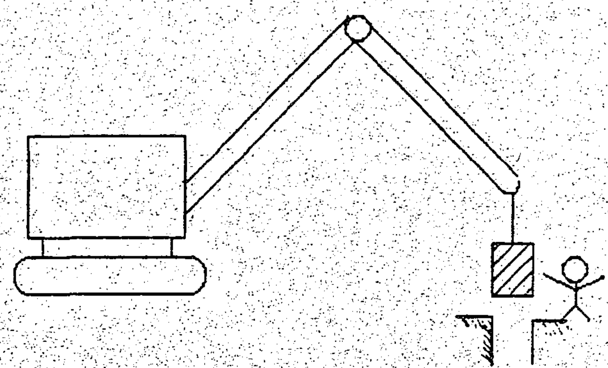


Figure 1 Crane -Human Coordination

1.2 Research Description

Control of this proposed topology is different from common dual arm control in some ways [5,6,7,8]. For example, the large arm is powerful and flexible, and the small arm is capable of precise positioning and is rigid. To take full advantage of such disparate features, special control schemes can be applied. So far, no prior work or study for those control schemes has been done. As a first approach to develop a control strategy for coordinating disparate manipulators, an advanced master/slave control scheme is proposed. In this scheme, the small rigid arm performs computed torque position control, and the large flexible arm uses robust force control to compensate for the internal forces due to relative kinematic motion errors between the two arms.

In the first phase of this research, the advantages of the crane-human configuration are analytically proven using the idea of a force ellipsoid from the serial chain redundant manipulator [9]. In the second phase, the dynamics of the disparate manipulators system is derived under the assumption that the large arm is flexible and the small arm is rigid. A large portion of the work has been devoted to design a control law for coordinating two arms which involves "non-collocated" force control for a flexible arm [10]. Furthermore, under the assumption that all dynamic parameters are not available, a robust force controller is extensively studied and applied to a flexible arm [11,12]. Also, simulation results are presented to prove that the proposed control scheme provides the disparate manipulators with better accuracy than the large arm and more payload capacity than the small arm. In the near future, experimental studies on the existing arrangement (RALF and SAM) are scheduled to verify the feasibility of the proposed concepts.

2. CHARACTERISTICS OF DISPARATE MANIPULATORS

2.1 Applications

Application scenarios of the proposed disparate manipulators are as follows. First, the large arm carries the small arm and a payload close to a destination position. While the large arm is traveling, the small arm may be used to generate inertial forces on the large arm to cancel the large arm's vibrations [13]. Second, the tip of the small arm is braced on a stationary frame using a tracking controller [14]. Thus, a closed kinematic chain is formed. Next, the joint where the small arm is mounted on the large arm becomes passive (one option). The proposed control scheme is applied to coordinate the two arms. Its objective is to provide the payload capacity of the large arm and the precise positioning of the small arm.

2.2 Kinematics

The proposed manipulator system in contact with their environment is characterized by closed chains in the structure and redundancy in actuation. In other words, the number of actuators typically exceeds the mobility. However, the general case of the Jacobian matrix can be applied to two disparate manipulators with kinematic constraints. We first assume that the joints are frictionless, that the link inertial forces are negligible, and that all the links are rigid to simplify kinematic analysis in this section.

Let us assume that neither arm is in a singular position and that $\mathbf{J}_i$ is a square matrix. Then we can obtain the joint rates required to perform a desired control task in the task coordinates.

$$\dot{\mathbf{q}}_i = \mathbf{J}_i(\mathbf{q}_i)^{-1}\,\dot{\mathbf{X}} \tag{2}$$

where $\mathbf{X}$ contains the task coordinates and $\mathbf{q}$ contains the joint coordinates, and $\mathbf{J}(\mathbf{q})$ is the Jacobian matrix. The following subscript, i, denotes either the large (L) or small (S) arms. If Eqn. (2) is combined with the kinematic constraint that $\dot{\mathbf{X}}$ is same for both arms, we can write the following form of a new Jacobian, $\mathbf{H}$, for the closed chain arm.

$$\begin{bmatrix} \dot{\mathbf{q}}_L \\ \dot{\mathbf{q}}_S \end{bmatrix} = \begin{bmatrix} (\mathbf{J}_L)^{-1} \\ (\mathbf{J}_S)^{-1} \end{bmatrix} \dot{\mathbf{X}} \tag{3}$$

or

$$\dot{\mathbf{q}} = \mathbf{H}\,\dot{\mathbf{X}} \tag{4}$$

where $\mathbf{q} = [(\mathbf{q}_L)^T, (\mathbf{q}_S)^T]^T$ and $\mathbf{H} = [\,[(\mathbf{J}_L)^{-1}]^T,\ [(\mathbf{J}_S)^{-1}]^T\,]^T$

Similarly, using the principle of virtual work in the static case, we can derive the following relationship.

$$\mathbf{F} = \mathbf{H}^T\,\mathbf{T} \tag{5}$$

$$= \left[\left[(\mathbf{J}_L)^{-1}\right]^T, \left[(\mathbf{J}_S)^{-1}\right]^T \right] \begin{bmatrix} \mathbf{T}_L \\ \mathbf{T}_S \end{bmatrix} \tag{6}$$

$$= \mathbf{F}_L + \mathbf{F}_S \tag{7}$$

where $\mathbf{T}_i$ is the vector of actuator torques, and $\mathbf{F}_i$ is the vector of forces and moments exerted by arm i on the end effector. Since the force in the task coordinates is the sum of the two arms' force, Eqn. (7) is true also.

2.3 Load Capacity

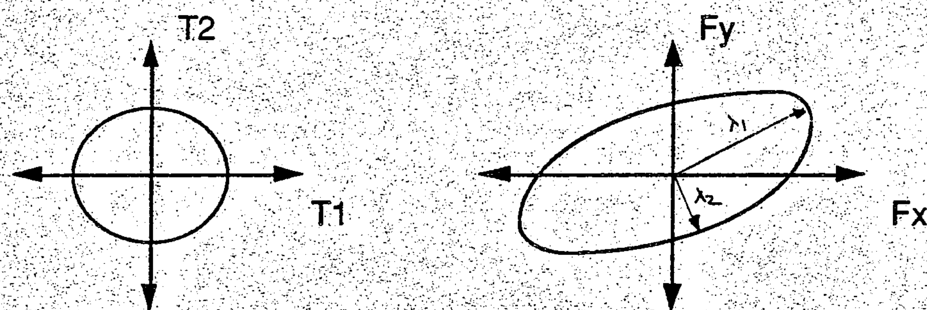
Intuitively we know that closed chain manipulators have greater load capacity than open chain manipulators. However, we need to prove analytically how much more load closed chains can handle than open chains. In this section, using the idea of a force ellipsoid from the serial chain redundant manipulator [15], an index for measuring the force transmission at a given posture for the closed chain manipulator is developed. Also, these indices for both closed and open chain arms are compared.

From Eqn.(5), we see that the Jacobian is simply a linear transformation that maps the joint torque $\mathbf{T}$ in R^m into a task force $\mathbf{F}$ in R^n. The unit sphere in R^m defined by

$$\mathbf{T}^T\mathbf{T} \leq 1 \tag{8}$$

is mapped into an ellipsoid in R^n defined by

$$\mathbf{F}^T (\mathbf{H}^T\mathbf{H})^{-1} \mathbf{F} \leq 1$$
$$\text{or} \quad \mathbf{F}^T \{(\mathbf{J}_L\mathbf{J}_L^T)^{-1} + (\mathbf{J}_S\mathbf{J}_S^T)^{-1}\}^{-1} \mathbf{F} \leq 1 \tag{9}$$



(a) unit sphere in joint space (b) force ellipsoid in task space

Figure 2 Force Ellipsoid

This ellipsoid has principal axes $\lambda_1\mathbf{u}_1, \cdots, \lambda_n\mathbf{u}_n$ where $\mathbf{u}_i \in R^n$ and λ_i is an eigenvalue of $(\mathbf{H}^T\mathbf{H})$. Chiu [15] call this a "force ellipsoid", and Yoshikawa[9] used the volume of a velocity ellipsoid as an index of manipulability for open chain redundant arms. Similarly, we can use the volume of the force ellipsoid as a measurement of a manipulator's payload capacity at a given posture. This volume gives the size of output for the given unit norm input. We use this transmission ratio (= output/input) as the payload capacity index. Thus, the payload capacity index for the closed chain is;

$$\begin{aligned} W_{L+S} &= \lambda_1 \cdots \lambda_n \\ &= \det(\mathbf{H}^T\mathbf{H}) \\ &= \det\{(\mathbf{J}_L\mathbf{J}_L^T)^{-1} + (\mathbf{J}_S\mathbf{J}_S^T)^{-1}\} \end{aligned}$$

This result can easily be obtained from a property of determinants [16]. Also, we know that $(\mathbf{J}_L\mathbf{J}_L^T)^{-1}$ and $(\mathbf{J}_S\mathbf{J}_S^T)^{-1}$ are positive definite. If we carry out the computation for W_{L+S},

$$W_{L+S} = \det\{(\mathbf{J}_L\mathbf{J}_L^T)^{-1} + (\mathbf{J}_S\mathbf{J}_S^T)^{-1}\}$$

$$= \det(J_L J_L^T)^{-1} \det \{ I + (J_L J_L^T)(J_S J_S^T)^{-1} \} \qquad (10)$$

Again, the matrix, $(J_L J_L^T)(J_S J_S^T)^{-1}$, is also positive definite and is diagonalizable with a unit similarity matrix. Furthermore, its diagonal terms are always positive. Thus, $\det \{ I + (J_L J_L^T)^{-1} + (J_S J_S^T)^{-1} \}$ is larger than 1. Now, we can say

$$\begin{aligned} W_{L+S} &= \det(J_L J_L^T)^{-1} \det \{ I + (J_L J_L^T)(J_S J_S^T)^{-1} \} \\ &> \det(J_L J_L^T)^{-1} = W_L \end{aligned}$$

where W_L is the payload capacity of the large arm itself. Physically, this means that the closed chain arm has a larger payload capacity than either the the large or small arm by the ratio of $\det\{ I + (J_L J_L^T)(J_S J_S^T)^{-1} \}$.

3 CONTROL OF DISPARATE MANIPULATORS

3.1 Dynamics

In the first phase of this work, the dynamics of the proposed disparate manipulators system is derived. Its topology can be considered as the large arm holding a payload and small arm helping the wrist of the large arm as shown in Figure 3. The motion of the manipulators is dynamically coupled because the generalized forces are interrelated through the passive joint **P**. Since joint **P** will be a ball joint, only forces can be transmitted through it. The forces on the large arm through joint **P** are denoted by a 3x1 vector $\mathbf{F}_L$, and the forces on the small arm through joint **P** are denoted by a 3x1 vector $\mathbf{F}_S$. We can decompose the system into three parts (large arm, small arm, and force sensor) and derive the equations of motion for each body. The payload dynamics is included in the large arm.

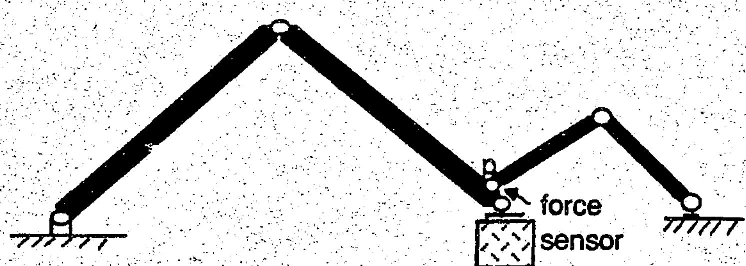


Figure 3 Two Disparate Manipulators Holding a Payload

First, modeling a large arm, which is assumed to be flexible, is complicated. The flexible deflection of the arm is approximated to be a finite series of separable functions which are the product of admissible shape functions $\phi_{ij}(x)$ and time dependent generalized coordinates $\eta_{ij}(t)$.

$$u_i(x,t) \approx \sum_{j=1}^{m} \phi_{ij}(x) \; \eta_{ij}(t) = \text{deflection of link } i \qquad (11)$$

The equation of the flexible arm motion can be derived from several techniques, but the Lagrange's formulation [17] is known for its simplicity and systematic approach. Using Jacobians [18] to obtain the velocity of a point, the kinematic and potential energies have been obtained by integrating the velocity and position of points over the total system. These energies were used in Lagrange's equations. For the large arm, $\mathbf{q}_L = [\,(\mathbf{q}_r)^T, (\mathbf{q}_f)^T]^T$

$$\begin{bmatrix} \mathbf{M}_{rr} & \mathbf{M}_{rf} \\ \mathbf{M}_{fr} & \mathbf{M}_{ff} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_r \\ \ddot{\mathbf{q}}_f \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{K} \end{bmatrix} \begin{bmatrix} \mathbf{q}_r \\ \mathbf{q}_f \end{bmatrix} + \text{Nonlinear Term} = \begin{bmatrix} \mathbf{I} \\ \\ \dfrac{\partial \phi_i}{\partial x_i} \end{bmatrix} \mathbf{T}_L - \begin{bmatrix} \mathbf{J}_r^T \\ \mathbf{J}_f^T \end{bmatrix} \mathbf{F}_L$$

where $\mathbf{X} = \mathbf{X}\,(\mathbf{q}_r, \mathbf{q}_f)$, $\mathbf{J}_r = \dfrac{\partial \mathbf{X}}{\partial \mathbf{q}_r}$, and $\mathbf{J}_f = \dfrac{\partial \mathbf{X}}{\partial \mathbf{q}_f}$. Again $\mathbf{q}_r$ contains the generalized rigid joint coordinates, and $\mathbf{q}_f$ contains the generalized flexible mode coordinates. The nonlinear term is negligible under the fine motion control.

For the small arm, which is assumed to be rigid, $\mathbf{q}_S$ contains the generalized joint coordinates. Its dynamics can be found in [19] as,

$$\mathbf{M}_S(\mathbf{q}_S)\ddot{\mathbf{q}}_S + \mathbf{N}_S(\mathbf{q}_S, \dot{\mathbf{q}}_S) = \mathbf{T}_S - \mathbf{J}_S^T \mathbf{F}_S \qquad (12)$$

where $\mathbf{M}_S$ is the inertia matrix of the manipulator, $\mathbf{N}_S$ is the nonlinear term including centrifugal force, Coriolis force and gravity term, and $\mathbf{T}_S$ is the joint torque.

If we model the force sensor as a linear spring $\mathbf{K}_{fs}$, we may obtain

$$\mathbf{F}_L = \mathbf{K}_{fs}\,(\mathbf{X}_L - \mathbf{X}_s)$$

where $\mathbf{X}_L$ is the position of the wrist $\mathbf{P}$ in the Cartesian coordinates, and $\mathbf{X}_s$ is the position of the tip of the small arm in the Cartesian coordinates. This force sensor measures internal forces in the closed chain.

3.2 Control Strategy

Various control approaches are available for dual cooperating arms. However, there are unique conditions for two disparate manipulators compared to common dual arm control. Those unusual characteristics of the proposed arms are :

* One arm is large, and the other is small.
* One is flexible, and the other is rigid.
* One is coarse (actuator and sensors), and the other is precise.
* One is strong (high forces and torques), and the other is less strong.

To take full advantage of the complimentary features of the large and small manipulators, the control strategy for each arm should be asymmetrical. In other words, each arm's role should be different rather than just sharing a payload. Thus, one proposed control strategy is an advanced master/slave scheme whereby the force-controlled slave arm (large arm) follows the position-controlled master arm (small arm). When using kinematic control of a closed chain that is redundant in actuation, geometric imperfections in the robot structures along with other sources of the kinematic error and feedback position control error results in undesired internal forces between two arms. Therefore, force control by feedback from the slave arm's tip mounted force sensors offers an approach to eliminating these unwanted forces. The rigid arm, which is capable of precise positioning, is position servoed and follows a preplanned trajectory. The flexible arm, which is capable of handling a massive payload, is force servoed to balance the load and accommodate any internal forces that may arise.

Since the actuators on the large (flexible) arm and the force sensor are physically located at different points, the performance of the force controller is severely limited as the feedback gain increases. Some people call this situation "non-collocated control" [23]. With only force feedback, the closed loop system becomes unstable with high gains. In addition, the flexibility in the system increases the number of poles in the system, and eventually these poles become unstable with high feedback gain, although those poles remain stable in collocated control. This is due to the output feedback control rather than perfect full state feedback. To solve this problem, additional sensors or a dynamic observer should be added. Thus, all the states become available, and the undesired poles can be moved to where the designer wants by full state feedback. If we install strain gages to monitor flexible motion, we can manipulate the position of closed loop poles using feedback gains.

The dynamics of the two arms are coupled, which make the system more complicated. However if we apply the master/slave scheme, the coupling force between two arms, which is an internal force, is force-controlled to be as small as possible. Also, since we can measure the uncontrolled internal force from the force sensor, two arms can be dynamically decoupled by feeding forward the coupling term using the small arm controller. This decoupling will be imperfect in any physical apparatus and must be evaluated with experiments and/or simulations. Therefore, robust control scheme will be necessary for sucessful decoupling of the two arms.

By reasoning about the master/slave scheme for the two disparate manipulators, two premises have been developed: (1) Make all the states available for flexible arm force control. Thus, the poles of the system can be moved to any desired location by all state feedback; (2) Decouple the system by feeding forward. Then, design a stable position control law without the consideration of the force control, and design a stable force control law by treating the large arm as an independent system.

3.2.1 Master Controller Design

The equations of motion of the system are derived in section 3.1. The equation for the small arm is shown in Eqn. (12). A position controller for a rigid arm is derived in this section. Since the orientation of the payload is controlled independently, only the position of P is considered. It is coupled through $\mathbf{F_s}$. However, $\mathbf{F_s}$ can be measured by the tip-mounted force sensor. Thus, we can decouple the system by feeding forward the undesired force $-\mathbf{J_s}^T \mathbf{F_s}$. Then the small arm becomes an independent nonlinear system from the large arm. To design a position controller for this system, many control laws can be applied depending on what kind of assumptions are made. One simple and practical controller uses, "computed torque method" [19,20]. With perfect knowledge of all the dynamic parameters , we can feed back computed nonlinear terms so that the dynamics of the small arm becomes linear, and then we can design a linear controller. The control law for the small arm is then:

$$\begin{aligned} T_s &= \text{decoupling force} + \text{nonlinear term feedback} + \text{linear controller} \\ &= -J_s^T F_s + N_s(q_s, \dot{q}_s) + \text{linear controller} \end{aligned}$$

Therefore, the error dynamics becomes $\ddot{e} + K_d \dot{e} + K_p e = 0$ where $e = q_d - q_s$. Thus, we can choose feedback gains K_d and K_p depending on the desired performance. Finally, the controller for the small arm is

$$T_s = -J_s^T F_s + N_s(q_s, \dot{q}_s) + M_s \{ \ddot{q}_d + K_d(\dot{q}_d - \dot{q}_s) + K_p (q_d - q_s) \}$$

3.2.2 Slave Controller Design

The purpose of the slave controller is to eliminate undesired internal forces and to follow the master arm by force control. In this section, the robust force controller for a flexible arm is used to satisfy both of those goals under the uncertainty of the system.See [22,23] for details. The proposed robust force control law for the flexible arm is

$$T_L = \hat{b}_1^{-1} \; (-K_v \dot{F}_L \; -K_p F_L + p) + \hat{T}_d + \hat{T}_c$$

where the controller gain K_v and K_p can be determined depending on the designer's choice. Also, they are diagonal matrices. If the small arm's acceleration, $\ddot{X}_s$, is estimated ahead, feeding forward may eliminate the unwanted disturbance input $T_d = b_1^{-1} \; (b_1 \; M_{X11} + b_2 \; M_{X21}) \ddot{X}_S$ directly. Also, by measuring F_L and q_f, computed input $T_c = b_1^{-1} \; \{(b_1 \; J_r^T + b_2 \; J_f^T) \; F_L + b_2 \; K \; q_f\}$ can decouple the system from flexible dynamics and make each joint independent. The hat," $\wedge$", means the estimated values. For example, $\hat{T}_c$ is the estimated value of T_c. The following robust control input p makes the internal force dynamic *globally practically stable.* This implies that the system solution is *uniformly bounded* and *uniformly ultimately bounded*. See [11,12] for details

$$p = - \mu \; \rho \; / \; \|\mu\| \; , \quad \text{if } \|\mu\| > \varepsilon$$
$$p = - \mu \; \rho \; / \; \varepsilon \; , \quad \text{if } \|\mu\| \le \varepsilon$$

where $\mu = B^T \; P \; x \; \rho$ and ε is a small positive constant (determined by the designer). Also, P is the solution of Lyapunov equation.

4 SIMULATION

As an illustrative example for the proposed control strategy, the coordination of RALF (Robotic Arm, Large and Flexible) and SAM (Small Articulated Manipulator) is simulated on the computer. We assume that the two arms form a kinematic closed chain as shown in Figure 3. Their dynamic parameters can be found in Refs [18,21,23]. The lowest natural frequency of RALF is 5.9 Hz, and the stiffness of the force sensor is assumed to be 10,000 N/m. SAM pulls RALF with the desired trajectory in the X direction only. i.e.

$$X_d(t) = \begin{Bmatrix} 0.1 \;\; (1+ \;\; \cos(2\pi t)), & 0 < t < 0.5 \\ 0, & t > 0.5 \end{Bmatrix}$$

RALF follows SAM by force control. The computed torque method is implemented on SAM with the knowledge of all the dynamic parameters. Several force controllers are applied to RALF for comparison. Figure 4 shows all the internal force responses measured at the tip force sensor in the one plot. The smaller internal force response indicates how well RALF tracks SAM. The larger internal force may cause failure in the mechanical bracing device.

The simulation results seek for small tracking error, zero internal force and good robustness with modeling error. First, a rigid arm force controller is examined on

RALF. This controller feeds back only F_L without information about flexible modes. This is a non-collocated control, and its performance is poor. Second, the proposed force controller with the perfect knowledge of all the dynamics shows very good results as expected. However, when there is 50 % uncertainty on the feed forward term, the response becomes worse. To overcome this uncertainty, the robust force controller is added. Consequently, some progress has been reached. Depending on the design parameter Q and ε, the performance of the robust controller could be improved further.

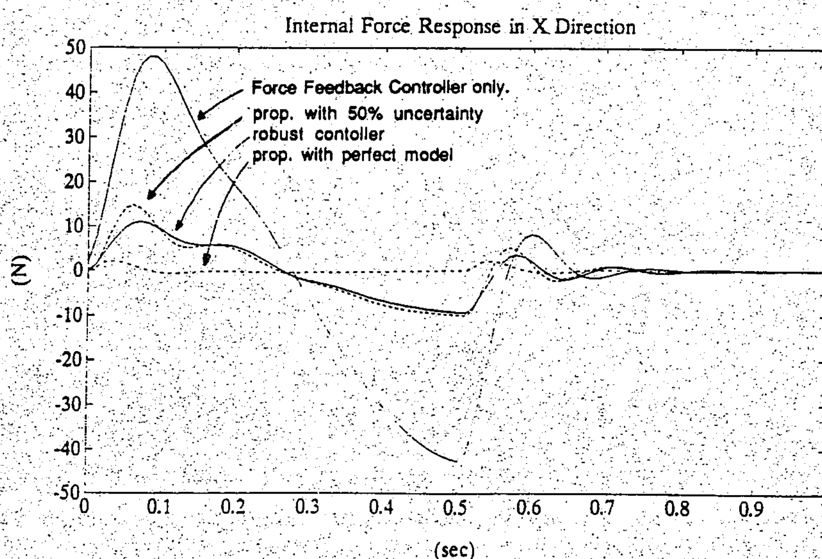


Figure 4 Internal Force Responses in the X Direction

5 CONCLUSION

In this research, the advantages of the crane-human configuration are first analytically proven using the idea of a force ellipsoid from the serial chain redundant manipulator. Second, the dynamics of the disparate manipulators is derived under the assumption that the large arm is flexible and the small arm is rigid. A large portion of the work has been devoted to design a control law for coordinating two arms which involves "non-collocated" force control for a flexible arm. Furthermore, under the assumptions that all dynamic parameters are not available, a robust force controller is extensively studied and applied to a flexible arm. Finally, the simulation results show the possibility that the disparate manipulators can have the accuracy of the small arm, and at the same time, it can have the payload capacity and large workspace of the large arm

6 EXPERIMENTS

A large experimental arm designated RALF (Robotic Arm, Large and Flexible) has been constructed and placed under computer control. Also a small experimental arm, SAM (Small Articulated Manipulator) has been constructed and is being tested under three parallel processing control. It will be mounted on the end of RALF. A similar task given in the illustrative example will be conducted. This experimental studies on the existing arrangement (RALF and SAM) are scheduled to verify the feasibility of the proposed concepts.

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