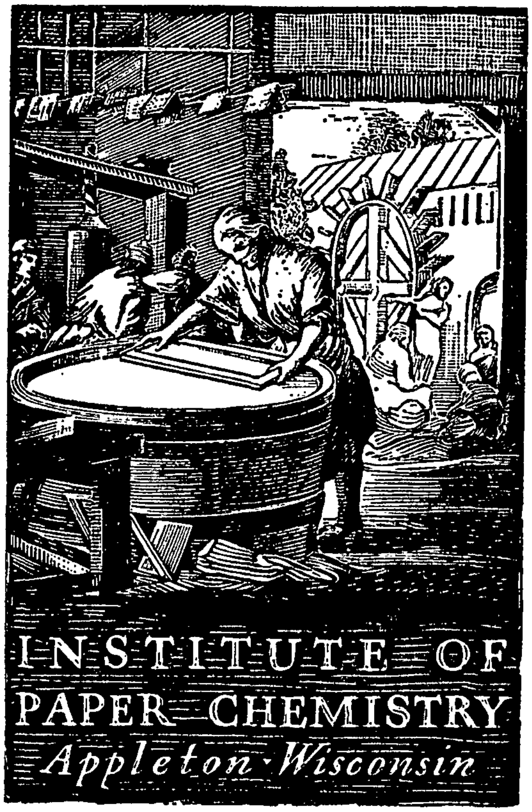

A STUDY OF THE SONIC PROPERTIES OF SACK PAPER

Project 2033

Report Forty

A Progress Report

to

MULTIWALL SHIPPING SACK PAPER MANUFACTURERS

January 3, 1967

THE INSTITUTE OF PAPER CHEMISTRY

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SUMMARY

The sonic properties of sack paper were studied as a part of a continuing attempt to better simulate in paper evaluation the behavior of sack paper during impact and converting. The term "sonic" arises in connection with a class of modern-day test methods which measure the response of a material to stress waves or pulses traveling through the specimen at the speed of sound.

When a force or displacement is applied at one point in a body, a short interval of time is required for the effect to be felt at a remote point in the body. The rate at which any mechanical disturbance propagates through the body is termed the sonic velocity, C; it is related to the modulus of elasticity, E, and the density, ρ, of the body approximately as follows:

$C = \sqrt{E'/\rho} \qquad E' = E\left(\frac{1-\mu}{1-\mu-2\mu^2}\right)$

$$C = \sqrt{E/\rho}$$

Sonic velocity appears to enter into the impact behavior of a sack because it is a property of the sack paper governing (along with other factors) the magnitude, distribution, and duration of impact strain. Sonic properties are attractive from the standpoint of materials testing because with present-day instruments they provide a rapid, nondestructive evaluation of modulus of elasticity (and hence stiffness of the paper) at dynamic test rates.

The sonic velocities and hence the "sonic moduli of elasticity" were evaluated for the samples of sack paper from the second fabrication program (12 samples of flat kraft and 14 samples of extensible kraft in the 50-lb. unbleached

grade). The measurements were made by means of a Morgan Dynamic Modulus tester Model PPM-5. This test device sends short pulses of stress between two points on a sheet of paper and measures the time of travel, from which sonic velocity may be calculated.

Among the conclusions reached in this phase of the study are the following:

1. The sonic velocity of these samples of sack paper ranged from 62,400 to 128,400 in./sec. (a 2-to-1 range) in keeping with the differences in modulus of elasticity between (a) flat and extensible kraft and (b) the principal directions of the sheet.

2. The precision of the estimate of sonic velocity of these samples by means of the Morgan Dynamic Modulus tester was $\pm$ 5%, on the average (as indicated by a 95% confidence interval). This degree of precision reflects variability of the paper and of the test method.

3. In a comparison of test methods on one sample of sack paper, the Morgan test estimate and an independent determination of sonic velocity by means of impact differed by no more than 13%. This is believed to be quite good agreement for tests of this nature.

4. The Morgan tester is apparently affected by the two-sidedness of paper. Determinations of sonic velocity made on the wire side of the sheet were 3.5% lower, on the average, than the felt-side determinations.

"Sonic modulus of elasticity," $\underline{\tilde{E}}$, for a sheet material is calculated from sonic velocity, $\underline{C}$, and density, ρ, according to $\underline{\tilde{E}} = \rho\underline{C}^2$. Through neglect of a factor involving Poisson ratio, sonic modulus is an overestimation of the true Young's modulus, theoretically by as much as 25% for sack papers. A study of the sonic moduli of the 26 samples of sack paper and comparison with moduli evaluated by means of an Instron testing machine at conventional, low rates led to the following conclusions:

5. Since the density of the samples of sack paper studied did not vary widely, sonic modulus is essentially proportional to the square of the sonic velocity and hence either property ranked the samples in about the same order.

6. The precision of the estimate of sonic modulus is about $\pm$ 10%, on the average, for these samples, as indicated by a 95% confidence interval. This is an optimistic estimate of precision because variability in the determination of density is not accounted for.

7. Sonic modulus (a dynamic property) and Instron modulus (a "static" property) were quite well correlated; correlation coefficients were 0.80 to 0.97, depending on type of paper and direction in the paper.

8. With extensible papers, sonic modulus exceeded Instron modulus by about 9%, on the average, which is in the direction and general magnitude anticipated from theory.

9. For flat kraft samples, sonic modulus was 2-1/2% less than Instron modulus, on the average. The sense of this difference is unexpected in view of the probable effect of difference in strain rate and the intrinsic overestimation associated with sonic modulus. A firm reason for the sense of the observed difference in moduli is lacking.

10. The ratio of sonic moduli in the two principal directions of the sheet (M.D./C.D.) was slightly greater than the directional ratio for the Instron moduli. Except for strain rate effects the ratios should be the same. A possible reason for the observed trend is that the machine direction is more rate sensitive than the cross direction, particularly with extensible papers.

The empirical relationship between sonic moduli and sack performance was studied in terms of (a) simple correlations involving the sonic moduli, and (b) multiple correlations with sonic moduli in conjunction with tensile energy absorption (T.E.A.). The conclusions may be stated as follows:

11. Machine-direction sonic modulus was significantly correlated with both progressive height face drop and butt drop for the combined flat and extensible data (correlation coefficients, $\underline{r}$, were -0.82 and -0.62, respectively). Modulus and sack performance display an inverse relationship, suggesting that high tensile stiffness (high modulus) is detrimental to sack impact performance. The aforementioned correlations are somewhat tenuous from physical considerations since it would not be expected that a rupture phenomenon such as impact failure is dependent solely on a prerupture property such as modulus of elasticity.

12. Cross-direction sonic modulus was not statistically significantly correlated with drop performance, except in the case of face drop of flat kraft sacks ($\underline{r}$ = 0.63).

13. Multiple regressions were calculated for face and butt drop performance on C.D. T.E.A. and M.D. sonic modulus on the assumption that the recognized importance of the first of these two paper properties may depend to some degree on the tensile stiffness in the other direction of the paper. This approach offered no improvement over prediction based on M.D. and C.D. T.E.A.

14. Multiple regression involving T.E.A. and sonic modulus in both principal directions (four factors) offered no overall improvement relative to the T.E.A.'s alone. One remarkably good four-factor predictive equation was obtained, however, for face drop of flat kraft sacks. The average prediction error was 5.1% – the lowest experienced in any phase of Project 2033. This result suggests that future research on sack impact might well be concerned with the possible importance of sack paper tensile stiffness (i.e., modulus of elasticity) as well as energy absorption.

INTRODUCTION

One objective of the research program of Project 2033 has been to develop methods for evaluating sack paper which are meaningful to its performance during converting and in service as a fabricated sack. Both of these types of performance involve application of stress to the sack paper at high time rates — for example, in the tubing and bottoming operations of sack fabrication and in impact of the sack on the filling line and during subsequent handling and transport.

The mechanical properties of sack paper (in common with many other materials) are rate sensitive. It is desirable, therefore, to have high-speed test methods for evaluation of the potential performance of the paper, or at least to enable study of the possible correlation between test properties at high rates and at the more traditional low rates of testing. A number of investigations of this type have been carried out in regard to energy absorption, tensile strength, stretch, and fatigue performance of sack paper. In general, the conclusion has been that these ultimate strength properties are quite well correlated at high and low test rates (1-3).

The present study extends this type of investigation to include the modulus of elasticity of sack paper. Modulus of elasticity is a prerupture property which can be expected to influence the magnitude of stress and strain in an impacted sack (4). Little or no attention has been given to the role of modulus in determining impact performance. While it seems likely that it plays a role secondary to rupture properties such as tensile energy absorption, it should not be overlooked as a possible contributing factor. Emergence of testing equipment for evaluation of modulus at dynamic rates has heightened interest in this mechanical property and its possible importance to sack impact behavior.

A class of test methods has evolved in recent years which is referred to as "sonic testing." This term arises because the methods generate a stress wave which travels with the speed of sound through the particular material under test. The velocity of the stress wave depends upon the internal structure of the material and more specifically on its moduli (elastic or viscoelastic), density, and Poisson ratios. The dynamic nature of these test methods makes them attractive where correlation with impact performance is of interest. The relative ease and rapidity of the test methods and the fact that they are nondestructive are of possible interest to process monitoring and control.

Sonic test methods have been used quite extensively in the textile industry, particularly in regard to yarns and other filamentary materials [see Reference (6)]. Pioneering work in sonic testing of paper has been done by Taylor and Craver (5,6). An outgrowth of both of the aforementioned areas of study has been the development by the H. W. Morgan Co. of a sonic pulse tester for paper and films (7).

The present study utilizes the Morgan tester to evaluate the sonic properties (wave velocity and associated modulus of elasticity) of the samples of flat and extensible kraft sack papers of the second fabrication program (8). Following a review of the theory of sonic testing, the results of the measurements of sonic modulus of the sack paper samples are examined from the standpoint of their relation to (a) modulus determined at a conventional test rate and (b) sack impact performance.

THEORY OF SONIC MEASUREMENTS

When a force or displacement is applied at one point of a body, a finite, though usually small, interval of time passes before the effect is felt at a remote point in the body. The initial disturbance propagates as a wave from one point to another in the body. Depending on the nature of the initial disturbance the wave may be comprised of tension-compression strain (longitudinal wave) or shear strain (transverse wave) or a combination of both.

The rate at which the wave travels is termed the velocity of propagation, $\underline{C}$. In the case of longitudinal waves (tension-compression) at low strains, the velocity of propagation is dependent upon the Young's moduli, Poisson ratios, and density of the material through which the wave travels (9). The longitudinal wave velocity is the velocity at which sound travels through the material and it is therefore frequently termed the sonic velocity. In the case of a transverse wave, the velocity depends upon the shear modulus (modulus of rigidity) and density of the material. The appropriate moduli and Poisson ratios at higher levels of strain, as in the inelastic region, are the subject of some controversy (10, 11) and are needful of further study beyond the scope of the present report.

The relationship between wave velocity and material properties has utility in materials testing because the material properties — moduli and Poisson ratios — may be calculated from suitable measurements of wave velocity and density. Determination of wave velocity is essentially a measurement of the time required for the disturbance to travel a known distance — a measurement rendered practical by modern-day electronic devices. Inasmuch as the determination of wave velocity is a dynamic test, the derived material properties may be meaningful in service applications of the material involving high rates of strain, for example, sack impact and converting. Moreover, the measurement of wave velocity is usually arranged to be nondestructive, which may be important to quality control or production monitoring applications [e.g., see Ref. (12)].

A third attribute cited for this approach to materials testing is that the wave velocity (and hence moduli and Poisson ratios) is an average over a sizable area or distance within the specimen and, therefore, does not reflect only the "weakest or least representative region" of the specimen as may occur in a static or slow speed test (5). It may be questioned whether dynamic tests are unique in this respect. While the tensile strength derived from a conventional tension test of paper may indeed reflect the weakest portion of the specimen, both strain (stretch) and Young's modulus are averages over the span of the specimen and thus they, like dynamic modulus, would appear to evaluate the sample as a whole. Another attribute of this type of dynamic testing is the speed and hence economy with which the testing may be carried out, although this factor is dependent on the apparatus and method of data analysis employed.

The relationship between longitudinal wave velocity (tension-compression) and the mechanical properties of the material supporting the wave depends upon the geometry of the specimen. The effect of geometry can be displayed most easily by citing the relationship for an isotropic material which, unlike machine-made paper, has the same value of Young's modulus and Poisson ratio in all directions. If the specimen is long in one dimension relative to the remaining two dimensions (for example, a long thin rod or bar) the velocity of propagation is (9):

$$C_1 = \sqrt{\frac{E}{\rho}} \qquad (1)$$

where C_1 = velocity of wave propagation, in./sec.

E = Young's modulus, lb./in.^{2}

ρ = mass density, lb. sec.^{2}/in.^{4}

The units are given in terms of the inch-pound-second system, although the relationship is valid in any other consistent system of units. Equation (1) reveals that the wave travels faster in a stiffer medium (high E) and in a lower density medium.

In a "planar" body, that is, one small dimension relative to the remaining dimensions, as in a sheet of isotropic paper or film, the velocity equation is (5):

$$C_2 = \sqrt{\frac{E}{\rho(1-v^2)}} \qquad (2)$$

where the newly introduced symbol is v = Poisson ratio. Inasmuch as v is less than unity, it is seen that $C_2 > C_1$; that is, the wave velocity is greater in the planar body than in the "one-dimensional" body. This may be explained on the grounds that lateral contraction or expansion in the plane of the sheet (perpendicular to the direction of longitudinal strain) is suppressed in the planar body, while it is not suppressed in the one-dimensional body; thus, the planar material is in effect stiffer than the one-dimensional body and accordingly the wave travels faster (9).

In a body whose dimensions are large and all of the same order of magnitude (strictly speaking, an infinite medium) which might here be called three-dimensional, longitudinal wave velocity is given by (9):

$$C_3 = \sqrt{\frac{E(1-v)}{\rho(1+v)(1-2v)}} \qquad (3).$$

The ratio of C_3 and C_2 is

$$\frac{C_3}{C_2} = \sqrt{1 + \frac{v^2}{1-2v}} \qquad (4).$$

If the Poisson ratio is between 0 and 0.5, as is true of many materials and reportedly true of paper (6), $C_3/C_2 > 1.0$, revealing that the wave velocity is higher in the infinite medium. Thus, for the stated conditions,

$$C_3 > C_2 > C_1 \tag{5}$$

In contrast, the velocity of a shear wave (transverse wave) is (9):

$$C_s = \sqrt{\frac{G}{\rho}} \tag{6}$$

where G = shear modulus, and this velocity is the same in the one-, two- and three-dimensional isotropic bodies. Equation (6) is also appropriate for an orthotropic planar body (6).

Equations (2) and (6) are of interest to isotropic paper — i.e., handsheets — and films. As a means of determining moduli, these equations may be written as follows:

$$E = \rho C_2^2(1-v^2) \tag{7}$$

$$G = \rho C_s^2 \tag{8}$$

Machine-made paper, however, is anisotropic (not isotropic) because of fiber orientation and unequal restraint during drying. The corresponding equations for the anisotropic sheet are three in number — one for modulus in the machine-direction (E_x), one for modulus in the cross-direction (E_y) and one for G_{xy} (6):

$$E_x = \rho \left[\frac{C_x^2 C_y^2 - C_{xy}^4}{C_y^2}\right] \tag{9}$$

$$E_y = \rho \left[\frac{C_x^2 C_y^2 - C_{xy}^4}{C_x^2}\right] \tag{10}$$

$$G_{xy} = \rho C_s^2 \tag{11}$$

where subscripts x and y refer to the machine- and cross-directions, respectively. C_{xy} is a complicated function of C_x, C_y, C_s and of the velocity C_{45} at 45° to the principal axes. It may be noted that, unlike the isotropic case, the equations for the anisotropic Young's moduli are coupled. That is, the x-direction modulus, for example, depends on the wave velocities in both principal directions. The inverse relationships are, of course, also coupled, meaning that the longitudinal wave velocity C_x, for example, depends on the Young's moduli in both principal directions of the sheet. The equation for G_{xy}, on the other hand, is not coupled with the remaining equations.

Two remarks should be made concerning the density ρ in Equations (1)-(3) and (6)-(11). First, it should be noted that this is mass density rather than weight density. Since mass equals weight divided by the acceleration due to gravity, g, the mass density ρ and weight density D are related as follows:

lb in⁴ sec $\quad \frac{lb/in^3}{in/sec^2}$

$$\rho = \frac{D}{g} \tag{12}$$

lb sec²/in⁴ $\quad$ 900

where g is approximately 386.4 in the inch-pound-second system of units.

Secondly, the choice between apparent sheet density and fiber density depends on the cross-sectional area for which modulus is to be calculated. If the modulus E is calculated on the basis of the apparent thickness of the sheet, then the apparent sheet density should be used. On the other hand, if the modulus is calculated on the basis of the solid fraction of cross-sectional area (i.e., the actual load-bearing area of the sheet cross section), then the fiber density should be used.

The underlying reason for this distinction may be readily seen from the derivation of the differential equation of wave motion in a slender rod (9):

$$A_\rho \rho dx \frac{\partial^2 u}{\partial t^2} = A_E E \frac{\partial^2 u}{\partial x^2} dx \tag{13}$$

where $\underline{u}$ is longitudinal displacement, $\underline{x}$ is the coordinate along the length of the rod, $\underline{t}$ is time, and $\underline{A}$ is cross-section area. The coefficient $\underline{A}_\rho \rho$ rises from the mass of a differential element of the rod and $\underline{A}_\rho$ is the area associated with the density ρ (either apparent density or fiber density). The coefficient $\underline{A}_{\underline{E}}\underline{E}$ pertains to the net longitudinal force on the element; $\underline{A}_{\underline{E}}$ is the area associated with the stress and hence the modulus $\underline{E}$. The wave velocity $\underline{C}$ is the square root of the quotient of the above-mentioned coefficients, namely

$$C = \sqrt{\frac{A_E E}{A_\rho \rho}} \qquad (14).$$

This reduces to Equation (1) if and only if $\underline{A}_{\underline{E}} = \underline{A}_\rho$, which is to say that $\underline{E}$ and ρ are referred to the same cross-section area – either the area of the solid fraction or of the fibers plus interfiber voids.

Returning to Equation (7) for isotropic planar materials, it is seen that Young's modulus can be estimated from wave velocity, density and the Poisson ratio. Unfortunately, an accepted and convenient method of measuring the Poisson ratio for paper is not available. Although the Poisson ratio can be evaluated by acoustical methods, Taylor and Craver (6) chose to avoid this measurement by defining and working with a sonic modulus $\tilde{E}$, as follows:

$$\tilde{E} = \rho C^2 \quad /kg/sec^2 \quad sec^2/cm^2 \qquad (15).$$

This amounts to ignoring the factor $(1-\underline{v})^2)$ in Equation (7) and in effect considers the paper as behaving as a thin rod or bar. The ratio of $\tilde{E}$ to $\underline{E}$ is $1/(1-\underline{v}^2)$ and thus the sonic modulus overestimates the true modulus.

Analogously, to avoid a tedious determination of $\underline{C}_{\underline{xy}}$, sonic moduli are defined (6) for the anisotropic planar material as:

$$\tilde{E}_x = \rho C_x^2 \tag{16}$$

$$\tilde{E}_y = \rho C_y^2 \tag{17}$$

and they also overestimate the true Young's moduli E_x and E_y. Taylor and Craver (6) analyzed the magnitude of the error in the anisotropic case; in theory the error is a function of (a) the ratio of the true Young's moduli and (b) the Poisson ratio v_x for the machine direction. For moduli ratios E_x/E_y in the range of 1.0 to 3.0 and the Poisson ratio in the range of 0.2 to 0.4, the error ranges from about 1-1/2 to 19%. The error decreases with increasing modulus ratio and increases with increasing Poisson ratio. Interestingly, the error is the same for sonic modulus in either principal direction of the sheet.

This analysis of error may be extended to include ratios of E_x/E_y less than unity, as occur in extensible papers. For example, Run 00 of the second fabrication program has a modulus ratio of 0.5. It may be calculated that the error is no greater than 25% (overestimation) when E_x/E_y is less than unity (as in extensible papers) and $v_x \leq 0.4$. This assumes that the paper is orthotropic ($E_x/E_y = v_x/v_y$). The maximum error occurs at $E_x/E_y = 0.8$ and $v_x = 0.4$.

It is also of interest to note that the ratio of sonic moduli in the two principal directions is equal to the ratio of the true moduli:

$$\frac{\tilde{E}_x}{\tilde{E}_y} = \frac{E_x}{E_y} \tag{18}$$

Thus, while the sonic moduli individually are systematically in error, the correct directional ratio of the sheet is preserved.

In summary on these points, it is important to realize that in theory the sonic modulus overestimates the true Young's modulus, but the directional ratio of the Young's moduli is given correctly by the ratio of sonic moduli.

TEST EQUIPMENT

Measurements of sonic velocity were made with a Morgan Dynamic Modulus tester Model PPM-5 (7), of which a photograph is shown in Fig. 1. This apparatus sends a series of short duration pulses through the test specimen and measures the time interval for the pulses to travel known distances through the specimen. This measurement is accomplished in the following way.

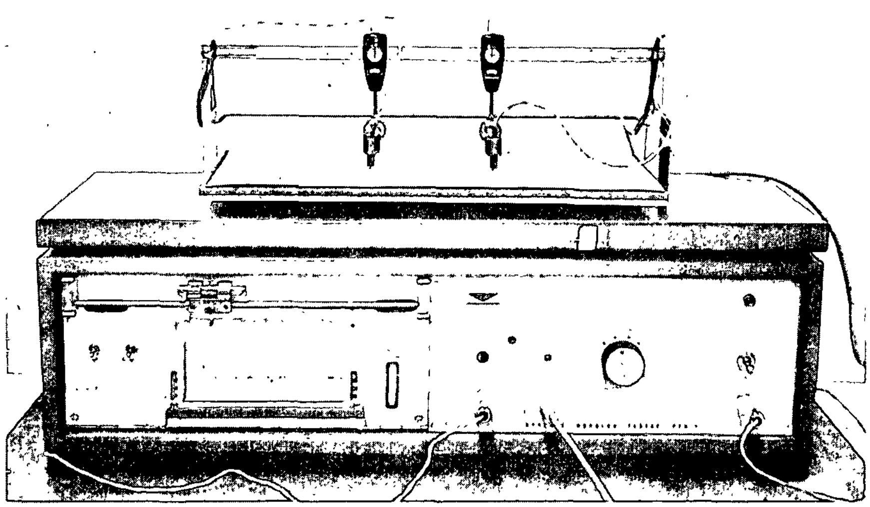

Figure 1. Photograph of Morgan Dynamic Modulus Tester, Model PPM-5

Two piezoelectric ceramic transducers are placed a known distance apart, as determined by a scale graduated in millimeters on the scanner assembly. Each transducer has a round-end probe which rests on the surface of the test specimen. The transducers are mounted in a silicone rubber damping material. A high voltage "spike" is applied to the "sender" transducer and sets the transducer probe into oscillation. The natural frequency is 5000 Hz.* The oscillation is highly damped so that it rapidly dies out. A second spike-voltage is applied 1/60 of a second after the first and generates a second pulse. This is done repeatedly at 60 Hz.

For measurement of longitudinal wave velocity, the piezoelectric transducers are aligned so that the direction of oscillation is parallel to the measured span in the test specimen. As indicated in Fig. 2, each pulsed oscillation sets up a longitudinal wave in the specimen which travels at velocity $\underline{C}_{\ell}$ from the sender crystal to the receiver crystal. Upon reaching the receiving crystal, the pulse sets the receiver crystal in oscillation. The sender crystal also sends out a rearward longitudinal wave and transverse (shear) waves, the latter propagating in a direction normal to the measured span at velocity $\underline{C}_{\underline{s}}$. The rearward longitudinal wave and the shear waves may eventually be reflected from specimen boundaries and travel toward the receiver crystal; however, it is intended that these waves will damp out before they reach the receiver crystal. Similarly, the main longitudinal pulse may be reflected from the boundary of the specimen after it has passed the receiver crystal, but it is expected to be sufficiently attenuated that it does not interfere significantly with the arrival of the second or succeeding main pulses generated by the sender crystal.

*1 Hz (Hertz) = 1 cycle per second.

Simultaneously with the generation of a pulse at the sender crystal, a linearly increasing "sweep" voltage is started by the measuring electronics of the instrument. When the longitudinal pulse in the specimen reaches the receiver crystal, the latter goes into oscillation and stops the sweep voltage. Thus, the maximum value of the sweep voltage is proportional to transit time of the pulse in the specimen. The maximum voltage is displayed on a 5-inch wide strip-chart recorder integral with the apparatus and calibrated in units of microseconds.

By making the abovementioned determination at two or more distances between probes, the transit times may be plotted against distance, as shown in Fig. 3. The sonic velocity is the reciprocal of the slope of the line of best fit. Because of inherent electrical and mechanical time delays in the apparatus, the line will not usually pass through the origin. The magnitude of the intercept on the time axis is said to be a function of the thickness of the specimen, the force exerted on the specimen by the transducer probes and the ambient temperature (13). It was found experimentally in the present study that both specimen thickness and width affected the time lag or delay. The intercept (time lag) decreased with decreasing thickness (90-lb. vs. 42-lb. linerboard) and with decreasing width (widths from 5 to 1/8 inch were studied in this regard). These effects are believed to be due to inertia of the specimen. There is provision for zeroing the strip-chart recorder to read zero at the intercept value, thereby permitting a one-point determination of velocity if desired.

Shear velocity, C_s, and hence shear modulus, G, may be measured by rotating the sender and receiver transducers so that they oscillate in a direction normal to the span between transducers, as illustrated schematically in Fig. 4.

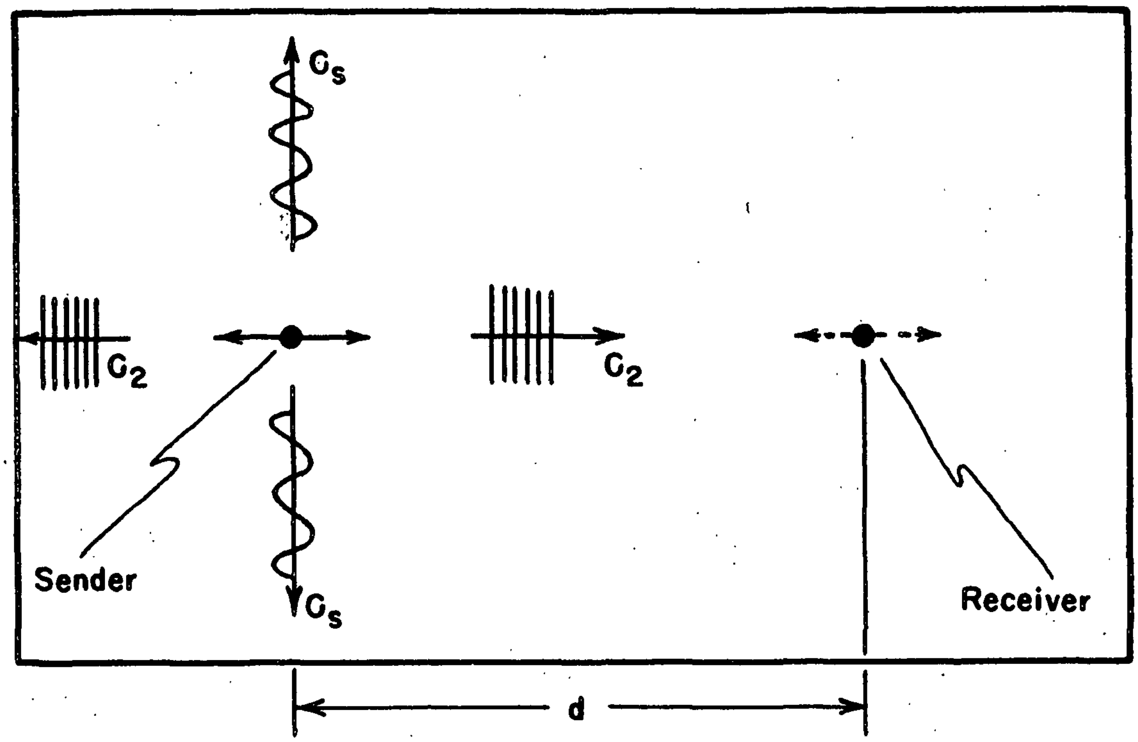

Figure 2. Generation of Pulses for Measurement of Longitudinal Wave Velocity, C

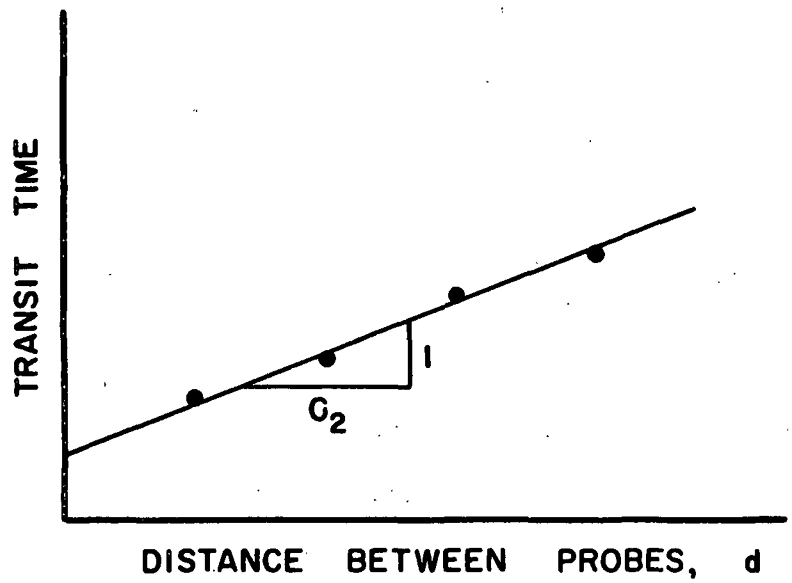

Figure 3. Graph of Transit Time vs. Distance between Probes for Determination of Sonic Velocity. Sonic Velocity is Reciprocal of Slope of Line

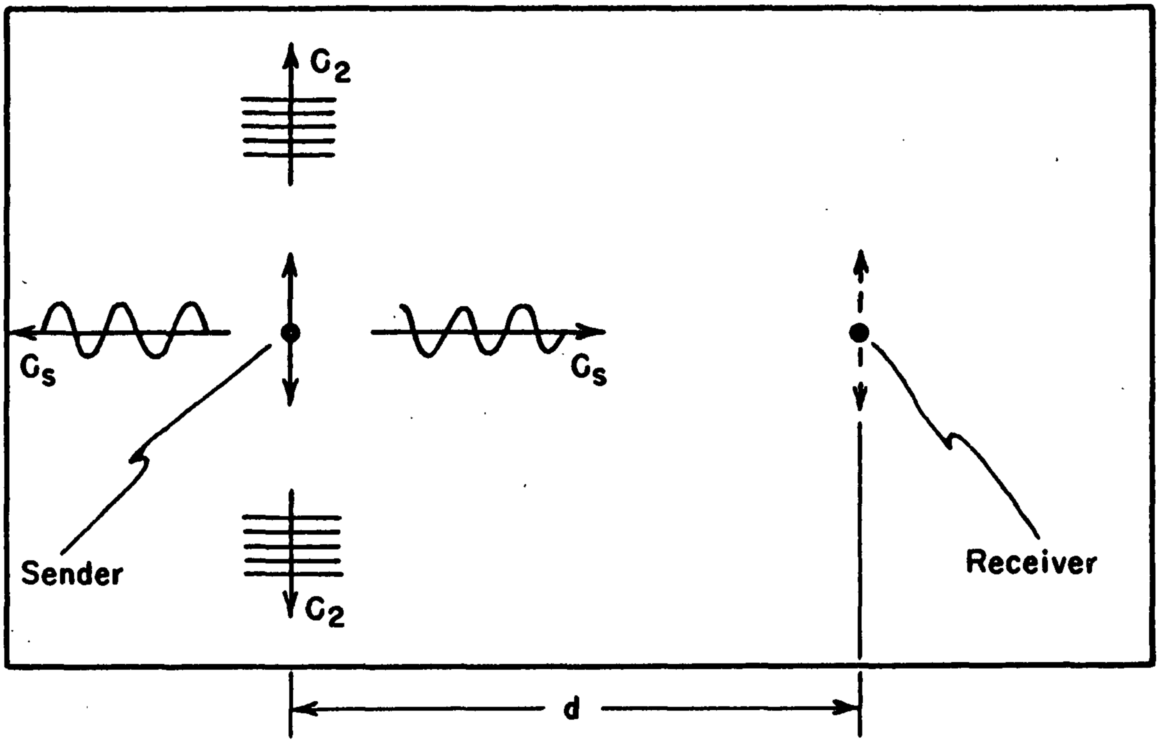

Figure 4. Generation of Pulses for Measurement of Shear Velocity, C_s

In this case the apparatus measures the transit time of the shear wave traveling with velocity C_s. Longitudinal waves are also generated in the lateral direction. The longitudinal waves travel faster than the shear waves (because $E > G$) and hence there is some danger of receiving a longitudinal wave traveling by an indirect path to the receiver crystal before the shear wave arrives. For this reason it is desirable to have the lateral dimension of the test specimen large relative to span d between the transducers for C_s measurements.

Several operational features of the Morgan Dynamic Modulus tester may be mentioned. The specimen rests on a urethane-foam padded turntable which may be rotated in increments of 15° to measure directional effects in the specimen. The transducers are mounted on counterbalanced arms to permit adjustment of contact pressure; the arms may be raised to insert a specimen. The separation between transducers may be varied in the range from 2 to 35 cm. Full-scale readings of 50, 100, 200, or 400 microseconds are available with the recorder. The manufacturer states that the accuracy of the instrument is within 1% of full scale and the reproducibility generally better than 1%.

The instrument used in this study was found to be very sensitive to external vibrations (at normal laboratory levels). It is required to provide additional isolation against vibration beyond that provided by the instrument manufacturer. Another problem with the instrument was malfunction of the recorder pen. This seemingly trivial item evidently continues to be the Achilles' heel of recording devices in our space age technology.

Comparison of moduli derived from wave velocity and moduli determined by other means (e.g., Instron tension) raises the question of the effect of strain rate on moduli. It is generally believed that Young's modulus increases with increasing strain rate, although perhaps less percentagewise than, say, tensile strength. The strain rate in the Morgan tester is thus a matter of interest. Strain rate in this tester is not specified, nor is it easily measurable. However, it may be helpful to make an order of magnitude estimate of strain rate for the purpose of placing the Morgan tester in approximate relationship to other tests.

Consider average strain rate to be the maximum strain (in./in.) divided by the time to reach maximum strain. As regards time, the sender crystal oscillates at approximately 5000 Hz and thus its period is 200 μ sec. The duration of time from zero to maximum strain at the sender probe is thus one-quarter of the period, namely, 50 μ sec. (assuming a sine-wave oscillation of the probe tip). As regards strain, it can only be guessed at this time what is the maximum strain induced in the specimen. (A direct measurement of strain or at least a measurement of the crystal probe motion would be helpful.) However, it is intended that the tester operate at very low strain levels such that the calculated modulus is the modulus at the origin of the stress-strain curve. On these grounds, it is believed that the strain probably does not exceed 0.1% (0.001 in./in.). The corresponding average strain rate is (0.001 in./in.)/50 μ sec. = 20 in./in./sec. In contrast, the strain rates of several other testing machines employed in Project 2033 studies are as follows:

Instrument	Strain Rate, in./in./sec.
Instron	0.0014
Van der Korput	0.55
IPC high-speed tensile	7
Plas-Tech high-speed tensile	2.8, 28, 42
Impulse tester	100

MATERIALS

The materials employed in this study were 26 samples of 50-lb. unbleached kraft sack paper procured in connection with the second fabrication program (8). The specimens were taken from butt rolls corresponding to the outer ply of sacks fabricated in that program. The samples are designated AA to LL for flat kraft and MM to ZZ for extensible kraft.

TEST PROCEDURE

The sack paper samples were preconditioned for at least 24 hours at less than 35% R.H. and 73 ± 3.5°F. and then conditioned for at least 48 hours and tested at 50 ± 2% R.H. and 73 ± 3.5°F.

Five specimens were cut from a given sample; the specimen dimensions were 38 cm. in the machine direction and 24 cm. in the cross direction. The transducers of the Morgan Dynamic Modulus tester were oriented for measurement of longitudinal wave velocity, with the left-hand transducer acting as the receiver. The specimen was placed felt side up on the specimen table such that the transducers were aligned in the machine direction and located 2.5 to 5 cm. from one edge of the specimen, as indicated in Fig. 5. Readings of transit time were taken successively at 4, 8, 12, and 16 cm. separation of the transducers. Then the specimen was rotated 180° and a second set of four transit times was taken along the opposite edge of the sheet. The remaining four specimens of the sample were tested in the same manner. This gave 10 readings of transit time at each of four separations. The 10 readings were subsequently averaged, giving four points from which to calculate the line of best fit (least squares) as illustrated in Fig. 3.

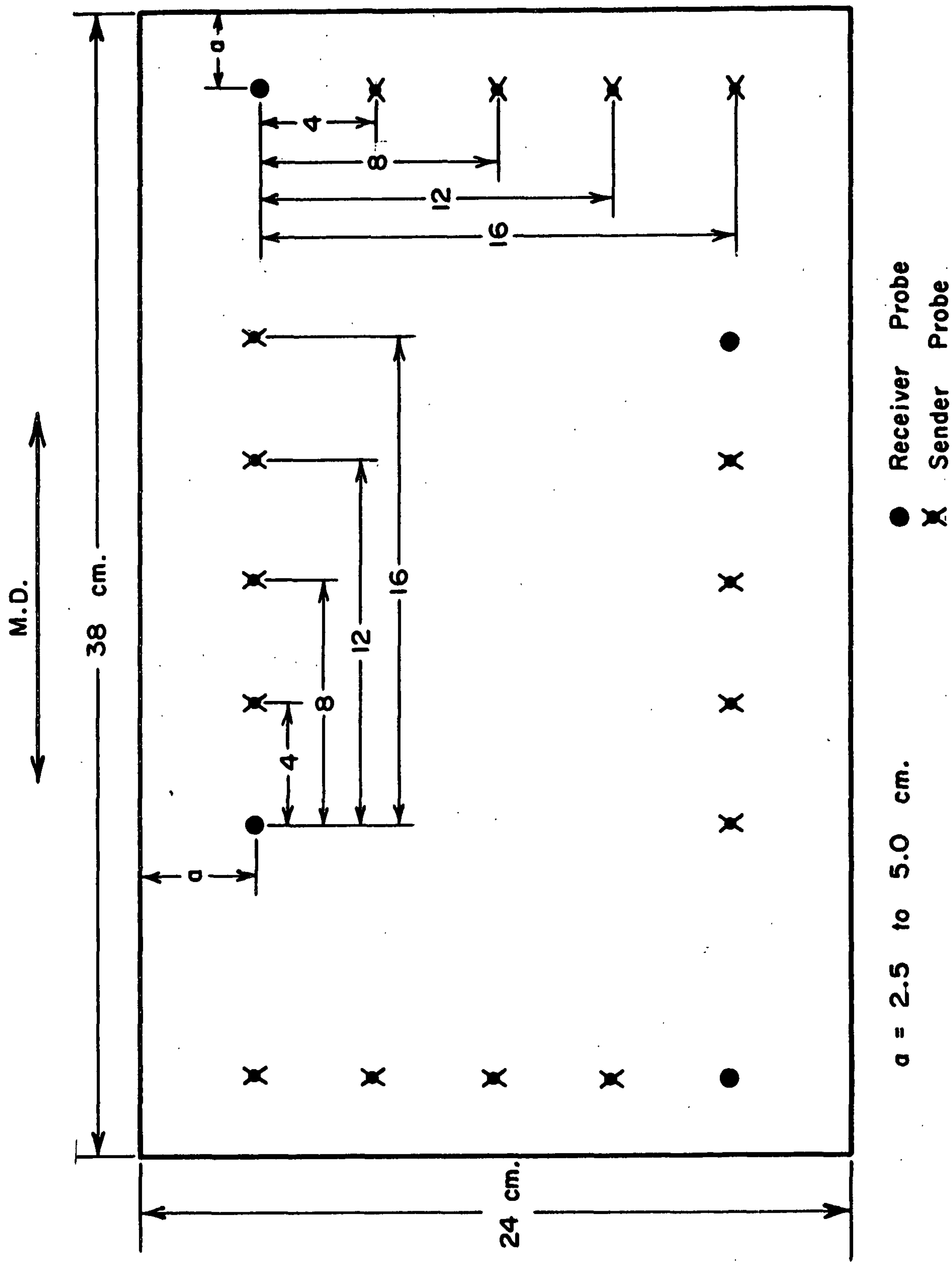

Figure 5. Location of Sonic Measurements in Sack Paper Specimen

A similar series of measurements were made in the cross-direction of each specimen, as indicated in Fig. 5.

Six of the paper samples were also tested with the wire side of the sheet uppermost, to enable studying the effect of sheet two-sidedness on sonic measurements.

The sheet sonic modulus, $\underline{E}$, was evaluated according to Equation (15), involving sheet density. Weight density in units of lb./in.^{3} was calculated from previously determined values of basis weight and caliper ($\underline{8}$) as follows:

$$\text{Weight density} = D = \frac{1}{432}\ \frac{\text{basis weight}}{\text{caliper in points}} \qquad (19).$$

Weight density was converted to mass density through division by 386.4, as discussed in THEORY.

For purposes of comparison, the Instron moduli of elasticity were evaluated for these samples of sack paper (outer ply samples). These moduli were calculated from the Instron tension curves obtained for the determination of tensile strength, stretch, and tensile energy absorption reported in Ref. ($\underline{8}$); the moduli have not been reported previously.

Modulus is proportional to the slope of the load-elongation curve at the origin of the curve. Specifically,

$$E' = \frac{PL}{ebt} \qquad (20)$$

where $\underline{E'}$ = Instron modulus of elasticity, lb./in.^{2}

$\underline{P/e}$ = slope of linear portion of load-elongation curve

$\underline{L}$ = span = 6 inches

$\underline{b}$ = width = one inch

$\underline{t}$ = sheet thickness, inch

Since $\underline{t}$ is sheet thickness, $\underline{E}'$ is the modulus of the sheet (rather than of the solid cross section) and thus is referred to the same cross-sectional area as the sonic modulus $\widetilde{\underline{E}}$. The same measurement of thickness is used in calculation of both $\underline{E}'$ and $\widetilde{\underline{E}}$.

Inasmuch as the load-elongation curves of these samples are straight for a substantial distance, the precision in determining slope is good and is not hampered by a nonlinear "toe" of the curve at the origin, as is sometimes the case in tensile testing.

500
500
250000

DISCUSSION OF RESULTS

SONIC VELOCITY

The sonic velocity (i.e., velocity of stress propagation) of 12 samples of flat kraft and 14 samples of extensible kraft 50-lb. sack paper was measured by means of the Morgan Dynamic Modulus tester. Pulse transit times were measured at four separations of the oscillator probes; these measurements were made twice on each of five specimens of sack paper from a given sample. The inverse slope of the straight line of best fit on a graph of time *vs.* distance (probe separation) is the estimate of sonic velocity.

The estimates of sonic velocity of these samples of paper are shown in Table I. It may be seen that the velocities of the several samples ranged from 62,400 to 128,400 in./sec. The velocities were highest in the machine direction of the flat kraft samples – 114,700 in./sec., on the average – primarily because of the higher modulus of elasticity in this orientation of this type of paper. The cross-direction velocity of the flat kraft samples was 83,600 in./sec., on the average.

The velocities in the two principal directions of the extensible paper are more nearly equal – 74,500 in the machine direction and 80,500 in the cross direction, on the average. The machine-direction velocity is generally lower than the cross-direction velocity and this is attributable to the lower machine-direction modulus. It may further be noted that sonic velocity in the machine direction of the extensible samples decreases with increasing degree of extensibility, as would be expected.

TABLE I

SONIC VELOCITY OF SACK PAPER
(50-Lb., unbleached Kraft)

Sample	Type	Machine Direction Sonic Velocity, in./sec.	Machine Direction 2 x Standard Error, in./sec.	Machine Direction 2 x Standard Error, %	Cross Direction Sonic Velocity, in./sec.	Cross Direction 2 x Standard Error, in./sec.	Cross Direction 2 x Standard Error, %
AA	Flat Kraft	105,600	9,400	8.8	79,600	3,600	4.5
BB	"	110,100	5,400	5.0	78,500	3,600	4.6
CC	"	120,800	1,200	1.0	76,500	2,200	3.0
DD	"	128,400	6,000	4.7	89,500	1,000	1.1
EE	"	117,300	8,800	7.6	86,000	3,000	3.6
FF	"	113,000	10,600	9.4	82,900	5,600	6.8
GG	"	108,700	5,000	4.6	81,500	3,200	4.0
HH	"	112,400	14,400	12.9	83,500	8,200	9.8
II	"	110,200	3,000	2.8	81,100	3,800	4.6
JJ	"	115,400	4,400	3.8	79,700	3,400	4.2
KK	"	111,100	9,400	8.5	90,900	6,600	7.3
LL	Flat Kraft	123,000	2,800	2.3	93,500	1,000	1.0
	Average	114,700	6,700	5.9	83,600	3,800	4.5
	Minimum	105,600	1,200	1.0	76,500	1,000	1.0
	Maximum	128,400	14,400	12.9	93,500	8,200	9.8
MM	6% Extensible	81,200	7,600	9.4	84,200	8,400	9.9
NN	9% "	75,000	3,200	4.4	84,300	6,000	7.1
OO	12% "	65,700	3,800	5.8	88,600	7,000	8.0
PP	6% "	81,800	3,000	3.6	78,500	3,800	4.7
QQ	9% "	79,100	2,800	3.6	80,400	2,200	2.8
RR	12% "	70,100	2,400	3.6	80,600	3,800	4.8
SS	6% "	89,500	1,200	1.4	81,600	2,200	2.6
TT	9% "	83,300	2,600	3.1	86,200	2,800	3.3
UU	12% "	72,600	1,200	1.7	80,200	1,000	1.1
VV	9% "	74,200	2,000	2.6	80,400	3,600	4.4
WW	9% "	67,300	3,600	5.4	70,700	4,600	6.6
XX	9% "	62,400	1,400	2.2	74,500	2,400	3.2
YY	9% "	72,200	4,200	5.8	75,600	4,600	6.2
ZZ	9% Extensible	68,600	5,200	7.6	81,200	6,600	8.2
	Average	74,500	3,200	4.3	80,500	4,200	5.2
	Minimum	62,400	1,200	1.4	70,700	1,000	1.1
	Maximum	89,500	7,600	9.4	88,600	8,400	9.9

In regard to the precision of these determinations, Table I also shows the two-standard-error limits of the velocities in terms of velocity units and also as a percent of the estimate of velocity. These limits are approximately the 95% confidence limits on velocity. For example, in the case of flat kraft Sample AA in the machine direction, there is 95% confidence that the true sonic velocity is in the interval 105,600 ± 9400 in./sec. — that is, between 96,200 and 115,000 in./sec. The confidence interval is ± 8.8% of the estimated velocity in this case.

The standard error of the velocity C was calculated in the following way (14): Inasmuch as $C = 1/b$, where b is the slope of the time vs. distance graph, the variance of C is $V(C) = V(b)/b^4$, where $V(b)$ is the variance of the slope of the line of best fit. The latter depends on the variance of the observations about the line of best fit, that is, $V(b) = \sigma^2/\Sigma(x-\bar{x})^2$, where σ^2 is variance of observations about the line and x is the distance between probes. Finally, the standard error of C is the square root of the variance of C. The standard error of C reflects variability in the paper and in the experimental method (operator, test instrument, etc.).

It may be seen in Table I that the two-standard-error limits range from ± 1.0 to ± 12.9% of the estimated velocities. On the average, this confidence interval is ± 5.0% of the velocity, indicating that sonic velocity is determined with quite good precision, in general.

In regard to the accuracy of the velocity determination, a comparison with a completely independent experimental method is available in the case of Sample YY, a 9% extensible paper. The sonic velocity of this sample was also measured by means of strain gages on a long impacted strip of paper in an earlier study (15). Conductive coating strain gages were spaced at 40-inch intervals on two 144 by 4-inch machine-direction strips of paper and at 10-inch intervals on two 34 by 4-inch cross-direction strips. The strips were suspended vertically and impacted at the lower end by a falling mass. Sonic velocity was determined by measuring the time for strain to propagate between gage locations.

The estimates of sonic velocity from the two experimental methods are shown in Table II. The strain gage determinations were made at both low and high stress levels; only the low level results are employed in this comparison inasmuch as the Morgan tester operates at a low stress level.

TABLE II

COMPARISON OF PULSE PROPAGATION AND IMPACT DETERMINATIONS OF SONIC VELOCITY

(Sample YY)

	Sonic Velocity, *C*, in./sec.	
	Machine Direction	Cross Direction
Morgan tester	72,200 ± 4200[a]	75,600 ± 4600[a]
Strain gage method	82,400 ± 1300[a]	75,000 ± 1600[a]
Diff., %, (based on mean)	13.2	0.8

[a]95% Confidence interval.

Sonic velocities determined by the two independent methods differed by 13.2% in the machine direction of the paper and by 0.8% in the cross direction. Clearly the cross-direction difference is not significant, in view of the 95% confidence intervals on the two determinations. On the other hand, the difference in velocities in the machine direction is highly significant, indicating some "systematic" difference in the determinations which is greater than can be accounted for by the intrinsic variability in the two test methods. Taken on balance, however, there appears to be reasonably good agreement between the two methods of velocity determination; a 13% difference does not seem to be extreme when it is considered that different instrumentation and paper sampling are involved. On the basis of this single comparison, there is no strong reason to doubt that either method accomplishes the intended measurement of sonic velocity of paper, although there may be a modest systematic difference between them.

The oscillator probes of the Morgan tester contact the specimen on one surface. It is natural to inquire whether the two-sidedness of paper affects the sonic modulus measurement. The measurements of sonic velocity in Table I were made with the probes contacting the felt side (denser side) of the sheet. For comparison, velocity determinations were made with the probes contacting the wire side of the specimen for seven of the samples of extensible paper, with the results shown in Table III.

It may be seen that, in general, the sonic velocity was lower when measured on the wire side of the sheet. On the average, the wire side determination was 3.1% lower than the felt side determination in the machine direction and 3.9% lower in the cross direction. In many instances the difference for individual samples is small and well within the nominal $\pm$ 5% uncertainty range of the velocity determination, which may lead one to question whether the difference is significant. On the other hand, the trend to a lower wire-side determination is evident in all instances except one, which suggests that the effect of two-sidedness is real. This is borne out by an analysis of variance of the data in Table III, where the effect of felt vs. wire side on sonic velocity is tested statistically against the average variance of the velocity determination ($[S.E.(C)]^2$, discussed above). In both directions of the sheet, the effect of felt side up vs. wire side up was significant (beyond the 0.025 level for machine direction and beyond the 0.01 level for the cross direction).

TABLE III

COMPARISON OF SONIC VELOCITY DETERMINATIONS
FOR FELT-SIDE VS. WIRE-SIDE OF SACK PAPER

	Sonic Velocity, in./sec.					
	Machine Direction			Cross Direction		
Sample	Felt	Wire	Diff., %[a]	Felt	Wire	Diff., %[a]
MM	81,200	81,500	+ 0.4	84,200	82,700	- 1.8
NN	75,000	72,300	- 3.6	84,300	83,200	- 1.3
OO	65,700	65,200	- 0.8	88,600	83,700	- 5.5
SS	89,500	85,600	- 4.4	81,600	76,000	- 6.9
TT	83,300	78,700	- 5.5	86,200	78,400	- 9.0
UU	72,600	68,100	- 6.2	80,200	78,200	- 2.5
WW	67,300	66,200	- 1.6	70,700	70,500	- 0.3
		Av.	- 3.1			- 3.9

[a] Based on felt side determination.

A possible reason for a difference between felt and wire side determinations is that the wave path favors the "half" of the sheet thickness adjacent to the oscillator probes. It is not evident, however, that this behavior would necessarily give rise to a difference in velocity for the following reason. While the felt side is probably denser than the wire side, it may also be true that the modulus of elasticity is higher near the felt side since the increased retention of fines at the felt side (which increases the density) probably also increases the degree of bonding and hence the modulus. Recalling that $\underline{C} = \sqrt{\underline{E}/\rho(1-\underline{v}^2)} \cong \sqrt{\underline{E}/\rho}$, there are grounds for expecting little or no difference in $\underline{C}$ — for example, if $\underline{E}$ is proportional to density ρ. The observed higher sonic modulus on the felt side implies that $\underline{E}$ increases relatively more than ρ, according to this line of reasoning involving wave path.

It should not be overlooked that perhaps another factor influencing the two-sidedness effect is a difference in surface finish of the felt and wire side of the sheet. Conceivably, surface finish may affect the contact of the oscillator probe and hence the "efficiency" of transfer of oscillator vibration to paper vibration and thereby the transit time of the pulse.

The above considerations are rather tenuous for lack of a clear understanding of pulse propagation in a fibrous mat and of the distribution of mechanical properties across the thickness of a sheet. Perhaps the most that should be said at this time is that, on the basis of experiment, the sonic velocity determined on the wire side of the sheet was 3.5% lower, on the average, than on the felt side of the sheet.

By way of summary of the above discussions of sonic velocity, it was found that flat kraft and extensible kraft samples exhibited nearly a 2-to-1 range in velocity, in keeping with the differences in modulus of elasticity. The precision of the estimate of sonic velocity by means of the Morgan tester, as indicated by a 95% confidence interval, was ± 5%, on the average. The Morgan test estimate and an independent determination of velocity by means of impact differed by no more than 13.2% with one sample of paper, which is believed to be satisfactory agreement for tests of this nature. The Morgan test is apparently affected by the two-sidedness of paper; the wire side determination was 3.5% lower, on the average, than the felt side determination.

SONIC MODULUS

The sonic modulus, $\tilde{E}$, of a sheet material such as paper is defined as ρC^2, where ρ is mass density and C is sonic velocity (i.e., velocity of wave propagation) (6). It is greater than the true Young's modulus by a factor $1/(1-v^2)$, where v is Poisson ratio.

The sonic moduli of the twenty-six samples of sack paper studied in this program are shown in Table IV. Inasmuch as the density factor (also shown in Table IV) does not vary widely, the sonic moduli rank the samples in approximately the same order as the sonic velocities in Table I. This may be seen in Fig. 6 which presents graphs of sonic modulus vs. sonic velocity for each type and orientation of paper. The curve in each graph is $\tilde{E} = \bar{\rho} C^2$ where $\bar{\rho}$ is the average density for that type of paper. Deviation of a plotted point from the curve reflects the deviation of the actual density from the average density. If all samples had identical density the points would fall exactly on the curve, and the ranking of samples by sonic modulus would be identical with the ranking by sonic velocity.

Regarding the precision of the sonic modulus determination, its standard error expressed as a percent of the mean (%S.E.) is given by:

$$\%S.E.(\tilde{E}) = \sqrt{4[\%S.E.(C)]^2 + [\%S.E.(W)]^2 + [\%S.E.(t)]^2} \qquad (21)$$

where W is basis weight and t is thickness. If basis weight and thickness are regarded as known exactly, then the percent standard error of $\tilde{E}$ is twice the percent standard error of sonic velocity C; the factor two is traceable to squaring the sonic velocity in the calculation of sonic modulus. Accordingly, the 95% confidence interval on sonic modulus is twice the interval on velocity, that is, about ± 10%, on the average. Actually, the interval is somewhat larger than ± 10% because of the experimental variability in the measurement of basis weight and thickness.

TABLE IV

SONIC MODULUS AND INSTRON MODULUS OF SACK PAPER SAMPLES

(50-Lb., Unbleached Kraft)

	Sample	Type	Apparent Density, lb./in.^{3}	Machine Direction Sonic Modulus, lb./in.^{2}	Machine Direction Instron Modulus, lb./in.^{2}	Machine Direction Diff., %[a]	Cross Direction Sonic Modulus, lb./in.^{2}	Cross Direction Instron Modulus, lb./in.^{2}	Cross Direction Diff., %[a]	Ratio, M.D. Modulus / C.D. Modulus: Sonic	Instron	Diff.
1	AA	Flat Kraft	0.0194	560,000	685,000	- 18.2	318,000	389,000	- 18.2	1.76	1.76	0.00
2	BB	"	0.0182	573,000	632,000	- 9.3	291,000	317,000	- 8.2	1.97	1.99	- .02
3	CC	"	0.0175	662,000	588,000	+ 12.6	265,000	230,000	+ 15.2	2.50	2.56	- .06
4	DD	"	0.0202	862,000	791,000	+ 9.0	418,000	358,000	+ 16.8	2.06	2.21	- .15
5	EE	"	0.0198	703,000	756,000	- 7.0	378,000	342,000	+ 10.5	1.86	2.21	- .35
6	FF	"	0.0192	635,000	594,000	+ 6.9	342,000	349,000	- 2.0	1.86	1.70	+ .16
7	GG	"	0.0200	612,000	698,000	- 12.3	344,000	392,000	- 12.2	1.78	1.78	+ .00
8	HH	"	0.0200	653,000	662,000	- 1.4	364,000	405,000	- 10.1	1.79	1.63	+ .16
9	II	"	0.0190	598,000	615,000	- 2.8	324,000	400,000	- 19.0	1.85	1.54	+ .31
10	JJ	"	0.0177	610,000	657,000	- 7.2	291,000	342,000	- 14.9	2.10	1.92	+ .18
11	KK	"	0.0218	696,000	687,000	+ 1.3	466,000	456,000	+ 2.2	1.49	1.51	- .02
12	LL	Flat Kraft	0.0228	892,000	839,000	+ 6.3	515,000	498,000	+ 3.4	1.73	1.68	+ .05
		Average	0.0196	671,000	684,000	- 1.8	360,000	373,000	- 3.0	1.90	1.87	+ .03
		Minimum	0.0175	560,000	588,000	- 18.2	265,000	230,000	- 19.0	1.49	1.51	- .35
		Maximum	0.0228	892,000	839,000	+ 12.6	515,000	498,000	+ 16.8	2.50	2.56	+ .31
13	MM	6% Extensible	0.0210	358,000	358,000	0.0	384,000	388,000	- 1.0	0.93	0.92	+ .01
14	NN	9% "	0.0214	311,000	294,000	+ 5.8	393,000	409,000	- 3.9	0.79	0.79	+ .07
15	OO	12% "	0.0213	238,000	205,000	+ 16.1	432,000	413,000	+ 4.6	0.55	0.50	+ .05
16	PP	6% "	0.0220	381,000	373,000	+ 2.1	351,000	344,000	+ 2.0	1.09	1.08	+ .01
17	QQ	9% "	0.0223	361,000	320,000	+ 12.8	373,000	358,000	+ 4.2	0.97	0.89	+ .08
18	RR	12% "	0.0223	284,000	249,000	+ 14.1	376,000	354,000	+ 6.2	0.76	0.70	+ .06
19	SS	6% "	0.0205	426,000	346,000	+ 23.1	354,000	312,000	+ 13.5	1.20	1.11	+ .09
20	TT	9% "	0.0222	399,000	303,000	+ 31.7	428,000	347,000	+ 23.3	0.93	0.87	+ .06
21	UU	12% "	0.0232	317,000	240,000	+ 32.1	387,000	362,000	+ 6.9	0.82	0.66	+ .16
22	VV	9% "	0.0208	297,000	271,000	+ 9.6	348,000	329,000	+ 5.8	0.85	0.82	+ .03
23	WW	9% "	0.0152	178,000	169,000	+ 5.3	197,000	164,000	+ 20.1	0.90	1.03	- .13
24	XX	9% "	0.0212	214,000	203,000	+ 5.4	305,000	333,000	- 8.4	0.70	0.61	+ .09
25	YY	9% "	0.0203	274,000	243,000	+ 12.8	300,000	280,000	+ 7.1	0.91	0.87	+ .04
26	ZZ	9% Extensible	0.0225	274,000	261,000	+ 5.0	384,000	385,000	- 0.3	0.71	0.68	+ .03
		Average	0.0212	308,000	274,000	+ 12.6	358,000	341,000	+ 5.7	0.86	0.82	+ .05
		Minimum	0.0152	178,000	169,000	0.0	197,000	164,000	- 8.4	0.55	0.50	- .13
		Maximum	0.0232	426,000	373,000	+ 32.1	432,000	413,000	+ 23.3	1.20	1.11	+ .16

[a]Based on Instron.

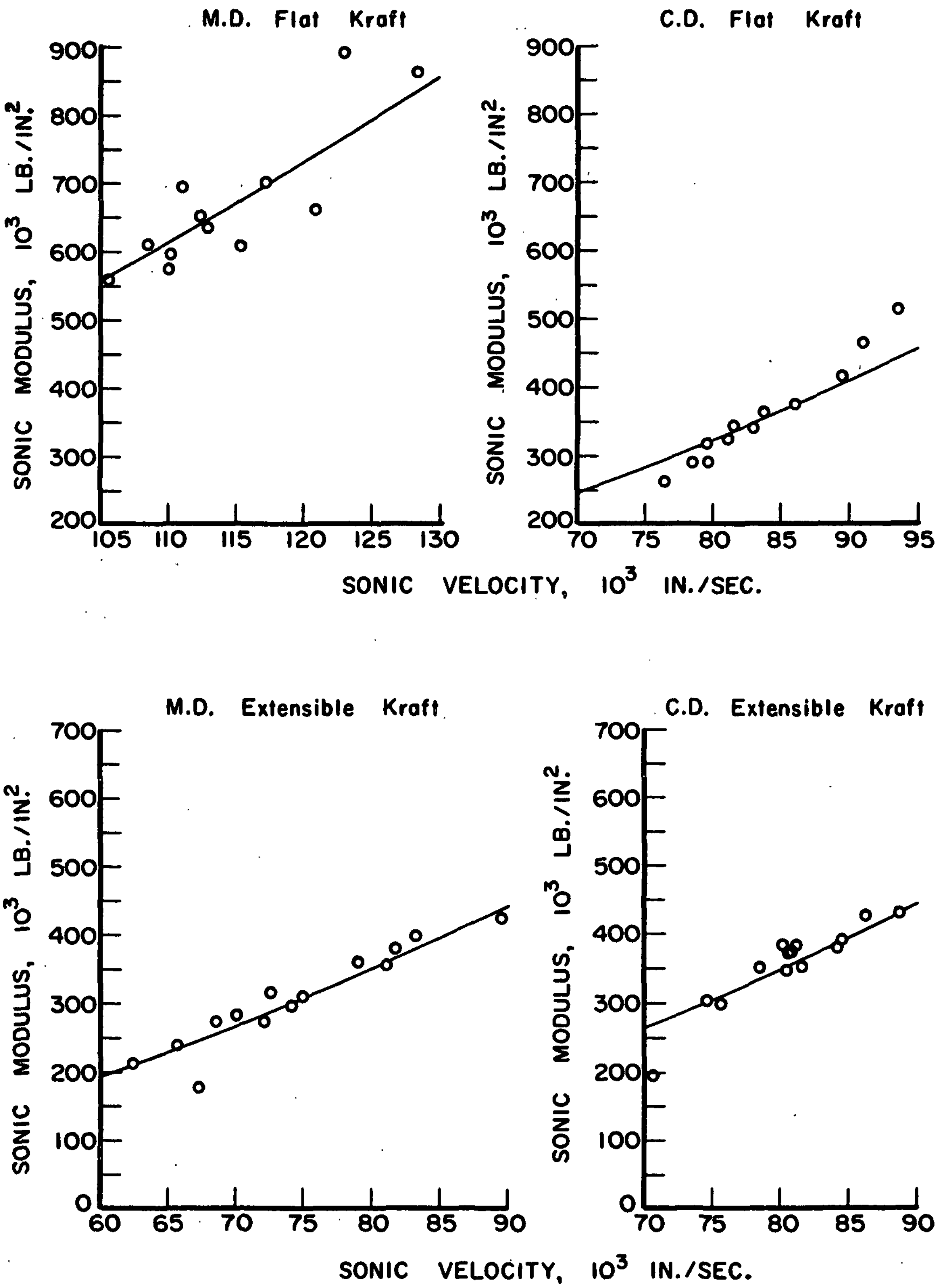

Figure 6. Correlation Between Sonic Modulus and Sonic Velocity (Ignoring Density)

The Instron modulus $\underline{E}'$ was evaluated for these samples from load-elongation curves obtained at a deformation rate of 0.5 in./min. The Instron modulus can be expected to be less than the true Young's modulus $\underline{E}$ at the dynamic rate of the sonic tests because of the effect of strain rate. Thus, it is to be expected that the three moduli of concern in this study bear the following relationship to one another:

$$E' < E < \tilde{E} \qquad (22)$$

where $\underline{E}'$ = Instron modulus (0.5 in./min. test rate)

$\underline{E}$ = true Young's modulus at sonic test rate

$\underline{\tilde{E}}$ = sonic modulus

A comparison of the sonic and Instron moduli is given in Table IV and graphically in Fig. 7 and 8. A measure of the degree of association between the two types of tests is given by the correlation coefficients in Table V.

TABLE V

CORRELATION COEFFICIENTS BETWEEN SONIC MODULUS AND INSTRON MODULUS

	Correlation Coefficient		
		Type of Paper	
	Flat	Extensible	Combined
Machine direction	0.798	0.910	0.971
Cross direction	0.803	0.908	0.827

The sonic modulus of the extensible samples is generally larger than the Instron modulus, as anticipated. On the average, the sonic modulus exceeds the Instron modulus by 12.6% in the machine direction and by 5.7% in the cross direction.

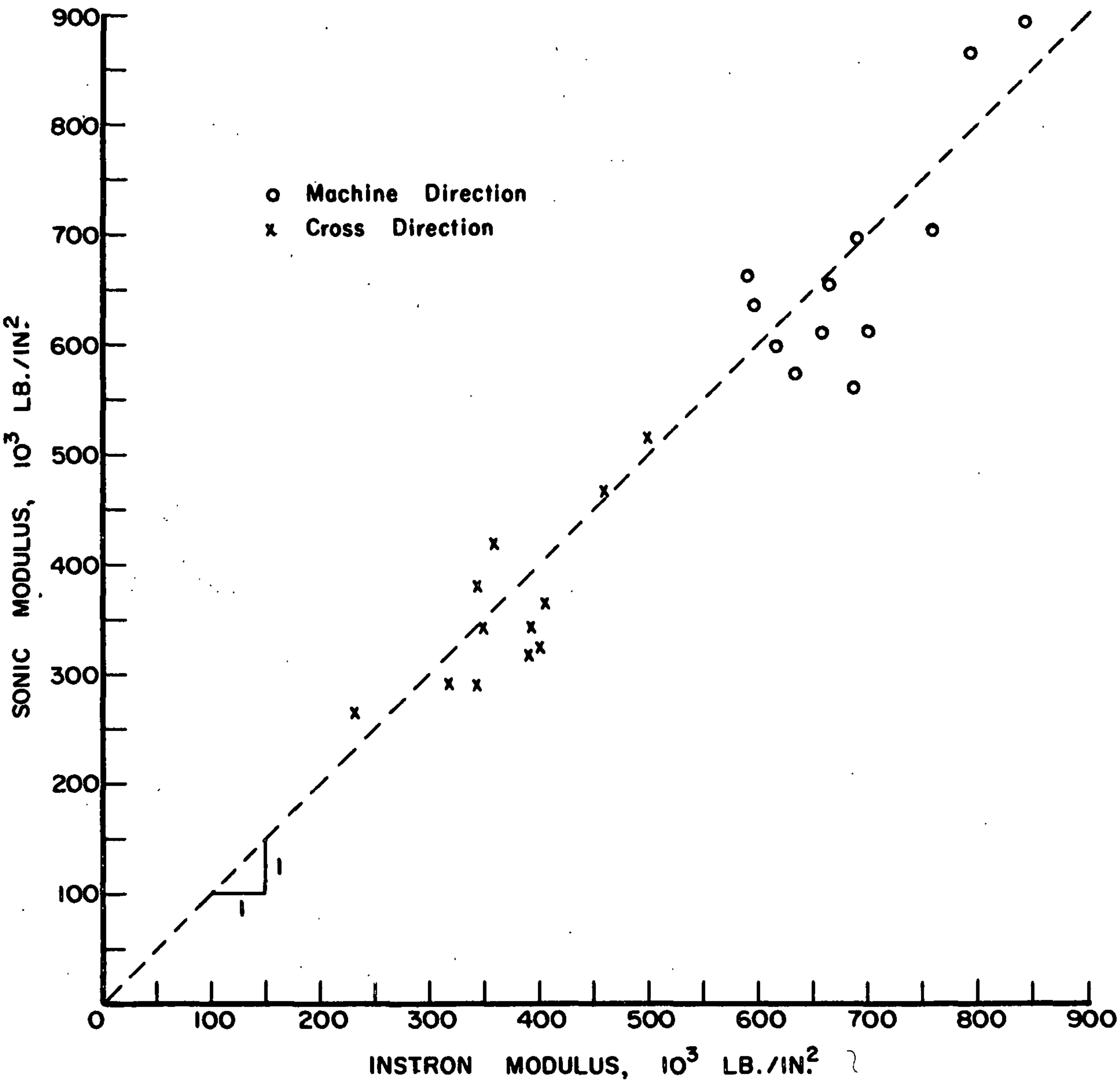

Figure 7. Relationship Between Sonic and Instron Moduli for Flat Kraft Samples

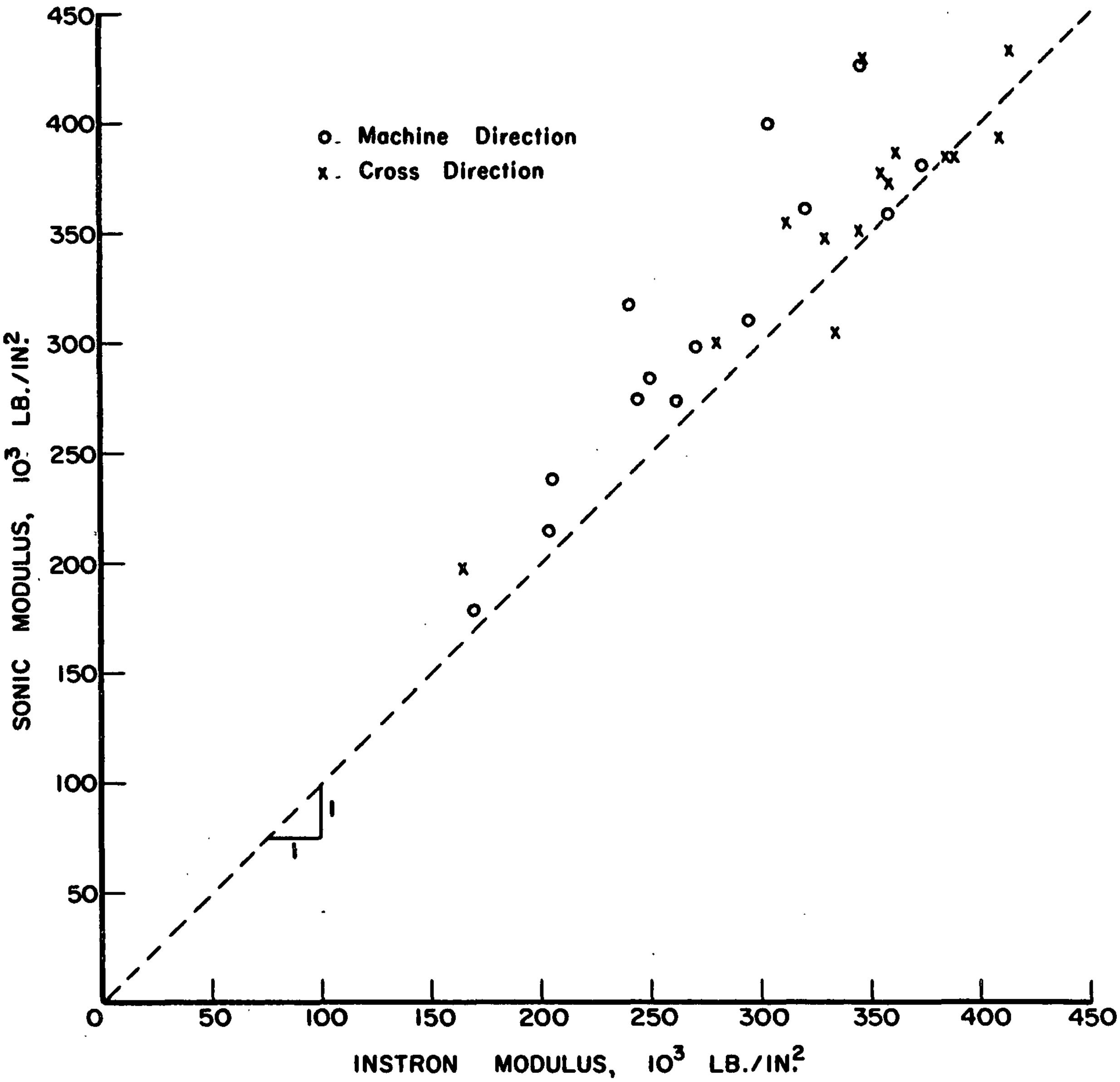

Figure 8. Relationship Between Sonic and Instron Moduli for Extensible Kraft Samples

Contrary to expectations, the sonic modulus was generally less than the Instron modulus for the regular sack papers — by 1.8% in the machine direction and by 3% in the cross direction, on the average. These results were quite unexpected; sonic modulus should exceed Instron modulus because of (a) rate effect and (b) through its definition sonic modulus is an intentional overestimation of the true Young's modulus. Apart from possible systematic error in measurement of time with the Morgan tester or of force and elongation with the Instron tester, it is difficult to conceive of a reason for the inversion of observed moduli. One possible reason, however, is that the Morgan sonic modulus estimate involves an average of tension and compression moduli of the paper because of the oscillatory pulse sent through the specimen. There is some evidence to support the belief that compression modulus is lower than the tension modulus (16). If so, the Morgan sonic modulus would be lower than the sonic modulus in tension and this may tend to offset the effects of rate and neglect of the Poisson ratio discussed above.

As mentioned in THEORY, the ratio of the sonic moduli in the two principal directions should be the same as the ratio of the true Young's moduli. This occurs because the ratio of sonic moduli does not depend, in theory, on the Poisson ratios which have been ignored through the definition of sonic modulus. The ratio of Instron moduli in the two principal directions should be the same as the ratio of Young's moduli and hence as the ratio of sonic moduli, except to the extent that strain rate effects may be different in the two directions of the sheet.

A comparison of the directional ratios for the sonic and Instron moduli is given in the right-hand columns of Table IV. In general, there is quite good agreement between these ratios. However, on the average, the sonic modulus ratio tends to run slightly higher than the Instron ratio, particularly for the extensible sack paper samples. This trend suggests that the machine-direction modulus may be more rate sensitive than the cross-direction modulus for these samples – particularly for the extensible papers.

In summary, the sonic moduli of these sack papers were calculated from sonic velocity measurements. The precision of the moduli are about ± 10%, on the average (95% confidence). This is an optimistic estimate of precision because it neglects the variability in the determination of basis weight and thickness. For extensible papers, the sonic modulus exceeded the Instron (low test rate) modulus by about 9%, on the average, which is in the direction anticipated from theory. With flat kraft papers, on the other hand, the sonic modulus was about 2-1/2% lower than the Instron modulus, a trend which is contrary to theory. The ratio of sonic moduli in the two principal directions of the paper was slightly greater than the directional ratio for Instron modulus; a possible reason for this trend is that the machine direction is more rate sensitive than the cross direction, particularly with extensible papers.

CORRELATION WITH SACK PERFORMANCE

The simple linear regressions relating sack performance and sonic modulus are shown graphically in Fig. 9-12 and in equation form in Table VI. Sack performance is in terms of progressive height face drop and butt drop. Performance data are given in the Appendix; they are taken from Ref. (8).

It may be seen in Table VI that only a few of the simple regressions are statistically significant. Both face drop and butt drop performance were significantly correlated with machine-direction sonic modulus when considered for the combined flat and extensible samples. As frequently happens in these studies, the correlation is enhanced by the relatively large range of a machine-direction property when flat and extensible papers are considered together.

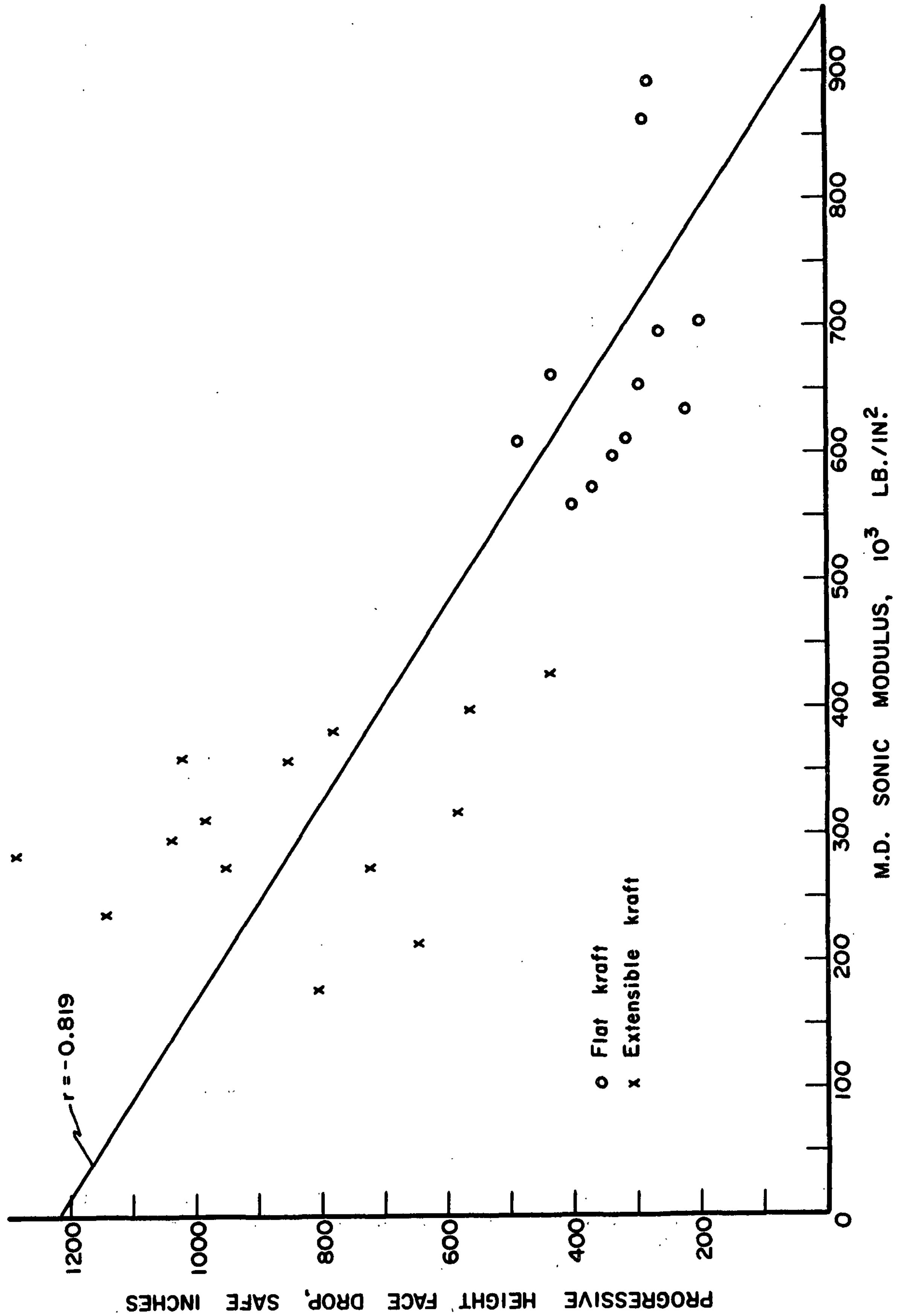

Figure 9. Relationship Between Progressive Height Face Drop Performance and Machine-Direction Sonic Modulus

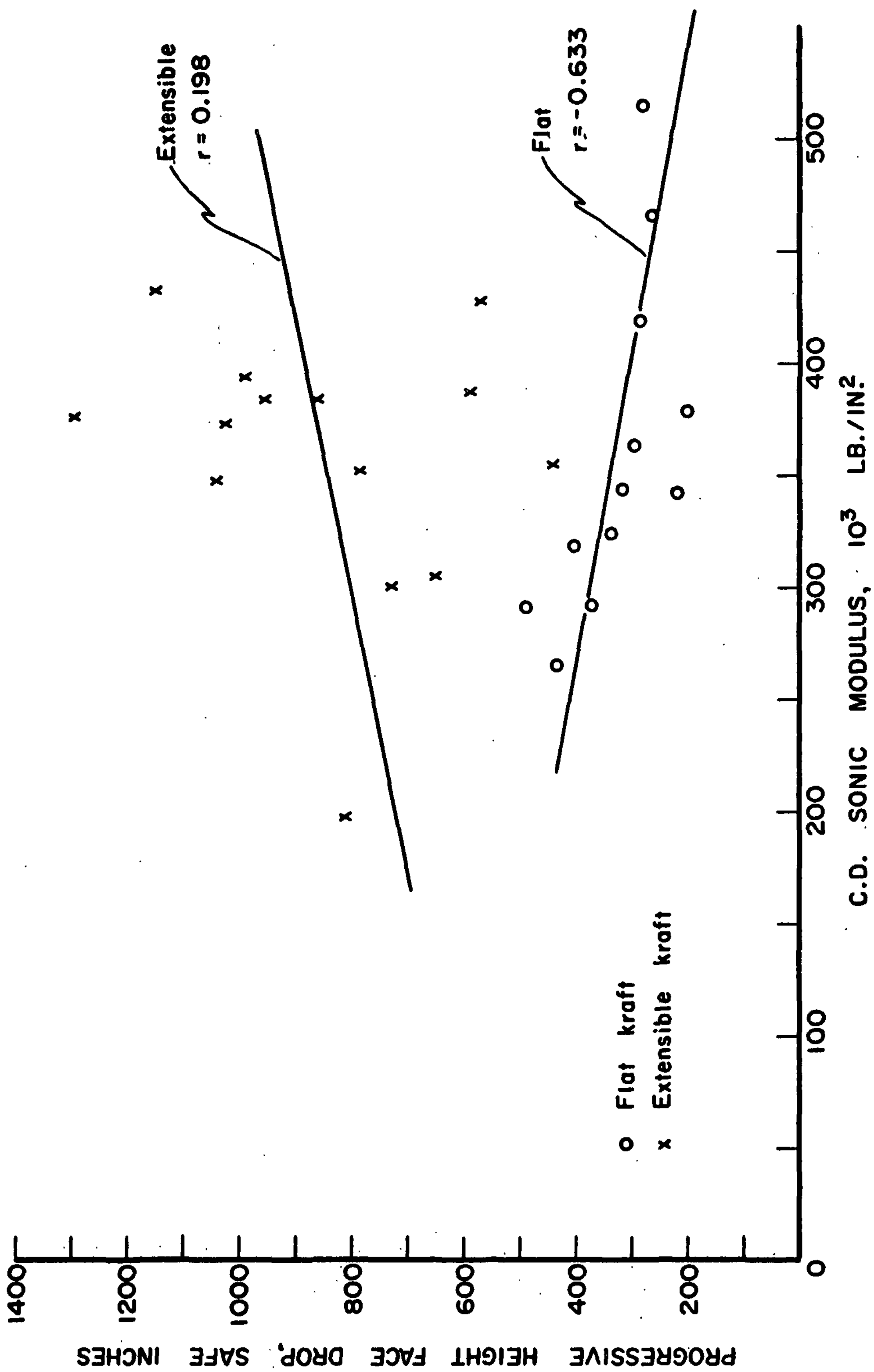

Figure 10. Relationship Between Progressive Height Face Drop Performance and Cross-Direction Sonic Modulus

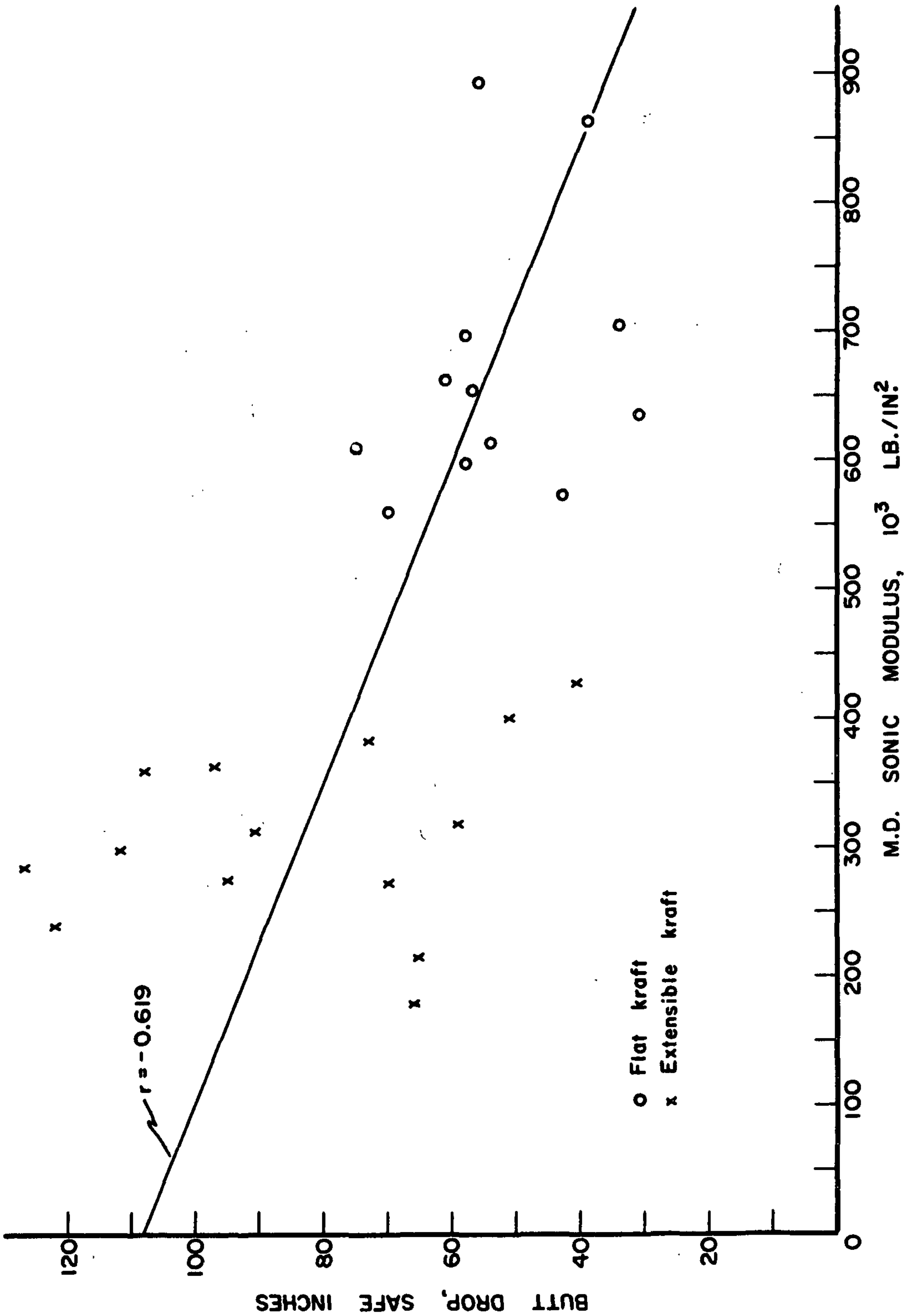

Figure 11. Relationship Between Butt Drop Performance and Machine-Direction Sonic Modulus

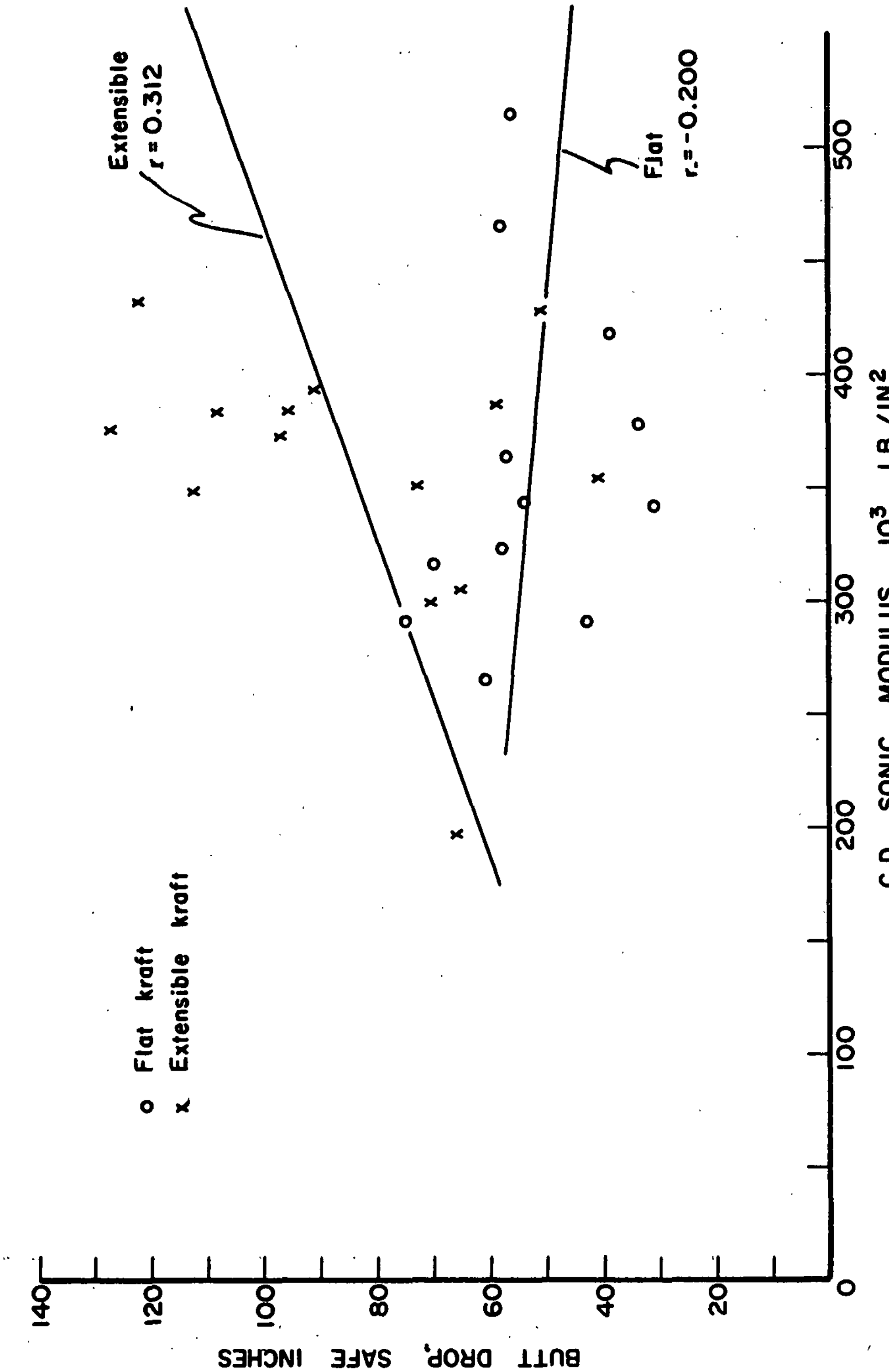

Figure 12. Relationship Between Butt Drop Performance and Cross-Direction Sonic Modulus

TABLE VI

SIMPLE REGRESSIONS OF SACK PERFORMANCE ON SONIC MODULUS

Type of Paper	Correlation Coefficient	Significant[a]	Equation
Progressive Height Face Drop (y) vs. M.D. Sonic Modulus (x)			
Flat	-0.424	No	$y = -0.000343x + 555$
Extensible	-0.358	No	$y = -0.001204x + 1216$
Combined	-0.819	Yes	$y = -0.001290x + 1219$
Progressive Height Face Drop (y) vs. C.D. Sonic Modulus (x)			
Flat	-0.633	Yes	$y = -0.000726x + 586$
Extensible	+0.198	No	$y = +0.000795x + 561$
Combined	-0.024	No	$y = -0.000120x + 648$
Butt Drop (y) vs. M.D. Sonic Modulus (x)			
Flat	-0.284	No	$y = -0.0000367x + 78$
Extensible	-0.276	No	$y = -0.0001046x + 116$
Combined	-0.619	Yes	$y = -0.0000810x + 108$
Butt Drop (y) vs. C.D. Sonic Modulus (x)			
Flat	-0.200	No	$y = -0.0000367x + 66$
Extensible	+0.312	No	$y = +0.0001414x + 33$
Combined	+0.091	No	$y = +0.0000372x + 56$

[a] At 0.05 level.

Cross-direction sonic modulus was not significantly related to sack performance, except in an isolated instance involving face drop of flat kraft samples.

It may be noted that, where significant correlations exist, the slope of the regression line is negative. That is, sack performance decreases with increasing modulus. This observation suggests that increasing stiffness (in the tensile sense) is detrimental to the performance of the paper.

In general, it may be concluded that there is at best a weak relationship between sack performance and sonic modulus of the paper. It is likely that a similar conclusion would be reached with Instron modulus in view of the moderately high correlation between Instron and sonic moduli ($\underline{r}$ = 0.80 to 0.91). These conclusions should not be surprising, because it would seem unusual for rupture of the sack to depend solely upon a prerupture property such as modulus.

On the other hand, it is conceivable that a stiffness property such as modulus in conjunction with an ultimate strength property (e.g., tensile energy absorption) may be correlated with sack performance. It might be conjectured, for example, that a sack paper which is relatively stiff in one direction would cause higher stresses to be developed in the other principal direction during sack impact. This consideration in conjunction with the ultimate strength of the paper in the critical direction may possibly relate to the performance of the sack.

This approach was studied in an empirical way by including sonic modulus in multiple regressions along with tensile energy absorption (T.E.A.). It may be recalled that machine- and cross-direction T.E.A. were found to be highly correlated with sack performance in previous studies (17,18). As shown in Table VII, the sonic moduli in both principal directions of the sheet were included with T.E.A.

TABLE VII

MULTIPLE REGRESSIONS INVOLVING TENSILE ENERGY ABSORPTION AND SONIC MODULUS

Type of Performance	Type of Paper	Regression Equation		Significant Properties[a]	Multiple Correlation Coefficient	Average Prediction Diff., %[b]
Progressive	Flat	F = 781 WI + 555 WC	- 191	WI, WC	0.84	12.1
Height		F = + 666 WC - 0.293 SMI	+ 211	WC	0.79	13.2
Face Drop	(n=12)	F = 448 WI + 609 WC + 0.210 SMI - 0.824 SMC	+ 49	WI, WC, SMC	0.98	5.1
Progressive	Extensible	F = 385 WI + 1676 WC	- 599	WC	0.73	17.9
Height		F = + 2069 WC - 1.535 SMI	+ 127	WC, SMI	0.80	13.4
Face Drop	(n=14)	F = 266 WI + 2074 WC - 1.067 SMI - 0.510 SMC	- 168	WC	0.82	13.8
Progressive	Flat and	F = 460 WI + 771 WC	- 169	WI, WC	0.89	17.4
Height	Extensible	F = 1440 WC - 0.937 SMI	+ 294	WC, SMI	0.91	22.0
Face Drop	(n=26)	F = 335 WI + 1253 WC - 0.232 SMI - 0.374 SMC	- 84	WC	0.92	18.3
Butt Drop	Flat	B = 64.0 WI + 121 WC	-24.4	WC	0.86	10.5
		B = + 129 WC - 0.0270 SMI	+10.7	WC	0.86	10.8
	(n=12)	B = 63.2 WI + 118 WC - 0.0357 SMI + 0.0250 SMC	- 7.8	WC	0.88	9.0
Butt Drop	Extensible	B = 31.9 WI + 233.4 WC	-90.4	WC	0.82	17.8
		B = + 268.8 WC - 0.1476 SMI	-25.3	WC, SMI	0.87	15.3
	(n=14)	B = 11.9 WI + 259.1 WC - 0.1371 SMI + 0.0045 SMC	-39.3	WC	0.88	14.8
Butt Drop	Flat and	B = 16.9 WI + 155.0 WC	-25.6	WI, WC	0.84	15.5
	Extensible	B = 187.9 WC - 0.0348 SMI	-12.4	WC, SMI	0.87	17.1
	(n=26)	B = 0.821 WI + 182.7 WC - 0.0358 SMI + 0.0143 SMC	-15.0	WC	0.87	16.9

[a] 0.05 Level.
[b] Based on observed performance.

Symbols:

Directional Properties	In	Cross
Tensile energy absorption	WI	WC
Sonic modulus x 10^{-3}, (i.e., $\tilde{E}$/1000)	SMI	SMC

Sack Performance	
Progressive height face drop	F
Butt drop	B

n = number of observations

in both principal directions. As a variation on this approach, sonic modulus in the machine direction only was paired with cross-direction T.E.A. This pairing was suggested by the observation that many of the sacks in the second fabrication program failed in what appears to be cross-direction tension and the severity of stress in this direction may depend in an inverse way on the stiffness in the machine direction.

In addition to the regression equations, Table VII lists (a) the paper properties (independent variables) which are significant in each regression, (b) the multiple correlation coefficient, and (c) the predictive ability of the regression equation as reflected in the average absolute percent difference between observed and predicted sack performance. It should be appreciated that the correlation coefficient and the average percent difference do not necessarily rank a series of regressions in identical order. The multiple correlation coefficient reflects the process of fitting the equation by the method of least squares, which minimizes the sum of squares of differences between observed and predicted values of performance. This process does not necessarily minimize the sum of the first power of the differences which are involved in percent differences of prediction.

A variable in the regression equation which is omitted in the "significant properties" column of Table VII is one whose regression coefficient (multiplying constant) is not significantly different from zero at the 0.05 level. It should be remembered that nonsignificance in this sense does not "prove" that the property has no bearing on sack performance, but merely that the data do not contradict the hypothesis that the variable has no effect. However, there are other hypotheses which the data also will not contradict, such as: the property has some small, though definite, effect on sack performance. Viewed in this light, it is seen that the test of significance (a go-no-go affair) is tyrannical. An equivalent statement of nonsignificance is that "the 95% confidence interval for the regression coefficient includes zero as a likely value." This admits the possibility that the coefficient is indeed nonzero but is determined very imprecisely from the data at hand. However, it may be reasoned that when the coefficient of a variable is so small that nonsignificance occurs in the presence of other variables of similar scale and range, then it is likely that the variable in question has at most a weak affect on sack performance and might be omitted with little sacrifice. Significance of a variable perhaps should best be interpreted, therefore, as an indication of the strength of the association rather than as "yes or no" matter (19).

One approach to studying the results in Table VII is to regard the regression containing M.D. and C.D. T.E.A. as a reference and then ascertain whether substitution or addition of sonic moduli improves prediction of sack performance. Following this approach, it may be seen that substitution of M.D. sonic modulus for M.D. T.E.A. (retaining C.D. T.E.A.) in most cases did not improve prediction of sack performance, despite modest increase in the correlation coefficient. There seems nothing to be gained, therefore, through introduction of M.D. sonic modulus in this manner.

Turning attention to the four-factor regressions involving T.E.A. and sonic modulus in both principal directions, all but one of these regressions offer no marked improvement over T.E.A. alone and, in general, show at best a weak dependence on sonic moduli. It may be noted that wherever sonic modulus is a significant variable in any of these multiple regressions, its coefficient is negative, in keeping with the results of the simple correlations discussed above.

There is one remarkably good prediction equation among all those studied, namely, progressive height face drop of flat kraft sacks. The predictive ability of the four-factor equation was 5.1%, on the average. All twelve flat kraft samples were within ± 20% and three-quarters of them were within ± 10%. This is the highest degree of precision ever experienced in studies of this type in Project 2033. The next best precision previously seen was 7.3% with tensile-stretch product and combined Elmendorf tear on flat kraft face drop (17).

The fact that the combination of T.E.A.'s and sonic moduli gave such favorable results in only one instance out of six considerably lessens its importance. However, the one favorable case hints that tensile stiffness of sack paper may be a factor in sack impact performance along with energy absorption capacity and indicates that future research might profitably pursue this matter further.

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APPENDIX

TABLE IX

PERFORMANCE OF MULTIWALL SACKS

Flat Kraft			Extensible Kraft		
Run	Progressive Height Face Drop, safe inches	Butt Drop, safe inches	Run	Progressive Height Face Drop, safe inches	Butt Drop, safe inches
AA	401	70	MM	855	108
BB	370	43	NN	987	91
CC	435	61	OO	1144	122
DD	288	39	PP	781	73
EE	201	34	QQ	1023	97
FF	222	31	RR	1288	127
GG	316	54	SS	438	41
HH	296	57	TT	565	51
II	338	58	UU	585	59
JJ	487	75	VV	1038	112
KK	262	58	WW	807	66
LL	281	56	XX	650	65
			YY	727	70
			ZZ	951	95