Preview of Award 0901355 - Final Project Report

Cover

Federal Agency and Organization Element to Which Report is Submitted:	4900
Federal Grant or Other Identifying Number Assigned by Agency:	0901355
Project Title:	Extremal Problems in Combinatorics and Their Applications
PD/PI Name:	Prasad Tetali, Principal Investigator
Submitting Official (if other than PD\PI):	N/A
Submission Date:	N/A
Recipient Organization:	Georgia Tech Research Corporation
Project/Grant Period:	06/15/2009 - 05/31/2013
Reporting Period:	06/01/2012 - 05/31/2013
Signature of Submitting Official (signature shall be submitted in accordance with agency specific instructions)	N/A

Accomplishments

* What are the major goals of the project?

Study several fundamental problems in extremal combinatorics, many of them motivated by problems in theoretical computer science.

* What was accomplished under these goals (you must provide information for at least one of the 4 categories below)?

Major Activities:	1. A Wowzer Type Lower Bound for the Strong Regularity Lemma
Specific Objectives: Significant Results:	The regularity lemma of Szemerédi asserts that one can partition every graph into a bounded number of quasi-random bipartite graphs. In some applications however, one would like to have a strong control on how quasi-random these bipartite graphs are. Alon, Fischer, Krivelevich and Szegedy obtained a powerful variant of the regularity lemma, which allows one to have an arbitrary control on this measure of quasi-randomness. However, their proof only guaranteed to produce a partition where the number of parts is given by the Wowzer function, which is the iterated version of the Tower function. We show here that a bound of this type is unavoidable by constructing a graph H, with the property that even

if one wants a very mild control on the quasi-randomness of a regular partition, then the number of parts in any such partition of H must be given by by a Wowzer-type function.

2. Color-Critical Graphs Have Logarithmic Circumference

A graph G is k-critical if every proper subgraph of G is (k-1)-colorable, but the graph G itself is not. We prove that every k-critical graph on n vertices has a cycle of length at least log(n)/100log(k), improving a bound of Alon, Krivelevich and Seymour from 2000. Examples of Gallai from 1963 show that the bound cannot be improved to exceed 2(k-1)log(n)/log(k-2). We thus settle the problem of bounding the minimal circumference of k-critical graphs, raised by Dirac in 1952 and Kelly and Kelly in 1954.

3. The Quasi-Randomness of Hypergraph Cut Properties

Let $a_1,...,a_k$ satisfy $a_1+...+a_k=1$ and suppose a k-uniform hypergraph on n vertices satisfies the following property; in any partition of its vertices into k sets $A_1, ..., A_k$ of sizes $a_1*n, ..., a_k*n$, the number of edges intersecting $A_1, ..., A_k$ is (asymptotically) the number one would expect to find in a random k-uniform hypergraph. Can we then infer that H is quasi-random? We show that the answer is negative if and only if $a_1 =...= a_k = 1/k$. This resolves an open problem raised in 1991 by Chung and Graham [J. AMS '91]. While hypergraphs satisfying the property corresponding to $a_1 =...= a_k = 1/k$ are not necessarily quasi-random, we manage to find a characterization of the hypergraphs satisfying this property. Somewhat surprisingly, it turns out that (essentially) there is a unique non quasi-random hypergraph satisfying this property. The proofs combine probabilistic and algebraic arguments with results from the theory of association schemes.

Key outcomes or
Other
achievements:

* What opportunities for training and professional development has the project provided?

PI Shapira was the PhD advisor of Subrahmanyam Kalyanasundaram who completed his PhD in 2011. The project also funded travel to many conferences where the PI gave lectures and interacted with other researchers.

* How have the results been disseminated to communities of interest?

Several project results have appeared in general mathematical journals that is, journals that publish results in all areas of mathematics. Lectures in many conferences and workshops were also given.

Products

Journals

A. Shapira, R. Yuster (3/20/10). Multigraphs (Only) Satisfy a Weak Triangle Removal Lemma+. *Electronic Journal of Combinatorics*. 16   N11.

Status = PUBLISHED; Acknowledgment of Federal Support = No ; Peer Reviewed = Yes

A. Nachmias. A. Shapira (11/23/09). Testing the Expansion of a Graph. *Information and Computation*. 208 (4),   309-314.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

A. Shapira, R. Yuster (10/21/09). The Effect of Induced Subgraphs on Quasi-Randomness. *Random Structures and Algorithms*. 36   90-109.

Status = PUBLISHED; Acknowledgment of Federal Support = No ; Peer Reviewed = Yes

A. Shapira (2/3/10). A Proof of Green's Conjecture Regarding the Removal Properties of Sets of Linear Equations. *Journal of the London Math Society*. 81   355-373.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

I. Benjamini, O. Schramm, A. Shapira (4/10/10). Every Minor-Closed Property of Sparse Graphs is Testable. *Advances in Math*. 223 (6),   2200-2218.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

E. Fischer, A. Matsliah, A. Shapira (7/8/10). Approximate Hypergraph Partitioning and Applications. *SIAM Journal on Computing*. 39 (7),   3155-3185.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

A. Shapira, R. Yuster (8/7/10). On the Density of a Graph and its Blowup. *Journal of Combinatorial Theory Ser. B*. 100 (6),   704-719.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

K. Costello, P. Tetali, A. Shapira (6/15/10). On Randomizing two Derandomized Greedy Algorithms. *Journal of Combinatorics*. 1 (3),   265-283.

Status = PUBLISHED; Acknowledgment of Federal Support = No ; Peer Reviewed = No

E. Even-Dar, A. Shapira (11/30/10). A Note on Maximizing the Spread of Influence in Social Networks. *Information Processing Letters*. 111 (4), 184-187.

Status = PUBLISHED; Acknowledgment of Federal Support = No ; Peer Reviewed = Yes

A. Shapira, R. Yuster, U. Zwick (6/3/09). All-Pairs Bottleneck Paths in Vertex Weighted Graphs. *Algorithmica*. 59 (4), 621-633.

Status = PUBLISHED; Acknowledgment of Federal Support = No ; Peer Reviewed = Yes

A. Shapira, R. Thomas (8/20/11). Color-Critical Graphs Have Logarithmic Circumference. *Advances in Math*. 227 (6), 2309-2326.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

L. Roditty, A. Shapira (9/1/11). All-Pairs Shortest Paths with a Sublinear Additive Error. *ACM Transactions on Algorithms*. 7 12 pp.

Status = PUBLISHED; Acknowledgment of Federal Support = No ; Peer Reviewed = Yes

R. Rubinfeld, A. Shapira (11/22/11). Sublinear Time Algorithms. *SIAM Journal on Discrete Math*. 25 (4), 1562-1588.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

A. Shapira, R. Yuster (3/30/11). The Quasi-Randomness of Hypergraph Cut Properties. *Random Structures and Algorithms*. 40 (1), 105-131.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

D. Dellamonica, S. Kalyanasundaram, D. Martin, V. Rodl, A. Shapira (1/3/12). A Deterministic Algorithm for the Frieze-Kannan Regularity Lemma. *SIAM Journal on Discrete Math*. 26 (1), 15-29.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

A. Bhattacharyya, E. Grigorescu, P. Raghavendra, A. Shapira (8/10/12). Testing Odd-Cycle-Freeness of Boolean Functions. *Combinatorics, Probability and Computing*. 21 (6), 835-855.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

S. Kalyanasundaram, A. Shapira (10/8/12). A Wowzer Type Lower Bound for the Strong Regularity Lemma. *Proc. of the London Math Society*. 106 (3), 621-649.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

S. Kalyanasundaram, A. Shapira (7/1/14). A Note on Even Cycles and Quasi-Random Tournaments. *Journal of Graph Theory*. TBD TBD.

Status = AWAITING_PUBLICATION; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

A. Czumaj, O. Goldreich, D. Ron, C. Seshadhri and C. Sohler (7/1/14). Finding Cycles and Trees in Sublinear Time. *Random Structures and Algorithms*. TBD TBD.

Status = AWAITING_PUBLICATION; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

A. Bhattacharyya, E. Grigorescu, A. Shapira (7/1/14). A Unified Framework for Testing Linear-Invariant Properties. *Random Structures and Algorithms*. TBD TBD.

Status = AWAITING_PUBLICATION; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

D. Dellamonica, S. Kalyanasundaram, D. Martin, V. Rodl, A. Shapira (7/1/14). An Optimal Algorithm for Computing Frieze-Kannan Regular Partitions. *Combinatorics, Probability and Computing*. TBD TBD.

Status = AWAITING_PUBLICATION; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

Books

Book Chapters

Thesis/Dissertations

Conference Papers and Presentations

Other Publications

Technologies or Techniques
Nothing to report.

Patents
Nothing to report.

Inventions
Nothing to report.

Licenses
Nothing to report.

Websites
Nothing to report.

Other Products

Nothing to report.

Participants

Research Experience for Undergraduates (REU) funding

What individuals have worked on the project?

Name	Most Senior Project Role	Nearest Person Month Worked
Asaf Shapira	PD/PI	2
Prasad Tetali	PD/PI	1

What other organizations have been involved as partners?
Nothing to report.

Have other collaborators or contacts been involved? N

Impacts

What is the impact on the development of the principal discipline(s) of the project?

Some important open problems were solved during the three years of the project. Some of them have been open for more than 20 years. A number of studies were initiated that have the potential of having many follow up papers.

What is the impact on other disciplines?
Nothing to report.

What is the impact on the development of human resources?

Under PI Tetali's direction, graduate students Ioannis Panageas and Arindam Khan were afforded the opportunity to attend conferences that included the Abel Conference at the IMA in honor of Endre Szemeredi and the ACM-SIAM annual Symposium on Discrete Algorithms.

What is the impact on physical resources that form infrastructure?
Nothing to report.

What is the impact on institutional resources that form infrastructure?
Nothing to report.

What is the impact on information resources that form infrastructure?
Nothing to report.

What is the impact on technology transfer?
Nothing to report.

What is the impact on society beyond science and technology?
Nothing to report.

Changes

Changes in approach and reason for change
Nothing to report.

Actual or Anticipated problems or delays and actions or plans to resolve them
Nothing to report.

Changes that have a significant impact on expenditures
Nothing to report.

Significant changes in use or care of human subjects
Nothing to report.

Significant changes in use or care of vertebrate animals
Nothing to report.

Significant changes in use or care of biohazards
Nothing to report.