Use of a Conceptual Sizing Tool for Conceptual Design of Tactical Missiles (U)

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ABSTRACT

This paper illustrates the use of a conceptual sizing tool for the design of tactical missiles. The sizing tool, called the Tactical Missile Design (TMD) spreadsheet, was developed to allow the user to quickly generate estimates of a missile configuration's performance and other measures of merit such as lethality. This capability allows the user to get a first order estimate of a missile's ability to meet a set of requirements and allows for fast trade-studies to quickly identify the performance drivers of a system.

In order to generate reasonable estimates for missile range and speed, a sizing tool must have analysis methods for aerodynamics, propulsion, weight, and trajectory. In order for the sizing tool to remain useful for conceptual tradeoffs, these analyses must be robust enough to handle a wide range of inputs, yet simple enough to be executed quickly. The analysis methods were constructed with these requirements in mind. The aerodynamics analysis is based upon several physics-derived analytical expressions as outlined in the text "Tactical Missile Design" by Eugene Fleeman. Propulsion uses a simplified cycle analysis to relate engine parameters (maximum inlet temperature, fuel heating value, expansion ratio) to overall specific impulse (I_{sp}). The propulsion analysis can handle either air-breathing and solid-rocket systems, or a combination thereof. For trajectory, a constant flight path is assumed, with a boost, cruise, and coast phase. These straightforward analysis methods combine to produce a very powerful, yet robust, conceptual missile design tool.

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The paper first lays out the analysis methods, assumptions, and limitations in the Tactical Missile Design spreadsheet. Next, a comparison is made between the results of the TMD spreadsheet and the performance of historical missile systems. In addition, the paper explores some example trade studies to identify the drivers of a rocket and a ramjet missile system. Finally, the TMD spreadsheet is used to show how easily a tactical missile can be optimized at the conceptual level.

INTRODUCTION

In early stages of conceptual design, sizing tools are needed that allow for quick tradeoffs among design parameters. For greatest effectiveness, these sizing tools must meet three requirements, they must accept wide variations in the first order parameters of components, they must accurately show the effects of parameter variation, and they must be capable of rapid, economical analysis[1]. Rapid analysis is required in conceptual design because the multi-dimensional nature of the design problem often results in a need to examine literally thousands of potential system configurations.

In the case of missile design, a tool is needed that can analyze a missile with a set of geometry, structural, and propulsion parameters and calculate the missile performance. In this case, the performance of the system may be summarized by the missile range, maximum velocity, and time-to-target. In many cases, these performance characteristics can be calculated from first-order, physics-based analyses. Often, these analyses can be represented in terms of straightforward analytical expressions that allow for easy calculation. The elegance of this approach is that with the linking of reasonably simple analytical expressions from multiple disciplines, a very powerful, yet useable tool can be developed that gives very fast and reasonably good estimates of missile system performance.

A spreadsheet developed in Microsoft Excel was used as the framework for this conceptual sizing tool. Microsoft Excel was chosen because of its wide availability and familiarity among both engineers and managers. Furthermore, Excel has all the functionality needed to handle the required calculations. Excel also

allows for the easy visualization of both interim and final numbers, thus increasing the overall tractability of the solution.

This paper first illustrates the Tactical Missile Design (TMD) spreadsheet, explaining the overall layout and governing equations. Later the paper looks at potential uses of the TMD tool, exploring single and then multiple dimension case studies. Finally, the paper explores missile optimization using the Tactical Missile Design spreadsheet.

TACTICAL MISSILE DESIGN SPREADSHEET

The Tactical Missile Design spreadsheet includes analyses for many disciplines. These disciplines are: aerodynamics, propulsion, trajectory, structure, warhead, radar, and dynamics. These disciplines include all the required disciplines for a complete conceptual design process. This process is illustrated in Figure 1. The spreadsheet is constructed so that each discipline is handled on an individual worksheet. This breakdown allows the user to focus on each discipline individually. Linking of the individual disciplines/worksheets is done through two master worksheets, one worksheet that is designed to handle user-inputs and one worksheet that holds results from the disciplinary worksheets. Part of the user-input worksheet is given in Figure 2. An additional output

worksheet also includes key figures and charts illustrating the current design's performance. Baseline inputs are included in the TMD spreadsheet for two missiles, a rocket system and an air-breather. These missiles give the user baseline systems from which to perform trade studies. The rocket is based on the Sparrow Medium-Range Air-to-Air Missile (MRAAM) while the air-breathing baseline is based upon the ramjet powered ASALM, or Advanced Strategic Air-Launched Missile.
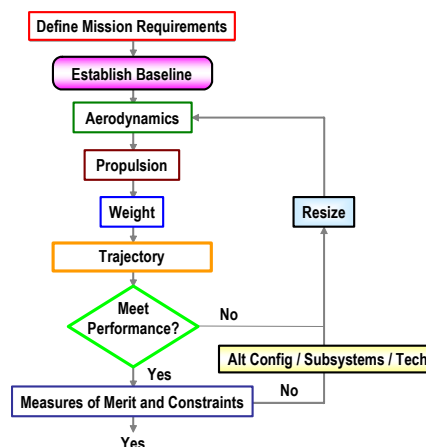

Figure 1: Process for Conceptual Missile Sizing and Synthesis[2]

Description	Variable Name	Value	ROCKET Default Value	RAMJET Default Value	Units	Additonal Information
Aerodynamics						
Missile Body Input						
Missile Length	l	125.29	143.90	171.00	*inches*	
Missile Boby Major Diameter	2a	8.74	8.00	20.38	*inches*	
Missile Body Minor Diameter	2b	0.00	0.00	0.00	*inches*	Zero defaults to circular cross-section
Nose Length	L_N	20.83	19.20	23.50	*inches*	
Nose Bluntness (0 = sharp nose, 1 = hemispherical nose)	D_{Hemi}	10%	10%	5%		
Missile Wing/Canard Input						
Number of wings (0,1,2)	N	2	2	0	*integer*	2 panels per wing
Wing Area	S_W	396.430906	367.2	0.0	*inches^2*	N=1 is a straight wing
Wing LE Thickness Angle		10	10	0	*deg*	N=2 is a cruiciform wing
Wing Airfoil Thickness to Cord, max	t/c,max	4.40%	4.40%	0.00%	%	
Wing Incident Angle	Waoa	0.0	0.0	0.0	*deg*	
Wing Aspect Ratio	War	2.82	2.82	0.00		
Station of Wing leading edge	Wle	60.8	60.8	0	*inches*	
Wing taper ratio	Wtr	0.175	0.175	0		
Missile Tail Input						
Number of Tail Wings		2	2	2	*integer*	
Mach Number for Tail Sizing	Mtail	1.5	1.5	1.5		
AOA for Tail Sizing	Taoa	0.0	0	0	*deg*	
Tail Aspect Ratio	Tar	2.59	2.59	1.64		
Tail Taper Ratio	Ttr	0	0	0.7		
Station of Tail LE	Tle	125.4	125.4	150.33	*inches*	
Tail Thickness-to-Chord, max	t/c,max	2.7%	2.7%	4.0%		
Tail LE Thickness Angle		6.2	6.2	9.1	*deg*	

Figure 2: Example Input for Tactical Missile Design Spreadsheet

Aerodynamics

The aerodynamics discipline requires inputs that define the missile geometry. These include nose fineness, length, diameter, number of wings, wing aspect ratio, taper ratio, wing area, etc. From these inputs, the aerodynamics discipline calculates the lift, drag, and center-of-pressure for the vehicle for various Mach numbers. Drag is calculated by doing a build-up of the

individual drag components: body friction, body wave, wing and tail friction, wing and tail wave, and base drag. These drag components are then added together to calculate the full drag coefficient of the vehicle. The drag calculations are based upon approaches given in references 2, 3, 4, 5, and 6. Linear wing theory, slender wing theory, and Newtonian impact theory are used for the calculations of the normal force on the wing and

tail. Slender body theory and crossflow theory are used in the calculations of the normal force on the body[3].

In addition to calculations of lift and drag, the aerodynamics section also includes tail sizing. Inputs such as tail aspect ratio, taper ratio, and distance from the nose are used to calculate the required tail area for neutral static stability at a user-specified Mach number. The contributions to drag and lift from the tail are then included in the total vehicle drag and lift calculations. An outline of the rocket baseline missile, with the given wing geometry and appropriately sized tail, is also provided to the user (see Figure 3).
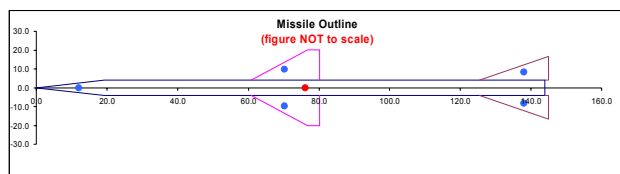

Figure 3: Missile Outline with Appropriately Sized Tail

Propulsion

The propulsion discipline is designed to handle a dual-thrust level missile. It has the capability of analyzing both rockets and air-breathing systems. The first thrust level is assumed to be a rocket-based boost motor. For the rocket, the user inputs the propellant-type, combustion chamber pressure, burn time, and expansion ratio. From this data, simple rocket relations are used to calculate the throat area, exit area, thrust, and specific impulse. The spreadsheet warns the user whenever the exit area is greater than the maximum diameter of the missile.

For the sustain thrust levels, the user selects between an air-breather and a rocket system. Rocket-based sustain motors are handled identically to the boost motor. For the air-breather, the user selects between several potential options: turbofan, turbojet, ramjet, or scramjet. For the turbofan, turbojet, and scramjet systems, a look-up table for I_{sp} versus Mach number is used. However, for the ramjet, the thrust and I_{sp} are explicitly calculated using a simple cycle analysis. This analysis takes the fuel type, fuel-to-air ratio, atmospheric and flight conditions, and an estimated pressure recovery to calculate the ramjet thrust and I_{sp}. Once the thrust and I_{sp} of the entire vehicle is known, along with the aerodynamics data, the vehicle trajectory can be calculated.

Trajectory

The trajectory discipline begins by assuming that the missile flies a constant flight path angle. This type of

flight-path is consistent with that of guided missile systems. The trajectory is broken into three phases: boost, sustain, and coast. The spreadsheet includes the option for the missile to decrease its empty weight between the boost and sustain motors. This decrease models the discarding of a boost motor or nozzle. To calculate the trajectory of the missile, the rocket equation is used to estimate the speed to which the rocket/boost motor accelerates the vehicle. This equation assumes a constant thrust and I_{sp} (provided by the propulsion discipline). In addition, an average drag over the boost stage is calculated by first calculating the drag at the initial flight condition, estimating the final boost velocity, and then calculating the drag at this new flight condition. The drag at the launch and burnout conditions are averaged together to obtain a drag estimate for Equation 1.

$$\Delta V = -g\, I_{sp}\, (1-D/T)*\ln(1 - W_P / W_L) \qquad \text{EQ 1}$$

The sustain motor's portion of the trajectory is handled separately depending on whether the system is rocket-based or air-breathing. For a rocket-based system, the sustain motor is handled identically to the boost motor, via Equation 1, with a similar estimate made for the average drag. For the air-breathing system, the velocity is assumed to remain constant throughout the cruise-portion of the flight. The lift-to-drag ratio for this condition is calculated by the aerodynamics discipline, which determines the required angle-of-attack for the missile. With the I_{sp} from propulsion and the lift-to-drag ratio determined by the aerodynamics, the cruise range of the missile is found using the Breguet Range Equation (Equation 2).

$$R = (L/D)\, I_{sp}\, V \ln[\,W_L / (W_L - W_P)\,],$$
$$\text{Breguet Range Equation} \quad \text{EQ 2}$$

There are two options in the modeling of the coast portion of flight. The first option is to continue assuming a constant flight path angle for the entire coast phase. This option uses a simple 1-DOF model, with an average drag value, to estimate the time and distance that the vehicle coasts. It assumes that the missile will coast down to a threshold Mach number set by the user. The 1-DOF equations used for the coast calculations are shown in Equation 3 and Equation 4.

$$V_{EC} / V_{BC} = 1 / \{ 1 + t_{coast} / [\, 2\, W_{BO} / (g\, \rho\, S_{Ref}\, C_{D0}\, V_{BC})]\} \quad \text{EQ 3}$$

$$R_{coast} / [\, 2\, W_{BO} / (g\, \rho\, S_{Ref}\, C_{D0})] =$$
$$\ln\{1 + t_{coast} / [\, 2\, W / (g\, \rho\, S_{Ref}\, C_{D0}\, V_{BC})]\} \quad \text{EQ 4}$$

The second option for the coast portion of flight is to assume that the vehicle maintains zero angle-of-attack and falls ballistically to a specified altitude. Again,

simple analytical expressions are developed for the ballistic trajectory which give the time and distance to impact, along with the impact velocity.

Plots, as generated by the Tactical Missile Design spreadsheet, are given in Figure 4 through Figure 7. These plots show the time-history of two missiles, the baseline rocket and baseline ramjet, and include both a co-altitude coast and the ballistic coast trajectory patterns for each system. In addition, Figure 8 shows the effects of launch altitude on the baseline rocket's range for a co-attitude flight.
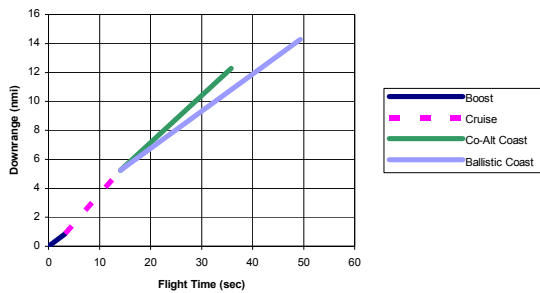

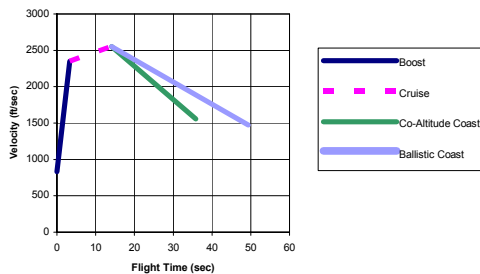**Figure 4: Missile Range vs. Flight Time for the Baseline Rocket at 20,000 feet**

Figure 5: Missile Velocity vs. Flight Time for the Baseline Rocket at 20,000 feet
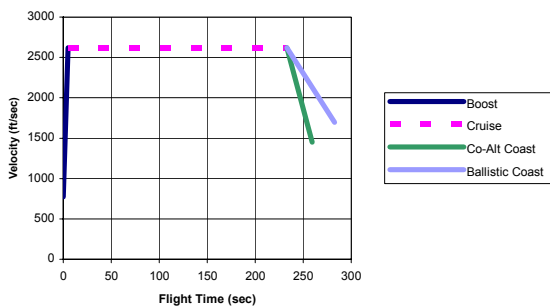

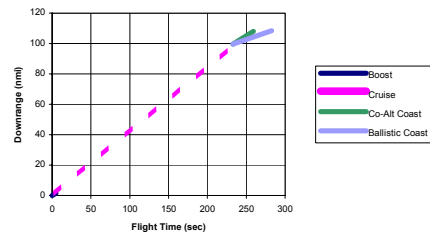**Figure 6: Missile Velocity vs. Flight Time for the Baseline Ramjet at 40,000 feet**

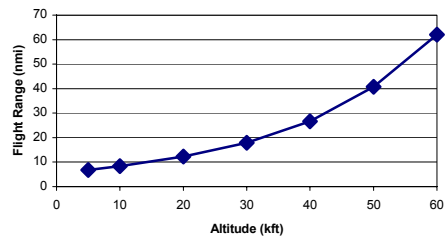**Figure 7: Missile Range vs. Flight Time for the Baseline Ramjet at 40,000 feet**

Figure 8: Missile Range vs. Launch Altitude for Baseline Rocket

Other Disciplines

The remaining disciplines are not intrinsically linked with the remainder of the Tactical Missile Design spreadsheet, i.e., there is no feedback from these disciplines into the missile sizing and synthesis, which basically consists of aerodynamics, propulsion, and trajectory fully coupled together. For example, the structures discipline does not calculate an empty weight that is used by the trajectory module. Instead, the structural worksheet exists as a stand-alone tool and can be used to estimate the required motor case dimensions and weight and the skin temperature of the missile. The analysis uses the expected loads on the missile, material properties, and the maximum Mach number from the trajectory discipline to calculate these values.

The warhead discipline is also an independent section. It takes into account the warhead dimensions, material, explosive weight, and impact velocity to determine the penetration depth of a hard target and the effective overpressure of the explosion. The worksheet can be used to simulate a variety of warheads, including a simple high-explosive warhead, a hit-to-kill warhead, or a combined penetrator/blast-frag warhead. The radar discipline uses the radar range equation to calculate the 3-dB beam-width of the system and estimates the detection range of various targets. The dynamics section is used to calculate the expected miss distance of a target in addition to dynamic considerations such

as horizontal turn radius. Miss distance is calculated by first estimating the total missile time constant and then accounting for flight time, target maneuverability, and initial heading error. The methods for these calculations are laid out in References 2, 3, 7, and 8.

VALIDATION

Verification and validation of the Tactical Missile Design spreadsheet was accomplished through comparisons with computer analysis codes and actual test data. For the baseline rocket case, the MRAAM missile was compared to wind tunnel data and a computer analysis program: Advanced Design of Aerodynamics Missiles (ADAM). The results of this comparison for one example are shown below, where for fixed launched conditions, it was desirable to see how quickly the rocket could travel 6.7 nautical miles at a flight altitude of 20,000 feet. As Table I shows, the calculated flight time of the missile and zero-lift drag coefficient compares well with the computer simulation (ADAM)[9], although there is some discrepancy with the wind tunnel data due to the much higher zero-lift drag coefficient estimated from the wind tunnel data.

Table I: Comparison of Missile Flight Time and CD_0 for 6.7 nmi Flyout

	Flight Time (sec)	Coast Cdo
ADAM	18	0.53
Wind Tunnel	21	1.05
TMD spreadsheet	**17.9**	**0.59**

Table II shows a comparison of a calculated trajectory from the Tactical Missile Design spreadsheet and MRAAM test data[10]. Note that the burnout velocity calculated by the TMD spreadsheet is higher than the actual data, and hence the ranges are higher, but overall the results compare favorably. Further TMD comparisons are planned against complete MRAAM and other missile wind tunnel data.

Table II: Comparison of TMD Predicted Missile Flight Time with Test Data

	Burnout Vel (ft/sec)	Burnout Range (nmi)	Total Range (nmi)
Test Data	2147	4.5	9
TMD spreadsheet	**2488**	**5.04**	**11.6**

ONE DIMENSION CASE STUDY

The Tactical Missile Design spreadsheet allows for the user to easily perform trade-studies. By changing input cells manually and tracking the results, the user can quickly do one-dimensional trade studies, searching for the optimal setting of any variable. The TMD spreadsheet was explicitly designed to give the user this type of capability. A quick example of this type of one-dimensional case study is given below. From the ramjet baseline system, the missile outer diameter was

varied from the original value of 20.38 inches to a minimum of 14 inches and a maximum of 24 inches. The total volume of the missile was held constant, so the length increased as the diameter decreased. Naturally, this type of length to diameter relationship would be contingent upon the subsystems being packageable into a smaller diameter missile and the missile maintaining launcher compatibility; but the relationship is sufficient for this level of analysis. A few key response parameters that were tracked are listed in Table III.

Table III: Case Study with Varying Missile Diameter

					Baseline		
Missile Diameter (in)	14	16	18	20	20.38	22	24
Burnout Mach	2.78	2.77	2.74	2.71	2.71	2.67	2.63
Flight Range (nmi)	257.7	195.4	148.2	113.5	108.0	88.2	69.7
Flight Time (sec)	601	459	351	272	259	214	172
Horizontal Turn Rad (ft)	14956	15471	15556	15296	15215	14783	14105

The results from the quick study can be represented graphically, as seen in Figure 9 and Figure 10. From these figures it is readily apparent that decreasing the diameter greatly enhances the maximum range of the missile. In addition, the user can see that the turn radius is benefited by either a large or small diameter, but for diameters near the baseline the turn radius performs poorly. These type of simple trade-offs can be readily made in the Tactical Missile Design spreadsheet environment.
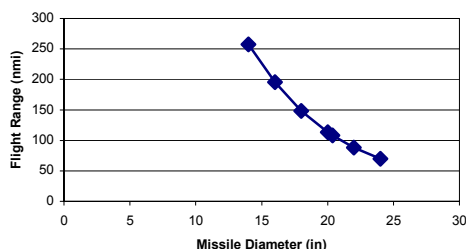

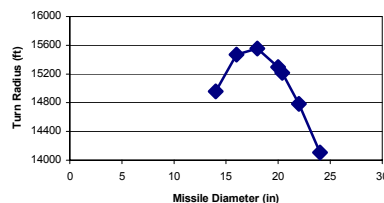
Figure 9: Variation of Flight Range with Missile Diameter

Figure 10: Variation of Missile Turn Radius with Diameter

MULTI-DIMENSIONAL CASE STUDY

The next study was accomplished by parametrically varying multiple design parameters. Seven design variables were chosen for this exploration, they are listed in Table IV. The ranges of the variables used in the study are also shown in the table. In order to study the effects of these variations, a full factorial Design of Experiments was used. A Design of Experiments, used here, is a statistical mechanism that identifies which experimental runs should be made to capture the most response behavior for the fewest number of total experimental runs[11]. For this application, an experimental run consists of a single setting of each of the design variables in the spreadsheet. In the study, 129 full-factorial runs were made, along with 21 random runs which were made by randomly setting each design variable. The results of these variations were tracked for 10 design outputs. The outputs tracked are given in Table V, along with the results from the rocket baseline and the minimum and maximum parameter values achieved in the study. Note that by no means did this study use a comprehensive set of design variables or outputs. Several other variables, such as propellant weight, could also have been used. These variables were simply chosen to be illustrative of the techniques and capabilities of the TMD spreadsheet.

Table IV: Ranges of Design Parameters in Parametric Study

	Launch Weight (lbm)	Diameter (in)	Nose Length (in)	Wing Area (in²)	Expansion Ratio	Boost Chamber Pressure (psi)	Sustain Chamber Pressure (psi)
Rocket Baseline	500	8	19.2	367.2	6	1769	300
Minimum	400	8	19	367.2	6	1769	300
Maximum	500	12	25	400	15	2500	1000

Table V: Minimum and Maximum Responses from Case Study

	Final Weight (lbm)	Maximum Velocity (ft/sec)	Burnout Range (nmi)	Final Range (nmi)	Final Time (sec)	Coast Cd0	Boost I_{sp} (sec)	Sustain I_{sp} (sec)	Horiz Turn Radius (ft)
Rocket Baseline	367	2537.7	5.22	12.06	35.24	0.607	270.5	252	4181
Smallest Response	267	2235	4.11	1.89	3.28	0.488	271	237	3160
Largest Response	367	2878	6.18	14.62	40.90	0.761	286	276	4218

One advantage of running a multi-dimensional study is that the primary drivers for each system level response, or performance metric, can be calculated. This concept is commonly referred to as the "Pareto Principle", which states that 80% of the response of the system is driven by only 20% of the variables. In general, this principle holds for most system responses, indicating that in general, only a few of the design variables tend to drive most of the variation in any one metric[12]. A statistical software package, JMP, was used to analyze the data from the 150 cases in this study. This software

was able to generate pareto charts for each response. These pareto charts show the relative impact of each design variable on the response. For instance, Figure 11 shows which variables contributed the most to the variation in maximum velocity. The design variables listed in the figure are shown in descending order, i.e., the design variable on top had the greatest contribution to the variation in maximum velocity. Thus, perturbing this variable would effect the greatest change upon maximum velocity. The impact of each variable is shown graphically in a bar chart, with the cumulative response shown with a line. Figure 11 indicates that of the seven design parameters, only three parameters, launch weight, missile diameter, and expansion ratio, contributed to over 90% of the variability in the maximum velocity performance metric. This information is very useful, since the user now knows that to have any appreciable effect on the missile velocity, the weight, diameter, or expansion ratio must be changed. Conversely, the decision-maker also knows that the other design variables may be changed without a significant impact on the missile velocity.
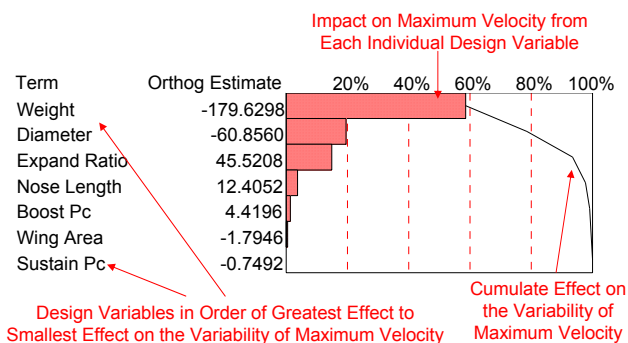

Figure 11: Pareto Chart Showing the Key Parameters Contributing to the Variability of Maximum Velocity

Another pareto chart, Figure 12, shows the key drivers in missile range. This chart also indicates that only three design variables contribute to the variation in missile range. Similar to the maximum velocity, these design variables, missile diameter, nose length (and hence nose fineness), and launch weight, account for nearly 90% of the total variability in missile range. Therefore, to affect the range, the decision-maker will need to alter one of these three key variables. Similar pareto charts can be constructed for all the performance metrics.
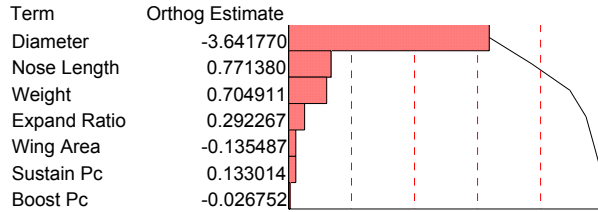
Term	Orthog Estimate
Diameter	-3.641770
Nose Length	0.771380
Weight	0.704911
Expand Ratio	0.292267
Wing Area	-0.135487
Sustain Pc	0.133014
Boost Pc	-0.026752

Figure 12: Pareto Chart Showing the Key Parameters Contributing to the Variability of Missile Range

Another powerful tool provided by the JMP statistical software package is the prediction profile. The prediction profile is a tool that shows how the design variables affect the overall system responses. A segment of the prediction profile is shown in Figure 13. This portion of the prediction profile relates three design variables, launch weight, missile outer diameter, and nose length to the final flight time and range. The center values along the x-axis indicate the current settings of the design variables, around these values are the minimum and maximum ranges of the design variables. In the JMP software package, these settings can be altered through the dynamical GUI. The center values on the y-axis show the system performance or system metrics resulting from the current design variable settings. With the illustrated settings of a 450 lb_m of weight, a 10 inch diameter, and a 22 inch nose length, the missile will travel approximately 8.9 nautical miles in about 26 seconds. Another beauty of this tool is that it shows the partial derivatives, or trendlines, of each of the design variables. These trendlines make the interactions between each design variable and performance characteristics clear. For instance, it is apparent from the figure that by increasing the missile diameter, both the range and time of flight of the missile are decreased. Similarly, by increasing the nose length, thereby increasing the nose fineness ratio, the user will increase both the flight range and time; however, this effect is much less pronounced than the effect of changing the outer diameter. Again, in the JMP software, the system is dynamic, so changing the value of a variable will immediately result in the user seeing the effects on the system response and on the individual trendlines.
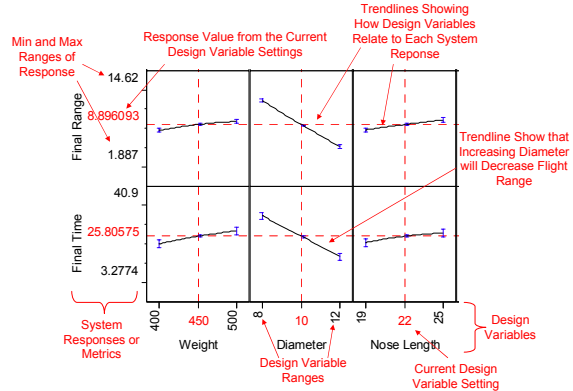

Figure 13: Prediction Profile Comparing Design Variables (x-axis) to the Flight Time and Range

These prediction profiles can be used in two ways. First, they provide insight into the behavior of the system. The user can examine the trendlines to determine if the system behaves as predicted, i.e., one would expect that for an underexpanded rocket plume, increasing the expansion ratio would have a dramatic effect on I_{sp}, and hence velocity. This expected trend can be verified by observing whether the trendlines in the prediction profile indicate this effect. The user can therefore use the prediction profile as a diagnostic tool to ensure that the appropriate trends are being captured in the analysis program. A prediction profile for the entire missile conceptual design case study is shown in Figure 14. From this figure the user can see that the system behaves as predicted and can readily identify key drivers to the system.

A second use of the prediction profile is for optimization. Since the prediction profile is set in a dynamical GUI environment, the user can use the computer mouse to alter the variable settings until an optimum is reached. In addition, the JMP software comes with an option that will allow for the automated optimization of the missile design variables. Thus, the combination of the Tactical Missile Design spreadsheet with the JMP statistical package greatly enhances the ability of the conceptual designer to make fast, accurate decisions about the missile design.
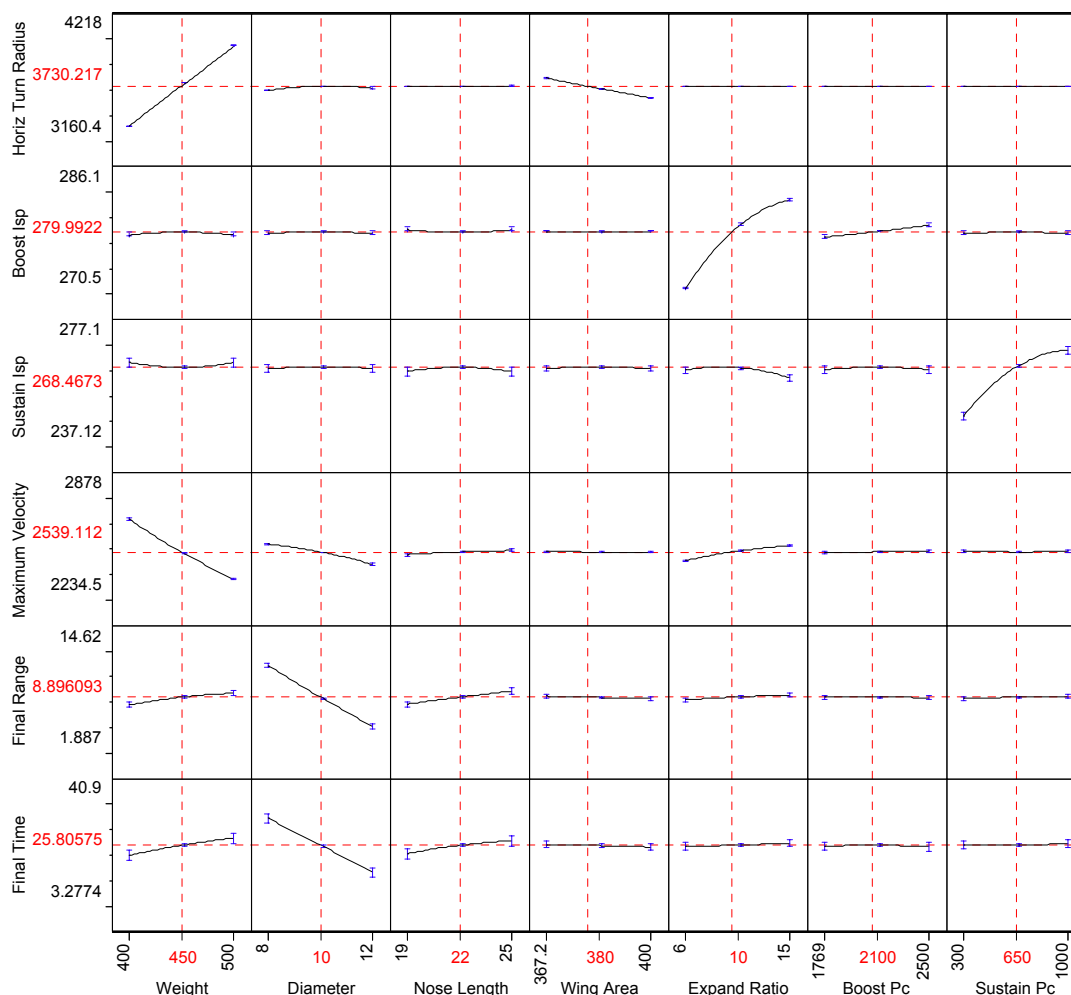
Figure 14: Prediction Profile for the Missile Showing the Linkages between Design Variables and System Responses

MONTE CARLO STUDY

Another application that can be combined with the Tactical Missile Design spreadsheet is called Crystal Ball. Crystal Ball is an add-in for Microsoft Excel. It can be used to run a Monte Carlo simulation over various cells, or inputs, and track the response of the output cells. These Monte Carlo studies are useful in conceptual design because they can illustrate for the user what type of performance can be achieved with the system[13].

Continuing with the previous example, a Monte Carlo simulation was run by altering the seven design variables that were used for the multi-dimensional case study given in Table IV. These variables were varied within the ranges shown in the table. A random value of each design variable was taken for each Monte Carlo trial, with 10,000 separate trials made over the variable ranges. The same outputs as the previous study, those listed in Table V, were also tracked in this study.

Figure 15 is the probability density function (PDF) for one of the missile outputs, the maximum Mach number; it shows how many of the 10,000 random cases resulted in a given Mach number. The figure shows that there were very few cases that had Mach numbers near 2.2 and very few cases that had Mach numbers near 2.7, but there was a flat distribution of Mach numbers between 2.3 and 2.6. The user can gleam from this information that only a few combinations of design variables result in Mach numbers near 2.2 and 2.7, but there is a wide range of design variables that can be chosen to generate a maximum Mach number anywhere within the range of 2.3 to 2.6. Therefore, using these design variables, a decision-maker would expect to have difficulty designing a missile to have a Mach number matching these extreme values but relatively little difficulty designing a system to meet one of the intermediate Mach numbers.
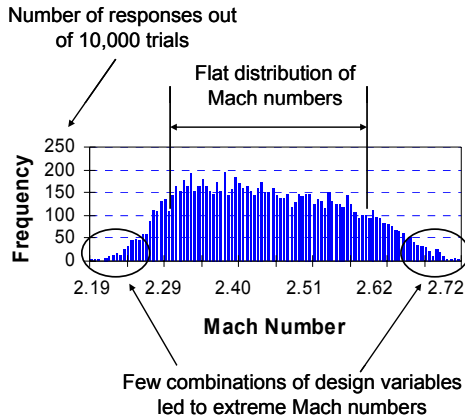
Number of responses out
of 10,000 trials

Flat distribution of
Mach numbers

Few combinations of design variables
led to extreme Mach numbers

**Figure 15: PDF Showing the Number of Responses for
each Mach Number**

Perhaps more useful than the PDF is the cumulative distribution function (CDF) shown in Figure 16 for the missile Mach number. The CDF shows what percent of the design space meets a given value or constraint[14]. For the example shown, approximately 30% of the designs, or 30% of the total design space, satisfies a hypothetical constraint for maximum Mach number greater than 2.5. By using this Monte Carlo technique in conjunction with the Tactical Missile Design spreadsheet, even with large numbers of variables (10-30 or more), the user can get a feel for what type of performance is possible for the system and the difficulty in meeting constraints.
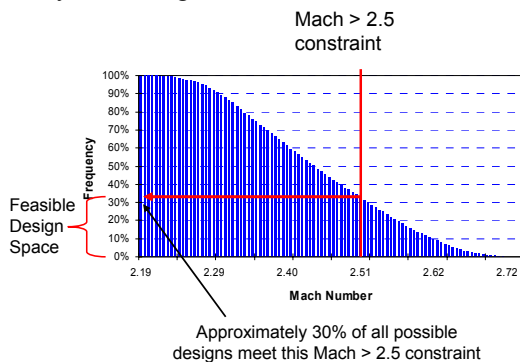

Mach > 2.5
constraint

Feasible
Design
Space

Approximately 30% of all possible
designs meet this Mach > 2.5 constraint

**Figure 16: CDF Showing the Portion of the Design Space
that exceeds each Mach Number Value**

Another example of a Monte Carlo response is given in Figure 17. This figure shows the response for turn radius. If a hypothetical constraint was given to the missile designer that required a turn radius less than 4,000 feet, then the Monte Carlo information informs the designer that over 90% of all possible design combinations satisfy this constraint. The designer then knows that this constraint is not going to be difficult to meet, unlike the Mach number constraint, where only 30% of potential designs satisfy the constraint.
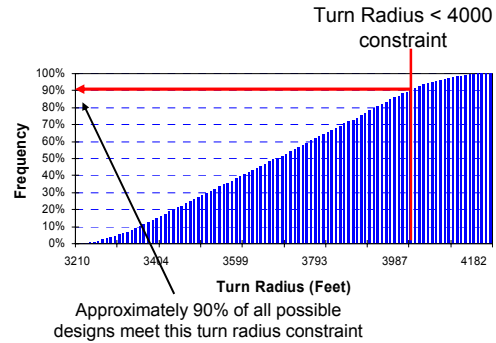

Turn Radius < 4000
constraint

Approximately 90% of all possible
designs meet this turn radius constraint

**Figure 17: CDF Showing the Response for Turn Radius
with a 4,000 ft Maximum Constraint**

Another important piece of information provided by Monte Carlo runs is when there are no feasible designs that satisfy a constraint. For instance, in Figure 17, if there was a design constraint of a turn radius less than 3,000 feet, then the designer immediately knows that this constraint is not feasible. The Monte Carlo results will have shown that not a single possible design variable combination satisfies this constraint. The designer can then go about solving the problem by relaxing the constraint, increasing the variable ranges, like increasing the maximum wing area, adding new design variables such as propellant weight or fuel fraction, or possibly infusing new technologies into the system such as thrust vector control.

MISSILE OPTIMIZATION

A design problem was posed to illustrate how the Tactical Missile Design spreadsheet can be used to optimize missile designs for particular missions. For the example design problem, the previous design variables and ranges (Table IV) were used, except for the missile weight, which was fixed at 500 lb_m, i.e., it was assumed that the user did not have control over empty weight. The objective of the optimization was to minimize the flight time of the missile subject to two constraints, a turn radius less than 4,000 feet and a flight range of at least 10 nautical miles.

Two separate optimizers were used for the optimization. The first optimizer was the automated solver that JMP uses in the prediction profile. This solver was able to minimize the flight time while meeting the two constraints. The other optimizer was the built-in solver function in Microsoft Excel. This solver is surprisingly powerful and, being a native feature of Excel, is fully integrated with the TMD spreadsheet.

The results of the two solvers are shown in Table VI and Table VII. These results are compared against the baseline MRAAM rocket. Table VI shows the missile

design variables selected by the two solvers. The values of the design variables are similar for both solutions, and the two missile designs had nearly identical performance. In fact, the difference in flight time was only 0.2 seconds between the two optimizers!

Table VI: Optimized Settings for Design Variables

	Min Allowed Value	Rocket Baseline	Max Allowed Value	JMP Solution	Excel Solution
Launch Weight (lbm) - *fixed*	500	500	500	500	500
Diameter (in)	8	8	12	9.2	9.6
Nose Length (in)	19	19.2	25	19.0	19.6
Wing Area (in^2)	367.2	367.2	400	400	400
Expansion Ratio	6	6	15	11.3	15.0
Boost Chamber Pressure (psi)	1769	1769	2500	2043	1769
Sustain Chamber Pressure (psi)	300	300	1000	742	1000

Table VII: Optimized Missile Performance Metrics

	Rocket Baseline	JMP Solution	Excel Solution
Final Time (sec)	35.2	29.4	29.2
Final Range (nmi)	12.06	10.05	10.00
Horiz Turn Radius (ft)	4181	3991	4000
Boost I_{sp} (sec)	270.5	281.7	283.3
Sustain I_{sp} (sec)	252.0	271.1	276.1
Maximum Velocity (ft/sec)	2538	2385	2443

One advantage of the prediction profile discussed earlier (shown in Figure 14) is that it helps illustrate the decisions made by the optimizers. For instance, if the user was curious as to why both optimizers chose the largest allowable wing area, a cursory glance at the trendlines in the prediction profile shows that increasing wing area has a dramatic effect on decreasing turn radius. The user could easily predict from this fact that increasing the maximum allowable wing area would lead to an even more optimal solution. Similarly, with the prediction profile the user can see that while decreasing the missile diameter improves the flight time, it also hurts the turn radius. Thus the user can understand the logic behind the optimizers' choice of a diameter near 9.4 inches; it is the smallest diameter that meets the turn rate requirement! The prediction profile provides invaluable assistance in visualizing the logic behind the muti-dimensional design optimization.

CONCLUSION

The use of simple, physics-based analyses can provide large amounts of design knowledge in the conceptual stages of design. The Tactical Missile Design spreadsheet can be used to quickly examine the performance of individual missile configurations. It can be used manually to perform trade-off studies through which an optimal parameter setting can be found. In addition, coupling the Tactical Missile Design spreadsheet with more powerful statistical packages allows great freedom in understanding the trade-offs and trends that exist simultaneously in multiple dimensions and drive the multi-disciplinary

nature of missile design. Through the use of these statistical packages with the TMD environment, trade-offs can be made in multiple dimensions and optimal settings of design variables for a specific mission can be found. In lieu of using additional software, the built-in optimizer in Excel is extremely powerful and can quickly generate fully optimized solutions.

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