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Frederick A. Kilpatrick

USE OF THE ELECTRONIC DIGITAL COMPUTER FOR THE DIRECT SOLUTION OF THE FLOW EQUATIONS FOR A NETWORK OF PIPES

A THESIS

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the Faculty of the Graduate Division

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USE OF THE ELECTRONIC DIGITAL COMPUTER FOR THE DIRECT SOLUTION OF THE FLOW EQUATIONS FOR A NETWORK OF PIPES

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ABSTRACT

Thorough analysis of pipe distribution systems is usually not accomplished because of the excessive number of computations involved. With the electronic digital computer, however, a means is available which promises to make thorough pipe-network analysis a simple, routine matter. The purpose of this study was to develop, with the aid of the electronic digital computer, a new method, readily applicable to all systems which may be encountered.

Essentially, the method consists of solving simultaneously the continuity and energy-loss equations which may be written for any pipe system. This is accomplished by linearizing the energy-loss equations by an approximation scheme and placing in matrix form the coefficients from both equations. Solution of the linear simultaneous equations is accomplished by inverting the matrix with the aid of an electronic digital computer. Repetitive solution of these equations necessitated by the linearization process is carried out until convergence of the correct flow values for the system is accomplished.

The method is particularly adaptable since it required only that the continuity and energy-loss equations of the system be written. To illustrate the adaptability of the method various types of systems were analyzed. These included a rather large system having 31 unknown flows; systems with elevated storage reservoirs which are being allowed to empty by either constant or free flow from the networks; and a system containing a pump, in which the head on the pump varies with flow.

The method proved to be simple and accurate. The excessive computer time required for analyzing large systems is the biggest deficiency of the method. A special computer program is not necessary because existing computer subroutines may be used in solving the simultaneous equations involved.

NOMENCLATURE

C = Hazen-Williams resistance coefficient;

D = inside diameter of a pipe;

f = Darcy-Weisbach resistance coefficient;

g = acceleration caused by gravity;

h_L = head loss in a pipe;

H_L = head loss in a series of pipes;

H = total head at a point;

k = measure of resistance to flow in a pipe;

K = kQ;

L = length of pipe;

Q = flow or discharge in cubic feet per second;

Q_o = initially assumed discharge in a pipe;

$\overline{Q}$ = average of the flow value obtained for an iteration with the value used in the iteration;

S = friction slope = h_L/L;

V = mean velocity and

ψ = k_a/k_b.

Superscripts: ' number of primes indicates the iteration involved.

CHAPTER I

INTRODUCTION

Thorough analyses of pipe distribution systems are usually not carried out because of the excessive computations required. This was remedied partially by the development of the electric analyzer (1). The event of the electronic digital computer naturally led to the programming of the Hardy Cross Method for pipe-network analysis (2, 3).

The purpose of this study was to develop, with the aid of the electronic digital computer, a new method of analyzing pipe distribution systems readily applicable to all cases which may be encountered.

Essentially, the method consists of solving simultaneously the continuity and energy-loss equations which may be written for any pipe system. This is accomplished by linearizing the energy-loss equations by an approximation scheme and placing in matrix form the coefficients of both types of equations. Solution of the linear simultaneous equations is accomplished by inverting the matrix with the aid of an electronic digital computer. Repetitive solution of these equations, necessitated by the linearization process, is carried out until convergence of the correct flow values for the system is accomplished.

This method is particularly adaptable since it requires only that one write the continuity and energy-loss equations descriptive of flow in a system. To illustrate the adaptability of the method various types of systems are analyzed. These include a rather large system having 31 unknown flows; systems with elevated storage reservoirs for cases of constant and free flow from their networks; and a system containing a pump in which the head on the pump varies with flow.

CHAPTER II

DEVELOPMENT OF THE METHOD

General.--With the development of the electronic computer, a vast number of mathematical methods previously impractical to use due to time and labor required have now become useable. This is particularly true when it comes to the solution of systems of linear algebraic equations. Computers now made are capable of inverting in a matter of minutes matrices which by hand would be virtually impossible.

The matrices of the following problems were solved using matrix-inversion subroutine ML 05 which was prepared by the International Business Machine Corporation. This subroutine is capable of inverting a matrix with a maximum of 43 unknowns. It was used on the IBM 650 digital computer located at the Rich Computer Center, Georgia Institute of Technology. It should be pointed out here that the writer did not have and, in fact, had no need to develop a program but merely used an existing subroutine as a tool in solving sets of simultaneous equations.

Laws governing flow distribution in a pipe network.--There are three basic laws which govern the distribution of flow in a pipe network. The flow of water into a pipe junction must equal the flow out, or the algebraic sum of the flows toward a junction must equal zero. Second, the algebraic sum of the head losses around any closed circuit of a pipe network must equal zero. The third law relates the head loss in a pipe to some power of the discharge. In the Darcy-Weisbach equation, the head-loss varies with the second power of the discharge, and in the Hazen-Williams equation the head loss varies with the 1.85 power of the discharge.

Basic principle.--The fact that linear simultaneous equations can now be solved on an electronic computer does not mean that the simultaneous equations for a pipe network are solvable. The same difficulty is encountered as that which hindered earlier efforts to use an electrical network as analogous to a pipe network (4). In an electrical network the voltage drop in a line is directly proportional to the first power of the current times the resistance rather than some higher power. This inexactness in the analogy was avoided by using a linear electrical circuit and by a process of successive approximations which involved changing the resistance of the circuit elements several times until the current and voltage were analogous to the head loss and flow of the pipe network. A somewhat similar approach is used here with the exception that the method is numerical.

Using the Darcy-Weisbach equation, the head loss in a straight pipe is

$$h_L = k\ Q^2$$

in which k is a function of the pipe properties and a measure of the resistance of the pipe to fluid flow. The continuity equations written for the pipe junctions of a network are linear algebraic equations and the head-loss equations are quadratic equations. It was reasoned that the head-loss equations might be linearized by letting

$$h_L = K\ Q$$

where

$$K = k\ Q.$$

The resulting linear simultaneous equations could then be solved. This would serve as a first approximation. The new values found in this first iteration on the computer would be used in a second approximation and the process repeated until convergence to the correct flow values was accomplished.

Theoretical examination of the method.--The examination of a single loop network using the principle outlined above will reveal the need for an additional step in order to bring about convergence.

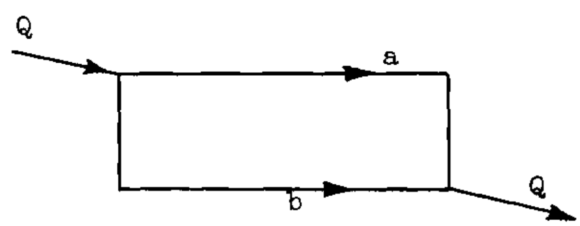


Fig. 1. A Simple Pipe Network

The simple loop shown is chosen for theoretical examination because the exact solution is readily obtainable. The respective continuity and head-loss equations for this simple loop are:

$$Q_a + Q_b = Q \qquad (1)$$

$$k_a Q_a^2 - k_b Q_b^2 = 0 \qquad (2)$$

from equation (2)

$$Q_b = Q_a \sqrt{\frac{k_a}{k_b}} = Q_a \sqrt{\psi}$$

in which

$$\psi = \frac{k_a}{k_b} .$$

From equation (1)

$$Q_a + Q_a \sqrt{\psi} = Q$$

and the exact solution for $\underline{Q_a}$ becomes

$$Q_a = Q \left[\frac{1}{1 + \sqrt{\psi}} \right]$$

and the exact solution for $\underline{Q_b}$ becomes

$$Q_b = Q - Q_a = Q \left[\frac{\sqrt{\psi}}{1 + \sqrt{\psi}} \right] .$$

The next step is to obtain solutions for $\underline{Q_a}$ and $\underline{Q_b}$ by the iterative procedure in which the head-loss equations are linearized. The respective continuity and linearized head-loss equations for the simple loop are:

$$Q_a + Q_b = Q$$

$$K_a'Q_a - K_b'Q_b = 0$$

in which

$$K_a' = k_aQ_a \quad \text{and} \quad K_b' = k_bQ_b$$

and the primes are added to indicate the number of the iteration. For a first approximation let

$$K_i' = k_iQ.$$

The continuity and energy loss equations become

$$Q_a' + Q_b' = Q$$

$$k_a\ Q\ Q_a' - k_b\ Q\ Q_b' = 0$$

or

$$Q_a' + Q_b' = Q$$

$$\psi Q_a' - Q_b' = 0$$

Solving for $\underline{Q_a}'$ and $\underline{Q_b}'$,

$$Q_a' = \left[\frac{1}{1 + \psi}\right] Q$$

$$Q_b' = \left[\frac{\psi}{1 + \psi}\right] Q$$

For a second approximation let

$$K_a'' = k_a\, Q_a' = k_a \left[\frac{1}{1 + \psi} \right] Q$$

and

$$K_b'' = k_b\, Q_b' = k_b \left[\frac{\psi}{1 + \psi} \right] Q$$

The continuity and head-loss equations for the second iteration are

$$Q_a'' + Q_b'' = Q$$

$$k_a \left[\frac{1}{1 + \psi} \right] Q\, Q_a'' - k_b \left[\frac{\psi}{1 + \psi} \right] Q\, Q_b'' = 0$$

Equation (2) then reduces to

$$\left[\frac{\psi}{1 + \psi} \right] Q_a'' - \left[\frac{\psi}{1 + \psi} \right] Q_b'' = 0$$

Here the iteration must stop with the obviously incorrect result that $Q_a'' = Q_b''$.

In order to obtain convergence, it becomes necessary to average the flow values obtained as a result of the first iteration with the originally assumed values. Thus for the second iteration,

$$K_a'' = k_a \left[\frac{Q + Q_a'}{2} \right] = \frac{k_a}{2} \left[1 + \frac{1}{1 + \psi} \right] Q$$

and

$$K_b'' = k_b \left[\frac{Q + Q_b'}{2}\right] = \frac{k_b}{2}\left[1 + \frac{1}{1 + \psi}\right] Q$$

The continuity and head-loss equations are

$$Q_a'' + Q_b'' = Q$$

and

$$\frac{k_a}{2}\frac{(2 + \psi)}{(1 + \psi)} Q\, Q_a'' - \frac{k_b}{2}\frac{(1 + 2\psi)}{(1 + \psi)} \quad Q\, Q_b'' = 0$$

Simplifying,

$$Q_a'' + Q_b'' = Q$$

and

$$\psi(2 + \psi)\; Q_a'' - (1 + 2\psi)\; Q_b'' = 0$$

Adding these two equations and solving,

$$Q_a'' = \left[\frac{1 + 2\psi}{1 + 4\psi + \psi^2}\right] Q$$

and

$$Q_b'' = \left[\frac{\psi(1 + 2\psi)}{1 + 4\psi + \psi^2}\right] Q$$

The flow values to be used in the third iteration would be the average of these new values and the values used in the second iteration.

Figure 2 is a plot of Q_a/Q versus k_a/k_b for the exact and the first and second approximate solutions obtained by averaging the flows. This reveals graphically how convergence occurs.

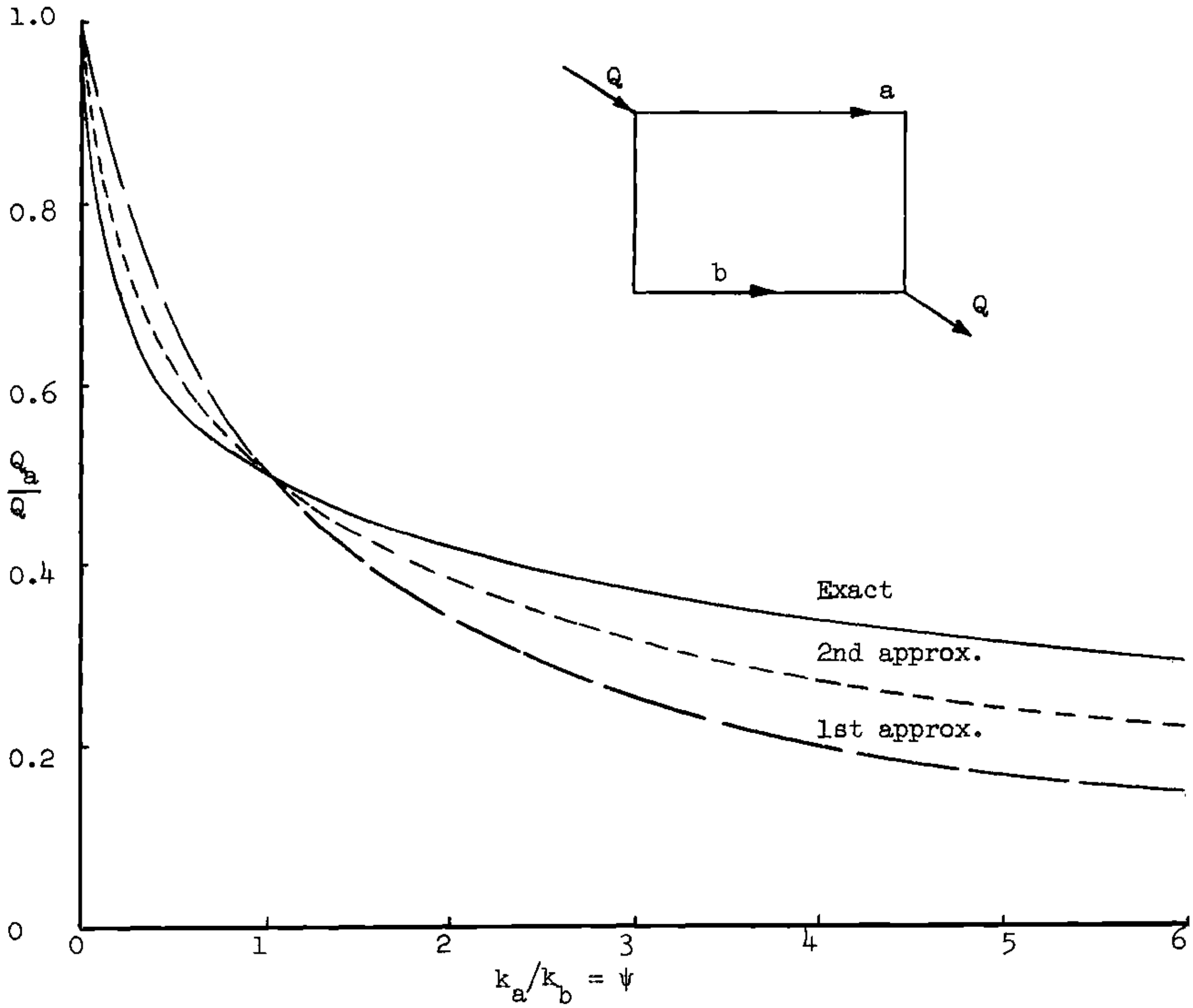


Fig. 2. Comparison of Exact and Successive Approximate Solutions for a Simple Loop Network.

Test case.--A three loop, ten-pipe network as shown in Figure 3 was selected for a test case. This figure shows the network with k values in parenthesis as computed for the Darcy-Weisbach equation. Appendix B contains the equation for the computation of k. Inflows and outflows from the system are considered to occur at junctions. As is the case in most methods of analysis, the direction and magnitudes of the initial flows, Q_o, are assumed and are shown in Figure 3. As may be seen, these flows are bad estimates in order to test the method thoroughly.

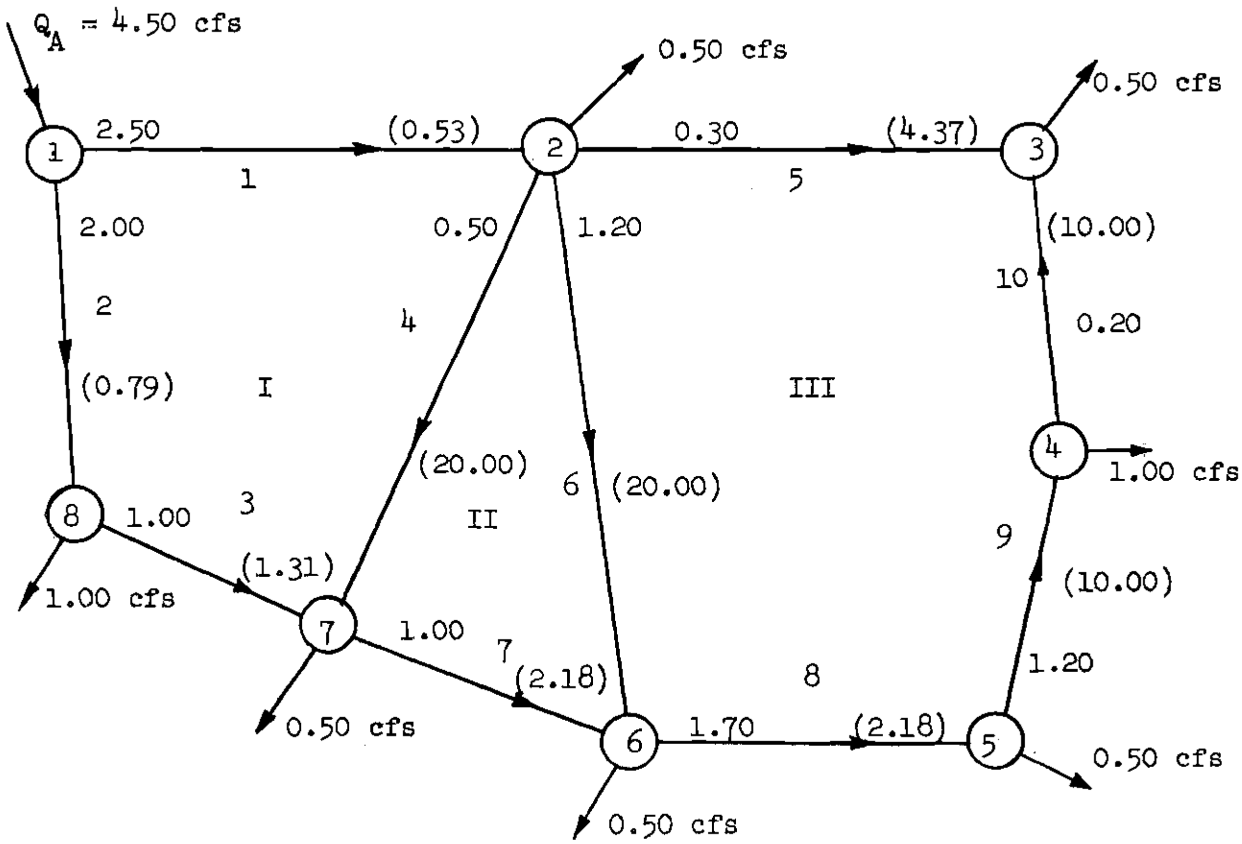


Fig. 3. Diagram of Network for the Initial Test Case.

The continuity and head-loss equations for this network are written assuming flows into a junction as positive and flows out as negative. Head loss is considered positive in the clockwise direction and negative if counterclockwise. The resulting continuity equations are

<u>Joint</u>

(1)	$-Q_1 - Q_2 + Q_A = 0$
(2)	$+Q_1 - Q_4 - Q_5 - Q_6 - 0.50 = 0$
(3)	$+Q_5 + Q_{10} - 0.50 = 0$
(4)	$+Q_9 - Q_{10} - 1.00 = 0$
(5)	$+Q_8 - Q_9 - 0.50 = 0$
(6)	$+Q_6 + Q_7 - Q_8 - 0.50 = 0$
(7)	$+Q_3 + Q_4 - Q_7 - 0.50 = 0$
(8)	$+Q_2 - Q_3 - 1.00 = 0$

and the three head-loss equations are

<u>Loop</u>

I	$+k_1 Q_1^2 - k_2 Q_2^2 - k_3 Q_3^2 + k_4 Q_4^2 = 0$
II	$-k_4 Q_4^2 + k_6 Q_6^2 - k_7 Q_7^2 = 0$
III	$+k_5 Q_5^2 - k_6 Q_6^2 - k_8 Q_8^2 - k_9 Q_9^2 - k_{10} Q_{10}^2 = 0$

The head-loss equations may now be linearized by substituting in their respective <u>K</u> values. The head-loss equations now may be written as

Loop

I $\qquad +K_1'Q_1 - K_2'Q_2 - K_3'Q_3 + K_4'Q_4 = 0$

II $\qquad -K_4'Q_4 + K_6'Q_6 - K_7'Q_7 = 0$

III $\qquad +K_5'Q_5 - K_6'Q_6 - K_8'Q_8 - K_9'Q_9 - K_{10}'Q_{10} = 0$

where the single prime indicates the values for the first iteration and is used only in this test case to clarify procedures.

The coefficients of the flows for the continuity and head-loss equations are now set up in matrix form as shown in Table 1 which contains

Table 1. Matrix for the Test Case

Junction or Loop	Flows											Constants
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_A	
1	-1	-1									+1	0
2	+1			-1	-1	-1						+0.50
3					+1					+1		+0.50
4									+1	-1		+1.00
5								+1	-1			+0.50
6						+1	+1	-1				+0.50
7			+1	+1			-1					+0.50
8		+1	-1									+1.00
I	$+K_1'$	$-K_2'$	$-K_3'$	$+K_4'$								0
II				$-K_4'$		$+K_6'$	$-K_7'$					0
III					$+K_5'$	$-K_6'$		$-K_8'$	$-K_9'$	$-K_{10}'$		0

eleven unknown flows and eleven equations. This matrix as shown may be inverted on the computer. However, it is obvious that the value of the incoming flow, $\underline{Q_A}$, may be found by merely summing up the outflows from the network. This operation results in a reduction of the matrix to ten unknown flows and ten equations. Since the continuity principle is used to determine the flow $\underline{Q_A}$, one of the junction continuity equations may be dropped. This matrix shown in Table 2 results from dropping the continuity equation for junction 2 and from inserting the computed value

Table 2. Reduced Matrix for the Test Case

Junction or Loop	Flows										Constants
	1	2	3	4	5	6	7	8	9	10	
1	-1	-1									-4.50
3					+1					+1	+0.50
4									+1	-1	+1.00
5								+1	-1		+0.50
6						+1	+1	-1			+0.50
7			+1	+1			-1				+0.50
8		+1	-1								+1.00
I	$+K_1'$	$-K_2'$	$-K_3'$	$+K_4'$							0
II				$-K_4'$		$+K_6'$	$-K_7'$				0
III					$+K_5'$	$-K_6'$		$-K_8'$	$-K_9'$	$-K_{10}'$	

of $\underline{Q_A}$. This matrix was inverted on the computer for the completion of the first iteration. The numerical values of the $\underline{K}'$ terms in the matrices were omitted only for clarity. The unfilled elements of the matrices are all zeroes.

Table 3 contains the complete tabular solution of the iterative process as well as the final results. The results of the third iteration are compared with an 11-iteration analysis of the network by the Hardy Cross method. The algebraic summation of the head losses around each loop at the end of the third iteration reveals the degree of exactness obtained. Each iteration required approximately 3 minutes of computer time.

As might be expected and as can be seen from Table 3, the flow in pipe 10 is reversed on the first iteration as is indicated by the negative sign. There is no need to alter the signs in the matrix for the next iteration in either the continuity or head-loss equations as a result of this flow reversal. The new K value for pipe 10 may be entered without altering the initially selected signs of any part of the matrix. The final flow value for pipe 10 retains the negative sign, indicating a flow in pipe 10 which is opposite in direction to the initially assumed direction.

In the example shown, the averaging of the flows and the computation of new trial K values was done manually although the whole problem could be programmed to carry out the entire series of iterations. However, a complete program would occupy space in the computer with the result that the maximum number of equations would be reduced. Since each successive iteration involves changing only the coefficients of the head-loss equations while the other seven continuity equations remain unaltered, the complete program is quite unnecessary. As mentioned above, even the initial sign designations may be retained for all iterations.

Table 3. Tabulation Showing Method and Results of Initial Test Case

Item	k	Initial Flow (assumed) Q_o	First Iteration: $K' = KQ_o$	First Iteration: Computed Flow, Q'	First Iteration: $\bar{Q}' = \frac{Q_o + Q'}{2}$
1	0.53	2.50	1.32	2.38	2.44
2	0.79	2.00	1.58	2.12	2.06
3	1.31	1.00	1.31	1.12	1.06
4	20.00	0.50	10.00	0.17	0.33
5	4.37	0.30	1.31	1.57	0.93
6	20.00	1.20	24.00	0.14	0.67
7	2.18	1.00	2.18	0.79	0.90
8	2.18	1.70	3.71	0.43	1.07
9	10.00	1.20	12.00	(-) 0.07	0.57
10	10.00	0.20	2.00	(-) 1.07	(-) 0.43

Second Iteration: $K'' = k\bar{Q}'$	Second Iteration: Computed Flow, Q''	Second Iteration: $\bar{Q}'' = \frac{\bar{Q}' + Q''}{2}$	Third Iteration: $K''' = K\bar{Q}''$	Third Iteration: Computed Flow, Q'''	Third Iteration: $\bar{Q}''' = \frac{\bar{Q}'' + Q}{2}$ *	Q_{HC}
1.29	2.32	2.38	1.26	2.37	2.37	2.37
1.63	2.18	2.12	1.68	2.13	2.13	2.13
1.39	1.18	1.12	1.47	1.13	1.13	1.13
6.68	0.33	0.33	6.62	0.35	0.34	0.34
4.08	1.19	1.06	4.63	1.07	1.06	1.06
13.42	0.31	0.49	9.80	0.44	0.47	0.47
1.95	1.00	0.95	2.07	0.98	0.97	0.97
2.32	0.81	0.94	2.05	0.93	0.94	0.94
5.66	0.31	0.44	4.40	0.43	0.44	0.44
4.34	(-) 0.69	(-) 0.56	5.60	(-) 0.57	(-) 0.56	(-) 0.56

*Final Values

$\Sigma\ H_L'''$ for Loop I $= +0.04$ ft.

$\Sigma\ H_L'''$ for Loop II $= -0.03$ ft.

$\Sigma\ H_L'''$ for Loop III $= +0.03$ ft.

CHAPTER III

ILLUSTRATIVE EXAMPLES

Analysis of the Warwick, Rhode Island distribution system.--The test case just analyzed was a comparatively small system and required only three iterations for convergence. To ascertain if a larger system might result in increased iterations to obtain convergence, the Warwick, Rhode Island System, as is found in the Handbook of Hydraulics by Davis (5), was selected for analysis.

Figure 4 is a diagram of this system. The initial flows in the lines are the same as assumed by Camp (5) in his analysis of the system by the Hardy Cross method, except that the flows are expressed in cubic feet per second. The initial flows are shown by their respective directional arrows. A Hazen-Williams C of 110 was used by Camp. Appendix B illustrates how the Hazen-Williams formula was converted into the form, $h_L = KQ^2$. The k values for the system are omitted from Figure 4 for the sake of clarity but are shown in Table 10 in Appendix A.

Table 4 is the matrix for the Warwick System. This system is seen to have 31 unknown flows and 7 loops with one loop containing 14 unknown flows. The continuity equation for junction 1 is arbitrarily dropped for reasons previously discussed.

Table 10 in Appendix A shows the iterative procedure as well as the final results. Again, only three iterations were necessary for convergence of the entire system. As may be seen from the small differences

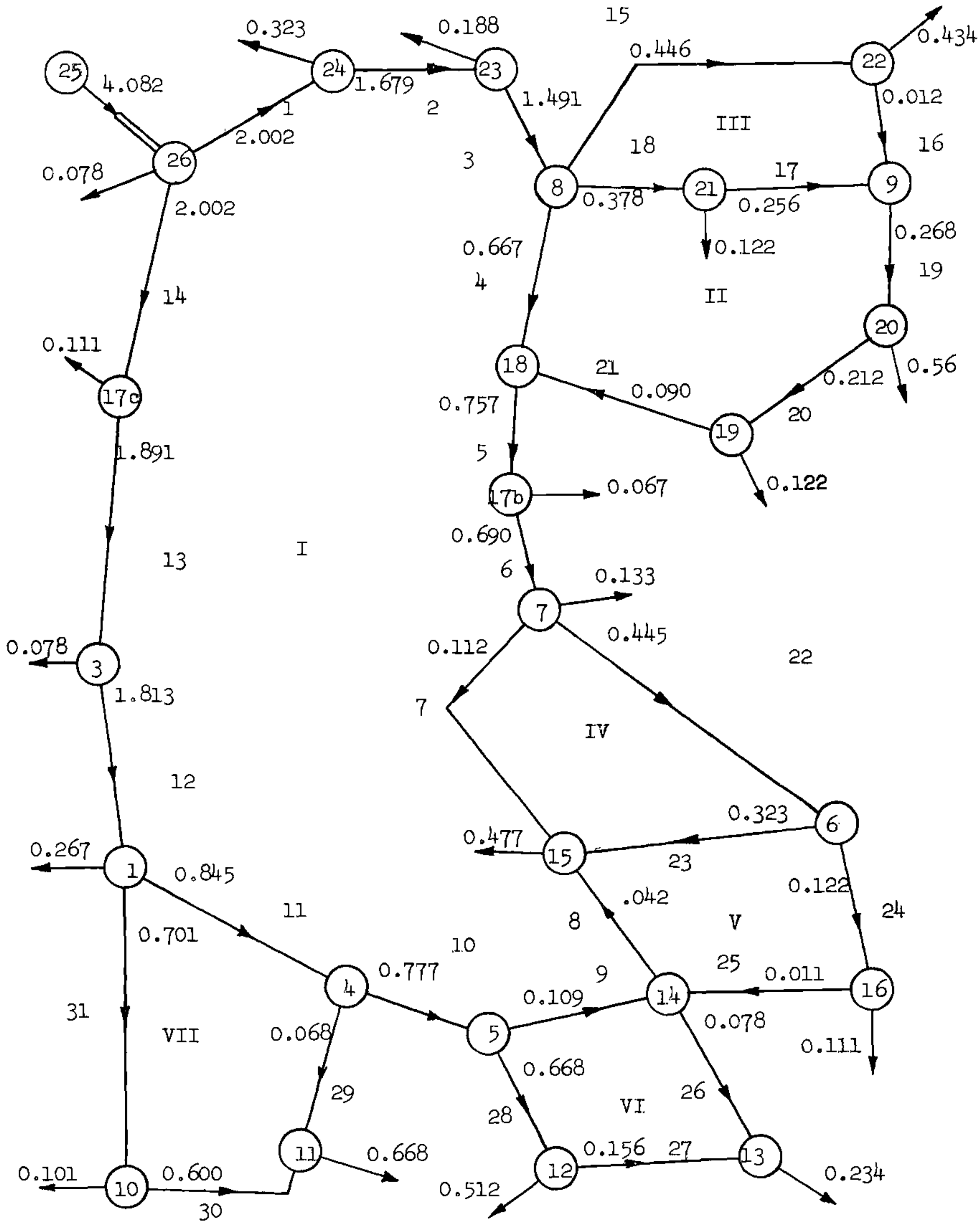


Fig. 4. Diagram of the Warwick, Rhode Island Distribution System Showing Initial Flow Conditions.

Table 4. Matrix for the Warwick System

Junction or Loop	Flows 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	C
26	-1													-1																		-4.004
24	+1	-1																														-0.323
23		+1	-1																													-0.188
8			+1	-1											-1			-1														0
18				+1	-1																+1											0
17b					+1	-1																										-0.067
7						+1	-1															-1										-0.133
15							+1	+1															+1									-0.477
14								-1	+1																+1	-1						0
5									-1	+1																		-1				0
4										-1	+1																		-1			0
3												-1	+1																			-0.078
17c													-1	+1																		-0.111
22															+1	-1																-0.434
9																+1	+1		-1													0
21																	-1	+1														-0.122
20																			+1	-1												-0.056
19																				+1	-1											-0.122
6																						+1	-1	-1								0
16																								+1	-1							-0.111
13																										+1	+1					-0.234
12																											-1	+1				-0.512
11																													+1	+1		-0.668
10																														-1	+1	-0.101
I	+K	+K	+K	+K	+K	+K	+K	-K	-K	-K	-K	-K	-K	-K																		0
II				-K													+K	+K	+K	+K	+K											0
III															+K	+K	-K	-K														0
IV							-K															+K	+K									0
V								+K															-K	+K	+K							0
VI									+K																	+K	-K	-K				0
VII											+K																		+K	-K	-K	0

Note: Superscripts and subscripts have been omitted in this matrix.

in the summation of the head losses around each loop, a high degree of accuracy was obtained. Computer time for each iteration was approximately 37 minutes. In Figure 5 the final flow values are compared with Camp's solution obtained by the Hardy Cross method. The small discrepancies may be accounted for by the method of converting the Hazen-Williams resistance coefficient.

Analysis of system containing elevated reservoirs for case of constant flow from the network.--A system containing elevated reservoirs is as readily analyzed as any other system. Essentially, the additional equations are those relating the total head (total energy) between reservoirs or reservoirs and points in the network. The ease and rapidity with which analyses may be made now makes it feasible to make more thorough investigations; in particular, where the elevated reservoirs are being allowed to empty.

A system containing two reservoirs with known areas and initial total heads is shown in Figure 6. As in previous problems, the k values are shown in parentheses and the flows for the lines are those initially assumed.

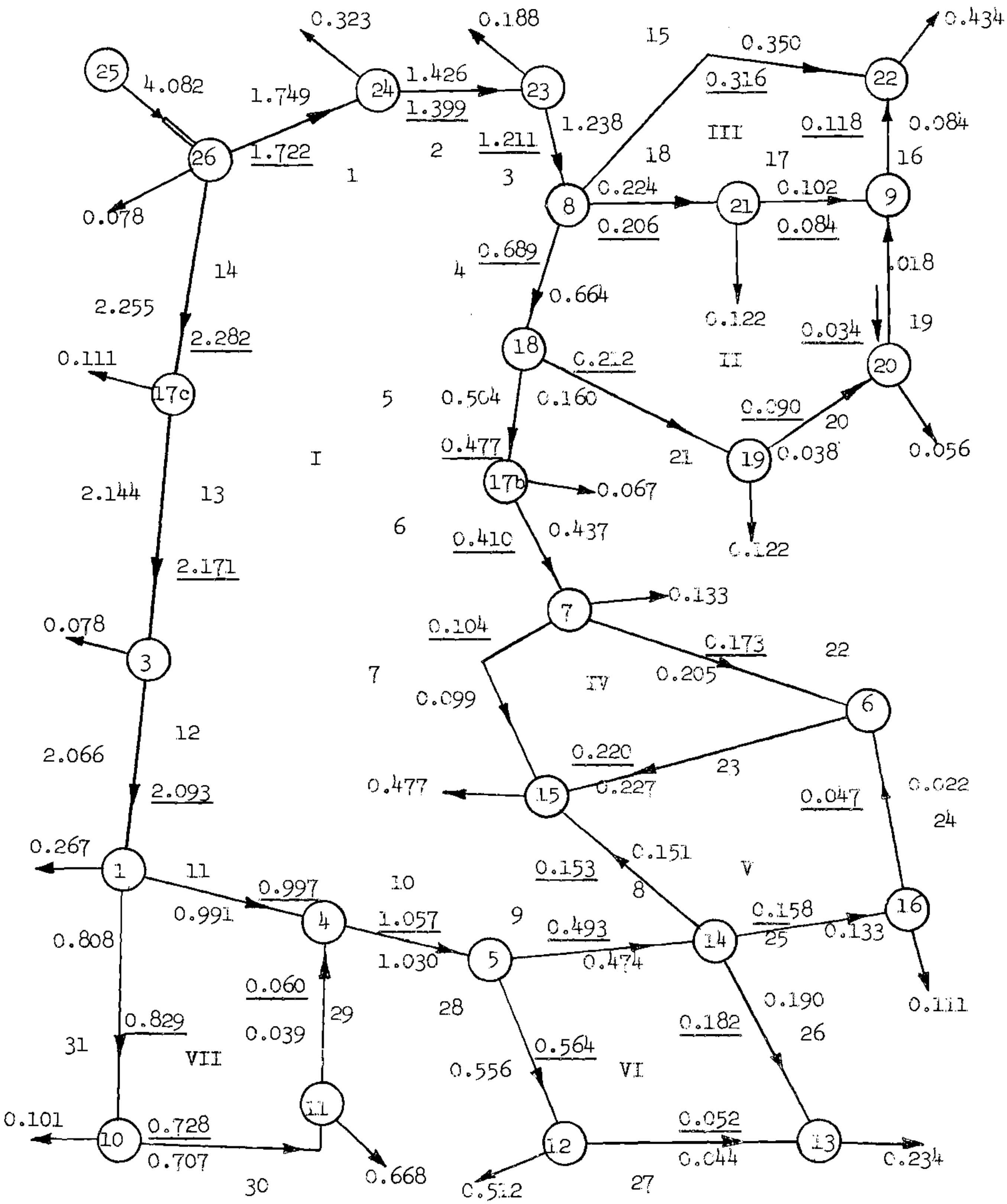
Table 5 is the matrix for this system. The equation relating the total head between reservoirs of Figure 6 is simply,

$$H_A - h_{L_{11}} - h_{L_1} - h_{L_5} + h_{L_{12}} = H_B$$

Rearranging,

$$-h_{L_1} - h_{L_5} - h_{L_{11}} + h_{L_{12}} = +H_B - H_A$$

which is the form shown in the matrix.



Note: The values obtained in this study are underlined.

Fig. 5. Final Values for the Warwick System Compared with Those Obtained by a Hardy Cross Analysis.

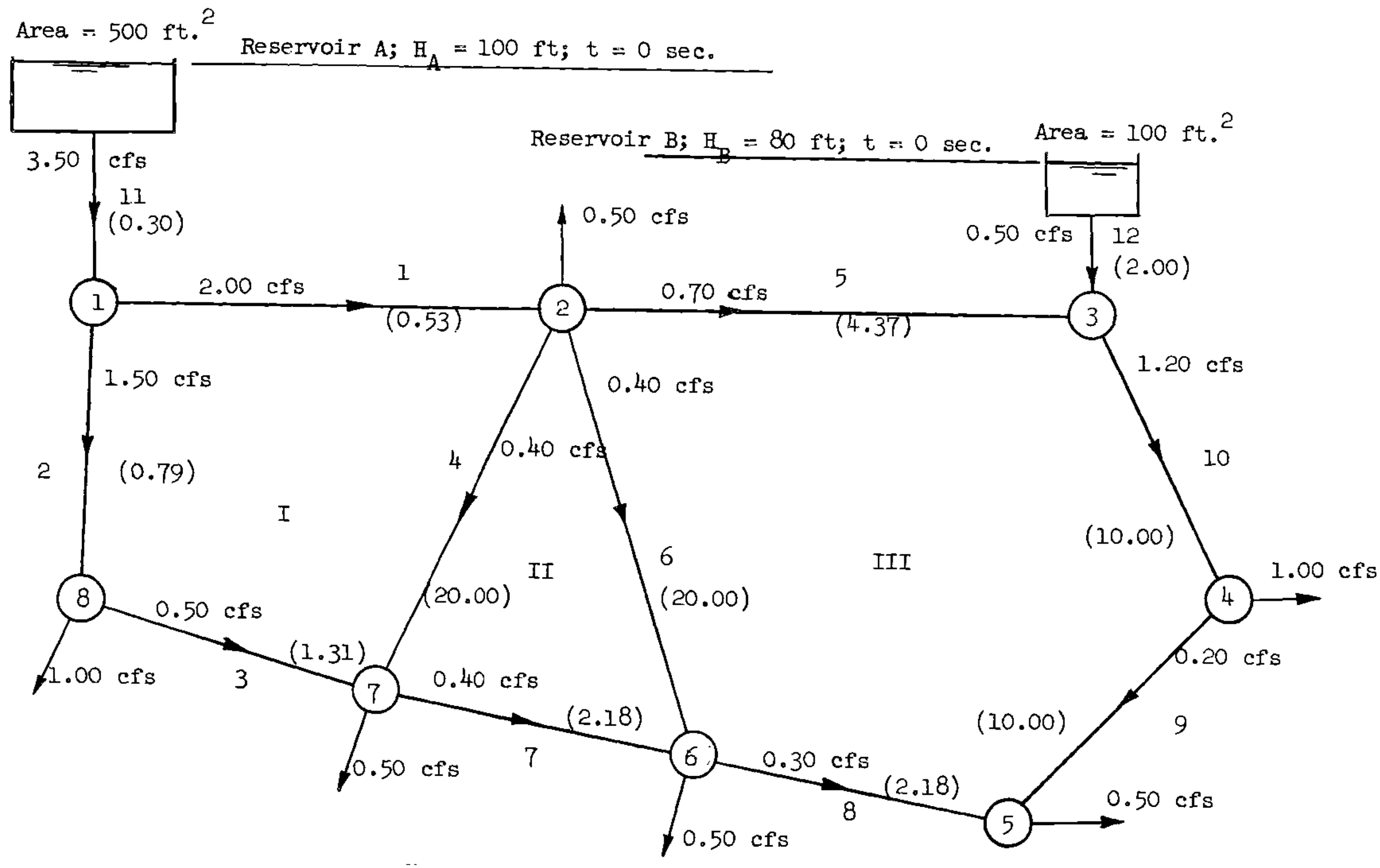


Fig. 6. Diagram of System Containing Elevated Reservoirs with Constant Flow from the Network.

Table 5. Matrix for the System Containing Elevated Reservoirs with Constant Flow from the Network

Junction or Loop	Flows 1	2	3	4	5	6	7	8	9	10	11	12	Constants
1	-1	-1									+1		0
2	+1			-1	-1	-1							+0.50
3					+1					-1		+1	0
4									-1	+1			+1.00
5								+1	+1				+0.50
6							+1	-1					+0.50
7			+1	+1			-1						+0.50
8		+1	-1										+1.00
I	$+K_1'$	$-K_2'$	$-K_3'$	$+K_4'$									0
II				$-K_4'$		$+K_6'$	$-K_7'$						0
III					$+K_5'$	$-K_6'$		$-K_8'$	$+K_9'$	$+K_{10}'$			0
A-B	$-K_1'$				$-K_5'$						$-K_{11}'$	$+K_{12}'$	H_B-H_A

As may be seen in Table 11 (Appendix A) three iterations were required to bring the system to convergence for the initial condition. Each iteration required approximately 3 minutes. It is interesting to note that there is an inflow into reservoir B instead of an outflow as initially assumed.

The next step is to analyze the system while the reservoirs are being allowed to empty. This is accomplished by computing new water levels in the reservoirs at specified times. On the basis of these new values of the total head, the distribution of flow in the system is again determined. Table 12 (Appendix A) contains the analysis of the system for the condition of constant flow from the network for a period of 4000 seconds. Succeeding K values were calculated using the final flow values obtained for the previous time increment.

Two iterations were required for complete convergence for the first two time increments. Succeeding time increments were shortened to make only one iteration for each time increment necessary. Time increments were lengthened as the rate of change of the total heads of the reservoirs decreased.

Figure 7 illustrates the typical characteristics of a system subject to constant flow from the network and emptying reservoirs. The flows in the systems and the difference in the total heads of the reservoirs eventually reach a steady state. The steady state is possible only because the outflows at the junctions 2, 4, 5, 6, 7, and 8 were assumed to be constant.

Analysis of system containing elevated reservoirs for the case of free flow from a point in the network.--This example is similar to the previous one except that free flow is allowed to occur at a point in the network while the reservoirs are allowed to empty.

Figure 8 is a diagram of a system with free flow occurring at point C. The matrix for this system, Table 6, is composed of 13 unknown flows and the coefficients of 13 equations. The free flow, Q_C, is the

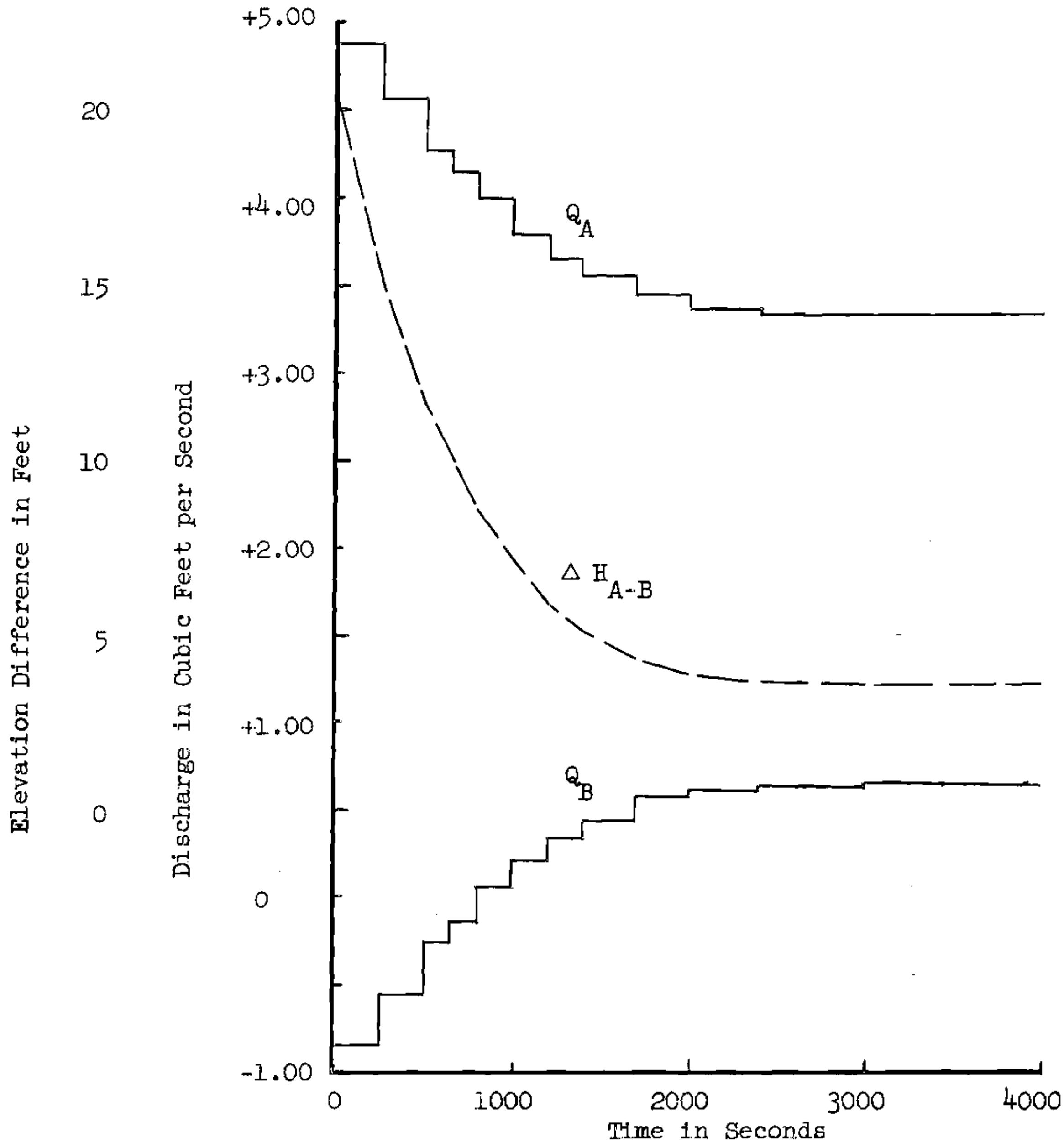


Fig. 7. Results of Analysis of System Subject to Constant Flow from the Network and Emptying Reservoirs.

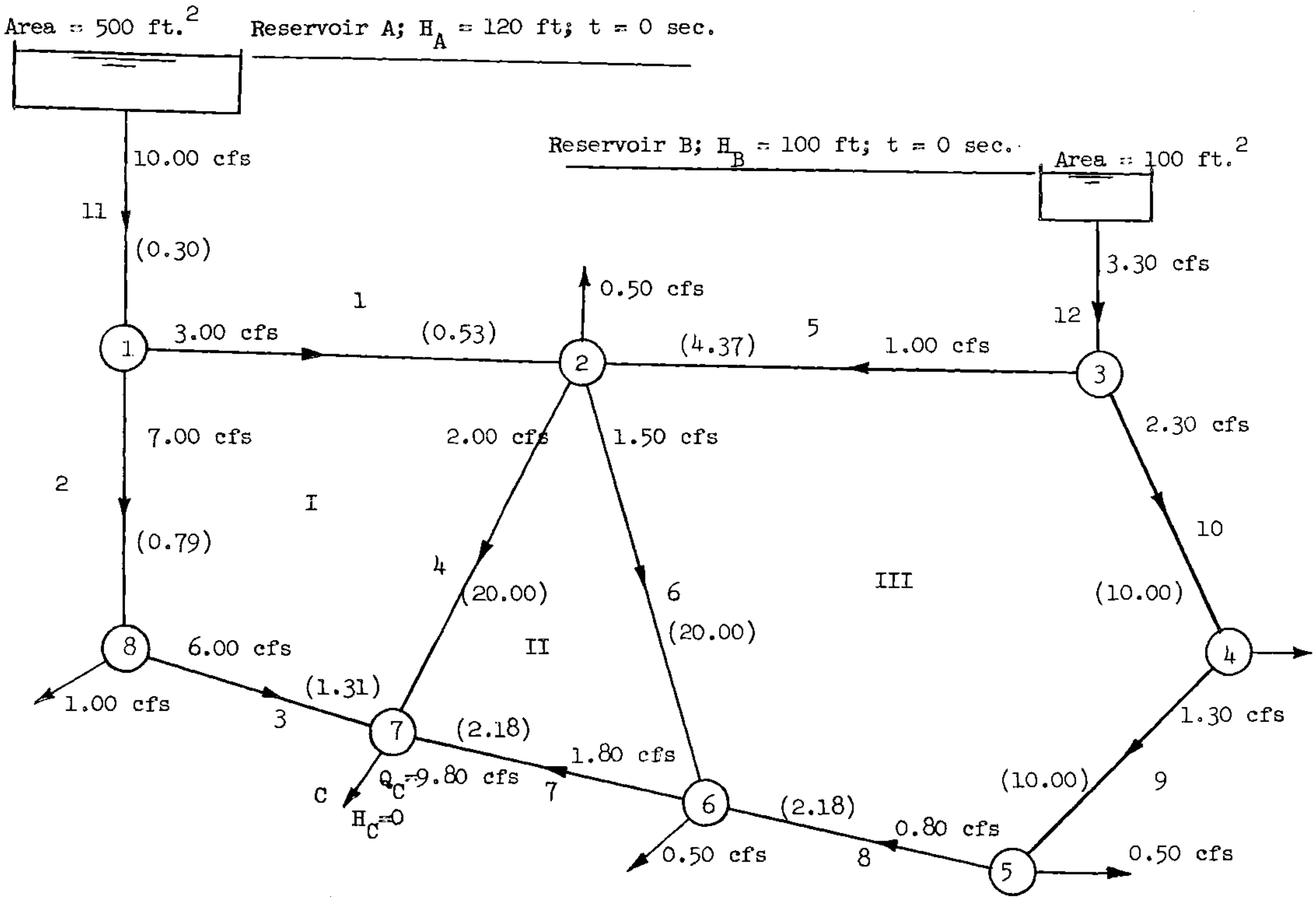


Fig. 8. Diagram of System Containing Elevated Reservoirs with Free Flow Occurring at a Point in the Network.

Table 6. Matrix for System Containing Elevated Reservoirs with Free Flow Occurring at a Point in the Network

Junction or Loop	Flows													Constants
	1	2	3	4	5	6	7	8	9	10	11	12	C	
1	-1	-1									+1			0
2	+1			-1	+1	-1								+0.50
3					-1					-1		+1		0
4									-1	+1				+1.00
5								-1	+1					+0.50
6						+1	-1	+1						+0.50
7			+1	+1			+1						-1	0
8		+1	-1											+1.00
I	$+K_1'$	$-K_2'$	$-K_3'$	$-K_4'$										0
II				$-K_4'$		$+K_6'$	$+K_7'$							0
III					$-K_5'$	$-K_6'$		$+K_8'$	$+K_9'$	$+K_{10}'$				0
A-C		$-K_2'$	$-K_3'$								$-K_{11}'$			$-H_A$
A-B	$-K_1'$				$+K_5'$						$-K_{11}'$	$+K_{12}'$		$-H_A+H_B$

additional unknown. The additional equation is that relating the head-loss from one of the reservoirs to the point of zero total head. The matrix was inverted on the computer to complete the first iteration.

As can be seen in Table 13 (Appendix A) again only three iterations were necessary to bring about convergence for the starting condition.

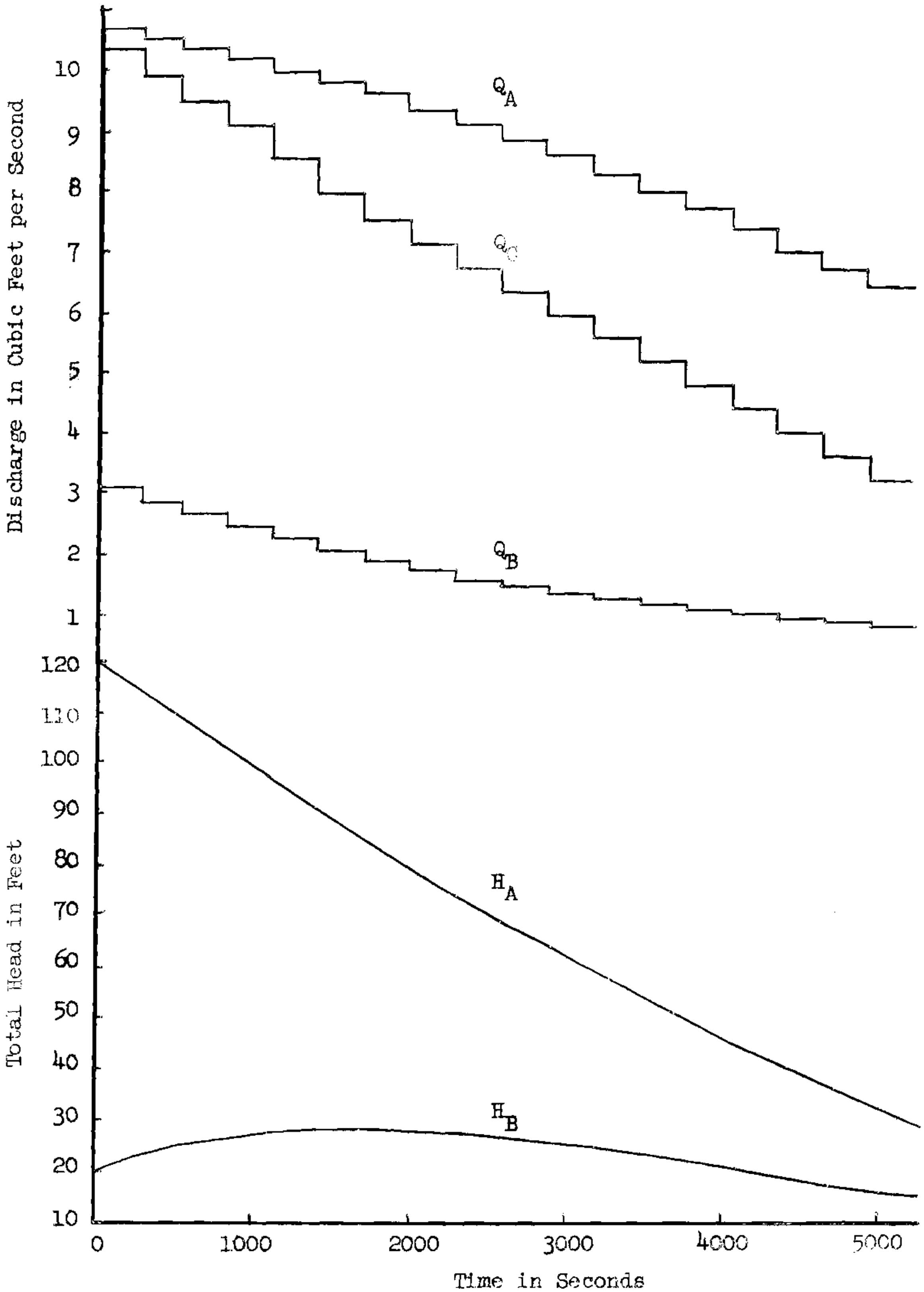


Fig. 9. Results of Analysis of System with Free Flow Occurring at a Point in the Network and Emptying Reservoirs.

Table 14 (Appendix A) contains the iterative procedure and results for analyzing the system for a period up to 5300 seconds while the reservoirs were being allowed to empty. Figure 9 shows the results of this analysis graphically.

<u>Analysis of a system containing a reservoir and a variable head pump</u>.--Networks fed by pumps are quite common. In the case of centrifugal pumps the head on the pump (total-head change at the pump) is a function of the pump discharge. Pump characteristic curves define this relationship. This type of system containing a variable-head source may be readily analyzed by the method developed.

Figure 11 is a diagram of a system containing a reservoir and a centrifugal pump. The characteristic curve relating head (total-energy change) and discharge for this pump is shown in Figure 10. Table 7 is

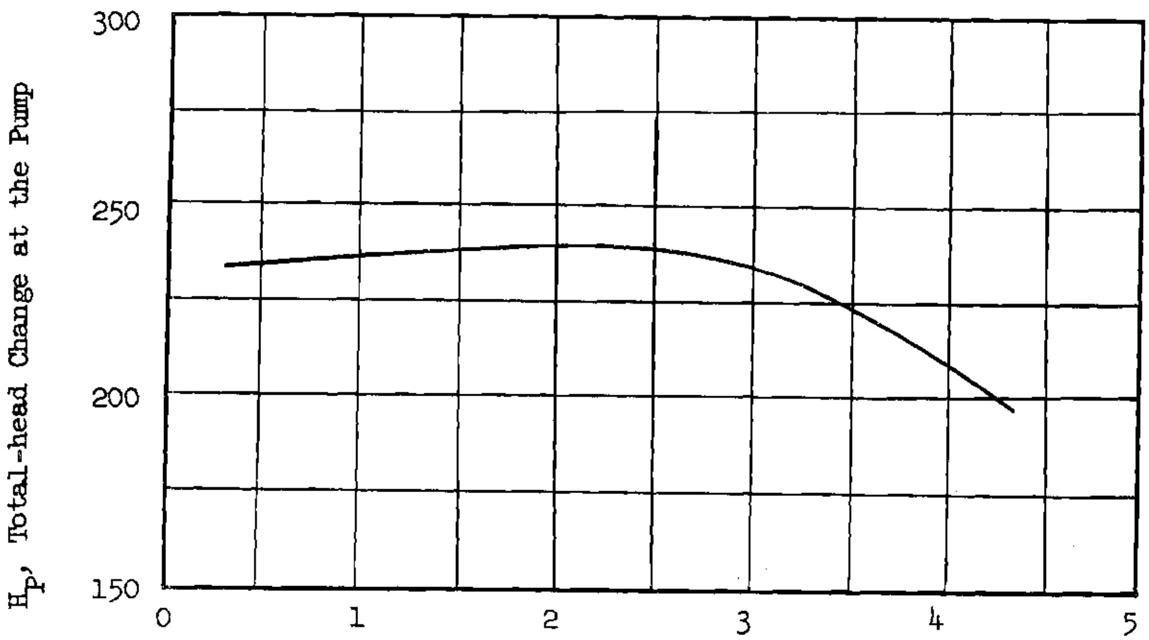
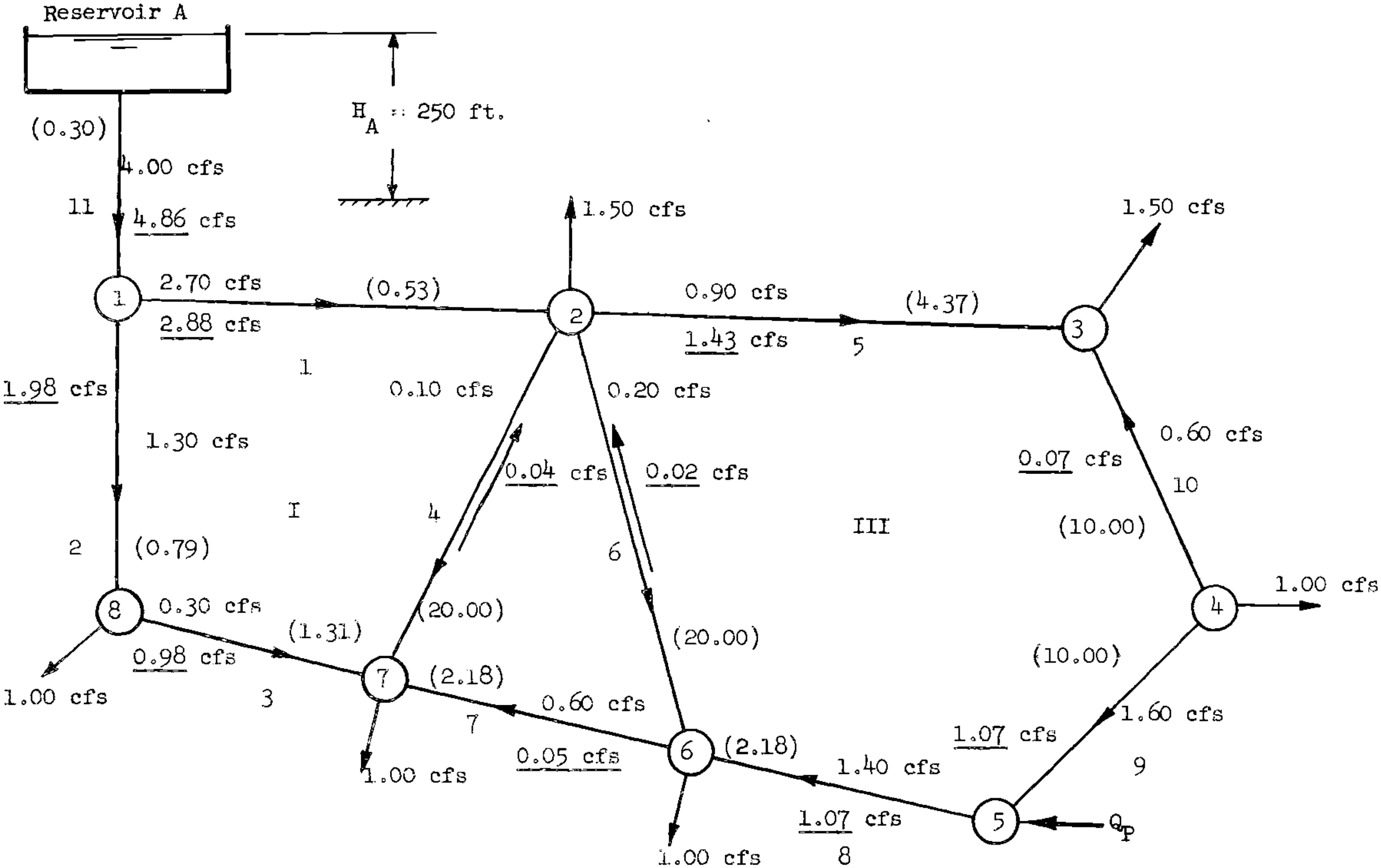


Fig. 10. Ordinary Pump-Characteristic Curve.



Note: Final values are underlined.

Fig. 11. System Containing an Elevated Reservoir and a Variable Head Pump.

Table 7. Matrix for System Containing an Elevated Reservoir and Variable Head Pump

Equation	Unknowns													Constants
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	Q_P	H_P	
1	-1	-1									+1			0
2	+1			-1	-1	-1								+1.50
3					+1					+1				+1.50
4									+1	-1				+1.00
5								-1	-1			+1		0
6						+1	-1	+1						+1.00
7			+1	+1			+1							+1.00
8		+1	-1											+1.00
I	$+K_1'$	$-K_2'$	$-K_3'$	$+K_4'$										0
II				$-K_4'$		$+K_6'$	$+K_7'$							0
III					$+K_5'$	$-K_6'$		$+K_8'$	$-K_9'$	$+K_{10}'$				0
A-P	$-K_1'$				$-K_5'$				$+K_9'$	$+K_{10}'$	$-K_{11}'$		-1	$-H_A$
Pump	Equation relating pump head and pump discharge													

the matrix for this system and contains not only twelve unknown flows, one of which is the pump discharge, but the head on the pump is an unknown also. As may be seen the equation for the characteristic curve relating pump head and pump discharge is not linear. This precludes placing the coefficients of such an equation into the matrix of Table 7.

There are two approaches to such a problem. First is to make the characteristic curve linear by making it a straight line with a constant slope <u>m</u>. Figure 12 is the linear pump characteristic curve. We see that the equation relating pump discharge and head on the pump may be expressed as:

$$H_P = h_P - mQ_P$$

or

$$H_P + mQ_P = h_P$$

This may now be placed in the matrix form shown in Table 8.

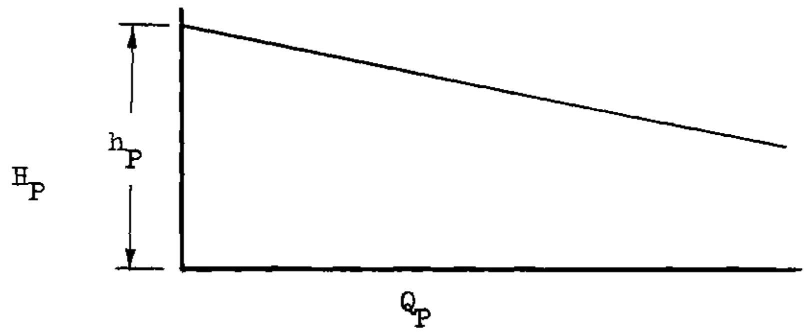


Fig. 12. Linear Pump Characteristic Curve.

The second approach, and the method of solution used, is not to enter the head on the pump as an unknown in the matrix but as a constant for a given iteration. A trial value of H_P may be selected from the pump characteristic curve of Figure 10 by assuming an initial pump discharge. Computed pump discharges are then used to pick the pump heads for succeeding iterations.

Table 8. Matrix for System Containing an Elevated Reservoir and Pump with a Linear Characteristic Curve

Equation	Unknowns													Constants
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	Q_P	H_P	
1	-1	-1									+1			0
2	+1			-1	-1	-1								+1.50
3					+1					+1				+1.50
4									+1	-1				+1.00
5								-1	-1			+1		0
6						+1	-1	+1						+1.00
7			+1	+1			+1							+1.00
8		+1	-1											+1.00
I	$+K_1'$	$-K_2'$	$-K_3'$	$+K_4'$										0
II				$-K_4'$		$+K_6'$	$+K_7'$							0
III					$+K_5'$	$-K_6'$		$+K_8'$	$-K_9'$	$-K_{10}'$				0
A-P	$-K_1'$				$-K_5'$				$+K_9'$	$+K_{10}'$	$-K_{11}'$		-1	$-H_A$
Pump												-m	-1	h_p

To solve the system shown in Figure 11 an initial pump discharge of 3.00 cubic feet per second is assumed and corresponds to a head on the pump of 235 feet. Table 9 contains the matrix of twelve unknown flows the coefficients of twelve equations.

Table 14 (Appendix A) contains the iterative procedure and results for analyzing this system. This time four iterations were necessary to

Table 9. Matrix for System Containing an Elevated Reservoir and a Pump with a Non-linear Characteristic Curve

Junctions or Loop	Flows 1	2	3	4	5	6	7	8	9	10	11	P	Constants
1	-1	-1									+1		0
2	+1			-1	-1	-1							+1.50
3					+1					+1			+1.50
4									+1	-1			+1.00
5								-1	-1			+1	0
6						+1	-1	+1					+1.00
7			+1	+1			+1						+1.00
8		+1	-1										+1.00
I	$+K_1'$	$-K_2'$	$-K_3'$	$+K_4'$									0
II				$-K_4'$		$+K_6'$	$+K_7'$						0
III					$+K_5'$	$-K_6'$		$+K_8'$	$-K_9'$	$-K_{10}'$			0
A-P	$-K_1'$				$-K_5'$				$+K_9'$	$+K_{10}'$	$-K_{11}'$		$-H_A+H_P$

obtain a solution. This undoubtedly resulted from the presence of the variable head of the pump. The final flow values are underlined in Figure 11.

As was the case for the example with free flow occurring at a point in the network; at the end of an iteration the flow from the pump could be evaluated by using the continuity equation at junction 5 and summing algebraically the flows at this point. This would allow us to reduce the matrix to eleven unknowns. If it is desired to reduce the size of the matrix by dropping a continuity equation, only that for junction 5 may be dropped for all the constant outflows must appear in the matrix.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

Conclusions.--With the aid of the electronic digital computer pipe distribution systems may be analyzed by solving simultaneously the head-loss and continuity equations descriptive of such systems. More thorough analysis of pipe distribution systems is now feasible.

Existing computer subroutines for inverting matrices may be used in solving the simultaneous equations. Computer time is excessive for large systems solved with existing subroutines. The number of repetitive calculations involved in analyzing systems containing elevated reservoirs which are emptying indicates the need for a complete computer program for such cases.

Recommendations.--It is suggested that efforts be taken to reduce the computer time required for the inversion of large matrices resulting from large systems. It is believed that advantage could be taken of the large number of zero terms which are seen to exist.

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APPENDIX A

Table 10. Tabulation Showing Iterative Procedure and Results for the Warwick System

Item	k	Q_o	First Iteration K	First Iteration Q	First Iteration $\bar{Q}$	Second Iteration K	Second Iteration Q	Second Iteration $\bar{Q}$	Third Iteration K	Third Iteration Q	Third Iteration $\bar{Q}$
1	3.34	2.002	6.690	1.449	1.726	5.760	1.719	1.722	5.750	1.721	1.722
2	4.07	1.679	6.830	1.126	1.402	5.705	1.396	1.399	5.690	1.398	1.399
3	1.22	1.491	1.820	0.938	1.214	1.480	1.208	1.211	1.478	1.210	1.211
4	2.77	0.667	1.847	0.523	0.595	1.648	0.780	0.688	1.905	0.690	0.689
5	1.63	0.757	1.232	0.204	0.480	0.782	0.474	0.477	0.778	0.476	0.477
6	1.79	0.690	1.235	0.137	0.414	0.741	0.407	0.410	0.734	0.409	0.410
7	40.83	0.112	4.570	0.045	0.078	3.180	0.141	0.105	4.240	0.103	0.104
8	16.52	0.042	0.694	0.340	0.191	3.160	0.119	0.155	2.560	0.152	0.153
9	3.67	0.109	0.400	0.860	0.484	1.775	0.501	0.492	1.804	0.494	0.493
10	3.26	0.777	2.530	1.330	1.054	3.440	1.060	1.057	3.443	1.058	1.057
11	7.58	0.845	6.400	1.145	0.995	7.550	0.998	0.996	7.551	0.998	0.997
12	1.16	1.813	2.102	2.364	2.088	2.420	2.094	2.091	2.425	2.094	2.093
13	0.80	1.891	1.522	2.442	2.166	1.740	2.172	2.169	1.742	2.171	2.171
14	0.14	2.002	0.282	2.553	2.278	0.321	2.283	2.281	0.322	2.283	2.282
15	17.06	0.446	7.610	0.266	0.356	6.075	0.267	0.312	5.320	0.321	0.316
16	5.22	0.012	0.063	-0.168	-0.078	0.408	-0.167	-0.122	0.637	-0.113	-0.118
17	58.20	0.256	14.900	0.026	0.141	8.200	0.039	0.090	5.230	0.077	0.084
18	29.10	0.378	11.000	0.148	0.263	7.650	0.161	0.212	6.160	0.199	0.206
19	10.58	0.268	2.835	-0.141	0.064	0.676	-0.128	-0.032	0.338	-0.036	-0.034
20	12.63	0.212	2.680	-0.197	0.008	0.101	-0.184	-0.088	1.111	-0.092	-0.090
21	4.60	0.090	0.414	-0.319	-0.114	0.524	-0.306	-0.210	0.965	-0.214	-0.212
22	1.48	0.445	0.657	-0.042	0.202	0.298	0.143	0.172	0.254	0.174	0.173
23	7.94	0.323	2.560	0.092	0.208	1.650	0.227	0.218	1.730	0.223	0.220
24	0.64	0.122	0.079	-0.133	-0.006	0.004	-0.084	-0.045	0.029	-0.049	-0.047
25	0.11	0.011	0.002	-0.244	-0.116	0.013	-0.195	-0.156	0.017	-0.160	-0.158

Table 10. Continued

Item	k	Q_o	First Iteration K	Q	$\bar{Q}$	Second Iteration K	Q	$\bar{Q}$	Third Iteration K	Q	$\bar{Q}$ *
26	1.46	0.078	0.114	0.276	0.177	0.258	0.187	0.182	0.265	0.182	0.182
27	60.45	0.156	9.420	-0.042	0.057	3.440	0.047	0.052	3.140	0.052	0.052
28	2.44	0.668	1.635	0.470	0.569	1.390	0.559	0.564	1.380	0.564	0.564
29	2.68	0.068	0.183	-0.185	-0.058	0.156	-0.062	-0.060	0.161	-0.060	-0.060
30	4.89	0.600	2.930	0.853	0.726	3.550	0.730	0.728	3.560	0.728	0.728
31	7.18	0.701	5.027	0.954	0.828	5.940	0.831	0.830	5.950	0.829	0.829

* Final Values

$\Sigma H_{L_I} = +0.01$ ft. $\quad \Sigma H_{L_{III}} = -0.02$ ft. $\quad \Sigma H_{L_V} = +0.01$ ft.

$\Sigma H_{L_{II}} = +0.02$ ft. $\quad \Sigma H_{L_{IV}} = -0.02$ ft. $\quad \Sigma H_{L_{VI}} = 0$ ft.

$\Sigma H_{L_{VII}} = 0$ ft.

Table 11. Tabulation Showing Iterative Procedure and Results for the System Containing Elevated Reservoirs with Constant Flow from the Network for t = 0 seconds and H_{AB} = 20.00 feet.

Item	k	Q_o	First Iteration			Second Iteration			Third Iteration		
			K	Q	$\overline{Q}$	K	Q	$\overline{Q}$	K	Q	$\overline{Q}$
1	0.53	2.00	1.06	3.39	2.69	1.42	2.62	2.66	1.41	2.62	2.64
2	0.79	1.50	1.18	3.06	2.28	1.80	2.16	2.22	1.75	2.23	2.23
3	1.31	0.50	0.66	2.06	1.28	1.68	1.16	1.22	1.60	1.23	1.23
4	20.00	0.40	8.00	0.17	0.28	5.60	0.38	0.33	6.60	0.33	0.33
5	4.37	0.70	3.06	2.35	1.52	6.65	1.15	1.33	5.81	1.32	1.33
6	20.00	0.40	8.00	0.36	0.38	7.60	0.60	0.49	9.80	0.47	0.48
7	2.18	0.40	0.87	1.73	1.06	2.31	1.04	1.05	2.29	1.06	1.06
8	2.18	0.30	0.65	1.59	0.94	2.05	1.14	1.04	2.27	1.03	1.04
9	10.00	0.20	2.00	-1.09	-0.44	4.40	-0.64	-0.54	5.40	-0.53	-0.54
10	10.00	1.20	12.00	-0.09	0.56	5.60	0.36	0.46	4.60	0.47	0.46
11	0.30	3.50	1.05	6.44	4.97	1.49	4.78	4.88	1.46	4.86	4.87
12	2.00	0.50	1.00	-2.44	-0.97	1.94	-0.78	-0.88	1.76	-0.86	-0.87

ΣH_{L_I} = -0.04 ft.　　$\Sigma H_{L_{III}}$ = -0.04 ft.

$\Sigma H_{L_{II}}$ = -0.02 ft.　　$\Sigma H_{L_{A-B}}$ = -0.04 ft.

Table 12. Tabulation Showing Iterative Procedure and Results for the System Containing Elevated Reservoirs with Constant Flow from the Network and with Reservoirs Being Allowed to Empty.

Item	t = 250 seconds			ΔH_{AB} = 15.38 feet			t = 500 seconds			ΔH_{AB} = 11.70 feet		
	K	Q	$\overline{Q}$	K	Q	$\overline{Q}$	K	Q	$\overline{Q}$	K	Q	$\overline{Q}$
1	1.40	2.23	2.44	1.29	2.40	2.42	1.28	2.01	2.22	1.18	2.17	2.19
2	1.76	2.07	2.15	1.70	2.14	2.14	1.69	2.02	2.08	1.64	2.07	2.08
3	1.61	1.07	1.15	1.51	1.14	1.14	1.49	1.02	1.08	1.41	1.07	1.08
4	6.60	0.34	0.34	6.80	0.33	0.34	6.80	0.35	0.34	6.86	0.34	0.34
5	5.81	0.94	1.14	4.98	1.08	1.12	4.85	0.72	0.92	3.99	0.86	0.89
6	9.60	0.45	0.46	9.20	0.48	0.47	9.40	0.45	0.46	9.16	0.46	0.46
7	2.31	0.91	0.99	2.16	0.98	0.98	2.14	0.86	0.92	2.01	0.92	0.92
8	2.27	0.87	0.95	2.07	0.95	0.95	2.07	0.81	0.88	1.92	0.88	0.88
9	5.40	-0.37	-0.45	4.50	-0.45	-0.45	4.50	-0.31	-0.38	3.80	-0.38	-0.38
10	4.60	0.63	0.55	5.50	0.55	0.55	5.50	0.69	0.62	6.20	0.62	0.62
11	1.46	4.31	4.59	1.38	4.54	4.56	1.37	4.03	4.30	1.29	4.24	4.27
12	1.74	-0.31	-0.59	1.18	-0.54	-0.56	1.13	-0.03	-0.30	0.59	-0.24	-0.27

Item	t= 650 sec, ΔH_{AB}=10.02ft			t=800sec, ΔH_{AB}= 8.59ft			t=1000sec, ΔH_{AB}=7.07ft			t=1200sec, ΔH_{AB}=5.95ft		
	K	Q	$\overline{Q}$	K	Q	$\overline{Q}$	K	Q	Q	K	Q	$\overline{Q}$
1	1.16	1.98	2.09	1.13	1.83	1.96	1.04	1.68	1.82	0.96	1.60	1.71
2	1.64	2.01	2.04	1.62	1.97	2.01	1.59	1.94	1.97	1.56	1.93	1.95
3	1.41	1.01	1.04	1.39	0.97	1.01	1.32	0.94	0.97	1.27	0.93	0.95
4	6.88	0.35	0.35	6.92	0.36	0.35	7.04	0.36	0.36	7.18	0.37	0.36
5	3.88	0.67	0.78	3.65	0.52	0.65	2.86	0.36	0.51	2.23	0.28	0.39
6	9.18	0.45	0.46	9.14	0.45	0.45	9.06	0.45	0.45	9.02	0.45	0.45
7	2.00	0.86	0.89	1.97	0.83	0.86	1.88	0.80	0.83	1.81	0.79	0.81
8	1.92	0.81	0.85	1.88	0.78	0.81	1.77	0.75	0.78	1.70	0.74	0.76
9	3.78	-0.31	-0.35	3.62	-0.28	-0.31	3.13	-0.25	-0.28	2.82	-0.24	-0.26
10	6.22	0.69	0.65	6.38	0.72	0.69	6.87	0.75	0.72	7.18	0.76	0.74
11	1.28	3.99	4.13	1.26	3.80	3.97	1.19	3.62	3.79	1.14	3.52	3.66
12	0.53	0.01	0.13	0.40	0.20	0.03	0.07	0.38	0.21	0.42	0.48	0.43

Table 12. Continued

Item	t=1400sec, ΔH_{AB}=5.19ft			t=1700sec, ΔH_{AB}=4.38ft			t=2000sec, ΔH_{AB}=3.96ft			t=2400sec, ΔH_{AB}=3.69ft		
	K	Q	$\overline{Q}$	K	Q	$\overline{Q}$	K	Q	$\overline{Q}$	K	Q	$\overline{Q}$
1	0.90	1.54	1.62	0.86	1.44	1.53	0.81	1.43	1.48	0.78	1.41	1.45
2	1.54	1.92	1.93	1.53	1.90	1.92	1.51	1.90	1.91	1.51	1.90	1.90
3	1.24	0.92	0.93	1.22	0.90	0.92	1.20	0.90	0.91	1.19	0.90	0.90
4	7.26	0.37	0.37	7.34	0.38	0.37	7.45	0.38	0.37	7.48	0.38	0.38
5	1.72	0.22	0.31	1.34	0.11	0.21	0.91	0.10	0.15	0.66	0.08	0.12
6	9.04	0.45	0.45	9.04	0.46	0.45	9.06	0.46	0.46	9.10	0.46	0.46
7	1.77	0.79	0.80	1.74	0.78	0.79	1.72	0.78	0.78	1.71	0.78	0.78
8	1.66	0.74	0.75	1.64	0.73	0.74	1.62	0.73	0.74	1.61	0.73	0.74
9	2.64	-0.24	-0.25	2.52	-0.23	-0.24	2.42	-0.23	-0.24	2.38	-0.23	-0.24
10	7.36	-.76	0.75	7.48	0.77	0.76	7.58	0.77	0.76	7.62	0.77	0.76
11	1.10	3.46	3.56	1.07	3.34	3.45	1.04	3.33	3.39	1.02	3.31	3.35
12	0.69	0.54	0.44	0.88	0.66	0.55	1.10	0.67	0.61	1.22	0.69	0.65

Item	t=3000sec, H_{AB}=3.56ft			t=4000sec, H_{AB}=3.58ft		
	K	Q	$\overline{Q}$	K	Q	$\overline{Q}$
1	0.77	1.41	1.43	0.76	1.43	1.43
2	1.50	1.90	1.90	1.50	1.90	1.90
3	1.18	0.90	0.90	1.18	0.90	0.90
4	7.52	0.38	0.38	7.54	0.38	0.38
5	0.50	0.08	0.10	0.42	0.10	0.10
6	9.12	0.46	0.46	9.12	0.46	0.46
7	1.70	0.78	0.78	1.70	0.78	0.78
8	1.60	0.73	0.74	1.60	0.74	0.74
9	2.36	-0.23	-0.24	2.35	-0.24	-0.24
10	7.64	0.77	0.76	7.65	0.76	0.76
11	1.00	3.31	3.33	1.00	3.34	3.33
12	1.30	0.69	0.67	1.34	0.66	0.67

H_{L_I} = 0 ft.

$H_{L_{II}}$ = -0.01 ft.

$H_{L_{III}}$ = +0.01 ft.

$H_{L_{A-B}}$ = +0.01 ft.

Table 13. Tabulation Showing Iterative Procedure and Results for the System Containing Elevated Reservoirs with Free Flow Occurring at a Point in the Newtork for t = 0 seconds and ΔH_{AB} = 20.00 feet.

Item	k	Q_o	First Iteration			Second Iteration			Third Iteration		
			K	Q	$\bar{Q}$	K	Q	$\bar{Q}$	K	Q	$\bar{Q}$
1	0.53	3.00	1.59	4.28	3.64	1.93	3.59	3.62	1.92	3.61	3.61
2	0.79	7.00	5.53	7.02	7.01	5.54	7.01	7.01	5.53	7.02	7.01
3	1.31	6.00	7.86	6.02	6.01	7.88	6.01	6.01	7.86	6.02	6.01
4	20.00	2.00	40.00	1.98	1.99	39.85	1.99	1.99	39.80	1.99	1.99
5	4.37	1.00	4.37	0.50	0.75	3.28	0.72	0.73	3.21	0.74	0.73
6	20.00	1.50	30.00	2.30	1.90	38.00	1.82	1.86	37.18	1.86	1.86
7	2.18	1.80	3.92	2.60	2.20	4.80	2.12	2.16	4.72	2.16	2.16
8	2.18	0.80	1.74	0.80	0.80	1.74	0.80	0.80	1.75	0.80	0.80
9	10.00	1.30	13.00	1.30	1.30	13.01	1.30	1.30	13.03	1.30	1.30
10	10.00	2.30	23.00	2.30	2.30	23.01	2.30	2.30	23.03	2.30	2.30
11	0.30	10.00	3.00	11.30	10.65	3.20	10.59	10.62	3.18	10.63	10.63
12	2.00	3.30	6.60	2.80	3.05	6.10	3.02	3.04	6.06	3.04	3.04
C		9.80		10.60	10.20		10.12	10.16		10.17	10.16

H_{L_I} = -0.02 ft.

$H_{L_{II}}$ = -0.07 ft.

$H_{L_{III}}$ = +0.09 ft.

$H_{L_{A-B}}$ = +0.04 ft.

$H_{L_{A-C}}$ = -0.10 ft.

Q_c = 10.16 cfs

Table 14. Tabulation Showing Iterative Procedure and Results for System Containing Elevated Reservoirs with Free Flow Occurring at a Point in the Network While the Reservoirs are Allowed to Empty.

Item	t=250sec, ΔH_{AB}=22.27ft			t=500sec, ΔH_{AB}=24.16ft			t=800sec, ΔH_{AB}=25.59ft			t=1100sec, ΔH_{AB}=27.18ft		
	K	Q	$\bar{Q}$	K	Q	$\bar{Q}$	K	Q	$\bar{Q}$	K	Q	$\bar{Q}$
1	1.92	3.66	3.64	1.93	3.69	3.66	1.94	3.71	3.68	1.95	3.71	3.70
2	5.54	6.68	6.84	5.40	6.52	6.68	5.28	6.27	6.48	5.11	6.06	6.27
3	7.88	5.68	5.84	7.65	5.52	5.68	7.44	5.27	5.48	7.18	7.06	5.27
4	39.80	1.88	1.93	38.65	1.82	1.88	37.55	1.73	1.80	36.15	1.66	1.73
5	3.21	0.48	0.61	2.65	0.34	0.47	2.07	0.17	0.32	1.41	0.03	0.18
6	37.15	1.76	1.81	36.20	1.71	1.76	35.25	1.64	1.70	34.07	1.58	1.64
7	4.71	1.95	2.06	4.48	1.84	1.95	4.25	1.70	1.82	3.98	1.57	1.70
8	1.75	0.69	0.74	1.62	0.63	0.69	1.50	0.55	0.62	1.35	0.49	0.55
9	13.02	1.19	1.24	12.45	1.13	1.19	11.88	1.05	1.12	11.20	0.99	1.05
10	23.02	2.19	2.24	22.45	2.13	2.19	21.88	2.05	2.12	21.20	1.99	2.05
11	3.19	10.34	10.48	3.14	10.21	10.34	3.10	9.98	10.16	3.05	9.78	9.97
12	6.08	2.66	2.85	5.70	2.47	2.66	5.33	2.22	2.44	4.89	2.02	2.23
Q_C		9.51	9.83		9.18	9.51		8.70	9.10		8.29	8.70

Item	t=1400sec, ΔH_{AB}=27.90ft			t=1700sec, ΔH_{AB}=28.11ft			t=2000sec, ΔH_{AB}=27.85ft			t=2300sec, ΔH_{AB}=27.26ft		
	K	Q	$\bar{Q}$	K	Q	$\bar{Q}$	K	Q	$\bar{Q}$	K	Q	$\bar{Q}$
1	1.96	3.72	3.71	1.96	3.71	3.71	1.97	3.64	3.68	1.95	3.57	3.62
2	4.95	5.86	6.06	4.80	5.64	5.85	4.62	5.45	5.65	4.46	5.25	5.45
3	6.90	4.86	5.06	6.64	4.64	4.85	6.36	4.45	4.65	6.09	4.25	4.45
4	34.66	1.59	1.66	33.25	1.52	1.59	31.87	1.45	1.52	30.48	1.39	1.46
5	0.76	-0.11	0.03	0.15	-0.23	-0.10	0.43	-0.29	-0.20	0.85	-0.34	-0.27
6	32.82	1.52	1.58	31.60	1.46	1.52	30.40	1.40	1.46	29.20	1.34	1.40
7	3.70	1.44	1.57	3.42	1.32	1.45	3.15	1.20	1.32	2.89	1.08	1.20
8	1.21	0.42	0.49	1.07	0.36	0.43	0.93	0.30	0.36	0.79	0.24	0.30
9	10.54	0.92	0.99	9.90	0.86	0.93	9.27	0.80	0.86	8.64	0.74	0.80
10	20.54	1.92	1.99	19.90	1.86	1.93	19.27	1.80	1.86	18.64	1.74	1.80
11	2.99	9.58	9.77	2.93	9.36	9.56	2.87	9.09	9.33	2.80	8.82	9.07
12	4.46	1.82	2.02	4.05	1.64	1.83	3.66	1.51	1.67	3.34	1.40	1.53
Q_C		7.89	8.30		7.48	7.89		7.10	7.50		6.72	7.11

Table 14. Continued

Item	t=2600sec, ΔH_{AB}=26.42ft			t=2900sec, H_{AB}=25.37ft			t=3200sec, ΔH_{AB}=24.17ft			t=3500sec, ΔH_{AB}=22.86ft		
	K	Q	$\bar{Q}$	K	Q	$\bar{Q}$	K	Q	$\bar{Q}$	K	Q	$\bar{Q}$
1	1.92	3.49	3.56	1.88	3.40	3.48	1.84	3.30	3.39	1.80	3.19	3.29
2	4.30	5.05	5.25	4.15	4.84	5.05	3.98	4.65	4.85	3.82	4.44	4.64
3	5.82	4.05	4.25	5.56	3.84	4.05	5.30	3.65	3.85	5.04	3.44	3.64
4	29.15	1.32	1.39	27.80	1.26	1.33	26.52	1.20	1.26	25.22	1.13	1.20
5	1.17	-0.38	-0.32	1.42	-0.41	-0.37	1.60	-0.43	-0.40	1.74	-0.44	-0.42
6	28.05	1.29	1.34	26.87	1.23	1.29	25.75	1.17	1.23	24.60	1.12	1.17
7	2.62	0.96	1.08	2.36	0.84	0.96	2.09	0.72	0.84	1.83	0.60	0.72
8	0.65	0.17	0.24	0.52	0.11	0.17	0.38	0.04	0.11	0.24	-0.02	0.04
9	8.00	0.67	0.74	7.37	0.61	0.67	6.73	0.54	0.61	6.09	0.48	0.54
10	18.00	1.67	1.74	17.37	1.61	1.67	16.73	1.54	1.61	16.09	1.48	1.54
11	2.72	8.54	8.81	2.64	8.24	8.53	2.56	7.94	8.24	2.47	7.64	7.94
12	3.07	1.29	1.41	2.82	1.20	1.31	2.61	1.12	1.21	2.42	1.04	1.12
Q_C		6.33	6.72		5.94	6.33		5.57	5.95		5.17	5.56

Item	t=3800sec, ΔH_{AB}=21.47ft			t=4100sec, ΔH_{AB}=20.02ft			t=4400sec, ΔH_{AB}=18.54ft			t=4700sec, ΔH_{AB}=17.03ft		
	K	Q	$\bar{Q}$	K	Q	$\bar{Q}$	K	Q	$\bar{Q}$	K	Q	$\bar{Q}$
1	1.74	3.08	3.19	1.69	2.96	3.08	1.63	2.84	2.96	1.56	2.71	2.83
2	3.66	4.24	4.44	3.57	4.03	4.24	3.35	3.83	4.03	3.18	3.63	3.83
3	4.77	3.24	3.44	4.51	3.03	3.24	4.24	2.83	3.03	3.97	2.63	2.83
4	23.94	1.07	1.13	22.66	1.01	1.07	21.40	0.94	1.01	20.14	0.88	0.94
5	1.84	-0.45	-0.44	1.90	-0.46	-0.45	1.95	-0.45	0.45	1.96	-0.44	-0.45
6	23.48	1.06	1.12	22.34	1.00	1.06	21.18	0.94	1.00	20.02	0.88	0.94
7	1.56	0.47	0.60	1.30	0.35	0.47	1.03	0.22	0.34	0.75	0.09	0.22
8	0.10	-0.09	-0.02	0.05	-0.15	-0.09	0.19	-0.22	-0.16	0.34	-0.30	-0.22
9	5.44	0.41	0.48	4.78	0.35	0.41	4.13	0.28	0.34	3.44	0.20	0.28
10	15.44	1.41	1.48	14.78	1.35	1.41	14.13	1.28	1.34	13.44	1.20	1.28
11	2.38	7.32	7.63	2.29	7.00	7.31	2.19	6.67	7.00	2.10	6.34	6.66
12	2.25	0.96	1.04	2.08	0.89	0.97	1.93	0.82	0.90	1.79	0.76	0.83
Q_C		4.78	5.17		4.39	4.78		3.99	4.39		3.60	3.99

Table 14. Continued

Item	t=5000sec, ΔH_{AB}=15.52ft			t=5300sec, ΔH_{AB}=14.01ft		
	K	Q	$\bar{Q}$	K	Q	$\bar{Q}$
1	1.50	2.57	2.70	1.43	2.43	2.57
2	3.02	3.42	3.62	2.86	3.21	3.42
3	3.71	2.42	2.62	3.44	2.21	2.42
4	18.88	0.82	0.88	17.62	0.76	0.82
5	1.95	-0.43	-0.44	1.92	-0.42	-0.43
6	18.84	0.82	0.88	17.62	0.76	0.82
7	0.47	-0.05	0.08	0.18	-0.19	-0.05
8	0.49	-0.37	-0.30	0.64	-0.44	-0.37
9	2.75	0.13	0.20	2.04	0.06	0.13
10	12.75	1.13	1.20	12.04	1.06	1.13
11	2.00	5.99	6.33	1.90	5.64	5.98
12	1.66	0.70	0.76	1.53	0.64	0.70
QC		3.19	3.58		2.78	3.19

ΣH_{L_I}= -0.26ft

$\Sigma H_{L_{II}}$= -0.01ft

$\Sigma H_{L_{III}}$= +0.07ft

$\Sigma H_{L_{AB}}$= +0.06ft

$\Sigma H_{L_{AC}}$= +0.10ft

Table 15. Tabulation Showing Iterative Procedure and Results for the System Containing a Reservoir and a Variable Head Pump.

Item	k	Q_o	First Iteration $Q_P = 3.00$cfs $H_P = 235$ ft			Second Iteration $Q_P = 1.60$cfs $H_P = 240$ ft		
			K	Q	$\bar{Q}$	K	Q	$\bar{Q}$
1	0.53	2.70	1.43	3.43	3.07	1.63	2.82	2.94
2	0.79	1.30	1.03	3.36	2.33	1.84	1.74	2.03
3	1.31	0.30	0.39	2.36	1.33	1.74	0.74	1.03
4	20.00	0.10	2.00	-0.26	-0.08	1.62	-0.07	-0.08
5	4.37	0.90	3.93	1.96	1.43	6.26	1.46	1.45
6	20.00	0.20	4.00	0.23	0.22	4.30	-0.07	0.07
7	2.18	0.60	1.31	-1.10	-0.25	0.54	0.34	0.04
8	2.18	1.40	3.05	-0.33	0.53	1.16	1.40	0.97
9	10.00	1.60	16.00	0.54	1.07	10.68	1.04	1.05
10	10.00	0.60	6.00	-0.46	0.07	0.68	0.04	0.05
11	0.30	4.00	1.20	6.80	5.40	1.61	4.56	4.98

Third Iteration $Q_P = 2.02$ cfs $H_P = 241$ ft			Fourth Iteration $Q_P = 2.13$ cfs $H_P = 241$ ft			
K	Q	Q	K	Q	Q	
1.56	2.83	2.89	1.53	2.87	2.88	
1.61	1.93	1.98	1.57	1.99	1.98	
1.35	0.93	0.98	1.28	0.99	0.98	$H_{L_I} = -0.01$ ft
1.52	-0.04	-0.06	1.16	-0.01	-0.03	
6.34	1.41	1.43	6.25	1.43	1.43	$H_{L_{II}} = +0.02$ ft
1.46	-0.05	0.01	0.26	-0.05	-0.02	
0.09	0.11	0.08	0.17	0.02	0.05	
2.11	1.16	1.06	2.32	1.07	1.07	$H_{L_{III}} = -0.01$ ft
10.52	1.09	1.07	10.69	1.07	1.07	
0.52	0.09	0.07	0.69	0.07	0.07	
1.49	4.76	4.87	1.46	4.86	4.86	$H_{L_{AP}} = -0.01$ ft

APPENDIX B

Computation of k using the Darcy-Weisbach formula.--The Darcy-Weisbach formula for flow in pipes may be written as

$$h_L = f \frac{L}{D} \frac{V^2}{2g}$$

in which,

f = Darcy-Weisbach resistance coefficient

L = length of the pipe

g = magnitude of the gravitational attraction

D = inside diameter of the pipe

When the equation is written in terms of the discharge, Q in cubic feet per second, it takes the form

$$h_L = \frac{8\ f\ L}{g\pi^2\ D^5}\ Q^2$$

or

$$h_L = k\ Q^2$$

in which,

$$k = \frac{8\ f\ L}{g\pi^2\ D^5}$$

In most pipe systems the resistance coefficient, f, for a given pipe may be considered constant. Thus, k values as computed from the above equation may be treated as constant also. The values for k in the text are in units of sec.^2 per ft.^5

<u>Computation of k using the Hazen-Williams formula</u>.--The Hazen-Williams formula is

$$V = 0.115\ C\ D^{0.63}\ S^{0.54}$$

in which

C = Hazen-Williams resistance coefficient

D = inside diameter of the pipe

S = friction slope $= \frac{h_L}{L}$

or

$$S = \frac{54.3\ V^2}{C^{1.85}\ D^{1.167}\ V^{0.15}}\ .$$

Substituting in h_L/L for S and solving for h_L gives

$$h_L = \frac{54.3\ L\ V^2}{C^{1.85}\ D^{1.167}\ V^{0.15}}\ .$$

If written in terms of the discharge, Q in cubic feet per second, with the exception of the velocity term in the denominator and with D in inches we obtain

$$h_L = \frac{1.82 \times 10^6\ L\ Q^2}{C^{1.85}\ D^{5.167}\ V^{0.15}}\ .$$

This equation is in the desired form

$$h_L = k\ Q^2$$

in which,

$$k = \frac{1.82 \times 10^6 L}{C^{1.85} D^{5.167} V^{0.15}} .$$

The velocity may vary from 3 to 8 feet per second without having any large influence on k. Thus, it is usually sufficient to assume an average value for the velocity and compute k as a constant for a given pipe.