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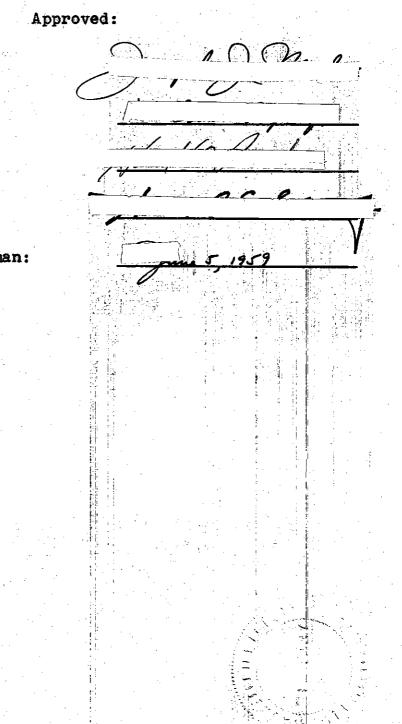
A THESIS

IN WORK CYCLES

A STUDY OF ELEMENT INDEPENDENCE

# A STUDY OF ELEMENT INDEPENDENCE

IN WORK CYCLES



Date Approved by Chairman:

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 $\{i_{1}, j_{1}\}$ 

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#### SUMMARY

This investigation was undertaken to gain further insight into the relationship among the elements of a work cycle, particularly to determine whether or not such elements are statistically independent.

The data were obtained from a micromotion study of two operators, one whose cycle times were statistically stable and one whose cycle times were statistically unstable. These operators performed a short cycle, manual assembly operation in an industrial plant. Work cycles containing major departures from the established work method were eliminated. A twelve element breakdown of the cycle was made for each operator. These data were subjected to Wilks' multivariate test of independence to test the null hypothesis that the elements of the cycle are statistically independent.

The results of the analysis led to an acceptance of this null hypothesis for the statistically stable operator and to a rejection of the null hypothesis for the statistically unstable operator. The only significant correlation found was between the last two elements of the cycle for the statistically unstable operator. From an examination of the films this was believed to be due to the operator's examining the product in order to determine the proper disposition of the completed assembly on those cycles where difficulty was encountered on the next to the last element.

### CHAPTER I

#### INTRODUCTION

There have been few changes in time study practices since its original tenents were developed. The establishment of time study was a part of "Scientific Management" which was later proposed by Taylor as a substitute for the management methods then employed (18).

There have been numerous criticisms of conventional practices in the field of time study. One analysis led Presgrave to conclude in 1945 that time study was in a "most unsatisfactory state," and that methods had been little improved in precision, manner, or uniformity (48).

Various approaches have been suggested by Sylvester (58), Davidson (18), Abruzzi (1), Lehrer (33), Wilkinson (64), Desmond (20), Gomberg (25), and others. Several of these advocate a statistical approach. This move toward a statistical frame of reference was made in an effort to improve the reliability of time study results.

Time study controversy seems to be concerned with two general assumptions: first, that time study elements are statistically independent; and second, that time study men can adequately appraise operator performance using normalizing techniques. This study is concerned with the assumption that time study elements are statistically independent.

The several conventional standard data systems are founded on the theory that manual activity may be sub-divided into readily identifiable elements of work and that the work content for a given element is the same regardless of where or by whom the element is performed. The theory further holds that the work content of a manual operation is the sum of the times for the individual elements comprising the operation.

An acceptable time study practice is to base work content on normal time, that is, the actual time which has been leveled. Actual time is proportional to normal time if the performance level is constant. Actual time was used in this study on the assumption that operator performance was constant. Hence, results based upon actual time values are presumed to be applicable to work content considerations.

If two different operations, (1) and (2), are divided into their individual elements, 1, 2, . . , they can be illustrated as follows:

> Operation (1) <u>1 2 "3" 4 5 6</u> Operation (2) <u>1 2 3 "4" 5 6 7 8</u>

Suppose element "3" of Operation (1) is the "same," by the definition used in standard data systems, as element "4" of Operation (2). Element 2 of Operation (1) is not the same as element 3 of Operation (2), and element 4 of Operation (1)

is not the same as element 5 of Operation (2). The observed time for element "3", on the ith cycle, of Operation (1) could be represented by:

> $y_1 = a + bx_{21} + cx_{41} + e_1$  and  $E(y_1) = \mu_y = a + b\mu_2 + e\mu_4$

a, b, and c are constants

 $x_{21}$  = observed time for element 2 on the ith cycle  $x_{41}$  = observed time for element 4 on the ith cycle E(y) = the expected value of y

 $\mu_2$  = mean of element 2

 $\mu_{4}$  = mean of element 4

The observed time for element "4" of Operation (2) could be represented in a similar manner as follows:

> $y_1 = a' + b'x_{31} + c'x_{51} + e_1$  $E(y_1) = \mu_y' = a' + b'\mu_3' + c'\mu_5$

If we accept the assumption of validity of standard data systems, then  $\mu_y = \mu'_y$ , and if the elements are independent, i.e., b = b' = c = c' = 0, then, this implies that a = a'. If, however, the elements are not independent, acceptance of  $\mu_y =$  $\mu'_y$  implies that  $a + b\mu_2 + c\mu_4 = a' + b'\mu'_3 + c\mu'_5$ . Since the latter is not plausible for a large variety of operations, establishing the independence of element times will add support to the use of standard data systems. This arguement could be extended to cover all of the elements in the work cycle, which is, in fact, what this study tested using Wilks' multivariate test of independence to show that time study elements are statistically independent.

#### CHAPTER II

#### REVIEW OF THE LITERATURE

Abruzzi (1) has stated that the assumption of independence is largely unjustified, and that relationships among parts of a motion cycle are partially a function of the operator who is observed. He also suggests that there is a relationship between degree of dependence and the number and size of parts into which the job cycle is analyzed.

Earlier work by Barnes (5) indicated an interaction between element times; that is, if distance was manipulated as a variable, the time required for the transport motions varied as would be expected, but the time required for grasping was also affected. Interaction appeared in both of these studies. This is, however, a dependence among the mean times, not in the element times within the individual cycles.

In a study from the Psychological Laboratories at the University of Wisconsin (55), the components of movement in an assembly type operation were investigated. In this motion pattern the subject was required to grasp a part, move it, position and release it, and finally reach back to the supply of parts. The results indicated that correlations may exist only for certain elements, perhaps between those involving difficult manipulations and between adjacent elements. Nadler and Denholm (41) Performed an experiment in which subjects had to reach to and manipulate rotary switches. The object of this study was to determine what effect the addition or elimination of an element of work would have on established therblig times within a cycle. Observers found that the original total cycle time and times for adjacent therbligs were significantly affected. The conclusion was that the division of an operation into therbligs for standard data purposes was unwarranted.

Ghiselli and Brown (23) performed a simple key-tapping experiment. They observed that by the elimination of two of the movements the cycle time was not reduced by as much as one would logically expect, thereby indicating interaction between the elements. They concluded that an operator works on an operation as a totality, and that each part of the operation affects all other parts. Buffa (13) in an experiment to gain further insight into the basic additivity of universal standard data elements, seemed to refute the results of the key-tapping experiment of Ghiselli and Brown (23). It should be pointed out, however, that the basis for Ghiselli and Brown's (23) study was element times while the basis for Buffa's (13) study was therblig times.

Barnes and Mundel (6, 7, 8, 9, 10) found, while studying the time required to position pins in bushings with beveled holes, that certain therbligs in the cycles were interrelated; hence, standard times for certain therbligs cannot be given

as independent values. Davidson (19) reported on experiments conducted by graduate students, Moffat and McClure. The experimenters used a simple task of the "post and washer" variety in reaching the same conclusions as did Barnes and Mundel. Gomberg (25) expressed his views on standard data as

<u>\_</u>\_\_\_\_

follows:

Basically, standard data systems may be divided into two categories: the macroscopic and the microscopic.

The macroscopic school generally formulates its data in terms of sizeable job elements that reappear in many operations. The microscopic school formulates its data in terms of minute muscular reactions, or therbligs.

There are several standard data systems currently in use. They are either the "element" type, the "motion" data type, or a combination of the two. Davidson (18) questioned the accuracy of all standard data systems on the basis that if the values of one system are accurate, then different values from other systems cannot be accurate. His findings do not establish which one of the systems is valid.

Balkcom's (3) study was an evaluation of results attained by three standard data systems. The results:

offered ample evidence to prove that there is significant difference in the ability of the three standard data systems under consideration to measure the time for a short manual operation. All three of the systems indicated a general lack of agreement between the elements of the synthesized cycle and the elements of the film cycle, as was exemplified by the fact that there was an absolute deviation of 27.86 per cent between the synthesized element times and the film element times.

Green's (28) investigation studied primarily the element-time distributions for an industrial operation; one of the secondary objectives was to investigate element independence. One of the results of this study showed evidence of independency among the elements of the operation. Green pointed out the limitations of his study by noting the small sample used (one operator, one method). He recommended that a rigorous study be made of element independence.

Perkins (47) followed Green's recommendation and proceeded to investigate the relationships among and between elements in the work cycle. The results were as follows:

It was found that there was evidence of correlation among the time values for the elements of a work cycle for both a five and two element breakdown. In addition, the following conclusions were drawn on the basis of the test results:

- 1. There was an indication that the degree of correlation among the elements of a cycle does not remain constant for the same operator during the work shift.
- 2. The nature and extent of correlation among the elements of a work cycle from period to period appeared to depend on the operator.
- 3. It appeared that in those film sequences where the degree of correlation was found to be the highest, there was a concentration of variables such as fumbles, slight delays in positioning the parts, and dropping extra parts.
- 4. There was an indication that the degree of correlation among the elements of the five element breakdown was decreased by combining those elements into a two element breakdown of the operation.
- 5. The grouping process did not decrease the degree of correlation among the elements to the same extent for the same operator or data for different shots.

6. The stable and unstable operators exhibited similar characteristics in regard to the degree and extent of correlation among the elements of the cycle.

Perkins recommended that a further analysis be made using a twelve element breakdown, treating them in a similar manner as he did the five and two element breakdown and comparing the results.

The studies cited above have some exploratory value although they cannot be considered conclusive because of certain inherent limitations. Many of the studies were performed under laboratory conditions, using highly motivated test subjects performing simple tasks. The results obtained under these conditions may not be comparable to those found in actual operations. In other studies the basic data were not evaluated statistically and no attempt was made to determine the statistical significance of the results.

The present investigation followed Perkins' (47) recommendation that there be employed the original twelve element breakdown of the industrial operation which he studied for element independency and that the results be statistically evaluated.

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#### CHAPTER III

#### OBJECTIVES

The reliability of the standard data concept depends largely on the hypothesis of additivity of element times. If the additivity hypothesis can be accepted generally, then considerable research remains to be done in developing a workable standard data system for all classes of motions comprising elements.

The objective of this investigation was to study the relationships among the elements of a work cycle to determine if they were statistically independent. The solution to the problem, therefore, involved accepting or refuting statistically the null hypothesis that the elements of the cycle are statistically independent.

#### PROCEDURE

CHAPTER IV

The data used in this investigation were available from a research project which began in 1951, under the direction of Dr. Lehrer and Dr. Moder, in the School of Industrial Engineering at the Georgia Institute of Technology. The overall purpose of this project was to contribute a better understanding of the characteristics of a worker's performance on manually controlled, repetitive operations.

These data available from the research project: 1) Were obtained from more than one operator, 2) Represented a sample size large enough to obtain valid statistical results, 3) Represented an operation established for more than three months, 4) Were obtained from experienced operators, 5) Were available on film so that a micromotion study could be made.

Taft (59) utilizing a high-speed camera (2,000 frames per minute), took 15,000 feet of film of nineteen operators at intervals of approximately one hour. He obtained from twelve to fifteen cycles per film sequence.

Taft made micromotion studies of each of the work cycles which were broken down into the following twelve elements:

1. Get barrel TE, ST, and G Barrels are in containers directly behind fixture and drive nuts.

- 3. Get writing units TE, ST, and G
- 4. Place writing units in barrels TL, P, A, and RL
- 5. Get drive nut TE, ST, and G
- 6. Place drive nut on unit TL, P, A, and RL
- 7. Get ferrule TE, ST, and G
- 8. Place ferrule over drive nut TL, P, A, and RL
- 9. Get complete unit TE and G
- 10. Place complete unit in staking device DA, TL, and P

placed in holding fixture, top opening up

Writing units are directly to left and right of holding fixture

Writing units are placed in barrels, point down

Drive nuts are in container directly behind fixture

Drive nuts are placed over top of writing unit

Ferrules are in containers directly in front of fixture

Ferrules are slipped over drive nuts

The complete units are grasped in order to be removed from the fixture

The complete units are removed from the holding fixture and placed in the staking device

- 11. Stake ferrule and remove unit from staking device A, H, and DA
- 12. Aside assembled unit to container TL and RL

The ferrule is staked and the complete unit is removed from the staker

The assembled unit is disposed to container on right of operator

Each element was broken down into therbligs and recorded by frame number. These recordings converted to times in minutes were used as the basic data for this investigation.

Summers (57), using the above data, undertook a study to evaluate the relationship between cycle time stability and the characteristics of the work time distribution. The statistical characteristics for this investigation were the mean time, the total variance, the variance between periods of observations, the skewness, the peakedness, the goodness of fit of the Normal Curve, the Log Normal Curve and the Pearson Type III Curve. The "Variance between Periods" was used as a measure of the level of stability for each operator. The results of these calculations (57), using data from Taft's (59) study, were used in the present investigation in the selection of operators for study, i.e. one whose cycle times were statistically stable and one whose cycle times were statistically unstable.

Two operators were chosen in accordance with the established level of stability criteria (57). The operators were designated by Summers (57) as Q and K. Operator Q represented the statistically stable operator and is hereafter called Operator One. Operator K represented the statistically unstable operator and is hereafter called Operator Two. The following results were obtained by Summers (57) in his analysis of variance:

Operato	r Standard Deviation between periods	Standard Devia- tion within				
		periods				
One	s <sub>m</sub> = 5.3	s <sub>w</sub> <u>-</u> 19.8				
Two	s <sub>m</sub> = 21.1	s <sub>w</sub> = 25.6				

In the present study an analysis was made of the data from the film analysis sheets for each operator in order to determine the variations in motion times which occurred in each cycle. Some of the cycles contained variables which could not be considered as a normal part of the work cycle. The variables which were selected and eliminated due to assignable causes by Perkins (47) were:

- 1. Inspection delay a prolonged visual or physical inspection of an assembled or a subassembly part. The workplace and required motions were methodized to such an extent and the parts were of such a uniform nature that assembly normally proceeded with little if any inspection.
- 2. Bad part occurred when an assembly operation could not be accomplished with the part originally selected necessitating the replacement of the part with another. This represented a major departure from the normal work cycle which to some extent could be corrected. Theoretically this could be eliminated by better quality control.
- 3. Part stuck in staker an occurrence which was due to the improper functioning of the mechanical staking device. This source of variation was readily apparent and subject to elimination.

4. Distraction - occurred when the operator's attention was purposely and noticeably directed to an object other than the assembly operation, talking to another person and reading while engaged in the assembly operation.

No attempt was made to eliminate the cycles which contained minor departures from the established method such as momentary fumbles, slight delays in positioning parts, and dropping extra parts, since they were considered to be an inherent part of the operation by virtue of their small size.

The element times used in this investigation were obtained by taking readings for the hand which was last to complete the preceding element and subtracting the reading for the hand which was last to complete the element being considered. The element time was taken directly from the film analysis sheets for operators one and two and recorded in thousandths of a minute.

Wilks' multivariate test of independence was used in this investigation to test the data from the film analysis sheets to determine whether variables (element times), in a given set, which are normally distributed, are mutually independent, i.e. the covariances  $\sigma_{i,j} = 0$  for  $i \neq j$  (65, p 242). Defining  $\sigma_{i,j}^{(0)}$  as follows:

$$\sigma_{1j}^{(o)} = \sigma_{1j}; 1 = j$$

we can test the mutual independence of the element times using the following null hypothesis:

The likelihood ratio for testing this hypothesis is:

$$\mathbf{L} = \frac{\left| \mathbf{T}_{1} \right| \frac{\mathbf{N}}{2}}{\left| \mathbf{T}_{2} \right| \frac{\mathbf{N}}{2}}$$

where L represents the likelihood ratio and N represents the sample size.  $\begin{vmatrix} T_1 \\ and \\ T_2 \end{vmatrix}$  represent determinants of the two "Covariance matrices."  $\begin{vmatrix} T_1 \\ T_1 \end{vmatrix}$  refers to the case where the presence of correlation is assumed; this is a symetric matrix.  $\begin{vmatrix} T_2 \\ T_2 \end{vmatrix}$  refers to the case where the absence of correlation is assumed; thus, the elements  $s_{ij}$  of  $\begin{vmatrix} T_2 \\ T_2 \end{vmatrix}$  are zero if  $i \neq j$  and are the time study element variances if i = j.

In the application of Wilks' (65) multivariate test of independence, the ratio is developed from the following matrices:

$$L = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1p} \\ s_{22} & \cdots & s_{2p} \\ & \ddots & & & \\ & & \ddots & & \\ & & & s_{pp} \end{bmatrix} \begin{bmatrix} s_{11} & s_{22} & 0 \\ & s_{33} & & \\ & & & s_{pp} \end{bmatrix}$$

where there are p variables (elements). The s terms are ij computed from

$$s_{1j} = \frac{\sum_{k=1}^{N} (x_{1k} - \overline{x}_1) (x_{jk} - \overline{x}_j)}{N}$$
$$= \frac{\sum_{k=1}^{N} x_{1k} x_{jk}}{N} - \overline{x}_1 \overline{x}_j$$

where  $X_{ik}$  and  $X_{jk}$  refer to the observed times for elements 1 and j on the kth cycle, respectively. When i = j, the formula reduces to

$$s_{11} = s_{1}^{2} = \frac{\sum_{k=1}^{N} (x_{1k} - \overline{x}_{1})^{2}}{N}$$
$$s_{1}^{2} = \sum_{k=1}^{N} (x_{1k})^{2} - (\overline{x}_{1})^{2}$$

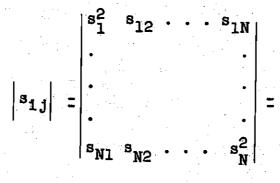
Each of the  $s_{ij}$  terms were computed by the IBM 650 Digital Computer located in the Rich Electronic Computer Center of the Georgia Institute of Technology, using Intercorrelation Program (ST - 09). The output card format for this program is:

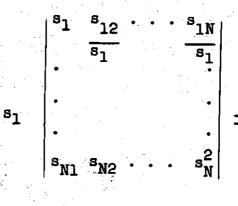
- Sums
   Cross product sums
   Means
   Covariances
- 5. Sigmas
- 6. Correlation coefficients
- 7.  $N_1$  the number of cycles

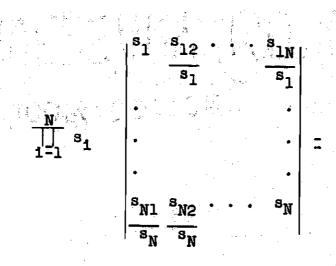
Output identification number four identified the covariances which were the matrix entries for the likelihood ratio determinant. The L value represents the likelihood that the hypothesis of independence is true as described by Wilks (65). It was, however, convenient to use  $L^* = -2 \log_e$ L, since  $-2 \log_e$  L has approximately the same distribution as Chi-square, for large samples (65), with p(p-1)/2 degrees of freedom, where p represents the number of elements. In this study, it was considered appropriate to reject the null hypothesis if the value of L\* exceeded the 95 per cent point of the Chi-square distribution, i.e., an  $\ll$  risk of five per cent was used.

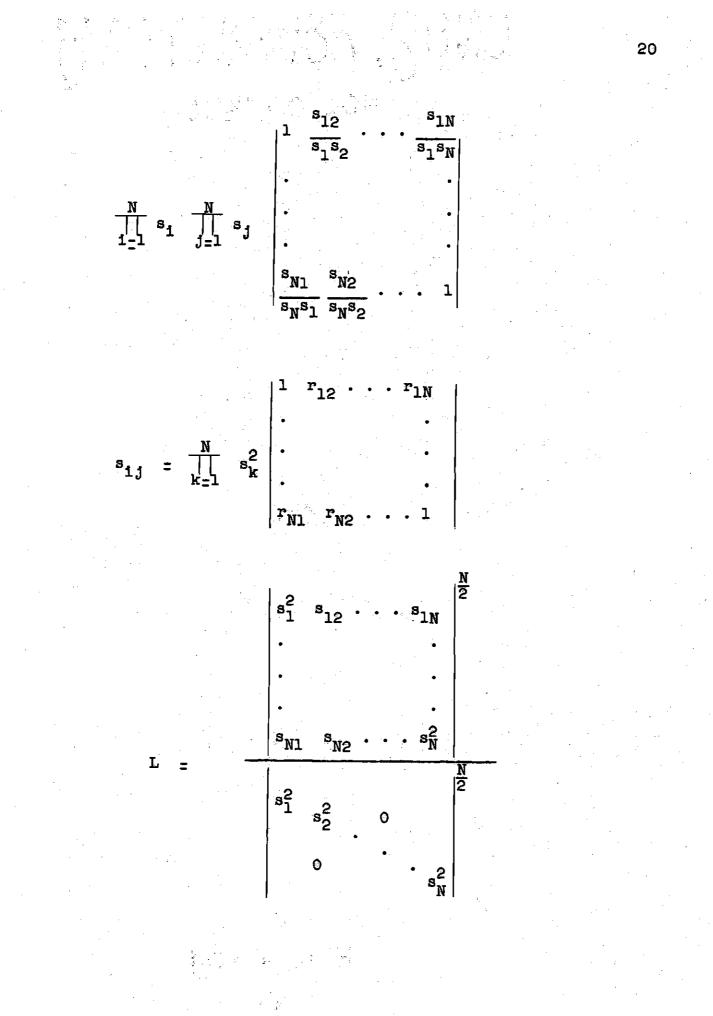
In order to compute L, the determinants  $|T_1|$  and  $|T_2|$ were evaluated. This evaluation was made using the IBM 650 and the FOR TRANSIT Program.

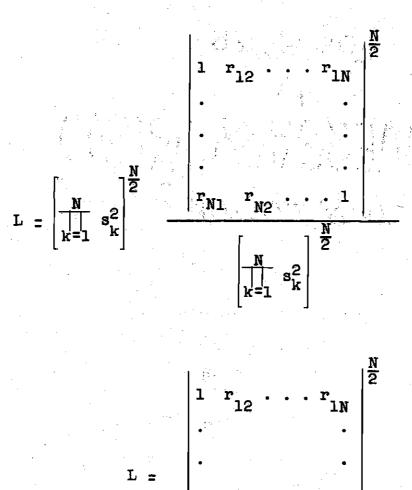
The capacity of the IBM 650 Digital Computer, using the FOR TRANSIT Program, was exceeded in the computation of the L value for each operator. The capacity of the computer, in the use of this program, is between the limits of  $10^{49}$  to  $10^{-51}$ . These limits were exceeded because of the extreme range of covariance values obtained. For this reason, the matrix of correlation coefficients was used in place of the covariancevariance matrices. The justification of this transformation is given below











r<sub>N1</sub>

The values of L were determined by computing the determinant of the matrix of correlation coefficients, again using the FOR TRANSIT Program on the IBM 650 Digital Computer. The sample calculations involved in computing L\* are shown in Figures two and three in the Appendix.

r<sub>N2</sub>

DISCUSSION OF RESULTS

CHAPTER V

The calculated L\* values which were obtained for operators One and Two were used to test for element independence, at the five per cent significance level. These calculated values are shown in Figures two and three of the Appendix.

An L\* value of 78.506 was obtained for Operator One, which corresponds to a significance level of approximately 14 per cent (46). Thus, the elements as originally defined were concluded to be statistically independent. Also noted was the fact that this same conclusion was reached from an inspection of the correlation coefficients which vary in the range expected for the sample sizes involved, under the null hypothesis that  $\rho$ , the universe correlation coefficient, is zero.

The L\* value of 93.239 was obtained for Operator Two, which corresponds to a significance level of approximately 0.2 per cent (46). Hence, the conclusion was that the elements involved here were correlated. The only significantly large simple correlation coefficient for this operation was 0.52, as shown in Table four of the Appendix, for elements eleven and twelve. The elements involved were

> Element Eleven - Stake ferrule and remove assembled unit from staker

Element Twelve - Aside assembled unit to container

These results indicate that correlation may exist only for some elements, perhaps between adjacent elements. These results agree with those of Abruzzi (1), Barnes and Mundel (6, 7, 8, 9), and Smader and Smith (55).

A re-examination of the films and the film analysis sheets for Operator One and Two indicated that many of the work cycles contained minor variations. These variations were due to the operator and were in the form of momentary fumbles, slight delays, and extraneous movements. They were found to occur very frequently and appeared to be a function of the small size of the parts involved in the assembly operation. The work cycles containing these variations were not eliminated since they were considered to be an inherent part of the operation and it was desired to preserve the actual work situation as closely as possible. Operator Two paused momentarily after each faulty staking operation (element eleven), examining her work to determine the proper disposition of the completed assembly (element twelve). This was due to a faulty staking machine and could be eliminated. An elimination of the faulty staking machine would probably eliminate the interdependency found between elements eleven and twelve.

A basic unit of motion always ends when its purpose is accomplished. Throughout the course of the motion, muscular control is directed toward completing the motion as required. No muscular control can be directed toward carrying out the motion that is to follow until the motion in progress is completed. As soon as it is completed, muscular control may be applied immediately to start the next motion. There can be no overlap in the use of muscular control between two successive motions. However, in performing a sequence of motions, the characteristics of a given motion may be influenced, at times, by the adjacent motions. The influence of the associated motions is such that the performance of a motion to carry out a staking or a placement action in one instance may not be delayed through being in the end of a series of motions requiring no visual attention. In another instance the staking motion may be the only one in a series of motions to require close visual attention; and therefore, it will be delayed.

Thus, in those cycles where a faulty staking operation occurred, visual attention was required in order to determine the proper disposition of the completed assembly. These results indicate that correlation may exist only for some elements, perhaps between adjacent elements.

The present study was based on an element breakdown. However, all elements were comparatively short, comprising two, three, or four therbligs; whereas, other investigations cited in this study were based on either longer elements or therbligs.

While the results of this investigation do not reject the hypothesis of statistical independence, neither do they prove that interdependence will not exist under certain circumstances. It could, for example, exist between certain

types of elements not represented in the task which was studied in this investigation. It may exist as a function of the way in which some, but not all, operators perform a given task. It may exist as a function of the way in which the task is defined into elements (i.e. the element breakdown). In fact, there is direct evidence for this latter conjecture in the fact that Perkins' (47) investigation on the same data used in this study did indicate the presence of interdependence when the total work cycle was described by means of a two-element, and a fiveelement breakdown.

The results of the present investigation using a twelve element breakdown are compared with those of Perkins (47) who used a two element breakdown and a five element breakdown. The results shown below indicate an increasing degree of dependence among the elements as their magnitudes are decreased. These results are in agreement with those of Abruzzi (1,p.156). Number of Elements in Probability of Obtaining L\*

the Breakdown

Probability of Obtaining L\* Value When the Elements Are Independent

	perator One	Operator Two
2	0.48	0.65
5	0.64	0.76
12	0.14	0.002

In addition, the following conclusions were drawn on the basis of the test results and agree with those of Perkins (47, pp. v,vi).

1. The nature and extent of correlation among the

elements of a work cycle from period to period appeared to depend on the operator.

2. It appeared that in those cycles in which the degree of correlation was found to be the highest, there was a concentration of variables.

#### CHAPTER VI

#### CONCLUSIONS AND RECOMMENDATIONS

The objective of this investigation was to gain further insight into the relationship between the elements of a work cycle, particularly to determine if they were statistically independent. The data used was taken from an actual work situation in industry.

The results cannot be considered conclusive, but must be viewed in the light of study limitations. The investigation covered only one assembly operation performed by two operators in one plant.

The null hypothesis of this investigation was that the elements of the cycle are statistically independent. The interpretation of the results indicate that the hypothesis can be accepted for the statistically stable operator and rejected for the statistically unstable operator.

The results of this investigation indicate that independency between elements:

1. May exist only for certain elements

2. Varies with the individual operator

Additional studies should be made of several manual operations in which many operators are employed and in which the sequence of elements of the operations are changed at random. The results of this proposed study should be compared with the findings of this investigation.



## ANALYSIS SHEET

FOR

T-600 BALL POINT PEN

Operator Lillie Time 3:40 PM Cycle 2 Film No. 26 Analyst G Date of Analysis June 12, 1953 Time Unit K

	Frame No.	1 1 1 1 N 1	acted me	Frame No.	Remarks
	LH	LH	RH	RH	
	482	•		491	
Get BblTE,ST, & G	475	05	23	468	. · ·
Place BblTL,P,A, & RL	458	17	31	437	
Get Unit-TE,ST, & G	437	21	31	406	
Place Unit-TL,P,A, & RL	409	28	58	348	
Get Dr. Nut-TE,ST, & G	395	14	16	332	
Place Dr. Nut-TL, P, A, & RL	331	64	28	304	
let Ferrule-TE,ST, & G	300	31	25	279	
Place Ferrule-TL, P, A, & RL	242	58	27	252	
Get Comp. Unit-TE, & G	239	03	09	243	
Place Comp. Unit-DA, TL & P	222	17	25	218	4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
Stake-A,H, & DA	117	105	101	117	
Aside-TL, & RL	102	15	12	94	Trans- ferred
					pens to RH

# Figure 1.

Sample Film Analysis Sheet

#### OPERATOR I

66 df

N = 61

 $\frac{N}{2}$ L = r L =  $(0.2757)^{\frac{N}{2}}$ Log<sub>10</sub> L =  $\frac{N}{2}$  Log<sub>10</sub> (0.2757) 2 Log<sub>e</sub> L = (61) (2.3) Log<sub>10</sub> (0.2757) 2 Log<sub>e</sub> L = (61) (2.3) Log<sub>10</sub> (0.2757) L\* = -2 Log<sub>e</sub> L = -(61) (2.3) Log<sub>10</sub> (0.2757) L\* = -(61) (2.3) (-1 + .44044) L\* = (-140.3) (-.55956) L\* = 78.506 (Significance level of approximately 14 per cent)

#### Figure 2.

Sample Calculations Involved in the Computation of the L, and L\*, Values for the 12 x 12 Matrix

### OPERATOR II

66 df

$$N = 73$$

$$L = r^{\frac{N}{2}}$$

$$L = (0.2784)^{\frac{N}{2}}$$

$$L_{00} = (0.2784)^{\frac{N}{2}}$$

$$Log_{10} = 1 + \frac{100}{2} \log_{10} (0.2784)$$

$$2 \log_{10} = 73 \log_{10} (0.2784)$$

$$2 \log_{e} = (73) (2.3) \log_{10} (0.2784)$$

$$L^{*} = -2 \log_{e} = 1 - (73) (2.3) \log_{10} (0.2784)$$

$$L^{*} = -(73) (2.3) (-1 + .44467)$$

$$L^{*} = -(167.9) (-.55533)$$

$$L^{*} = 93.239 \text{ (Significance level of approximately 0.2 per cent)}$$

1.5

# Figure 3.

Sample Calculations Involved in the Computation of the L, and L\* Values for the 12 x 12 Matrix

						5775 		erek.			•	· · · · ·
Cycle				· •••	]	Elem	ents	ы.				· ·
. <u></u>	<u> </u>	2	3	4		6	_7	8	9	10	11	12
1234567891111111111222222222223333335678901 12345678901234567890123456789012345678901	018 026 033 028 017 018 019 018 019 018 019 018 019 019 019 019 019 019 027 0220 020 019 019 03216 019 019 019 019 019 019 019 019 019 019	026 018 018 016 016 016 016 016 016 016 016 016 016	027999561815585622064503022520024829325 0100000000000000000000000000000000000	000263415546254649414558662223333473 0002634155462554649414558662223333473	01512 012 012 0137 014 016 0154 0134 30 016 016 016 017 017 017 017 017 017 017 017 017 017	445437780452335681030548129739955033663356352503356810300000000000000000000000000000000000	025 0192 036 0437 0162 0162 0162 0162 0162 0162 0162 0162	0348160685109343174542959505834848555776226 03430000000000000000000000000000000000	004 0034 0004 0004 0004 0004 0004 0000 0000 0000 0000 0000 0000 0000 0000	$\begin{array}{c} 021\\ 024\\ 017\\ 029\\ 017\\ 029\\ 017\\ 029\\ 015\\ 015\\ 015\\ 015\\ 015\\ 015\\ 029\\ 029\\ 027\\ 026\\ 038\\ 018\\ 039\\ 026\\ 038\\ 046\\ 026\\ 014\\ 014\\ 021\\ 025\\ 015\\ 025\\ 015\\ 025\\ 015\\ 025\\ 025\\ 025\\ 025\\ 025\\ 025\\ 025\\ 02$	$\begin{array}{c} 101\\ 991\\ 925\\ 924\\ 786\\ 43991\\ 114\\ 566\\ 1214\\ 3999\\ 738\\ 1020\\ 920\\ 924\\ 920\\ 922\\ 922\\ 922\\ 922\\ 922\\ 922\\ 922$	012 024 018 017 026 014 013 016 013 016 013 016 013 016 013 016 013 016 013 016 013 016 013 016 014 011 013 016 014 011 012 016 014 011 012 016 014 011 012 016 014 011 012 016 014 011 012 016 014 011 012 016 014 011 016 014 011 016 014 011 016 014 011 016 014 011 016 014 011 016 014 011 016 014 011 016 014 011 016 014 011 016 014 011 016 014 011 016 014 011 016 014 011 016 014 011 016 014 011 016 014 011 016 014 011 016 014 011 017 016 014 011 017 016 014 011 017 016 014 011 013 016 014 011 013 016 014 011 013 016 014 011 013 016 014 011 013 016 014 011 013 016 014 011 011 013 016 014 011 011 011 011 011 011 011 011 011

Table 1. Element Times in Decimal Minutes for Operator One

# Table 1. Element Times in Decimal Minutes for Operator One

(Continued)

Cycle	ycle Elements													
	1	2	3	4	5	6	7	8	9	10	11	12		
43456789012345678901	018 019 023 016 017 019 013 023 013 024 016 016 016 017 014 017 014 019 014 024	026 016 013 023 013 017 017 017	014 012 017 012 019 014 017 022	042 034 027 034 037 021 037 021 035 044 95 044 05 044 05 045 045 05 05 05 05 05 05 05 05 05 05 05 05 05	013 022 016 012 020 012 017 014 012 013 011 024 027 017 021	030 026 037 045 028 037 045 028 039 055 037 055 037 055 037 055 037 070	022 039 021 024 015 017 021 013	021 024 022 040	009 008	021 023 013 014 016 027 015 018 019 017 017 016 018 020 036 017 021 019 025 037	021 041 025 024 024 024 024 024 024 026 035 024 026 035 027 036 035 029 035	016 027 006 012 015 011 026 011 016 014 026 010 011 020 011 020 011 012 014 032		

# Table 2. Element Times in Decimal Minutes for Operator Two

4

79.5

- 3

Cycle Elements													
	1	2	3	4	<u>5</u>	6	7	<u> </u>	<u>9</u>	10	<u>, 11</u>	12	
123456789111111111122222222222233333333333344	014 013 013 014 014 013 013 014	031 036 037 026 027 023 023 023 023 023 023 023 023 023 023	027 027 020 025 025 025 021 025 021 027 022 022 022 022 022 022 022 022 022	0364194416525111192652762579116701904636 0000000000000000000000000000000000	0246 017 018 019 010 018 018 018 018 018 018 018 018 018	0 0 0 0 0 0 0 0 0 0 0 0 0 0	0156 0194 0162 0130 0130 0130 0122 0130 0120 0120 012	0228 0228 0228 02239 0000000000	$\begin{array}{c} 007\\ 008\\ 008\\ 008\\ 008\\ 008\\ 008\\ 007\\ 007$	023 024 034 023 023 028 021 002 022 022 022 022	0318 0330 0300 0300 0300 0332 0332 0332 033	011 010 010 010 010 010 012 013 012 014 013 012 014 013 012 014 015 014 015 014 015 014 015 017 010 010 010 010 010 010 010 010 010	

# Table 2. Element Times in Decimal Minutes for Operator Two

(Continued)

Cycle		· · ·				Elem	ents				· ·	
<u> </u>	1	2	3	4	5	6	_7_	8	9	10		12
42 43 44 45 46 47 49 51 52	017 012 012 011 012 013 013 014	027 036 029 030 025 023 027 031 023 039	019 020	042 063 031 038 037 040 040 036 038 040	019 023 017 013 018 012 015 015 015 016 020	040 077 037 064 051 043 042 042 042	027 014 018 017 014 017 017 017 016 020 020	024 031 022 027 021 023 021	007 008 010 007 006 010 007 008 008 008	018 032 022 023 020 021 021 024 021 041 021	040 030 037 053	012 010 012 011 012 012 012 015 013 013 013 018
53 55 55 55 57 55 55 50 61	015 020 011 013 014 011 012 015 013	027 028 022 028 028 028 028 028 023 023	014 013 025 021 016 020 015 015 017	033 045 038 038 038 039 048 048 033	012 019 013 013 013 011 015 013 010 011	042 044 057 053 051 052 055	013 016 028 022 015 017 014 024 015	025 027 028 023 025 021 024 018 024	008 005 008 009 009 009 009 009 009	025 022 024 022 022 022 022 022 021 020	047 027 040 026 042 027 032 029 026	015 012 009 010 011 010 010 012 011
62 63 65 66 66 66 70 12 73 73	011 012 013 022 015 014 015 013 015 013 017	035 025 025 025 025 025 025 025 025 025 02	022 014 015 017 013 015 015 015 020 018 030	037 039 048 034 034 034 031 038 034 037 033 040	010 018 014 017 014 011 014 013 012 014 012 026	049 060 038 050 033 049 050 057 040 057 063 066	027 017 018 016 033 013 013 014 013 014 016 017	023 022 026 024 017 025 024 025 029 023	006 008 014 008 009 007 007 007 006 010 008 010 005	020 026 034 024 024 023 023 023 023 023 023 023 0225	029 030 029 027 034 031 034 030 030 030 030 030 030	009 010 009 011 010 010 013 009 018 010 010 010 009

## Table 3. Correlation Coefficient - 12 x 12 Symmetrical Matrix for Operator One

1.0000 -.2320 -.0042 -.0571 .0017 .0720 -.0832 -.1106 .0748 -.0013 -.1905 .0325 1.0000 -.0551 -.0347 .3103 .1250 .0508 .1616 -.0386 .2166 .1948 .3072 1.0000 -.2302 .1807 -.0231 .0343 -.7330 -.1059 .0636 .0184 .1112 1.0000 -.0838 .1453 .1111 -.0053 -.2288 .0195 .1218 -.2270 1.0000 -.0871 .0003 .1013 -.0898 .0342 .0025 .0024 1.0000 .2316 -.0751 .0597 -.0168 -.1479 -.0432 1.0000 .3113 -.0757 -.1354 .1610 -.0257 1.0000 -.0333 -.1693 .1171 .0059 1.0000 -.0418 -.0409 .0009 1.0000 -.2929 .1088

1.0000 .1644

1.0000

## Table 4. Correlation Coefficient - 12 x 12 Symmetrical Matrix for Operator Two

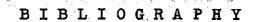
	1.0000	.0538	.2481	1135	.3295	.0440	0.0000	0602	0547	0418	.1499	.1240
		1.0000	0693	0403	.0191	0305	0087	0490	2496	1774	.1512	.0962
	· · ·		1.0000	.1288	.0507	.0971	.1055	.2139	0611	0440	.0318	1521
		u	· · ·	1.0000	.0199	.1053	0404	.0571	0037	0387	.0663	0810
•	· . ·				1.0000	.0519	0556	0428	.0265	.0660	.0243	.2356
						1.0000	1295	.0721	.0266	1783	0148	.0680
	· · ·			· :		· · · · · · · · · · · · · · · · · · ·	1.0000	2055	.0852	.0585	.0035	2335
	·				· · · ·		· · · · ·	1.0000	0832	0310	0919	2397

1.0000 .0328 -.1068 -.0845

1.0000 .0893 .1234

1.0000 .5164

1.0000



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