Final Report for Period: 09/2008 - 12/2008

Submitted on: 03/18/2009

Principal Investigator: Fekri, Faramarz .

Award ID: 0430964

Organization: GA Tech Res Corp - GIT

Submitted By:

Fekri, Faramarz - Principal Investigator

Title:

Low Density Parity Check Coding: Applications and New Challenges

Project Participants

Senior Personnel

Name: Fekri, Faramarz

Worked for more than 160 Hours: Yes

Contribution to Project:

He direct the research and advise graduate research assistance involved in the project.
A quarter of a month of the PI salary was charged in FY05.
He is also partially supported by NSF Career award, AFOSR and from the School of ECE (via NSF Career matching).

Post-doc

Graduate Student

Name: Pishro-Nik, Hossein

Worked for more than 160 Hours: Yes

Contribution to Project:

Hossein was also supported by Funding from other somewhat related grant from AFOSR. He finished his PhD in Aug. 2005.

Name: Rahnavard, Nazanin

Worked for more than 160 Hours: Yes

Contribution to Project:

Supported also by internal funding to partially work on this project.

Name: Vellambi R., Badri

Worked for more than 160 Hours: Yes

Contribution to Project:

Suppported by this grant and by the startup money from the school of ECE.

Name: Sardari, Mohsen

Worked for more than 160 Hours: Yes

Contribution to Project:

Research on unequal error protection

Name: Torabkhani, Nima

Worked for more than 160 Hours: Yes

Contribution to Project:

Coding and information theory applications

Name: sartipi, Mina

Worked for more than 160 Hours: Yes

Contribution to Project:

She was supported by another grant. However, she worked on a closely related project namely 'distributed source-coding using LDPC codes'.

Undergraduate Student

Technician, Programmer

Other Participant

Research Experience for Undergraduates

Organizational Partners

Airforce Office of Scientific Research

We had a project on high density holographic storage from AFOSR (expired in Dec. 2004). This project involved LDPC code design for nonuniform channels. The idea developed in this project helped us to make significant progress in the unequal error protection and nonuniform LDPC project for NSF.

Georgia Tech Broadband Institute

Other Collaborators or Contacts

1. Prof. Hossein Pishro-Nik (University of Massachusetts at Amherst).
2. Prof. Mina Sartipi (University of Tennessee at Chattanooga).
3. Prof. Nazanin Rahnavard (Oklahoma State University).

Activities and Findings

Research and Education Activities:

On research, we worked on several projects:

1. LDPC codes for nonuniform channels.
2. Asymptotic study of punctured LDPC codes.
3. Unequal error protection using LDPC codes.
4. Punctured LDPC codes for the finite length cases.
5. Stopping sets and improved decoding algorithm for low-density parity check codes over the binary erasure channel.
6. Unequal error protection via rateless codes and their generalizations.
7. Analysis of maximum-Likelihood decoding of finite-Length rateless codes.
8. Bounds and improved Decoding Algorithm for Low-Density Parity-Check Codes over the Binary Erasure Channel.
9. Application of rate-compatible and non-uniform LDPC codes in distributed source coding.

The results of these activity have been appeared or submitted to more than twenty journal and conference papers.

On education, the PI trains three graduate students on these activities, including a female student.

Findings:

The results of our research activity on this grant have been appeared or submitted to more than twenty conferences and competitive journals such as IEEE Transactions on Information Theory and IEEE Transactions on Communications. The highlight of these findings are as follows. For technical details, please refer to the publications.

1. LDPC codes for nonuniform channels: This work introduced a general framework to design and analyze LDPC codes over nonuniform channels. This includes LDPC codes for channels with non-uniform noise distributions, rate-compatible coding, and unequal error protection. In particular, we investigate the design of LDPC codes over a set of parallel subchannels (i.e., each subset of bits passes through one of the subchannels). One trivial approach is to design a separate error correcting code for each of the channels. However, using the prior knowledge of which subset of coded bits passes through which channel, we

developed a single LDPC code that is optimal for these multichannels. We presented the asymptotic analysis of the performance of the corresponding ensemble of LDPC codes.
Collectively, our method offers the following benefits: First, the design procedure is very simple since we do not need to perform the high-complexity degree optimization algorithms that are necessary for conventional LDPC codes. Second, using the proposed method, we can find codes that have near Shannon-limit performance and have lower error floor. Third, for the applications that the code length cannot be large (finite length codes), the proposed codes can have better performance than the ordinary LDPC codes.
As an application, we considered holographic storage in which the signal to noise ratio (SNR) decreases from the center to the corner of the page. This storage system can virtually be viewed as a set of parallel subchannels. It was shown that our LDPC codes have a superior performance to that of Reed-Solomon (RS) Codes and regular LDPC counterparts. Simulation results showed that the maximum storage capacity of holographic memories is increased by more than 50 percent.

2. Next, rate-compatible punctured LDPC codes (the asymptotic case) were studied as a special case of the proposed design methodology for the nonuniform channels. We proved that punctured LDPC codes have a threshold effect and compute the threshold for different methods of puncturing. We specifically showed that arbitrary rates are achievable via puncturing. We then discussed the optimality of punctured LDPC codes. The work presented a method to avoid the loss of performance when the puncturing ratio is high. For the BEC, much stronger results were obtained. For example, using only one encoder and decoder, we showed that we can achieve the capacity of BEC on arbitrary set of rates. In other words, we showed that there is no performance loss due to random puncturing (in comparison to optimized puncturing) over the binary erasure channel. We discussed design of good puncturing schemes for LDPC codes and we proposed a simple rule for constructing rate-compatible LDPC codes. The proposed method prevents the performance degradation for the high rates that was previously observed by other researchers. Finally we considered the open research problem of capacity achieving sequences for general memoryless binary-input output-symmetric (MBIOS) channels. We proved that if capacity achieving sequences of LDPC codes exist when the rate of the codes approaches zero, then capacity achieving LDPC codes exist for all rates.

3. In this project, we also studied a method to generate LDPC codes with unequal error protection property, where the highly protected bits (or packets) can be decoded without requiring the entire word to be decoded. This has many applications. For example, in a collaborative environment with heterogenous devices, some portions of data may need more protection than others. As an example, in multimedia delivery, a ``scene'' is composed of various media objects and each object is encoded into multiple layers of importance starting with the base layer and followed by multiple enhancement layers. Moreover, according to the context and the layout of the scene, each object is assigned a weight of importance. All these parameters were combined together to assign an importance level to the packet stream to be reliably delivered to other clients by a single code. We developed unequal error protecting (UEP) rateless codes to meet this constraint. Note that classical unequal protection schemes split data to be protected into classes which are encoded independently. Unfortunately, employing two separate codes would be suboptimal due to the poor performance of the random codes for decoding short/moderate number of packets. We derived density evolution formulas for the unequal error protecting LDPC ensembles over the binary erasure channel (BEC). Using the density evolution formulas, we optimized the codes. For the finite-length cases, we compare our codes with some other LDPC codes, the time-sharing method, and a previous work on UEP using LDPC codes. Simulation results indicate the superiority of the proposed design methodology for unequal error protection.

4. In this project, we investigated punctured LDPC codes for the finite length cases. We presented a novel method of estimating lower and upper bounds on the expected performance of randomly punctured codes of finite length in terms of the performance of their parent code. The bounds are easy to evaluate and provide reasonably good estimates for the average deterioration suffered due to puncturing.
We then presented a simple rate-compatible puncturing scheme for finite length low-density parity-check codes. The proposed scheme is easy to implement and allows for a lot of improvement based on the parent code to be implemented. By simulations it was seen that, over BEC, the performance deterioration induced by puncturing is almost equal to the corresponding increase in rate due to puncturing.
The puncturing scheme is also directly applicable to irregular parent ensembles. By simulations, the proposed rate-compatible puncturing scheme is shown to be superior to the existing puncturing methods for both regular and irregular LDPC codes over BEC and Additive White Gaussian Noise (AWGN). However, the range of achievable rates under the proposed scheme is limited.

5. Stopping sets and improved decoding algorithm for LDPC codes over the binary erasure channel: We derived a necessary and sufficient condition for the improved decoding algorithm to successfully complete decoding when the decoder is set to guess a pre-determined number of guesses after the standard message-passing terminates at a stopping set. We first successfully analyzed the improved decoding algorithm proposed by Pishro-Nik and Fekri to derive necessary and sufficient conditions for g-solvability of stopping sets. We also presented lower and upper bounds on the minimum number of guesses required to completely decode a (ML-decodable) stopping set. The upper bound was noted to be superior to that given by Pishro-Nik and Fekri. We then presented some observations regarding the structure of stopping sets that we

justified qualitatively and verified by simulations.
We used our observations to introduce a fast improved decoding algorithm over the binary erasure channel that is easily applicable to codes of all lengths. The proposed algorithm combines the fact that a considerable fraction of unsatisfied check nodes in the neighborhood of a stopping set are of degree two with the concept of guessing bits to perform intuitive and simple graph-theoretic manipulations on the Tanner graph. We demonstrated by simulations that at reasonable BERs of about 10^{-2}, our algorithm had an average running time of no more than twice that of the standard MP decoder. We established by simulations that at lower bit error rates, we obtain about three orders of magnitude improvement in certain ensembles. It was also shown by simulations that our proposed algorithm outperforms the present improved decoding algorithm by almost an order of magnitude for regular ensembles even though we use significantly smaller number of guesses. It was verified that in almost all ensembles, our algorithm outperforms that of Pishro_Nik et. al. even at very high BERs. Lastly, we would like to add that since the design of finite-length LDPC codes is still an open problem, our proposed algorithm provides an alternative approach to improve the performance of these codes over BEC.

6. We also proposed and developed a generalization of rateless codes (such as LT codes, online codes, Raptor codes) for unequal error protection. The methods we developed for LDPC codes can carry well to these rateless codes. Therefore, we expanded our research on unequal error protection on these types of codes. The proposed codes can provide unequal error protection (UEP) property. We derived upper and lower bounds on maximum-likelihood (ML) decoding error probabilities of finite-length LT and Raptor codes with the unequal error protection property. We further verified our work with simulations. Simulation results indicate that the proposed codes have strong UEP property. Then, we studied rateless codes that provide unequal error protection (UEP) property under iterative decoding. We examined asymptotic properties of these codes. We further verified our work with simulations. Simulation results indicate that the proposed codes have strong UEP property. Moreover, the UEP property does not have a considerable drawback on the overall performance of the code.

7. Analysis of Maximum-Likelihood Decoding of Finite-Length Rateless Codes: In this work, we focused on finite-length rateless codes and derived upper and lower bounds on bit error rates of Maximum Likelihood decoding. The presented bounds are tight for small error rates and depict the fact that Raptor codes can have much smaller bit error rates than LT codes. These bounds can specially be useful for optimizing degree distributions of finite-length rateless codes when very small error probabilities are needed.

8. Bounds and improved Decoding Algorithm for Low-Density Parity-Check Codes over the Binary Erasure Channel: In this work, we first investigated some analytical aspects of the recently proposed improved decoding algorithm for Low-Density Parity-Check (LDPC) codes over the binary erasure channel (BEC). We derived a necessary and sufficient condition for the improved decoding algorithm to successfully complete decoding when the decoder is initialized to guess a pre-determined number of guesses after the standard message-passing terminates at a stopping set. Furthermore, we presented improved bounds on the number of bits to be guessed for successful completion of the decoding process when a stopping set is encountered. Under suitable conditions, we derived a lower bound on the number of iterations to be performed for complete decoding of the stopping set. We then presented a superior, novel improved decoding algorithm for LDPC codes over the Binary Erasure Channel (BEC). The proposed algorithm combines the observation that a considerable fraction of unsatisfied check nodes in the neighborhood of a stopping set are of degree two, and the concept of guessing bits to perform simple and intuitive graph-theoretic manipulations on the Tanner graph. The proposed decoding algorithm has a complexity similar to previous improved decoding algorithms. Finally, we presented simulation results of short-length codes over BEC that demonstrate the superiority of our algorithm over previous improved decoding algorithms for a wide range of bit error rates.

9. Application of rate-compatible and non-uniform LDPC codes in distributed source coding: In this work, we developed a single systematic LDPC code for distributed source coding of correlated sources. In particular, we considered LDPC codes with finite lengths (less than 10000 bits) that can achieve every arbitrary coding rate on the Slepian-Wolf rate region. We simplified the distributed source coding problem to the rate-compatible LDPC code design with an unequal error protection property. At the decoder, each source is decoded independently (only part of information bits are exchanged) which prevents the propagation of errors. The distributed source coding is not supported by this award, however, the unique coding approach we developed is benefited from this NSF award.

Training and Development:

The research seeks to grow a research area on the boundary of applied mathematics (analysis and graph theory), computer science (algorithms), and communications (coding theory). The work provides new connections, and new interpretations. The PI trains graduate students on coding. Graduates with research and education skills in this area of telecommunication are in huge demand. One of the PhD students; Hossein Pishro-Nik; who was supported by this award has already graduated in Aug. 2005 and took an Assistant Professorship position at the University of Massachusetts at Amherst. Moreover, the PI inspires women students in the college to assess and enhance their research and educational experience. The PI has already gradauted two PhD female students, Mina Sartip and Nazanin Rahnavard, on error control codes and their applications. Mina Sartip took a faculty position at University of Tennessee in Fall 2006. Nazanin Rahnavard (supported by this grant) finished her PhD thesis in coding theory and applications and took an Assistant Professor position starting fall 2007 in Oklahoma State University. The fourth PhD student, Badri Vellambi, is still working on his PhD on this topic.

Outreach Activities:

1. The PI trained two female PhD student, Nazanin Rahnavard and Mina sartipi, on the LDPC/rateless coding and their applications. Mina Sartipi, who graduated in July 2006 (thesis topic: Modern codes and their applications to distributed source coding) and took an offer of the Assistant Professor Position at the University of Tennessee   Chattanooga Computer Science & Electrical Engineering.
Nazanin Rahnavard, who graduated in July 2007, (thesis topic: Coding for Wireless Ad-Hoc and Sensor Networks: Unequal Error Protection and Efficient Data Broadcasting) and took an offer of the Assistant Professor Position at the Oklahoma State University.

Journal Publications

H. Pishro-Nik, N. Rahnavard, and F. Fekri, "Non-uniform error correction using low-density parity check codes", IEEE Trans. Inform. Theory , p. 2702, vol. 51, (2005). Published,

H. Pishro-Nik and F. Fekri, "On graphs of LDPC codes", Proc. of 42nd Annual Allerton Conference on Communication, Control, and Computing, p. CDROM, vol. , (2004). Published,

H. Pishro-Nik and F. Fekri, "Results on punctured LDPC codes", Proc. 2004 IEEE Information Theory Workshop, p. CDROM, vol. , (2004). Published,

B. N. Vellambi and F. Fekri, "On stopping sets and an improved decoding algorithm for low-density parity-check codes over the binary erasure channel", Proc. of 42nd Annual Allerton Conference on Communication, Control and Computing, p. CDROM, vol. , (2004). Published,

H. Pishro-Nik and F. Fekri, "On some graph theoretic properties of LDPC codes", Proc. American Mathematical Society Special Meeting on Codes and Applications, p. 56, vol. , (2004). Published,

B. N. V. Ravisankar and F. Fekri, "An improved decoding algorithm for low-density parity check codes over the binary erasure channel", IEEE Global Telecommunications Conference (GLOBECOM 2005), p. CDROM, vol. , (2005). Published,

N. Rahnavard and F. Fekri, "Finite-length unequal error protection via rateless codes: Design and analysis", IEEE Global Telecommunications Conference (GLOBECOM 2005), p. 1353, vol. 3, (2005). Published,

N. Rahnavard and F. Fekri, "Generalization of Rateless Codes for Unequal Error
Protection and Unequal Recovery Time: Asymptotic Analysis", IEEE nternational Symposium on Information Theory (ISIT 2006), Seattle, Washington, p. CDROM, vol. , (2006). Published,

N. Rahnavard and F. Fekri, "Bounds on Maximum-Likelihood Decoding of Finite-Length Rateless Codes", 39th Annual Conference on Information Sciences and Systems (CISS?5), Baltimore, MD., p. CDROM, vol. , (2005). Published,

N. Rahnavard and F. Fekri, "New Results on Unequal Error Protection Using LDPC Codes

", IEEE Communications Letters, p. 43, vol. 10, (2006). Published,

Badri N. Vellambi R. and Faramarz Fekri, "Rate-Compatible Puncturing of Finite-Length Low-Density Parity-Check Codes", IEEE International Symposium on Information Theory, Seattle., p. CDROM, vol. , (2006). Published,

Pishro-Nik and F. Fekri, "Performance of low-density parity-check codes with linear minimum distance", IEEE Trans. Inform. Theory, p. 292, vol. 52, (2006). Published,

H. Pishro-Nik and F. Fekri,, "On Raptor Codes", Proc. IEEE ICC Conference, Istanbul, Turkey., p. CDROM, vol. , (2006). Published,

M. Sartipi and F. Fekri, "Distributed source coding of unknown correlated sources using LDPC codes: lossy and lossless cases", -Third Annual Allerton Conference on Communication, Control and Computing, Urbana-Champaign, IL, October 2005,(invited paper)., p. CDROM, vol. , (2005). Published,

M. Sartipi and F. Fekri, "Distributed source coding in wireless sensor networks using LDPC coding: a non-uniform framework", Proc. of IEEE Data Compression Conference, p. 477, vol. , (2005). Published,

M. Sartipi and F. Fekri, "Distributed source coding in wireless sensor networks using LDPC coding: The entire Slepian-Wolf rate region", IEEE Wireless Communications and Networking Conference, p. 1939, vol. , (2005). Published,

H. Pishro-Nik and F. Fekri, "On punctured low-density parity-check codes and improved iterative decoding techniques", IEEE Trans. Inform. Theory, p. 599, vol. 53, (2007). Published,

N. Rahnavard, B. Vellambi and F. Fekri, "Rateless codes with unequal error protection property", IEEE Trans. Inform. Theory, p. 1521, vol. 53, (2007). Published,

Badri N. Vellambi R. and Faramarz Fekri, "Results on the Improved Decoding Algorithm for Low-Density Parity-Check Codes over the Binary Erasure Channel", IEEE Trans. Inform. Theory, p. 1510, vol. 53, (2007). Published,

B. N. Vellambi and F. Fekri, "Finite-length rate-compatible LDPC codes: a novel puncturing scheme", IEEE Trans. on Communications, p. , vol. , (2008). Accepted,

M. Sartipi and F. Fekri, "Distributed Source Coding using Finite-Length Rate-Compatible LDPC Codes: The Entire Slepian-Wolf Rate Region", IEEE Transactions on Communications, p. 400, vol. 56, (2008). Published,

B. N. Vellambi, and F. Fekri, "On estimation of the performance of randomly punctured short-length LDPC codes", IEEE Trans. on Communications, p. , vol. , (2008). Submitted,

N. Rahnavard, H. Pishro-Nik, and F. Fekri, "Unequal error protection using Partially-Regular LDPC codes", IEEE Transactions on Communication, p. 387, vol. 55, (2007). Published,

Books or Other One-time Publications

Web/Internet Site

Other Specific Products

Contributions

Contributions within Discipline:

Low-density parity-check codes are considered breakthrough in information theory for achieving the fundamental limits in reliable communication over noisy channels. The PI is developing a program that systematically explores the applications of the low-density parity-check codes to the key problems in data transmission. The work introduced a framework to extend the known results and to design and analyze LDPC codes for channels with non-uniform noise distributions, rate-compatible coding, and unequal error protection. These fundamental problems arise in several applications. Specifically, support from NSF resulted in non-uniform design of LDPC codes for volume holographic memory (VHM) systems for which the noise distribution is nonuniform. The work concluded that nonuniform codes have superior performance to that of Reed-Solomon (RS) Codes and conventional LDPC counterparts. Simulation results showed that the maximum storage capacity of holographic memories may increased by more than 50 percent. The framework also provided insight to the design of rate-compatible punctured LDPC codes. In particular, the research presented a method to avoid the loss of performance when the puncturing ratio is high. In other words, the LDPC code can now adapt to the time-varying wireless channels without losing performance. Finally, the

nonuniform framework led to a methodology for construction of LDPC codes with unequal error protection property, where the highly protected bits can be decoded without requiring the entire word to be decoded. These are potentially very useful codes for networking applications where the packets have to travel through multiple hops and their headers should be recovered at the intermediate routers with low complexity. These codes are also very powerful in the design of distributed source coding via LDPC codes where the correlation channel and the actual wireless channel can be viewed as a nonuniform channel which require unequal error protection.
Collectively, the work demonstrates how novel mathematical approaches can offer new solutions to engineering problems, which may lead to profound benefits for society on the technology and commercial products.

This work has significant impacts on both basic science and engineering. We develop new theories and design tools for good code design. Although part of the activity is basic research, it is oriented toward solving practical problems concerning the storage and transmission of the information. The research is ultimately oriented toward designing good codes, reducing decoding complexity, and improving the decoding performance. These contributions specifically include:

1. In a mobile adaptive network, the channel is time varying, and different types of data require different error protection needs. Therefore, we need to change the coding rate during transmission to maintain an acceptable quality of service. In channels with time-varying erasures, it is desirable to have a single encoder-decoder pair to enable encoding at rates that can be adapted according to the changes in the channel. Although at the time of this work, considerable work existed on the asymptotic puncturing of LDPC codes, the exploration of issues in the area of short-length puncturing had just started. We considered two research directions in this broad area -- the design of a family of rate-compatible codes obtained by puncturing a particular code and the estimation of the average performance (BER) of punctured ensembles of a given LDPC code. During the first step in our research, we noticed the positive correlation of the minimum distance between punctured nodes with a certain measure of optimality. We then observed that maximizing the average distance could be loosely translated to a simple and generic criterion for the selection of bits to be punctured. We applied this criterion to the synthesis of the two-stage puncturing algorithm proposed. The BERs of punctured codes constructed from the proposed puncturing scheme was verified to be superior to the BERs of those obtained from other design methodologies for short-length puncturing not only over the BEC but also over the additive white Gaussian noise (AWGN) channel model.

2. In some applications it is desirable for certain bits to be protected more than the rest. For these applications we need unequal error protection (UEP). We have developed both LDPC codes and rateless codes (another family of random codes) that have the unequal error protection property. We have developed a framework to design random codes with unequal error protection properties. These UEP codes are optimized for the number of importance levels in the content and their relative importance for given bit (packet) budget and loss process parameters to maximize the end-to-end content delivery quality. For example, we jointly optimized the degree distributions of codes with the adaptive content presentation (layering) for broadcast over heterogeneous client population to meet their latency and presentation quality requirements. Moreover, these codes can be used for unequal-recovery time (URT).

3. Improving the performance of the iterative decoding algorithm is particulary important for short-length LDPC codes for which a good optimization methodology is lacking. We have developed improved decoding algorithms that outperform standard iterative decoding by two orders of magnitude with subtle computaional cost. We first develop the improved decoding technique for the erasure channel. Then, the study considers several other types of channels such as BSC and AWGN.

4. We developed a technique to design good LDPC codes for volume holographic memory (VHM) systems for which the noise distribution is nonuniform. More specifically, the signal to noise ratio (SNR) decreases from the center to the corner of the page. To increase the storage capacity, the knowledge about the nonuniform error pattern in holographic memories was utilized in designing as well as decoding of the code. It was shown that these codes have a superior performance to
that of Reed-Solomon (RS) Codes and regular LDPC counterparts. Simulation results showed that the maximum storage capacity of holographic memories is increased by more than 50 percent.

5. Our research in non-uniform and rate-compatible LDPC codes resulted in new approaches in distributed source coding (with applications in sensor networks). For the first time, we take both the correlation channel and the actual channel and formulate the source coding problem to the code design over non-uniform channels. This is in contrast with the previous work which designs the LDPC code for the equivalent channel. First, we investigate a system of two correlated signals. We present a methodology that involves construction of non-uniform LDPC codes and rate-compatible LDPC codes with an unequal error protection
property. Our approach to distributed source coding via these new types of LDPC codes provides improved compression ratio. This is ongoing research.

Contributions to Other Disciplines:

The PI's research on the low density parity check codes opened
up several theoretical and practical research possibilities in

this coding technique where the PI believes there is a great potential for further research. The research seeks to grow a research area on the boundary of applied mathematics (analysis and graph theory), computer science (algorithms), and communications (coding theory). The work brings together researchers from these areas, and provides new connections, and new interpretations. For example, the knowledge we earned on random graphs and probabilistic methods during the investigation of random codes has lead to new results in the connectivity study of the geometric graphs, which is a computer science problem. We introduced a random graph theoretic model to study the properties of sensor networks, which incorporates geometrical considerations, node distribution and node and link failures. We considered graph theoretic properties of the above model, such as connectivity and k-connectivity. We analyzed this model and obtained optimal threshold for communication range. The result entitled 'On connectivity properties of large-scale wireless sensor networks,' (H. Pishro-Nik, K. Chan, and F. Fekri) is accepted to appear in Wireless Networks, Kluwer Academic Press.

Contributions to Human Resource Development:

The PI trains graduate students on coding. Graduates with research and education skills in this area of telecommunication are in huge demand. Moreover, The PI motivates women to achieve full potential in their careers as engineers. He has already graduated three female students: Maneli Noorkami (with MS thesis) who was supported by another NSF grant, Mina Sartipi who graduated with PhD degree in Summer 2006, and Nazanin Rahnavard who graduated with PhD degree in Summer 2007. The later two worked on this project.

Contributions to Resources for Research and Education:

One of the top priorities of the PI is to train strong human resources for our education system. Although the PI has been a faculty member for a relatively short period of time, he has been very successful in training teachers and researchers for Academia. Thus far, he has graduated four PhDs who all have gone to the academia. His first Graduate student, Hossein Pishro-Nik, who was supported by this award and was graduated in Aug. 2005 became an Assistant Professor at the University of Massachusetts at Amherst. The second graduate student of the PI, Mina Sartipi, who was supported by another NSF project on the use of coding theory for distributed source coding graduated Summer 2006 and she has became an Assistant Professor at UT Chattanooga Computer Science & Electrical Engineering. The third student, Nazanin Rahnavard, who was also supported by this grant, was graduated summer 2007 and beame an Assistant Professor in Oklahoma State University. Further, the fourth student of the PI, Farshid Delgosha, who was supported by another NSF grant took an Assistant Professorship position in NYIT in fall 2007.

Contributions Beyond Science and Engineering:

Categories for which nothing is reported:

Any Book

Any Web/Internet Site

Any Product

Contributions: To Any Beyond Science and Engineering

Final Report

<u>NSF Project Information:</u>

Project Title: *Low Density Parity Check Coding: Applications and New Challenges*
PI: Faramarz Fekri
Institution: Georgia Institute of Technology
Award Number: CCF-0430964
Start Date: 01/01/2005 (award came fall 2004)
Expiration Date: 12/31/2008
Project Type: Research Project

This final report summarizes the accomplishments in Dr. Fekri's research group at Georgia Institute of Technology through the NSF CCF-0430964 award. Major achievements are listed in this report. Detailed information can be found in the publications or can be directly requested from Dr. Fekri.

The Award was very important in supporting the initial steps of the research. Several projects are still ongoing as the application of the developed theory. Thus far, the research resulted in 10 published Journal papers, an additional journal paper pending decision, and 13 conference papers. In the following, we list student guidance, the course development, and all the publications/products as a result of this research activity:

<u>A. Students Guidance</u>

A1. Ph.D. students Graduated (funded by this project fully or partially)

1. Hossein Pishro-Nik
   PhD Thesis Title: Application of Random Graphs to Design and Analysis of LDPC Codes and Woreless Sensor Networks
   Graduation Date: Fall 2005
   Position and place of first employment: Assistant Professor at University of Massachuetts at Amherst

2. Mina Sartipi (female)
   PhD Thesis Title: Modern Error Control Codes and Applications to Distributed Source Coding
   Graduation Date: Fall 2006
   Position and place of first employment: Assistant Professor at University of Tennessee, Chattanooga

3. Nazanin Rahnavard (female)
   PhD Thesis Title: Coding for Wireless Ad-Hoc and Sensor Networks: Unequal Error Protection and Efficient Data Broadcasting

Graduation Date: Fall 2007
Position and place of first employment: Assistant Professor at Oklahoma State University

4. Badri Narayanan Vellambi
PhD Thesis Title: Application of Graph-Based Codes in Networks: Analysis of Capacity and Design of Improved Decoding Algorithms
Graduation Date: Fall 2008
Position and place of first employment: Research Fellow, University of South Australia

A2. Current Ph.D. students (partially funded by this project)

1. Mohsen Sardari
PhD Thesis Title: Modern Error Control Coding for IPTV
Expected Graduation Date: Summer 2012

2. Nima Torabkhani
PhD Thesis Title: Network Coding with Finite Buffers
Expected Graduation Date: Spring 2012

A4. Visitors (not funded by the project)

1. Professor Faramarz Hendessi
period: Oct. 2007- Sept. 2008.
Project: Dissemination of Multimedia in Vehicular Networks Using Rateless Codes

B. Scholarly Accomplishments

B1. Refereed Journal Papers

[1] H. Pishro-Nik, N. Rahnavard, and F. Fekri, "Non-uniform error correction using low-density parity check codes", *IEEE Trans. Inform. Theory* , vol. 51, pp. 2702-2714, July 2005.
[2] N. Rahnavard and F. Fekri, "New Results on Unequal Error Protection Using LDPC Codes ", *IEEE Communications Letters*, vol.10, no. 1, pp. 43- 45, Jan 2006.
[3] Pishro-Nik and F. Fekri, "Performance of low-density parity-check codes with linear minimum distance", *IEEE Trans. Inform. Theory*, Vol. 52, No. 1, pp. 292-300, Jan. 2006.
[4] H. Pishro-Nik and F. Fekri, "Results on punctured low-density parity-check codes and improved iterative decoding techniques", *IEEE Trans. Inform. Theory*, Vol. 53, no. 1, pp. 387-391, January 2007.
[5] N. Rahnavard, B. Vellambi and F. Fekri, "Rateless codes with unequal error protection property", *IEEE Trans. Inform. Theory*, vol. 53, (2007), p. 1521.,

[6] Badri N. Vellambi R. and Faramarz Fekri, "Results on the Improved Decoding Algorithm for Low-Density Parity-Check Codes over the Binary Erasure Channel", *IEEE Trans. Inform. Theory*, vol. 53, no. 4., pp. 1510 - 1520, April 2007.
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B2. Refereed Conference Papers with Proceedings

[1] H. Pishro-Nik and F. Fekri, "On graphs of LDPC codes", *Proc. of 42nd Annual Allerton Conference on Communication, Control, and Computing*, 2004, (CDROM).
[2] H. Pishro-Nik and F. Fekri, "Results on punctured LDPC codes", *Proc. 2004 IEEE Information Theory Workshop*, 2004, (CDROM).
[3] B. N. Vellambi and F. Fekri, "On stopping sets and an improved decoding algorithm for low-density parity-check codes over the binary erasure channel", *Proc. of 42nd Annual Allerton Conference on Communication, Control and Computing*, 2004, (CDROM).
[4] H. Pishro-Nik and F. Fekri, "On some graph theoretic properties of LDPC codes", *Proc. American Mathematical Society Special Meeting on Codes and Applications*, 2004 (abstract published).
[5] B. N. V. Ravisankar and F. Fekri, "An improved decoding algorithm for low-density parity check codes over the binary erasure channel", *IEEE Global Telecommunications Conference (GLOBECOM 2005)*, 2005, (CDROM).
[6] N. Rahnavard and F. Fekri, "Finite-length unequal error protection via rateless codes: Design and analysis", *IEEE Global Telecommunications Conference (GLOBECOM 2005)*, 2005, (CDROM).
[7] N. Rahnavard and F. Fekri, "Generalization of Rateless Codes for Unequal Error Protection and Unequal Recovery Time: Asymptotic Analysis", *IEEE International Symposium on Information Theory (ISIT 2006), Seattle, Washington*, 2006, (CDROM).
[8] N. Rahnavard and F. Fekri, "Bounds on Maximum-Likelihood Decoding of Finite-Length Rateless Codes", *39th Annual Conference on Information Sciences and Systems (CISS05), Baltimore, MD.*, 2005, (CDROM).

[9] Badri N. Vellambi R. and Faramarz Fekri, "Rate-Compatible Puncturing of Finite-Length Low-Density Parity-Check Codes", *IEEE International Symposium on Information Theory, Seattle.*, 2006, (CDROM).

[10] H. Pishro-Nik and F. Fekri,, "On Raptor Codes", *Proc. IEEE ICC Conference, Istanbul, Turkey.*, 2006, (CDROM).

[11] M. Sartipi and F. Fekri, "Distributed source coding of unknown correlated sources using LDPC codes: lossy and lossless cases", *-Third Annual Allerton Conference on Communication, Control and Computing, Urbana-Champaign, IL, October 2005,(invited paper),* 2005, (CDROM).

[12] M. Sartipi and F. Fekri, "Distributed source coding in wireless sensor networks using LDPC coding: a non-uniform framework", *Proc. of IEEE Data Compression Conference*, 2005, (CDROM).

[13] M. Sartipi and F. Fekri, "Distributed source coding in wireless sensor networks using LDPC coding: The entire Slepian-Wolf rate region", *IEEE Wireless Communications and Networking Conference*, 2005, (CDROM).

B3. Invited Talks /Special Presentations

[1] H. Pishro-Nik and F. Fekri, "On some graph theoretic properties of LDPC codes", *Proc. American Mathematical Society Special Meeting on Codes and Applications*, 2004.

C. Research Activities and findings:

On research, we worked on several projects:

- LDPC codes for nonuniform channels.
- Asymptotic study of punctured LDPC codes.
- Unequal error protection using LDPC codes.
- Punctured LDPC codes for the finite length cases.
- Stopping sets and improved decoding algorithm for low-density parity check codes over the binary erasure channel.
- Unequal error protection via rateless codes and their generalizations.
- Analysis of maximum-Likelihood decoding of finite-Length rateless codes.
- Bounds and improved Decoding Algorithm for Low-Density Parity-Check Codes over the Binary Erasure Channel.
- Application of rate-compatible and non-uniform LDPC codes in distributed source coding.

The results of our research activity on this grant have been appeared or submitted to more than twenty four conferences and competitive journals such as IEEE Transactions on Information Theory and IEEE Transactions on Communications. The highlight of these findings are as follows. For technical details, please refer to the publications.

1. LDPC codes for nonuniform channels: This work introduced a general framework to design and analyze LDPC codes over nonuniform channels. This includes LDPC codes for channels with non-uniform noise distributions, rate-compatible coding, and unequal error

protection. In particular, we investigate the design of LDPC codes over a set of parallel subchannels (i.e., each subset of bits passes through one of the subchannels). One trivial approach is to design a separate error correcting code for each of the channels. However, using the prior knowledge of which subset of coded bits passes through which channel, we developed a single LDPC code that is optimal for these multichannels. We presented the asymptotic analysis of the performance of the corresponding ensemble of LDPC codes. Collectively, our method offers the following benefits: First, the design procedure is very simple since we do not need to perform the high-complexity degree optimization algorithms that are necessary for conventional LDPC codes. Second, using the proposed method, we can find codes that have near Shannon-limit performance and have lower error floor. Third, for the applications that the code length cannot be large (finite length codes), the proposed codes can have better performance than the ordinary LDPC codes. As an application, we considered holographic storage in which the signal to noise ratio (SNR) decreases from the center to the corner of the page. This storage system can virtually be viewed as a set of parallel subchannels. It was shown that our LDPC codes have a superior performance to that of Reed-Solomon (RS) Codes and regular LDPC counterparts. Simulation results showed that the maximum storage capacity of holographic memories is increased by more than 50 percent.

2. Asymptotic study of punctured LDPC codes: Next, rate-compatible punctured LDPC codes (the asymptotic case) were studied as a special case of the proposed design methodology for the nonuniform channels. We proved that punctured LDPC codes have a threshold effect and compute the threshold for different methods of puncturing. We specifically showed that arbitrary rates are achievable via puncturing. We then discussed the optimality of punctured LDPC codes. The work presented a method to avoid the loss of performance when the puncturing ratio is high. For the BEC, much stronger results were obtained. For example, using only one encoder and decoder, we showed that we can achieve the capacity of BEC on arbitrary set of rates. In other words, we showed that there is no performance loss due to random puncturing (in comparison to optimized puncturing) over the binary erasure channel. We discussed design of good puncturing schemes for LDPC codes and we proposed a simple rule for constructing rate-compatible LDPC codes. The proposed method prevents the performance degradation for the high rates that was previously observed by other researchers. Finally we considered the open research problem of capacity achieving sequences for general memoryless binary-input output-symmetric (MBIOS) channels. We proved that if capacity achieving sequences of LDPC codes exist when the rate of the codes approaches zero, then capacity achieving LDPC codes exist for all rates.

3. Unequal error protection using LDPC codes: In this project, we studied a method to generate LDPC codes with unequal error protection property, where the highly protected bits (or packets) can be decoded without requiring the entire word to be decoded. This has many applications. For example, in a collaborative environment with heterogenous devices, some portions of data may need more protection than others. As an example, in multimedia delivery, a ``scene" is composed of various media objects and each object is encoded into multiple layers of importance starting with the base layer and followed by multiple enhancement layers. Moreover, according to the context and the layout of the scene, each

object is assigned a weight of importance. All these parameters were combined together to assign an importance level to the packet stream to be reliably delivered to other clients by a single code. We developed unequal error protecting (UEP) rateless codes to meet this constraint. Note that classical unequal protection schemes split data to be protected into classes which are encoded independently. Unfortunately, employing two separate codes would be suboptimal due to the poor performance of the random codes for decoding short/moderate number of packets. We derived density evolution formulas for the unequal error protecting LDPC ensembles over the binary erasure channel (BEC). Using the density evolution formulas, we optimized the codes. For the finite-length cases, we compare our codes with some other LDPC codes, the time-sharing method, and a previous work on UEP using LDPC codes. Simulation results indicate the superiority of the proposed design methodology for unequal error protection.

4. Punctured LDPC codes for the finite length cases: In this project, we investigated punctured LDPC codes for the finite length cases. We presented a novel method of estimating lower and upper bounds on the expected performance of randomly punctured codes of finite length in terms of the performance of their parent code. The bounds are easy to evaluate and provide reasonably good estimates for the average deterioration suffered due to puncturing. We then presented a simple rate-compatible puncturing scheme for finite length low-density parity-check codes. The proposed scheme is easy to implement and allows for a lot of improvement based on the parent code to be implemented. By simulations it was seen that, over BEC, the performance deterioration induced by puncturing is almost equal to the corresponding increase in rate due to puncturing. The puncturing scheme is also directly applicable to irregular parent ensembles. By simulations, the proposed rate-compatible puncturing scheme is shown to be superior to the existing puncturing methods for both regular and irregular LDPC codes over BEC and Additive White Gaussian Noise (AWGN). However, the range of achievable rates under the proposed scheme is limited.

5. Stopping sets and improved decoding algorithm for low-density parity check codes over the binary erasure channel: Stopping sets and improved decoding algorithm for LDPC codes over the binary erasure channel: We derived a necessary and sufficient condition for the improved decoding algorithm to successfully complete decoding when the decoder is set to guess a pre-determined number of guesses after the standard message-passing terminates at a stopping set. We first successfully analyzed the improved decoding algorithm proposed by Pishro-Nik and Fekri to derive necessary and sufficient conditions for g-solvability of stopping sets. We also presented lower and upper bounds on the minimum number of guesses required to completely decode a (ML-decodable) stopping set. The upper bound was noted to be superior to that given by Pishro-Nik and Fekri. We then presented some observations regarding the structure of stopping sets that we justified qualitatively and verified by simulations. We used our observations to introduce a fast improved decoding algorithm over the binary erasure channel that is easily applicable to codes of all lengths. The proposed algorithm combines the fact that a considerable fraction of unsatisfied check nodes in the neighborhood of a stopping set are of degree two with the concept of guessing bits to perform intuitive and simple graph-theoretic manipulations on the Tanner graph. We demonstrated

by simulations that at reasonable BERs of about 10^{-2}, our algorithm
had an average running time of no more than twice that of the standard MP decoder. We established by simulations that at lower bit error rates, we obtain about three orders of magnitude improvement in certain ensembles. It was also shown by simulations that our proposed algorithm outperforms the present improved decoding algorithm by almost an order of magnitude for regular ensembles even though we use significantly smaller number of guesses. It was verified that in almost all ensembles, our algorithm outperforms that of Pishro_Nik et. al. even at very high BERs. Lastly, we would like to add that since the design of finite-length LDPC codes is still an open problem, our proposed algorithm provides an alternative approach to improve the performance of these codes over BEC.

6. Unequal error protection via rateless codes and their generalizations: We also proposed and developed a generalization of rateless codes (such as LT codes, online codes, Raptor codes) for unequal error protection. The methods we developed for LDPC codes can carry well to these rateless codes. Therefore, we expanded our research on unequal error protection on these types of codes. The proposed codes can provide unequal error protection (UEP) property. We derived upper and lower bounds on maximum-likelihood (ML) decoding error probabilities of finite-length LT and Raptor codes with the unequal error protection property. We further verified our work with simulations. Simulation results indicate that the proposed codes have strong UEP property. Then, we studied rateless codes that provide unequal error protection (UEP) property under iterative decoding. We examined asymptotic properties of these codes. We further verified our work with simulations. Simulation results indicate that the proposed codes have strong UEP property. Moreover, the UEP property does not have a considerable drawback on the overall performance of the code.

7. Analysis of Maximum-Likelihood Decoding of Finite-Length Rateless Codes: In this work, we focused on finite-length rateless codes and derived upper and lower bounds on bit error rates of Maximum Likelihood decoding. The presented bounds are tight for small error rates and depict the fact that Raptor codes can have much smaller bit error rates than LT codes. These bounds can specially be useful for optimizing degree distributions of finite-length rateless codes when very small error probabilities are needed.

8. Bounds and improved Decoding Algorithm for Low-Density Parity-Check
Codes over the Binary Erasure Channel: In this work, we first investigated some analytical aspects of the recently proposed improved decoding algorithm for Low-Density Parity-Check (LDPC) codes over the binary erasure channel (BEC). We derived a necessary and sufficient condition for the improved decoding algorithm to successfully complete decoding when the decoder is initialized to guess a pre-determined number of guesses after the standard message-passing terminates at a stopping set. Furthermore, we presented improved bounds on the number of bits to be guessed for successful completion of the decoding process when a stopping set is encountered. Under suitable conditions, we derived a lower bound on the number of iterations to be performed for complete decoding of the stopping set. We then presented a superior, novel improved decoding algorithm for LDPC codes over the Binary Erasure Channel (BEC). The proposed algorithm combines the observation that a considerable fraction of unsatisfied check nodes in the neighborhood

of a stopping set are of degree two, and the concept of guessing bits to perform simple and intuitive graph-theoretic manipulations on the Tanner graph. The proposed decoding algorithm has a complexity similar to previous improved decoding algorithms. Finally, we presented simulation results of short-length codes over BEC that demonstrate the superiority of our algorithm over previous improved decoding algorithms for a wide range of bit error rates.

9. Application of rate-compatible and non-uniform LDPC codes in distributed source coding: In this work, we developed a single systematic LDPC code for distributed source coding of correlated sources. In particular, we considered LDPC codes with finite lengths (less than 10000 bits) that can achieve every arbitrary coding rate on the Slepian-Wolf rate region. We simplified the distributed source coding problem to the rate-compatible LDPC code design with an unequal error protection property. At the decoder, each source is decoded independently (only part of information bits are exchanged) which prevents the propagation of errors. The distributed source coding is not supported by this award, however, the unique coding approach we developed is benefited from this NSF award.

D. Training and Contributions to Education and Human Resources:

The research sought to grow a research area on the boundary of applied mathematics (analysis and graph theory), computer science (algorithms), and communications (coding theory). The work provided new connections, and new interpretations. The PI trained graduate students on coding and their applications to networks. Graduates with research and education skills in this area of telecommunication are in a great demand. One of the PhD students; Hossein Pishro-Nik; who was supported by this award has already graduated in Aug. 2005 and took an Assistant Professorship position at the University of Massachusetts at Amherst. Moreover, the PI inspires women students in the college to assess and enhance their research and educational experience. The PI has gradauted two PhD female students, Mina Sartip and Nazanin Rahnavard, on error control codes and their applications. Mina Sartip took a faculty position at University of Tennessee in fall 2006. Nazanin Rahnavard (supported by this grant) finished her PhD thesis in coding theory and applications and took an Assistant Professor position starting fall 2007 in Oklahoma State University. The fourth PhD student, Badri Vellambi, who worked on Graph-Based Codes graduated in fall 2008 and became a Research Fellow in the University of South Australia.

E. Contributions within Discipline:

Low-density parity-check codes are considered breakthrough in information theory for achieving the fundamental limits in reliable communication over noisy channels. The PI is developing a program that systematically explores the applications of the low-density parity-check codes to the key problems in data transmission. The work introduced a framework to extend the known results and to design and analyze LDPC codes for channels with non-uniform noise distributions, rate-compatible coding, and unequal error protection. These fundamental problems arise in several applications. Specifically, support from NSF resulted in non-uniform design of LDPC codes for volume holographic memory (VHM) systems for which the noise distribution is nonuniform. The work concluded that nonuniform codes have superior performance to that of Reed-Solomon (RS) Codes and conventional LDPC counterparts. Simulation results showed that the maximum storage capacity of holographic memories may increased by more than 50 percent. The framework also provided insight to the design of rate-compatible punctured LDPC codes. In particular, the research presented a method to avoid the loss of performance when the puncturing ratio is high. In other words, the LDPC code can now adapt to the time-varying wireless channels without losing performance. Finally, the nonuniform framework led to a methodology for construction of LDPC codes with unequal error protection property, where the highly protected bits can be decoded without requiring the entire word to be decoded. These are potentially very useful codes for networking applications where the packets have to travel through multiple hops and their headers should be recovered at the intermediate routers with low complexity. These codes are also very powerful in the design of distributed source coding via LDPC codes where the correlation channel and the actual wireless channel can be viewed as a nonuniform channel which require unequal error protection. Collectively, the work demonstrates how novel mathematical approaches can offer new solutions to engineering problems, which may lead to profound benefits for society on the technology and commercial products.

This work has significant impacts on both basic science and engineering. We develop new theories and design tools for good code design. Although part of the activity is basic research, it is oriented toward solving practical problems concerning the storage and transmission of the information. The research is ultimately oriented toward designing good codes, reducing decoding complexity, and improving the decoding performance. These contributions specifically include:

1. In a mobile adaptive network, the channel is time varying, and different types of data require different error protection needs. Therefore, we need to change the coding rate during transmission to maintain an acceptable quality of service. In channels with time-varying erasures, it is desirable to have a single encoder-decoder pair to enable encoding at rates that can be adapted according to the changes in the channel. Although at the time of this work, considerable work existed on the asymptotic puncturing of LDPC codes, the exploration of issues in the area of short-length puncturing had just started. We considered two research directions in this broad area -- the design of a family of rate-compatible codes obtained by puncturing a particular code and the estimation of the average performance (BER) of punctured ensembles of a given LDPC code. During the first step in our research, we noticed the positive correlation of the minimum distance between punctured nodes with a certain measure of optimality. We then observed that maximizing the average distance could be loosely translated to a simple and generic criterion for the selection of bits to be punctured. We applied this criterion to the synthesis of the two-stage puncturing algorithm proposed. The BERs of punctured codes constructed from the proposed puncturing scheme was verified to be superior to the BERs of those obtained from other design methodologies for short-length puncturing not only over the BEC but also over the additive white Gaussian noise (AWGN) channel model.

2. In some applications it is desirable for certain bits to be protected more than the rest. For these applications we need unequal error protection (UEP). We have developed both LDPC codes and rateless codes (another family of random codes) that have the unequal error protection property. We have developed a framework to design random codes with unequal error protection properties. These UEP codes are optimized for the number of importance levels in the content and their relative importance for given bit (packet) budget and loss process parameters to maximize the end-to-end content delivery quality. For example, we jointly optimized the degree distributions of codes with the adaptive content presentation (layering) for broadcast over heterogeneous client population to meet their latency and presentation quality requirements. Moreover, these codes can be used for unequal-recovery time (URT).

3. Improving the performance of the iterative decoding algorithm is particulary important for short-length LDPC codes for which a good optimization methodology is lacking. We have developed improved decoding algorithms that outperform standard iterative decoding by two orders of magnitude with subtle computational cost. We first develop the improved decoding technique for the erasure channel. Then, the study considers several other types of channels such as BSC and AWGN.

4. We developed a technique to design good LDPC codes for volume holographic memory (VHM) systems for which the noise distribution is nonuniform. More specifically, the signal to noise ratio (SNR) decreases from the center to the corner of the page. To increase the storage capacity, the knowledge about the nonuniform error pattern in holographic memories was utilized in designing as well as decoding of the code. It was shown that these codes have a superior performance to that of Reed-Solomon (RS) Codes and regular LDPC counterparts. Simulation results showed that the maximum storage capacity of holographic memories is increased by more than 50 percent.

5. Our research in non-uniform and rate-compatible LDPC codes resulted in new approaches in distributed source coding (with applications in sensor networks). For the first time, we take both the correlation channel and the actual channel and formulate the source coding problem to the code design over non-uniform channels. This is in contrast with the previous work which designs the LDPC code for the equivalent channel. First, we investigate a system of two correlated signals. We present a methodology that involves construction of non-uniform LDPC codes and rate-compatible LDPC codes with an unequal error protection property. Our approach to distributed source coding via these new types of LDPC codes provides improved compression ratio. This is ongoing research.

F. Contributions to Other Disciplines:

The PI's research on the low density parity check codes opened up several theoretical and practical research possibilities in this coding technique where the PI believes there is a great potential for further research. The research lies in the intersection of several fields such as applied mathematics (analysis and graph theory), computer science (algorithms), and communications (coding theory). The work brings together researchers from these areas, and provides new connections, and new interpretations. For example, the knowledge we earned on random graphs and probabilistic methods during the investigation of random codes has lead to new results in the connectivity study of the geometric graphs, which is a computer science problem. We introduced a random graph theoretic model to study the properties of sensor networks, which incorporates geometrical considerations, node distribution and node and link failures. We considered graph theoretic properties of the above model, such as connectivity and k-connectivity. We analyzed this model and obtained optimal threshold for communication range. The result entitled 'On connectivity properties of large-scale wireless sensor networks,' (H. Pishro-Nik, K. Chan, and F. Fekri) is accepted to appear in Wireless Networks, Kluwer Academic Press.