Size Switching in Multi-agent Formation Control

A Thesis Presented to the Academic Faculty

by

Samuel D. Coogan

in Partial Fulfillment of Institute Requirements for the
Undergraduate Research Option in the School of Electrical
and Computer Engineering

Approved by

Dr. Magnus B. Egerstedt, Faculty Advisor
Professor
School of Electrical and Computer Engineering

Dr. Jeff S. Shamma
Professor, Julian T. Hightower Chair in Systems & Controls
School of Electrical and Computer Engineering

Dr. Douglas B. Williams
Professor, Associate Chair for Undergraduate Affairs
School of Electrical and Computer Engineering

School of Electrical and Computer Engineering
Georgia Institute of Technology

May, 2010

Acknowledgements

My sincerest thank you to Dr. Magnus Egerstedt, whose teaching and mentorship has helped me grow tremendously as a student and a researcher. He agreed to mentor my undergraduate research with little incentive, and provided invaluable advice and lessons about more than just my research. It is largely because of him that I have decided to continue my education towards a PhD.

I'd also like to thank Dr. Jeff Shamma, who helped me finalize my thesis in his official role as my second reader. But, perhaps more importantly, Prof. Shamma assisted me greatly while I was selecting graduate schools and exposed me to a number of topics in the field, both in the classroom and in his office.

Although he did not have a direct role in the completion of my thesis, I would like to acknowledge Dr. David Taylor, who also provided indispensable graduate school advise and research mentorship as I completed the Research Option and worked to identify my research interests.

Contents

1 Introduction

Multi-agent robot systems are emerging as plausible technology platforms that can solve a number of application problems including distributed sensing, establishing mobile *ad-hoc* communication networks, and robotic search and rescue applications. A multi-agent system is any system in which two or more autonomous units interact to achieve a certain goal, such as maintaining formation. In the context of this thesis, the autonomous units will usually be mobile robots and are referred to as "agents." A simple and practical example would be multiple robots searching a building for an intruder. Effective control strategies are challenging to develop for such systems because the agents often have limited communication and sensing capabilities and mobility [1]. For example, a robot may only be able to communicate with other robots within a certain distance, or may have no communication capabilities and can only sense the presence of robots directly ahead within a "field of vision," or the presence of communications links may follow a stochastic model. Each of these scenarios present unique engineering challenges which have been the focus of increasing research in the last 15-20 years.

Despite these obstacles, multi-agent systems offer certain enticing advantages. A well-designed multi-agent system may be robust against communication or robot failures, meaning that the global objective of the system can still be obtained even with the failure of certain communication links or an individual robot. In addition, many methods and results related to multi-agent systems are scalable, meaning that the results are not dependent on the size of the system. This is an important feature, as many practical multi-agent systems could have as few as two agents or as many as thousands of agents. These advantages have motivated increasing research in the field of multi-agent robot systems [2].

A common requirement of multi-agent systems is to maintain formation while achieving other tasks. Examples include a convoy of autonomous vehicles moving along a road in a line, or a group of unmanned aerial vehicles (UAVs) maintaining a tight formation while flying. Another example is a team of robots attempting to spread out across a specific area to ensure that the area is clear of hazards, a common task in military applications. Formation control has seen significant research in recent years, and certain aspects of formation control are well understood.

Numerous formation control strategies exist to accomplish certain tasks. For example, some control strategies allow the formation size to grow or shrink, known as scaling. Others allow the formation to move and rotate, in which case the formation is considered *rotationally invariant*. In addition, some strategies require communication between robots, while other strategies only require that each robot can sense or measure certain aspects of other robots. Strategies allowing communication may be more flexible or allow for increased complexity, but are often more difficult to implement. Communication among robots requires additional physical hardware, which could be expensive or large, depending on the application. In addition, communication requires that the robots operate on a common protocol for communication, which may be a difficult requirement. Control strategies requiring communication are also susceptible to communications failures, interference, or interception of communications. On the other hand, strategies that do not require communication are sometimes simpler to implement, but may not have the same flexibility as strategies allowing inter-agent communication.

While many formation control strategies do exist, certain formation control tasks remain underdeveloped, and this thesis attempts to solve a specific formation control task. Assume a team of mobile robots are moving in a two-dimensional space from some point A to some point B in a formation. Also assume that it is desirable for the team of robots to maintain the shape of the formation, although variations in size and rotation are permissible. If there are obstacles or boundaries along the formation's path, the formation can then grow and shrink as necessary to accommodate and avoid these obstacles. This thesis presents a strategy for allowing the formation to change size among several discrete possibilities in a coordinated fashion using only inter-agent sensing without inter-agent communication.

The proposed strategy brings together several aspects of multi-agent control theory. First, a method for driving the mobile robots into formation is required. In addition, the scale of the formation needs to be adjustable over time. Another requirement is for the robots to agree on a formation scale. This problem can be viewed as a separate multi-agent problem in which the agents need to achieve consensus on a scaling factor. It will be assumed that each agent possesses a scaling factor which is a continuous variable, and that several discrete possibilities for a formation size are desired. This objective requires an understanding of target rendezvous. Finally, a strategy for allowing the formation to move within a space is required.

Multi-agent research typically relies on a combination of simulation, theoretical development, and, when appropriate, implementation. The presented thesis will follow this path of research. The simulation stage relies heavily on MATLAB to demonstrate the feasibility of the proposed control strategy, and the theoretical stage develops a robust mathematical understanding of the control strategy.

The presented research develops a strategy for allowing a team of agents to move through an environment while maintaining formation with the ability to change formation scale. The presented research does have some limitations, however. Specifically, only a discrete number of possible formation scales are considered, so certain narrow passages or large obstacles may not be avoidable. In addition, it is assumed that each robot can observe the relative speed and position of certain other robots. This assumption plays an important role in determining formation scale. Finally, it may not be possible for the team of robots to successfully navigate certain environments, particularly environments with obstacles without convex boundaries.

Formation control is an important area of multi-agent research not only because it is of theoretical importance, but also because it fulfills many practical needs. A formation control strategy which allows the formation scale to change as necessary could be of great practical important in scenarios where previous formation control does not suffice. For example, suppose a convoy of vehicles is traversing unfamiliar territory. It may be desired that the vehicles maintain a certain formation for security purposes, yet paths of travel may be wide in some areas, and narrow in other areas. Also, obstacles may be present along the path of travel. Other similar examples exist, such as a team of autonomous robots moving through a building looking for intruders or certain hazards. Again, it may be desired that the robots maintain a certain formation for survey purposes, but obstacles may require that the formation scale change. These examples demonstrate the practical importance of the conducted research.

Section 2 is a review of the relevant literature on decentralized multi-agent systems, graph-theoretic approaches to control, formation control, and target identification. Section 3 outlines the methods and assumptions of the conducted research. Section 4 reports on the research findings. Section 5 introduces relevant discussion points, and Section 6 presents the conclusions of the conducted research.

2 Review of Literature

2.1 Decentralized Multi-agent Robotic Systems

There are many methods for controlling system of multiple agents, falling mainly into two categories: centralized control strategies in which one or more agents are designated as leaders, and decentralized control strategies in which each robot operates without a leader. Decentralized control strategies have become prevalent in recent years as communication and sensing hardware have become less expensive and easier to implement on teams of mobile agents, and these control strategies offer certain advantages discussed subsequently. These systems are extending the range of autonomy in robotics and, consequently, new control strategies are required to achieve the desirable range of complex operating modes. While traditional, centralized control strategies can accomplish a variety of tasks, they ultimately are unable to fully meet the advantages of a decentralized multi-agent system [1].

Decentralized control strategies are leaderless and use information local to each agent. Usually, it is assumed that this information is only a subset of the overall network topology [2]. Network topology refers to the interaction of an agent with other agents. For example, a particular control strategy may only use information about the neighbors within a certain spatial vicinity of a robot. In some cases, the information flow between agents is bidirectional, meaning that if agent i is capable of receiving or measuring information from agent j, then agent j is capable of measuring or receiving information from agent i. Sometimes information flow is unidirectional, meaning that it is possible for agent i to receive or measure information from agent j, but agent j cannot receive information from agent i.

An example of a simple model in which the information flow is bidirectional is the so-called δ-disk proximity model. In this model, agent i can receive or measure information from agent j only if agent i and agent j are within a distance δ from each other. Assume agent i is at position $x_i \in \mathbb{R}^n$ and agent j is at position $x_j \in \mathbb{R}^n$. it is easy to see that if

$$|x_i - x_j| \leq \delta,$$

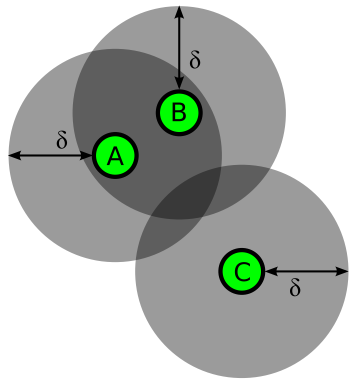

Figure 1: *The δ-disk proximity model. Each agent is capable of exchanging information with other agents within a distance δ.*

then

$$|x_j - x_i| \leq \delta,$$

i.e., the information flow must be bidirectional. Figure 1 shows this model pictorially. In the figure, agents A and B are capable of information exchange, while agent C cannot exchange information with either agents A or B.

Likewise, an example of a simple model in which the information flow is unidirectional is a model in which an agent i can only measure information from another agent j if agent j is within a certain "field of view" of agent i. Unlike the previous example, if agent j is within agent i's field of view, it does not necessarily follow that agent i is within agent j's field of view. Therefore, the information flow need not be bidirectional.

Information exchange in multi-agent systems can either involve active communication between agents or passive sensing. While communication between agents can be more powerful and flexible, it requires communication hardware (often wireless), and a protocol for handling inter-agent communications, which can be expensive and difficult to implement. The alternative is to use sensors for information exchange. For example, it may be possible to measure the relative position and speed of neighboring agents without active communication. This method is often cheaper and easier to implement. In addition, many control strategies exist that rely only on sensors and do not require communication. This research assumes the agents are only capable of sensing relative positions and speeds of agents.

Despite the apparent limitations of some multi-agent systems, decentralized control strategies can accomplish a vast array of tasks including location rendezvous [1], formation control [3], and target assignment [4]. In location rendezvous, the robots attempt to arrive at the same location in space. It can be shown that the robots will achieve location rendezvous under certain conditions, and these conditions change depending on whether information flow is unidirectional or bidirectional [5]. In addition, the required conditions are relatively easy to satisfy, meaning that it is not difficult to develop a multi-agent system capable of location rendezvous. Consequently, significant research has been conducted in location rendezvous in recent years, and the problem is well-understood. Formation control is investigated in Section 2.3.

When designing decentralized control strategies for multi-agent systems, it is necessary to incorporate information such as communication capabilities of the agents and sensing ranges. However, this can be incorporated at a lower level and abstracted to a simple definition of which "links" between agents exists over time. This type of approach is known as the graph-theoretic approach to multi-agent control design and is prevalent in many control applications.

2.2 Graph-Theoretic Approach to Multi-agent Control

While decentralized control strategies have proven advantages over centralized strategies, they can be difficult to implement due to the nature of the communication links between the robotic agents. Depending on specific hardware implementations, the ability of one agent to communicate or sense another agent is often geometrically constrained. For example, a robot that uses a camera may be limited to line-of-sight sensing, while a robot that uses wireless communication may have a proximity disk associated with its communication capabilities, and a robot equipped with radar will often have an even more complicated geometric constraint on its sensing capabilities [6].

Without simplifying these geometric constraints, the problem of developing decentralized control strategies can be prohibitively difficult. Fortunately, it is usually sufficient to know simply whether an information link exists between two agents rather than the complex geometric relationship between the two agents. Because of this, it is often beneficial to

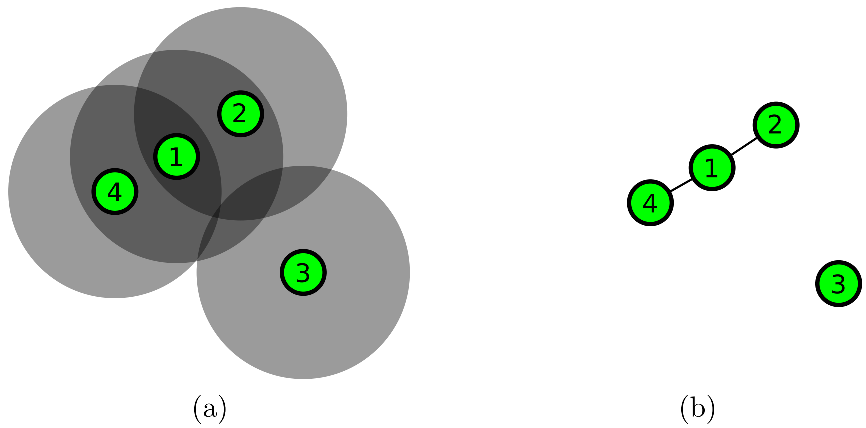

Figure 2: *(a) Four agents with δ-disk sensing ranges, and (b) their corresponding proximity graph.*

model the system in a graph theoretic framework in which the communication links are captured in the combinatorial objects associated with a graph such as the edge and vertex sets [1]. After doing so, well-established algebraic graph theory can be applied to the resulting graph model which leads to meaningful conclusions about the robotic system [7, 6, 8].

Using graph theory to capture the network topology of a multi-agent robotic system has now become standard. Common graph models include a static graph in which the communications links (and the corresponding edge set) do not change over time. While this is a simple model, it often does not follow reality. Another common model is a random graph in which communication links are modeled using stochastic processes. In a proximity graph, a communication link exists only if the distance between the two agents satisfies a geometric proximity constraint [6]. This type of graph was introduced briefly in the previous section The presented research mostly employs a static graph or a proximity graph to model the network topology. Figure 2 shows an example of a proximity graph. Each robot corresponds to a node in the graph, and each information link corresponds to an edge in the graph.

2.3 Formation Control

In many respects, formation control is closely related to location rendezvous. In location rendezvous, it is desired that the agents move towards a common point in space. In formation control, it is desired that the agents maintain certain inter-agent distances. The methods used to enforce these inter-agent distances are similar to the methods used in location rendezvous.

Formation control of multiple robots has been an active topic of recent research. Analysis of the stability properties of formations has been investigated by Fax et al. [9], which relates the eigenvalues of the formation graph Laplacian to formation stability. In addition, the formation graph and eigenvalues of the Laplacian give rise to a Nyquist plot, which can be used to quantify stability [5]. Hybrid control strategies in which the robots switch between different formations has also received some attention.

In [10], Axelsson et al. present a strategy for switching formation modes by introducing a *formation error* associated with each possible formation. The robots autonomously decide to execute the formation which results in the smallest error. Each robot uses a hybrid automaton to decide which formation to execute, and state transitions are triggered by minimizing a cost function associated with formation error.

2.4 Target Identification

A common task for multi-agent robotic systems is to interact with specific objects or locations in their environment. These "targets" may be other robots (such as leader robots), desired formation scales, or objects whose locations are unknown. In the classical *Assignment Problem* as applied to multi-agent systems, there are m targets and $n \leq m$ agents. Each target must be assigned one agent. A sense of optimality is obtained by associating a cost function with the mapping of each agent to each target. It has been shown that a suboptimal solution can be obtained if the target assignment is obtained centrally, and then the agents are deployed and may travel decentrally to their respective targets. This solution applies even if rotation and translation of the targets is allowed and is particularly useful for formation assignment [11].

Zavlanos and Pappas demonstrate a decentralized solution that optimally assigns the agents to the targets and drives the agents to the target locations [4]. This solution allows the agents to be assigned dynamically and does not require a centralized, predetermined assignment. The authors present both a solution in which inter-agent communication is assumed [12] and a solution in which only inter-agent sensing is assumed [13].

Mesbahi and Egerstedt offer some algebraic methods for analyzing such systems. It is typical to associate a Laplacian matrix with a multi-agent network topology which captures information about the neighbors of each node. Mesbahi and Egerstedt demonstrate how to partition the Laplacian matrix such that the follower agents can be separated from the target agents, offering an algebraic framework for analyzing equilibrium positions in certain scenarios [14].

3 Research Methods

Several simplifying assumptions were made for the purpose of theory and simulation design in this research. First, the agents are assumed to obey single-integrator dynamics and are capable of instantly changing direction, i.e.,

$$\dot{x}_i = u_i$$

where $x_i \in \mathbb{R}^2$ is the position of robot i and u_i is the control input to the vehicles. Alternatively, the agents could have been modeled with *unicycle* dynamics as in [10]. In this model, the state of each agent consists of its position and its orientation. The control inputs include the translational and angular velocities of the agent. In this model, the dynamics of the agents are

$$\begin{aligned}\dot{z}_{i1} &= u_{i1}\cos(z_{i3})\\ \dot{z}_{i2} &= u_{i2}\sin(z_{i3})\\ \dot{z}_{i3} &= u_{i3},\end{aligned}$$

where $(z_{i1}, z_{i2}) \in \mathbb{R}^2$ is the position of the robot, $z_{i3} \in [0, 2\pi)$ is the orientation of the robot, and u_{i1}, u_{i2} are the translational and rotational velocities, respectively. This model is arguably more realistic in that it captures properties typically found in actual robotic platforms, but it introduces a level of complexity not needed for the main results of this thesis.

The inter-agent communications are modeled as a static, undirected graph. This implies that a communication link between two agents persists throughout the duration of the executed control strategy. Alternatives include modeling the communication links using dynamic graphs, such as a δ-disk proximity graph, or modeling the links using a stochastic process. These models may provide interesting insight into the problem, but increase the complexity of the problem and are outside the scope of this thesis.

It is assumed that each agent is capable of sensing the relative position and speed of other agents within its sensing graph. The agents are not capable of sharing any other information with each other. It is also assumed that each agent is equipped with a sensor capable of measuring the clearance and girth of obstacles and boundaries along the direction of travel of the formation. MATLAB was used extensively to simulate the developed control strategies.

This thesis focuses on formation control strategies and not motion planning strategies. Consequently, a simple motion planning algorithm is used in the simulations. Each agent is endowed with the same globally-known and constant x-velocity. Each agent uses a simple gradient decent algorithm to remain approximately centered in the y-direction.

4 Findings

4.1 Robot Formations

In order to control a team of mobile robots in formation, we first define the multi-agent network and then define a formation. We assume that there are N agents, each evolving in a d-dimensional space. Then $x_i \in \mathbb{R}^d$ is the state of agent i. Furthermore, assume that the agents operate with single integrator dynamics, i.e., $\dot{x}_i = u_i$ where $u_i \in \mathbb{R}^d$ is the input associated with robot i.

We associate an undirected graph $\mathcal{G} = (V, E)$ with the agents, where V is the vertex set of all agents, and E is the edge set defining the sensing links between the agents where $E \subseteq V \times V$. By *undirected* graph, we mean $(v_i, v_j) \in E \Leftrightarrow (v_j, v_i) \in E$. A sensing link (v_i, v_j) implies that agent i can measure agent j's relative position and speed. We call $\mathcal{G}$ the *sensing graph.*

A desired robot formation is specified as a set, D, of inter-agent distances, i.e.,

$$D = \{d_{ij} \in \mathbb{R} \mid d_{ij} > 0,\ i, j = 1, \ldots, N,\ i \neq j\} \tag{1}$$

with $d_{ij} = d_{ji}$, where it is assumed that D is a feasible formation. By feasible formation, we mean that there exists a set of points Ξ where

$$\Xi = \{\xi_1, \xi_2, \ldots, \xi_N\},\ \xi_i \in \mathbb{R}^p,\ i = 1, \ldots, N, \tag{2}$$

such that $||\xi_j - \xi_i|| = d_{ij},\ i, j = 1, \ldots N,\ i \neq j.$

This formation specification is encoded in a weighted graph $\mathcal{G}_f = (V, E_f, w)$ where V is the vertex set of all agents, $E_f \subseteq V \times V$ is the set of edges specifying which inter-agent distances are defined by the formation, and edge weight w encodes the desired inter-agent distances, i.e $w : E_f \mapsto \mathbb{R}_+$ and $w(v_i, v_j) = d_{ij}$. We will call $\mathcal{G}_f$ the *formation graph.* It is usually desirable for $\mathcal{G}_f$ to only specify necessary inter-agent distances. Usually, "necessary" edges are those which are required to form a rigid formation so that $\mathcal{G}_f$ is not complete but does require that $|x_i - x_j| = d_{ij}\ \forall\ i, j.$

From this formation specification, we can derive a control strategy for driving the agents to a rotationally invariant formation encoded by $\mathcal{G}_f$ and a target location set Ξ as defined above. By *rotationally invariant*, we mean a formation which is a translation and rotation of Ξ.

We define a potential function $V : \mathbb{R}^{Nd} \mapsto [0, \infty)$ in terms of desired inter-agent distances D and the states of the agents:

$$V(x_1, \ldots, x_N) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j \in N_i^f} \left(||x_i - x_j|| - \alpha_i d_{ij} \right)^2 , \tag{3}$$

where α_i is an agent-specific scaling factor and N_i^f is the formation neighborhood of agent i defined as

$$N_i^f = \{ v_j \in V \ : \ (v_i, v_j) \in E_f \}. \tag{4}$$

Determination of α_i is discussed in Section 4.2. Note that, by using this formulation for V, V is positive semi-definite and only equal to 0 when the agents have agreed on a formation scaling factor and have achieved the correct inter-agent distances.

Now that we have defined V, we can establish a gradient descent control law for the agents which reduces V to zero. We set

$$\dot{x}_i = -\frac{\partial V(\mathbf{x})}{\partial x_i}, \tag{5}$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_N)^\mathsf{T}$ and

$$\frac{\partial V(\mathbf{x})}{\partial x_i} = \sum_{j \in N_i^f} \left(||x_i - x_j|| - \alpha_i d_{ij} \right) \frac{x_i - x_j}{||x_i - x_j||} \tag{6}$$

if α_i is constant. While α_i is, in general, not a constant, we will show that it is constant for relatively long periods of time, and during those times these results hold. It is then evident that

$$\frac{dV(\mathbf{x})}{dt} = \frac{\partial V(\mathbf{x})}{\partial \mathbf{x}}^\mathsf{T} \dot{\mathbf{x}} = - \left|\left| \frac{\partial V(\mathbf{x})}{\partial \mathbf{x}} \right|\right|^2 \tag{7}$$

is negative or zero if $x \in \mathrm{Ker}(V)$, which implies that the control law drives the formation

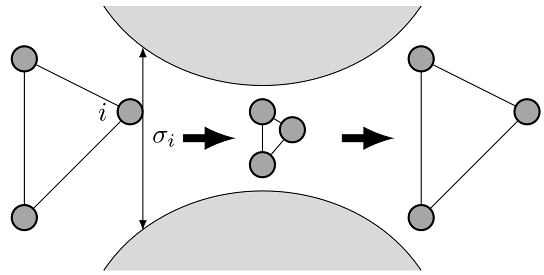

Figure 3: *Three agents maintaining shape but changing size while passing through a narrow passage. Agent i is capable of measuring the clearance σ_i.*

error to zero.

4.2 Determining Formation Scale

Suppose two formation sizes are desired and are specified by D_{big} and D_{small} such that

$$D_{\text{big}} = \alpha_{\text{big}} D$$
$$D_{\text{small}} = \alpha_{\text{small}} D,$$

where $\alpha_{\text{big}}, \alpha_{\text{small}} \in \mathbb{R}_+$ and $\alpha_{\text{big}} > \alpha_{\text{small}}$. Define $\alpha_i \in [\alpha_{\text{small}}, \alpha_{\text{big}}]$ and suppose that $j \in N_i^f$ implies that agent i can infer α_j.

We will show in the next section that α_j is indeed observable from i if the sensing graph G meets certain requirements. We can then establish a strategy for the evolution of α_i.

Assume that each agent moves through an environment with boundaries on either side of the direction of travel and is capable of measuring the distance between these boundaries. Define this measured clearance to be $\sigma_i(x_i(t))$, which is illustrated in Figure 3. Define a threshold clearance $\bar{\sigma}$ that allows the formation to pass through a narrow passage, i.e., $\bar{\sigma} > \max(D_{\text{big}})$. The control law for the evolution of α_i is consensus over all α values with a flow term towards either α_{big} or α_{small}, which depends on σ_i:

$$\dot{\alpha}_i = -\sum_{j \in N_i^f} (\alpha_i - \alpha_j) + \gamma_i \tag{8}$$

where γ_i is a flow term directing α_i either towards α_{big} or α_{small} such that

$$\gamma_i = \begin{cases} -c & , \sigma_i < \bar{\sigma} \\ c & , \sigma_i > \bar{\sigma}, \end{cases} \tag{9}$$

where $c \in \mathbb{R}_+$ is the magnitude of the flow term γ_i.

In addition, α_i has saturation limits such that $\alpha_i \in [\alpha_{\text{small}}, \alpha_{\text{big}}]$. Note that it is possible for $\alpha = [\alpha_1 \ \alpha_2 \ \dots \ \alpha_n]^\mathsf{T}$ to reach an equilibrium where $\alpha_i \notin \{\alpha_{\text{small}}, \alpha_{\text{big}}\} \ \forall \ i$, as can be seen by investigating

$$\sum_i \dot{\alpha}_i = \sum_i \sum_{j \in N_i^f} (\alpha_i - \alpha_j) + \sum_i \gamma_i. \tag{10}$$

It can easily be shown that $\sum_i \sum_{j \in N_i^f} (\alpha_i - \alpha_j) = 0$ by noting that, because $j \in N_i^f$ implies $i \in N_j^f$, for each $(\alpha_i - \alpha_j)$ term there is a corresponding $(\alpha_j - \alpha_i)$ term. Since each γ_i is dependent on σ_i, it is possible that $\sum_i \gamma_i = 0$ if N is even. This would cause $\sum_i \dot{\alpha}_i$ to equal 0, regardless of α, i.e., the centroid remains static and therefore the formation scales, α_i, do not tend towards α_{small} or α_{big}. However, if a small perturbation term γ_i^ϵ is added to each γ_i, it can be ensured that $\sum_i \dot{\alpha}_i \neq 0$, i.e., the centroid of the formation scales tends towards α_{small} or α_{big}. In practice, α_i will typically tend towards α_{big} or α_{small} for all i as the formation moves through an environment. The following subsection considers the dynamics of the α terms.

4.3 Formation Scale Dynamics

Define $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_N]^\mathsf{T}$ and $\gamma = [\gamma_1, \gamma_2, \dots, \gamma_N]^\mathsf{T}$. Define A to be the $N \times N$ adjacency matrix of the formation graph such that

$$a_{ij} = \begin{cases} 1 & \text{if } (v_i, v_j) \in E_f \\ 0 & \text{if } (v_i, v_j) \notin E_f. \end{cases}$$

Define the degree matrix D to be the $N \times N$ diagonal matrix where $d_{ii} = |N_i^f|$. Define the graph Laplacian matrix to be $\mathcal{L} = D - A$. Then (10) can be written as

$$\dot{\alpha} = -\mathcal{L}\alpha + \gamma. \tag{11}$$

If there were no flow term, i.e., $c = 0$ and $\gamma = \mathbf{0}$, then $\sum_i \dot{\alpha}_i = 0$. This can be easily seen by noting that the column sums of L are 0 and therefore

$$\begin{aligned}\mathbf{1}^\mathsf{T}\dot{\alpha} &= \mathbf{1}^\mathsf{T}(-\mathcal{L}\alpha)\\ &= -(\mathbf{1}^\mathsf{T}L)\alpha\\ &= \mathbf{0}.\end{aligned}$$

In the system dynamics of (10), in general $\sum_i \dot{\alpha}_i \neq 0$ and it is evident that $\sum_i \dot{\alpha}_i = \sum_i \gamma_i$.

Theorem 1. *In the system dynamics of* (10), $\sum_i \dot{\alpha}_i = \sum_i \gamma_i$.

Proof.

$$\begin{aligned}\sum_i \dot{\alpha}_i = \mathbf{1}^\mathsf{T}\dot{\alpha} &= \mathbf{1}^\mathsf{T}(-\mathcal{L}\alpha + \gamma)\\ &= -\mathbf{1}^\mathsf{T}\mathcal{L}\alpha + \mathbf{1}^\mathsf{T}\gamma\\ &= \mathbf{1}^\mathsf{T}\gamma\\ &= \sum_i \gamma_i. \end{aligned} \tag{12}$$

□

However, if $\sum_i \gamma_i = 0$, then $\sum_i \dot{\alpha}_i = 0$. For example, if $|\gamma_i| \triangleq c$ where c is some constant, there are an even number of agents N, and the agents are evenly divided between the two target formation sizes, then $\sum_i \dot{\alpha}_i = 0$. Furthermore, the agents will move towards an equilibrium where $\dot{\alpha} = \mathbf{0}$, but, in general, $\alpha_i \notin \{\alpha_\mathrm{s}, \alpha_\mathrm{b}\}$ for all $i = 1 \dots N$.

Define $|\gamma_i| \triangleq c + \epsilon_i$ where ϵ_i is a random number such that $\epsilon_i \ll c$. Then $\sum_i \gamma_i \neq 0$ with probability 1.

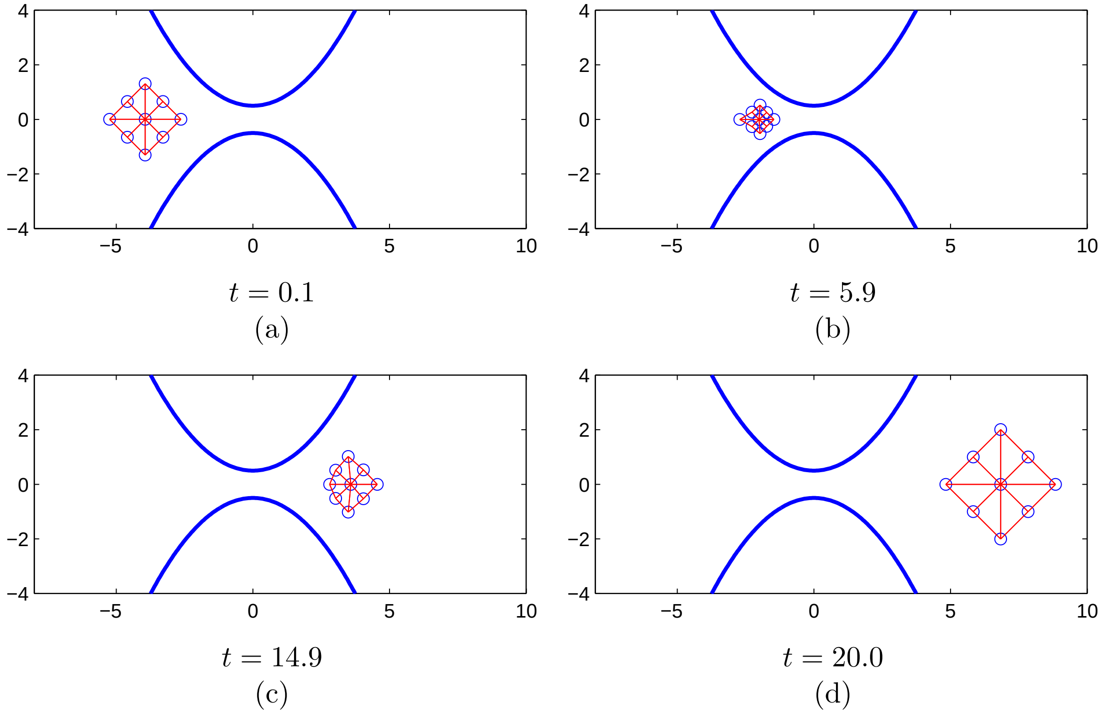

Figure 4: *(a)–(d) show a progression of nine agents in formation traversing through a narrow passage with no inter-agent communication using a decentralized control strategy.*

Further define $\gamma = \gamma^c + \gamma^\epsilon$ where $\gamma_i^c = \pm c$ and $\gamma_i^\epsilon = \pm\epsilon_i$, depending on the direction of flow of each agent such that $\gamma_i = \gamma^c + \gamma^\epsilon$. In some sense, the scenario in which N is even and the agents are evenly divided between the two formation sizes is a worst case scenario due to the symmetry. In such a case, $\sum_i \dot{\alpha}_i = \sum_i \gamma_i^\epsilon$:

$$\begin{aligned}
\mathbf{1}^\mathsf{T}\dot{\alpha} &= -\mathbf{1}^\mathsf{T} L\alpha + \mathbf{1}^\mathsf{T}\gamma \\
\mathbf{1}^\mathsf{T}\dot{\alpha} &= -\mathbf{1}^\mathsf{T} L\alpha + \mathbf{1}^\mathsf{T}\gamma^c + \mathbf{1}^\mathsf{T}\gamma^\epsilon \\
&= \mathbf{1}^\mathsf{T}\gamma^c + \mathbf{1}^\mathsf{T}\gamma^\epsilon \\
&= \mathbf{1}^\mathsf{T}\gamma^\epsilon.
\end{aligned}$$

Since $\epsilon_i \ll c$, $\dot{\alpha}$ will be dominated by the $-L\alpha + \gamma^c$ terms until $L\alpha \approx \gamma^c$. At this point, the agents will appear to be in a formation that "drifts" with velocity $\sum_i \gamma_i^\epsilon$. Note that in this scenario, $\sum_i \gamma_i^\epsilon = \sum_i \gamma_i$, and therefore it is not necessary to know each ϵ_i explicitly.

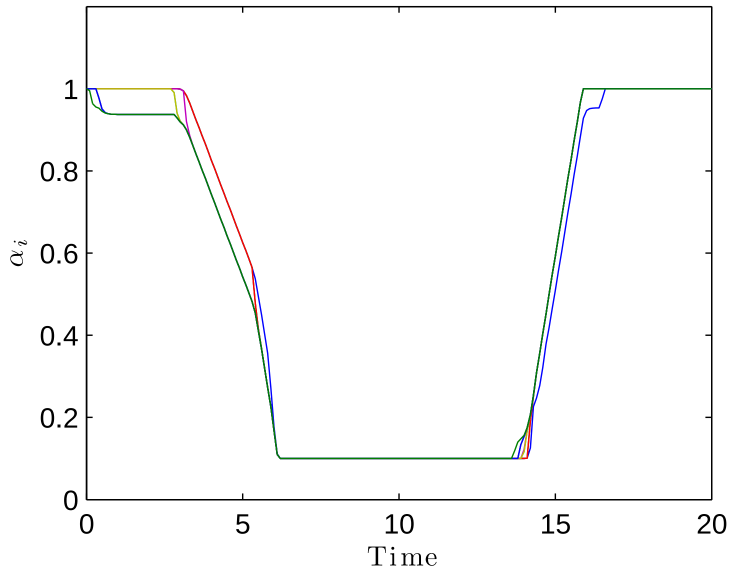

Figure 5: *Progression of the α_i terms over time for the maneuver show in Figure 4.*

4.4 Observability of α

It is possible to determine α_i of neighboring agents if the sensing graph meets certain requirements. Namely, if $(v_k, v_i) \in E_f$ and $(v_i, v_e) \in E_f$ then $(v_k, v_i) \in E$ and $(v_k, v_e) \in E$. In other words, all one-hop and two-hop neighbors of k in the formation graph are neighbors of k in the sensing graph. Suppose agent k is attempting to observe α_i. Then, because of the restrictions on the sensing graph, k can measure $(x_i - x_j)\ \forall\ j \in N_i^f$. Also, k can measure $(\dot{x}_k - \dot{x}_j)\ \forall\ j \in N_i^f$. Since $\dot{x}_k$ is known, it can then be inferred that k can measure $\dot{x}_j\ \forall\ j \in N_i^f$.

Theorem 2. *Assume if $(v_k, v_i) \in E_f$ and $(v_i, v_e) \in E_f$ then $(v_k, v_i) \in E$ and $(v_k, v_e) \in E$. Then α_i is observable from k and*

$$\alpha_i = \frac{e_l^{\mathsf{T}}(\dot{x}_i + \sum_{j \in N_i^f}(x_i - x_j))}{e_l^{\mathsf{T}} \sum_{j \in N_i^f} \frac{x_i - x_j}{||x_i - x_j||} d_{ij}}, \qquad l = 1, 2, \ldots, d$$

where $e_l, l = 1, 2, \ldots, d$ are the base vectors.

Proof. We begin with the control law established in the preceding section,

$$\dot{x}_i = -\sum_{j \in N_i^f} \left(||x_i - x_j|| - \alpha_i d_{ij} \right) \frac{x_i - x_j}{||x_i - x_j||}. \tag{13}$$

For ease of notation, we will define the following:

$$\psi_i = \sum_{j \in N_i^f} (x_i - x_j) \tag{14}$$

$$\tau_i = \sum_{j \in N_i^f} \frac{x_i - x_j}{||x_i - x_j||} d_{ij}, \tag{15}$$

and we can rewrite the control law as

$$\dot{x}_i = -\psi_i + \alpha_i \tau_i. \tag{16}$$

It is therefore possible to determine α_i of neighboring agents,

$$\alpha_i = \frac{e_l^{\mathsf{T}} (\dot{x}_i + \psi_i)}{e_l^{\mathsf{T}} \tau_i}, \qquad l = 1, 2, \ldots, d, \tag{17}$$

where $e_l,\ l = 1, 2, \ldots, d$ are the base vectors of the d-dimensional space. □

4.5 Obstacle Avoidance

The technique developed in Section 4.2 allowing a formation to contract when passing through a narrow passage can also be used to expand around obstacles by simply altering the flow term γ_i. Assume it is possible for each agent to detect an obstacle ahead of the formation's trajectory or near the agent. Then, if agent i detects an obstacle ahead or nearby, denoted by the Boolean term $\zeta_i \in \{\text{true}, \text{false}\}$, the flow term γ_i^{flow} will direct α_i towards α_{bigger} where $\alpha_{\text{bigger}} \in \mathbb{R}$ and $\alpha_{\text{bigger}} > \alpha_{\text{big}}$. Let

$$\gamma_i = \begin{cases} c & , \zeta_i = \text{true or } \alpha < \alpha_{\text{big}} \\ -c & , \text{else} \end{cases} \tag{18}$$

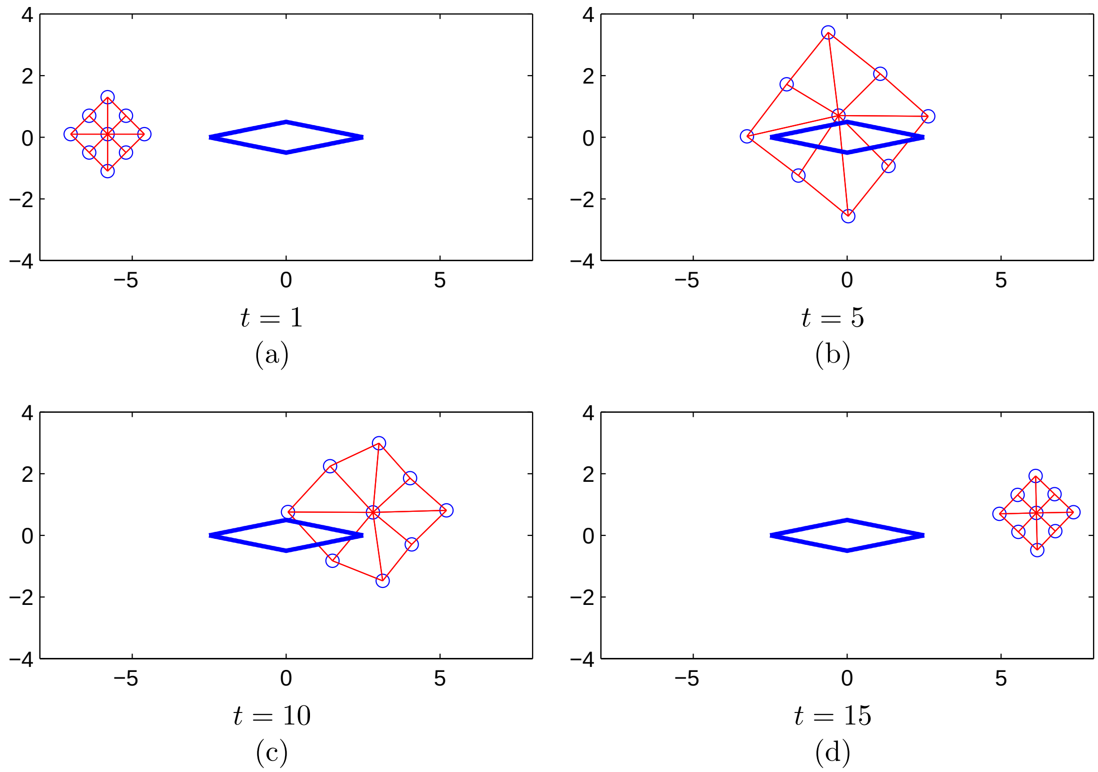

Figure 6: *(a)–(d) show a progression of nine agents in formation expanding around an obstacle.*

where, as before, $c \in \mathbb{R}_+$ is the magnitude of the flow term. In addition, saturation limits are required to ensure that $\alpha_i \leq \alpha_{\text{bigger}}$.

Figure 6 shows nine agents expanding to pass to accommodate an obstacle as they move from left to right.

4.6 Simulations

Figure 8 shows five agents traversing an environment with narrow passages and an obstacle. The simulation includes a simple controller to keep the formation centered in the y-direction, resulting in slight vertical distortion. Note that, even in the presence of this distortion, which is not accounted for in observing α, the control of formation size works reasonably well.

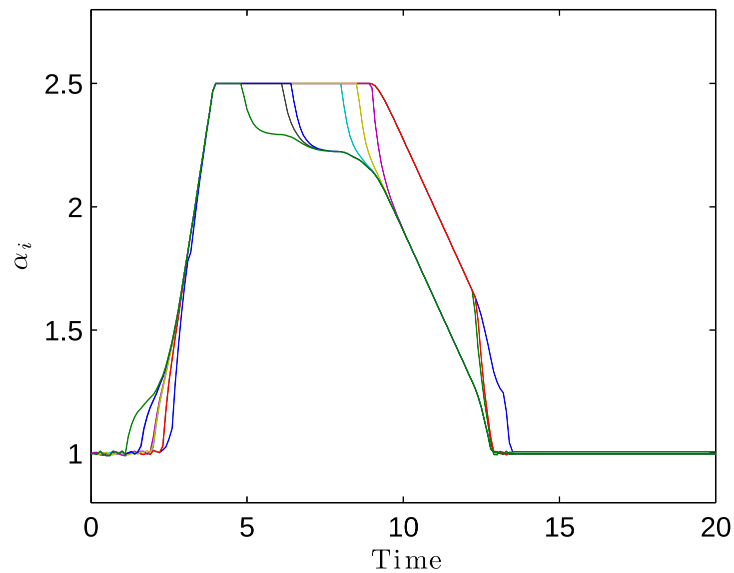

Figure 7: *Progression of the α_i terms over time for this maneuver show in Figure 6.*

5 Discussion

The presented formulation relies on certain assumptions which may be rather restrictive. This discussion addresses some of these restrictions and points toward directions for potential future research.

To allow contraction through narrow passages, it is assumed that each agent can measure the clearance σ_i perpendicular to the overall direction of travel. It is not necessary to assume that the agents can measure an arbitrarily large clearance. It is sufficient if the agents can measure clearance of at least σ_{thresh}, and measurements outside the range of the sensors can be treated as larger than σ_{thresh}. In fact, the true accuracy of the measured σ_i over all sensing ranges is not important as a long as the sensor can accurately distinguish clearances that are less than $\sigma_{\text{threshold}}$ and clearances greater than $\sigma_{\text{threshold}}$.

It is assumed that each agent can measure the clearance perpendicular to the overall motion of the formation, however this may be difficult to achieve in practice. Because the formation size is expected to change, it is not reasonable to assume that the agents will always, or even usually, head in the direction of the overall formation travel. It may

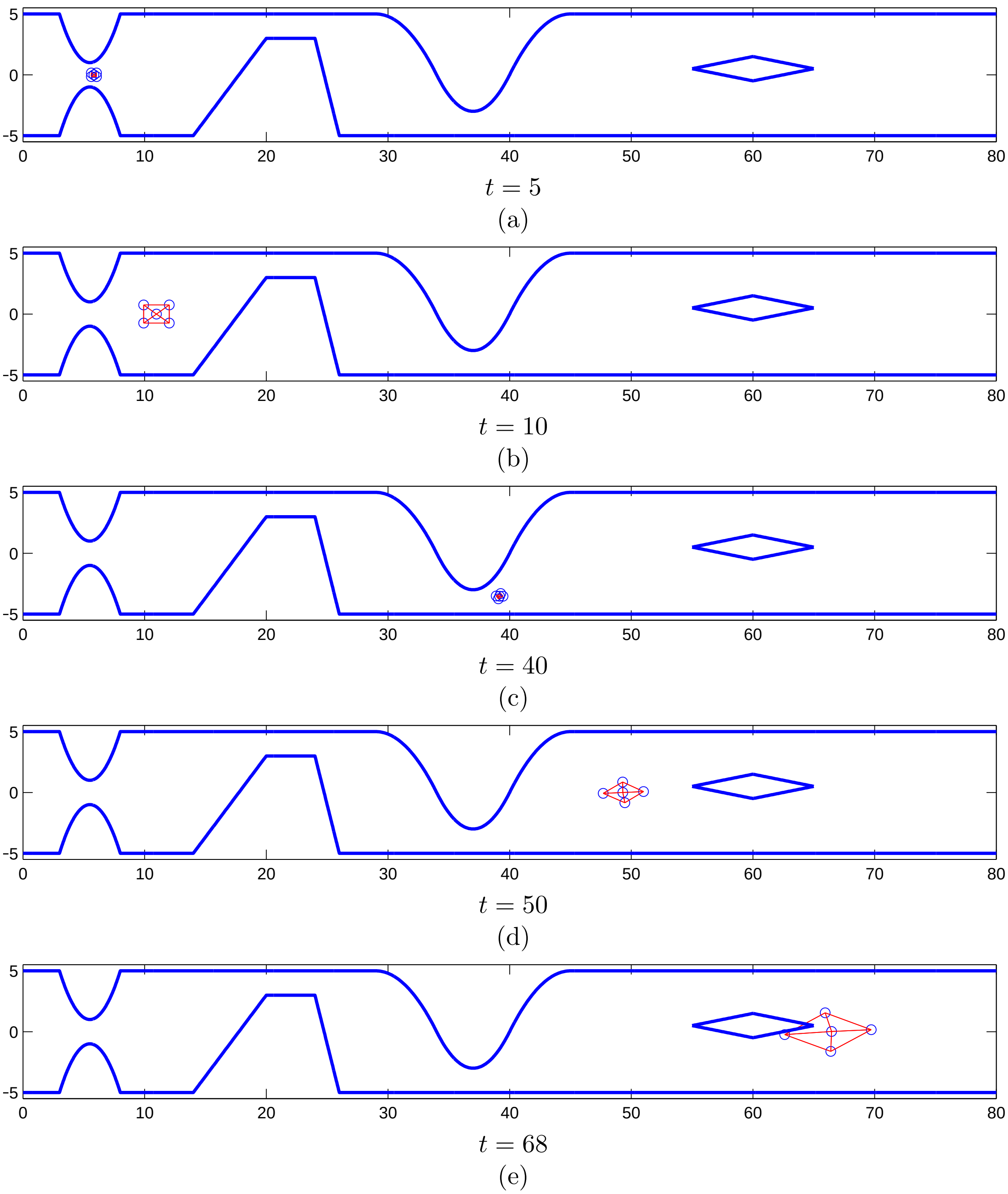

Figure 8: *5 agents traversing an environment from left to right. The agents adjust to a smaller α to traverse narrow passages and adjust to a larger α to avoid obstacles.*

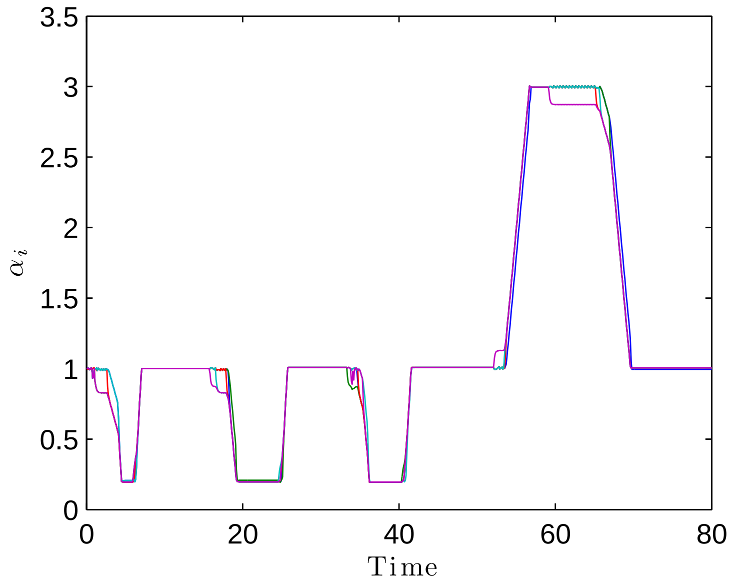

Figure 9: *Progression of the α_i terms over time for this maneuver show in Figure 8.*

be potentially valuable to investigate the scenario in which each agent possesses so-called unicycle dynamics where the agents have a velocity and heading, and control inputs correspond to forward velocity and rotational velocity. In such a model, it may be assumed that each agent can measure clearance perpendicular to its heading and a new formulation for γ_i would be necessary. However, the presented methods can be easily adapted to algorithms with different notions of clearance.

Perhaps the most obvious deficiency of the developed control strategy is the restrictions imposed by a finite number of formation sizes. This assumption is reasonable in certain cases, as it it easy to imagine practical scenarios in which the agents must be in or near one of only several possible formation sizes. However, if a passage is more narrow than D_{small} will allow, or if an obstacle is too large, the presented control strategy will not work. It is natural to then ask if it is possible to allow for continuous formation scaling. By adjusting the γ_i term, it is possible to obtain different results. For example, one agent may be designated as the formation size leader. This agent can use the measured clearance σ_i to determine a formation scale. The other agents then use the measured α_i values of neighboring agents to reach consensus on this formation scale. However, this scenario loses some of the decentralized aspects which are attractive in the presented algorithm.

While the final simulation presented in Section 4.6 does include both contraction and expansion of the formation, the agents are not determining the difference between environment boundaries and obstacles. Rather, the agents are separately aware of the boundaries and the obstacles. Future research could focus on appropriate methods for allowing the agents to sense obstacles which are small enough to be expanded around, obstacles which are two large and need to be avoided, and environment boundaries.

6 Conclusion

This thesis presents a decentralized control strategy for allowing a team of mobile agents in formation to change formation size while maintaining formation shape to accommodate boundaries and obstacles in the environment without inter-agent communication. The agents possess a local, continuous formation scaling factor which is adjusted based on the sensed relative position and velocity of neighboring agents and local perception of obstacles and boundaries. In this way, the team of agents tends towards a discrete number of formation sizes to accommodate certain environmental hazards. First, we present a method for allowing the agents to contract to a smaller formation size when navigating through a narrow passage. Then, we make slight modifications to allow the agents to expand around an obstacle.

We have shown that mild constraints on the ability of an agent to sense other agents' positions and velocities ensures observability of the formation scaling factor employed by its formation neighbors. We have presented a decentralized control strategy for allowing the agents to arrive at a consensus of the formation size, and have discussed the conditions for when a consensus will be reached.

In addition to a theoretical treatment of the problem, we have presented simulations showing the effectiveness of the algorithm in a variety of environments.

References

[1] M. Egerstedt, "Graph-theoretic methods for multi-agent coordination," *ROBOMAT*, Sept. 2007.

[2] J. Cortes, S. Martinez, and F. Bullo, "Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions," *IEEE Transactions on Automatic Control*, vol. 51, no. 8, pp. 1289–1298, 2006.

[3] B. Smith, A. Howard, J. McNew, and M. Egerstedt, "Multi-robot deployment and coordination with embedded graph grammars," *Autonomous Robots*, vol. 26, pp. 79–98, January 2009.

[4] M. Zavlanos and G. Pappas, "Dynamic assignment in distributed motion planning with local coordination," *IEEE Transactions on Robotics*, vol. 24, pp. 232–242, February 2008.

[5] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007. Compilation and indexing terms, Copyright 2009 Elsevier Inc. 20091612036842 Consensus algorithms Cooperative control Flocking Graph Laplacians Networked control systems.

[6] M. Egerstedt, 2007. Lecture notes from Networked Control System graduate course. KTH Royal Institute of Technology, Stockholm, Sweden.

[7] C. Godsil and G. Royle, *Algebraic graph theory.* Springer, 2001.

[8] M. Ji and M. Egerstedt, "Distributed coordination control of multi-agent systems while preserving connectedness," *IEEE Transactions on Robotics*, vol. 23, pp. 693–703, Aug. 2007.

[9] J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1465–1476, 2004.

[10] H. Axelsson, A. Muhammad, and M. Egerstedt, "Autonomous formation switching for multiple, mobile robots," 2003.

[11] M. Ji, S. Azuma, and M. Egerstedt, "Role-assignment in multi-agent coordination," *International Journal of Assistive Robotics and Mechatronics*, vol. 7, pp. 32–40, Mar. 2006.

[12] M. Zavlanos and G. Pappas, "Sensor-based dynamic assignment in distributed motion planning," *Proc. IEEE Int. Conf. Robot. Autom.*, pp. 3333–3338, April 2007. Rome, Italy.

[13] M. Zavlanos and G. Pappas, "Dynamic assignmnet in distributed motion planning with limited information," *Proc. Am. Control Conf.*, pp. 1173–1178, July 2007. New York, NY.

[14] M. Mesbahi and M. Egerstedt, *Graph Theoretic Methods for Multi-Agent Networks.* Princeton University Press. To be published.