Final Summary Report

for the

U.S. Army Operational Test and Evaluation Agency

on Sponsored Thesis Research Under Contract

"Study to Evaluate Results of Operational Tests and Evaluations of Complex Command and Control Systems"

DAAG39-75-C-0095

Conducted by

The School of Industrial and Systems Engineering
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I. Nature of the Research Program

A. Background: The School of Industrial and Systems Engineering of the Georgia Institute of Technology began to offer Operations Research/ Systems Analysis courses at the graduate level in the mid 1950's. A small number of officers and civilians from the Department of Defense who were pursuing graduate degrees in established areas enrolled in these courses. In 1969 the U.S. Army developed a core curriculum for a formal graduate program in OR/SA, and selected Georgia Tech as one of the two civilian institutions for concentrated use in meeting Army graduate educational needs in this area. In 1972 the School was authorized to award a graduate degree in operations research, MSOR. A number of joint reviews have been made in improving the Army OR/SA program requirement with the latest in April 1974 (Incl. 1). Sixteen Army personnel entered the program in 1969, and by 1973, 35 students were in residence with approximately 20 graduating a year. At present 15 are in residence with a forecasted level of 20 in residence and an output of 10 a year.

B. The Theses Problem

For almost all Master's degree candidates, the identification and definition of a Thesis topic of interest both to the student and to his research advisor requires a disproportionate amount of time when compared with the course requirements or thesis research. One of the important objectives to be realized in this program is the development of readily available research topics relevant to Army needs and objectives and potentially interesting to Army personnel, and of competent, involved research advisors. These availabilities are critical if the Army personnel are to complete an acceptable thesis within the time constraint of their tenure in the program. A review of theses by Army officers prior to 1974 indicates a small percentage related to Army needs and problem areas (Incl. 2). This situation was highlighted by Dr. Wilbur Payne, Deputy Under Secretary of the Army in October 1973 in a letter to Georgia Tech approving the revised curriculum programs (Incl. 3) when he stated:

> "I was very interested in the comments you received from the officer students in response to your Proposal Review memorandum. Of particular interest were their remarks concerning the lack of adequate communication between the Army and students, and the resulting scarcity of appropriate military related thesis topics. This has for some time also been a concern of mine. I believe that something can be done to improve this situation, and would be delighted to work with the Institute toward that goal."

C. Theses Support Program

During the fall of 1973 and spring of 1974 a number of conferences and seminars were held between Georgia Tech faculty and Army agents to improve the relevancy of thesis research. In June 1974 the Army Materiel Systems Analysis Agency contracted to support three officers and in the fall of 1974 the U.S. Army Operational Test and Evaluation Agency agreed to sponsor seven officers under two separate contracts. These contracts support the officer students by providing special office space, leased computer terminals, and other logistic support at Tech, TDY travel funds, and data sources within the sponsoring agency. The contracts have also covered approximately 1/4 time salaries, overhead and limited travel for three faculty members for efforts beyond what would otherwise be required for their faculty duties. Actual thesis topics are developed between the individual student, the faculty and the sponsor to assure relevance and academic quality.

D. General Method of Approach

Literature search and problem definition in the two areas above began in the summer of 1974 even though the contracts were not awarded until December 1974. The three faculty members met frequently with individual students and began to collect background material from OTEA, USAMSAA, Command and General Staff College, the Army Logistic Management Agency, and other Army agencies as well as from the Georgia Tech Library. Frequent seminars and conferences between all the students and faculty were held from the end of September until development of individual thesis topics in January 1975. After the Phase I briefing for OTEA in February 1975 and the individual officers worked primarily as individuals with their own thesis advisor and committee until June graduation.

E. Scope of Report

This report provides a final summary for work done for the U.S. Army Operational Test and Evaluation Agency under contracts awarded in the fall of 1974 in the following area:

"Study to Evaluate Results of Operational Tests and Evaluation of Complex Command and Control Systems" DAAG39-75-C-0095

II. Development of Command and Control Evaluation Research Area

Project Objective: Three theses were directed towards the objective of developing improved procedures or methodologies to assist OTEA in planning and evaluating operational tests for tactical command and control systems.

Definitions and Concepts: At least twenty different definitions and types of systems were identified as related to the class of "Army command and control systems." In the developmental area they range from the AN/MSQ-19 system developed by the Signal Corps in the 1950's to the TSQ075 air defense fire direction system, TACFIRE artillery fire control system, to variations of the Tactical Operations System (TOS). At the operational level the Integrated Battlefield Control System and the Revised Army Training Tests for Division Command Posts provide additional variations of conceptual schemes for defining, modeling and evaluating command and control systems. For purpose of their theses, the three officers (one Infantry, one Armor and one Signal Corps) used the standard definition:

> "An arrangement of personnel, facilities, and the means for information acquisition, processing, and dissemination employed by a commander in planning, directing and controlling tactical operations."

Existing Operational Evaluation and Test Procedures

The Modern Army Selected Systems Evaluation and Review Agency (MASSTER) at Fort Hood, Texas has emerged as the primary center for field testing of division level command and control systems. Consequently that agency played a major role in providing copies of plans of tests, reports and field data. The Division Command Post Test, FM286, was selected as a typical test and evaluation methodology used by OTEA. The attribute structure, operational issues and data from this test were not classified, and the test was recently conducted in January 1975.

Research Questions and Approach: Each of the student officers began by asking the basic question: "How can the size and scope of an operational test of a division level command and control system be reduced without reducing the significance of the test results?" Williams approached the problem by looking for a rational basis to reduce the number of critical attributes in subsequent tests employing the same evaluation structure. Rankin sought to develop a methodology for use in the test planning stage which would identify the relative importance of various configurations of components, personnel and sub-systems when evaluating a single critical issue. Finally Burnett examined the application of Multivariate Analysis of Variance (MANOVA) in lieu of conventional ANOVA practice in test design in reaching the same statistical significance and power levels but with smaller sample size.

III. Review of Theses

"A Comparison of the Applicability and Effectiveness of ANOVA with MANOVA for Use in the Operational Evaluation of Command and Control Systems"; by Thomas N. Burnette, Jr., Captain, Infantry.

The Problem

Many Army Operational test designs and evaluations presently rely on the Analysis of Variance (ANOVA) statistical technique which does not take into account the correlation or dependence between critical issue attributes or MOE. This is particularly true in the case of operational tests for command and control systems which utilize a complex large hierarchical attribute structure.

Approach and Methodology

Multivariate analysis of variance (MANOVA) offers a powerful statistical technique for subjectively or objectively taking into account the correlation or dependence between attributes. This technique has not been widely used because MANOVA techniques require lengthy and specialized computer programs, and there is no convenient and usable form for determining the statistical power of the test, i.e., the probability of rejecting the null hypothesis when it is false. This research overcomes these two limitations by adaptation of the BIOMEDICAL COMPUTER PROGRAMS (BMD) to the computational constraint, and by the development and validation of a new and efficient Monte Carlo procedure to determine the power of the tests.

In comparing the applicability and effectiveness of ANOVA with MANOVA the following factors were considered:

a. The powers of the tests versus correlation, sample size, and the probability of type I error

b. The validity of probability statements concerning system parameters

These factors provided the basis for a 6 step methodology for the comparative evaluation of ANOVA and MANOVA.

Summary of the Methodology

A summary of the methodology for comparing the effectiveness of ANOVA with MANOVA under the assumption that the system in question meets the required assumptions for each model is as follows:

1. Determine the correlation matrix for the measures of effectiveness.

2. Separate the measures of effectiveness into mutually independent sets of independent measures, I, and correlated measures, C_i, $i=1,\ldots,k$.

3. Determine the probability of Type I error, α, and the power of the test, $(1 - \beta)$, to be utilized.

4. For each measure of effectiveness, determine the maximum sample size permitted, n_{max}, and the univariate departure to be detected, D.

5. For each measure of effectiveness, determine the sample size, n_{anova}, required to achieve the required power. If $n_{anova} > n_{max}$, reconcile the difference by adjusting D and/or n_{max}.

6. For each set of correlated measures of effectiveness, C_i, $i=1,\ldots,k$, perform the following.

a. For each measure of effectiveness, Y^j, $j=1,\ldots,p_i$, determine the sample size, $n_{manova\ j}$, required to achieve the desired MANOVA power with the measure under consideration departure set at D_j, and all remaining measure departures selected from Uniform $(0,D_j/R_i)$ where R_i is the ratio chosen by the testor.

b. If the $n_{manova\ j}$ are less than or equal to the n_{min} = min ($n_{anova\ j}$) for the desired power, stop; MANOVA is more effective than ANOVA for the measures in the set.

c. If the $n_{manova\ j}$ are greater than the n_{min} for one or more measures in the set, remove from the set the measure corresponding to the n_{min}. If more than one measure corresponds to the n_{min}, remove from the set the measure with the lowest power which corresponds with the n_{min}. Renumber all measures in the set which remain; set $p_i = p_i-1$. If $p_i = 1$, stop; ANOVA is more effective than MANOVA for all original measures in the set C_i. If $p_i > 1$, repeat steps a through c.

Demonstration of the Methodology

A comparative evaluation of two systems in OT-2 is assumed with three scenarios. Seven critical issues or measures of effectiveness are designated MOE-1 through MOE-7. A completely crossed two-factor experiment with equal numbers of observations per cell is assumed. Based on the utilization of the same seven MOE during OT-1 an objective correlation matrix is known (Step 1)

	1	2	3	4	5	6	7
1	1.00	.00	-.06	-.12	.00	-.17	.16
2	.00	1.00	.01	-.11	.01	-.04	.76
3	-.06	.01	1.00	.68	-.49	.56	.07
4	-.12	-.11	.68	1.00	-.21	.72	-.04
5	.00	.01	-.49	-.21	1.00	-.26	-.11
6	-.17	-.04	.56	.72	-.26	1.00	-.08
7	.16	.76	.07	-.04	-.11	-.08	1.00

As required in (Step 2) the MOE are separated into mutually independent sets of independent measures I and correlated measures C_i using both subjective and analytical means with the result that

$$I = \text{MOE-1}$$

$$C_1 = \{\text{MOE-2, MOE-7}\}$$

$$C_2 = \{\text{MOE-3, -4, -5, -6}\}$$

The appropriate correlation matrices for C_1 and C_2 are as follows:

C_1:	2	1.00	.76		
	7	.76	1.00		
C_2:	3	1.00	.68	-.49	.56
	4	.68	1.00	-.21	.72
	5	-.49	-.21	1.00	-.26
	6	.56	.72	-.26	1.00

ANOVA is appropriate for MOE-1 the sole member of set I. As required in (Step 3) for investigating the correlation structures in C_1 and C_2 the following parameters for selected for both ANOVA and MANOVA.

Probability of Type I error	= 0.05
Power of Test $(1 - \beta)$	= 0.75
Max sample size	$= n_{max}$
Departure to be Detected	= D

Using standard statistical techniques the minimum sample size is determined for ANOVA for each MOE (Step 4, Step 5)

Table 1. MOE Sample Sizes for Required Power

MOE	Maximum Sample Size n_{max}	Departures To Detect D	Minimum Sample Size n_{anova}
1	6	1.5	5
2	6	1.5	5
3	4	2.0	4
4	6	1.5	5
5	6	1.5	5
6	7	1.0	7
7	6	1.5	5

Per step 6, for set C_1, use the MANOVA Power Generator with $n_{min} = 5$, $= \min(n_{A2}, n_{A7})$. The power of 0.762 is obtained which is greater than for ANOVA.

For set C_2 with $n_{min} = n_{A3} = 4$ the results were

Table 2. MOE MANOVA Power I

MOE	MANOVA Sample Size n_{manova}	Departure To Detect D	Power
3	4	2.0	0.614
4	4	1.5	0.482
5	4	1.5	0.496
6	4	1.0	0.452

The n required to obtain the desired power 0.75 is greater than min $n_A = 4$ so remove from the set MOE-3 and determine the new n_{min} with the reduced set C_2 {-4, -5, -6}. n_{min} is now equal to 5 which produces the powers below.

Table 3. MOE MANOVA Power II

MOE	Manova Sample Size n_{manova}	Departure To Detect D	Power
4	5	1.5	0.686
5	5	1.5	0.646
6	5	1.0	0.632

Since the desired power is still not obtained, remove the MOE from the set which has the lower power and for which n_{anova} is 5. The final run obtained the desired power,

Table 4. MOE MANOVA Power III

MOE	Manova Sample Size n_{manova}	Departure To Detect D	Power
4	5	1.5	0.782
6	5	1.0	0.758

It is therefore concluded that ANOVA is more effective than MANOVA for MOE-1, -3, -5 and MANOVA for C_1 = {MOE-2, -7} and C_2 = {MOE-4, -6}

Comments

This research did not examine in detail the assumptions required in the basic ANOVA and MANOVA models or the effects of departures from the required assumptions as a basis for comparison. It was limited by the assumption of two factor, fixed-effect, crossed models with equal sample sizes per cell. Its major contribution was the development of a MANOVA Power Generator and the observations that

1. Power is a decreasing function of dimension of the multiresponse.
2. Power is an increasing function of the size of the departure from the null hypothesis.
3. Power is an increasing function of sample size.
4. Power is an increasing function of the probability of a Type I error.
5. Power is an increasing function of log $|\underset{\sim}{P}|$ where $\underset{\sim}{P}$ is the correlation matrix of the multiresponse.

"An Application of Fault Tree Analysis to Operational Testing" by Gordon Lee Rankin, Captain, Signal Corps.

Objective

To develop a methodology to be used to detect the factors within the complete system that are most likely to contribute to the failure of an operational issue. This was done by means of an adaptation of fault tree analysis. Fault tree analysis is a fairly well known technique which has been used primarily in system reliability analysis. A number of modifications to the technique had to be made before it could be used for evaluation of failures modes of an operational nature such as those in a command and control system as opposed to a "hardware" type system.

Fault tree analysis has a number of characteristics which seem just as desirable to the evaluation of a command and control system as to a hardware system. Some of these are:

1. Failures are deductively identified.
2. All system characteristics relevant to a particular type of failure must be determined.
3. The procedure provides a visual aid to system understanding.
4. The technique is useful for both qualitative and quantitative analysis of system failures.
5. An analyst must study one failure mode at a time. This may be a drawback to use of this technique.
6. The analyst gains an excellent insight into system behavior.

Definitions and Symbols Used

Before reviewing the developments of this thesis some pertinent definitions and symbols should be introduced. Two types of symbols are included here, logic symbols and event symbols. The pertinent definitions are:

Component Configuration: Description of the component states where the component may have several operating states none of which are necessarily failed.

Fault Event: A failure situation which results from the logical interaction of basic component faults or primary failures.

Branch: The decomposition of any fault event results in a branch of the fault tree.

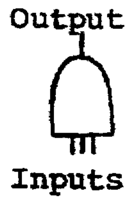
Base Event of the Branch: The fault event which developed leads to the branch.

Domain: Every event in a branch is in the domain of the base event.

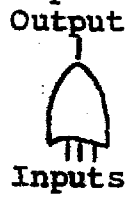
Gate: The Boolean logic symbol that shows the action between inputs to the gate and the output.

Minimal Cut Set: The smallest set of primary events which must happen to cause the top event.

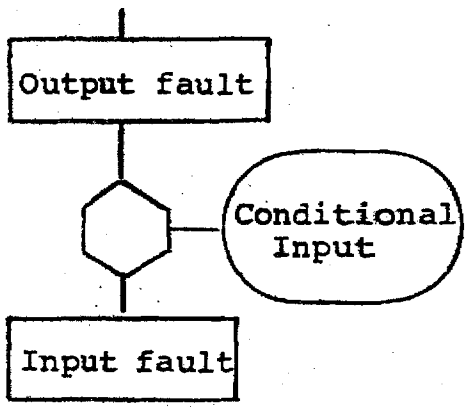
The fault tree logic symbols are:



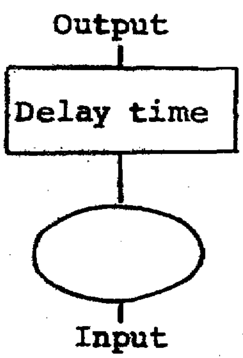
AND Gate: Coexistence of all inputs is required to produce output.



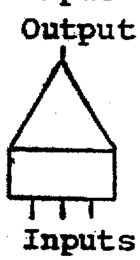
OR Gate: Output will exist if at least one input is present.



INHIBIT Gate: Input produces output directly when conditional input is satisfied.

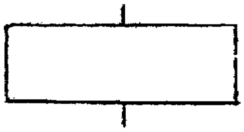


DELAY Gate: Output occurs after specified delay time has elapsed.

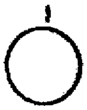


Matrix Gate: Output is related to one or more unspecified combinations of undeveloped inputs.

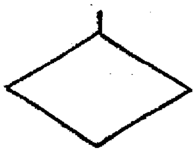
The fault tree event symbols are:



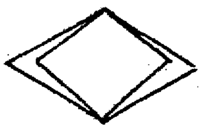
Rectangle: A fault event usually resulting from the combination of more basic faults acting through logic gates.



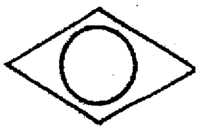
Circle: A basic component fault, an independent event.



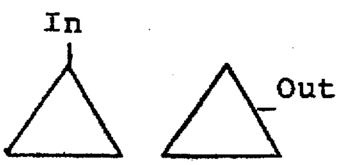
Diamond: A fault event not developed to its cause.



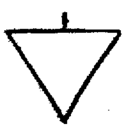
Double Diamond: A significant undeveloped fault event that requires further development to complete the tree.



Circle-Diamond: A fault event, independent of the rest of the tree, was developed separately. Treated as a component.



Triangle: A connecting or transfer symbol.



Upside Down Triangle: A similarity transfer--the input is similar but not identical to the like identified input.

House: An event that is normally expected to occur. Also useful as a "trigger event" for logic structure changes within the tree.

Methodology

The methodology is best described by using an illustrative hypothetical example. The example system is a relatively simple one called SIMGUN which consists of a firing device, projectile, target, control panel, operator and power source. The control panel is connected to the firing device and power source by means of cables.

Details of the system must first be determined. These include its purpose, functions, subsystems and components, and boundary conditions. For this system these are as follows:

Functional Purpose:

What: Eliminate or disable a moving or static armored vehicle at a range of 5,000 meters with a 95 percent probability and a 90 percent hit probability.

When: 5 minutes allowed from target sighting to hit.

Where: in a combat environment.

These system bounds can be as elaborate as necessary. As a minimum those characteristics that may affect the fault tree analysis should be specified.

Sub-systems and Components:

Projectile: propulsion device
homing device
radio element

Firing device: firing device

Control panel: control panel
power supply

Operator: operator

<u>System Boundary Conditions:</u>

TOP Event: miss target

Initial condition: System checks operable

No component has more than one operating state

Not-allowed events: Cable failures

Failures due to effects external to system

Existing effects: None

The fault tree for this simple example is shown in Figure 1. Figure 2 shows the same fault tree with events coded and a logic (Boolean) equation describing the equation at the bottom.

The next step in the methodology is to determine the minimal cut set. Several procedures for doing this are discussed in the thesis. The first, the Fussell Method, uses the relationship

$$I_k = \frac{P(A)}{P(S)}$$

where I_k = Probability mode A is causing system failure

$P(A)$ = Probability mode A has failed

$P(S)$ = Probability system has failed

For SIMGUN example; assume the probability of all component failures is equal to 0.05.

$$P(S) = P(B) + P(H) + P(C) + P(DE) + P(DF) + P(DG) = 0.1575$$

$$I_B = I_H = I_C = 0.32$$

$$I_{DE} = I_{DF} = I_{DG} = 0.02$$

Minimal cut sets are B, H, C since I_k is largest for these failure modes. Note that an assumption of equally likely failure modes was made. This procedure requires either knowledge or an assumption of these probabilities.

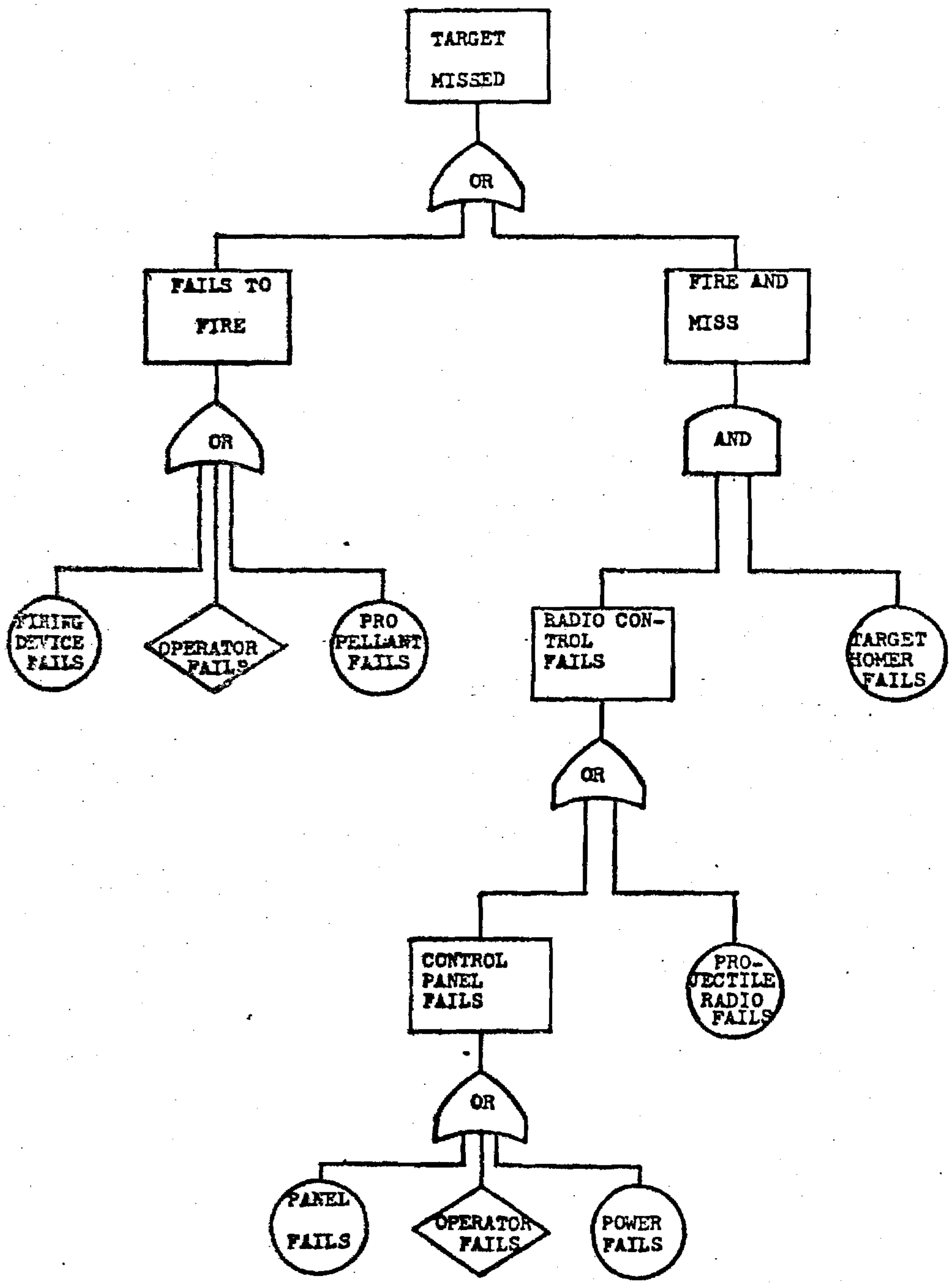
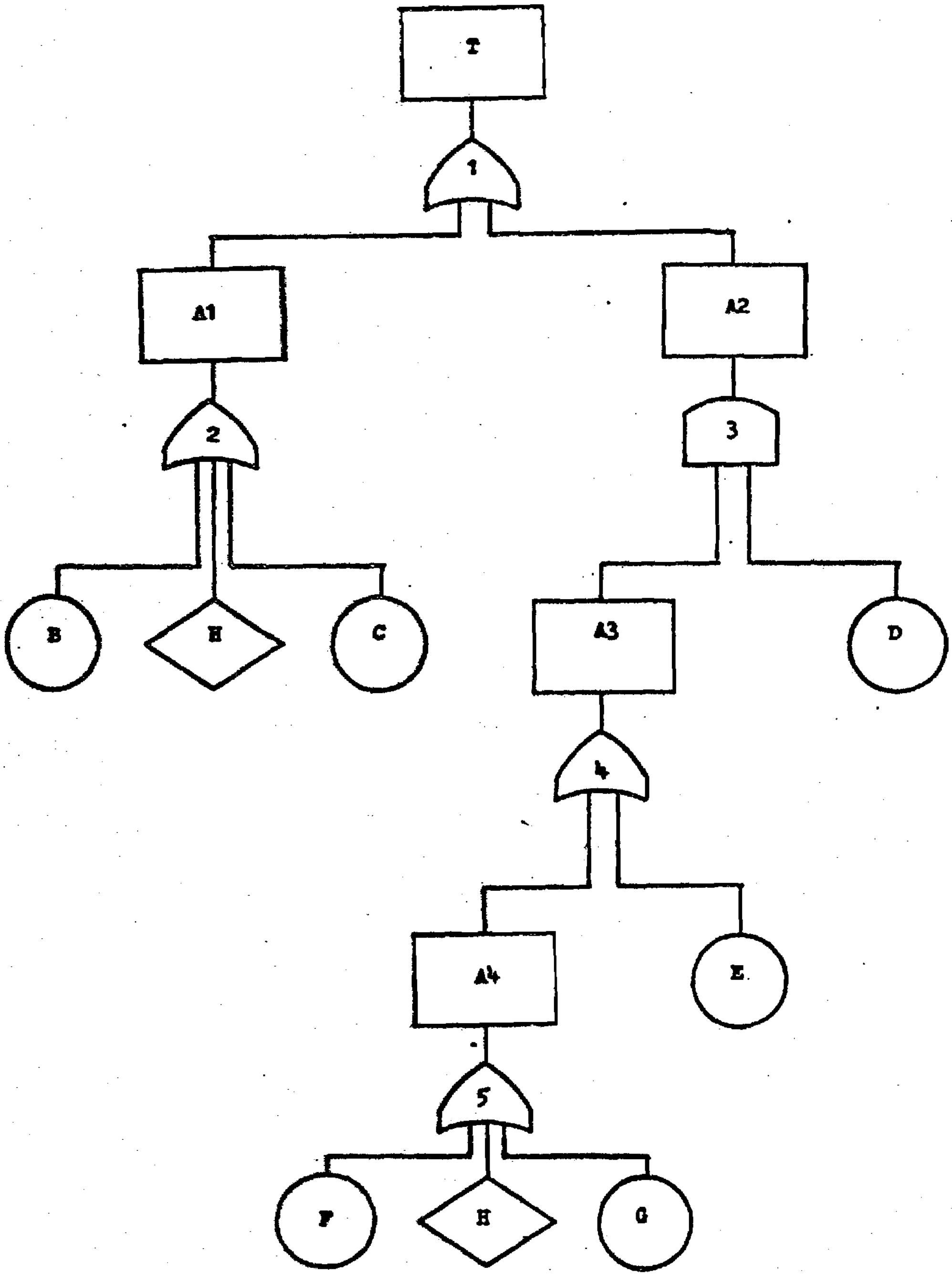


Figure 1. SIMGUN Fault Tree



$$T = B + H + C + FD + GD + ED$$

Figure 2. Coded SIMGUN Fault Tree

A second method which does not require such knowledge is also included in the thesis. This method, here called the Barlow and Prochan method makes use of the relationship,

$$I_k = \sum_{i \in K} \int_0^1 h(1_i, \underline{0}^{K-\{i\}}, p)\,(1-p)^{k-1} dp$$

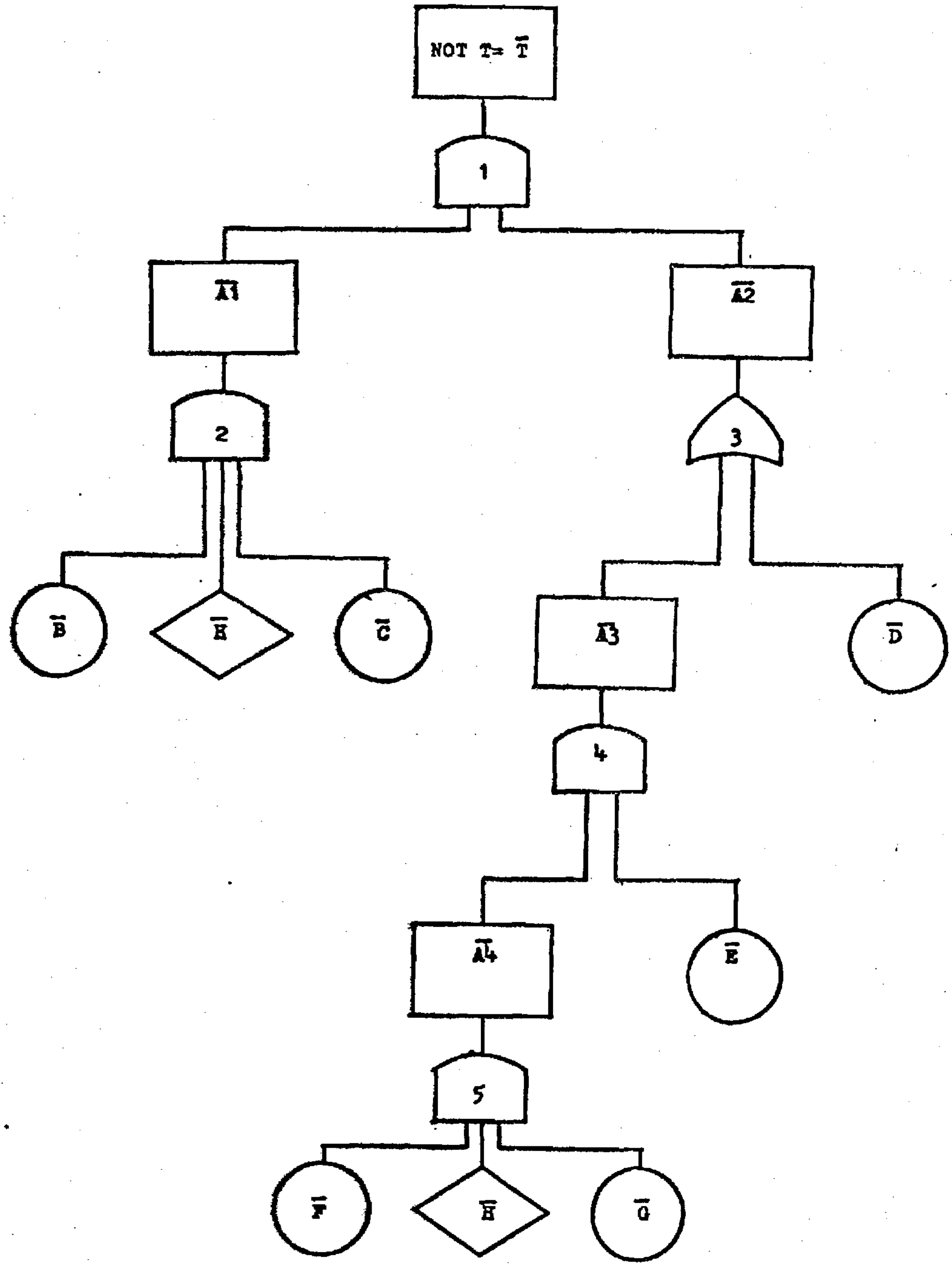
where k = no. of components in cut set K

i = component i of cut set K

$h(1_i, \underline{0}^{K-\{i\}}, p)$ = probability i is critical

$(1-p)^{k-1}$ = probability remaining (k-1) components in K have failed.

In order to simplify analysis by this method a dual fault tree is developed. This is shown, for the SIMGUN, in Figure 3.



$(\bar{T})$ = (BHCD) + (BHCEFG) - (BHCDEFG)

Figure 3. Dual Fault Tree

The minimal cut sets are then determined in the following manner:

Let H = 1, all other components = p

$$I_H = \int_0^1 (p^3 + p^5 - p^6)(1-p)^0 dp$$

$$I_H = 0.274$$

Similarly

$$I_B = I_C = 0.274$$

$$I_{DE} = I_{DF} = I_{DG} = 0.074$$

Again the minimal cut sets are seen to be sets B, C, and H.

A procedure is also presented which will determine the relative importance of components within the cut sets of failure modes. The first procedure for measuring component importance, due to Fussell, makes use of the relationship,

$$I(i) = \sum_{j=1}^{n} I_j$$

where n = no. of minimal cut sets containing component i

I_j = importance of jth cut set

$I(i)$ = Importance of component i

$I(B) = I_B = .32 = I(H) = I(C)$

$I(E) = I_{DE} = .02 = I(F) = I(G)$

$I(D) = I_{DE} + I_{DF} + I_{DG} = 3(.02) = .06$

Since cut sets B, H and C were single component sets it is not surprising to see that these three failure modes are the most important. Since D appears in more than one cut set it is seen to be more important than E, F and G. Several other methods for determining component importance were also presented in the thesis.

Demonstration of Methodology

The thesis finally proceded to illustrate the methodology with the command and control system shown in Figure 4. The fault tree for this system is shown in Figure 5. A complete analysis of this system is presented in the thesis using the methodology developed.

Discussion

This technique is very useful for determining the most likely causes of failure of a system. Procedures are given for using it when good reliability data is available and when little or no data is present. As discussed earlier, it is quite helpful to the analyst to draw the fault tree in that it will help him to understand the system. However, it should also be stated that it may be quite time consuming to draw such a tree for every system to be analyzed.

This type of analysis considers only one type of failure at a time. That is a new fault tree must be developed for each type of system failure. Another limitation of the technique is that it considers only binary failures. That is, it cannot consider a situation in which one or more components have not failed but are not doing quite what they are supposed to do. For this type of analysis each component either fails or works as it should.

If these limitations are not too serious the methodology should be quite useful to OTEA for their consideration of various test configurations to be used to test a system.

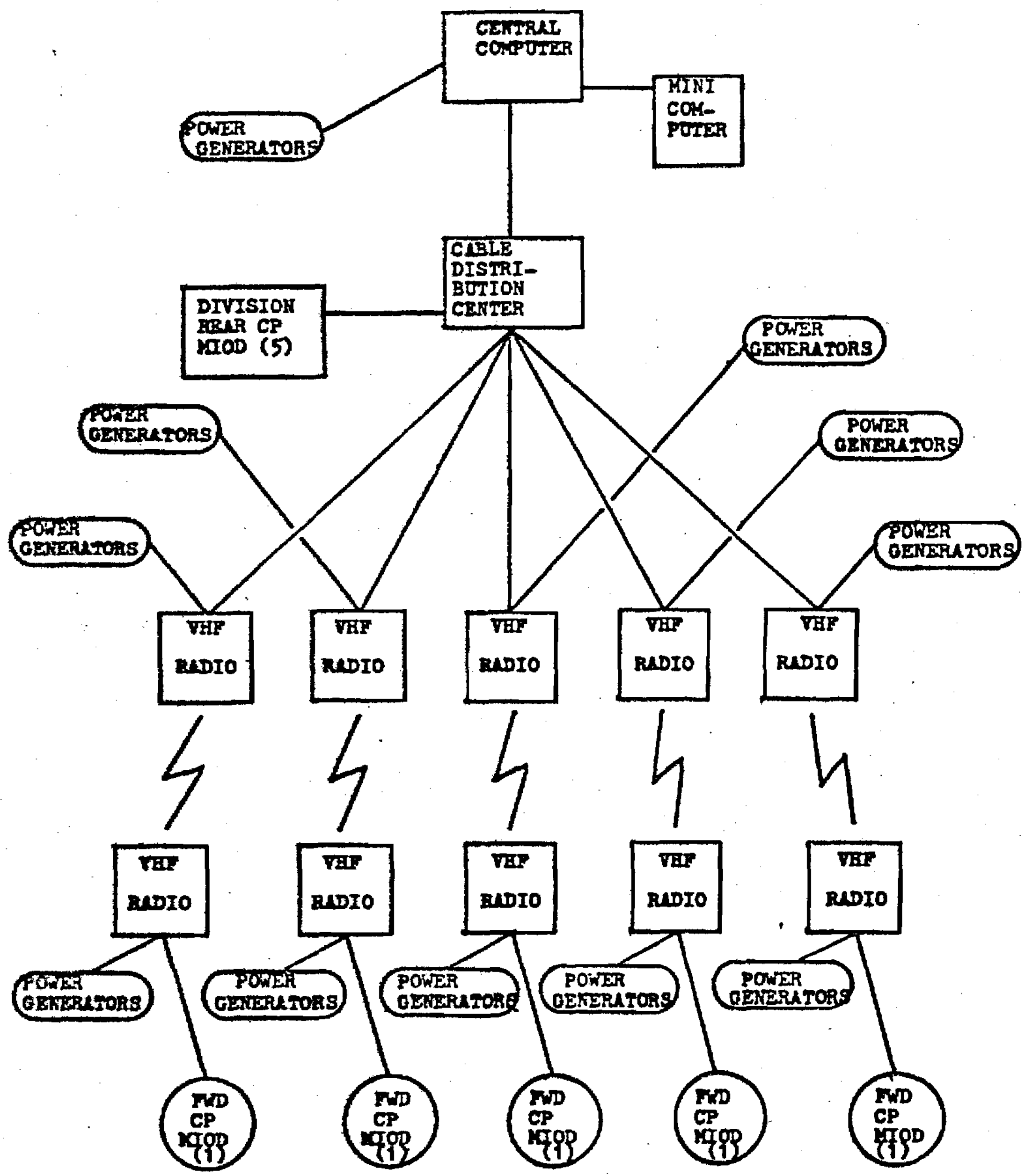


Figure 4. An Example of a Command and Control System

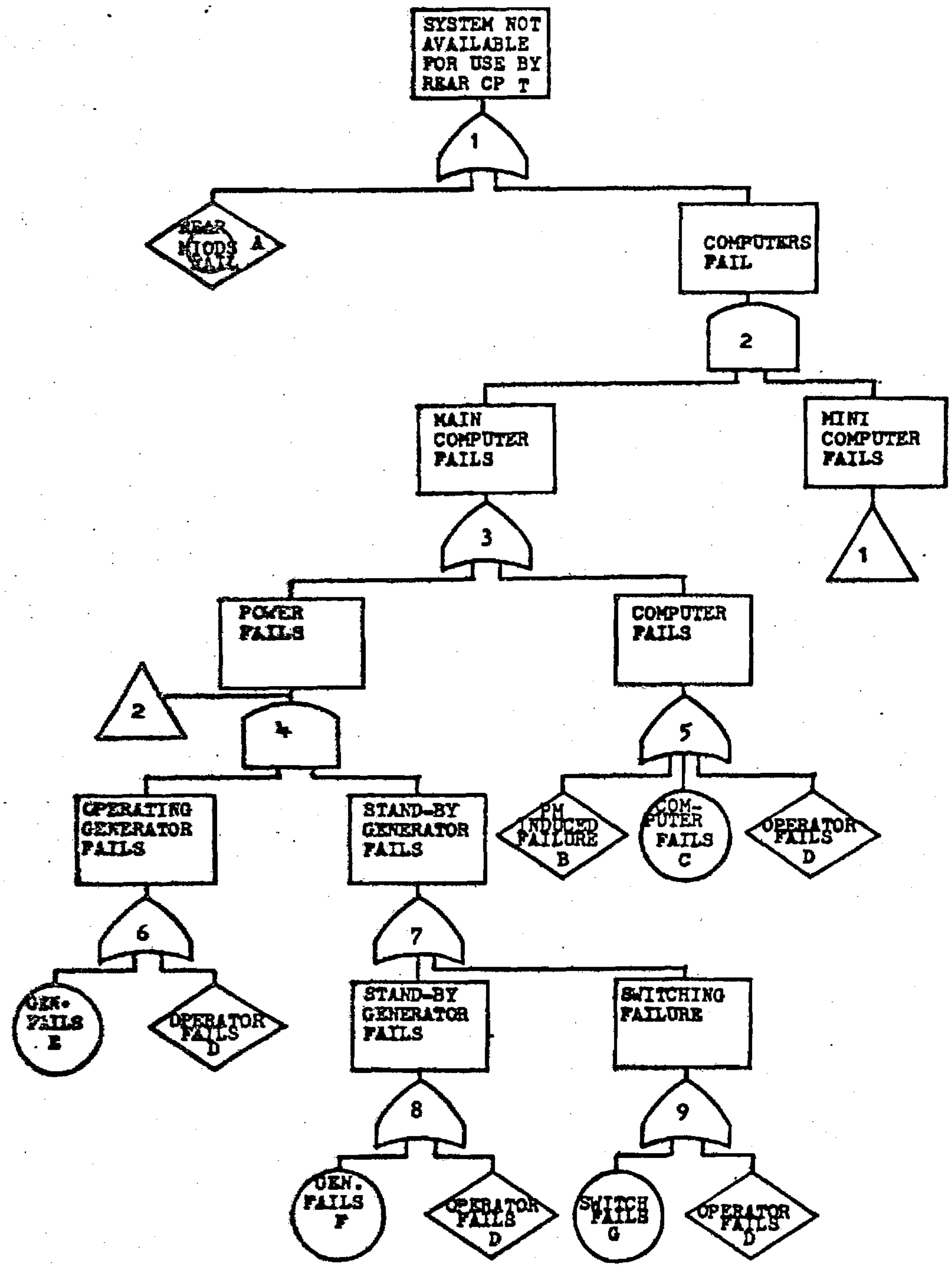


Figure 5. Fault Tree for C & C System

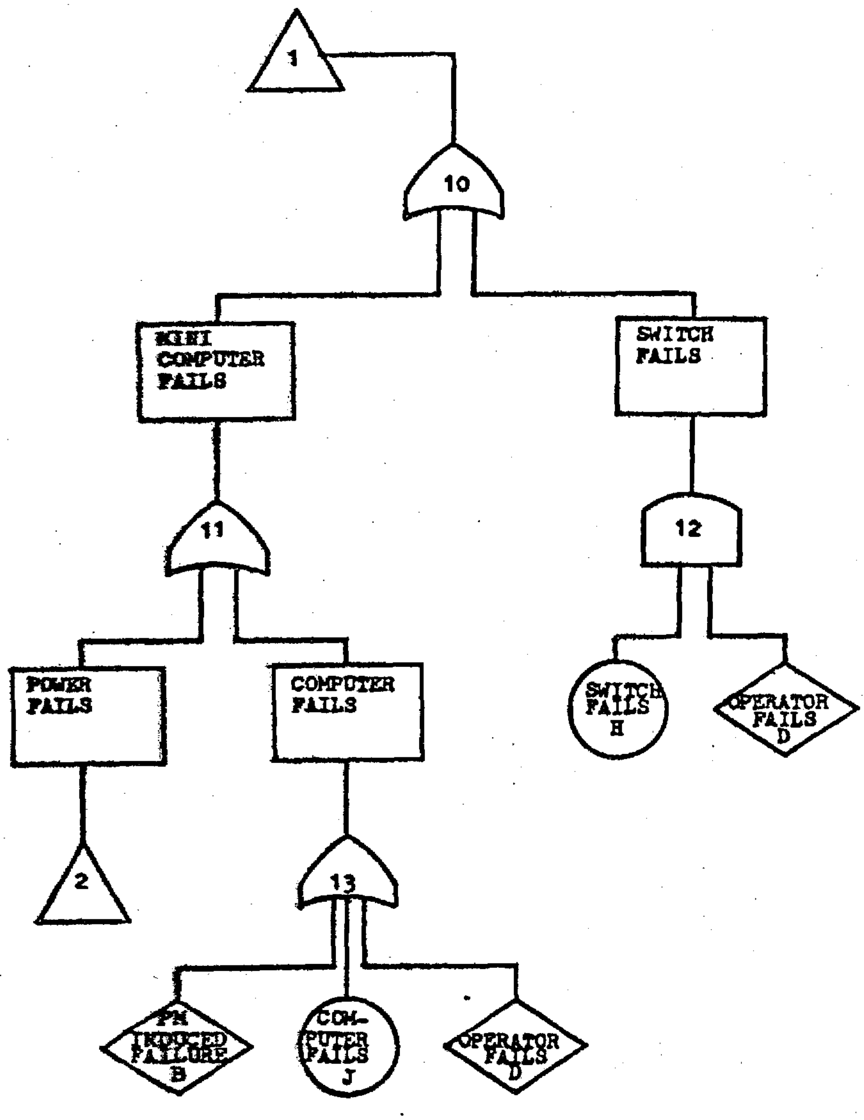


Figure 5. Fault Tree for C & C System, Continued

"A Methodology to Establish the Criticality of Attributes in Operational Tests" by Gary S. Williams, Captain, Armor.

Objective

To develop a methodology which will provide a basis for the selection of critical attributes of complex command and control systems.

For the purpose of this thesis, attributes were defined as measurable characteristics for which test data may be obtained. Critical attributes are those attributes which impart the most information regarding system evaluation.

Three approaches to accomplishing this objective were considered. The first was completely subjective. This was rejected as being infeasible. The number of attributes to be considered along with the number of associated variables is usually so large that too many people would be required. Furthermore no statistical inferences could be made following such an approach. The second was completely objective. This too was rejected as being infeasible. The procedures would require the use of multivariate statistical techniques but the tests considered were of such a nature that replicate tests were impossible to obtain.

The third approach consisted of a combination of the first two. Subjective procedures were used to obtain input to an objective multivariate analysis procedure.

Methodology

A covariance matrix and mean vector are determined subjectively. The estimates are then used to generate the required multivariate data. A combination stepwise regression and linear discriminant analysis procedure (called stepwise discriminant analysis) is used to analyze the resulting data and to select the critical attributes. A six step procedure was developed:

1. Examination and preparation of data. Here the data is examined and grouped into frequency distributions. A test for marginal normality is made using the Kolmogrov-Smirnov test for goodness of fit.
2. Determination of covariance matrix. Estimates are made of the variances of the marginal distributions of each attribute along with their covariances. In order to obtain the last, estimates of the correlation coefficients between pairs of attributes must be subjectively determined. A procedure for accomplishing this is suggested.
3. Determination of the mean vector. This consists of arbitrarily chosen values of attribute means which can be called acceptable and other values which can be called unacceptable. That is, two values of the mean result for each attribute are selected and two mean vectors are thus determined.
4. Generation of multivariate normal observation. Two sets of MVN observations are generated using a computer simulation program included in the thesis. One set uses the acceptable mean vector while the other uses the unacceptable vector. Both use the covariance matrix developed in step 2.
5. Stepwise discriminant analysis. A BIMD library computer program is used for this analysis. The relative ability of each attribute to discriminate between the acceptable and unacceptable conditions are determined.

6. Analysis of results. Here the final selection of the set of critical attributes is made.

Demonstration of Methodology

For purposes of a demonstration of this procedure, data from an already completed and documented test was used. This was the Division Command Post Test, FM 286. The purpose of this test was to evaluate the efficiency of the command post in the command and control of division tactical operations and to evaluate command post vulnerability.

A set of measurable attributes were first selected for consideration using the proposed procedure. These are shown in Table 1 along with the frequency of each rating given by test evaluators.

Table 1. Data Used for Demonstration

VARIABLE	RATING CATEGORY					TOTAL
	1	2	3	4	5	
Relevency of Information	88	73	14	0	2	177
Accuracy of Information	60	95	18	4	0	177
Timeliness of Information	13	49	38	23	5	176
Chg. of Com Loc	62	38	22	3	11	137
Organ. Concept	24	69	33	10	20	156

K-S tests showed the above distributions to be significant departures from normality. Also logarithmic and square root transformations were non-normal. This was to be expected since rating type data could seldom be expected to take on a normal appearing pattern. The demonstration of the methodology was continued anyway since its only purpose is to determine which of the five attributes are most critical.

STEP 2: Estimate $\hat{\rho}_{ij}$ for each i, j, $i \neq j$ to obtain the correlation matrix in Table 2.

Table 2. Correlation Matrix

	VARIABLES				
VARIABLES	A	B	C	D	E
A	1.00	0.65	0.36	0.58	0.25
B	0.65	1.00	0.47	0.50	0.50
C	0.36	0.47	1.00	0.42	0.65
D	0.58	0.50	0.42	1.00	0.80
E	0.25	0.50	0.65	0.80	1.00

Compute $S_{ij} = \hat{\rho}_{ij} S_i S_j$ to obtain the covariance matrix in Table 3.

Table 3. Covariance Matrix

	VARIABLES				
VARIABLES	A	B	C	D	E
A	0.533	0.335	0.387	0.512	0.221
B	0.335	0.497	0.488	0.426	0.426
C	0.387	0.488	2.168	0.748	1.156
D	0.512	0.426	0.748	1.463	1.169
E	0.221	0.426	1.156	1.169	1.460

STEP 3: Choose mean vector of 1.5 for an acceptable population and 2.5 for unacceptable results.

STEP 4: Generate 2 sets of data with computer routine,
1 set with mean 1.5
1 set with mean 2.5

STEP 5: Stepwise regression analysis using BIMD program 07M. Table 4 is a summary of the results obtained.

Table 4. Results of Analysis

Step Number	Variable Entered	Value to Enter	Number of Variables Modeled
1	2	111.2828	1
2	1	14.0945	2
3	5	5.8616	3
4	4	3.4448	4
5	3	1.4660	5

A linear discriminant function was next developed using variables 1, 2 and 5.

STEP 6: Analysis of Results

Attributes B, A, E were determined to be critical. That is, they best discriminated between the acceptable and unacceptable populations.

Evaluation

This thesis was written assuming that the basic test design was fixed and could not be changed. Even though the data were discrete, the multivariate normal distribution was used for the analysis. Since the basic data are in the form of ratings, a nonparametric approach might have been more appropriate. This type of approach would also be simpler to use and, since the purpose of the analysis is only to screen the attributes, it should be as powerful as the approach used in this thesis.