Particle Transport and Density Gradient Scale Lengths in the Edge Pedestal

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Abstract

A new flux-gradient relation for the ion particle flux in the edge pedestal is derived from continuity and momentum balance, taking into account atomic physics, and cast in the form of a generalized 'diffusion-pinch' transport relation. This flux-gradient relation is used to derive a new expression for a first-principles calculation of the ion density gradient scale length.

1. Introduction

The importance of the edge pedestal region in establishing and maintaining high confinement mode (H-mode) plasmas is now well established (e.g. Refs. 1-3). While factors that determine the gradients and widths of the edge pedestal have been the subject of intensive investigation for a number of years (e.g. Refs. 4-6), the 'first-principles' determination of the structure of the edge pedestal remains elusive.

We have previously suggested[7-9] that temperature and density gradient scale lengths [$L_x \equiv -x/(dx/dr)$] in the edge pedestal are determined by transport constraints, at least between or in the absence of ELMs. Expressions for calculating (L_{Te}, L_{Ti} , L_n) in terms of local particle and heat fluxes from the core, local transport coefficients, atomic physics effects associated with recycling neutral atoms and impurities have been presented. These expressions have been based on the conventional heat conduction closure relation $q = nT\chi L_T^{-1}$ and the pinch-diffusion particle flux relation $\Gamma = nDL_n^{-1} + v_p$. The heat conduction relation is theoretically well founded and lends itself to comparison of L_T resulting from various theoretical expressions for χ with directly measured L_T. However, the pinch-diffusion relation is heuristic and does not lend itself readily to comparison of theory with experiment. The purpose of this paper is 1) to present the derivation of a theoretically well founded pinch-diffusion model for particle fluxes in the edge pedestal and 2) to employ this model to develop an expression for the first-principles calculation of density gradient scale lengths in the edge pedestal.

2. Generalized 'Pinch-Diffusion' Particle Flux Relations

The particle continuity equation for ion species 'j' is

$$\nabla \cdot n_j \boldsymbol{\upsilon}_j = S_j \tag{1}$$

where $S_j(r,\theta) = n_e(r,\theta) n_{j0}(r,\theta) <\sigma\upsilon>_{ion} \equiv n_e(r,\theta)\nu_{ion}(r,\theta)$ is the ionization source rate of ion species 'j' and n_{j0} is the local concentration of neutrals of species 'j'. Taking the flux surface average of this equation yields $<(\nabla \cdot n_j \boldsymbol{\upsilon}_j)_r> \; = \; <S_j>$ because $<\nabla \cdot n_j \boldsymbol{\upsilon}_j)_\theta> \; = 0$ identically and $<\nabla \cdot n_j \boldsymbol{\upsilon}_j)_\phi> \; = 0$ by axisymmetry, which allows Eq. (1) to be written

$$\left(\nabla \cdot n_j \boldsymbol{\upsilon}_j\right)_\theta = S_j - \left\langle S_j \right\rangle \equiv \tilde{S}_j \tag{2}$$

Integration of this equation, in toroidal (r, θ, ϕ) coordinates, yields

$$n_j \upsilon_{\theta j} = \frac{K_j \bar{B}_\theta + r\bar{B}_\theta \int_o^\theta \left(1+\varepsilon cos\theta\right)\tilde{S}_j}{1+\varepsilon cos\theta} \equiv \left[K_j\left(r\right) + I_j\left(r,\theta\right)\right] B_\theta\left(r\right) \tag{3}$$

where $K_j = \left\langle n_j \upsilon_{\theta j} \right\rangle \big/ \bar{B}_\theta \approx \bar{n}_j \bar{\upsilon}_{\theta j} \big/ \bar{B}_\theta$ and the overbar denotes the average value over the flux surface.

Subtracting $m_j \upsilon_j$ times Eq. (1) from the momentum balance equation for ion species 'j' and noting that $\left(\nabla \cdot n_j \boldsymbol{\upsilon}_j\right)_r \; \square \; \left(\nabla \cdot n_j \boldsymbol{\upsilon}_j\right)_\theta$ leads to

$$n_j m_j \left(\boldsymbol{\upsilon}_j \cdot \nabla\right)\boldsymbol{\upsilon}_j + \nabla p_j + \nabla \cdot \boldsymbol{\pi}_j = n_j e_j \left(\boldsymbol{\upsilon}_j \times \boldsymbol{B}\right) + n_j e_j \boldsymbol{E} + \boldsymbol{F}_j + \boldsymbol{M}_j - n_j m_j \nu_{at}^j \upsilon_j - m_j \tilde{S}_j \boldsymbol{\upsilon}_j \tag{4}$$

where $\mathbf{F}_j$ represents the interspecies collisional friction, $\mathbf{M}_j$ represents the external momentum input rate, and the last two terms represent the momentum loss rate due to elastic scattering and charge exchange with neutrals of all ion species 'k'[$\nu_{atj} = \Sigma_k n^c_{k0}(<\sigma\upsilon>_{el} + <\sigma\upsilon>_{cx})_{jk}$] and due to the introduction of ions with no net momentum via ionization of a neutral of species 'j'. Only the 'cold' neutrals that have not already suffered an elastic scattering or charge-exchange collision in the pedestal are included in ν_{atj}.

Taking the cross product $\mathbf{B}\times$Eq. (4) yields a 'radial' ($\mathbf{n}_r$) component equation

$$\begin{aligned} n_j e_j \upsilon_{rj} B^2 = B_\theta &\left\{ n_j m_j \left[\left(\boldsymbol{\upsilon}_j \cdot \nabla\right)\boldsymbol{\upsilon}_j\right]_\phi + \left[\nabla \cdot \boldsymbol{\pi}\right]_\phi - M_{j\phi} - F_{j\phi} - n_j e_j E_\phi + m_j\left(n_j \nu_{atj} + \tilde{S}_j\right)\upsilon_{\phi j}\right\} - \\ &B_\phi \left\{ n_j m_j \left[\left(\boldsymbol{\upsilon}_j \cdot \nabla\right)\boldsymbol{\upsilon}_j\right]_\theta + \left[\nabla \cdot \boldsymbol{\pi}\right]_\theta + \frac{\partial p_j}{\partial \ell_\theta} - M_{j\theta} - F_{j\theta} - n_j e_j E_\theta + m_j\left(n_j \nu_{atj} + \tilde{S}_j\right)\upsilon_{\theta j}\right\} \end{aligned} \tag{5}$$

and a 'perpendicular' ($B_\phi \mathbf{n}_\theta$ - $B_\theta \mathbf{n}_\phi$) component equation

$$n_j m_j \left[\left(\boldsymbol{\upsilon}_j \cdot \nabla\right)\boldsymbol{\upsilon}_j\right]_r + \left[\nabla \cdot \boldsymbol{\pi}_j\right]_r + \frac{\partial p_j}{\partial r} = n_j e_j \left(\upsilon_{\theta j} B_\phi - \upsilon_{\phi j} B_\theta + E_r\right) \tag{6}$$

and taking the scalar product $\mathbf{B}\cdot$Eq. (4) yields a third, independent parallel momentum balance equation

$$\begin{aligned} &B_\phi \left\{ n_j m_j \left[\left(\boldsymbol{\upsilon}_j \cdot \nabla\right)\boldsymbol{\upsilon}_j\right]_\phi + \left[\nabla \cdot \boldsymbol{\pi}\right]_\phi - M_{j\phi} - F_{j\phi} - n_j e_j E_\phi + m_j\left(n_j \nu_{atj} + \tilde{S}_j\right)\upsilon_{\phi j}\right\} \\ &= -B_\theta \left\{ n_j m_j \left[\left(\boldsymbol{\upsilon}_j \cdot \nabla\right)\boldsymbol{\upsilon}_j\right]_\theta + \left[\nabla \cdot \boldsymbol{\pi}\right]_\theta + \frac{\partial p_j}{\partial \ell_\theta} - M_{j\theta} - F_{j\theta} - n_j e_j E_\theta + m_j\left(n_j \nu_{atj} + \tilde{S}_j\right)\upsilon_{\theta j}\right\} \end{aligned} \tag{7}$$

The quantity E_ϕ^A is the induced toroidal field due to transformer action.

Using Eq. (7) in Eq. (5), multiplying the result by R and taking the flux surface average leads to an expression for the flux surface average radial particle flux

$$\bar{R}\bar{B}_\theta e_j \left\langle n_j \upsilon_{rj} \right\rangle \simeq R B_\theta e_j \bar{n}_j \bar{\upsilon}_{rj} = \left\langle R^2 \nabla\phi \cdot n_j m_j \left(\boldsymbol{\upsilon}_j \cdot \nabla\right) \boldsymbol{\upsilon}_j \right\rangle + \left\langle R^2 \nabla\phi \cdot \nabla \cdot \boldsymbol{\pi}_j \right\rangle - \left\langle RM_{j\phi} \right\rangle - \left\langle RF_{j\phi} \right\rangle - \left\langle R n_j e_j E_\phi^A \right\rangle + \left\langle R m_j \left\langle n_j \nu_{atj} + \tilde{S}_j \right\rangle \upsilon_{\phi j} \right\rangle \tag{8}$$

The first (inertial) term on the right vanishes identically. The remaining terms on the right represent the transport fluxes in response to the toroidal viscous force, the (beam) momentum input, the interspecies collisional momentum exchange, the inductive toroidal electric field, and the momentum loss due to interactions with neutral particles, respectively.

Neglecting the viscous and inertial terms in Eq. (6), using Eq. (3), and assuming that the radial electric field is electrostatic leads to an expression for the flow velocity of ion species 'j' in the flux surface

$$n_j \boldsymbol{\upsilon}_j = \left(K_j + I_j\right) \boldsymbol{B} - \frac{1}{e_j B_\theta} \left(\frac{\partial p_j}{\partial r} + n_j e_j \frac{\partial \phi}{\partial r} \right) \boldsymbol{n}_\phi \tag{9}$$

Flux surface averaging this equation yields an expression for the average toroidal rotation over the flux surface in terms of the average poloidal rotation and radial gradients of the pressure and electrostatic potential

$$\bar{\upsilon}_{\phi j} = f_p^{-1} \bar{\upsilon}_{\theta j} - \left(\bar{P}_j' + \bar{\Phi}' \right) \tag{10}$$

where

$$f_p \equiv B_\theta / B_\phi, \quad \bar{P}_j' \equiv \frac{1}{\bar{n}_j e_j \bar{B}_\theta} \frac{\partial \bar{p}_j}{\partial r}, \quad \bar{\Phi}' \equiv \frac{1}{\bar{B}_\theta} \frac{\partial \phi}{\partial r} = -\frac{\bar{E}_r}{\bar{B}_\theta} \tag{11}$$

The particle fluxes within and across the flux surface are determined by Eqs. (9) and (8), respectively. In order to evaluate these fluxes it is necessary to specify the models for the viscosity and collisional friction, to know the constant K_j (equivalently the average value of the poloidal velocity), and to know the radial electric field.

Using the Lorentz approximation for the collisional friction

$$\boldsymbol{F}_j = -n_j m_j \sum_{k \neq j} \nu_{jk} \left(\boldsymbol{\upsilon}_j - \boldsymbol{\upsilon}_k \right) \tag{12}$$

Eqs. (8) may be reduced to

$$\Gamma_{rj} \equiv \bar{n}_j \bar{\upsilon}_{rj} = \frac{1}{e_j \bar{B}_\theta} \Big[-\left(\bar{M}_{\phi j} + \bar{n}_j e_j \bar{E}_\phi^A \right) + \bar{n}_j m_j \sum_{k \neq j} \bar{\nu}_{jk} \left(\bar{\upsilon}_{\phi j} - \bar{\upsilon}_{\phi k} \right) + \bar{n}_j m_j \bar{\nu}_{dj}^* \bar{\upsilon}_{\phi j} \Big] \tag{13}$$

where the total 'drag' frequency $\nu_{dj}*$ is given by

$$\bar{\nu}_{dj}^* \equiv \bar{\nu}_{dj} + \bar{\nu}_{atj} + \bar{\nu}_{ionj} \xi_j \tag{14}$$

which consists of a cross-field viscous momentum transport frequency formally given by

$$\bar{\nu}_{dj} \equiv \left\langle R^2 \nabla\phi \cdot \nabla \cdot \boldsymbol{\pi}_j \right\rangle \big/ \bar{R} \bar{n}_j m_j \bar{\upsilon}_{\phi j} \tag{15}$$

and of the two atomic physics momentum loss terms discussed previously, with the neutral ionization source asymmetry characterized by

$$\xi_j \equiv \left\langle R^2 \nabla \phi \cdot m_j \tilde{S}_j \upsilon_{\phi j} \right\rangle \Big/ \bar{R} m_j \bar{S}_j \bar{\upsilon}_{\phi j} \tag{16}$$

Writing

$$\bar{P}_j' = -\frac{T_j}{e_j B_\theta}\left(L_{nj}^{-1} + L_{Tj}^{-1}\right) \tag{17}$$

the 'perpendicular' component of the momentum balance given by Eq. (10) can be used to eliminate the toroidal velocity in the 'radial' component given by Eq. (13) to obtain a generalized pinch-diffusion equation for each ion species present

$$\Gamma_j = n_j D_{jj}\left(L_{nj}^{-1} + L_{Tj}^{-1}\right) - n_j D_{jk}\left(L_{nk}^{-1} + L_{Tk}^{-1}\right) + n_j \upsilon_{pj} \tag{18}$$

where the diffusion coefficients are given by

$$D_{jj} \equiv \frac{m_j T_j \left(\nu_{dj}^* + \nu_{jj}\right)}{\left(e_j B_\theta\right)^2} \quad , \quad D_{jk} \equiv \frac{m_j T_k \nu_{jk}}{e_j e_k B_\theta^{\;2}} \tag{19}$$

the pinch velocity is given by

$$n_j \upsilon_{pj} \equiv -\frac{\bar{M}_{\phi j}}{e_j B_\theta} - \frac{n_j \bar{E}_\phi^A}{B_\theta} + \frac{n_j m_j \nu_{dj}^*}{e_j B_\theta}\left(\frac{E_r}{B_\theta}\right) + \frac{n_j m_j f_p^{-1}}{e_j B_\theta}\left(\left(\nu_{jk} + \nu_{dj}^*\right)\bar{\upsilon}_{\theta j} - \nu_{jk}\bar{\upsilon}_{\theta k}\right) \tag{20}$$

and where a sum over the 'k' terms is understood when more than two ion species are present. Note that the 'self-diffusion' coefficient D_{jj} involves the atomic physics and viscous momentum transfer rates as well as the interspecies collisional momentum exchange frequency. The v_θ can be obtained by solving the poloidal momentum balance equations[10] (the left side of Eqs. (7) set equal to zero) numerically.

3. Density Gradient Scale Length

Since the temperature gradient scale lengths for each ion species in principle can be determined from the heat conduction relations $q_j = n_j T_j \chi_j L_{Tj}^{-1}$, the set of Eqs. (18) can be recast as a coupled set of inhomogeneous equations that can be solved for density gradient scale lengths in terms of these temperature gradient scale lengths, the local particle fluxes, Γ_j, the terms appearing in the generalized pinch velocities given by Eq. (20) and the diffusion coefficients given by Eqs. (18)

$$D_{jj} L_{nj}^{-1} - D_{jk} L_{nk}^{-1} = \frac{\Gamma_j}{n_j} - \upsilon_{pj} - D_{jj} L_{Tj}^{-1} + D_{jk} L_{Tk}^{-1} \equiv \alpha_j \tag{21}$$

There are as many coupled Eqs. (21) as there are ion species present, and again the 'k' terms are understood to represent sums over species $k \neq j$.

For a two-species (ion-i, impurity-I) model these equations can be solved explicitly for the ion density gradient scale length

$$L_{ni}^{-1} = \frac{\left(\dfrac{m_I T_I \nu_{Ii}}{\left(e_I B_\theta\right)^2}\right)\left(1+\dfrac{\nu_{dI}^*}{\nu_{Ii}}\right)\alpha_i + \left(\dfrac{m_i T_i \nu_{iI}}{e_i e_I B_\theta^2}\right)\alpha_I}{\left(\dfrac{m_I T_I \nu_{Ii}}{\left(e_I B_\theta\right)^2}\right)\left(\dfrac{m_i T_i \nu_{iI}}{\left(e_i B_\theta\right)^2}\right)\left[\left(1+\dfrac{\nu_{dI}^*}{\nu_{Ii}}\right)\left(1+\dfrac{\nu_{di}^*}{\nu_{iI}}\right)-1\right]} \tag{22}$$

and a similar expression for the impurity density gradient scale length with the 'i' and 'I' interchanged.

In order to more clearly display the physics involved, we use Eqs. (17) and (10) to eliminate the toroidal velocity only in the last term in Eq. (13), leading to an expression for the density gradient scale length of ion species 'j'

$$\begin{aligned} L_{nj}^{-1} = \frac{e_j B_\theta}{n_j m_j \nu_{dj}^* T_j}\Big[& e_j B_\theta \Gamma_j + M_{\phi j} + n_j e_j E_\phi^A - n_j m_j \nu_{jk}\left(\upsilon_{\phi j} - \upsilon_{\phi k}\right) \\ & - n_j m_j \nu_{dj}^*\left(f_p^{-1}\upsilon_{\theta j} + \frac{Er}{B_\theta}\right)\Big] - L_{Tj}^{-1} \end{aligned} \tag{23}$$

The coupling among species has not disappeared; it is present in the friction term, which may be small, and in the poloidal rotation velocity and radial electric field dependence on all species. The density gradient scale length not only depends on the particle flux, as would be intuitively expected, but also on the poloidal rotation velocity and the radial electric field, which latter in turn depends on the toroidal and poloidal rotation velocities because of toroidal momentum balance. Note that these expressions result from momentum balance and are independent of the mechanisms causing the particle transport (e.g. of whether local or non-local phenomena are involved).

We have applied Eq. (23) to predict the average density gradient scale length in several DIII-D shots[11] covering a wide range of edge parameters. The particle flux was calculated from particle balance, the beam momentum input was calculated directly, E_φ^A, L_{Tj}, n_j and T_j were taken from experiment, E_r was calculated from toroidal momentum balance using the experimental pressure gradient[11], ν_{dj} was calculated from neoclassical gyroviscous theory[12], $\upsilon_{\theta j}$ and the density asymmetries needed to evaluate ν_{dj} were calculated from poloidal momentum balance[10], and ν_{atj} and ν_{ionj} were calculated using a 2D neutral recycling code[13]. The friction term involving differences in main ion and carbon υ_φ was neglected. The density gradient scale lengths calculated by this procedure are compared with the values directly measured by Thomson scattering in Table 1. Although there is some roll-over in the experimental data at the separatrix and the top of the pedestal, extraction of an average value of the density gradient scale length is relatively unambiguous, and Eq. (22) corresponds to the theoretical average value.

Table 1: Calculated and Measured Density Gradient Scale Lengths (cm) in the Pedestal of DIII-D H-Mode Shots[11]

Shot	93045	87085	97979	106005	106012	92976	98893
Exp. L_n	2.8	4.3	3.3	2.7	2.4	6.0	1.5
Calc. L_n	2.7	3.3	2.4	1.9	1.8	3.3	0.8

4. Summary

We have derived from particle continuity and momentum balance a fundamental relationship among the radial particle flux and gradients in the density and temperature for each ion species present in the edge pedestal. This expression may be cast in the form of a generalized 'diffusion-pinch' relation. This relation has been used to construct an expression for the calculation of ion density gradient scale lengths in the edge pedestal, which are found to be in good agreement with experiment for a representative set of DIII-D H-mode shots. When combined with the previously presented[7-9] expressions for the temperature gradient scale lengths based on heat conduction, $L_{Tj}^{-1} = q_j/n_jT_j\chi_j$, these new expressions provide a first-principles calculation of gradient scale lengths in the edge pedestal from edge transport constraints involving particle fluxes, atomic physics and local transport coefficients.

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