HYDROMECHANICAL IMPACT BREAKER

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HYDROMECHANICAL IMPACT BREAKER





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NOMENCLATURE

A _o	= orifice area of valve	[in ²]
۹ م	= effective piston area	[in ²]
в	= viscous damping coefficient	[lb-sec/in]
BPM	= blows per minute	[] ⁻¹
С	= conversion factor	[] ⁻¹
°ce	= contraction coefficient	۲ ^۲ (۲
ca	= discharge coefficient	[] ¹
° _{ep}	= external leakage coefficient	[in ³ /sec/psi]
c_{ip}	= internal leakage coefficient	[in ³ /sec/psi]
с _н	= hydraulic conductance	[in ⁴ //lb sec]
c _v	= velocity coefficient	[] ⁻¹ ,
D	= piston diameter	[in]
E	= energy per impact	[ft-lb _f]
FL	= external load	[1b _f]
$\mathbf{H}^{\mathbf{L}}$	= head losses	[lb _f /lb.in]
к	= spring constant	[lb/in]
ĸ	= valve flow pressure	$[in^3/sec/in]$
к _q	= valve flow gain	[in ³ /sec/in]
Kp	= losses coefficient	[] ⁻¹
K s	= losses coefficient	[] ⁻¹
\mathbf{L}	= total hose length	[ft]
г ^о	= piston length	[in]
MT	= total mass piston	[lb.in]

x _v	= valve displacement	[in]
∆P	= pressure drop	[1b/in ²]
β	= effective bulk modulus	[1b/in ²]
ψ	= viscous strain	[sec] ⁻¹
ø	= hose diameter	[in]
P	= mass density	[lb/sec ² /in ⁴]
٩	= weight density	[lb.in/in ³]
μ	= absolute viscosity	[lb-sec/in ²]
e	= excentricity	[in]
δ	= damping ratio	[] ⁻¹
ω	= hydraulic natural frequency	[rad/sec]
τ	= shearing stress	[1b/in ²]

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N	= describing function notation	-
\mathtt{P}_{L}	= pressure load	[lb/in ²]
P _S	= pressure supply	[1b/in ²]
ନ୍	= volumetric flow rate	[in ³ /sec]
v ₁	= volume forward chamber	[in ³]
v ₂	= volume return chamber	[in ³]
vo	= initial volume	[in ³]
v ₇	= total volume	[in ³]
a	= amplitude	
c	= circumference	[in]
e _v	= velocity of sound in fluid	[in/sec]
đ	= piston shaft diameter	[in]
đ _v	= valve sleeve diameter	[in]
е	= exponential	
f	= friction factor	[] ⁻¹
g	= acceleration of gravity	[in/sec ²]
h	= radial clearance	[in]
i	= imaginary operator	
P	= pressure strain	[lb/in ²]
r_{e}	= radial clearance between spool and sleeve	[in]
ន	= Laplace Transform operator	
ī	= mean velocity in hose	[in/sec]
vp	= instantaneous velocity	[in/sec]
$\mathbf{v}_{\mathbf{x}}$	= fluid velocity	[in/sec]
W	= value area gradient	[in ² /in]
х _р	= piston displacement	[in]

х

SUMMARY

Presently available pneumatic jackhammers do not meet the long range Federal Noise Level Standards for 1974. A definite need exists for a long-term solution to decrease the noise pollution by such a device. This study examines a closed hydromechanical system to alleviate the problem.

A dynamic analysis of the hydromechanical components is performed with linear approximations adopted for most parts. The resulting system is examined with special attention to the working specifications of existing pneumatic jackhammer systems.

The non-linearity in the system due to the switching effect of the valve is included in the dynamic analysis. Stability aspects of the non-linear behavior are examined by the Describing Function method. Computer simulations of the system are performed. It has been shown that the proposed system will operate; but great care must be kept in assigning values to design parameters in order to assure stability; also fluid momentum effects are present, due to the switching motion of the piston. A continued investigation in this latter point was recommended, in order to measure the degree of pressure surge due to stoppage of fluid, and establish its direct effects on the system performance.

CHAPTER I

1

INTRODUCTION

1.1 The Problem of Noise Reduction

It has been established that noise and vibration does affect the population in a number of ways and it may, if sufficiently loud or intense, damage hearing or health; interfere with work tasks and adversely affect speech communication. It also can affect interroom privacy, course general annoyance and interrupt sleep.

Noise reduction can be achieved by controlling its source, its path of propagation, or by the combination of the two.

The first method is the most effective measure for products still in the design stage and also the least expensive in the long run. The second is a corrective measure for an existing noise control problem utilizing changes in the path of sound propagation by diverting or absorbing sound. The third method is employed when a complete redesign of the existing product is too expensive to accommodate. The alternative is to make minor inexpensive changes or additions to the actual design in conjunction with manipulation of the sound propagation path. Important considerations in adopting the specific scheme for noise reduction are the demands of the receiver, which are subject to individuals, and their reasons for reducing noise in a particular environment.

In the case of the pneumatic jack hammer, it is at best questionable if noise levels emanating from this tool used presently, does comply with the current government regulations. Readings taken on typical jackhammers used for road construction in the metropolitan Atlanta* area and data from other documents [13] show that the exhaust sound level does exceed government standards and although public opinion would call for its silencing, but as the jackhammer is not a consumer product, companies could hold back on redesigning expenses. Currently available jackhammer designs have been modified both from the point of view of controlling the amount of noise propagation as well as some control at the source level via modified designs. This study concentrates on a more fundamental change at the source of the noise generation from such a tool.

1.2 A Review of Previous Work

There does not exist a large volume of information on a valvepiston combination used as an impact device. In fact, an extensive literature search revealed no significant analytical or experimental work on the subject matter. On the other hand not much can be added to the vast literature available on servovalves in particular and hydraulic fluid power control technology, in general. So a short account of what has been done to reduce noise caused by a conventional pneumatic jackhammer will perhaps be of help in establishing the need for this work.

In the late 1950's, people were already concerned about noise pollution by such devices as pneumatic jackhammers; this, as mentioned before, not being a consumer product, little attention was given to the See Appendix A.

problem. Pneumatic contractor's tools were mentioned by the Wilson Committee [7] noise report in 1963. Later on the Building Research station of the Ministry of Technology of the United Kingdom concluded that noise by these devices was unnecessary. It was thought then that this would entice legislation restricting the use of unsilenced breaker and other similar annoying pneumatic tools. A report carried out by the Building Research Station [7] shows a comparative analysis of noise, work, output and performance of unsilenced and muted breakers. Currently in effect, noise is restricted as set forth by the U.S. Government Occupational Safety and Health Act of 1970. It is a foregone conclusion that regulations against noise pollution will get progressively stiffer and, consequently, permissible noise levels will be lowered, whereby manufacturers will have to comply with new regulations.

The Environmental Protection Agency is at present preparing a report regulating every apparatus, vehicle and device on the market as well as those already in use. This report will be concluded soon and is expected to be enforced by the latter part of 1974.

The Building Research Station of the Ministry of Technology performed their tests employing four different techniques to bring down the noise level in current pneumatic concrete breakers. These are:

1. Use of strap-on mufflers,

 Redesigning of the pneumatic jackhammer body to integrate muffling,

3. Diverting exhaust through existing compressor mufflers, and

4. Putting an acoustic screen around the breaker.

The strap-on muffler has been applied by several manufacturers, Ingersol-Rand being one of them. Several makes of mufflers are currently available. A typical design is a light; durable, inexpensive (average cost \$30.00) device that fits over the body of the tool. Excessive loss of working efficiency can result from a badly designed muffler. For example, the base of the muffler has to be designed to release the exhaust air, but inevitably the muffler sets up some degree of back pressure and it is thus which is more than likely to impair performance. Typical noise reduction with the use of a strap on muffler is moderate at seven decibels, and power loss is approximately between 20 percent and 45 percent. It seems that any further government restrictions would necessitate a complete redesigning of the pneumatic tool at a considerable expense.

The integrated muffler design tried out by several manufacturers achieves a sound reduction of about 25 decibels, but is more expensive (average 25 percent more over total cost) and power losses fluctuate between 30 percent and 40 percent.

Diverting the exhaust air back to the compressor muffler has an advantage in that it uses a larger muffler, but also at considerable power loss.

In other words, muffling has brought down sound levels. In general terms, the best mufflers can offer a 50 percent total sound reduction but provide only an adequate short-range solution to the problem.

The fourth and last method employed for noise reduction is the acoustic screen around the tool and compressor. Clearly this detracts in no way the efficiency of the breaker. The only drawback of this method, (apart from the cost of the screen material which includes a three sided light weight structure, with two inch thick internal lining of polyurethane foam) is that it has to be portable to follow the job progress. The combined effect of the muffler and enclosure can reduce the sound level by as much as 20 decibels, or 75 percent total noise reduction with the corresponding trade-off in power loss due to silencer. There is also the inconvenience of moving the enclosure.

Back pressure created by the muffler causes extra loading on the compressor for a specific work output which increases compressor noise levels and which have been established to be as significant as 20 percent over normal use.

The actual breaking of concrete contributes approximately 0.1 percent* to the total noise created by the tool and compressor. It is thus quite important to consider a complete redesign of the jackhammer.

Ingersol Rand has at present on the market their new "Hobgoblin" impulse breaker which mounts on a back-hoe or fork lift. This blockbuster employs hydraulic fluid to move the hammer and piston of the tool which in turn compresses nitrogen gas at a high pressure. At a pre-set hydraulic pressure the energy stored in the compressed gas is

*See Appendix A, Figure A.1.

released causing the hammer to strike the tool. The work-blow delivered by the tool can be varied between 135 (ft-1b) to 1200 (ft-1b).

1.3 Statement of the Problem

The object of the present study is to examine a closed hydromechanical system employing a hydraulic pump to supply energy to an ' impact breaker. It is proposed to incorporate the conventional combination of a servo value and piston respectively employed as the oscillating device and hydraulic power element.

Hydraulic losses due to friction, valving and leakage from the pump to the power piston must be determined in order to evaluate their influence on the overall design. The basic approach will be to assume a constant supply pressure and to develop governing equations for each component. Solutions of such equations will render the dynamic characteristics of the system. Also pressure-flow curves will be obtained. Due to the mathematical complexity of the problem, it is anticipated that linearization must be adopted for most parts.

The two most important characteristics of employing hydraulic fluid as a measure of energy transmission: "Stiffness and pressure" will be exploited to best advantage.

Temperature rises occurring due to friction and shearing upon the oil can be as high as 120° to 180° F. for large velocities. Since one of the characteristics of a hydraulic fluid power system is to dissipate excessive heat from the power source by the fluid stream itself; inclusion of an adequate heat exchanger in the sump or in another appropriate location keeps the oil at a reasonable working

temperature. Hence temperature rises will not be considered in this work.

Finally, stability which probably is the most important performance characteristic of a servo system, will be analysed by comparing various methods for determining absolute stability of a system. The non-linearity present in the system due to the switching effect of the valve will be examined by the Describing Function technique and by graphical methods.

CHAPTER II

SYSTEM CONFIGURATION

2.1 Basic Considerations

Conventional demolition tools, such as concrete breaker utilize compressed air to displace a piston, which acts as a hammer, to strike a tool or chisel with a high impact energy. This impact is transmitted through the tool to the concrete. It has been established by manufacturers that the energy per impact for small portable type pneumatic concrete breakers is between 100 - 200 Ft-lb $[1,2,1^4]$. The total energy output can be varied by controlling the frequency of impact.

This analysis adopts some of the working specifications such as blows per minute and power output per stroke of the existing pneumatic system. However, the physical parameters, specifically; piston weight, piston stroke and cylinder bore of the power element, need to be modified to suit hydraulic system design criteria. In a pneumatic system part of the stroke is used to compress the air, whereas in a hydraulic system the entire piston stroke goes into moving the piston. As a result the hydraulic system needs a considerably shorter piston stroke to attain a specified maximum velocity for the piston. This can be further supported by using the continuity equation to write expressions of velocity of the piston in the hydraulic and pneumatic case. These are:

Liquid:
$$V_p = \frac{dx}{dt} + X \frac{\rho_i}{\beta} \frac{dP}{dt}$$
, (2.1.1)

and

Gas: $V_p = \frac{dxp}{dt} + \frac{x_p^{\rho}}{P} \frac{dP}{dt}$, (2.1.2)

where:

Vp	=	velocity of piston,
X p	=	displacement of piston,
ρ	=	fluid density,
ρ ₁	=	fluid density at zero pressure,
β	=	bulk modulus of elasticity, and,
Р	=	fluid pressure

It may be noted that the isothermal bulk modulus of elasticity of a perfect gas equals its pressure, whereas for liquids it is independent of pressure. Therefore piston velocities as given by Equation 2.1.1 are reached at a much faster rate than the corresponding values from Equation 2.12. The gas behaves like a nonlinear spring compared to the linear spring characteristic of a liquid. This can also be seen in Figure 1. Appendix B contains details of development of the equations for piston velocities.

Inherently, the shorter stroke considered in this analysis cuts down the volumetric flow rate. This is of considerable importance since it directly affects the choice of the hydraulic pumps? which provides the power element with the necessary fluid energy.

So far as the power element is concerned, several configurations





_ _ _ _ _ _ _ _

are possible. Basically, it consists of a main body which encloses the cylinder where the piston is displaced up and down, striking the tool when the piston reaches its lowest position. It is necessary to have a controlling element to switch the fluid flow from one end of the cylinder to the other to produce the oscillatory piston motion. Several schemes of achieving this need to be considered to find the most appropriate one.

2.2 Possible Configurations

A qualitative as well as a quantitative analysis must be performed for a number of possible schemes as candidate configurations to achieve oscillatory piston motion. This leads to a criterion to select the appropriate method compatible with the specifications desired per impact. Special attention must be paid to the kinetic energy of impact with a minimum of power consumption and adequate response time.

The primary difference between the basic configurations shown in Figure 2, is that in Figure 2.1 the piston is displaced down by the supply pressure P_s and is returned by a spring force, whereas in Figures 2.2 and 2.3 the piston is driven by the supply pressure in both directions. The energy stored in the spring at the end of the full forward Stroke of configuration 2.1 would have to meet the specifications at impact.

The following mathematical analysis will consist of obtaining the equations of motion for the total cycle for each of configurations 2.1, 2.2, and 2.3. It will further consist of finding suitable



Figure 2. Possible Configurations.

expressions which relate these dynamic equations to the total kinetic energy delivered at impact. Next, a power consumption equation, together with expressions for potential and kinetic energies, all in terms of a nondimensionalized time, will be obtained. Through the aid of adequate substitutions these expressions will be combined to form a canonical relation describing the response of the system. Naturally such a relation will be in terms of the potential energy the nondimensional time and the work done during the cycle. Work being the function to be minimized, the Hamiltonian principle of dynamics can be applied, and a nondimensionalized plot can be constructed containing the minimum nondimensional work values corresponding to different values of the nondimensionalized time. Such a plot will yield a criteria for minimum work consumption per impact, to be used toward optimization of candidate configurations.

It may be noted that as far as power consumption is concerned, configurations 2.2 and 2.3 are identical. The only difference between configurations 2.2 and 2.3 concern the switching time, which will be examined later:

2.2.1 Power Consumption Analysis for Configuration of Figure 2.1.

The preliminary analysis consists of obtaining a simplified forward and return dynamic equations for configuration 2.1.

For small piston motion, i.e., with $|A_pX_p| \ll V_o$, where V_o is the volume of forward chamber, A_p the piston cross sectional area and x_p the piston displacement; and for negligible leakage, the load flow rate may be written as

$$q_{\rm L} = A_{\rm p} x_{\rm p} + \frac{V_{\rm o}}{\beta} \dot{p}_{\rm L} , \qquad (2.1)$$

where

 $q_L \simeq load flow$

 $\dot{\mathbf{p}}_{L}$ = load pressure; the dot indicates the time rate of change

In Laplace transformed notation thus becomes

$$Q_{\rm L} = A_{\rm p} \, s \, X_{\rm p} + \frac{V_{\rm o}}{\beta} \, s \, P_{\rm L} \quad , \qquad (2.2)$$

where s, is the Laplace operator.

For a spring-mass system, the simplified equation of piston motion is

$$A_{p}P_{L} = Ms^{2}X_{p} + \frac{P_{s}}{P_{s}}A_{p}^{2}sX + kX_{p}$$
, (2.3)

where,

M = total piston mass, $P_s = \text{supply pressure,}$ $Q_s = \text{supply flow, and}$ K = spring constant.

If the pressure output is assumed linear and if line dynamics as well as delay due to length of line are neglected, then

$$\mathbf{p}_{\mathrm{L}} = \mathbf{p}_{\mathrm{s}} - \frac{\mathbf{p}_{\mathrm{s}}}{\mathbf{q}_{\mathrm{s}}} \mathbf{q}_{\mathrm{L}} \quad . \tag{2.4}$$

It has been shown in Appendix B that pressure losses due to friction and changes in cross sectional area of lines in this systems are very small and added to the fact that the length of the lines ()) connecting the value and ram are very short, (three to five inches) it is justifiable to use Equation 2.4.

Combination of Equations 2.2, 2.3 and 2.4 gives the forward stroke dynamic equation of the system as

$$Ms^{2}X_{p} + \frac{P_{s}}{Q_{s}} \left(A_{p}^{2}X_{p} + \frac{A_{p}}{\beta} V_{o}P_{L} \right) s + KX_{p} = A_{p}P_{s}$$
(2.5)

For very large bulk modulus, β , of the fluid, $A \underset{p \ o}{V}/\beta \approx 0$, which simplifies Equation 2.5 to

$$X_{p} = \frac{A_{p}P_{s}}{s(M_{s}^{2} + \frac{P_{s}}{Q_{s}}A_{p}^{2}s + K)}$$
(2.6)

At the instant of impact of the piston with the tool, i.e. at t = T, $x_p = X_p$, the position of the piston is fed back either mechanically or by electrical impulses to the actuating servovalve which in turn connects the drain port to the cylinder chamber, which releases the pressure on the piston and returns it to its original position by the energy stored in the compressed spring.

The return motion is described by

$$Ms^{2}X_{p} + KX_{p} = 0$$
, $X(T) = 0$

(2.7)

Since the duration of the impact is very short it is reasonable to assume an inelastic impact.

The constraint requiring a specified energy of impact can be described relating the potential energy an kinetic energy at impact to the dynamic equations of the forward and return motion of the piston.

The kinetic energy $K\!\underline{E}_{\underline{\mu}}$ and the potential energy PE at t = T are respectively given by

$$KE = \frac{1}{2} MV_p^2$$
 , (2.8)

$$PE = \frac{1}{2} K x_p^2 .$$
 (2.9)

From Equation 2.8

$$\sqrt{2KE} = \sqrt{MX_{p}} = z(t)$$
, (2.10)

where z(t) is a function of time.

Equation 2.10 in Laplace transformed notation is

$$x_{p} = \frac{Z(s)}{s\sqrt{M}}$$
(2.11)

Substitution of Equation 2.11 into Equation 2.6 yields

$$Z(s) = \frac{A_{p}P_{s}}{Ms^{2} + \frac{P_{s}}{Q_{s}}A_{p}^{2}s + K}, \qquad (2.12)$$

The inverse Laplace Transform of Equation 2.12 evaluated at t = T, for the case of complex roots gives [See Appendix D for details]

$$z(\mathbf{T}) = \frac{A_{\mathbf{p}}^{\mathbf{p}} \mathbf{s}^{\mathbf{e}}}{\sqrt{\frac{\mathbf{p}}{M}} \sqrt{\frac{\mathbf{k}}{\mathbf{M}} - \left(\frac{\mathbf{p}}{2\mathbf{Q}_{\mathbf{S}}^{\mathbf{M}}}\right)^{\mathbf{T}}}} \sqrt{\frac{\mathbf{k}}{M} - \left(\frac{\mathbf{p}}{2\mathbf{Q}_{\mathbf{S}}^{\mathbf{M}}}\right)^{\mathbf{T}}} \mathbf{T} , \qquad (2.13)$$

where T is the time for the forward stroke. Equation 2.12 can also be evaluated at t = T for the case of real and equal roots, by first noting that critical damping occurs when

$$\frac{{}^{P} {}^{A} {}^{2}}{{}^{Q} {}^{M}_{S}} = 2 \sqrt{\frac{K}{M}} . \qquad (2.14)$$

Use of Equation 2.14 into 2.12 yields

$$Z(s) = \frac{\frac{A_{p}P_{s}}{\sqrt{M}}}{s^{2} + 2\sqrt{\frac{K}{M}} s + \frac{K}{M}}$$
(2.15)

Inverse transform of Equation 2.15 yields at t = T, for critical damping

$$z(T) = \frac{\frac{P_{A}T}{s}}{\sqrt{M}} \cdot e^{-\sqrt{\frac{K}{M}}T}$$
(2.16)

For the case of real and unequal roots of Equation 2.12, the

following conditions must hold

$$\begin{bmatrix} \mathbf{P} \mathbf{A}^2 & \mathbf{P} \\ \mathbf{S} & \mathbf{p} \\ \mathbf{Q} & \mathbf{M} \end{bmatrix} > \frac{\mathbf{4}K}{\mathbf{M}} \quad . \tag{2.17}$$

The Inverse Laplace Transform of Equation 2.12 with condition 2.17 gives the overdamped response evaluated at t = T, as

$$z(T) = P_{s}^{A} e^{-\left(\frac{P_{s}^{A}^{2}}{Q_{s}^{M}}\right)T} \frac{\sqrt{\frac{K}{M} - \frac{P_{s}^{A}^{2}}{Q_{s}^{M}}} T \qquad (2.18)$$

Equations 2.13, 2.16 and 2.18 are piston responses for the forward stroke evaluated at t = T. These equations can be examined more conveniently in terms of a total response time, or can be expressed in terms of a nondimensionalized time, τ which is the ratio of the total cycle response time T_T to the forward stroke response time, T. This simplification can be achieved by several straight forward substitu-

tions as outlined in the procedure that follows:

$$T_{\rm c} = T_{\rm m} - T ,$$

where

Define

 $T_S = return stroke time,$ $T_T = total time per cycle, and$ T = forward stroke time.

Obtain the solution to the return stroke equation as

$$X_{p}(t_{s}) = X_{p}(T) \cos \sqrt{\frac{K}{M}} t_{s}$$
, (2.20)

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(2.19)

where ts is the instantaneous return stroke time, $T < t_{_{\rm S}} < T_{_{\rm T}}.$

Let $t_s = t - T$ and evaluate Equation 2.20 at $t = T_T$ to give

$$X_{p}(T_{T}-T) = X_{p}(T) \cos \sqrt{\frac{K}{M}} (T_{T}-T)$$
 (2.21)

Note that $t = T_T$, $X_p(T_T) = 0$. Equation 2.21 will admit this only when

$$\frac{\pi}{2} = \sqrt{\frac{\tilde{K}}{M}} \left(T_{\rm T} - T \right) \qquad (2.22)$$

Equation 2.22 can be expressed as

$$\tau = \frac{\pi}{2T} \sqrt{\frac{M}{K}} + 1$$
, (2.23)

where

$$\mathbf{T} = \mathbf{T}_{\mathbf{m}} \quad . \tag{2.24}$$

Before Equations 2.13, 2.16 and 2.18 can be expressed in terms of the dimensionless time, τ , it is necessary to express the cycle work in terms of τ , since it is desired to minimize the energy consumption at the same time delivering a specified energy per impact.

The power supplied to the piston is

$$HP = P_{s} Q_{s} k , \qquad (2.25)$$

where k is a conversion factor to express lb-in/sec in horsepower, HP. For one complete cycle the work done, W, is

$$W = \hat{P}_{s} Q_{s} T_{T}$$
 (2.26)

Use of Equation 2.24 into Equation 2.26 yields

$$W = P_{S} Q_{S} T_{T}$$
 (2.27)

At this point it needs to be pointed out that Equation 2.27 must be optimized subject to a fixed energy described by Equation 2.8, as well as the response dynamics of the forward stroke given by Equation 2.12 and that of the return stroke given by a suitably modified form of Equation 2.20.

To perform this optimization the forward stroke response, Equations 2.13, 2.16 must be manipulated to incorporate the cycle work, W, and the dimensionless time, τ . This is attained through the use of Equation 2.27 in the response equation, resulting in the expressions

$$z(\tau) = \frac{A_{p}P_{s}e}{\sqrt{M}} \frac{\left(\frac{P_{s}A_{p}^{2}}{2Q_{s}M}\right) \frac{W}{P_{s}Q_{s}\tau}}{\sqrt{\frac{K}{M}} - \left(\frac{P_{s}A_{p}^{2}}{2Q_{s}M}\right)} \sin \sqrt{\frac{K}{M}} - \left(\frac{P_{s}A_{p}^{2}}{2Q_{s}M}\right)} \frac{W}{P_{s}A_{s}\tau} \qquad (2.28)$$

for the underdamped case,

$$z(\tau) = \frac{P_{s}A_{p}W e}{\sqrt{M}P_{s}Q_{s}\tau}$$
(2.29)

for the critically damped case, and

$$z(\tau) = \frac{\frac{P_{s}A^{2}}{Q_{s}M}}{\sqrt{\frac{K}{M} - \left(\frac{P_{s}A^{2}}{Q_{s}M}\right)}} \frac{W}{\frac{P_{s}Q_{s}\tau}{s_{s}r}} \sinh \sqrt{\frac{K}{M} - \left(\frac{P_{s}A^{2}}{Q_{s}M}\right)} \frac{W}{\frac{P_{s}Q_{s}\tau}{Q_{s}M}} (2.30)$$

for the overdamped case.

These expressions for the response must be further manipulated to incorporate the potential energy, PE, and the kinetic energy, KE. This is attained by substituting suitable modified combination of Equations 2.8, 2.9, 2.23 and 2.27 into the Response Equations. Development of such a combination proceeds as follows:

The force upon a spring is given by

$$F = X_{p}K$$
 . (2.31)

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This same force, F, in terms of pressure and area yields

$$P_{s p} = X_{p} K$$
 (2.32)

By substituting X from Equation 2.32 into Equation 2.9, an p expression for potential energy is obtained as

$$2PE = \frac{P^2 A^2}{K} .$$
 (2.33)

The kinetic energy given by Equation 2.8 can be written introducing the substitutions,

$$\dot{x}_{p} = \frac{Q_{s}}{A_{p}}, \qquad (2.34)$$

which combined into Equation 2.8 results in

$$2KE = \frac{MQ_s^2}{A_p^2} .$$

Equations 2.23, 2.27, 2.33 and 2.35 can be combined to give

$$KE = \frac{2W^2(\tau-1)^2}{\tau^2 PE \tau^2} .$$
 (2.36)

Equation 2.36 is a very useful relation in that it shows the functional relationship between kinetic and potential energies, the work done per cycle and the ratio of total time for one cycle to its forward stroke time. Substitution of Equation 2.36 into Equation 2.28 gives for the underdamped case

$$z(\tau) = \frac{(\underline{PE})^{\frac{1}{2}} e^{-(\frac{\tau^{2} - \underline{PE} - \tau}{\underline{\mu}W(\tau-1)^{2}})}}{(1 - \frac{(\underline{PE})^{2}\pi^{2}\tau^{2}}{8W^{2}(\tau-1)^{2}})^{\frac{1}{2}}} \sin \left[\frac{\tau}{2(\tau-1)} \left(1 - \frac{\pi^{2}(\underline{PE})^{2}\tau^{2}}{8W^{2}(\tau-1)^{2}}\right]\right] (2.37)$$

Equation 2.37 is a function of three variables W, PE and τ , so 2.37 can be rewritten as follows

$$F(W, PE, \tau) = \frac{(PE)^{\frac{1}{2}}}{S} e^{-N} \sin \frac{\pi S}{2(\tau-1)} - z(\tau) , \qquad (2.38)$$

where

$$N = \frac{\pi^2 PE \tau}{4W(\tau-1)^2} , \qquad (2.39)$$

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(2.35)

and

$$S = \left(1 - \frac{\pi^2 (PE)^2 r^2}{8W^2 (\tau - 1)^2}\right)^{\frac{1}{2}}.$$
 (2.40)

Similar analysis can be performed on Equations 2.29 and 2.30 for the critically damped and overdamped cases respectively.

For the critically damped case this gives

$$\mathbf{F}(\mathbf{W}, \mathbf{PE}, \mathbf{\tau}) = \frac{2\mathbf{W}}{(\mathbf{PE})^{\frac{1}{2}}\mathbf{\tau}} \mathbf{e} - \mathbf{z}(\mathbf{\tau}) , \qquad (2.41)$$

where the relation $(PE)^{\frac{1}{2}} = 2(KE)^{\frac{1}{2}}$ for relating potential and kinetic energies at critical damping has been utilized.

For the overdamped case

$$F(W, PE, \tau) = \frac{(PE)^{\frac{1}{2}}}{H} e^{-N} \sinh \left[\frac{\pi H}{2(\tau-1)}\right] - z(\tau) , \qquad (2.42)$$

where

$$H = \left(\frac{\pi^2 (PE)^2 \tau^2}{8W^2 (\tau^{-1})^2} - 1\right)^{\frac{1}{2}} . \qquad (2.43)$$

Equations 2.38, 2.41 and 2.42 represent expressions for work and potential energy and it can be seen from these equations, that W is a function of PE for specific values of the nondimensional time τ . To find the minimum for the function W for a specific value of KE, the criteria

$$\frac{\mathrm{d}W}{\mathrm{d}PE} = -\frac{\mathrm{\partial}F/\mathrm{\partial}PE}{\mathrm{\partial}F/\mathrm{\partial}W} = 0 , \qquad (2.44)$$

must be applied to Equations 2.38, 2.41 and 2.42. In other words, for minimum work.

$$\frac{\mathrm{dW}}{\mathrm{dPE}} = 0 \ , \ \mathrm{or} \tag{2.45}$$

$$\frac{\partial \mathbf{F}}{\partial \mathbf{PE}} = 0 , \qquad (2.46)$$

and

$$\frac{\mathrm{d}\mathbf{F}}{\mathrm{d}\mathbf{W}} = \mathbf{U} \quad , \tag{2.47}$$

where U is a finite positive value different from zero.

Differentiation as indicated by Equations 2.46 and 2.47 can be performed on Equations 2.38, 2.41 and 2.42. After suitable rearrangement this gives, respectively, for $\partial f/\partial PE = 0$

$$\tan \frac{\pi S}{2(\tau-1)} = \frac{1}{\frac{16W^2(\tau-1)^3 S}{\pi^3 (PE)^2 \tau^2} - \frac{4W(\tau-1)S}{PE\pi\tau} + \frac{2(\tau-1)}{\pi S}}$$
(2.48)

representing the underdamped case,

$$\frac{4W}{PE\tau} = 1 \tag{2.49}$$

for the critically damped case, and

$$\tanh \frac{\pi H}{2(\tau-1)} = \frac{1}{\frac{4W(\tau-1)H}{PE-\pi\tau} - \frac{16W^2(\tau-1)^3H}{\pi^3(PE)^2\tau^2} + \frac{2(\tau-1)}{\pi H}}$$
(2.50)

for the overdamped condition.
For $\partial f/\partial W = U$, the corresponding results are

$$\frac{\partial F}{\partial W} = \frac{e^{-N} \pi^2 (PE)^{3/2} \tau}{S 8W^2 (\tau-1)^2} \left[\sin \frac{\pi S}{2(\tau-1)} - \frac{PE \tau}{2S^2 W} \sin \frac{\pi S}{2(\tau-1)} + \frac{PE \tau \pi}{2W(\tau-1)S} \cos \frac{\pi S}{2(\tau-1)} \right] = U , \qquad (2.51)$$

for the underdamped solution,

$$\frac{\partial F}{\partial W} = e^{-\left(\frac{\partial W}{PE_{T}}\right)} \left[\frac{2}{(PE)^{1/2}\tau} + \frac{4W}{(PE)^{3/2}\tau^{2}}\right] = U , \qquad (2.52)$$

for the critically damped case, and

$$\frac{\partial F}{\partial W} = \frac{e^{-N}}{H} \frac{\pi^2 (PE)^{3/2} \tau}{8W^2 (\tau - 1)^2} \left[\sinh \frac{\pi H}{2(\tau - 1)} + \frac{PE \tau}{2H^2 W} \sinh \frac{\pi H}{2(\tau - 1)} - \frac{PE \tau \pi}{2W(\tau - 1)H} \cosh \frac{\pi H}{2(\tau - 1)} \right] = U \quad .$$
(2.53)

for the overdamped case.

Numerical values for the underdamped and overdamped cases are obtained by a combined numerical and graphical procedure. Both sides of Equations 2.48 and 2.50 are functions of PE/W. Values for PE/W that satisfy Equations 2.48 and 2.50 for different values of τ are found graphically. Table 2.1, for different values of τ and corresponding values for PE/W, is constructed from this graph. The values from this table are subsequently substituted into Equations 2.39,

2.40 and 2.43 to obtain solution for N, S and H, respectively.

Consequently these values of N, S and H are substituted into Equations 2.38 and 2.42 to obtain PE for the underdamped and the overdamped case as a function of τ . By use of Table 2.1 the values for $W(\min)$ different values of τ are obtained as a function of z(t), and plotted in Figure 3. The expression for minimum work for the critically damped case is:

$$W(\min) = [z(\tau)]^2 e \cdot \tau \qquad (2.55)$$

This equation has been obtained by combining Equations 2.27, 2.33 and 2.35 and substituting into Equation 2.16 together with the expression of Equation 2.49. Details of development of Equation 2.55 are provided in Appendix D.

The term τ in Equation 2.55 can be found by combining Equations 2.23 and 2.27, which yields

$$\tau = \frac{\pi PE \tau}{4W} + 1 \quad . \tag{2.56}$$

By substituting Equation 2.49 into Equation 2.56, 7 becomes

$$\tau = \pi + 1$$
 (2.57)

Finally Equation 2.57 is substituted into Equation 2.55, to

give

$$W(\min) = 11.256 [z(T)]^2$$
 (2.58)

for critical damping.

It is quite simple to demonstrate that Equation 2.47 is

satisfied. Substitution of Equations 2.49 and 2.57 into Equation 2.52 yields the expression

$$\frac{\partial F}{\partial W} = \frac{.146}{\sqrt{PE}}$$

PE is always a finite number consequently, Equation 2.47 is satisfied. When similar manipulations are performed on Equations 2.51 and 2.53 it is easy to show that the basic Equation 2.47 is again satisfied.

The results of Equations 2.48, 2.50 and 2.58 are plotted in Figure 3, where it can be observed that there is a minimum value for W at a corresponding $\tau = 2.8$. This indicates an underdamped region. No real values are obtained from Equations 2.48 and 2.50 in the range $3.6 < \tau < 5$, which indicates a transition range between the two extreme values of damping.

The values plotted in Figure 3 are nondimensional and can be employed for any configuration similar to Figure 2. To obtain the dimensional value of work, W, say in 1b-in, the corresponding value of dimensionless W(min) from the plot of Figure 3 must be multiplied by $[z(t)]^2$, which, in turn is obtained from Equation 2.10.

2.2.2 Power Consumption Analysis for Configurations 2.2 and 2.3.

Similar analysis as performed on configuration 1 is developed in order to optimize the work consumption per impact.

Neglecting friction, flow coefficients and fluid compressibility, an equation of motion for configuration represented in Figure 2.2 is

<u> </u>	PE/W	(PE/W) ²
1.6	.36	. 1296
1.8	.23	.0529
2.0	٠33	, 1089
2.2	٠53	.2809
2,4	•57	.3249
2.6	•59	.3481
2.8	• 75	.5625
3.0	.80	.6400

Table 2.1. Underdamped Case.

Table 2.2 Underdamped Case.

	S		(PE) ^{1/2}
1.6	.567	1.96	4.07
1.8	.913	• 79	2.06
2.0	.855	.81	2,27
2.2	.646	.99	2.33
2.4	.644	.855	2.29
2.6	.660	.732	2.27
2.8	.409	•79	2.43
3.0	• 341	. 738	2,75

$$M \ddot{x}_{p} + \frac{P_{s}A^{2}}{q_{s}} \dot{x}_{p} = P_{s}A_{p} , \qquad (2.59)$$

with initial conditions

$$x(0) = 0$$

 $\dot{x}(0) = 0$ (2.60)

This case reflects the fact that the piston is driven in both directions by P(s), from which is concluded that for a full cycle $\tau = 2$ and from Equation 2.27

$$W = 2P_{S}Q_{S}T$$
, (2.61)

which is the function to be minimized solution of Equation 2.59 gives

$$z(T) = \sqrt{M} \frac{Q_s}{A_p} \left(1 - e^{-\frac{P_s A^2}{Q_s M} T}\right), \qquad (2.62)$$

where

$$z(T) = \sqrt{M} \dot{x} p(T)$$
 (2.63)

Substitution of 2.61 and 2.35 into Equation 2.62 yields

$$z(T) = R - R e^{-\frac{W}{2R^2}}$$
, (2.64)

where

$$R = \sqrt{2KE}$$

(2.65)

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From Equation 2.64 it can be shown that

$$\frac{W}{2R^2} = \ln \frac{R}{R-Z(T)}$$
(2.66)

Due to simplicity of Equation 2.66. Minimizing W with respect to the function R, $\frac{dW}{dR} = 0$, 2.67 gives

$$2 \ln \left[\frac{1}{1 - z(T)/R} \right] = \frac{1}{1 - z(T)/R} - 1 , \qquad (2.68)$$

From which a value for z(T)/R can very easily be obtained with the aid of Numerical calculations, and results in

$$\frac{z(T)}{R} = .655$$
 (2.69)

Substituting 2.69 into Equation 2.66, a nondimensional solution as a function of $[z(T)]^2$ is obtained for the minimum value of W for configurations of Figures 2.2 and 2.3 as

$$W(\min) = 4.94 [Z(T)]^2$$
 (2.70)

This corresponds to a value which is 61 percent of that for configuration 2.1.

2.3 Selection of Configurations

It is apparent from Figure 3 that from minimum power requirements point of view, configurations of Figures 2.2 and 2.3 are superior to that of Figure 2.1, the former resulting in an approximate saving of 39 percent energy consumption over the latter. For





Configuration of Figure 2.1, τ is varied, which implies varying the return stroke time for a given forward stroke time. In Configuration 2.2, τ is a constant as it is assumed that forward stroke time is equal to the return stroke time. This corresponds to a singular point at $\tau = 2$.

Other considerations such as dynamic performance of Configuration 2.1 versus that for Configurations 2.2 and 2.3 should also be taken into account.

The spring effect in Configuration 2.1 will undoubtedly introduce frequency lags which will slow down the systems response. This aspect will be further examined in the next chapter.

The only difference between configurations shown in Figures 2.2 and 2.3 is in the number of lands of the spool value. It can be seen that in the three land configuration flow to the piston will be reversed as the spool moves half way between the extremes. For the scheme shown in Figure 2.3 flow reversal occurs at the end of each stroke of the spool. It may be noted parenthetically that the practical convenience of Configuration 2.3 over Configuration 2.2 will become apparent later when the type of feedback to perform the switching on the spool valve displacement is considered. Therefore, for the purpose of performing the dynamic analysis either, Configuration 2.2 or 2.3 might be equivalently employed.

2.4 Selection of Switching Configuration

Selection of an appropriate scheme for feedback from the piston to the servovalve to reverse the displacement of the piston itself,





must be based on such practical considerations as that it be of simple construction and inexpensive. Four different configurations are shown in Figure 4. Cases "a" and "b" with kinetmatic linkages as feedback elements provide a weak design against fatigue failure. This particularly so, due to the high frequency at which the system oscillates. Following McAdams [16], a quantitative estimate of the fatigue strength of the pins can be obtained. McAdams $\lceil 16 \rceil$ performed various tests on carbon and alloy steels to determine the relation between energy per blow and number of blows necessary to cause fracture. He used a modified Charpy method supporting specimens of different diameters as beams fixed at the ends. He then subjected these to repeated impacts by a falling hammer. Using various hammers at different heights of drop the number of blows necessary to cause fracture could be varied from about 500 to several millions. He thus plotted curves of "Energy gering" per blow" (ft-lb) against the number of blows to fracture. Some of his results using a 0.5 inch diameter high carbon showed that at impact of 200 ft-lb., one blow was sufficient for rupture; at an impact of 10 ft-lb. rupture occurred approximately after 100 blows. At an impact of 0.1 ft-lb. for a similar specimen, fracture was evident after approximately 10,000,000 blows. These results can be related to the pins supporting the connecting arm between the piston and spool valve of Figures 4.a and 4.b. If the pins are subjected to an impact force of 0.98 ft-lb. at the end of each piston stroke. Referring to McAdams graph; again for an 0.5 inch diameter pin, this corresponds to approximately 100,000 blows at rupture, which means the pins supporting the feedback arms would have to be replaced every 69 minutes.

Case C employs limit switches or solenoids activated by the piston at its extreme positions. The resulting pulse is fed to an amplifier activating an actuator to displace the servovalve to the left or to the right.

By keeping in mind that the hydraulic jackhammer for which this analysis is performed demands a highly abrasive field use, such a precise and expensive scheme to generate its oscillations appears to be very inappropriate.

Finally, Case d appealed mostly because of the ample possibilities of design using the same fluid media as a feedback from the piston to the spool. This last case where simplicity of construction and low cost are at a great advantage when comparing it to Case C; seems to be the likeliest candidate to be considered. However, several design changes need to be made to make it suitable for the present application and the final configuration can be seen in Figure 5.

2.5. Operation of Selected Configuration

Referring to Figure 5, when main spool 9 is ported to 1, supply pressure P_s displaces piston 11 to the top of the cylinder. As piston reaches the end of its top stroke, the feedback access 15 ports supply pressure P_{s6} to 8. Thus displacing spool 9 to right (positive direction) which closes port P_1 to P_s and opens it to drain port P_{D1} . At the same time port P_2 is opened to P_s and closed to drain port P_{D2} . As main spool is being displaced in positive direction by P_{s6} entering 8; oil from space 7 is drained. When piston 11; seven is connected through shaft drain passway four to $P_{D3,3}$.



· _

LEGEND FIGURE NO.

1 Lower chamber

2 Upper chamber

3 Valve actuation drain line

4 shaft drain passway

5 supply line for actuating left displacement of valve

6 supply line for actuating right displacement of valve

7 right valve chamber

8 left valve chamber

9 valve body

10 tool

11 piston (RAM)

12 hammer

13 tool lock in pins

14 feedback access-valve to left.

15 feedback access-valve to right

16 drainport

With P_s ported to chamber 2 and piston in top stroke position. The flow into chamber 2 displaces piston to bottom of cylinder. When feedback shaft 17 is moving down, both chamber 8 and 7 are sealed off to pressures P_{s5} , P_{s6} and drain P_{D3} . Thus the main spool maintains a fixed position until reaches opposite end stroke and connects through the corresponding ports. As piston reaches bottom, stroke position, hammer 12 strikes the tool 10. Just prior to the impact, feedback shaft 17 ports P_{s5} , five through 14 to chamber 7, displacing main spool 9 to left (negative direction). At the same time feedback-shaft 17 ports chamber 8 which must drain by connecting eight through four to P_{D3} . From here the process described is repeated producing an oscillating action.

Not considered in this analysis, a three way spool valve manually operated valve located before the main spool, series to divert flow of oil back to sump when system is not in use.

All connecting lines are sufficiently short allowing friction losses, log to be considered negligible.

2.6. Hydromechanical Components

A brief summary of the details of the hydromechanical components of the optimized configuration for the oscillator is provided here for ready reference.

With reference to Figure 5 a four way - four land spool value hydraulically operated 9, with the following specifications: A maximum displacement of 1.0 in. \pm 0.001 inch in either direction, minimum flow of 38 G.P.M. and rectangular ports of area equal to

.785 in . The pressure-flow characteristics are as in Figure C.2, Appendix C. It may be noted that symmetry is most important as it improves the linearity characteristics of the spool valve.

The piston is a reversible ram with an effective area of 3 in^2 , ram length 1 in., and a one inch stroke.

CHAPTER III

DYNAMIC ANALYSIS AND PERFORMANCE OF THE SYSTEM

3.1. Scope of the Analysis

A dynamic analysis of the valve-piston configuration arrived at in Chapter II can be performed by considering the continuity equation, Newton's second law of motion, and the Pressure-Flow Relation for the valve. In what follows, a linearized, lumped parameter description of the system is obtained. Various system parameters as well as valve coefficients are determined, together with a brief analysis of the rationale for a linearized analysis. Pressure transients due to sudden stoppage of fluid masses are also analyzed.

The latter part of this chapter is devoted to the dynamic stability analysis of the valve-piston configuration based on the Bode and Nyquist criteria for stability of linear systems. The stability analysis also incorporates the Describing Function analysis for nonlinear components.

3.2. System Governing Equations

A two-land, four-way, critical-center value is considered in this analysis on account of its linear flow gain characteristics as shown in the sketch.

The high precision employed in the manufacture of spool valves, with optimized performances and minimal losses with reference to the overall volumetric flow and working pressure, allows the following



Two-land, Four Way Critical Center Valve Characteristics

simplifying assumptions commonly adopted in hydraulic analyses [4]:

a. Radial clearance between the spool value and value body is taken to be zero, and metering edges are taken to be perfectly sharp and oriented.

b. The flow through each orifice is assumed to be based on steady state value of value coefficients, and all flow changes are assumed to take place instantaneously with variations in either orifice area or in pressure drop across the orifice.

c. Connecting lines are considered short enough in length and large enough in diameter to result in negligible inertial effects.

d. Friction losses in connecting lines are considered negligible.

e. Leakage flow past the piston is assumed to be laminar.

f. Supply pressure and fluid bulk modulus are assumed constant throughout.

g. System discharge is considered to be at atmospheric pressure.



1.

Figure 6. Flow and Pressure Drop in Piston.

3.2.1 Dynamic Analysis for the Piston

Consider Figure 6, which is a simplified modification of the piston-cylinder configuration of Figure 5 for the purpose of identifying the different variables in this analysis. The continuity of flow through chambers 1 and 2 may be expressed respectively, as, [4]

$$\Delta q_1 - C_{ip} \Delta (p_1 - p_2) - C_{ep} \Delta p_1 = \frac{d\Delta V_1}{dt} + \frac{\Delta V_1}{\beta} \frac{d\Delta p_1}{dt}, \qquad (3.1)$$

and

$$C_{ip} \Delta(p_1 - p_2) - C_{sp} \Delta p_2 - \Delta q_2 = \frac{d\Delta V_2}{dt} + \frac{\Delta V_2}{\beta} \frac{d\Delta p_2}{dt} , \qquad (3.2)$$

where C_{ip} and C_{ep} are, respectively, internal and external leakage coefficients, P and φ , respectively, represent pressure and flow, the subscripts 1 and 2, respectively refer to the forward and return chamber and the Δ indicates a change of the variable it precedes. It is seen that

$$\Delta V_{l} = V_{o} + \Delta_{p} \Delta x_{p}, \text{ and}$$
(3.3)

$$\Delta V_2 = V_0 - A_p \Delta x_p, \qquad (3.4)$$

where V is the volume and the subscript O represents the volume of either chamber at will.

Equations 3.1 and 3.2 can be combined with help of the relation

$$\frac{d\Delta v}{dt} = A_{p} \frac{\frac{d\Delta x}{dt}}{dt}, \qquad (3.5)$$

$$2 V_{o} = V_{1} + \frac{1}{2} = V_{T}$$
, (3.6)

and

$$\frac{d\Delta V_1}{dt} = -\frac{d\Delta V_2}{dt} , \qquad (3.7)$$

to obtain,

$$\Delta q_{1} + \Delta q_{2} - 2C_{1p}\Delta(p_{1}-p_{2}) - C_{ep}\Delta(p_{1}-p_{2})$$

$$= \frac{d\Delta v_{1}}{dt} - \frac{d\Delta v_{2}}{dt} + \frac{v_{o}}{\beta} \left(\frac{d\Delta p_{1}}{dt} - \frac{d\Delta p_{2}}{dt} \right)$$

$$+ \frac{A_{p}\Delta x_{p}}{\beta} \left(\frac{d\Delta p_{1}}{dt} + \frac{d\Delta p_{2}}{dt} \right)$$
(3.8)

Since the load flow and pressure are given by

$$\Delta q_{L} = \frac{\Delta q_{1} + \Delta q_{2}}{2} , \qquad (3.9)$$

and

۰.,

$$\Delta p_{\rm L} = \Delta p_1 - \Delta p_2 \quad , \tag{3.10}$$

Equation 3.8 may written as

$$\Delta q_{\rm L} = \Delta p_{\rm L} \left(C_{\rm 1p} + \frac{C_{\rm ep}}{2} \right) + A_{\rm p} \frac{d\Delta x_{\rm p}}{dt} + \frac{V_{\rm o}}{2\beta} \left[\frac{d\Delta p_{\rm L}}{dt} \right] + \frac{A_{\rm p} \Delta x_{\rm p}}{2\beta} \left[\frac{d\Delta p_{\rm 1}}{dt} + \frac{d\Delta p_{\rm 2}}{dt} \right]$$

$$(3.11)$$

Since

$$\frac{\mathrm{d}\Delta p_1}{\mathrm{d}t} = -\frac{\mathrm{d}\Delta p_2}{\mathrm{d}t} , \qquad (3.12)$$

the last term in 3.11 vanishes.

The Laplace transform of Equation 3.11 is

$$\Delta Q_{\rm L} = \Delta P_{\rm L} \left(C_{\rm ip} + \frac{C_{\rm sp}}{2} \right) + A_{\rm p} s \Delta X_{\rm p} + \frac{V_{\rm o}}{2\beta} s \Delta P_{\rm L} \quad . \tag{3.13}$$

The upper case variable indicating Laplace transformed.

$$\Delta Q_{\rm L} = \Delta P_{\rm L} C_{\rm TP} + A_{\rm p} \le \Delta X_{\rm p} + \frac{V_{\rm T}}{4\beta} \le \Delta P_{\rm L} , \qquad (3.14)$$

where

$$C_{TP} = C_{ip} + \frac{C_{ep}}{2} \qquad (3.15)$$

Next, a force balance on the piston gives

$$\Delta \mathbf{f}_{gg} = M_{\mathrm{T}} \frac{\mathrm{d}^{2} \Delta x_{\mathrm{p}}}{\mathrm{d}t^{2}} + B \frac{\mathrm{d} \Delta x_{\mathrm{p}}}{\mathrm{d}t} + K \Delta x_{\mathrm{p}} + \Delta \mathbf{f}_{\mathrm{L}} , \qquad (3.16)$$

also,

$$\Delta \mathbf{f}_{g} = \mathbf{A}_{p} \Delta \mathbf{P}_{L} \quad . \tag{3.17}$$

The Laplace transform of Equation 3.16 gives

 $\Delta F_{q} = M_{T} S^{2} \Delta X_{\rho} + B S \Delta X_{p} + K \Delta X_{\rho} + \Delta F_{L} , \qquad (3.18)$

where

 F_g = force on piston, M_T = total mass of piston and shaft, B = viscous damping coefficient, K = spring constant, and F_T = external load.

Equation 3.14 relates the load pressure with load flow through pertinent parameters. Equation 3.18, which represents a force balance on the piston, together with Equation 3.14 form the basic dynamic model for the piston.

3.2.2. Linearized Analysis for the Spool Valve

In order to complete the basic dynamic model of the piston-valve arrangement, linearized equations describing the pressure-flow curves for the spool valve must be developed. In steady-state, the load flow to the valve is a function of the load pressure and the valve position. In other words

$$Q_{L} = Q_{L}(X_{v}, P_{L})$$
 (3.19)

A linearized version of equation 3.19 about the operating point is

$$\Delta Q_{\rm L} = \frac{\partial Q_{\rm L}}{\partial X_{\rm V}} \Delta X_{\rm V} + \frac{\partial Q_{\rm L}}{\partial P_{\rm L}} \Delta P_{\rm L} , \qquad (3.20)$$

 \mathbf{or}

$$\Delta Q_{\rm L} = Kq \quad \Delta X_{\rm V} - K_{\rm c} \Delta P_{\rm L} , \qquad (3.21)$$

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where

$$Kq = \frac{\partial Q_{L}}{\partial x_{v}} \text{ is the flow gain,} \qquad (3.22)$$

and

$$K_{c} = -\frac{\partial Q_{L}}{\partial P_{L}}, \qquad (3.23)$$

is the flow pressure coefficient.

For any value configuration $\partial Q_{\rm I} / \partial P_{\rm L}$ is negative. This can be shown by differentiating the orifice equation, which describes the flow through orifices with respect to the load pressure $P_{\rm L}$.

A combination of dynamic Equations 3.14 and 3.18 for the piston, and Equation 3.23 for the value yields,

$$\Delta X_{p} = \frac{K_{q}\Delta X_{v} - \frac{K_{ce}}{A_{p}} \left(1 + \frac{V_{t}}{4\beta K_{ce}} S\right) \Delta F_{L}}{\frac{V_{t}M_{t}}{4\beta A_{p}} s^{3} + \left[\frac{K_{ce}M_{t}}{A_{p}} + \frac{BV_{t}}{4\beta A_{p}}\right] s^{2} + \left[\frac{BK_{ce}}{A_{p}} + \frac{KV_{t}}{4\beta A_{p}} + A_{p}\right] s + \frac{K_{ce}K}{A_{p}} + (3.24)$$

where $K_{ce} = K_c + C_{ip} + C_{ep}/2$, total losses coefficients.

For the purpose of simplifying notation the Δ of Equation 3.24 is dropped from here on. It must be kept in mind though, that in linear dynamic system analyses, it is a standard procedure to analyze a system with respect to deviations of its variables from an equilibrium point or null position of its components. Equation 3.24 is the dynamic equation for piston position for variable valve positions and load force functions for predominantly inertia type load. The first term in the numerator is related to the no load speed and the second term to the speed reduction due to load. Since the load does not include a mechanical spring, the quantity $(KV_T/4\beta A_p)$ in Equation 3.24 vanishes. Furthermore, it also may be noted that

$$\frac{\frac{BK_{ce}}{A_p^2}}{A_p^2} \ll 1 = O(A_p).$$

With these assumptions, Equation 3.24 can be reduced to the form.

$$x_{p} = \frac{KqX_{v} - \frac{K_{ce}}{A_{p}} \left(1 + \frac{V_{T}}{A\beta K_{ce}} s\right)F_{t}}{\frac{V_{T}M_{T}}{4\beta A_{p}} s^{3} + \left[\frac{K_{ce}M_{T}}{A_{p}} + \frac{V_{T}}{4\beta A_{p}}\right]s^{2} + A_{p}\hat{s}}$$
(3.25)

or,

$$X_{p} = \frac{\frac{K_{q}}{A_{p}} \times \frac{K_{ce}}{A_{p}^{2}} \left(1 + \frac{V_{T}}{4\beta K_{ce}} s\right) F_{L}}{s\left[\frac{V_{T}M_{T}}{4\beta A_{p}^{2}} s^{2} + \left[\frac{K_{ce}M_{T}}{A_{p}^{2}} + \frac{BV_{t}}{4\beta A_{p}^{2}}\right] s + 1\right]}$$
(3.26)

Considering the inelasticity of the piston impact against the tool without rebound, and that the switching of the piston takes place at the instant of impact, it is reasonable for the purpose of dynamic stability analysis of the system, to disregard the influence of the load disturbance on the piston displacement. Thus the effective equation for further analysis and design is

$$\frac{P}{F} = \frac{\frac{Kq}{A}}{\frac{p}{\omega}}, \qquad (3.27)$$

$$s\left(\frac{s^2}{\omega} + \frac{2\delta}{\omega}s_z + 1\right)$$

where

X X X

$$\omega = \sqrt{\frac{4\beta A_{\rm p}^2}{V_{\rm T}M_{\rm T}}} , \qquad (3.28)$$

and

$$\delta = \frac{K_{ce}}{A_{p}} \sqrt{\frac{\beta M_{T}}{V_{T}}} + \frac{B}{4A_{p}} \sqrt{\frac{V_{T}}{\beta M_{T}}} \quad . \tag{3.29}$$

It may be noted that \boldsymbol{w} is the undamped natural frequency, and $\boldsymbol{\delta}$ is the damping ratio.

3.3 Valve and Piston Coefficients

i) Expressions for Null Value of Valve Coefficients

It must be pointed out that most critical values of various valve coefficients from the stability standpoint are defined at Null. It is, therefore, appropriate to analyze the system dynamics in terms of null values of Kq and K_c , namely Kq_o and K_{co} , respectively.

Earlier the flow gain, Kq, was defined as

$$Kq = \frac{\partial Q_L}{\partial x_v} . \qquad (3.22)$$

 $\partial Q_L^{\prime}/\partial x_v$ is obtained by differentiating the general pressure-flow and the seneral pres

equation [Equation C.2, Appendix C] for a four way critically centered value. This yields, for $\frac{x_v}{|x_v|} = 1$,

$$Kq = C_{d} W \sqrt{\frac{1}{\rho} (P_{s} - P_{L})} . \qquad (3.30)$$

Evaluating at null position [Q $_{\rm L}$ = P $_{\rm L}$ = x $_{\rm v}$ = 0] yields the null flow-gain coefficient as,

$$Kg_{o} = C_{d} W \sqrt{\frac{P_{s}}{\rho}} , \qquad (3.31)$$

where

C_d = discharge coefficient, W = area gradient, and o = means density

The O subscript indicates value at valve null position.

It may be noted that leakage flow in the general pressure-flow equation has been neglected in order to simplify the analysis. This does not alter Equation C-l as $Q_L/Q_{leak} \gg 1$, Q leakage being of the order of 10⁻³ units. Furthermore, the leakage at null position of the spool valve is constant because P_s is constant and $P_L = 0$. Its partial derivative with respect to x_v is zero which makes Equation 3.31 accurate.

The flow-pressure coefficient, K_c, was earlier defined as

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$$K_{c} = -\frac{\partial Q_{L}}{\partial P_{L}} \quad . \tag{3.23}$$

By differentiating Equation C.2 (Appendix C) and evaluating at null yields, $K_{co} = 0$ which is an unacceptable value. It may be noted that since Q_L is zero at null position of a critically centered spool value, the leakage flow becomes guite significant.

Thus, the null expression for the flow-pressure coefficient can be obtained after the leakage flow is specified. An expression for leakage flow can be obtained by considering laminar flow through a sharp-edged rectangular orifice.

With reference to Figure 7.A and 7.B, the leakage flow equation is given by

$$\frac{Q_c}{2} = \frac{\pi r_c^2 W P_s}{32 \mu 2},$$

(3.32)

where

 $r_{c} \ll 1$,

and

 $\Delta W >> r_{\rm e}$.

Equation 3.32 describes the flow through a sharp-edged orifice formed by the radial clearance, r_e , when the spool value is at null position. The pressure drop and leakage flow in a four way spool value configuration are $P_g/2$ and $Q_c/2$, respectively, for each orifice, where Q_c is the leakage flow when the spool value is at null position.

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Differentiation of Equation 3.32 with respect to P_s and use of the definition, $\partial Q_s / \partial P_s = K_c$, yields

$$K_{co} = \frac{\pi W \frac{w^2}{c}}{32\mu}, \qquad (3.33)$$

where

 K_{co} = null pressure-flow coefficient, and $W = \frac{dA}{dx_v}$ = area gradient.

The pressure sensitivity at null is the ratio of the null flow-gain coefficient over the null flow-pressure coefficient. A numerical estimate of the value of pressure sensibility can be made by assigning typical numbers to various parameters. For petroleum base fluids, the fluid density and viscosity are [4], approximately,

$$\rho = 0.78 \cdot 10^{-4} \text{ lb sec}^2/\text{in}^4$$
, and
 $\mu = 1.8 \cdot 10^{-6} \text{ lb sec}/\text{in}^2$.

For sharp-edged orifices, regardless of the specific geometry, the discharge coefficient [4] is, approximately, 0.60. A typical value for radial clearance in hydraulic spool values is about $2. \cdot 10^{-4}$ inches.

Thus

$$K_{po} = \frac{Kq_{o}}{K_{co}} = \frac{C_{d}W\sqrt{\frac{P_{s}}{\rho}}}{\frac{\pi}{2}\frac{W}{c_{c}^{2}}} = \frac{C_{d}^{32\mu}\sqrt{P_{s}}}{\pi \cdot x_{c}^{2}\sqrt{\rho}} = 31,800\sqrt{P_{s}} . \qquad (3.34)$$

For a supply pressure of 1000 Psi, it isn't unusual to obtain pressure sensitivities of the order of 10^6 psi/in. This is an important coefficient, as it indicates the ability of a particular value to overcome STICTION forces. The higher the pressure sensitivity, the quicker the response. It may be noted, parenthetically, that the null pressure sensitivity is independent of value area gradient, and, therefore, independent of value size.

ii) Expression for Leakage Coefficients for Piston (C_{ip} and C_{ep})

The leakage coefficients are obtained by assuming leakage flow linearly proportional to pressure drop across the piston. With reference to Figure 7C the flow through an annulus is

$$Q = \frac{\pi D_{\rm p} h^3}{12 \mu L_{\rm o}} \Delta P ,$$

 \mathbf{or}

$$\frac{1}{C_{ip}} = \frac{\Delta P}{Q} = \frac{12\mu L_{o}}{\pi Dh^{3}} . \qquad (3.35)$$

Note that C_{ip} is the annulus internal leakage coefficient. Equivalently, with reference to the Figure 7c, external annulus leakage through the shaft and walls is given by

$$\frac{\Delta P}{Q} = \frac{1}{C_{ep}} = \frac{12\mu \ell}{\pi d h_0^3} \quad . \tag{3.36}$$

iii) The Viscous Damping Coefficient for the Piston

By definition,

 $B_{p} = \frac{F}{V_{p}} = \frac{\tau A_{p}}{V_{p}} ,$

where

I = viscous force, B_p = viscous damping coefficient, V_p and A_p are the piston velocity and cross sectional area, respectively, and

$$\boldsymbol{\tau} = \boldsymbol{\mu} \boldsymbol{\psi} , \qquad (3.39)$$

where

 τ = shear stress, and ψ = viscous strain rate.

The viscous strain rate is defined as

$$\psi = \frac{\partial V_p}{\partial h} \approx \frac{V_p}{h} \quad . \tag{3.40}$$

Note that h is the radial clearance of the piston. By combining Equations 3.38, 3.39, and 3.40 an expression for viscous damping is obtained as

$$B_{p} = \mu \pi D_{p}$$
, (3.41)

where

 D_{p} = piston diameter.

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(3.38)

(3.42)

(3.43)

Similarly, for the shaft

$$B_{S} = \mu \pi d$$
;

where

 B_g = viscous damping coefficient of the shaft,

and

Referring to Figure 7c. the total damping coefficient for the piston and shaft is

$$B = B_{p} + B_{g}$$

3.4. <u>Numerical Estimates for Values</u> of Various Systems Parameters

In order to perform a frequency response analysis of the dynamic transfer function,

$$\frac{x_p}{x_v} = \frac{\frac{kq}{A}}{s\left(\frac{s^2}{\omega} + \frac{2\delta}{\omega}s + 1\right)},$$
(3.27)

it is necessary to assign numerical values to ω and δ , respectively defined by Equation 3.28 and 3.29. Since it is intended to compare the performance of the present hydraulic oscillator with the more commonly available pneumatic device, typical values of the physical dimensions of the pneumatic piston cylinder arrangement are taken from reference [14]. It may be observed from Equations 3.28 and 3.29 that in order to obtain numerical values of these expressions it is necessary to have the physical values of various parameters of the system, including various value and piston coefficients and fluid properties.

From Reference [14], some of the dimensions chosen are (See Figure 7c) the area of the piston, $A_p = 3$ inches, length of the piston $L_o = 2$ inches, mass of the piston $M_T = 4$ lbin., diameter of the shaft, d = 1 inch, and length of the shaft passage, $\ell = 2$ inches.

The fluid properties forma typical hydraulic fluid are taken from Reference [4], and in addition to the values for mass density, ρ , and absolute viscosity, μ , quoted previously, it is necessary to include the bulk modulus, β , a fluid property that affects both ω and δ in Equations 3.28 and 3.29. A typical value of β for hydraulic fluids including entrapped air is, $\beta = 10^5 \text{ lb/in}^2$. The discharge coefficient, C_d , is approximately 0.60 and the area gradient, W, is taken as 0.785 in²/in because this particular value multiplied by the full displacement of the value ($x_v = 1$ in.) yields a cross sectional area of 0.785 in², which is consistent with that used in Reference [14].

Substituting these numerical values, into Equations 3.31, 3.33, 3.35, 3.36 and 3.43 yields

$$\begin{split} & \mathrm{Kq}_{\mathrm{o}} = 3,765 \ \mathrm{in}^{3}/\mathrm{sec}/\mathrm{in}, \\ & \mathrm{K}_{\mathrm{co}} = 68 \cdot 10^{-4} \ \mathrm{in}^{3}/\frac{\mathrm{sec}}{\mathrm{in}} \ [\text{with } \nu_{\mathrm{c}} = 2 \cdot 10^{-3} \ \mathrm{in}], \\ & \mathrm{C}_{\mathrm{ip}} = .159 \cdot 10^{-3} \ \frac{\mathrm{in}^{3}}{\mathrm{sec}}/\mathrm{psi}, \\ & \mathrm{C}_{\mathrm{ep}} = .073 \cdot 10^{-3} \ \frac{\mathrm{in}^{3}}{\mathrm{sec}}/\mathrm{psi}, \end{split}$$

م موجعين and

 $B = 18.1 \cdot 10^{-6}$ lb-sec/in.

Furthermore, use of the definition

$$K_{ce} \simeq K_{co} + C_{ip} + C_{ep} , \qquad (3.44)$$

gives the total value coefficient as

$$K_{ce} = 0.007 \frac{in^3/sec}{in}$$
[with $r_c = 2.10^{-3}$ in]

Determination of the hydraulic natural frequency (w) and the dimensionless damping ratio is performed by simply substituting the preceding numerical values into Equations 3.28 and 3.29, respectively.

From Equation 3.33 it can be seen that K_{co} is directly proportional to the square of the radial clearance r_c . An increase in r_c increases the damping ratio, since K_{ce} is linearly relates to K_{ce} as seen from Equation 3.44. Furthermore, K_{ce} is directly proportional to δ (Equation 3.29).

Variation of δ as a function of x_{0}^{2} , is plotted in Figure 6a. and these values will be determining the solution of Equation 3.27 as a function of δ .

The hydraulic natural frequency thus obtained is

$$\omega = 10,766 \text{ rad/sec}$$
.

The damping ratio varies as a function of the parameter r_c . This is seen in Figure 8. Discussion of the use of Figure 8 in radial clearance, r_c 10⁴ in. 3 ;9 .5 1.1 1.3 .3 .7 damping ratio, 5





Effects of Parameter Variations on the Characteristic Transfer Function.

where

n,

$$T = 1/\omega,$$

$$\delta = K_{ce}/A_{p} (\beta M_{t}/V_{t})^{1/2}; \qquad K_{\Delta f} = K_{ce}/A_{p}^{2},$$

and

$$T_1 = V_t / 4\beta K_{ce}$$
calculating system response is postponed until a later section.

3.5. Effects of Variation of Parameters on the Characteristic Equation

It is of considerable interest to determine how different parameters affect the system characteristic equation. This is explained via a scheme illustrated in Chart 1. It is seen from this chart, for example, that as the total volume, $V_{\rm T}$, is increased, the period of oscillation, T, increases and the damping ratio, δ , decreases. Also coefficient T₁ in Equation 3.26 increases, whereas there is no influence on coefficient K_{Δf} of Equation 3.26. In a similar fashion the influence of variation of other system parameters on the system characteristic function may be ascertained from the chart.

3.6 Validity of Linearization

Up to this point the analysis of the servovalve and piston combination has been made assuming very small displacement of the valve. Also the valve coefficients Kq and K_c are evaluated at null as this is the operating point at which stability is most critical. The validity of such a linearization remains to be established. This may be accomplished by first deriving the more accurate equation of motion for the valve-piston system.

A simple lumped mass load representation of the piston motion as in Equation 2.3 thus neglecting the spring motion term gives

$$\mathbf{P}_{\mathrm{L}} \mathbf{A}_{\mathrm{p}} = \mathbf{M}_{\mathrm{T}} \frac{\frac{\mathrm{d}^{2} \mathbf{x}_{\mathrm{p}}}{-\frac{\mathrm{p}}{\mathrm{d}t^{2}}}.$$
 (3.44)

Combination of Equations 3.14 and C.2 (Appendix C) and

substitution of P_L from 3.44 yields

$$\frac{C_{d}}{A_{p}^{\circ}} \left(\frac{P_{s}}{\rho}\right)^{1/2} wx_{v} \left(1 - \frac{x_{v}}{|x_{v}|} \frac{M_{T}}{P_{s}^{A}p} \frac{d^{2}x_{p}}{dt^{2}}\right)^{1/2}$$
$$= \frac{V_{T}M_{T}}{4gA_{p}^{2}} \frac{d^{3}x_{p}}{dt^{3}} + \frac{C_{tp}Md^{2}x_{p}}{A_{p}^{2}dt^{2}} + \frac{dxp}{dt} \qquad (3.45)$$

This is a nonlinear differential equation describing the value piston system. The pressure flow equation for the value piston system [Equation C.17] is derived in Appendix C and is also plotted in Figure C.2. A first look at this figure indicates that these characteristics are highly nonlinear. However, a closer look at Figure C2 indicates a linear operating range which corresponds to $P_L/P_S \leq 2/3$. This is in fact, the design condition for maximum power transfer (Reference [4]). If it is assumed that $P_L/P_S < 1$ then

$$\left(1 - \frac{x_{v}}{|x_{v}|} \frac{P_{L}}{P_{S}}\right)^{1/2} = 0 \left[1 - \frac{1}{2} \frac{x_{v}}{|x_{v}|} \frac{P_{L}}{P_{S}}\right], \qquad (3.46)$$

where O[], indicates of the order of.

Subsequently, Equation 3.45 becomes

$$\frac{C_{d}}{A_{p}} W \left(\frac{P_{s}}{\rho}\right)^{1/2} X_{v} = \frac{d}{dt} \left[\frac{1}{\omega^{2}} \frac{d^{2}x_{p}}{dt^{2}} + \frac{2\delta}{\omega} \frac{dx_{p}}{dt} + 1\right], \quad (3.47)$$

In Equation 3.47, ω remains the same as that defined for the linear model by Equations 3.27 and 3.28, whereas the damping ratio is redefined as

$$\delta = \left[\frac{C_{d} W |x_{v}|}{2P_{s}} \sqrt{\frac{P_{s}}{p}} + C_{tp}\right] \frac{M\omega}{2A_{p}^{2}}.$$
 (3.48)

This is the equation of motion valid for the operating range of the valve-piston system, namely $P_L \le \frac{2}{3} P_S$.

It can be observed from Equation 3.28 that the hydraulic natural frequency is fixed by four parameters, namely β , A_p , V_t , and M_T ; and is in no way affected by the operating point of the valve. Thus ω is independent of the operating point defined by the valve position. This is not the case for the damping ratio, which depends strongly on x_v . In order to simplify the nonlinear Equation 3.47 it is necessary to obtain an effective value for X_v . Toward this end, first consider a sinusoidal motion of the valve, defined by

$$X_{\mu} = A \sin \omega t \quad . \tag{3.49}$$

The effective displacement of the motion is defined as

$$X_{v} \text{ eff} = \sqrt{X_{v}^{2}} , \qquad (3.50)$$

where the overline indicates the time average. Since the time average of the squared sine wave is $\frac{1}{2}$, Equation (3.50) simplifies to

$$X_v \text{ eff} = \frac{A}{\sqrt{2}} \qquad (3.51)$$

Equation 3.52 can be used to replace $|x_v|$ in Equation 3.47. This modifies Equation 3.47 to yield

$$\frac{\mathrm{Kq} \mathrm{A}}{\sqrt{2} \mathrm{A}_{\mathrm{p}}} = \frac{\mathrm{d}}{\mathrm{dt}} \left[\frac{1}{\omega^2} \frac{\mathrm{d}^2 \mathrm{x}_{\mathrm{p}}}{\mathrm{dt}^2} + \frac{2\delta}{\omega} \frac{\mathrm{dx}}{\mathrm{dt}} + 1 \right] , \qquad (3.52)$$

where

$$Kq = WC_d \left(\frac{P_s}{\rho}\right)^{1/2}$$
, as defined earlier.

The Laplace Transform of Equation 3.52 for zero initial conditions is

$$\frac{\mathbf{v}_{\mathbf{p}}}{A/\sqrt{2}} = \frac{\frac{Kq}{A}}{s\left(\frac{s^{2}}{2} + \frac{2\delta}{w}s + 1\right)}.$$

(3.53)

Concluding, the quadratic representation of the hydraulic power system may be used over a large range of operation which validates the previously performed approximate linear analysis. The hydraulic natural frequency is independent of valve piston motion, but the damping ratio varies with valve position. Thus the validity of linearization is justifiable. Furthermore, the system stability analysis shows that the linearized model satisfies more stringent stability criteria than the actual nonlinear system.

3.7. Dynamic Response of Three-way Vis-a-vis Four-way Valves

As has been pointed out earlier in section 2.2, the four way valve configuration offers a better switching design characteristic for the present system. A simple comparison of the natural frequency

and damping ratio for a three-way valve with those for the four-way valve must be considered to confirm this observation.

The undamped natural frequency and the damping ratio of a three-way value controlled piston for constant supply pressure can be derived by considering the value flow equation developed in section 3.2.2. For the three-way value $P_L = P_H$ = head pressure on piston, giving,

$$Q_{\rm L} = KqX_{\rm v} - K_{\rm c}P_{\rm H} \quad . \tag{3.54}$$

Use of the continuity equation to the control volume of Figure 9, yields

$$\Delta q_{L} + Ci \Delta (p_{s} - p_{H}) = \frac{d\Delta V_{H}}{dt} + \Delta V_{H} \frac{d\Delta p_{H}}{dt} , \qquad (3.55)$$

where

 $C_i = leakage coefficient, and <math>V_H = head$ chamber volume.

The head chamber volume is given by

$$\mathbf{V}_{\mathrm{H}} = \mathbf{V}_{\mathrm{o}} + \mathbf{A}_{\mathrm{p}} \Delta \mathbf{x}_{\mathrm{p}} \quad , \tag{3.56}$$

where

 V_{o} = initial head chamber volume, and A_{p} = head side area.

Combination of Equation 3.55 and 3.56 with $|A_{p,p}| \ll V_{o}$, gives after





Laplace transformation and dropping the Δ before the variables,

$$Q_{L} + C_{i}P_{S} = A_{p} + C_{i}P_{H} + \frac{V_{o}}{\beta} s P_{H}$$
 (3.57)

The force balance on the piston gives

$$P_{H}A_{p} = P_{s}A_{s} = M_{T}s^{2} X_{p}$$
, (3.58)

where

 $A_s = area$ on shaft side of piston.

Combination of Equation 3.54, 3.57 and 3.58 results in the system model.

$$\frac{X_{p}}{X_{v}} = \frac{\frac{Kq}{A}}{\frac{s^{3}}{\omega^{3}} + \frac{2\delta}{\omega}s^{2} + s}, \qquad (3.59)$$

where

$$\delta = \frac{K_c}{2A_p} \sqrt{\frac{\beta M_T}{V_o}} , \text{ and} \qquad (3.60)$$

$$w = \sqrt{\frac{\beta A_p^2}{V_o M_T}} \quad . \tag{3.61}$$

The corresponding expressions for a four-way valve controlled piston, respectively given by Equations 3.28 and 3.29, are:

$$\delta = \frac{K_{ce}}{A_p} \sqrt{\frac{\beta M_T}{V_T}} + \frac{B}{4A_p} \sqrt{\frac{V_T}{\beta M_T}} , \text{ and}$$



It can be seen that both ω and δ for the three-way value controlled piston are lowered by a factor of $\frac{1}{\sqrt{2}}$ as against those for the four-way value. This is because in a three-way value only one line is controlled, leading to a single volume and oil spring. This is vis-a-vis two lines in a four-way value. The four way value also controls two volumes and oil springs. Keeping all other parameters constant, the dynamic response of a four-way value controlled piston is superior, because the ω of four-way value is greater, which in turn makes the period, T, shorter.

3.8. Stability of the Linearized System

The linearized system stability may be examined through the dynamic Equation 3.27. This equation was solved on a digital computer for a damping ratio varying between 0.04 and 1.2 with the valve near the null position. These results are shown in Figure 13 as Nyquist's plots. Figure 15 shows the same results in the form of Bode plots.

It can be seen that for $\delta = 0.0^4$ numerator dynamics has a pronounced effect on the resulting Nyquist plot leading to a very unstable system. As the damping ratio is increased the effect of numerator dynamics decreases and the system stabilizes. This can be seen from the curve for $\delta = 0.6$.

The same conclusion may be made from the Bode diagram of Figure 15. For a damping ratio of $\delta = 0.04$ it can be seen that the magnitude ratio decreases from + 80 db at $\omega = 0.1$ rad/sec to - 10 db

at w = 5,000 rad/sec. At an w = 10,500 there is a pronounced peak. At the same time the phase angle drops sharply from - 90° to - 270° . It can also be seen that the phase angle curve crosses the magnitude ratio curve at an angle, less than - 180° and at the magnitude ratio greater than 0 db. Thus indicates that the system is unstable. This point also corresponds to the - 1 < - 180° critical point of the Nyquist diagram as expected. It is seen from the Bode diagram that the crossover frequency w_c is approximately equal to the velocity constant, that is

$$w_c \approx Kq/A_p$$
 (3.62)

Furthermore, it is also noted that the resonant peak of the quadratic occurs at approximately 10.500 rad/sec., which corresponds to the hydraulic natural frequency \boldsymbol{w} . By definition, the gain level of the asymptotic curve is $\boldsymbol{w}_c/\boldsymbol{w}$, or using Equation 3.62 the gain level is $Kg/A_p/\boldsymbol{w}$ at the frequency \boldsymbol{w} . Also by definition, the amplification factor for the quadratic, at resonance is $\frac{1}{2\delta}$.

This means that the gain level at resonant peak is $\frac{\frac{Kq}{2}}{2\delta \omega}^{p}$, which must be less than unity for stability. This yields

$$\frac{Kq/A_p}{\omega} < 2\delta \qquad (3.63)$$

This result can also be obtained by taking the equation

$$1 + G(jw) = 0$$
, (3.64)

and performing the Routh test for stability.

Equation 3.62 sets a rule for a permissible crossover frequency. It can also be noted from this, that large velocity constants, Kq/A_p , require large hydraulic natural frequencies and also large damping ratios. It is worth noting that values of thus computed damping ratios are only slightly lower than those measured experimentally.

The Bode diagrams of Figure 15 were constructed with the various damping ratios. As mentioned previously, the effects of numerator dynamics disappear completely for $\delta \ge 0.6$, but is still noticeable for $\delta = 0.4$.

It is definitely not desired to choose damping ratios higher than $\delta = 0.6$ as these values correspond to operating points close to null position of the valve. At full displacement of the valve the damping ratio is larger, and since it is not desired to have a slow responding system, damping ratios in the range $0.4 < \delta < 0.6$ are appropriate for this system. The value $\delta = 0.6$ will be used in the subsequent text.

The damping ratio in this analysis was varied by changing the values of the leakage through the valve. This ensured no changes in other system parameters, as would be the case if lapping of the servo-valve lands had been varied to adjust the damping ratio (see Figure 8 and section 3.4). It was also noticed in section 3.3, that use of different viscosity and density values (different oils), had little or no effect on the systems damping ratio.

The system stability at off-null position is automatically guaranteed if the system is stable at Null position. This is due to

the fact that the damping ratio at an operating point away from the null position is higher in proportion to the distance of valve movement. For example, in Figure 14, a $\delta = 1.0$ at null, corresponds to a $\delta = 4.84$ at $x_v = 0.5$ in. This renders the system very stable but too stiff to be useful. Therefore, if the system is stable for very small valve displacements from null, the system will be stable for all other operating position.

The magnitude of the system's open loop gain constant Kq/A $_p$, for which this system is critically stable, is readily found by considering the system transfer function

$$G(s) = \frac{\frac{Rq}{A}}{s\left(\frac{s}{2} + 2\delta \omega_{n} s + 1\right)} .$$
(3.27)

Separation of the imaginary and real parts gives

Ka

$$G(\mathbf{j}\boldsymbol{\omega}) = \frac{\frac{\mathbf{k}q}{\mathbf{k}p}}{\left[\left(-2\delta\boldsymbol{\omega}_{n}\boldsymbol{\omega}^{2}\right)^{2} + \left(\boldsymbol{\omega} - \frac{\boldsymbol{\omega}^{3}}{\boldsymbol{\omega}_{n}^{2}}\right)^{2}\right]^{\frac{1}{2}}} \quad (3.65)$$

The phase angle is

$$\phi = -\tan^{-1} \left[\frac{\omega_{1}}{-26\omega_{10}} \right].$$
 (3.66)

For marginal stability $\phi = -180^{\circ}$ and $w = w_{c}$. Thus Equation 3.66

becomes

$$1 - \frac{w^2}{2} - 180 = -\tan^{-1} \left[\frac{w_n}{-2\delta w_n w} \right], \qquad (3.67)$$

$$1 - \frac{w_c^2}{2}$$

$$0 = -\frac{w_h}{-2\delta w_h \omega} . \qquad (3.68)$$

with

$$\delta = 0.6$$
, and

$$\mathbf{u}_{n} = 10.766 \text{ rad/sec}$$

Equation 3.68 yields

$$w_{c} = 10.766$$

For marginal stability $G_0 = 1$

i.e.
$$[G_{o}] = \frac{\frac{kq}{A}}{[(.2\delta w_{n} w^{2})^{2} + (w - w^{3} / w_{n}^{2})^{2}]^{\frac{1}{2}}} = 1 \qquad (3.69)$$

By substitution of the value $w_c = 10,766$ for w in Equation 3.69 yields

$$\frac{Kq}{A} = 1.2 \cdot 10^{12} . \qquad (3.70)$$

Therefore, it can be concluded that the system is stable for

 $Kq/A_p < 1.2 \cdot 10^{12}$.

3.9. Describing Function Analysis

A canonical transformation of the system block diagram of Figure 10 is shown in Figure 11. This may be represented in mathematical form as

$$\frac{C(j\omega)}{R(j\omega)} = \frac{N(a) * G(j\omega)}{1 + N(a) * G(j\omega)} . \qquad (3.71)$$

For the present problem this may be written as

$$\frac{x_{p}}{x_{y}} = \frac{N(a)}{1 + N(a)} \frac{\phi G(j\omega)}{\phi G(j\omega)} , \qquad (3.72)$$

where the linear frequency dependent G(jw) is of the form described by Equation 3.27. The nonlinear part of the transfer function may be represented by a relay with hysteresis of the form $N(a) \not - \phi$. Note that the latter is, of course, amplitude dependent and has a phase lag due hysteresis.

Stability of this system is examined by considering different methods. Stability from the Nyquist plot is based on the characteristic equation which is obtained by equating the denominator of Equation 3.71 to zero. Thus

$$1 + (\mathbf{N}(\mathbf{a}) \mathbf{\omega} \phi) \mathbf{G} (\mathbf{j} \mathbf{w}) = 0 \quad . \tag{3.73}$$

A polar plot of $G(j_{\omega})$ $N(a) \not = \emptyset$ must be examined with reference to the critical point $-1 + j_0$. Rearrangement of Equation 3.73 in the





Figure 11. Reduced Canonical Block Diagram.





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. .

















form

$$G(\mathbf{j}\boldsymbol{w}) = -\frac{1}{N(\mathbf{a})} \not = \phi \quad , \qquad (3.74)$$

separatés the linear-transfer function factor from the non-linear factor. $G(j_W)$ and $-1/N(a) \not = \emptyset$ are plotted on the same plot obtaining a frequency locus and an amplitude locus. If these loci intersect, there exists a sustained oscillation in the system described as a limit cycle. It can be added that a stable limit cycle is one which after being subject to a disturbance will return to its original frequency of sustained oscillations. On the other hand, an unstable limit cycle is one which after an applied disturbance, will either increase its frequency of oscillation to infinity or decrease and eventually die out. The nature of this systems limit cycle will be examined through a gain-phase describing function plot.

From Figure 12 the relay characteristic may be represented by

$$\phi = -\sin\frac{h}{a} , \qquad a \ge h ,$$

$$N(a) = \frac{4M}{\pi a} \ge -\sin^{-1}\frac{h}{a} ,$$

$$(3.75)$$

where

$$u = a \sin \omega t$$

The Describing Function Equation 3.74 becomes

$$-\frac{1}{N(a)} = \frac{1}{\frac{4}{M}} \sum_{n=1}^{\infty} \sin^{-1} \frac{h}{a} , \qquad (3.76)$$

$$= \frac{\pi a}{4M} \left(\cos \phi - i \sin \phi\right) \quad . \tag{3.77}$$

Substitution of Equation 3.75 into 3.77 yields

$$\frac{1}{\bar{N}(a)} = \frac{\pi}{4M} \left[\sqrt{a^2 - h^2} + ih \right]. \qquad (3.78)$$

Equation 3.78 can be divided into the real and imaginary parts

$$Im = \frac{\pi h}{4M} , \text{ and}$$

$$Re = \frac{\pi}{4M} \sqrt{a^2 - h^2} . \qquad (3.79)$$

From here the solution is obtained by graphical methods. From physical characteristics of the systems variables maximum displacement x_p and x_y . The relay is defined by h = 0.5 and M = 1. Thus gives

 $-i\frac{\pi}{4}h = -i0.3927$ (3.80)

(3.81)

A line drawn parallel to the real axis at a distance on the negative imaginary axis of 0.3927 units intersects with the Nyquist diagram at 0.1 units from the negative real axis for $\delta = 0.6$ (See Figure 13.A).

Thus,

$$-\frac{\pi}{4}\sqrt{a^2-h^2}=-0.139 ,$$

 \mathbf{or}

as

and

 \mathbf{or}

$$\boldsymbol{\omega} = 4350 \, \frac{\mathrm{rad}}{\mathrm{sec}} \quad . \tag{3.83}$$

The range at which the imput amplitude "a" varies with respect to δ , is found to be .5159 $\leq a \leq .6782$, which happen to be very small. In summary, from the describing function analysis, it can be mentioned that the system will oscillate at an amplitude of a = 0.52 and a frequency of $\omega = 4350$ rad/sec.

a = 0.5159 🐆

Figure 18 shows a gain-phase plot as well as the relay characteristics drawn to the same scale. The gain phase curve is obtained directly from the Bode diagram (Figure 15) and the describing function curve for the relay characteristic is obtained from Figure 17, which has been reproduced from Reference [10]. The intersection of both curves yields the amplitude and frequency of oscillation of the systems limit cycle, which corresponds approximately to the amplitude range and frequency obtained by the describing function method. It can be seen from this figure that as the damping ratio is increased, the amplitude is also increased. The direction of the arrows indicate on increasing values of M/a and ω , respectively. At the intersection of the two curves, the arrows are pointing in the same direction, thus, indicating a stable limit cycle (Reference [11]).







Figure 18. Phase-Gain, Describing Function.

CHAPTER IV

ANALOG COMPUTER SIMULATIONS

4.1 Introduction

Computer simulations of the system described by the block diagram of Figure 11 to verify the validity of the linear part with respect to stability and to reproduce the complete systems output were performed on a Systron Downer SD 10/20 Analog Computer. The simulations response to step disturbances and the complete system output was displayed on an oscilloscope; additionally, for permanent record, an x-y plotter was hooked up to the computer's output.

It is often necessary to time and magnitude scale, the real system variables before programming into the analog computer. Because the real time solution of this particular system would be too fast for the computer output display units to respond adequately, it is necessary to slow down the computer solution. Also, it is necessary to magnitude scale this configuration to increase the value of the small voltages that represent the system real variables, to levels where computer noise and error are minimized. The amplifiers for the SD 10/20 admit voltages within the range of + 100 volts.

4.2 Dynamic Stability

The system described by the block diagram in Figure 11 was taken as the model for the computer simulation. The general form for the closed loop linear block is

input Xv(tc)



Figure 19. Analog Computer Set-up.

•

-output

$$a_{o} \ddot{x}_{p} + a_{1} \dot{x}_{p} + a_{a} \dot{x}_{p} + a_{a} x_{p} = \frac{K_{q}}{\Delta_{p}} x_{v}(t)$$
 (4.1)

where a_0 , a_1 , a_2 and a_3 are the systems coefficients and the dot over the variable x_p denotes the order of the derivative of the variable with respect to time (i.e. \ddot{x}_p third order, \ddot{x}_p second, etc.). Rearrangement for analog computer programming, together with the corresponding time and magnitude scaling yields

$$\ddot{x}_{p} = -\left[\frac{a_{1}}{a_{o}} \text{ 10 } T_{c} \, \ddot{x}_{p} + \frac{a_{2}}{a_{o}} \text{ 10 } T_{c}^{2} \, \dot{x}_{p} + \frac{a_{3}}{a_{o}} \text{ 10 } T_{c}^{3} \, x_{p} - \frac{K_{q}}{4_{p}} \frac{T_{c}^{3}}{X_{a_{o}}} \, x_{v}(t_{c}) \right],$$

where

 $T_c = time scale constant,$

X = Magnitude scale constant, and,

t = computer time.

In order to determine a suitable T_c . The following two criterias are employed.

a)

$$T_{c} \leq K_{o} \frac{a_{o}}{a_{1}},$$

$$T_{c} \leq \left(K_{o} \frac{a_{o}}{a_{2}}\right)^{1/2},$$

$$T_{c} \leq \left(k_{o} \frac{a_{o}}{a_{3}}\right)^{1/3},$$

$$(4.3)$$

(4.2)

where

and

$$1 \ge K \le 100$$
 and (4.4)

b) A sinusoidal forcing function of frequency ω (Rad/sec) is

$$x_v(t_c) = a \sin \omega(T_c t_c)$$
 (4.5)

For

$$w T_{c} = 1$$
, and $w = 4356 \frac{Rad}{sec}$, (4.6)

this gives

$$T_c = .23 \cdot 10^{-3}$$
 (4.7)

This value of \mathbf{T}_c should also be consistent with the criterion described by Equation 4.3.

Since

$$a_{0} = 86 \cdot 10^{-10}$$

$$a_{1} = a\delta b/\omega_{b}$$

$$a_{2} = 1$$

$$a_{3} = \left(\frac{K_{g}}{A} + 1\right)$$
(4.8)

Substituting the corresponding values into equations 4.3 and taking $K_{c} = 7$. This yields

$$K_{o} \frac{a_{o}}{a_{1}} = .539 \cdot 10^{-3}, \text{ for } \delta = 0.6$$

 $\left(K_{o} \frac{a_{o}}{a_{2}}\right)^{1/2} = .246 \cdot 10^{-3}, \text{ and}$

$$\left(\mathbb{E}_{o} \frac{\mathbf{a}_{o}}{\mathbf{a}_{3}}\right)^{1/3} = .363 \cdot 10^{-3}.$$

The smallest value is $.246 \cdot 10^{-3}$, from the second equality of Equations 49. The variation of the variation of the damping ratio, δ , in the first equality of Equations 4.9, which is included in the term a_1 ; between 0.1 < δ < 1.4 yields values of $K_0 \frac{a_0}{a_1}$ that fluctuate between 3.24 \cdot 10⁻³ for a δ = 0.1, gradually decreasing to 0.231 \cdot 10⁻³ for a value of δ = 1.4. Therefore it is acceptable to use $T_c = 0.23 \cdot 10^{-3}$ sec.

The final scaled equation becomes

$$\ddot{x} = - [49.7\delta b \ddot{x}_{p} + 61.6 \dot{x}_{p} + 17.5 x_{p} - 1.75 x_{v}(t_{c})]$$
 (4.9)

The first term to the right of the equality sign of Equation 4.9 contains the damping ratio which can be varied conveniently via a potentiometer on the computer.

Substituting numerical values in Equation 4.5 yields

$$x_v(t_c) = .5 \sin t_c$$
 (4.10)

(4.9)







Figure 22. Response to 0.1 Step Disturbance, $\delta = 0.4$, 0.6.

j.


Figure 23. Response to 0.1 Step Disturbance, $\delta = 0.8 - 1.2$.







_____fi _____

This forcing function equation needs to be scaled for reason already mentioned. A convenient magnitude scale factor is $X = \frac{1}{34.28}$. This yields

$$\varphi = 30 \sin t_{c},$$

$$\dot{\varphi} = 30 \cos t_{c}, \text{ and },$$

$$\dot{\varphi} = -30 \sin t_{c} = -\varphi.$$
(4.11)

The analog computer setup for Equation 4.9 and the generation of the sinusoidal forcing function is shown in Figure 19. Equation 4.9 set up is subjected to a step disturbance in order to determine the closed loop stability conditions. The results are plotted in Figures 20 through 24 for various damping ratios. The experimental results seem to substantiate the response obtained by the analytical methods in Chapter III. From these figures a plot of percent overshoot versus damping ratio, δ , is constructed and can be seen in Figure 25.

4.2 System Simulation

A relay is incorporated to the system as originally described in Figure 11 in order to examine the switching effect of the piston. A forcing sinusoidal signal with amplitude a = 0.5, and frequency of $\omega = 4,356 \frac{\text{Rad}}{\text{sec}}$, respectively, is fed into the system. This sinusoidal signal is integrated as it passes through the system and activates the relay which in turn switches the direction of the voltage.

It may be noted that the systems response to switching is highly dependent on the mechanical characteristics of the relay being

employed. In this case it was noted that the relay was slow in reacting to changing in voltage, probably due the physical gap between the relay contacts. The output desired was displayed on the oscilloscope.

With adequate scaling, it was recorded via the x-y plotter. The original recording can be seen in Figure 26. The damping ratio used for this display was, $\delta = 0.6$. One inch displacement of the plotter marker in x_p direction corresponds to 0.5[in] of the real magnitude; similarly, one inch displacement in the t_c direction corresponds to 1.6 real seconds. From this same figure it can be seen that switching occurs within approximately 0.02228 real seconds. Considering that maximum return transient peak pressure due to sudden stopping of piston, occurs approximately at $\pi/2\omega[\sec]$ after the valve is closed [4], or in this system 0.15 \cdot 10⁻³(sec); it is evident that peak transfent pressure would present in the system. The peaks that can be seen at the end of the vertical strokes of the plotter marker are attributed plotters mass inertia and in no way should be confused with pressure surges, as the plot represent x_p displacement only.

In order to avoid transient pressures in the cylinder, the valve should reverse the flow in the lines at

$$t < \frac{\pi}{2\omega}$$
(4.12)

Controlling deceleration to reduce the peak pressure is not practical. The only other solution, if switching time requirements can not be met, would be to install a relief value between lines.



This would operate just above the supply pressure, P_s . The relief value can serve to by-pass the flow back to the drain port in the event of a sudden pressure surge, and at the same time prevent cavitation on the opposite side of the piston.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

An extensive literature search turned up little or no information on any study on hydraulically powered oscillator systems to drive construction equipment or even to drive light weight tools. On the basis of this study, however, it is found to be quite feasible to implement such a device using hydraulic fluid to drive the tool. In the meantime, it has been established that the need to design quieter tools has never been greater.

The conclusions drawn from this analytical investigation are:

1. A hydraulically operated jackhammer system can attain higher evergies per impact and thus increase the overall performance of the device.

2. The overall physical size of such a device can be decreased by at least 30 percent due to the shorter stroke needed by the piston to attain a maximum velocity.

3. The pneumatic system employs the potential energy stored in a spring for the return stroke of the piston, whereas the proposed hydraulic configuration uses the supply pressure, P_s for the forward and the return stroke of the piston. It is shown that the latter, controlling two oil lines to the cylinder employed only 61 percent of the work energy used to perform the same cycle by the one line spring-return system.

4. Response time in the hydraulic system is increased by a factor of $\sqrt{2}$ over the conventional pneumatic system.

5. The damping ratio, δ is the most significant and important parameter in this hydraulic system as it directly controls stability characteristic of the hydraulic device. This must be kept in mind in the design of any hydraulic valve piston configuration. It is shown that other parameters affect the system stability to lesser degree since very large gain constants are necessary to jeopardize the system stability.

6. The system stability analysis shows that the linearized approximation satisfies more rigid stability criteria than the actual nonlinear model.

On the basis of the study carried out in this investigation it is recommended that further work be conducted along the following lines:

1. A scaled laboratory model of the system analysed needs to be tested for purposes of verifying the analysis, as well as to develop parametric design criteria.

2. This paper increased the leakage through the valve in order to stabilize the system by varying the damping ratio. Investigation towards other methods of optimizing the system performance should be developed, such as valve design, or even an entirely different method of producing the oscillating effect desired. Leakages should be kept small to prevent unnecessary power losses.

3. The computation of transient response due to load was omitted, because in system design it is not critical and of particular interest, as it cannot be modified appreciably by design. Furthermore it does not affect system stability. Pressure transients due to sudden stoppages of fluid by the piston might be an interesting topic to develop. The alternate occurrence of high pressure on one side of the piston and cavitation on the other, leading to deterioration on the corresponding components, can be examined. The analysis of these peak pressure transients can be analysed on an Analog Computer, by solving the pressure-flow equations for each line to the piston. In reference [6] a superficial analysis of these pressure transients are performed, and may be of help as comparison data.

⁴. It may be of interest to examine the adaptability of this type of hydraulic servo-valve and piston combination to other present pneumatic tools, such as nail driver, tool stripper, etc.

5. There is no doubt that the sound pressure level emanating from the hydraulic version of a jackhammer will be considerably lower. It would be interesting to determine just how much indefinite figures, and compare with actual figures. (Appendix A)

6. A different mathematical model to represent the switching effect of system might be used and examined, such as the relay used in this paper, but including a deadband characteristic. Saturation might also be another alternative.

7. Finally, adding to recommendation 1, the laboratory model subjected to different loads (asphalt, concrete, etc.) would enable to determine how these loads affect the frequency of oscillation of the system. Presence of subharmonics could also be determined and analysed as to its effects on the systems performance.

APPENDIX A

FIELD DATA OF SOUND PRESSURE LEVELS ON EXISTING PNEUMATIC JACKHAMMERS

In taking the necessary SPL measurements on the pneumatic jackhammer, the codes set fourth by Reference 1 were followed. The instrument used was a portable sound-level meter (Brüel and Kjaer) with octave-band filter set. Readings in db-A were taken in the near field, at approximately 3.5 feet from the source. Two different measurements were taken, one with the jackhammer actually breaking concrete, and the other with tool held in suspense. The measurements were recorded in Tables 1 and 2, respectively, and all plotted in Figure A.1. It may be noted that no corrections were performed on the original readings to account for background sound or environmental considerations.

The jackhammer was located in an unobstructive space area setting aside any possibility of the meter picking up reflected sound waves. There is little or no difference in S.P.L. noted between the jackhammer breaking concrete and just in suspense. This goes to show that the main source of noise emanates from the air exhaust from the tool.





HZ	OBSPL A	CORRECTIVE	RESPONSE
31.5	. 90	- 39.4	51.6
63	94	- 26	7 2
125	95	- 16.1	79-9
250	95	- 8.6	82.4
500	1.00	- 3.2	98.8
1000	101	0	101
2000	104	+ 1.2	105
4000	102	+ 1	103
8000	97	- 1.1	75.9

Table A.1. Typical SPL of Pneumatic Jackhammer Breaking Concrete.

 \approx Total 109%9 dBA.

Maximum Exposure to this SPL approximately 45 minutes.

ĦŻ	OBSPL	CORRECTIVE	RESPONSE
31.5	89	- 39.4	50.5
63	92	- 26	66.0
125	94	- 16.1	77.9
250	96	- 8.6	87.4
500	97	- 3.2	93.8
1000	97	0	97.0
2000	100	+ 1.2	101.2
4000	104	+ 1	105.0
8000	103	- 1.1	101.9
		·	Total 108.7 dbA.

Table A.2. Typical SPL of Pneumatic Jackhammer No Load.

APPENDIX B

ESTIMATE OF SUPPLY PRESSURE REQUIREMENTS

The Flow Reynolds Number

For a total volume, V_r , of the cylinder with N blows per minute, the volumetric flow rate, Q, is

$$Q = V_{\rm T} \cdot N \tag{B.1}$$

For the present configuration $V_T = 6 \text{ in}^3$ and there are 1,450 blows per minute; this gives a flow rate of 3766 gal/min. The mean velocity in the hose is given by

$$\bar{\mathbf{v}} = 0.408 \frac{Q}{\phi^2}$$
, (B.2)

where ϕ is the hose diameter in inches and Q is in gpm.

Thus, the average velocity in the one inch diameter hose is 15.38 ft/sec. This gives the flow Reynolds number (@90F⁰) of

$$Re = 8,000$$
, (B.3)

indicating turbulent flow.

The supply pressure at the pump exit must provide for the losses in the system. The total pressure at the pump, referring to Figure B.1, is given by

$$P_{T} = \Delta P_{f} + \Delta P_{v} + \Delta P_{p} + \Delta P_{L} , \qquad (B.4)$$



Figure B-1. Overall System and Corresponding Pressure Requirements.

where $P_{\rm up}$ = total required supply pressure at pump,

 ΔP_{f} = pressure drop due to frictional losses, ΔP_{v} = pressure drop across the valve, ΔP_{p} = pressure drop across the piston, and ΔP_{t} = pressure drop across the load.

At the Re of 8000 in the smooth hose, the friction factor, f, from the Moody chart is 0.034. For a hose of 50 feet length this gives a pressure drop of

$$\Delta P_{f} = \frac{f L \rho v^{2}}{\rho \cdot 2} = 27.1 \text{ psi}$$
 (B.5)

The pressure drop due to the value arrangement is estimated by considering a sudden expansion from a hose diameter ϕ of one inch to the value diameter dv of two inch, and a contraction back to the hose diameter.

$$\Delta P_{v} = \rho_{w} \frac{\frac{k_{g}v}{2g}}{2g} , \qquad (B.6)$$

where the loss coefficient $\boldsymbol{K}_{\mathrm{g}}$ for expansion is given by

$$K_{\rm s} \, \exp \, = \left(1 - \frac{\phi^2}{dv^2}\right)^2 \, .$$
 (B.7)

This gives for an cil density of 0.0301 $lbin/in^3$, a pressure loss of

$$\Delta P_{v_1} = 0.748 \text{ psi}, \qquad (B.8)$$

for a sudden expansion, and

$$\Delta P_{v_2} = 0.32 \text{ psi}$$
, (B.9)

for a sudden contraction, where the expression for the loss coefficient K_s for a sudden contraction is given by

$$K_{s \text{ cont}} = \frac{1}{2} \left(1 - \frac{\phi}{dv} \right)$$
 (B.10)

In addition to the losses due to sudden area changes, there is a pressure drop across the value due to flow forces. Let this be denoted by $\Delta P_{\rm V3}$. This can be examined with reference to the following sketch



A balance of forces yields

$$F_{1} = 2 C_{d} C_{v} A_{0} (P_{1} - P_{2}) \cos \theta , \qquad (B.11)$$

where

 $C_{d} = discharge coefficient, and$

$C_v = velocity coefficient.$

The force F_1 is also given by

$$F_1 = -F_j \cos \theta , \qquad (B.12)$$

where F_j is the jet force, which acts normal to the plane of fluid at the vena contracta. This force can also be written as

$$F_{j} = \rho \frac{Q^{2}}{C_{c}A_{o}},$$
 (B.13)

where

$$A_{o} = w \cdot x_{v} , \qquad (B.14)$$

and w is the area gradient of the rectangular port of the valve.



Equation B.11 yields

$$- (P_1 - P_2) = \frac{\rho Q^2}{c_c A_0^2 2 c_d C_v} , \qquad (B.15)$$



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where

$$C_{c} = \frac{C_{d}}{C_{v}} \qquad (B.16)$$

Assigning the values

$$C_d = 0.611$$
 and $C_v = 0.98$ (B.17)

gives

$$C_{c} = 0.62$$
 (B.18)

Equation (B.15) thus becomes

$$- (P_1 - P_2) = \frac{.78 \cdot 10^{-4} \frac{.3}{445^2}}{0.62 \cdot \sqrt{0.785} \cdot 2 \cdot 0.61 \cdot 0.98} = 3.64 \frac{1b}{in^2}$$
(B.19)

Thus $\Delta P_{v3} = 3.64$ psi, and, hence

$$\Delta \mathbf{P}_{\mathbf{v}} = \Delta \mathbf{P}_{\mathbf{v}_{1}} + \Delta \mathbf{P}_{\mathbf{v}_{2}} + \Delta \mathbf{P}_{\mathbf{v}_{3}} = 4.708 \text{ psi} . \tag{B.20}$$

Pressure drop due to leakage through value is negligible as it is of the order O(0.1) and does not affect ΔP_v significantly.

Simular calculations of pressure drops due to sudden changes in cross sectional area in the piston yields

$$\Delta P_{p_{1}} = 0.843 \text{ psi}$$
, (B.21)

for sudden expansion from connecting hose of diameter, $\phi = \mathbf{1}^n$ to the

cylinder diameter D = 232". Also for contraction from the cylinder to the hose

$$\Delta P_{p_2} = 0.328 \text{ psi} . \tag{B.22}$$

Losses due to leakage flow through piston and cylinder must be considered. The flow configuration is similar to a fully developed laminar flow between two parallel plates, the bottom plate moving at a uniform velocity while the top plate remains stationary, as shown by the sketch.



the equation of motion is

$$\mu \frac{dv}{dy} = \frac{dP}{dx} y + C , \qquad (B.23)$$

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integrating with respect to y and assuming velocity of fluid at the boundary equal to the velocity of the moving plate, the conditions

$$v = V @ y = h$$
, and
 $v = 0 @ y = 0$. (B.24)

The velocity profile is obtained as a function of dP/dx as

$$V_x = \frac{V_p Y}{h} - \frac{1}{2\mu} \frac{dP}{dx} (hy - y^2)$$
, (B.25)

where

 μ = fluid viscosity,

x = distance along passage,

h = height of passage,

 $L_{o} = passage length,$

 V_p = velocity of piston, and V_x = velocity of fluid.

The flow rate through the passage is

$$Q = \int_{\Omega}^{h} V x dy = \frac{V p h}{2} - \frac{h^3}{12\mu} \frac{dP}{dx}$$
(B.26)

The instantaneous velocity of the piston, V_p , is determined approximately assuming no losses in velocity due to friction. This yields $V_p = Q/A_p = 48.3$ in/sec. Assuming a working pressure of 2000 psi, $L_o = 2$ inches, and piston circumference

$$C = \pi D = 6.9 \text{ in.}$$
, (B.27)

Equation B.26 yields

$$Q_{\text{leakage}} = 0.148 \frac{\text{in}^3}{\text{sec}}$$
(B.28)

It is now necessary to determine the pressure drop through the clearance. Use of Equation

$$\frac{\mathrm{dP}}{\mathrm{dx}} = \frac{12\mu Q}{\pi \mathrm{Dh}^3 \left[1 + 1.5 \left(\frac{\mathbf{e}}{\mathrm{b}}\right)^2\right]}, \qquad (B.29)$$

which describes a steady flow in annulus between circular piston and cylinder. Assuming, ϵ , the eccentricity to be zero, Equation B.29 becomes by the use of Equation B.28 for Q,

$$= \frac{dP}{dx} = 463 \frac{1b}{in^3}$$
 (B.30)

The shearing stress between two moving plates separated by a small clearance is given by

$$-\tau_{s} = \mu \frac{V}{h} + \frac{h}{2} \frac{dP}{dx} , \qquad (B.31)$$

Taking $\frac{dP}{dx}$ from Equation B.30 and substituting into Equation B.31 and taking numerical values gives

$$-\tau_{c} = 0.318 \text{ psi}$$
 (B.32)

Thus $\Delta P_{p3} = 0.318$ psi, and hence

$$\Delta P_{p} = \Delta P_{p1} + \Delta P_{p2} + \Delta_{p3} = 1.489 \text{ psi}$$
 (B.33)

To calculate the pressure requirement at the load, it is necessary to estimate the energy per impact.

The numerical values for the following calculation are taken from Reference 2. For an impact energy of 200 ft-lb, with, a piston displacement of $x_p = 1$ inch, and piston area of $A_p = 3 \text{ in}^2$. The load pressure is

$$P_{L} = \frac{E \cdot 12}{X + A} = 800 \text{ psi}$$
 (B.34)

To determine the maximum power transfer to the load with a servovalve controlled actuator, consider a critically centered valve, where the flow rate is given by

$$Q_{L} = Cd w x_{v} \left[\frac{1}{\rho} \left(P_{S} - P_{L}\right)\right]^{1/2} . \qquad (B.35)$$

The horsepower delivered at the load is

$$hP = P_{L}Q_{L} = CdWx_{v} \left[\frac{1}{\rho} (P_{S} - P_{L})\right]^{1/2} \cdot P_{S} \frac{P_{L}}{P_{S}} , \qquad (B.36)$$

the maximum of which is found by taking the derivative of Equation B.36 with respect to $P_{T_{L}}$ and setting it equal to zero. This yields

$$P_{\rm L} = \frac{2}{3} P_{\rm S}$$
, (B.37)

provided that the load is relatively constant over the complete cycle. Although it is possible that high acceleration rates during sudden transients can cause P_L to exceed 2/3 P_S , these conditions are

most likely for short duration and need not be taken into consideration.

Therefore, the required supply pressure is obtained by combining Equations B.37 and B.34 to yield

$$P_{\rm S} = 1200 \, \frac{1b}{{\rm in}^2} \, .$$
 (B.38)

In summary the major losses in pressure are $\Delta P_f = 27.15 \frac{1b}{1n^2}$, the minor losses, or losses in valving, $\Delta P_v = 4.7 \frac{1b}{1n^2}$; and losses in piston, $\Delta P_p = 1.49 \frac{1b}{1n^2}$. The total required supply pressure at the pump exit is, therefore

 $P_{s \text{ pump}} = 1200 + 27.15 + 4.7 + 1.49 = 1233.4 \frac{1b}{in^2}$ (B.39)

APPENDIX C

DEVELOPMENT OF PRESSURE-FLOW CURVES FOR THE SERVO-VALVE

Pressure-Flow Curves

The general form for the orifice equation is obtained by combining Bernoulli's and continuity equation to yield

$$Q = C_{d} A_{o} \sqrt{\frac{p}{\rho}} P - P_{o} , \qquad (C.1)$$

where

• •

....

b

3

1.41

 C_d = discharge coefficient, A_o = vena contracta area, P = pressure upstream, P_o = pressure downstream, and, ρ = mass density.

Referring to Figure C.1 and applying Equation C.1, the general equation for the pressure-flow curves of a four way ideal critical center valve with matched, symmetrical orifices and rectangular ports is:

$$Q_{L} = C_{d} W_{x_{v}} \sqrt{\frac{1}{\rho} \left(P_{S} - \frac{x_{v}}{|x_{v}|} P_{L}\right)}, \qquad (C.2)$$

where w = area gradient for each port.



In order to develop a more suitable nondimensionalized

equation for the pressure-flow relationship and plot in a coordinate system, the following analysis need to be performed. From Figure C.1

$$P_{L} = P_{1} - P_{2}$$
, and, (C.3)

$$P_{\rm S} = P_1 + P_2$$
 (C.4)

Solving Equations C.3 and C.4 simultaneously to yield

$$P_{1} = \frac{P_{S} + P_{L}}{2} , \qquad (C.5) .$$

$$P_2 = \frac{P_S - P_L}{2} , \qquad (C.6)$$

and expressing $\mathbf{P}_{\rm L}$ in terms of $\mathbf{P}_{\rm S}$ and $\mathbf{P}_{\rm 2}$ to yield

$$P_{\rm L} = P_{\rm S} - 2P_2$$
, (C.7)

which is the expression sought. In order to express Equation C.7 terms of P_S and a flow through the value orifice, the following equations are obtained referring to Figure C.1.

$$Q_1 = C_d A_1 \sqrt{\frac{2}{\rho} (P_1 - P_0)}$$
, (C.8)

$$Q_2 = C_d A_2 \sqrt{\frac{2}{\rho}} (P_s - P_1)$$
, (C.9)

$$Q_3 = C_d A_3 \sqrt{\frac{2}{\rho}(P_s - P_2)}$$
, (C.10)

$$Q_{4} = C_{d}A_{4} \sqrt{\frac{2}{\rho}(P_{2} - P_{0})}$$
, and, (C.11)

usually the drain pressure $\boldsymbol{P}_{_{\boldsymbol{O}}}$ is zero.

By multiplying and dividing the second term to the right of the equality sign in Equation C.7 by $A_{j4}C_{\rm d}~\sqrt{\frac{2}{\rho}}$, and rearranging to yield

$$P_{L} = P_{S} - 2 \frac{\left(A_{4}C_{d} \sqrt{\frac{2}{\rho}} \sqrt{P_{2}}\right)^{2}}{\left(A_{4}C_{d} \sqrt{\frac{2}{\rho}}\right)^{2}} , \qquad (C.12)$$

Substituting the expression of Equation C.11 into Equation C.12 to get

$$P_{\rm L} - P_{\rm S} - \frac{2Q_{\rm h}^2}{(C_{\rm H})^2}$$
, (C.13)

where

$$C_{\rm H} = C_{\rm d} A \sqrt{\frac{2}{\rho}} \quad . \tag{C.14}$$

Nondimensionalizing Equation C.13 by using

$$\mathcal{P}_{L} = \frac{P_{L}}{P_{S}}$$
, and, (C.15)

dropping the subscription on Q

$$\varphi_{\rm L} = \frac{Q}{C_{\rm d}^{\rm A} \sqrt{\rho} \sqrt{\rho_{\rm S}^{\rm P}}} . \qquad (C.16)$$

Substituting Equations C.15 and C.16 into Equation C.13 to

obtain

$$\mathcal{P}_{\rm L} = 1 - \frac{2\varphi_{\rm L}^2}{\gamma^2}$$
, (C.17)

where

17²⁷

$$\gamma = \frac{A}{A_{o}} = \frac{w_{X}}{w_{X}}$$
(C.18)

Equation C.17 represents a family of parabolas. A computer program for solutions to Equation C.17 is written up, and the results can be seen plotted on Figure C.2.





APPENDIX D

DEVELOPMENT OF EQUATIONS

i.

Details of Derivation of Equations 2.13, 2.16 and 2.55.

Equation 2.12 gives the Laplace transform of the piston

$$Z(s) = \frac{\frac{P_{s} A_{p}}{\sqrt{M}}}{\sqrt{M}} \frac{1}{s^{2} + \frac{P_{s} A^{2}}{Q_{s} M} s + \frac{k}{M}}$$
(D.1)

This may be rearranged as

$$Z(s) = \frac{\frac{P_{s}}{p}}{\sqrt{M}} \frac{1}{\left(s + \frac{\frac{P_{s}}{s}}{\frac{P_{s}}{p}}\right)^{2} + \left[\frac{K}{M} - \left(\frac{\frac{P_{s}}{s}}{\frac{P_{s}}{2Q_{s}}}\right)^{2}\right]},$$
 (D.2)

and further rearrangement

$$Z(s) = \frac{P_{s}A_{p}}{\sqrt{M}} \frac{\sqrt{\frac{P_{s}A_{p}^{2}}{2Q_{s}M}}}{\sqrt{\frac{K}{M} - (\frac{P_{s}A_{p}^{2}}{2Q_{s}M})^{2}}} \left[\left[s + \left(\frac{P_{s}A_{p}^{2}}{2Q_{s}M}\right)^{2} + \left(\frac{K}{M} - \left(\frac{P_{s}A_{p}^{2}}{2Q_{s}M}\right)^{2}\right] \right] \right]$$
(D.3)

This equation can be transformed by use of the Laplace transform tables according to the form

$$f(s) = \frac{k}{(s+2)^2 + k^2}$$
, $F(t) = e^{-2t}$ sinkt. (D.4)

 \mathbf{or}

$$z(t) = \frac{P_{s}A_{p}}{\sqrt{K_{s} - M(\frac{P_{s}A_{p}^{2/2}}{2Q_{s}M})}} e^{-\frac{P_{s}A_{p}^{2}}{2Q_{s}M}T} sin\sqrt{\frac{K}{M} - (\frac{P_{s}A_{p}^{2/2}}{2Q_{s}M})} T$$
(D.5)

ii. Derivation of Equation 2.16 taking Equation 2.15

$$Z(s) = \frac{\frac{A_{p}P_{s}}{\sqrt{M}}}{s^{2} + 2\sqrt{\frac{K}{M}} s + \frac{K}{M}} , \qquad (D.6)$$

rearranging

$$Z(s) = \frac{A_p P_s}{\sqrt{M}} \cdot \frac{1}{\left(s + \sqrt{\frac{K}{M}}\right)^2} \quad . \tag{(b)}$$

Taking the Laplace transform according to the form

$$f(s) = \frac{n!}{(s-a)^{n+1}}$$
; $f(t) = t^n e^{at}$, (D.8)

where n = 1,2,3 This yields

$$z(T) = \frac{A_{p}P_{s}}{\sqrt{M}} \cdot T e^{-\sqrt{\frac{K}{M}}T} . \qquad (D.9)$$

iii. Development of Equation 2.55.

A combination of Equations 2.27, 2.33, 2.35 in terms of the time T at impact yields,

$$T = \frac{W}{\sqrt{PE2KE}} \quad (D.10)$$

Substituting Equation D.10 and 2.33 into Equation D.9, and squaring yields

$$[z(T)]^{2} = \frac{W^{2}}{\tau^{2} 2 KE} e^{-\frac{2W}{\sqrt{PE2KE_{T}}}}.$$
 (D.11)

Using the fact, that for critical damping the expression

$$\sqrt{PE} = 2\sqrt{2kE}$$
 , (D.12)

and by further combining with Equation 2.49 to simply Equation D.11 to yield

$$[z(T)]^2 = \frac{W}{T} e^{-1} . \qquad (D.13)$$

APPENDIX E

COMPUTER PRINTOUT

Computer Print Outs and Programs are available upon request to Dr. P. V. Desai, Room 204, Space Science Technology Building #1.

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REFERENCE VALUE OF PARAMETERS

A _o	=	.785	[in ²]
A p	=	3.	[in ²]
B. P. M.	=	1450	
Bp	=	12.4410 ⁻⁶	[lb-sec/in]
B _s	=	5.66 * 10 ⁻⁶	[lb-sec/in]
в	=	18.1×10^{-6}	[lb-sec/in]
D	=	2.2	[in]
Е	=	200	[ft-lb]
Kc	=	variable	
K _c	=	Variable	
к _р	=	$1.1 * 10^{-6}$	[psi/in]
K g	=	3,766	[in ³ /sec/in]
Lo	=	2.	[in]
м _т	=	4	lb.in
Ps	=	1.200	[lb/in ²]
\$	=	37.66	GPM
v _T	=	3	[in ³]
g	=	386.4	[in/sec ²]
h	=	10 ⁻³	[in]
rc	=	0.0002 (standard)	[in]
W	=	.785	[in ²]
x _p	=	1	[in]

β	= 100,000
*p	$= 0.78 \cdot 10^{-4}$
۴μ	$= 1.8 \cdot 10^{-6}$
ω	= 10,766



*Values taken from Reference 4. Petroleum base fluids.

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