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RESEARCH PROJECT INITIATION

Date: April 11, 1973

Project Title: Recovery of Fleets Based on Statistical Methods, Fracture Mechanics & Fatigue Theories

Project No: E-16-631

Principal Investigator Dr. S. V. Hanagud

Sponsor: NASA - Langley Research Center; Hampton, Virginia

Agreement Period: From April 1, 1973 Until March 31, 1974

Type Agreement: Grant No. NGR 11-002-169

Amount: $12,494 - NASA Funds (E-16-631)
1,238 - GIT Contribution (E-16-329)
$13,732 - Total

Reports Required:
Semi-Annual Status Report; Final Technical Report

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Project No: E-16-631

Project Director: Dr. S. V. Hanagud   Grant NGR 11-002-169

Sponsor: NASA - Langley

Effective Termination Date: 6/30/76

Clearance of Accounting Charges: 6/30/76

Grant/Contract Closeout Actions Remaining:

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DANIEL GUGGENHEIM SCHOOL
OF AERONAUTICS

RECOVERY OF FLEETS BASED ON STATISTICAL METHODS, FRACTURE MECHANICS AND FATIGUE THEORIES

Semi Annual Status Report
NASA Grant NGR-11-002-169
Georgia Tech Project No. E-16-631

October 31, 1973

Principal Investigator: Dr. S. Hanagud
School of Aerospace Engineering

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I. Maximum Method of Parameter Estimation

Research program on NASA Grant NGR-11-002-169 was initiated during April 1973. The first step in the analysis was to investigate if the two parameter Weibull distribution can be used model the available inspection data on a specific fleet of aircraft provided by NASA.

In order to carry out this investigation a computer program has been written to estimate the parameters of a 2-parameter Weibull distribution by the maximum likelihood method from the available inspection data on fatigue cracks. The Weibull density function [1]* is as follows.

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right] \tag{1}$$

where α and β are shape and scale parameter respectively. Following Cohen [2] the density function can also be rewritten as follows:

$$f(x) = \frac{\alpha}{\theta} x^{\alpha-1} \exp\left(\frac{x}{\theta}\right)^{\alpha} \tag{2}$$

where $\theta = \beta^{\alpha}$.

Equation (2) results in a computer program which takes less time. Maximum likelihood equations derived from (1) and (2) are equivalent. Appendix I shows the Maximum Likelihood Equations and the computer programs for two cases. In Case I all observed data points in a sample are failure points. In Case II some data points of the sample may represent failure at certain number of hours while others may not have suffered any failure during the observed period. α and β obtained from the program should be corrected for finite sample size. The parent distributions can be obtained by following order statistics procedure similar to Eggwertz [3]. The computer programs have been checked out for several samples from References [2] and [3]. This version of the program is in Fortran IV applicable to UNIVAC computer available at Georgia Institute of Technology.

* Numbers inside [ ] indicate references.

II. Goodness of fit test

Two computer programs have been written to test the goodness of fit. A preliminary version of these programs can be seen in Appendix II. Program A can be used to calculate the statistic needed in χ^2 test and Program B is for Kolmogorov-Smirnov test. Even though Kolmogorov-Smirnov test is strictly valid only when the model is hypothesized wholly independently of the data the test if often used [4]* for cases in which parameters have been estimated from the same data. The computer programs can be used to calculate the needed statistic. Comparison, acceptance or rejection of the hypothesis will have to be done manually.

Appendix III contains the listing of the computer programs which have been rewritten in basic language for use in Hewlett Packard Computer HP2115 which is available in the School of Aerospace Engineering. This change was done to have flexibility in usage time and reduction in computer costs. This program is capable of estimating the Weibull parameters from the supplied crack data on time to observation of cracks and perform the appropriate goodness of fit tests without any necessity of reprogramming.

III. Analysis of Fatigue Data from 'B' Series

Before using the computer programs to estimate the parameters by the Maximum Likelihood Method, graphical procedures have been used to estimate the parameters of Weibull distributions for time to first failure. In addition to providing the approximate range for the numerical values to be expected in the output of computer programs which use more accurate methods, these graphical procedures give a better feeling for the data to be studied and the range of outliers to be considered.

Table No. 1 illustrates the ordering of the data as needed in graphical procedures. The first column corresponds to the number of hours to first failure of each of the airplanes in the observed fleet. These numbers represent the number of hours to the observation of first crack in each aircraft irrespective of the location of the crack. Column 2 shows its order as counted from the lowest number of hours to failure. Table No. 2 shows data selected for plotting. This represents every seventh point in

* Numbers inside [ ] indicate references.

the ordered data. This selection was necessary to keep the plots from becoming over crowded. Figure 1 illustrates the plot of data points in column 2 of Table No. 2 versus the percent failed on a Weibull paper. Most of the data appear to fit the Weibull distribution with the exception of points marked X. These points are considered to be "outliers" and as those belonging to a different sample. The term "outlier" has been used to denote those points where failure has been observed but belong to a different sample. The term "runout" will be used to describe the portion of the data that do not represent failure in the observed period.

This graphical plot suggests that a suitable Weibull distribution can be obtained by censoring those points which represent failure above 6000 hours or below 3000 hours. Outliers in this case are then that portion of the data which correspond to failure above 6000 hours or below 3000 hours. Detailed calculations by MLE method and the subsequent chi-square and Kolmogorov goodness of fit tests confirm the significance of these observations. Figure 2 illustrates a plot of the data excluding outliers on a Weibull paper with an expanded scale. The values for estimated parameters are $\alpha = 9.3$ and $\beta = 4670$. Because these values of α appeared to be larger than the value of 4 used by other investigators, the graphical procedure was repeated on different scales. These values of α, approximate as they are, were found to lie in the range of 8.85 to 9.3.

Furthermore, figure 1 shows that the data may be approaching an asymptote of 3400 hours if the outliers are censored. Then a three parameter Weibull model may be more suitable. Figure 3 illustrates the plot of the same data on Weibull paper with 3400 as the lower bound location parameter in the three parameter Weibull distribution. The result was still not a perfect straight line on Weibull paper. Three other lower bound location parameters were tried as shown in Figures 4 and 5. The best fit was for location parameter of 2500 hours, α equal to 3.58 and the value of β equal to 4620 hours. Figure 6 illustrates the comparison of two and three parameter Weibull cumulative density functions as obtained from the same data. Both distributions have very nearly the same C.D.F. plot. Outliers have been censored in both cases.

IV. Maximum Likelihood Method of Parameter Estimation and Goodness of Fit Tests

An examination of the inspection data on B series reveals that time to first failure listed in Table No. 1 need not be always at the same location. Of the 92 locations selected in the center box wing for inspections, some locations such as stations 89, 90, 91, 92, 73, 74, 75, and 76 exhibit very frequent failure and are responsible for many observed first failures. Some stations (1, 2 etc.) exhibit very little failure.

In addition, some airplanes assigned to particular bases exhibit more frequent failure when compared to other base assignments. This can be attributed to the type of use in that particular base. For a thorough analysis the data were considered in two sets. Set A consists of all the airplanes irrespective of the assigned bases. Set B consists of only those airplanes assigned to Clark Air Base.

In terms of location on the center box wing the following sets were considered:

1. First failure of the center box wing irrespective of the location.
2. First failure at any one of the location stations 89 to 92.
3. First failure at station 89.
4. First failure at station 90.
5. First failure at station 91.
6. First failure at station 92.
7. First failure at any one of the location stations 73 to 76.
8. First failure at any one of the location stations 9 to 10.
9. First failure at any one of the location stations 11 to 16.
10. First failure at stations 18 to 28.
11. First failure at stations 48 to 48.
12. First failure at stations 3 to 8.

Table No. 3 illustrated the following information: Shape parameter and scale parameter for each set (A or B) and location selection. Also indicated in the table are the number of degrees of freedom for χ^2 test, acceptable level for χ^2 test, decision regarding acceptance of rejection, and the acceptable level for Kolmogorov-Smirnov tests. This table does not contain all the results because we are still in the process of investigation. The results in these tables include runout data points which could not be considered by the graphical method, but could be included in the MLE procedures.

The following conclusions can be drawn from the results:

1. Even with base restriction (Set B) 2 parameter Weibull distribution should be rejected when the high outliers (data greater than 6000 hours censored) are included in the analysis.
2. Two parameter models with censored high outliers are acceptable.
3. Models with censored outliers at high and low level (data greater than 6000 hours and less than 3000 hours censored) show better acceptability.
4. In the acceptable distribution α and β are close to the range suggested by graphical analysis (α = 8.4 to 9.2 and β in the range of 4500 to 5000).
5. Selected stations 89 to 92 and 73 to 76 have values of α and β very close to that for the entire fleet, but exhibit some scatter.
6. Most of the acceptable values of α are in the range of 8 and 9.3. compared to the value of 4 used by other investigators.

Final conclusions will be drawn after completing the calculation at other locations and groupings based on the base assignments. The following different classification of the crack data will be considered. Classifications are described for B series data.

1. B (complete data)
2. B (classified by bases and groups of bases)
3. B (classified by stations)
4. B (classified by stations, bases)
5. B (classified by censoring)
6. B (classified by stations, censoring)
7. B (classified by stations, bases and censoring)
8. B (classified by runout considerations)
9. B (classified by bases and runouts)
10. B (classified by stations and runouts)
11. B (classified by stations, bases, runouts, and censoring)
12. B (three parameter models)

V. Consideration of Observed Data which Include Cracks of Varying Length

Usually inspection data such as those on B series are used to obtain the parameters of the hypothesized distribution for time to first failure. Data to be used are number of hours at which the first cracks are observed in different aircraft of the fleet. However, these cracks are usually of varying lengths. At present these varying lengths are not taken into analysis. A simple model has been proposed to take into account varying lengths of observed cracks. This needs the consideration of crack initiation and growth.

It will be assumed that cracks can grow in quantum lengths of magnitude ΔL. Depending on the problem we can assign the value of ΔL. $P(t;k)$ is defined as the probability that at time t we have a crack of length k (ΔL). As a first step in the analysis let us consider the probability that at time $t + \Delta t$ the length of crack is $k(\Delta L)$. This probability can be denoted by $P[t+\Delta t;k]$. At time $t + \Delta t$ we can have a length of crack $k(\Delta L)$ in the following two ways:

A. The length of the crack was $(k-1)\Delta L$ at time t and the length of the crack increased by ΔL during Δt. Then the length of the crack at $(t + \Delta t)$ will be $k(\Delta L)$.

B. The length of the crack was $k(\Delta L)$ at time t and the length of the crack did not increase during Δt. Then, again, the length of the crack will be $k(\Delta L)$ at time $(t + \Delta t)$.

Δt is assumed to be so short that these are the only possibilities and the two events are exhaustive and mutually exclusive. Then,

$$P[t + dt;\ k(\Delta L)] = P(A) + P(B) \tag{3}$$

As a very simple assumption let the probability of rate of crack growth be ν. This will be modified to consider ν to be a function of its present length and other parameters.

P(A) requires that the length of the crack be $(k-1)\Delta L$ at t and the length increases by ΔL in Δt. P(A) is then the joint occurrence of these events.

Probability of crack being of length $(k-1)\Delta L = P[t;\ (k-1)\Delta L]$

Probability of length increasing by ΔL in time $\Delta t = \nu\Delta t$

$$P(A) = \nu dt\ P[t;(k-1)] \tag{4}$$

Similarly

$$P(B) = (1 - \nu dt)\ P[t;k] \qquad (5)$$

and

$$P[t+\Delta t;k] = \nu\Delta t\ P[t;k-1] + (1 - \nu\Delta t)\ P[t;k] \qquad (6)$$

that is,

$$\frac{P[t+\Delta t;k] - P(t;k)}{\Delta t} = \nu\ P[t;\ k-1] - \nu\ P[t;k] \qquad (7)$$

Taking limits as Δt tends to zero and denoting

$$\frac{dP[t;k]}{dt} = P'[t;k], \qquad (8)$$

$$P'(t,k) = \nu\ P[t;k-1] - \nu\ P[t;k]\ . \qquad (9)$$

The case k = 1 (length of the crack is ΔL) is considered to be crack initiation. k = 1 is possible in the following two ways:

A_1: crack did not exist at t and a crack appeared during Δt

B_1: a crack of length $1(\Delta L)$ did exist at t and crack did not grow during Δt.

$P(A_1) = P$ (crack initiation or time to first failure)

$P(B_1) = (1 - \nu\Delta t)\ P(t;k)$

$P(t+\Delta t;1) = f_c(t)\Delta t + (1-\nu\ \Delta t)\ P(t;k)$

$f_c(t)$ is the Weibull or equivalent density function for crack initiation.

$$\frac{P'(t+\Delta t;1) - P(t;1)}{\Delta t} = f_c(t) - \nu\ P(t;1) \qquad (10)$$

$$P'(t;1) + \nu P(t,1) = f_c(t) \qquad (11)$$

Other equations are as follows:

$$P'(t;2) + \nu(t;2) = \nu\ P(t;1) \qquad (12)$$

$$P'(t,n) + \nu\ P(t,N) = \nu\ P(t;N-1) \qquad (13)$$

These are similar to the equations for simple stream in Queing Theory. Further work involves solution of these equations and application of the results to observed data.

VI. Work In Progress

(1) Further work on parameter estimation and goodness of fit for B series data by using MLE and related computer programs.

(2) Analysis of E series data by use of Maximum Likelihood Method and graphical procedures.

(3) Alternate ways of deriving the equations for consideration of cracks of varying sizes, their solutions in simple cases and applications.

(4) Development of game plans for future inspection will be considered after completing data analysis as outlined in items 1 and 2.

References

1. Whittaker, I. C. and Besuner, P. M., "A Reliability Analysis Approach to Fatigue Life Variability of Aircraft Structures," Technical Report AFML TR-69-65, 1969.

2. Cohen, A. D., "Maximum Likelihood Estimation in the Weibull Distribution Based on Complete and Censored Samples," Technometric Vol. 7, No. 6, 1965, pp. 579-588.

3. Eggwertz, S., "Investigation of Fatigue Life and Residual Strength of Wing Panel for Reliability Purposes," Probabilistic Aspects of Fatigue, ASTM STP 511, American Society of Testing of Materials, 1972, pp. 75-105.

4. Benjamin, J. R. and Cornell, A. C., "Probability, Statistics and Decision for Civil Engineers," McGraw-Hill, 1970.

5. Tribus, M., "Rational Descriptions, Decisions and Design," Pergamon Press, 1969.

TABLES

TABLE 1

Ordered Data (Hours to First Crack)

Number of Hours To Failure	Order Number	Number of Hours To Failure	Order Number
0,468	1	4,349	45
1,887	2	4,352	46
2,741	3	4,384	47
2,768	4	4,387	48
3,430	5	4,392	49
3,433	6	4,403	50
3,464	7	4,450	51
3,467	8	4,468	52
3,663	9	4,484	53
3,686	10	4,516	54
3,750	11	4,516	55
3,757	12	4,519	56
3,772	13	4,539	57
3,773	14	4,542	58
3,791	15	4,548	59
3,803	16	4,557	60
3,818	17	4,574	61
3,858	18	4,574	62
3,877	19	4,587	63
3,888	20	4,594	64
3,954	21	4,594	65
4,005	22	4,618	66
4,011	23	4,619	67
4,059	24	4,623	68
4,091	25	4,630	69
4,096	26	4,653	70
4,108	27	4,656	71
4,111	28	4,657	72
4,147	29	4,666	73
4,148	30	4,682	74
4,148	31	4,690	75
4,162	32	4,691	76
4,190	33	4,738	77
4,191	34	4,818	78
4,196	35	4,831	79
4,196	36	4,851	80
4,224	37	4,853	81
4,238	38	4,880	82
4,239	39	4,888	83
4,246	40	4,904	84
4,258	41	4,914	85
4,273	42	4,918	86
4,283	43	4,921	87
4,292	44	4,927	88

TABLE 1 Cont.

Number of Hours To Failure	Order Number	Number of Hours To Failure	Order Number
5,043	89	6,189	98
5,062	90	6,197	99
5,099	91	6,277	100
5,142	92	6,292	101
5,154	93	6,406	102
5,198	94	6,448	103
5,396	95	6,455	104
5,927	96	6,884	105
6,023	97		

Table No. 2: Plotted Data

1	2	3	4	5	6	7	8
Hours	Order Number	% Failed	Hours 2500	Hours 2800	Hours 3400	Hours 3200	Hours 3000
468	1	0.74					
1,887	2	1.88					
2,741	3	2.83					
2,768	4	3.77					
3,430	5	4.71	930	630	30	230	430
3,433	6	5.66	933	633	33	233	433
3,464	7	6.6	964	664	64	264	464
3,773	14	13.2	1273	973	373	573	773
3,954	21	19.8	1454	1154	554	754	954
4,111	28	26.4	1611	1311	711	911	1111
4,196	35	33.0	1696	1396	796	996	1196
4,273	42	39.6	1773	1473	873	1073	1273
4,392	49	46.2	1892	1592	992	1192	1392
4,519	56	52.8	2019	1719	1119	1319	1519
4,587	63	59.4	2087	1787	1187	1387	1587
4,653	70	66.0	2153	1853	1253	1453	1653
4,738	77	72.6	2238	1938	1338	1538	1738
4,904	84	79.2	2404	2104	1504	1704	1904
5,099	91	85.8	2599	2299	1699	1899	2099
6,189	98	92.5					
6,884	105	99.0					

TABLE NO. 3. Two Parameter Weibull Model for B Series

Base Assignment Clark only (B) Entire Fleet (A)	Outliers Censored	Stations	Shape Parameter by MLE Method	Scale Parameter by MLE Method	Number of d.o.f (χ^2-test)	Accept or Reject at 5% (χ^2-test)	Acceptable level (χ^2-test) % sig.lev.	Accept or Reject Kol-Smir test (at 10%)	Acceptable level Kol-Smir test	Runout Data Included
A	None	1st crack at any location	5.085	4828	7	Reject	—	Reject	—	Yes
A	None	"	"	"	12	Reject	0.007%	Reject	—	Yes
A	>6000	"	7.784	4497	7	Accept	42.8%	Accept	>10%	No
B	None	"	5.644	4997	7	Reject	0.004%	Reject	1.0%	Yes
B	>6000	"	9.045	4738	7	Accept	≈10%	Accept	>10%	No
B	{<3000 >6000}	"	8.366	4727	&	Accept	≈10%	Accept	>10%	No
B	<4000	"	4.216	4734	7	Reject	—	Reject	—	Yes
A	None	9-10	2.694	8.988	}					
B	None	9-10	2.9	10.166	} Work	to be	completed			
A	None	61-72	3.104	9.989	}					
A	None	48-58	2.235	11.211	7	Reject	—	Reject	—	Yes
A	>5500	48-58	7.841	4502	7	Accept	24%	Accept	>10%	No
A	None	89-92	3.219	6006	7	Reject	—	Reject	—	Yes
B	None	89-92	6.666	5003	7	Reject	≈0.1%	Reject	1%	Yes
A	None	73-76	3.306	5906	7	Reject	—	Reject	—	Yes
A	>7000	73-76	5.378	4885	7	Reject	0.001%	Reject	—	Yes
A	>6000	73-76	11.641	4542	7	Accept	40%	Accept	>10%	No
A	None	11-16	2.236	11768	7	Reject	—	Reject	—	Yes
A	>6000	11-16	6.358	4666	7	Accept	45%	Accept	>10%	No
A	None	18-28	2.728	10436	7	Reject	—	Reject	—	Yes
A	>6000	18-28	9.129	4676	7	Accept	1%	Accept	>10%	No
A	None	3-8	2.604	11074	7	Reject	—	Reject	—	Yes
A	>5000	3-8	8.446	4865	7	Accept	18%	Accept	>10%	No

FIGURES

BY.......... DATE.......... SUBJECT.......... SHEET NO. ..... OF .....
CHKD. BY.......... DATE.......... JOB NO. ....

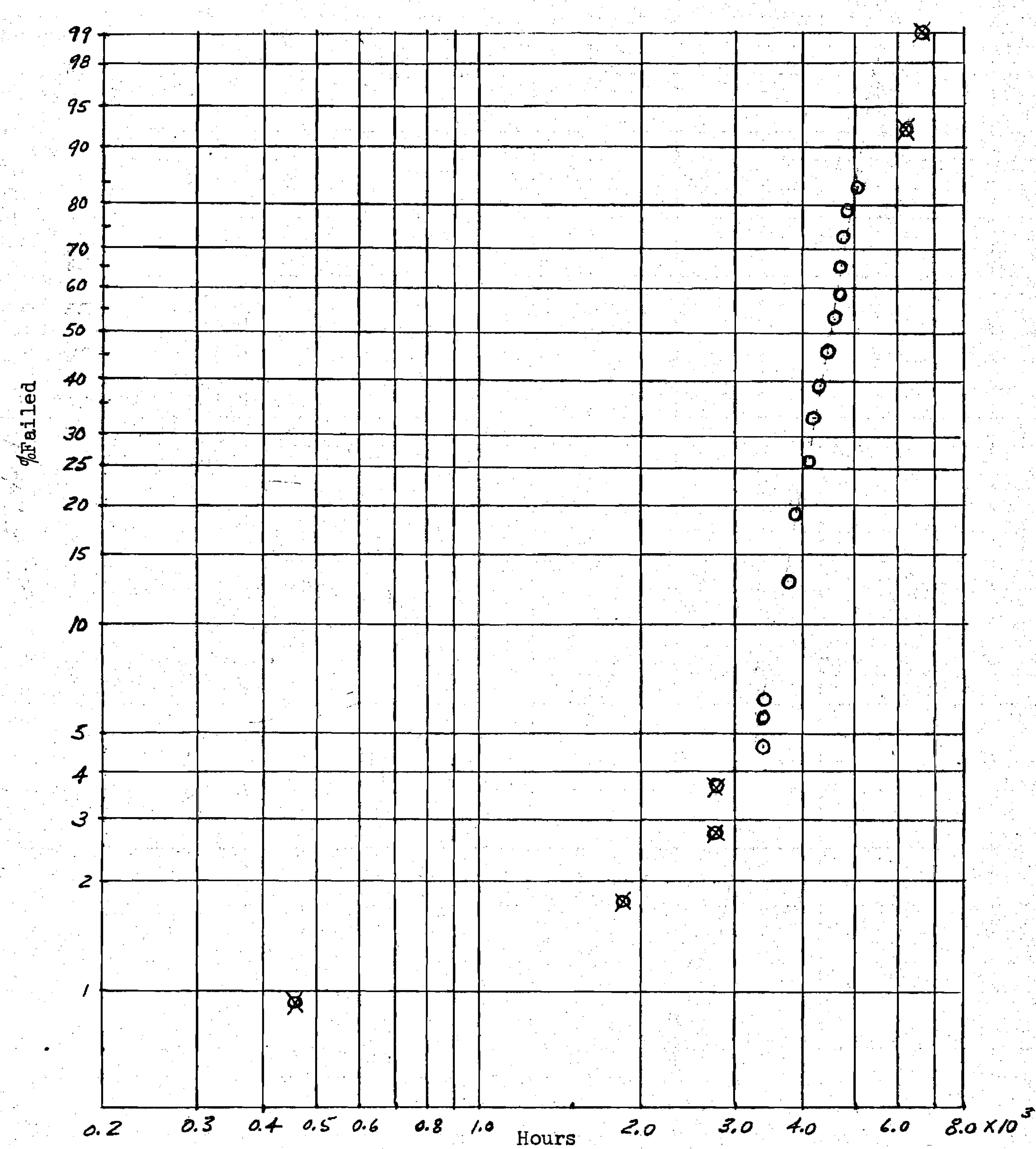

Figure 1. Weibull plot of B series data.

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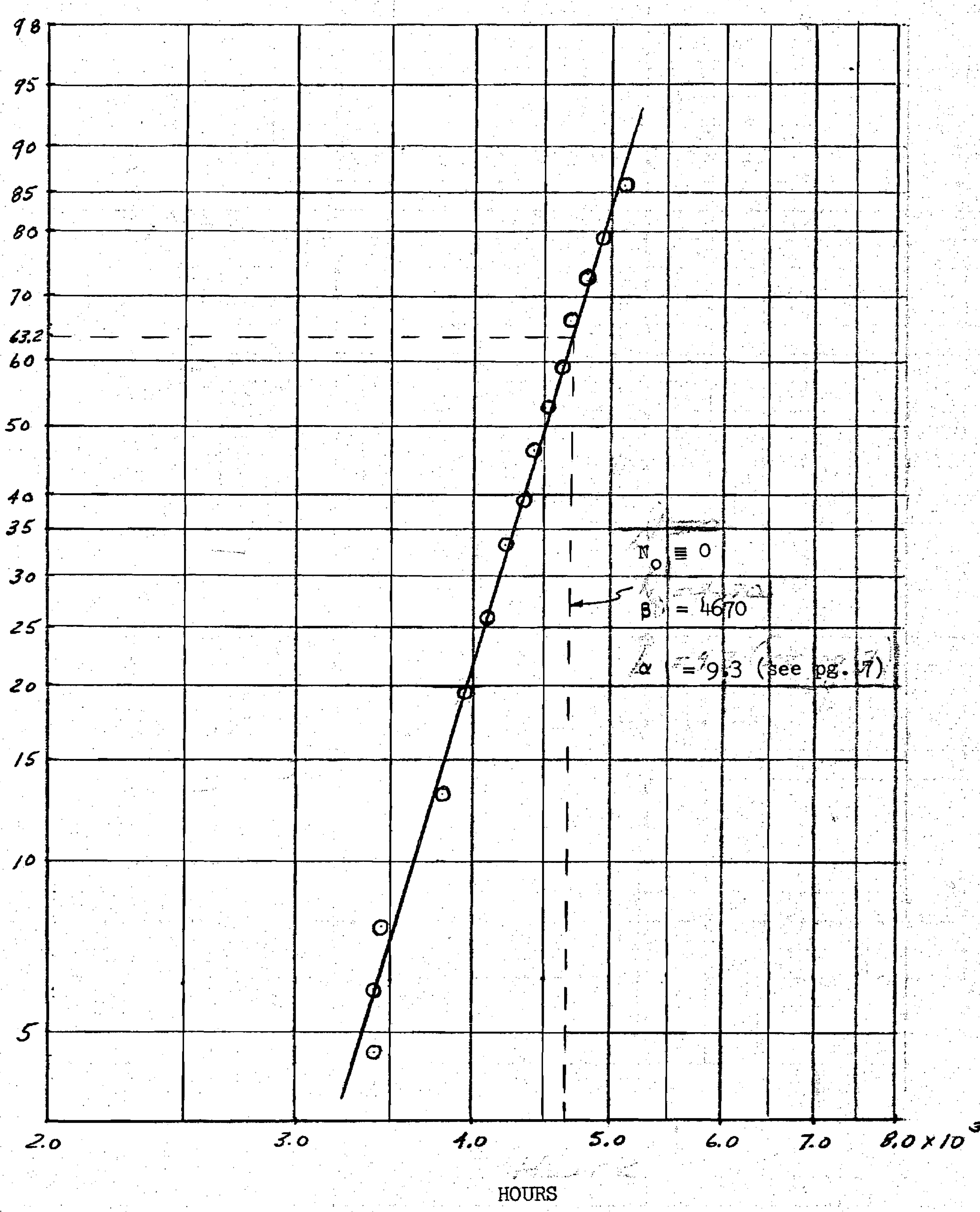


Figure 2: Weibull plot of B series data on expanded scale.

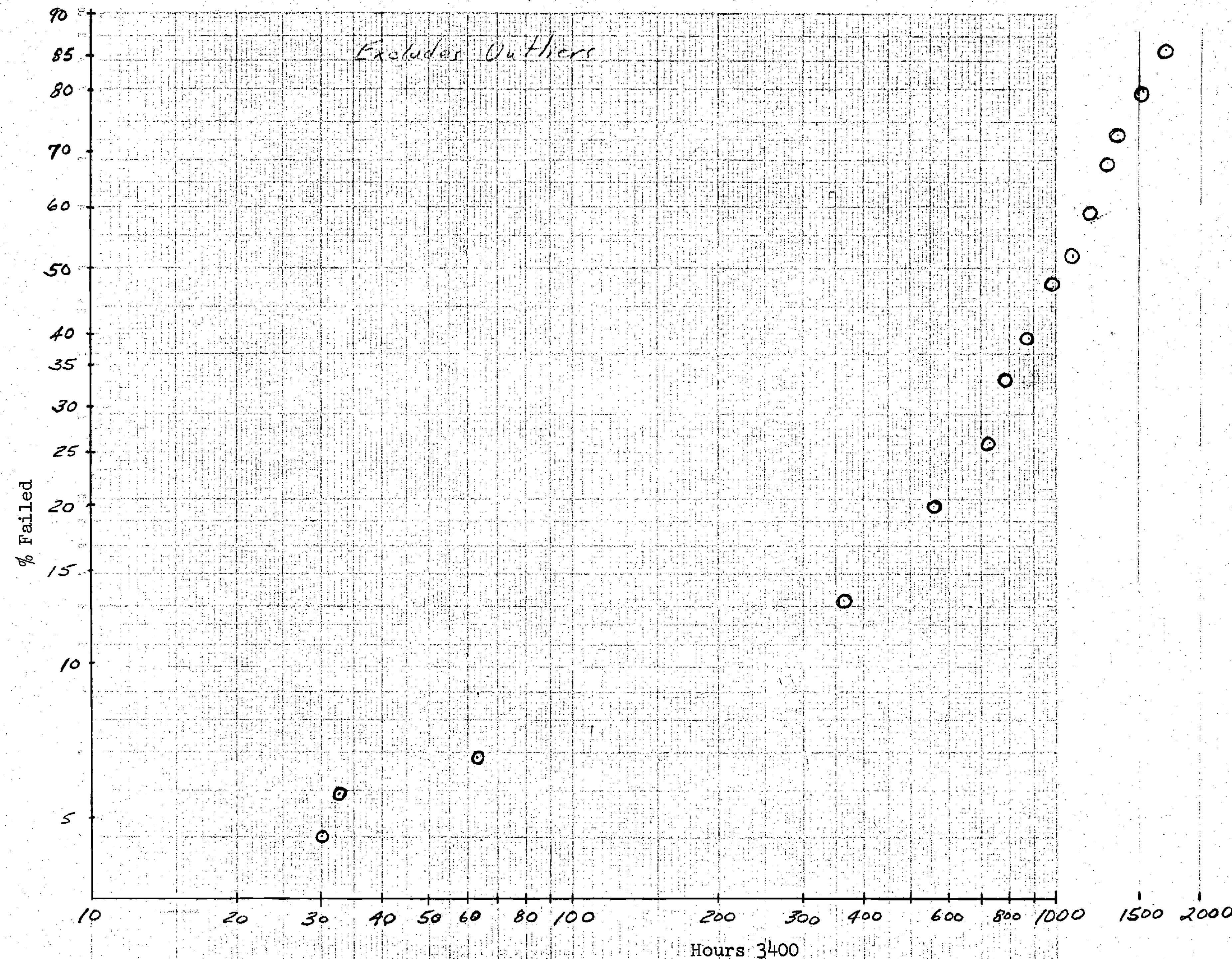


Figure 3. Three parameter Weibull plot: location parameter=3400 hours.

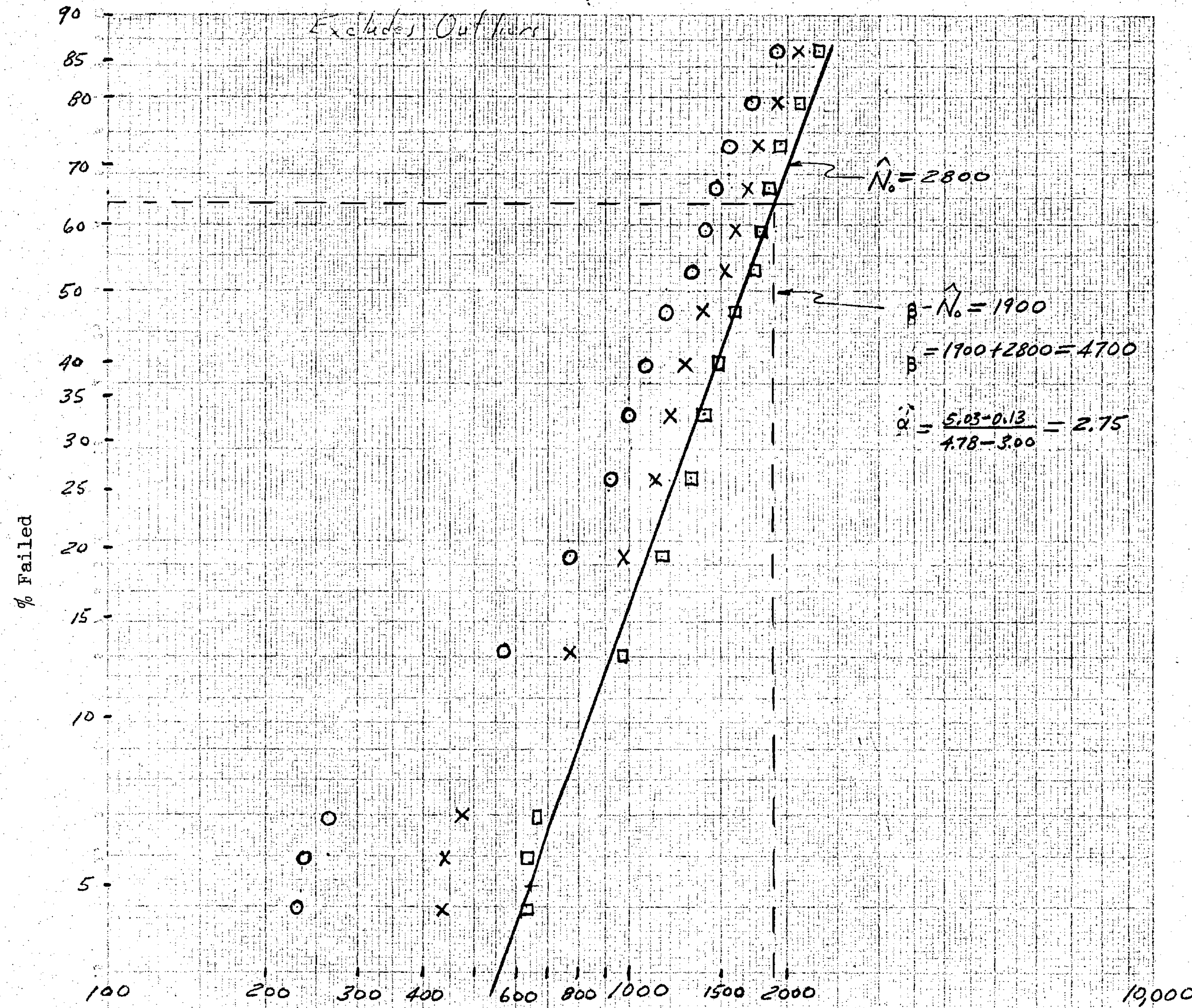


Figure 4. Three parameter Weibull plots

o Hours - 3200 (location parameter)

X Hours - 3000 (location parameter)

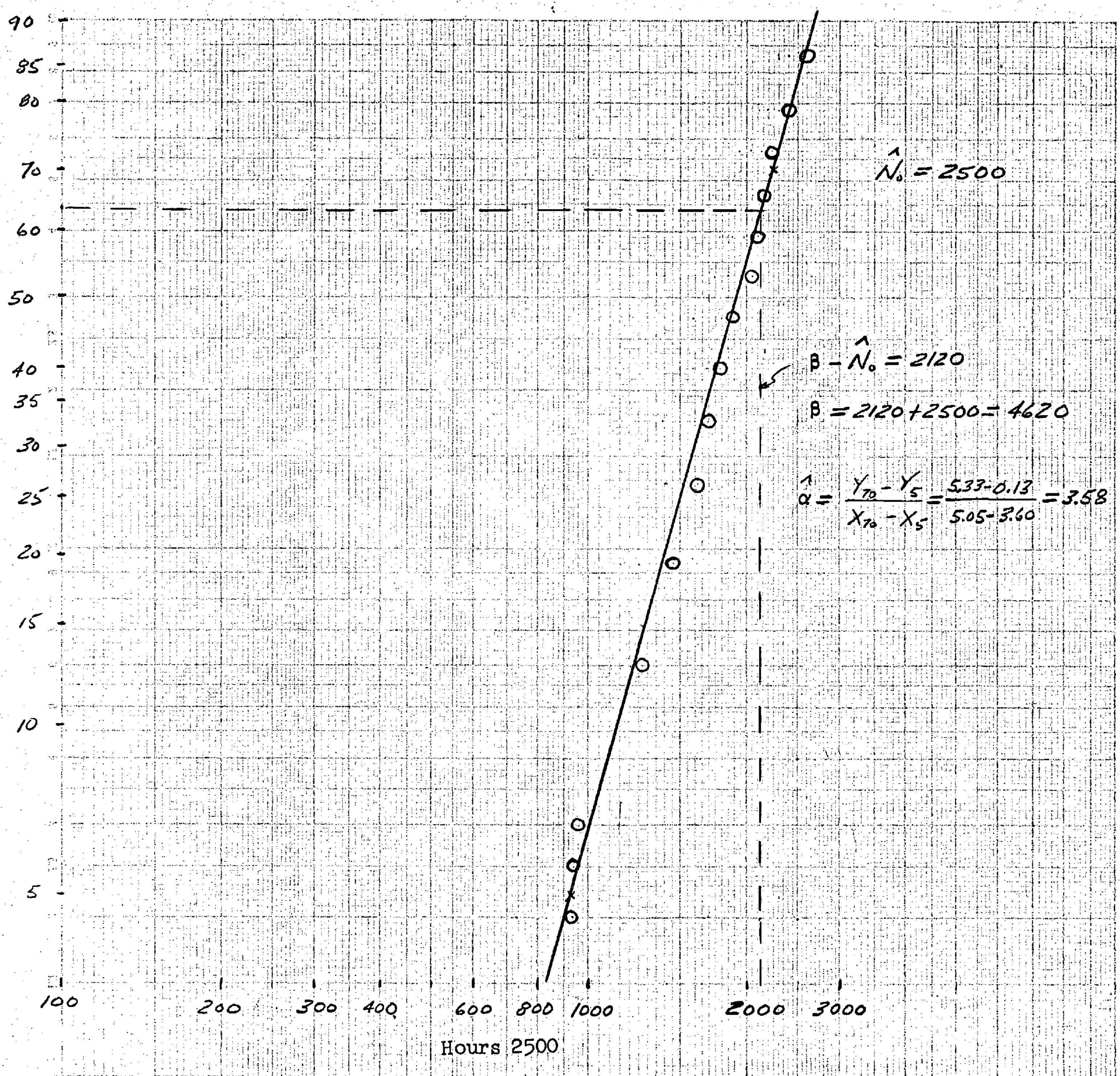
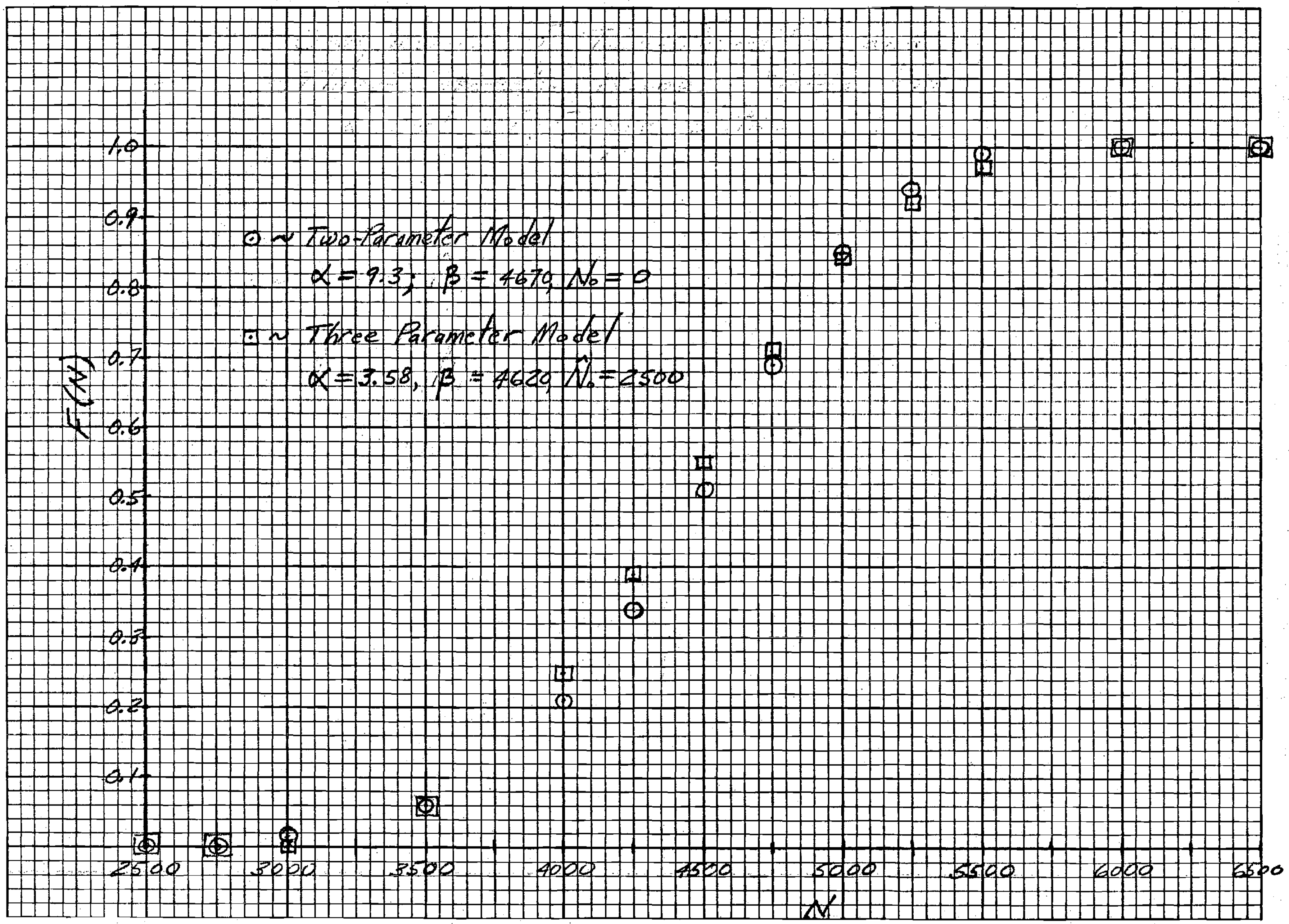


Figure 5. - Three-parameter Weibull plot. Location parameter 2500 hours.



6 Comparison of two-parameter and three-parameter Weibull

APPENDIX I

Likelihood functions can be written as follows:

For

$$i=1 \ldots k$$

observed X_i's are equal to failure times x_i. Then the likelihood function

$$L(\theta/X_1 = x_1 - X_k = x_k) = \prod_{i=1}^{k} f(x_i/\theta). \tag{1}$$

For

$$i=k+1, \ldots n$$

the observed X_i's are less than the failure load xi

$$P(X_i > x_i) = 1-F_X(xi). \tag{2}$$

Then

$$L(\theta/X_{k+1} > x_k, \ldots) = \prod_{i=k+1}^{n} (1-F_x(x_i)). \tag{3}$$

In the preceding equation θ is the vector of parameters $[\alpha,\beta]$.

In general

$$L(\theta/X_1 = x_1, \ldots X_k - x_k; X_{k+1} > x_k \ldots X_n > x_n)$$

$$= \prod_{i=1}^{k} f(x_i)_{X_i} \prod_{j=k+1}^{n} (1-F_{xj}(xj). \tag{4}$$

In case of two parameter Weibull distribution

$$L = \prod_{i=1}^{k} \frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1} \exp\left[\left(\frac{-x_i}{\beta}\right)^{\alpha}\right] \prod_{k+1}^{n} \exp\left[\left(\frac{-x_i}{\beta}\right)^{\alpha}\right]. \tag{5}$$

$$\ln L = \sum_{i=1}^{k} \left[\ln\left(\frac{\alpha}{\beta}\right) + (\alpha-1)\ln\left(\frac{x_i}{\beta}\right) - \left(\frac{x_i}{\beta}\right)^{\alpha}\right]$$

$$- \sum_{k+1}^{n} \left[\frac{x_i}{\beta}\right]^{\alpha}. \tag{6}$$

Maximizing ℓn L the following equations are obtained for α and β:

$$\frac{k}{\alpha} + \sum_{i=1}^{k} \ell n \frac{x_i}{\beta} - \sum_{i=1}^{n} \left(\frac{x_i}{\beta}\right)^{\alpha} \ell n \frac{x_i}{\beta} = 0 \tag{7}$$

$$- \frac{k\alpha}{\beta} + \frac{\alpha}{\beta}\sum_{i=1}^{n} \left(\frac{x_i}{\beta}\right)^{\alpha} = 0 \tag{8}$$

These equations can be simplified by eliminating β and obtaining an an equation for α.

$$\frac{1}{\alpha} + \frac{1}{k} \sum_{i=1}^{k} \left[\ell n\, x_i - \alpha\, \ell n \left(\frac{1}{k} \sum_{i=1}^{n} x_i^{\alpha}\right)\right]$$

$$- \sum_{i=1}^{n} \left[\frac{x_i^{\alpha}}{\sum_{i=1}^{n} x_i}\, \alpha \left\{\ell n\, x_i - \alpha\, \ell n \left(\frac{1}{k} \sum_{i=1}^{n} x_i^{\alpha}\right)\right\}\right] . \tag{9}$$

Equation (9) can be solved for $1/\alpha$ by Newton's iteration technique. Value of β can then be obtained from equation (8). Computer programs have been written to solve these equations.

Case I. Weibull Parameters by Maximum Likelihood Estimation

```
C.....'X' IS THE RANDOM VARIABLE REPRESENTING THE TIME TO FIRST FAILURE
C.....'N1' IS MAXIMUM NUMBER OF ITERATIONS
C.....'E' IS ERROR ALLOWED IN ITERATION
C.....U IS WEIBULL SHAPE PARAMETER
C.....N IS NUMBER OF X'S
      DIMENSION
      N1=1000
      E=1E-3
      J=0
      G=0.1
      N=18
  4   READ (5.1) (X(I).I=1.N)
   1  FORMAT (10F5.3)
      WRITE (6.60)(X(I),I=I,N)
 60   FORMAT (1X,10(F5.3,4X))
      K=0
   15 A1=0
      B1=0
      C1=0
      C4=0
      D) 10 I=1,N
      A=ALOG(X(I))
      B=X(I)**(1.0/G)
      C2=A*B
      C3=A*A*B
      A1=A1+A
      B1=B1+B
      C1=C1+C2
      C4=C4+C3
   10 CONTINUE
      FG=C1/B1-(A1/N)-G
      FGP=(-B1*C4+C1*C1)/(G*G*B1*B1)-1
      B2=(B1/N)**G
C.....'NEWTIT' IS A SUB ROUTINE AVAILABLE IN THE LIBRARY FOR ITERATION USING
C     NEWTON-RAPHSON ITERATION METHOD.
C.....K IS AN INDEX IN THE 'NEWTIT' PROGRAM
      CALL NEWTIT(G,FG,FGP,E,N1,K)
      U=1.0/G
      GO TO (15,20,30,50,50,50),K
   20 WRITE (6,25)U,B2,FG,FGP,E
   25 FORMAT (/,5F15.5)
 50   J=J+1
      IF (J.LT.2) GO TO 4
      GO TO 999
   30 WRITE (6.35)U,B2,FG,FGP,E
 35   FORMAT(/,5(F15.5,5X),/5X,52H ROOTS DO NOT CONVERGE WITHIN SPECIFIED ITERATIONS
      GO TO 50
 40   WRITE (6,45)U,B2,FG,FGP,E
 45   FORMAT(/,5(F15.5,5X),/,5X,34H
```

```
      GO TO 50
C.....OUTPUT:  DATA IS PRINTED FIRST. THE 5COLUMNS RESPECTIVELY REPRESENT SHAPE
C     PARAMETER, SCALE PARAMETER, FUNCTION, DERIVATION OF THE FUNCTION AND THE
C     ERROR
```

Case II. Weibull Parameters by Maximum Likelihood Estimation Considering Censored Samples

```
C.....'X' IS THE RANDOM VARIABLE REPRESENTING THE TIME TO FIRST FAILURE OR TIME
C     OF CONCLUDING THE EXPERIMENT IF THE SPECIMEN DID NOT FAIL.
C.....N IS TOTAL NUMBER OF SPECIMENS
C.....K IS NUMBER OF RAILED SPECIMENS
C.....N1 IS MAXIMUM NUMBER OF ITERATIONS
C.....K1 IS AN INDEX IN'NEWTIT'
C.....E IS ERROR ALLOWED FOR ITERATION
C.....U IS WEIBULL SHAPE PARAMETER
      DIMENSION
      E=1E-3
      N=19
      K=18
      J=0
 4    READ (5.1)(X(I),I=1,N)
   1  FORMAT (10F5.3)
      WRITE (6.60)(X(I),I=1,N)
 60   FORMAT (1X,10(F5.3,4X))
      G=1.8
      N1=1500
   15 A1=0
      K1=0
      A2=0
      B1=0
      C1=0
      D1=0
      DO 10 I=1,N
      A=ALOG(X(K))
      B=X(I)**(1.0/G)
      C2=A*B
      D=A*A*B
      B1=B1+B
      C1=C1+C2
      A2=A2+A
      D1=D1+D
 10   CONTINUE
      DO 11 I=1,K
      A=ALOG(X(I))
      A1=A1+A
 11   CONTINUE
      FG=G+(A1/K)-(C1/B1)
      FGP=1.0-(C1*C1/(G*B1)-D1/G)/(G*B1)
C.....'NEWTIT' IS AN ITERATION SUB-ROUTINE PROGRAM AVAILIABE IN THE LIBRARY USING
C     NEWTON RAPHSON ITERATION METHOD.
      CALL NEWTIT(G,FG,FGP,E,N1,K1)
      B2=(B1/K)**G
      GO TO (15,20,30,40,40,40)
   20 WRITE (6,25)U,B2,FG,FGP,E
   25 FORMAT (/,5F15.5)
```

```
 50   J=J+1
      IF (J.LT.3) GO TO 4
      GO TO 999
  30  WRITE (6,35) U,B2,FG, FGP,E
 35   FORMAT(/,5(F15.5,5X),/,5X,52H ROOTS DO NOT CONVERGE WITHIN SPECIFIED
C     ITERATIONS
      GO TO 50
 40   WRITE (6,45) U,B2,FG,FGP,E
 45   FORMAT(/,5(F15.5,5X),/,5X,36H UNDERFLOW?OVERFLOW OCCURS IN QUO
      GO TO 50
C.....OUTPUT:- DATA: X'S THE 5 COLUMNS REPRESENT SHAPE PARAMETER, SCALE PARAMETER,
C     VALUE OF THE FUNCTION, VALUE OF THE DERIVATIVE OF THE FUNCTION, AND THE
C     ERROR RESPECTIVELY.
```

APPENDIX II

```
C.....'CHI-SQUARE' GOODNESS-OF-FIT TEST FOR WEIBULL MODEL
C.....'AX' IS EQUAL PROBABILITY INTERVAL DIVISION TIME
C.....'INT' IS NUMBER OF ITEMS IN INTERVAL
C.....'IN' IS CUMULATIVE NUMBER OF ITEMS IN INTERVALS
C.....'INN' IS NUMBER SPECIMENS
C.....'Y' IS DATA - TIME TO FIRST FAILURE
C.....'P' IS SHAPE PARAMETER
C.....'R' IS SCALE PARAMETER
C.....'N' IS NUMBER EQUAL PROBABILITY INTERVALS
      DIMENSION AS(20), INT(10), IN(10), Y(20), P(10),R(10)
      READ (5,3)(P(K),K=1,3)
 3    FORMAT(3F6.4)
      READ(5,12)(R(K),K-1,3)
 12   FORMAT(3F6.4)
      K=0
 14   K=K+1
      G=P(K)
      B2=R(K)
      WRITE(6,11)P(K)
 11   FORMAT(/,2X,F6.4)
      WRITE(6,13)R(K)
 13   FORMAT(/,2X,F6.4)
      A=1.0
      N=5.0
      NN=N-1
      DO 10 M=1,NN
      X=B2*(-ALOG(1-A/5.0))**(1.0/G)
      AX(M)=X
      A=A+1
   10 CONTINUE
      WRITE (5,4)(AX(M),M=1,NN)
 5    FORMAT (/,5X4(F5.3,5X))
      I=1
      J=1
      INN=18
 4    READ (5,1) (Y)I),I=1,INN)
 1    FORMAT(10.F5.3)
      WRITE(6.60)(Y(I),I=1,INN)
 60   FORMAT (1X,10(F5.3,4X))
 15   YA=Y(I)
      X=AX(J)
      IF (YA.LE.X) GO TO 120
      IF *YA.GT.X) GO TO 130
 120  I=I+1
      GO TO 15
 130  B=I-1
      IN(J)=B
      J=J+1
      IF(J.EQ.N) GO TO 140
      GO TO 15
```

```
 140  IN(J)=INN
      INT(1)=IN(1)
      DO 150 I=1,NN
 150  INT(I+1)=IN(I+1)-IN(I)
      WRITE(6,160)(INT(I),I=1,N)
 160  FORMAT (/,5X,5(I3,5X))
      D=0
      AP=FLOAT(INN)/FLOAT(N)
      DO 170I=1,N
      C=(INT(I)-AP)**2
      D1=C/AP
      D=D+D1
 170  CONTINUE
      WRITE(6,190)D
 190  FORMAT (/,2X,F10.5)
      IF(K.LT.3) GO TO 14
      GO TO 999
C.....OUTPUT:- IN ORDER REPRESENTS SHAPE PARAMETER, SCALE PARAMETER, EQUAL
C     PROBABILITY DIVISION TIMES, DATA POINTS, NUMBER OF ITEMS IN INTERVAL, MEAN
C     SQUARED DVIATION.
```

```
C.....'KOLMOGOROV-SMIRNOV' GOODNESS-OF-FIT TEST FOR WEIBULL MODEL


C.....X IS THE RANDOM NUMBER REPRESENTING THE TIME TO FIRST FAILURE OR TIME
C     OF CONLCUSION OF THE EXPERIMENT IF THE SPECIMEN DID NOT FAIL
C.....'P' and 'R' ARE SHAPE AND SCALE PARAMETERS RESPECTIVELY
C.....'FREQ' IS CUMULATIVE FREQUENCY OF OCCURRENCE OF FAILURE
C.....D IS THE DEVIATION OF OBSERVED CUMULATIVE FREQUENCY AND THEORETICAL
C     PROBABILITY
      DIMENSION X(100), FREQ(50),P(L),R(10)
      READ (5,3)(P(K),K=1,3)
 3    FORMAT(3F6.4)
      READ(5,12)(R,K,K=1,3)
 12   FORMAT(3F6.4)
      K=1
 14   G=P(K)
      B=R(K)
      WRITE(6,11)P(K)
 11   FORMAT(2X,F6.4)
      WRITE(6,13)R(K)
 13   FORMAT(2X,F6.4)
      J=0
      N=18
      READ (5,1)(X(I),I=1,N)
 1    FORMAT(10F5.3)
      I=1
 15   AX=X(I)
      BX=X(I+1)
      IF(AX.EQ.BX)GO TO 100
      IF(AX.LT.BX)GO TO 200
 100  I=I+1
      IF(I.EQ.N)GO TO 200
      GO TO 15
 200  WRITE(6,20)AX
 20   FORMAT(/,2X,F5.3)
      J=J+1
      FREQ(J)=FLOAT(I)/FLOAT(N)
      WRITE(6,25)FREQ(J)
 25   FORMAT(/,2X,F10.4)
      F=1-EXP(-(AX/B)**G
      WRITE(6,30)F
 30   FORMAT(/,2X,F10.4)
      D=FREQ(J)-F
      I=I+1
      IF(I.LT.N)GO TO 50
      IF(I.EQ.N)GO TO 40
      IF(I.GT.N)GO TO 45
 50   GO TO 15
 40   AX=BX
      GO TO 200
```

```
 45   K=K+1
      IF(K.LT.4) GO TO 14
      GO TO 999
C.....OUTPUT:  IN ORDER RESPECTIVELY REPRESNTS SHAPE, SCALE PARAMETERS,
C     AX,FREQ(J),F AND D
999   STOP
      END
```

APPENDIX III

The computer program listed in this appendix has been developed from the original program written for UNIVAX 1108 in Fortran language. This program is in Basic language and is capable of executing estimation procedures for two Weibull parameters by the method of Maximum Likelihood, Chi-Square test and Kolmogorov tests.

```
READY
LIST
10  PRINT "MLE METHOD OF WEIBULL PARAMETER ESTIMATION"
20  PRINT "E-SERIES, STNS 9-10"
25  DIM X[100]
30  READ N
32  READ K
33  PRINT N,K,
34  FOR I=1 TO N
35  READ X[I]
36  NEXT I
40  FOR I=1 TO N
44  PRINT X[I],
45  NEXT I
46  LET E=1.00000E-03
47  LET N1=50
48  LET G=.2
49  LET K1=0
50  LET A1=0
55  LET A2=0
60  LET B1=0
65  LET C1=0
70  LET D1=0
75  LET I=1
80  LET A=LOG(X[I])
85  LET B=X[I]↑(1/G)
90  LET C2=A*B
95  LET D=A*A*B
100  LET B1=B1+B
105  LET C1=C1+C2
110  LET A2=A2+A
115  LET D1=D1+D
120  LET I=I+1
125  IF I <= N THEN 80
130  LET L=1
135  LET A=LOG(X[L])
140  LET A1=A1+A
145  LET L=L+1
150  IF L <= K THEN 135
155  LET F1=G+(A1/K)-(C1/B1)
160  LET F2=1-(C1*C1/(G*B1)-D1/G)/(G*B1)
165  GOSUB 310
170  LET B2=(B1/K)↑G
175  LET U=1/G
180  IF K1=1 THEN 50
```

```
181  IF K1=2 THEN 185
182  IF K1=3 THEN 205
183  IF K1=4 THEN 215
184  IF K1=5 THEN 215
185  PRINT U,B2,F1,F2,
200  GOTO 441
205  PRINT U,B2,F1,F2,"NO CONVERGENCE WITHIN N MAX"
210  GOTO 999
215  PRINT U,B2,F1,F2,"UNDER/OVER FLOW OCCURS IN QUOTIENT"
220  GOTO 999
305  PRINT "NEWTIT"
310  IF K1>0 THEN 335
315  LET J=K1
320  LET K1=1
325  LET N2=N1
330  LET N1=0
335  LET T=F1
340  IF J=0 THEN 355
345  IF ABS(G)<1.00000E-34 THEN 360
350  LET T=T/G
355  IF ABS(T) <= E THEN 400
360  IF N1=N2 THEN 420
365  IF ABS(F2) <= 0 THEN 435
370  LET T=G
375  LET G=G-F1/F2
380  LET N1=N1+1
385  IF ABS(T-G) <= 0 THEN 430
390  RETURN
400  LET K1=2
410  RETURN
420  LET K1=3
425  RETURN
430  LET K1=4
434  RETURN
435  LET K1=5
440  RETURN
441  PRINT " X-2 TEST"
442  DIM A[6],I[6],J[6]
444  LET A=1
446  LET N2=5
447  LET N3=N2-1
448  LET X=B2*(-LOG(1-A/N2))↑(1/U)
450  LET A[A]=X
452  LET A=A+1
454  IF A <= N3 THEN 448
456  FOR A=1 TO N3
458  PRINT A[A]
460  NEXT A
462  LET I=1
464  LET J=1
466  LET Y1=X[I]
468  LET X=A[J]
470  IF Y1<X THEN 474
472  IF Y1 >= X THEN 478
474  LET I=I+1
475  IF I>N THEN 478
476  GOTO 466
```

```
478  LET B3=I-1
480  LET J[J]=B3
482  LET J=J+1
484  IF J=N2 THEN 488
486  GOTO 466
488  LET J[J]=N
490  LET I[1]=J[1]
492  PRINT I[1]
494  FOR I=1 TO N3
496  LET I[I+1]=J[I+1]-J[I]
498  PRINT I[I+1]
500  NEXT I
502  LET D3=0
504  LET A4=N/N2
506  FOR I=1 TO N2
508  LET C=(I[I]-A4)↑2
510  LET D2=C/A4
512  LET D3=D3+D2
514  NEXT I
516  PRINT D3
518  PRINT "KOL-SMIRN TEST"
524  LET I=1
526  LET A5=X[I]
528  LET B4=X[I+1]
530  IF A5=B4 THEN 534
532  IF A5<B4 THEN 540
534  LET I=I+1
536  IF I=N THEN 540
538  GOTO 526
540  PRINT A5
544  LET F1=I/N
546  PRINT F1
548  LET F=1-EXP(-(A5/B2)↑U)
550  PRINT F
552  LET D4=F1-F
554  PRINT D4
556  LET I=I+1
558  IF I<N THEN 526
560  IF I=N THEN 540
562  IF I>N THEN 999
600  DATA 51,51
```

E-16-631

GEORGIA INSTITUTE OF TECHNOLOGY

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SCHOOL OF AEROSPACE ENGINEERING

404-894-3000

DANIEL GUGGENHEIM SCHOOL OF AERONAUTICS

Semi Annual Status Report

Reliability Based Recovery of Floors

NASA Grant NGR 11-002-169

(E-16-631)

Principal Investigator: S. Hanagud, Professor

School of Aerospace Engineering

Period Ending: April, 1974

Introduction

Fatigue damage is one of the causes responsible for the deterioration of the reliability of the aircraft structure with use. This fatigue damage can be detected and corrected by selected time inspections and maintenance of these engines. The time between such inspection depends on the fatigue behavior of the concerned structure and the desired reliability standards. More frequent inspections assure higher reliability standards but result in increased cost due to down time and inspection expenses. Cost benefits can be realized by optimizing the time between inspections. However, any methodology for the development of such optimization procedures needs an understanding and quantitative representation of the fatigue behavior of engines.

The present state of the art does not provide any reliable techniques for estimating the fatigue behavior of the full-scale engine structures at the design stage. Usually the fatigue behavior of the engine is estimated from the available inspection data on the particular engine, similar engines or test results. This reference is concerned with the development of models for fatigue behavior from the inspection data.

Inspection data are obtained by checking for fatigue cracks at critical regions of the engine that are specially prone to fatigue damage. These regions can be identified before or after the fleet has been put into operation. Such critical areas are called "location stations" in this report. Typical inspection data contain the identification numbers of the aircraft, identification numbers of the location stations, the

the number of flight hours at the inspection time, length and orientation of the observed cracks. Fatigue crack lengths at the same location station of a given fleet vary from engine to engine and exhibit a random behavior.

Deterministic models are not in general suitable to analyze the inspection data and to develop quantitative models for the fatigue behavior from the data. Many attempts[1-17] have been made in the past to develop probabilistic models to describe fatigue failure qualitatively and quantitatively. Most of the investigations, however, have been restricted to the results of coupon tests. Some works, including reference (14), are concerned with the analysis of data from full scale aircraft wings.

In the usual development of probabilistic models, failure time has been defined as the number of the flight hours corresponding to that inspection time at which at least one crack, regardless of its size, is observed at a location station. This definition of failure has been used in Part I of the analysis. Variations in the lengths of observed cracks are not included in the analysis. These variations can be attributed to the random character of fatigue crack initiation time and the differing flight hours at the time of inspection.

Part I

The possibility of hypothesizing a 2-parameter Weibull distribution for time to failure of B and E fleets were examined during this phase of research. As explained before, the definition of failure time is that time at which at least one crack, regardless of its size was observed at least one of the inspection stations. Some of the important conclusions from this investigation are as follows:

1. No accurate representation of the inspection data for the entire fleet was possible with a 2-parameter Weibull model.

2. By censoring those data that indicate failure at number of figure hours (high outliers) exceeding 6000 and those that did not indicate failure during the scheduled inspections (runout data), an acceptable 2-parameter Weibull distribution can be obtained for B-fleet. For this case, the shape parameter α was equal to 7.34 and the scale parameter β was equal to 4520 hours. The scale parameter represents the characteristic time to failure. The value of α obtained here for the full scale aircraft is much greater than the value of 4.0 as suggested and used by other investigators[4-14]. No acceptable 2-parameter Weibull model was obtained for E-fleet under these conditions.

3. The data from B-fleet was later analyzed on the basis of classification into certain location station groupings. This analysis provided acceptable 2-parameter Weibull distributions when the number of flight hours to failure exceeding 6000 hours were censored. The value of the shape parameter varied from 4.2 to 11.0. The characteristic time to failure varied from 4000 to 6000 flight hours. Again, the value of alpha was not restricted to 4.0.

In a similar analysis for E-fleet, it was necessary to censor those failure times that were below 2500 hours and those that were above 5000 hours to obtain acceptable 2-parameter Weibull distributions. Censoring low times to failure can result in serious error when making decisions on recovery of fleets and optimizing inspection schedules.

Part II

In Part II, the feasibility of development of a probabilistic model that can incorporate the varying sizes of cracks at selected inspection times was considered. It was attempted to describe the complete stochastic process of crack initiation and growth. Fundamental concepts of Queing Theory was used to obtain a set of differential equations for the probability $P(t;k\Delta L)$. The quantity $P(t,k\Delta L)$ represents the probability that for $t \leq T$ the length of the crack is equal to $k\Delta L$. It was assumed that the length of the crack was in terms of integer multiples of ΔL.

The derived set of differential equations were solved for hypothesized probability distributions for crack initiation and known distributions for crack growth times. By using these solutions, a method has been developed for estimation of parameters of the hypothesized crack initiation distribution from the observed inspection data that include the location station, number of flight hours at inspection and the length of cracks. Numerical example has been worked out by using the exponential distribution for crack initiation.

Part III

This part of the investigation is concerned with cold working process of stress coining that is used to provide the improvement of the fatigue behavior at the fastner holes in aircraft structures. The cold working process of stress coining is responsible for the radial flow of the metal. The residual stresses resulting from the stress coining provide protection against fatigue damage by opposing the applied tensile stresses at the edges of fastnerhholes. However, the investigation in this part of the report stresses that in addition to the compression stresses surrounding the hole, there are tensile stresses that result from the coining operation. These residual stresses can have a deleterious effect on the stress corrosion susceptibility of the post coined structure. Theoretical and experimental studies have been conducted on AISI 7075-T651 aluminum alloy.

Reports and Papers

Two NASA technical notes are being prepared on the basis of work reported in Parts I to III. A paper based on the work in Part III and entitled "The Stress Corrosion Susceptibility of Stress Coined Fastner Holes in Aircraft Structures" has been submitted to the AIAA Journal for publication.

An M. S. thesis was written by Mr. Aubrey Carter on the basis of research work done in Part III of the program. The thesis was entitled "Stress Corrosion Susceptibility of Stress Coined Aluminum Alloy Structures". This M. S. thesis won the Sigma-Xi Research Award at the Georgia Institute of Technology for the year 1973-1974.

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E-16-631

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404-894-3000

May 2, 1975

SEMI ANNUAL PROGRESS REPORT
NASA GRANT NGR 11-002-169 (E-16-631)

RECOVERY OF FLEETS AND RELIABILITY-
BASED FATIGUE DESIGN PROCEDURES

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Period Ending: April 1, 1975

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Tasks Completed

1. Fatigue failure data from the center wing of the fleet C-130 were analyzed. The purpose was to investigate if a Weibull model can be used to probabilistically describe the fatigue of aircraft. The Weibull model was found suitable under the following conditions.

   - Only one fatigue critical region was considered for each hypothesized Weibull model.

   - The data were reduced to yield the number of flight hours required for the development of a "standard length of crack" such as 0.1 inch or 0.05 inch. The data reduction was done by regression type of analysis.

   The conclusions from this task can be summarized as follows:

   - Any probabilistic model for fatigue must include the concept of crack growth. The probability that for time $t \leq T$ the fatigue crack length $a \leq A$ should be considered. This will need a model for the stochastic process of fatigue crack growth from the initial "micro-sized" flaw (a_o) to critical crack length (a_c).

2. Following the conclusions of task 1, a probabilistic model for the stochastic process of fatigue crack growth was developed by assuming continuous variation of time and discrete space of crack lengths. Time was measured either in terms of flight hours or number of cycles.

3. Some methods for the quantitative estimation of the developed probabilistic model for the stochastic process of fatigue were investigated.

4. The work on the development of a reliability-based fatigue design procedures has been initiated.

Papers Presented At Meetings

1. "Reliability-Based Optimum Inspection and Maintenance Procedure" Symposium on Propulsion Systems Structural Integration and Engine Integrity, Monterey, California, September 1974.

2. "Decision Theory in Structural Reliability" Annual Reliability and Maintainability Symposium, Washington, D. C., January 1975.

Papers Accepted For Presentation

1. "Stochastic Model for Fatigue and Cost Effective Design Decisions" AIAA Structures, Structural Dynamics and Materials Conference, Denver, Colorado, May 27-29, 1975.

2. "Interference Fits and Optimum Design for Stress Corrosion and Fatigue" Stress Corrosion Symposium, ASTM Annual Meeting, Montreal, Canada, June 1975.

Papers Published

1. "Decision Theory in Structural Reliability" Proceedings of 1975 Annual Reliability and Maintainability Symposium, pp. 255-262.

Papers Accepted For Publication

1. "Reliability-Based Optimum Inspection and Maintenance Procedures" Journal of Aircraft (Tentative, July 1975).

2. "Stress Corrosion Susceptibility of Stress Coined Aluminum Alloy Structures" AIAA Journal (Tentative, July 1975).

E-16-631

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GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GEORGIA 30332

Semi-Annual Report NGR-11-002-169
E-16-631 (NASA Grant)

RELIABILITY BASED FATIGUE DESIGN AND MAINTENANCE PROCEDURES

July 1, 1976

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This is the final status report on the grant. This report describes the different topics of investigations that were carried out during the project, the papers published, the papers presented at technical meetings, papers pending publication and the thesis that were generated during the project.

Investigations

(1) Analysis of fatigue failure data from a specific fleet of aircraft to evaluate the state of art and determine the needed developments.

(2) Development of a simple stochastic model for fatigue by using the concept of a varying hazard rate and a birth process.

(3) Quantitative estimation of the parameters of the stochastic model by using fracture mechanics considerations.

(4) Quantitative estimation of the parameters of the stochastic model from available data.

(5) Application of the stochastic model to develop a reliability-based, cost effective fail-safe design procedure.

(6) Development of procedures for devising optimum inspection and maintenance schemes.

(7) Application of statistical decision theory to select appropriate test options and safety factors subject to reliability restraints.

(8) Investigations of alternate methods of improving fatigue life and fatigue reliability by using interference fit techniques and the associated stress corrosion considerations.

(9) Application of the principles of analysis of variance to study the significance of present methods of grouping fatigue failure data.

As a bi-product of the above investigations, an improved mathematical technique has been developed. This technique and its application can be described as follows:

(10) An improved numerical technique of multiple integration with respect to one independent variable.

(11) Application of new technique of integration to develop a procedure for the study of some random vibration problems.

Papers Published

1. Hanagud, S., and Uppaluri, B. "Reliability-Based Optimum Inspection and Maintenance Procedure", Journal of Aircraft, p. 403, 1975.

2. Thomas, J. M., Hanagud, S., and Hawk, J. D. "Decision Theory in Structural Reliability" Proceedings of 1975 Annual Reliability and Maintainability Symposium, p. 255.

3. Carter, A. E., and Hanagud, S. "Stress Corrosion Susceptibility of Stress Coined Fastner Holes in Aircraft Structures", AIAA Journal, 13, p. 858, 1975.

4. Hanagud, S., and Carter, A. E. "Interference Fits and Stress Corrosion Failure", Paper accepted for publication in ASTM STP 610 (expected to be published in 1976).

5. Hanagud, S., and Uppaluri, B. "A Reliability-Based Cost Effective Fail Safe Design Procedure", Proc. 17th S.D.M. Conference, Valley Forge, 1976.

6. Hanagud, S. and Uppaluri, B. "Stochastic Model for Fatigue and Cost Effective Fatigue Design Decisions", Proc. of 16th S.D.M. Conference, Denver, Colorado, p. 195.

Papers Pending Publication

1. Uppaluri, B., and Hanagud, S. "An Improved Numerical Technique of Multiple Integration with Respect to One Independent Variable"

2. Hanagud, S., and Uppaluri, B. "A Study of Random Vibration by Using a Technique of Multiple Integration".

3. Hanagud, S., and Uppaluri, B. "Significance of Subgroups of Aircraft Fatigue Failure Data". (To be published).

4. Hanagud, S., and Uppaluri, B. "Significance Tests for Estimated Fatigue Model from Data Containing Crack Lengths and Cycles" (To be published).

5. Hanagud, S., and Uppaluri B. "Decision Theory and Material Selection by Using Probabilistic Model for Fatigue Failure".

Papers Presented at Technical Meetings

1. "Reliability-Based Optimum Inspection and Maintenance Procedures," Symposium on Propulsion System Structural Integration and Engine Integrity, Sponsored Jointly by Navy Air System Command, U.S. Army Material Command, NASA, AFOSR, AFFDL, ONR, AFML and Aero Propulsion Laboratory, Monterey, California, 1974.

2. "Decision Theory in Structural Reliability" 1975 Annual Reliability and Maintainability Symposium, Washington, D.C., 1975.

3. "Stochastic Model for Fatigue and Cost Effective Design Decisions", 16th AIAA S.D.M. Conference, Denver, Colorado, 1975.

4. "Interference Fits and Stress Corrosion Failure" ASTM Annual Meeting, Stress Corrosion Symposium, Montreal, Canada, June, 1975.

5. "Reliability-Based Cost-effective Fail-Safe Design Procedure" 17th AIAA S.D.M. Conference, Valley Force, Pennsylvania.

6. "Subjective Options and Selection of Test Options," AIAA Aerospace Conference, 1975.

Thesis

Partial support was provided toward the following theses.

1. "Reliability-Based Econometers of Aerospace Structures: Design Criteria and Test Options", Ph.D. Thesis of J. M. Thomas, 1974.

2. "Stochastic Model for Fatigue and Procedures for Optimum Design and Maintenance", Ph.D. Thesis of B. Uppaluri (expected to be completed by December 1976.)

3. "Stress Corrosion Susceptability of Stress Coined Aluminum Alloy Structures," M.S. Thesis of A. E. Carter, January 1976.

E-16-631

GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GEORGIA 30332

Final Report NGR-11-002-169:
E-16-631 (NASA Grant)

RELIABILITY BASED FATIGUE DESIGN AND MAINTENANCE PROCEDURES

July 1, 1976

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ABSTRACT

Demand for light weight aircraft structures results in the use of as small a safety margin as is practical. As a consequence of the small safety margin and other uncertainties, cracks or partial damages are likely to occur before the economical life of the aircraft is expended. Fatigue is one of the principal causes for the cracks. Fatigue loading and fatigue crack growth also contain uncertainties.

The susceptibility of the aircraft structure to crack or partial damage during the useful life of the structure imposes the requirement that the structure should be capable of supporting the service loads with these cracks. Furthermore, it must be possible to detect these cracks before they extend to critical sizes and cause catastrophic failure of the structure. Therefore, any fail safe design that can achieve this objective needs a knowledge of the probability of the presence of a crack of a certain length at a given location after certain number of flight hours. A stochastic model has been developed to describe such a probability for fatigue process by assuming a varying hazard rate. This stochastic model can be used to obtain the desired probability of a crack of certain length at a given location after certain number of cycles or time.

Quantitative estimation of the developed model has also been discussed. Application of the model to develop a procedure for reliability-based cost-effective fail-safe structural design has been discussed. This design procedure includes the reliability improvement due to inspection and repair. Methods of obtaining optimum inspection and maintenance schemes have also been discussed.

Alternate methods of fatigue reliability improvement by cold working processes have been discussed. The associated stress corrosion problem has been studied. Application of statistical decision theory to select suitable test options and safety factors subject to a reliability constraint have also been investigated.

Most of the investigations under this project have either been published in journals and conference proceedings or pending publication.

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INTRODUCTION

It is now generally accepted that all structural materials are not "flaw-free". Sometimes, a maximum acceptable flaw-size can be specified as a part of the structural specifications. Thus, an initial flaw size (a_o) and the associated probability distribution characterize the structure. Due to fatigue loading, these initial "micro-sized" flaws grow to detectable sizes. Time or number of cycles required for this growth to detectable size of crack length is often called the "crack initiation time." In many cases, this growth time amounts to a significant function of the total fatigue life of the structure. Due to further fatigue loading, crack sizes increase until they attain critical sizes. These critical sizes depend upon the critical stress intensity factors and the external loads. Thus, the probability distribution for crack sizes is changing continuously with time or number of cycles at all locations of the structure. Therefore, the probabilistic description of fatigue process can be expressed as the probability that for time $t \leq T$, the crack size $a \leq A$. This is a shochastic process.

In most of the reported works [1-8], the varying crack lengths "a" associated with the fatigue process are ignored. In these works, stochastic process is not considered. The entire fatigue process is described by a single random variable "t" which is the time for fatigue failure. The quantitative description consists of the probability that for time $t \leq T$ fatigue failure took place. Because of the simplicity of the model, probability distributions such as the Weibull Distribution have been used to describe the time for fatigue failure.

The use of such a description that uses a single random variable is very limited because such a model neglects many important aspects of fatigue process. For example, one question that needs an answer is as follows. What is the length of the crack that corresponds to the defined failure time? Is this length the initiation length or critical crack length or some arbitrarily chosen length? Initiation length can vary depending on the available non-destructive inspection capability. Furthermore, such a model does not provide any information for optimizing repair threshold crack length, crack arresting devices, N.D.I. capabilities and different loading process. Another arguement used by the users of a single random variable is to assume that the effect of varying crack length is negligable and a stochastic process is not needed. In order to verify if such a statement could be true fatigue data from specific fleet of aircraft are analyzed as a first step of the investigation. As explained in later sections, these investigations demonstrated that a stochastic model is necessary to describe fatigue of structure. Further investigations during the project period is described as follows:

a) Development of a simple stochastic model for fatigue by using the concept of a varying hazard rate and a birth process[9-10].
b) Quantitative estimation of the parameters of the stochastic model by using fracture mechanics considerations[9].
c) Quantitative estimation of the parameters of the stochastic model from available data[11,12].

d) Application of the stochastic model to develop a reliability-based, cost effect fail-safe design procedure [9].

e) Development of procedures for devising optimum inspection and maintenance schemes [11, 12].

f) Application of statistical decision theory to select appropriate test options and safety factors subject to reliability restraints [13].

g) Investigation of alternate methods of improving fatigue life and fatigue reliability by using interference fit techniques and the associated stress corrosion considerations [14,15].

h) Application of the principles of analysis of variance to study the significance of present methods of grouping fatigue failure data [16].

As a bi-product of the above investigations, an improved mathematical technique has been developed. This technique and its application can be described as follows:

i) An improved numerical technique of multiple integration with respect to one independent variable [17].

j) Application of new technique of integration to develop a procedure for the study of some random vibration problems [18].

Analysis of Fatigue Failure Data

In order to investigate if fatigue provess can be described by a single random variable "t" that denotes time for fatigue, fatigue data from two specific fleets were analyzed. A typical inspection record contained the following information.

1. Identification number of the airplane
2. Number of flight hours completed before the inspection
3. Inspection date
4. Number of reinspection(s)
5. The command
6. The base
7. Facility of inspection
8. Crack location by numbers of the critical regions as has been previously identified
9. Number of such cracks in a given region
10. Direction of crack growth
11. Crack length
12. Information as to whether the crack has been repaired

A two parameter weibull model was hypothesized for the fatigue failure time "t".

$$F(t) = 1 - \exp\left[-(t/\beta)^{\alpha}\right] \qquad (1)$$

In the equation, α and β are shape and scale parameters respectively. These parameters were estimated from the data by using the method of maximum likelihood[19].

The chi-square and Kolmogrov[20] tests were used to verify the goodness of fit of the estimated parameters. The following conclusions were reached.

(1) For a given critical location or a selected group of critical locations, no acceptable Weibull distribution was obtained unless the data were censored in some way. In general, censoring of both high-level outliers and low level outliers were needed. Low-level outliers refer to those fatigue failure times that lie below a selected failure time for purposes of censoring. Similarly, high-level outlier refers to those failure times that lie above a time corresponding to high-censoring level. Use of a low level outlier was not conservative. Any model derived by the use of low level censoring can result in serious errors in decisions concerning design and maintenance. Similarly, models derived by the use of high level censoring can result in increased weight and cost.

(2) A three parameter weibull distribution or a log-normal distribution did not improve the results.

(3) However, when the observed failure times at a given location were reduced by regression techniques to correspond to the time for initiation of crack of a given length acceptability of the two-parameter weibull model improved in many cases. Probability distribution was different for different crack lengths. Necessity for a stochastic model was evident.

Development of A Stochastic Model for Fatigue

It is assumed that a single crack is present in a fatigue critical region. Multiple cracks can be treated by order statistics or other procedures. Then, the variation of crack length with time is qualitatively of the type shown in Figure 1. This consists of a continuous variation of crack length with continuous variation of time or number of cycles. The corresponding model for the stochastic process for fatigue crack sizes involves the consideration of continuous state space of crack lengths and continuous time. It is difficult to develop such a model. The development of the model is simplified by considering the state space of crack length to be discrete as shown in Figure 1. Accuracy can be increased by decreasing the magnitude of '$\Delta \ell$' of discrete crack length increments. This process of considering the state space of crack length can also accomodate consideration of crack initiation i.e. probability of a crack of length a_i initiating at time t less than or equal to t_i as shown in Figure 2. Even though the crack lengths are assumed to increase in discrete steps the mean crack growth rate can vary continuously as a function of time. Because the resulting process is nonstationary, the probability that a crack of length $k(\Delta \ell)$ i.e., k times $\Delta \ell$, is present at a time $t \leq t_k$ depends on the initial value of time t_o. This is denoted by $P(k, t_o, t_k)$.

By considering the different ways in which the event of the development of a crack of length $k(\Delta \ell)$ can occur in time interval t_o to $t + \Delta t$ the following equation can be written

$$P(k,t_0,t+\Delta t) = \sum_{i=0}^{\infty} P_i(k,t_0,t)\,P(k-i,t_0,\ t+\Delta t) \qquad (2)$$

by assuming orderliness of crack growth i.e.,

$$\lim_{\Delta t\to 0} \frac{S_2(t,t+\Delta t)}{\Delta t} = 0 \qquad (3)$$

and

$$\lim_{\Delta t\to 0} \frac{S_1(t,t+\Delta t)}{\Delta t} = 0 \qquad (4)$$

where

$$S_1(t) = \sum_{k=1}^{\infty} P(k,t_0,t) = 1 - P(0,t_0,t) \qquad (5)$$

and

$$S_2(t) = \sum_{k=2}^{\infty} P(k,t_0,t) = 1 - P(0,t_0,t) - P(1,t_0,t) \qquad (6)$$

It can be shown that the following differential equations are for $k > 1$

$$\frac{\partial}{\partial t}\left[P(k,t_0,t)\right] = E[\dot{a}(t)]\,P(k-1,t_0,t) - E[\dot{a}(t)]\,P(k,t_0,t) \qquad (7)$$

In this equation, $E[\dot{a}(t)]$ is the mean crack growth rate at t. For $k = 1$, the equation (7) takes the following special form.

$$\frac{\partial}{\partial t}\left[P(1,t_0,t)\right] = f_e(t) - E[\dot{a}(t)]\,P(1,t_0,t) \qquad (8)$$

where $f_e(t)$ is the probability density for crack initiation. These equations can be solved by methods similar to those discussed in Reference (11). However, P(k, t) can be obtained only if $E[\dot{a}(t)]$ and $f_e(t)$ can be estimated if probability distribution for initial flaw sizes are known. This procedure will be discussed in a separate note. The method of obtaining $E[\dot{a}(t)]$ is discussed in the next section.

Mean Crack Growth Rate

Knowledge of the mean crack growth rate is essential to estimate the crack length at a given time. According to Forman[21] the rate of crack growth is given by

$$\frac{da}{dN} = \frac{C_1 (\Delta K)^n}{(1-r)K_{1c} + \Delta K} \qquad (9)$$

where C_1 and n are material constants, ΔK is the range of stress intensity factors, K_{1c} is the critical-stress intensity factor, r is the ratio of minimum stress intensity factor to the maximum stress intensity factor 'a' is the half crack length and 'N' is the number of cycles. For a stiffened panel the range ΔK is given by

$$\Delta K = \Delta L \left(\pi a \, f\left(\frac{a}{w}\right)\right)^{\frac{1}{2}} C_R(a, b) \qquad (10)$$

where ΔL is the range of applied loads at a given time, $f(\frac{a}{w})$ is the finite width correction factor, C_R (a, b) is the tip stress reduction factor, and b is half the stringer spacing. For a fixed value of 'a', $\frac{da}{dN}$ is a function of the random load parameters ΔL and r. Thus at a given crack length say a - a_1, the growth rate is a random variable. If ΔL (N) and r(N) are assumed to be independent stationary stochastic processes with known density functions, then the expected value of the growth rate is given by

$$E\left[\frac{da}{dN}\right]_{a=a_1} = \int_{R_{\Delta L}} \int_{R_r} \left(\frac{da}{dN}\right)_{a=a_1} f(r) \, g(\Delta L) \, dr \, d(\Delta L) \qquad (11)$$

where f(r) and g(ΔL) are the density fucntions of the random variables r and ΔL respectively. $R_{\Delta L}$ and R_r are the range spaces of ΔL and r respectively. Equation (11), thus gives the mean crack growth rate at any value of crack length under the random loading. This quantity expressed in terms of the discrete length units ΔL is required in the equation for P(k, t) of previous section

The mean crack growth rate as given by Equation (11) is a complicated integral to be solved and does not have a closed form solution. Hence, numerical methods have been used to solve the equation. However, for a special cases where r and ΔL are stationary Gaussian processes, Taylor's series expansion has been used to obtain approximation. Then $E[\hat{a}]$ at any value of a_1 is given by the following equation.

$$E\left[\frac{da}{dN}\right]_{a=a_1} = A\mu_{\Delta L}\left\{\mu_{\Delta L}^2 + 3\sigma_{\Delta L}^2\right\} / \left[B(1-\mu_r) + C\mu_{\Delta L}\right] - 3A\cdot C\cdot\mu_{\Delta L}^2\,\sigma_{\Delta L}^2 / \left[B(1-\mu_r) + C\mu_{\Delta L}\right]^2 + A\mu_{\Delta L}^3\left\{C^2\sigma_{\Delta L}^2 + B^2\sigma_r^2\right\} / \left[B(1-\mu_r) + C\mu_{\Delta L}\right]^3 \quad (12)$$

Alternate Method of Estimation of Parameters

An alternate method of estimation of parameters is to use the fatigue failure data from the same fleet, similar fleet or from tests. Such a method requires the following steps.

(1) The first required step is the solution of equation (7) and (8) to obtain P(k, t). This could be left in the form of quadratures.

(2) The next step needed is the normalization to a realistic maximum crack length N (ΔL).

(3) If the parameters are to be estimated by the method of maximum likelihood it is necessary to formulate the likelihood function from the results of steps (1) and (2).

(4) The next step will be to maximize the likelihood function to obtain the parameters.

This work has been carried out as a part of the project investigation. Preliminary results are published in references 11 and 12. These papers include the consideration of data from a specific fleet supplied by NASA. A detailed analysis including the model verification will be published[12].

Applications of the Developed Model

One of the applications of the developed model is to develop a reliability-based, cost-effective design procedure. This method has been developed and reported by the investigators in reference 18. Some of the significant items and example problems are discussed here.

Problem Setting

The problem setting can be best explained by considering an example. In this report, the deisgn of a built-up structure such as a sheet-stiffener combination is considered. Figure 3 illustrates the stiffened panel. The panel is of width w and thickness t. The panel is assumed to be made of a specific material and the particular structure is assumed to be a sub-assembly of an aircraft structure. It is also assumed that large number of aircraft will be produced as a result of this design. Even though the discussed methodology considers a specific material, an optimum choice among several candidate materials can be made by following a similar procedure and statistical decision theory. External loading F consists of a sustained loading F_1, and a random fatigue loading F_2. It is assumed that the random fatigue loading has been quantified probabilistically. Thus, the total loading F is specified probabilistically.

For a particular choice of the thickness t, the stringer spacing 2b, and the choice of the material, the initial ultimate load carrying capacity F_u is known. If the initial microsized flaws or cracks are specified by a probabilistic distribution, the initial load carrying capacity F_u is characterized by an appropriate probabilistic distribution which depends on the initial flaw size distribution, the material and the dependence of the load carrying capacity of the structure on the flaw size and other dimensions.

On the other hand, if it is assumed that the effect of initial flaw size distribution can be described by a crack initiation probability distribution, the load carrying capacity F_u can be expressed as a deterministic quantity if the material properties are also assumed to be deterministic. The corresponding initial ultimate stress is defined to be σ_u. Similarly, for a given thickness, stress corresponding to external loading is denoted by σ_L. If σ_u and σ_L are deterministic, the initial safety margin i.e., before fatigue effects are present, is given by the ratio of σ_u to σ_L. As explained earlier, both σ_u and σ_L have uncertainties and need probabilistic representation. Then the initial reliability can be considered as a safety measure. This can be represented by the probability that σ_u / σ_L is greater than 1. Due to the presence of fatigue loading, cracks grow in size. Crack growth rates and the crack sizes depend on the material properties, stress and the number of cycles. The presence of a crack of size a_i reduces the ultimate strength from σ_u to σ_{ui}. Then the reliability which is defined by the probability that the ratio σ_{ui} to σ_L is greater than 1 is also reduced. Consequently, the probability of failure which is the probability of the ratio σ_{ui} to $_L$ is less than 1 is increased. The probability of failure increases as the crack lengths increase to such an extent (a_{cr}) that the strenghth is reduced below the externally applied load. The probability of failure can be reduced by increasing the initial margin of safety or reliability. This of course, increases the weight of the structure. Another way of decreasing the probability of failure is to inspect the structure at selected times so that the cracks can be detected and repaired before they reach their critical sizes. In this process, allowable initial margin of safety can be small because cracks are not allowed to grow to their critical sizes. This process however, increases the cost due to inspection. Increasing weight also increases the initial cost and the cost of operation. Therefore, the required design procedure consists of selecting the design variables such as the thickness, stiffener spacing, and inspection frequency during the projected service life so as to minimize the total expected cost or weight. The cost and weight can be considered as interchangeable functions that can be optimized. Many a time it is easy to express the objective function to be optimized as an equivalent weight function. This entire procedure, however, is subjected to the restraint that the margin of safety or reliability does not fall below an acceptable limit during the projected life of the structure.

Therefore, reliability-based fail-safe fatigue design procedure consists of selecting specified design variables including inspection frequency, subject to constraints, so as to minimize the expected cost or weight function while the probability of failure is kept below specified limits during the projected life of the structure. In order to make the design procedure acceptable to a designer who is not familiar with the statistical methods, the reliability or probability of failure can be related to a 'variable' safety factor or safety margin.

Methodology

The following are the steps that need to be followed in the methodology for the reliability-based fail-safe fatigue design procedure discussed in this paper.

- The first step consists of specifying the design variables and constraints. This step identifies the design variables that can be selected by the designer to minimize the objective-function (weight or cost).
- The second step is to specify the probabilistic distribution of the external loading. This can be a stochastic process.
- The third step is to formulate the objective function. This can be a weight or cost function and is related to the probability of failure, the projected life of the structure, the specified and selectable design variables, and external loading.
- The fourth step is to select trial design variables and obtain the initial margin of safety or reliability.
- The next step is to obtain the variation of crack size and crack growth probabilities with time. A stochastic model for crack growth developed by the authors is used in this report to obtain the probabilistic description of crack sizes. This probability depends on the material, load description and the number of cycles.
- From this knowledge of the probability distribution of crack sizes reduction in strength and probability of failure is estimated. The inspection and repair frequency during the projected design life is included in this estimate of the probability of failure.

- The seventh step is to substitute all the information in to the cost or weight that was formulated in the third step. This yields the cost or weight due to the particular selection of the trial design variables.
- Steps two to seven are repeated with different trial variables to minimize the objective function by search method.
- The final design variables are selected subject to restraints such as reliability bounds, minimum spacing, etc.

These are the general steps that are necessary in the design procedure developed in this report. This needs the description of a stochastic model for fatigue crack growth and crack sizes, methods of estimation of the probability of failure, methods of including the effects of inspection and repair frequency during the projected design life in the probability of failure, and an objective function in terms of cost or weight. The stochastic model and the estimation of the parameters of the model are already discussed in previous sections. The estimation of the probability of failure, reliability improvement due to inspection and repair, formulation of the objective function and its minimization are discussed in the following sections.

Probability of Failure

In this section, method of estimating probability of failure is discussed. The improvement in reliability due to inspection, repair and consequent renewal and the estimation of this reliability improvement are not discussed in this section. These are discussed in the next section.

The first step in estimation of probability of failure is to identify the possible failure modes. In addition to the fatigue failure mode, other failure modes such as the sudden over stress or buckling are possible. If the event of fatigue failure is denoted by E_f, the event of sudden over stress by E_s and the event of buckling failure by E_b. The probability of failure P_f is given by the union of the all the possible events of failure.

$$P_f = P[E_f \cup E_s \cup E_b] \qquad (13)$$

Probability of occurrence of each of these events depends on the strength of the structure to resist that particular type of failure and the probability of occurrence of the load that can result in that particular type of failure. Because the discussions of the paper are primarily restricted to fatigue failure, it will be assumed that only fatigue failure are possible. This means that only failure mode possible is due to the growth of fatigue cracks and consequent reduction in strength.

Before discussing the probability of failure under conditions of uncertainty, a deterministic design procedure is briefly reviewed here. This review is useful in identifying the different probabilistic fatigue failure modes. Consider the stiffened panel shown in Figure 3. Let it be assumed that a central crack is likely to develop in this structure due to fatigue. For given w and assumed length between stiffeners 2b, the variation of the residual strength σ_u with half the length of the central crack[29-31] is shown in Figure 4. The value of the maximum external load L is precisely known in deterministic design. Then, for a particular choice of the initial safety margin S, the thickness t and the corresponding stress σ_L, critical crack length a_c can be obtained. These are shown in Figure 4. As the fatigue cracks initiate and grow, failure is not possible until the crack attains a length of a_c. The length of a_c can also be obtained analytically from the following formula in the case of a stiffened panel.

$$K_c = \frac{L}{t}\sqrt{\pi a_c f\left(\frac{a_c}{w}\right)}\; C_R(a_c/b) \tag{14}$$

In this equation $f(a_c/w)$ is the width correction factor[22], C_R is the tip stress reduction factor[23], K_c is the fracture toughness of the material.

Because the maximum load L is known precisely in a deterministic case, the stresses due to external load never exceed the residual strength for crack lengths $a < a_c$. Alternately, it can be stated that probability of failure is zero for crack lengths $a < a_c$ and the probability of failure is one for $a \geq a_c$.

In reality, the external load is not precisely known. The load is usually characterized by a random variable. This is the case in which reliability based design procedures are needed. In this paper, external loading is assumed to be characterized by a stationary stochastic process. Even in this case, a value of a_c can be selected in the Figure 4. This curve is assumed to be known deterministically. This means that for a given width of the panel w and a choice of stiffener spacing 2b, a value of critical crack length a_c is chosen. This value of a_c corresponds to a definite value of σ_L on the curve in Figure 4. But, the external loading is not known precisely as in the deterministic case. Therefore, the value of a_c and σ_L cannot be related to initial safety margin and choice of thickness t. However, the probabilistic description of the external loading L is known. As will be shown later, the choice a_c, σ_L, and thickness t can be related to reliability or probability of failure. From a knowledge of the specified bounds on reliability, a_c and t can be chosen.

Alternately, the following procedure can be used instead of starting with a choice, a_c. A value of $\bar{\sigma}$ is selected such that

$$\bar{\sigma} = \mu\left(\frac{L}{t}\right) + \bar{\alpha}\,\Sigma\left(\frac{L}{t}\right) \tag{15}$$

where $\mu\left(\frac{L}{t}\right)$ is the mean value of external load divided by the choice of thickness t and $\Sigma\left(\frac{L}{t}\right)$ is the corresponding variance. The quantity $\bar{\alpha}$ is constant which is similar to safety margin in a deterministic design. However, $\bar{\alpha}$ is not arbitrary. The quantities $\bar{\alpha}$, t and a_c are related to reliability. They can be selected on the basis of the prescribed reliability bounds. As can be seen in the figure, a selected value of corresponds to a value of which corresponds to a value of a_c.

Unlike the case of deterministic loading, failure may take place even for values of crack sizes smaller than a_c. Such a failure is possible because the externally induced stress (L/t) has a probability distribution and does not represent the absolute maximum possible stress. For values of $a < a_c$, fatigue failure is possible if the externally induced stress exceeds the residual strength at any time during the service life of the aircraft. This failure is defined as static fatigue failure P_{sf}. In order to define absolute safety limits, the structure is assumed to fail definitely when crack length exceeds a_c. This is defined as the critical crack size fatigue failure P_{fc}. Then, the total fatigue failure at any time t is due to union of these two events.

$$P_f = P(a \geq a_c, t) + P_{sf} \qquad (16)$$

It is to be noted that P_{fc} is given by

$$P_{fc} = P(a \geq a_c, t) \qquad (17)$$

For a given $\bar{\sigma}$, a_c can be obtained from equation (14) by replacing $\frac{L}{t}$ by $\bar{\sigma}$ and using the appropriate value for fracture toughness k_c for the material.

The probability of critical crack size fatigue failure needed in Equation (20) can be obtained from the developed stochastic model. In terms of discrete crack sizes a_c corresponds to $k_c(\Delta L)$ where k_c is an integer and ΔL is the size of discrete crack sizes. Then

$$P(a \geq a_c, t) = P[(k \cdot \Delta l) \geq k_c(\Delta l), t] \qquad (18)$$

The probability of static fatigue failure P_{sf} can be obtained by a method discussed in the Appendix.

Reliability Improvement Due to Inspection and Repair

If no inspections are done during the projected design life, the probability of critical crack size fatigue is given by

$$P_{fc} = P(a \geq a_c, t) = P[k \geq k_c, t] = \sum_{k=k_c}^{N_{max}} P(k\Delta l, T_D) \qquad (19)$$

In this equation T_D is the projected design of the structure. The probability of critical crack size fatigue failure P_{fc} can be improved due to inspections. This change in probability of failure and hence in reliability can be obtained in the following way.

The projected design life is still assumed to be T_D number of hours or cycles. It is assumed that one inspection is done at T_o number of hours or cycles. If only periodic inspections are considered $2T_o = T_D$. At the time of inspection, if cracks of length $k(\Delta L) \geq k_r(\Delta L)$ are observed, the cracks are repaired. The quantity $k_r(\Delta l)$ is the repair threshold crack length. It is further assumed that structure is as good as new after repair. This means any further crack initiation and growth are to be calculated as though the structure is put into service at $t = T_o$ and not at $t = o$. It is also to be noted that only structures with $k_r < k < k_c$ are repaired because the structures with $k(\Delta l) \geq k_c(\Delta l)$ have failed due to critical size fatigue failure. It is implicit that the cracks of $k(\Delta l) < k_r(\Delta l)$ are not repaired.

There is still another quantity to be considered. This is the probability of detecting a crack by nondestructive inspection techniques if a crack exists. In the first step of the derivation, it will be assumed that the repair threshold crack length $k_r(\Delta L)$ is chosen that the detection probability is one. Then, the probability of critical size fatigue failure in the two intervals can be obtained as follows.

The probability of failure P(1) in the first interval corresponding to $0 < t \leq T_o$ is given by

$$P(1) = \sum_{k=k_c}^{N_{max}} P[k(\Delta l), T_o] \qquad (20)$$

By referring to Figure 4, the probability of survival in $0 < t < T_o$ is $1 - P(1)$ because there is the probability P(1) that structures fail in $0 < t \leq T_o$. For $t \leq T_o$,

$$P[k < k_r, T_o] + P[k_r \leq k < k_c, T_o] + P[k \geq k_c, T_o] = 1 \qquad (21)$$

and the probability of repair P_R is given by

$$P_R = P[k_r \leq k < k_c, T_o] \qquad (22)$$

Then the total probability of critical crack size fatigue failure in $0 < t \leq 2T_o = T_D$ can be written as follows:

$$P_{fc} = P(1) + P_R\, P(1) + F_1 \left[P(2) - P(1)\right] \Big/ \left[1 - P(1)\right] \qquad (23)$$

where $\qquad F_1 = P\left[k < k_r, T_o\right] \qquad (24)$

and $\qquad P(2) = P\left[k \geq k_c, 2T_o\right]$

Equation (23), for the probability of failure under one inspection is obtained by considering the three mutually exclusive and exhaustive events F_1, P_R and P(1) [see Equation (21)]. The quantity in the parenthesis of the last term of Equation (30) is the conditional probability that the structures will fail in $T_o < t \leq 2T_o$ given that they survived $0 < t \leq T_o$. This expression for P_{fc} satisfies all the limiting conditions. For example, when $P_R = 0$, P_{fc} reduces to P(2), as expected.

When $P_R = 1$ and hence $P(1) = 0$, P_{fc} becomes zero. Similarly, the probability of failure under any number of inspections can be obtained.

If the crack detection probability due to nondestructive inspection techniques is considered, the probability of repair P_R changes. The repair is now possible only if a crack of size $k_r(\Delta L) < k(\Delta L) \leq k_c(\Delta L)$ exists and is detected by the NDI capability, with a probability D(k). Here, D(k) is the probability of detecting a crack of size k(1). Then, the unconditional probability of detecting and repairing cracks of size $k_r(\Delta L) < k(\Delta L) \leq k_c(\Delta L)$ at T_o is given by

$$\bar{P}_R = \sum_{k=k_r}^{k_c - 1} P\left[k, T_o\right] D(k) \qquad (25)$$

Then, of the repairable aircraft given by P k_r k k_c, T_o , only $\bar{P}_R$ are repaired and the others are not repaired. Now, equations similar to (30) can be written with detection probability for cracks included.

Total Weight Function

Every optimization problem involves the so-called objective function which is a function of the design variables appropriate to the problem at hand[24-28]. The optimum values of the design variables are obtained by finding the stationary locations of the objective function subject to the design constraints[24-28].

For aircraft structures "weight" is the most crucial consideration in design. In the present context, the weight of the stiffened panel is considered to be minimized. The design variables are the thickness of the sheet and the width of the stringer spacing. The total "weight function" comprises of the deterministic weight of the panel and the expected loss of weight due to the probability of failure. The expected loss of weight is given by the product of the probability of failure under a given number of inspections and the deterministic weight of the panel. The deterministic weight of the panel consists of the weight of the panel consists of the weight of the sheet and the stringers. Expressed mathematically, the total weight function is given by

$$W(b,t) = (1 + P_f)\,[w t h \gamma + N_{st} W_{st}] \qquad (26)$$

where w = total width of the sheet

t = thickness of the sheet

h = breadth of the sheet

p = density of the sheet material

N_{st} = number of stringers

$2b$ = stringer spacing

W_{st} = weight of one stringer

Equation (26) is the proper objective function for the minimization of the weight. The effect of increasing the thickness is to reduce to expected loss of weight because of the reduction in the probability of failure. On the other hand, the deterministic weight is increased by increasing the thickness. Thus, a balance has to be found between the two. Stringer spacing has the opposite effect on the different weights.

The minimization is carried out by the search method. The total weight function is calculated for a set of thicknesses and stringer spacings. It is then plotted versus thickness with stringer spacing a the parameter. Then, the lowest weight is selected. The thickness and the stringer spacing corresponding to the minimum weight are the optimum values if the reliability constraint is satisfied at these values. The weight can be expressed in terms of equivalent costs.

Total Cost Function:

If the problem at hand is the determination of the optimum number of the periodic inspections, then the total weight function may not be the proper objective function. Then the total cost function concept has to be introduced. The total cost function comprises of the expected cost of failure and the deterministic cost of the periodic inspections. The expected cost of failure is given by the product of the probability of failure under the given number of inspections and the deterministic cost of structure. The deterministic cost of inspections is proportional to the number of inspections. The mathematical expression for the total cost function is given as follows:

$$C_T(j) = P_f C_s + C_I$$

where P_f is the probability of failure under j inspections

C_s is the cost of new structure

C_I is the cost of one inspection

J is the number of inspections

Equation (34) gives the proper objective function because as the number of inspections increases, the expected cost of failure decreases while the cost of inspections increases. The minimum value of the total cost function is found by the search method. The minimization is subject to the reliability constraint.

Illustrative Example

In order to illustrate the developed method, two examples have been considered. The first problem is that of a minimum weight design of 7075-T6 alloy. The problem has been deliberately kept simple for purposes of illustration. A more detailed problem is discussed in the Appendix II.

The design life is supposed to be 15,000 cycles with two periodic inspections made during the design life. The reliability is to be 99.5%. The design variables to be selected are the thickness t and the spacing of the stringers 2b. The following data is assumed to be known.

$w = 24''$

$\alpha = 8.9$

$\mu_{\Delta L} = 1000$ lb/in

$\mu_r = 0.5$

$\gamma = 0.09$ lb/in^3

$K_{IC} = 68000$ lb/in$^{3/2}$

$C_1 = 5 \times 10^{-13}$

$h = 12''$

$\beta = 5000$ cycles

$\sigma^2_{\Delta L} = 100.0$ lb^2/in^2

$\sigma^2_r = 0.01$

$W_{st} = 0.118$ lb

$m = 3$

As outlined in the preceding sections, the solution procedure is carried out. As a first step, the residual strength-critical crack length diagrams are obtained for a choice of number of stringers, e. g. 3, 5, 7, 9, 11, etc. (Fig. 6). As the number increases the stringer spacing decreases. As one might expect, the rate of growth decreases with the number of stringers. The tip stress reduction factor $C_R(a/b)$ h which is required in the expression for the residual strength is obtained from references (29-30) as shown in Figure 6.

The variation of the static reliability with residual strength and thickness is shown in Figure 7. For a given loading, in order to maintain the same static reliability, the thickness has to increase as the design residual strength decreases and vice versa.

In Figure 8, the relation between the probability of static failure, fatigue failure, and total failure is delineated.

The total weight functions are calculated in the manner explained previously for fixed R - 0.9996 and $N_{stringer}$ = 3, 5, 7, 9, Figure 9 depicts the minimization curves. From these curves, the minimum W for each curve can be obtained, and then compared with other minima of other curves. The overall minum in Figure 10 occurs for a thickness of 0.106 inches, $N_{stringer}$ = 7.

Figure 10 represents the minimization curves for $R_s = 0.9997$. As expected, the minimum values are now changed, and occur at different thicknesses. The minimum now occurs for $N_{stringer} = 7$ and thickness $t = 0.1044$ inches. From Figure 11, for R - 0.9998, the overall minimum decreases to 3.554 and at $N_{stringer} = 7$ and t - 0.1052.

Then the static reliability R_s is increased further to R_s = -.9999 the overall minimum is higher than before, i.e. $W_{min} = 3.630$ and occur for $N_{stringer} = 7$ and thickness t - 0.1052, Figure 12.

Thus comparing all the minima over the various variables, the minimum most is $W_{min} = 3.554$ for $R_s = 0.9997$, t - 0.1044 inches and $N_{stringer} = 7$. This corresponds to an overall reliability of 0.99765 and a design residual strength = 15,500 psi. The reliability constraint is satisfied since 0.99765 $R_b = 0.995$.

Check on the Initial Factor of Safety:

The mean and standard deviation of the maximum load L_{max}, are obtained. Then, considering different numbers of standard deviations above the mean-maximum load L, the initial factors of safety are obtained. For example, for one standard deviation above L, the initial factor of safety of the optimum design, based on yield strength is found to be 3.067. When two and three standard deviations are employed, the corresponding factors of safety are 2.60 and 2.32 respectively. This is indicative of the adquency of the optimum design obtained above for an equivalent deterministic design.

Cost Optimization:

To demonstrate cost optimization, the designed stiffened panel is considered. The only variable now, is the number of periodic inspections or the inspection interval. Since the panel is of a given configuration, its weight is fixed. Hence the total cost function C_T Equation (34) is the proper objective function to be considered in the present context.

As a first step, the probability of fatigue failure under j inspections, j - 0, 1, 2, 3, 4, . . .etc. is calculated. These values are graphically depicted in Figure 13. Corresponding to each of these numbers of inspections the total cost function C_T is calculated from Equation (34)., Figure 15. This is repeated for various values of the ratio of the cost of one inspection C_I to the cost of the structure C_S. When $C_I/C_S = 0.1$, the minimum occurs for one inspection. Decreasing C_I/C_S to 0.01, 0.005, 0.001 renders the minimum to occur at two inspections, three inspections and four inspections respectively as delineated in Figure 14.

Alternate Methods of Improving Fatigue Life Fatigue Reliability

The models for fatigue discussed in the preceding sections do not apply to cases for which residual stresses are present near fastner holes due to a cold working process such as stress coining. The purpose of stress coining is to improve the fatigue life of the structure. A simple method of stress coining in aluminum alloy is to expand the fastner hole of the structural member by drawing an oversized mardrel hydraulically through the fastner hole. Many similar processes are available for cold working fastner holes.

Such cold working processes result in a radial flow of the material. This results in residual stresses. Residual compressive stresses surrounding the hole provide protection against the fatigue damage by opposing the applied tensile stresses, However, as shown in the investigation, there is a zone of sustained residual tensile stresses located at a short distance from the hole. The maximum tensile stress usually occurs at the elastic-plastic boundary. Although the tensile stresses are not critical in the point of view of fatigue life of the structure, they can cause stress corrosion under certain conditions.

Therefore, the reliability of a stress coined structure needs the consideration of both the fatigue improvement and stress corrosion sustibility. The first step in such a study is to assess the residual stresses and stress coining susceptibility in such structures. The investigations carried out in the project have been published in references 14 and 15.

Bi-Products From the Project

As a bi-product of the investigations, the following have been developed. An improved numerical technique was needed in quantitative estimation of the parameters of the stochastic model. This has been discussed in Appendix III. An application of the technique has been done to random vibration problems. The purpose of the application was to verify the accuracy of the technique.

Another bi-product is the application of the principles of analysis of variance to study the significance of the present methods of grouping fatigue failure data. Preliminary work in the field has been discussed in Appendix IV.

Conclusions and Recommendations

It has been demonstrated that an accurate description of fatigue is possible by means of stochastic model. A simple model has been developed This model can be quantitatively estimated. The model has been applied to develop a procedure for a reliability-based cost-effective fail-safe design for aircraft structures. In particular, reliability improvement due to inspection and maintenance has been considered.

Deterministic design procedures that do not consider the involved uncertainties usually result in an over design. This results in an increased weight that affects both cost and performance. Furthermore, risks involved in a deterministic design are not known. On the other hand, the reliability-based design that uses a stochastic model considers the uncertainties that are consistent with the model. Risks in a design can be assessed. consistent with the model considered. Such a procedure usually results in lower weight than deterministic designs. This results in low operating cost and better performance of the aircraft . A very costly item in owning and operating an aircraft is the inspection and maintenance during the life of the aircraft. As has been demonstrated in the project an optimum schemes can be developed by using a stochastic model for fatigue and considering the reliability improvement due to inspection and repair. Methods of including such reliability improvement at the design stage has also been discussed.

The following further investigations are suggested in the point of view of the practical application of the developed procedures.

1. Development of different types of stochastic models so that the user has a choice depending on the particular application. It is necessary that all uncertainties be properly included in the model. Different and more accurate methods of quantitative estimation and verification of the model are needed.
2. It is also necessary to develop simple optimization techniques to include the combination of discrete inspection costs with other costs. This is necessary to avoid the difficulty with local minimums and provide a simple practical procedure.
3. The developed procedures should be modified to include multiple locations, and multiple cracks.
4. It appears as though cold working process will be used to improve the fatigue life of most existing and future metal aircraft. Probabilistic model for failure of such structures that includes both the life improvement and the stress corrosion susceptibility has not yet been developed. Such models are essential to fully take advantage of the cost and weight savings potential offered by the cold working processes.
5. In the point of view of increasing fuel costs, present levels of performance can be maintained only by using a material that has a higher strength to weight ratio than that offered by present aircraft structural materials. Advanced composites have such a potential. Mechanical behavior and failure modes of these advanced composites are different from that of metals. Instead of developing a deterministic design procedure and then modifying the procedures to develop probabilistic procedures,

reliability-based design procedure should be developed from the very beginning. By such a process the weight saving potential of advanced composites can be explored completely. This needs modification of the project to adopt to failure modes of composites.

6. Development of more accurate cumulative damage estimation techniques are essential for both metal and composite aircraft.

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Appendix I

In this appendix, a method of estimation of the static fatigue failure P_{fs} has been discussed. This fatigue failure is possible when the external loading exceeds the residual strength of the structure and the crack size a is less than a_c. By defining a quantity 's' in the following way

$$S = \sigma_R / \sigma_L \quad , \quad \sigma_L = L_{max} / t \tag{1}$$

The probability of static fatigue failure can be defined as the probability of s being less than or equal to 1. Alternately, reliability against the static fatigue failure can be defined as R_s

$$R_S = P[S \geq 1] = P[\sigma_R / \sigma_L \geq 1] \tag{2}$$

This probability can be evaluated from the following integral if the marginal probability density functions of σ_R and σ_L are given.

$$R_S = \int_1^\infty \int_{R_Z} \left| Z/S^2 \right| f(Z) \, g(Z/S) \, dZ \, dS \tag{3}$$

In this equation, f and g are the marginal probability density functions of σ_R and σ_L respectively, Z is an auxiliary variable and R_Z is the range space of Z. The integral given in Equation (3) is difficult to evaluate.

Instead of evaluating the integral of Equation (3), the following alternate procedure can be adopted to evaluate the static-fatigue reliability R_s[29-33]. The generalized Chebychev inequality is employed to determine the reliability R_s. For any shape of density function h(s), the probability that the random variable s lies within a range $(d - \delta) \leq s \leq (d + \delta)$ is given by the following inequality[34]:

$$P[(d-\delta) \leq s \leq (d+\delta)] \geq 1 - \frac{1}{\delta^2} E[s-d]^2 \tag{4}$$

in this equation $E[\cdot]$ denotes the expectation operation, 2δ is the width of the strip and d is any particular value of s. The lower limit of s namely, $(d-\delta)$ is unity, i.e. $\delta = d - 1$. Substituting these limits in Equation (4),

$$P[1 \leq s \leq 2d-1] \geq 1 - \frac{1}{(d-1)^2}\left[E(s^2) - 2dE(s) + d^2\right] \quad (5)$$

Now, recognizing that

$E(S) = \bar{S}$, the mean value of S, and

$E(S^2) = \sigma_S^2 + \bar{S}^2$

the equation(5) reduces to the following form after using Equation (2):

$$R_s \geq 1 - \frac{1}{(d-1)^2}\left[\sigma_S^2 + (\bar{S} - d)^2\right] \quad (6)$$

For R_s to be a maximum it is necessary that[29, 32]

$$\frac{\partial R_s}{\partial d} = 0 \quad , \quad \frac{\partial^2 R_s}{\partial d^2} < 0 \quad (7)$$

From the first of Equation (7),

$$\hat{d} = \bar{S} + \sigma_S^2/(\bar{S}-1) \quad (8)$$

From the second of Equation (7) and (8)

$$\frac{\partial^2 R_s}{\partial d^2} = -2\,(\bar{S}-1)^4/[(\bar{S}-1)^2 + \sigma_S^2] \quad (9)$$

which is negative for all $\bar{S}$ and σ_S^2.

Substituting for d from Equation (8) in Equation (6) it follows that

$$R_s \geq \frac{(\bar{S}-1)^2}{\sigma_S^2 + (\bar{S}-1)^2} \quad (10)$$

Appendix II

Numerical Example:

The problem is to design a stiffened panel subjected to a given random loading. The panel can have a central crack extending through the thickness. Also, the panel will be subjected to periodic maintenance inspections with attendant repairs of the crack when possible. Thus, the design variables involved can be categorized as follows:

(1) Material parameters

(2) Geometrical parameters and

(3) Maintenance parameters.

The design problem therefore consists of (1) selecting the optimum material from a given set of different materials, (2) selecting the optimum stringer spacing and thickness, and (3) selecting the optimum number of periodic inspections.

The following are assumed to be given and the designer has no choice in these variables

$\omega = 20.0''$

$h = 15.0$

$\alpha = 7.5$

$\beta = 48{,}000$ cycles

$T_D = 6.0 \times 10^5$ cycles

$R_b = 99.95\%$

In the above set 'ω' and 'h' are the overall dimensions of the panel. The quantity α and β characterize the Weibull model for crack initiation of 0.005 inches. The design life T_D is to be 6.0×10^5 cycles. The reliability restriant R_b should be 0.9995. The material properties are as follows:

For 7075T6 Aluminum Alloy

$$K_{1c} = 68,000 \text{ Nb/in}^{3/2}$$

$$C = 5 \times 10^{-13}$$

$$n = 3$$

For 2024-T3 Aluminum Alloy

$$K_{1c} = 83,000 \text{ lb/in}^{3/2}$$

$$C = 3 \times 10^{-13}, \ n = 3$$

A computer program has been written to obtain the probability of failure for each selected thickness, stringer spacing, material and the number of periodic inspections N during the design life. This information is later used in another computer program to obtain the expected cost or weight function. The design variables that meet the minimum expected cost or weight function subject to reliability constraints are selected. The following tables illustrate representative results and the selected design variables and the material.

For the first material, i.e., 2026-T3 the overall minimum occurs for 6 periodic inspections, 3.3" stringer spacing and sheet thickness of 0.105". For 7075-T6, the overall minimum occurs for 6 periodic inspections, 3.5" stringer spacing and 0-103" thickness when both minimums were compared, 7075-T6 has the lower minimum weight at 6 inspections, 3.3" stringer spacing and 0.103" sheet thickness. Hence, 7075-T6 would be the selected material. All the details of the calculations will be published in the Ph.D. thesis of Mr. B. Uppaluri and in a journal.

Appendix III

Multiple integration with respect to one independent variable was needed in integrating the equations (7) and (8) of the main text to obtain P(k$\Delta\ell$, t). Such a technique is also needed in many other engineering problems. Hunter[35] developed a method of numerical multiple integration and called it "the integrating matrix method". He applied the technique to forced vibration problem of helicopter rotor blades. In Hunter's method, the derivation of the integrating matrix consisted of dividing the range of integration into N intervals of equal size and N+1 points. At each of the N+1 points, the values of the integrand were obtained and represented in a column matrix $\{f_r\}$. The functional variation of the integrand in each interval was represented by an r^{th} degree polynomial. In order to obtain the values of the integral, an r^{th} degree integrating matrix $[I_r]$ was constructed b using Newton's interpolation formula. By multiplying the integration matrix and the integrand column matrix, the values of the integral were obtained. For multiple integration, the integrand matrix f_r was repeatedly multiplied by the integrating matrix $[I_r]$. For example,

$$\int_0^{x_1}\int_0^{x}\int_0^{x} f(x)\,dx\,d\bar{x}\,d\hat{x} \simeq [I_r][I_r][I_r]\{f_r\} \quad (1)$$

Improvement

The mathematical motivation for the improvement is the fact that when an 'r^{th}' degree polynomial is integrated an '$(r + 1)^{th}$' degree polynomial is obtained. Thus, the improvement suggested is that the degree of the integrating matrix be increased by one after each integration is a multiple integral. For example,

$$\int_0^{X_1}\int_0^{x}\int_0^{x} f(x)\,dx\,d\bar{x}\,d\hat{x} \simeq [I_{n+2}][I_{n+1}][I_n]\{f_n\} \quad (2)$$

The improved method was applied for the following problems:

(i) Multiple Integration of an algebraic function $0 < x < 20$

(ii) Forced Vibration response of a Canilever beam

(iii) Free vibration of Canilever beams

The results were compared with the exact solutions.

In the first example, a constant function f(x) = 1.0 was successively integrated four times using a second degree integrating matrix and number of divisions N = 20. The percent error ranged from 200.0 at x = 1 to 0.5 at x=20. The improved method was employed with the same N = 20, but with integrating matrices of degree 2, 3, 4, and 5 successively. The percent error was zero all through the range of integration.

For the forced vibration problem the span was divided into five equal intervals (N = 6) and a second degree integrating matrix is employed four times consecutively. The percent error ranged from 6.4 at 1/5 span to 0.3 at 5/5 span. The improved technique with the same N = 6 but increasing degree of integrating matrix from two resulted in a maximum percent error of only 0.03.

For the free vibration problem less than 1 percent error in natural frequency and/or less mean square error in mode shape was obtained at a lower number of spanwise divisions than in the case when the integrating matrix was not altered. Also, the mean square error in the modeshape compared to the exact mode shape for any mode was less in the improved method than in the method of Hunter.

The difference between the two methods decreased as the degree of the starting integrating matrix is increased. All the results will be published. At present the manuscript is being prepared.

Appendix IV

Introduction

Analysis of variance is a means of determinimg the homogeneity of a large collection of data that have been formed by lumping together several small groups of data. The small groups are denoted as "sub groups" and the variation between them as "variation between subgroups". The name, analysis of variance, itself stems from an analysis in which the total variation in the entire data is partitioned into component parts. These components are used to develop a test statistic.

The total variation is expressed by the total corrected sum of squares, i.e.

$$SS_T = \sum_{i=1}^{a} \sum_{j=1}^{n_i} X_{ij}^2 - T_{..}^2/N \qquad (1)$$

In this equation

a is the number of treatments

X_{ij} is the data point

$T_{..}$ is the total sum of data points

N is the total number of data points, and

n_i is the number of data points in 'i'th treatment.

The total variation SS_T can be split up into two components as follows:

$$SS_T = SS_A + SS_E, \qquad (2)$$

The term SS_A is variation between subgroups and SS_E is variation within subgroups. Then, the following table is constructed to facilitate the analysis of variance.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Sum of Squares	F
Between sub-groups	SS_A	a-1	$SS_A/(a-1)$	$\frac{SS_A/(a-1)}{SS_E/(N-a)}$
Within sub-groups	SS_E	N-a	$SS_E/(N-a)$	
Total	SS_T	N-1		

The value in the last column is compared with the critical F value at a given percent of significance and degrees of freedom of (a-1) and (N-a) respectively. The data is homogeneous if the F value is less than the critical F value[36].

If the above analysis of variance indicates that the data is non-homogeneous, then it is desirable to find out which of the subgroups form a homogeneous set of data. For this purpose, Duncan's multiple range test[37] can be employed. It consists of comparing the modified difference between the various means $(m_i - m_j)'$ with the corresponding critical value R'_p. The modified means are calculated from the following expressions

$$(m_i - m_j)' = (m_i - m_j)\, a_{ij} \qquad (3)$$

$$a_{ij} = \left(\frac{2r_i r_j}{r_i + r_j}\right)^{1/2} \qquad (4)$$

where r_i, r_j are the number of replications in each group. The critical values can be calculated from Table II of Duncan's[37] paper. Then all the possible groups are subjected to Duncan's test and those groups whose modified mean does not exceed the critical value of R'_p belong to one homogeneous set of data.

Application

The procedure that has been discussed in the preceding paragraph is used to analyze the fatigue failure data from a specific fleet of aircraft. The objective is to investigate if the fatigue failure data from several critical regions can be lumped together. If it is possible to lump the data together a small number of probability distributions can be used to describe the fatigue failure of the entire structure. It is also possible to use the system of lumping to do large number of inspections at a few representative locations.

The particular aircraft under consideration has 92 fatigue critical regions. Investigations show that the station group (2 to 15), (33 to 38), (41 to 46) and (89 to 92) can be lumped together. Analysis of variance tests indicate that these subgroups form a homogeneous set of fatigue data. The station groups (1-92), (61-70) and (71-80) cannot be lumped together because the test results show that their data varies significantly. These results are quantitatively presented in the following table.

Group	Variance		Result
	Within Groups	Between Groups	
(2-15)	0.9490	0.7785	No Significant Variation
(33-38)	0.2680	0.5972	No Significant Variation
(41-46)	0.5229	0.4460	No Significant Variation
(41-46, 89-92)	0.8026	0.3890	No Significant Variation
(89-92)	0.8457	0.4224	No Significant Variation
(61-70)	0.6846	3.7367	Data Varies Significantly
(71-80)	0.6753	1.8720	Data Varies Significantly
(33-38, 41-46, 61-72, 89-92)	0.7488	3.8284	Data Varies Significantly
1-92	0.7651	1.4761	Data Varies Significantly

Complete details will be published in a Journal.

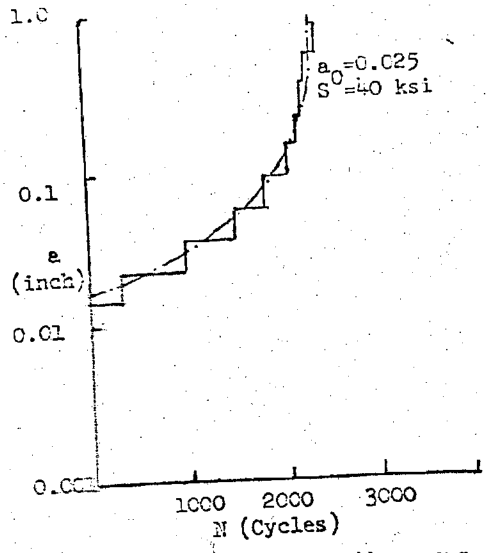

Figure 1 Crack Length versus Growth Cycles

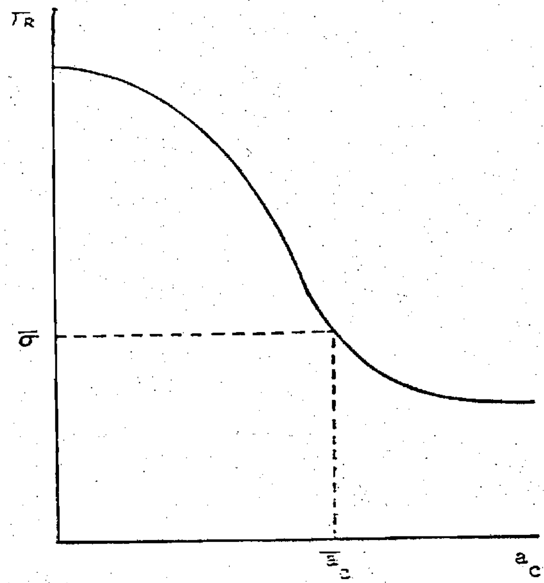

Figure 4 Residual Strength Variation

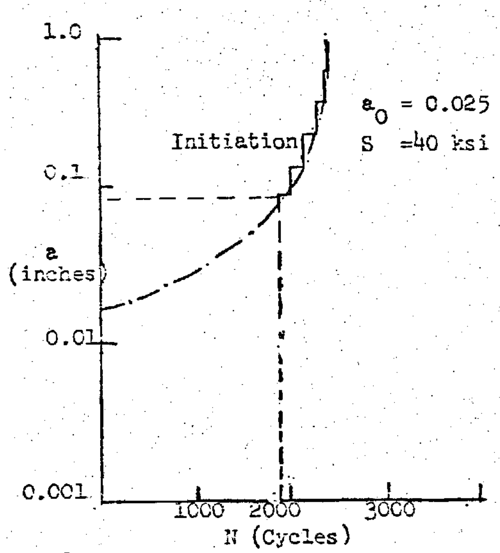

Figure 2 Crack Length versus Growth

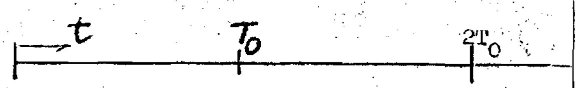

Figure 5 Inspection Schedule

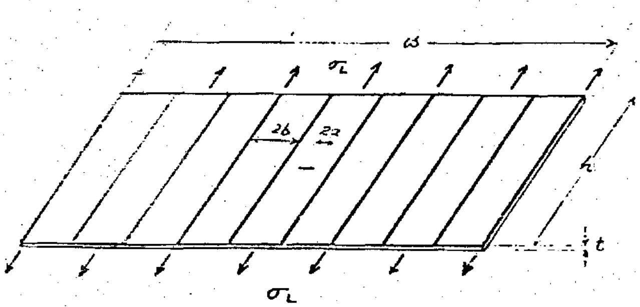

Figure 3 Panel Configuration

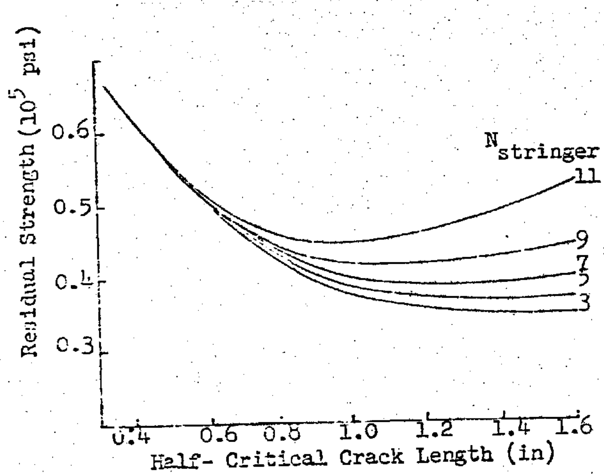

Figure 6
Residual Strength Variation

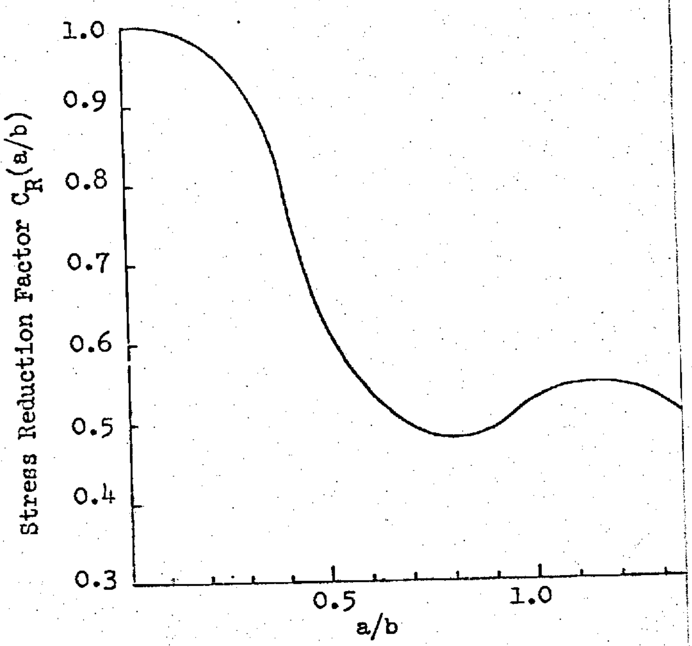

Figure 7 Stress Reduction Factor

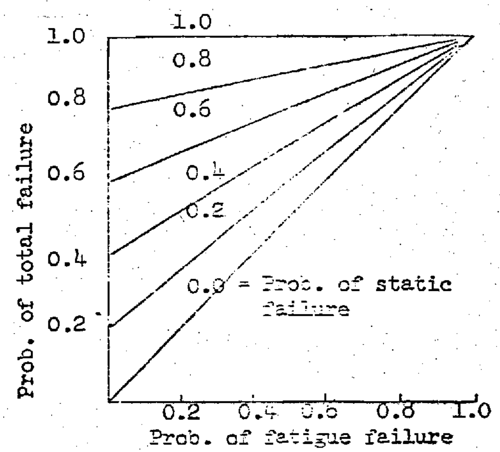

Figure 9 Static failure-Fatigue failure Relationship

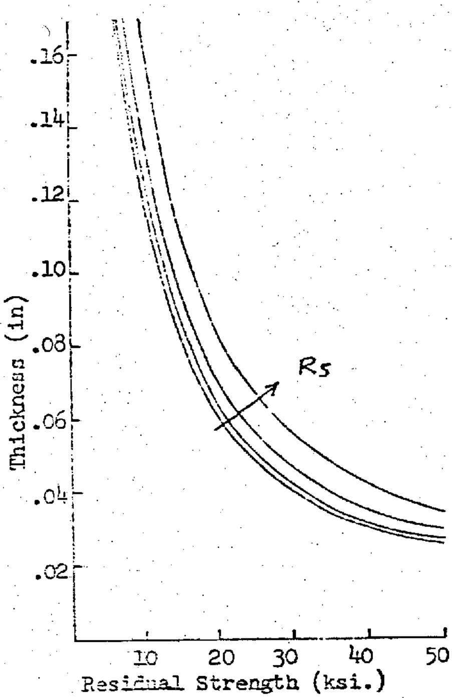

Figure 8 Static Reliability Variation

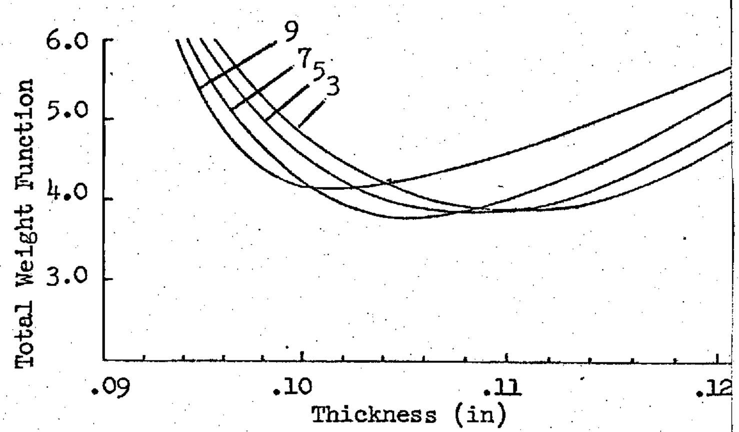

Figure 10 Minimization Curves

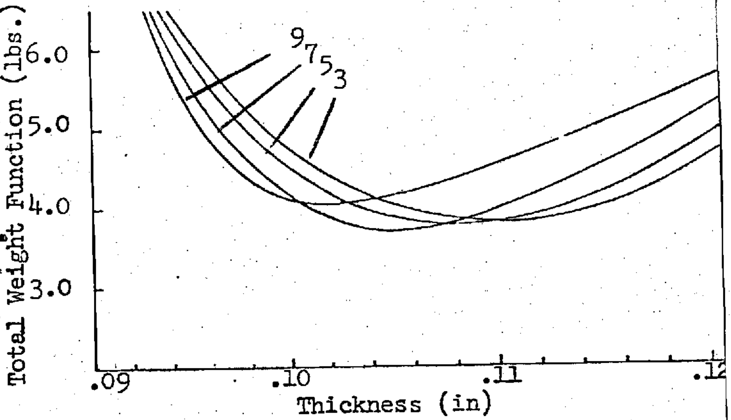

Figure 11 Minimization Curves

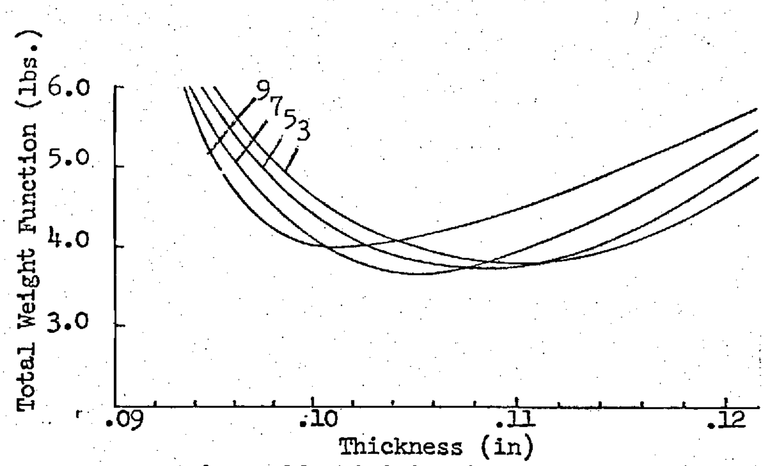

Figure 13 Minimization Curves

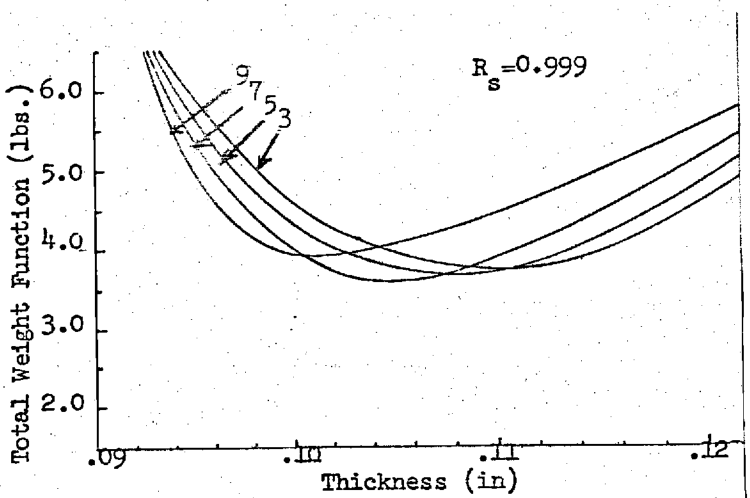

Figure 12 Minimization Curves

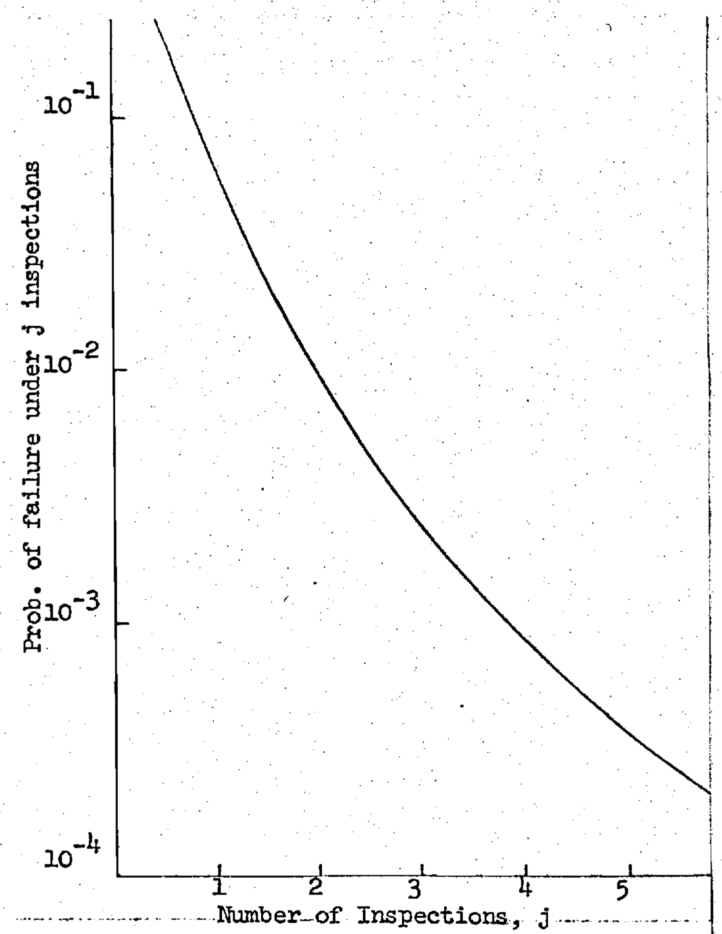

Figure 14 Reliability Variation with Number of Inspections

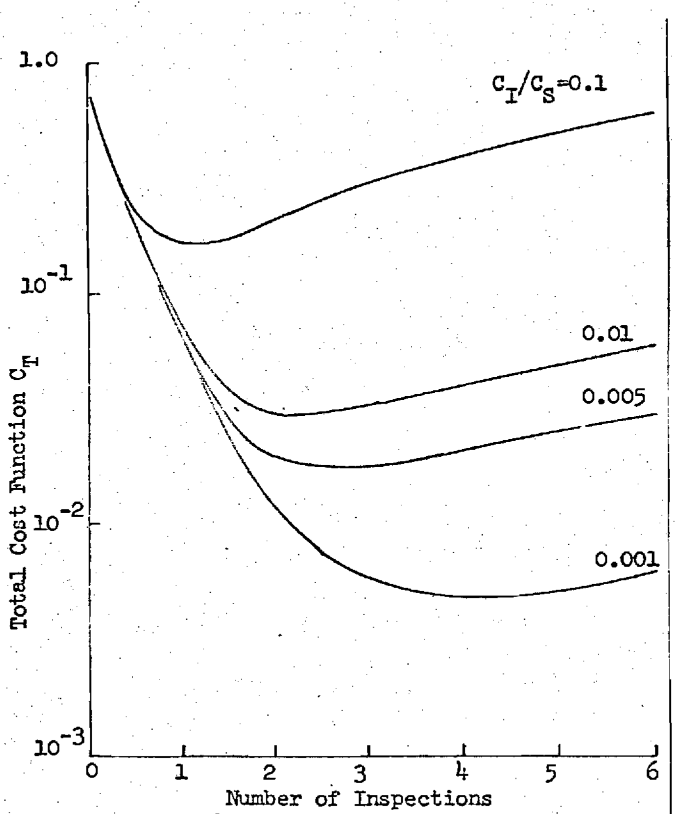

Figure 15 Minimization Curves